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Trigonometric Functions in the Biangular Plane: Part 1

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1. Introduction

Throughout this paper, biangular relations will be those that relate the angles ϕ and θ in some type of formula. These relations, however, will be converted to rectangular coordinates and plotted within the rectangular plane. We employ the rectangular plane for this study, because plotting biangular relationships in the biangular plane, in which the axes are labeled θ and ϕ as opposed to x and y, delivers identical graphs to the rectangular counterparts of the Cartesian plane. The shapes of these graphs are familiar. Instead, our interest lies in examining the transformed image of the biangular relations in the Cartesian plane. Our goal is to explore the properties of the trigonometric functions cosine and sine from the biangular plane by examining them within the scope of the rectangular plane.

2. The Biangular Coordinate System

Definition

Two familiar coordinate systems are the Cartesian coordinate system which employs two distances, x and y, and the polar coordinate system which employs one distance and one angle, r and θ, to specify points in space. A complement to these systems is the biangular coordinate system, which employs two angles, θ and ϕ, to denote points. In this system, we fix two points A and B as poles [2, p. 34]. We let A and B equal the points (0,0) and (1,0) of the xy-plane respectively. This is a convenient choice of poles for conversion. The horizontal line (i.e. the x-axis) passing through

these two points is called the polar axis [1, p. 29]. We assign our poles to points in the rectangular plane because it makes conversion between the two systems possible.

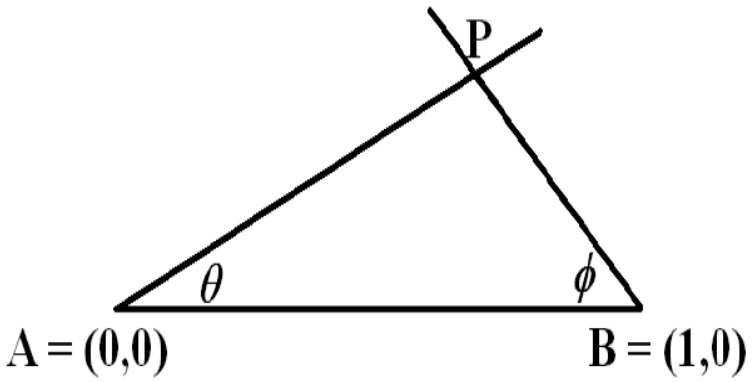

Figure 1 [1]

We specify the point P in the biangular plane by the intersection of two rays formed by the angles θ and ϕ. The first ray originates at the point $A = (0, 0)$ and is at an angle of θ measured counterclockwise from the polar axis. The second ray originates from the point $B = (1, 0)$ and is at an angle of ϕ measured clockwise from the polar axis. Where these two rays intersect is the point P, notated as $(\theta, \phi)_b$ in biangular coordinates (see figure 1) [1].

Plotting Techniques in *Mathematica*

The small amount of research done on the topic provides a reason why biangular coordinates are not prevalent in any particular field. We simply do not know enough about the system's limitations and benefits. A major hindrance is that during the 1800's when the bulk of research on the topic was being performed, computers and their mathematical plotting programs were non-existent. Plotting numerous relations in the biangular coordinate system by hand can be difficult. The technical aid provided by a program such as *Mathematica* allows us to observe a myriad of diverse graphs with relative ease.

Even though we now have many plotting programs readily available to us, most of them are programmed to plot in rectangular coordinates. Many

of these programs have applications for plotting in polar coordinates and other coordinate systems, but biangular coordinates are excluded. Biangular coordinates are not included within these systems because of the system's unfamiliarity. Thus we are compelled to find a conversion technique between the two systems.

Derivation and Introduction of the Conversion

Consider a triangle with ϕ and θ as angles and an altitude $\overline{PQ}$ as displayed in figure 4. We can use this to determine the coordinates of the point P in a biangular relation. Notice in the right triangle $\triangle AQP$ the line segments $\overline{AQ}$ and $\overline{PQ}$ are the coordinates x and y, respectively, of the point P in the rectangular plane. We need only use some simple trigonometric relationships to solve for these coordinates. Although this derivation is not provided in their 2010 article, it is consistent with the findings of Naylor and Winkel [1].

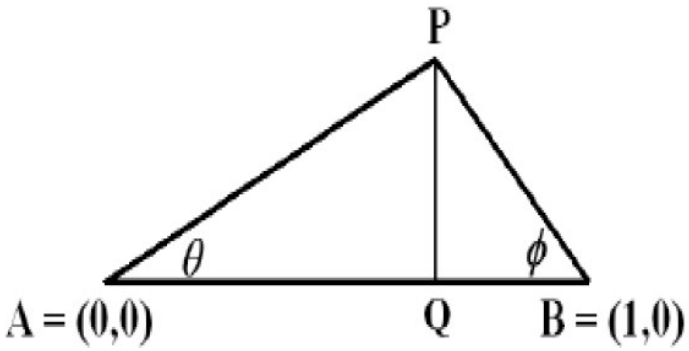

Figure 2 [1]

From this figure we have

$$\tan(\theta) = \frac{\overline{PQ}}{\overline{AQ}} \tan(\phi) = \frac{\overline{PQ}}{\overline{BQ}}$$

and thus we have

$$\begin{aligned} \overline{AQ} &= \frac{\overline{PQ}}{\tan(\theta)} = \frac{\overline{PQ}\cos(\theta)}{\sin(\theta)} \\ \overline{BQ} &= \frac{\overline{PQ}}{\tan(\phi)} = \frac{\overline{PQ}\cos(\phi)}{\sin(\phi)} \end{aligned}$$

Because $\overline{AB}$ is equal to 1 we can conclude that $\overline{BQ} = 1 - \overline{AQ}$.

$$\begin{aligned}\frac{\overline{PQ}\cos(\theta)}{\sin(\theta)} &= 1 - \frac{\overline{PQ}\cos(\phi)}{\sin(\phi)} \\ &= \frac{\sin(\phi) - \overline{PQ}\cos(\phi)}{\sin(\phi)}\end{aligned}$$

From this we can solve for $\overline{PQ}$

$$\begin{aligned}& \overline{PQ}\cos(\theta)\sin(\phi) = \sin(\phi)\sin(\theta) - \overline{PQ}\cos(\phi)\sin(\theta) \\ \Rightarrow\ & \overline{PQ}[\cos(\theta)\sin(\phi) + \cos(\phi)\sin(\theta)] - \sin(\phi)\sin(\theta) = 0 \\ \Rightarrow\ & \overline{PQ}\sin(\phi+\theta) - \sin(\phi)\sin(\theta) = 0 \\ \Rightarrow\ & \overline{PQ} = \frac{\sin(\phi)\sin(\theta)}{\sin(\phi+\theta)}\end{aligned}$$

Substituting this into our equation for $\overline{AQ}$:

$$\begin{aligned}\frac{\frac{\sin(\phi)\sin(\theta)}{\sin(\phi+\theta)}\cos(\theta)}{\sin(\theta)} &= \frac{\sin(\phi)\sin(\theta)\cos(\theta)}{\sin(\phi+\theta)\sin(\theta)} \\ &= \frac{\sin(\phi)\cos(\theta)}{\sin(\phi+\theta)}\end{aligned}$$

Thus we have the coordinates

$$x = \frac{\sin(\phi)\cos(\theta)}{\sin(\phi+\theta)}, y = \frac{\sin(\phi)\sin(\theta)}{\sin(\phi+\theta)}$$

that will convert a biangular relation into rectangular coordinates. With this conversion, it is easy to plot biangular relationships with a rectangular plotting program such as *Mathematica*.

3. Trigonometric Functions in the Biangular Plane

Function Family: $\phi = \cos(\theta)$

The biangular relation $\phi = \cos(\theta)$ has some unusual qualities when displayed in the rectangular plane. Most people are familiar with $y = \cos(x)$ in the rectangular plane. It resembles a valley, starting at $y = 1$ when $x = 0$, decreasing to $y = -1$ when $x = \pi$, and then increasing to $y = 1$ when $x = 2\pi$. This pattern repeats as x continues on to infinity. Thus, it is common knowledge that $y = \cos(x)$ has a period of 2π.

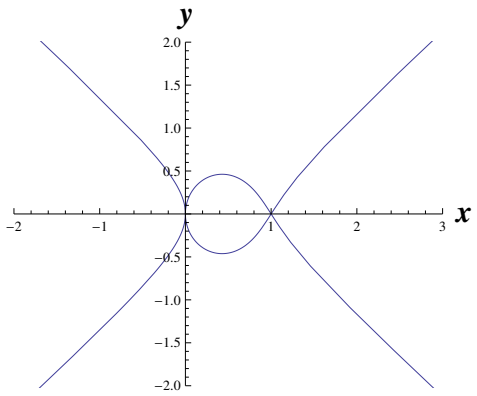

Figure 3

The relationship $\phi = \cos(\theta)$, displayed in the rectangular plane, resembles a bow-tie with a central loop surrounded by sweeping arms on both sides. It bears no resemblance to its rectangular counterpart, $y = \cos(x)$. They do, however, share the characteristic of a period of 2π. It is interesting to note that if the bow-tie shape resulted from a rectangular relationship it would not be considered a function within the rectangular plane. It would, in fact, fail the vertical line test in an infinite number of locations. The vertical line test, however, is not appropriate for the biangular relations displayed in the rectangular plane. Many of these relations contain loops and curves, which if plotted in rectangular coordinates would be clumsy and determined non-functions. This is a benefit to the biangular coordinate system. We allow for the possibility of loops and curves.

We are familiar with certain characteristics that the rectangular relation $y = \cos(x)$ possesses when we change specific aspects of the equation. We now investigate if these characteristics translate when we map a biangular relation into the Cartesian plane.

Adding a Constant Outside the Argument: $\phi = \cos(\theta) + k$

What would happen if we were to add a constant to $\cos(\theta)$ outside of the argument of the function? In rectangular coordinates, adding a constant of k in this position of the equation would simply shift the graph k units in the vertical direction. For instance, if we were to compare the relationships of y=$\cos(x)$ and y=$\cos(x) + 1$, we would see that the second relationship was shifted one unit upward. It remains the same shape and period, but all of its points are shifted 1 unit upward (see figure 4).

