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The Mathematics of Juggling

Iago Montero Núñez, *student*

North Carolina Zeta

Catawba College
Salisbury, NC, 28144

Abstract

During the 1980's, a group of mathematicians started to develop different theorems and methods to study the art of juggling. Shannon's Juggling Theorem was the first theorem invented with this purpose, but it was not until the creation of a combinatorial notation known as siteswap that the study of juggling became really meaningful. This notation set a turning point regarding the study of juggling, it allowed us to easily count juggling patterns, determine which sequences can be juggled, and develop, learn and communicate new tricks. Using siteswap as a basis, a new set of theorems and techniques became a reality. Arguably the most important one is what we know as juggling states, a system that allows jugglers to create transitions from one trick to another.

Introduction

Juggling is one of the oldest forms of entertainment known to man. Its most traditional form, which is the one we are going to focus on for this project, can be defined as follows: continuous tossing into the air and catching of a number of objects so as to keep at least one in the air while handling the others. For the purposes of this paper we are going to call these objects "balls" and we are going to assume that they are being juggled by two hands. There exist three basic ways of juggling a particular number of balls. The first one is the pattern that is shown in cartoons and which most people learn by themselves before being taught other ways of juggling; it consists of a hand making high throws to the opposite hand while this one passes the balls back making a circular pattern. This is called the ***shower*** (Figure 1). However, the standard way of juggling an odd number of balls is having them follow each other in a figure 8 pattern in a way that they cross in the middle, between the two hands. This pattern receives the name of ***cascade*** (Figure 2). When the number of balls is even, half of them are juggled in circles in each hand and this is what is known as a ***fountain*** (Figure 3).

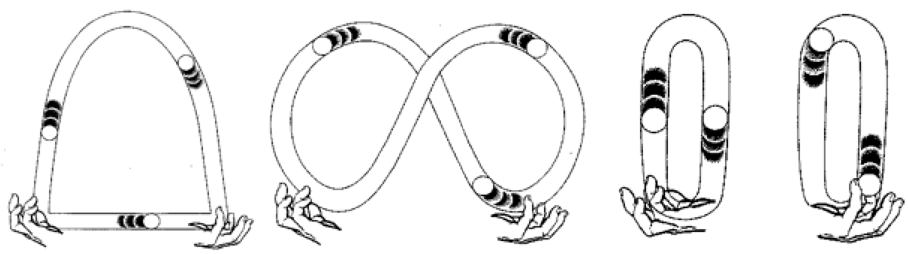

Figure 1 Figure 2 Figure 3
[1], used with permission

One observation that has been made by many people in past years is that a significant percentage of all jugglers, and particularly the most famous ones, are also interested in mathematics, physics and computer science [1]. This is not a coincidence, since all these fields, and in particular mathematics, are very closely related to juggling as we will see in the following pages. However, despite this close relationship, the mathematics of juggling did not become popular among jugglers until the 1980's [3], when a series of theorems and methods began to develop in order to further study the art of juggling. The first theorem with this purpose is named after Claude E. Shannon, an American mathematician and juggler who used his knowledge of both disciplines to come up with what we know as Shannon's Juggling Theorem.

Definition 1 *A **uniform juggle** is a juggling pattern in which the time a ball spends in a hand, the time a ball spends in the air and the time a hand spends empty stay constant throughout the juggle (*[4] *page 4).*

Theorem 1 (Shannon's Juggling Theorem) *(*[4] *page 4) Let F be the time a ball spends in the air (Flight), D be the time a ball spends in a hand (Dwell), V be the time a hand spends empty (Vacant), N be the number of balls and H be the number of hands. Then for a uniform juggle*

$$\frac{F+D}{V+D} = \frac{N}{H}.$$

Proof. We can measure the time T it takes for a juggling cycle to be completed in two different ways:

1. Considering the perspective of the ball, time can be divided into Flight time (F) and Dwell time (D). Since we add this up for all balls, we have to divide by the Number of balls (N). Thus $T = \frac{F+D}{N}$.

2. Considering the perspective of the hand, time can be divided into Vacant time (V) and Dwell time (D). Since we add this up for all hands, we have to divide by the number of Hands (H). Thus $T = \frac{V+D}{H}$.

Thus, since the time measured is the same in both cases, $\frac{F+D}{N} = \frac{V+D}{H}$ and so $\frac{F+D}{V+D} = \frac{N}{H}$. ■

Shortly after Shannon's findings, several undergraduate students began to develop a combinatorial notation to describe juggling patterns. This notation is what we know today as siteswap.

Siteswap

Definition 2 ***Siteswap*** *is a juggling notation used to describe or represent juggling patterns.*

The Siteswap Notation provides a foundation for people to count juggling patterns, determine which sequences are valid, and develop, learn and communicate new juggling tricks. For this notation we assume that time is broken up into a sequence of *beats* (throws) and we denote in siteswap a sequence $T = (t_1, t_2, \ldots, t_n)$ of nonnegative integers. Each of these integers is assigned to a throw and the integer itself is the number of beats that occur before the ball is thrown again (*height*). For siteswap, we assume that the juggler has being juggling for an infinite amount of time and he will continue to do so. We do this in order to identify the shortest repeating sequence, which is what we ultimately refer to as siteswap. Examples of this are the sequence (…,4,1,5,2,4,1,5,2,4,1,5,2,…) which we would write as 4152 and the sequence (…,4,4,1,3,4,4,1,3,4,4,1,3,…) which we would write as 4413. Other examples that we have already seen are (…,5,1,5,1,5,1,…) or just 51 for the shower (Figure 1), (…,3,3,3,3,3,…) or just 3 for the cascade (Figure 2) and (…,4,4,4,4,4,…) or just 4 for the fountain (Figure 3). To help us understand the concept, we can graph these sequences in the following way:

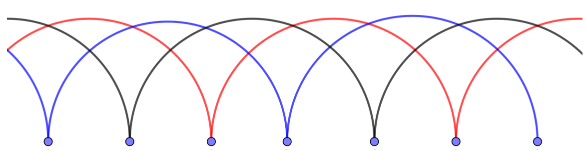

Figure 4: graph of the sequence 3

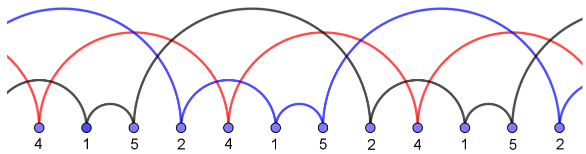

Figure 5: graph of the sequence 4152

Definition 3 *A sequence $T = (t_1, t_2, \ldots, t_n)$ where two or more balls never come down at the same time (on the same beat) is known as a **juggling sequence** or **juggling pattern**.*

If that condition was not met, it would not be possible for the sequence to be juggled. To find out if this condition is met in a sequence we use the following theorem (Theorem 2).

Theorem 2 *The sequence $T = (t_1, t_2, \ldots, t_n)$ is a juggling sequence if and only if the quantities $i + t_i$ (mod n) are distinct for $1 \leqslant i \leqslant n$.*

Proof. We know that t_i denotes the height of the ball, the number of beats that occur before the ball is caught and thrown again (note that these two actions always happen in the same beat). A ball thrown at height t_i and at beat i is caught at beat $i+t_i$ by definition of height. The modular condition simply makes sure that only one ball falls on each beat. This, as we have seen before, is the condition that makes a pattern a juggling pattern. ■

The integer n is what we call the ***period*** of the pattern. For example, the pattern 51 has period 2, and the pattern 4152 has period 4. Using this theorem, we see that 4152 is a juggling pattern as all the quantities are distinct: 1 + 4 (mod 4) = 1; 2 + 1 (mod 4) = 3; 3 + 5 (mod 4) = 0; 4 + 2 (mod 4) = 2. However, we see that 4251 is not a juggling sequence as 1 + 4 (mod 4) = 1 = 4 + 1 (mod 4); 2 + 2 (mod 4) = 0 = 3 + 5 (mod 4). We can also see that when we try to graph a sequence that does not meet this condition, at some point we get 2 balls falling in the same beat (Figure 6) making the sequence impossible to juggle.

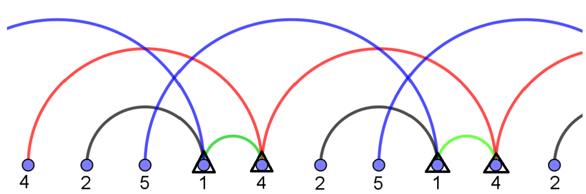

Figure 6: graph of the sequence 4251

In order to find the number of balls needed to juggle a pattern we use the next theorem (Theorem 3).

Theorem 3 (Average Theorem) *If $T = (t_1, t_2, \ldots, t_n)$ is a juggling sequence with period n, then:*

$$\frac{1}{n}\sum_{i=1}^{n} t_i = N,$$

where N is the number of balls needed to juggle the particular pattern.

Proof. The sum of all heights that each ball uses to go from t_1 to tn equals n since it is the distance it has to cover. There are N balls, thus the sum of all heights is $N \cdot n$. On the other hand, the sum of all the heights can also be expressed as $t_1 + t_2 + \cdots + t_n = \sum_{i=1}^{n} t_i$. Therefore $t_1 + t_2 + \cdots + t_n = \sum_{i=1}^{n} t_i$, i.e. $\frac{1}{n}\sum_{i=1}^{n} t_i = N$. ■

From this theorem we can see that both 51 and 4152 juggling patterns are juggled with 3 balls, as (5+1)/2=3 and (4+1+5+2)/4=3. However, 534 would be juggled with 4 balls as (5+3+4)/3=4. If using the Average Theorem with a particular sequence does not result in an integer, we can conclude that that sequence does not give us a juggling pattern. If the result is indeed an integer, we call this a ***qualifying sequence***.

Creating New Juggling Patterns

Even though we commonly use the term siteswap to refer to the type of notation we just saw, the word was originally invented to name a method for creating new juggling patterns. This method simply took existing juggling sequences and "swapped" their landing "sites". We can do this by drawing the graph of one juggling pattern such as 441 and swapping, for example, the landing sites of the first and second throw. The result would look as we see in Figure 7 and it would be notated as follows: $(4, 4, 1)^{1,2} = (5, 3, 1)$.

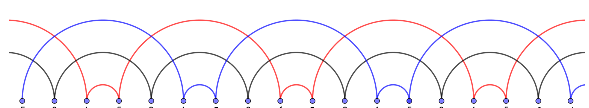

Figure 7: graph of $(4, 4, 1)^{1,2} = (5, 3, 1)$

If we were to change the landing sites of the second and third throw, the result would look as we see in Figure 8 and it would be notated $(4, 4, 1)^{2,3} = (4, 5, 0)$.

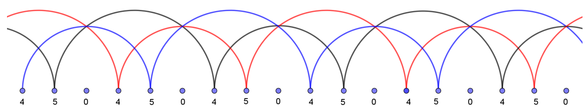

Figure 8: graph of $(4, 4, 1)^{2,3} = (4, 5, 0)$

We can also apply this method without drawing graphs. We let $T = (t_1, t_2, \ldots, t_n)$ be a juggling pattern. We create a new sequence $t^{x,y}$ called the ***siteswap of*** t ***at beats*** x ***and*** y, where $1 \leqslant x < y \leqslant n$, by interchanging the landing sites of the throws t_x and t_y. We do this by letting ${t_x}^{x,y} = t_x + y - x$, ${t_y}^{x,y} = t_y + x - y$, and ${t_z}^{x,y} = t_z$ for all $1 \leqslant z \leqslant n$ where $z \neq x, y$.

Applying this method, we see how $(4, 4, 1)^{1,2} = (5, 3, 1)$ as 4 + 2 – 1 = 5 and 4 + 1 – 2 = 3. Also, $(4, 4, 1)^{2,3} = (4, 5, 0)$ as 4 + 3 – 2 = 5 and 1 + 2 – 3 = 0.

Juggling States

When studying juggling sequences, we call a ***state*** the sequence $v = (v_1, v_2, \ldots, v_h)$ that indicates the time (beat) at which a ball is going to land. If a ball is landing at a particular beat v_i, we set v_i = 1. Otherwise, if no ball is landing at beat v_i, we set $v_i = 0$. For a juggling sequence $T = (t_1, t_2, \ldots, t_n)$, we say that its ***juggling state*** is the state immediately after the last throw t_n. Knowing this, we have that the juggling pattern 4152 has a juggling state 1101 (Figure 9).

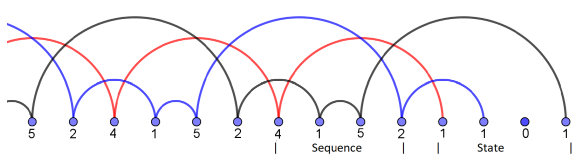

Figure 9: juggling state of the siteswap 4152

If the first value of the state is 0, so $v = (0, v_2, \ldots, v_h)$, the next throw to be made is a throw of height 0 (no throw in that beat). After executing this throw, we get a new state w of the form $w = (w_2, w_3 \ldots, w_h, 0)$. If the first value is 1, so $v = (1, v_2, \ldots, v_h),$, the next throw to be made is a throw of height t. After executing this throw, we get a new state w of the form $w = (w_2, \ldots 1, \ldots, w_h)$ where the 1 is placed at the t^{th} position. For example, using the state 110010, and using throws of heights 3 and then 5 we get the juggling states 101100 and 011010 (Figure 10)

110010 → 3 → **101100** → 5 → **011010**

Figure 10: transitions of juggling states

Juggling states are particularly useful for jugglers because they give them the ability to create new transitions between juggling patterns. By having these transitions, jugglers are able to perform several different tricks one after another without having to stop and catch all the balls before moving into a new pattern. In Figure 11 we can see all the states and transitions that are possible when juggling 3 balls with a maximum height of 4 and in Figure 12 we have the diagram we get by increasing the maximum height to 5. As we can see, the options increase very quickly when we increase the height or the number of balls. This gives jugglers almost endless possibilities when developing a juggling routine, which was always very hard to do before the invention of these types of notations.

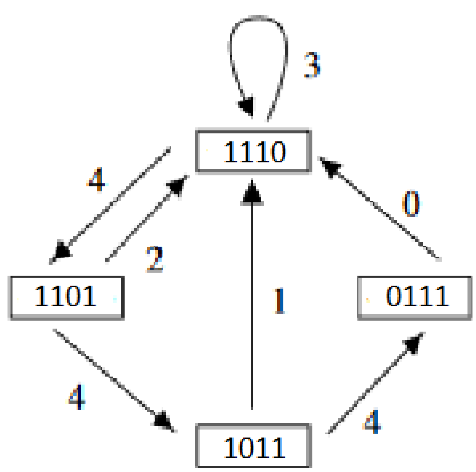

Figure 11: state diagram for 3 balls and maximum height 4
[6], used with permission

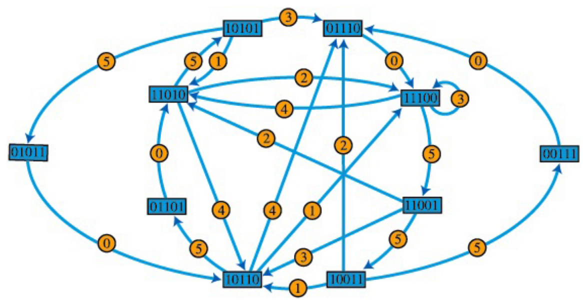

Figure 12: state diagram for 3 balls and maximum height 5
[5], used with permission

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Integer triangles with sides and angles in harmonic progression

Ioannis D. Sfikas

National and Kapodistrian University of Athens Faculty of Primary Education

Abstract

In this paper, several theorems are proved in elementary number theory establishing that there are no integer triangles whose angles form a harmonic progression, and there are no Heronian triangles with rational angles whose lengths form a harmonic progression. Finally, we present some properties of integer triangles with sides or angles in harmonic progression.

Introduction

A triangle whose sides and area are rational numbers is called a *rational triangle*. If the rational triangle is right-angled, it is called a *right-angled rational triangle* or a *rational Pythagorean triangle* or a *numerical right triangle*. If the sides of a rational triangle are of integer length, it is called an *integer triangle*. If, further, these sides have no common factor greater than unity, the triangle called a *primitive integer triangle*. If the integer triangle is right-angled, it is called a *Pythagorean triangle*. A *Heronian triangle* (named after Heron of Alexandria) is an integer triangle with the additional property that its area is also an integer [1]. A Heronian triangle is called a *primitive Heronian triangle* if the sides have no common factor greater than unity. In the 7th century, the Indian mathematician Brahmagupta studied the special case of triangles with consecutive integer sides [2].

