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Contents

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An Exploration of Tropical Mathematics

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This paper was presented at the 2013 National Convention
and given the "People's Choice Award" by the Awards Committee.

1. Introduction

The idea of tropical mathematics is much like that of standard mathematics only with the definitions of addition and multiplication changed. In tropical mathematics, the operations addition and multiplication are symbolized as $\oplus$ and $\odot$.

The operation of $\oplus$ is the **tropical addition** of real numbers where $a \oplus b$ is the minimum of a and b, denoted $\min(a, b)$. An example: $7 \oplus 4 = \min(7, 4) = 4$. **Tropical multiplication** $\odot$ is defined as standard addition, or $a \odot b = a + b$. An example is $7 \odot 4 = 7 + 4 = 11$.

These operations create a tropical semiring over the real numbers $(\mathbb{R} \cup \{\infty\}, \oplus, \odot)$. A **semiring** is defined to be a ring without the requirement of each element having an additive inverse. It is easy to show that tropical mathematics is in fact a semiring. To complete the requirements of a ring, the set $(\mathbb{R} \cup \{\infty\}, \oplus, \odot)$ satisfies the following properties:

1. *Addition is associative:*
   $(x \oplus y) \oplus z = \min(x, y, z) = x \oplus (y \oplus z)$.
2. *Addition is commutative:*
   $(x \oplus y) = \min(x, y) = \min(y, x) = (y \oplus x)$.
3. *Additive Identity:*
   $x \oplus \infty = x$ as the $\min(x, \infty) = x$.
4. *Multiplication is associative:*
   $(x \odot y) \odot z = x \odot (y \odot z)$,
   which follows from standard addition being associative.

5. *Distribution:*
   $$\begin{aligned} x \odot (y \oplus z) &= x + \min(y, z) \\ &= \min(x + y, x + z) \\ &= (x \odot y) \oplus (x \odot z). \end{aligned}$$
   [Note that when switching the operations of tropical addition and multiplication, distribution does not hold: there exist $x, y, z \in \mathbb{R}$ such that $x \oplus (y \odot z) \neq (x \oplus y) \odot (x \oplus z)$ as $\min(x, y + z) \neq \min(x + y) + \min(y + z)$.]
6. *Closure:*
   For every $x, y \in \mathbb{R}$, $x \oplus y \in \mathbb{R}$ and $x \odot y \in \mathbb{R}$.

Other characteristics to notice about the set are:

1. *Multiplication is commutative:*
   $x \odot y = y \odot x$ since $x + y = y + x$.
2. *Multiplicative Identity:* $x \odot 0 = x$.

Note that as multiplication is commutative and there is a multiplicative identity, the semiring is commutative with unity.

The requirement of a ring that is not satisfied is that of the **additive inverse**. For any element x, an additive inverse of x is an element such that the addition results in the additive identity. Remember, in tropical mathematics, the additive identity is ∞. But for any element x, there is no element y such that $x \oplus y = \infty$, as $\min(x, y) \neq \infty$ because ∞ is the largest element in the semiring. An example of how this causes subtraction to be undefined can be shown. When looking at $x \oplus 4 = 10$, there is no value for x such that $\min(x, 4) = 10$.

Tropical mathematics has a characteristic where powers distribute, often referred to as the **freshman's dream**. For example: $(x \oplus y)^3 = x^3 \oplus y^3$. This can be proven quite easily:

$$\begin{aligned} (x \oplus y)^3 &= (x \oplus y) \odot (x \oplus y) \odot (x \oplus y) \\ &= (x \odot x) \oplus (x \odot y) \oplus (y \odot x) \oplus (y \odot y) \odot (x \oplus y) \\ &= (x \odot x) \oplus (x \odot y) \oplus (y \odot y) \odot (x \oplus y), \\ &\qquad \text{as } \min(x + y, y + x) = x \odot y \\ &= (x \odot x \odot x) \oplus (x \odot x \odot y) \oplus (x \odot y \odot y) \oplus (y \odot y \odot y) \\ &= x^3 \oplus x^2 y \oplus x y^2 \oplus y^3 \\ &= \min(x^3, x^2 y, x y^2, y^3), \end{aligned}$$

where terms can be combined and written using powers. It is just necessary to remember they are still tropical operations.

Now, assume without loss of generality that $x \geq y$. Then $x + x + x \geq$

$x + x + y \geq x + y + y \geq y + y + y$. Thus, the minimum is y^3. As $x + x + y$ and $x + y + y$ are always sandwiched between x^3 and y^3, $\min(x^3, x^2y, xy^2, y^3) = \min(x^3, y^3) = (x^3 \oplus y^3)$. It follows that the minimum is determined by $x \geq y$ and powers can distribute.

2. Tropical Polynomials and Lines

In the previous example of the freshman's dream, note that it was possible to rewrite $x \odot x \odot x$ as x^3. Using this notation, it is possible to examine tropical monomials.

A tropical monomial appears much like a standard monomial, only with the operator consisting of tropical multiplication verses standard multiplication. A **tropical monomial** is the tropical multiplication of a constant and various unknown variables. An example of a tropical monomial $3 \odot x \odot x \odot x \odot y \odot y \odot z = 3x^3y^2z$ can be written in standard notation as $3x^3y^2z = 3+x+x+x+y+y+z = 3+3x+2y+z$. Note that a tropical monomial can be written as a linear function with integer coefficients.

The next step is to create tropical polynomials. A **tropical polynomial** is a combination of tropical monomials, connected with tropical addition. So, if we take $3x^3y^2z$ from before and tropically add x^2z^4, we have created the tropical polynomial $3x^3y^2z \oplus x^2z^4$, which in standard notation can be written as: $\min\{(3 + 3x + 2y + z), (2x + 4z)\}$. Note that any tropical polynomial can be written as the minimum of a finite combination of linear functions.

Using these finite combinations of linear functions, it is possible to graph tropical polynomials on a coordinate axis. We will look at the example $p(x) = x^3 \oplus 2 \odot x^2 \oplus 4 \odot x \oplus 7$. This tropical polynomial can be written using four linear functions: $3x$, $2x + 2$, $x + 4$, and 7. These linear functions can be graphed on a coordinate axis as any regular function $f(x)$ (see Figure 1). Now, remember these are tropical functions and we need to locate the minimum of them. When looking at the graph, the tropical polynomial is the lower envelope of functions. Highlighted yellow in the figure, the envelope consists of the functions with the lowest y values. Note that at times, not all the functions may be used. In this case, $2x + 2$ only has one point located on the tropical polynomial, $(2, 6)$. However, this point is also found on $x + 4$ and $3x$, so, if we were to graph the polynomial without $2x + 2$, we would have the same polynomial. Hence, $x^3 \oplus 2 \odot x^2 \oplus 4 \odot x \oplus 7 = x^3 \oplus 4 \odot x \oplus 7$, and graphical representations of tropical polynomials can depict more than one polynomial.

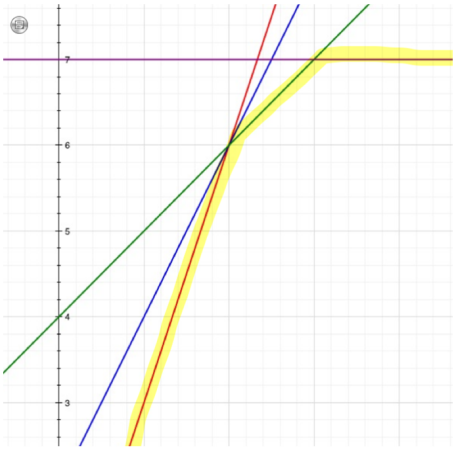

Figure 1

The points at which the function "breaks" are called the **roots** of the function. These roots are the points at which a minimum is attained more than once. The roots make up the set that is known as a **hypersurface**, denoted $H(p)$. For the above example, $H(p) = \{(2, 6), (3, 7)\}$.

We can now use these ideas to define a **tropical line**. The equation of a tropical line is $p(x, y) = a \odot x \oplus b \odot y \oplus c$, or in a more standard form, $\min(a + x, b + y, c)$. The graph of a tropical line is the set of points that belong to the hypersurface, where now instead of single points being the roots in the hypersurface, the points make up **half rays**. Recall, the hypersurface is where at least two of the terms must equal the minimum with the remaining term being greater than or equal to the minimum. It follows that the equations of the half rays of a tropical line are:

$$\begin{aligned} a + x &= b + y \leq c \\ b + y &= c \leq a + x \\ a + x &= c \leq b + y \end{aligned}$$

Visualizing the half rays is easier when the right hand side of the inequality is dropped and the equations are written as a linear function.

$$\begin{aligned} y &= x + (a - b) \\ y &= c - b \\ x &= c - a \end{aligned}$$

The **center of a line**, located at $(c - a, c - b)$, is where the minimum is attained 3 times, and is therefore where all the half rays meet. As these rays are where the minimum is attained at least twice, they extend in three different directions from the center point. To picture this, it is necessary to look at the minimum y values, similar to a tropical polynomial. Approach-

ing the center from the left side on a coordinate axis, the ray $y = x+(a-b)$ has a slope of one and is headed in the southwest direction, as for every value of x, it has the lowest y values. When it reaches the center point, all three rays contain the lowest y value. $x = c - a$ is a vertical ray on the y-axis in the positive direction. It does not extend below the center point as $a+x = c \leq b+y$ and below the center point, $a+x = c \geq b+y$. Finally, moving past the center in the positive direction, $y = c - b$ is horizontal on the x-axis. It also cannot travel in the negative direction past the center point due to the restrictions by the inequalities.

As every tropical line has this structure, if two lines share a center point, they are in fact, the same line. This can be proved in the following way.

Theorem: If two lines share a center point, they are the same line.

Proof: Let $p(x, y) = a \odot x \oplus b \odot y \oplus c$ and $q(x, y) = d \odot x \oplus e \odot y \oplus f$ be two tropical lines such that their centers are the same. Note the center of $p(x, y)$ is $(c - a, c - b)$ and the center of $q(x, y)$ is $(f - d, f - e)$, and therefore, $c - a = f - d$ and $c - b = f - e$. Since the tropical line $p(x) = \min(a+x, b+y, c)$, the hypersurface is defined as 3 rays for each line:

$p(x, y)$:	$q(x, y)$:
$a + x = b + y \leq c$	$d + x = e + y \leq f$
$a + x = c \leq b + y$	$d + x = f \leq e + y$
$b + y = c \leq a + x$	$e + y = f \leq d + x$

First, the third ray of $p(x, y)$, $b + y = c \leq a + x$, implies $y = c - b$ and the third ray for $q(x, y)$, $e + y = f \leq d + x$, implies $y = f - e$. It follows from the assumption that $c - b = f - e$ and the horizontal half rays are equal. Now, the second ray of $p(x, y)$, $a + x = c \leq b + y$, implies $x = c - a$. Similarly, the second ray of $q(x, y)$, $d + x = f \leq e + y$, implies $x = f - d$. Again from the assumption, $c - a = f - d$ and thus, the vertical half rays are equal.

Finally, the diagonal rays are determined by the first ray of $p(x, y)$ and $q(x, y)$. $a+x = b+y \leq c$ can be rewritten as $y = x + (a - b)$, and $d + x = e + y \leq f$ as $y = x + (d - e)$. Note they both have a slope of one and pass through the center points, which are equal, making the diagonal half rays the same. Thus, if $c - a = f - d$ and $c - b = f - e$, the two tropical lines $p(x, y)$ and $q(x, y)$ are the same line. ■

Tropical lines share many characteristics with standard Euclidean lines. An example of this is the fact that two general tropical lines only cross at one point. Here the term "general" refers to two tropical lines such that the centers of two tropical lines share no common values, nor lie together on a line parallel to the $x = y$ line.

This property can be viewed by inspection by situating the center of one line in various locations around the center point of another (see Figure 2), but it can also be proved.

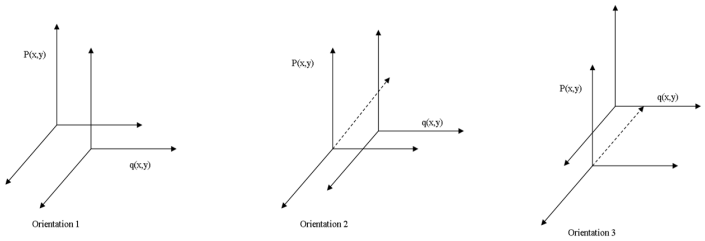

Figure 2

Theorem: Two general tropical lines only cross at one point.

Proof: Let p and q be two tropical lines such that the half rays of each line are:

$p(x, y)$:	$q(x, y)$:
$a + x = b + y \leq c$	$d + x = e + y \leq f$
$a + x = c \leq b + y$	$d + x = f \leq e + y$
$b + y = c \leq a + x$	$e + y = f \leq d + x$

Note that there are three different ways to orient the center of $q(x, y)$ around the center of $p(x, y)$.

Orientation 1: Assume $c - a < f - d$, then the center of p is to the left of the center of q. Also, let $f - e < c - b$, which implies that the center of p is above the center of q. Note that because the lines are in a general position, parallel rays do not intersect. Therefore, $a + x = b + y$ does not intersect $d + x = e + y$; $a + x = c$ does not intersect $d + x = f$; and $b + y = c$ does not intersect $e + y = f$. This leaves six cases left to test.

Case 1: Consider $a + x = b + y$ and $d + x = f$. Then note that all x values of $a + x = b + y$ are less than $c - a$ by the definition of a tropical line. Also note that $x = f - d$ and as $c - a < f - d$, the two half rays share no x values. Thus, the half rays do not intersect.

Case 2: Consider a$+x = b+y$ and $e+y = f$. It follows from the definition of a tropical line that all the x values of $a+x = b+y$ are less than $c-a$. Also, all the x values of $e+y = f$ are greater than $f-d$. Now, as $c-a < f-d$, the two half rays share no x values, and they cannot intersect.

Case 3: Consider $a+x = c$ and $d+x = e+y$. Note that all points on $a+x = c$ have y values greater than $c-b$. Also note that the y values of $d+x = e+y$ are less than $f-e$. As $f-e < c-b$, these two half rays do not share any y values and thus, they do not intersect.

Case 4: Consider $a+x = c$ and $e+y = f$. Note that all points on $a+x = c$ have y values greater than $c-b$. Also note that the y values of $e+y = f$ equal $f-e$. As $f-e < c-b$, the half rays do not have any points in common and thus, they do not intersect.

Case 5: Consider $b+y = c$ and $d+x = e+y$. Note that all points on $b+y = c$ have y values equal to $c-b$ and all points on $d+x = e+y$ have y values less than $f-e$. As $f-e < c-b$, these two half rays do not share any y values and thus, they do not intersect.

Case 6: Consider $b+y = c$ and $d+x = f$. Note $y = c-b$ and the x values are greater than $c-a$. As $f-d > c-a$, one of the points on the line $b+y = c$ is $(f-d, c-a)$. Now, it follows from $d+x = f$ that $x = f-d$. Also note that the y values of this half ray are greater than $f-e$. As $f-e < c-b$, one such y value will be $f-e$. Thus, there is some point on the half ray with coordinate $(f-d, c-a)$. Thus, as each half ray shares a common point, they intersect at one point.

Therefore, two general tropical lines $p(x,y)$ and $q(x,y)$ where the center of p is to the left and above the center of q, intersect at one and only one point.

Orientation 2: Let $c-a < f-d$ and $c-b < f-e < x+(a-b)$. Here the center of p is to the left of q and q is between the line $y = c-b$ and $y = x+(a-b)$. This case is much like the first and for the purpose of this paper will not be explained in detail.

Orientation 3: Let $c-a < f-d$ and $f-e > x+(a-b)$. Here the center of p is to the left of q and q is above the line $y = x+(a-b)$. This case will also reason similarly to the first case.

Note that there are no more than three orientations because if the center of q is to the left of the center of p, it is merely a repeat of the

first orientation without loss of generality. Therefore, two general lines $p(x, y)$ and $q(x, y)$ intersect and one and only one point. ■

Another similarity between tropical and Euclidean lines is the theorem that states that any two points lie on exactly one line. This is true for tropical lines as well; although, the two points must not share an x or y value as well as lie on a line parallel to the $x = y$ line. In such cases, infinitely many lines contain the two points in question. However, it is possible to prove that two general points have one and only one line in common.

Theorem: Any two general points lie on exactly one line.

Proof: Let (s, t) be a general point that sits in the Cartesian plane. Note there are three relevant lines passing through (s, t): $y = t$, $x = s$ and $y = t - s + x$ (see Figure 3). Let (v, w) be another point in the Cartesian plane. There are three different orientations of (v, w) in relation to (s, t).

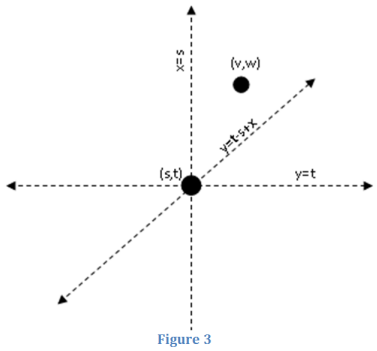

Figure 3

Orientation 1: Let (v, w) be oriented such that (1) $v > s$, and (2) $w > t - s + v$. Now, for tropical line $P(x, y) = a_1 \odot x \oplus b_1 \odot y \oplus c_1$ to pass through (s, t) and (v, w), (s, t) and (v, w) will lie on half rays beginning at the center point $(c_1 - a_1, c_1 - b_1)$. Note that it only takes two half rays to determine a tropical line. The third ray is determined by the other two. Since $P(x, y)$ is determined by the points where $\min(a_1 + x, b_1 + y, c_1)$, the half rays are where:

$$\begin{aligned} a_1 + x &= b_1 + y < c_1 \\ b_1 + y &= c_1 < a_1 + x \\ a_1 + x &= c_1 < b_1 + y \end{aligned}$$

Putting (s,t) and (v,w) in for (x,y), six equations for half rays are found:

$$a_1+s=b_1+t<c_1 \qquad a_1+v=b_1+w<c_1$$
$$b_1+t=c_1<a_1+s \qquad b_1+w=c_1<a_1+v$$
$$a_1+s=c_1<b_1+t \qquad a_1+v=c_1<b_1+w$$

Because each point that is not the center point may only lie on one half ray, only one statement on the left and one statement on the right may be true at the same time. This creates 9 cases.

Case 1: Let $a_1+s=b_1+t<c_1$ and $a_1+v=b_1+w<c_1$ both be true. Then $a_1+s-t=b_1$ and $a_1+v-w=b_1$, so $a_1+s-t=a_1+v-w$ implies $s-t=v=w$, which can be written as: $w=v-s+t$. This is a contradiction to $w>t-s+v$. Therefore this case is not valid.

Case 2: Let $a_1+s=b_1+t<c_1$ and $b_1+w=c_1<a_1+v$ both be true. Then $b_1+w<a_1+v$ and $v-w<a_1-b_1$. Also, $a_1+s=b_1+t$ implies $a_1-b_1=t-s$. Then $w-v<t-s$ shows $w<t-s+v$, which is a contradiction to $w>t-s+v$. Therefore this case is not valid.

Case 3: Let $a_1+s=b_1+t<c_1$ and $a_1+v=c_1<b_1+w$ both be true. Then $a_1+s<a_1+v$ follows from $a_1+s<c_1=a_1+v$. Now, $s<v$, so condition (1) holds. Also, $a_1+s=b_1+t$ shows $a_1-b_1=t-s$. Note, $a_1+v<b_1+w$ can be rewritten as $a_1+b_1<-v+w$, and it follows that $t-s<w-v$, which implies $v+t-s<w$, so condition (2) holds. Thus, this clam is valid.

Case 4: Let $b_1+t=c_1<a_1+s$ and $a_1+v=b_1+w<c_1$ both be true. Then $a_1+v<c_1<a_1+s$ shows $a_1+v<a_1+s$ which reduces to $v<s$. We have now reached a contradiction to $v>s$. Therefore this case is not valid.

Case 5: Let $b_1+t=c_1<a_1+s$ and $b_1+w=c_1<a_1+v$ both be true. Then $b_1+t=c_1=b_1+w$ implies $b_1+t=b_1+w$, which reduces to $t=w$. Since condition (1) states $v>s$, $v-s>0$. Therefore, $t+(v-s)>w$ is a contradiction to $w>t-s+v$. Therefore this case is not valid.

Case 6: Let $b_1+t=c_1<a_1+s$ and $a_1+v=c_1<b_1+w$ both be true. Then $a_1+v<a_1+s$ follows from $a_1+v=c_1<a_1+s$, and then $v<s$. Now, we have reached a contradiction to $v>s$. Thus, this case is not valid.

Case 7: Let $a_1+s=c_1<b_1+t$ and $a_1+v=b_1+w<c_1$ both be true. Then $a_1+v<c_1=a_1+s$ implies $a_1+v<a_1+s$, which reduces to $v<s$. This is a contradiction to $v>s$. Thus, this

case is not valid.

Case 8: Let $a_1 + s = c_1 < b_1 + t$ and $b_1 + w = c_1 < a_1 + v$ both be true. Then $a_1 + s < b_1 + t$ shows $a_1 - b_1 < t - s$. Also $b_1 + w < a_1 + v$ implies $w - v < a_1 - b_1$. It follows from $w - v < t - s$ that $w < t - s + v$. We now have a contradiction to $w > t - s + v$. Therefore this case is not valid.

Case 9: Let $a_1 + s = c_1 < b_1 + t$ and $a_1 + v = c_1 < b_1 + w$ both be true. Then $a_1 + s = c_1 = a_1 + v$ and $a_1 + s = a_1 + v$ reduces to $s = v$. We have a contradiction to $v > s$. Thus, this case is not valid.