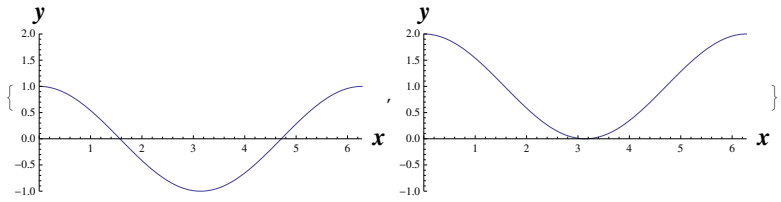

Figure 4

When we convert the biangular relation ϕ = $\cos(\theta) + 1$ into the rectangular plane, we do not see a uniform shift as we did with rectangular coordinates. Instead, we see that our loop has changed form and position and we no longer have the bow-tie shape that we possessed beforehand. The graph could actually be misinterpreted as an entirely new relationship (see figure 5).

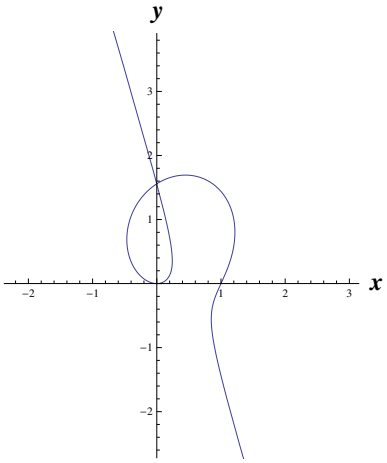

Figure 5

When we add a constant to the right side of our equation, what actually happens is that instead of shifting upward as it did in the rectangular plane, the graph actually begins a rotation. In figure 6, the values of $k = 0.1, 0.3, 0.5, 0.7, 0.9,$ and 1 correspond to the A, B, C, D, E, and F graphs respectively.

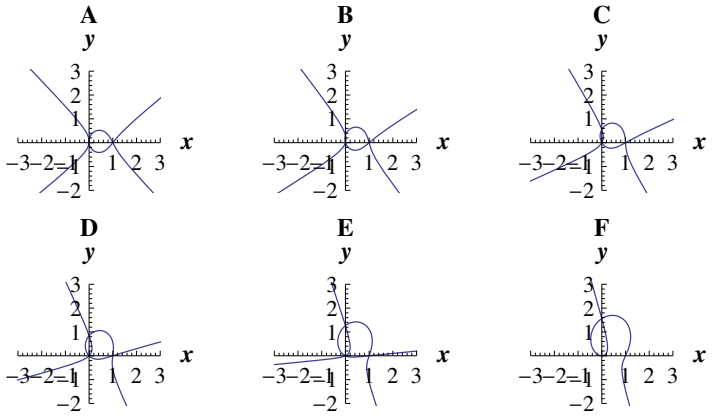

Figure 6

As k grows from 0 to 1, the top of our loop rises and the arms rotate towards the right (see figure 6), until eventually when $k = 1$ (F graph), we no longer have the double arms and our loop is of an entirely different shape (see figure 5). For $\phi = \cos(\theta)$ adding a constant of 1 does not deliver a vertical shift, but a rotation and a new graph with a new shape.

When we plot the relationship $\phi = \cos(\theta) + \pi$, we find that we get the same graph as $\phi = \cos(\theta)$. Thus, the rotation itself appears to have a period of π. It appears that adding a constant variable results not in a vertical shift, but in a rotational periodic characteristic of the graph. We can prove analytically that this is indeed the case.

Theorem 1 *The biangular relation $\phi = \cos(\theta) + k$ (or $\phi = \sin(\theta) + k$) delivers the same graph as $\phi = \cos(\theta) + \pi k$ (or $\phi = \sin(\theta) + \pi k$). That is, the rotation for the relations $\phi = \cos(\theta) + k$ and $\phi = \sin(\theta) + k$ have a period of π.*

Proof.

To show that the rotation has a period of π, we must show that $\phi = \cos(\theta) + \pi$ delivers the same graphs as $\phi = \cos(\theta)$. We know that converting the biangular relation $\phi = \cos(\theta)$ into the Cartesian plane results in the coordinates

$$\left(\frac{\cos(\theta)\sin(\cos\theta)}{\sin(\theta+\cos(\theta))}, \frac{\sin(\theta)\sin(\cos\theta)}{\sin(\theta+\cos(\theta))}\right).$$

Then considering $\phi = \cos(\theta) + \pi$, we obtain an abscissa of

$$\frac{\cos(\theta)\sin(\cos(\theta)+\pi)}{\sin(\theta+\cos(\theta)+\pi)},$$

and by using the trigonometric identity for the sine of a sum, we find that

$$\begin{aligned}
& \frac{\cos(\theta)\sin(\cos(\theta)+\pi)}{\sin(\theta+\cos(\theta)+\pi)} \\
= \; & \frac{\cos(\theta)\left[\sin(\cos(\theta))\cos(\pi)+\cos(\cos(\theta))\sin(\pi)\right]}{\sin(\theta+\cos(\theta))\cos(\pi)+\cos(\theta+\cos(\theta))\sin(\pi)} \\
= \; & \frac{-\cos(\theta)\sin(\cos(\theta))}{-\sin(\theta+\cos(\theta))} \\
= \; & \frac{\cos(\theta)\sin(\cos(\theta))}{\sin(\theta+\cos(\theta))}.
\end{aligned}$$

Our ordinate is then

$$\begin{aligned}
& \frac{\sin(\theta)\sin(\cos(\theta)+\pi)}{\sin(\theta+\cos(\theta)+\pi)} \\
= \; & \frac{\sin(\theta)\left[\sin(\cos(\theta))\cos(\pi)+\cos(\cos(\theta))\sin(\pi)\right]}{\sin(\theta+\cos(\theta))\cos(\pi)+\cos(\theta+\cos(\theta))\sin(\pi)} \\
= \; & \frac{-\sin(\theta)\sin(\cos(\theta))}{-\sin(\theta+\cos(\theta))} \\
= \; & \frac{\sin(\theta)\sin(\cos(\theta))}{\sin(\theta+\cos(\theta))}.
\end{aligned}$$

The proof for $\phi = \sin(\theta) + k$ follows similarly. ■

So we have found that the coordinates obtained for $\phi = \cos(\theta)$ and $\phi = \cos(\theta) + \pi$ are equal and thus our rotation has a period of π. Because of these characteristics, we can conclude that this relationship is biperiodic.

Negative values of k are valid in the biangular plane. The biangular relationship $\phi = \cos(\theta) - k$ transformed in the rectangular plane is the graph of $\phi = \cos(\theta) + k$ reflected over the x-axis (see figure 7).

A reflection characteristic like this occurs for rectangular relationships, but when we multiply by a negative number. For a biangular relation the reflection occurs by subtracting a number outside of the argument.

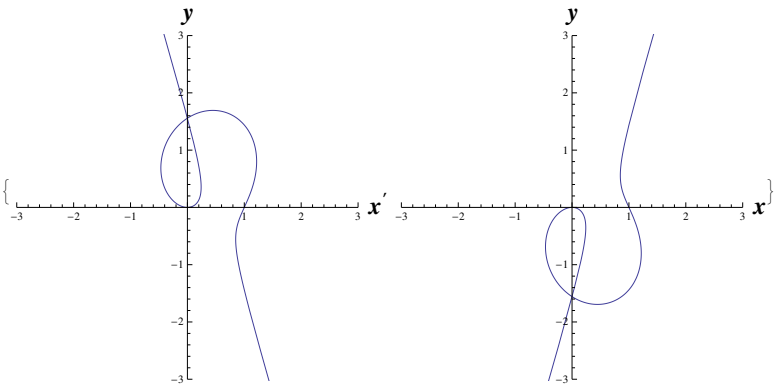

Figure 7

Adding a Constant within the Argument: $\phi = \cos(\theta + k)$

With rectangular relationships, adding a constant within the argument of the cosine function shifts the graph horizontally to the right or left by k units (right if k is negative, left if k is positive) (see figure 8).

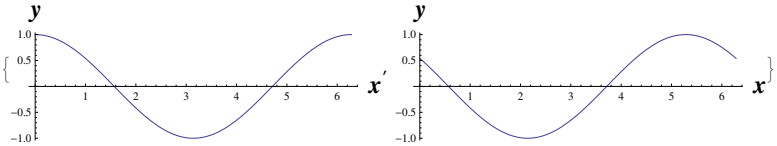

Figure 8

For biangular relationships, we do not observe a set horizontal shift such as this when we convert into the rectangular plane.

When we added a constant outside of the function, we observed that the central loop grew in size by rising upward and that the arms of the graph rotated in a clockwise direction. Now when we add the constant k within the argument of cosine, we see that the loop increases in size like before, but by going in a downward direction (see figure 9).

The values of $k = 0.1, 0.3, 0.5, 0.7, 0.9,$ and 1 again correspond to the A, B, C, D, E, and F graphs respectively. This figure displays the same periodicity of the rotation. When $k = \pi$, we again return to the original $\phi = \cos(\theta)$ graph. Adding a constant within the brackets also results in a biperiodic relationship. The characteristic for rectangular relationships that results in a horizontal shift, appears to translate into a rotation for biangular relationships.

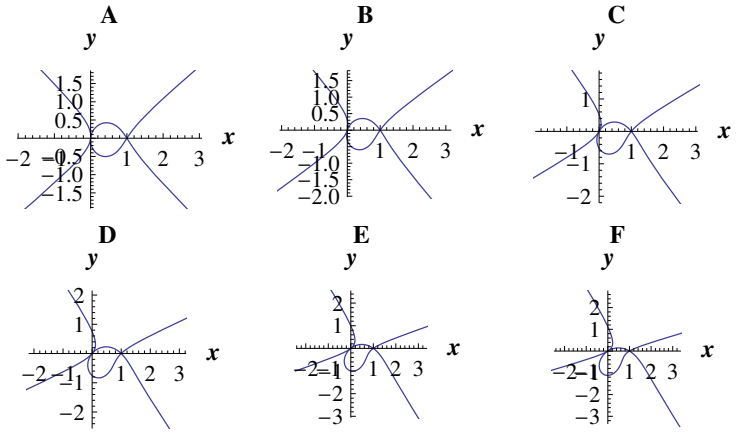

Figure 9

Multiplying by a Constant Within the Argument: $\phi = \cos(k\theta)$

We observe again another property of rectangular relationships before reconsidering it with biangular relationships. When the argument is multiplied by a constant k such that $|k| > 1$, we know that the period of $y = \cos(kx)$ is smaller than $y = \cos(x)$. In figure 10, we can observe that for $k = 2$ our new graph has a shorter period than when $k = 1$.