A *harmonic progression* (or *harmonic sequence*) is a progression formed by taking the reciprocals of an arithmetic progression. In other words, it is a sequence $\{\, a_n |\, n \in \mathbb{N}\}$ such that $\frac{1}{a_{n+1}} - \frac{1}{a_n} = r$, where r is a positive real number. A harmonic progression has:

1. the n^{th} term $a_n = \frac{1}{\frac{1}{a_1}+(n-1)r}$, and

2. recursive relation $a_{n+1} = \frac{1}{\frac{1}{a_n}+r}$.

Equivalently, a harmonic progression is a sequence of real numbers such that any term a_n in the sequence is the *harmonic mean*

$$\frac{2}{\frac{1}{a_{n-1}} + \frac{1}{a_{n+1}}}$$

of its two neighbors. Finally, a *rational angle* is one such that $\theta = \frac{m}{n}\pi$ for some integers m and n.

The study of geometric shapes has been fundamental in number theory. For example, the study of Pythagorean triples began centuries before the time of Pythagoras (590-490 B.C.) since there are Babylonian tablets (Tablet Plimpton 322) containing lists of such triples including (3,4,5). Triangles have two specific elements, angles and sides. The purpose of this article is to consider triangles from these two perspectives, and consists of two parts: (i) establishing that there are no integer triangles whose angles form a harmonic progression (Corollary 1), and (ii) that there are no Heronian triangles with rational angles, whose lengths form a harmonic progression (Corollary 2). That means we investigate properties of this class of triangles, which is a countable subset of the entire class of triangles. Problems of integer triangles with sides having lengths in harmonic progression is also discussed (Theorem 3). The results may be generalized to ask whether there exist cyclic quadrilaterals having integer area and integer sides in harmonic progression.

Integer triangles with angles in harmonic progression

Let us consider a Heronian triangle whose angles (A, B, C) are in harmonic progression. Using (1) above, this means that the angles $(A, B, C) = \left(A, \frac{A}{rA+1}, \frac{A}{2rA+1}\right)$, for some $r > 0$. Niven's theorem states that the only rational angles $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ for which the $\sin\theta$ is also a rational number are: $\sin\left(-\frac{\pi}{2}\right) = -1, \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}, \sin 0 = 0, \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$, and $\sin\left(\frac{\pi}{2}\right) = -1$ (see [3]). Theorem 1 presents an elementary proof for a version of Niven's theorem regarding the cosine function (see [4], [5]).

Theorem 1 *If θ is a rational angle and if $\cos\theta$ is a rational number, then* $\cos\theta \in \left\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right\}$.

Proof. The addition formula for cosine implies $\cos(2\theta) = 2\cos^2\theta - 1$, or $2\cos(2\theta) = 4\cos^2\theta - 2$. If we assume $2\cos\theta = \frac{a}{b}$, with $a, b \in \mathbb{Z}$, $b \neq 0$ and $\gcd(a, b) = 1$, then $2\cos(2\theta) = \frac{a^2}{b^2} - 2 = \frac{a^2-2b^2}{b^2}$. Similarly,

$2\cos(4\theta) = \frac{4a^4-8a^2b^2+2b^2}{b^4}$. If $\gcd(a^2-2b^2, b^2) \neq 1$, let p be a prime dividing both. Then $p|b^2 \Rightarrow p|b$ and then $p|(a^2-2b^2) \Rightarrow p|a$, contradicting $\gcd(a,b)=1$. Therefore, if $b \neq \pm 1$, then in $2\cos\theta, 2\cos(2\theta), 2\cos(4\theta), \ldots$ the denominators $b^2, b^4, \ldots$ increase. Now, if $\theta = \frac{m}{n}\pi$ is a rational angle, since the cosine function has period 2π, the sequence $\{\, 2\cos(2k\theta) |\, k \in \mathbb{N}\}$ may admit at most n different values – those for which $0 \leqslant km \leqslant (n-1)$. This contradicts the note that the denominators of the sequence must increase to infinity. Therefore , $b = \pm 1$ and the values $-1, -\frac{1}{2}, 0, \frac{1}{2}$ and 1 are the only rational values of cosine at rational angles. ■

We now apply this result to obtain the following theorem:

Theorem 2 *Any integer triangle with rational angles must be equilateral.*

Proof. By the Law of Cosines, every angle of an integer triangle has a rational cosine so, if the angle A is rational, by Theorem 1

$$\cos A \in \left\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right\}.$$

In fact, it must be the case that the cosine of every angle in the triangle must be such that $\cos A \in \left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$ and this can only occur when each angle is $\frac{\pi}{3}$. ■

Corollary 1 *There are no integer triangles with rational angles whose angles form a harmonic progression.*

Proof. By Theorem 2, an integer triangle with rational angles must be equilateral so the angles form a constant sequence. Such a sequence cannot be harmonic as, in the definition of a harmonic sequence, $r > 0$. ■

An analogous argument yields:

Corollary 2 *There are no integer triangles with rational angles whose sides form a harmonic progression.*

Further Results

There are many interesting theorems about triangles and harmonic progressions. Many of the proof techniques discussed can be applied to obtain further results about integer triangles. Recall that for a triangle with sides of length a, b and c, the *semiperimeter* is defined to be $s = \frac{a+b+c}{2}$. An *incircle* is an inscribed circle of a triangle, i.e. a circle that is tangent to each of the triangle's sides. An incircle exists and is, moreover, unique.

The radius of the incircle is the *inradius*; the center of the incircle is the *incenter*.

Theorem 3 *If the sequence of the side lengths a, b, c of an integer triangle forms a harmonic progression then:*

1. *the sequence* $\sin^2\left(\frac{A}{2}\right), \sin^2\left(\frac{B}{2}\right), \sin^2\left(\frac{C}{2}\right)$ *is also a harmonic progression, i.e.* $\frac{2}{\sin^2\left(\frac{B}{2}\right)} = \frac{1}{\sin^2\left(\frac{A}{2}\right)} + \frac{1}{\sin^2\left(\frac{C}{2}\right)}$;
2. *the straight line* $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ *always passes through the fixed point* $(1, -2)$*;*
3. *the altitudes of the integer triangle are in arithmetic progression.*

Proof. (1): If the sides a, b, c form a harmonic sequence, then:

$$
\begin{aligned}
\frac{2}{b} &= \frac{1}{a} + \frac{1}{c} \\
&\Rightarrow 2ca = ab + bc \\
&\Rightarrow 8ca = 4ab + 4bc \\
&\Rightarrow -4ab - 4bc + 4ca = -4ac \\
&\Rightarrow 2(a^2+b^2+c^2) - 4ab - 4bc + 4ca = 2(a^2+b^2+c^2) \\
&\qquad + (-2ab + 2bc - 2ca) + (2ab - 2bc - 2ca) \\
&\Rightarrow 2(a+c-b)^2 = (b+c-a)^2 + (a+b-c)^2 \\
&\Rightarrow 2\left[\frac{1}{2}(a+c-b)\right]^2 = \left[\frac{1}{2}(b+c-a)\right]^2 + \left[\frac{1}{2}(a+b-c)\right]^2 \\
&\Rightarrow 2\left[\frac{1}{2}(a+b+c) - b\right]^2 = \left[\frac{1}{2}(a+b+c) - a\right]^2 \\
&\qquad + \left[\frac{1}{2}(a+b+c) - c\right]^2 \\
&\Rightarrow 2(s-b)^2 = (s-a)^2 + (s-c)^2.
\end{aligned}
$$

Therefore $\frac{2(s-b)^2}{\rho^2} = \frac{(s-a)^2}{\rho^2} + \frac{(s-c)^2}{\rho^2}$ where ρ is the inradius. Now, by the Law of Cotangents,

$$
\frac{(s-b)^2}{\rho^2} = \cot^2\left(\frac{B}{2}\right), \frac{(s-a)^2}{\rho^2} = \cot^2\left(\frac{A}{2}\right), \frac{(s-c)^2}{\rho^2} = \cot^2\left(\frac{C}{2}\right)
$$

so we have

$$2\cot^2\left(\frac{B}{2}\right) = \cot^2\left(\frac{A}{2}\right) + \cot^2\left(\frac{C}{2}\right)$$
$$\Rightarrow 2\left(\csc^2\left(\frac{B}{2}\right) - 1\right)$$
$$= \csc^2\left(\frac{A}{2}\right) - 1 + \csc^2\left(\frac{C}{2}\right) - 1$$
$$\Rightarrow \frac{2}{\sin^2\left(\frac{B}{2}\right)} = \frac{1}{\sin^2\left(\frac{A}{2}\right)} + \frac{1}{\sin^2\left(\frac{C}{2}\right)}.$$

Thus $\sin^2\left(\frac{A}{2}\right), \sin^2\left(\frac{B}{2}\right), \sin^2\left(\frac{C}{2}\right)$ is also a harmonic progression, i.e. $\frac{2}{\sin^2\left(\frac{B}{2}\right)} = \frac{1}{\sin^2\left(\frac{A}{2}\right)} + \frac{1}{\sin^2\left(\frac{C}{2}\right)}$ form a harmonic progression.

(2): If the sides a, b, c are in harmonic progression, then $b = \frac{a}{1+ra}, c = \frac{a}{1+2ra}$ and it follows that $r = \frac{a-b}{ab}$ so $c = \frac{ab}{2a-b}$. The equation of the line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ becomes

$$\frac{x}{a} + \frac{y}{b} + \frac{2a-b}{ab} = 0 \text{ or } \frac{x-1}{a} + \frac{y+2}{b} = 0. \tag{1}$$

Equation (1) holds when $x - 1 = 0$ and $y + 2 = 0$ and hence the line always contains the point $(x, y) = (1, -2)$.

(3): If a, b, c are the sides of the integer triangle and h_a, h_b, h_c are the altitudes upon a, b, c respectively, then, the area Δ of the triangle is $\frac{1}{2}ah_a = \frac{1}{2}bh_b = \frac{1}{2}ch_c$. Then $a = \frac{2\Delta}{h_a}, b = \frac{2\Delta}{h_b}$, and $c = \frac{2\Delta}{h_c}$. It follows that if a, b and c form a harmonic progression, then

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$
$$\Rightarrow \frac{2h_b}{2\Delta} = \frac{h_a}{2\Delta} + \frac{h_c}{2\Delta}$$
$$\Rightarrow 2h_b = h_a + h_c,$$

i.e. the altitudes form an arithmetic progression with

$$r = -\frac{1}{2}h_a + \frac{1}{2}h_c.$$

■

Theorem 4 *If the squares of the sides a, b, c of an integer triangle form an arithmetic progression, then:*

1. *the tangents of its angles form a harmonic progression;*
2. *the middle angle has to be smaller than $\frac{\pi}{3}$;*
3. $\cos A = \frac{c}{b} - \frac{b}{2c}$.

Proof. (1): If h_b is the altitude to side b of triangle ABC, then $\cot A + \cot C = \frac{b}{h_b}$ but we also have $h_b = \frac{2\Delta}{b}$, where Δ is the area of the triangle. Thus $\cot A + \cot C = \frac{b^2}{2\Delta}$. Similarly

$$\cot B + \cot C = \frac{a^2}{2\Delta} \text{ and } \cot A + \cot B = \frac{c^2}{2\Delta}.$$

Since the squares of a, b and c form an arithmetic progression we have $2b^2 = a^2 + c^2$. Replacing a^2, b^2 and c^2 using the equations above and simplifying, we obtain

$$2\cot B = \cot A + \cot C. \tag{2}$$

It follows that the cotangents form an arithmetic progression and therefore the reciprocals, the tangents, form a harmonic progression (see [6]).
(2): Since $A+B+C = \pi$ we have $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ or $\cot A \cot B + \cot B \cot C + \cot A \cot C = 1$. Solving this last equation for $\cot C$ and replacing this in (2), we see that

$$\cot^2 A - 2\cot A \cot B + 1 - 2\cot^2 B = 0.$$

As $\cot A$ is real, the discriminant

$$(2\cot B)^2 - 4(1 - 2\cot^2 B) = 4(3\cot^2 B - 1) \geqslant 0.$$

Thus $\cot^2 B \geqslant \frac{1}{3}$ or $\tan^2 B \leqslant 3$. If $\cot B < 0$, then by (2) at least one of $\cot A, \cot C$ must be negative, but this would imply that one of them must be greater than $\pi/2$, which is impossible. So $\cot B > 0$ and hence $B < \pi/2$, i.e. $\tan B > 0$. Therefore, since $\tan^2 B \leqslant 3$, we have $0 < \tan B \leqslant \sqrt{3}$ and so $B \leqslant \pi/3$.
(3): Suppose a is the shortest side of the triangle and let $b^2 = a^2 + d$ and $c^2 = a^2 + 2d$. By the Law of Cosines, $\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{a^2+3d}{2bc}$. Now, $a^2 + 3d = 2(a^2 + 2d) - (a^2 + d) = 2c^2 - b^2$ so

$$\cos A = \frac{2c^2 - b^2}{2bc} = \frac{c}{b} - \frac{b}{2c}.$$

Similarly

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{c}{4a} - \frac{a}{4c}$$

and

$$\cos C = \frac{b^2 + a^2 - c^2}{2ab} = \frac{2a^2 - b^2}{2ab} = \frac{a}{b} - \frac{b}{2a}.$$

■

The arithmetic, geometric, and harmonic means of two positive real numbers a and b, denoted by A_2, G_2 and H_2 respectively, are defined as follows:

$$A_2 = \frac{a+b}{2}, G_2 = \sqrt{ab} \text{ and } H_2 = \frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a+b}.$$

These means satisfy two interesting properties:

$$A_2 \geqslant G_2 \geqslant H_2 \text{ and } G_2^2 = A_2 H_2.$$

Theorem 5 *The arithmetic, geometric and harmonic means of two positive real numbers are the lengths of the sides of a right triangle if and only if the ratio of the arithmetic to the harmonic mean is the golden ratio.*

Proof. Let A_2, G_2 and H_2 be the lengths of the sides of a triangle. By the inequality above it follows that A_2 must be the hypotenuse and hence $A_2^2 = G_2^2 + H_2^2$. Using the equality above, we then have$A_2^2 = A_2 H_2 + H_2^2$ or $A_2^2 - A_2 H_2 - H_2^2 = \left(\frac{A_2}{H_2}\right)^2 - \frac{A_2}{H_2} - 1 = 0$. Therefore $\frac{A_2}{H_2} = \frac{1+\sqrt{5}}{2}$, the golden ratio. Note that all of these implications reverse, so we also have the converse (see [7]). ■

Theorem 6 *Three positive numbers A_3, G_3 and H_3 are respectively the harmonic, geometric and arithmetic means of the sides a, b, c of an integer triangle if and only if the following inequality holds*

$$\frac{A_3^3}{G_3^3} + \frac{G_3^3}{H_3^3} + 1 \leqslant \frac{3}{4}\left(1 + \frac{A_3}{H_3}\right)^2. \tag{3}$$

Proof. By definition if A_3, G_3 and H_3 are the sated means, we have $a + b + c = A_3$, $abc = G_3^3$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{3}{H_3}$. Let

$$P(v) = v^3 + c_1 v^2 + c_2 v + c_3 = 0$$

be a cubic polynomial with roots a, b and c. From Vieta's formulas, we have

$$-c_1 = a + b + c = 3A_3$$
$$c_2 = ab + ac + bc = \frac{3G_3^3}{H_3}$$
$$-c_3 = abc = G_3^3,$$

So $P(v) = v^3 - 3A_2v^2 + \frac{3G_3^3}{H_3}v - G_3^3$ and there are only positive solutions to $P(v) = 0$ since A_3, G_3 and H_3 are positive. We can rewrite $P(v) = 0$ as

$$(v - A_3)^3 - \left(3A_3^2 - \frac{3G_3^3}{H_3}\right)(v - A_3) + A_3\left(\frac{3G_3^3}{H_3} - 3A_3^2\right) + A_3^3 - G_3^3 = 0.$$

Substituting $u = v - A_3$ we have

$$u^3 - \left(3A_3^2 - \frac{3G_3^3}{H_3}\right)u + \left(-2A_3^3 + \frac{3A_3^2}{H_3} - G_3^3\right) = 0.$$

All solutions of the equation $u^3 - pu + q = 0$ are real numbers if and only if $\frac{q^2}{4} \leqslant \frac{p^3}{27}$ so we must have

$$\left(-A_3^3 + \frac{3A_3^2}{2H_3} - \frac{G_3^3}{2}\right)^2 \leqslant \left(A_3^2 - \frac{G_3^3}{H_3}\right)^3.$$

Multiplying these out gives the inequality in (3). [8] ■

Theorem 7 1. *In any integer triangle, the radius of the incircle is one-third the harmonic mean of the altitudes.*

2. *Let x be the side of a square inscribed within a triangle and having one side lying along the base b of the triangle. Then, x is half the harmonic mean of the base of the triangle and the altitude of the triangle on the base.*

3. *In a right triangle with legs a and b and altitude h from the hypotenuse to the right angle, h^2 is half the harmonic mean of a^2 and b^2.*

Proof. (1) The area Δ of a triangle ABC with sides a, b, c and incircle of radius r is $\Delta = rs$ where $s = \frac{a+b+c}{2}$ is the semiperimeter. Then $r = \frac{2\Delta}{a+b+c}$. We have also seen that the altitudes from a, b and c are given by $a = \frac{2\Delta}{h_a}, b = \frac{2\Delta}{h_b}$, and $c = \frac{2\Delta}{h_c}$, so $a + b + c = 2\Delta\left(\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}\right)$. Therefore

$$r = \frac{2\Delta}{a + b + c} = \frac{1}{\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}},$$

which is one-third the harmonic mean of the altitudes.
(2) The area of the entire triangle is $\frac{1}{2}bh_b$, the area of the small triangle with vertex B is $\frac{1}{2}x(h_b - x)$ and the triangle obtained by sliding the two smaller triangles standing on the base is $\frac{1}{2}x(h_b - x)$ (Figure 1).

