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Some Different Ways to Tackle the Basel Problem

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A version of this paper was presented
on Friday, April 10, 2015 at the 40th Biennial Convention/
Central Florida Undergraduate Mathematics Conference
at Embry-Riddle Aeronautical University in Daytona Beach, FL
and was one of the awarded presentations by the Awards Committee.

1. Introduction

In 1644, Pietro Mengoli posed the famous Basel problem. Named after the hometown of the great Leonard Euler, the Basel problem asked for the exact summation of the reciprocals of the squares of the natural numbers

$$\sum_{n=1}^{\infty}\frac{1}{n^2}=\lim_{n\to+\infty}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}\right).$$

The Basel problem withstood attacks by many outstanding mathematicians of the time including the efforts of the Bernoulli brothers. It was not until almost a hundred years later, in 1734, that Euler finally tamed the problem, showing that it was exactly equal to $\pi^2/6$. Euler's solution, published in 1735, involved mathematical manipulations that were not justified at the time, and it was not until 1741 that he published a rigorous proof of his solution.

Euler's ideas were later taken up by Bernhard Riemann who defined his namesake zeta function as

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s},$$

where s is any complex number with real part greater than one. Riemann's zeta function is one of the most important functions in mathematics because of its intimate relationship with the Prime Number Theorem.

Ever since Euler first solved it, mathematicians from a variety of different fields have found new and exciting ways of solving the Basel problem. In this paper, we investigate several different ways of solving the Basel problem. We start with the classic Eulerian solution, and then we look at more modern solutions including those involving Fourier series and double integrals. Join us in learning some different ways to sum a series and discover a bit of the history behind computing $\zeta(2)$.

2. Euler's Solution

In 1735, Euler presented the first solution to the Basel problem. To solve it as he did, we start with the Maclaurin series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots .$$

Dividing both sides of the given Maclaurin series by x gives us

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots . \tag{1}$$

The left side is undefined at $x = 0$, but since $\lim_{x\to 0} \frac{\sin x}{x} = 1$, the singularity is removable. The roots of $\frac{\sin x}{x}$ occur at $x = n\pi$ where $n = \pm 1, \pm 2, \pm 3, \ldots$. Treating the infinite series as if it were a polynomial, we factor it into its roots to get

$$\frac{\sin x}{x} = \left(1 + \frac{x}{\pi}\right)\left(1 - \frac{x}{\pi}\right)\left(1 + \frac{x}{2\pi}\right)\left(1 - \frac{x}{2\pi}\right)\cdots . \tag{2}$$

Using the difference of two squares, we can rewrite the equality as

$$\frac{\sin x}{x} = \left(1 - \frac{x^2}{\pi^2}\right)\left(1 - \frac{x^2}{4\pi^2}\right)\left(1 - \frac{x^2}{9\pi^2}\right)\cdots$$

Multiplying it out and gathering like terms gives us

$$\frac{\sin x}{x} = 1 - \left(\frac{1}{\pi^2} + \frac{1}{4\pi^2} + \frac{1}{9\pi^2} + \cdots\right)x^2 + O(x^4).$$

Notice that the coefficient of x^2 is now an infinite series. However, recall from (1) that the coefficient of x^2 is $-\frac{1}{3!}$. Euler equates the coefficients and concludes that

$$-\frac{1}{3!} = -\left(\frac{1}{\pi^2} + \frac{1}{4\pi^2} + \frac{1}{9\pi^2} + \frac{1}{16\pi^2} + \cdots\right).$$

Multiplying both sides by $-\pi^2$, gives us our solution

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \blacksquare$$

Euler's approach to solving the Basel problem was ingenious, but his assumption that infinite series can be treated like finite polynomials was unjustified at the time. It was not until Karl Weierstrass developed his namesake factorization theorem a hundred years later that equation (2) was justified.

3. Using Fourier Series

In [2], Mark Bridger and Andrei Zelevinsky investigated positive integer lattice points–those points on a coordinate system with positive integer coordinates. Their goal was to calculate the fraction of those points that are visible from the origin, that is, the fraction of those lattice points with no other lattice points directly between them and the origin of the coordinate system. Bridger and Zelevinsky proved that the density of lattice points visible from the origin is $\frac{6}{\pi^2}$, and in the process, they provided us with another solution to the Basel problem. Their solution involved Fourier series, and we will take a look at it here.

Recall that the generalized Fourier series for $f(x)$ over $[-\pi, \pi]$ is

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right), \tag{3}$$

where

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\, dx,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\, dx.$$

For conditions for which (3) is valid, see Silverman's translation of Tolstov's *Fourier Series*.

In our case, we start with the odd function $f(x) = x$. Evaluating the formula for a_0, we find that $a_0 = 0$. To find a_n, we use $f(x) = x$ and evaluate by parts. The terms cancel, and we get $a_n = 0$. To find b_n, we use $f(x) = x$ in the appropriate formula and evaluate by parts to get $b_n = -\frac{2}{n}\cos(\pi n)$. Notice that $-\cos(\pi n)$ oscillates between -1 and 1, so we can rewrite b_n as $b_n = (-1)^{n-1}\frac{2}{n}$. Combining them in (3), we get the Fourier series of x on $[-\pi, \pi]$

$$x = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{2}{n}\sin(nx). \tag{4}$$

Now we turn to Parseval's theorem, which relates the inner product of a function to the sum of the squares of the coefficients of its Fourier series as follows

$$\frac{1}{L}\int_{-L}^{L} f^2(x)\,dx = \frac{a_0^2}{2} + \sum_{n\geq 1}(a_n^2 + b_n^2).$$

Parseval's theorem is a special case of Bessel's inequality, and a complete description of it can be found in Zygmund's *Trigonometric Series*.

Applying Parseval's theorem to (4) gives us

$$\frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx = \sum_{n=1}^{\infty}\left[(-1)^{n-1}\frac{2}{n}\right]^2 = \sum_{n=1}^{\infty}\frac{4}{n^2}.$$

The definite integral on the left evaluates to $\frac{2\pi^2}{3}$. Dividing both sides by 4 gives us

$$\sum_{n\geq 1}\frac{1}{n^2} = \frac{\pi^2}{6}. \blacksquare$$

4. A Complex Approach

This solution was given by Russell in [6]. Russell utilized the complex definition of the cosine function and a power series to evaluate a definite integral that yielded a solution to the Basel problem.

Trying Russell's solution, we start with the improper integral

$$I = \int_0^{\frac{\pi}{2}} \ln(2\cos x)\,dx.$$

The complex extension of the cosine function is $\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$, which implies that $2\cos x = e^{ix}(1 + e^{-2ix})$. Using this substitution, our integral becomes

$$I = \int_0^{\frac{\pi}{2}} \ln(e^{ix}(1 + e^{-2ix}))\,dx.$$

Using the properties of logarithms, we can split it into two integrals, after which the first is evaluated easily, and we are left with

$$I = \frac{i\pi^2}{8} + \int_0^{\frac{\pi}{2}} \ln(1 + e^{-2ix})\, dx. \tag{5}$$

Using the power series

$$\ln(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \cdots\,,$$

we can rewrite (5) as

$$I = \frac{i\pi^2}{8} + \int_0^{\frac{\pi}{2}} \left(e^{-2ix} - \frac{e^{-4ix}}{2} + \frac{e^{-6ix}}{3} - \frac{e^{-8ix}}{4} + \cdots \right) dx.$$

Integrating term–by–term and evaluating each term from 0 to $\frac{\pi}{2}$, we get

$$I = \frac{i\pi^2}{8} - \frac{1}{2i} \left(e^{-i\pi} - 1 - \frac{e^{-2i\pi} - 1}{2^2} + \frac{e^{-3i\pi} - 1}{3^2} - \frac{e^{-4i\pi} - 1}{4^2} + \cdots \right).$$

From Euler's identity, we know that $e^{-i\pi} = -1$, $e^{-2i\pi} = 1$, $e^{-3i\pi} = -1$, and so on. All the terms with even squares in the denominator disappear, and we are left with

$$I = \frac{i\pi^2}{8} - \frac{1}{2i} \left(-2 - \frac{2}{3^2} - \frac{2}{5^2} + \cdots \right).$$

Factoring out -2 and rationalizing the i in the denominator, we get

$$I = \frac{i\pi^2}{8} - i \left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \right). \tag{6}$$

Inside the parentheses, we now have the sum of the odd terms of the Basel problem.

Examining the Basel problem, we find that the sum of the even terms is exactly one-fourth of the sum of all the terms

$$\sum_{n\geq 1} \frac{1}{(2n)^2} = \frac{1}{4} \sum_{n\geq 1} \frac{1}{n^2},$$

which implies that the odd terms must amount to three–fourths of the entire series

$$\sum_{n\geq 0} \frac{1}{(2n+1)^2} = \frac{3}{4} \sum_{n\geq 1} \frac{1}{n^2}.$$

Returning to (6), we have

$$I = \frac{i\pi^2}{8} - i\frac{3}{4} \sum_{n\geq 1} \frac{1}{n^2}. \tag{7}$$

Recall that in our integral I, we are integrating a real–valued function, hence the result should be a real number. Our result in (7) is a complex number. A real number can equal a complex number only when the imaginary part of the latter is zero. Setting (7) equal to zero and factoring out i, we get

$$i\left(\frac{\pi^2}{8} - \frac{3}{4}\sum_{n\geq 1}\frac{1}{n^2}\right) = 0,$$

which implies that the expression in parentheses is equal to zero. Simplifying, we get

$$\sum_{n\geq 1}\frac{1}{n^2} = \frac{\pi^2}{6}. \blacksquare$$

5. A Solution Involving Double Integrals

The use of the double integral presented here was first given as a problem exercise by Leveque in [5] and rediscovered by Apostol in [1]. We start with the geometric series

$$\frac{1}{1-xy} = \sum_{n=0}^{\infty}(xy)^n, \qquad \text{if } |xy| < 1. \tag{8}$$

This series converges absolutely, which allows us to integrate both sides of the equation. Integrating both sides of (8) from 0 to 1 with respect to x and y gives us

$$\int_0^1\int_0^1 \frac{1}{1-xy}\,dx\,dy = \int_0^1\int_0^1\sum_{n=0}^{\infty}(xy)^n\,dx\,dy. \tag{9}$$

Interchanging integration and summation on the right side and evaluating the double integral gives us

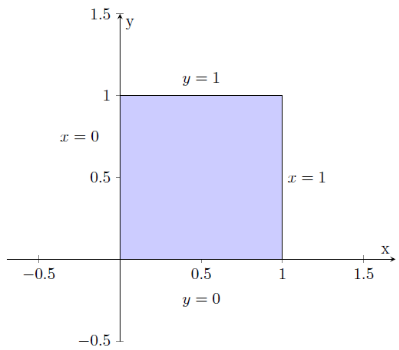
$$\sum_{n=0}^{\infty}\int_0^1\int_0^1 x^n y^n\,dx\,dy = \sum_{n=0}^{\infty}\frac{1}{n+1}\int_0^1 y^n\,dy = \sum_{n=0}^{\infty}\frac{1}{(n+1)^2} = \sum_{n=1}^{\infty}\frac{1}{n^2}.$$

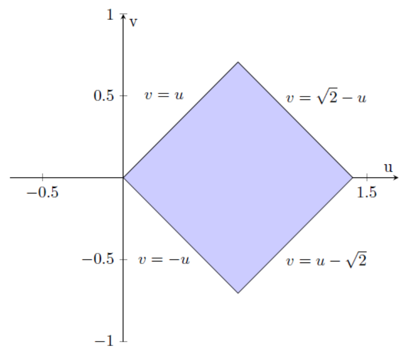
In the final step, we are simply adjusting the index of summation from $n = 0$ to $n = 1$.

We turn our attention now to the other double integral, namely,

$$\int_0^1\int_0^1\frac{1}{1-xy}\,dx\,dy. \tag{10}$$

To evaluate this double integral we make the transformation $x = \frac{u-v}{\sqrt{2}}$ and $y = \frac{u+v}{\sqrt{2}}$, which corresponds to a $\frac{\pi}{4}$ radians clockwise rotation of the region of integration as illustrated here.





The Jacobian for this transformation is computed as

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} = 1,$$

which implies that $dx\,dy = dv\,du$. Our integral (10) after the transformation is

$$\int_0^{\frac{1}{\sqrt{2}}} \int_{-u}^{u} \frac{2}{2 - u^2 + v^2}\,dv\,du + \int_{\frac{1}{\sqrt{2}}}^{\sqrt{2}} \int_{u-\sqrt{2}}^{\sqrt{2}-u} \frac{2}{2-u^2+v^2}\,dv\,du. \quad (11)$$

If we let $a^2 = 2 - u^2$, we can evaluate the inside integrals as inverse tangents, and (11) becomes

$$2\int_0^{\frac{1}{\sqrt{2}}} A\Big|_{v=-u}^{u}\,du + 2\int_{\frac{1}{\sqrt{2}}}^{\sqrt{2}} A\Big|_{v=u-\sqrt{2}}^{\sqrt{2}-u}\,du, \quad (12)$$

where

$$A = \frac{1}{\sqrt{2-u^2}} \tan^{-1}\left(\frac{v}{\sqrt{2-u^2}}\right).$$

When we evaluate the integrands, we make use of the fact that inverse tangent is an odd function, which allows us to factor a negative out of its argument so that

$$A\Big|_{-u}^{u} = \frac{2}{\sqrt{2-u^2}} \tan^{-1}\left(\frac{u}{\sqrt{2-u^2}}\right),$$

and

$$A\Big|_{u-\sqrt{2}}^{\sqrt{2}-u} = \frac{2}{\sqrt{2-u^2}} \tan^{-1}\left(\frac{\sqrt{2}-u}{\sqrt{2-u^2}}\right).$$

Next, we make the substitution $u = \sqrt{2}\sin\theta$ and $du = \sqrt{2}\cos\theta\,d\theta$. Then $\sqrt{2-u^2} = \sqrt{2}\cos\theta$. Many of the pieces cancel out and (12) becomes

$$4\int_0^{\frac{\pi}{6}} \theta\, d\theta + 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \tan^{-1}\left(\frac{1-\sin\theta}{\cos\theta}\right) d\theta.$$

Now we can evaluate the first integral easily. We evaluate the second integral by integrating by parts–differentiating the entire integrand and integrating 1. Our result is $\frac{\pi^2}{6}$.

In summary, we showed that the right side of (9) becomes $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and the left side evaluates to $\frac{\pi^2}{6}$, proving that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \blacksquare$$

6. Using Trigonometric Manipulations

Our next solution is particularly interesting in that it uses the manipulation of trigonometric functions rather than advanced mathematical concepts. It was first presented by Chen in [3]. We start by looking at the fraction

$$A = \frac{1}{\sin^2 x}.$$

Using the double angle identity, we have that $\sin^2 x = 4\sin^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)$. Making this substitution in the denominator and then letting the numerator equal $\sin^2\left(\frac{x}{2}\right) + \cos^2\left(\frac{x}{2}\right)$, we can decompose the fraction to get

$$A = \frac{1}{4}\left[\frac{1}{\sin^2\left(\frac{x}{2}\right)} + \frac{1}{\cos^2\left(\frac{x}{2}\right)}\right]. \tag{13}$$

Next, using the sum formula $\sin(a+b) = \sin a\cos b + \sin b\cos a$, we have that $\sin\left(\frac{\pi}{2}+\frac{x}{2}\right) = \sin\left(\frac{\pi}{2}\right)\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\cos\left(\frac{\pi}{2}\right)$, which simplifies to $\sin\left(\frac{\pi}{2}+\frac{x}{2}\right) = \cos\left(\frac{x}{2}\right)$. Squaring both sides, we can substitute this into (13), to get

$$A = \frac{1}{4}\left[\frac{1}{\sin^2\left(\frac{x}{2}\right)} + \frac{1}{\sin^2\left(\frac{\pi}{2}+\frac{x}{2}\right)}\right].$$

Letting $x = \frac{\pi}{2}$, we have that

$$1 = \frac{1}{4}\left[\frac{1}{\sin^2\left(\frac{\pi}{4}\right)} + \frac{1}{\sin^2\left(\frac{3\pi}{4}\right)}\right].$$

Noticing that $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ are supplementary angles, we can simplify this further to

$$1 = \frac{2}{4}\left[\frac{1}{\sin^2\left(\frac{\pi}{4}\right)}\right]. \tag{14}$$

If we repeat our process on (14) of expanding the denominators using the double angle identity, replacing the cosine terms using the sum formula for sine, and combining supplementary angles, our next result is

$$1 = \frac{2}{16}\left[\frac{1}{\sin^2\left(\frac{\pi}{8}\right)} + \frac{1}{\sin^2\left(\frac{3\pi}{8}\right)}\right]. \tag{15}$$

Repeating it yet again, using (15), gives us

$$1 = \frac{2}{64}\left[\frac{1}{\sin^2\left(\frac{\pi}{16}\right)} + \frac{1}{\sin^2\left(\frac{3\pi}{16}\right)} + \frac{1}{\sin^2\left(\frac{5\pi}{16}\right)} + \frac{1}{\sin^2\left(\frac{7\pi}{16}\right)}\right].$$

We find that the n-th iteration of this process gives us

$$1 = \frac{2}{4^n}\sum_{k=0}^{2^{n-1}-1}\frac{1}{\sin^2\left(\frac{(2k+1)\pi}{2^{n+1}}\right)} = 2\sum_{k=0}^{2^{n-1}-1}\frac{1}{(2^n)^2\sin^2\left(\frac{(2k+1)\frac{\pi}{2}}{2^n}\right)}. \tag{16}$$

We arrive at the summation on the right by substituting $(2^n)^2 = 4^n$ in the fraction on the outside, pulling the denominator into the summation and then rearranging the argument of the sine function.

Our goal is to transform (16) into an infinite series. To do that, we digress a bit to look at the limit $\lim_{n\to\infty} N\sin\left(\frac{x}{N}\right) = x$. Because direct substitution gives $\infty \cdot 0$, the equality is proven by rearranging the argument and applying L'Hospital's Rule. Next, we square both sides of the equation, which is allowed because the limit of a product is the product of limits, and then we make the substitutions $N = 2^n$ and $x = (2k+1)\frac{\pi}{2}$. Finally, by the properties of limits, we know that the limit of the reciprocal of a function is the reciprocal of the limit of the function, and we arrive at the important result

$$\lim_{n\to\infty}\frac{1}{(2^n)^2\sin^2\left(\frac{(2k+1)\frac{\pi}{2}}{2^n}\right)} = \frac{1}{\left((2k+1)\frac{\pi}{2}\right)^2}. \tag{17}$$

Tannery's Theorem allow us to swap limits and sums when certain conditions are satisfied. It states that if $\lim_{n\to\infty} f_m(n) = f_m$, then

$$\lim_{n\to\infty}\sum_{m=0}^{a(n)} f_m(n) = \sum_{m=0}^{\infty} f_m.$$

There are actually several more conditions, but we can ignore them. In our case they are all satisfied.

Returning to (16), we take the limit of both sides

$$\lim_{n\to\infty} 1 = 2 \lim_{n\to\infty} \sum_{k=0}^{2^{n-1}-1} \frac{1}{(2^n)^2 \sin^2\left(\frac{(2k+1)\frac{\pi}{2}}{2^n}\right)},$$

then by Tannery's Theorem and the fact that our limit in (17) satisfies its conditions, we get

$$\lim_{n\to\infty} 1 = 2 \sum_{k=0}^{\infty} \frac{1}{\left((2k+1)\frac{\pi}{2}\right)^2},$$

which simplifies to

$$1 = \frac{8}{\pi^2} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2}.$$

Multiplying both sides by $\frac{\pi^2}{8}$, we get

$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} = \frac{\pi^2}{8},$$

which gives the sum of the odd terms in the Basel problem. Because the sum of all terms is the sum of the odd terms plus the sum of the even terms, we have that

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{8} + \sum_{k=1}^{\infty} \frac{1}{(2k)^2}.$$

Pulling $\frac{1}{4}$ out of the denominator of the sum of the right and then combining like terms gives us

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}. \blacksquare$$

7. Beyond the Basel Problem

In this paper, we proved using a number of techniques that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

We started with Euler's solution, and then we showed solutions that use more modern techniques like Fourier series and double integrals. The story of this problem and its solution does not end here. In addition to solving the Basel problem, or $\zeta(2)$, when expressed in the notation of Riemann's zeta function, Euler went on to generalize the solution for all even positive integers.

The odd integers are more challenging. For the case $\zeta(1)$, we simply have the divergent harmonic series. However, for $\zeta(3)$, $\zeta(5)$, $\zeta(7)$, and so on, we do not have anything in closed form. For example, known as *Apery's constant*, the exact value of

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3}$$

has yet to be found. We leave it as an exercise for the reader.

8. Notes and Acknowledgments

These solutions to the Basel problem were studied by the authors and other students at State College of Florida in a unique mathematics course titled "Integration, Sequences, and Series." That course was taught by Associate Professor C. Altay Özgener in the 2014 fall semester.

This paper was first presented at the 39th Annual 2014 MAA-FL Suncoast Conference at University of South Florida-Bradenton. It was later presented at the 2015 KME National Convention held at Embry-Riddle Aeronautical University where it received a best presentation award.

Interested readers are encouraged to read Dan Kalman's "Six Ways to Sum a Series," published in the November 1993 issue of *College Mathematics Journal*. Kalman's paper offers a similar overview of different ways to solve the Basel problem.

Thanks are due the referees for alerting us to some of the papers and books cited in this paper and for many other helpful suggestions.

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Triphos: An Alternative Coordinate System

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and was one of the awarded presentations by the Awards Committee.

Abstract

In this paper, we investigate characteristics and properties of the Triphos coordinate system, an alternative, two-dimensional coordinate plane consisting of three axes evenly spaced $120°$ apart. We will examine similarities and differences between the Triphos system and the Cartesian coordinate system, explore its algebraic and geometric properties, and discuss some advantages and applications of this nonconventional coordinate system.

1. Introduction

Since its inception in the seventeenth century, the Cartesian coordinate system has revolutionized the areas of algebra, geometry, and calculus, providing an abstract environment in which to manipulate and visualize mathematics. What if we introduce another coordinate system that can accomplish the very same things in a whole new way? This paper serves as an introduction to the Triphos coordinate system, a new, alternative coordinate system with its own collection of properties and applications. We hope that this system will be a useful tool for various fields of mathematics.

In April of 2014, we attended the Kansas sectional meeting of the Mathematical Association of America at Emporia State University. While there, we heard a talk by a student, Keely Grossnickle, introducing and discussing some characteristics of the Triphos coordinate system. Thus, the set up of the coordinate system, as well as a few of the properties discussed below, come from Grossnickle [2]. We had access to a limited amount of her work, so while the work presented below is based on the foundation laid by Grossnickle, we independently arrived at many of the conclusions outlined below.

2. Set Up of Triphos Coordinate System

As defined in [2], the Triphos coordinate system consists of three axes, called red, green, and blue, that lie in the two-dimensional plane. These axes are rays which originate at the origin and travel radially outward. Each of these rays corresponds to a positive real number line. We assume that there is a corresponding negative real number line for each axis that extends in the opposite direction. In Triphos, negative coordinates are allowable but unnecessary, so the negative axes are not explicitly shown in the coordinate system. Each positive ray is situated 120° apart from the other two rays, dividing the plane into three equal sections called *tridants*.

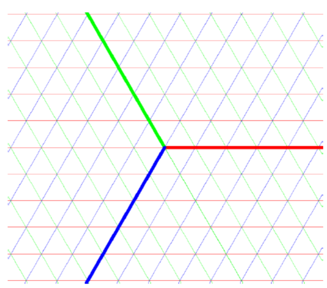

Figure 1: Triphos Plane

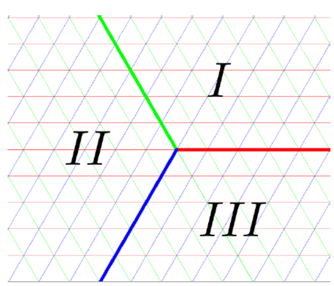

Figure 2: Tridants

These tridants, reminiscent of the quadrants in the Cartesian plane, are numbered I, II, and III as shown in Figure 2.

There are two types of notation used to describe the location of a point on the Triphos coordinate plane. The first notation, favored by [2], lists the respective red, green, and blue coordinates in the following format:

$$ {}^{g}_{b}\mathrm{r}\,. $$

This is useful as an aid in remembering which coordinate corresponds to each axis, as the placement of each coordinate in the notation indicates the relative position of the red, green, and blue axes in the plane. However, for the remainder of this paper, we will use the notation

$$ (r, g, b). $$

Here, we list the red, green, and blue coordinates, respectively, as an ordered triple as shown.