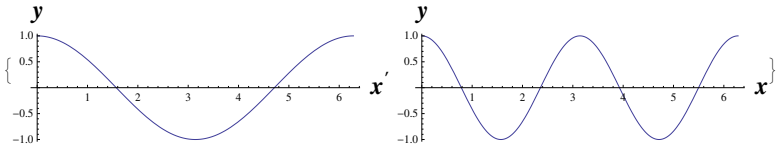

Figure 10

This trait does not occur for biangular relationships mapped into the Cartesian plane. When we multiply our argument by 2 we get the graph displayed in figure 11. Notice that we have lost our sweeping double arms and that our central loop is elongated and completely below the polar axis. This is akin to our other graphs that we have seen so far: we have only ever seen one loop in each graph. When $k = 3$, however, we notice that this property does not hold.

Multiplying by a constant of $k = 3$ has delivered six loops as opposed to the one loop to which we are accustomed. Note that some of our loops can not be completely viewed on our graph. The tips of these loops stretch out to infinity and one of the tips actually reappears on the opposite side of the graph. The resemblance between these two graphs is minimal. They

look like entirely different relationships. When we multiply by $k = 2$ and $k = 3$, ϕ fluctuates more as θ goes from 0 to 2π. ϕ alternates between increasing and decreasing more rapidly while θ is always increasing. Numerical evidence proposes that it is this alternation that results in the loops. A higher value of k means that there is more fluctuation for ϕ (i.e. shorter period), which results in more loops. As we continue to multiply by larger and larger constants an interesting quality arises, however. We plot the graphs for $k = 4$ and $k = 5$ below.

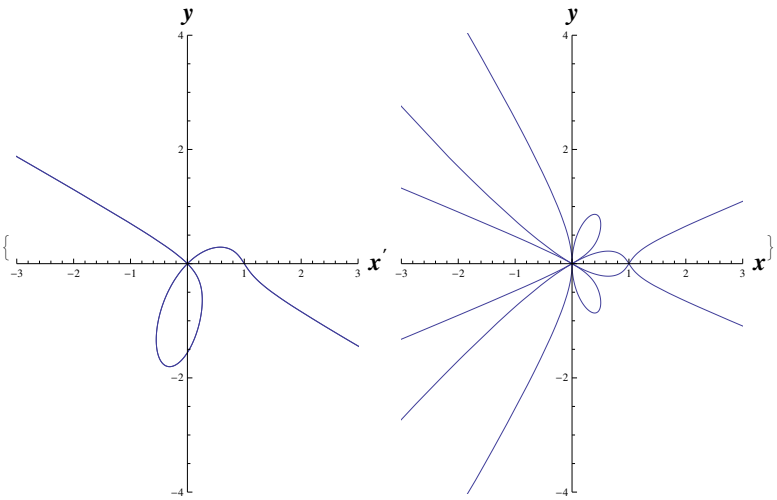

Figure 11

While we had six loops for $k = 3$, for $k = 4$ we have only three, but for $k = 5$ we have 10 loops (not all can be completely seen on the graph, see figure 12). This pattern continues as we try different values of k. Computational evidence suggests that for even integer values of k, the graph possesses $k - 1$ loops, but for odd values of k, our graph possesses $2k$ loops. Why is this the case? The difference lies in the overall periods of the graphs. Our original $\phi = \cos(\theta)$ relationship has a period of 2π and for the relationship $\phi = \cos(k\theta)$ with non-even integer values of k this quality stays the same. The relationships with even integer values of k, however, have a period of π. The proof of this is as follows.

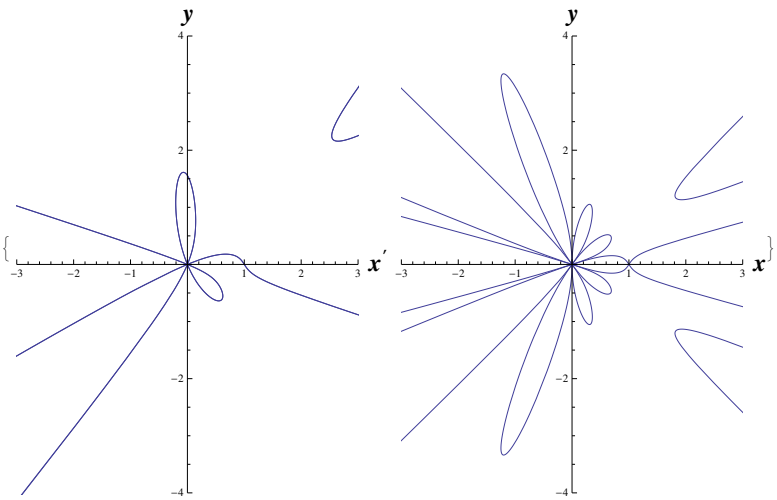

Figure 12

Theorem 2 *For even integer values of k, the biangular relation $\phi = \cos(k\theta)$ (or $\phi = \sin(k\theta)$) has a period of π. Other values of k (i.e. odd integers and non-integer values in $\mathbb{R}$) do not have a period of π.*

Proof. For $m, n \in \mathbb{Z}$, consider the following cases
Case 1: k is an even integer, $k = 2n$.
We must show that the coordinates for the relationships $\phi = \cos(2n\theta)$ and $\phi = \cos(2n(\theta + \pi m))$ are equal.
Consider the abscissa:

$$\begin{aligned} &\frac{\cos(\theta)\sin(\cos(2n\theta + \pi m))}{\sin(\theta + \cos(2n(\theta + \pi m)))} \\ &= \frac{\cos(\theta)\sin[\cos(2n\theta + 2n\pi m)]}{\sin[\theta + \cos(2n\theta + 2n\pi m)]} \\ &= \frac{\cos(\theta)\sin[\cos(2n\theta)\cos(2n\pi m) - \sin(2n\theta)\sin(2n\pi m)]}{\sin[\theta + \cos(2n\theta)\cos(2n\pi m) - \sin(2n\theta)\sin(2n\pi m)]} \\ &= \frac{\cos(\theta)\sin[\cos(2n\theta)(1) - \sin(2n\theta)(0)]}{\sin[\theta + \cos(2n\theta)(1) - \sin(2n\theta)(0)]} \\ &= \frac{\cos(\theta)\sin(\cos(2n\theta))}{\sin(\theta + \cos(2n\theta))} \end{aligned} \tag{1}$$

which is what we wished to show. The ordinate follows under a similar argument.

Case 2: k is not even and not an odd integer k, $k \neq 2n$ and $k \neq 2n+1$

By way of contradiction assume that the coordinates for the relationships $\phi = \cos(k\theta)$ and $\phi = \cos(k(\theta+\pi))$ are equal. That is, assume that the period for this value of k is π.
Then

$$\frac{\cos(\theta)\sin(\cos(k\theta))}{\sin(\theta+\cos(k\theta))} = \frac{\cos(\theta)\sin[\cos(k(\theta+\pi))]}{\sin[\theta+\cos(k(\theta+\pi))]}$$

But we know from a trigonometric identity that

$$\begin{aligned} &\frac{\cos(\theta)\sin[\cos(k(\theta+\pi))]}{\sin[\theta+\cos(k(\theta+\pi))]} \\ &= \frac{\cos(\theta)\sin[\cos(k\theta+k\pi)]}{\sin[\theta+\cos(k\theta+k\pi)]} \\ &= \frac{\cos(\theta)\sin[\cos(k\theta)\cos(k\pi)-\sin(k\theta)\sin(k\pi)]}{\sin[\theta+\cos(k\theta)\cos(k\pi)-\sin(k\theta)\sin(k\pi)]} \end{aligned} \tag{2}$$

and by assumption that $k \neq 2n\pi$. Thus, $\cos(k\theta) \neq 1$. Therefore,

$$\frac{\cos(\theta)\sin(\cos(k\theta))}{\sin(\theta+\cos(k\theta))} \neq \frac{\cos(\theta)\sin[\cos(k(\theta+\pi))]}{\sin[\theta+\cos(k(\theta+\pi))]}$$
$$\Rightarrow\Leftarrow$$

Therefore, for values of k that are not even or odd integers, the period can not be π. The period is actually 2π and can be easily shown.

Case 3: k is an odd integer

Again by contradiction assume that for this value of k, the period is π. Then,

$$\frac{\cos(\theta)\sin(\cos(k\theta))}{\sin(\theta+\cos(k\theta))} = \frac{\cos(\theta)\sin[\cos(k(\theta+\pi))]}{\sin[\theta+\cos(k(\theta+\pi))]}$$

but we know that

$$\begin{aligned} &\frac{\cos(\theta)\sin[\cos(k(\theta+\pi))]}{\sin[\theta+\cos(k(\theta+\pi))]} \\ &= \frac{\cos(\theta)\sin[\cos(k\theta+k\pi)]}{\sin[\theta+\cos(k\theta+k\pi)]} \\ &= \frac{\cos(\theta)\sin[\cos(k\theta)\cos(k\pi)-\sin(k\theta)\sin(k\pi)]}{\sin[\theta+\cos(k\theta)\cos(k\pi)-\sin(k\theta)\sin(k\pi)]} \\ &= \frac{\cos(\theta)\sin[\cos(k\theta)(-1)-\sin(k\theta)(0)]}{\sin[\theta+\cos(k\theta)(-1)-\sin(k\theta)(0)]} \\ &= \frac{-\cos(\theta)\sin(\cos(k\theta))}{\sin(\theta-\cos(k\theta))} \end{aligned} \tag{3}$$

Thus,

$$\frac{\cos(\theta)\sin(\cos(k\theta))}{\sin(\theta+\cos(k\theta))} \neq \frac{-\cos(\theta)\sin(\cos(k\theta))}{\sin(\theta-\cos(k\theta))}$$
$$\Rightarrow\Leftarrow$$

Therefore, for odd integer values of k the period can not be π. The period is actually 2π and can also be easily shown.
The proof for $\phi = \sin(k\theta)$ follows under a similar argument. ■

The shorter period results in a graph displaying fewer loops, $k-1$. This is quite an unusual characteristic.