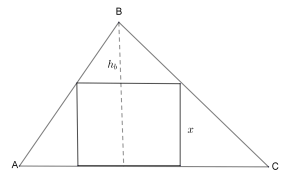

Figure 1

Therefore we have

$$\frac{1}{2}bh_b = \frac{1}{2}x(b-x) + \frac{1}{2}x(h_b - x) + x^2.$$

or $bh_b = x(b+h_b)$. Then $x = \frac{bh_b}{b+h_b} = \frac{1}{2}\left(\frac{2}{\frac{1}{b}+\frac{1}{h_b}}\right)$.

(3) Given a right triangle with legs of length a and b, hypotenuse of length c, and altitude of length h from the right angle to the hypotenuse, let x and y be as in Figure 2

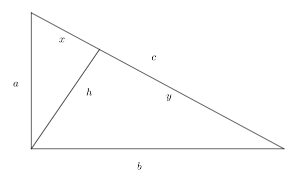

Figure 2

Then

$$a^2 + b^2 = (x+y)^2 = x^2 + 2xy + y^2, a^2 = x^2 + h^2$$

and $b^2 = y^2 + h^2$ so

$$a^2 + b^2 = x^2 + y^2 + 2h^2.$$

It follows that

$$a^2 = h^2 + x^2 = xy + x^2 = x(y+x) = xc.$$

Similarly $b^2 = yc$ and hence

$$h^2 = \frac{a^2b^2}{c^2} = \frac{a^2b^2}{a^2+b^2} = \frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}.$$

■

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before November 1, 2019. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2019 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051).

NEW PROBLEMS 829 - 839

Problem 829. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Let $\Omega_n = \binom{n}{7} + 2\binom{n-1}{7} + 3\binom{n-2}{7} + \cdots (n-6)\binom{7}{7}$ for all $n \geqslant 7$. Find $\Omega = \lim_{n\to\infty} \sqrt[n]{\Omega_n}$.

Problem 830. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

If $x \in (0, \frac{\pi}{2})$, prove that $2(\sin x)^{1-\sin x} \cdot (1-\sin x)^{\sin x} \leqslant 1$.

Problem 831. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

If $\Delta ABC \sim \Delta A'B'C'$, prove that

$$\sum \frac{(a'+b')(a'+c')}{b'c'} + 3 \geqslant \frac{15(b+c)(c'+a')(a'+b')}{8ab'c'}.$$

Problem 832. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Prove that in any triangle ABC the following holds:

$$\frac{a}{a+b+c} \geqslant \frac{2\sqrt{3}}{9}\sin A.$$

Problem 833. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Show that the equation $x^6 - 5x^5 - 6x^4 + 2x^3 + 9x^2 - 17x + 1 = 0$ has no negative roots.

Problem 834. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let (F_n) be the Fibonacci sequence, i.e. $F_0 = 0, F_1 = 1$, for all $n \geqslant 0$. If $e_n = \left(1 + \frac{1}{n}\right)^n$, prove that $\left(\sum\limits_{k=1}^{n} e_k F_{2k-1}\right)\left(\sum\limits_{k=1}^{n} \frac{F_{2k-1}}{e_k}\right) \leqslant \frac{(e+2)^2}{8e} F_{2n}^2$.

Problem 835. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let (L_n) be the Lucas sequence, i.e. $L_0 = 2, L_1 = 1$, for all $n \geqslant 0$. Prove that $n^{n-2}(n-1)\sum\limits_{k=1}^{n} L_k^n + n^{n-1}\prod\limits_{k=1}^{n} L_k > (L_{n+2} - 3)^n$ for all $n \geqslant 2$.

Problem 836. *Proposed by Abhijit Bhattacharjee (student), Banaras Hindu University, India.*

Prove that the equation $1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} = 0$ has exactly one real root if n is odd and no real root if n is even.

Problem 837. *Proposed by Pedro H.O. Pantoja, University of Campina Grande, Brazil.*

Evaluate $\lim\limits_{n\to\infty} \int\limits_{-1}^{1} \left(x^{2n+1} + \frac{1}{x^{2n+1}}\right) \ln(1 + e^{nx})dx$.

Problem 838. *Proposed by Mathew Cropper, Eastern Kentucky University, Richmond, KY.*

Let A_n be the number of n-bit strings of zeros and ones that contain at least one sequence of three consecutive ones (111) and no sequence of four or more consecutive ones. The sequence starts

$$A_1 = 0, A_2 = 0, A_3 = 1, A_4 = 2.$$

Using the well-known tribonacci sequence

$$T_0 = 0, T_1 = 0, T_2 = 1,$$

which counts the number of $(n-3)$-bit strings that contain NO sequence of three consecutive ones, develop a recursive formula for A_n and use it to compute A_{15}.

Problem 839. *Proposed by the editor.*

A recurrence is defined in the following way: $c_1 = 3$, $c_n = 4 + \sum_{i=1}^{n-1} c_i$ for all $n \geqslant 2$. Find a formula for c_n for $n \geqslant 2$ that just involves n.

SOLUTIONS TO PROBLEMS 808 - 819

Problem 808. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu - Severin, Mehedinti, Romania.*

Prove that if $a, b, c \in [1, \infty)$ then

$$\frac{e^{a+b+c}}{e^{b/a+c/b+a/c}} \leqslant a^b b^c c^a \leqslant \frac{e^{ab+bc+ca}}{e^{a+b+c}}.$$

Solution *by Brent Dozier, North Carolina Wesleyan College, Rocky Mount, NC.*

Let $a \in (1, \infty)$. Using the fact that $f(x) = \ln x$ is concave on $[1, a]$, we have

$$f'(a) = \frac{1}{a} \leqslant \frac{\ln a - \ln 1}{a - 1} \leqslant 1 = f'(1),$$

which yields

$$1 - \frac{1}{a} \leqslant \ln a \leqslant a - 1.$$

Raising e to all three parts of this inequality produces $\frac{e}{e^{1/a}} \leqslant a \leqslant \frac{e^a}{e}$. (Note that this is true for $a = 1$.) Raising this inequality to the power b, we get $\frac{e^b}{e^{b/a}} \leqslant a^b \leqslant \frac{e^{ab}}{e^b}$. Similarly, using the pairs b, c and c, a we have

$\frac{e^c}{e^{c/b}} \leqslant b^c \leqslant \frac{e^{bc}}{e^c}$ and $\frac{e^a}{e^{a/c}} \leqslant c^a \leqslant \frac{e^{ca}}{e^a}$. Multiplying all corresponding sides of the three inequalities gives the desired inequality.

Also solved by Ioannis Sfikas, National and Kapodistrian University of Athens, Greece; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Soumava Chakraborty, Soft Web Technology, Kolkata, India; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; and the proposer.

Problem 809. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu - Severin, Mehedinti, Romania.*

Prove that if $a, b, c \in (2, \infty)$ then

$$\sqrt{2}\sum\left(\sqrt{a(b-2)}+\sqrt{b(a-2)}\right) < 3\sqrt{abc}.$$

Solution *by Marian Ursărescu, National College "Roman Voda", Roman, Romania.*

The inequality is equivalent to

$$\sqrt{2}\sum\left(\sqrt{a}(\sqrt{b-2}+\sqrt{c-2}\right) < 3\sqrt{abc}$$

$$\Leftrightarrow \sum\left(\sqrt{a}(\sqrt{2(b-2)}+\sqrt{2(c-2)}\right)) < 3\sqrt{abc}.$$

Now we use Mahler's inequality:

$$\sqrt{x_1x_2}+\sqrt{y_1y_2} \leqslant \sqrt{(x_1+y_1)(x_2+y_2)}, \forall x_i, y_i > 0$$

which says that

$$\sqrt{2(b-2)}+\sqrt{(c-2)\cdot 2} < \sqrt{c\cdot b}$$

$$\Rightarrow \sqrt{a}(\sqrt{2(b-2)}+\sqrt{2(c-2)} < \sqrt{abc}.$$

Similarly,

$$\sqrt{b}(\sqrt{2(a-2)}+\sqrt{2(c-2)} < \sqrt{abc}$$

and

$$\sqrt{c}(\sqrt{2(a-2)}+\sqrt{2(b-2)} < \sqrt{abc}.$$

Summing the three inequalities gives the desired result.

Also solved by Soumava Chakraborty, Soft Web Technologies, Kolkata, India; Amit Dutta, College NIT, Jamshedpur, India; Ioannis Sfikas, National and Kapodistrian University of Athens, Greece; and the proposer.

Problem 810. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu - Severin, Mehedinti, Romania.*

Compute $L = \lim\limits_{n\to\infty} \frac{1}{n} \int_1^n \frac{x^4+4x^3+12x^2+9x}{(x+3)^5-x^5-243} dx$.

Solution *by Andrea Fanchini, Cantù, Italy.*

$$I = \int_1^n \frac{x^4+4x^3+12x^2+9x}{(x+3)^5-x^5-243}dx = \int_1^n \frac{x(x+1)(x^2+3x+9)}{15x(x+3)(x^2+3x+9)}dx$$

$$= \frac{1}{15}\int_1^n \frac{x+1}{x+3}dx$$

$$= \frac{1}{15}\int_1^n 1 - \frac{2}{x+3}dx = \frac{1}{15}(n - 2\log(n+3) - 1 + \log 16).$$

Finally, $\frac{1}{15}\lim\limits_{n\to\infty} \frac{n-2\log(n+3)-1+\log 16}{n} = \frac{1}{15}$.

Also solved by Brent Dozier, North Carolina Wesleyan College, Rocky Mount, NC; Benjamin Junkins (student), Jacksonville State University, Jacksonville, AL; Ioannis Sfikas, National and Kapodistrian University of Athens, Greece; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Brian Beasley, Presbyterian College, Clinton, SC; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; Julio Mohnsam, Mateus Lucas (student), and Oscar Schmitt(student), IFSUL Campus Pelotas-RS, Brazil; Pedro Pantoja, University of Campina Grande, Brazil; Amit Dutta, Jamshedput, India; Lazaros Zachariadis, Thessaloniki, Greece; Nawar Alasadi, Babylon, Iraq; Remus Florin Stanca, Romania; Sagar Kumar, Kolkata, India; Shivam Sharma, New Delhi, India; and the proposer.

Problem 811. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let a, b, c be three positive integers. Prove that

$$\sqrt[ab+bc+ca]{a^{bc}b^{ca}c^{ab}} \leqslant \sqrt[3]{abc}.$$

Solution *by Henry Ricardo, Westchester Area Math Circle, Purchase, NY.*

Maclaurin's inequality gives us $\sqrt{\frac{ab+bc+ca}{3}} \geqslant \sqrt[3]{abc}$ or $\frac{3}{ab+bc+ca} \leqslant \frac{1}{(abc)^{2/3}}$. Applying the weighted AM-GM inequality, we see that

$$\begin{aligned} & a^{\frac{bc}{ab+bc+ca}} \cdot b^{\frac{ca}{ab+bc+ca}} \cdot c^{\frac{ab}{ab+bc+ca}} \\ & \qquad \leqslant \frac{bc}{ab+bc+ca} \cdot a + \frac{ca}{ab+bc+ca} \cdot b + \frac{ab}{ab+bc+ca} \cdot c \\ & \qquad = \frac{3abc}{ab+bc+ca} \leqslant \frac{abc}{(abc)^{2/3}} = \sqrt[3]{abc}. \end{aligned}$$

Also solved by Ioan Viorel Codreanu, Satulung, Maramures, Romania; Pedro Pantoja, University of Campina Grande, Brazil; Marian Urs/varescu, National College "Roman Voda", Roman, Romania; Soumitra Mandal, "Scottish Church College", Chandar Nagore, India; Ioannis Sfikas, National and Kapodistrian University of Athens, Greece; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; and the proposer.

Problem 812. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let S be a finite set. Consider three partitions of S, each one with n elements:

$$A_1, \ldots, A_n; B_1, \ldots, B_n; C_1, \ldots, C_n.$$

If for all $1 \leq i, j, k \leq n$, it holds that

$$|A_i \cap B_j| + |B_j \cap C_k| + |C_k \cap A_i| \geqslant n,$$

then prove that $|S| \geqslant \frac{n^3}{3}$. When does equality hold?

Solution *by Ioannis Sfikas, National and Kapodistrian University of Athens, Greece.*

We will fix some A_i, B_j. Summing over all C, we have

$$|A_i| + |B_j| + n\,|A_i \cap B_j| \geqslant n^2.$$

Summing over all B_j, we have $2n\,|A_i| + |S| \geqslant n^3$. Summing over all $|A_i|$, we have $3n\,|S| \geqslant n^4$. Here we are repeatedly using the fact that the sum of the cardinalities of the intersection of a set with disjoint sets is not more than the cardinality of that set. The equality case holds in every inequality.

The proposer gave the following scheme for $n = 6$:

$$\begin{array}{cccccccc}
 & C_2 & C_3 & C_4 & C_5 & C_6 & & \\
A_1 & A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & C_1 \\
A_2 & A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & C_2 \\
A_3 & A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & C_3 \\
A_4 & A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & C_4 \\
A_5 & A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & C_5 \\
A_6 & A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} & C_6 \\
 & B_1 & B_2 & B_3 & B_4 & B_5 & B_6 &
\end{array}$$

Also solved by the proposer.

Problem 813. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Prove that if $a, b, c \in (1, \infty)$ then

$$\log_{ab^2c^2} a + \log_{a^2bc^2} b + \log_{a^2b^2c} c \geqslant \frac{3}{5}.$$

Solution *by Ángel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

The inequality may be written as $\sum\limits_{cyclic} \frac{\ln a}{\ln a + 2\ln b + 2\ln c} \geqslant \frac{3}{5}$ or $\sum\limits_{cyclic} \frac{x}{x+2y+2z} \geqslant \frac{3}{5}$ where $x = \ln a,\ y = \ln b,\ z = \ln c$ are positive numbers. Since the last inequality is homogeneous, we may assume that $x + y + z = 1$ and the inequality becomes $\sum\limits_{cyclic} \frac{x}{2-x} \geqslant \frac{3}{5}$. Finally, since the function $f(x) = \frac{x}{2-x}$ is convex for $x \in (0, 1)$, the conclusion follows by Jensen's inequality.

Also solved by Marian Ursărescu, National College "Roman Voda", Roman, Romania; Rovsen Pirguliyev, "Sumgait High School", Sumgait, Azerbaidian; Soumitra Mandal, "Scottish Church College", Chandar Nagore, India; Ioannis Sfikas, National and Kapodistrian University of Athens, Greece; and the proposers.