Now, given that case 3 ($a_1 + s = b_1 + t < c_1$ and $b_1 + w = c_1 < a_1 + v$ both true) is the only valid case, point (s, t) lies on half ray $a_1 + x = b_1 + y$ and point (v, w) lies on half ray $a_1 + x = c_1$. It follows that the third ray is $b_1 + y = c_1$, and the center of the line $P(x, y) = a_1 \odot x \oplus b_1 \odot y \oplus c_1$ is at $(v, t - s + v)$. If a separate line $Q(x, y) = a_2 \odot x \oplus b_2 \odot y \oplus c_2$ passes through (s, t) and (v, w), it similarly follows that (s, t) is on the half ray $a_2 + x = b_2 + y$ and (v, w) is on the half ray $a_2 + x = c_2$. The center of the line is also at $(v, t - s + v)$. As the centers are at the same point, it follows from the previous theorem that the lines $P(x, y)$ and $Q(x, y)$ are the same line and for any two general points with these orientations, there exists one unique tropical line.

Now, there are two other orientations for point (v, w). The first is $v > s$ and $t < w < t - s + v$, and the second is $v > s$ and $w < t$. Both of these orientations results in a set of nine cases each, but because the reasoning will be much like the first set of constraints, the rest of the proof will not be shown. Also, note that if $v < s$, the constraints will be the same by stating $s > v$, then $t > w - v + s$ or $w < t < w - v + s$ or $t < w$, and the reasoning of the first three constraints is used to determine which line lies on the two points. ■

3. Tropical Matrices

A **tropical matrix** $[A]$ appears as any other matrix with numerical entries in rows and columns, with the operations redefined in a tropical manner. As in standard multiplication, if two matrices $[A]$ and $[B]$ are being multiplied together, there must be the same number of columns in $[A]$ as there are rows in $[B]$. If $[A]$ is $n \times m$ and $[B]$ is $m \times l$, the resulting matrix, $[AB]$ will be a $n \times l$ matrix.

When completing the process of standard matrix multiplication of two matrices, each individual entry x_{jk} is found by completing the sum of the products of each pair of entries in row j, column k. So,

$$x_{jk} = j_1k_1 + j_2k_2 + \cdots + j_nk_n.$$

Tropical matrix multiplication is defined similarly to standard matrix multiplication, only with the operations in tropical form. Multiplication is tropical multiplication and addition is tropical addition. So, to find each entry of $[A] \odot [B]$, the entries in the rows of $[A]$ must be added to the entries in the columns of $[B]$. Then the minimum of these is determined in order to state the value of each individual entry. Therefore,

$$x_{jk} = j_1 \odot k_1 \oplus j_2 \odot k_2 \oplus \cdots \oplus j_n \odot k_n.$$

For example, for the 2×2 matrix $[A]$ and the 2×3 matrix $[B]$, let

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } [B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}.$$

Then,

$$[A] \odot [B] = \begin{bmatrix} ab_{11} & ab_{12} & ab_{13} \\ ab_{21} & ab_{22} & ab_{23} \end{bmatrix},$$

where

$$ab_{11} = a_{11} \odot b_{11} \oplus a_{12} \odot b_{21} = \min(a_{11} + b_{11}, a_{12} + b_{21}).$$

The rest of the entries are computed in a similar manner.

Tropical matrix multiplication is useful when considering adjacency matrices of graphs. For a tropical matrix, all graphs must be simple graphs with no loops or multiple edges between two vertices. If there are multiple edges between a pair of vertices, all edges but the shortest may be removed. Weighted graphs are usually used, but if there are no weights, a value of 1 may be used for each edge.

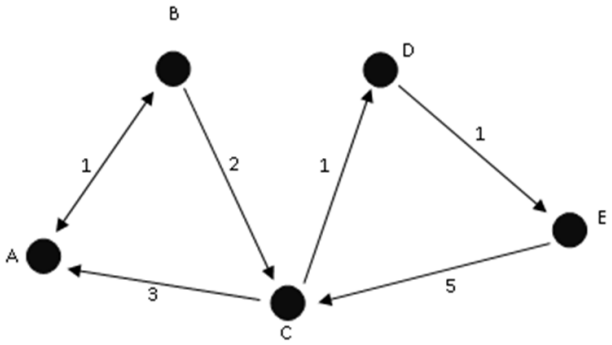

Figure 4: Graph G

Now, an **adjacency matrix** $[A]$ is a $n \times n$ matrix with a main diagonal consisting of entries equaling 0. If there is an edge between two vertices j and k, the length of the edge is noted in matrix entry in the j^{th} row and k^{th} column. If there is no edge extending from j adjacent to k, the entry in the j^{th} row and k^{th} column of the matrix will be ∞. This is different than adjacency matrices using standard mathematics, which would have the value of 0 if there is no edge. Adjacency matrices can be constructed for both graphs and digraphs; although, note that the adjacency matrix for a digraph will not necessarily be symmetrical.

We look at an example concerning the graph G (refer to Figure 4). The adjacency matrix is

$$[M] = \begin{bmatrix} 0 & 1 & \infty & \infty & \infty \\ 1 & 0 & 2 & \infty & \infty \\ 3 & \infty & 0 & 1 & \infty \\ \infty & \infty & \infty & 0 & 1 \\ \infty & \infty & 5 & \infty & 0 \end{bmatrix}.$$

For any simple graph, it is possible to determine the shortest path between any two vertices in the graph using the tropical adjacency matrix. To do so, tropically multiply the $n \times n$ adjacency matrix by itself $(n-1)$ times. After the product is complete, to find the shortest length of a path from one vertex to another, it is only necessary to reference the entry in the row of the beginning vertex and the column of the ending vertex.

We return to our example G, where to find the shortest path between any two points, it is necessary to compute $[M]^4$,or $M \odot M \odot M \odot M$. Now,

$$\begin{aligned} [M]^2 &= \begin{bmatrix} 0 & 1 & \infty & \infty & \infty \\ 1 & 0 & 2 & \infty & \infty \\ 3 & \infty & 0 & 1 & \infty \\ \infty & \infty & \infty & 0 & 1 \\ \infty & \infty & 5 & \infty & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & \infty & \infty & \infty \\ 1 & 0 & 2 & \infty & \infty \\ 3 & \infty & 0 & 1 & \infty \\ \infty & \infty & \infty & 0 & 1 \\ \infty & \infty & 5 & \infty & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 3 & \infty & \infty \\ 1 & 0 & 2 & 3 & \infty \\ 3 & 4 & 0 & 1 & 2 \\ \infty & \infty & 6 & 0 & 1 \\ 8 & \infty & 5 & 6 & 0 \end{bmatrix} \end{aligned}$$

To clarify how to tropically multiply these matrices, let us look at how the entry in row A and column C was computed. First, entries in row A are

tropically multiplied to entries in column C. The minimum is then found.

$$\begin{aligned} M_{AC} &= 0 \odot \infty \oplus 1 \odot 2 \oplus \infty \odot 0 \oplus \infty \odot \infty \oplus \infty \odot 5 \\ &= \min(\infty, 3, \infty, \infty, 5) = 3 \end{aligned}$$

In the entries where ∞ is the result, the tropical multiplication results in five ∞ values and $\min(\infty, \infty, \infty, \infty, \infty) = \infty$. Now, to continue the multiplication process:

$$[M]^3 = \begin{bmatrix} 0 & 1 & 3 & \infty & \infty \\ 1 & 0 & 2 & 3 & \infty \\ 3 & 4 & 0 & 1 & 2 \\ \infty & \infty & 6 & 0 & 1 \\ 8 & \infty & 5 & 6 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & \infty & \infty & \infty \\ 1 & 0 & 2 & \infty & \infty \\ 3 & \infty & 0 & 1 & \infty \\ \infty & \infty & \infty & 0 & 1 \\ \infty & \infty & 5 & \infty & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 3 & 4 & \infty \\ 1 & 0 & 2 & 3 & 4 \\ 3 & 4 & 0 & 1 & 2 \\ 9 & \infty & 6 & 0 & 1 \\ 8 & 9 & 5 & 6 & 0 \end{bmatrix}$$

$$[M]^4 = \begin{bmatrix} 0 & 1 & 3 & 4 & \infty \\ 1 & 0 & 2 & 3 & 4 \\ 3 & 4 & 0 & 1 & 2 \\ 9 & \infty & 6 & 0 & 1 \\ 8 & 9 & 5 & 6 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & \infty & \infty & \infty \\ 1 & 0 & 2 & \infty & \infty \\ 3 & \infty & 0 & 1 & \infty \\ \infty & \infty & \infty & 0 & 1 \\ \infty & \infty & 5 & \infty & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 3 & 4 & 5 \\ 1 & 0 & 2 & 3 & 4 \\ 3 & 4 & 0 & 1 & 2 \\ 9 & 10 & 6 & 0 & 1 \\ 8 & 9 & 5 & 6 & 0 \end{bmatrix}$$

Now, although a direct path from vertex A to vertex E may not be apparent in the graph, by referencing row A and column E, there is a path that can be traveled with a total distance of 5.

To determine how this algorithm works, it is necessary to think of the matrix as consisting of steps from one vertex to another. In an adjacency matrix, the entry in row A and column B states the distance of the step between the two vertices A and B. So, it is possible to depict a matrix as

$$[D] = \begin{bmatrix} A \to A & A \to B & A \to C \\ B \to A & B \to B & B \to C \\ C \to A & C \to B & C \to C \end{bmatrix}.$$

Now, when squaring this matrix tropically, every possible connection among vertices is being compared. For example, to find the shortest path from A to C, it is necessary to compute the entry for row A and column C. This is done tropically by

$$D_{AA} \odot D_{AC} \oplus D_{AB} \odot D_{BC} \oplus D_{AC} \odot D_{CC}.$$

Or, stating this in standard notation: $\min(D_{AA} + D_{AC}, D_{AB} + D_{BC}, D_{AC} + D_{CC})$. Note that the connections are what are being compared. $D_{AB} + D_{BC}$ is determining if there is a connection from $A \to B$ and then $B \to C$, or a path from $A \to B \to C$ and finding the length of that path. This value is then being compared to the lengths of the other possible pathways, $A \to A \to C$ and $A \to C \to C$. As these matrices have a main diagonal of 0 entries, $A \to A$ and$C \to C$ are always 0. Therefore, if the distance from $A \to C$ is shorter than the distance from $A \to B \to C$, that is the value that appears in the final matrix, or vice versa.

In the example with graph G, it is also interesting to note what the computation of $[M]^4$ shows. By only computing $[M]^2$, all of the two step paths are found. Note that the entry in row A and column D for $A \to D = \infty$ as there is no two step path from A to D. However, compute $[M]^3$ and the value of that entry has changed to 4, meaning there is a three step path from A to D with a distance of 4.

Also, as this matrix has only 5 vertices, the shortest path from any one vertex to another must be within four steps, else the path will return to a previously visited vertex. Therefore, if $[M]^5$ is computed, it is the same as $[M]^4$ as there are no paths of 5 steps that are shorter than any of the 4 step paths. One can go further to say that for an $n \times n$ matrix, $[A]^{n-1} = [A]^k$ for all $k \geq n$.

This matrix strategy can be applied to any situation where there are multistep choices and an optimal route is desired. The choices can be represented in a graph with distances between vertices. Note that the distances do not necessarily have to be measured in length. Time, weight, and cost may be measurements used in applications.

As an example, we consider the problem of finding a route that takes the shortest driving time in a city setting from one location A to another location G. There are multiple roads and intersections between the starting and ending location that may be taken, so the situation is represented as a graph with intersections depicted as vertices and roads marked as edges. The edge from one vertex to another is the distance between the two intersections that the vertices represent.

As it is necessary to determine the time from one intersection to another, a number of assumptions must be made. The first is that the traffic flow is normal, with the average speed being that of the speed limit. Thus, the length of an edge can be replaced with a time value calculated using a distance and rate ratio. It is also necessary to take into consideration the time spent at traffic lights. In this specific situation from A to G, the average stop light is red for 45 seconds according to the Kansas Department of Transportation. Thus, any stoplight that the path travels from adds 45 seconds to the time it would take to travel to the connecting vertex.

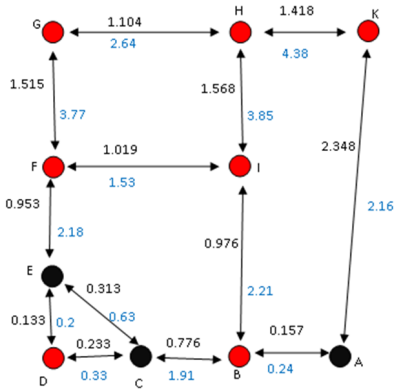

Figure 5

Using these assumptions, it is possible to determine the amount of time in minutes it takes to travel between each pair of adjacent vertices. The adjacencies are put into a 10×10 adjacency matrix. Using the matrix strategy discussed, the matrix is now tropically multiplied by itself nine times.

During the multiplication process, some interesting events occur. While finding the square of the matrix, the first comparisons of path lengths are made by the algorithm. One of the first paths where the minimum has an effect is from I to G. The first choice travels through F and the second travels through H. The time through H is 7.99 and the time through F is 6.8. With a minute difference, the algorithm picks the path to travel through F rather than H.

	A	B	C	D	E	F	G	H	I	K
A	0	0.24	∞	∞	∞	∞	∞	∞	∞	2.16
B	0.99	0	2.66	∞	∞	∞	∞	∞	2.96	∞
C	∞	1.91	0	0.33	0.63	∞	∞	∞	∞	∞
D	∞	∞	1.18	0	0.95	∞	∞	∞	∞	∞
E	∞	∞	0.63	0.2	0	2.18	∞	∞	∞	∞
F	∞	∞	∞	∞	2.93	0	4.52	∞	2.28	∞
G	∞	∞	∞	∞	∞	4.52	0	3.39	∞	∞
H	∞	∞	∞	∞	∞	∞	3.39	0	4.6	5.13
I	∞	2.96	∞	∞	∞	2.28	∞	4.6	0	∞
K	2.16	∞	∞	∞	∞	∞	∞	5.13	∞	0

Adjacency Matrix

A second event deals with the starting and ending points A and G. After the matrix is cubed, there is a path found from A to G that lasts 10.68 minutes. This path is found at this point in the algorithm as it contains only three edges. A to K, K to H, and H to G. But, as the algorithm is not complete, it is necessary to multiply the cubed matrix by the original. During this iteration the value from A to G changes to a lesser time of 10 minutes. This tells us that there is a path with more edges, but the edges are associated with shorter times. As the algorithm process is completed, this time no longer changes, stating the shortest path from A to G is 10 minutes and it can be completed in 4 steps. To determine this path it is only a matter of traveling backwards through the multiplication process to determine the connections that were chosen. The resulting path is A to B to I to F to G.

	A	B	C	D	E	F	G	H	I	K
A	0	0.24	0.29	2.32	3.53	5.48	10	7.29	3.2	2.16
B	0.99	0	2.66	2.99	3.29	5.24	9.76	7.56	2.96	3.15
C	2.9	1.91	0	0.33	0.63	2.81	7.33	9.47	4.87	5.06
D	4.08	3.09	1.18	0	0.95	3.13	7.65	10.01	5.41	6.24
E	3.53	2.54	0.63	0.2	0	2.18	6.7	9.06	4.46	5.69
F	6.23	5.24	3.56	3.13	2.93	0	4.52	6.88	2.28	12.01
G	10.68	9.76	8.08	7.65	7.45	4.52	0	3.39	6.8	8.52
H	7.29	7.56	10.22	10.19	9.81	6.88	3.39	0	4.6	5.13
I	3.95	2.96	5.62	5.41	5.21	2.28	6.8	4.6	0	9.73
K	2.16	2.5	5.06	5.39	7.51	12.01	8.52	5.13	9.13	0

4^{th} Iteration

While it is interesting in this case to note the nuances of the algorithm, in this problem situation, there is only a difference of a matter of seconds from 10 minutes to 10.68. In an actual driving situation, road conditions, work zones and traffic flow may affect these times, causing the actual route time to vary and there really be no difference between the path choices. In other situations, this may not be the case and it could be possible to find the shortest route using this method.

This is the idea of mathematical modeling. It is possible to take a complex situation and represent it using mathematical language. A conclusion may be reached about the situation using equations and calculations, but this conclusion may not be perfect as assumptions made may not be correct or some factors in the situation cannot be represented mathematically. However, the first model of a situation is merely a start and can be corrected and perfected over time as assumptions are corrected and new mathematical strategies are used.

A mathematical model must be used by corporations such as *Google Maps*, *Mapquest*, and *Garmin* to determine an optimal driving route. As the maps used must be much larger than the situation modeled above, it is impossible to say what model is being used.

Another interesting application of tropical matrices is phylogenetics. **Phylogenetics** is a biological study to determine the difference between two or more species. The tropical matrices that are important in this application are called **metrics**, or matrices that when multiplied by themselves, result in the original matrices. Note that these are similar to standard idempotent matrices. A theorem exists that can help prove if a matrix represents a metric.

Theorem: Let $[D]$ be a matrix such that d_{ij} are entries in $[D]$. Then $d_{ik} \leq d_{ij}+d_{jk}$ holds for all indices i, j, k, if and only if $D \odot D = D$.

Proof: Let $D = (d_{ij})$ be a symmetric $n \times n$ matrix with zeros on the diagonal and positive off diagonal entries. If $d_{ik} \leq d_{ij} + d_{jk}$ for all indices i, j, k, then $d_{ik} \oplus d_{ij} \odot d_{jk} = d_{ik}$. Note that to find the value of the matrix entry in the i^{th} row and k^{th} column when taking $D \odot D$, it is necessary to tropically multiply the values in row i with the values of column k, then find the minimum. So, the entry in the i^{th} row and k^{th} column is $\min(d_{ii} + d_{ik}, d_{ij} + d_{jk}, d_{ik} + d_{kk})$. As all the diagonal entries are 0, both d_{ii} and d_{kk} are 0, $d_{ii} + d_{ik} = d_{ik} + d_{kk} = d_{ik}$. Now, as $d_{ik} \leq d_{ij} + d_{jk}$, the minimum is d_{ik} and the value of the entry in the i^{th} row and k^{th} column is d_{ik}, the original value. Thus, $D \odot D = D$.

Let $D \odot D = D$. Note that this means the original entries in

D are the same as the result of $D \odot D$. Then, by tropical matrix multiplication, the value of the entry in the i^{th} row and k^{th} column is $d_{ii} \odot d_{ik} \oplus d_{ij} \odot d_{jk} \oplus d_{ik} \odot d_{kk}$, which equals d_{ik} as it was the original entry in the i^{th} row and k^{th} column. Thus, $\min(d_{ii} + d_{ik}, d_{ij} + d_{jk}, d_{ik} + d_{kk}) = d_{ik}$. As all the diagonal entries are 0, both d_{ii} and d_{kk} are 0, $d_{ii} + d_{ik} = d_{ik} + d_{kk} = d_{ik}$. In order for the minimum to be d_{ik}, d_{ik} must be less than or equal to $d_{ij} + d_{jk}$. And thus, for $D \odot D = D$, $d_{ik} \leq d_{ij} + d_{jk}$ for all indices i, j, and k. ■

For further information on phylogenetics, refer to [3].

As with lines, it is interesting to compare tropical matrices with standard matrices. However, tropical matrices are a little more different than their standard counterparts. Take for example, the determinant. In a standard matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $ad - bc$. Applied to larger matrices, it is the summation of the product of all the regular diagonals, minus the summation of all the products of the reverse diagonals. The problem is that this method cannot be translated directly into tropical operations, as there is no tropical counterpart for subtraction. So, instead of subtraction, the tropical determinant is defined using addition. Thus for a tropical matrix, the determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a \odot d \oplus b \odot c = \min(a + d, b + c)$. The same pattern from a larger standard determinant can be used to find the determinant of larger tropical matrices, only instead of using subtraction, addition is always used.

With very large matrices, that computation can be very tedious. Luckily, there is an algorithm known as the Hungarian algorithm that can shorten this calculation. This algorithm was originally designed to solve the assignment problem and as such, it is not a proponent of tropical mathematics. The assignment problem is generally described to be when there is a number of jobs to be completed and the same number of people to complete them, but the cheapest way of assigning each person to each job is desired. An $n \times n$ matrix can be created with this scenario with people being assigned to the rows and jobs being assigned to the columns. Harold Kuhn first published the Hungarian algorithm which solved this combination in 1955 [2, p. 14].

4. Conclusions and Open Questions

A major component of this work was to explore the ideas behind tropical lines. In this section, the definition of a hypersurface was introduced to be the set of points where the minimum was obtained more than once. Further research can look into how these apply to other tropical curves such as quadratic and cubic curves. Also, what applications are there of tropical curves? The tropical line appears to share many characteristics with the standard line, but quadratics and cubic curves have a much different graphic structure than their standard counterparts. It is assumed that their applications are quite different.

Another direction for future research is with creating classroom enrichment opportunities using tropical mathematics. Redefining addition and multiplication enforces the idea that definitions are subjective to the situation they are being used in. Working with tropical mathematics may give students a chance to explore definitions and theorems for themselves. It would be necessary to determine an appropriate connection to currently studied mathematics and the level at which it could be introduced.

5. Note

For this exploration, the proofs are original works of the author, although they have been previously explored by other mathematicians. Any similarities in reasoning or word choice are coincidence.

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Optimal Numbers and Solutions in the Euclidean Algorithm

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1. History of the Euclidean Algorithm

The Euclidean Algorithm is an ancient algorithm used to determine the greatest common divisor (GCD) of two natural numbers. The Euclidean Algorithm is an intricate part of the RSA algorithm, which is used to protect electronic commerce. It is further used to find solutions to Linear Diophantine Equations, such as multiple congruences and the multiplicative inverses of a field. In general, the Euclidean Algorithm is a key topic in modern number theory. [4]

The algorithm is named after Euclid, who described it in Books VII, for integers, and Book X, for line segments, in his *Elements*. This makes it one of the oldest numerical algorithms that is still used by mathematicians today. There is evidence that suggests that Euclid was not the founder of the algorithm, that in fact Euclid took the algorithm from a text written by mathematicians in the school of Pythagoras. The algorithm may also have been used by Eudoxus of Cnidus, who was born nearly a century before Euclid. Euclid's algorithm may even go farther back than Eudoxus. [2]

Centuries after the publication of *Elements*, the Euclidean Algorithm was independently discovered in both India and China where it was used to solve problems in Astronomy. Qin Jiushao published the algorithm in 1247 in his book *Shushu Juzhang* with his general solution to the Chinese remainder theorem. In 1624, the Euclidean Algorithm was first published in Europe in Bachet's second edition of *Pleasant and Enjoyable Problems*. Gauss wrote on his application of the Euclidean Algorithm in his *Disquisitiones Arithmeticae*, published in 1801, where he explained its use to find the continued fraction representation of a rational number. [5]

It was not until Dirichlet that the Euclidean Algorithm became a ba-

sis for number theory. Dirichlet's lectures were edited and extended by Richard Dedekind, who used Euclid's algorithm to study algebraic integers and the Euclidean domain. [1]

In this paper, we turn our attention to a result proved by Gabriel Lamé in the early 1800's. Lamé's Theorem demonstrates the relationship between the Euclidean Algorithm and Fibonacci numbers.