Multiplying by a Constant Outside of the Function: $\phi = k\cos(\theta)$

When we multiply by k outside of the argument of the function, we see some unusual shapes for smaller levels of k. The larger our constant, k, becomes, however, the more arms and loops our graph possesses. Notice the difference between $k = 2$ and $k = 10$ in figure 13.

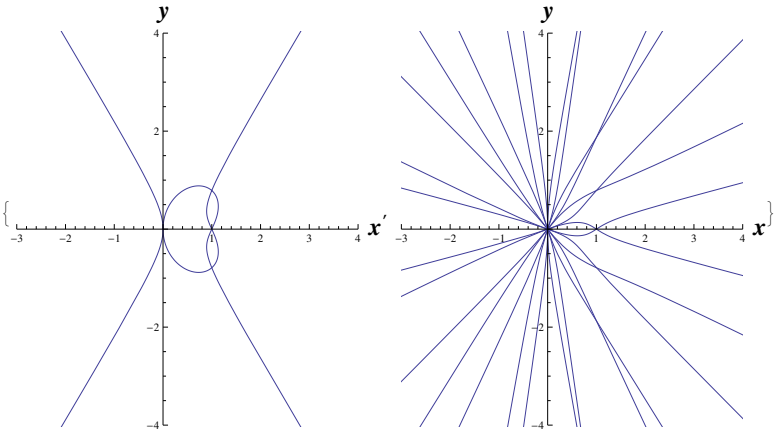

Figure 13

As the constant grows, more arms and loops develop. With so many features contained within one graph, it is challenging to observe a relationship between ϕ and θ. This results from the large difference in growth between ϕ and θ. When we multiply $\cos(\theta)$ by a large constant, ϕ increases at a greater rate than θ and it forms arms with such a steep slope that they leave our viewing screen rapidly (off to infinity) and then return on another side of the graph only to leave the viewing screen promptly again. With all of these arms, it is hard to keep track of how the relationship is progressing. Earlier, when we multiplied by a constant within the function, we did not get this result because the range of the cosine function

kept ϕ bounded between -1 and 1 no matter the magnitude of k. Thus, ϕ was not increasing or decreasing at such a great rate and the relationship was easier to observe on the graph.

An interesting feature occurs when we multiply by a negative constant. For rectangular trigonometric relationships, multiplication by a negative number results in the graph reflecting over the x-axis. For biangular coordinates, the graph of $\phi = -k\cos(\theta)$ is the same as $\phi = k\cos(\theta)$. The difference is where the graph starts when $\theta = 0$. As θ goes from 0 to 2π, the graph $\phi = \cos(\theta)$ starts at the loop and then draws the arms. The graph $\phi = -\cos(\theta)$, however, starts by drawing all of the arms and then finishes with the loop. In the end, if θ is allowed to fluctuate from 0 to 2π, the resulting graphs are the same, but the corresponding abscissa and ordinate points are not the same. We obtain the same graph, but not the same ordered pairs.

Raising the Relationship to a Power: $\phi = \cos(\theta)^k$

Raising $\cos(\theta)$ to a power enhances a characteristic between even and odd numbers that we saw in the previous section.

If we let $k = 2$ we get a graph that has a shape resembling half of our original $\phi = \cos(\theta)$ graph (see figure 14 left). When we let $k = 3$, however, the resulting graph returns to the bow-tie shape, but the loop is flatter and less circular (see figure 14 right).

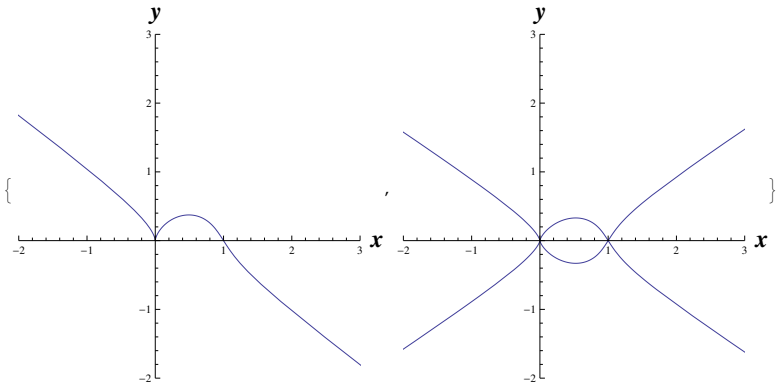

Figure 14

As was the reason in the previous section, the periods are different for these graphs depending on the parity of their power. Even integer powers have a period of π, while odd powers have a period of 2π. We can show this analytically.

Theorem 3 *When k is an even integer, the biangular relation $\phi = \cos(\theta)^k$ (or $\phi = \sin(\theta)^k$) has a period of π.*

Proof. For $n \in \mathbb{Z}$, let k be an even integer, $k = 2n$.
We must show that the coordinates for the relationships $\phi = \cos(\theta)^k$ and $\phi = \cos(\theta + \pi)^k$ are equal. Consider the abscissa for $\phi = \cos(\theta + \pi)^k$

$$\begin{aligned}
&\frac{\sin(\phi)\cos((\cos(\theta+\pi))^{2n})}{\sin(\phi+(\cos(\theta+\pi))^{2n})} \qquad (4)\\
&= \frac{\sin(\phi)\cos((\cos^2(\theta+\pi))^{n})}{\sin(\phi+(\cos^2(\theta+\pi))^{n})}\\
&= \frac{\sin(\phi)\cos((\frac{1+\cos(2\theta+2\pi)}{2})^{n})}{\sin(\phi+(\frac{1+\cos(2\theta+2\pi)}{2})^{n})}\\
&= \frac{\sin(\phi)\cos((\frac{1+\cos(2\theta)\cos(2\pi)-\sin(2\theta)sin(2\pi)}{2})^{n})}{\sin(\phi+(\frac{1+\cos(2\theta)\cos(2\pi)-\sin(2\theta)sin(2\pi)}{2})^{n})}\\
&= \frac{\sin(\phi)\cos((\frac{1+\cos(2\theta)(1)-\sin(2\theta)(0)}{2})^{n})}{\sin(\phi+(\frac{1+\cos(2\theta)(1)-\sin(2\theta)(0)}{2})^{n})}\\
&= \frac{\sin(\phi)\cos((\frac{1+\cos(2\theta)}{2})^{n})}{\sin(\phi+(\frac{1+\cos(2\theta)}{2})^{n})}\\
&= \frac{\sin(\phi)\cos((\cos^2(2\theta))^{n})}{\sin(\phi+(\cos^2(2\theta))^{n})}\\
&= \frac{\sin(\phi)\cos((\cos(2\theta))^{2n})}{\sin(\phi+(\cos(2\theta))^{2n})}
\end{aligned}$$

by applying the trigonometric identity for the square of cosine. This is what we wished to show. The ordinate of this relationship follows similarly.
It is easy to show that for $k \neq 2n$ the period is 2π. ■

But how does half the graph just disappear with a change of power? When k progresses from 1 to 2, the arms from quadrants I and III curl in and form a circle below the polar axis. As k gets closer to 2, this circle becomes smaller and moves closer to the polar axis, where it eventually crosses the polar axis and forms a circle above the axes.

This second circle grows until it reaches the next integer power (in this case $k = 2$) at which point the period changes and we end up with half of the bow-tie shape (see figure 15).

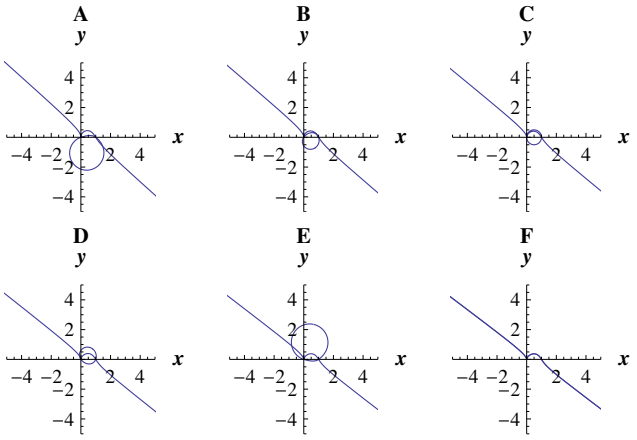

Figure 15

Converting the biangular relation $\phi = \cos(\theta)$ into the Cartesian plane reveals new and familiar characteristics. There is an interesting trait with the parity of multipliers and powers. When we added a constant k, we saw that the graph possessed two periods. One period is for the overall graph (which results from the rotation of the angles) and another for the rotation that occurs when we add k. We saw that the reflection property still exists when we convert the biangular relationships, but it results from the sign of an added constant outside of the argument, not the sign of the multiplier. The sign of the multiplier for biangular relationships changes the ordered pairs (ϕ, θ), but not the overall shape of the graph. We see some commonalities between the two systems but also different ways of executing them.

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A Mathematical Study of the Game of Sprouts and Similar Games

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1. Introduction

The game of Sprouts was created in the 1960s by John Horton Conway and Michael Stewart Paterson [6, p. 71]. It is a two-player, pencil-and-paper game in which the last player capable of making a move is the winner.

In this paper not only will the game of Sprouts be discussed, but also some similar games. The games are simple to play; however, they offer some interesting mathematical questions and require a more difficult analysis in the mathematics behind them.

2. Sprouts

Rules of the Game

Each game of Sprouts begins with n dots, $n \geq 1$. Each move is made by:

- drawing a path either connecting two dots or looping back to the same dot and
- drawing a new dot on the newly drawn path.

Additional rules are:

- paths may not cross;
- each dot can have a maximum of three edges; and
- the last player that can accomplish a full move wins the game [1, p.598].

As an example, consider the two-dot game in Figure 1 that will be used later as well. The two initial dots are labeled I and II. The newest dot to be added is white-filled, whereas the previously added dots are gray-filled.