The red, green, and blue coordinates, each a real number, indicate units of movement parallel to the red, green, and blue axes, respectively, starting at the origin. To plot a point, (r, g, b), we begin at the origin, move r units along the red axis, g units parallel to the green axis, and b units parallel to the blue axis to arrive at the location of the point (r, g, b) in the plane. If a coordinate is negative, the coordinate is plotted by moving opposite the direction indicated by its respective ray.

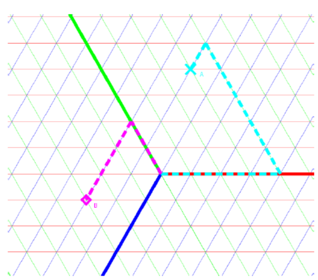

Figure 3: Plotting points

Example 1 *Given points* $A = (4, 5, 1)$ *and* $B = (0, 2, 3)$*, we plot point* A *by moving along the light blue path as indicated in Figure* 3*. Starting at the origin, move four units along the red axis, five units parallel to the green axis, and one unit parallel to the blue axis, arriving at the point* A *(indicated with an* X*). Similarly, point* B *is plotted by moving along the magenta path. Starting at the origin, move zero units along the red axis, two units along the green axis, and three units parallel to the blue axis, arriving at the point* B *(indicated with a diamond).*

Definition 1 *The unit vectors, $\hat{r}$, $\hat{g}$, and $\hat{b}$, of the Triphos coordinate system are the vectors of Euclidean length one unit that extend from the origin to the points* $(1, 0, 0)$, $(0, 1, 0)$ *and* $(0, 0, 1)$, *respectively.*

We will write the unit vectors as column vectors such that

$$\hat{\mathbf{r}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ \hat{\mathbf{g}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ \text{ and } \hat{\mathbf{b}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

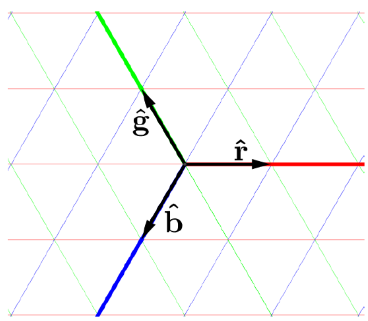

Figure 4: Unit Vectors

These unit vectors allow conversion from Triphos coordinates to Cartesian coordinates and, likewise, from Cartesian coordinates to Triphos coordinates. This conversion is made clear when looking at the Cartesian and Triphos planes superimposed on each other and examining the trigonometry involved (see Figures 5 and 6). We know $\hat{\mathbf{x}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\hat{\mathbf{y}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ as Cartesian vectors.

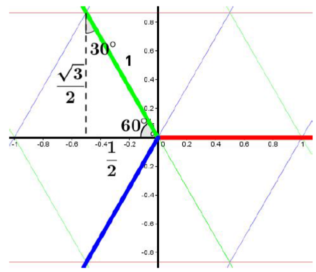

Figure 5: The Green Unit Vector

Clearly, $\hat{\mathbf{r}}$ will equal $\hat{\mathbf{x}}$, or $\hat{\mathbf{r}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as a Cartesian vector. From Figure 5, given a $30° - 60° - 90°$ triangle is formed, we can see that $\hat{\mathbf{g}}$ will correspond to $-\frac{1}{2}$ along $\hat{\mathbf{x}}$ and $\frac{\sqrt{3}}{2}$ along $\hat{\mathbf{y}}$, or $\hat{\mathbf{g}} = \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$ as a Cartesian vector. Similarly, $\hat{\mathbf{b}}$ will correspond to $-\frac{1}{2}$ along $\hat{\mathbf{x}}$ and $-\frac{\sqrt{3}}{2}$ along $\hat{\mathbf{y}}$, or $\hat{\mathbf{b}} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix}$ as a Cartesian vector.

Thus, given an ordered triple, (r, g, b), converting from Triphos to Cartesian is equivalent to the matrix multiplication:

$$[r, g, b] * \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix},$$

or just, $(r - \frac{1}{2}g - \frac{1}{2}b, \frac{\sqrt{3}}{2}g - \frac{\sqrt{3}}{2}b)$.

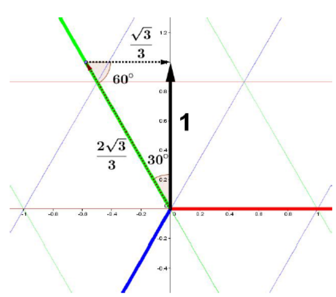

Figure 6: The $\hat{\mathbf{y}}$ Unit Vector

Similarly, to convert from Cartesian coordinates to Triphos coordinates, looking at Figure 6, we recognize that $\hat{\mathbf{x}}$ corresponds to $\hat{\mathbf{r}}$ and $\hat{\mathbf{y}}$ corresponds to $\frac{\sqrt{3}}{3}\hat{\mathbf{r}} + \frac{2\sqrt{3}}{3}\hat{\mathbf{g}}$, with no dependence on the blue axis. Thus, given an ordered pair, (x, y), converting from Cartesian to Triphos is equivalent to the matrix multiplication:

$$[x, y] * \begin{bmatrix} 1 & 0 & 0 \\ \frac{\sqrt{3}}{3} & \frac{2\sqrt{3}}{3} & 0 \end{bmatrix},$$

or just $(x + \frac{\sqrt{3}}{3}y, \frac{2\sqrt{3}}{3}y, 0)$.

3. Equivalent Representations

As a consequence of the construction of the Triphos system, which uses a set of three vectors to describe a two-dimensional plane, it is immediately apparent that any point on the plane can inherently be represented by an infinite number of ordered triples.

Example 2 *The ordered triples* $(3, 2, 5)$ *and* $(1, 0, 3)$ *both represent the same point in the plane. In Figure 7, the path described by* $(3, 2, 5)$ *is represented by the purple dotted line, and the path described by* $(1, 0, 3)$ *is represented by the light blue line. Additionally, there are infinitely many more equivalent ordered triples that represent this point shown. To understand this fact, we begin with the following lemma.*

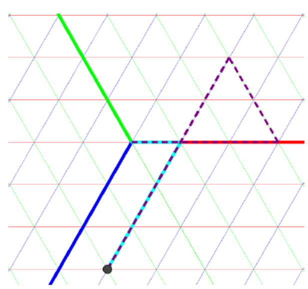

Figure 7: Equivalent Ordered Triples

Lemma 1 *Given any real number,* c*, the triple* (c, c, c) *represents the origin,* $(0, 0, 0)$.

Proof: Recall the conversion from Triphos coordinates to Cartesian coordinates given in Section 2. That is, given an ordered triple, (r, g, b), the corresponding point in the Cartesian plane is given by

$$\left(r-\frac{1}{2}g-\frac{1}{2}b,\frac{\sqrt{3}}{2}g-\frac{\sqrt{3}}{2}b\right).$$

Let c be a real number. The triple, (c,c,c) can be converted to the following Cartesian ordered pair:

$$\left(c-\frac{1}{2}c-\frac{1}{2}c,\frac{\sqrt{3}}{2}c-\frac{\sqrt{3}}{2}c\right)=(0,0).$$

Also, recall the conversion from Cartesian coordinates to Triphos coordinates from Section 2. That is, given an ordered pair, (x,y), the corresponding point in the Triphos plane is given by $\left(x+\frac{\sqrt{3}}{3}y,\frac{2\sqrt{3}}{3}y,0\right)$. Then the Cartesian origin, $(0,0)$, corresponds to the Triphos ordered triple:

$$\left(0+\frac{\sqrt{2}}{3}\cdot 0,\frac{2\sqrt{3}}{3}\cdot 0,0\right)=(0,0,0).$$

So using these coordinate conversions, we see that the triple (c,c,c) corresponds to the origin $(0,0,0)$. ∎

Geometric intuition corroborates the conclusion reached in Lemma 1. Due to the construction of the Triphos system, plotting a triple of the form (c,c,c), with c being a constant, creates an equilateral triangle of side-length c. Thus any triple of this form corresponds to the Triphos origin as illustrated

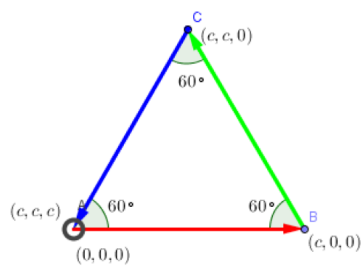

Figure 8: The Triple, (c,c,c), Returns to the Origin

Lemma 2 *Given any real number, c, the triple* $(r+c,g+c,b+c)$ *corresponds to the triple* (r,g,b).

Proof: Let c be a real number, and let (r,g,b) be an arbitrary ordered triple. Using the same coordinate conversions as in Lemma 3.1, we have $(r+c,g+c,b+c)$ corresponds to the Cartesian ordered pair:

$$\left((r+c)-\frac{1}{2}(g+c)-\frac{1}{2}(b+c),\frac{\sqrt{3}}{2}(g+c)-\frac{\sqrt{3}}{2}(b+c)\right)$$
$$=\left(r-\frac{1}{2}g-\frac{1}{2}b,\frac{\sqrt{3}}{2}g-\frac{\sqrt{3}}{2}b\right),$$

which we note is exactly the triple (r, g, b) given in its corresponding Cartesian coordinates. ∎

Using the fact that any point of the form (c, c, c) represents the origin and $(r + c, g + c, b + c)$ represents (r, g, b), intuition suggests that the following relation can be used to define equivalent representations of any point in the plane.

Theorem 1 *The relation defined by* $(r, g, b) \sim (r+c, g+c, b+c)$*, where* c *is a real number, is an equivalence relation. That is, it preserves the reflexive, symmetric, and transitive properties.*

Proof: Reflexive: Let (r, g, b) be an arbitrary ordered triple and $c = 0$. Then, $(r, g, b) \sim (r + 0, g + 0, b + 0) = (r, g, b)$.
Symmetric: Let (r_1, g_1, b_1) and (r_2, g_2, b_2) be arbitrary ordered triples. Suppose
$(r_1, g_1, b_1) \sim (r_2, g_2, b_2)$. Then there exists $c \in \mathbb{R}$ such that $(r_2, g_2, b_2) = (r_1 + c, g_1 + c, b_1 + c)$. Then, $(r_2, g_2, b_2) \sim (r_1 + c + (-c), g_1 + c + (-c), b_1 + c + (-c)) = (r_1, g_1, b_1)$. Thus $(r_2, g_2, b_2) \sim (r_1, g_1, b_1)$.
Transitive: Suppose $(r_1, g_1, b_1) \sim (r_2, g_2, b_2)$ and $(r_2, g_2, b_2) \sim (r_3, g_3, b_3)$. Then there exists $c \in \mathbb{R}$ such that $(r_1, g_1, b_1) \sim (r_1 + c, g_1 + c, b_1 + c) = (r_2, g_2, b_2)$, and there exists $d \in \mathbb{R}$ such that $(r_2, g_2, b_2) \sim (r_2 + d, g_2 + d, b_2 + d) = (r_3, g_3, b_3)$. Then, $(r_1, g_1, b_1) \sim (r_1 + c + d, g_1 + c + d, b_1 + c + d) = (r_3, g_3, b_3)$ ∎

Definition 2 *By reason of the equivalence relation defined above, each ordered triple* (r, g, b) *has an* equivalence class *of ordered triples* $\{(r + c, g + c, b + c) | c \in \mathbb{R}\}$*, denoted* $[(r, g, b)]$.

Fact 1 *Each ordered triple in an equivalence class corresponds to the same point on the plane.*

Example 3 *The ordered triples* $(3, 2, 5)$ *and* $(1, 0, 3)$ *both represent the same point in the plane. This is because* $(3, 2, 5) \in [(1, 0, 3)]$*, since*

$$(3, 2, 5) \sim (3 - 2, 2 - 2, 5 - 2) = (1, 0, 3),$$

as illustrated in Figure 9. (Also see Example 2.)

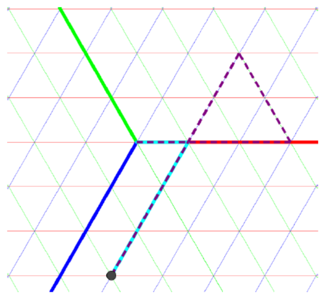

Figure 9: Equivalent Ordered Triples

Since there are an infinite number of equivalent ordered triples representing a given point, we define a *standard form* for equivalence classes, so that each point in the plane can be represented by a unique, standard ordered triple. This standard triple will be a representative of its equivalence class.

Definition 3 *An ordered triple is in* standard form *if all coordinates are non-negative, with at least one coordinate being equal to* 0*.*

To reduce an ordered triple to standard form, subtract the value of the smallest coordinate from all coordinates in the triple to arrive at an equivalent ordered triple.

Example 4 *To reduce the point* $(2,6,5)$ *to standard form, we subtract the smallest coordinate value,* 2*, from all coordinates in the triple. This reduces to another triple in the same equivalence class. Thus,*

$$\begin{aligned}(2,6,5) &\sim (2-2,6-2,5-2)\\ &= (0,4,3).\end{aligned}$$

Therefore, $(0,4,3)$ *is the reduced, standard form of the triple* $(2,6,5)$*. In Figure 10, the path defined by* $(0,4,3)$ *is denoted by the light blue solid line, and the path defined by* $(2,6,5)$ *is denoted by the purple dotted-line.*

The standard form of a point can tell us several things about the point. For example, from the standard form, we know the location of a point in the plane. Recall that we have divided the plane into three tridants, I, II, and III, whose boundaries are the red, green, and blue axes, as shown in Figure 11.

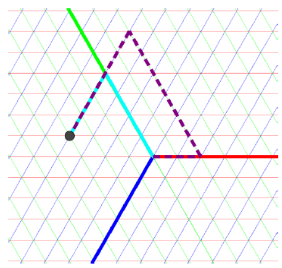

Figure 10: Standard Form

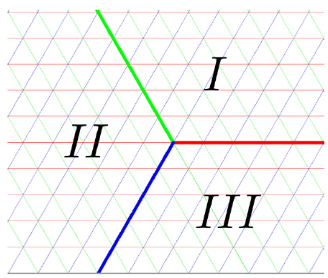

Figure 11: Tridants

Using the standard form of a point, we can tell immediately which tridant this point will lie in.

Fact 2 *Let $a, b > 0$. Then,*

Points of the form $(a, b, 0)$ will lie in Tridant I.
Points of the form $(0, a, b)$ will lie in Tridant II.
Points of the form $(a, 0, b)$ will lie in Tridant III.
Points of the form $(a, 0, 0)$ will lie on the red axis.
Points of the form $(0, a, 0)$ will lie on the green axis.
Points of the form $(0, 0, a)$ will lie on the blue axis.
The point $(0, 0, 0)$ is the origin.

Prior to further exploration into standard form, there are some important facts to note regarding linear algebra. First, we will show that any two of the Triphos unit vectors are linearly independent.

Definition 4 ([5]) *A set of vectors $\mathbf{v}_1, \mathbf{v_2}, \ldots, \mathbf{v_p}$ in $\mathbb{R}^n$ is said to be* linearly independent *if the vector equation*

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_p\mathbf{v}_p = \mathbf{0}$$

has only the trivial solution. That is, $x_1 = x_2 = \cdots = x_p = 0$.

An alternative method uses the following theorem.

Theorem 2 ([5]) *A set of n vectors in $\mathbb{R}^n$ is linearly independent if and only if the determinant of the matrix formed by taking the vectors as its columns is non-zero.*

Note that the determinant of a 2 x 2 matrix is defined by

$$det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc.$$

Lemma 3 *Of the unit vectors, $\hat{\mathbf{r}}, \hat{\mathbf{g}}$, and $\hat{\mathbf{b}}$, any two unit vectors, converted to Euclidean vectors, are linearly independent.*

Proof: Refer to Definition 1. The unit vectors, $\hat{\mathbf{r}}$, $\hat{\mathbf{g}}$, and $\hat{\mathbf{b}}$, are the vectors extending from the origin to the Triphos points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, respectively. We convert these points to Cartesian points and write them as vectors (see conversion in Section 2). Thus,

$$\hat{\mathbf{r}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \hat{\mathbf{g}} = \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \hat{\mathbf{b}} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix}.$$

We use Theorem 2 to prove that any two unit vectors are linearly independent.

Case 1: Take vectors $\hat{\mathbf{r}}$ and $\hat{\mathbf{g}}$. Using these as column vectors in a matrix, the determinant is

$$det \begin{bmatrix} 1 & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{bmatrix} = \frac{\sqrt{3}}{2} \neq 0.$$

Thus, $\hat{\mathbf{r}}$ and $\hat{\mathbf{g}}$ are linearly independent.

Case 2: Take vectors $\hat{\mathbf{r}}$ and $\hat{\mathbf{b}}$. Using these as column vectors in a matrix, the determinant is

$$det \begin{bmatrix} 1 & \frac{-1}{2} \\ 0 & -\frac{\sqrt{3}}{2} \end{bmatrix} = -\frac{\sqrt{3}}{2} \neq 0.$$

Thus, $\hat{\mathbf{r}}$ and $\hat{\mathbf{b}}$ are linearly independent.

Case 3: Take vectors $\hat{\mathbf{g}}$ and $\hat{\mathbf{b}}$. Using these as column vectors in a matrix, the determinant is

$$det \begin{bmatrix} \frac{-1}{2} & \frac{-1}{2} \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} = \frac{\sqrt{3}}{2} \neq 0.$$

Thus, $\hat{\mathbf{g}}$ and $\hat{\mathbf{b}}$ are linearly independent. ∎

This fact is used to prove Theorem 3.

Theorem 3 *If point $P = [(r_1, g_1, b_1)]$ and point $Q = [(r_2, g_2, b_2)]$ are defined by the triples (r_1, g_1, b_1) and (r_2, g_2, b_2), respectively, such that the triples are in standard form with*

$$r_1 \neq r_2 \, or g_1 \neq g_2 \, or b_1 \neq b_2,$$

then $[(r_1, g_1, b_1)] \neq [(r_2, g_2, b_2)]$. So P and Q represent different points in the plane.

Proof: We will use contradiction. Suppose that P and Q are the same point in the plane. Then $P = [(r_1, g_1, b_1)]$ and $P = [(r_2, g_2, b_2)]$. P is defined by the triples (r_1, g_1, b_1) and (r_2, g_2, b_2) with the property $r_1 \neq r_2$ or $g_1 \neq g_2$ or $b_1 \neq b_2$. Note that (r_1, g_1, b_1) and (r_2, g_2, b_2) are in standard form. Now, in standard form, at least one coordinate in an ordered triple must be 0. Without loss of generality, let $r_1 = 0$. Then P is in Trident II or on the blue or green axes. This means that, since P is also represented by the point (r_2, g_2, b_2) in standard form, $r_2 = 0$ to ensure this location of P. So the location of P depends only on the coordinates g_1 and b_1, or g_2 and b_2. Now, g_1 and g_2 define movement parallel to the green axis, and b_1 and b_2 define movement parallel to the blue axis. Using the unit vectors, $\hat{\mathbf{g}}$ and $\hat{\mathbf{b}}$, we can write the point P as a linear combination of vectors as shown:

$$P = g_1\hat{\mathbf{g}} + b_1\hat{\mathbf{b}} = g_2\hat{\mathbf{g}} + b_2\hat{\mathbf{b}}.$$

This implies that

$$g_1\hat{\mathbf{g}} + b_1\hat{\mathbf{b}} - g_2\hat{\mathbf{g}} - b_2\hat{\mathbf{b}} = \mathbf{0},$$

and, by factoring,

$$(g_1 - g_2)\hat{\mathbf{g}} + (b_1 - b_2)\hat{\mathbf{b}} = \mathbf{0}.$$

Since $\hat{\mathbf{g}}$ and $\hat{\mathbf{b}}$ are linearly independent, by Definition 3.3, the only solution to this equation is the trivial solution. Hence,

$$g_1 - g_2 = 0$$

and

$$b_1 - b_2 = 0.$$

This implies that $g_1 = g_2$ and $b_1 = b_2$. We already know that $r_1 = r_2 = 0$ so this is a contradiction to the hypothesis that $r_1 \neq r_2$ or $g_1 \neq g_2$ or $b_1 \neq b_2$. This concludes this proof by contradiction. We reject our hypothesis, so P and Q are distinct points in the plane. Thus, $[(r_1, g_1, b_1)] \neq [(r_2, g_2, b_2)]$. ∎

4. Properties

In this section, we will discuss some algebraic properties of the coordinates in the Triphos system. These include properties of addition and properties of multiplication. The culmination of these properties is the fact that the set of Triphos equivalence classes is a *field*.

Definition 5 ([2]) Addition *in Triphos corresponds to adding vectors. That is,*

$$[(r_1, g_1, b_1)] + [(r_2, g_2, b_2)] = [(r_1 + r_2, g_1 + g_2, b_1 + b_2)].$$

Theorem 4 *Addition in Triphos is well-defined.*

Proof: Let $[(r_1 + x, g_1 + x, b_1 + x)] = [(r_1, g_1, b_1)]$ and $[(r_2 + y, g_2 + y, b_2 + y)] = [(r_2, g_2, b_2)]$, where $x, y \in \mathbb{R}$. It suffices to show that

$$[(r_1+x, g_1+x, b_1+x)]+[(r_2+y, g_2+y, b_2+y)] = [(r_1, g_1, b_1)]+[(r_2, g_2, b_2)].$$

By definition of addition,

$$\begin{aligned}&[(r_1 + x, g_1 + x, b_1 + x)] + [(r_2 + y, g_2 + y, b_2 + y)]\\ &\quad = [(r_1 + r_2 + x + y, g_1 + g_2 + x + y, b_1 + b_2 + x + y)]\end{aligned}$$

The term $x + y$ is constant so

$$\begin{aligned}&[(r_1 + r_2 + x + y, g_1 + g_2 + x + y, b_1 + b_2 + x + y)]\\ &\qquad = [(r_1 + r_2, g_1 + g_2, b_1 + b_2)]\\ &\qquad = [(r_1, g_1, b_1)] + [(r_2, g_2, b_2)].\end{aligned}$$

Thus, addition is well-defined on equivalence classes. ∎

Theorem 5 *Addition in Triphos is commutative.*

Proof: It is sufficient to show

$$[(r_1, g_1, b_1)] + [(r_2, g_2, b_2)] = [(r_2, g_2, b_2)] + [(r_1, g_1, b_1)].$$

We have

$$\begin{aligned}&[(r_1, g_1, b_1)] + [(r_2, g_2, b_2)]\\ &\qquad = [(r_1 + r_2, g_1 + g_2, b_1 + b_2)]\\ &\qquad = [(r_2 + r_1, g_2 + g_1, b_2 + b_1)],\\ &\qquad\qquad \text{since real numbers commute}\\ &\qquad = [(r_2, g_2, b_2)] + [(r_1, g_1, b_1)].\end{aligned}$$

∎

Theorem 6 *Addition in Triphos is associative.*

Proof: It is sufficient to show that

$$([(r_1, g_1, b_1)] + [(r_2, g_2, b_2)]) + [(r_3, g_3, b_3)]$$
$$= [(r_1, g_1, b_1)] + ([(r_2, g_2, b_2)] + [(r_3, g_3, b_3)])\,.$$

We have

$$\begin{aligned}
&([(r_1, g_1, b_1)] + [(r_2, g_2, b_2)]) + [(r_3, g_3, b_3)]\\
&= [(r_1 + r_2, g_1 + g_2, b_1 + g_2)] + [(r_3, g_3, b_3)]\\
&= [(r_1 + r_2 + r_3, g_1 + g_2 + g_3, b_1 + b_2 + b_3)]\\
&= [(r_1, g_1, b_1)] + [(r_2 + r_3, g_2 + g_3, b_2 + b_3)]\\
&= [(r_1, g_1, b_1)] + ([(r_2, g_2, b_2)] + [(r_3, g_3, b_3)])\,.
\end{aligned}$$

∎

Theorem 7 ([2]) *The additive identity in Triphos is* $[(0, 0, 0)]$.

Proof: It is sufficient to show $[(r, g, b)] + [(0, 0, 0)] = [(r, g, b)]$. Clearly,

$$[(r, g, b)] + [(0, 0, 0)] = [(r + 0, g + 0, b + 0)] = [(r, g, b)].$$

∎

Theorem 8 *Given a point* $[(r, g, b)]$*, the additive inverse is* $[(-r, -g, -b)]$.

