

## THE PENTAGON

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# ***Pisano Arrays and the Merit of Looking at Sequences in Two Dimensional Arrays***

Joseph Shoulak

## **1. Introduction**

In this paper, we will take a look at the Fibonacci numbers taken modulus  $m$  and arrange them in a 2-dimensional array to better discuss the patterns we will see, which was previously considered in [2]. All numbers are integers.

To begin, we review the Fibonacci numbers. The Fibonacci numbers  $F_n$  are given by the recursive definition  $F_n = F_{n-1} + F_{n-2}$ . Starting with  $F_0 = 0$  and  $F_1 = 1$ , we get

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 \dots$$

The numbers were originally popularized by Leonardo de Pisa (aka Fibonacci) when he used them to describe rabbit populations, but turn up often and hold interesting enough properties themselves. One of these properties is that as you divide one Fibonacci number by the one just previous, you approach the Golden Ratio, such that

$$\lim_{n \rightarrow \infty} \frac{F_n}{F_{n-1}} = \phi$$

Next, we review the modulus operator. A number  $n$  taken mod  $m$  returns the remainder of  $\frac{n}{m}$ , or for some number  $j$ ,

$$n \bmod m = k \leftrightarrow n = jm + k.$$

## **2. Pisano Arrays**

We first consider the following two theorems.

**Theorem 1 (Lagrange, 1774, [1])**  $F_n \bmod 10 = F_{n+60} \bmod 10$ .

**Theorem 2 (Wall, 1960)**  $F_n \bmod m = F_{n+\pi(m)} \bmod m$ , where  $\pi(m)$  is the period of the Fibonacci numbers modulo  $m$ .

We see that Theorem 2 generalizes Theorem 1 for modulus  $m$ . In particular, we see that  $\pi(10) = 60$ . When the Fibonacci numbers are taken mod  $m$ , that is, when we look at the integer series

$$f_n = F_n \bmod m,$$

we find it to be periodic with period  $\pi(m)$ , called the Pisano Period. This means

$$f_n = f_{n+k\pi(m)}.$$

A Pisano Array is an array of descending columns composed of the numbers  $f_n$ , with  $n$  ranging from 0 to  $\pi(m) - 1$ , and so the total number of entries in the array would be  $\pi(m)$ . This means a Pisano array must be an  $h \times w$  array such that  $hw = \pi(m)$ . Since there are possibly multiple Pisano arrays for a given  $m$  (the number of factor pairs of  $\pi(m)$ , and their reverses), a Pisano Array can be defined by what modulus we're taking ( $m$ ) and the height of the array ( $h$ ). Therefore, the notation for a Pisano array is  $P(m, h)$ . Hence,

$$P(m, h) = \begin{bmatrix} F_0 \bmod m & F_h \bmod m & \dots & F_{(w-1)h} \bmod m \\ F_1 \bmod m & F_{h+1} \bmod m & \dots & F_{(w-1)h+1} \bmod m \\ \vdots & \vdots & \ddots & \vdots \\ F_{h-1} \bmod m & F_{2h-1} \bmod m & \dots & F_{hw-1} \bmod m \end{bmatrix}$$

This paper takes a specific look at  $P(10, 5)$ . Let  $f_n = F_n \bmod 10$ . Above, we saw that  $\pi(10) = 60$ , and so the number of columns is  $w = \frac{\pi(10)}{h} = \frac{60}{5} = 12$ . Hence, we have

$$\begin{aligned} P(10, 5) &= \begin{bmatrix} f_0 & f_5 & f_{10} & f_{15} & f_{20} & f_{25} & f_{30} & f_{35} & f_{40} & f_{45} & f_{50} & f_{55} \\ f_1 & f_6 & f_{11} & f_{16} & f_{21} & f_{26} & f_{31} & f_{36} & f_{41} & f_{46} & f_{51} & f_{56} \\ f_2 & f_7 & f_{12} & f_{17} & f_{22} & f_{27} & f_{32} & f_{37} & f_{42} & f_{47} & f_{52} & f_{57} \\ f_3 & f_8 & f_{13} & f_{18} & f_{23} & f_{28} & f_{33} & f_{38} & f_{43} & f_{48} & f_{53} & f_{58} \\ f_4 & f_9 & f_{14} & f_{19} & f_{24} & f_{29} & f_{34} & f_{39} & f_{44} & f_{49} & f_{54} & f_{59} \end{bmatrix} \\ &= \begin{bmatrix} 0 & 5 & 5 & 0 & 5 & 5 & 0 & 5 & 5 & 0 & 5 & 5 \\ 1 & 8 & 9 & 7 & 6 & 3 & 9 & 2 & 1 & 3 & 4 & 7 \\ 1 & 3 & 4 & 7 & 1 & 8 & 9 & 7 & 6 & 3 & 9 & 2 \\ 2 & 1 & 3 & 4 & 7 & 1 & 8 & 9 & 7 & 6 & 3 & 9 \\ 3 & 4 & 7 & 1 & 8 & 9 & 7 & 6 & 3 & 9 & 2 & 1 \end{bmatrix} \end{aligned}$$

### 3. Propositions in $P(10, 5)$

Each proposition will be demonstrated on an array composed of arbitrary consecutive columns from  $P(10, 5)$ . In the following propositions, we let  $P(10, 5) = A = (a_{i,j})$ , where  $0 \leq i \leq 4$  and  $0 \leq j \leq 11$ . Before continuing, it is worth mentioning the Lucas numbers  $L_n$ , an integer sequence similar to the Fibonacci numbers in that their rule is the same, but they have different starting numbers, namely  $L_0 = 2$  and  $L_1 = 1$ . While the Fibonacci sequence is

$$0, 1, 1, 2, 3, 5, \dots,$$

the Lucas sequence is

$$2, 1, 3, 4, 7, 11, \dots$$

**Proposition 1** In the Pisano array  $A = (a_{i,j})$ , for  $1 \leq i \leq 3$ ,  $a_{i,j} = (a_{i-1,j} + a_{i-2,j}) \bmod 10$ .

**Proof.** Consider a column of a Pisano array  $A$ :

$$\begin{bmatrix} \dots \\ a_{i-2,j} \\ a_{i-1,j} \\ a_{i,j} \\ \dots \end{bmatrix}$$

Since a column in a Pisano array is composed of Fibonacci numbers, it follows that  $a_{i,j} = (a_{i-1,j} + a_{i-2,j}) \bmod 10$ . ■

Proposition 1 is a good example of how we can describe patterns using arrays.

**Proposition 2** In the Pisano array  $A = (a_{i,j})$ , for  $0 \leq i \leq 4$ ,  $2 \leq j \leq 11$ ,  $a_{i,j} = (a_{i,j-1} + a_{i,j-2}) \bmod 10$ .

**Proof.** Consider the Pisano array  $A$  with the given row below:

$$\begin{bmatrix} \dots & \dots & \dots & \dots & \dots \\ \dots & a_{i,j-2} & a_{i,j-1} & a_{i,j} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

We must show that  $a_{i,j} = (a_{i,j-1} + a_{i,j-2}) \bmod 10$ . This is the same pattern as in Proposition 1, but now horizontal. Note that

$a_{i,j} = F_{i+5j} \bmod 10$  for  $0 \leq i \leq 4, 0 \leq j \leq 11$ . Let  $k = 5F_{n-9} + 3F_{n-10}$ . Multiplying by 10, we have

$$\begin{aligned} 10k &= 50F_{n-9} + 30F_{n-10} \\ &= 50F_{n-9} + 31F_{n-10} - F_{n-10}. \end{aligned}$$

Using the definition of Fibonacci numbers, we obtain

$$10k = 31F_{n-8} + 19F_{n-9} - F_{n-10}.$$

A repeated application of this definition yields

$$\begin{aligned} 10k &= 5F_{n-4} + 2F_{n-5} - F_{n-10} \\ &= 5F_{n-4} + 3F_{n-5} - F_{n-5} - F_{n-10} \\ &= 3F_{n-3} + 2F_{n-4} - F_{n-5} - F_{n-10} \\ &= 2F_{n-2} + F_{n-3} - F_{n-5} - F_{n-10} \\ &= F_{n-1} + F_{n-2} - F_{n-5} - F_{n-10} \\ &= F_n - (F_{n-5} + F_{n-10}). \end{aligned}$$

And, so letting  $n = i + 5j$ , we have

$$\begin{aligned} F_{i+5j} &= (F_{i+5j-5} + F_{i+5j-10}) \bmod 10 \\ &= (F_{i+5(j-1)} + F_{i+5(j-2)}) \bmod 10, \end{aligned}$$

or equivalently,

$$a_{i,j} = (a_{i,j-1} + a_{i,j-2}) \bmod 10,$$

as desired. ■

**Proposition 3** In the Pisano array  $A = (a_{i,j})$ , for  $0 \leq j \leq 11$ ,  $a_{0,j} = 5k$ , for some  $k \in \mathbb{Z}$ . In other words,  $a_{0,j} \bmod 10 \in \{0, 5\}$ .

$$\begin{bmatrix} a_{0,j} \\ \dots \\ \dots \\ \dots \\ \dots \end{bmatrix}$$

**Proof.** Recall that  $a_{i,j} = F_{i+5j} \bmod 10$  for  $0 \leq i \leq 4, 0 \leq j \leq 11$ , and so  $a_{0,j} = F_{5j} \bmod 10$  for  $0 \leq j \leq 11$ . We will prove  $F_{5j} = 5k$ , for some  $k \in \mathbb{Z}$ , by induction on  $j$ .

Base Step:  $j = 0$ .  $F_{5j} = F_{5(0)} = F_0 = 0 = 5k$ , with  $k = 0$ .

Inductive Step: Assume  $F_{5j} = 5k$ , for some  $k \in \mathbb{Z}$ . We will show  $F_{5(j+1)} = 5m$ , for some  $m \in \mathbb{Z}$ . Using the definition of

Fibonacci numbers, we have

$$\begin{aligned}
 F_{5(j+1)} = F_{5j+5} &= F_{5j+4} + F_{5j+3} \\
 &= F_{5j+3} + F_{5j+2} + F_{5j+2} + F_{5j+1} \\
 &= F_{5j+2} + F_{5j+1} + 2(F_{5j+1} + F_{5j}) + F_{5j+1} \\
 &= F_{5j+1} + F_{5j} + 4F_{5j+1} + 2F_{5j} \\
 &= 5F_{5j+1} + 3F_{5j} \\
 &= 5F_{5j+1} + 3(5k) \text{ by the inductive hypothesis} \\
 &= 5(F_{5j+1} + 3k) = 5m, \text{ for some } m \in \mathbb{Z}.
 \end{aligned}$$

Hence, by induction, for all  $j$ ,  $a_{0,j} = 5k$ , for some  $k \in \mathbb{Z}$ . ■

**Proposition 4** In the Pisano array  $A = (a_{i,j})$ , for  $2 \leq j \leq 11$ ,  $a_{3,j-2} = (a_{2,j-1} + a_{1,j}) \bmod 10$ .

$$\begin{bmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & a_{1,j} \\ \cdots & a_{2,j-1} & \cdots \\ a_{3,j-2} & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

**Proof.** Let  $n = j - 2$  for  $2 \leq j \leq 11$ . Then, by Proposition 3,  $F_{5n} = 5m$  for some  $m \in \mathbb{Z}$ . Let  $k = 10F_{5n+1} + 31m$ . Multiplying by 10, we have

$$\begin{aligned}
 10k &= 100F_{5n+1} + 310m \\
 &= 100F_{5n+1} + 62(5m) \\
 &= 100F_{5n+1} + 62F_{5n}.
 \end{aligned}$$

Repeatedly applying the definition of Fibonacci numbers, we obtain

$$\begin{aligned}
 10k &= 24F_{5n+4} + 14F_{5n+3} \\
 &= 24F_{5n+4} + 15F_{5n+3} - F_{5n+3} \\
 &= 6F_{5n+7} + 3F_{5n+6} - F_{5n+3} \\
 &= 3F_{5n+8} + 2F_{5n+7} + F_{5n+7} - F_{5n+3} \\
 &= 2F_{5n+9} + F_{5n+8} + F_{5n+7} - F_{5n+3} \\
 &= F_{5n+10} + F_{5n+9} + F_{5n+7} - F_{5n+3} \\
 &= F_{5n+11} + F_{5n+7} - F_{5n+3}.
 \end{aligned}$$

Letting  $n = j - 2$ , we have

$$\begin{aligned}
 10k &= F_{5j+1} + F_{5j-3} - F_{5j-7} \\
 &= F_{5j+1} + F_{5(j-1)+2} - F_{5(j-2)+3}.
 \end{aligned}$$



Hence, modulo 10, we obtain

$$F_{5(j-2)+3} = (F_{5j+1} + F_{5(j-1)+2}) \bmod 10,$$

or equivalently,

$$a_{3,j-2} = (a_{2,j-1} + a_{1,j}) \bmod 10,$$

as desired. ■

**Proposition 5** In the Pisano array  $A = (a_{i,j})$ , for  $2 \leq j \leq 11$ ,  
 $a_{4,j-2} = (a_{2,j} - a_{3,j-1}) \bmod 10$ .