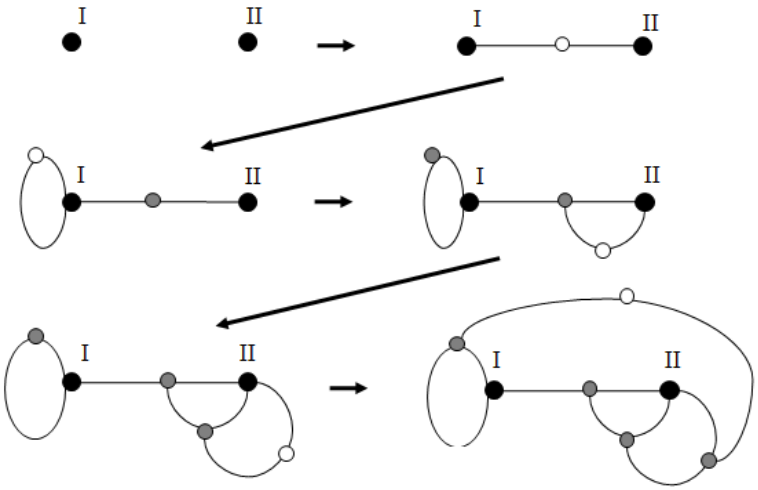

Figure 1

Terminology

We now define some terms that will be used throughout this paper. Many of the terms are the same as those found in graph theory.

Each game can be represented by a graph where each dot is a vertex of the graph. The edges of the graph come from the paths, where each path generates two edges. In the graphical representation of the game of Sprouts, each vertex can have a maximum of three edges.

The degree of a dot has the same definition as the degree of a vertex in graph theory; that is, the degree is the number of edges emanating from the dot. A dead dot is a dot that has the maximum possible degree as allowed in the rules. In Sprouts, a dot is dead when it has degree three. If a dot has a degree less than three, then it is a live dot.

We say that each dot on the graph has three doors. An open door is a possible place for an edge to intersect a dot. A closed door denotes a place where an edge already intersects a dot. For instance, a live dot has at least one open door, and a dead dot has exactly three closed doors.

A dead game is a game that has no possible moves left. The length of a game is the number of moves played to reach a dead game.

A dot can interact with a dot (including itself) if there is a possible move connecting the dots. If there is a live dot that cannot interact with any dot, then this dot has been isolated.

The term face is used as in graph theory. That is, paths border faces. Note that the exterior region of the graph is also a face. To clarify what is meant by a face of the graph, Figure 2 shows the final stage of the game from Figure 1 with its five faces numbered.

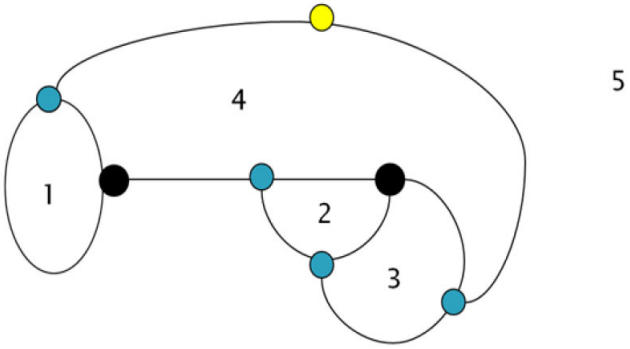

Figure 2

One-Dot Game

We begin by analyzing the one-dot game. The only possible first move is to loop from the initial dot back to itself and draw a new dot on that loop as in Figure 3. Note that each dot at this stage is of degree two.

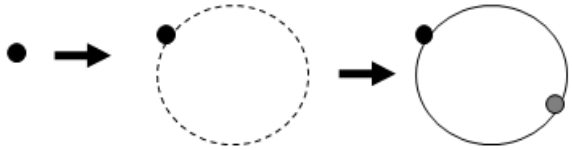

Figure 3

The only possible remaining move is to connect these two dots with a path. However, this very short and simple move raises an important question: Is a path drawn inside a region the same as a path drawn outside a region? These two possibilities are shown in Figure 4. The issue of equivalent moves and games will be addressed later.

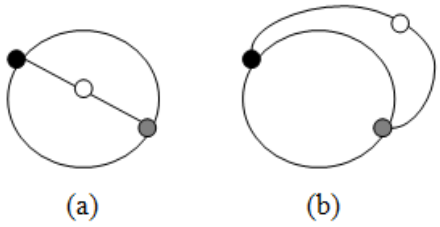

Figure 4

In this case, the second path being drawn inside or outside does not affect the outcome of the game. This is because either way there is only one dot remaining that is not dead, and this dot is already of degree two. Thus, there are no more moves possible in either situation.

Two-Dot Game

Let us consider a two-dot game to further explore paths being drawn inside or outside of regions. Let us play the games shown in Figure 5.

As in the game in Figure 1, the initial dots are labeled I and II. With each move, the new dot is shown in white and numbered as to reflect in which move the dot appeared. After the third move, we reach an interesting dilemma in the game. If one decides to connect dot 3 with dot II, one could do this in two ways. One way is to draw the line between the two paths connecting dot 1 and dot II. The second way is to draw the line outside of these two paths as shown.

Using the first method, the game ends with the newly added dot (dot 4). By playing inside a face, no opportunity is left for the remaining live dots (dots 2 and 4) to interact with each other. In other words, they are isolated from opportunities to further the length of game.

Using the second method, there is one more possible move left—to connect dot 4 with dot 2 and to add dot 5 on this path. The exact same dots are connected as in the first method, but by playing outside, the newly added dot 4 is able to interact with another live dot, namely dot 2.

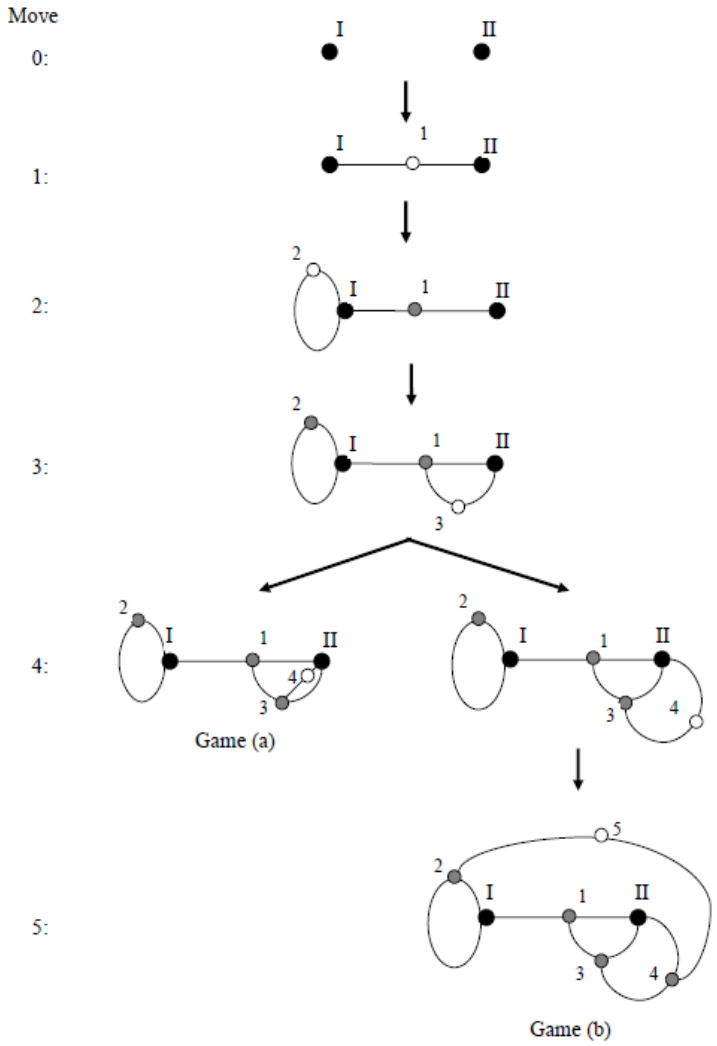

Figure 5

Game (a) ends after 4 moves because the 4th path was drawn inside a region. If the path is drawn outside of this region, as in Game (b), then there is still one more move available as pictured. The original dots are the black ones labeled I and II; the newly-added dot is white, and the previously-added dots are gray for each step.

Equivalent Games

Let's consider again the question of equivalent moves and games. Notice from the examples in Figure 5 that graph isomorphism is not a sufficient condition to show whether two games are equivalent or not. At move 4, both examples are isomorphic, but one noteworthy difference is the length of the games. With that alone, we can call these non-equivalent games. What if, however, there was no such obvious difference as the length of the game? How could one distinguish between moves?

Since the literature on Sprouts gives little explanation on game equivalence we will define our own standards. Two games will be considered equivalent if at some stage one graph can be transformed into another graph by:

- stretching, compressing, or bending the edges without touching other edges or dots,
- rotating the graph, and
- reflecting the graph over a line of symmetry.

This is analogous to continuous deformations in algebraic topology.

Maximum Length of Game

In the one-dot game, the maximum and minimum lengths of games are equal in value. No matter how the game is played, there are exactly 2 moves. That is, the second player always wins.

The two-dot game, however, is not so simple because there are many different outcomes. Let us consider a game in which no live dots will be isolated. Hence, at the end of the game, every dot will be dead except one, which will have only one open door available. Now let us examine such a game move by move.

At the beginning of the two-dot game, which will be referred to as the zeroth (0th) move, there are 6 doors total—none are closed. In the first move, the path drawn closes two of the existing doors. Completing the move creates a new dot with three doors, two of which are automatically closed. There are now nine doors total—four are closed and five are open.