2. Background

The Euclidean Algorithm:

Given two natural numbers a and b, find natural numbers q_1 and r_1 such that

$$a = q_1 b + r_1 \quad 0 \leq r_1 < b$$

If $r_1 \neq 0$, find natural numbers q_2 and r_2 such that:

$$b = q_2 r_1 + r_2 \quad 0 \leq r_2 < r_1$$

Continue in this manner: at each step, find the previous step's divisor in terms of its remainder. When a remainder of 0 is obtained, stop.

Theorem 1 *When performing the Euclidean Algorithm on $a, b \in \mathbb{N}$ the last non-zero remainder is $GCD(a, b)$.*

Proof. A proof can be found in [4]. ■

Example 1 *Find $GCD(21, 13)$ using the Euclidean Algorithm.*

Applying the Euclidean Algorithm to 21 and 13 gives

$$\begin{aligned} 21 &= 1(13) + 8 \\ 13 &= 1(8) + 5 \\ 8 &= 1(5) + 3 \\ 5 &= 1(3) + 2 \\ 3 &= 1(2) + 1 \\ 2 &= 2(1) + 0. \end{aligned}$$

This implies that the GCD$(21, 13) = 1$.

Given two natural numbers, a and b, the *number of steps* it takes for the Euclidean Algorithm to reach a remainder of 0 is denoted $T(a, b)$. For example, in Example 1 we find that $T(21, 13) = 6$. Note that the remainders in Example 1 are Fibonacci numbers, which give some indication of their role in determining $T(21, 13)$. This is a central concept in Lamé's research and is a major focus of this article.

Definition 1 *The **Fibonacci numbers** are generated by the additive sequence:*

$F_o = 0$
$F_1 = 1$
$F_{n+1} = F_n + F_{n-1}$ *where* $n \geq 1$.

Fibonacci numbers can be explicitly determined by the closed formula

$$F_n = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}},$$

where $\varphi = \frac{1+\sqrt{5}}{2}$.[4] The Fibonacci numbers can be used to determine an upper bound for $T(a, b)$.

Lemma 1 *Let* $a, b \in \mathbb{N}$. *If* $b < F_{n+2}$ *for some* $n \in \mathbb{N}$ *then* $T(a, b) \leq n$.

Proof.

Suppose $b < F_{n+2}$ for some $n \in \mathbb{N}$ and suppose to the contrary that $T(a, b) = k$ where $k \geq n + 1$. Performing the Euclidean Algorithm on a and b yields

$$\begin{array}{rll}
a & = q_1(b) + r_1 & 0 < r_1 < b \\
b & = q_2(r_1) + r_2 & 0 < r_2 < r_1 \\
r_1 & = q_3(r_2) + r_3 & 0 < r_3 < r_2 \\
 & \cdots & \cdots \\
r_{k-4} & = q_{k-2}(r_{k-3}) + r_{k-2} & 0 < r_{k-2} < r_{k-3} \\
r_{k-3} & = q_{k-1}(r_{k-2}) + r_{k-1} & 0 < r_{k-1} < r_{k-2} \\
r_{k-2} & = q_k(r_{k-1}) + r_k & 0 < r_k < r_{k-1} \\
r_{k-1} & = q_{k+1}(r_k) + 0. &
\end{array}$$

We have $0 < r_k$ implying $r_k \geq 1$. Likewise $r_k < r_{k-1}$, implying $r_{k-1} \geq 2$. In general

$$\begin{aligned}
r_{k-i} &= q_{k-(i-2)}(r_{k-(i-1)}) + r_{k-(i-2)} \\
&\geq r_{k-(i-1)} + r_{k-(i-2)}.
\end{aligned}$$

Since $r_{k-i} \geq r_{k-(i-1)} + r_{k-(i-2)}$ where $r_k \geq 1 = F_1$ and $r_{k-1} \geq 2 = F_2$ we find

$$r_{k-i} \geq F_{i+1}.$$

By letting $b = r_0 = r_{k-k}$, we get $b \geq F_{k+1}$. Since we assumed $k \geq n+1$, it follows that $b \geq F_{n+2}$, which is a contradiction. Therefore, $T(a, b) \leq n$. ■

Remark 2 *In the proof of Lemma 1, we showed that when $T(a,b) = k$, $r_{k-i} \geq F_{i+1}$ where $T(a,b) = k$ for $0 \leq i \leq k$. It follows directly that*

$$r_i \geq F_{k-i+1}.$$

Lemma 2 $F_{n+1} < \varphi^n < F_{n+2}$, *for all* $n \in \mathbb{N}$.

Proof.

We will prove the upper bound of our inequality. For the case of $n = 1, 2$ we find that $\varphi < F_3$ and $\varphi^2 < F_4$. Let $n \in \mathbb{N}$ and suppose that $\varphi^k < F_{k+2}$ for all $k < n + 2$. We want to show that $\varphi^{n+1} < F_{n+3}$. Observe

$$\varphi^n + \varphi^{n-1} < F_{n+2} + F_{n+1},$$

which implies that

$$\varphi^{n+1}\left(\frac{1}{\varphi} + \frac{1}{\varphi^2}\right) < F_{n+3}.$$

Therefore,

$$\varphi^{n+1} < F_{n+3}.$$

We can similarly show that $F_{n+1} < \varphi^n$. ■

For $b \in \mathbb{N}$, let

$$N(b) = \left\lceil \frac{\ln(b)}{\ln(\varphi)} \right\rceil.$$

We now prove that $N(b)$ is an upper bound for $T(a,b)$.

Theorem 3 ***(Lamé's Bound)*** *Given $b > 1$, $T(a,b) \leq N(b)$.*

Proof. Suppose $a, b \in \mathbb{N}$ where $b > 1$. Let $n = \left\lceil \frac{\ln(b)}{\ln(\varphi)} \right\rceil$. It follows that $\log_\varphi b \leq n$, so that $b \leq \varphi^n$. However, by Lemma 2, $\varphi^n < F_{n+2}$. Therefore, $b < F_{n+2}$. Applying Lemma 1, we find that $T(a,b) \leq n$. ■

Definition 2 *A natural number, b, is **optimal** if there exists a natural number, a, such that $T(a,b) = N(b)$. In this case, the pair (a,b) is called an **optimal solution**.*

In Example 1, we showed that $T(21,13) = 6 = N(13)$. Therefore, 13 is optimal and $(21,13)$ is an optimal solution.

Example 2 *There exists no natural number a such that $T(a,6) = N(6)$. In other words, $b = 6$ is not optimal.*

Notice that $N(6) = \left\lceil \frac{\ln(6)}{\ln(\varphi)} \right\rceil = 4$. In general we find that

$$\begin{aligned} a &= q_1 \cdot 6 + r_1 \\ 6 &= q_2 \cdot r_1 + r_2 \\ r_1 &= q_3 \cdot r_2 + r_3 \end{aligned}$$

r_1	q_2	r_2	q_3	r_3	*number of steps*
0	–	–	–	–	1
1	6	0	–	–	2
2	3	0	–	–	2
3	2	0	–	–	2
4	1	2	2	0	3
5	1	1	5	0	3

Therefore $T(a, 6) \leq 3 < N(6)$ *implying that* 6 *is not optimal.*

3. Optimal Numbers

We have shown that 13 is optimal but 6 is not. We now determine which natural numbers are optimal. We begin by proving that all Fibonacci numbers are optimal. We will then show that Lucas numbers of odd index are also optimal. We conclude this section by proving that these are the only natural numbers which are optimal.

Theorem 4 $N(F_{n+1}) = n$ *for all* $n \in \mathbb{N}$ *where* $n \in \mathbb{Z}^n$.

Proof. By Lemma 2, $F_{n+1} < \varphi^n < F_{n+2}$. It follows that

$$\varphi^{n-1} < F_{n+1} < \varphi^n,$$

so that

$$\ln(\varphi^{n-1}) < \ln(F_{n+1}) < \ln(\varphi^n),$$

which implies

$$n - 1 < \frac{\ln(F_{n+1})}{\ln(\varphi)} < n.$$

Therefore, $N(F_{n+1}) = \left\lceil \frac{\ln(F_{n+1})}{\ln(\varphi)} \right\rceil = n$. ■

Theorem 4 and $n = 6$ give $N(13) = N(F_7) = N(F_{6+1}) = 6$, which was the result from Example 1.

Now that we have proven that the maximum number of steps for a given Fibonacci number is one less than its index, we can use Lamé's Theorem to prove that all Fibonacci numbers are optimal.

Theorem 5 ***Lamé's Theorem****: For* $n \geq 1$*, let* a *and* b *be integers with* $a > b > 0$ *such that the Euclidean Algorithm applied to* a *and* b *requires exactly* n *division steps and such that* a *is as small as possible satisfying these conditions. Let* F_k *denote the* k*th Fibonacci number, then* $a = F_{n+2}$ *and* $b = F_{n+1}$.

Proof. This proof is similar to that of Lemma 1. ■

Theorem 6 *F_n is optimal for all $n \in \mathbb{N}$.*

Proof. Suppose $b = F_n$ for some $n \in \mathbb{N}$. Theorem 4 implies that $N(F_n) = n - 1$; furthermore, by Lamé's Theorem, we know that

$$T(F_{n+1}, F_n) = n - 1.$$

Therefore, $T(F_{n+1}, F_n) = N(F_n)$. Thus, F_n is optimal by definition. ■

We will now examine the Lucas numbers. The Lucas sequence is an additive sequence which is closely related to the Fibonacci sequence.

Definition 3 *The **Lucas numbers** are generated by the additive sequence:*
$L_o = 2$
$L_1 = 1$
$L_{n+1} = L_n + L_{n-1}$ *where* $n \geq 1$.

Lucas numbers can be explicitly determined by the closed formula

$$L_n = \varphi^n + (-\varphi)^{-n}. \text{ [4]}$$

Lemma 3 *$L_n \leq F_{n+2}$ for $n \in \mathbb{N}$.*

Proof. Let $n \in \mathbb{N}$. Consider the case when $n = 1$, it follows that $L_1 = 1 < 3 = F_3$. Likewise when $n = 2$ we have $L_2 = 3 = F_4$. Now suppose that $L_k \leq F_{k+2}$ for all $k < n$ where n is fixed. We find

$$L_n = L_{n-1} + L_{n-2} \leq F_{n+1} + F_n = F_{n+2}.$$

We have shown that $L_n \leq F_{n+2}$ for $n \in \mathbb{N}$. ■

By applying Lemma 1, we find that $T(L_{n+1}, L_n) \leq n$. We now determine a closed formula for $N(L_n)$ based on the parity of n.

Lemma 4 *$\varphi^{2n} < L_{2n} < L_{2n+1} < \varphi^{2n+1}$ for all $n \in \mathbb{N}$.*

Proof. Let $n \in \mathbb{N}$. Then

$$\begin{aligned} L_{2n} &= \varphi^{2n} + (-\varphi)^{-2n} \\ &= \varphi^{2n} + (-1)^{2n} \cdot \varphi^{-2n} \\ &= \varphi^{2n} + \frac{1}{\varphi^{2n}} \\ &> \varphi^{2n}. \end{aligned}$$

Similarly,

$$\begin{aligned} L_{2n+1} &= \varphi^{2n+1} + (-\varphi)^{-(2n+1)} \\ &= \varphi^{2n+1} + (-1)^{2n+1} \cdot \varphi^{-(2n+1)} \\ &= \varphi^{2n+1} - \frac{1}{\varphi^{2n+1}} \\ &< \varphi^{2n+1}. \end{aligned}$$

Furthermore, we can say that $L_{2n} < L_{2n+1}$ since the Lucas sequence is an increasing sequence. Hence, $\varphi^{2n} < L_{2n} < L_{2n+1} < \varphi^{2n+1}$. ■

Theorem 7 $N(L_{2n}) = 2n + 1 = N(L_{2n+1})$ *for all* $n \in \mathbb{N}$.

Proof. We begin with the statement of Lemma 4.

$$\varphi^{2n} < L_{2n} < L_{2n+1} < \varphi^{2n+1}$$

which implies

$$\ln(\varphi^{2n}) < \ln(L_{2n}) < \ln(L_{2n+1}) < \ln(\varphi^{2n+1})$$

so that

$$2n < \frac{\ln(L_{2n})}{\ln(\varphi)} < \frac{\ln(L_{2n+1})}{ln(\varphi)} < 2n + 1.$$

Therefore,

$$\left\lceil \frac{\ln(L_{2n})}{\ln(\varphi)} \right\rceil = \left\lceil \frac{\ln(L_{2n+1})}{ln(\varphi)} \right\rceil = 2n + 1.$$

■

Example 3 *We have for* $n = 4$ *in Theorem 7,*

$N(47) = N(L_8) = N(L_{2\cdot 4}) = 2(4)+1 = N(L_{2\cdot 4+1}) = N(L_9) = N(76).$

Theorem 8 *Given* $L_n \neq F_m$, L_n *is optimal if and only if* n *is odd.*

Proof. First suppose that n is odd. Then $n = 2k + 1$ for some $k \in \mathbb{Z}^+$. By Theorem 7, $N(L_n) = 2k + 1$. Consider the Euclidean Algorithm performed on $L_{n+1} = L_{2k+2}$ and $L_n = L_{2k+1}$:

$$\begin{aligned} L_{2k+2} &= q_1(L_{2k+1}) + r_1 \\ &= L_{2k+1} + L_{2k} \\ L_{2k+1} &= L_{2k} + L_{2k-1} \\ &\cdots \\ L_3 &= L_2 + L_1 \\ L_2 &= 3(L_1) + 0. \end{aligned}$$

Thus, $T(L_{n+1}, L_n) = 2k + 1$ as desired, implying that L_n is optimal if n is odd.

Instead, suppose that n is even. Then $n = 2k$ for some $k \in \mathbb{N}$. Applying Lemma 3 we find that $L_n \leq F_{n+2}$. We assumed that $L_n \neq F_m$. Thus $L_n < F_{n+2}$. By Lemma 1, $T(a, L_n) \leq n$ for any $a \in \mathbb{N}$. Furthermore, by Theorem 7, $N(L_n) = N(L_{2k}) = 2k + 1 = n + 1$. Thus, $T(a, L_n) \leq n < n + 1 = N(L_n)$, implying that L_n is not optimal when n is even. ■

We have shown that all Fibonacci numbers and Lucas numbers of odd index are optimal. The remainders we have considered thus far have been generated by an additive sequence where each nonterminating remainder does not divide the preceding remainder. We prove that, in fact, when using the Euclidean Algorithm to compute $T(a, b)$ if there exists a quotient $q_i > 1$ for some i where $1 < i < T(a, b)$ then (a, b) is not an optimal solution. To do so, we represent a and b in terms of Fibonacci numbers. We consider the case of $r_0 = b$ when applicable.

Lemma 5 *Consider the Euclidean Algorithm performed on $a, b \in \mathbb{N}$. If there exists $i \in \mathbb{N}$ such that for all $j \leq i$ we have $q_j = 1$, then*

$$a = F_i(r_{i-2}) + F_{i-1}(r_{i-1}) \text{ and}$$
$$b = F_{i-1}(r_{i-2}) + F_{i-2}(r_{i-1}).$$

Before proving Lemma 5, we will given an example.

Example 4 *We use the Euclidean Algorithm to find $T(60, 38)$ and then use Lemma 5 to find an equivalent $T(a', b')$ involving Fibonacci numbers and remainders.*

$$\begin{aligned} 60 &= (1)38 + 22 \\ 38 &= (1)22 + 16 \\ 22 &= (1)16 + 6 \\ 16 &= (2)6 + 4 \\ 6 &= (1)4 + 2 \\ 4 &= (2)2 + 0. \end{aligned}$$

Thus, $T(60, 38) = 6$. Notice that $q_4 > 1$ and $q_1, q_2, q_3 = 1$, thus $i = 4$. We can now apply Lemma 5 to determine a' and b':

$$\begin{aligned} a' &= F_i(r_{i-2}) + F_{i-1}(r_{i-1}) \\ &= F_4(r_2) + F_3(r_3) \\ &= 3(16) + 2(6) = 60. \end{aligned}$$

$$\begin{aligned} b' &= F_{i-1}(r_{i-2}) + F_{i-2}(r_{i-1}) \\ &= F_3(r_2) + F_2(r_3) \\ &= 2(16) + 1(6) = 38. \end{aligned}$$

Therefore,

$$\begin{aligned} 60 &= F_4(r_2) + F_3(r_3) \\ 38 &= F_3(r_2) + F_2(r_3). \end{aligned}$$

We also find that $N(38) = 8$, *so* $(60, 38)$ *is not an optimal solution.*

Proof. Suppose that $i = 2$. This implies that,

$$\begin{aligned} a &= q_1 b + r_1 \\ &= b + r_1 \\ &= r_0 + r_1 \\ &= r_{i-2} + r_{i-1}. \end{aligned}$$

Furthermore,

$$\begin{aligned} b &= q_2 r_1 + r_2 \\ &= r_1 + r_2 \\ &= r_{i-1} + r_i. \end{aligned}$$

Thus, $T(a, b) = T(b + r_1, b) = T(F_2(b) + F_1(r_1), F_1(b) + F_0(r_1))$.

Now suppose, for some $k \in \mathbb{N}$,

$$T(a, b) = T(F_k(r_{k-2}) + F_{k-1}(r_{k-1}), F_{k-1}(r_{k-2}) + F_{k-2}(r_{k-1})),$$

We will first show that $a = F_{k+1}(r_{k-1}) + F_k(r_k)$. By assumption

$$\begin{aligned} a &= F_k(r_{k-2}) + F_{k-1}(r_{k-1}) \\ &= F_k(r_{k-1} + r_k) + F_{k-1}(r_{k-1}) \\ &= F_k(r_{k-1}) + F_k(r_k) + F_{k-1}(r_{k-1}) \\ &= (F_k + F_{k-1})(r_{k-1}) + F_k(r_k) \\ &= F_{k+1}(r_{k-1}) + F_k(r_k). \end{aligned}$$

Again, by our inductive assumption we have

$$\begin{aligned} b &= F_k(r_{k-1}) + F_{k-1}(r_k) \\ &= F_{k-1}(r_{k-1} + r_k) + F_{k-2}(r_{k-1}) \\ &= F_{k-1}(r_{k-1}) + F_{k-1}(r_k) + F_{k-2}(r_{k-1}) \\ &= (F_{k-1} + F_{k-2})(r_{k-1}) + F_{k-1}(r_k) \\ &= F_k(r_{k-1}) + F_{k-1}(r_k). \end{aligned}$$

Therefore,

$$a = F_{k+1}(r_{k-1}) + F_k(r_k)$$
$$b = F_k(r_{k-1}) + F_{k-1}(r_k).$$

■

Theorem 9 ***(The Shifting Property)*** $F_{m+n} = F_m(F_{n+1}) + F_{m-1}(F_n)$, *where* $m, n \in \mathbb{N}$.

Lemma 6 $F_{n+1} = 2F_{n-1} + F_{n-2}$, *for all* $n \in \mathbb{N}$.

Proof. This can be shown using a proof by strong induction. ■

Lemma 7 *Given natural numbers* $a < m \leq n < b$ *where* $m = a + k$ *and* $n = b - k$, $mn > ab$.

Proof.

Consider the product

$$\begin{aligned} mn &= (a + k)(b - k) \\ &= ab - ak + bk - k^2 \\ &= ab - k(a - b + k) \end{aligned}$$

Since $b > m = a+k$ we find $a-b+k < 0$. Therefore, $-k(a-b+k) > 0$, which implies that $ab < ab - k(a - b + k) = mn$. ■

Theorem 10 *If, when the Euclidean Algorithm is performed on natural numbers* a *and* b, *we have* $q_i > 1$ *for some* i *such that* $1 < i < T(a, b)$ *and* $q_j = 1$ *for all* $j < i$, *then* (a, b) *is not an optimal solution.*

Proof. Suppose to the contrary that (a, b) is an optimal solution. Then $T(a, b) = N(b)$. By Lemma 5 we know that

$$b = F_{i-1}(r_{i-2}) + F_{i-2}(r_{i-1}).$$

Since

$$r_{i-2} = (q_i)r_{i-1} + r_i$$

and $q_i > 1$, we can conclude that $r_{i-2} \geq 2r_{i-1} + r_i$. Thus,

$$\begin{aligned}
b &\geq F_{i-1}(2(r_{i-1}) + r_i) + F_{i-2}(r_{i-1}) \\
&= r_{i-1}(2F_{i-1} + F_{i-2}) + F_{i-1}(r_i) \\
&= r_{i-1}(F_{i+1}) + F_{i-1}(r_i) \text{ (by Lemma 6)} \\
&= (F_i + F_{i-1})(r_{i-1}) + F_{i-1}(r_i) \\
&= F_i(r_{i-1}) + F_{i-1}(r_i) + F_{i-1}(r_{i-1}) \\
&\geq F_i(F_{(T(a,b)-(i-2))}) + F_{i-1}(F_{(T(a,b)-(i-1))}) + F_{i-1}(F_{(T(a,b)-(i-2))}) \\
&\qquad \text{(by Remark 2)} \\
&= F_{(T(a,b)+1)} + F_{i-1}(F_{T(a,b)-(i-2)}) \text{ (by The Shifting Property)} \\
&\geq F_{(T(a,b)+1)} + F_{T(a,b)} \text{ (by Lemma 7)} \\
&= F_{(T(a,b)+2).}
\end{aligned}$$

By Theorem 4, we have

$$N(b) \geq N(F_{(T(a,b)+2)}) = T(a,b) + 1.$$

However, by assumption, $N(b) = T(a,b)$. By contradiction, we must conclude that (a,b) is not an optimal solution. ■

Corollary 1 *For all optimal solutions (a,b), where $a < 2b$, there exists an additive sequence $\langle S_n \rangle$ such that $a = S_{T(a,b)}$ and $b = S_{T(a,b)-1}$.*

Proof.