Problem 814. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let the sequence $(a_n)_{n\geqslant 1}$, defined by $a_1 = 1, a_{n+1} = (n+1)!a_n$ for any positive integer n. Compute

$$\lim_{n\to\infty} \frac{\sqrt[2n]{n!}}{\sqrt[n^2]{a_n}}.$$

Solution *by Rovsen Pirguliyev, Sumgait High School, Sumgait, Azerbaidian.*

$$\begin{aligned} &a_{n+1} = (n+1)!a_n \Rightarrow \frac{a_{n+1}}{a_n} = (n+1)! \Rightarrow \frac{a_{n+1}^2}{a_n^2} = ((n+1)!)^2 \\ &\Rightarrow \ln \frac{a_{n+1}^2}{a_n^2} = \ln\left((n+1)!\right)^2 \Rightarrow \ln a_{n+1}^2 - \ln a_n^2 = \ln\left((n+1)!\right)^2. \end{aligned} \tag{1}$$

It is known by the Stolz-Cesaro theorem that

$$\lim \frac{a_n}{b_n} = \lim \frac{a_{n+1} - a_n}{b_{n+1} - b_n}. \tag{2}$$

It is also well known that

$$\lim_{n\to\infty} \sqrt[n]{x_n} = \lim_{n\to\infty} \frac{x_{n+1}}{x_n}. \tag{3}$$

Let $x = \lim\limits_{n\to\infty} \frac{\sqrt[2n]{n!}}{\sqrt[n^2]{a_n}}$. Then

$$\begin{aligned} \ln x &= \lim_{n\to\infty} (\ln \sqrt[2n]{n!} - \ln \sqrt[n^2]{a_n}) \\ &\Rightarrow \ln x = \lim_{n\to\infty} \left(\frac{1}{2n} \ln n! - \frac{1}{n^2} \ln a_n \right) \\ &= \lim_{n\to\infty} \frac{n \ln n! - 2\ln a_n}{2n^2} \\ &= \lim_{n\to\infty} \frac{n \ln n! - \ln a_n^2}{2n^2}. \end{aligned}$$

Using (2), we have

$$\begin{aligned}
\ln x &= \lim_{n\to\infty} \frac{(n+1)\ln(n+1)! - \ln a_{n+1}^2 - n\ln n! + \ln a_n^2}{2((n+1)^2 - n^2)} \\
&= \lim_{n\to\infty} \frac{(n+1)\ln(n+1)! - n\ln n! - (\ln a_{n+1}^2 - \ln a_n^2)}{2(2n+1)} \\
&= \lim_{n\to\infty} \frac{(n-1)\ln(n+1)! - n\ln n!}{2(2n+1)} \quad \text{(from (1))} \\
&= \lim_{n\to\infty} \frac{n\ln(n+1)! - \ln(n+1)! - n\ln n!}{2(2n+1)} \\
&= \lim_{n\to\infty} \frac{n\ln(n+1) - \ln(n+1)!}{2(2n+1)} = \lim_{n\to\infty} \frac{1}{4}\sqrt[n]{\frac{(n+1)^n}{(n+1)!}}.
\end{aligned}$$

Let $x_n = \frac{(n+1)^n}{(n+1)!}$. Using (3)

$$\begin{aligned}
\lim_{n\to\infty} \sqrt[n]{x_n} &= \lim_{n\to\infty} \frac{(n+2)^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{(n+1)^n} \\
&= \lim_{n\to\infty} \left(\frac{n+2}{n+1}\right)^n = \lim_{n\to\infty} \left(1 + \frac{1}{n+1}\right)^n = e
\end{aligned}$$

Finally, $\ln x = \frac{1}{4}\ln e \Rightarrow x = e^{1/4} = \sqrt[4]{e}$.

Also solved by Soumitra Mandal, "Scottish Church" College, Chandar Nagore, India; Ioannis Sfikas, National and Kapodistrian University of Athens, Greece; and the proposers.

Problem 815. *Proposed by Marius Drăgan, National College Mircea cel Bătran, Bucharest, Romania, and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let a, b, c be positive real numbers such that $a^4 + b^4 + c^4 = 3$. Show that

$$2(a^2+b^2)(b^2+c^2)(c^2+a^2) \geqslant (a^2b^2 + b^2c^2 + c^2a^2 + abc)^2.$$

Solution *by Soumava Chakraborty, Soft Web Technology, Kolkata, India.*

We start with $\Pi(x+y) \geqslant \frac{8}{9}\left(\sum x\right)\left(\sum xy\right)$ for all $x, y > 0$. So

$$2(a^2+b^2)(b^2+c^2)(c^2+a^2) \geqslant \frac{16}{9}\left(\sum a^2\right)\left(\sum a^2b^2\right).$$

We want to show: $\frac{16}{9}\left(\sum a^2\right)\left(\sum a^2b^2\right) \geqslant \left(\sum a^2b^2 + abc\right)^2$, or

$$16\left(\sum a^2\right)\left(\sum a^2b^2\right) \geqslant 9\left(\sum a^2b^2\right)^2 + 18abc\left(\sum a^2b^2\right) + 9a^2b^2c^2. \tag{1}$$

Using the AM-GM, $\left(\sum a^2\right)\left(\sum a^2b^2\right) \geqslant 9\sqrt[3]{a^2b^2c^2}\sqrt[3]{a^4b^4c^4} = 9a^2b^2c^2$. From this, (1) follows if

$$\begin{aligned}5\left(\sum a^2\right)\left(\sum a^2b^2\right) \geqslant& 3\left(\sum a^2b^2\right)^2 + 6abc\left(\sum a^2b^2\right)\\ &\Leftrightarrow 5\left(\sum a^2\right) \geqslant 3\left(\sum a^2b^2\right) + 6abc.\end{aligned} \tag{2}$$

Now $3 = \sum a^4 \geqslant \frac{1}{27}\left(\sum a\right)^4$ by Chebyshev's sum inequality, so

$$\begin{aligned}&3 \geqslant \sum a \geqslant 3\sqrt[3]{abc} \text{ (by AM-GM)}\\ &\quad\Rightarrow 3\sum a^2 \geqslant 3\sqrt[3]{abc}\left(\sum a^2\right) \geqslant 9\sqrt[3]{abc}\sqrt[3]{a^2b^2c^2} = 9abc \text{ (by AM-GM)}\\ &\quad\Rightarrow \sum a^2 \geqslant 3abc \Rightarrow 2\sum a^2 \geqslant 6abc.\end{aligned}$$

So to show (2), it suffices to show:

$$2\sum a^2 \geqslant 2\sum a^2b^2 = \left(\sum a^2\right)^2 - \sum a^4 = \left(\sum a^2\right)^2 - 3.$$

If we let $t = \sum a^2$, the above is equivalent to

$$t^2 - 2t - 3 \leqslant 0 \Leftrightarrow (t+1)(t-3) \leqslant 0. \tag{3}$$

Now $3 = \sum a^4 \geqslant \frac{1}{3}\left(\sum a^2\right)^2 \Rightarrow \sum a^2 = t \leqslant 3$ and so (3) is true.

Also solved by Ioannis Sfikas, National and Kapodistrian University of Athens, Greece; and the proposers.

Problem 816. *Proposed by Stanescu Florin, Serban Cioclescu School, Gaesti, Romania.*

Determine the largest real number α with the property that for any function $f : [0,1] \to [0,\infty)$ where the following hold:

1. f is convex and $f(0) = 0$;
2. there exists $\epsilon > 0$ with f differentiable on $[0,\varepsilon)$ and $f'(0) \neq 0$;

the following inequality holds:

$$\int_0^1 \frac{x^2}{\int_0^x f(t)dt}dx \geqslant \int_0^1 (x+\alpha)\cdot\frac{f(x)}{\left(\int_0^1 f(t)dt\right)^2}dx.$$

Solution *by the proposer.*

For $f(x) = x, \forall x \in [0,1]$, we obtain that $\alpha \leqslant \frac{1}{3}$. We will show that

$$\int_0^1 \left[\frac{x^2}{\int_0^x f(t)\,dt}\right]dx \geqslant \int_0^1 \left[\left(x+\frac{1}{3}\right)\cdot\frac{f(x)}{\left(\int_0^1 f(t)\,dt\right)^2}\right]dx.$$

a) We will prove the inequality: $c^2\int_0^c f(x)dx \geqslant \int_0^c f(x)dx, \forall x \in [0,1]$.

Proof. Since f is convex on $[0,1]$, then

$$f(\lambda x + (1-\lambda)y) \leqslant \lambda f(x) + (1-\lambda) f(y),$$

$\forall x, y \in [0,1]$ and $\forall \lambda \in [0,1]$. Now, for $y = 0$ there is

$$\lambda = c \in [0,1] \Rightarrow f(cx) \leqslant cf(x)$$

$$\Rightarrow \int_0^1 f(cx)\,dx \leqslant c\int_0^1 f(x)dx$$

$$\underset{t=cx}{\Rightarrow} \int_0^c f(x)dx \leqslant c^2\int_0^1 f(x)dx, \forall c \in [0,1].$$

■

b) Now we will prove that if

$$f, g : [a,b] \to \mathbb{R}$$

are two continuous functions, then:

$$\int_a^b f(x)\left(\int_a^x g(t)\right)dx = \int_a^b g(x)\left(\int_x^b f(t)\right)dx.$$

Proof. We can write

$$\begin{aligned}\int_a^b f(x)\left(\int_a^x g(t)\right)dx &= \int_a^b -\left(\int_x^b f(t)\,dt\right)'\left(\int_a^x g(t)\right)dx \\ &= \int_a^b g(x)\left(\int_x^b f(t)\right)dx.\end{aligned}$$

■

Thus $\int_0^x f(t)dt \leqslant x^2 \cdot \int_0^1 f(t)dt$. If there exists $x_0 \in (0,1)$ such that $\int_0^{x_0} f(t)\,dt = 0$ and f is continuous at 0 and f is convex, then f is continuous on $[0, x_0]$, so the mean value theorem says there is $y_0 \in (0, x_0)$ such that $f(y_0) = 0$. If $x \in (0, y_0)$, there is $\alpha \in (0,1)$, such that

$$\begin{aligned}x = 0\cdot\alpha + (1-\alpha)\,y_0 \Rightarrow f(x) &= f(0\cdot\alpha + (1-\alpha)\,y_0) \\ &\leqslant \alpha f(0) + (1-\alpha)\,f(y_0) = 0 \\ &\Rightarrow f(x) = 0, \forall x \in [0, y_0) \Rightarrow f'(0) = 0,\end{aligned}$$

a contradiction. Thus $\int_0^x f(t)dt > 0, \forall x \in (0,1)$, and now

$$\begin{aligned}\int_0^1 f(t)dt \geqslant \int_0^x f(t)dt > 0, \forall x \in (0,1) \\ \Rightarrow \int_0^x f(t)dt > 0, \forall x \in (0,1].\end{aligned}$$

We define the function

$$\varphi : [0,1] \to \mathbb{R}, \varphi(x) = \begin{cases} \frac{x^2}{\int_0^x f(t)dt}, x \in (0,1] \\ \frac{2}{f'(0)}, x = 0 \end{cases}$$

Now, f' has the property of Darboux and is increasing (because it is convex), so f' is continuous on $[0, \varepsilon)$. According to L 'Hospital's rule, we have: $\lim_{x\searrow 0} \varphi(x) = \lim_{x\searrow 0} \frac{2x}{f(x)} = \lim_{x\searrow 0} \frac{2}{f'(x)} = \frac{2}{f'(0)}$, so φ is contiuous and integrable on $[0,1]$. Further, from $\int\limits_0^x f(t)dt \leqslant x^2 \cdot \int\limits_0^1 f(t)dt$ we obtain

$$\frac{t^2}{\int\limits_0^t f(y)\,dy} \geqslant \frac{1}{\int\limits_0^1 f(x)\,dx}$$

$$\Rightarrow \int\limits_0^x \left[\frac{t^2}{\int\limits_0^t f(y)\,dy}\right] dt \geqslant \frac{x}{\int\limits_0^1 f(x)\,dx}$$

$$\Rightarrow f(x) \int\limits_0^x \left[\frac{t^2}{\int\limits_0^t f(y)\,dy}\right] dt \geqslant \frac{x f(x)}{\int\limits_0^1 f(x)\,dx}$$

$$\Rightarrow \int\limits_0^1 \left(f(x) \int\limits_0^x \left[\frac{t^2}{\int\limits_0^t f(y)\,dy}\right] dt \right) dx \geqslant \frac{1}{\int\limits_0^1 f(x)\,dx} \int\limits_0^1 x f(x) dx$$

$$\Rightarrow \int\limits_0^1 \left(\frac{x^2}{\int\limits_0^x f(t)\,dt} \cdot \left[\int\limits_x^1 f(t)\,dt\right] dx \right) \geqslant \frac{1}{\int\limits_0^1 f(x)\,dx} \int\limits_0^1 x f(x) dx$$

$$\Rightarrow \int\limits_0^1 \left(\frac{x^2}{\int\limits_0^x f(t)\,dt} \cdot \left[\int\limits_0^1 f(t)\,dt - \int\limits_0^x f(t)\,dt\right] dx \right) \geqslant \frac{1}{\int\limits_0^1 f(x)\,dx} \int\limits_0^1 x f(x) dx$$

$$\Rightarrow \int\limits_0^1 f(x)\,dx \cdot \int\limits_0^1 \left[\frac{x^2}{\int\limits_0^x f(t)\,dt}\right] dx \geqslant \int\limits_0^1 x^3 dx + \frac{1}{\int\limits_0^1 f(x)\,dx} \int\limits_0^1 x f(x) dx$$

$$\Rightarrow \int\limits_0^1 \left[\frac{x^2}{\int\limits_0^x f(t)\,dt}\right] dx \geqslant \int\limits_0^1 \left[\left(x + \frac{1}{3}\right) \cdot \frac{f(x)}{\left(\int\limits_0^1 f(t)\,dt\right)^2}\right] dx.$$

Thus $\alpha = \frac{1}{3}$.

Problem 817. *Proposed by Stanescu Florin, Serban Cioclescu School, Gaesti, Romania.*

Consider the complex numbers a, b, c, d, all of modulus one, which have the following properties:

1. $\arg a < \arg b < \arg c < \arg d$;
2. $2\sqrt{|(b+ai)(c+bi)(d+ci)(a+di)|} - \sqrt{\frac{[(a-c)(b-d)]^2}{abcd}} = 4.$

Show that $\max\left\{\left|1 - \frac{i+4a^2}{i+8a(b-d)}\right|; \left|1 - \frac{i+4b^2}{i+4b(a-c)}\right|\right\} \geqslant \frac{4}{\sqrt{17}}$.

Solution *by the proposer.*

It is clear that $|(b+ai)(c+bi)(d+ci)(a+di)| \neq 0$ and $a \neq b \neq c \neq d \neq a$. We consider the Cartesian points A, B, C, D corresponding to a, b, c respectively d. From $|a| = |b| = |c| = |d| = 1$ it is clear that the four points are on the circle with center O and radius 1. Also, from the first condition it is clear that $ABCD$ is a convex quadrilateral. Thus, A, B, C, D are oriented in a trigonometric sense. We calculate:

$$\begin{aligned}
\frac{AC \cdot BD}{2} &= \frac{|a-c|\,|b-d|}{2} \\
&= \frac{\sqrt{(a-c)\left(\overline{a-c}\right)(b-d)\left(\overline{b-d}\right)}}{2} \\
&= \frac{1}{2}\sqrt{\left[2 - \left(\frac{a}{c} + \frac{c}{a}\right)\right]\left[2 - \left(\frac{b}{d} + \frac{d}{b}\right)\right]}\sqrt{\frac{[(a-c)(b-d)]^2}{abcd}}.
\end{aligned}$$

Further, we can write:

$$\begin{aligned}
4 &= 2\sqrt{|(b+ai)(c+bi)(d+ci)(a+di)|} - \sqrt{\frac{[(a-c)(b-d)]^2}{abcd}} \\
&= 2\sqrt[4]{|b+ai|^2|c+bi|^2|d+ci|^2|a+di|^2} - \sqrt{\frac{[(a-c)(b-d)]^2}{abcd}} \\
&\leqslant 2 \cdot \frac{|b+ai|^2 + |c+bi|^2 + |d+ci|^2 + |a+di|^2}{4} - \sqrt{\frac{[(a-c)(b-d)]^2}{abcd}}
\end{aligned}$$

So

$$4 \leqslant \frac{|a|^2 + a\overline{b}i - \overline{a}bi + |b|^2 + |b|^2 + b\overline{c}i - \overline{b}ci + |c|^2}{2}$$
$$+ \frac{|c|^2 + c\overline{d}i - \overline{c}di + |d|^2 + |d|^2 + d\overline{a}i - \overline{d}ai + |a|^2}{2}$$
$$- \sqrt{\frac{[(a-c)(b-d)]^2}{abcd}}$$
$$= 4 + \frac{1}{2i}\left(\overline{a}b + \overline{b}c + \overline{c}d + \overline{d}a - \overline{b}a - \overline{c}b - \overline{d}c - \overline{a}d\right) - \sqrt{\frac{[(a-c)(b-d)]^2}{abcd}}$$
$$= 4 + \operatorname{Im}\left(\overline{a}b + \overline{b}c + \overline{c}d + \overline{d}a\right) - \sqrt{\frac{[(a-c)(b-d)]^2}{abcd}}$$

This last as $\frac{z-\bar{z}}{2i} = \operatorname{Im} z$. Then

$$\frac{1}{2}\sqrt{\frac{[(a-c)(b-d)]^2}{abcd}} \leqslant \frac{1}{2}\operatorname{Im}\left(\overline{a}b + \overline{b}c + \overline{c}d + \overline{d}a\right)$$
$$\Rightarrow \frac{AC \cdot BD}{2} \leqslant S_{ABCD}.$$

We used the formula of Kiril Docev for the area of a quadrilateral:

$$S_{ABCD} = \frac{1}{2}\operatorname{Im}\left(\overline{a}b + \overline{b}c + \overline{c}d + \overline{d}a\right),$$

with A, B, C, D oriented in a trigonometric sense . This results in

$$\frac{AC \cdot BD}{2} \leqslant S_{ABCD} \leqslant \frac{AC \cdot BD}{2}\cos(AC, BD) \Rightarrow AC \perp BD$$

and the above inequalities become equal. So:

1. $ABCD$ is an orthodiagonal quadrilateral;
2. $|b+ai| = |c+bi| = |d+ci| = |a+di|$.