Each move thereafter will follow the same pattern. Thus, in general, each move closes two doors and creates a new dot with three new doors. Two of those will be automatically closed, and one will be left open. This leaves us with one less open door with each move. For a summary of each move, see Table 1. Note that by the end of the fifth move, there is only one open door. Therefore, there are no more possible moves. So the

maximum length of the two-dot game is 5. As an example of a two-dot game achieving a length of 5 moves, one can look back at Figure 5 (b).

move	total dots	total doors	closed doors	open doors
0	2	6	0	6
1	3	9	4	5
2	4	12	8	4
3	5	15	12	3
4	6	18	16	2
5	7	21	20	1

Table 1

Table 2 summarizes the maximum length of the three-dot game and shows that the maximum length is 8 moves using similar reasoning as with the two-dot game.

move	total dots	total openings	closed openings	available openings
0	3	9	0	9
1	4	12	4	8
2	5	15	8	7
3	6	18	12	6
4	7	21	16	5
5	8	24	20	4
6	9	27	24	3
7	10	30	28	2
8	11	33	32	1

Table 2

Figure 6 gives an example of the final stage of a three-dot game lasting for exactly 8 moves. Again, the black dots are the initial dots; the gray dots are the intermediate dots; and the white dot is the last dot added with only two edges.

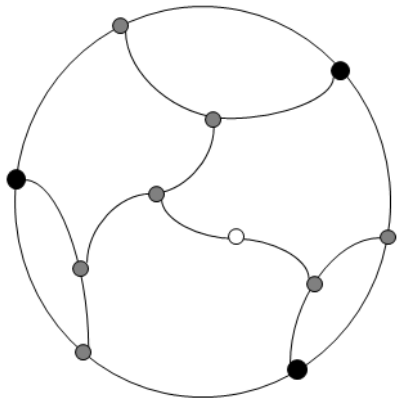

Figure 6

In general, the multi-dot game follows the same pattern. A game with n original dots begins with $3n$ open doors. Each move reduces the total number of open doors by one. Since the game of maximum length ends when there is exactly one opening (from the last drawn dot), we conclude as in [1, p. 598; 5, p. 116] that there must be a maximum of $3n - 1$ moves for an n-dot game. It is therefore justified that:

Theorem 4 *(Maximum Length) For an n-dot game of Sprouts, the maximum length of a game is $3n - 1$ moves.*

To show that a game of $3n - 1$ moves exists, one can use induction to prove:

Theorem 5 *(Existence) For an n-dot game of Sprouts, there exists a game of length $3n - 1$ moves.*

For this proof, a similar technique is used as that found in [2, p. 479].

Proof. For the base case of a 1-dot game, consider Figure 4 (b) where the final open door is on a dot that borders the exterior face. The total number of moves is $3(1) - 1 = 2$.

For clarity, consider the 2-dot game. Separate the game into two 1-dot games played as described within the base case. This gives a total of four moves. The remaining two live dots both border the exterior face and can, therefore, be connected with a new path. This gives $3(2) - 1 = 5$ total moves and is demonstrated in Figure 7 below.

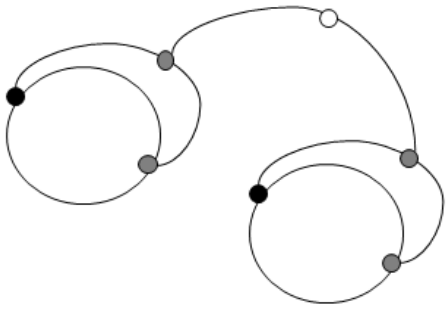

Figure 7

For the inductive step, assume that with k initial dots there is a game of length $3k - 1$ where the remaining live dot borders the exterior face. Now consider a game with $k + 1$ initial dots. This can be separated into the k-dot game with the assumed length and the 1-dot game as in the base case. The total number of moves at this point is $(3k - 1) + 2 = 3k + 1$. Note that there are exactly two live dots which both border the exterior face. Thus, a path can be drawn between them which gives a total of $3k - 1 + 3(1) - 1 + 1 = 3(k + 1) - 1$ moves. ■

Minimum Length of Game

The literature [1, p. 598; 2, p. 478] finds that the minimum number of moves for an n-dot game is actually $2n$. The proofs of this are very detailed, and what is shown here still remains in agreement with the literature; however, our theorem is not as specific.

One can learn about the minimum number of moves in an n-dot game using Euler's characteristic equation. Euler's characteristic equation states that $v + f - e = 2$, where $v \equiv$ number of vertices, $f \equiv$ number of faces, $e \equiv$ number of edges [3, p. 231]. Utilizing this, a claim similar to that made in [2, p. 478] is made:

Theorem 6 *(Minimum Length): For an n-dot game of Sprouts, the number of moves is at least $2n - 1$.*

Proof. Recall that for each move, a path is drawn and then split into two edges by the newly added dot. Since each move generates two edges, the total number of edges in a dead game is given by: $e = 2m$, where m is the number of moves.

Each game begins with n dots. An additional dot is created for each move. Thus, the total number of dots (vertices in a graphical representation) is given by: $v = n + m$.

The lower bound for the number of faces can now be found. First note that in any game of Sprouts, there are $3n - m$ open doors at the end of the game. This is because there are $3n$ open doors at the beginning of any game, and as explained previously, each move reduces the total number of open doors by one. With m moves, $3n$ is reduced by m.

Now, since there are $3n - m$ open doors at the end of the game, there are $3n - m$ live dots at the end of the game. This is because no live dot in a dead game can have more than one open door, for if a dot had two open doors, an additional move could be made drawing a loop from the dot back to itself.

Furthermore, note that in a dead game no face can have more than one live dot along its border. Otherwise, the game is unfinished because the dots can still interact with one another. Clearly each live dot will border at least one face. Thus, there are at least $3n - m$ faces at the end of a game.

Plugging in the equivalencies for the number of edges and vertices and the lower bound for the number of faces into Euler's characteristic equation, we have:

$$\begin{aligned} & e = f + v - 2 \\ \Longrightarrow \quad & 2m = f + (n + m) - 2 \\ \Longrightarrow \quad & 2m \geq (3n - m) + (n + m) - 2 \\ \Longrightarrow \quad & 2m \geq 4n - 2 \\ \Longrightarrow \quad & m \geq 2n - 1. \end{aligned}$$

This gives that the lower bound for an n-dot game is $2n - 1$. ■

As previously noted, the literature shows that the actual minimum length of the game of Sprouts is $2n$. To show that a game of length $2n$ exists, simply play the one-dot game in Figure 4 (a) with each initial dot.

3. Brussels Sprouts

Rules of the Game

Brussels Sprouts [4, p. 489] is a variation of Sprouts that begins with n, $n \geq 1$, crosses (each drawn as an X), where each cross is a vertex with four arms protruding from it. An arm with no path drawn from or to it will be called open. Otherwise, it is closed. The first move is made by drawing a path from one open arm to another open arm and then drawing a stripe on this path. Each stripe must be such that exactly one new open arm is on either side of the path it crosses. All subsequent moves are similarly done with the restriction that paths may not cross. The game ends when there are no possible moves remaining.

As with Sprouts, each game can be represented by a graph where each cross (be it from an initial cross or a path-stripe intersection) is a vertex of the graph. The edges of the graph originate from the paths, where each path generates two edges. In the graphical representation of the game of Brussels Sprouts, each vertex can have a maximum of four edges, one per arm. Once a vertex has four edges it is dead. The last player that can accomplish a full move wins the game.

As an example, an $n = 2$ game is shown in Figure 8. The two initial crosses are the large, bold crosses, I and II. Each new cross is numbered to correspond with the move from which it was created. Some interesting things to note are:

- unlike Sprouts, a dead game of Brussels Sprouts must be a connected graph;
- each move closes two arms and opens two arms; and
- each face in a dead game has exactly one open arm within it.

These three statements hold true for any game of Brussels Sprouts and will help us find the length of an n-cross game.

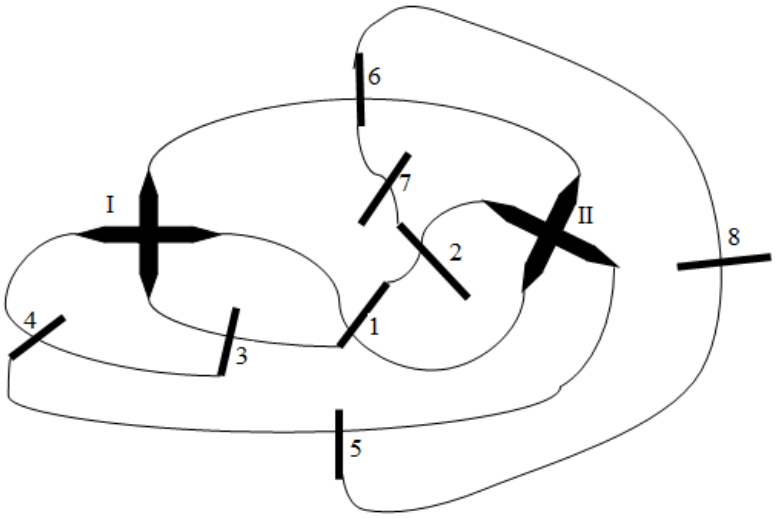

Figure 8

Length of Game

As with Sprouts, Euler's characteristic equation can be used. With n original crosses and m moves one can find expressions for the number of vertices, v; number of faces, f ; and number of edges, e, for a game of Brussels Sprouts.

Similar to the reasoning for Sprouts, a game of Brussels Sprouts beginning with n crosses and lasting for m moves will have $e = 2m$ edges and $v = n + m$ vertices.

Now, one only needs to find an expression for the number of faces in a dead game. Since each move draws a path, with a stripe then drawn across that path, each face that borders this new path will have a new open arm in it. Note that in a dead game, it is not possible for there to be a face with two open arms inside of it. Otherwise, these two arms could be connected and crossed.

Since there is exactly one open arm in any face within a dead game, the number of faces in the end is the same as the number of open arms. The question then becomes: How many open arms are there in a dead game?

At the beginning of any game of Brussels Sprouts, there are n crosses and thus $4n$ open arms. Each move will close two arms and create two new, open arms. Thus the number of open arms stays constant throughout the game. Hence, a dead, n-cross game will have $4n$ open arms. Since the total number of faces in a dead game is equal to the number of open arms at any point in the game, $f =$ the number of open arms $= 4n$.

Finally, these n- and m-dependent expressions can be substituted into Euler's characteristic equation.

$$\begin{aligned} & v + f - e = 2 \\ \Longrightarrow \quad & (n + m) + (4n) - (2m) = 2 \\ \Longrightarrow \quad & 5n - m = 2 \\ \Longrightarrow \quad & 5n - 2 = m \end{aligned}$$

Therefore, the following, which is consistent with [4, p. 491], has been shown.