Proof: It is sufficient to show $[(r, g, b)] + [(-r, -g, -b)] = [(0, 0, 0)]$. Clearly,

$$\begin{aligned}
[(r, g, b)] + [(-r, -g, -b)] &= [(r + (-r), g + (-g), b + (-b))]\\
&= [(0, 0, 0)].
\end{aligned}$$

Thus, in adding an ordered triple and its additive inverse, the result is the additive identity, as one would expect. ∎

Now we will examine multiplication in the Triphos system. Unfortunately, there is not a nice vector analog for multiplication, as there was for addition. However, as hinted in [2], Triphos multiplication can be defined so as to mimic multiplication with complex numbers. To see this, we must first make a brief divergence into the complex numbers.

To begin, we must note that any complex number can be represented in the form $a+bi$, where $a, b \in \mathbb{R}$. If, for example, we had the number $2+2i$, then $a = 2$ and $b = 2$. Alternatively, if we had the number $-1 + 0.35i$, then $a = -1$ and $b = 0.35$. These numbers can then be plotted in the *complex plane*, with values of a along the Real (horizontal) axis and values of b along the Imaginary (vertical) axis (see Figure 12). These points can then be represented by vectors from the origin to the point. The product of two complex vectors has the characteristics such that the angles from the positive real axis to the vectors add and the magnitudes of the vectors multiply [Boa]. The key property that we want to preserve in Triphos multiplication is that *angles add* and *magnitudes multiply*.

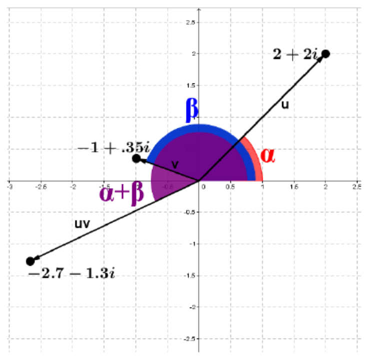

Figure 12: Complex Multiplication

We can use this property to find the products of our unit vectors. Note that the unit vector, $\hat{\mathbf{r}}$ is $0°$ from the horizontal axis, $\hat{\mathbf{g}}$ is $120°$ from the horizontal axis, and $\hat{\mathbf{b}}$ is $240°$ from the horizontal axis. The Euclidean magnitude of each unit vector is equal to 1. Thus, using the fact that angles add and magnitudes multiply,

$$\hat{\mathbf{r}}\hat{\mathbf{r}} = \hat{\mathbf{r}},\ \hat{\mathbf{r}}\hat{\mathbf{g}} = \hat{\mathbf{g}},\ \hat{\mathbf{r}}\hat{\mathbf{b}} = \hat{\mathbf{b}},\ \hat{\mathbf{g}}\hat{\mathbf{g}} = \hat{\mathbf{b}},\ \hat{\mathbf{g}}\hat{\mathbf{b}} = \hat{\mathbf{r}},\ \text{and } \hat{\mathbf{b}}\hat{\mathbf{b}} = \hat{\mathbf{g}}.$$

Then, we can write triples (r_1, g_1, b_1) and (r_2, g_2, b_2) in vector form:

$$r_1\hat{\mathbf{r}} + g_1\hat{\mathbf{g}} + b_1\hat{\mathbf{b}} \text{ and } r_2\hat{\mathbf{r}} + g_2\hat{\mathbf{g}} + b_2\hat{\mathbf{b}}.$$

Expanding the product of the triples (r_1, g_1, b_1) and (r_2, g_2, b_2) in this form will yield

$$r_1r_2\hat{\mathbf{r}} + r_1g_2\hat{\mathbf{g}} + r_1b_2\hat{\mathbf{b}} + g_1r_2\hat{\mathbf{g}} + g_1g_2\hat{\mathbf{b}} + r_1b_2\hat{\mathbf{r}} + b_1b_2\hat{\mathbf{b}} + b_1g_2\hat{\mathbf{r}} + b_1b_2\hat{\mathbf{g}},$$

which reduces to the following formula for multiplication.

Definition 6 ([2]) Multiplication *in Triphos is defined as*

$$[(r_1, g_1, b_1)] * [(r_2, g_2, b_2)]$$
$$= [(r_1r_2 + g_1b_2 + b_1g_2,\ r_1g_2 + g_1r_2 + b_1b_2,\ r_1b_2 + g_1g_2 + b_1r_2)].$$

Given this definition of multiplication, we can discuss properties of multiplication.

Theorem 9 *Multiplication in Triphos is well-defined.*

Proof: Let
$$[(r_1 + x, g_1 + x, b_1 + x)] = [(r_1, g_1, b_1)]$$
and
$$[(r_2 + y, g_2 + y, b_2 + y) = [(r_2, g_2, b_2)],$$
where $x, y \in \mathbb{R}$. It suffices to show that
$$[(r_1 + x, g_1 + x, b_1 + x)] * [(r_2 + y, g_2 + y, b_2 + y)] \\ = [(r_1, g_1, b_1)] * [(r_2, g_2, b_2)].$$
By definition of multiplication,
$$\begin{aligned} & [(r_1 + x, g_1 + x, b_1 + x)] * [(r_2 + y, g_2 + y, b_2 + y)] \\ = \quad & [(r_1r_2 + r_1y + r_2x + xy + g_1b_2 + g_1y + \\ & \quad b_2x + xy + b_1g_2 + b_1y + g_2x + xy, \\ & r_1g_2 + r_1y + g_2x + xy + g_1r_2 + g_1y + \\ & \quad r_2x + xy + b_1b_2 + b_1y + b_2x + xy, \\ & r_1b_2 + r_1y + b_2x + xy + g_1g_2 + g_1y + \\ & \quad g_2x + xy + b_1r_2 + b_1y + r_2x + xy)]. \end{aligned}$$
Let $m = r_1y + g_1y + b_1y + r_2x + g_2x + b_2x + 3xy$. Then
$$\begin{aligned} & [(r_1 + x, g_1 + x, b_1 + x)] * [(r_2 + y, g_2 + y, b_2 + y)] \\ = \quad & [(r_1r_2 + g_1b_2 + b_1g_2 + m, r_1g_2 + g_2r_1 + \\ & \quad b_1b_2 + m, r_1b_2 + g_1g_2 + b_1r_2 + m)] \\ = \quad & [(r_1r_2 + g_1b_2 + b_1g_2, r_1g_2 + g_2r_1 + b_1b_2, r_1b_2 + g_1g_2 + b_1r_2)] \\ = \quad & [(r_1, g_1, b_1)] * [(r_2, g_2, b_2)]. \end{aligned}$$
∎

Theorem 10 ([2]) *Multiplication in Triphos is commutative.*

Proof: It is sufficient to show that
$$[(r_1, g_1, b_1)] * [(r_2, g_2, b_2)] = [(r_2, g_2, b_2)] * [(r_1, g_1, b_1)].$$
We have
$$\begin{aligned} & [(r_1, g_1, b_1)] * [(r_2, g_2, b_2)] \\ = \quad & [(r_1r_2 + g_1b_2 + b_1g_2,\ r_1g_2 + g_1r_2 + b_1b_2,\ r_1b_2 + g_1g_2 + b_1r_2)] \\ = \quad & [(r_2r_1 + g_2b_1 + b_2g_1, r_2g_1 + g_2r_1 + b_2b_1, r_2b_1 + g_2g_1 + b_2r_1)] \\ = \quad & [(r_2, g_2, b_2)] * [(r_1, g_1, b_1)]. \end{aligned}$$
∎

Theorem 11 ([2]) *Multiplication in Triphos is associative.*

Proof: It is sufficient to show that

$$[(r_1,g_1,b_1)]*([(r_2,g_2,b_2,)]*[(r_3,g_3,b_3)]) \\ =([(r_1,g_1,b_1)]*[(r_2,g_2,b_2)])*[(r_3,g_3,b_3)].$$

We have

$$\begin{aligned} & [(r_1,g_1,b_1)]*([(r_2,g_2,b_2,)]*[(r_3,g_3,b_3)]) \\ = \; & [(r_1,g_1,b_1)]* \\ & \quad [(r_2r_3+g_2b_3+b_2g_3, r_2g_3+g_2r_3+b_2b_3, r_2b_3+g_2g_3+b_2r_3)] \\ = \; & [(r_1r_2r_3+r_1g_2b_3+r_1b_2g_3+g_1r_2b_3+g_1g_2g_3+ \\ & \quad g_1b_2r_3+b_1r_2g_3+b_1g_2r_3+b_1b_2b_3, \\ & r_1r_2g_3+r_1g_2r_3+r_1b_2b_3+g_1r_2r_3+g_1g_2b_3+ \\ & \quad g_1b_2g_3+b_1r_2b_3+b_1g_2g_3+b_1b_2r_3, \\ & r_1r_2b_3+r_1g_2g_3+r_1b_2r_3+g_1r_2g_3+g_1g_2r_3+ \\ & \quad g_1b_2b_3+b_1r_2r_3+b_1g_2b_3+b_1b_2g_3)] \\ = \; & [((r_1r_2+g_1b_2+b_1g_2)r_3+(r_1g_2+g_1r_2+b_1b_2)b_3+ \\ & \qquad (r_1b_2+g_1g_2+b_1r_2)g_3, \\ & (r_1r_2+g_1b_2+b_1g_2)g_3+(r_1g_2+g_1r_2+b_1b_2)r_3+ \\ & \qquad (r_1b_2+g_1g_2+b_1r_2)b_3, \\ & (r_1r_2+g_1b_2+b_1g_2)b_3+(r_1g_2+g_1r_2+b_1b_2)g_3+ \\ & \qquad (r_1b_2+g_1g_2+b_1r_2)r_3)] \\ = \; & [(r_1r_2+g_1b_2+b_1g_2, r_1g_2+g_1r_2+b_1b_2, r_1b_2+g_1g_2+b_1r_2)] \\ & \quad *[(r_3,g_3,b_3)] \\ = \; & ([(r_1,g_1,b_1)]*[(r_2,g_2,b_2)])*[(r_3,g_3,b_3)]. \end{aligned}$$

∎

Theorem 12 ([2]) *The multiplicative identity in Triphos is* $[(1,0,0)]$.

Proof: It is sufficient to show $[(r,g,b)]*[(1,0,0)]=[(r,g,b)]$. So

$$\begin{aligned} & [(r,g,b)]*[(1,0,0)] \\ = \; & [(r*1+g*0+b*0,\ r*0+g*1+b*0,\ r*0+g*0+b*1)] \\ = \; & [(r,g,b)]. \end{aligned}$$

∎

Theorem 13 *The multiplicative inverse of a point* $[(r,g,b)]$*, where* $[(r,g,b)] \neq [(0,0,0)]$*, is*

$$\left[\left(\frac{r}{r^2+g^2+b^2-rb-rg-gb}, \frac{b}{r^2+g^2+b^2-rb-rg-gb}, \frac{g}{r^2+g^2+b^2-rb-rg-gb}\right)\right].$$

Proof: It is sufficient to show that

$$[(r,g,b)]*\left[\left(\frac{r}{r^2+g^2+b^2-rb-rg-gb}, \frac{b}{r^2+g^2+b^2-rb-rg-gb}, \frac{g}{r^2+g^2+b^2-rb-rg-gb}\right)\right]$$
$$= [(1,0,0)].$$

We have

$$[(r,g,b)]*\left[\left(\frac{r}{r^2+g^2+b^2-rb-rg-gb}, \frac{b}{r^2+g^2+b^2-rb-rg-gb}, \frac{g}{r^2+g^2+b^2-rb-rg-gb}\right)\right]$$
$$=\left[\left(\frac{r^2+g^2+b^2}{r^2+g^2+b^2-rb-rg-gb}, \frac{rb+rg+gb}{r^2+g^2+b^2-rb-rg-gb}, \frac{rb+rg+gb}{r^2+g^2+b^2-rb-rg-gb}\right)\right]$$
$$=\left[\left(\frac{r^2+g^2+b^2-rb-rg-gb+rb+rg+gb}{r^2+g^2+b^2-rb-rg-gb}, \frac{rb+rg+gb}{r^2+g^2+b^2-rb-rg-gb}, \frac{rb+rg+gb}{r^2+g^2+b^2-rb-rg-gb}\right)\right]$$
$$=\left[\left(1+\frac{rb+rg+gb}{r^2+g^2+b^2-rb-rg-gb}, \frac{rb+rg+gb}{r^2+g^2+b^2-rb-rg-gb}, \frac{rb+rg+gb}{r^2+g^2+b^2-rb-rg-gb}\right)\right].$$

Note that

$$\frac{rb+rg+gb}{r^2+g^2+b^2-rb-rg-gb}$$

is a constant real number. So

$$[(r,g,b)]*\left[\left(\frac{r}{r^2+g^2+b^2-rb-rg-gb}, \frac{b}{r^2+g^2+b^2-rb-rg-gb}, \frac{g}{r^2+g^2+b^2-rb-rg-gb}\right)\right]$$
$$= [(1,0,0)].$$

∎

Theorem 14 *Using Triphos multiplication and addition, the distributive property holds.*

Proof: It is sufficient to show that

(a) $[(r_1,g_1,b_1)] * ([(r_2,g_2,b_2,)] + [(r_3,g_3,b_3)])$
$= ([(r_1,g_1,b_1)] * [(r_2,g_2,b_2)]) + ([(r_1,g_1,b_1)] * [(r_3,g_3,b_3)])\,,$

and

(b) $([(r_1,g_1,b_1)] + [(r_2,g_2,b_2,)]) * [(r_3,g_3,b_3)]$
$= ([(r_1,g_1,b_1)] * [(r_3,g_3,b_3)]) + ([(r_2,g_2,b_2,)] * [(r_3,g_3,b_3)])\,.$

We will first deal with the first case. We have

$$[(r_1,g_1,b_1)]*([(r_2,g_2,b_2,)] + [(r_3,g_3,b_3)])$$
$$= [(r_1,g_1,b_1)] * [(r_2+r_3, g_2+g_3, b_2+b_3)]$$

$$
\begin{aligned}
&= [(r_1r_2 + r_1r_3 + g_1b_2 + g_1b_3 + b_1g_2 + b_1g_3,\\
&\quad r_1g_2 + r_1g_3 + g_1r_2 + g_1r_3 + b_1b_2 + b_1b_3,\\
&\quad r_1b_2 + r_1b_3 + g_1g_2 + g_1g_3 + b_1r_2 + b_1r_3)]\\
&= [(r_1r_2 + g_1b_2 + b_1g_2 + r_1r_3 + g_1b_3 + b_1g_3,\\
&\quad r_1g_2 + g_1r_2 + b_1b_2 + r_1g_3 + g_1r_3 + b_1b_3,\\
&\quad r_1b_2 + g_1g_2 + b_1r_2 + r_1b_3 + g_1g_3 + b_1r_3)]\\
&= [(r_1r_2 + g_1b_2 + b_1g_2, r_1g_2 + g_1r_2 + b_1b_2, r_1b_2 + g_1g_2 + b_1r_2)]\\
&\quad +[(r_1r_3 + g_1b_3 + b_1g_3, r_1g_3 + g_1r_3 + b_1b_3, r_1b_3 + g_1g_3 + b_1r_3)]\\
&= ([(r_1, g_1, b_1)] * [(r_2, g_2, b_2)]) + ([(r_1, g_1, b_1)] * [(r_3, g_3, b_3)])\,.
\end{aligned}
$$

The second case follows by the commutative property. ∎

Theorem 15 *The set of all equivalence classes in Triphos is a field.*

Proof: For a set to be a field, eleven properties must hold [4]. These are:

1. Addition is closed. Based on our definition of addition, it is clear that adding two Triphos triples will yield another Triphos triple.
2. Addition is associative. (See Theorem 6.)
3. Addition is commutative. (See Theorem 5.)
4. An additive identity is included in the set. (See Theorem 7.)
5. An additive inverse exists for each equivalence class. (See Theorem 8.)
6. Multiplication is closed. Based on our definition of multiplication, it is clear that multiplying two Triphos triples will yield another Triphos triple.
7. Multiplication is associative. (See Theorem 11.)
8. The distributive law holds. (See Theorem 14.)
9. Multiplication is commutative. (See Theorem 10.)
10. A nonzero multiplicative identity is included in the set. (See Theorem 12.)
11. A multiplicative inverse exists for each nonzero equivalence class. (See Theorem 13.)

Each of these properties has been shown to hold. Thus, the set of equivalence classes in Triphos coordinates is a field. ∎

5. Metric

A fundamental aspect of geometry on a coordinate pane is the definition of a distance between two points. A familiar distance on the Cartesian coordinate plane is the Euclidean distance between two points, (x_1, y_1) and (x_2, y_2), given by the function

$$D((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This function is called a *metric*.

Definition 7 ([7]) *Let* $\mathbf{P}$ *be a set of points. A* metric *is a function* $D : \mathbf{P} \times \mathbf{P} \mapsto \mathbb{R}$ *such that*

1. $D(P, Q) = D(Q, P)$ *for every* $P, Q \in \mathbf{P}$*,*
2. $D(P, Q) \geq 0$ *for every* $P, Q \in \mathbf{P}$*,*
3. $D(P, Q) = 0$ *if and only if* $P = Q$*, and*
4. $D(P, Q) \leq D(P, R) + D(R, Q)$*,for every* $P, Q, R \in \mathbf{P}$.

In the Triphos coordinate system, we define a metric that exploits the inherent properties of the system, in the hopes that this definition will be useful in applications of the system.

Definition 8 *Let* $\mathbf{T}$ *be the Triphos plane. The* hexa-metric *function* $D : \mathbf{T} \times \mathbf{T} \mapsto \mathbb{R}$ *between points* $[(r_1, g_1, b_1)]$ *and* $[(r_2, g_2, b_2)]$ *is defined by*

$$\begin{aligned} &D([(r_1, g_1, b_1)], [(r_2, g_2, b_2)]) \\ &\quad = \min\{|(r_1 - b_1) - (r_2 - b_2)| + |(g_1 - b_1) - (g_2 - b_2)|, \\ &\qquad |(r_1 - g_1) - (r_2 - g_2)| + |(b_1 - g_1) - (b_2 - g_2)|, \\ &\qquad |(g_1 - r_1) - (g_2 - r_2)| + |(b_1 - r_1) - (b_2 - r_2)|\}. \end{aligned}$$

Theorem 16 *The hexa-metric is well-defined. That is, given*

$$[(r_1 + c, g_1 + c, b_1 + c)] = [(r_1, g_1, b_1)]$$

and

$$[(r_2 + d, g_2 + d, b_2 + d)] = [(r_2, g_2, b_2)]$$

where $c, d \in \mathbb{R}$*,*

$$\begin{aligned} &D([(r_1, g_1, b_1)], [(r_2, g_2, b_2)]) \\ &\quad = D([(r_1 + c, g_1 + c, b_1 + c)], [(r_2 + d, g_2 + d, b_2 + d)]). \end{aligned}$$

Proof: From the hexa-metric formula:

$$\begin{aligned}
&D([(r_1, g_1, b_1)], [(r_2, g_2, b_2)]) \\
&\quad = \min\{|(r_1 - b_1) - (r_2 - b_2)| + |(g_1 - b_1) - (g_2 - b_2)|\,, \\
&\qquad\quad |(r_1 - g_1) - (r_2 - g_2)| + |(b_1 - g_1) - (b_2 - g_2)|\,, \\
&\qquad\quad |(g_1 - r_1) - (g_2 - r_2)| + |(b_1 - r_1) - (b_2 - r_2)|\} \\
&\quad = \min\{|(r_1 + c - (b_1 + c)) - (r_2 + d - (b_2 + d))| + \\
&\qquad\qquad |(g_1 + c - (b_1 + c)) - (g_2 + d - (b_2 + d))|\,, \\
&\qquad\quad |(r_1 + c - (g_1 + c)) - (r_2 + d - (g_2 + d))| + \\
&\qquad\qquad |(b_1 + c - (g_1 + c)) - (b_2 + d - (g_2 + d))|\,, \\
&\qquad\quad |(g_1 + c - (r_1 + c)) - (g_2 + d - (r_2 + d))| + \\
&\qquad\qquad |(b_1 + c - (r_1 + c)) - (b_2 + d - (r_2 + d))|\} \\
&\quad = D([(r_1 + c, g_1 + c, b_1 + c)], [(r_2 + d, g_2 + d, b_2 + d)]).
\end{aligned}$$

∎

Geometrically, the hexa-metric function of two points returns the length of the shortest possible path between the two points, while restricting movements to be parallel to the red, green, and blue axes.

Example 5 *(See Figure 13.) Below are three possible paths between the two points shown while restricting movements. Each path moves parallel to the red, green, and/or blue axes. In our formula, the output will yield the length of the shortest possible path, here denoted by the solid path. In this case, the hexa-metric output is 7.*

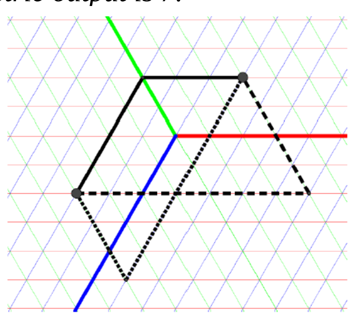

Figure 13: Hexa-Metric

Theorem 17 *The hexa-metric is a metric.*

Theorem 17 follows directly from Lemmas 4-7 below.

Lemma 4 *The symmetric property holds for the hexa-metric. That is,* $D(P, Q) = D(Q, P)$ *for every* P *and* Q.

Proof: Let P and Q be points with $P = [(r_1, g_1, b_1)]$ and $Q = [(r_2, g_2, b_2)]$. Using the formula from Definition 8 and properties of absolute values,

$$\begin{aligned} D(P,Q) &= \min\{|(r_1 - b_1) - (r_2 - b_2)| + |(g_1 - b_1) - (g_2 - b_2)|, \\ &\quad |(r_1 - g_1) - (r_2 - g_2)| + |(b_1 - g_1) - (b_2 - g_2)|, \\ &\quad |(g_1 - r_1) - (g_2 - r_2)| + |(b_1 - r_1) - (b_2 - r_2)|\} \\ &= \min\{|(r_2 - b_2) - (r_1 - b_1)| + |(g_2 - b_2) - (g_1 - b_1)|, \\ &\quad |(r_2 - g_2) - (r_1 - g_1)| + |(b_2 - g_2) - (b_1 - g_1)|, \\ &\quad |(g_2 - r_2) - (g_1 - r_1)| + |(b_2 - r_2) - (b_1 - r_1)|\} \\ &= D(Q,P). \end{aligned}$$

∎

Lemma 5 *The nonnegative distance property holds for the hexa-metric. That is, $D(P,Q) \geq 0$ for every P and Q.*

Proof: Let P and Q be points with $P = [(r_1, g_1, b_1)]$ and $Q = [(r_2, g_2, b_2)]$. By definition, $|a| \geq 0$ for all $a \in \mathbb{R}$. Since the metric gives a distance as the sum of two absolute values, $D(P,Q) \geq 0$ for every P and Q. ∎

Lemma 6 *The coincidence axiom holds for the hexa-metric. That is, $D(P,Q) = 0$ if and only if $P = Q$.*

Proof: Let P and Q be points with $P = [(r_1, g_1, b_1)]$ and $Q = [(r_2, g_2, b_2)]$. First, suppose $D(P,Q) = 0$. Then the minimum of

$$|(r_2 - b_2) - (r_1 - b_1)| + |(g_2 - b_2) - (g_1 - b_1)|,$$
$$|(r_2 - g_2) - (r_1 - g_1)| + |(b_2 - g_2) - (b_1 - g_1)|$$

and

$$|(g_2 - r_2) - (g_1 - r_1)| + |(b_2 - r_2) - (b_1 - r_1)|$$

must equal 0.

Case 1: Suppose the first element is the minimum, so that

$$|(r_2 - b_2) - (r_1 - b_1)| + |(g_2 - b_2) - (g_1 - b_1)| = 0.$$

Then $|(r_2 - b_2) - (r_1 - b_1)|$ and $|(g_2 - b_2) - (g_1 - b_1)|$ must each equal 0, since absolute values are non-negative. This implies that

$$r_2 - b_2 = r_1 - b_1$$

and

$$g_2 - b_2 = g_1 - b_1.$$

Using algebra, we see that

$$r_2 = r_1 + (b_2 - b_1)$$

and
$$g_2 = g_1 + (b_2 - b_1).$$
Clearly,
$$b_2 = b_1 + (b_2 - b_1).$$
Let $c = b_2 - b_1$. Then we see that
$$(r_2, g_2, b_2) = (r_1 + c, g_1 + c, b_1 + c).$$
Thus (r_1, g_1, b_1) and (r_2, g_2, b_2) are in the same equivalence class so $P = Q$.