Suppose (a,b) is an optimal solution where $a < 2b$. By Theorem 10, $q_i = 1$ for $0 < i < T(a,b)$. Furthermore, $q_0 = 1$ since $a < 2b$. Using this fact, we can write a and b as consecutive terms in the additive sequence $\langle S_n \rangle$ where

$$\begin{aligned}
S_0 &= r_{T(a,b)-1} \\
S_1 &= r_{T(a,b)-2} \\
S_n &= S_{n-1} + S_{n-2}.
\end{aligned}$$

It follows by our construction of S that $a = S_{T(a,b)}$ and $b = S_{T(a,b)-1}$. ■

Corollary 1 gives us an important characterization of optimal and non-optimal solutions. Another crucial insight is that Fibonacci and Lucas numbers are the slowest growing additive sequences which have positive generating terms. We will use both of these facts to prove that Fibonacci numbers and Lucas numbers of odd index are the only optimal numbers.

Theorem 11 *Given $b \in \mathbb{N}$, b is optimal if and only if $b = F_n$ or $b = L_{2n+1}$ where $n \in \mathbb{N}$.*

Proof.

By Theorem 6, $b = F_n$ is optimal. Similarly, by Theorem 8, $b = L_{2n+1}$ is optimal. Suppose to the contrary that $b \neq F_n, L_n$ is optimal. Then there exists $a \in \mathbb{N}$ where $a < 2b$ such that (a, b) is an optimal solution. Furthermore, let $T(a, b) = k = N(b)$. By Corollary 1, there exists an additive sequence $\langle S_n \rangle$ where $a = S_k$ and $b = S_{k-1}$.

Since $\langle S_n \rangle \neq \langle F_n \rangle, \langle L_n \rangle$ we find that the smallest, relatively prime, generating values for $\langle S_n \rangle$ are $S_0 = 1$ and $S_1 = 4$ which generate the sequence

$$\langle S_n \rangle = \{1, 4, 5, 9, 14, 23, 37, 60, ...\}.$$

We can show that $S_n \geq F_{n+3}$ for $n > 1$ using a proof similar to that of Lemma 3. It follows that $S_{k-1} \geq F_{k+2}$, which implies that $N(S_{k-1}) \geq N(F_{k+2})$. By Theorem 4, $N(F_{k+2}) = k + 1$. Therefore,

$$T(a, b) = T(S_k, S_{k-1}) = k < k + 1 \leq N(S_{k-1}) = N(b).$$

Thus $T(a, b) < N(b)$ which is a contradiction. Since $k > 1$ was chosen arbitrarily, we find that (S_{n+1}, S_n) is not optimal for all $n \in \mathbb{N}$. Given another additive sequence generated by relatively prime terms, $\langle S'_n \rangle \neq \langle F_n \rangle, \langle L_n \rangle$, we find that $S'_n \geq S_n$ for all $n \in \mathbb{N}$. We similarly conclude that (S'_{n+1}, S'_n) is not an optimal solution. Therefore, the only optimal numbers are of the form $b = F_n$ or $b = L_{2n+1}$. ■

4. Optimal Solutions

We have already shown that (F_{n+1}, F_n) and (L_{2n}, L_{2n-1}) are optimal solutions. We now provide another infinite set of optimal solutions.

Corollary 2 $T(F_{n+1}(1 + mF_n), F_n) = N(F_n)$, *where* $m, n \in \mathbb{N}$.

Proof.

Let $a = F_{n+1}(1 + mF_n)$ and $b = F_n$. Observe that

$$\begin{aligned}
\frac{a}{b} &= \frac{F_{n+1}(1 + mF_n)}{F_n} \\
&= \frac{(F_n + F_{n-1})(1 + mF_n)}{F_n} \\
&= \frac{F_n(1 + mF_n + mF_{n-1}) + F_{n-1}}{F_n} \\
&= 1 + mF_n + mF_{n-1} + \frac{F_{n-1}}{F_n}.
\end{aligned}$$

Thus, $a = bq + r = F_n(1 + mF_n + mF_{n-1}) + F_{n-1}$. By Theorem 6, $(F_{n+1}(1 + mF_n), F_n)$ is an optimal solution. ■

Corollary 3 $T(L_{2n+2}(1+mL_{2n+1}), L_{2n+1}) = N(L_{2n+1}), where\, m, n \in \mathbb{N}$.

Proof. Proof similar to Corollary 2. ■

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before October 1, 2014. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2014 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051).

NEW PROBLEMS 730-739

Problem 730. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

Find two primes such that neither divides any of the numbers 2^a+2^b+1 where a and b are positive integers.

Problem 731. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

1. Solve the Diophantine equation $1+2p = y^3$ where p may be any prime.
2. Solve the Diophantine equation $1+4p = y^3$ where p may be any prime.

Problem 732. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

By PPT we mean a primitive Pythagorean triple (a, b, c) such that $a^2 + b^2 = c^2$ and $\gcd(a, b) = 1$. Prove the following three things.

1. There are infinitely many multiples of 5 that occur as the hypotenuse (the third value) of a PPT.

2. There are infinitely many multiples of 5 that occur as the odd leg of a PPT.

3. Every odd multiple of 5 occurs as either the hypotenuse or odd leg of a PPT.

Problem 733. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

Prove that every square number $4, 9, 16, \ldots$ is the leg of some PPT by providing an explicit construction.

Problem 734. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA..*

Let $T_n = \frac{n(n+1)}{2}$ and $P_n = \frac{n(3n-1)}{2}$ be the n^{th} triangular and pentagonal numbers, respectively. Prove that there are infinitely many positive integers a, b, c such that
$P_a P_b = 2T_c$.

Problem 735. *Proposed by Ovidiu Furdiu, Technical University of Cluj-Napoca, Cluj, Romania.*

Let $n \geq 1$ be an integer. Calculate the integral

$$\int_0^{2\pi} \sin^2(x) \sin^2(2x) \cdots \sin^2(nx)\, dx.$$

Problem 736. *Proposed by D.M. Batinetu-Siurgiu, "Matie Basarab" National College, Bucharest and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.*

Let $a, b \in \mathbb{R}$ with $a < b$ and continuous functions $f, h : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(a+b-x) = -f(x), h(a+b-x) = -h(x), \forall x \in \mathbb{R}$. Prove that

$$\int_a^b f(x) \arctan(x) \ln(1 + e^{h(x)})\, dx = \frac{1}{2} \int_a^b f(x)h(x) \arctan(x)\, dx.$$

Problem 737. *Proposed by Jose Luis Diaz-Barrero, Technical University of Catalonia (BARCELONA TECH), Barcelona, Spain.*

Let α, β, γ be the measures of the angles of an acute triangle ABC. Prove that

$$\cos^2 \alpha \sin \beta + \cos^2 \beta \sin \gamma + \cos^2 \gamma \sin \alpha < \frac{5}{4}.$$

Problem 738. *Proposed by Jose Luis Diaz-Barrero, Technical University of Catalonia (BARCELONA TECH), Barcelona, Spain.*

Let x, y, z be positive real numbers such that $xyz = 1$. Prove that

$$\sqrt[4]{x\left(\frac{x+y}{x^2+y^2}\right)^2}+\sqrt[4]{y\left(\frac{y+z}{y^2+z^2}\right)^2}+\sqrt[4]{z\left(\frac{z+x}{z^2+x^2}\right)^2} \leq \sqrt{3(x+y+z)}.$$

Problem 739. *Proposed by Ed Wilson, Eastern Kentucky University, Richmond, KY.*

A set of seven positive integers form a geometric sequence. The median of the set is 168 and the mean is 381. What is the first term of the sequence and what is the common ratio?

SOLUTIONS TO PROBLEMS 711-721

Problem 711. *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA.*

Prove that there are infinitely many squares that are each the sum of two cubes and infinitely many cubes that are each the sum of two squares.

Solution *by Lindsey Conrotto (student), Eastern Kentucky University, Richmond, KY.*

Consider powers of 2. Let $k \in \mathbb{N}$. Then $2^{3(2k-1)}$ is a cube and

$$2^{3(2k-1)} + 2^{3(2k-1)} = 2 * 2^{3(2k-1)} = 2^{6k-2} = 2^{2(3k-1)}$$

which is a square. Since k is arbitrary, there are infinitely many squares that are the sum of two cubes.

Let $k \in \mathbb{N}$. Then 2^{6k-4} is a square and

$$2^{6k-4} + 2^{6k-4} = 2 * 2^{6k-4} = 2^{6k-3} = 2^{3(2k-1)}$$

which is a cube. Since k is arbitrary, there are infinitely many cubes that are the sum of two squares.

Also solved by Nathan Russell (student), Eastern Kentucky University, Richmond, KY; Carl Libis, Lane College, Jackson, TN; Bill Yankosky, North Carolina Wesleyan College, Rocky Mount, NC; and the proposer.

Problem 712. *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA.*

Find, with proof, all positive integer powers of 2 that can be written as the sum of two squares (of positive integers). Do the same for powers of 5.

Solution *by Carl Libis, Lane College, Jackson, TN.*

For each natural number n, 2^{2n+1} is the sum of two squares since it is the sum $(2^n)^2 + (2^n)^2$. Thus all odd powers of 2 can be written as the sum of two squares.

For each natural number n, 5^{2n+1} is the sum of two squares since it is the sum $(5^n)^2 + (2 \cdot 5^n)^2$. For each natural number n, 5^{2n} is the sum of two squares since it is the sum

$$\left(3 \cdot 5^{n-1}\right)^2 + \left(4 \cdot 5^{n-1}\right)^2 = 9 \cdot 5^{2n-2} + 16 \cdot 5^{2n-2} = 25 \cdot 5^{2n-2} = 5^{2n}.$$

Also solved by Nathan Russell (student), Eastern Kentucky University, Richmond, KY; Bill Yankosky, North Carolina Wesleyan College, Rocky Mount, NC (odd powers of 2 and even powers of 5); and the proposer.

Problem 713. *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA.*

Show that every positive integer power of 2 greater than 1 is either the sum of two triangular numbers or the sum of a triangular number and a pentagonal number.

Solution *by Steven Sly (student), Northeastern State University, Tahlequah, OK.*

We prove a more general result: Every number which is a square or twice a square is either the sum of two triangular numbers or the sum of a triangular number and a pentagonal number.

Consider $2n$ for n a natural number. We know that n is either even or odd, namely, $n = 2k$ or $n = 2k + 1$. In the even case, $2^n = 2^{2k} = (2^k)^2$ which is a square. In the odd case, $2^n = 2^{2k+1} = 2 \cdot (2^k)^2$ which is twice a square.

Now consider any square, say m^2 with m a natural number. We have

$$m^2 = \frac{m(m-1)}{2} + \frac{m(m+1)}{2} = T_{m-1} + T_m$$

which is the sum of two triangular numbers. Now consider two times any

square, say $2s^2$ with s a natural number. We have

$$2s^2 = \frac{s(s+1)}{2} + \frac{s(3s-1)}{2} = T_s + P_s$$

which is the sum of a triangular number and a pentagonal number.

Also solved by Michael Crockett (student), Northeastern State University, Tahlequah, OK; Wade Combs (student), Eastern Kentucky University, Richmond, KY; and the proposer.

Problem 714. *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA.*

The triangular numbers are 1, 3, 6, 10,... and satisfy the formula $T_n = \frac{n(n+1)}{2}$.

a) Prove that the squares of infinitely many triangular numbers can be written as the sum of a square and a cube.

b) Prove that an infinite sequence of squares of nontrivial multiples of triangular numbers can be written as the sum of a square and a cube.

Solution *by the proposer*.

a)Note that $1^2 + 2^3 = T_2^2$, and $3^2 + 3^3 = T_3^2$. In general, we have

$$T_{n-1}^2 + n^3 = \frac{n^2(n^2-2n+1)}{4} + \frac{4n^3}{4} = \frac{n^2(n^2+2n+1)}{4} = T_n^2,$$

for all $n \geq 2$.

b)Take $b > 1$ and set $a = b^2 - 1$. Then

$$a^2 + b^3 = a^2(1+a) = a^2b^2 = (ab)^2.$$

Now, $ab = (b^2-1)b = (b-1)(b+1)b = 2(b-1)T_b$, and so we have $(2(b-1)T_b)^2 = a^2 + b^3$. Hence the sequence $(2T_2)^2, (4T_3)^2, (6T_4)^2, ...$ is a sequence that works.

Also, part a) was proved by Neculai Stanciu and Titu Zvonaru, Romania.

Problem 715. *Proposed by D. M. Batinetu-Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil Palade Secondary School, Buzau, Romania.*

For $t > 0$, define

$$x_n(t) = n^{1-t}\left(\frac{\left(\sqrt[n+1]{(n+1)!}\right)^{2t}}{(n+1)^t} - \frac{\left(\sqrt[n]{n!}\right)^{2t}}{n^t}\right).$$

Calculate $\lim_{n\to\infty} x_n(t)$.

Solution *by the proposers.*

We have

$$\begin{aligned} x_n(t) &= n^{1-t}\cdot\frac{\left(\sqrt[n]{n!}\right)^{2t}}{n^t}\cdot(u_n-1) \qquad (1)\\ &= \left(\frac{\sqrt[n]{n!}}{n}\right)^{2t}\cdot n\cdot\frac{u_n-1}{\ln u_n}\cdot\ln u_n\\ &= \left(\frac{\sqrt[n]{n!}}{n}\right)^{2t}\cdot\frac{u_n-1}{\ln u_n}\cdot\ln u_n^n, \end{aligned}$$

for all $n \geq 2$, where we denote

$$u_n = \left(\frac{\sqrt[n+1]{(n+1)!}}{\sqrt[n]{n!}}\right)^{2t}\cdot\left(\frac{n}{n+1}\right)^t,$$

for all $n \geq 2$. We have $\lim_{n\to\infty} u_n = 1$, and then, $\lim_{n\to\infty}\frac{u_n-1}{\ln u_n} = 1$. We also have

$$\begin{aligned} \lim_{n\to\infty} u_n^n &= \lim_{n\to\infty} e^{-t}\left(\frac{\sqrt[n+1]{(n+1)!}}{\sqrt[n]{n!}}\right)^{2t}\\ &= \lim_{n\to\infty} e^{-t}\left(\frac{(n+1)!}{n!}*\frac{1}{\sqrt[n]{n!}}\right)^{2nt}\\ &= e^{-t}\lim_{n\to\infty}\left(\frac{n+1}{\sqrt[n+1]{(n+1)!}}\right)^{2t}\\ &= e^{-t}e^{2t} = e^t. \end{aligned}$$

Finally, by (1), we have $\lim_{n\to\infty} x_n(t) = e^{-2t}\cdot 1\cdot\ln(e^t) = t\cdot e^{-2t}$.

Also solved by G.C. Greubel (graduate student), Old Dominion University, Norfolk, VA.

Problem 716. *Proposed by D. M. Batinetu-Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil*

Palade Secondary School, Buzau, Romania.

Prove that if $m \geq 0$, then in any triangle ABC, the following holds:

$$\cot^{m+1}\frac{A}{2} + \cot^{m+1}\frac{B}{2} + \cot^{m+1}\frac{C}{2} \geq 3^{(m+3)/2}.$$

Solution *by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

Since the angles of the triangle sum up to $180°$, we have $A + B + C = 180°$. Since the function $f(x) = \cot^{m+1} x$ is convex on the interval $[0°, 90°]$ (the interval where the angles $\frac{A}{2}$, $\frac{B}{2}$, $\frac{C}{2}$ lie), the Jensen inequality says the left-hand side of the inequality satisfies the inequality

$$\begin{aligned} LHS \geq 3\cot^{m+1}\left(\frac{\frac{A}{2}+\frac{B}{2}+\frac{C}{2}}{3}\right) &= 3\cot^{m+1} 30° \\ &= 3\left(\sqrt{3}\right)^{m+1} = 3^{(m+3)/2}. \end{aligned}$$

Also solved by Ion Viorel Codreanu, Satulung, Maramures, Romania; and the proposers.

Problem 717. *Proposed by Pedro H. O. Pantoja (student), Natal-RN, Brazil.*

Let A and B be 4×4 matrices with integer entries such that $\det A + \det B = 0$. Prove that

$$|\det(A + 2B) + 15\det A - 3\det(A + B) - \det(A - B)|$$

is a multiple of 6.

Solution *by the proposer.*

Let $\det(A + xB) = \det A + ax + bx^2 + cx^3 + (\det B)x^4$, where $a, b, c \in \mathbb{Z}$. Since $\det A + \det B = 0$, we have $\det(A + B) = a + b + c$, and $\det(A-B) = -a+b-c$. Adding and subtracting these two equations, we have

$$\det(A + B) + \det(A - B) = 2b$$

and

$$\det(A + B) - \det(A - B) = 2(a + c).$$

Then

$$\det(A + 2B) = -15\det A + 2a + 4b + 8c,$$

since $\det B = -\det A$. Now, we also have

$2a+4b+8c$

$$\begin{aligned} &= 2(a+c)+4b+6c \\ &= \det(A+B)-\det(A-B)+2(\det(A+B)+\det(A-B))+6c \\ &= 3\det(A+B)+\det(A-B)+6c. \end{aligned}$$

Finally,

$$\det(A+2B)+15\det A-3\det(A+B)-\det(A-B)=6c$$

Taking absolute values,

$$|\det(A+2B)+15\det A-3\det(A+B)-\det(A-B)|=6|c|.$$

Problem 718. *Proposed by Jose Luis Diaz-Barrero, BARCELONA TECH, Barcelona, Spain.*

Let x, y, z be positive real numbers and n a positive integer. Show that

$$\frac{1}{2}\left(\sum_{\text{cyclic}}\frac{x^{n+2}+y^{n+2}}{x^n+y^n}\right)\geq\sum_{\text{cyclic}}\frac{x^2y^2}{x^2+y^2}.$$

Solution *by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

By Chebyshev's sum inequality, $x^{n+2}+y^{n+2}\geq\frac{1}{2}(x^n+y^n)(x^2+y^2)$, and therefore

$$\frac{1}{2}\frac{x^{n+2}+y^{n+2}}{x^n+y^n}\geq\frac{1}{4}\left(x^2+y^2\right).$$

We also have

$$\frac{1}{4}\left(x^2+y^2\right)\geq\frac{x^2y^2}{x^2+y^2}$$

because it is equivalent to $(x^2+y^2)^2\geq 4x^2y^2$, or $(x^2-y^2)^2\geq 0$. The solution follows from this.

Also solved by the proposer.

Problem 719. *Proposed by Jose Luis Diaz-Barrero, BARCELONA TECH, Barcelona, Spain.*

Find, with proof, all real solutions to the equation

$$3\cdot 1331^x+4\cdot 363^x=34\cdot 99^x+77\cdot 27^x.$$

Solution *by Ioan Viorel Codreanu, Satulung, Maramures, Romania.*

Rewrite the equation in the form

$$3 \cdot 11^{3x} + 3 \cdot 3^x \cdot 11^{2x} - 34 \cdot 3^{2x} \cdot 11^x - 77 \cdot 3^{3x} = 0$$

or

$$3 \cdot \left(\frac{11}{3}\right)^{3x} + 4\left(\frac{11}{3}\right)^{2x} - 34\left(\frac{11}{3}\right)^{x} - 77 = 0.$$

Let $t = \left(\frac{11}{3}\right)^x$. The equation becomes

$$3t^3 + 4t^2 - 34t - 77 = (3t - 11)(t^2 + 5t + 7) = 0.$$

Because $t^2 + 5t + 7 > 0$, we deduce that $t = \frac{11}{3}$ and then $x = 1$.

Also solved by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Steven Sly (student), Northeastern State University, Tahlequah, OK; and the proposer.

Problem 720. *Proposed by Jeremy Wade, Pittsburg State University, Pittsburg, KS.*

Let $\{n_k\}$ be an increasing sequence of positive integers. Prove that

$$\sum_{k=1}^{\infty} \frac{n_{k+1} - n_k}{n_{k+1}} = \infty.$$

Solution *by the proposer.*

Let $b_k = \frac{n_{k+1} - n_k}{n_{k+1}}$. Since b_k is positive for all k, if $\lim\limits_{k\to\infty} b_k \neq 0$, then the series diverges to ∞. Now, consider the case that $\lim\limits_{k\to\infty} b_k = 0$. Then, there exists a number K so that

$bk < \frac{1}{2}$ for all $k > K$. Then, for $k > K$, we have

$$\frac{n_{k+1} - n_k}{n_{k+1}} < \frac{1}{2} \leftrightarrow n_{k+1} - n_k < \frac{1}{2} n_{k+1} \leftrightarrow n_{k+1} < 2n_k \leftrightarrow \frac{1}{n_{k+1}} > \frac{1}{2n_k}.$$

It follows that

$$\sum_{k=1}^{\infty} \frac{n_{k+1} - n_k}{n_{k+1}} > \sum_{k=K+1}^{\infty} \frac{n_{k+1} - n_k}{2n_k} > \sum_{k=K+1}^{\infty} \int_{n_k}^{n_{k+1}} \frac{1}{2x}\, dx = \infty.$$

Also solved by Mark Buckles, Northeastern State University, Tahlequah, OK; and Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.