(1): Since $ABCD$ is an orthodiagonal quadrilateral we get

$$AB^2 + CD^2 = BC^2 + AD^2$$
$$\Rightarrow |a-b|^2 + |c-d|^2 = |b-c|^2 + |a-d|^2$$
$$\Rightarrow \frac{a}{b} + \frac{b}{a} + \frac{c}{d} + \frac{d}{c} = \frac{b}{c} + \frac{c}{b} + \frac{a}{d} + \frac{d}{a}$$
$$\Rightarrow (ac+bd)\left(\frac{1}{bc} + \frac{1}{ad} - \frac{1}{cd} - \frac{1}{ab}\right) = 0,$$

a contradiction, because $a \neq b \neq c \neq d \neq a$. Thus $ac + bd = 0$. Now we show that $|ac + (a-c)(b-d)|^2 - |(a-c)(b-d)|^2 = 1$. We have

$$\begin{aligned}
&abcd\left(|ac + (a-c)(b-d)|^2 - |(a-c)(b-d)|^2 - 1\right) \\
&= abcd\,(ac + (a-c)(b-d))\left(\frac{1}{ac} + \frac{(a-c)(b-d)}{abcd}\right) \\
&\qquad - abcd\,(a-c)(b-d)\,\frac{(a-c)(b-d)}{abcd} - 1 \\
&= (ac + (a-c)(b-d))\,(bd + (a-c)(b-d)) \\
&\qquad - (a-c)^2(b-d)^2 - abcd \\
&= ac\,(a-c)(b-d) + bd\,(a-c)(b-d) \\
&= (a-c)(b-d)(ac+bd) = 0.
\end{aligned}$$

Analogously, $|ac + (a-c)(b-d)|^2 - |(a-c)(b-d)|^2 = 1$.
(2): Now

$$\begin{aligned}
|b+ai|^2 = |c+bi|^2 &\Rightarrow -\frac{b}{a} + \frac{a}{b} = -\frac{c}{b} + \frac{b}{c} \\
&\Rightarrow \frac{a^2-b^2}{a} = \frac{b^2-c^2}{c} \\
&\Rightarrow b\left(\frac{1}{a} + \frac{1}{c}\right) = \frac{1}{b}(a+c) \\
&\Rightarrow a + c = 0
\end{aligned}$$

or $b^2 = ac \overset{ac+bd=0}{\Rightarrow} b^2 = -bd \Rightarrow b + d = 0$.
(2.1) We assume that $a+c=0$. A similar argument shows that if $a+c=0$ and $ac+bd=0$, then $|b+ai| = |c+bi| = |d+ci| = |a+di|$. Thus, we have

$$|ac + (a-c)(b-d)| = \sqrt{1 + |(a-c)(b-d)|^2}$$

$$= \sqrt{\frac{1^2}{1} + \underbrace{\frac{\left(\frac{|(a-c)(b-d)|}{4}\right)^2}{1} + \frac{\left(\frac{|(a-c)(b-d)|}{4}\right)^2}{1} + \ldots + \frac{\left(\frac{|(a-c)(b-d)|}{4}\right)^2}{1}}_{16-times}}$$

$$\geqslant \sqrt{\frac{\left(1 + \underbrace{\frac{|(a-c)(b-d)|}{4} + \frac{|(a-c)(b-d)|}{4} + \ldots + \frac{|(a-c)(b-d)|}{4}}_{16-ori}\right)^2}{17}}$$

$$= \frac{1}{\sqrt{17}} \left(1 + 4 \left|(a-c)(b-d)\right|\right)$$
$$= \frac{1}{\sqrt{17}} \left(|i| + 4 \left|(a-c)(b-d)\right|\right) \geqslant \frac{1}{\sqrt{17}} \left|i + 4(a-c)(b-d)\right|$$
$$\Rightarrow \left| \frac{|ac + (a-c)(b-d)|}{i + 4(a-c)(b-d)} \right| \geqslant \frac{1}{\sqrt{17}}$$
$$\Rightarrow \left| 1 - \frac{i - 4ac}{i + 4(a-c)(b-d)} \right| \geqslant \frac{4}{\sqrt{17}} \overset{c=-a}{\Rightarrow} \left| 1 - \frac{i + 4a^2}{i + 8a(b-d)} \right| \geqslant \frac{4}{\sqrt{17}}.$$

(2.2) For $b + d = 0$, analogously we obtain that

$$\left| 1 - \frac{i + 4b^2}{i + 8b(a-c)} \right| \geqslant \frac{4}{\sqrt{17}}$$

from where

$$\max \left\{ \left| 1 - \frac{i + 4a^2}{i + 8a(b-d)} \right| ; \left| 1 - \frac{i + 4b^2}{i + 4b(a-c)} \right| \right\} \geqslant \frac{4}{\sqrt{17}}.$$

Problem 818. *Proposed by Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade", Buzău, Romania.*

Determine all positive integers a, b, c, d, x, y, z, t such that $a \neq b \neq c \neq d$ and $a + b + c = td$, $b + c + d = xa$, $c + d + a = yb$, and $d + a + b = zc$.

Solution *by Ioannis Sfikas, National and Kapodistrian University of Athens, Greece.*

Assume the following matrices

$$A = \begin{bmatrix} -x & 1 & 1 & 1 \\ 1 & -y & 1 & 1 \\ 1 & 1 & -z & 1 \\ 1 & 1 & 1 & -t \end{bmatrix} ; x = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}.$$

The problem will have the form $Ax = 0$ which is a homogeneous system of linear equations. The determinant of the matrix A is

$$\det A = (xt - 1)(yz - 1) - (x + t + 2)(y + z + 2).$$

Since $a \neq b \neq c \neq d$, the homogeneous system of linear equations has a nontrivial solution. Subtracting pairwise the equations, one obtains

$$a : b : c : d = \frac{1}{1+x} : \frac{1}{1+y} : \frac{1}{1+z} : \frac{1}{1+t}.$$

Multiplying the right-hand side with $(1 + x)(1 + y)(1 + z)(1 + t)$ results in the following integer solutions:

$$\begin{aligned} a &= (1+y)(1+z)(1+t) \\ b &= (1+x)(1+z)(1+t) \\ c &= (1+x)(1+y)(1+t) \\ d &= (1+x)(1+y)(1+z). \end{aligned}$$

One can easily check that substitution of these values into the homogeneous system results in the expression for det A, i.e. 0. Though it is not immediately obvious, by brute force one finds six (x, y, z, t) solutions exist:

$$\begin{array}{ccc} (1,2,6,41) & (1,2,7,23) & (1,2,8,17) \\ (1,2,9,14) & (1,3,4,19) & (1,3,5,11)\,. \end{array}$$

The corresponding vectors (a, b, c, d) can be computed from the above equations. For example, $(x, y, z, t) = (1, 3, 5, 11)$ gives

$$(a, b, c, d) = (4*6*12, 2*6*12, 2*4*12, 2*4*6)\ (6, 3, 2, 1).$$

Also solved by the proposers.

Problem 819. *Proposed by the editor.*

Billy has created a snowman with a base, torso, and a head all of which are spheres where the torso is smaller than the base and the head is smaller than the torso. The radius of each piece is a positive integer with the base having a 12 inch radius. He decides that he wants to use the snow to make 3 similar and identical snowmen for his younger siblings. He discovers that he can do this. What are the radii of the original and smaller snowmen?

Solution *by Brian Beasley, Presbyterian College, Clinton, SC.*

We seek positive integers a, b, w, x, and y with

$$12 > a > b, 12 > w > x > y,$$

and

$$\frac{4\pi}{3}(12^3 + a^3 + b^3) = 3 \cdot \frac{4\pi}{3}(w^3 + x^3 + y^3)$$

or equivalently $12^3 + a^3 + b^3 = 3(w^3 + x^3 + y^3)$. Then $7 \leqslant w$ since $w \leqslant 6$ would imply $3(w^3 + x^3 + y^3) \leqslant 3(6^3 + 5^3 + 4^3) < 12^3$. Also $w < 11$, as otherwise $a^3 + b^3 > 3 * 11^3 - 12^3$ would imply $a = 11$ and $b = 10$ and thus $x^3 + y^3 = 22$, which is impossible. If $w = 7$, then $3(x^3 + y^3) > 12^3 - 3 * 7^3$ so $x = 6$. Thus $51 + a^3 + b^3 = 3y^3$ which yields $y \in \{3, 4, 5\}$ and hence $a^3 + b^3 \in \{30, 141, 324\}$ which is impossible. Similarly, $w = 8$ leads to impossibilities. If $w = 9$, then

$a^3 + b^3 > 3 * 9^3 - 12^3$ so $a \in \{7, 8, 9, 10, 11\}$. If $a = 7$, then $b^3 = 3x^3 + 3y^3 + 116$ so $b \equiv 2 (\mathrm{mod} 3)$ and $b^3 > 116$ implies $b = 5$. Thus $x^3 + y^3 = 3$, which is impossible. Similarly, $a = 8, 9$, and 11 lead to impossibilities. When $a = 10$, $b^3 + 541 = 3x^3 + 3y^3$ so $b \equiv 2 (\mathrm{mod} 3)$ implies $b \in \{2, 5, 8\}$. Thus $x^3 + y^3 \in \{183, 222, 351\}$. While 183 and 222 are not possible, 351 is possible for $x = 7$ and $y = 2$. The case of $w = 10$ also leads to impossibilities. Hence the unique solution is

$$(a, b, w, x, y) = (10, 8, 9, 7, 2);$$

that is, the original torso radius is 10 inches, the original head radius is 8 inches, the small base radius is 9 inches, the smaller torso is 7 inches, and the smaller head radius is 2 inches.

Also solved by Brad Meyer (student), Missouri State University, Springfield, MO; and the proposer.

Kappa Mu Epsilon News

Edited by Cynthia Huffman, Historian
Updated information as of June 2018

News of chapter activities and other noteworthy KME events should be sent to

Cynthia Huffman, KME Historian
Pittsburg State University
Mathematics Department
117 1701 S. Broadway
Pittsburg, KS 66762
or to
cjhuffman@pittstate.edu

KAPPA MU EPSILON
Installation Report
New York Sigma, College of Mount Saint Vincent
The Bronx, New York

The New York Sigma Chapter of Kappa Mu Epsilon was installed at 6:00 p.m. on April 30, 2018 at a ceremony held on the campus of the College of Mount Saint Vincent in The Bronx, New York. The meeting was conducted by Dr. Peter Luthy. KME national president, Dr. Brian Hollenbeck, served as the installing officer.

Five students and three faculty members were initiated as the charter members of the New York Sigma Chapter. The three faculty are Assistant Professors of Mathematics Dr. Peter Luthy and Dr. Victor Miroshnikov, and Associate Professor of Mathematics, Dr. Amir Niknejad. The five students initiated are Maria Aceituno, Savannah George, Weily Lang, Sean Rice, and Laurén Tate. The first officers of the chapter were installed: Weily Lang, President; Laurén Tate, Vice President; Sean Rice, Secretary; Savannah George, Treasurer; Peter Luthy, Corresponding Secretary; and Victor Miroshnikov, faculty sponsor.

Following the installation ceremony, Dr. Hollenbeck presented a talk titled, When the “Best” Strategy Fails, during which attendees played and analyzed the dice game, Kyboi. The evening concluded with a meal. Below is a picture of the charter members of New York Sigma.

New York Sigma

Installation Report
Pennsylvania Upsilon, Seton Hill University
Greensburg, PA

The installation of the Pennsylvania Upsilon Chapter of Kappa Mu Epsilon was held in the JoAnne Woodyard Boyle Health Sciences Center on the campus of Seton Hill University on Tuesday, May 8, 2018, at 5:00 PM.

Professor Jared Burns, corresponding secretary and faculty co-sponsor, organized the event, provided the first welcome, and served as master of the ceremony. The next welcome remarks were provided by Dr. Bernadette Fondy, Dean of the School of Natural and Health Sciences and Professor of Biology, and Sr. Susan Yochum, Provost and Professor of Chemistry. KME Great Lakes Regional Director Pete Skoner offered remarks about the history of Kappa Mu Epsilon, and then led the installation and initiation ceremony. After the ceremony, Professor Joshua Sasmor presented an interesting talk on "Mathematics Education: Are We Relevant?".

Participating in the ceremony were the charter officers: Rachael Boggs, President; Cecilia Ducar, Vice President, and Kristen Barczynsky; Secretary. Other officers installed were Daniel William Stariha, Treasurer, and Morgan Vincent, President-elect. Each officer heard the responsibilities of the office, and each chose to accept those responsibilities. After Secretary Barczynski described the KME crest, the organization was declared to be the Pennsylvania Upsilon Chapter of Kappa Mu Epsilon and the chapter's crest was presented to chapter president Boggs.

In addition to the faculty mentioned, the other faculty member initiated was Abigail Bogdan. The other student charter members of Pennsylvania Upsilon are Jordan Barzensky, Rachael Ditzler, Margaret Gerthoffer, Brittney Racioppo, and Abbie Thrower. Each initiate signed the Pennsylvania Upsilon Chapter Roll, and received a certificate, membership card, program, and KME jewelry pin.

A cookie and punch reception followed the ceremony and then students were off to study for final exams. A total of 28 people attended, including some parents and friends of initiates. Congratulations to the charter and future members of Pennsylvania Upsilon.

Pennsylvania Upsilon

Chapter News

AL Alpha — Athens State University

Chapter President - Kassidy McCormick; 20 Current Members; 10 New

Members

Other Spring 2018 Officers: Nahanna Corbin, Vice President; Katelyn Sudduth, Secretary; Dr. Patricia Glaze, Corresponding Secretary and Faculty Sponsor.

New Initiates – Nahanna Corbin, Eric Curt Day, Christopher Brett Godbee, Yafei Jiang, Ilish Kane, Jake E. Loring, Clarissa Dawn McCarty, Kassidy Liana McCormick, Tomi Whitaker Stapler, and Katelyn Hope Sudduth.

AL Beta – University of North Alabama

Corresponding Secretary – Ashley Johnson; 11 New Members

New Initiates – John Batson, Kristopher Gibson, Magdelene Gruber, Nathan Hansen, Elizabeth Haygood, Walker Mattox, Mallory Morris, Jacob Phillips, Elicia Springer, Aaron Thrasher, and Joshua Tittle.

AL Gamma — University of Montevello

Chapter President – Drew Mullinax; 22 Current Members; 9 New Members

Other Spring 2018 Officers: Elizabeth Blue, Vice President; Drew Carman, Secretary; Dr. Scott Varagona; Corresponding Secretary and Faculty Sponsor.

AL Theta — Jacksonville State University

Chapter President – Daniel Miradakis; 50 Current Members; 26 New Members

Other Spring 2018 Officers: Blake Jackson, Vice President; Becky Peters, Secretary; Holly Sparkman, Treasurer; and Dr. David Dempsey, Corresponding Secretary and Faculty Sponsor.

On April 24, 2018, the Alabama Theta chapter initiated 26 new student members. New members received their certificates, pins, and honor cords in a ceremony held in room 355 Ayers Hall. The event had been scheduled for March 30; however, JSU was closed for three weeks following the March 19 tornado that damaged much of the campus. Nevertheless, our chapter met approximately biweekly before the tornado; activities included hosting a game night, attending the JSU musical “Guys and Dolls”, and breaking out of an escape room! Despite the university closure, 7 students and 1 faculty member attended the KME South Eastern Regional Convention at Georgia Gwinnett College; 3 of our students presented, and Blake Jackson won the best presentation award. We are proud of all of our students and we look forward to getting back into regular activities in the fall semester.