The other cases are similar.

Now, suppose that $P = Q$. We must show that $D(P,Q) = 0$. Since $P = Q$,
$$(r_1, g_1, b_1) = (r_2 + c, g_2 + c, b_2 + c)$$
where $c \in \mathbb{R}$. So, $r_1 = r_2 + c$, $g_1 = g_2 + c$, and $b_1 = b_2 + c$. Thus,

$$\begin{aligned}
&|(r_2 - b_2) - (r_1 - b_1)| + |(g_2 - b_2) - (g_1 - b_1)| \\
&\quad = |(r_2 - b_2) - (r_2 + c - (b_2 + c))| + |(g_2 - b_2) - (g_2 + c - (b_2 + c))| \\
&\quad = |0| + |0| \\
&\quad = 0.
\end{aligned}$$

Therefore,
$$\begin{aligned}
D(P,Q) &= \min\{|(r_1 - b_1) - (r_2 - b_2)| + |(g_1 - b_1) - (g_2 - b_2)|, \\
&\qquad |(r_1 - g_1) - (r_2 - g_2)| + |(b_1 - g_1) - (b_2 - g_2)|, \\
&\qquad |(g_1 - r_1) - (g_2 - r_2)| + |(b_1 - r_1) - (b_2 - r_2)|\} \\
&= 0.
\end{aligned}$$

∎

Lemma 7 *The triangle inequality holds for the hexa-metric. That is,*
$$D(P,Q) \leq D(P,R) + D(R,Q).$$

Proof: Recall that the hexa-metric distance between points $[(r_1, g_1, b_1)]$ and $[(r_2, g_2, b_2)]$ is defined as

$$\begin{aligned}
&D([(r_1, g_1, b_1)], [(r_2, g_2, b_2)]) \\
&\qquad = \min\{|(r_1 - b_1) - (r_2 - b_2)| + |(g_1 - b_1) - (g_2 - b_2)|, \\
&\qquad\qquad |(r_1 - g_1) - (r_2 - g_2)| + |(b_1 - g_1) - (b_2 - g_2)|, \\
&\qquad\qquad |(g_1 - r_1) - (g_2 - r_2)| + |(b_1 - r_1) - (b_2 - r_2)|\}
\end{aligned}$$

We can call the first element
$$|(r_1 - b_1) - (r_2 - b_2)| + |(g_1 - b_1) - (g_2 - b_2)|$$

the yellow distance, D_{yellow}, between $[(r_1, g_1, b_1)]$ and $[(r_2, g_2, b_2)]$ as it is the length of the shortest path between the two points while moving parallel to the red and green axes only. (Note: Red light plus green light makes yellow light.) Similarly, the second element

$$|(r_1 - g_1) - (r_2 - g_2)| + |(b_1 - g_1) - (b_2 - g_2)|$$

is the magenta distance, $D_{magenta}$, between the two points, moving parallel to the red and blue axes. The third element

$$|(g_1 - r_1) - (g_2 - r_2)| + |(b_1 - r_1) - (b_2 - r_2)|$$

is the cyan distance, D_{cyan}, between the two points, moving parallel to the green and blue axes. Now, there are several cases to consider based on which element is the minimum of the formula.

Case 1: Suppose $D(P, R)$ is the yellow distance and $D(R, Q)$ is the yellow distance. By the equivalence relation, we can force any coordinate in a triple to be zero. Let $P = [(0, g_1, b_1)]$, $Q = [(r_2, g_2, 0)]$, and $R = [(r_3, g_3, 0)]$. Then,

$$\begin{aligned} &D(P, R) + D(R, Q) \\ &\quad = D_{yellow}(P, R) + D_{yellow}(R, Q) \\ &\quad = |-b_1 - r_3| + |g_1 - b_1 - g_3| + |r_3 - r_2| + |g_3 - g_2| . \end{aligned}$$

Also, the yellow distance between points P and Q is

$$D_{yellow}(P, Q) = |-b_1 - r_2| + |g_1 - b_1 - g_2| .$$

We will show $D_{yellow}(P, Q) \leq D(P, R) + D(R, Q)$. By the triangle inequality,

$$|-b_1 - r_3 + r_3 - r_2| \leq |-b_1 - r_3| + |r_3 - r_2| .$$

This implies

$$|-b_1 - r_2| \leq |-b_1 - r_3| + |r_3 - r_2| .$$

Similarly,

$$|g_1 - b_1 - g_3 + g_3 - g_2| \leq |g_1 - b_1 - g_3| + |g_3 - g_2| .$$

This implies

$$|g_1 - b_1 - g_2| \leq |g_1 - b_1 - g_3| + |g_3 - g_2| .$$

Therefore

$$\begin{aligned} &|-b_1 - r_2| + |g_1 - b_1 - g_2| \\ &\qquad \leq |-b_1 - r_3| + |g_1 - b_1 - g_3| + |r_3 - r_2| + |g_3 - g_2| . \end{aligned}$$

Thus, $D_{yellow}(P, Q) \leq D(P, R) + D(R, Q)$. Since the hexa-metric distance is defined as the minimum of the yellow, magenta, and cyan distances, we know $D(P, Q) \leq D(P, R) + D(R, Q)$.

Cases 2 and 3: Suppose $D(P,R)$ is the cyan distance and $D(R,Q)$ is the cyan distance (or both magenta). Let $P=[(0,g_1,b_1)]$, $Q=[(r_2,g_2,0)]$, and $R=[(r_3,g_3,0)]$. The argument in these cases is similar to the argument in Case 1.

Case 4: Suppose $D(P,R)$ is the yellow distance and $D(R,Q)$ is the magenta distance. Let $P=[(r_1,g_1,0)]$, $Q=[(r_2,g_2,0)]$, and $R=[(r_3,g_3,0)]$. Then

$$\begin{aligned} D(P,R)+&D(R,Q) \\ &= D_{yellow}(P,R)+D_{magenta}(R,Q) \\ &= |r_1-r_3|+|g_1-g_3|+|r_3-g_3-r_2+g_2|+|g_3-g_2|\,. \end{aligned}$$

By the triangle inequality,

$$\begin{aligned} D(P,R)+&D(R,Q) \\ &\geq |(r_1-r_3)+(r_3-g_3-r_2+g_2)|+|g_1-g_3|+|g_3-g_2| \\ &= |r_1-g_3-r_2+g_2|+|g_1-g_3|+|g_3-g_2|\,. \end{aligned}$$

For the sake of simplicity, let $A=r_1-g_3-r_2+g_2$, let $B=g_1-g_3$, and let $C=g_2-g_3$. Then

$$D(P,R)+D(R,Q)\geq |A|+|B|+|C|.$$

Now, we'll examine the three possible distances between points P and Q. We see that

$$\begin{aligned} D_{magenta}&(P,Q) \\ &= |r_1-g_1-r_2+g_2|+|-g_1+g_2| \\ &= |(r_1-g_3-r_2+g_2)-(g_1-g_3)|+|(g_2-g_3)-(g_1-g_3)| \\ &= |A-B|+|C-B|\,. \end{aligned}$$

Also,

$$\begin{aligned} &D_{cyan}(P,Q) \\ &=|r_1-g_1-r_2+g_2|+|-r_1+r_2| \\ &=|(r_1-g_3-r_2+g_2)-(g_1-g_3)|+|(g_2-g_3)-(r_1-g_3-r_2+g_2)| \\ &=|A-B|+|C-A| \\ &=|B-A|+|C-A|\,. \end{aligned}$$

Lastly,

$$\begin{aligned} D_{yellow}(P,Q)=|r_1-r_2|+&|g_1-g_2| \\ &= |(r_1-g_3-r_2+g_2)-(g_2-g_3)|+|(g_1-g_3)-(g_2-g_3)| \\ &= |A-C|+|B-C|\,. \end{aligned}$$

Now, we prove that

$$D(P,Q) \leq |A| + |B| + |C| \leq D(P,R) + D(R,Q).$$

We show that in every case, at least one of the colored distances between P and Q (yellow, magenta, or cyan) is less than the sum of the hexa-metric distances between P and R and between R and Q. Since the hexa-metric distance between P and Q is the minimum of its yellow, magenta, and cyan distances, this would imply that, in all cases,

$$D(P,Q) \leq D(P,R) + D(R,Q).$$

Thus, we will show in any case at least one of the following is true:

- $|A - C| + |B - C| \leq |A| + |B| + |C|$
- $|A - B| + |C - B| \leq |A| + |B| + |C|$
- $|B - A| + |C - A| \leq |A| + |B| + |C|$.

There are 24 cases to consider, namely all possible orders of A,B,C, and 0 (eg. $0 \leq A \leq B \leq C$.) It is fairly straightforward to prove that, in each case, at least one of those listed above is true.

Cases 5-9: The remaining cases can be proven using the same method as in Case 4. In each, it suffices to show that at least one of three bulleted cases listred above is true. ∎

Several visual examples of the triangle inequality using hexa-metric distances are shown in Figures 14 and 15.

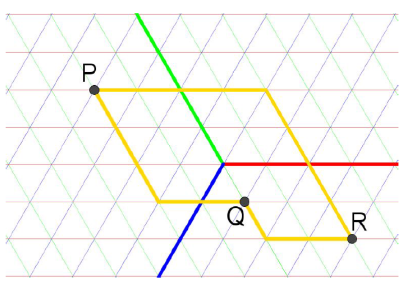

Figure 14: Yellow Distances Between Points

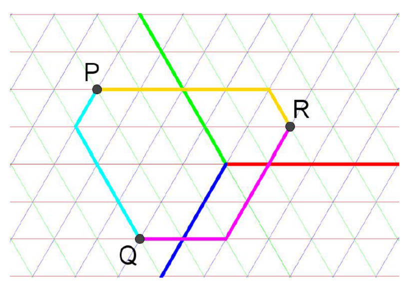

Figure 15: Yellow, Magenta, and Cyan Distances Between Points

Conjecture 1 *Under the hexa-metric, a circle of radius* 1 *centered at the origin is a regular hexagon with Euclidean side lengths of* 1*, as shown in Figure 16.*

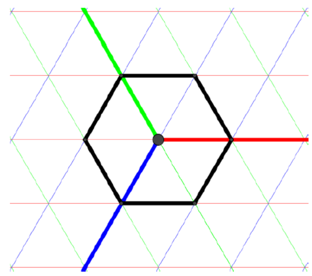

Figure 16: Conjectured Unit Circle in Hexa-Metric

6. Applications

Perhaps the most readily-useful application of the Triphos coordinate system is as a learning tool for middle and early high school students. Students at this age have likely only been exposed to the Cartesian coordinate system, but do not yet have the skills to understand other common coordinate systems, such as the complex system. The Triphos system provides another simple, understandable system for the purpose of comparison with the Cartesian coordinate system. Students can explore the two systems, discovering the relative strengths and weaknesses of each.

The Triphos system can challenge students' preconceived notion that there is only one way to solve a problem. Many will have never considered the origin of the Cartesian system, nor its inherently useful properties. This investigation will help students explore mathematics in a new way, with particular emphasis on how our familiar systems were developed, and why they are useful. In defining new operations and a metric, students can recognize the freedom of mathematical creativity, as opposed to the rigid structure so often equated with mathematical thinking. Rather than encouraging students to passively absorb preexisting mathematical ideas, the Triphos system can introduce them to new ways of thinking, empowering them to make their own discoveries or propose new mathematical models. As a middle-school math teacher, the second author has utilized this as a learning tool in her classroom with great results.

Additionaly, the properties of the Triphos coordinate system suggest potential for use in the real world. Here, we will explore a few practical applications for this system, though this is certainly not an exhaustive list.

The first application was suggested in [2]. We can use the Triphos coordinate system to model the addition of light. By labeling the three axes in the Triphos coordinate system as *red*, *green*, and *blue*, the basic properties of the system set us up for this application since red, green, and blue are the three primary colors of light. Adding equal amounts of red, green, and blue light results in white light.

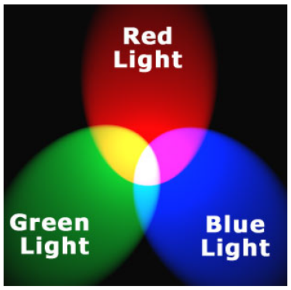

Figure 17: Color Addition

In this application, we can think of the origin as white light. Then it makes sense that an ordered triple of the form (c, c, c) would represent the origin, because it implies that we are adding equal amounts of red, green, and blue light, which results in white light. We believe the Triphos coordinates would be more applicable to the combination of pigments, where only the ratios of the added colors affect the resulting color. In light, the intensity of each color affects the brightness of the resulting color of light.

A similar application would arise in the field of computer science. Here, colors are represented by *RGB values*. These are ordered triples which assign a value between 0 and 255 for each coordinate, representing the intensity of red, green, and blue light in a given color. For example, the RGB value $(255, 0, 255)$ represents a color with a high intensity of red and blue light, resulting in a bright magenta color. All RGB values could be plotted on the Triphos coordinate system, which could potentially be useful for computers. However, since the intensity of light affects the brightness of the resulting color, we would have all gray-scale colors represented at the origin. This application will need further investigation because of this issue.

A final potential application of the Triphos system arises in the study of quantum chromodynamics. Quantum chromodynamics is a theory describing the strong force between the quarks, which make up protons and neutrons, wherein quarks are assigned a “color charge" of red, green, or blue that is conserved in every particle. It is an area of quantum field theory, called non-abelian gauge theory, with a group of symmetries given by $SU(3)$, the special unitary group of degree 3 [6]. The $SU(3)$ group has been well-studied by mathematicians and it is plausible to believe that there is some correlation between the red, green, and blue of the Triphos coordinate system and the red, green, and blue quarks and the associated $SU(3)$ group.

7. Future Work and Conclusion

This paper has just brushed the surface of the Triphos coordinate system, and there is still much more to discover. Natural areas for further exploration include:

- definition of a line in the hexa-metric;
- properties of parallel and perpendicular lines;
- using functions to further understand the interaction of variables in the Triphos plane;
- a study of trigonometry in this system;
- a study of calculus and its potential applications in Triphos;
- defining a three dimensional Trihpos system and exploring its properties and applications.

In conclusion, this paper has formally defined and characterized the Triphos coordinate system. In addition to introducing the system, we have explored its basic algebraic properties and proven that the set of equivalence classes in Triphos is a field. Finally, we have investigated a few of the basic geometric properties, such as a metric and unit circle, of the Triphos coordinate system.

8. Acknowledgements

This research was conducted in Fall 2014 under the direction of Dr. Leah Childers in fulfillment of Directed Research requirements at Benedictine College in Atchison, KS. Dr. Childers, you continue to inspire us. We would like to thank Dr. Brian Hollenbeck of Emporia State University and Keely Grossnickle for their contributions to our project.

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before September 1, 2016. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2016 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051).

NEW PROBLEMS 769-779

Problem 769. *Proposed by the Northwest Missouri State University Problem Solving Group, Maryville, MO.*

Let $T_k = \frac{k(k+1)}{2}$ be the k^{th} triangular number.

1. Under what condition(s) on $n \in \mathbb{N}$ does 13 divide $2(T_{3^n} - 1)$?
2. Under what condition(s) on $n \in \mathbb{N}$ does 13 divide $2T_{3^n} + 1$?

Problem 770. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let $f : [0, 1] \to \mathbb{R}$ be a continuous concave function. Prove that

$$\frac{3}{4}\int_0^{1/7} f(t)\,dt + \frac{1}{12}\int_0^{2/7} f(t)\,dt \leq \frac{2}{3}\int_0^{3/14} f(t)\,dt.$$

Problem 771. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let $a < b$ be positive real numbers and let $f : [a, b] \to \mathbb{R}$ be a continuous function. Prove that there exists $c \in (a, b)$ such that

$$2f(c) = \frac{1}{\sqrt{c}}\left[\frac{\sqrt{a}+\sqrt{c}}{a-c} + \frac{\sqrt{b}+\sqrt{c}}{b-c}\right]\int_a^c f(t)\,dt.$$

Problem 772. *Proposed by Marcel Chirita, Bucharest, Romania.*

Solve in positive integers the equation $x^2 - 97y! = 2015$.

Problem 773. *Proposed by Marcel Chirita, Bucharest, Romania.*

Let a, b, c be real numbers greater than or equal to 3. Prove that

$$\min\left(\frac{a^2b^2+3b^2}{b^2+27}, \frac{b^2c^2+3c^2}{c^2+27}, \frac{c^2a^2+3a^2}{a^2+27}\right) \le \frac{abc}{9}.$$

Problem 774. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

If both x and y are positive real numbers, then find y as a function of x, provided

$$y' + (y+1)\ln(y+1)[1 - (\ln(y+1))^{-2}((1/4)x^{-1} + x^{1/2})]x^{1/2} = 0.$$

Problem 775. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Determine y explicitly as a function of x provided

$$x(1+\sin x)y' + [(x^2+y^2+4) - (-3+\sin x)y - 2(1+\sin x)] = 0,$$

with $y \neq -2$ and $x \neq k\pi$.

Problem 776. *Proposed by Natanael Karjanto, University College, Suwon, Republic of Korea.*

Show that for $\alpha > 0$ and $n \in \mathbb{N}$, the harmonic number H_n can be represented by the following integral:

$$\begin{aligned} H_n &= \sum_{k=1}^{n} \frac{1}{k} \\ &= \frac{1}{2} \sum_{k=1}^{n} \int_{-\infty}^{\infty} e^{-\alpha|x|} \operatorname{sech}^{k+1} x \, dx \\ &\qquad + \frac{1}{2} \sum_{k=1}^{n} \frac{\alpha - (k-1)}{n} \int_{-\infty}^{\infty} e^{-(\alpha+1)|x|} \operatorname{sech}^{k} x \, dx. \end{aligned}$$

Problem 777. *Proposed by Robert Gardner and William Ty Frazier (graduate student), East Tennessee State University, Johnson City, TN.*

Let $[x]$ represent the floor (or greatest integer) function. Let $n, m \in \mathbb{N}$ with $2 \leq m \leq n-1$ and $k \in \{0, 1, 2, \ldots, m-1\}$. Use the floor function to express the smallest integer N greater than or equal to n which is congruent to k modulo m.

Problem 778. *Proposed by Thomas Chu (graduate student), Western Illinois University, Macomb, IL.*

Let p_1 and p_2 be distinct odd primes both congruent to 1 or $3 \bmod 4$. Prove that

$$\gcd\left(\frac{p_1 + p_2}{2}, \frac{|p_1 - p_2|}{4}\right) = 1.$$

Problem 779. *Proposed by the editor.*

Use all the digits 1, 2, 3, $\ldots$, 9 without repeats to create two primes such that their product is a maximum. Each digit should be used in only one of the two numbers.

SOLUTIONS TO PROBLEMS 749-759

Problem 749. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

The pentagonal numbers are $1, 5, 12, 22, \ldots$ and are given by

$$P_n = \frac{n(3n-1)}{2}, \forall n \geq 1.$$

Prove that every positive even power of 2 is expressible as the difference of two pentagonal numbers.

Solution *by Jeremiah Bartz, Francis Marion University, Florence, SC.*

Let $n \geq 1$. We exhibit two pentagonal numbers whose difference is 2^{2n}. Define $k_n = \frac{2^{2n}-1}{3}$. This is an integer because $2^{2n}-1 = 4^n - 1 \equiv 1 - 1 \equiv 0 (\mathrm{mod}\, 3)$. The consecutive pentagonal numbers

$$P_{k_n} = \frac{k_n(3k_n-1)}{2} = \frac{\left(2^{2n}-1\right)\left(2^{2n}-2\right)}{6} = \frac{\left(2^{2n}-1\right)\left(2^{2n-1}-1\right)}{3}$$

and

$$\begin{aligned} P_{k_n+1} &= \frac{(k_n+1)\left(3\left(k_n+1\right)-1\right)}{2} \\ &= \frac{\left(2^{2n}+2\right)\left(2^{2n}+1\right)}{6} \\ &= \frac{\left(2^{2n-1}+1\right)\left(2^{2n}+1\right)}{3} \end{aligned}$$

have a difference of

$$\begin{aligned} P_{k_n+1} - P_{k_n} &= \frac{\left(2^{2n-1}+1\right)\left(2^{2n}+1\right)}{3} - \frac{\left(2^{2n}-1\right)\left(2^{2n-1}-1\right)}{3} \\ &= \frac{2^{4n-1}+2^{2n}+2^{2n-1}+1}{3} - \frac{2^{4n-1}-2^{2n}-2^{2n-1}+1}{3} \\ &= \frac{2\left(2^{2n}+2^{2n-1}\right)}{3} = \frac{2^{2n}\left(2+1\right)}{3} = 2^{2n}. \end{aligned}$$

Also solved by Henry Ricardo, New York Math Circle, NY; Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania; Ioan Viorel Codreanu, Maramures, Romania; Missouri State University Problem Solving Group Missouri State University, Springfield, MO; and the proposer.

Problem 750. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

It is fairly well known that if (a, b, c) is a primitive Pythagorean triple (PPT), then the product abc is divisible by 60. Find infinitely many PPTs (a, b, c) such that abc is divisible by 120.

Solution *by Ed Wilson, Eastern Kentucky University, Richmond, KY.*

Every primitive Pythagorean triple (a, b, c) can be generated by the equations

$$a = u^2 - v^2, \qquad b = 2uv, \qquad c = u^2 + v^2,$$

given the conditions $u > v$, $\gcd(u, v) = 1$ and exactly one of u or v is even. If u is even and v = 1, then they satisfy the required conditions to generate a PPT. The generated PPT would be $a = u^2 - 1$, $b = 2u$, $c = u^2 + 1$. Choose $u = 60k$ where k is any natural number. Then $b = 2u = 120k$ and the product abc is divisible by 120.

Also solved by Neculai Stanciu, "George Emil Palade" School, Buzau, Romania and Titu Zvonaru, Comanesti, Romania; Suparno Ghoshal (student), Heritage Institute of Technology, Kolkata, India; Missouri State University Problem Solving Group Missouri State University, Springfield, MO; and the proposer.

Problem 751. *Proposed by Iuliana Trasca, Olt, Romania.*

Prove that if $a, b, c > 0$, then

$$a^{11} + b^{11} + c^{11} \geq a^4 b^4 c^4 \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right).$$

Solution *by Ioan Viorel Codreanu, Satulung, Maramures, Romania.*

The inequality is equivalent to

$$\sum \frac{a^8}{b^3 c^3} \geq \sum ab$$

or

$$\sum a^2 \cdot \frac{a^6}{b^3 c^3} \geq \sum ab.$$

Since the numbers (a^2, b^2, c^2) and $\left(\frac{a^6}{b^3c^3}, \frac{b^6}{a^3c^3}, \frac{c^6}{a^3b^3} \right)$ are in the same order, the Chebyshev Inequality gives

$$\sum a^2 \cdot \frac{a^6}{b^3c^3} \geq \frac{1}{3}\left(\sum a^2\right)\left(\sum \frac{a^6}{b^3c^3}\right).$$

By the AM-GM Inequality, we get

$$\sum \frac{a^6}{b^3c^3} \geq 3\sqrt[3]{\prod \frac{a^6}{b^3c^3}} = 3.$$

Using the well-known inequality $\sum a^2 \geq \sum ab$, the conclusion follows.

Also solved by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Neculai Stanciu, "George Emil Palade" School, Buzau, Romania and Titu Zvonaru, Comanesti, Romania; D.M. Batinetu-Giurgiu, "Matie Basarab" National College, Bucharest, Romania; Suparno Ghoshal (student), Heritage Institute of Technology, Kolkata, India; and the proposer.