$$\begin{bmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & a_{2,j} \\ \cdots & a_{3,j-1} & \cdots \\ a_{4,j-2} & \cdots & \cdots \end{bmatrix}$$

**Proof.** Let  $n = j - 1$  for  $2 \leq j \leq 11$ . Then, by Proposition 3,  $F_{5n} = 5m$  for some  $m \in \mathbb{Z}$ . Let  $k = 9m + F_{5n-1}$ . Multiplying by 10, we have

$$\begin{aligned} 10k &= 90m + 10F_{5n-1} \\ &= 18(5m) + 10F_{5n-1} \\ &= 18F_{5n} + 10F_{5n-1} \\ &= 18F_{5n} + 11F_{5n-1} - F_{5n-1}. \end{aligned}$$

Repeatedly applying the definition of Fibonacci numbers, we obtain

$$\begin{aligned} 10k &= 3F_{5n+4} + F_{5n+3} - F_{5n-1} \\ &= 3F_{5n+4} + 2F_{5n+3} - F_{5n+3} - F_{5n-1} \\ &= 2F_{5n+5} + F_{5n+4} - F_{5n+3} - F_{5n-1} \\ &= F_{5n+6} + F_{5n+5} - F_{5n+3} - F_{5n-1} \\ &= F_{5n+7} - F_{5n+3} - F_{5n-1}. \end{aligned}$$

Letting  $n = j - 1$ , we have

$$10k = F_{5j+2} - F_{5(j-1)+3} - F_{5(j-2)+4}.$$

Hence, modulo 10, we obtain

$$F_{5(j-2)+4} = (F_{5j+2} - F_{5(j-1)+3}) \bmod 10,$$

or equivalently,

$$a_{4,j-2} = (a_{2,j} - a_{3,j-1}) \bmod 10,$$

as desired. ■

**Proposition 6** For any row  $i \neq 0$ , the entries in the Pisano array  $A = (a_{i,j})$  are consecutive Lucas numbers modulo 10:

$$L_n \bmod 10 = 2, 1, 3, 4, 7, 1, 8, 9, 7, 6, 3, 9, \dots,$$

with different starting points depending on the row.

$$A = \begin{bmatrix} 0 & 5 & 5 & 0 & 5 & 5 & 0 & 5 & 5 & 0 & 5 & 5 \\ 1 & 8 & 9 & 7 & 6 & 3 & 9 & \boxed{2} & 1 & 3 & 4 & 7 \\ 1 & 3 & 4 & 7 & 1 & 8 & 9 & 7 & 6 & 3 & 9 & \boxed{2} \\ \boxed{2} & \mathbf{1} & \mathbf{3} & \mathbf{4} & \mathbf{7} & \mathbf{1} & \mathbf{8} & \mathbf{9} & \mathbf{7} & \mathbf{6} & \mathbf{3} & \mathbf{9} \\ 3 & 4 & 7 & 1 & 8 & 9 & 7 & 6 & 3 & 9 & \boxed{2} & 1 \end{bmatrix}$$

We see that all rows (but the first) are horizontal translations of the first twelve terms of the Lucas sequence modulo 10. Since  $a_{3,0} = L_0 = 2$ , we will prove that row  $i = 3$  is equal to these twelve Lucas numbers modulo 10. However, before doing so, we state two lemmas, the first of which we prove in detail.

**Lemma 1**  $L_{5n} - L_n = 10k$  for some  $k \in \mathbb{Z}$ .

**Proof of Lemma 1.** We will prove that  $L_{5n} - L_n = 10k$  for some  $k \in \mathbb{Z}$ , by strong induction on  $n$ .

Base Steps:

$n = 0$ .  $L_{5n} - L_n = L_{5(0)} - L_0 = L_0 - L_0 = 2 - 2 = 0 = 10k$ , with  $k = 0$ .

$n = 1$ .  $L_{5n} - L_n = L_{5(1)} - L_1 = L_5 - L_1 = 11 - 1 = 10 = 10k$ , with  $k = 1$ .

Inductive Step: Assume our result is true for all nonnegative integers less than or equal to an arbitrary  $n$ . In particular, assume  $L_{5(n-1)} - L_{n-1} = 10q$  for some  $q \in \mathbb{Z}$ , and  $L_{5n} - L_n = 10m$  for some  $m \in \mathbb{Z}$ . We will show  $L_{5(n+1)} - L_{n+1} = 10k$  for some  $k \in \mathbb{Z}$ . Repeated application of the definition of Lucas numbers yields

$$\begin{aligned} L_{5(n+1)} - L_{n+1} &= L_{5n+5} - L_{n+1} \\ &= L_{5n+4} + L_{5n+3} - L_{n+1} \\ &= L_{5n+3} + L_{5n+2} + L_{5n+3} - L_{n+1} \\ &= 2L_{5n+3} + L_{5n+2} - L_{n+1} \\ &= 5L_{5n+1} + 3L_{5n} - L_{n+1} \\ &= 5L_{5n+1} + 2L_{5n} - (L_n + L_{n-1}) + L_{5n}. \end{aligned}$$

Then, by the inductive hypothesis, we have

$$\begin{aligned} L_{5(n+1)} - L_{n+1} &= 5L_{5n+1} + 2L_{5n} - L_{n-1} + (L_{5n} - L_n) \\ &= 5L_{5n+1} + 2L_{5n} - L_{n-1} + 10m. \end{aligned}$$

Again, a repeated application of the definition of Luca numbers gives

$$\begin{aligned} L_{5(n+1)} - L_{n+1} &= 5L_{5n} + 5L_{5n-1} + 2L_{5n} - L_{n-1} + 10m \\ &= 7L_{5n} + 5L_{5n-1} - L_{n-1} + 10m \\ &= 50L_{5n-4} + 31L_{5n-5} - L_{n-1} + 10m \\ &= 50L_{5n-4} + 30L_{5n-5} + L_{5n-5} - L_{n-1} + 10m. \end{aligned}$$

Then, by the other piece of the inductive hypothesis, we have

$$\begin{aligned} L_{5(n+1)} - L_{n+1} &= 50L_{5n-4} + 30L_{5n-5} + (L_{5(n-1)} - L_{n-1}) + 10m \\ &= 50L_{5n-4} + 30L_{5n-5} + 10q + 10m \\ &= 10(5L_{5n-4} + 3L_{5n-5}) + 10q + 10m \\ &= 10j + 10q + 10m, \text{ where } j = 5L_{5n-4} + 3L_{5n-5} \\ &= 10k, \text{ where } k = j + q + m. \end{aligned}$$

Hence, by strong induction, for all  $n \geq 0$ ,  $L_{5n} - L_n = 10k$  for some  $k \in \mathbb{Z}$ . ■

The following lemma is a well-known fact in number theory relating the Lucas and Fibonacci numbers.

**Lemma 2**  $L_n = F_{n-1} + F_{n+1}$ .

We are now prepared to prove Proposition 6 for row  $i = 3$ .

**Proof of Proposition 6.** We will prove that in the Pisano array  $A = (a_{i,j})$ , for  $0 \leq j \leq 11$ ,  $a_{3,j} = L_j \bmod 10$ . By Lemma 1,  $L_{5j} - L_j = 10q$  for some  $q \in \mathbb{Z}$ . By Proposition 3,  $F_{5j} = 5m$  for some  $m \in \mathbb{Z}$ . Let  $k = m + q$ . Multiplying by 10, we have

$$\begin{aligned} 10k &= 10m + 10q \\ &= 2(5m) + 10q \\ &= 2F_{5j} + (L_{5j} - L_j) \\ &= 2F_{5j} + (F_{5j-1} - F_{5j+1}) - L_j, \text{ by Lemma 2.} \end{aligned}$$

By applying the definition of Fibonacci numbers, we have

$$\begin{aligned}
 10k &= F_{5j+1} + (F_{5j} + F_{5j-1}) + F_{5j} - L_j \\
 &= F_{5j+1} + F_{5j+1} + F_{5j} - L_j \\
 &= F_{5j+1} + F_{5j+2} - L_j \\
 &= F_{5j+3} - L_j.
 \end{aligned}$$

Hence, modulo 10, we obtain

$$F_{5j+3} = L_j \pmod{10},$$

or equivalently,

$$a_{3,j} = L_j \pmod{10},$$

as desired. ■

#### 4. Conclusion

Looking at integer sequences in two-dimensional arrays allows visual patterns to be observed that would otherwise be lost, or only be observed by looking at every  $n$  numbers in the sequence. A further study into Pisano Arrays, more specically the general array, could yield more substantive results.

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# *Nonnegative Definiteness of Chebyshev Polynomials and Its Application*

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## 1. Introduction

Chebyshev polynomials are a pearl of analysis. These polynomials have remarkable properties. We prove the little-known property of nonnegative definiteness for Chebyshev polynomials. Based on this property, we give a very simple proof of the Sidelnikov inequality for the unit vectors on the plane. Conditions for the inequality to become an equality are given.

For vectors  $x = (x^{(1)}, x^{(2)})$ ,  $y = (y^{(1)}, y^{(2)})$  in  $\mathbb{R}^2$  we use the inner product  $\langle x, y \rangle = x^{(1)}y^{(1)} + x^{(2)}y^{(2)}$  and the norm  $\|x\| = \sqrt{\langle x, x \rangle}$ . Let  $S^1 = \{x \in \mathbb{R}^2 : \|x\| = 1\}$  be the unit circle.

We establish the property of nonnegative definiteness for Chebyshev polynomials

$$T_n(t) = \cos(n \arccos t). \quad (1)$$

In section 2, we derive the formula (1).

**Theorem 1** *Let  $m$  be a natural number. For any vectors  $x_1, \dots, x_m \in S^1$  the following inequality holds:*

$$\sum_{i=1}^m \sum_{j=1}^m T_n(\langle x_i, x_j \rangle) \geq 0. \quad (2)$$

**Proof.** Formula (1) yields the identity

$$T_n(\cos \theta) = \cos n\theta. \quad (3)$$

Write the vector  $x_i \in S^1$  in the form  $x_i = (\cos \theta_i, \sin \theta_i)$ . Then

$$\langle x_i, x_j \rangle = \cos(\theta_i - \theta_j). \quad (4)$$

Let  $S_n(X)$  denote the sum in the left-hand side of equation (2). From equations (3) and (4) we obtain

$$\begin{aligned} S_n(X) &= \sum_{i,j=1}^m \cos n(\theta_i - \theta_j) = \sum_{i,j=1}^m [\cos n\theta_i \cos n\theta_j + \sin n\theta_i \sin n\theta_j] = \\ &= \left[ \sum_{i=1}^m \cos n\theta_i \right]^2 + \left[ \sum_{i=1}^m \sin n\theta_i \right]^2. \end{aligned} \quad (5)$$

Therefore, we have

$$\sum_{i,j=1}^m T_n(\langle x_i, x_j \rangle) = \sum_{i,j=1}^m T_n(\cos(\theta_i - \theta_j)) = S_n(X) \geq 0.$$

■

Similarly we can prove that

$$\sum_{i=1}^m \sum_{j=1}^m T_n(\langle x_i, x_j \rangle) a_i a_j \geq 0$$

for all  $x_1, \dots, x_m \in S^1$  and for any real  $a_1, \dots, a_m$ .

As an application of inequality (2) we establish the Sidelnikov inequality for vectors  $x_1, \dots, x_m$  in  $S^1$ .

**Theorem 2** *Let  $k, m$  be natural numbers. For any  $x_1, \dots, x_m \in S^1$ , we have*

$$\sum_{i=1}^m \sum_{j=1}^m \langle x_i, x_j \rangle^{2k} \geq \frac{(2k-1)!!}{(2k)!!} m^2, \quad (6)$$

where  $(2k-1)!! = (2k-1) \cdot (2k-3) \cdots 3 \cdot 1$ , and  $(2k)!! = (2k) \cdot (2k-2) \cdots 4 \cdot 2$ .

V. M. Sidelnikov proved a more general inequality for vectors in the sphere in  $n$ -dimensional space. His proof [2] is rather sophisticated. Goethals, Seidel [1] and B. B. Venkov [3] gave simpler proofs. We can give a straightforward proof of inequality (6) in the case of vectors in  $S^1$  if properties of Chebyshev polynomials, including property (2), are involved.

## 2. Some Properties of Chebychev polynomials

In 1853 Chebyshev studied the problem on the polynomial  $Q(t) = t^n + a_1 t^{n-1} + \dots + a_n$  with least deviation from zero on segment  $[-1, 1]$ . Chebyshev proved that polynomial  $Q^*(t)$  of least deviation has  $n+1$  points  $-1 = t_0 < t_1 < \dots < t_n = 1$  of alternation and least deviation is equal to  $1/2^{n-1}$ , so that  $Q^*(t_i) = (-1)^{n+i}/2^{n-1}$ ,  $i = 0, 1, \dots, n$ .