Problem 721. *Proposed by the editor.*

The Northeastern Section meeting of the MAA in November 2012 was devoted to mathematical problems and problem solving. It was such a good meeting that the following alphametic was created. Find the two solutions base 8 to the alphametic. (Each different letter is replaced by a different digit so that the summation is true.)

$$\begin{array}{cccc} & N & E & S \\ + & M & A & A \\ \hline G & O & O & D \end{array}$$

Solution *by students Andrea Morgan, Joanna Billups, and Cheyenne Phillips, Northeastern State University, Tahlequah, OK.*

Assuming $G = 1$ in order to get a carry, we get

$$\begin{array}{cccc} & 4 & 6 & 5 \\ + & 3 & 2 & 2 \\ \hline 1 & 0 & 0 & 7 \end{array}$$

Since the N and M are interchangeable, we also have

$$\begin{array}{cccc} & 3 & 6 & 5 \\ + & 4 & 2 & 2 \\ \hline 1 & 0 & 0 & 7 \end{array}.$$

Also solved by Steven Sly (student), Northeastern State University, Tahlequah, OK; Cindy Jeffcoat and Kong Vang (students), Northeastern State University, Tahlequah, OK; Lindsey Conrotto (student), Eastern Kentucky University, Richmond, KY; and the proposer.

Report of the 39th Biennial Convention

Kappa Mu Epsilon
April 11-13, 2013
Washburn University, Kansas Delta
Topeka, Kansas

Thursday, April 11, 2013

On Thursday evening, April 11, a mixer with games and refreshments was held in the Ruth Garvey Fink Convocation Hall in the Bradbury Thompson Center. Participants were able to pick up registration packets at this mixer. At 8:00 p.m., the National Council, the Regional Directors and *The Pentagon* Editor and Business Manager held their biennial business meeting in Morgan Hall 279.

Friday, April 12, 2013

Friday's activities began at 7:30 am with breakfast and continued registration outside Washburn A/B Rooms in the Memorial Union. At 8 am, the Awards Committee held a meeting in the Vogel Room in the Memorial Union. At 8:30 a.m. in the Washburn A/B Rooms in the Memorial Union. The first general session began, with KME President Ron Wasserstein presiding.

Dr. Gordon McQuere, Dean of Washburn University's College of Arts & Sciences, welcomed participants. Mark S. Hamner, KME Secretary, then called the roll. There were 17 chapters in attendance from eight states (Alabama, Kansas, Louisiana, Michigan, Missouri, New York, Pennsylvania, and Texas). Nineteen talks were given and eighty-eight registrants participated in the convention. Chapters represented were Alabama Theta, Jacksonville State University; Kansas Alpha, Pittsburg State University; Kansas Beta, Emporia State University; Kansas Delta, Washburn University; Louisiana Delta, University of Louisiana – Monroe; Michigan Beta, Central Michigan University; Missouri Alpha, Missouri State University; Missouri Beta, University of Central Missouri; Missouri Theta, Evangel University; Missouri Iota, Missouri Southern State University; Missouri Lambda, Missouri Western State University; New York Eta, Niagara University; New York Omicron, Saint Joseph's College; New York Rho, Molloy College; Pennsylvania Mu, Saint Francis University; Texas Alpha, Texas Tech University; and Texas Gamma, Texas Woman's University.

The following three new chapters installed during the 2011-2013 biennium were recognized:

- Pennsylvania Tau at DeSales University, April 29, 2012
- Tennessee Zeta at Lee University, November 5, 2012
- Rhode Island Beta at Bryant University, April 3, 2013

Rhonda McKee, KME President-Elect of Missouri Beta conducted the filing of delegates. There was no old business.

Introduction of new business began with an update from the KME President, Ron Wasserstein, on the attempts from the Nominating Committee to obtain two candidates to run for President Elect, as is customary, prior to the National KME Convention. These two candidates would ordinarily have been presented to the voting delegates at the convention, and an election held. However, after considerable effort, the committee had only one candidate. The National Council decided to postpone the election until after the convention in hopes of getting two candidates, as per our Constitution. Thus, an email election to all corresponding secretaries was held in June of this year to determine the choice for President-Elect. The National Council had one highly qualified candidate run for president-elect, Brian Hollenbeck, Kansas Beta. By a unanimous vote, Brian Hollenbeck was ratified as President-Elect.

Rhonda McKee, KME President-Elect presided over the student paper presentations, which commenced at 9:00 a.m. in the Washburn A/B Rooms in the Memorial Union.

First Paper Session

- 1-1: *Chromatic Number of Intersecting Subgraphs*, by Rachael Sachs, Kansas Alpha, Pittsburg State University
- 1-2: *A Quantitative Approach to Predicting College Experiences and Outcomes*, by Lauren Beaudoin, Jeffery Behounek, Jennifer Rice, New York Omicron, St. Joseph's College
- 1-3: *Eigenvector Ranking of Sports Teams*, by Brittany Street, Missouri Alpha, Missouri State University
- 1-4: *The Collatz Conjecture*, by Amanda Kovacs, New York Rho, Molloy College

At 10:15 a.m. there was a 15-minute Refreshment Break in the hallway outside the Washburn A/B Rooms in the Memorial Union.

Session #2 of the Student Presentations commenced at 10:30 a.m. in the Washburn A/B Rooms in the Memorial Union.

Second Paper Session

- 2-1: *Why Do I Have to Take Modern Algebra If I Am Going to Teach in High School*, by Joanna Mantone, New York Rho, Molloy College
- 2-2: *Stochastic Approach to Interest Theory*, by Brad Isom and Sam Wold, Missouri Lambda, Missouri Western State University
- 2-3: *Tree Decompositions of Regular Graphs and Multigraphs*, by Marie Ermete, Michigan Beta, Central Michigan University
- 2-4: *Mathematical Research in the Classroom: Building and Inspiring Minds*, by Joel Jeffries, Missouri Beta, University of Central Missouri

After the paper session, at 11:45 a.m., a Group Picture was taken on the main stairway in the Bradbury Thompson Center

Lunch was held at 12 noon in the Ruth Garvey Fink Convocation Hall in the Bradbury Thompson Center (BTC) followed by two meetings of the National KME Auditing Committee, Brian Hollenbeck, Kansas Beta, Chair (in the Forum Room, BTC) and the National KME Resolutions Committee, Steve Klassen, Missouri Lambda, Chair (in the Ora Wade Baker Room, BTC)

Session #3 of the Student Presentations commenced at 1:00 p.m. in the Washburn A/B Rooms in the Memorial Union.

Third Paper Session

- 3-1: *The Truth Tree Method of Proof*, by Tony Pochini, Michigan Beta, Central Michigan University
- 3-2: *Using Polygon Triangulation to Generate Catalan Numbers*, by Morgan Russell, Missouri Lambda, Missouri Western State University
- 3-3: *Triphos: An Alternate Numbering System*, by Keely Grossnickle, Kansas Beta, Emporia State University

At 2 p.m., after the paper session, section meetings were held: the Student Section in the Washburn A/B Rooms in the Memorial Union, and the Faculty Section in the Vogel Room in the Memorial Union

Session #4 of the Student Presentations commenced at 2:45 p.m. in the Washburn A/B Rooms in the Memorial Union.

Fourth Paper Session

- 4-1: *Illuminating Biological Phenomena using Orthogonal Polynomials*, by Saba Nafees, Texas Alpha, Texas Tech University
- 4-2: Cancelled
- 4-3: *Complex Variables: What Can We See?*, by Brandon Marshall, Kansas Delta, Washburn University

Between 3:45 p.m. and 4 p.m. there was a 15-minute break for refreshments.

Session #5 of the Student Presentations commenced at 4 p.m. in the Washburn A/B Rooms in the Memorial Union.

Fifth Paper Session

- 5-1: *Persistence of Water Hyacinths in Northeast Louisiana*, by Jesse Lee Pope, Louisiana Delta, University of Louisiana at Monroe
- 5-2: *Krylov Subspaces and Adaptive Arnoldi Methods in Page Rank Computations*, by Alex Thomson, Missouri Alpha, Missouri State University

The convention banquet was held in the in the Ruth Garvey Fink Convocation Hall in the Bradbury Thompson Center at 7:00 p.m. Rhonda McKee, KME President-Elect, served as emcee. KME President Ron Wasserstein presented the George R. Mach Distinguished Service Award to Robert Bailey, 38-year KME charter member of the New York Eta chapter at Niagara University, who served as corresponding secretary and faculty advisor of that chapter from 1968-2006. Dr. Bailey also served as a KME national officer for sixteen years, eight as National Secretary (1987-1995), four as President-Elect (1997-2001) and four as President (2001-2005). He and his chapter hosted the 1993 KME Convention. Following dinner, Richard De Veaux of Williams College, Williamstown, Massachusetts gave the keynote address. His topic was *Data Mining: Fool's Gold or the Mother Lode?*

Saturday, April 13, 2013

Saturday's activities began between 7:30 a.m. and 8:15 a.m. with Registration and Breakfast in the hallway outside the Washburn A/B Rooms in the Memorial Union.

Session #6 of the Student Presentations commenced at 8:30 a.m. in the

Washburn A/B Rooms in the Memorial Union.

Sixh Paper Session

- 6-1: *Classifying Extensions of a Characteristic p Local Field*, by Alfeen Hasmani, New York Rho, Molloy College
- 6-2: *An Exploration of Tropical Mathematics*, by Stacy Rottinghaus, Kansas Delta, Washburn University
- 6-3: *Queuing System Analysis of Math Tutoring*, by Kristen Beall, Kansas Delta, Washburn University
- 6-4: *Statistical Analysis of a Batting Average Ranking Algorithm*, by Shunya Miyatake, Kansas Beta, Emporia State University

At 9:45 a.m. a Refreshment Break was held in the hallway outside the Washburn A/B Rooms in the Memorial Union. The Awards Committee, Pedro L. Muíño, Chair met in the Vogel Room in the Memorial Union.

At 10 a.m. in the Washburn A/B Rooms in the Memorial Union there was a Career Panel Discussion. Panelists included Carolyn Covington, Washburn University (2006), Tamela Lake, Washburn University (2008), Cara Lance, Pittsburg State University (1996), and Simone Westermayer, University of Central Missouri (2011).

Following at 10:45 a.m., also in the Washburn A/B Rooms in the Memorial Union, the Second General Session began with President Ron Wasserstein presiding. Convention Evaluation Forms were distributed and collected.

For the Continuation of New Business, the following national officers made reports: Charles Curtis, Editor, *The Pentagon*; Peter Skoner, Historian; Cynthia Woodburn, Treasurer; Mark Hamner, Secretary; Rhonda McKee, President-Elect; and Ron Wasserstein, President.

Following the national officer reports were reports from the Section Meetings, the Auditing Committee and the Resolutions Committee. All reports are given below.

Rhonda McKee of Missouri Beta, University of Central Missouri completed her term as President Elect and inherits the role of National KME President. Rhonda McKee was installed as National KME President by the outgoing President, Ron Wasserstein.

The report of the Awards Committee and presentation of awards were made. The top four papers were:

- 1-1: *Chromatic Number of Intersecting Subgraphs*, by Rachael Sachs, Kansas Alpha, Pittsburg State University

- 1-3: *Eigenvector Ranking of Sports Teams*, by Brittany Street, Missouri Alpha, Missouri State University
- 3-2: *Using Polygon Triangulation to Generate Catalan Numbers*, by Morgan Russell, Missouri Lambda, Missouri Western State University
- 6-3: *Queuing System Analysis of Math Tutoring*, by Kristen Beall, Kansas Delta, Washburn University

These students were awarded a $100 check for their respective winning paper. In addition, $50 for travel was given to each chapter for each presenter. The "People's Choice Award," which is selected by submitted votes of the attending KME student members, was presented to Stacy Rottinghaus. The "People's Choice Award" recipient received $50.

The convention concluded with the National Treasurer Cynthia Woodburn presenting checks for travel allowances to each chapter present.

Mark Hamner
National Secretary

Report of the National President

The final two years of my term as President of Kappa Mu Epsilon have flown by. Thanks to the wonderful work of my colleagues on the National Council, they have been two interesting and productive years.

Highlights of our activities between the April 2011 convention in St. Louis and this 2013 convention in Topeka include:

- I installed three new chapters:
  - Pennsylvania Tau at DeSales University, April 29, 2012
  - Tennessee Zeta at Lee University, November 5, 2012
  - Rhode Island Beta at Bryant University, April 3, 2013
- I had the privilege of installing nine chapters in eight states during my term as President. Each one was a delight, a reminder of why we have a national mathematics honor society.
- The National Council met regularly (almost every month) by conference call. These regular meetings not only allowed us to get more business done more quickly, but also alleviated the need for a face to face meet between conventions, which reduced our expenses.
- The National Council met regularly (almost every month) by conference call. These regular meetings not only allowed us to get more business done more quickly, but also alleviated the need for a face to face meet between conventions, which reduced our expenses.
- As noted in the Treasurer's Report, we have done well financially, which puts us in a position to experiment with some new approaches to the National Convention. The biggest experiment is the plan to move from a national convention in odd-numbered years and regional conventions in even-numbered years to an annual national convention. Many adjustments will be required as we make this transition, but the next set of officers is fully aware of the steps that need to be taken, and two chapters have volunteered to serve as the next two national convention hosts. Alabama Theta at Jacksonville State U. will host us in 2014, and New York Omicron at St. Joseph's College will be the host in 2015.
- Speaking of conventions, the Career Panel Session is an experiment at this convention, one we are likely to continue.
- The National Council is taking steps to put us in a better position to get grant funding for the annual conventions.

- Increasing attendance at the convention has been and will continue to be a top priority of the National Council.
- We have come to the end of eight years of outstanding service by Chip Curtis as editor of *The Pentagon*, and are happy to welcome Brother Daniel Wisniewski as the new editor.
- We named an outstanding volunteer as the recipient of the George Mach Distinguished Service Award.
  - Robert Bailey has served Kappa Mu Epsilon with distinction, grace, and humor for 38 years. He was a charter member of the New York Eta chapter at Niagara University, and served as corresponding secretary and faculty advisor of that chapter from 1968-2006. He served as a national officer for sixteen years, eight as National Secretary (1987-1995), four as President-Elect (1997-2001) and four as President (2001-2005). He and his chapter hosted the 1993 KME Convention. Working with Bob was a pleasant and productive experience for everyone who had the pleasure. No one who did so will forget the joy with which he did his work, or the twinkle in his eye that let you know that another pun was about to be spoken. The spirit of committed service that George Mach embodied has been carried forward by Bob Bailey. KME is proud to award Bob Bailey the 2013 George R. Mach Distinguished Service Award.
- We have continued to maintain close and productive ties with our colleagues at Pi Mu Epsilon. I hope this will continue over the coming years.

As I noted in my report two years ago, a list of bullet points is appropriate for a report such as this, but it belies the amount of effort made by the national officers, who serve as unpaid volunteers of this society. Indeed, one of the most rewarding aspects of serving as your President has been working with President-Elect Rhonda McKee, Treasurer Cynthia Woodburn, Secretary Mark Hamner, and Historian Pete Skoner on the National Council. They work hard, but do so cheerfully and professionally. It has also been a great privilege to serve with Chip Curtis, who does a wonderful job as Pentagon editor, and Don Tosh, past president, business manager of the Pentagon. Don is a good friend and tremendous source of encouragement. The Regional Directors (Kevin Charlwood, David Dempsey, Vince Dimiceli, Beth Mauch, and Pedro Muiño) contribute so much to KME, and I am deeply grateful for their service as well. Finally, thank you to all the faculty sponsors and corresponding secretaries who are the heart and soul of KME.

Of course, the most gratifying aspect of this work is the impact that KME has on the lives of thousands of mathematics students nationwide each year. This is what brings us all together, and is the reason why we do this work.

Respectfully,
Ron Wasserstein
National President

Report of the National President-Elect

This convention marks the end of my tenure as President Elect. I have certainly enjoyed serving in this position. Working with the other members of the National Council is very pleasant work. I have especially enjoyed the monthly conference calls that have been made possible by President Ron Wasserstein. These calls have proved to be a very efficient way to get much accomplished. I hope to be able to continue holding conference calls as I transition to president.

This past spring and summer, a committee consisting of Pete Skoner, Mark Hamner, Cynthia Huffman Woodburn, Pedro Muíño and myself prepared an NSF grant proposal which would have supported our national conventions by way of funds for student travel. Unfortunately, the proposal was not funded, but we learned a lot in the preparation of the proposal and have made a few changes in our convention as a result. Two such changes are the career panel discussion, which will take place Saturday morning, and the evaluation form to be filled out by participants on Saturday. We plan to submit another proposal to NSF in one to two years.

2013National Convention

Kappa Mu Epsilon's 39th Biennial Convention is being held this weekend, April 11-13, 2013 in Topeka, Kansas. Our host chapter is Kansas Delta at Washburn University. There are 17 chapters in attendance from eight states (Alabama, Kansas, Louisiana, Michigan, Missouri, New York, Pennsylvania, and Texas). Nineteen talks will be given over the course of the next two days. Ninety-nine people are registered for the convention.

By way of comparison, in 2011, 16 chapters from nine states (Indiana, Kansas, Kentucky, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas) participated in the convention in St. Louis, Missouri. Eighteen papers were presented. Eighty-seven people attended.

In 2009, 16 chapters from nine states (Georgia, Kansas, Maryland, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas) participated in Philadelphia, PA. Sixteen students presented papers. Seventy-five people attended.

In 2007, 14 chapters from five states (Kansas, Missouri, New York, Oklahoma, and Tennessee) participated in Springfield, Missouri.

In 2005 (Schreiner U., Kerrville, TX), there were 17 chapters from nine states (California, Kansas, Missouri, Michigan, New York, Oklahoma, Pennsylvania, Tennessee, and Texas) and 15 student presentations.

In 2003 (ORU, Tulsa, OK), there were 19 chapters from 9 states (Iowa, Kansas, Michigan, Missouri, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). Thirteen student papers were presented.

In 2001 (Washburn U., Topeka, KS), there were 20 chapters from 10 states (Colorado, Iowa, Kansas, Kentucky, Missouri, New York, Oklahoma, Ohio, Pennsylvania, and Tennessee)

The following chapters have participated in the last seven conventions:

Colorado Delta (2001)
Georgia Alpha (2009)
Indiana Delta (2011)
Iowa Alpha (2001, 2003)
Iowa Gamma (2001)
Kansas Alpha (2001, 2003, 2005, 2007, 2009, 2011, 2013)
Kansas Beta (2001, 2003, 2005, 2007, 2009, 2011, 2013)
Kansas Gamma (2001, 2003, 2007)
Kansas Delta (2001, 2003, 2005, 2007, 2009, 2011, 2013)
Kansas Epsilon (2001)
Kentucky Alpha (2001, 2011)
Louisiana Delta (2013)
Maryland Beta (2009)
Maryland Epsilon (2009)
Michigan Beta (2003, 2005, 2009, 2011, 2013)
Missouri Alpha (2001, 2003, 2007, 2013)
Missouri Beta (2001, 2003, 2005, 2007, 2009, 2011, 2013)
Missouri Iota (2001, 2003, 2005, 2007, 2009, 2011, 2013)
Missouri Kappa (2001, 2003, 2005, 2007)
Missouri Lambda (2013)
Missouri Mu (2011)
Missouri Theta (2001, 2003, 2005, 2007, 2009, 2011, 2013)
New York Eta (2001, 2003, 2005, 2007, 2013)
New York Lambda (2003, 2005)
New York Omicron (2005, 2009, 2011, 2013)
New York Rho (2011, 2013)
Ohio Alpha (2001)
Oklahoma Alpha (2003, 2007)
Oklahoma Gamma (2001, 2003, 2007)
Oklahoma Delta (2001, 2003, 2005, 2007, 2009, 2011)
Pennsylvania Lambda (2003, 2009)
Pennsylvania Mu (2005, 2009, 2011, 2013)
Pennsylvania Theta (2001)
Tennessee Gamma (2001, 2003, 2005, 2007)
Texas Alpha (2013)
Texas Gamma (2003, 2005, 2009, 2011, 2013)
Texas Mu (2003, 2005, 2009, 2011)

Thus, in the 2000's, 38 different chapters have participated. Five have participated in all seven conventions. Three chapters are participating for the first time, or at least the first time in a long time, this year.

Again this year the AMS and the ASA both contributed $500 each, which will be used to help defray the cost of student travel to the convention. We are certainly grateful for their support.

Kevin Charlwood and his committee at Kansas Delta at Washburn University have done a tremendous job of organizing this convention. They have worked hard and taken care of many details. Their behind-the-scenes work has made the convention run very smoothly.

Regional Convention

One regional convention was held in the spring of 2012. The North Central Regional Convention was hosted by Missouri Theta at Evangel University in Springfield, Missouri. Eleven student presentations were made. Eight chapters participated and 55 people attended.

Rhonda McKee
President-Elect

Report of the National Secretary

Kappa Mu Epsilon, National Mathematics Honor Society initiated 2,453 new members in 122 chapters during the 39th Biennium that ended March 15, 2013. This represents a decrease of 441 new members compared to the last biennium. The 39th Biennium was about two months shorter than the last biennium, which is a primary reason for the decrease. Twenty-Two active chapters did not report any initiates during the 39th biennium. The total membership of KME is 80,874.

As National Secretary, I receive all initiation reports from chapters, make a record of those reports, up-date mailing list information for corresponding secretaries and forward copies of the reports to other officers. At the beginning of each new biennium, I prepare a new KME brochure. During an academic year, I send out supplies to each chapter. The supplies include information brochures and membership cards. When a college or university petitions for a new chapter of KME, I send out a summary of the petition, prepared by the president, to each chapter and receive the chapter ballots.

Mark Hamner
National Secretary

Report of the National Historian

It continues to be a pleasure for me to serve as the National Historian of the Kappa Mu Epsilon National Mathematics Honor Society for a second biennium. I enjoy the continual communication and friendship with the national and regional officers, and with the corresponding secretaries. I also enjoy contributing to this great organization as we celebrate its 82 years of history.