Problem 752. *Proposed by Iuliana Trasca, Olt, Romania.*

Prove that if $x, y, z > 0$, then

$$\frac{2x+2y+4z}{4x+4y+3z} + \frac{2x+4y+2z}{4x+3y+4z} + \frac{4x+2y+2z}{3x+4y+4z} \geq \frac{24}{11}.$$

Solution *by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

Since all terms are homogeneous, we may assume WLOG that $x + y + z = 1$. Then the left-hand side becomes $\sum \frac{2+2x}{4-x}$. The function $f(x) = \frac{2+2x}{4-x}$ is convex since $f''(x) > 0$ for $0 < x < 1$. Then by Jensen's Inequality

$$\sum \frac{2+2x}{4-x} \geq 3 \cdot \frac{2 + 2 \cdot \frac{x+y+z}{3}}{4 - \frac{x+y+z}{3}} = 3 \cdot \frac{2+\frac{2}{3}}{4-\frac{1}{3}} = \frac{24}{11}.$$

Also solved by Henry Ricardo, New York Math Circle, NY; Titu Zvonaru, Comanesti, Romania; Ioan Viorel Codreanu, Satulung, Maramures, Romania; D.M. Batinetu-Giurgiu, Bucharest and Neculai Stanciu, Buzau, Romania; and the proposer.

Problem 753. *Proposed by D.M. Batinetu-Giurgiu, "Matie Basarab" National College, Bucharest and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.*

If $a, b, c > 0$ and $m \geq 0$, then prove that

$$(a+b+c)^{2m}\left(\frac{1}{(ab)^m}+\frac{1}{(bc)^m}+\frac{1}{(ca)^m}\right) \geq 3^{2m+1}.$$

Solution *by the proposers.*

We have

$$\begin{aligned}
&(a+b+c)^{2m}\left(\frac{1}{(ab)^m}+\frac{1}{(bc)^m}+\frac{1}{(ca)^m}\right)\\
&\quad = \quad (a+b+c)^{2m}(a^m+b^m+c^m)\cdot\frac{1}{(abc)^m}\\
&\quad \geq \quad \frac{\left(3\cdot\sqrt[3]{abc}\right)^{2m}\cdot 3\cdot\sqrt[3]{(abc)^m}}{(abc)^m}\\
&\quad = \quad 3^{2m+1}.
\end{aligned}$$

Also solved by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Henry Ricardo, New York Math Circle, NY; and Ioan Viorel Codreanu, Satulung, Maramures, Romania.

Problem 754. *Proposed by D.M. Batinetu-Giurgiu, "Matie Basarab" National College, Bucharest and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.*

Prove that if $a, b > 0$, then

$$\frac{4}{\sqrt{ab(a+b)}} \leq \frac{1}{a}+\frac{1}{b}+\frac{1}{ab}.$$

Solution *by Titu Zvonaru, Comanesti, Romania.*

The given inequality is equivalent to

$$
\begin{aligned}
&\frac{4}{a+b}-\frac{4}{\sqrt{ab(a+b)}}+\frac{1}{ab}+\frac{a+b}{ab}-\frac{4}{a+b}\geq 0\\
\Leftrightarrow\quad &\left(\frac{2}{\sqrt{a+b}}-\frac{1}{\sqrt{ab}}\right)^2+\frac{(a+b)^2-4ab}{ab\,(a+b)}\geq 0\\
\Leftrightarrow\quad &\left(\frac{2}{\sqrt{a+b}}-\frac{1}{\sqrt{ab}}\right)^2+\frac{(a-b)^2}{ab\,(a+b)}\geq 0.
\end{aligned}
$$

The equality holds when $a=b$ and $2\sqrt{ab}=\sqrt{a+b}$ which occurs when $a=b=1/2$.

Also solved by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Henry Ricardo, New York Math Circle, NY; and Ioan Viorel Codreanu, Satulung, Maramures, Romania; and the proposers.

Problem 755. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Sara has a total number of 26 paper bills in denominations of $\$1$, $\$5$, $\$10$ and $\$20$ in her purse. The number of $\$5$ bills is 4 times the number of $\$10$ bills, and the number of $\$1$ bills is 1 less than twice the number of $\$5$ bills. She remembers that the total amount is less than $\$100$ and more than $\$90$. Furthermore, she remembers that the total amount is an odd number, but it is not 91 or 99. How much money does she have and what is the number of each denomination?

Solution *by Gennifer Farrell (student), Slippery Rock University, Slippery Rock, PA.*

Let A, B, C, D be nonnegative integers which represent the number of 1, 5, 10, 20 dollar bills in Sarah's purse respectively. We are given $B=4C$, $A=2B-1$, $A+B+C+D=26$. By substitution, $8C-1+4C+C+D=26$. So $C=0$, 1, or 2. Assume the total value is T where T can be $\$93$, $\$95$, or $\$97$ with $T=A+5B+10C+20D$. We solve the above system by back substitution for the following three cases.

Case 1: When$C=0$, $A=0$, $B=0$, $D=26$, $T=520$.

But $T\neq 520$, so $C\neq 0$.

Case 2: When $C=1$, $A=7$, $B=4$, $D=14$, $T=317$.

But $T\neq 317$ so $C\neq 1$.

Case 3: When $C=2$, $A=15$, $B=8$, $D=1$, $T=95$.

This is the only way to satisfy the conditions. Sarah must have fifteen $\$1$ bills, eight $\$5$ bills, two $\$10$ bills, and one $\$20$ bill in her purse.

Also solved by Neculai Stanciu, "George Emil Palade" School, Buzau, Romania and Titu Zvonaru, Comanesti, Romania; Henry Ricardo, New York Math Circle, NY; Ed Wilson, Eastern Kentucky University, Richmond, KY; Misty Megee, Northeastern State University, Tahlequah, OK; Madison Estabrook (student), Missouri State University, Springfield, MO; Alexander Joyce (student), Francis Marion University, Florence, SC; and the proposer.

Problem 756. *Proposed by Jose Luis Diaz-Barrero, Technical University of Catalonia (BARCELONA TECH), Barcelona, Spain.*

Find all triples of nonzero real numbers x, y, z such that

$$36\left(\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}\right)=1, \quad 36\left(\frac{x}{yz}+\frac{y}{zx}+\frac{z}{xy}\right)=49,$$
$$36^2\left(\frac{1}{x^2y^2}+\frac{1}{y^2z^2}+\frac{1}{z^2x^2}\right)=49.$$

Solution *by Mary Mulholland (student), Francis Marion University, Florence, SC.*

Clear the denominators to simplify the system to:

$$\begin{aligned} 36(x+y+z) &= xyz \\ 36(x^2+y^2+z^2) &= 49xyz \\ 36^2(x^2+y^2+z^2) &= 49(xyz)^2 \end{aligned}$$

Substituting the second into the third gives $xyz = 36$. This simplifies the equations to:

$$\begin{aligned} xyz &= 36 \\ x+y+z &= 1 \\ x^2+y^2+z^2 &= 49 \end{aligned}$$

Use the first and second equation to eliminate x and obtain $36/z = -y^2 + y - yz$. Isolate x in the second equation to insert it into the third equation. Rearrange to get $-y^2 + y - yz = z^2 - z - 24$. Combining these two new equations, we obtain $z^3 - z^2 - 24z - 36 = 0$. This factors as $(z+3)(z+2)(z-6) = 0$. Solving gives $z = -3, -2, 6$. Each value of z yields two solutions through back substitution. So the (x, y, z) solutions are:

$$(6,-2,-3),(6,-3,-2),(-2,6,-3),$$
$$(-2,-3,6),(-3,6,-2),(-3,-2,6).$$

Also solved by Neculai Stanciu, "George Emil Palade" School, Buzau, Romania and Titu Zvonaru, Comanesti, Romania; Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; and the proposer.

Problem 757. *Proposed by Jose Luis Diaz-Barrero, Technical University of Catalonia (BARCELONA TECH), Barcelona, Spain.*

Let a, b be two complex numbers lying on the circle $|z| = 1$. Prove that

$$\left(\frac{a+b}{1+ab}\right)^2 + \left(\frac{a-b}{1-ab}\right)^2 \geq 1.$$

Solution *by Henry Ricardo, New York Math Circle, NY.*

We will prove the generalization

$$\left(\frac{a+b}{r^2+ab}\right)^2 + \left(\frac{a-b}{r^2-ab}\right)^2 \geq \frac{1}{r^2}$$

for $|a| = |b| = r > 0$. This is equivalent to

$$\left(\frac{r\,(a+b)}{r^2+ab}\right)^2 + \left(\frac{r\,(a-b)}{r^2-ab}\right)^2 \geq 1.$$

Letting $a = re^{ix}$ and $b = re^{iy}$, $r > 0$, we have

$$\begin{aligned}
\frac{r\,(a+b)}{r^2+ab} &= \frac{r^2\left(e^{ix}+e^{iy}\right)}{r^2\left(1+e^{i(x+y)}\right)} \\
&= \frac{e^{i\left(\frac{x+y}{2}\right)}\left(e^{i\left(\frac{x-y}{2}\right)}+e^{-i\left(\frac{x-y}{2}\right)}\right)}{e^{i\left(\frac{x+y}{2}\right)}\left(e^{i\left(\frac{x+y}{2}\right)}+e^{-i\left(\frac{x+y}{2}\right)}\right)} \\
&= \frac{\cos\left(x-y\right)}{\cos\left(x+y\right)}.
\end{aligned}$$

Similarly,

$$\frac{r\,(a-b)}{r^2-ab} = \frac{\sin\left(x-y\right)}{\sin\left(x+y\right)}.$$

Therefore

$$\begin{aligned}\left(\frac{r\left(a+b\right)}{r^2+ab}\right)^2+\left(\frac{r\left(a-b\right)}{r^2-ab}\right)^2 &= \frac{\cos^2\left(x-y\right)}{\cos^2\left(x+y\right)}+\frac{\sin^2\left(x-y\right)}{\sin^2\left(x+y\right)}\\ &\geq \cos^2\left(x-y\right)+\sin^2\left(x-y\right)\\ &= 1.\end{aligned}$$

Also solved by Neculai Stanciu, "George Emil Palade" School, Buzau, Romania and Titu Zvonaru, Comanesti, Romania; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; and the proposer.

Problem 758. *Proposed by the editor.*

Prove that the sequence $11231, 1012301, 100123001, 10001230001...$ (i.e., each number starts and ends with 1 and has $k \geq 0$ zeroes on either side of 123) has an infinite subsequence of all composite numbers.

Solution *by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO.*

It is easy to see that each term in the sequence is of the form $a_k = 10^{2k+4} + 123 * 10^k + 1$. If k is even, then since $10 \equiv -1 (\bmod 11)$,

$$\begin{aligned}10^{2k+4}+123*10^k+1 &\equiv (-1)^{2k+4}+123*(-1)^{k+1}+1\\ &\equiv 1-123+1\equiv -121\equiv 0(\bmod 11).\end{aligned}$$

So a_k is divisible by 11 for any k which is even.

Also solved by Neculai Stanciu, "George Emil Palade" School, Buzau, Romania and Titu Zvonaru, Comanesti, Romania; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Henry Ricardo, New York Math Circle, NY; Ioan Viorel Codreanu, Satulung, Maramures, Romania; and the proposer.

Problem 759. *Proposed by Marcel Chirita, Bucharest, Romania.*

Show how to evaluate the definite integral

$$\int_0^1 \frac{(x-x^2)\arctan x}{(1+x)(1+x^2)}\,dx.$$

Solution *by Suparno Ghoshal (student), Heritage Institute of Technology, Kolkata, India.*

Let $x = \tan\theta$, so $dx = \sec 2\theta \; d\theta$ and the new endpoints are 0 and $\pi/4$. The integral is

$$\begin{aligned}\int_0^1 \frac{(x-x^2)\arctan x}{(1+x)(1+x^2)}\,dx &= \int_0^{\pi/4} \frac{(\tan\theta - \tan^2\theta)\cdot\theta}{(1+\tan\theta)(1+\tan^2\theta)} \cdot \sec 2\theta \; d\theta \\ &= \int_0^{\pi/4} \frac{(\tan\theta - \tan^2\theta)\cdot\theta}{(1+\tan\theta)}\,d\theta \\ &= \int_0^{\pi/4} \theta \tan\theta \tan(\pi/4-\theta)\; d\theta.\end{aligned}$$

Let I be this last integral. Using the property that

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx,$$

we have

$$\begin{aligned} I &= \int_0^{\pi/4} (\pi/4-\theta)\tan(\pi/4-\theta)\tan(\theta)\; d\theta \\ &= \int_0^{\pi/4} (\pi/4)\tan(\pi/4-\theta)\tan(\theta)\; d\theta - I.\end{aligned}$$

Hence,

$$2I = \int_0^{\pi/4} (\pi/4)\tan(\pi/4-\theta)\tan(\theta)\; d\theta.$$

Then

$$\begin{aligned} I &= (\pi/8)\int_0^{\pi/4} \frac{\tan\theta - \tan^2\theta}{1+\tan\theta}\,d\theta \\ &= (\pi/8)\int_0^{\pi/4} \frac{(1+\tan\theta) - \left(1+\tan^2\theta\right)}{1+\tan\theta}\,d\theta \\ &= (\pi/8)\left[\int_0^{\pi/4} d\theta - \int_0^{\pi/4} \frac{\sec^2\theta}{1+\tan\theta}\,d\theta\right] \\ &= (\pi/8)\left[\frac{\pi}{4} - \ln 2\right].\end{aligned}$$

Also solved by D.M. Batinetu-Giurgiu, "Matie Basarab" National College, Bucharest and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania; Angel Plaza, Universidad de Las Palmas de Gran Canaria, *Spain; Henry Ricardo, New York Math Circle, NY; Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; and the proposer.*

[It was pointed out that this problem appeared in the *Romanian Mathematical Gazette*,Volume CIX, No. 9/2004 p. 361 as Problem 25155. Its solution appeared in 2005.]

Citation for Ron Wasserstein

The George R. Mach Distinguished Service Award Recipient

April 11, 2015

Ron Wasserstein began his service to KME as a student in the Kansas Delta chapter at Washburn University, serving as president in 1976-1977. He was awarded first place for a paper he presented at a regional convention in 1978.

Later, he returned to Washburn University as a faculty member and was a consistently active participant in chapter activities throughout his years as a faculty member and administrator, often driving the van that took students to regional and national conventions. During this time he also helped in organizing and hosting KME national conventions in 1989 and 2001 and a regional convention in 1996. He gave the keynote address at the banquet in 2001.

From 2005 to 2009, Ron served as National President Elect for KME and as President from 2009 to 2013. In 2007, he was named the Executive Director of the American Statistical Association. Despite this demanding career change, he continued his commitment to KME. His leadership in KME was marked by vision and innovation as he led the organization to try new ways of doing things that were attractive to today's students.

In recognition of his long-term commitment and service, KME is proud to present Ron Wasserstein with the 2015 George R. Mach Distinguished Service Award.

Report of the 40th Biennial Convention/ Central Florida Undergraduate Mathematics Conference

Kappa Mu Epsilon
April 9-11, 2015
Embry-Riddle Aeronautical University, Florida Delta
Daytona Beach, Florida

This 40th Biennial Convention/Central Florida Undergraduate Mathematics Conference, sponsored in part by the American Mathematical Society and American Statistical Association, and partially funded by a grant from the National Science Foundation, was held during April 9-11, 2015 at Embry-Riddle Aeronautical University in Daytona Beach, Florida, Host Chapter Florida Delta.

Thursday, April 9, 2015

On Thursday evening, April 9th from 7-9 p.m. an Ice Cream Social and SET© [card game] Tournament was held in the Henderson Center, Duva Hall, Ballrooms 1-2. Participants were able to pick up registration packets at this event. From 8-9 p.m., the National Council held a business meeting in the Henderson Center, Duva Hall, Ballroom 3.

Friday, April 10, 2015

Friday April 10th's activities began at 8 a.m. to 9 a.m. with coffee and continued registration in the Atrium of Henderson Center. From 8:30-8:55 a.m., members of the Awards Committee held a meeting in the Henderson Center, Duva Hall, Ballroom 3. At 9 a.m. in the Henderson Center, Duva Hall, Ballrooms 1-2 the first general session began, with KME President Rhonda McKee presiding.

KME President Rhonda McKee and Greg Spradlin, Embry-Riddle Aeronautical University Mathematics Department and host of the Central Florida Undergraduate Mathematics Conference, welcomed participants. Mark S. Hamner, KME Secretary, then called the roll. In attendance were 43 students and 18 faculty members, a total of 61 registrants, representing 16 chapters from eight states (Alabama, Florida, Kansas, Michigan, Missouri, Pennsylvania, Texas, and West Virginia). Participants in the Central Florida Undergraduate Mathematics Conference represented four Florida universities: Florida Gulf Coast University, State College of Florida, University of Central Florida, and Bethune-Cookman University.

Chapters represented were Alabama Theta, Jacksonville State University; Florida Delta, Embry-Riddle Aeronautical University; Kansas Alpha, Pittsburg State University; Kansas Beta, Emporia State University; Kansas Gamma, Benedictine College; Kansas Delta, Washburn University; Michigan Beta, Central Michigan University; Missouri Alpha, Missouri State University; Missouri Beta, University of Central Missouri; Missouri Theta, Evangel University; Missouri Lambda, Missouri Western State University; Pennsylvania Mu, Saint Francis University; Pennsylvania Tau, DeSales University; Texas Alpha, Texas Tech University; Texas Gamma, Texas Woman's University; and West Virginia Alpha, Bethany College.

Sixteen talks, a 7-member career panel discussion, three workshops (each presented twice), an Integral Bee, and a tour and demonstration of Embry-Riddle's Nonlinear Wave Lab were on the program.

There were no new chapters in attendance. Brian Hollenbeck KME President-Elect of Kansas Beta conducted the filing of delegates.

There was no old business.

Introduction of new business included the following items:

- Introduction of candidate for National Secretary: Mark S. Hamner, Texas Gamma, was nominated for re-election as KME Secretary.
- Introduction of candidates for National Treasurer: Dianne Twigger, Missouri Theta, and David Dempsey, Alabama Theta, were nominated for KME Treasurer.
- David Gardner, Texas Gamma, KME Webmaster, unveiled the new KME Website.

Rhonda McKee, KME President, and Brian Hollenbeck, KME President-Elect, presided over the student paper presentations. Student paper presentations were held in the Henderson Center, Duva Hall, Ballroom 2 (Rhonda McKee) and Ballroom 3 (Brian Hollenbeck) in the College of Arts and Sciences Building, Room 203.

The following Student papers were presented during the morning Session #1, 9:30 a.m.-9:55 a.m.:

- (Ballroom 2-Henderson Center) *Dynamics and Bifurcations of Cholera Epidemiology Model*, by Meagan Leppien and Mena Whalen, Missouri Alpha, Missouri State University
- (Ballroom 3-Henderson Center) *The Brachistochrone Challenge: Traveling from Here to There in the Shortest Time*, by Grace Chester, Missouri Lambda, Missouri Western State

The following Student papers were presented during the morning Session #1, 9:55 a.m.-10:20 a.m.:

- (Ballroom 2-Henderson Center) *Spatial Filtering on Digital Images*, by T.J. Huettenmueller, Kansas Beta, Emporia State University
- (Ballroom 3-Henderson Center) *Factoring Quartic Polynomials*, by Brionna Benjamin, Bethune-Cookman University

The following Student papers were presented during the morning Session #1, 10:20 a.m.-10:45 a.m.:

- (Ballroom 2-Henderson Center) *Mathematical Modeling on an Open Limestone Channel*, by David Wolfe, Pennsylvania Mu, Saint Francis University
- (Ballroom 3-Henderson Center) *Generalizations of the Comparison Test*, by Alexis M. Taylor, Bethune-Cookman University

Between 11:00 a.m. and 11:50 a.m. participants attended one of three workshops (prior signup required):

- (COAS Building Room 203) *Iterative Constructions Using GeoGebra*, by Steve Klassen, Missouri Lambda, Missouri Western State University
- (COAS Building Room 301.39) *LaTeX Workshop* presented by C Altay Ozgener, Robert Shollar, and Leon Hostetler, State College of Florida
- (Ballroom 2-Henderson Center) *The Mathematics Behind Cryptography*, by Joe Yanik, Kansas Beta, Emporia State University

Between 12 noon-12:15 p.m. a Group Photograph was taken in the Henderson Center Atrium.

Lunch was held between 12:15 p.m.-1:05 p.m. on the Flight Deck in the Student Center. Concurrently the KME Resolutions Committee met in the Mathematics Department Conference Room, 3d floor, in the College of Arts and Sciences [COAS] Building. In addition, the KME Auditing Committee met concurrently in the College of Business [COB] Building, Room 268.

From 1:15 p.m-2:05 p.m. participants attended one of three workshops (prior signup required):

- (Ballroom 3-Henderson Center) *Iterative Constructions Using GeoGebra*, by Steve Klassen, Missouri Lambda, Missouri Western State University
- (COAS Building Room 301.39) *LaTeX Workshop* presented by C Altay Ozgener, Robert Shollar, Leon Hostetler, State College of Florida

- (Ballroom 2-Henderson Center) *The Mathematics Behind Cryptography,* by Joe Yanik, Kansas Beta, Emporia State University.

The following Student papers were presented during the afternoon Session #2, 2:15 p.m.-2:40 p.m.:

- (Ballroom 2, Henderson Center) *The Black-Scholes Formula in Risk Management,* by Casey Cornelius, Missouri Alpha, Missouri State University
- (Ballroom 3, Henderson Center) *Some Different Ways to Sum a Series*, by Leon Hostetler and Robert Shollar, State College of Florida

The following papers were presented during the afternoon Session #2, 2:40 p.m.-3:05 p.m.:

- (Ballroom 2, Henderson Center) *Japanese as a Set Theory Construct: Verb Conjugation,* by Naimul Chowdhury, State College of Florida
- (Ballroom 3, Henderson Center) *Triphos: An Alternative Coordinate System,* by Paula Egging, Kansas Gamma, Benedictine College

From 3:15 p.m.-4:00 p.m., section meetings were held, The Faculty Section meeting was held in the Henderson Center Ballroom 3. The Student Section meeting was held in the College of Arts and Sciences [COAS] Building Room 126.

From 4:05 p.m.-5:15 p.m. an Integral Bee was held in the College of Arts and Sciences [COAS] Building Room 303. Prior signup was required. Upon conclusion of the event, winners of the Integral Bee were announced:

1st place: Melinda Geisel, State College of Florida
2rd place: Annabel Offer, Texas Tech University, Texas Alpha
3rd place: Yountong Chen, State College of Florida
4th place: James Mixco, University of Central Missouri, Missouri Beta
5th place: Jean Merone, Florida Gulf Coast University

From 4:15 p.m.-4:45 p.m., Dr. Andrei Ludu, Professor of Mathematics, Embry-Riddle Aeronautical University led a tour and demonstration of Embry-Riddle's Nonlinear Wave Lab in the College of Arts and Sciences [COAS] Building. Prior signup was required.

From 7:00 p.m.-9:30 p.m. the convention banquet was held in the in the Henderson Center's Ballrooms 1 and 2. Brian Hollenbeck, KME President-Elect, served as emcee. Following dinner, Liz McMahon, Professor of Mathematics at Lafayette College, Easton, Pennsylvania, gave the keynote address: Mathematics in the Game of SET©.

Saturday, April 11, 2015

Saturday April 11th's activities began between 7:30 a.m. and 8:15 a.m. with Registration and Breakfast in the Henderson Center Atrium.

Session #3 of the Student Presentations commenced at 8:30 a.m. in the Henderson Center's Ballrooms 2 and 3. The following Student papers were presented during Session #3, 8:30 a.m.-8:55 a.m.:

- (Ballroom 2, Henderson Center) *NFL Combine Correlation Analysis*, by Brian Mosier, Kansas Beta, Emporia State University
- (Ballroom 3, Henderson Center) *Factorization Theory in* $\mathbb{Z}/n\mathbb{Z}$, by James Mixon, Missouri Beta, University of Central Missouri

The following Student papers were presented during Session #3, 8:55 a.m.-9:20 a.m.:

- (Ballroom 2, Henderson Center) *Classification of Instances in a National Climactic Data Center Precipitation Dataset*, by David Armas, Embry-Riddle Aeronautical University
- (Ballroom 3, Henderson Center) *Making Hard Problems Easy: On a New Integral Transform, Its Properties, and Applications*, by John Vastola, University of Central Florida

The following Student papers were presented during Session #3, 9:20 a.m. -9:45 a.m.:

- (Ballroom 2, Henderson Center) *Modeling Hantavirus Cytokine Activity with Stochastic Differential Equation*, by Annabel Offer, Texas Alpha, Texas Tech University
- (Ballroom 3, Henderson Center) *Permanent of a Matrix*, by Nitish Aggarwal, Embry-Riddle Aeronautical University

Between 9:45 a.m. and 10:40 a.m. the KME Awards Committee met in the Mathematics Department Conference Room, 3rd floor, in the College of Arts and Sciences Building.