Introduce Chebyshev polynomial  $T_n(t) = 2^{n-1}Q^*(t)$ . We have  $T_n(t_i) = (-1)^{n+i}$ ,  $i = 0, 1, \dots, n$ , and  $|T_n(t)| \leq 1$  on  $[-1, 1]$ .

Interior points  $t_1, \dots, t_{n-1}$  are roots of  $T'_n(t)$  and double roots of polynomial  $1 - T_n^2(t)$ . Hence,

$$T_n^{n-1}(t - t_1) \dots (t - t_{n-1}),$$

$$1 - T_n^2(t) = 2^{n-1}(1+t)(t-t_1)^2 \dots (t-t_{n-1})^2(1-t).$$

For function  $y = T_n(t)$  we obtain the differential equation

$$\frac{(y')^2}{1-y^2} = \frac{n}{1-t^2}, \quad y(1) = T_n(1) = 1.$$

In addition,  $y' > 0$  on  $(t_{n-1}, 1]$ , whence

$$-\frac{y'}{\sqrt{1-y^2}} = -\frac{n}{\sqrt{1-t^2}}, \quad t \in (t_{n-1}, 1).$$

We multiplied by  $-1$  to obtain the solution  $y(t)$  in an easier way. We have  $\arccos y = n \arccos t + C$ . The condition  $y(1) = 1$  gives  $C = 0$  and  $y = \cos(n \arccos t)$ . This equality is proved only for  $t \in (t_{n-1}, 1]$ . The function  $\phi_n(t) = \cos(n \arccos t)$  is a polynomial. Indeed, from identity  $\cos(n+1)\theta = 2 \cos(\theta) \cos n\theta - \cos(n-1)\theta$  we have  $\phi_{n+1}(t) = 2t\phi_n(t) - \phi_{n-1}(t)$ ,  $\phi_0(t) = 1$ ,  $\phi_1(t) = t$ . If polynomials  $T_n$  and  $\phi_n$  are equal on  $(t_{n-1}, 1]$  then  $T_n(t) \equiv \phi_n(t)$ .

We have established the following properties of the Chebyshev polynomials:

$$T_n(t) = \cos(n \arccos t), \quad t \in [-1, 1], \quad (7)$$

$$\begin{aligned} T_{n+1}(t) &= 2tT_n(t) - T_{n-1}(t), \quad n = 1, 2, \dots \\ T_0(t) &= 1, \quad T_1(t) = t. \end{aligned} \quad (8)$$

In particular, this recurrence yields  $T_2(t) = 2t^2 - 1$ . Also, equation (7) implies the orthogonality property:

$$\int_{-1}^1 \frac{T_n(t)T_m(t)}{\sqrt{1-t^2}} dt = 0, \quad n \neq m,$$

in particular

$$\int_{-1}^1 \frac{T_n(t)}{\sqrt{1-t^2}} dt = 0, \quad n \neq 0. \quad (9)$$

We also need the expansion of  $t^{2k}$  in terms of Chebyshev polynomials. Applying

$$tT_n(t) = \frac{1}{2}T_{n+1}(t) + \frac{1}{2}T_{n-1}(t) \quad (10)$$

in the case of  $n = 1$ , we obtain  $t^2 = \frac{1}{2}T_2(t) + \frac{1}{2}T_0(t)$ . Multiplying this by  $t$  and applying equation (10) again, we get

$$t^3 = \frac{1}{2} \left[ \frac{1}{2}T_3(t) + \frac{1}{2}T_1(t) \right] + \frac{1}{2}T_1(t) = \frac{1}{4}T_3(t) + \frac{3}{4}T_1(t).$$

Continuing in the same way, we obtain

$$t^{2k} = \sum_{l=0}^k c_l T_{2k-2l}(t), \quad (11)$$

where  $c_l > 0$  for  $l = 0, 1, \dots, k$ . We need the explicit value of the last coefficient  $c_k$ . To find it, we multiply (11) by  $1/\sqrt{1-t^2}$  and integrate over  $[-1, 1]$ ; in view of (9), this gives

$$\int_{-1}^1 \frac{t^{2k}}{\sqrt{1-t^2}} dt = c_k \int_{-1}^1 \frac{dt}{\sqrt{1-t^2}}.$$

The second integral is equal to  $\pi$ , while the first is easily computed by substitution  $t = \sin \theta$ :

$$\int_{-1}^1 \frac{t^{2k}}{\sqrt{1-t^2}} dt = 2 \int_0^{\pi/2} \sin^{2k} \theta d\theta = \frac{(2k-1)!!}{(2k)!!} \pi.$$

Finally we have  $c_k = (2k-1)!!/(2k)!!$ .



### 3. Proof of Theorem 2

**Proof.** Let the system  $X = \{x_1, \dots, x_m\}$  be a subset of  $S^1$ . As before, denote

$$S_n(X) = \sum_{i,j=1}^m T_n(\langle x_i, x_j \rangle).$$

Taking  $t = \langle x_i, x_j \rangle$  in (11) and summing over  $i, j \in 1 : m$ , we obtain

$$S := \sum_{i,j=1}^m \langle x_i, x_j \rangle^{2k} = \sum_{l=0}^k c_l S_{2k-2l}(X), \quad (12)$$

where  $c_l > 0$  and  $S_{2k-2l} \geq 0$ . Hence,  $S \geq c_k S_0(X) = c_k m^2$ . ■

We can prove the inequality

$$\sum_{i,j=1}^m \langle x_i, x_j \rangle^{2k} \geq c_k \left( \sum_{i=1}^m \|x_i\|^{2k} \right)^2$$

for any  $x_1, \dots, x_m \in \mathbb{R}^2$  using the above method with minor variations.

### 4. When the Sidelnikov inequality becomes an equality

**Lemma 1** *For the system  $X = \{x_1, \dots, x_m\}$ , where  $x_j = (\cos \theta_j, \sin \theta_j)$ , the equality*

$$\sum_{i,j=1}^m \langle x_i, x_j \rangle^{2k} = c_k m^2 \quad (13)$$

*holds iff the following conditions hold:*

$$\sum_{j=1}^m \cos n\theta_j = 0, \quad \sum_{j=1}^m \sin n\theta_j = 0, \quad n = 2, 4, \dots, 2k. \quad (14)$$

**Proof.** Rewrite equation (12) in the form

$$S = \sum_{l=0}^{k-1} c_l S_{2k-2l}(X) + c_k S_0(X),$$

where  $c_l > 0$ ,  $S_{2k-2l}(X) \geq 0$ ,  $S_0(X) = m^2$ .

If  $S = c_k m^2$  then  $S_{2k-2l}(X) = 0$ ,  $l \in 0 : k - 1$ , or  $S_n(X) = 0$ ,  $n = 2, 4, \dots, 2k$ . By equation (5), it follows that conditions (14) hold. Conversely, if the equations in (14) hold, then  $S_n(X) = 0$ ,  $n = 2, 4, \dots, 2k$ , whence  $S = c_k m^2$ . ■

Conditions (14) are valid for  $m \geq k + 1$  and for the half of the vertices of a regular  $2m$ -gon:

$$x_j = (\cos j\pi/m, \sin j\pi/m), \quad j \in 1 : m.$$

### References

- [1] J. M. Goethals and J. J. Seidel, "Spherical designs," *Proc. Symp. Pure Math. A.M.S.* **34** (1979): 255–272.
- [2] V. M. Sidelnikov, "New estimates for the closest packing of spheres in  $n$ -dimensional Euclidean space," *Mat. Sb. (N.S.)* **95 (137)** (1974): 148–158.
- [3] B. Venkov, "Réseaux et designs sphériques," in *Réseaux euclidiens, designs sphériques et formes modulaires*, Monographie de L'Enseignement Mathématique **37** (2001): 10–86.

## ***The Problem Corner***

Edited by Pat Costello

*The Problem Corner* invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before June 15, 2015. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2016 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051).

### NEW PROBLEMS 760-768

**Problem 760.** *Proposed by Mathew Cropper, Eastern Kentucky University, Richmond, KY.*

Consider a  $5 \times 5$  grid. There are  $\binom{25}{13}$  ways to fill the grid with exactly 13 X's and 12 O's. How many of these have at least one row, one column, or one diagonal with 5 X's?

**Problem 761.** *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Cluj, Romania.*

Let  $n \geq 0$  be an integer and let  $T_{2n}$  denote the  $2n^{\text{th}}$  Taylor polynomial corresponding to the cosine function at 0:

$$T_{2n}(x) = \sum_{k=1}^{n+1} (-1)^{k-1} \frac{x^{2k-2}}{(2k-2)!}$$

Let

$$I_n = \int_0^{\infty} \frac{T_{2n}(x) - \cos x}{x^{2n+2}} dx.$$

1. Prove that

$$I_n = -\frac{1}{(2n+1)(2n)} I_{n-1}, n \geq 1.$$

2. Calculate  $I_n$ .

**Problem 762.** *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, IN.*

If  $x \neq 0$ ,  $x \neq 1$ ,  $y > 0$ , and  $y \neq 1$ , then find  $y$  as a function of  $x$  provided

$$y' + y(\ln y)^2 - \frac{1}{x(x-1)} y \ln y = 0.$$

**Problem 763.** *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade", Buzau, Romania.*

Let  $x \in \mathbb{Z}$  and

$$A(x) = \begin{pmatrix} x+1 & 1 & 1 & 1 \\ 1 & x+1 & 1 & 1 \\ 1 & 1 & x+1 & 1 \\ 1 & 1 & 1 & x+1 \end{pmatrix}.$$

Compute the matrix product  $A(0) * A(1) * A(2) * A(3)$ .

**Problem 764.** *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade", Buzau, Romania.*

Calculate

$$\lim_{n \rightarrow \infty} \sqrt{n} \left( ((n+1)!)^{1/2(n+1)} - (n!)^{1/2n} \right).$$

**Problem 765.** *Proposed by Marcel Chirita, Bucharest, Romania.*

Let  $a, b, c$  be the lengths of the sides of a triangle in which  $b^2 + c^2 = a^2$ . Prove that

$$b^3 + c^3 < 3abc + (2\sqrt{2} - 1) a^3.$$

**Problem 766.** *Proposed by Marcel Chirita, Bucharest, Romania.*

Let  $x$  be an integer. Prove that if  $x^5 + 5x^3 + 15x^2 > 21x$ , then  $x^5 + 5x^3 + 15x^2 - 21x \geq 30$ .

**Problem 767.** *Proposed by the editor.*

Prove that the number  $13^{17}$  cannot be written as the sum of a square and a fifth power of another integer.

**Problem 768.** *Proposed by the editor.*

Calculate the value of the series

$$\sum_{n=0}^{\infty} \frac{(1/2)^n}{(n+2)(n+4)}.$$

## SOLUTIONS TO PROBLEMS 740-748

**Problem 740.** *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

Let  $\tau(n)$  be the number of different divisors of the positive integer  $n$  and let  $\varphi(n)$  be Euler's totient function (the number of integers less than  $n$  that are relatively prime to  $n$ ).

1. Prove that there are infinitely many  $n$  such that  $\tau(n) = \tau(\tau(2015n))$ .
2. Prove that there are infinitely many  $n$  such that  $\tau(n) = \varphi(\tau(2015n))$ .

**Solution** *by the proposer.*

1. Let  $n = 12p$  where  $p$  is a prime  $> 31$ . Then

$$\tau(n) = \tau(12p) = \tau(12)\tau(p) = 12$$

and

$$\begin{aligned} \tau(\tau(2015n)) &= \tau(\tau(24180p)) \\ &= \tau(\tau(2^2 * 3 * 5 * 13 * 31p)) \\ &= \tau(\tau(2^2 * 3 * 5 * 13 * 31)\tau(p)) \\ &= \tau(48 * 2) = \tau(96) = 12. \end{aligned}$$

2. Let  $n = 1625p = 5^3 * 13p$  where  $p$  is a prime  $> 31$ . Then

$$\tau(n) = \tau(5^3 * 13p) = \tau(5^3 * 13)\tau(p) = 4 * 2 * 2 = 16$$

and

$$\begin{aligned}\varphi(\tau(2015n)) &= \varphi(\tau(5 * 13 * 31 * 5^3 * 13)\tau(p)) \\ &= \varphi(\tau(5^4 * 13^2 * 31) * 2) \\ &= \varphi(5 * 3 * 2 * 2) = \varphi(60) = 16.\end{aligned}$$

**Problem 741.** *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

Let  $O_n = n(n + 1)$  be the  $n^{th}$  oblong number, for  $n = 1, 2, 3, \dots$ . Show that there are infinitely many pairs of distinct oblong numbers  $O_a$ ,  $O_b$  and  $O_c$ ,  $O_d$  with  $c > d$ , such that  $O_a + O_b = O_c - O_d$ .