Theorem 7 *The number of moves for any n-cross game of Brussels Sprouts is $5n - 2$.*

4. Stars and Stripes

Rules of the Game

Stars and Stripes [1, p. 603] is played similarly to Brussels Sprouts. The difference is that where Brussels Sprouts begins solely with crosses, Stars and Stripes begins with N different types of stars. A star is a vertex with some number of arms projecting from it. Stars are the same type if and only if they have the same number of arms. Additionally, there are n_i stars each with k_i arms for $i = 1, 2, \ldots, N$. As in Brussels Sprouts, moves are made by drawing a path between any two open arms and drawing a stripe across the path.

Length of Game

Euler's characteristic equation will be used again to find the number of moves for a game of Stars and Stripes.

Theorem 8 *For a game of Stars and Stripes, the total number of moves for N total types of stars is given by*

$$\sum_{i=1}^{N} \left[(k_i + 1)\, n_i\right] - 2,$$

where k_i is the number of arms and n_i is the number of vertices for type i of the star ($i = 1, 2, ..., N$).

Proof. We will first consider the number of edges. Each newly created edge will be split in two by each new cross. Thus, each move will generate two edges. Since there are initially no edges in the game, this means that $e = 2m$. Concerning the number of vertices, the same logic as in Brussels Sprouts can be used to find that each completed game will have

$$n_1 + n_2 + \cdots + n_N + m = \left(\sum_{i=1}^{N} n_i\right) + m$$

vertices. Again, finding the number of faces is done similarly as in Brussels Sprouts. In Stars and Stripes, the game begins with $k_i n_i$ arms for each type of star. Thus, there is a total of $\sum_{i=1}^{N} k_i n_i$ arms at the beginning of the game. Using the same reasoning as in Brussels Sprouts, in a dead game of Stars and Stripes, each face will have one and only one open arm. This means that there are $\sum_{i=1}^{N} k_i n_i$ faces in a dead game.

We can now substitute the values for v, f, and e into Euler's characteristic equation $v + f - e = 2$, obtaining

$$\left[\left(\sum_{i=1}^{N} n_i\right) + m\right] + \sum_{i=1}^{N} k_i n_i - 2m = 2,$$

and thus

$$\sum_{i=1}^{N} \left[\left(k_i + 1\right) n_i\right] - 2 = m,$$

as claimed. ■

With a quick calculation, one can find that Brussels Sprouts is actually a special case of Stars and Stripes where there is only one type of star ($N = 1$) which has four arms projecting from it ($k = 4$). This gives us Theorem 4 again.

5. Final Comments

In this discussion of the game of Sprouts, Brussels Sprouts, and Stars and Stripes, we found the following:

- Theorem 1 (Maximum Length): For an n-dot game of Sprouts, the maximum length of a game is $3n - 1$ moves.
- Theorem 2 (Existence) For an n-dot game of Sprouts, there exists a game of length $3n - 1$ moves.
- Theorem 3 (Minimum Length): For an n-dot game of Sprouts, the number of moves is at least $2n - 1$.
- Theorem 4: The number of moves for any n-cross game of Brussels Sprouts is $5n - 2$.
- Theorem 5: For a game of Stars and Stripes, the total number of moves for N total types of stars is given by

  $$\sum_{i=1}^{N} \left[\left(k_i + 1\right) n_i\right] - 2,$$

  where k_i is the number of arms and n_i is the number of vertices for type i of the star ($i = 1, 2, ..., N$).

These results were arrived at independent of the literature, but were later compared to existing results. The only difference is the aforementioned Theorem 3 where the literature proves a better estimate of $2n$.

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An Application of Semigroup Theory to the Sierpinski Triangle

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1. Introduction

In this paper we will discuss some of our research pertaining to semigroup theory, with an emphasis on the application of semigroup theory that we explored: the partial symmetries of the Sierpinski triangle. Prior to delving into the intricacies of this application, we will define basic vocabulary and provide supporting examples. The examples included were put forth by Christopher Hollings' in his article titled "Some First Tantalizing Steps into Semigroup Theory," which served as one of our primary sources for research. From there, we will shift the focus of the article to the application of the partial symmetries of the Sierpinski triangle. We will begin our discussion by introducing the original notation and formulas we developed for the partial symmetries of the Sierpinski triangle. From there we will prove that the set of partial symmetries of the Sierpinski triangle is a subsemigroup and a semigroup.

2. Semigroup Definition and Examples

Definition 1 Let $\mathcal{S}$ be a nonempty set and let $*$ be an operation on $\mathcal{S}$. The pair $(\mathcal{S}, *)$ is a *semigroup* if $*$ is associative.

Let's look at examples that satisfy this definition as a means of familiarizing ourselves with the structure of a semigroup.

Example 1 Let $\mathbb{N}$ denote the set of positive integers and $+$ ordinary addition. Then $(\mathbb{N}, +)$ forms a semigroup because $+$ is an associative operation.

Example 2 Let $A \times B$ be the Cartesian product of two nonempty sets A and B. Define an operation $*$ on $A \times B$ by $(a, b) * (c, d) = (a, d)$. The pair, consisting of the set $A \times B$, together with the operation $*$, is called as a *rectangular band*. It forms a semigroup.

Proof. Let $A \times B$ be a rectangular band. Suppose (a, b), (c, d), and (e, f) are fixed but arbitrarily chosen elements of $A \times B$. Then

$$\begin{aligned}[(a, b) * (c, d)] * (e, f) &= (a, d) * (e, f) \\ &= (a, f)\end{aligned}$$

and

$$\begin{aligned}(a, b) * [(c, d) * (e, f)] &= (a, b) * (c, f) \\ &= (a, f)\,.\end{aligned}$$

Thus, $*$ is associative and $A \times B$ is a semigroup. ■

3. Subsemigroup Definition and Examples

In set theory, subsets are an important structure, because they allow one to forge "relationships" between sets [5]. In semigroup theory, the structure that is analogous to that of a subset is called a subsemigroup.

Definition 2 A non-empty subset T of a semigroup S is called a *subsemigroup* if it forms a semigroup under the same operation as S.

Let us now consider examples of a subsemigroup.

In the example that follows, let E^+ be the set of positive even integers.

Example 3 *$(E^+, +)$ is a subsemigroup of $(\mathbb{N}, +)$.* First, $E^+ \subseteq \mathbb{N}$ and E^+ is closed under $+$. Further, $(E^+, +)$ inherits associativity from $\mathbb{N}$. Hence, E^+ is a semigroup under $+$. Therefore, $(E^+, +)$ is a subsemigroup of $(\mathbb{N}, +)$.

Example 4 *A semigroup S is a subsemigroup of itself.*

Proof. Assume we are given an arbitrary semigroup S with operation $*$. Since every group is a subset of itself, $S \subseteq S$. Suppose that a, b, and c are fixed but arbitrarily chosen elements of S. Then,

$$\begin{aligned}(a * b) * c &= a * b * c \\ a * (b * c) &= a * b * c\end{aligned}$$

Thus, $*$ is associative. Since a, b, and c were arbitrarily chosen elements of S, $*$ is associative for all a, b, and c in S. Therefore, a semigroup S is always a subsemigroup of itself. ■

4. Application: The Partial Symmetries of the Sierpinski Triangle

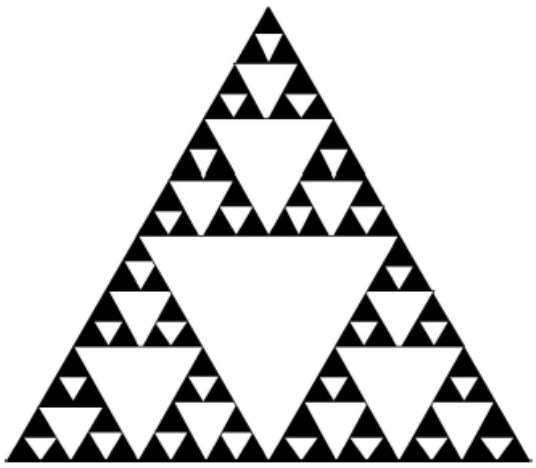

Figure 1

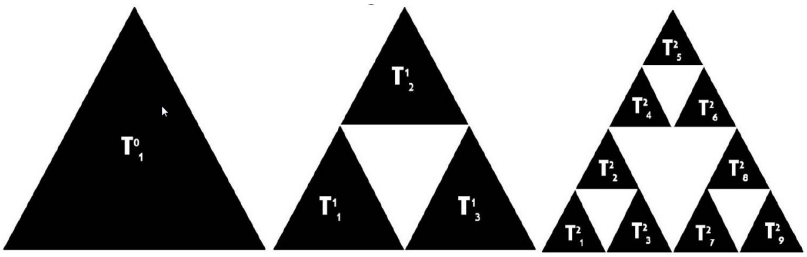

Figure 2

The Sierpinski triangle, also referred to as the Sierpinski gasket, is the fractal that results from repeatedly subdividing and deleting portions of an equilateral triangle at infinitum (refer to Figure 1). Before we can investigate the Sierpinski triangle through the lens of semigroup theory, we must first discuss one area of semigroup theory that has not yet been included in this document: the study of partial symmetries.

A *symmetry* is a transformation that preserves the shape and contents of a structure. For example, consider two structures: an equilateral triangle and the Sierpinski triangle. There are six symmetries of an equilateral triangle: (1) the identity; (2) the rotation by 120° about the center of the triangle; (3) the rotation by 240° about the center of the triangle; and (4-6) the reflections about the altitudes. (Refer to Figure 3 above for a depiction

of three of the aforementioned symmetries.) Even though an equilateral triangle and the Sierpinski triangle are quite different structures, the group of symmetries of the Sierpinski triangle is identical to that of an equilateral triangle [2].