Between 10:10 a.m. and 10:40 a.m. a Career Panel Discussion was held in the Henderson Center, Ballrooms 1 and 2. Panelists included:

- Sergey Drakunov, Associate Dean for Research and Graduate Studies, Embry-Riddle Aeronautical University
- Shan Guruyadoo, Mathematics Instructor/Graduate Student , Bethune-Cookman University/University of Central Florida (UCF)
- Brad Hansen, Teacher, Island Coast High School, Cape Coral, Florida

- Andrew Hawkins, Product Quest Manufacturing
- Jeff Sanders, Professor, Physical Sciences Department , Embry-Riddle Aeronautical University
- Steven Thompson, Graduate Student, University of Central Florida Institute for Simulation and Training
- Rebecca Wood, Senior Financial Analyst, Spirit Airlines

Following at 10:40 a.m., also in the Henderson Center, Ballrooms 1 and 2, the Second General Session began with President Rhonda McKee presiding. Convention Evaluation Forms were distributed and collected.

Elections were held for the positions of KME National Secretary and KME National Treasurer.

- Mark S. Hamner, Texas Gamma, Texas Woman's University, was reelected as KME National Secretary by unanimous acclamation.
- David Dempsey, Alabama Theta, Jacksonville State University, Jacksonville, AL, was announced as being elected the new KME Treasurer and installed in this position.

The recipient of the Mach Award was announced. This year's recipient was Ron Wasserstein, Kansas Delta.

For the Continuation of New Business, the following national officers made reports:

- Dan Wisniewski, Editor, *The Pentagon*
- Peter Skoner, Historian
- Cynthia Huffman, outgoing Treasurer
- Mark S. Hamner, Secretary
- Brian Hollenbeck, President-Elect
- Rhonda McKee, President

Following the national officer reports were reports from the Section Meetings and the Resolutions Committee. All reports are given below.

The report of the Awards Committee and presentation of awards were made. The awarded papers were:

- *Mathematical Modeling on an Open Limestone Channel*, by David Wolfe, Pennsylvania Mu, Saint Francis University
- *Some Different Ways to Sum a Series*, by Leon Hostetler and Robert

Shollar, State College of Florida

- *Triphos: An Alternative Coordinate System*, by Paula Egging, Kansas Gamma, Benedictine College
- *NFL Combine Correlation Analysis*, by Brian Mosier, Kansas Beta, Emporia State University

These students were awarded a $100 check for their respective winning paper. The “People’s Choice Award,” which is selected by submitted votes of the attending KME members, was presented to James Mixon from Missouri Beta, University of Central Missouri. The “People’s Choice Award” recipient received $50.

The convention concluded with the National Treasurer Cynthia Huffman presenting checks for travel allowances to each chapter present.

Mark Hamner
National Secretary

Report of the National President

This has been a great year for KME, and a busy one! Here are some highlights.

- The national council has continued to meet regularly (monthly, with a few exceptions) via teleconference.
- The new chapter petition form was updated to an electronic form and to more accurately provide needed information.
- KME sent a letter to Pi Mu Epsilon, congratulating them on their centennial, which they celebrated at Math Fest 2014. Cynthia Huffman, KME Treasurer delivered the letter at Math Fest.
- The National Council met in Warrensburg, Missouri in June to sort through file cabinets and boxes of documents and other KME items. Some items were disposed of, but many were saved with the intent of submitting them to the Math Archives in Texas at some point. The hope is to scan all past initiation records in order to have a digital record.
- Possible regional directors were discussed, and it was decided that we would hold off appointing regional directors until the issue of annual vs biennial conventions has been decided.
- David Gardner, web editor showed the National Council some possible web page layouts. After receiving input, David designed the new KME web page, which will be unveiled at this convention.
- Historian, Pete Skoner and I created a survey that was sent to all Corresponding Secretaries via Google Forms. The purpose of the survey was to solicit input from the chapters regarding preferences for annual or biennial conventions. Forty-three chapters responded to the survey. A small majority (25) favor biennial conventions.
- I appointed a nominating committee in October, so that they could complete their work in time for the convention.
- In February, the KME Regional Directors and Pentagon staff members were invited to join the National Council on a conference call. Several good suggestions came from this meeting, including putting the regional directors on the email list with corresponding secretaries and publishing the Problem Corner of the Pentagon separately on the web page.
- I sent letters to six chapters who have not submitted initiation reports in the last four to six years. Four chapters responded with interest or

intent to hold an initiation this year.

- Installations of new chapters (installing officer in parentheses):
  - Iowa Delta, Central College, April 30, 2014 (Cynthia Huffman)
  - California Eta, Fresno Pacific University, March 24,2015 (Rhonda McKee)
  - Ohio Theta, Capital University, Petition approved. Installation scheduled for April 24, 2015. (Pete Skoner)
  - Georgia Zeta, Georgia Gwinnett College, Petition approved. Installation scheduled for April 28, 2015. (David Dempsey)
- And, of course, the National Council has spent quite a lot of time, organizing this conference. Brian Hollenbeck, President Elect, and Greg Spradlin sponsor of the host chapter have done a lot of work and done a great job of organizing and planning this convention. One good challenge has been organizing the KME convention along with the Regional Undergraduate Mathematics Conference. Greg and Brian have ironed out all the details and we are all benefitting from their hard work.

Working with the National Council, Regional Directors and Pentagon staff is always a pleasant job. My sincerest thanks go to all of them. I'm also always grateful to the many corresponding secretaries and faculty sponsors who keep this organization running.

Rhonda McKee
National President

Report of the National President-Elect

This is my second year as President-Elect. It continues to be a pleasure working with the rest of the National Council. In particular, I would like to wish Cynthia Huffman the best as her term as treasurer is coming to an end. I look forward to the next treasurer continuing her tradition of excellence, not to mention her attention to detail and collegiality.

Following the tradition of recent President-Elects, let me mention some statistics about recent conferences.

2015 National Convention

Kappa Mu Epsilon's 40th Biennial Convention, which is KME's 41st national convention overall, is being held this weekend, April 9-11 in Daytona Beach, Florida. Our host chapter is Florida Delta at Embry-Riddle Aeronautical University. There are 16 chapters in attendance from eight states (Alabama, Florida, Kansas, Michigan, Missouri, Pennsylvania, Texas, and West Virginia). 11 presentations will be given by students from these chapters. In a departure from tradition, we are also pleased to have four universities joining us as part of the Central Florida Undergraduate Mathematics Conference. Their participation will allow us to feature a total of 16 talks in two parallel sessions which is also a first for KME conventions of recent memory. There will also be a panel discussion, integral bee, and three workshops given over the course of the next two days. 121 people are registered for the convention, including 71 members of KME.

By way of comparison, in 2014 the host chapter was Alabama Theta at Jacksonville State University. There were 17 chapters in attendance from nine states (Alabama, Kansas, Michigan, Missouri, Pennsylvania, Rhode Island, South Carolina, Texas, and West Virginia). Eight talks and a panel discussion were given as well as three workshops, which was a new feature of the KME convention. This was also the first time that KME held a national convention during an even-numbered year. Eighty-nine people attended the convention.

In 2013, the host chapter was Kansas Delta at Washburn University and there were 17 chapters in attendance from eight states (Alabama, Kansas, Louisiana, Michigan, Missouri, New York, Pennsylvania, and Texas). Nineteen talks and a panel discussion were given over the course of two days. Ninety-nine people attended.

In 2011, 16 chapters from nine states (Indiana, Kansas, Kentucky, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas) participated in the convention in St. Louis, Missouri. Eighteen papers were presented. Eighty-seven people attended.

In 2009, 16 chapters from nine states (Georgia, Kansas, Maryland, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas) participated in Philadelphia, PA. Sixteen students presented papers. Seventy-five people attended.

In 2007, 14 chapters from five states (Kansas, Missouri, New York, Oklahoma, and Tennessee) participated in Springfield, Missouri.

In 2005 (Schreiner U., Kerrville, TX), there were 17 chapters from nine states (California, Kansas, Missouri, Michigan, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). There were 15 student presentations.

In 2003 (ORU, Tulsa, OK), there were 19 chapters from 9 states (Iowa, Kansas, Michigan, Missouri, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). Thirteen student papers were presented.

In 2001 (Washburn U., Topeka, KS), there were 20 chapters from 10 states (Colorado, Iowa, Kansas, Kentucky, Missouri, New York, Oklahoma, Ohio, Pennsylvania, and Tennessee).

The following chapters have participated in at least one of the last nine conventions:

Alabama Alpha (2014)
Alabama Beta (2014)
Alabama Epsilon (2014)
Alabama Zeta (2014)
Alabama Theta (2013, 2014, 2015)
California Epsilon (2005)
Colorado Delta (2001)
Georgia Alpha (2009)
Florida Delta (2015)
Indiana Delta (2011)
Iowa Alpha (2001, 2003)
Iowa Gamma (2001)
Kansas Alpha (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015)
Kansas Beta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015)
Kansas Gamma (2001, 2003, 2007, 2015)
Kansas Delta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015)
Kansas Epsilon (2001)
Kentucky Alpha (2001, 2011)
Louisiana Delta (2013)
Maryland Beta (2009)
Maryland Epsilon (2009)
Michigan Beta (2003, 2005, 2009, 2011, 2013, 2014, 2015)
Missouri Alpha (2001, 2003, 2007, 2013, 2015)
Missouri Beta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015)

Missouri Theta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015)
Missouri Iota (2001, 2003, 2005, 2007, 2009, 2011, 2013)
Missouri Kappa (2001, 2003, 2005, 2007)
Missouri Lambda (2013, 2015)
Missouri Mu (2011)
New York Eta (2001, 2003, 2005, 2007, 2013)
New York Lambda (2003, 2005)
New York Omicron (2005, 2009, 2011, 2013)
New York Rho (2011, 2013)
Ohio Alpha (2001)
Oklahoma Alpha (2003, 2007)
Oklahoma Gamma (2001, 2003, 2007)
Oklahoma Delta (2001, 2003, 2005, 2007, 2009, 2011)
Pennsylvania Theta (2001)
Pennsylvania Lambda (2003, 2009)
Pennsylvania Mu (2005, 2009, 2011, 2013, 2014, 2015)
Pennsylvania Tau (2014, 2015)
Rhode Island Beta (2014)
South Carolina Delta (2014)
Tennessee Gamma (2001, 2003, 2005, 2007)
Texas Alpha (2013, 2015)
Texas Gamma (2003, 2005, 2009, 2011, 2013, 2014, 2015)
Texas Mu (2003, 2005, 2009, 2011)
West Virginia Alpha (2014, 2015)

Thus, in the 2000's, 48 different chapters have participated. Five have participated in all nine conventions. One chapter is participating for the first time this year.

Again this year the AMS and the ASA both contributed $500 each, which will be used to help defray the cost of student travel to the convention. Due to the efforts of Greg Spradlin, the NSF grant DMS-0241090 was secured which helped pay for student travel and food expenses. We are certainly grateful for their support.

Finally, I would like to extend special thanks to Greg Spradlin and the Florida Delta chapter for hosting this convention. It was an ambitious task for a KME chapter to be willing to host a national convention in their first year of existence. Their efforts will make this convention a successful and unique experience.

Brian Hollenbeck
President-Elect

Report of the National Secretary

Kappa Mu Epsilon, National Mathematics Honor Society initiated 2,573 new members in 123 chapters during the 40th Biennium that ended March 15, 2015. This represents an increase of 120 new members compared to the last biennium. Twenty-Seven active chapters did not report any initiates during the 40th biennium. The total membership of KME is 83,447.

As National Secretary, I receive all initiation reports from the chapters, make a record of those reports, up-date mailing list information for corresponding secretaries and forward copies of the reports to other officers. At the beginning of each new biennium, I prepare a new KME brochure. During an academic year, I send out supplies to each chapter. The supplies include information brochures and membership cards. When a college or university petitions for a new chapter of KME, I send out a summary of the petition, prepared by the president, to each chapter and receive the chapter ballots.

Mark Hamner
National Secretary

Report of the National Historian

It continues to be a pleasure to serve as the National Historian of the Kappa Mu Epsilon National Mathematics Honor Society for a third biennium. Many thanks for the continual communication and friendship with the national and regional officers, and with the corresponding secretaries and faculty sponsors of individual chapters. The passion and time all of you give contributes to this great organization that will celebrate the 84th anniversary of our founding since April 18, 1931 next week on April 18, 2015, as we close our 42nd biennium, and as we gather for our 41st national convention, our 13th prime convention. Just to explain, this would be our 42nd national convention but two were not held during WWII, and then for the first time in 2014 we held a convention in an even-numbered year.

The primary function of the national historian continues to be soliciting, collecting, maintaining and compiling records of chapter activities, installation of new chapters, and other society activities that have historical significance. Distribution e-mail messages solicit Chapter News reports after every fall and spring semester. This regular communication provides the opportunity to learn when a corresponding secretary changes; and maintains communication between the national society and the local chapters, hopefully helping to maintain the local leadership for each chapter. The work of the National Historian is impossible without the aid of the corresponding secretaries for each chapter. Thank you for all that you do in serving the students at your institution, your local Kappa Mu Epsilon chapter, and the national organization.

During the past biennium from March 2013 to March 2015, 73 chapters responded at least once to the chapter news request, an increase from 69 in the previous biennium. Special mention goes to the following 18 chapters for their cooperation in responding to all four inquiries: AL Theta, HI Alpha, IA Alpha, MA Beta, MD Delta, MI Delta, MO Alpha, MO Theta, NE Delta, NJ Delta, NY Omicron, OH Alpha, OK Alpha, PA Beta, PA Kappa, PA Mu, PA Tau, and RI Beta.

At the 39th national convention at Washburn University, the 82nd Anniversary KME History and Information Booklet was distributed. Since then, several sections of that document have been added to the KME home page, and updates are made periodically. Another printed copy is expected in 2017.

A special thank you goes to the editor of *The Pentagon*, Bro. Daniel Wisniewski, O.S.F.S., of the Pennsylvania Tau chapter. The edited Chapter News section is sent to the editor each semester, and Brother Daniel has been great to work with. A special thank you also to our Saint Francis University executive assistant Melita O'Donnell and work study student Megan Gatto who scanned all the documents from 9 boxes into electronic files stored in a separate folder for every chapter. Since 2009, all new history files have been stored electronically. The hope eventually is to have a searchable electronic database of chapter records, and to move the paper files into a national mathematics archive.

Peter R. Skoner
National Historian

Report of the National Treasurer

40th Biennium (March 16, 2013 – March 15, 2015)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows biennium assets of $90,048.36. The Cash Flow Report shows that we have an asset gain of $7,047.15 this biennium. A National Council goal to maintain an asset base of at least $40,000 has been met.

BIENNIUM ASSET REPORT

Total Assets (March 16, 2013)		$88,900.25
Current Assets		
Kansas Teachers Community Credit Union		
Checking	42,439.43	
Share Account	43,608.93	
CD17014	10,000.00	
Total Current Assets		$96,048.36

BIENNIUM CASH FLOW REPORT

Receipts		
Initiation fees received	51,540.00	
Installation fees received	245.00	
Interest income	682.89	
Gifts & misc. income	2,540.00	
Total Biennium Receipts		$55,007.89
Expenditures		
Association of College Honor Soc	956.00	
Administrative expenses	3,042.81	
National Convention expenses	28,906.97	
Regional Convention expenses	0.00	
Council Meetings travel	2,265.08	
Certificates, jewelry & shipping	11,995.06	
Bank charges	0.00	
Pentagon expenses	762.25	
Miscellaneous	32.57	
Total Biennium Expenses		$47,960.74
Biennium Cash Flow		**$7,047.15**

The cash flow last biennium (11-13) was **$18,533.69**, which was the maximum cash flow for a biennium, partially due to *The Pentagon* moving to an electronic format. The National Council passed on part of this increase to the students and chapters by adding extra to travel allocations for the national conventions in 2013 and 2014. National convention expenses were higher this biennium since there were two conventions rather than one. Each chapter is also receiving extra in their travel allocation for the 2015 convention which will show up on the next biennium report.

We have easily continued to meet our goal of maintaining assets of at least $40,000. The financial condition of Kappa Mu Epsilon is sound.

This is my last biennium as national treasurer. I want to offer sincere thanks to the dedicated, talented, hard-working professionals of the National Council (Rhonda McKee, Brian Hollenbeck, Mark Hamner, Pete Skoner, and David Gardner) and past National Presidents (Ron Wasserstein and Don Tosh). In addition, a big thanks to the work of the corresponding secretaries who maintain such a vital role in Kappa Mu Epsilon. It has been an amazing opportunity to serve the past 8 years with such outstanding individuals in encouraging and recognizing students for accomplishments in mathematics.

Cynthia Huffman
National Treasurer

Report of *The Pentagon* Editor

Introduced in 1941, *The Pentagon*, is the official publication of Kappa Mu Epsilon. Publication of student papers continues to be the focus of *The Pentagon*. Following tradition, papers given "top" status and other recognition by the Awards Committee at the KME National Convention are guaranteed an opportunity to be published. *The Pentagon* is now completely all-electronic and available for free online via the KME website: www.kappamuepsilon.org.

I have been the Editor of *The Pentagon* since August 2013. During the past year, I have been corresponding with authors of potential articles for submission, and facilitating referee feedback and author corrections for upcoming issues. Since last year's convention, two issues (Fall 2013 and Spring 2014) of *The Pentagon* have been published and made available on the website. While overdue, the Fall 2014 issue is near completion, awaiting final article revisions from authors. In the meantime, the new problems from its Problem Corner have been made available on the KME website; a valuable and much sought-after resource, the availability of these problems in advance of publishing the full issue has been well-received.

I continue to communicate with potential authors who seek publication information from the website. This summer, I plan to review and update the current list of referees on file, seeking to widen the pool of referees qualified to review articles in over twenty different specialty areas of mathematics.

Finally, I am grateful to the Associate Editors: Pat Costello, who organizes the Problem Corner for each issue, and Pete Skoner, who collects and prepares the KME News Items, as well as Don Tosh, who serves as Business Manager of *The Pentagon*. I especially appreciate their assistance, cooperation and patience in producing a fine journal.

Daniel P. Wisniewski, O.S.F.S.
Editor, *The Pentagon*

Report of the Audit Committee

The Audit Committee consisted of David Dempsey, faculty member from the Alabama Theta chapter, Pedro Muíño, faculty member from the Pennsylvania Mu chapter, Jacob Orell, student member from the Missouri Beta chapter, and Dianne Twigger, faculty member from the Missouri Theta chapter.

Audit Process

1. Prior to the national convention Treasurer Cynthia Huffman mailed biennium financial summary data to the committee chair to facilitate verification of asset account totals prior to the convention. At the convention, Treasurer Huffman provided the committee with a heavy box with relevant biennium summary and detailed documentation for receipt and payment transactions, along with monthly bank and savings account reconciliation documentation.
2. Before the national convention, the Audit Committee Chair contacted the Kansas Teachers Community Credit Union (Pittsburg, KS) by telephone. The account balances for the Kappa Mu Epsilon checking and savings accounts and 30-month CD were verified to correspond exactly to associated totals found on Treasurer Huffman's biennium reports. This verification was conducted on March 26, 2015 for the balances as submitted by Treasurer Huffman on March 19, 2015.
3. At the national convention, Treasurer Huffman provided the Audit Committee Chair with bank statements, expense reports, receipts, income information, and her own reports for the full biennium. The chair shared these documents with the rest of the committee.
4. At the national convention committee members interviewed President Rhonda McKee and Secretary Mark S. Hamner to determine their impressions of the accuracy and completeness of the recording of the financial transactions throughout the biennium. The committee also perused the financial documentation, income sheets and receipts provided by Treasurer Huffman to the committee and interviewed her.

Findings

1. The bank information provided by Treasurer Huffman as of March 19, 2015 was verified by the committee chair on March 26 via a phone conversation with Mr. Jamie Canada, of the Kansas Teachers Community Credit Union.

2. The committee spot checked the Secretary's report and corresponding computer generated Treasurer's report and found no inconsistencies throughout. We talked to the Secretary who reported perfect compliance by Treasurer Huffman on this subject.

3. The committee spot checked the expense payment reports and receipts provided and found no inconsistencies. The President expressed total satisfaction with the integrity of the process.

4. The committee inspected monthly financial institution statements, quarterly interest statements and CD interest reports. We compared those to the Treasurer quarterly reports. We found complete consistency between bank statements throughout the biennium. A couple of minor discrepancies on dates of transactions were found in Quicken for the share account. We emphasize that the bank data was self-consistent throughout the biennium and perhaps the next treasurer should be aware that Quicken may revert from a manually entered date to the current date when switching accounts and to watch for this when preparing reconciliation reports since it can cause a small discrepancy between a quarterly report and an end of quarter bank statement.

Recommendations

1. Information forwarded by the Treasurer to the committee chair prior to the national convention provides the opportunity for verification of assets in a careful, yet relaxed manner and should be continued for future audit committees.

2. The internal checks built into the regular financial processing between the Treasurer and the President and Secretary provide an important safeguard to the integrity of the office of the Treasurer and help avoid the necessity of an expensive external audit. These ongoing internal audit processes should be continued and updated by the National Council as needed.

3. In view of item 4 of our findings, we recommend that the next Treasurer is mindful to double-check the dates in Quicken, as Quicken sometimes reverts from a date that has been manually entered to the current date.

4. The organization files an electronic tax notecard annually even though no taxes are required and Treasurer Huffman has provided the committee with copies of the IRS filing receipts. The committee recommends that this practice continue.

Commendations

1. The committee commends Professor Cynthia Huffman for her excellent maintenance, management and presentation of the financial records and for her generous time commitment through eight years of service as Treasurer for Kappa Mu Epsilon.

2. We further commend her for her valuable input to this Audit Committee and her helpful detailed written guidelines for the Audit Committee.

3. The committee commends the national President, Secretary and Treasurer for the manner in which they communicate and cooperate to maintain the internal checks which preserve the integrity of the office of Treasurer.

4. The committee commends the work of the previous audit committees and is thankful for the sample reports provided by the Treasurer.

Pedro Muíño
Chairperson, Audit Committee

Report of the Resolutions Committee

The Resolutions Committee consisted of Stephanie Albert, student member from Kansas Beta chapter, Paula Egging, student member from Kansas Gamma chapter, Casey Cornelius, student member from Missouri Alpha chapter, Dale Bachman, faculty member from Missouri Beta chapter, Lydia Mignogna, student member from Pennsylvania Mu chapter, Annabel Offer, student member from Texas Alpha chapter, and Adam Fletcher, faculty member from West Virginia Alpha chapter.

This Committee hereby proposes the following resolutions. "Whereas the success of any undertaking relies heavily upon the dedication and ability of its leaders, be it resolved:

1. That this Fortieth Biennial National Convention express its gratitude
   a. to Rhonda McKee (national president), Brian Hollenbeck (president-elect), Mark S. Hamner (national secretary), and Pete Skoner (national historian);
   b. to Cynthia Huffman for eight years of hard work and dedication to the office of national treasurer;
   c. to David Gardner for constructing the new streamlined website, increasing its functionality,and serving as national KME webmaster;
   d. to Dan Wisniewski for his service as editor of *The Pentagon*; and
   e. to Pedro Muíño, David Dempsey, and Kevin Charlwood for their service as regional directors.
2. That this Convention acknowledge the participation of the students and faculty who served on the Auditing, Awards, Nominating, and Resolutions committees, which is so essential for the success of the meeting.

"Whereas this organization has benefited for many years from his hours of dedication and work, be it further resolved that this Convention thank Dr. Don Tosh of Missouri Theta at Evangel University in Springfield, Missouri.

- Dr. Tosh has served Kappa Mu Epsilon as:
  - corresponding secretary for thirty years,
  - national historian for four years,
  - president-elect for four years,
  - national president for four years, and
  - most recently, *The Pentagon* Business Manager for nine years.

- The National Council has accepted Dr. Tosh's recommendation to eliminate the business manager position because the online format no longer requires the work that was once needed. Therefore, Dr. Tosh is resigning his position, and this Convention hereby thanks Dr. Don Tosh for his excellent service as *The Pentagon* Business Manager.