**Solution** by Elizabeth Cobb (student), George Moore (student), and Bill Yankosky, North Carolina Wesleyan College, Rocky Mount, NC.

We first note that if  $n$  is a natural number, then using the definition of  $O_n$  we have the following:

$$\begin{aligned}O_n &= n(n + 1), & O_{n-1} &= (n - 1)n, \\ O_{n^2} &= n^2(n^2 + 1), & O_{n^2-1} &= (n^2 - 1)n^2.\end{aligned}$$

Now

$$O_n + O_{n-1} = n(n + 1) + (n - 1)n = n^2 + n + n^2 - n = 2n^2.$$

But also,

$$O_{n^2} - O_{n^2-1} = n^2(n^2 + 1) - (n^2 - 1)n^2 = n^4 + n^2 - n^4 + n^2 = 2n^2.$$

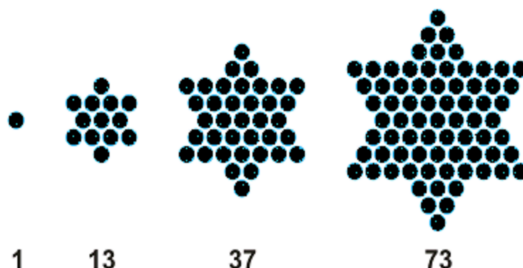
Thus for all  $n$ ,

$$O_n + O_{n-1} = O_{n^2} - O_{n^2-1}.$$

*Also solved by the Missouri State Problem Solving Group, Missouri State University, Springfield, MO; Neculai Stanciu, "George Emil Palade," Buzau, Romania, and Titu Zvonaru, Comanesti, Romania; Corneliu Manescu-Avram, Transportation High School, Ploiesti, Romania; Frank P. Battles, Plymouth, MA; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Ioan Viorel Codreanu, Satulung, Maramures, Romania; Carl Libis, University of Tennessee at Martin, Martin, TN; and the proposer.*

**Problem 742.** *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

The star numbers are given by the formula for nonnegative integers. These begin 1, 13, 37, 73, ... and are called star numbers because they can be pictured like we show here for the first few of them:



1. Prove that infinitely many of these numbers are divisible by 11 and infinitely many are divisible by 13 but none are divisible by 17 and none by 19.
2. Prove that infinitely many star numbers are the odd legs of primitive Pythagorean triples (PPTs).

**Solution** *by the Missouri State Problem Solving Group, Missouri State University, Springfield, MO.*

1. We will show more generally that if  $p$  is a prime and  $p \equiv \pm 1 \pmod{12}$  then there are infinitely many star numbers divisible by  $p$ , and if  $p$  is not congruent to  $\pm 1 \pmod{12}$ , there are no star numbers divisible by  $p$ . If  $p = 2$  or  $3$ , then dividing a star number by  $p$  leaves a remainder of 1. Note that for all other primes  $p \equiv \pm 1 \pmod{12}$  or  $p \equiv \pm 5 \pmod{12}$ . Suppose that  $6n^2 + 6n + 1 \equiv 0 \pmod{p}$ . Then  $(6n + 3)^2 \equiv 3 \pmod{p}$ . This means that 3 is a square mod  $p$  or the Legendre symbol  $(3/p) = 1$ . By the Law of Quadratic Reciprocity,  $(3/p) = (-1)^{(p-1)/2} (p/3)$ . If  $p \equiv 1 \pmod{12}$ , then  $(p/3) = (1/3)$  and  $(3/p) = 1$ . If  $p \equiv 11 \pmod{12}$ , then  $(p/3) = (2/3) = -1$  and  $(3/p) = (-1)(-1) = 1$ . In both of these cases, there must be  $u$  so that  $u^2 \equiv 3 \pmod{p}$ . We can then solve for  $v > 0$  such that  $6v + 3 \equiv u \pmod{p}$ . Taking  $n = v + pt$  with  $t \in \mathbb{N}$  gives

$$(6n + 3)^2 \equiv 3 \pmod{p} \Leftrightarrow 6n^2 + 6n + 1 \equiv 0 \pmod{p}.$$

This yields an infinite number of star numbers divisible by  $p$ . If  $p \equiv 5 \pmod{12}$ , then  $(p/3) = (2/3) = -1$  and  $(3/p) = -1$ . If  $p \equiv$

$7 \pmod{12}$ , then  $(p/3) = (1/3) = 1$  and  $(3/p) = -1$ . So star numbers are not divisible by primes (like 17 and 19) of the form  $\pm 5 \pmod{12}$ .

2. Since every odd integer  $2k + 1$  greater than 1 is the leg of a PPT (e.g.  $(2k + 1, 2k^2 + 2k, 2k^2 + 2k + 1)$ ) and all star numbers are odd, every star number greater than 1 is the leg of a PPT.

*Also solved by Neculai Stanciu, "George Emil Palade," Buzau, Romania, and Titu Zvonaru, Comanesti, Romania; Corneliu Manescu-Avram, Transportation High School, Ploiesti, Romania; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; and the proposer.*

**Problem 743.** *Proposed by Jose Luis Diaz-Barrero, BARCELONA TECH, Barcelona, Spain.*

Find all real solutions of the equation  $8^x + 15^x = 17^x$ .

**Solution** *by Ioan Viorel Codreanu, Satulung, Maramures, Romania.*

The given equation can be written in the form

$$\left(\frac{8}{17}\right)^x + \left(\frac{15}{17}\right)^x - 1 = 0.$$

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by

$$f(x) = \left(\frac{8}{17}\right)^x + \left(\frac{15}{17}\right)^x - 1.$$

Since  $8/17 < 15/17 < 1$ , the function  $f$  is strictly decreasing. Because  $f(2) = 64/289 + 225/289 - 1 = 0$ , it follows that  $x = 2$  is the unique solution.

*Also solved by Neculai Stanciu, "George Emil Palade", Buzau, Romania, and Titu Zvonaru, Comanesti, Romania; Corneliu Manescu-Avram, Transportation High School, Ploiesti, Romania; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Frank P. Battles, Plymouth, MA; Steven Sly (student), Northeastern State University, Tahlequah, OK; and the proposer.*

**Problem 744.** *Proposed by Jose Luis Diaz-Barrero, BARCELONA TECH, Barcelona, Spain.*

Let  $n \geq 3$  be a positive integer. Prove that



$$\frac{1}{2} \left( \frac{F_n + L_n}{P_{n+1}^2 + P_{n+1}P_n + F_nL_n} \right) + \frac{1}{2} \left( \frac{L_n + P_n}{P_{n+1}^2 + P_{n+1}F_n + L_nP_n} \right) + \frac{1}{2} \left( \frac{P_n + F_n}{P_{n+1}^2 + P_{n+1}L_n + P_nF_n} \right) < \frac{1}{P_{n+1}},$$

where  $F_n$ ,  $L_n$ , and  $P_n$  are the  $n^{\text{th}}$  Fibonacci, Lucas, and Pell numbers, respectively.

**Solution** by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.

The left-hand side, LHS, has the property that

$$\begin{aligned} \text{LHS} &< \left( \frac{\frac{1}{2}(F_n + L_n)}{P_{n+1}^2} + \frac{\frac{1}{2}(L_n + P_n)}{P_{n+1}^2} + \frac{\frac{1}{2}(P_n + F_n)}{P_{n+1}^2} \right) \\ &= \frac{F_n + L_n + P_n}{P_{n+1}^2} < \frac{1}{P_{n+1}}, \end{aligned}$$

where the last inequality is true since for  $n \geq 3$ , one has

$$F_n + L_n + P_n < P_{n+1}.$$

Also solved by the proposer.

**Problem 745.** Proposed by Robert Gardner and William Frazier (student), East Tennessee State University, Johnson City, Tennessee.

Consider the group  $G$  being the reals under addition. Define the set  $X = \{10^{-n} \mid n \in \mathbb{N}\}$ . Denote the subgroup of  $G$  generated by  $X$  as  $F$ . Describe the elements of  $F$ . Describe the subsets of  $X$  which also generate  $F$  and justify your answer. You may assume that  $F$  is the group  $\{\cap H \mid X \subseteq H, H \text{ is a subgroup of } G\}$  and that the elements of  $F$  are exactly the finite sums of elements of  $X$  and additive inverses of elements of  $X$ .

**Solution** by the proposers.

We claim that  $F$  consists precisely of those rational numbers with a finite decimal expansion. For any  $10^{-n} \in X$  and  $z \in \mathbb{N}$  we have  $z$  written as a sum of  $z \cdot 10^n$  copies of  $10^{-n}$ . So  $\mathbb{N} \subset F$ . Every additive subgroup of  $\langle \mathbb{R}, + \rangle$  containing  $z \in \mathbb{N}$  must also contain  $-z$ . Also, every additive subgroup must contain the identity 0. Therefore,  $\mathbb{Z} \subset F$ . We now consider the positive noninteger rational numbers with terminating decimal representations. We need a notation for such a number and adopt the following (where  $m \geq 0$  and  $n \geq 0$ ):

$$f = f_m * 10^m + f_{m-1} * 10^{m-1} + \dots + f_1 * 10^1 + f_0 * 10^0 + f_{-1} * 10^{-1} + \dots + f_{-n} * 10^{-n},$$

where the  $f_k$  are between 0 and 9,  $f_m \neq 0$  and  $f_{-n} \neq 0$ . We refer to  $n$  as the length of the decimal part of  $f$ . For such a number, we have

$$f = \left( \sum_{i=0}^{m+n} f_{-n+i} * 10^i \right) * 10^{-n}.$$

So  $f$  is a sum of copies of  $10^{-n}$ , and so  $f \in F$ . As above, any group containing  $f$  must also contain  $-f$  and so  $-f \in F$ . So  $F$  contains all rational numbers with a finite decimal expansion.

Next, the elements of  $F$  are finite sums of elements of  $X$  and additive inverses of elements of  $X$ . The sum of a finite number of these elements will have a decimal part of length at most the length of the largest-length decimal part of each element in the sum. So the finite sum represents a rational number with a finite decimal expansion. That is, the only element of  $F$  are those rational numbers with a finite decimal expansion. Hence  $F$  consists of exactly these rational numbers.

**Problem 746.** *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

On January 1, 2014, Camran borrowed \$50,000 from three of his best friends, Jacob, Jeff, and Brock, to start his own small company. He agreed to pay back the entire amount to each of his friends on December 31, 2014. The annual rate of interest on each of these loans is 0.9%, 1%, and 1.2%, respectively. According to his calculation, he will be paying 1.04% interest on the total amount borrowed. If the amount that he borrowed from Jeff is  $2\frac{1}{2}$  times the amount that he borrowed from Jacob, how much did he borrow from each of his friends?

**Solution** *by Frank P. Battles, Plymouth, MA.*

Let  $x$ ,  $y$ , and  $z$  represent the amounts borrowed from Jacob, Jeff, and Brock, respectively. We have  $x + y + z = 50000$ ,

$$0.009x + 0.01y + 0.012z = 0.0104(50000) = 520,$$

or  $9x + 10y + 12z = 520000$ , and  $y = 2.5x$ . Using the latter equation to eliminate  $y$  in the other two equations gives  $3.5x + z = 50000$  and  $34x + 12z = 520000$ . Solving this pair for  $x$  and  $z$ , and substituting

back to obtain  $y$ , we have that the amounts borrowed from Jacob, Jeff, and Brock are \$10,000, \$25,000, and \$15,000 respectively.

*Also solved by Neculai Stanciu, "George Emil Palade", Buzau, Romania, and Titu Zvonaru, Comanesti, Romania; Steven Sly (student), Northeastern State University, Tahlequah, OK; Christopher Blankenship (student), Northeastern State University, Tahlequah, OK; and the proposer.*

**Problem 747.** *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Cluj, Romania.*

Let  $f : [0, 1] \rightarrow \mathbb{R}$  be the function  $f(x) = \sqrt{\frac{1+x}{2}}$ .

1. Determine the function  $f^n = f \circ f \circ \cdots \circ f$ .
2. Calculate  $\lim_{n \rightarrow \infty} 4^n(1 - f^n(x))$  for  $x \in [0, 1]$ .

**Solution** by Corneliu Manescu-Avram, Transportation High School, Ploiesti, Romania.

1. From  $x \in [0, 1]$ , it follows that there is a number  $t \in [0, \pi/2]$  such that  $x = \cos t$ . Then we have

$$f(x) = f(\cos t) = \sqrt{\frac{1 + \cos t}{2}} = \cos \frac{t}{2},$$

and inductively

$$f^n(x) = \cos \frac{t}{2^n} = \cos \frac{1}{2^n} (\arccos x).$$

2. We have

$$\begin{aligned} \lim_{n \rightarrow \infty} 4^n(1 - f^n(x)) &= \lim_{n \rightarrow \infty} \frac{1 - \cos \frac{t}{2^n}}{\frac{1}{2^{2n}}} \\ &= \lim_{\substack{y \rightarrow 0 \\ y > 0}} \frac{1 - \cos ty}{y^2} \\ &= \lim_{\substack{y \rightarrow 0 \\ y > 0}} \frac{t \sin ty}{2y} = \frac{t^2}{2} = \frac{(\arccos x)^2}{2}, \end{aligned}$$

by applying L'Hopital's Rule.