The primary function of the national historian continues to be soliciting, collecting, maintaining and compiling records of chapter activities, installation of new chapters, and other society activities that have historical significance. Most of these records are gathered from individual chapters, who receive several electronic mail requests beginning in January and May of each year asking for a report of chapter activities from the previous semester. With the low cost and ease of group distribution e-mail messages, several requests are sent each semester. Each sent message typically results in additional reports being received. An additional benefit is the opportunity to learn when the corresponding secretary changes; this helps maintain communication between the national society and the local chapters, and hopefully helps maintain the local leadership for each chapter. The work of the National Historian is impossible without the aid of the corresponding secretaries for each chapter. Thank you for all that you do in serving the students at your institution, your local Kappa Mu Epsilon chapter, and the national organization.

During the past biennium from 2011 to 2013, 69 of the active chapters responded at least once to the chapter news request. Special mention goes to the following 17 chapters for their cooperation in responding to all four inquiries: IA Alpha, IL Zeta, MD Delta, MD Epsilon, MI Delta, MI Epsilon, MO Alpha, MO Theta, NE Beta, NE Delta, NJ Delta, NY Lambda, NY Nu, NY Omicron, OH Epsilon, OK Alpha, and TN Gamma.

A special thank you also goes to the editor of *The Pentagon*, Charles (Chip) Curtis of Missouri Iota chapter. The edited Chapter News section is sent to him each semester, and he has been great to work with.

The national historian participated this year in helping write a National Science Foundation grant that was submitted to provide travel support to the national convention. Though the grant was not approved, it was a good effort, and hopefully can be funded in the future.

This is the 82nd Anniversary of KME, and this is the 39th national convention. In honor of these important dates, the historian and other national officers updated the *KME History and Information* booklet from the 60th Anniversary Edition that was distributed at the 1991 national convention

by then Historian Mary S. Elick. The 82nd Anniversary edition will be available on the national home page, and paper copies will be distributed at the national convention.

Peter R. Skoner,
National Historian

Report of the National Treasurer

39th Biennium (April 15, 2011 – March 15, 2013)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows end-of-biennium assets of $88,900.25. The Cash Flow Report shows that we had an asset gain of $18,533.69 during the biennium. A National Council goal to maintain an asset base of at least $40,000 has been met.

BIENNIUM ASSET REPORT

Total Assets (April 14, 2011)		$70,366.56
Current Assets		
Kansas Teachers Community Credit Union		
Checking	35,870.01	
Share Account	3,030.24	
CD15229	10,000.00	
CD15261	10,000.00	
CD15288	10,000.00	
CD17014	10,000.00	
CD17015	10,000.00	
Total Current Assets		$88,900.25

BIENNIUM CASH FLOW REPORT

Receipts		
Initiation fees received	49,040.00	
Installation fees received	300.00	
Interest income	867.77	
Gifts & misc. income	500.00	
Total Biennium Receipts		$50,707.77
Expenditures		
Association of College Honor Soc	901.00	
Administrative expenses	3,685.58	
National Convention expenses	9,914.40	
Regional Convention expenses	628.26	
Council Meetings travel	0.00	
Certificates, jewelry & shipping	16,297.71	
Installation expenses	0.00	
Bank charges	40.00	
Pentagon expenses	707.13	
Total Biennium Expenses		$32,174.08
Biennium Cash Flow		$18,533.69

The cash flow last biennium (09-11) was $3,289.05. Although receipts dropped this past biennium, expenses dropped by even more, resulting in the cash flow being more than 5 times higher. The National Council is making an effort to return this to the students by extra funding for chapter travel and reimbursing convention registration for presenters. We initiated fewer members and installed fewer new chapters this biennium than the last one. Pentagon expenses continued to drop with the move to electronic publication. National convention expenses were much lower in 2011 than in 2009, although it is expected with the increase in the mileage reimbursement rate and the extra funding for chapter travel, that the national convention expenses will be higher in 2013 than in 2011.

We have maintained our goal of maintaining assets of at least $40,000. The financial condition of Kappa Mu Epsilon is sound.

I want to offer sincere thanks to the dedicated, talented, hard-working professionals of the National Council (Ron Wasserstein, Rhonda McKee, Mark Hamner, and Pete Skoner). Also, special thanks to Don Tosh and Chip Curtis for their efforts in moving *The Pentagon* to an electronic format. In addition, a big thanks to the work of the corresponding secretaries who maintain such a vital role in Kappa Mu Epsilon.

Cynthia Woodburn
National Treasurer

Report of the Pentagon Business Manager, April 2013

I took over as business manager of The Pentagon in December, 2006. My first issue was the Fall, 2006 issue (Volume 66, Number 1). During the 2009-2011 biennium we went to an electronic only edition of The Pentagon. We now provide electronic copies of The Pentagon for every issue of The Pentagon ever printed. The location is kappamuepsilon.org and you can follow the link to find the issues of The Pentagon. The site is updated as new issues come out. This past biennium we printed three issues: Spring, 2011 (Vol 70 Num 2) to Spring, 2012 (Vol 71 Num 2).

While the publication of The Pentagon has gone entirely digital, we have still been printing approximately 100 hard copies of each issue to fulfill paid subscriptions that were made before we switched to the new format. This number will decrease and should soon be eliminated as those subscriptions expire. We do not accept subscriptions any longer.

Because of this policy, we were able to go into a standby mode to use up the previous balance that was on hand to meet printing and postage needs. The plan is to work with a balance of $0.00 and invoice KME for expenses incurred in the printing and postage of the outstanding subscriptions. This financial need has dropped dramatically and should eventually become 0, unless there is a decision made to print a nominal number of copies for some reason.

As you can see from the table, the beginning balance at the start of the biennium was $1249. This paid for almost two complete issues of The Pentagon before there was a need to invoice for outstanding balances. So the total cost of printing and distributing The Pentagon has dropped from several thousand dollars each biennium to around $1500, and this should drop even farther in the next biennium until it eventually disappears.

On a personal note, it has been a tremendous pleasure working with the editor, Chip Curtis, and I would like to thank him for the time and effort he has put into The Pentagon. He has become a good friend over the last several years of working together.

I am looking forward to meeting and working with the new editor as he takes over the production of this magazine.

Don Tosh
Business Manager, The Pentagon

Pentagon receipts/Expenses 4/01/11 - 4/13/13				
Beginning Balance 4/01/11	**$1249.00**		**Ending balance 4/13/13**	**$0.00**
Receipts			**Expenses**	
Vol 70 Num 2 S11 printing	$0.00		Vol 70 Num 2 S11 printing	$436.79
Vol 70 Num 2 S11 postage	$0.00		Vol 70 Num 2 S11 postage	$179.38
Vol 71 Num 1 F11 printing	$0.00		Vol 71 Num 1 F11 printing	$504.70
Vol 71 Num 1 F11 postage	$170.86		Vol 71 Num 1 F11 postage	$298.99
Vol 71 Num 2 S12 printing	$415.27		Vol 71 Num 2 S12 printing	$415.27
Vol 71 Num 2 S12 postage	$121.00		Vol 71 Num 2 S12 postage	$121.00
Total Receipts 4/01/11 - 4/13/13	**$707.13**		**Total expenses 4/01/11 - 4/13/13**	**$1,956.13**

Report of the Audit Committee

Audit Committee Members

- Brian Hollenbeck, Kansas Beta, Chair
- Amy Bretches, Kansas Beta
- David Dempsey, Alabama Theta

Audit Process

1. Prior to the national convention, Treasurer Cynthia Woodburn mailed biennium financial summary data to Brian Hollenbeck, the committee chair. At the convention, Treasurer Woodburn provided the audit committee with biennium summary and detail documentation of receipts and payment transactions, as well as bank and savings account reconciliation documentation.
2. Prior to the national convention, Brian Hollenbeck contacted the Kansas Teachers Community Credit Union. He discovered that the account balances for the Kappa Mu Epsilon checking and savings accounts match the totals from Treasurer Woodburn's biennium reports, after adjusting for uncleared transactions.
3. At the national convention, committee members spoke to President Ron Wasserstein and Secretary Mark Hamner to determine their impressions of the accuracy and completeness of the recording throughout the biennium. The committee examined the financial documentation provided by Treasurer Woodburn.

Recommendations

1. Information forwarded by the Treasurer to the committee chair prior to the national convention provides the opportunity for verification of assets in a careful manner and should be continued in the future.
2. The internal checks built into the regular financial processing between the Treasurer and the President and Secretary provide an important safeguard to the integrity of the office of the Treasurer and help avoid the necessity of an expensive external audit. These ongoing internal audit processes should be continued and updated by the National Council as needed.
3. Finally, the Audit Committee recommends the acceptance of the financial records and reports of Kappa Mu Epsilon for the 2011-2013 biennium as presented by the Treasurer Cynthia Woodburn.

Commendations

1. The committee commends Treasurer Woodburn for her exemplary maintenance, management and presentation of the financial records and for her generous donation of time as Treasurer for Kappa Mu Epsilon.
2. We further commend her for her valuable input to this Audit Committee and her helpful guidelines for the Audit Committee.
3. The committee commends President Ron Wasserstein, Secretary Mark Hamner and Treasurer Cynthia Woodburn for the manner in which they communicate and cooperate to maintain the internal checks which preserve the integrity of the office of Treasurer.
4. The committee commends the work of the previous audit committees and gratefully received reports provided by the Treasurer.

Report of the Resolutions Committee

The Resolutions Committee consisted of Steven Klassen, faculty member from the Missouri Lambda chapter, Morgan Russell, student member from the Missouri Lambda chapter, Carl W. Baurle III, student member from the New York Omicron chapter, Amanda Kovacs, student member from the New York Rho chapter, Joanna Mantone, student member from the New York Rho chapter, Alfeen Hasmani, student member from the New York Rho chapter, Elise Lofgren, student member of Pennsylvania Mu, Sivaram Narayan, faculty member of Michigan Beta, and Marie Ermete, student member of Michigan Beta. The committee proposed the following resolutions.

"Whereas the success of any undertaking relies heavily upon the dedication and ability of its leaders, be it resolved:

1. that this Thirty-ninth Biennial National Convention express its gratitude to (a) Cynthia Woodburn, for her 8 years of faithful service as national treasurer; (b) Mark Hamner, for his past 4 years of faithful service as national secretary; (c) Pete Skoner, for his past 4 years of faithful service as national historian and 4 more with his re-election; (d) Chip Curtis, for his continuing service as editor of the Pentagon; and to (e) Ron Wasserstein and Rhonda McKee for their efforts in guiding Kappa Mu Epsilon as its president and president-elect, and

2. that this Convention acknowledge the participation of the students and faculty who served on the Auditing, Awards, Nominating, Paper Selection and Resolutions committees, which is so essential for the success of the meeting.

"Whereas the primary purpose of Kappa Mu Epsilon is to encourage participation in mathematics and the development of a deeper understanding of its beauty, be it further resolved:

1. that this convention commends the following chapters for their participation: Alabama Theta, Kansas Alpha, Kansas Beta, Kansas Delta, Louisiana Delta, Michigan Beta, Missouri Alpha, Missouri Beta, Missouri Theta, Missouri Iota, Missouri Lambda, New York Eta, New York Omicron, New York Rho, Pennsylvania Mu, Tennessee Gamma, Texas Alpha, Texas Gamma,

2. that, in particular, special commendation is given to Kansas Alpha, Kansas Beta, Kansas Delta, Louisiana Delta, Michigan Beta, Missouri Alpha, Missouri Beta, Missouri Lambda, New York Omicron, New York Rho, and Texas Alpha by this Thirty-ninth Biennial Convention

for contributing enthusiastic, creative, and interesting student presentations,

3. that this Convention express thanks to Carolyn Covington, Tamela Lake, and Cara Lance, and Simone Westermayer for their informative Career Panel Discussion, and to Richard De Veaux of Williams College for his captivating keynote address "Data Mining: Fool's Gold? or the Mother Lode" at the Friday night banquet, and
4. that this Convention express its gratitude to Washburn University for hosting the Thursday night social activities and the Friday night banquet.

"Finally, whereas Washburn University and the surrounding community of Topeka have provided this Convention with gracious hospitality, be it resolved:

1. that this Thirty-ninth Biennial Convention express its heartfelt appreciation to all the members of Kansas Delta, who devoted their time and talents to ensure the success of this meeting,
2. that this convention give special recognition to Kevin Charlwood of Washburn University; along with Sarah Cook, Hwa Chi Liang and Jennifer Wagner, for organizing dining arrangements; Mike Mosier and Bill Gahnstrom, for assistance with technology; Donna Lalonda and Hee Seok Nam, for assistance with registration; and Evelyn Pitts, for organizing hospitality, and
3. that this Convention recognize and thank Jerry Farley, President of Washburn University, as well as Gordon McQuere, Dean of the College of Arts and Sciences, who welcomed this convention to their beautiful campus."

Respectfully submitted.
Steven Klassen, Chairperson

Report of the Awards Committee

The Awards Committee met to select the four award winners. These are:

- 1-1: *Chromatic Number of Intersecting Subgraphs*, by Rachael Sachs, Kansas Alpha, Pittsburg State University
- 1-3: *Eigenvector Ranking of Sports Teams*, by Brittany Street, Missouri Alpha, Missouri State University
- 3-2: *Using Polygon Triangulation to Generate Catalan Numbers*, by Morgan Russell, Missouri Lambda, Missouri Western State University
- 6-3: *Queuing System Analysis of Math Tutoring*, by Kristen Beall, Kansas Delta, Washburn University

Respectfully submitted.
Pedro L. Muíño, Chairperson

Kappa Mu Epsilon News

Edited by Peter Skoner, Historian

Updated information as of December 2013

Send news of chapter activities and other noteworthy KME events to

Peter Skoner, KME Historian
Saint Francis University
117 Evergreen Drive, 313 Scotus Hall
Loretto, PA 15940
or to
pskoner@francis.edu

Chapter News

AL Alpha – Athens State University
Chapter President – Chris Vaughn; 15 Current Members; 9 New Members
Other Spring 2013 Officers: Tanya Sanderson, Vice President; Sarah Cox, Secretary; Patricia Glaze, Corresponding Secretary and Faculty Sponsor

AL Gamma – University of Montevallo
Corresponding Secretary – Scott Varagona; 12 New Members
New Initiates - Jacob Brown, Amanda Choat, Stephanie Dorough, Brittany Easterling, Alan Evans, Jared Kornegay, Madeline Mechum, Anthony Melton, Nathaniel Miller, Frank Sciandra, Nandi Stokely, and Linh Tong.

AL Zeta – Birmingham-Southern College
Chapter President – Nino Yu Tiamco; 22 Current Members; 13 New Members
Other Spring 2013 Officers: Melanie Short, Vice President; Huda Qureshi, Secretary; Courtney Mauck and Andrew Conner (Executive Council), Treasurers; Maria Stadnik, Corresponding Secretary and Faculty Sponsor
Kappa Mu Epsilon welcomed Dr. Hugh Howards from Wake Forest University as an invited speaker this spring. Howards, the Mathematical Association of America Southeastern Section Lecturer, gave a talk entitled "Knots vs. Graphs: An Epic Tale of Survival." This talk introduced the topological concepts of knots and links and explained how links can be studied in graph theory. Kappa Mu Epsilon also held its annual initiation ceremony in which thirteen new members were honored.
New Initiates - Taylor Bassett, Ryan Deveikis, Hope Ervin, Xuejiao Feng, Shay Gibson, Chase Hoffman, Yuting Liu, Tho Nguyen, Miles Nordgren, Taylor O'Daniel, Allison Rus-

sell, Magdalena Teske, and Xili Zhou.

AL Theta – Jacksonville State University

Chapter President – Shannon Bolton; 60 Current Members; 22 New Members

Other Spring 2013 Officers: Kristen Carlisle, Vice President; Kaitlyn Ledbetter, Secretary; Noel Overton, Treasurer; and Dr. David Dempsey, Corresponding Secretary and Faculty Sponsor

On March 20, 2013, the Alabama Theta chapter initiated 22 new members—21 students and 1 faculty. New members received their certificates, pins, and honor cords in a ceremony held on the 11th floor of Houston Cole Library. Spring activities included several Wednesday night study sessions in the student lounge, as well as a pizza/game night on April 5. New officers were elected during the April meeting.

CA Gamma – Cal Poly

Corresponding Secretary – Jonathan Shapiro; 3 New Members

New Initiates - Michael Campbell, Kelly Johnson, and Katherine Eng.

CA Delta – Cal Poly Pomona

Corresponding Secretary – Patricia Hale; 24 New Members

New Initiates - Sevana Aghasanian, Alicia Arrua, Carissa Cardenas, Shant Danielian, Kristin Dettmers, Saie Eum, Megan Ford, Eric La Fevers, Charles Frayne, Natalie Gasca, Megan Hans, Darren Lai, Jon Lopez, Brandon Marian, Matthew McDonough, Lisa Ngay, Kara Rotunno, Vanessa Salvary, Patricia Valle, Brad Vidal, Jesse Waite, Andrea Wuethrich, Adam Yee, and John Zaheer.

CA Epsilon – California Baptist University

Corresponding Secretary – James Buchholz; 40 New Members

New Initiates - Elizabeth Baker, Nicholas Braden, Marlene Busch, Heather Crews, Patrick Dietz, Daniel Donaldson, Bertrand Dushime, Brandon Fawson, Desire Giraneza, Christopher Grimm, Brenna Hahn, Paulin Hakizimana, Joy Higley, Vanessa Hlebowski, Maria Kangabe, Timothy Kiefer, Allison Krickl, Nathan Lawrence, Andrew Linder, Michael Magiera, Valerienne Maltemps, Robert Maystrovich, Justin Mueller, Modeste Muhire, Alex Muganza, Valentine Muhawenimana, Patrick Nsengiyumva, Christina Osborne, Kirsten Overholt, Brent Pino, Adriana Quezada, Catherine Reeves, Scott Rendel, Tim Replogle, Ezekiel Short, Joshua Smith, Joseph Sugira, Dustin Taniguchi, Taylor Welch, and Kayla Zetsch.

CT Beta – Eastern Connecticut State University

Corresponding Secretary – Christian Yankov; 21 New Members

Other Spring 2013 Officer: Mizan R. Khan, Treasurer

New Initiates - Richard Buckland, Sarah Byrnes, Sydney Cothran, Christine DeFilippo, Taylor Driscoll, Richard Magner, Robert McDonald, Krystine Murtha, Max Olsen, Joseph Perreault, Ashley Pine, Nicole Prasser, Cory Quinet, Colleen Slattery, Katelyn Smachetti,

Samantha Stevens, Catherine Ventura, Stephanie Venuti, Katherine Wilson, Nicole Woodcock, and James Young.

FL Beta – Florida Southern College

Corresponding Secretary – Dr. Susan Serrano; 19 New Members

New Initiates - Grace Anne Beggs, David Josue Blanco, Derek Edward Deibert, Monika Christie Dunn, James Earl Ecker, Jessica Kate Fischer, Desiree Rose Ippolito, Jaime Nicole LaMar, Nicole Maida, Carolyn Jane Mays, Anna Maria McIntyre, Christopher Clark Morgan, Madison Paige Ouellette, Benoit Paradis, Megan Allyse Sandoval, Katherine Rose Saunders, Aaron David Valdivia, Rachel Marie Viglione, and Jennifer Suzanne Yudichak.

FL Gamma – Southeastern University

Corresponding Secretary – Dr. Berhane Ghaim; 4 New Members

New Initiates - Mathew Chenoweth, Benjamin Goodberry, Sarah Schillinger, and Melody Weaver.

GA Alpha – University of West Georgia

Corresponding Secretary – Scott R. Sykes; 9 New Members

New Initiates - Kyle Carter, Dana Collins, Meredith Foster, Hannah Hurst, Stephen Sanders, Detong Song, Courtney Stowe, Jared Thacker, and Ariadne Watson.

HI Alpha – Hawai'i Pacific University

Chapter President – Matt Troglia; 12 Current Members; 4 New Members

Other Spring 2013 Officer: Tara Davis, Corresponding Secretary and Faculty Sponsor

We had an initiation ceremony during spring 2013 semester at a local restaurant and 12 members were in attendance.

New Initiates - Christina Bowen, Keila Elderts, Madison Jupitz, and Jason M. Plozin.

IA Alpha – University of Northern Iowa

Chapter President – Adam Feller; 35 New Members; 5 New Members

Other Spring 2013 Officers: Elizabeth Johnson, Vice President; Shaina Steger, Secretary; Lucas Thomas, Treasurer; Mark Ecker, Corresponding Secretary and Faculty Sponsor

Our first spring KME meeting was held on March 26, 2013 at Professor Mark Ecker's residence where student member Lucas Thomas talked about his KME paper entitled "Analysis of NFL Team Statistics." Student member Adam Feller addressed the spring initiation banquet with "An Analysis of Collision Fatalities by State" on April 30, 2013. Our banquet was held at Godfather's Pizza in Cedar Falls, where five new members were initiated.

New Initiates - Brooke Becker, Benjamin Castle, Paige Hageman, Benjamin Hellman, and Bridget Klein.

IA Gamma – Morningside College

Corresponding Secretary – Chris Spicer; 11 New Members

New Initiates - Andrew Christen, Elise Darlington, Jeffrey Hamblen, Carly Heese, Kayla Krom, Shaun Meyer, Heather Penning, Oswaldo Sanchez, Logan Schaefers, Joshua Schreiner,

and Alyssa Turnquist.

IA Delta – Wartburg College

Chapter President – Adam Kucera; 38 New Members; 18 New Members

Other Spring 2013 Officers: Bailey Wilson, Vice President; Sarah White, Secretary; Alyssa Hanson, Treasurer; Brian Birgen, Corresponding Secretary; and Dr. Joy Becker, Faculty Sponsor

In March, eighteen new initiates were welcomed at our annual banquet and initiation, which included the initiation for Sigma Pi Sigma physics honors society. Our speaker was Chad Feldmann, Wartburg alum and KME member; he was recently recognized in the Cedar Valley Business Monthly 20 under 40 series as one of the founders of Far Reach Technologies. In May, together with the Physics and Computer Science clubs, we hosted the end of the year picnic.