"Whereas the primary purpose of Kappa Mu Epsilon is to encourage participation in mathematics and the development of a deeper understanding of its beauty, be it further resolved:

1. That students Meagan Leppien, Mena Whalen, Grace Chester, T.J. Huettenmueller, Brionna Benjamin, David Wolfe, Alexis M. Taylor, Casey Cornelius, Leon Hostetler, Robert Shollar, Naimul Chowdhury, Paula Egging, Brian Mosier, James Mixco, David Armas, John Vastola, Annabel Offer, and Nitish Aggarwal, who prepared, submitted, and then presented their papers, be given special commendation by this Fortieth Biennial Convention for their enthusiasm and dedication, and

2. That this Convention express its thanks to:
   a. career panelists Dr. Sergey Drakunov, Shan Guruvadoo, Brad Hansen, Andrew Hawkins, Jeff Sanders, Steven Thompson, and Rebecca Wood;

   b. Steve Klassen for "Iterative Constructions Using GeoGebra;"

   c. C. Altay Ozgener, Robert Shollar, and Leon Hostetler for "LaTeX Workshop;"

   d. Joe Yanik for "The Mathematics Behind Cryptography;" and

   e. Liz McMahon for her keynote address "Mathematics in the Game of SET©" at the Friday night banquet.

"Finally, whereas Embry-Riddle Aeronautical University and the surrounding community of Daytona Beach, Florida have provided this Convention with gracious hospitality, be it resolved:

1. That this Fortieth Biennial Convention express its heartfelt appreciation to the KME Florida Delta chapter for the thorough arrangements they have planned and carried out so successfully, and

2. to the American Mathematical Society, the Mathematical Association of America, the American Statistical Association, and the National Science Foundation through whose grant support the operations of this Convention were augmented;

3. to the participants in the Central Florida Undergraduate Mathematics Conference, for their attendance and contributions to this Convention; and

4. that this Convention recognize and thank Dr. Nirmal Aggarwal, Chair of the Mathematics Department at Embry-Riddle Aeronautical University, as well as Greg Spradlin, together with all the other members of the KME Florida Delta Chapter , who devoted their time and talents to ensure the success of this meeting."

Respectfully submitted,
Adam C. Fletcher, Chairperson

Report of the Awards Committee

The awarded papers were:

- *Mathematical Modeling on an Open Limestone Channel*, by David Wolfe, Pennsylvania Mu, Saint Francis University
- *Some Different Ways to Sum a Series*, by Leon Hostetler and Robert Shollar, State College of Florida
- *Triphos: An Alternative Coordinate System*, by Paula Egging, Kansas Gamma, Benedictine College
- *NFL Combine Correlation Analysis*, by Brian Mosier, Kansas Beta, Emporia State University

These students were awarded a $100 check for their respective winning paper. The "People's Choice Award," which is selected by submitted votes of the attending KME members, was presented to James Mixon from Missouri Beta, University of Central Missouri. The "People's Choice Award" recipient received $50.

Respectfully submitted,
Brian Hollenbeck, Chairperson

Report of the Faculty Sectional Meeting

President Rhonda McKee opened up the meeting about the purpose of section meetings: to share ideas.

Participants introduced themselves.

- Feedback on parallel faculty/student sessions.
  - Good feedback on increase in number of students presenting.
  - At the same time, downside is that students end up presenting at the same time: which session should one attend?
  - More options need to be explored that will reduce fatigue but at the same time provide for greater sharing together.
- Workshops: Not a lot of comments, except…
  - Break up the day
  - General comments positive (there were some technical glitches at the workshops)
  - Continue to make workshops interactive
- How get chapters involved and more students involved with KME
  - Convenience of locations will encourage more students to attend
  - Faculty can encourage students to get published
  - Money/Funding
  - Publicity aspect of getting students to know about National KME and KME chapters
  - Getting students exposed to KME and showing that they can participate in KME activities
  - Contact non-participating chapters
  - Have students create messages about KME to post on Social Media: with live feed
  - Active KME students contact “Non-Active” chapters
  - Keep convention at reasonable price
  - Involve chapter student officers at a national level to get the word out to other students or chapter’s
  - Welcome back incentives for non-active chapters

- Annual vs. Biennial Conventions:
  - Rhonda informed participants that National council members voted to return having regional conventions during their business meeting
  - Advertise regional convention nationally to help attendance
  - Regional directors could help find a hosting chapter by contacting student presidents in their region.
  - Regions could help other regions. Have regional partnerships, if necessary.

Rhonda McKee concluding remarks focused on thanking the Corresponding Secretaries.

Report of the Student Sectional Meeting

- Event Ideas
  - Annual Picnic & kickball (3)
  - Ice cream social
  - Math-a-palooza
  - Calculator Training (2)
  - Math movies
  - Having food at meetings
  - Game night/game theories (Poker tournament) (2)
  - Bowling tournament
  - YouTube videos/math movies
  - T-shirts
  - Trips (Daytona) (Historical Library)
  - Chess tournament
  - Speakers
  - Book Sale
  - Free pie for pi day
  - Field day
  - Lunch together
- Fundraising/Service Project Ideas
  - math/science day
  - book sales (4)
  - sell KME shirts (2)
  - Pi your professor (2)
  - Tutoring (2)
  - Restaurant benefit
  - 50/50 raffle
  - pink flamingos in faculty lawns

  - clean up after sports events
  - sell example finals
- Parallel Sessions or all talks at once?
  - split the talks by subject
  - workshops are great
  - more talks over features
  - fewer but longer
  - like interactive workshops – avoid lectures (3)
  - like options
  - more with a variety of topics
  - a bit of conflict about seeing a few at the same time (2) maybe post recordings online?
  - liked it all
  - parallel talks prevent bigger groups

Kappa Mu Epsilon News

Edited by Peter Skoner, Historian

Updated information as of November 2015

Send news of chapter activities and other noteworthy KME events to

Peter Skoner, KME Historian
Saint Francis University
117 Evergreen Drive, 313 Scotus Hall
Loretto, PA 15940
or to
pskoner@francis.edu

Chapter News

AL Alpha – Athens State University

Chapter President – Matthew Crosswhite; 15 Current Members; 7 New Members

Other Spring 2015 Officers: Victoria Counce, Vice President; Alison Partlow, Secretary; and Patricia Glaze, Corresponding Secretary and Faculty Sponsor

New Initiates – Victoria Lynn Counce, Matthew T. Crosswhite, Brooke Lea Kuykendall, Alison Blair Partlow, Charles Sanford, Blakely West, and Marlene Williams.

AL Beta – University of North Alabama

Corresponding Secretary – Ashley Johnson; 17 New Members

New Initiates – Christian Bayesn, Kyle Black, Daniel Branscomb, Morgan Burcham, Vincent Chiriaco, John Carr, Matthew Cooper, Victoria Davis, Joseph Dawson, Elyse Eckl, Kaitlin Eckl, Jordan Frederick, Brianna Hillman, Emily Malone, Amberly Smith, Anna Kate West, and Tomoyuki Yara.

AL Epsilon – Huntingdon College

Corresponding Secretary – Dr. William Young; 10 New Members

New Initiates – Lauren Coe, Alex Dunkel, Lindsay Grinstead, Nick Haas, Alex Layson, Brandon Mattox, Wes Nail, Logan Smith, Aaron Williams, and James Worthington.

AL Zeta – Birmingham-Southern College

Chapter President – Chase Hoffman; 18 Current Members; 16 New Members

Other Spring 2015 Officers: Ryan Deveikis, Vice President; Sam Crowder, Secretary; Nirja Patel, Treasurer; and Maria Stadnik, Corresponding Secretary and Faculty Sponsor

Our KME chapter was busy this spring. At our spring initiation on April

2, 2015, we welcomed 16 new members to KME. On April 15, Dr. Chun Schiros from Regions Bank, came and gave an interesting talk entitled "Clinical data analysis using statistical model." Lastly, On April 20, 2015, several KME members headed down the road to nearby Samford University to participate in a Math Jeopardy tournament with a pizza party. The Samford team won; but, everyone had a good time and enjoyed the pizza.

AL Theta – Jacksonville State University

Chapter President – Shannon Bolton; 60 Current Members; 35 New Members

Other Spring 2015 Officers: Paitra Onkst, Vice President; James Tucker Davis, Secretary; Shawanna Roper, Treasurer; and Dr. David Dempsey Corresponding Secretary and Faculty Sponsor

On February 23, 2015, the Alabama Theta chapter initiated 35 new members, all students. New members received their certificates, pins, and honor cords in a ceremony held on the 11th floor of Houston Cole Library. Spring activities included a semester-opening game night and a Super Pi Day celebration on 3/14/15. Four students accompanied me to the biennial convention in April. New officers were elected during the April meeting, which included an ice cream social and games.

CA Epsilon – California Baptist University

Corresponding Secretary – James Buchholz; 25 New Members

New Initiates – Chrystal Alfaro, Jason Alvarez, Kendall Barkley, Wyatt Carr, Timothy Decious, Ethan Depledge, Deborah Donaldson, Joshua Flaherty, Jeremiah Gotts, Jacob Graff, Kimberly Howell, Brian Hughes, Kristin McCord, Meghan Ostrosky, Jasmine Pang, Lauren Rither, Alyssa Rowe, Lea Sarabwe, Yoana Silva, Corey Stein, Mitchell Sweetman, Daley Thomale, Stephanie Urban, Isaac Weber, and Samantha Wiza.

CO Delta – Colorado Mesa University

Corresponding Secretary – Erik Packard; 22 New Members

New Initiates – Rachael Alvir, Jacob Edminston, Brandon Gracey, Jamie Johnson, Kayla Jubert, Elizabeth Kanaly, Danae Lanigan, Amelia Metz, Tom Morrison, Erin Nissen, Chris Orr, Kelly Regimbal, Elizabeth Reimer, Austin Ruybal, Kayla Schaffer, Eric Sisneros, Carson Snart, Jamie Stephens, Orion Stranger, Dirk Terpstra, Allison Theobold, and Danny Weller.

CT Beta – Eastern Connecticut State University

Corresponding Secretary – Mehdi Khorami; 443 Current Members; 10 New Members

New Initiates – Kaylee Alberti, Kyle Burke, Molly Hankard, Megan Heenehan, Alexandria Holmes, Daniel Kowalsky, Jeremy MacDonough, Mate Magyar, Daniel Shenkle, and Andrew St. Jean.

FL Beta – Florida Southern College

Chapter President – Chris Morgan; 40 Current Members; 9 New Members

Other Spring 2015 Officers: Virginia Machado, Vice President; Wiresh Punwasi, Secretary and Treasurer; and Lisa De Castro, Corresponding Secretary and Faculty Sponsor

New Initiates – Emily Elizabeth Crowe, Nina Desrosiers, Jazmine L. Esparza, Silvano Falcao, Lisa Marie Bettina Gentil, Anthony W. Martino, Alex J. McClanahan, Selene A. Nelson, and Blake Matthew Weil.

FL Delta – Embry Riddle Aeronautical University

Corresponding Secretary – Dr. Jayathi Raghavan; 8 New Members

New Initiates – Nitish Aggarwal, Sarah Brodeur, Luca De Beni, Armando R Collazo Garcia III, Eric Andrew Lee Sabol, Daniel Tellez, Bruno Malo Torres Trueba, and Pablo Valdivieso.

GA Alpha – University of West Georgia

Corresponding Secretary – Scott Sykes; 3 New Members

New Initiates – Sarah Martino, Steven Redolfi, and Hailey Swafford.

GA Zeta – Georgia Gwinnett College

Corresponding Secretary – Dr. Jamye Curry; 34 New Members

New Initiates – Kodwo Annan, Alvina Atkinson, Tee Barron, Andrea Robles Benitez, Patrick Brooks, Bess Burnett, Jamye Curry, Mai Bao Doan, Amy H. Erickson, Keith Erickson, James Frye, Boyko Gyurov, Christina Holt, Amanda Iduate, Paula Krone, Zhongxiao Li, Christopher Lohrmann, Audrey Lynn, William Machamer, Heather McAfee, Tayeba Mohammed, Ekaterina Nathanson, Junkoo Park, Daniel Pinzon, Katherine Pinzon, Daniel Pragel, Joshua Roberts, Lee Ann Roberts, Michael Saum, Jennifer Sinclair, Marty Thomas, Shawn Sanderlin, Shahriyar Amy Warner, and Roshan Zamir.

HI Alpha – Hawaii Pacific University

Chapter President – Keila Elderts; 15 Current Members; 1 New Member

Other Spring 2015 Officers: Joshua Troglia, Vice President; and Tara Davis, Corresponding Secretary and Faculty Sponsor

We had our initiation dinner in April.

New Initiate – Nathan Miyano.

IA Alpha – University of Northern Iowa

Chapter President –Parash Upreti; 35 Current Members; 6 New Members

Other Spring 2015 Officers: Ben Castle, Vice President; Jacob Oswald, Secretary; Katy Goodmundson, Treasurer; and Mark D. Ecker, Corresponding Secretary and Faculty Sponsor

Our first spring KME meeting was held on February 23, 2015 at Professor Mark Ecker's residence where student member Parash Upreti talked about his KME paper entitled "Does Religion have an Impact on the Economic Growth of a Country?" Our second meeting was held on March 24, 2015 at Professor Syed Kirmani's residence where student member Ben

Castle talked about his semester abroad studying math in Europe. Student member Parash Upreti addressed the spring initiation banquet with "Factors that Affect Per Capita Growth of Developing Countries" on April 28, 2015. Our banquet was held at Godfather's Pizza in Cedar Falls, where six new members were initiated.

New Initiates – Lucas Beving, Morgan Bigbee, Emily Herbst, Julie Kirkpatrick, Amber Kracht, and Jacob Snyder.

IA Beta – Drake University

Corresponding Secretary – Lawrence Naylor; 3 New Members

New Initiates – Daniel Deeter, David Mascharka, and Katherine Roth.

IA Gamma – Morningside College

Chapter President – Jessie Byrnes; 18 Current Members

Other Spring 2015 Officers: Alyssa Turnquist, Vice President; Ashley Fiedler, Secretary; Crysta Brewer, Treasurer; and Chris Spicer, Corresponding Secretary and Faculty Sponsor

IA Delta – Wartburg College

Chapter President – Kelsey Miner; 53 Current Members; 19 New Members

Other Spring 2015 Officers: Ben Bogard, Vice President; Jennifer Fossum, Secretary; Ashlyn Bagge, Treasurer; Brian Birgen, Corresponding Secretary; and Dr. Joy Becker, Faculty Sponsor

In March, nineteen new initiates were welcomed at our annual banquet and initiation ceremony. Our speaker was Jill Seeba, a 2007 Wartburg Alum and KME member. Jill is a Fellow with the Society of Actuaries who works with Global Atlantic Financial Group in Des Moines. In May, we hosted the departmental end of the year picnic.

New Initiates – Ashlyn Bagge, Robert Blackburn, Cody Carlson, Jennifer Fossum, Matthew Hageman, Amanda Halvorson, Alex Harrison, Ryan Iverson, Ana Albino Julante, Breanna Lien, Kelsey Jean Miner, Megan Neuendorf, Eric Puls, Stetson Shook, Andrew Sorenson, Samuel Van Fleet, Breanna Walczyk, Mark Weber, and Hunter Westhoff.

IL Zeta – Dominican University

Chapter President – Maria Beltran; 24 Current Members; 4 New Members

Other Spring 2015 Officers: Katie Holden, Vice President; Radhika Patel, Secretary; and Aaron Zerhusen, Corresponding Secretary and Faculty Sponsor

We initiated 4 new members, and hosted a talk by Professor David Reimann from Albion College.

New Initiates – Chandler Allen, Jacob Nunez, Jose Rangel, Beata Rapacz, and Mason Eli Sherman.

IL Eta – Western Illinois University
Corresponding Secretary – Amy Ekanayake; 1 New Member
New Initiate – Oghenerabome P. Asama.
IL Theta – Benedictine University
Corresponding Secretary – Dr. Thomas Wangler; 1 New Member
New Initiate – Abigail Rodriguez.
IN Beta – Butler University
Corresponding Secretary – Bill Johnston; 3 New Members
New Initiates – Carly Allen, Kate Nikodym, and Sarah Stoops.
KS Alpha – Pittsburg State University
Chapter Co-Presidents – Audrey Gilbreath and Ashlee Hashman; 25 Current Members; 11 New Members
Other Spring 2015 Officers: Sarah Nistler, Vice President; Ashley Willis, Secretary; Caitlin Ogden, Treasurer; Dr. Tim Flood, Corresponding Secretary; and Dr. Cynthia Huffman, Faculty Sponsor
KS Delta – Washburn University
Chapter President – Jonathan Tyler; 18 Current Members; 7 New Members
Other Spring 2015 Officers: Branden Childers, Vice President and Secretary; Paige Eslick, Treasurer; and Kevin Charlwood, Corresponding Secretary and Faculty Sponsor
The Kansas Delta chapter of KME met with Club Mathematica three times each semester for pizza lunches. Often, we featured a speaker from off campus, to discuss their careers as secondary mathematics teachers or as actuaries at local firms. We held our annual initiation ceremony on March 2, 2015, where we initiated 7 new members. We had one faculty member attend the national KME convention held at Embry-Riddle Aeronautical University April 9 – 11, 2015.
LA Delta – The University of Louisiana at Monroe
Corresponding Secretary – Dr. Brent Strunk; 6 New Members
New Initiates – Allison Crotwell, Brittany Crowe, Evan Lay, Jasmine Nguyen, Similoluwa Ogundare, and Adip Raut.
MA Beta – Stonehill College
Chapter President – Jeanette Hogan; 16 Current Members; 5 New Members
Other Spring 2015 Officers: Molly Neubauer, Vice President; Mei-Lin McCarthy, Secretary; and Timothy Woodcock, Corresponding Secretary and Faculty Sponsor
Massachusetts Beta, at Stonehill College, held its annual initiation ceremony in April, with a buffet dinner enjoyed by our student members, their families, and faculty as well. As a service to the College, our student members rounded out the semester during the final exam week by staffing

drop-by help sessions that were open to all calculus students.

MD Beta – McDaniel College

Corresponding Secretary – Spencer Hamblen; 2 New Members

New Initiates – Anna Cooke and Morgan Stanback.

MD Delta – Frostburg State University

Chapter President – Chris Colwander; 35 Current Members; 15 New Members

Other Spring 2015 Officers: Michelle Weich, Vice President; Olivia Elisio, Secretary; Michelle Welch, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet and Justin Dunmyre, Faculty Sponsors

The Initiation Ceremony this March saw a record number of fifteen new members join the Maryland Delta Chapter. Faculty sponsor Dr. Marc Michael presented an interesting lecture involving origami and mathematics. Later in the month, the chapter conducted our annual fundraisers, namely, the Pi-Day Bake Sale and the sale of candy Easter Eggs. We had two meetings in April at the beginning and end of the month. These featured a two part presentation on the quadrature of the Spiral of Archimedes by faculty sponsor Dr. Mark Hughes. One lecture went through the proof due to Archimedes while the other showed Cavalieri's approach using indivisibles. Other business was also conducted during these meetings, namely, the choosing of a design for a new chapter T-shirt and the election of new officers. The new officers are Dustin Ullery as President, Morgan Allman as Vice President, Tyler Ram as Treasurer, and Amanda Monahan as Secretary. A good time was had by all at our end of the semester picnic. Finally, we offer best wishes to graduating seniors Jake Wigfield, Michelle Welch, Sara Zachritz, and Adam Witmer-Bosley.

MI Delta – Hillsdale College

Chapter President – Ayla Meyer; 32 Current Members; 7 New Members

Other Spring 2015 Officers: Joshua Mirth, Vice President; Arena Govier, Secretary; JoAnna Waterman, Treasurer; and Dr. David Murphy, Corresponding Secretary and Faculty Sponsor

This spring the Michigan Delta chapter of KME initiated seven new members and participated in the college's Honorama Bowling Tournament for honorary societies.

MO Alpha – Missouri State University

Chapter President – Steven Cornelius; 34 Current Members, 9 New Members

Other Spring 2015 Officers: Julie Barnum, Vice President; Ashley Kingston, Secretary; Meagan Leppien, Treasurer; and Jorge Rebaza, Corresponding

Secretary and Faculty Sponsor

As every semester, we had three seminars. Seminar 1 was Thursday February 5, 2015. Dr. Michael Reed (Physics Department), talked about "Using Mathematics to Translate Starlight into Physical Quantities." Pizza and soda were served. Seminar 2 was Monday March 16. Dr. Patrick Sullivan (Mathematics Department), talked about "Division: The Case of Zero." Pizza and soda were served. The event was very special because we celebrated "Pi-Day" with participation of faculty and students, and lots of pies on faces! Seminar 3 was Tuesday April 21. Two students from the Senior Seminar class (MTH 497) presented their papers: "The mathematics behind Sudoku puzzles," by Daytona Davis, and "Eliminating risk in financial markets," by Steven Cornelius. Also, KME Public Relations Officer, Mena Whalen, talked about her experience at participating at math conferences; in particular about her participation at the Nebraska Conference for Undergraduate Women in Mathematics. Pizza and soda were served. We also had an end-of-semester party on Thursday May 7, 2015 the last day of classes. We had lots of games music, food, drinks, and desserts.

New Initiates – Daniel Ayasse, Mallory Chapman, Patrick Johnson, Serge Ndjana, Michelle Sims, John Talarico, Morgan Whitaker, Christopher Wilson, and Joshua Yamdogo.

MO Epsilon – Central Methodist University

Corresponding Secretary and Faculty Sponsors – Pam Gordy; 5 New Members

New Initiates – Allan Richard Anderson, Kathleen Nicole Dozier, Rachel Marie Howieson, Jennifer Alexis Long, and Denise Katlyn Weigand.

MO Eta – Truman State University

Corresponding Secretary – David Garth; 3 New Members

New Initiates – Jonathan Deneke, Khala Schulte, and Adam Venneman.

MO Theta – Evangel University

Chapter President – Kevin Grimes; 17 Current Members; 11 New Members

Other Spring 2015 Officers: Kaitlyn Hong, Vice President; and Don Tosh, Corresponding Secretary and Faculty Sponsor

Meetings were held monthly. In January we initiated 11 new members and elected new officers. In April, Dr. Tosh, Dianne Twigger and four students attended the national convention at Embry Riddle Aeronautical University in Daytona Beach, Florida. Also in April we had our end-of-year barbeque at the home of Dianne Twigger, where honor cords were presented to graduating members.

MO Iota – Missouri Southern State University

Corresponding Secretary – Dr. Charles Curtis; 14 New Members

New Initiates – Connor Ames, Evan Bass, Katelyn Brewster, Jaired Collins, Aji Fatou

Dibba, Jessica Heston, Andrew Hoffman, Kurt Housh, Michael Manley, Nicklas Polizzi, Shaw Robertson, Claire Roarty, Cori Smith, and Derek Stokes.

MO Kappa – Drury University

Corresponding Secretary – Dr. Carol Browning; 21 New Members

New Initiates – Seth Adu Amankrah, Blake Andrews, Weston P. Buchanan, James Collins, Barrett Cummins, Matt Diekemper, Eriq Gaskill-Kristek, Ashley Hesterberg, Cynthia Heyse, Jessica Kjeldgaard, Kevin Liles, Derek Louderbaugh, Corey Marquardt, Stephanie Mazzoni, Sarah Metz, William Pearl, Josef Polodna, Victoria Robinson, Alex Schrader, Anjan Shrestha, and Breanna Tuhlei.

MO Lambda – Missouri Western State University

Corresponding Secretary – Dr. Steve Klassen; 10 New Members

New Initiates – Tiffany Adams, Adam Ambrosius, Michael Brown, Michael Frojd, Janolin Higgins, Kasey Maag, Haden McDonald, Taylor Schaben, Matthew Scholz, and Aaron Westlake.

MS Delta – William Carey University

Corresponding Secretary – Janie Bower; 8 New Members

New Initiates – Jamie Amacker, Joshua Deaton, Emily Harvell, Prashant Kharki, Ryan Lott, Dr. Ben Dribus, Dr. Jalynn Roberts, and Samuel Starnes.

MS Delta – Delta State University

Corresponding Secretary – Paula Norris; 2 New Members

New Initiates – John Downs and Brenda Ruth Smith.

NC Zeta – Catawba College

Corresponding Secretary – Doug Brown; 8 New Members

New Initiates – Jevgenji Gamper, Fernando Guerrera, Ivan Jimenez, Dominque Karriker, Yolanda Martinez, Alicia Richards, Declan Stimson, and Christian Watts.

NE Beta – University of Nebraska Kearney

Corresponding Secretary – Dr. Katherine Kime; 5 New Members

New Initiates – Aspen Clements, Brett Klima, Tim Marx, Trevor Rosno, and Adam Zheng.