*Also solved by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; and the proposer.*

**Problem 748.** *Proposed by the editor.*

Find the line that goes through the point  $(1, 1)$  which has area under the line in the first quadrant equal to  $\frac{5}{2}$ .

**Solution** by Steven Sly (student), Northeastern State University, Tahlequah, OK.

Let  $y = mx + b$  be a line containing the point  $(1, 1)$  and having area under the line in the first quadrant of  $\frac{5}{2}$ . So we know  $1 = m + b$ . This implies  $m = 1 - b$ . The line is then  $y = (1 - b)x + b$ . This intersects the  $x$ -axis at  $x = -b/(1 - b)$ . Since the area under the line is  $\frac{5}{2}$ ,

$$\begin{aligned} \frac{5}{2} &= \int_0^{-b/(1-b)} (1-b)x + b \, dx \\ &= \left[ (1-b)\frac{x^2}{2} + bx \right]_0^{-b/(1-b)} = -\frac{b^2}{2(1-b)}. \end{aligned}$$

This gives the quadratic equation  $b^2 - 5b + 5 = 0$ . The quadratic formula gives  $b = \frac{5 \pm \sqrt{5}}{2}$ . There are two lines that work; they are

$$y = \left( \frac{-3 - \sqrt{5}}{2} \right) x + \left( \frac{5 + \sqrt{5}}{2} \right)$$

and

$$y = \left( \frac{-3 + \sqrt{5}}{2} \right) x + \left( \frac{5 - \sqrt{5}}{2} \right).$$

Also solved by Carl Libis, University of Tennessee at Martin, Martin, TN; Elizabeth Cobb (student), George Moore (student), and Bill Yankosky, North Carolina Wesleyan College, Rocky Mount, NC; the Intro to Proof class, Northeastern State University, Tahlequah, OK; Corneliu Manescu-Avram, Transportation High School, Ploiesti, Romania; Frank P. Battles, Plymouth, MA; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Neculai Stanciu, "George Emil Palade" School, Buzau, Romania and Titu Zvonaru, Comanesti, Romania; Caleb Garscia (student), Eastern Kentucky University, Richmond, KY; and the proposer.

## ***Kappa Mu Epsilon News***

Edited by Peter Skoner, Historian

**Updated information as of May 2015**

Send news of chapter activities and other noteworthy KME events to

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### **Installation Report**

California Eta Chapter  
Fresno Pacific University



The installation of the California Eta chapter of Kappa Mu Epsilon was held at 5:00 P.M. on March 24, 2015 in the BC Lounge of North Hall on the campus of Fresno Pacific University in Fresno, California. Dr. Rhonda McKee, KME president, served as the installing officer, and Dr. Ronald Pratt, a KME member and member of the FPU faculty, served as conductor. Light refreshments were served as initiates and guests gathered for the event. Dr. Karen Cianci, dean of the College of Natural Sciences, welcomed the guests and congratulated the initiates. Dr. Marshall Johnston, director of the University Scholars Program, spoke to the initiates about the importance of the fellowship of honor societies such as KME.

The following charter members were initiated during the ceremony.  
(Those who were also installed as officers are noted below.)

Chase Conklin (Secretary)	Kristalyn Patzkowski
Lindsey Ham	Alfredo Ramirez (Vice President)
Katherine Lee	Elijah Roth (Treasurer)
David Maes	Colton Wilson (President)
Alexander Ours	Cho Fai Wong
Jonathan Passmore	Michelle Yi

Dr. Terrence Yi was installed as the corresponding secretary and Dr. Ronald Pratt as the faculty sponsor. Following the installation ceremony, Dr. McKee gave a talk titled “Spirograph Math,” and then the group enjoyed a wonderful dinner at DiCicco’s restaurant. About 30 people attended the installation.

## Installation Report

Ohio Theta Chapter  
Capital University



On Friday, April 24, 2015 at 6:30 P.M., the Ohio Theta chapter of Kappa Mu Epsilon was installed at Capital University, a liberal arts college in the small town of Bexley near Columbus, Ohio. Sixteen new members and their families and friends joined the mathematics faculty in the Weiler Suites of the Harry C. Moores Student Union for the ceremony.

Dr. Pete Skoner, National Historian, shared the history of Kappa Mu Epsilon and served as installing officer. Dr. Pedro Muíño, Great Lakes Regional Director, gave a great presentation entitled “Pi is a Harsh Mistress: A Tale of Irrational Passion,” which interested both math and non-math attendees alike. The faculty sponsor and conducting officer Dr. Jonathan Stadler was initiated into Kappa Mu Epsilon when he was an undergraduate student at the Ohio Alpha Chapter at Bowling Green State University.

He also gave a presentation at the 32nd Biennial National Convention in 1999 at Florida Beta at Florida Southern College in Lakeland, FL. Dr. Paula Federico was installed as corresponding secretary of Ohio Theta, along with President Abigail Neininger, Vice President Julia Kunkel, Secretary Jaime Ashworth, and Treasurer Oscar O'Flaherty.

The following student charter members were initiated during the installation.

Jaime Ashworth	Abigail Neininger
Patrick Donahue	Brittany Nicholson
John Einsweiler	Oscar O'Flaherty
Rachel Fountain	Ashley Pallone
Matthew Jennell	Jessica Potts
Julia Kunkel	Otto Shaw
Alex Lambert	

The three faculty charter members of Ohio Theta are Dr. Federico, Dr. Leigh Johnson, and Dr. Patrick Shields. Congratulations, photographs, fellowship, and refreshments were enjoyed by all following the ceremony.

## Installation Report

Georgia Zeta Chapter  
Georgia Gwinnett College



The installation of the Georgia Zeta Chapter of Kappa Mu Epsilon was held at 2:00 P.M. on Tuesday, April 28, 2015, in the Library's Heritage Room on the campus of Georgia Gwinnett College in Lawrenceville, GA. The meeting was conducted by Dr. Stan Perrine and Dr. James Price, Associate Professors of Mathematics at GGC. Dr. David Dempsey, KME South Eastern Regional Director, served as the installing officer. Dr. Tom Mundie, Dean of the School of Science and Technology, welcomed and

congratulated the initiates.

The following student charter members were initiated during the installation. (Those who were also installed as officers are noted below.)

Patrick Brooks (Vice President)	William Machamer
Bess Burnett	Heather McAfee
Mai Bao Doan	Tayeba Mohammed
James Frye	Andrea Robles Benitez
Christina Holt	Shahriyar Roshan Zamir (Secretary)
Amanda Iduate (Treasurer)	Shawn Sanderlin
Christopher Lohrmann	Amy Warner
Audrey Lynn (President)	

And the following faculty were initiated as charter members. (Those who were also installed as officers are noted below.)

Alvina Atkinson	Junkoo Park
Kodwo Annan	Daniel Pinzon
Tee Barron	Katherine Pinzon
Jamye Curry	Daniel Prager
Amy H. Erickson	Joshua Roberts
Keith Erickson	Lee Ann Roberts
Boyko Gyurov	Michael Saum
Paula Krone	Jennifer Sinclair
Zhongxiao Li	Marty Thomas
Ekaterina Nathanson	

Dr. Jamye Curry was installed as the corresponding secretary and Drs. Jenny Sinclair and Livy Uko as the faculty co-sponsors. The afternoon concluded with a reception and refreshments. About 50 people attended the event.



## Chapter News

### **AL Gamma – University of Montevallo**

Chapter President – Stephanie Dorough; 17 Current Members; 14 New Members

Other Fall 2014 Officers: Brandon McMahan, Secretary; Dr. Scott Varagona, Corresponding Secretary; and Dr. Michael Sterner, Faculty Sponsor

### **AL Zeta – Birmingham-Southern College**

Chapter President – Chase Hoffman; 18 Current Members; 16 New Members

Other Fall 2014 Officers: Ryan Deveikis, Vice President; Sam Crowder, Secretary; Nirja Patel, Treasurer; and Maria Stadnik, Corresponding Secretary and Faculty Sponsor

We kicked off the fall semester with a Movie Night viewing of Good Will Hunting on Wednesday, September 10. The fall KME colloquium was Thursday, November 20. Ryan Davis from the University of Mississippi gave a math finance lecture entitled "Canary in a Coal Mine? One-share Orders and Trades." Ryan is a doctoral student at Ole Miss and taught here at BSC from 2009-2012. We also had a few problem solving sessions throughout the fall semester to get some of our students ready for the Putnam Mathematical Competition.

New Initiates – Kristen Archer, Catherine Bowers, Brittany Hand, Jesse Handlon, Yuqing Huang, Erez Kaminski, Jessica Kariuki, Andrew Garrison Arthur Linn, Huasiyuan Liu, Caitlin McCurdy, Avery Newcomb, Adam Pratt, Anna Marie Ronderos, Austin West, Austin Williams, and Xiaokun Ye.

### **AL Theta – Jacksonville State University**

Chapter President – Shannon Bolton; 50 Current Members; 35 New Members

Other Fall 2014 Officers: Paitra Onkst, Vice President; Tucker Davis, Secretary; Jeremy Moses, Treasurer; and Dr. David Dempsey, Corresponding Secretary and Faculty Sponsor

The Alabama Theta chapter opened the 2014-15 academic year with an Ice Cream Social and Game Night on August 29; meetings have been held biweekly thereafter. Members met for dinner and a local dramatic production of Spamalot on September 26. October activities included disc golf on October 22 and a costume/pumpkin-carving party on October 24. On November 19, members met for dinner and then traveled to a local game center for trampoline fun. A large Christmas party was held on December 5. Spring activities include a dinner/game night in January and a definite Super Pi Day activity. The annual Spring Initiation Ceremony is planned for February 23, 2015; there will be 35 new members. After planning last

year's convention, our members are excited about traveling to Daytona Beach for the 2015 Convention!

New Initiates – Roxanne Grace Bailey, Jasmine Amanda Beaudette, Victoria Masako Beitel, Kameron Nicole Blair, Morgan Blakeli Bolt, Joanna Lee Burke, Cindy Linh Chung, David Edward Crawford, Tara Gabrielle Farmer, Ashley Galloway, Timothy Garrett, Sadie Gregory, Faith L. Griffith, Natalie Michele Gunter, Daniel Trey Hilburn, William M.

Hollingsworth, Bobbie Hoskins, Chandria E. McCurdy, Paige Nicole Moon, Emilee Parkerson, Savannah Payne, Brandon Lawrence Phillips, Shawanna N. Roper, Jacquelyn Kateri Rush, Sari Sabouh, Luis Segura Hidalgo, Torey Adison Smith, Katherine Stedham, Bryan K. Sutton, Jr., James Darin Thompson, Jr., Scott M. Troutman, Dexter Lee Vernon, Joshua Goodwin Walker, Johnathan Watts, and Whitney Blair Wood.

#### **AR Beta – Henderson State University**

Chapter President – Erin Yancey; 12 Current Members; 3 New Members  
Other Fall 2014 Officers: Sara Watkins, Vice President; Jacob Woodall, Secretary; Spencer Scroggins, Treasurer; Fred Worth, Corresponding Secretary; and Carolyn Eoff, Faculty Sponsor

New Initiates – Molly L. Pate, Zachary M. Winfield, and Carmen M. Wise.

#### **CA Eta – Fresno Pacific University**

Corresponding Secretary – Terence Yi; 12 New Members

New Initiates – Chase Conklin, Lindsey Ham, Katharine Lee, David Maes, Alexander Ours, Jonathan Passmore, Kristalyn Patzkowski, Alfredo Ramirez, Elijah Roth, Colton Wilson, Cho Fai Wong, and Michelle Yi.

#### **FL Beta – Florida Southern College**

Chapter President – Christopher Morgan; 46 Current Members; 7 New Members

Other Fall 2014 Officers: Virginia Machado, Vice President; Wiresh Punwasi, Secretary and Corresponding Secretary; and Dr. Lisa DeCastro, Faculty Sponsor

We are still getting ourselves started so we have not had much going on. Our last initiation was in the Spring of 2014 and we are preparing for another initiation in March 2015. We hope to generate interest and create some momentum this semester. We also hope to send a group to the convention in April.

#### **FL Gamma – Southeastern University**

Corresponding Secretary – Dr. Berhane Ghaim; 4 New Members

New Initiates – Taylor Burrell, Kayla Rebecca Connors, Katharina Nagassima, and Kaitlyn G. Ricci.