New Initiates – Jacob Anthony Battles, Summer Brianna Burton, Caden Garrett, Benjamin Junkins, Brittney Mae Langley, Mendi Rose Lash, Robi Ledbetter, Hannah Leonard, Haley Elizabeth Long, Blake Evan Noah, Callie Mae Payne, Jalyn Perine, Alison Leeanne Powell, Kyle Anthony Roberson, Alexander James Roberts, Kerry Brendan Roberts, Emily

Claire Robertson, Marcus L. Shell, Amber Faith Smith, Cassidy Blu Stephens, Luke Christian Tatum, Lexie Hope Tisdale, Anna Traylor, Kalei Rae Whitson, Tiffany Williams, and Joshua Alexander Williamson.

CA Epsilon — California Baptist University

Corresponding Secretary – James Buchholz; 13 New Members

New Initiates – Grayson Boese, Keira Burgeson, Anna Crosby, Austin Eatinger, Lesley Hobgood, Brittany Meraz, Makenna Nishimoto, Jazmine Perez, Jennifer Romero, Marcos Salido-Lopez, Christina Seaton, Kristine-Joy Sellona, and Haley Smith.

CO Delta — Colorado Mesa University

Corresponding Secretary – Eric Packard; 5 New Members

New Initiates – Kayley Archuleta, Matthew Keaney, Araceli Martinez-Garcia, Hannah Naftanel, and Katelynn Olsen.

CT Beta — Eastern Connecticut State University

Corresponding Secretary – Dr. Mehdi Khorami; 475 Current Members; 12 New Members

New Initiates – Catherine Falvey, Tracey Ferguson, Allison Gagliano, Sean Gilmartin, Joshua Langdon, James Malizia, Bryce Mase, Emily Menendez, Connor Moran, James Rowley, David Whiting, and Brian Wisniewski.

FL Gamma — Southeastern University

Corresponding Secretary – Dr. Berhane Ghaim; 7 New Members

New Initiates – Cailey Cantrell, Shane Coris, Melissa Fuentes, Charis Howell, Jessica Jastrzebski, Tori Scheffler, and Marlena Schmidt.

GA Beta — Georgia College & State University

Corresponding Secretary – Rodica Cazacu; 6 New Members

New Initiates – Joshua C. Ballard-Myer, Matthew P. Dallas, Amy B. Edwards, Maranda N. Fulco, Cain A. Gantt, and Hanna L. Kagele.

GA Gamma – Piedmont College

Corresponding Secretary – Hope Menzel; 5 New Members

New Initiates – Courtney Bell, Rebecca Bowen, William Foster, Morgan Ivey, and Kaitlyn Loesel.

GA Zeta — Georgia Gwinnett College

Chapter President – Bess Burnett; 42 Current Members; 8 New Members
Other Spring 2018 Officers: Antoinette Miezan, Vice President; Matthew Stiller, Treasurer; Dr. Jamye Curry, Corresponding Secretary; and Drs. Jenny Sinclair and Livy Uko, Faculty Sponsors.

The Georgia Zeta Chapter hosted the Spring 2018 South Eastern KME Convention this year at Georgia Gwinnett College. The program of the convention is included below.

New Initiates – – Lanjing Andreasen, Tonya DeGeorge, Patrick Graham, Stacy Jones, Jose Ramos Rodriguez, Carmen Roland, Edgar Vasquez, and Alexandra Yon.

GA Eta — Atlana Metropolitan State College

Chapter President – Lanessa Northcut; 30 Current Members; 10 New Members

Other Spring 2018 Officers: Amanuel Belete, Vice President; Joanan Destin, Secretary; Joseph Daniels, Treasurer; Dr. Mulugeta Markos, Corresponding Secretary, and Dr. Kwan Lam, Faculty Sponsor.

In the spring of 2018 we initiated 10 new members. “The Beauty of Mathematics” speech was delivered by Dr. Markos in the initiation ceremony.

New Initiates – Belete Amanuel, Elijah Billings, Joanan Destin, Jashua Green, Dongheon Han, Brianna Horton, Tameka Little, Karaus Miller, Bryan Mitchell, and Kerese Moore.

HI Alpha — Hawaii Pacific University

Corresponding Secretary – Tara Davis; 17 Current Members; 9 New Members

In April 2018 we had an initiation dinner that included math faculty, previously initiated members, and the new members.

New Initiates – Mollie Asbury, Kianna Bagdon, Lundy Burge, Celeste Kawena Lydia Cravalho, Jake Ryan DeBosschere, Alexxis De Lamere, Marisa Gomez, Mikaela Gray, Petra Keiser, and Louiegy Martin Ringor Ponce.

IA Alpha — University of Northern Iowa

Chapter President – Jake Weber; 25 Current Members; 5 New Members

Other Spring 2018 Officers: Isaac Neppel, Vice President; Destiny Leitz, Secretary; Nicole Mlodzik, Treasurer; and Dr. Mark D. Ecker, Corresponding Secretary and Faculty Sponsor.

Our first spring KME meeting was held on February 27, 2018 at Professor Mark Ecker’s residence, where student member Isaac Neppel presented his paper entitled “Optimal Pricing for Promoted Spirits.” Our second meeting was held on March 27, 2018 at Professor Syed Kirmani's residence where student Taylor Olson talked about his undergraduate research paper entitled "Using Twitter to Analyze U.S. Positions on Popular Ethical Issues." Student member Destiny Leitz addressed the spring initiation banquet on April 24, 2018 with her honors thesis entitled "Money Matters: An Analysis of Campaign Finance in the U.S. House of Representatives Elections from 2010 to 2016." Our banquet was held at Godfather's Pizza in Cedar Falls, where five new members were initiated.

New Initiates – Joshua De La Bruere, Lincoln Fuhs, Christopher Merck, Jaclyn Miller, Isaac Neppel, Eric Nichols, and Stephanie Peiffer.

IA Gamma — Morningside College

Chapter President – Kelsey Brenner; 26 Current Members; 0 New Members

Other Spring 2018 Officers: Billy Salber, Vice President; David Swerev, Secretary; Sam Anderson, Treasurer; and Dr. Chris Spicer, Corresponding Secretary and Faculty Sponsor.

IA Delta — Wartburg College

Corresponding Secretary – Brian Birgen; 5 New Members

New Initiates – Grant Gingerich, Shahzeb Jadoon, Allison Limke, Jason Niccolls, and Codey Olson.

IA Epsilon — Central College

Chapter President – Joseph Eilers; 30 Current Members; 10 New Members

Other Spring 2018 Officers: Sydney Pratt, Vice President/Secretary; Dr. Russell E. Goodman, Corresponding Secretary and Faculty Sponsor.

We are proud to have initiated 11 new members this spring, bringing our total membership to 41; our KME chapter is wrapped inside of our MATH Club, so we've become a steady strong group here at Iowa Epsilon!

New Initiates – Elise Askelsen, Edward Ginnan, Macin Harvey, D.J. Imoehl, Colin Jones, Lauren Kriegel, Allison Reinbold, Sarah Sponder, Lauren Vahlkamp, Jodie Viers, and Nicholas Vogel.

IL Zeta — Dominican University

Corresponding Secretary and Faculty Sponsor – Aliza Steurer; 25 Current Members; 7 New Members

In April, we were excited to induct seven new members, all of whom are active around campus. Three are teaching assistants for calculus and mathematics for elementary education courses, two are mathematics tutors, one is a Math Club officer, and one is an architectural engineering student in our joint program with Illinois Institute of Technology. At the induction ceremony, Robert Vallin from Lamar University gave a great plenary address, "The Other Side of Recreational Mathematics" that new members and their family and friends enjoyed. He spoke about how puzzles, riddles, card tricks, and other types of recreational mathematics can create deep research problems.

New Initiates – Meyahueltzin Angeles, Jairo Duarte, Francesco Greco, Romeo A. López-Rodríguez, Antonio Morales, Daniela Rios, and Yesenia Soto.

IL Eta — Western Illinois University

Corresponding Secretary – Amy Ekanayake; 10 New Members

New Initiates – Md Mostak Ahammed, Latifah Alharbi, Lauren Bender, Omoniyi Celestine Enikanlogbon, Audrey Fletcher, Eddah Mauti, Nathan Miller, Jonathan Njeunje, Shaikh Obaidullah, and Jerold E. Young.

IL Kappa — Aurora University

Corresponding Secretary – Sebastian Wyman; 4 New Members

New Initiates – Jordan Brassil, Alyssa Daum, Myrna Garcia, and Ariel Torres.

IN Beta — Butler University

Corresponding Secretary – Chris Wilson; 9 New Members

New Initiates – Lauren Bauerle, Rachel Burke, Abby Craig, Michelle Jensen, Emily Knaub, Christopher Koch, Dillon O'Donovan, David Purdum, and Abigale Wynn.

KS Alpha — Pittsburg State University

Chapter Presidents – Georgette Searan; 25 Current Members; 13 New Members

Other Spring 2018 Officers: Jordan Bailey, Vice President; Zachary Bowen, Secretary; Peyton Burlingame, Treasurer; Dr. Tim Flood, Corresponding Secretary; and Dr. Scott Van Thuong, Faculty Sponsor.

The Kansas Alpha chapter once again had a very active semester, with three monthly meetings in February, March, and April. Lively presentations about RSA encryption, braid groups, and geodesy were enjoyed by all with pizza and pop. Kansas Alpha sent three faculty and three students to the North/South Central regional conference hosted by Emporia State. Our very own Dr. Cynthia Huffman delivered the keynote address on the mathematical treasures to be found at the Linda Hall Library. Also, our student Jordan Bailey, sponsored by Dr. Leah Childers, received a top presentation award for his novel method of solving certain Diophantine equations with triangular numbers.

New Initiates – Yaser Almuarifi, Shelby Bicknell, Alison Davidson, Noelle Dooley, Elena Flott, Jonathan Harris, Camille Holman, Levi Humbard, Allie Moody, Jordan Morrison, Kelsey Pennington, Samuel Randall, and Patrick Sarwinski.

KS Beta — Emporia State University

Chapter President – Anthony Gagliano; 40 Current Members; 6 New Members

Other Spring 2018 Officers: Nicklaus Mathias, Vice President; Natalie Leffler, Secretary; Jake Klausner (Fall) / Henry Weiner (Spring), Trea-

surer; Tom Mahoney, Corresponding Secretary; and Brian Hollenbeck, Faculty Sponsor.

Emporia State University hosted the 2018 North Central-South Central Regional KME Convention, which had 67 attendees. The KME Student Lounge continues to be a popular gathering space for students and a location for game nights.

New Initiates – Mohammad Daouk, Jake Klausher, Nicklaus Mathias, Austin McCosh, Tyler Ross, and Victoria Wilson.

KS Delta — Washburn University

Chapter President – Laura Crosswhite; 24 Current Members; 13 New Members

Other Spring 2018 Officers: Jacob Talkin, Vice President; Mary Greene, Secretary; Benjamin Nelson, Treasurer; Kevin Charlwood, Corresponding Secretary and Faculty Sponsor.

The Kansas Delta chapter of KME met three times with our Math Club for pizza lunches, and to hear presentations. One faculty member presented on the cantor ternary set in February, we watched the DVD video of "Colours of Infinity" on Pi Day in March, and our Math Club student President and KME chapter Vice President gave a talk on origami complete with hands-on demonstration. We initiated 13 new KME members at our March initiation dinner and ceremony (12 students, 1 faculty member).

New Initiates – Jacob Andrews, Rahasya Bharaniah, Mary P. Greene, Ryan Haller, Jade Hodge, David Knaack, Denise Meltz, Bridget Minellono, Benjamin Nelson, Samantha Schmitz, Jirani Smith, Jacob Talkin, and Angela Crumer (Faculty).

KS Eta — Sterling College

Corresponding Secretary – Pete Kosek; 10 New Members

New Initiates – Tiana Bonde, Laura Clark, Lucas Garrett, Pete Kosek, Amy Kosek, Aaron Pinkerton, Shelby Stowe Jenna Weimer, Mikaela Wells, and Zachary Williams.

MA Alpha — Assumption College

Corresponding Secretary – Joseph Alfano; 8 New Members

New Initiates – Sara Amato, Colleen Barry, Olivia Burke, Kelliann Keaney, David Kenney, Melanie LeBlanc, Zane Lynch, and Shannon Mulvaney.

MA Beta — Stonehill College

Corresponding Secretary – Timothy Woodcock; 6 New Members

New Initiates – Samantha Claveau, JeanneAnne Cosgrove, Laura Darr, Sydney Martin, Samantha Mauro, and Catherine Tierney.

MD Alpha — Notre Dame of Maryland University

Chapter President – Chinwendu Nwokeabia; 13 Current Members; 6 New Members

Other Spring 2018 Officers: Bhavya Bhardwaj, Vice President; Justice Walrath, Secretary Hannah Woodworth, Treasurer; Charles Buehrle, Cor-

responding Secretary and Faculty Sponsor.

Below are items from MD Alpha.

New Initiates – Gerriza Balmes, Gayle Caldamo, Abigail Dibert, Lesly Mendoza, Gabriella Shahine, and Marion Smedberg.

MD Beta — McDaniel College

Corresponding Secretary – Spencer Hamblen; 8 New Members

New Initiates – Timothy Banks, Darby Bortz, Alexis Comeau, Maia Hanlon, Dimitri Lezcano, Francisco Marques Dos Santos Vieira, Angel Tuong, and Sarah Wright.

MD Delta — Frostburg State University

Chapter President – Sarah Sparks; 25 Current Members; 8 New Members

Other Spring 2018 Officers: Demetrick McDonald, Vice President; Zach Kline, Secretary; Emma Seibert, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor.

The Maryland Delta Chapter had a very active spring semester. Eight new members were welcomed to the chapter at our Induction Ceremony held on March 4. The ceremony featured a lecture on the life and work of Sofya Kovalevsky delivered by faculty member Debbie Devlin. Our annual Pi-Day Bake Sale was quite successful as was a second bake sale held in late April. Eight chapter members (five students and three faculty) attended the KME Great Lakes Regional Convention held on April 7 at Wheeling Jesuit University. Three of the students gave presentations, namely, Emma Seibert on tessellations and the golden ratio, Sarah Sparks on Euler's Polyhedral Formula and Alissa Whiteley on Elliptic Curve Cryptography. At our meeting in late April, a new set of officers was elected. Demetrick Mc-

Donald will serve as President, with Braden Ebersole as Vice President, Jordan Thomas as Secretary and Zach Kline as Treasurer. We ended the semester with a picnic (held indoors due to rain!). We offer best wishes to graduating Maryland Delta members and note with pride that four of these students will be starting graduate studies in mathematics in the fall (Rebecca Lee, Sarah Sparks, Jimmy West and Alissa Whiteley). Finally, we are exciting to relate that our home base, Frostburg State University, will be the location for the 2019 KME National Convention.

New Initiates – Amber Eure, Alyssa Malanik, Kimera Peterson, Gabrielle Sandy, Emily Shatley, Chad Shumaker, Katelynn Suesse, and Jordan Thomas.

MD Epsilon — Stevenson University

Chapter President – Rebecca Gruver; 151 Current Members; 8 New Members

Other Spring 2018 Officers: Courtney Hohn, Vice President; Lindsey Weishaar, Secretary; Jessica Rega, Treasurer; and Dr. Benjamin Wilson, Corresponding Secretary and Faculty Sponsor.

In Spring 2018, 8 new members were inducted at the 13th initiation of the Maryland Epsilon Chapter of KME at Stevenson University. Events sponsored by KME this spring included a Pi day celebration and a math movie night which included a showing of the film Hidden Figures.

New Initiates – ETCOlivia Apicella, Alex Dangel, Cara Habicht, William Heidel, Darian Hileman, Cassandra Morris, Alayna Roesener, and Lindsey Weishaar.

MI Beta — Central Michigan University

Chapter President – Yejean Han; 18 Current Members; 13 New Members

Other Spring 2018 Officers: Brianne Becker, Vice President; Natalie DeVos, Secretary; Jared Williams, Treasurer; and Dr. Ben Salisbury, Corresponding Secretary and Faculty Sponsor.