New Initiates - Caitlin Anderson, Jalen Bergman, Brian Crow, Kelsie Durscher, Ashley Fisher, Holly Halstead, Robert King, Sarah Langston, Matthew Partridge, Hailee E. Reist, Jordan David Richards, Robert Seubert, Sabrina Raeann Sieg, Kyle Sprain, Daniel VanGroningen, Philip J. Wessels, Sarah Nicole White, and Bailey Wilson.

IL Zeta – Dominican University

Chapter President – Lisa Gullo; 30 Current Members; 10 New Members

Other Spring 2013 Officers: Ivonne Machuca, Vice President; Claudia Ramirez, Secretary; Azucena Bahena, Treasurer; and Sara Quinn, Corresponding Secretary and Faculty Sponsor

The Illinois Zeta Chapter of KME operates together with Dominican University's Math Club. Thus, the above officers are members of KME or Math Club (or both) and the below activities were prepared by both groups. In Spring 2013 the math club and KME sponsored three mathematics lectures given by mathematicians from outside of Dominican. We also held a career panel giving students information about careers using mathematics. As always, we passed out free slices of pie on Pi Day. We initiated 10 new members into KME in April, and our initiation ceremony included a wonderful lecture by Dr. Judith Covington from LSU Shreveport.

New Initiates - Cynthia Ayala, Stephanie Cappiello, Rebecca Fritz, Mary Heard, Geraldine Holmes, Peter Mielcarek, Anthony Perri, Mohit Shekhar, Jacob Smith, and Kevin Tednes.

IL Theta – Benedictine University

Chapter President – Natalia Poniatowska; 275 Current Members; 8 New Members

Other Spring 2013 Officers: Betsy Williams, Vice President; Jennifer Cagney, Fall Secretary; Trisha Russo, Spring Secretary; Sandra Tovalin-Schmidt, Fall Treasurer; Gabe Rodriguez, Spring Treasurer; Dr. Thomas Wangler, Corresponding Secretary; and Dr. Anthony DeLegge, Faculty Sponsor

New Initiates - Fatjona Aliaj, Dana Cairns, Xinwei Chen, Marihan Hegazy, Kessler Kerr,

Andrew Litoborski, Andrew Nordbye, and Maryam Zafar.

IN Beta – Butler University

Corresponding Secretary – Bill Johnston; 7 New Members

New Initiates - Aliah Ajamoughli, Matthew Blandford, Danielle Edgeworth, Lauren Hannah, Tyler Pollock, Aaron Schlenker, and Kevin Wolf.

IN Gamma – Anderson University

Corresponding Secretary – Courtney Taylor; 3 New Members

New Initiates - Sambed Adhikari, Samriddi Adhikari, and Chan Mi Park.

IN Delta – University of Evansville

Corresponding Secretary – Adam Salminen; 18 New Members

New Initiates - Vaughn Ahlf, Cody Baker, Ian Bashor, Allison Deford, Alexander DiBenedetto, Sarah Dory, Sandra Dunn, Tiffani Emery, Allison Gray, Chelsea Hillenberg, Sarah Joseph, Adam Joyce, Tyler Lawson, Alyssa Matthews, Dakota Roberts, Ernesto Rodrigo Rojas Romero, McIntyre Watts, and Luke Zarnoth.

KS Alpha – Pittsburg State University

Corresponding Secretary – Tim Flood; 26 New Members

New Initiates - Ahmad Alalyani, Trenton Biggerstaff, Philip Brackett-Oetinger, Minh Bui, Jeanette Campbell, Jeremy Chapman, Anthony Craig, Jeremy Donahue, Mark Flood, Cody Franklin, Cody Frieden, Audrey Gilbreath, Craig Gilmore, Charles Harrison, Ashlee Hisey, Justin Johnson, Matthew Johnson, Sarah Nistler, Samantha Pendleton, Lizeth Rojo, Eric Rothmier, Danielle Russell, Zhiwen Wang, Ashley Willis, Xinuo Zhao, and William Zimmerman.

LA Delta – University of Louisiana at Monroe

Chapter President – Jesse Pope; 256 Current Members; 17 New Members

Other Spring 2013 Officers: Blaine Stroud, Vice President; Demi Morvant; Secretary; Dr. Brent Strunk, Corresponding Secretary; and Dr. Youssef Dib, Faculty Sponsor

New Initiates - Adebanjo Adedoja, Lucy Beebe, Indya Bruce, Lauren Chandler, Cody Grimsley, Brittany Guillory, Raisa Howard, Morgan Huber, Jeremy Katz, Kwogie Ly, Sanjay Manandhar, Suraj Sapkota, Amanda Schorr, Ashleigh Simmons, Heather Terrell, Christopher Tyler, and Sailesh Wagle.

MA Beta – Stonehill College

Chapter President – Lauren Hinchey; 25 Current Members, 9 New Members

Other Spring 2013 Officers: Kelsey Roberts, Vice President; Olivia Almeida, Secretary; and Timothy Woodcock, Corresponding Secretary and Faculty Sponsor

Massachusetts Beta welcomed nine new student members at their annual initiation ceremony, held on April 12. The banquet included a buffet-style dinner, and chocolate cake for dessert, which has become a perennial favorite. The gathering was enjoyed by students, families, and faculty alike.

As the semester drew to a close, several of our student members gave their time to staff calculus review sessions, during the final exam period.

MD Beta – McDaniel College

Corresponding Secretary – Spencer Hamblen; 6 New Members

New Initiates - Gina Mayonado, Brandon Myers, Amanda Owens, Huy Phan, Sam Whitfield, and Emma Wingerd.

MD Delta – Frostburg State University

Chapter President – Kevin Loftus; 33 Current Members; 10 New Members

Other Spring 2013 Officers: DeVonte' McGee, Vice President; Justin Zimmerman, Secretary; Meghan Voelkel, Treasurer; and Mark Hughes, Corresponding Secretary and Faculty Sponsor

Ten new members joined the Maryland Delta Chapter at our March 3 Initiation Ceremony. The ceremony featured a presentation by faculty sponsor Dr. Mark Hughes on some historical applications of Cavalieri's Principle. In mid-March we had a couple of successful fund raising events, namely a bake sale on Pi-Day and a sale of candy Easter Eggs. Our March meeting after spring break involved some math related videos. During our last meeting of the semester in late April, chapter president Kevin Loftus gave a presentation on his independent study entitled "Group Theory and Rubik's Cube." This meeting also saw the election of new officers for the coming year. Chris Colwander will serve as President with Jen Scudder elected as Vice President and Nick Torgerson as Secretary. Meghan Voelkel will continue on as Treasurer. In May we had our annual end-of-semester cookout where a great time was had by all. We also note that participating in the Second Annual Undergraduate Research Symposium here at Frostburg State University were two chapter members, namely, Kevin Loftus and Debbie Wiles. Finally, we offer best wishes to graduating seniors Kevin Loftus, Aaron Littlejohn and Justin Zimmermann.

MD Epsilon – Stevenson University

Chapter President – Kellie Forsyth

Other Spring 2013 Officers: Harriet Adutwum, Vice President; Rebecca Wong, Secretary; Bud Schuster, Treasurer; and Dr. Christopher E. Barat, Corresponding Secretary and Faculty Sponsor

At the School of the Sciences' May Celebration, the graduating senior members of KME received honor cords. The Fall 2013 KME Initiation is tentatively scheduled for late September.

MI Alpha – Albion College

Corresponding Secretary – Mark Bollman; 4 New Members

New Initiates - Cara Delaney, Rebecca Guntz, Jennifer Polinski, and Stephanie Sanders.

MI Delta – Hillsdale College

Chapter President – Megan Moss; 48 Current Members, 11 New Members

Other Spring 2013 Officers: Abigail Loxton, Vice President; Andrea Hay, Secretary; Viktor Rozsa, Treasurer; and Dr. David Murphy, Corresponding Secretary and Faculty Sponsor

During the Spring 2013 semester, we initiated 11 new members. A few of us attended this year's Michigan Undergraduate Mathematics Conference at Siena Heights University in Adrian, MI on February 23. We held our Spring 2013 Initiation on Pi Day, March 14, and several members made pies and one attempted to construct a Menger Sponge cake. That Saturday, March 16, some of our members volunteered to supervise or assist in events for the Michigan Region 9 Science Olympiad tournament hosted at Hillsdale College. On April 5, we had a fondue party, which was a fun opportunity to socialize. The following Tuesday, April 9, we hosted Dr. John Bourke (visiting assistant professor, Kalamazoo College), who presented his talk "Flirting with Contradiction" to students and faculty in the library as part of Math Awareness Month in April; the library also put up a display during the month. The semester closed with our annual math department faculty/student lunch recognizing our graduating majors on Thursday, May 9.

MO Alpha – Missouri State University

Chapter President – Rebecca Wood; 32 Current Members; 13 New Members

Other Spring 2013 Officers: Rachel Sieman, Vice President; Sarah Kramer, Secretary; Marissa Mullen, Treasurer; and Jorge Rebaza, Corresponding Secretary and Faculty Sponsor

As every semester, we had three seminars. The first was on Monday February 11th and Matthew Knepper, actuarial analyst III from ANPAC, talked about "Actuarial Role in Catastrophe Modeling," and about careers in mathematics and statistics; pizza and soda were served. The second seminar was Wednesday March 30th where Professors Redd and Young from the Physics Department at MSU talk about "Advances in Neural Network Research;" again, pizza and soda were served. And the third seminar was Tuesday April 16th and two students from the Senior Seminar class (MTH 497) presented their papers: "The Chinese Remainder Theorem," by Miles Collins, and "Analyzing the Run Time of a Red Black Tree," by Nelson Caberto. Also, KME president, Rebecca Wood, and KME secretary, Sarah Kramer, talked about their experiences at participating at math conferences; in particular about their participation at the Nebraska Conference for Undergraduate Women in Mathematics, and at the National KME Convention. Pizza and soda were served. We also had an end-of-semester

party on Thursday May 9th, the last day of classes; we had lots of games music, food, drinks, and desserts.

New Initiates - Ali Bajwa, Maureen Birchler, Bethany Brown, Steven Cornelius, Ryan Fabro, Brandon Fitzlaff, Thom Hutchison, Trevor Knoblauch, William Meyers, Tyler Miller, Matthew O' Dell, Matthew Swinney, and Bailey West.

MO Beta – University of Central Missouri

Chapter President – Amy Billups; 25 Current Members, 5 New Members

Other Spring 2013 Officers: Matthew Slack, Vice President; Leigh Ann Sherfey, Secretary; Alex Card, Treasurer; Tifini Gast, Historian; Rhonda McKee, Corresponding Secretary and Faculty Sponsor; and Dale Bachman and Steve Shattuck, Faculty Sponsors

The Missouri Beta chapter of KME had a great spring semester. We were well-represented at the national convention in Topeka, KS, as 14 students and three faculty members attended. This year marks the 75th anniversary of the installation of the Missouri Beta Chapter, so we are working with our Alumni Office to organize a reunion for the fall.

MO Epsilon – Central Methodist University

Corresponding Secretary – Pam Gordy; 12 New Members

New Initiates - JoeRyan Anderson, Cole Brower, Tabitha Dickey, Kayla Esquivel, Kayla Heidbrink, Tabitha Hoback, Jesse Marks, Luke Mayer, Jordan Meyer, Melissa Nienhuis, James Schwieter, and Michael Utlaut.

MO Eta – Truman State University

Corresponding Secretary – David Garth; 13 New Members

New Initiates - Josh Bright, Vindya Hettiarachchige de Silva, Cayce Duda, Melissa Fulton, Stefanie Hughes, Rebecca Hutson, Donna Jennings, Megan Mange, Peter Rankin, Hannah Snyder, Anna Sumner, Adam Wagenknecht, and Nathan Wikle.

MO Theta – Evangel University

Chapter President – Hope Moorhead; 14 Current Members; 5 New Members

Other Spring 2013 Officers: David Annas, Vice President; Don Tosh, Corresponding Secretary and Faculty Sponsor

Meetings were held monthly. In January we initiated 8 new members and elected new officers. In April Dr. Tosh and four students attended the national convention at Washburn University. Also in April we had our end-of-year ice cream social at the home of Don Tosh.

MO Iota – Missouri Southern State University

Corresponding Secretary – Charles Curtis; 11 New Members

Other Spring 2013 Officers: Rich Laird and Grant Lathrom, Faculty Sponsors

New Initiates - Luke Barr, Kathryn Goins, Lacie Hale, Neil Huffaker, Adam Mallady, Joshua Mason, Gretchen Moran, Hollie Murray, Jonathan Scofield, Gregg VanLoon, and

Jessica Woodward.

MO Kappa– Drury University

Corresponding Secretary – Carol Browning; 4 New Members

New Initiates - Cameron Duneman, McKenna Feltes, Lauren Himmelberg, and Maurilinn Waneka.

MO Lambda – Missouri Western State University

Chapter President – Cody Beyers; 26 Current Members; 24 New Members

Other 2013 Officers: Brad Isom, Vice President; Jon Guilkey, Secretary; Dr. Steve Klassen, Corresponding Secretary and Co-Sponsor; Dr. Jennifer Hegeman, Faculty Sponsor

In Spring 2013, three MO Lambda members, along with our two faculty co-sponsors, attended the KME National Convention; two students presented and one brought home an award for a top presentation. This was our first participation at a national convention since 1993. Our 24 new initiates is one of the largest in our chapter history. We hope to continue momentum in the Fall.

New Initiates - Ryan Barnett, Grace Chester, Joshua Chester, Amanda Estep, Janna Frederick, Joel Henningsen, Julie Hodson, Nathan Jones, Joseph Kinney, Portia Hahn Leat, Alex Luke, Dr. Lori McCune, Alexis McGhee, Alexander Moore, Darien Morrow, Aaron Munsell, David Myers, Virginia Perkins, Donald Pierre, Sachitch Polpitiyaarachchige, Jaclyn Ray, Brooke Schaben, Ammon Sindt, and Michael Stewart.

MS Alpha – Mississippi University for Women

Corresponding Secretary – Joshua Hanes; 2 New Members

New Initiates - Heather R. Hanes and Maggie E. Leake.

NC Zeta – Catawba College

Chapter President – Jordan Hunsaker; 13 Current Members; 9 New Members

Other Spring 2013 Officers: Cameron Beard, Vice President; Suzanne Williams, Secretary; Lori Fraley, Treasurer; and Douglas Brown, Corresponding Secretary and Faculty Sponsor

New Initiates - Frank Villa Hernandez, Matthew Hodgson, Sydney Hyder, Lindsay C. King, Ekaterina Kruglov, Jacob Regensburger, Cristin Ritchie, Brinsley Stewart, and Elizabeth J. White.

NC Eta – Johnson C. Smith University

Chapter President – Merischia Griffin; 8 Current Members

Other Spring 2013 Officer: Dr. Brian Hunt, Corresponding Secretary and Faculty Sponsor

NE Alpha – Wayne State College

Chapter President – Tim Wiese; 14 Current Members; 6 New Members

Other Spring 2013 Officers: Alex Asche, Vice President; Jesse Smidt, Secretary; Kyle Martin and Mackenzie Masching, Treasurers; Dr. Jennifer

Langdon, Corresponding Secretary and Faculty Sponsor
The Nebraska Alpha Chapter had a productive Spring 2013 semester. We added six new members in December 2012. In the spring of 2013 our chapter sponsored a speaker, Dr. Dan Van Peursem, who spoke on the Pulfrich Effect. We also had a pie day fundraiser, wrote math Jeopardy questions and helped with the college's math competition day for high school students.

NE Beta – University of Nebraska Kearney
Corresponding Secretary – Dr. Katherine Kime; 5 New Members
New Initiates - Sara Ambrose, William Broeckelman, Dustin Broeder, Kayla McMahon, and Stephanie Rudder.

NE Delta – Nebraska Wesleyan University
Chapter President– Alex Whigham; 13 Current Members; 6 New Members
Other Spring 2013 Officers: Laura Booton, Vice President; Jayme Prenosil, Secretary and Treasurer; and Melissa Erdmann, Corresponding Secretary and Faculty Sponsor
Our activities included viewing "Donald Duck in Mathmagicland," about 50 running in a Pi Mile Fun Run, a student talk on topology, and initiating 6 new members. Initiation was followed by a joint picnic with computer science and physics students and professors. Alex Whigham, our president, juggled torches and knives after the picnic.
New Initiates - Elizabeth Dorwart, Ben Harvey, Keenan Hawekotte, Frankie Smith, Brock Taute, and Michael Watson.

NH Alpha – Keene State College
Corresponding Secretary – Vincent J. Ferlini; 10 New Members
New Initiates - Alec Antonioni, Derek Blunt, Vincent Bradley, Rebecca Brady, Jacob Cheverie, Kaitlyn Conor, Jeffrey Garand, Kayla Lanagan, Brittany Rexford, and Cory Valentine.

NJ Delta – Centenary College of New Jersey
Chapter Presidents – Johanne Barthelemy and Brittany Howell, 10 Current Members; 4 New Members
Other Spring 2013 Officers: Linda Ritchey, Treasurer; and Kathy Turrisi, Secretary, Corresponding Secretary, and Faculty Sponsor
NJ Delta initiated three students and one new faculty member on April 20. The Mathematics Tutoring Center (MTC) was open in February by Kathy Turrisi, Assistant Professor of Mathematics; it offers free math tutoring to the community. Members of the NJ Delta Chapter and Delta Kappa Gamma volunteer their time for this cause. The MTC is an experiential learning experience for the Kappa Mu Epsilon students and has sparked the students in the community to enjoy mathematics. If any Kappa Mu Epsilon Chapter is interested in opening up a tutoring center for the community,

please contact Kathy Turrisi at Turrisik@centenarycollege.edu.

New Initiates - Courtney Lorraine Ferris, Rob Linepensel, Heidi O'Neill, and Cory Vernon.

NJ Epsilon – New Jersey City University

Corresponding Secretary – Beimnet Teclezghi; 25 New Members

New Initiates - Teko Teko-Agbo, Marquis Alvararadous, Ali Aryakia, Carolina Barone, Ometa Besanta, James Cena, Robert Daily, Eugenia Editoiu, Simone Estefanos, Faith Gelb, Eric Ginzburg, Malika Harraki, Daisy Hernandez, Alicia Markle, Ingi Nakla, Daniel Ondieki, Jacilia Otero, David Paladines, David Scalcione, Naa-Deidei Solomon, Brianna Taylor, Holly Terracciano, John Wojuikewicz, Gisela Vivanco, and Rachid Zguiouar.

NY Iota – Wagner College

Corresponding Secretary – Dr. Zohreh Shahvar; 15 New Members

New Initiates - Kristina Abruzzo, Nadia Asfar, Marcella Biordi, Danielle N. Bruscella, Marisa A. Carmello, Daniela DiMeglio, Caitlin Donovan, Taylor Fredrickson, Stephen Galazzo, Alanna Gibbons, Carrie Holt, Sidney Hoshko, Kaitlin Pinkos, Demi Poznak, and Juliana Todeschi.

NY Kappa – Pace University

Corresponding Secretary – Lisa Fastenberg; 3 New Members

New Initiates - Michael D'Avella, Sofya Sternik, and Fiona Jeanine Yu.

NY Lambda – C.W. Post Campus of Long Island University

Chapter President – Daniel Barone; 21 New Members

Other Spring 2013 Officers: Elyse Capozza, Vice President; Anthimos Michael, Secretary; Thomas Fallon, Treasurer; and Dr. James B. Peters, Corresponding Secretary

The New York Lambda chapter held its initiation ceremony at its annual KME Banquet, on April 14. Thirteen new members were initiated, half of whom were graduate students. Including faculty and former students, there were over 80 in attendance at the Banquet. Our guest speaker was a former student who is teaching mathematics and physics at a Nassau County high school.

New Initiates - Brianna Damadian, Philip Demeri, Jennifer Dileo, Shuxian Gao, Russell Hoenig, Paul Kreinbihl, Eda Kuscakoglu, Yael Landau, Anthimos Michael, Syphax Mouloudj, Ashley Oracewski, Christopher Pierce, Erik Raessler, Michael Riggs, Gregory Sanzari, Ryen Seymour-McClean, Brad Turow, Brendan Walter, Yiran Zhang, Shiming Zheng, and Yiao Zheng.

NY Nu – Hartwick College

Chapter President – Nathan Nichols; 18 Current Members; 11 New Members

Other Spring 2013 Officers: Jessica Bentley, Vice President; Aaron Parisi, Secretary; Steve Grzeskowiak, Treasurer; and Ron Brzenk, Corresponding Secretary and Faculty Sponsor

New Initiates - Dane Alexander, Victor Angeline, Ashley Curtis, Kyle Murray, Hannah

Placzek, Brian Redder, Molly Schewe, Susan A. Schwarz, Martina Shorkey, Amanda Wood, and Hongji Zhu.

NY Omicron – St. Joseph's College

Chapter President – Stephen Bates; 40 Current Members, 22 New Members

Other Spring 2013 Officers: Janece Guerra, Vice President; Daniel G. Ferguson, Secretary; Carl W. Baurle, III, Treasurer; Dr. Elana Reiser, Corresponding Secretary; and Dr. Donna Marie Pirich, Faculty Sponsor

This semester the NY Omicron chapter held an initiation for 22 new members. Our members volunteered Saturday mornings in the math clinic tutoring local high school students. Our chapter also ran several fundraisers, including an Easter basket donation. We donated raised funds to help victims of hurricane Sandy and to help a fellow KME member undergoing chemo.

NY Rho – Molloy College

Chapter President – Amanda Kovacs; 97 Current Members, 15 New Members

Other Spring 2013 Officers: Natalie Khouryawad, Vice President; Joanna Mantone, Secretary; Alfeen Hasmani, Treasurer; and Dr. Manyiu Tse, Corresponding Secretary; and Deborah Upton, Faculty Sponsor

We held elections. Our new officers (effective Fall 2013) are President: Lauren Kenedy; Vice President: Kerry Murphy; Secretary: Erin Fay; and Treasurer: Jessica Buonfrisco. We congratulate the following KME members who will be going to graduate school in mathematics this fall - Amanda Kovacs (President of KME) - SUNY Albany; Alfeen Hasmani (Treasurer of KME) – Cornell; and Joanna Mantone (Secretary of KME) - Central Michigan University.