NE Delta – Nebraska Wesleyan University

Chapter President – Leanne Hinrichs; 11 Current Members;5 New Members

Other Spring 2015 Officers: Connor Bohklen, Vice President; Sheridan Mason, Secretary and Treasurer; and Melissa Erdmann, Corresponding Secretary and Faculty Sponsor

This spring our chapter took part in numerous fun events. Brian Albright, a math professor from the nearby Concordia University, gave a talk about the mathematics of the game Spot-It. We hosted a Pi Mile fun run/walk on March 14, 2015 in which 50 runners/walkers took part. Each participant received a pie. In April we played ultimate Frisbee. In May, we initiated five new members into Kappa Mu Epsilon and enjoyed a celebratory picnic thereafter.

New Initiates – Adam Braegelman, Derek Hedges, Trevor Leiting, Spencer Randazzo, and William Reimer.

NH Alpha – Keene State College

Corresponding Secretary – Vincent J. Ferlini; 15 New Members

New Initiates – Lindsey Cioffi, Cara Colotti, Joshua Comstock, Michael DeMarco, Meegan Ellis, Megan Fisk, Brooke Hatanaka, Erin Hilow, Kegan Landfair, Katherine Marinoff, Nicole Marrero, Heather Paight, Christopher Rand, Anna Stephens, and Alyssa Valladares.

NJ Delta – Centenary College of New Jersey

Corresponding Secretary – Kathy Turrisi; 3 New Members

Other Spring 2015 Officer: Linda Ritchie, Treasurer

NJ Delta continues to assist students in the local community at the free math tutoring center (MTC) located at the Downtown Centenary location.

New Initiates – Jessika Beahm, Brian Edward DeParis, and Nicholas Perkalis.

NY Kappa – Pace University

Corresponding Secretary – Shamita Dutta Gupta; 9 New Members

New Initiates – Samantha Arato, Ryan Barone, Richard Brun Jr., Janice Caimares, John Chapeton, Matthew Hyatt, Kimberly Persaud, Donald Stone, and Shing So.

NY Lambda – LIU Post

Chapter President – Rebecca Greenberg; 10 New Members

Other Spring 2015 Officers: Rebecca Phillips, Vice President; Erica Gershkowitz, Secretary; Marta Szpak, Treasurer; and Corbett Redden, Corresponding Secretary and Faculty Sponsor

The NY Lambda chapter of KME held its annual banquet and initiation on April 19, 2015. Ten students were initiated.

New Initiates – David Alini, Clifford Clark, Lauren Gould, Samantha Hart, Elizabeth Hartmann, John O'Rourke, Natalie Post, Deena Prevete, Marta Szpak, and Christina Wolf.

NY Nu – Hartwick College

Corresponding Secretary and Faculty Sponsor – L. Gerald Hunsberger; 10 New Members

New Initiates – Chad Barhydt, Jia-Min Chu, Gina DeVoto, Tohmas Heritage, Stavros Kerchoulas, Eric Moore, Ryan O'Malley, Jessica Pimental Almonte, Sarajane Roenke, and Kit Tregear.

NY Omicron – St. Joseph's College

Chapter President – Stephen A. Bates; 50 Current Members; 14 New Members

Other Spring 2015 Officers: Carl Baurle, Vice President; James Young, Secretary; Jessica Alessi, Treasurer; Dr. Elana Reiser, Corresponding Secretary; and Dr. Donna Marie Pirich, Faculty Sponsor

This semester we had an initiation ceremony for 14 members. We also put together Easter baskets to donate to a local charity. Members also volunteered to tutor local high schools students at our math clinic.

NY Rho – Molloy College

Chapter President – Mary-Kate Michels; 127 Current Members; 15 New Members

Other Spring 2015 Officers: Samantha Novak, Vice President; Santiago Vargas, Secretary and Treasurer; and Manyiu Tse and Deborah Upton, Corresponding Secretaries and Faculty Sponsors

OH Eta – Ohio Northern University

Chapter President – Michelle Haver; 16 Current Members; 5 New Members

Other Spring 2015 Officers: Michael Potter, Vice President; Kayla Hummell, Secretary; Tyler Bernardy, Treasurer; and Dr. Ryan Rahrig, Corresponding Secretary Faculty Sponsor

OH Theta – Capital University

Chapter President – Abigail Neininger; 16 Current Members; 16 New Members

Other Spring 2015 Officers: Julia Kinkel, Vice President; Jaime Ashworth, Secretary; Oscar O'Flaherty, Treasurer; Paula Federico, Corresponding Secretary; and Jonathan D. Stadler, Faculty Sponsor

We held our installation ceremony on Friday, April 24, 2015. Capital University is a small, liberal arts college in small-town Bexley near Columbus, Ohio. Sixteen new members and their families and friends joined the mathematics faculty in the Student Union to install a new chapter, Ohio Theta, and to initiate new members. Even Capital's Provost and Dean attended! Dr. Pete Skoner, National Historian, shared the history of Kappa Mu Epsilon, especially in the states surrounding Ohio. Dr. Pedro Muino, Regional Director, then gave a great presentation entitled "Pi is a Harsh Mistress: A Tale of Irrational Passion," which interested both math and non-math attendees alike. The faculty sponsor and conducting officer Dr. Jonathon Stadler was initiated into Kappa Mu Epsilon when he was an undergraduate student at Bowling Green State University, which shows that Kappa Mu Epsilon has a close-knit community and shared history. Dr. Paula Federico was installed as corresponding secretary of Ohio Theta, along with President Abigail Neininger, Vice President Julia Kunkel, Secretary Jaime Ashworth, and Treasurer Oscar O'Flaherty. While Ohio Theta is a brand new chapter, it is already quite active. Since installation, Ohio Theta has teamed up with Capital's chapter of the Mathematical Association of America to raise money to buy math supplies for kindergarten classrooms in the area, and plans on hosting more charity and social events together this coming fall.

New Initiates – Jaime Ashworth, Patrick Donahue, John Einsweiler, Paula Federico, Rachel

Fountain, Matthew Jennell, Leigh Johnson, Julie Kunkel, Alex Lambert, Abigail Neininger, Brittany Nicholson, Oscar O'Flaherty, Ashley Pallone, Jessica Potts, Otto Shaw, and Patrick Shields.

OK Alpha – Northeastern State University

Chapter President – Cindy Jeffcoat; 75 Current Members, 13 New Members

Other Spring 2015 Officers: Caleb Stubbs, Vice President; Natalie Mayberry, Secretary; John Moore, Treasurer; and Dr. Demitri Plessas, Corresponding Secretary and Faculty Sponsor

Our spring initiation brought twelve students into our chapter. At the first meeting of the semester, Dr. Elwin Davis, Pittsburg State University, spoke on "What Did Columbus Know, and When Did He Know It?" We then had a talk by Dr. Nathan Bloomfield, NSU, on "Phrasal Templates for Fun and Profit." Kappa Mu Epsilon then had a joint meeting with the NSU Society of Physics Students where Dr. Rui Zhang, NSU, spoke on "From Newton's Calculus to Feynman's Path Integral: Application of Mathematical Analysis in Physics." We ended the semester with our annual ice cream social, where Dr. Buckles, NSU, gave a talk on "Visualizing Group Theory." We also had two student presentations at this meeting. Natalie Mayberry spoke on "Alpha-Beta Communities in Twitter-like Structures," and Skylar Wapato spoke on "Disqualifying Candidates for Odd Perfect Squares Using Modular Arithmetic."

New Initiates – Christopher R. Blankenship, Gentry L. Blankenship, Marisa R. Girdner, Geba I. Hawkins, Lacie J. Hensley, Benjamin J. Kruger, Erinn L. Lawson, Lizzie I. Lightning, Richard Ly, Tracy M. Pruitt, Jonathan M. Wilhelm, Jennifer C. Wilkie, and Susan Yang-Her.

OK Epsilon – Oklahoma Christian University

Chapter President – Kaylee Edwards; 14 Current Members; 7 New Members

Other Spring 2015 Officers: Josh Bilello, Vice President; Aubrey Gonzalez, Secretary; Dr. Jennifer Bryan, Corresponding Secretary; and Dr. Craig Johnson, Faculty Sponsor

PA Alpha – Westminster College

Corresponding Secretary – Pamela Richardson; 7 New Members

New Initiates – Charles Cratty Anthony J. Groves, Kia Howe, Sumer A. Kassim, Isaiah Morgenstern, Mark Patton, and Christijana Vucenovic.

PA Beta – La Salle University

Chapter President – Carmen Esposito; 30 Current Members

Other Spring 2015 Officers: Mary-Elizabeth Voss, Vice President; Austin Anderson, Secretary; Katherine Boligitz, Treasurer; and Janet Fierson,

Corresponding Secretary and Faculty Sponsor
In the spring semester of 2015, members of La Salle's KME chapter attended the Moravian College Student Mathematics Conference, the spring meeting of the Eastern Pennsylvania and Delaware section of the MAA at Franklin & Marshall College, the Nebraska Conference for Undergraduate Women in Mathematics, and the Philadelphia Undergraduate Mathematics Conference Series at Temple University; chapter president Carmen Esposito presented talks on Polya's enumeration theorem at two of these conferences. The group also sponsored a university-wide late-night Pi Day event, which included various board games and a contest to solve a mathematical puzzle. In April, the chapter celebrated Mathematics Awareness Month by distributing interesting mathematical tidbits to people passing through the student union building. They also held a 24 game contest and donated half of the proceeds to St. Athanasius School, a local elementary school where chapter members continue to volunteer weekly to share their mathematical talents and enthusiasm.

PA Gamma – Waynesburg University

Corresponding Secretary – James R. Bush; 8 New Members

New Initiates – Caley Blankenbuehler, Larissa Anne Bray, Ashley Marie Farber, Renee Filippelli, Taylor Mae Garrett, Sarah A. Hathaway, Jessica M. Marabello, and Jonathan D. Sandoval.

PA Zeta – Indiana University of Pennsylvania

Chapter President – Derek Hanely; 17 Current Members; 5 New Members
Other Spring 2015 Officers: Melissa Reinhardt, Vice President; Shawn Mosley, Secretary and Treasurer; and Gary Stoudt, Corresponding Secretary and Faculty Sponsor
On April 22, 2015 PA Zeta initiated four new student members and accepted the transfer membership of one faculty member, Dr. Rachelle Bouchat, who joined the IUP Mathematics Department faculty this year. May 6, 2015 marked the 50th Anniversary of the installation of the PA Zeta Chapter.

New Initiates – Dr. Rachelle Bouchat, Katherine Browe, Justin Charles, James Faucette, and Melissa Reinhardt.

PA Eta – Grove City College

Corresponding Secretary – Dale L. McIntyre; 18 New Members

New Initiates – Natalie Blandino, Michael Boom, Laura Brestensky, Ryan Brown, Jessica Fenton, Rachael Gehman, Ethan Gelpi, Amanda Johnson, Ethan Johnson, Sera Lopez, Lauren Miller, Megan Obley, Matthew Peffer, Devon Powley, Grace Prensner, Peter Richards, Natalie Smith, and Rachael Zdaniewicz.

PA Iota – Shippensburg University

Chapter President – Emily Owens; 749 Current Members; 8 New Members

Other Spring 2015 Officers: Pat Wells, Vice President; Lara John, Secretary; Tyler Garrett, Treasurer; Dr. Paul Taylor, Corresponding Secretary; and Dr. Ji Young Choi, Faculty Sponsor

New Initiates – Julie Fuhrman, Tyler Garrett, Lara John, Luis Melara, Emily Owens, Jordan Unger, Brandon Weiser, and Pat Wells.

PA Kappa – Holy Family University

Chapter President – Rebecca Gaetani; 3 Current Members; 6 New Members

Other Spring 2015 Officer: Sister Marcella Wallowicz, Corresponding Secretary and Faculty Sponsor

The PA Kappa Chapter initiated 6 new members during the School of Arts & Sciences Honor Society program on April 10, 2015. Two PA Kappa members presented their senior research at the annual SEPCHE Honors Conference, held on March 28, 2015 at Immaculata University. Jared DeLeo presented "Aerodynamics – A Look Into Basics and Stunts!" A brief glimpse into the realm of aerodynamics, backed by the wonderful world of Mathematics! A look into the evolution of aircraft design and how they take off, stay airborne, and then land. A discussion into stunt capabilities is included as we journey into the skies with both math and aircraft! Rebecca Gaetani presented "Knot Theory and the Trefoil Knot." Knot theory is a relatively new topic in mathematics and a lot of information is still being discovered about different types of mathematical knots. This presentation will focus on the trefoil knot and how the left and right trefoil knots are distinct. Important vocabulary relating to knot theory and the trefoil knot.

PA Lambda – Bloomsburg University of Pennsylvania

Corresponding Secretary – Elizabeth Mauch; 11 New Members

New Initiates – Kayla Brady, Kristi Brittain, Brandon Fairchild, Michael Gensel, Nathan Henry, Rachel Livingston, Doug Lockard, Noah Long, Britainy Ritz, Hannah Shriver, and Michael Unitis.

PA Mu – Saint Francis University

Chapter President – Kaitlyn Waldron; 45 Current Members; 25 New Members

Other Spring 2015 Officers: David Wolfe, Vice President; Kelly Beegle, Secretary; Cathleen Fry, Treasurer; Dr. Peter Skoner, Corresponding Secretary; and Dr. Brendon LaBuz, Faculty Sponsor

The annual Pi Day celebration was held on Friday, March 13, 2015;.faculty, students, and staff enjoyed taste testing an assortment of "pi" served by members of Kappa Mu Epsilon throughout the day. Initiation ceremonies were held on Thursday, January 29, 2015 in DiSepio 213; The evening began with a prayer by chapter chaplain and member Fr. Joseph Chancler, T.O.R., was followed with dinner, continued with a talk "Unexplained Mysteries from the Fourth Dimension," by Mr. Joseph Wilson, Instructor of Mathematics at the University of Pittsburgh at Johnstown, continued with the initiation ceremony for the 25 new members, and concluded with remarks and a closing prayer by Professor of Chemistry and Great Lakes regional director Dr. Pedro Muíño. Two faculty members and four students attended the National Convention held April 9-11, 2015 at Embry-Riddle Aeronautical University in Daytona Beach, Florida; David Wolfe, junior mathematics and chemistry major, received the award for the best presentation in applied mathematics for his presentation, "Mathematical Modeling on a Limestone Channel." KME students and faculty served as judges for the 2015 Pennsylvania Statistics Poster Competition, hosted for the seventh year by Saint Francis University. A large number of posters (584) were received, cash awards were given for first through fourth place in each of four grade level categories, and winning posters were submitted to the National Statistics Poster Competition, coordinated by the American Statistical Association.

PA Xi – Cedar Crest College

Corresponding Secretary – Marie Wilde; 1 New Member

New Initiate – Alyssa Babecki.

PA Sigma – Lycoming College

Chapter President – Rachel Duncan; 29 Current Members; 10 New Members

Other Spring 2015 Officers: Madalyn Confer, Vice President; Amanda Ferster, Secretary; Tung Nguyen, Treasurer; Santu de Silva, Corresponding Secretary; and Dr. Eileen Peluso, Faculty Sponsor

The academic year was mostly uneventful, except that for the first time we were able to raise an unusual volume of funds for the activities next year, so we may look forward to something new.

New Initiates – Jeremy Chobot, Jason Coles, Madalyn Confer, Devon Dietrich, Rachel Duncan, Amanda Ferster, Emily Hiller, Dylan McDonald, Craig Needhammer, and Daniel Zebrine.

PA Tau – DeSales University

Chapter President – Jacob M. Kean; 11 Current Members; 11 New Members

Other Spring 2015 Officers: Alison Malatesta, Vice President; Joshua Brobst, Secretary and Treasurer; and Bro. Daniel P. Wisniewski, Corresponding Secretary and Faculty Sponsor

On Sunday, April 19, 2015, the PA Tau Chapter of Kappa Mu Epsilon at DeSales University (DSU) initiated eleven new KME members. The event included a presentation entitled "STANDards" by Mr. Kevin C. Lutz, Mathematics Teacher at Archbishop Ryan High School (Philadelphia, PA) and a Consultant for the Penn Literacy Network at University of Pennsylvania, who received his B.S. in mathematics in 2001 from DSU. In attendance were family and friends of the new and current KME members, as well as three KME alumni.

New Initiates – Erik A. Alderiso, Mei Ying Carbonaro, Brian Cruts, Julie A. Farmer, John J. Flynn, III, Katherine L. Frederick, Christine Holmes, Kevin C. Lutz, Theresa Marlin, Zachary Moser, and Lee E. Orzol.

RI Alpha – Roger Williams University

Corresponding Secretary – Robert Jacobson; 18 New Members

New Initiate – Amanda Becotte, Jean-Luc Bergeron, Molly Brash, Alexandra Brown, Joshua Castigliego, Kellie Dean, Kelly Demolles, Maxwell Eichberg, Allex Gourlay, Sarah Krasnecky, Heather Larsen, Torrie Lewine, Brittany McMullen, Alexa Paparelli, Jill Resh, Alison Slaughter, Morgan Stafford, and Andrea Wright.

RI Beta – Bryant University

Corresponding Secretary – John Quinn; 7 New Members

New Initiates –Emily Genereux, William Kelley, Blayne Michalski, Jonathan Mudge, Jonathan Skaza, Courtney Stine, and Emma Wieduwilt.

SC Gamma – Winthrop University

Corresponding Secretary – Jessie A. Hamm; 12 New Members

New Initiates – Lindsay Bradley, Katelyn Evans, C. Matthew Farmer, Lisa Farmer, Justin Groves, Kristin Hinson, MaLyn Lawhorn, Kristen Melton, Alexander Middleton, Brianna Rae, Corey Riley, and Alison Tighe.

SC Delta – Erskine College

Chapter President – Kelly Walker; 5 Current Members

Other Spring 2015 Officers: Paris Hanvey, Vice President; and Dr. Art Gorka, Corresponding Secretary and Faculty Sponsor

The Chapter organized a couple of math talks for math students and the general public. The meetings were preceded by a social with snacks and drinks.

SC Epsilon – Francis Marion University

Chapter President – Taylor Evan Burch; 24 Current Members

Other Spring 2015 Officers: Rigel F. Lochner, Vice President; Cody Tyler McKenzie, Secretary and Treasurer; Jeremiah Bartz, Corresponding Secretary and Faculty Sponsor

TN Beta – East Tennessee State University

Chapter President – James "Dustin" Chandler; 40 Current Members; 18 New Members

Other Spring 2015 Officers: William "Ty" Frazier, Vice President; Austin Patrick, Secretary; Nathan Molder, Treasurer; and Dr. Robert Gardner,

Corresponding Secretary and Faculty Sponsor
Our chapter webpage is http://faculty.etsu.edu/gardnerr/KME/KME.html. A webpage of spring 2015 activities is here: http://faculty.etsu.edu/gardnerr/KME/Spring-2015/Spring2015.html. One spring meeting was held on April 9, 2015. New officers were elected (as listed above) and TN Beta faculty co-advisor Dr. Robert Gardner gave a presentation on Euclid's Elements of Geometry (an online version of which is available at http://faculty.etsu.edu/gardnerr/Geometry-History/abstract.html. TN Beta chapter funds were used to provide the approximately 20 attendees with pizza from the local Italian Pizza Pub. On April 16, 2015, the Tennessee Beta chapter of Kappa Mu Epsilon held the initiation ceremony for new members. The ceremony was held at the ETSU Department of Mathematics and Statistics' annual Honors Banquet. The keynote speaker was Brian K. Post of the Manufacturing Systems Research Group at Oak Ridge National Laboratory. He gave a presentation on "additive manufacturing," also known as 3D printing.

New Initiates – Maria Avila, David Burton, William "Ty" Frazier, Erica Fugate, Erika Hale, Caleb Ignace, Samuel Kakraba, Ciana Kissel, John Lagergren, Nathan Molder, Jessica Osborne, Brandi Persons, Austin Patrick, Alex John Quijano, Haley Russell, Thomas Torku, Colleen Walls, and Robert Whitinger.

TN Gamma – Union University

Chapter President – Megan Mouser; 23 Current Members; 9 New Members

Other Spring 2015 Officers: Nicole Bantz, Vice President; Vicki Searl, Secretary and Treasurer; Michael Kelly, Webmaster and Historian; George Moss, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor

New Initiates – Rachel Brewer, Lindsey Chesak, Emily Easter, Dillon Lisk, Cherish Lo, Whitney Moskovitz, William Pierce, Joshua Stucky, and Tom Trabue.

TN Delta – Carson-Newman University

Corresponding Secretary – Kenneth Massey; 7 New Members

New Initiates –Mitchell Benjamin, Ryan Eberle, John Echols, Rachel Harper, Joseph Hughes, Kendra Ivins, and Lauren Morelock.

TN Zeta – Lee University

Corresponding Secretary – Caroline Maher-Boulis; 6 New Members

New Initiates – Adam Carty, Joshua Crumbliss, Danielle Lin, Sarah Minucci, Jon Weeks, and Amy Wells.

TX Alpha – Texas Tech University

Corresponding Secretary – Magdelena Toda; 5 New Members

New Initiates – Edgar Hernandez, Nitish Mittal, Aaron Trusty, Regan Willmann, and Beth Zabilka.

TX Kappa – University of Mary Hardin-Baylor

Corresponding Secretary – Peter H. Chen; 7 New Members

New Initiates – Brianna Brown, Thao Giang, Adam Janecka, Sangheetha Kannan, Madeline Necessary, Stephanie Quellette, and Rostislav Radchenko.

TX Lambda – Trinity University

Chapter President – Alyssa Fink; 246 Current Members; 11 New Members

Other Spring 2015 Officers: Ailie Vuper, Vice President; Rachel Hure, Secretary; and Dr. Hoa Nguyen, Corresponding Secretary and Faculty Sponsor

New Initiates – Avva Bassiri, Camille DeMars, Hannah Kuhl, Evan LeGros, Lu Liu, Shelby Luikart, Hoa Nguyen, David Stroud, Ashley Tessnow, Zacharia Tuten, and Eliza Wright.

TX Mu – Schreiner University

Corresponding Secretary – Stefan Mecay; 19 New Members

New Initiates – Diana Aguire, Dr. Brian Bernard, Ariana DeLeon, Kaleen Dean, Soledad Diaz, Elliot Frey, Leanna Haynes, Brandon Higgins, Brittany Hoadley, Michael Hormuth, Ulises Jasso, Nathaniel Johnston, Heather McCain, Callen McCauley, Erick McCollum, Gisela Meza, Nicole Roberts, Brandy Vaclavick, and Robert Vastano.

VA Delta – Marymount University

Chapter President and Secretary – Lucy Ogbole; 34 Current Members; 3 New Members

Other Spring 2015 Officers: Bernadette Wunderly, Vice President and Treasurer; William Heuett, Corresponding Secretary; and Elsa Schaefer, Faculty Sponsor

New Initiates – Will Heuett, Alice Petillo, and Bernadette Wunderly.

VA Gamma – Liberty University

Corresponding Secretary – Dr. Tim Van Voorhis; 16 New Members

New Initiates – Daniel Adams, Justin Branham, Bradley Buckland, Dae Yeon Cho, Daniel Freese, Andrew Freese, Jess Gregory, Adriane Guy, Alesha Hagerty, Ximan Huang, Aaron Hultstrand, Stephen Jacobsen, Naser Mansour, Zachary Minner, Lauren Spahr, and Nathan Wakefield.

WI Alpha – Mount Mary University

Corresponding Secretary –Roxanne Back; 7 New Members

New Initiates – Cortisha Breber, Erica Hill, Agnieszka Mosio, Brooklynne Slamka, Tara Strook, Mary Wallace, and Tong Xiong.

WV Alpha – Bethany College

Corresponding Secretary and Faculty Sponsor – Adam C. Fletcher; 7 New Members

New Initiates – Samuel W. Duvall, Daniel E. Faix, Julia A. Mouch, Tess L. Parry, Dayanara Ramos, Jacob T. Riddell, and Brandon A. Trinh.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, Jonesboro	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
KS Zeta	Southwestern College, Winfield	14 Apr 1998
TN Epsilon	Bethel College, McKenzie	16 Apr 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 Mar 1999
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
NJ Gamma	Monmouth University, West Long Branch	21 Apr 2002
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008

CA Zeta	Simpson University, Redding	4 Apr 2009
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011
PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresnp Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015