#### **FL Delta – Embry Riddle Aeronautical University**

Chapter President – Ajay Raghavendra; 20 Current Members; 10 New Members

Other Fall 2014 Officers: Tianyuan Zhao, Vice President; Sara Huey, Sec-

retary; Benjamin Dillahun, Treasurer; Dr. Jayathi Raghavan, Corresponding Secretary and Faculty Sponsor; and Greg Spradlin, Faculty Sponsor. The KME FL Delta Chapter was installed on April 22, 2014, by the National President Dr. Rhonda McKee, Dr. Jayathi Raghavan Professor of Mathematics at Embry-Riddle as the corresponding secretary, and Mr. Ajay Raghavendra and Mr. Benjamin D. Dillahun as the chapter's founding president and treasurer. The chapter also conducted its second initiation ceremony on September 2, 2014 where ten additional member applications were reviewed and initiated. With the active participation from the majority of members, the FL Delta Chapter achieved a successful Fall 2014 semester and is striving for a successful Spring 2015 semester. Activities fair at Embry-Riddle is a campus event at the beginning of every Fall semester for all student organizations to advertise their presence on campus and the best time to recruit new members into the organization. During the Fall 2014 activities fair, our chapter's booth displayed the ongoing research projects by chapter members. Students were also shown interesting mathematical facts using display boards and on-site illustrations. In this event, we had more than 30 students showing their interests in joining our chapter. Another event of our chapter in the Fall semester was having a field trip to Miami. In this trip, members visited the world's biggest wave research tank and other facilities in the University of Miami's Rosenstiel School of Marine and Atmospheric Science. Over lunch, our members sat together with university's faculty and scientists to discuss information on past projects, recent discoveries and future outlooks on the marine and atmospheric studies. In the afternoon, we visited the National Oceanic and Atmospheric Administration's Atlantic Oceanographic and Meteorological Laboratory. The director and scientists at AOML gave us several speeches on their history and current research projects to further our understanding of tropical cyclones. During presentations, members understood the importance of mathematics in oceanic and atmospheric studies. The chapter would like to recognize Dr. Stefan Mancas and Dr. Harihar Khanal for their interest and helping to make this trip a success. Also, this trip was fully funded by ERAU's Student Government Association. Last but not least was the community service project. Members joined the yard work in Lilian's Place, a project of The Heritage Preservation Trust of Volusia County. This place was the oldest house in Daytona's beachside. Through three hours of yard work, members learned the history of this place, on the other hand, this event was highly recognized by the local community. In Spring 2015, we will recruit more members into the organization. We are also planning to participate in the National Conference of SIAM, the ASPIRE '15 conference and many more community service project. Finally

our chapter will be proudly hosting the KME National Conference at our campus.

New Initiates – Matthew J. Burchfield, Stephen P. Cronin, Remelisa P. Esteves, Kristopher R. Gaffey, Kevin Leong, Carlos Marquez, Joshua J. Meyers, Jacky H. Qi, Gregory Spradlin, and Thomas W. Stone.

#### **GA Beta – Georgia College and State University**

Corresponding Secretary – Rodica Cazacu; 7 New Members

New Initiates – Samantha F. Clapp, Meghann Moriah Gibson, Megan L. McGurl, Kirsten D. Morris, Matthew W. Pearson, Rhett M. Roberts, and Cuyler D. Warnock.

#### **GA Gamma – Piedmont College**

Corresponding Secretary – Hope Menzel; 4 New Members

New Initiates – Elizabeth Gale, Maxwell T. Oberkofler, Jennifer Caitlin Pitt, and Jenny Whitmire.

#### **HI Alpha – Hawaii Pacific University**

Chapter President – Keila Elderts; 20 Current Members; 5 New Members

Other Fall 2014 Officer: Marnick Pigao, Vice President; Hailey Walters, Secretary; Josh Troglia, Treasurer; and Tara Davis, Corresponding Secretary and Faculty Sponsor

New Initiates – Hailey Brianne Lynn Collins, Taylor N. Martin, Mar Dominick Pigao, Joshua David Troglia, and Marissa Mika Yamamoto.

#### **IA Alpha – University of Northern Iowa**

Chapter President – Elizabeth Johnson; 35 Current Members; 6 New Members

Other Fall 2014 Officers: Ben Castle, Vice President; Allysha Whitsell, Secretary; Paige Hageman, Treasurer; and Mark D. Ecker, Corresponding Secretary and Faculty Sponsor

Our first fall KME meeting was held on October 8, 2014 at Professor Mark Ecker's house where student member Luke Peterson presented "In Defense of Defense: A Statistical Look at Roster Construction, Coaching Strategy and Team Defense in the NBA." Student member Jacob Oswald demonstrated his Visual Basic program that he wrote to play Yahtzee at our second meeting on November 12, 2014 at Professor Doug Mupasiri's home. Student member Aaron Manternach addressed the fall initiation banquet with "Analysis Factors that Influence the Number of Wins in the NFL." Our fall banquet was held at Peppers Grill and Sports Pub in Cedar Falls on December 10, 2014, where six new members were initiated.

New Initiates – Emily Bachmeier, Katy Goodmundson, Toby Maggert, Hannah Miller, Julia North, and Lindsey Pederson.

#### **IA Epsilon – Central College**

Corresponding Secretary – Russell E. Goodman; 8 New Members

New Initiates – Morgan Ernst, Joseph Heath, Matthew Keppen, Drew Lindaman, Parker

Smith, Blake Thingstad, Annie Weaver, and Paige Wilkin.

### **IL Zeta – Dominican University**

Chapter President – Kaitie Holden; 25 Current Members

Other Fall 2014 Officers: Maria Beltran, Vice President; Theodora Rondstadt, Secretary; Radhika Patel, Treasurer; Aliza Steurer, Corresponding Secretary and Faculty Sponsor

The Illinois Zeta Chapter of KME operates together with Dominican University's Math Club. Thus, the above officers are members of KME or Math Club (or both) and the below activities were prepared by both groups. During fall 2014, we arranged a Research Opportunities and Internship Panel, open to the Dominican University community, where five of our students discussed and answered questions about their experiences with REUs, research projects, and internships in mathematics or mathematics-related fields. Also, Dr. Marco Martinez from nearby North Central College gave a great talk about using mathematics to control gypsy moth populations. Lastly, throughout the semester, we held biweekly meetings during which students played games or worked on fun problems from logic, geometry, and other subjects.

### **IL Eta – Western Illinois University**

Corresponding Secretary – Amy Ekanayake; 7 New Members

New Initiates – Julie Herek, Amanda Meiners, Michael Power, Mitchell Riley, Nicole Szabo, Charlotte Wilson, and Jacob Winters.

### **IN Delta – University of Evansville**

Corresponding Secretary – Adam Salminen; 14 New Members

New Initiates – Ashley L. Bauer, Meghan E. Becker, Alexandra N. Billhartz, Travis A. Chatham, Joshua Alexander Kunju Herrera, Eugene Nii Akwei Hoffman, Ethan L. Lehmann, Reagin Taylor McNeill, Mary Therese Padberg, Aaron Daniel Reynolds, Maria Christina Schilling, Brenna R. Siscoe, Abigail N. Smith, and Daniel Vibbert.

### **KS Alpha – Pittsburg State University**

Corresponding Secretary – Tim Flood; 26 New Members

New Initiates – Nassir Alkhan, Ahlam Alzharani, Michael Anderson, Amie Beggs, Colton Beitzinger, Zachary Botkin, Jayden Bowen, Zachary Bowen, Shelby Brennon, Margaret Detrick, Garrett Eckols, Andrew Ferrell, Jason Fenske, Jordan Garbin, Louis Handte, Ashley Keller, Susan Martin, Caitlin Ogden, Haley Reynolds, Melissa Ryan, Georgette Searan, Jennifer Sexton, Jacqueline Smoot, Esther Thuo, Whitlea Ulrich, and Peter Villa.

### **KS Beta – Emporia State University**

Corresponding Secretary – Connie Schrock; 13 New Members

New Initiates – Stephanie Albert, Makenzie Blitz, Tingua Chen, Sarah Clark, Michelle Foster, Rachel Hull, Jeana Johnson, Luke Langston, Zijian Lui, Asia Mackey, Brian Mosier, Elliot McNelly, and Dallas Shafer.

**KS Delta – Washburn University**

Chapter President – Jonathan Tyler; 15 Current Members; 7 New Members  
 Other Fall 2014 Officers: Branden Childers, Vice President and Secretary;  
 Paige Eslick, Treasurer; and Kevin Charlwood, Corresponding Secretary  
 and Faculty Sponsor

The Kansas Delta chapter held its spring initiation banquet on March 3, 2014 to initiate 6 new initiates. Our chapter also attended the national convention held at Jacksonville State University April 3 – 5 sending two students and one faculty member. Our presenter, Brandon Marshall, won a “top 2” award for his presentation, “Colorfully Complex; Visualizing Complex Numbers using Domain Coloring.” Our KME chapter met three times during the fall semester along with our math club, Club Mathematica.

New Initiates – Ryan DeLong, Jalen Dickson, Huimin Liu, Beth McNamee, Katelynn Robinson, Katelyn Rollins, and Leanna Willer.

**KY Beta – University of the Cumberland**

Corresponding Secretary – Jonathan Ramey; 15 New Members

New Initiates – Terra Baker, Bethany Bargo, Ashley Bee, E. Nicole Braden, Emily Campbell, Mollie Creech, Anthony DeRocchis, Thomas Adam Green, Rufus Deron Higgins, Samuel Huntley, Sheena Jackson, Cedric Muteshi, Kallye Joanah Renner, Breanne Schroeder, and Xiaohang Zhou.

**MA Beta – Stonehill College**

Corresponding Secretary – Timothy Woodcock; 11 Current Members; 5 New Members

Massachusetts Beta marked the closing of the fall semester, now by tradition, by sponsoring a pizza party for all mathematics majors on the last day of classes. Along with great food and camaraderie, mathematical stories were traded, and several interesting problems were discussed over the course of the gathering.

New Initiates – Julia Afeltra, David Lawson, Siobhan McAlister, Abigail Soraci, and Emily Wiley.

**MD Delta – Frostburg State University**

Chapter President – Chris Colwander; 25 Current Members; 15 New Members

Other Fall 2014 Officers: David Foerster, Vice President; Olivia Elisio, Secretary; Michelle Welch, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet and Justin Dunmyre, Faculty Sponsors

Our September and October meetings were devoted to watching some math related videos and planning a couple of activities. We participated in the university’s annual Majors Fair and had a successful food and bake

sale in November. Discussions were held on designing a new T-shirt for members and obtaining funding for their purchase. The highlight of the semester was Dr. Frank Barnett's lecture on mathematical paradoxes during our last meeting of the semester. We offer congratulations to graduating senior Phil Rose.

New Initiates – Morgan Allman, Blake Bender, Ryan Diehl, Caeman Feller, Steven Messmer, Amanda Monahan, Tyler Ram, Isaac Robinson, Chris Santina, Michael Shannon, JT Singo, Alex Treasure, Dustin Ullery, Kerwin Yoder, and Sara Zachritz.

### **MI Beta – Central Michigan University**

Corresponding Secretary, Dr. Sivaram K. Narayan; and Faculty Sponsor, Dr. Meera G. Mainkar; 5 New Members

During the academic semesters KME met once every two weeks. Five new members were initiated in the spring of 2014. During February members raised money through a used book sale held jointly with other student organizations in the department. KME members sold travel mugs and t-shirts for Pi day (March 14). A fundraising event was organized through the Italian Oven restaurant in April 2014. The money raised from these events and membership dues was used for buying pizza on meeting days and for conducting an initiation ceremony. Five KME members and faculty advisor Narayan attended the Michigan Undergraduate Mathematics Conference (MUMC) at Eastern Michigan University on March 8, 2014. Professor Hugh Montgomery, University of Michigan, gave a talk on March 6, 2014; the title of his talk was "Peg Solitaire." Dr. Ben Salisbury, CMU, gave a talk titled "Representation of the Symmetric Group" on March 20, 2014. Four KME members attended the KME National Convention at Jacksonville State University, Jacksonville, Alabama from April 9-12, 2014. Professor Robert L. Devaney, Professor of Mathematics at Boston University and the President of Mathematical Association of America gave a talk on April 16, 2014; the title of his talk was "The Fractal Geometry of the Mandelbrot Set." KME members raised money to support the "Special Olympics" event by selling medallions. On Friday, April 25 (Grace Friday), 1-5 pm, KME members held a Math-a-palooza event and tutored undergraduate students before the final exams. Dr. Narayan gave 15-20 minute talks on various topics in mathematics and its applications during many meetings in both the spring and fall semesters. A DVD show, "The Great Pi/e Debate: Which is the better number" was held on October 2, 2014. Dr. Tibor Marcinek, CMU, gave a talk on October 16, 2014; it was titled, "Model-Centered Learning: Creating Visual Mathematical Models with GeoGebra." Five members took part in the 20th Annual Michigan Autumn Take Home (MATH) Challenge on November 1, 2014. On November 13, 2014, Rachel Domagalski gave a talk on her summer 2014 REU

research project on “Frames in Finite Dimensions;” it was also an informational meeting about summer research opportunities. Math-a-palooza took place on December 5, 2014 from 1-5 p.m.; KME members tutored undergraduate students to prepare for their final exams. On December 6, 2014, three KME members took part in the William Lowell Putnam Mathematics Competition.