The first meeting of the Spring 2018 semester was on January 25. The meeting conducted some ice breakers to get acquainted with new attendees and played mathematical bingo. On February 8, KME member Jared Williams gave a LaTeXworkshop. Mathematical *Jeopardy!* took place on February 22. KME held a competition amongst its members for a Pi Day T-shirt design. The winning design was submitted by Jean Han. Shirts were sold in the department from March 12 until March 15. By the completion of the sale, the shirts were sold out. On March 15, CMU undergraduate student Johan Branding gave a talk entitled, "Why?: An Introduction to the Philosophy of Mathematics." With spring break looming, a movie night was held on March 29. The movie was "The Theory of Everything," in honor of Stephen Hawking who had recently passed away. KME members raised money to support the Special Olympics event on CMU campus by selling medallions in April 2018. The overall campaign took place from March 19 until May 5 in preparation for the games held May 31 to June 2. KME had a table set up in the department selling medallions from April 9 until April 12. The final KME meeting of the semester was held on April 19. Students who were unable to make it to the initiation ceremony were inducted at this meeting. Elections for the upcoming year's executive board were also held. Lastly, graduate student Jordan Gill led a discussion about the preparation for the MTTC and/or GRE. KME was a co-organizer, led by the AMS Graduate Student chapter and along with Gamma Iota Sigma, of the Pi Miles-5K event on April 22. KME held its 2018 initiation ceremony on April 22; the originally scheduled date of April 15 was postponed due to inclement weather. Eleven new members were inducted into the society (including those inducted at the April 19 meeting). Sivaram Narayan, Professor of Mathematics at CMU, gave a talk entitled "MATH… What is it good for?". *Math-a-palooza* was again co-organized by KME, with both the AMS Graduate Student Chapter and Gamma Iota Sigma. The Spring 2018 event was held on April 27.

MI Delta — Hillsdale College

Chapter President – Tanner Orion Wright; 48 Current Members; 3 New Members

Other Spring 2018 Officers: Abigail Trouwborst, Vice President; Gill West, Secretary; Emma Clifton, Treasurer; and Dr. David Gaebler, Corresponding Secretary and Faculty Sponsor.

The Michigan Delta chapter of KME initiated three new members on March 29, 2018. We also hosted a movie night with a showing of Hidden Figures; attendance was small (around a half-dozen) but a good time was had by all.

New Initiates – Ben Becker, Alan Kotlyar, and Trevor Vogel.

MO Beta — University of Central Missouri

Chapter President – Christina Duerr; 25 Current Members; 5 New Members

Other Spring 2018 Officers: Mackenzie Snyder, Vice President; Matthew Enlow, Secretary; Juliana Ortiz, Treasurer; Rhonda McKee, Corresponding Secretary; and Steve Shattuck, Faculty Sponsors.

The Missouri Beta Chapter enjoyed several mathematical presentations on various topics during the spring semester. Programs included creating Platonic solids using Zome tools, and a presentation from Mr. Andrew Buchholz, who spoke about his study abroad experience through Semesters in Budapest. He also shared information regarding federal career opportunities for math majors. Eleven students, one alumnus and two faculty members from MO Beta attended the North Central Regional convention. Two MO Beta members gave presentations on their research. We always enjoy the conventions and encourage all chapters to attend a regional or national convention every year.

New Initiates – Justin Bao Doung, Amanda Horvath, Callie Ruffener, John Sifuentes, and Mickayla Lee Strother.

MO Epsilon — Central Methodist University

Chapter President – Marlow Case; 15 Current Members; 4 New Members

Other Spring 2018 Officers: Arnold Mukisa, Vice President; Miranda Weigand, Secretary; Alexandra Huddelson, Treasurer; and Pam Gordy, Corresponding Secretary and Faculty Sponsor.

New Initiates – Joe Bruemmer, Ripken Dodson, Arnold Musicka, and Josh Schroer.

MO Eta — Truman State University

Corresponding Secretary – Megan Perry; 13 New Members

New Initiates – Tatsuya Akiba, Anna Bucheit, Amanda Claywell, Kimberlyn Eversman, Benjamin Forsythe, Jamie Harper, Patrick Howe, Kelsey Lieberman, Wyatt Mansell, Vince Mazzola, Nicole Orf, Karie Schmitz, and Mengqing Zhang.

MO Theta — Evangel University

Chapter President – Rebekah Chase; 19 Current Members; 5 New Members

Other Spring 2018 Officers: Heather Culbertson, Vice President; and Don Tosh, Corresponding Secretary and Faculty Sponsor.

Meetings were held monthly. In January we initiated 5 new members and elected new officers. In April Don Tosh and 5 students attended the regional convention of KME held at Emporia State University. In April we had our end-of-year meeting at the home of Dianne Twigger, where honor cords were presented to graduating members.

New Initiates – Laura Leigh Armstrong, Jacob Crews, Nicholas Fleece, Isaac Moore, and Brice Todd.

MO Iota — Missouri Southern State University

Corresponding Secretary – Dr. Charles Curtis; 8 New Members

New Initiates – Karen Anderson, Michael Coberley, Amber Idleburg, Trista Lee, James Nguyen, Toby Pederson, Hayden Rogler, and Monica White.

MO Kappa – Drury University

Corresponding Secretary – Dr. Carol Browning; 12 New Members

New Initiates – Meagan Carmack, Puja Chhetri, Miriam Colligan, William M Goolsby, Michelle Herman, Sacha Mourier, Jacob Nass, Jae Hwi Park, Sydney Quaid, Allison Schiltz, Carter Sifferman, and Jacob Willhoite.

MO Nu — Columbia College

Corresponding Secretary – Kenny Felts; 2 New Members

New Initiates – Kelsey Perry and Sarah Walters.

MO Xi — William Woods University

Corresponding Secretary – Dr. Chris Schneider; 9 Current Members; 3

New Members
The Missouri Xi chapter held their third initiation ceremony on February 20, 2018. Three new inductees, students Mikayla Maple Laburay and Desi DesBouillons II were inducted, as well as new Mathematics faculty member Julie Davenport. They joined returning members Briley Browning, James Rogers, Kelsey Scherder, and Bailey Ward. Mathematics faculty Chris Schneider and Raymond Hune were also present. After the initiation, refreshments were enjoyed by all.

New Initiates – Julie R. Davenport, Glen DesBouillons II, and Mikayla Maple Laburay.

MS Gamma — The University of Southern Mississippi

Chapter President – Yumi Maharjan; 16 Current Members; 6 New Members

Other Spring 2018 Officers: Hamas Tahir, Vice President; Amit Tripathi, Secretary; Gokul Bhusal, Treasurer; and Dr. Zhifu Xie, Corresponding Secretary and Faculty Sponsor.

Report: KME MS Gamma chapter has its new officers for the new year of 2018-2019, which was elected in a general KME meeting on April 13, 2018. New officers proposed a series of activities for the Fall semester of 2018 and KME MS Gamma chapter became an active student organization at the University of Southern Mississippi again.

New Initiates – Gokul Bhusal, Yumi Maharjan, Aaditya Paudel, Nishchal Sapkota, Hamas Tahir, and Amit Tripathi.

NC Epsilon — North Carolina Wesleyan College

Corresponding Secretary – Bill Yankosky; 11 New Members

New Initiates – Morgan MacKenzie Branham, Victoria Burkhart, Catelyn Jo Cink, Ivan Cockman, Bikash Ghimire, Akash Giri, Jessie Harbour, Abigail Hering, JaWuanna Harielle McAllister, Gailyn Minchew, and Nabin Singh.

NC Zeta — Catawba College

Chapter President – Marcia-Mariel Erhart; 21 Current Members; 8 New Members

Other Spring 2018 Officers: Matt Hefner, Vice President; Cody Bennett, Secretary; Kerry Aitken, Treasurer; and Dr. Doug Brown, Corresponding Secretary and Faculty Sponsor.

The NC Zeta Chapter initiated 8 new members on April 26 and said farewell to 4 graduating seniors. Dr. Zerger gave a talk on solutions and open questions regarding certain Diophantine equations entitled "McClane and McNuggets."

New Initiates – Dr. Katherine Baker, Federico Borrego-Higareda, Chase Cummins, Keenan

D'Arcy, Brandon Davis, Patrick Dissosway, Morgan Hester, and Devan Shell.

NE Beta – University of Nebraska Kearney

Chapter President – Candy Smith; 11 Current Members; 5 New Members
Other Spring 2018 Officers: Kyle Halsted, Vice President; Alexis Stockton, Secretary; Stephanie Slayden; and Dr. Katherine Kime, Corresponding Secretary and Faculty Sponsor.

This spring, we sponsored an invited speaker, Dr. Daniel Mowrey. Dr. Mowrey got his bachelor's at UNK and then received his Ph.D. in statistics from Iowa State. He had a career at Eli Lilly, in research concerning animal pharmaceuticals. He spoke of his experiences and his views on the state of the profession. The talk was attended by about 20 faculty and students, including the Dean of our College. The Department of Mathematics and Statistics provided support for Dr. Mowrey's dinner following the talk, and one student and three faculty members ate with him at local restaurant. In April, four student KME members participated in the Big Event, which is a day of community service. The students "spring-cleaned" the yard of a Kearney resident, who furnished a reward of donuts. Four students and one faculty member attended the 2018 Regional Convention at Emporia State. Students were inspired by the talks and came back with a number of ideas. The Dean of our College generously supported an additional night's stay for the students in Kansas, as a late season snow storm made immediate return too hazardous. Four KME members graduated: Candy Smith, Stephanie Slayden, Alexis Stockton, and Shelby Study. Alexis was honored at the CNSS Honors Convocation by the Math/Stat Department (presented by Dr. Kime) and Stephanie was honored at the Convocation by the Computer Science and Information Technology Department. Alexis has a position with her national sorority next year, as a visiting advisor to chapters around the country. Stephanie will attend graduate school in computer science at Kansas State. Candy is applying for positions in Kearney and Shelby has taken a job teaching high school in Nebraska. Alexandria Linnerson, a KME member who graduated in December 2017, has also taken a job teaching high school in Nebraska.

New Initiates – Joshua Garcia. Julie Kent. Evan Olson. Corbin Snow, and Kaitlynn Thomas.

NE Delta — Nebraska Wesleyan University

Chapter President – Carter Lyons; 19 Current Members; 13 New Members
Other Spring 2018 Officers: Mackenzie Maschka, Vice President; Trevor McKeown, Secretary/ Treasurer; and Melissa Erdmann, Corresponding Secretary and Faculty Sponsor.

In March we sponsored our 8th annual Pi Mile Fun Run. Officers chalked jokes along the way including: "Why was 6 afraid of 7? Because 7 8 9.

Why did 7 eat 9? To get 3^2 meals." Our initiation ceremony followed by a joint picnic with the Physics Club was a great success.

New Initiates – Delaney Bartling, Elijah Beed, Jessica Belfiore, Drew Damme, Braxtyn DeGolyer, Austin Hall, An Ho, Alex Kerr, Yesenia Lopez, Austin Mohr, Morgan Nishida, Jared Pohlmann, and Samantha Wright.

NH Alpha —Keene State College

Corresponding Secretary – Vincent J. Ferlini; 7 New Members

New Initiates – Emily Bradway, Nicole DeAlmeida, Hannah Melanson, Erin Morley, Sarah Otterbeck, Cameron Spiess, and Jennifer Weston.

NY Iota — Wagner College

Corresponding Secretary – Marisa Scarpa; 10 New Members

New Initiates – Jessica Aletta, Emily Austein, Brandon Kuczabski, Armend Lajka, Jennifer Lloyd, Olivia Silvestri, Piper Skinner, Kelly Sohmer, Addison Williams, and Jeanine Woody.

NY Kappa — Pace University

Corresponding Secretary – Shamita Dutta Gupta; 21 Current Members; 4 New Members

Our 45th annual induction ceremony for Kappa Mu Epsilon was held on May 1, 2018. Our guest speaker was Dr. Emilio Fernandez, Assistant Professor, Mathematics Department, PLV, and the title of the talk was "Having fun while thinking mathematically."

New Initiates – Michael Capezzuto, Emilio R. Fernandez, Eric Huang, and Emma Nolan.

NY Lambda — LIU Post

Chapter President – Christopher Guevara; 23 Current Members; 18 New Members

Other Spring 2018 Officers: Michelle Arenella, Vice President; Ashok Prabhu, Secretary; Clifford Clark, Treasurer; and Dr. Corbett Redden, Corresponding Secretary and Faculty Sponsor

The NY Lambda chapter attended the KME Northeast Regional Conference, held at St. Joseph's College on April 7, 2018. Three chapter members (Christopher Guevara, Bushra Kazmi, and Samuel McCrosson) gave talks. Our chapter also held its annual banquet and initiation ceremony on April 15, 2018.

New Initiates – Mr. Bassam Albalawi, Mr. Anthony Caruso, Ms. Hillevi Ekloew , Ms. Megan Frey, Mr. Alexander Hailes, Ms. Bushra Kazmi, Ms. Julissa Larkin, Ms. Elizabeth Law, Ms. Jessica Marosz, Mr. Samuel McCrosson, Ms. Susanna Mirabelli, Mr. Richard Montchal, Mr. Rokib Morad, Mr. Michael Palmaccio, Mr. Amun Patel, Mr. Paul Tredwell, Mr. Alessandro Tripoli, and Mr. Timoleon Ziogkas.

NY Mu — St. Thomas Aquinas College

Corresponding Secretary – Dr. Heather A. Rave; 5 New Members

New Initiates – Isabella Rose Diaz, Courtney J. Dumain, Paul M. Rhodes, III, Natalia

Dorota Swiecki, and Valerie Winalski.

NY Nu – Hartwick College

Corresponding Secretary – Gerald Hunsberger; 10 New Members

New Initiates – Yury Chernyak, Meghan Fothergill, Lauren Heinz, Heng Li , Matthew Mackenzie, Asher Mendez, Emma Reining, Jesse Rodriguez, John Senko, and Joseph Tecklenburg.

NY Omicron — St. Joseph's College

Chapter President – Robert V. Dilworth; 15 Current Members; 0 New Members

Other Spring 2018 Officers: Kathrine A. Miller, Vice President; Frank D. Loglisci, Secretary; Christiana R. Morante, Treasurer; Dr. Elana Reiser, Corresponding Secretary; and Dr. Donna Pirich, Faculty Sponsor.

We held the New England Regional Convention of KME on our campus April 7, 2018. Four chapters were in attendance. Our KME members volunteered in the math clinic for tutoring to local high school students.

NY Pi — Mount Saint Mary College

Corresponding Secretary – Lee Fothergill; 7 New Members

New Initiates – Vita Bosco, Brigid C. Gauthier, Benjamin Lindenauer, Francisco Mojica III, Cameron Todd Pagán, Alexander Harley Perlak, and Emily Salgado.

NY Rho — Molloy College

Corresponding Secretary – Manyiu Tse; 7 New Members

New Initiates – Alyssa Amodio, Arianna Anzano, Marina Foley, Ashley Liebegott, Marissa Marinaccio, Nikoletta Markoulli, and Sean Moran.

OH Alpha — Bowling Green State University

Corresponding Secretary – Steven M. Seubert; 3 New Members

New Initiates – Samuel Carolus, Michael Gulas, and Todd Romutis.

OH Gamma — Baldwin Wallace University

Chapter President – Corrinne Horvath; 16 New Members

Other Spring 2018 Officers: Stephen Osborn, Vice President; James Corkish, Secretary; David Calvis, Corresponding Secretary and Faculty Sponsor.

In Spring 2018 we received 16 new initiates into membership. Our officers for 2018-19 will be President: Corrinne Horvath (returning), Vice-President: Katherine Wohlford, and Secretary: Andrew McBride.

New Initiates – Jeremiah R. Allis, Mary T. Carlo, Katherine V. Cendrowski, Jeremiah R. Hamlin, Daniel J. Jesensky, Gabrielle E. Knauer, Briana M. Laszlo, Jonathan S. Mahan, Andrew E. McBride, Abigail C. McElroy, Kyle J. Ray, Taylor A. Stevens, Emily J. Timko, Ian M. Walton, Sophia R. Weidner, and Rachel R. Womeldorf.

OH Epsilon — Marietta College

Corresponding Secretary – John Tynan; 27 Current Members; 26 New

Members

New Initiates – Ali AlMeshari, Omar AlMjmad, Alec Bianchi, Matthew Chih, Madison Crawford, Joshua Daniell, Cody Dobbs, Bryce Emerick, Austin Fligor, YiFei Guo, Stephen Harper, Brian Hofmeier, Mitchell Houston, Quinci Jones, Johnathan Kungle, Carter Lang, Morgan McElwain, Diana McGraw, Jason Miller, Rachel Miller, Gwyneth Nelson, Cole Pappas, Alexander Petrovski, KeXin Shao, Nicholas Trocano, and Xuan Zhu.

OH Zeta — Muskingum University

Corresponding Secretary – Richard Daquila; 3 New Members

New Initiates – Samuel Hvizdos, Ryan Moffo, and Courtney Rutter.