OH Alpha – Bowling Green State University

Corresponding Secretary – Steven M. Seubert; and Co-advisors, Dr. James Albert and Dr. Gordon Wade

The Bowling Green State University chapter of KME has been dormant for the past several years. During the past semester, we did confirm with the KME National Officers that our chapter is not officially dormant. We are now in the process of resurrecting it, and plan to initiate new members in the Spring semester of the academic year 2013-2014. We are also planning various planned service activities during the year.

OH Gamma – Baldwin Wallace University

Corresponding Secretary – Dr. David Calvis; 20 New Members

New Initiates - Kelsey L. Amidon, Deanna M. Biesan, Nicholas E. Boron, Daniel N. Candrea, Anisa Cobaj, Catherine A. DeBlauw, Jared W. Early, Aric F. Gady, Robert W. Johansen, Nicholas A. Klufas, Brian J. McLaughlin, Ryan W. Mitchell, Gilbert I. Montague,

Scott J. Robinson, Jordan M. Saxton, Kyle D. Schifano, Matthew T. Shaniuk, Tessa M. Wagner, Jessica N. Walter, Alec J. Weisman, and Jessica R. Ziegler.

OH Epsilon – Marietta College

Chapter President – Misty Hussing; 35 Current Members; 14 New Members

Other Spring 2013 Officer: John Tynan, Corresponding Secretary and Faculty Sponsor

New Initiates - Andrew Bushman, Jacob Double, Hannah Duncan, Jacob Eicher, Alexandra Jurgens, Ethan Kelley, Jeffrey Lucas, Sarah Mayo, Kellen Quinn, Nicole Schaly, Chen Yu Song, Robert Thaxton, Vance Turnewitsch, and Kaifeng Xia.

OH Zeta – Muskingum University

Corresponding Secretary – Richard Daquila; 9 New Members

New Initiates - Jessica Barak, Matthew Connor, Dr. Matt Davis, Keyang He, Jason Holman, Sean Lally, Changqing Li, Zhixuan Li, and Stacy Russell.

OH Eta – Ohio Northern University

Corresponding Secretary – Donald Hunt; 14 New Members

New Initiates - Derek Sheridan Blubaugh, Katherine Bowman, Camden C. Brown, Kerry DuLaney, Morgan Hammer, Jason Michael Robert Luthman, Matthew McCandless, Benjamin A. McKinniss, Zach Myers, Donald J. Pleshinger, Thomas E. Steinberger, Matthew Tolley, Levi Topp, and Jeremy Gregory Wilson.

OK Alpha – Northeastern State University

Chapter President –.Rho Middleton; 61 Current Members; 9 New Members

Other Spring 2013 Officers: Blane Burge, Vice President; Anna Faina, Secretary; Ryan Berkley, Treasurer; Dr. Joan E. Bell, Corresponding Secretary and Faculty Sponsor

Our spring initiation brought five new members into our chapter. At our meetings we worked on problems from The Pentagon. We had several speakers this year. Dr. Elwyn Davis, Pittsburg State University, gave a very interesting talk about "How to Draw a Straight Line." Toni Slagle, a Collinsville High School mathematics teacher who graduated from NSU and was the OK Alpha president for several years, presented "Presenting a Math Lesson Reflecting the Common Core Standards." Dr. Mark Buckles spoke on "The Mathematical Basis for Microchip Design." And James Sherrell, KME member, presented his Honors Program research in "RISK: Developing Tools to assist in the Strategic Decision-Making Process." Our fund raiser for the year was a bake sale in March. We also sold thirteen sets of honor cords to KME members who graduated spring 2013. The last meeting of the year was the annual ice cream social for KME members, math majors, and math faculty.

New Initiates - Cherokee B. Anderson, Joanna F. Billups, Margaret J. Homer, Cindy J.

Jeffcoat, Briana L. Ketcher, Lauren E. Lowe-Thompson, Allan A. Porras-Romero, and Farah E. Toler.

PA Alpha – Westminster College

Corresponding Secretary – Jeffrey Boerner; 5 New Members

New Initiates - Stephanie Burkus, Brittany Erbe, Michael Girata, John Griebel, and Morgan Swartz.

PA Beta – La Salle University

Chapter President – Dominick Macaluso; 19 Current Members

Other Spring 2013 Officers: David Comberiate, Vice President; Olivia Shoemaker, Secretary; Daniel Bowers, Treasurer; and Dr. Stephen Andrilli, Corresponding Secretary and Faculty Sponsor

At La Salle (Pennsylvania Beta Chapter), many of our KME Activities are done in conjunction with our Math Club, whose moderator is Dr. Janet Fierson. This Spring (2013), two of our math students attended and presented talks on graph theory at the Nebraska Conference for Undergraduate Women in Mathematics (1/25-1/27/13). Also, two math students competed as a team in the COMAP Mathematical Contest in Modeling from 1/31-2/4/13. In addition, seven math students attended a Student Mathematics Conference at Moravian College in Bethlehem, PA on 2/16/13, at which two students presented talks. Finally, the math students held a Valentine's Day Fundraiser (selling cupcakes and candygrams).

PA Gamma – Waynesburg University

Corresponding Secretary – James R. Bush; 8 New Members

New Initiates - Julie L. Collins, Kevin M. Graham, Elaine M. Hasek, Nathaniel Duane Huffman, Elizabeth A. LeCain, Karen J. Moyer, Bethany L. Orndoff, and Marc B. Timmons.

PA Epsilon – Kutztown University

Corresponding Secretary – R.S. Schaeffer; 9 New Members

New Initiates - Christopher Amici, Jessie Bailey, Joshua Fulmer, Brandi Lees, Jacob LeVan, Jenna Marrazzo, Sarah McQueen, Zachary Moran, and Michael Rex.

PA Eta – Grove City College

Corresponding Secretary – Dale L. McIntyre; 4 New Members

New Initiates - Annie Laurie Holfelder, Ashley Lindow, Jennifer Toth, and Sarah Whiting.

PA Kappa – Holy Family University

Chapter President – Gidget Montelibano; 8 Current Members; 5 New Members

Other Spring 2013 Officer: Sister Marcella Wallowicz CSFN, Corresponding Secretary and Faculty Sponsor

Five students were initiated in Spring 2013. Prior to initiation, candidates usually engaged in a mathematics service project. This year's group continued the traditional project of a "drop-in" math tutoring lunchtime pro-

gram for students enrolled in a full range of math courses from developmental math through Calculus I. Chapter President Gidget Montelibano made the local news when her boyfriend, Matt Murray, proposed to her during Holy Family's Commencement ceremonies on May 18 at the Academy of Music. A wedding is planned for next year.

New Initiates- Rebecca Gaetani, Sheridan Goodwill, Timothy McCarthy, Benjamin Savidge, and Livia Yang.

PA Lambda – Bloomsburg University

Corresponding Secretary – Elizabeth Mauch; 4 New Members

New Initiates - Danielle Goss, Ryan Oravec, Megan Paul, and Darrell Ritsick.

PA Mu – Saint Francis University

Chapter President – Phuong Minh Do; 47 Current Members; 13 New Members

Other Spring 2013 Officers: Ryan Knee, Vice President; Elise Löfgren, Secretary; James Shiring, Treasurer; Dr. Peter Skoner, Corresponding Secretary; Dr. Katherine Remillard, Faculty Sponsor; and Fr. Joe Chancler, T.O.R., Chaplain

The annual Pi Day celebration was held on March 14; it began with A Graphical Presentation of the Millennium Development Goals and their Progress by students in Dr. Kate Remillard's Math 101 class. Through the use of numbers and graphics, students illustrated present-day conditions and challenges related to: ending poverty and hunger, achieving universal primary education, promoting gender equality and empowering women, reducing child mortality, improving maternal health, combating HIV/AIDS/Malaria, ensuring environmental sustainability and developing a global partnership for development. And, faculty, students, and staff enjoyed taste testing an assortment of "pi" served by KME members. A free will offering was taken for CAMFED (Campaign for Female Education) whose mission is to educate girls and support young women to help tackle poverty in rural communities, especially in sub-Saharan Africa. Initiation ceremonies were held on Thursday, January 31, 2013 in DiSepio 213. The evening began with a prayer by chapter member Dr. Pedro Muíño; followed with dinner; followed by a talk "Metrics in a Plane," by Dr. Brendon LaBuz; followed by the initiation ceremony for the thirteen new members; and concluded with a closing prayer from Dr. Muíño. Two faculty members and two students attended the 39th Biennial National Convention of held April 12-14, 2013 at Washburn University in Topeka, Kansas. Students, faculty, and staff from Saint Francis University served at Try-Math-A-Lot, a mathematics competition for more than 200 sixth and seventh grade students from area schools on Wednesday, May 1, 2013, conducting mathematics quiz bowl competitions in three different rooms during three

different time periods.

PA Nu – Ursinus College

Corresponding Secretary – Jeff Neslen; 7 New Members

New Initiates - Riley Acton, Hongli Chen, Frank DeVone, Heather Hodges, Anna LoPresti, Alexander Mellus, and Cara Sulyok.

PA Pi – Slippery Rock University

Chapter Co-Presidents – Shane Mattson and Christina Rajchel; 12 Current Members; 6 New Members

Other Spring 2013 Officers: Shane Mattson, Secretary; Christina Rajchel, Treasurer; Elise Grabner, Corresponding Secretary and Faculty Sponsor, and Dr. Richard Marchand, Faculty Sponsor

New Initiates - Joseph Cortez, Stephen Galloway, Shane Mattson, Christina Rajchel, Brad Spangler, and Steven Ways.

PA Rho – Thiel College

Corresponding Secretary – Max Shellenbarger; 11 New Members

On Sunday, April 7, 2013, eleven new members were initiated into the Pennsylvania Rho Chapter of Kappa Mu Epsilon at Thiel College, Greenville, PA. Professor Max Shellenbarger, Corresponding Secretary, and Kayce Grimm, current President, served as the Masters of Ceremony. Dr. Russell Richins gave a presentation about fractals. The new members were then initiated and the new officers installed; both pictures are below.

New Initiates - Becky Crawford, Keeley Criswell, Katelyn Downey, Ben Hellner-Burris, Eric King, Kristin Patrick, Emily Petrak, Dr. Russell Richins, Trevor Ridilla, Christina Ryan, and Joshua Stewart.

PA Sigma – Lycoming College

Corresponding Secretary – Santu de Silva; 3 New Members

New Initiates - Ryan Boccagno, Brittany Coppola, and Jamy Erin Fite.

PA Tau – DeSales University

Chapter President – Michael P. Russo; 7 Current Members; 8 New Members

Other Spring 2013 Officers: Kelsey R. Foster, Vice President; Joseph A. Marlin, Secretary; Tripty Modi, Treasurer; and Bro. Daniel P. Wisniewski,

O.S.F.S., Faculty Sponsor and Corresponding Secretary

On the afternoon of Tuesday, March 19, 2013, the PA Tau Chapter of Kappa Mu Epsilon sponsored a Pizza Party Roundtable featuring DSU alumna and founding charter member of the PA Tau Chapter of KME, Carrie Caswell, who is currently a graduate student in the Applied Statistics Program at Villanova University (Villanova, PA). Carrie gave a talk on coding theory and shared her experiences at graduate school; she also answered questions and discussed how she had prepared for and applied to graduate school. Fifteen students and four faculty members enjoyed pizza, hoagies, and fresh baked cookies during the event. On Sunday, April 21, 2013, the PA Tau Chapter of Kappa Mu Epsilon at DeSales University initiated eight new members. The event included a presentation entitled "Wavelets: No Surfboards Required" by Beth Campbell Hetrick, Associate Professor of Mathematics at Gettysburg College (Gettysburg, PA). In attendance were family and friends of current and new KME members, as well as four KME alumni.

New Initiates – Deena C. Blazka, Keith T. Crozier, Jaquelin M. Pastor, Zachary Sikanowicz, Morgan A. Treffinger, Angela M. Ulrich, Beth Campbell Hetrick, and Christopher Micklewright.

RI Alpha – Roger Williams University

Chapter President – Emma Long; 15 Current Members; 12 New Members

Other Spring 2013 Officer: Corresponding Secretary – Dr. Robert Jacobson, Corresponding Secretary and Faculty Sponsor

New Initiates - Hailey Brignac, Sean Damico, Hy Dinh, Kevin Franca, Lauren Formanski, Robert Jacobson, Hubert Noussi Kamdem, Emma Long, Bronson Martin, Manahil Naqvi, Aaron Schrank, and Helena Zimmermann.

RI Beta – Bryant University

Chapter President – James Wood; 42 Current Members; 42 New Members

Other Spring 2013 Officers: Andrew DiFronzo, Vice President; Delaney Carr, Secretary; Summer Lyons, Treasurer; John Quinn, Corresponding Secretary; and Dr. Alan Olinsky, Faculty Sponsor

We formed the Rhode Island Beta chapter of Kappa Mu Epsilon at an installation ceremony on April 3, 2013.

New Initiates - Caitlin Allen, Billie Anderson, Samantha Andrews, Taylor Bell, James Bishop, Joseph Capalbo, Delaney Carr, Marissa Cohen, Elise Daniell, Robert Davis, Andrew DiFronzo, Joan Graham, Thomas Hartl, Louise Hasenfus, Kristin Kennedy, Olivia Jankins, Amanda Jillson, Michael Leonard, Corinne Loiacono, Summer Lyons, Megan Mapp, Robert Muksian, Huy Nhat Ho, Jessica Nolan, Kyle Nyskohus, Alan Olinsky, Chester Piascik, Jeffrey Pierro, John Quinn, Kerry Quirk, Ariana Ricci, Michael Salzillo, Phyllis Schumacher, Sarah Segill, Paul Sheard, Richard Smith, Stephanie Smyers, Bryan

Start, Matthew Tomlinson, Alyssa Westfall, Lindsey Wilson, and James Wood.

SC Beta – South Carolina State University

Corresponding Secretary – Cynthia Davis; 6 New Members

New Initiates - Edwin K. Maina, Daudi Muhamed, Tempest Neal, Eva Nesmith, Shilindria S. Rivers, and Angela Self.

SC Gamma – Winthrop University

Corresponding Secretary – Matthew R. Clark; 8 New Members

New Initiates - Tyler Brooks, Courtney Cabaniss, Gabrielle Epelle, Ruth Fujino, Harrison Morris, Grandy Nguyen, Lindsey Saunders, and Hannah Swan.

SC Delta – Erskine College

Chapter President – Holly Anderson; 2 Current Members; 6 New Members

Other Spring 2013 Officers: Kelly Walker, Vice President; Tiffany Mills, Secretary; Rachel Whitmire, Treasurer; and Dr. Art Gorka, Corresponding Secretary and Faculty Sponsor

The SC Delta chapter held monthly joint meetings with the Krazy Math Enthusiasts Math Club and organized a few successful events and activities, including: 1. Creation of club t-shirts for the members of KME (from design to order to print to sale); 2. Organizing a college convocation event "Are you better than a Fifth-grader," involving inviting fifth graders from a local school to Erskine to compete against college students to solve math problems and win a pizza party; 3. Attending a MAA SE section math conference (together with Math Department) at Winthrop University in Rock Hill, SC; 4. Planning the events for next semester. On May 7 an initiation ceremony was held in the Math Department, followed by a snack and drink informal gathering. The initiates along with Dr. Gorka (left) and Dr. Kokou Abalo (right) are pictured below.

New Initiates - Holly Anderson, Emily Brown, Tiffany Mills, Kelly Walker, Catherine White, and Rachel Whitmire.

SD Beta – Black Hills State University

Corresponding Secretary – Kristel Ehrhardt; 10 New Members

New Initiates - Kelsey Dalzell, Kristel Ehrhardt, Jason Gifford, Keenan Justice, Hui Ma, Shalayne Mowry, Daluss Siewert, Rachel Solano, Jill Trimble, and Zachary Zenk.

TN Beta – East Tennessee State University

Corresponding Secretary – Robert Gardner; 21 New Members

New Initiates - Benedict Adjogah, Kristen Bales, Catherine Below, Andrew Boghozian, Andre Campbell, Laura Chambers, James Dustin Chandler, David Elliot, Jenna Brooke Estep, Shelley Goodson, Aaron Gray, Samuel Green, Zachary Helbert, Jennifer Houser, Joseph McNeil, Brittany Munro, Geophrey Odero, Lisa Stacy, Qi Tang, Elizabeth Williams, and Inna Vasylieva.

TN Gamma – Union University

Chapter President – Rachel Carbonell; 22 Current Members; 10 New Members

Other Spring 2013 Officers: Caroline McConnell, Vice President; Alexandra Archer, Secretary; David Clark, Treasurer; Dr. Michelle Nielsen, Corresponding Secretary; and Dr. Matt Lunsford, Faculty Sponsor

New Initiates - Michael Cochran, Lydia DeWolf, McKenzie Dorris, Brittany Hagler, Timothy Johnson, Zakary Johnson, Evan Lewoczko, Chris Love, Megan Mouser and Cara Nance.

TX Alpha – Texas Tech University

Chapter President – Saba Nafees; 13 Current Members; 12 New Members

Other Spring 2013 Officer: Magdalena Toda, Corresponding Secretary

On March 24, 2013, the Texas Alpha Chapter held an Initiation and Orientation Meeting. This meeting was coordinated by the Advisor and Secretary, Dr. Toda. She invited the new initiates and gave a small speech regarding the importance of being in a group that promotes Mathematics such as KME. Her generosity is evident as she bought tens of copies of How Math Can Save Your Life by James Stein and refreshments for the new members. Pins and membership cards were distributed. All members introduced themselves and described what their future plans were. She gave a brief history of KME and encouraged students to become more involved and attend the KME National Conference. She commended Saba Nafees for being accepted to attend the 2013 Conference. Election was held to determine the position of Saba Nafees as Interim President. Upon voting, the new members unanimously decided to promote Ms. Nafees to President. Voting also took place to determine the winner of the Professor of the Year Award. Dr. Edward Allen was unanimously voted as the recipient of this award. This award was to be given at the Department Banquet in April. The orientation meeting ended rather joyfully. Saba Nafees was the sole student from the grand state of Texas to attend the KME Na-

tional Conference on April 11-13, 2013. She presented her research on Cystic Fibrosis and Orthogonal Polynomials. Next year, she hopes to take more members to the event. On April 19-20, 2013, the newly formed TTU Mathematics Ambassadors volunteered their time to recruitment of potential Math majors at the regional UIL High School meet. Most of this year's KME members are also Math Ambassadors and volunteered many hours during this event. On April 23, 2013, at the Department of Mathematics and Statistics Banquet, Ms. Nafees proposed a new award to be given by KME every year. This award is called Outstanding Mathophile Award. It was named by Dr. Edward Allen and is given to someone who shows extreme spirit and passion for the art and study of Mathematics. This unique individual dedicates their lifetime to the advancement and service of Mathematics. This individual can be any undergraduate or graduate student, any faculty or staff. The first recipient of this award, which will be given annually, was named Dr. Udaya Jayatilake. He received his Ph.D. in Mathematics in May, 2013 from Texas Tech University. He is well deserving of this award and accepted it humbly by his graduate advisor, Dr. Roger Barnard, http://www.math.ttu.edu/~ujayatil/. The 2012-2013 KME Professor of the Years award was given to the wonderful Dr. Edward Allen. KME members when voting all exclaimed how effective of a teacher Dr. Allen is and how he has positively and profoundly impacted their understanding of Mathematics. He is always there to help, before, after, at any time. He is the teacher who loses sleep at night if he feels he didn't teach something correctly. His dedication is immense and he was well deserving of this award. More about Dr. Allen can be found at http://www.math.ttu.edu/~edallen/. On July 1, 2013, entries to the KME Quiz given during the National Conference in April were submitted. The Texas Alpha Chapter was one of the winners and received $50. This money will be allocated for award plaques and other chapter needs.

TX Kappa – The University of Mary Hardin-Baylor

Chapter President - Kelsey Janis; 10 Current Members; 2 New Members

Other Spring 2013 Officers: Chad Manns, Vice President; Loren Watson, Secretary; Peter H. Chen, Corresponding Secretary; and Maxwell M. Hart, Faculty Sponsor

New Initiates – Brandon Heller and Chad Manns.

TX Lambda – Trinity University

Corresponding Secretary – Peter Olofsson; 6 New Members

New Initiates - Rebecca Victoria Dias, Alyssa Ann Fink, Rachel Elizabeth Hure, Catherine Claire Norman, Carolyn Anne Rice, and Ailie Marie Vuper.

VA Gamma – Liberty University

Corresponding Secretary – Dr. Tim Van Voorhis; 14 New Members

New Initiates – Lina Baquero, Cassandra Black, Kenneth Bruce, Rebecca Carpenter, Lindsay Collins, Brett Hemric, Amanda King, Gail McGowan, Brandi Morgante, Minh Nguyen, Charles Snyder, Bethany Thigpen, Randolph Tomkins, and Bailu Zhang.

VA Delta – Marymount University

Corresponding Secretary – William Heuett; 1 New Member

New Initiate - Myriam Joga.

WV Alpha – Bethany College

Corresponding Secretary – Professor Adam C. Fletcher; 13 New Members

New Initiates - William Edward Baldinger Bambury, Cameron Avery Cooper, Hallee Paige Cramer, Alexander Vincent DelGiorno, Jacob Emerson Fischer, Thal Trac-Doanh Kratzke, Dustin A. Maiolo, Corey Dane Mays, Tyler Nelson Pannebaker, Lisa Marie Reilly, Chelsea Claire Sroka, Keisten Ann Sroka, and Joseph William Walsh.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, Jonesboro	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
KS Zeta	Southwestern College, Winfield	14 Apr 1998
TN Epsilon	Bethel College, McKenzie	16 Apr 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 Mar 1999
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
NJ Gamma	Monmouth University, West Long Branch	21 Apr 2002
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008

CA Zeta	Simpson University, Redding	4 Apr 2009
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011
PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013