New Initiates – James Asmus, Rachel Domagalski, Jocelyn Faydenko, Elmo Harris, and Ronald Klingler.

### **MI Delta – Hillsdale College**

Chapter President – Ayla Meyer; 25 Current Members; 7 New Members  
Other Fall 2014 Officers: Joshua Mirth, Vice President; Arena Govier, Secretary; JoAnna Waterman, Treasurer; and Dr. David Murphy, Corresponding Secretary and Faculty Sponsor

During the Fall 2014 semester, we initiated 7 new members, though the formal initiation ceremony will take place this spring when all of this year’s new members officially join KME. We cosponsored an event with the Classics Honorary, Eta Sigma Phi, in which we watched the NOVA documentary “Infinite Secrets” about the lost palimpsest of Archimedes that was recently recovered (October 6). Next, with the physics department, we ran an REU information session (October 14). We then ran the Lyceum society’s Honorary Colloquium discussing Alfred North Whitehead’s essay, “Mathematics as an Element of Human Thought” (November 7). Besides these events, our KME chapter enjoyed several math talks including “Coconuts, Fingerprints, and Polyhedra” by Peter Ulrickson (Notre Dame graduate student), “When Does the Maximizing Hill Have One Peak?—Concavifying the Quasiconcave” by Dr. Eric Rasmusen (Indiana University), and “Fractals, p-adics, and a problem of Erdős” by Dr. Will Abram (Hillsdale College).

### **MO Alpha – Missouri State University**

Chapter President – Steven C. Cornelius; 39 Current Members; 14 New Members

Other Fall 2014 Officers: Julie Barnum, Vice President; Ashley Kingston, Secretary; Megan Leppien, Treasurer; and Jorge Rebaza, Corresponding Secretary and Faculty Sponsor

As every semester, we had three seminars: Seminar 1: Wednesday, September 24, 2014—Dr. Stan Adamson, faculty member in the Insurance Program at MSU, talked about “Actuarial Mathematics and a Minor in Insurance can make a Major Difference,” and about careers in actuarial mathematics. Pizza and soda were served. Seminar 2: Thursday, October 30, 2014—Dr. Troy Hicks, Emeritus Professor from the Missouri University of Science and Technology talked about “Teaching Mathematics and



Interesting Mathematical Problems.” Pizza and soda were served. During this seminar we also initiated 13 new KME members. Seminar 3: Tuesday, November 18, 2014. Three students from the Senior Seminar class (MTH 497) presented their papers: “Group Theory of the Rubik’s Cube”, by Andrew Ashley, “The RSA Algorithm” by Andrew Riggs, and “Prime Number Conjectures”, by John Sutton. Pizza and soda were served. As in every fall semester, we organized a picnic on Wednesday, September 10, 2014, starting at 5:00 PM at Phelps Grove Park. As usual, we had a great turnout of students, faculty, and their families! We also had an end-of-semester party on Thursday, December 5, 2014, the last day of classes. We had lots of games music, food, drinks, and desserts. The Secret Santa gift exchange was a great success!

New Initiates – Spencer Adams, Andrea Bagwell, Jennifer Barnes, Benjamin Borgstede, Paige Buchmueller, Bethany Carter, Chelsea Cozort, Rebecca Crow, Kimberlee Davis, Riker Douglas, Steven Harrelson, Hayley Hutson, Ashley Kingston, and Lauren Kopf.

#### **MO Beta – University of Central Missouri**

Corresponding Secretary – Rhonda McKee

New Initiates – Katherine A. Bicker, Daniel Carey, Joshua Covey, Christina Duerr, Samuel Golomeke, Layne Jackson, Lauren Langkamp, James Mixco, Nicholas Purcell, Josiah L. Reynolds, Katie Ringhausen, Jessica Sanders, Roland Smith, Nicole Turner, and Anthony Willett.

#### **MO Zeta – Missouri University of Science and Technology**

Corresponding Secretary – Dr. Vy K. Le; 38 New Members

New Initiates – Fuad Aaden, Kelsey Bardell, Max Bevell, Arielle Bodine, Hayden Cayse, Julia Clements, Matt Courtney, Chelsea Diestelkamp, Adam Dwilewicz, Hannah Hill, Holden Hershenson, John Hoffmann, Josip Juric, Cory Karle, Glenn Konersmann, Jason Mao, Rachel McCormick, Austin Miller, Zach Miller, Jamie Mucalo, Alex Mundahl, Katherine Overend, Sahilkumar Parikh, Juan David Remolina, Kyle Nocona Sanders, Payne Sawyer, Delaney Sexton, Sarah Shelburg, Kimberly Shrine, Maegan Shults, Keith Sponsler, Joshua Sutter, Christian Teske, Andrew Thorn, Joseph Volpe, Eric Walter, Ian Wilson, and Connor Wittmaier.

#### **MO Eta – Truman State University**

Corresponding Secretary – David Garth; 9 New Members

New Initiates – Amanda Clevenger, Valerie Free, Melinda Mathews, Miranda Meyer, Stephen Paul, Raquel Rhoads, Amanda Schlaker, James Sorsen, and Andrew Stratmann.

#### **MO Theta – Evangel University**

Chapter President – Kaitlyn Hong; 12 Current Members; 11 New Members

Other Fall 2014 Officers: Bethany VanderMolen, Vice President; and Don Tosh, Corresponding Secretary and Faculty Sponsor

Meetings were held monthly. In November we held an Ice Cream Social

at the home of Don Tosh.

New Initiates – Barak Barton, Kevin Grimes II, Ethan Grumke, Lindsay Jenkins, Christopher Knoth, Rebecca Grace Oord, Samantha Breanna Orr, Jeremy Christian Sparks, Sarah Specter, John Mark Vallelonga, and Kara Walla.

#### **MO Nu – Columbia College**

Corresponding Secretary – Kenny Felts; 3 New Members

New Initiates – Ryan Frappier, Joshua Giliberto, and Amanda Lutz.

#### **MS Alpha – Mississippi University for Women**

Chapter President – Britny Sarver; 15 Current Members; 4 New Members

Other Fall 2014 Officers: Maggie Leake, Vice President; Sumitra Karki, Secretary; Dr. Joshua Hanes, Treasurer, Corresponding Secretary and Faculty Sponsor

We had one of our larger initiation classes this past fall. We anticipate having even more new members this spring. We participated in Operation Christmas Child in November, packing shoe boxes which are sent all around the globe to children in need. We plan on walking to support Multiple Sclerosis research later this semester.

New Initiates – Mandi Elam, Ciara Peoples, Audra Polk, and Birendra Sharma.

#### **NC Eta – Johnson C. Smith University**

Corresponding Secretary – Brian Hunt; 3 New Members

New Initiates – Radijah Hudson, Jheanelle Linton, and Keturah Wallace.

#### **NE Beta – University of Nebraska-Kearney**

Chapter President – Stephanie Rudder; 11 Current Members

Other Fall 2014 Officers: Sara Ambrose, Vice President; Lindsey Pearson, Secretary; Natalie Hanisch, Treasurer; and Dr. Katherine Kime, Corresponding Secretary and Faculty Sponsor

This past fall, KME planned the visit of Dr. Jeffrey Weeks, MacArthur Fellow, to the UNK campus. Funding was obtained from the UNK student government. Dr. Weeks gave two public talks, on January 14 and 15, 2015. The talks were titled “Shape of Space” and “Visualizing the Fourth Dimension.” About eighty people—students, faculty and others—attended each night. We also had a “Meet and Greet” one afternoon, and a group dinner one evening. Dr. Weeks featured the KME crest in his presentation the first night and commented on the hand-painted crest that we have displayed here in the Math/Stat Dept. He also commented several times on how impressed he was that the event had been organized by the students of KME, particularly the President, Stephanie Rudder.

#### **NE Delta – Nebraska Wesleyan University**

Chapter President – Leanne Hinrichs; 11 Current Members

Other Fall 2014 Officers: Sheridan Mason, Vice President; Connor Bohlken, Treasurer; and Melissa Erdmann, Corresponding Secretary and Faculty

**Sponsor**

In Fall 2014 we had a math game night, an ultimate Frisbee event, an evening where math professors discussed their research, and a holiday party with homemade chili and math/physics carols. Fun times!

**NJ Delta – Centenary College**

Corresponding Secretary – Kathy Turrisi; and Faculty Sponsor – Linda Ritchie

The New Jersey Delta Chapter continues to have an outreach to assist students in math.

**NJ Epsilon – New Jersey City University**

Corresponding Secretary – Beimnet Teclezghi; 27 New Members

New Initiates – Edwin Abreau, Sucel Arias, Aubrey Campbell, Mustack Chowdhury, Angela Herrera, Timothy Hollis, Yelena Jacome, Starlette Kirby, Uyen Konn, Aziz Manur, Margaret Massett, June McIntyre, Oscar Mendez, Louis Nikrumah, Joel Nunez, Ugochukwu Onyemgba, Jennifer Rak, Michael Reigle, Siara Rodriguez, Joe Roman, Mousa Saadeh, Unnati Shah, Shobika Sivaram, Paige Trillo, Anh Truong, Matthew Wegman, and Jeffrey Woo.

**NY Iota – Wagner College**

Corresponding Secretary – Marisa Scarpa; 25 New Members

New Initiates – Ann Attardi, Dardan Bajraktari, Emma Bartlett, James Bodeker, James Catalano, Brittany Ciampi, Allison Mary Cooper, Ava DiLeo, Leobardo Nieto Dominguez, Jacquelyn Gandelman, Brian Gehring, Michelle Greenough, Olivia Iacono, Taylor Krantz, Paul Mammino, Ashley Marchese, Lauren Christine McGoldrick, Sandra Gabriela Minchala, Jarrod Molzon, Megan Elizabeth Moore, Pamela Pangaro, Courtney Lyn Titus, Melanie Trinkwald, Rea Ulaj, and Colleen Withers.

**NY Kappa – Pace University**

Corresponding Secretary – Shamita Dutta Gupta; 3 New Members

The induction ceremony was held on 5/9/2014. Three members were initiated. They are Kalli Bader, Kevin C. Hankins and Kaylee Pina. The ceremony was well attended. Our guest speaker was Kaylee Pina. The title of her talk was A Deeper Look into Projective Geometry.

**NY Mu – St. Thomas Aquinas College**

Corresponding Secretary – Heather A. Rave; 9 New Members

New Initiates – Samantha Bader, Diane France, Lindsey Heiberger, Samantha Irizarry, Duncan Kasibante, Christopher Panepinto, Michelle Roux, Nicole Sullivan, and Jessica Winter.

**NY Omicron – St. Joseph's College**

Chapter President – Stephen A. Bates; 50 Current Members; 14 New Members

Other Fall 2014 Officers: Carl W. Baurle, III, Vice President; James T. Young, Secretary; Jessica Lynn Alessi, Treasurer; Dr. Elana Reiser, Cor-

responding Secretary; and Dr. Donna Marie Pirich, Faculty Sponsor

We had members volunteer at our Saturday morning math clinic, which is free for local high school students to get tutored in math. We also assembled Easter baskets for a local charity.

New Initiates – Daniel S. Capell, Nicole E. Cestare, Daniel E. Cisek, Justine Marie Ciullo, Nicole M. Coffey, Tiana A. Cureton, Paulina Duque, Alexander M. Fisher, Paige M. Marro, Giovanni Mayo, Michael C. Palmaccio, Ryan T. Stephens, Kristen H. Strickland, and Rebecca G. Vogel.

#### **NY Pi – Mount Saint Mary College**

Corresponding Secretary – Lee Fothergill; 5 New Members

New Initiates – Christina Antico, Shelbi Babcock, Catherine Anne Corry, Jonathan Garofalo, and Melanie Rose Hofbauer.

#### **NY Rho – Molloy College**

Chapter President – Mary-Kate Michels; 127 Current Members; 12 New Members

Other Fall 2014 Officers: Samantha Novak, Vice President; Santiago Vargas, Secretary and Treasurer; Corresponding Secretaries and Faculty Sponsors, Manyiu Tse and Deborah Upton

New Initiates – Marissa Cirillo, Alyssa Costello, Kelsey Crapo, Susan Daily, Eric Haslbauer, Nicole Hein, Sara Jacklin, Matthew Marsala, Magdalene Milonakis, Vijay Racktoo, Christina Rukstele, Matthew Vento, and Alec Volpe.