OH Eta — Ohio Northern University

Corresponding Secretary – Ryan Rahrig; 4 New Members

New Initiates – Samuel Alan Inbody, Takumi Kijima, Megan Meyer, and Shannon Tefft.

OH Theta — Capital University

Chapter President – Nick Hernandez; 13 Current Members; 9 New Members

Other Spring 2018 Officers: Sage Conger, Vice President; Tiffany Kempthorne, Secretary; Lisa Lotz, Treasurer; Paula Federico, Corresponding Secretary; and Jonathan Stadler, Faculty Sponsor.

During the spring semester our chapter held one meeting and welcomed new members. Our Induction Ceremony was on Sunday March 18th 2018. Our guest speaker was Mr. Chris Bolognese from Columbus Academy. His interactive presentation was entitled “Between the Folds: Paper folding as a means of problem posing.” All attendees had fun folding strips of paper following a set of rules. After his presentation, nine new members were inducted into our Chapter of KME. The chapter will select new officers for the 2018-2019 academic year at the beginning of the Fall semester.

New Initiates – Andrew Decker, Nicholas Ehrenborg, Jerrod Estell, Jordan Fike, Taylor Manivanh, John Scott, Krista Taulker, Elli Wachtman, and Jarrett Williams.

OK Gamma — Southwestern Oklahoma State University

Corresponding Secretary – Dr. Tom McNamara; 11 New Members

New Initiates – Terin C. Brownen, Cameron Lee Cinnamon, Mariah Cook, Kevin Freeman, Kendra Gift, Tyler Kehn, Dalton Orahood, Smith Pease, Jaxon Taylor, Hein Tram To, and Boubacar Wane.

OK Epsilon — Oklahoma Christian University

Chapter President – Tristan Minor; 4 New Members

Other Spring 2018 Officers: Kara Conway, Vice President; Chaunicie Ehrlich, Secretary; Prof. Don Leftwich, Faculty Sponsor; and Dr. Jennifer Bryan, Corresponding Secretary.

New Officers for the 2018-19 school year are Chapter President: Alex

Edwards, Vice President: Chaunicie Ehrlich and Secretary: Tristan Minor.

New Initiates – Morgan Arledge, Taylor Glover, Jonathan Hartnett, and Addison Schwamb.

PA Alpha — Westminster College

Corresponding Secretary – Pamela Richardson; 2 New Members

New Initiates – Stephanie Bell and George Huncik.

PA Epsilon — Kutztown University

Corresponding Secretary – Dr. Lyn McQuaid; 27 New Members

New Initiates – Shannon Carey, Madison Carlucci, Kelsey Conroy, Nicholas DeGennaro, Brooke Eshbach, Amanda Ferraro, Maria Ferrizzi, Grant Fickes, Grace Fusciardi, Rebecca Graham, Kolten Hendershot, Tamara Jennings, Jacob McCann, Bryan McNally, Gabriel Olah, Nicole Prantow, Ryan Rauenzahn, Michael Rex, Joseph Ritzko, Asja Saito, Elyse Sellers, Mikayla Stengel, Bethany Tirrell, Angelo Vardaxis, Andrew Venzie, Laura Wetzel, and Taylor Worthington.

PA Zeta — Indiana University of Pennsylvania

President – Victoria Jakicic; 16 Current Members; 4 New Members

Other Spring 2018 Officers: Melissa Reinhardt, Vice President; Sarah Settlemire, Secretary and Treasurer; and Gary Stoudt, Faculty Sponsor and Corresponding Secretary.

New Initiates – Jessica A. Jones, Mariah M. Murray, Heidi L. Plant, and Bridget C. Scanga.

PA Theta — Susquehanna University

Corresponding Secretary – Kenneth Brakke; 7 New Members

New Initiates – Justice Bufford, Hannah Cooper, Unique Divine, Adam Fritz, Shawn Khanna, Anne Loeliger, and Adeline Wolfe.

PA Kappa — Holy Family University

Chapter President – Walter Webb; 5 Current Members; 2 New Members

Other Spring 2018 Officer: Sister Marcella Louise Wallowicz CSFN, PhD, Corresponding Secretary and Faculty Sponsor.

The PA Kappa held its Induction Ceremony on Friday, April 13. Two new members were initiated.

New Initiates – Melissa Cahill and Tuyen Le.

PA Lambda — Bloomsburg University

Corresponding Secretary – Elizabeth Mauch; 11 New Members

New Initiates – Danielle Bickelman, Ian Birdwell, Courtney Danials, Vrunda Desai, Elizabeth Drumm, Riley Hughes, Alyssa Lopatka, Alison Martin, Ashley Sebastian, Daniel Staros, and Eric Zalewski.

PA Mu — Saint Francis University

Chapter President – Arlan (AJ) Zelenky; 52 Current Members; 14 New Members

Other Spring 2018 Officers: Vanessa Valovage, Vice President; Ry Gallagher, Secretary; Hannah Boyd, Treasurers; Dr. Peter Skoner, Corre-

sponding Secretary; and Dr. Brendon LaBuz, Faculty Sponsor.

The annual Pi Day celebration was held on Monday, March 14, 2018; faculty, students, and staff enjoyed taste testing an assortment of "pi" served by members of Kappa Mu Epsilon throughout the day. Initiation ceremonies were held on Tuesday, February 2, 2016 in DiSepio 213. The evening began with a prayer by chapter chaplain and member Fr. Joseph Chancler,T.O.R., was followed with dinner, continued with a talk "Sudoku Solutions in Mupad Using Gröbner Basis," by Aj Zelenky and Ry Gallagher, continued with the initiation ceremony led by faculty sponsor Dr. Brendon LaBuz for the 14 new members, and concluded with remarks by corresponding secretary Dr. Peter Skoner. Two faculty members and four students attended the Great Lakes Regional Convention, held at Wheeling Jesuit University on April 7, 2018; AJ Zelenky was selected as one of the best talks. KME students and faculty served as judges for the 2018 Pennsylvania Statistics Poster Competition, hosted for the tenth year by Saint Francis University. A large number of posters (474) were received, cash awards were given for first through fourth place in each of four grade level categories, and winning posters were submitted to the National Statistics Poster Competition, coordinated by the American Statistical Association.

New Initiates – Tinsae Alemu, Ryan Alu, Katherine Augustine, Tim Bartowiak, Michelle Karpinsky, Franz Luna , Lavidalie, Logan Madison, Melinda Novak, Erin Oneill, Andrew Potopa, Jacob Schulte, Rakeb Tafesse, Connor Witman, and Guofang Zheng.

PA Nu — Ursinus College

Corresponding Secretary – Nicholas Scoville; 2 New Members

New Initiates –Jason Bennett and Shichao Li.

PA Xi — Cedar Crest College

Corresponding Secretary – Joshua Harrington; 5 New Members

New Initiates – Magdalena Dammer, Lina Koseki., Kirsten Replogle, Kirin Uzar, and Madison Wellen.

PA Pi — Slippery Rock University of Pennsylvania

Chapter President – John Yannotty; 10 Current Members; 16 New Members

Other Spring 2018 Officers: Justin Long, Vice President; Rebecca Nicholson, Secretary; Bradley Schweitzer, Treasurer; Dr. Richard Marchand, Faculty Sponsor; and Dr. Elise Grabner, Corresponding Secretary.

New Initiates – William Davis, Kaila DeChristofaro, Warren Geither, Jordan Goodrick, Rebecca Himes, Dean Ketterer, Jessica Lefler, Justin Long, David Mishler, Rebecca Nicholson, Sarah Roudybush, Bradley Schweitzer, Parker Servello, Dr. Joshua Ballew, Dr. Nicholas

Hurl, and Dr. Kirk McDermott.

PA Rho — Thiel College

Chapter President – Rebecca Adams; 5 New Members

Other Spring 2018 Officers: Sarah McConnell, Vice President; Dugan Paxton, Secretary; John Thiel, Treasurer; Dr. Jie Wu, Faculty Sponsor; and Dr. Russell Richins, Corresponding Secretary.

The members of KME organized a Pi Day festivities with different pi themed activities. We held our annual induction for 5 new inductees and their families. We elected new officers for the next school year.

New Initiates – Brett Eckroate, Taylor Guth, Courtney Harriman, Breanna Mesich, and Jingnan Xie.

PA Sigma — Lycoming College

Corresponding Secretary – Christopher Reed; 10 New Members

New Initiates – Kaitlyn Adams, Carter Alexander, Samantha Barrett, Bryce Dias, Andrew Dion, Son Pham, Crystal Riebe, Ziqi Su, Lucas Wilkins, and Cheng Yang.

PA Tau — DeSales University

Chapter President – Matthew J. Reeder; 6 Current Members; 6 New Members

Other Spring 2018 Officers: Nicholas P. Speranza, Vice President; Adam N. Desseyn, Secretary/Treasurer; and Brother Daniel P. Wisniewski, O.S.F.S., Corresponding Secretary and Faculty Sponsor.

On Sunday, April 29, 2018, the PA Tau Chapter of Kappa Mu Epsilon at DeSales University (DSU) initiated six new KME members. The event included a presentation entitled "I Can See Clearly Now" by Mr. Joseph Marlin, Systems Developer at Saucon Technologies (Bethlehem, PA), who received his B.S. in mathematics and computer science in 2013 from DSU. Mr. Marlin was a member of the inaugural cohort of KME initiates of the PA Tau Chapter in 2012. In attendance were family and friends of the new and current KME members, as well as several KME alumni.

New Initiates – Emily M. Bennett, Michael V. Biase, Alexander N. Cominsky, Brendan G. Graham, Sigrid I. Gulbis, and Ilya Shvindlerman.

RI Beta — Bryant University

Chapter President – Owen Wrinn; 39 Current Members; 27 New Members

Other Spring 2018 Officers: Danica Butler, Vice President; Tyler Talbot, Secretary; John Quinn, Corresponding Secretary; and Alan Olinsky, Faculty Sponsor.

Three students made presentations about their research projects at the KME regional convention on Saturday, April 7, 2018 at St. Joseph's College in Patchogue, NY (New York Omicron). We held our KME initiation ceremony on Thursday, April 26, 2018. We inducted 23 students and 4 faculty members into KME.

SC Gamma — Winthrop University

Chapter President – Colin Frazier; 17 Current Members; 9 New Members
Other Spring 2018 Officers: Christy Knight, Vice President; Sydney McCall, Secretary; Victoria Nidiffer, Treasurer; and Dr. Jessie Hamm, Corresponding Secretary and Faculty Sponsor.

This spring semester our chapter initiated 9 new members. Below is a picture of our new members along with our faculty advisor and officers at the induction ceremony.

We had a lot of fun activities this semester. We had our spring service project in March. We went to a local elementary school and hosted their "Math Day". For this we created 12 fun activities/stations for over 200 students to participate in. We had a great time sharing our love and passion for mathematics with the public!

In addition to our service project we also celebrated Math Appreciation Month with a pizza party, Pie-A-Professor (luckily not me), math game day, and a movie night! We're already looking forward to all the fun we will have together next year!

TN Gamma — Union University

Chapter President – Amy Murdaugh; 8 New Members

Other Spring 2018 Officers: Andrew Edmiston, Vice President; Yoo Jin Moon, Secretary and Treasurer; Cole LeMahieu, Webmaster and Historian; Bryan Dawson, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor.

The TN Gamma chapter held its annual initiation banquet at the Old Country Store on April 16. The banquet speaker was 2011-2012 chapter president Emilie Huffman, with former chapter vice president Kim Lukens also in attendance. TN Gamma reached a milestone with its 500^{th} inductee. We also have begun ordering honor cords for graduates.

New Initiates – Christian Brown, Josephine Carrier, Jenna Marie Dula, Ainsley Noelle Duncan, Taylor Katherine Lewelling, John Walter Mayer, Davina Amaris Norris, and Renee Maureen Seavey.

TN Delta — Carson-Newman University

Corresponding Secretary – Kenneth Massey; 7 New Members

New Initiates – Elaine I. Brickey, Abigail L. Fiessinger, Branda E. Moore, Kaylib A. Powell, Jasmine N. Rizk, Tucker N. Williams, and Samuel D. Wilson.

TN Zeta — Lee University

Corresponding Secretary – Caroline Maher-Boulis; 8 New Members

New Initiates – Emily Brock, Precious Enwere, Kelsey Fowler, Julianna Gay, Daniel Costa Prata, Meredith Sheeks, Judianne Speech, and Rachel Wood.

TX Iota — McMurry University

Corresponding Secretary – Dr. Kelly L. McCoun; 3 New Members

New Initiates – Ananda Lewis, Richard Nygaard, and David Winski.

TX Kappa — University of Mary Hardin-Baylor

Corresponding Secretary – Peter H. Chen; 12 New Members

New Initiates – Colton Center, Noah Crosby, Hao Guo, Ryan Hogan, Maya Kovalik, XueQi Lu, Amy Nguyen, Olumide Odusanya, Marlon Orta, Callie Peebles, John Snow, and Robert Woods.

TX Lambda — Trinity University

Chapter President – Alice Von Ende-Becker; 280 Current Members; 17 New Members

Other Spring 2018 Officers: Emma Williams, Vice President; Grace Corley, Secretary and Treasurer; Dr. Hoa Nguyen, Corresponding Secretary and Faculty Sponsor.

New Initiates – Zachary Acevedo, Emily Babcock, Zachary Carter, David Clark, Arthur Feeney, Dupri Grimes, Connor Hahn, Daniel Henkes, Mica Jarocki, Abigail Jones, Jordan Koeller, Kara S. Poole, Charles Stein, Lutfi Sun, Robin Tran, Abraham Ybarra, and Gabriel Zighelboim.

TX Mu – Schreiner University

Corresponding Secretary – Clint Coles; 14 New Members

New Initiates – Lindsey Ann Fitz, Keenan N. Gumbs, Kevin Hannay, Wesley James Joseph, Jamison Lee, Colton J. Lund, Alyssa Ann Maki, Mackenzie Palmer, Dalton Reid, Jacob D. Schreiber, Katie Smith, Tyler L. Stefek, Tristan D. Teplicek, and Jacob Lee Woytek.

VA Beta — Radford University

Chapter President – Cameron Leo; 18 Current Members; 8 New Members

Other Spring 2018 Officer: Eric P. Choate, Corresponding Secretary and Faculty Sponsor.

New Initiates – Dylan Bobbit, Morgyn Church, Collier Crisanti, D. Nicole Hodges, Alexandra Largen, Amanda McClelland, John Saad, and Christopher Shimp.

WI Gamma — University of Wisconsin – Eau Claire

Corresponding Secretary – Dr. Carolyn Otto; 13 New Members

Other Fall 2017 Officers: ETC

New Initiates – Jonah Amundsen, Lucas Buchanan, Sara Ericson, Grant Keane, Brendan Kwick, Clara Lambrecht, Noah Morris, Caroline Palen, Dawn Paukner, Molly Petersen, McKenzie Scanlan, Makenna Shaske, and Seth William.

WV Alpha — Bethany College

Chapter President – Alyssa K. Smydo; 16 Current Members; 5 New Members

Other Spring 2018 Officers: Jacob D. Stemmerich, Vice President; Joseph S. Sawyer, Secretary/Treasurer; and Dr. Adam C. Fletcher, Corresponding Secretary and Faculty Sponsor.

It has been a busy academic year for the West Virginia Alpha chapter! In the fall semester, the vice president of the chapter, Jacob Stemmerich, traveled with the Mathematics and Computer Science Club to present on sports analytics at Undergraduate Mathematics Day at the University of Dayton. The chapter helped the club host the twelfth annual Math/Science Day on campus this spring. West Virginia Alpha also sponsored Bethany College's induction ceremony of two new members into the Upsilon Pi Epsilon international computing sciences honor society in April, as well as welcoming five new members into its own ranks. Along with West Virginia Beta (Wheeling Jesuit University), the chapter co-hosted the Great Lakes Regional Convention this April, and had the president of the chapter, Alyssa Smydo, present on fractals and fractal dimension.

New Initiates – Rachel E. Gantzer, Alexandra Courtney Metz, Robert A. Newhart, Jr., Austin G. Paul-Orecchio, and Abigail E. Turner.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961

NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
TN Gamma	Union University, Jackson	24 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987

NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011

PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresno Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015
MO Xi	William Woods University, Fulton	17 Feb 2016
IL Kappa	Aurora University, Aurora	3 May 2016
GA Eta	Atlanta Metropolitan University, Atlanta	1 Jan 2017
CT Gamma	Central Connecticut University, New Britan	24 Mar 2017
KS Eta	Sterling College, Sterling	30 Nov 2017
NY Sigma	College of Mount Saint Vincent, The Bronx	4 Apr 2018
PA Upsilon	Seton Hill University, Greensburg	5 May 2018