#### **OH Alpha – Bowling Green State University**

Chapter President – Mike Hughes; 6 Current Members

Other Fall 2014 Officers: John Maddrey, Vice President; Mark Medwed, Treasurer; Steve Seubert, Corresponding Secretary; and Jim Albert, Faculty Sponsor

#### **OH Gamma – Baldwin Wallace University**

Chapter President – Kyle Schifano; 45 Current Members; 12 New Members

Other Fall 2014 Officer: Corresponding Secretary and Faculty Sponsor, David Calvis

On Sunday March 23, 2014, the Ohio Gamma chapter initiated students into membership. The initiation was held as part of a larger ceremony that included initiation into our local chapter of computer science honorary Upsilon Pi Epsilon, as well as Departmental honors and awards.

New Initiates – Jeffrey Bregar, Anthony Calvitti, Sean Hoehn, Alan Jankowski, Russell Kurak, Devan MacDonald, Lyndsey Marshall, Akshya Pandey, Paul Matthew Switlyk, Luke Szumyckj, Kevin Thompson, and Remy Yovanno.

**OH Zeta – Muskingum University**

Corresponding Secretary – Richard Daquila; 2 New Members

New Initiates – Leah E. Buck and Jacob D. Miller.

**OH Eta – Ohio Northern University**

Corresponding Secretary – Donald Hunt; 5 New Members

New Initiates – Brendan Joseph Graziano, Michelle Haver, Kayla Hummell, Jared A. Orr, and Nicholas R. Weiner.

**OK Alpha – Northeastern State University**

Chapter President – Cindy Jeffcoat; 63 Current Members; 4 New Members

Other Fall 2014 Officers: Caleb Stubbs, Vice President; Natalie Mayberry, Secretary; John Moore, Treasurer; and Dr. Demitri Plessas, Corresponding Secretary and Faculty Sponsor

Our fall initiation brought four students into our chapter. Our chapter ran an activity center for elementary school students during the Northeastern State University Halloween Carnival. During the second meeting of the semester, Dr. Plessas, NSU, spoke on “My Favorite Unsolved Problems.” The last meeting of the semester was dedicated to learning and playing combinatorial games.

New initiates – Cally J. Bond, Elizabeth R. Cagle, Whitney L. James, and Frances B. Millspaugh.

**OK Gamma – Southwestern Oklahoma State University**

Corresponding Secretary – Tom McNamara; 13 New Members

New Initiates – Maggie Chang, Steven Tyler Gorshing, Emily Hartley, Sawyer Johnston, Tina Le, Katherine Martin, Mary Dossanna Jean Miller, Alyssa Olson, Kimmi Sawatzky, Allison Truong, Kelsey Wall, John Paul Woods, and Gray Zhen.

**PA Beta – La Salle University**

Chapter President – Carmen Esposito; 30 Current Members

Other Fall 2014 Officers: Mary-Elizabeth Voss, Vice President; Austin Anderson, Secretary; Katherine Boligitz, Treasurer; and Janet Fierson, Corresponding Secretary and Faculty Sponsor

Last semester, members of La Salle’s KME chapter attended the fall meeting of the Eastern Pennsylvania and Delaware section of the MAA at University of the Sciences in Philadelphia. They also hosted a mathematical movie night, a board game night, and a potluck holiday dinner.

**PA Delta – Marywood University**

Corresponding Secretary – Thomas Kent; 1 New Member

New Initiate – Jamie Lee.

**PA Zeta – Indiana University of Pennsylvania**

Chapter President – Shawn Mosley; 10 Current Members

Other Fall 2014 Officers: Jenna Scherrah, Vice President; Derek Hanely, Secretary and Treasurer; and Gary Stoudt, Corresponding Secretary and

**Faculty Sponsor**

The PA Zeta chapter at Indiana University of Pennsylvania reactivated in the spring semester of 2014 with 7 new student members and 3 new faculty members. Students: Janell Connelly, Emily Downs, Derek Hanely, Sarah Letender, Matthew McBurney, Shawn Mosley, Jenna Scherrah. Faculty: Francisco Alarcón, Timothy Flowers, Gary Stoudt. We are proud to be back in the KME fold, given that PA Zeta's history includes two former National Presidents of KME, William R. Smith from 1973-1977 and the late Ida Z. Arms from 1981-1985.

**PA Eta – Grove City College**

Corresponding Secretary – Dale L. McIntyre; 11 New Members

New Initiates – Joanna Brooks, Emma Cinatl, Samantha Condie, Jessica Fortney, Samuel Gill, Holly Lueers, Amy Noll, Kimberly Schlabach, Nicholas Wargo, Chelsea Weber, and Zachary Wheeler.

**PA Theta – Susquehanna University**

Corresponding Secretary – Kenneth Brakke; 3 New Members

New Initiates – Margaret Crann, Colin Eberhardt, and Emily Reich.

**PA Kappa – Holy Family University**

Chapter President – Rebecca Gaetani; 7 Current Members; 6 New Members

Other Fall 2014 Officer: Sister Marcella Wallowicz, Corresponding Secretary and Faculty Sponsor

On October 30, 2014, KME members joined the math club members at the Evening of Mathematical Suspense, a mathematical murder mystery "dinner theater" which takes place on campus each year near Halloween. Participants solve math problems in order to obtain clues which will help them to determine "who dunnit."

New Initiates – Hana Britland, Mahbuba Choudhury, Dominic McAllister, Nicole Pyle, Jason Seeton, and Jenna Souyack.

**PA Lambda – Bloomsburg University of Pennsylvania**

Chapter President – Thyme Greenfield; 20 Current Members; 14 New Members

Other Fall 2014 Officers: Annya D'Amato, Vice President; Paige Kavanaugh, Secretary; Katerina Custis, Treasurer; Dr. Eric Kahn, Corresponding Secretary; and Dr. William Calhoun, Faculty Sponsor

New Initiates – Kaleigh Brady, Lara Cesco-Cancian, Margaret Erdman, Laurie Grodziak, Joshua Halbfoerster, Grant Klinger, Matthew Leshner, Morgan Lewis, Kristen Morris, Laura Peiffer, Conor Renn, Riley Renn, Colin Swingle, and Nathan Weed.

**PA Mu – Saint Francis University**

Chapter President – Katie Waldron; 42 Current Members; 25 New Members

Other Fall 2014 Officers: Dave Wolfe, Vice President; Kelly Beegle, Secretary; Cathleen Fry, Treasurer; Travis Schofield, Historian; Dr. Peter Skoner, Corresponding Secretary; and Dr. Brendon LaBuz, Faculty Sponsor

New Initiates – Amanda Bancroft, Sylvia Joy Bintrim, Kasey Butts, Derek Civis, Margaret Creciun, Chelsea Easter, Casey Gallaher, Brandon Herman, Jeffrey Hogan, Benjamin Jones, Timothy Keith, Karla Kozak, James Krug, Ariana Leeper, Michelle Lipski, Jessica Mazzur, Rebecca Meyer, Lydia Mignogna, Katelyn Nusbaum, Hannah Patton, Benjamin Pillot, Jacob Spryn, Kevin Tomkowski, Joshua Vinglish, and Ruby Whorl.

**PA Nu – Ursinus College**

Corresponding Secretary – Nicholas Scoville; 10 New Members

New Initiates – Rose Blanchard, Sean Gregory, Kevin Hamilton, Danielle Kritz, Jonathan Kustina, Andrew Merves, Ian Rand, Matthew Rink, Collin Takita, and Andrew Trachtman.

**PA Rho – Thiel College**

Corresponding Secretary – Max Shellenbarger; 9 New Members

New Initiates – Amanda Callahan, Julia Fink, Megan Kidder, Sara Matczak, Sean McCarthy, Daniel McFetridge, Kale Postlewait, Nicole Richins, and Jennifer Rickens.

**PA Tau – DeSales University**

Chapter President – Jacob M. Kean; 10 Current Members

Other Fall 2014 Officers: Alison Malatesta, Vice President; Joshua Brobst, Secretary and Treasurer; and Br. Daniel P. Wisniewski, Corresponding Secretary and Faculty Sponsor

**RI Beta – Bryant University**

Chapter President – Andrew DiFronzo; 30 Current Members

Other Fall 2014 Officers: Stephen Lamontagne, Vice President; Allison Orr, Treasurer; Jerlyn Crowley, Secretary; and Billie Anderson, Corresponding Secretary

**SC Epsilon – Francis Marion University**

Chapter President – Taylor Burch; 25 Current Members; 15 New Members

Other Fall 2014 Officers: Rigel Lochner, Vice President; Cody McKenzie, Secretary and Treasurer; and Jeremiah Bartz and Damon Scott, Corresponding Secretaries

New Initiates – Margaret Smolinka Adams, Preston Alexander, Christian Belk, Alvina B. Blanks, Taylor Evan Burch, Dorothy Dickson-Vandervelde, A. Reneé Dowdy, Corey A. Greene, Jason M. Harrington, Rigel F. Lochner, Kathy McCoy, Cody Tyler McKenzie, Nicholas Newman, Sharon K. O'Kelley, and George E. Schnibben.

**TN Alpha – Tennessee Technological University**

Corresponding Secretary – Andrew Hetzel; 12 New Members

New Initiates – Katherine Brown, David Bynum, Amanda Carpenter, Jacob Cleveland, Jennifer Hudson, James Kent, Maria Long, Jamie Lundy, Jeremy Moffett, Robert Ponder, Joshua Swaim, and Joseph Tippit.

**TN Beta – East Tennessee State University**

Chapter President – Dustin Chandler; 25 Current Members; 14 New Members

Other Fall 2014 Officers: Derek Kiser, Vice President; Cory Ball, Secretary; Thomas Robacker, Treasurer; Robert Gardner, Corresponding Secretary; and Robert Beeler, Faculty Sponsor

Our chapter webpage is <http://faculty.etsu.edu/gardnerr/KME/KME.html>.

New Initiates – Cory Ball, Carrie Elliot, Russell Harper, Katie Heidt, Miranda Hilton, Amanda Justus, Derek Kiser, David Lewis, Leanna Murdock, Thomas Robacker, Brett Shields, Alex Smyth, Rebekah White, and Alyssa Williams.

**TN Zeta – Lee University**

Chapter President – Lindsay Holdman; 11 Current Members

Other Fall 2014 Officers: Lauren Todd, Vice President; Maria Medrano, Secretary; Casey Taylor, Treasurer; Blayne E. Carroll, Sr., Corresponding Secretary; and Caroline Maher-Boulis, Faculty Sponsor

Tennessee Zeta had one planning meeting of the officers at the beginning of the semester.

**TX Gamma – Texas Woman's University**

Corresponding Secretary – Mark Hamner; 7 New Members

New Initiates – Jaquelynn Abamu, Dinora Gonzalez, Hanna Gustrin, Valeria Perez, Katherine Resa, Enrique Saldana, and Sabre Schmidt.

**TX Lambda – Trinity University**

Chapter President – Katy Norman; 12 Current Members; 7 New Members

Other Fall 2014 Officers: Ailie Vuper, Vice President; Molly Zumbro, Secretary; and Dr. Hoa Nguyen, Corresponding Secretary

**VA Delta – Marymount University**

Chapter President and Secretary – Marina Romadan; 34 Current Members

Other Fall 2014 Officers: Lucy Ogbole, Vice President and Treasurer; Will Heuett, Corresponding Secretary, and Elsa Schaefer, Faculty Sponsor

**WI Gamma – University of Wisconsin-Eau Claire**

Corresponding Secretary – Carolyn Otto; 30 New Members

New Initiates – Alyssa Dalsky, Marcus Berends, Adam Bubolz, Sara DeBranbender, Emily Degner, Rachel Franczyk, Melissa Franson, Michelle Gebert, Christopher Gomes, Christopher Graewin, Emily Janzig, Kelsey Jensen, Ellen Junko, Joowon Kim, Daniel Klingensmith, Megan Linton, Hengzhou Liu, Jessica Lowe, Sarah Luman, Chris Magyar, Lucy Pepin, Chelsea Robach, Kensli Rollman, Kelly Rowe, Sierra Snapp, Meron Tefera, Lee Thompson IV, Elizabeth Thooft, Thao Tran, and Cara Wiskow.

**WV Alpha – Bethany College**

Chapter President – Tyler N. Pannebaker; 6 Current Members

Other Fall 2014 Officers: James A. Long, Jr., Vice President; and Adam C. Fletcher, Corresponding Secretary



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In the Fall 2014 semester, West Virginia Alpha chapter and the Bethany College Mathematics and Computer Science Club participated in both the Virginia Tech Regional Mathematics Contest and the William Lowell Putnam Mathematical Competition. Additionally the two groups took a trip to Washington, D.C. to visit the Smithsonian Institution museums.

## *Active Chapters of Kappa Mu Epsilon*

*Listed by date of installation*

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, Jonesboro	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
KS Zeta	Southwestern College, Winfield	14 Apr 1998
TN Epsilon	Bethel College, McKenzie	16 Apr 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 Mar 1999
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
NJ Gamma	Monmouth University, West Long Branch	21 Apr 2002
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008

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CA Zeta	Simpson University, Redding	4 Apr 2009
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011
PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresno Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015