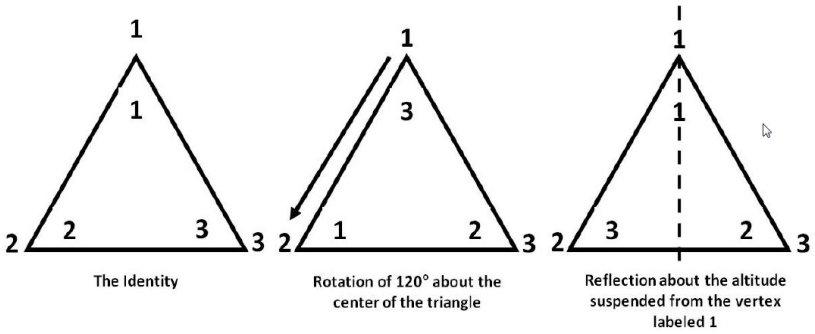

Figure 3

Semigroup theory can be used to differentiate between the Sierpinski triangle and an equilateral triangle. In what follows, we will first introduce a new class of transformations called partial symmetries. Then, we will show that a subset R of the set of partial symmetries is closed and is a semigroup. It is important to note that the notation and formulas derived in this section are novel. In addition, the proofs in the latter part of this document are original.

Partial Symmetries

Definition 3 A *partial symmetry* on a set $\mathcal{S}$ is a one-to-one mapping from one subset of $\mathcal{S}$ to another.

An equilateral triangle has a finite number of partial symmetries. As we will see, the Sierpinski triangle has infinitely many partial symmetries. The operation that is associated with partial symmetries is composition, $\circ$. One important note is that when we write $\mathcal{S} \circ \mathcal{T}$ throughout this paper, we mean first apply $\mathcal{S}$, then apply $\mathcal{T}$ (which is the reverse of what $\mathcal{S} \circ \mathcal{T}$ often means). We use this notation because it is consistent with Hollings' paper. In general, the composition $\mathcal{S} \circ \mathcal{T}$ of two partial symmetries $\mathcal{S}$ and $\mathcal{T}$, is defined only if the intersection of the range of $\mathcal{S}$ and the domain of $\mathcal{T}$ does not equal the empty set. If the intersection is empty, then the composition is the empty transformation $\emptyset_R$.

Partial Symmetries of the Sierpinski Triangle

In our analysis of the Sierpinski triangle, we first developed notation for the inner triangles. In general, each inner triangle is labeled as T_k^n where n is a nonnegative integer that represents the iterate being investigated and k is a positive integer that is less than or equal to 3^n. The triangles were labeled from left to right starting in the bottom left corner of the structure in a clockwise manner (see Figure 2). With this notation of the triangles established, each iterate can be constructed by taking the union of the inner triangles that results after subdividing and deleting the middle portion(s) from the previous iterate. Thus, we will label the nth iterate G_n, where

$$G_n = \bigcup_{k=1}^{3^n} T_k^n.$$

Using this notation for the iterates, the 0th iterate, which consists of the equilateral triangle, will be denoted G_0, while the first iterate, which consists of the equilateral triangle with the middle triangle deleted, will be denoted G_1. With the notation for the inner triangles and the notation for the iterates established, the following notation for the Sierpinski triangle, S, holds:

$$S = \bigcap_{n=1}^{\infty} G_n = \bigcap_{n=1}^{\infty} \left(\bigcup_{k=1}^{3^n} T_k^n \right).$$

This notation helped us consider three types of partial symmetries: (1) rescaling transformations, (2) translational symmetries, and (3) combinations of both (1) and (2). Rescaling transformations are mappings that take a triangle within the structure and either enlarge it or shrink it to a different iterate. For example, in Figure 2, the mapping $T_1^2 \rightarrow T_1^1$ is the rescaling transformation that enlarges T_1^2 into T_1^1. Translation symmetries are mappings that take a triangle within the structure and move it to a different part of the structure. For instance, in Figure 2, the mapping $T_1^1 \rightarrow T_2^1$ is the translational symmetry that moves T_1^1 to T_2^1. There also exist partial symmetries that are combinations of both of these that essentially take a piece of the structure and move it to another piece and then enlarge it or shrink it, or vice versa. An example of this type of partial symmetry, is the mapping $T_1^2 \rightarrow T_2^1$ which enlarges T_1^2 and moves it to T_2^1.

We then constructed the set ***R*** that was inclusive of all of these partial symmetries:

$$R = \left\{ T_k^n \to T_l^m \ : \begin{array}{l} n \in \mathbb{N} \cup \{0\} \\ m \in \mathbb{N} \cup \{0\} \\ k \in \mathbb{N} \text{ and } k \leq 3^n \\ l \in \mathbb{N} \text{ and } l \leq 3^m \end{array} \right\} \cup \{\emptyset_R\}$$

where $\emptyset_R$ denotes the empty transformation and T_k^n and T_l^m are arbitrary triangles that lie within the Sierpinski triangle. It is important to note that R contains infinitely many elements.

As previously stated, the operation associated with the set R is composition. Let's look at two examples in which we compose two elements of R.

Example 5 The composition of $T_1^0 \to T_1^1$ and $T_1^1 \to T_1^2$ is as follows:

$$(T_1^0 \to T_1^1) \circ (T_1^1 \to T_1^2) = (T_1^0 \to T_1^2)$$

Example 6 The composition of $T_1^0 \to T_1^2$ and $T_1^1 \to T_2^2$ is as follows:

$$(T_1^0 \to T_1^2) \circ (T_1^1 \to T_2^2) = (T_1^0 \to T_1^2) \circ (T_1^2 \to T_4^3) = (T_1^0 \to T_4^3)$$

Supplementary Note: In example 6, since the range of $T_1^0 \to T_1^2$ was a subset of the domain of $T_1^1 \to T_2^2$, we restricted the domain of $T_1^1 \to T_2^2$, namely T_1^1, to the range of $T_1^0 \to T_1^2$, namely T_1^2. Further, since we restricted the domain of $T_1^1 \to T_2^2$ to T_1^2, we needed to restrict the range of $T_1^1 \to T_2^2$ to be inclusive of only the image of T_1^2, namely T_4^3. Thus, in the compositions, we replaced the mapping $T_1^1 \to T_2^2$ with $T_1^2 \to T_4^3$. Once this substitution was made the composition was straightforward.

Example 7 The composition of $T_1^0 \to T_3^1$ and $T_9^2 \to T_{27}^3$ is as follows:

$$(T_1^0 \to T_3^1) \circ (T_9^2 \to T_{27}^3) = (T_3^1 \to T_9^2) \circ (T_9^2 \to T_{27}^3) = (T_3^1 \to T_{27}^3)$$

Supplementary Note: In example 7, since the domain of $T_9^2 \to T_{27}^3$ was a subset of the range of $T_1^0 \to T_3^1$, we restricted the range of $T_1^0 \to T_3^1$, namely T_3^1, to the domain of $T_9^2 \to T_{27}^3$, namely T_9^2. Further, since we restricted the range of $T_1^0 \to T_3^1$ to T_9^2, we needed to restrict the domain of $T_1^0 \to T_3^1$ to be inclusive of only the pre-images of T_9^2, namely T_3^1. Thus, in the compositions we replaced the mapping $T_1^0 \to T_3^1$ with $T_3^1 \to T_9^2$. Once this substitution was made the composition was straightforward.

Example 8 The composition of $T_1^1 \to T_1^2$ and $T_2^1 \to T_6^2$ is as follows:

$$(T_1^1 \to T_1^2) \circ (T_2^1 \to T_6^2) = \emptyset_R$$

Notice that in Example 5 the composition of the two elements of R resulted in a defined transformation because the range of $T_1^0 \to T_1^1$ intersected with the domain of $T_1^1 \to T_1^2$ is non-empty. In contrast, in Example 8, the range of $T_1^1 \to T_1^2$ intersected with the domain of $T_2^1 \to T_6^2$ is empty, thus producing the empty transformation.

Besides constructing this set R which is a subset of the set of all partial symmetries of the Sierpinski triangle, we observed that a transformation between any two triangles contained within the Sierpinski triangle can be established via a composition of transformations that maps the triangle back to the 0th iterate, or an equilateral triangle, one iterate at a time, and then forwards one iterate at a time to the desired triangle. Let $\lfloor a \rfloor$ denote the integer floor of the real number a. Each composition in the strand of compositions is defined in one of two ways: (1) $T_k^n \to T_l^{n-1}$, where $n \in \mathbb{N} \cup \{0\},\ k \in \mathbb{N}$ and $k \leq 3^n,$ and $l \in \mathbb{N}$ and $l = \lfloor \frac{(k-1)}{3} \rfloor + 1$, defines a composition in the first part of the strand before the obtainment of the 0th iterate; and (2) $T_v^{m-1} \to T_s^m$, where $m \in \mathbb{N} \cup \{0\},\ s \in \mathbb{N}$ and $s \leq 3^n,$ and $v \in \mathbb{N}$ and $v = \lfloor \frac{(s-1)}{3} \rfloor + 1$, defines a composition in the second part of the strand after the obtainment of the 0th iterate. For example, the transformation that maps the triangle T_{27}^3 into the triangle T_5^2 can be obtained via the following compositions:

$$(T_{27}^3 \to T_9^2) \circ (T_9^2 \to T_3^1) \circ (T_3^1 \to T_1^0) \circ (T_1^0 \to T_2^1) \circ (T_2^1 \to T_5^2).$$

(R,∘) is a Semigroup

To establish that R is a semigroup under the operation of composition, we need to show two properties: closure and associativity of $\circ$.

Theorem 9 *The set R of translational symmetries and rescaling transformations of the Sierpinski triangle is closed.*

Proof. Assume that $T_a^p \to T_b^q$ and $T_r^c \to T_s^d$ are fixed but arbitrarily chosen elements of R. This implies that $T_a^p,\ T_b^q,\ T_r^c,$ and T_s^d are triangles contained within an equilateral triangle. Consider the composition of these two mappings, $(T_a^p \to T_b^q) \circ (T_r^c \to T_s^d)$. The result will be a transformation only if the range of $T_a^p \to T_b^q$ intersected with the domain of $T_r^c \to T_s^d$ does not equal the empty set. If the intersection is empty, then the compo-

sition of these two mappings will generate the empty transformation, $\emptyset_R$, which is an element of R. Provided that the range of $T_a^p \to T_b^q$ intersected with the domain of $T_r^c \to T_s^d$ does not equal the empty set, then either $T_b^q = T_r^c$, T_b^q is contained in T_r^c, or T_r^c is contained in T_b^q. In general, when the intersection of two sets is non-empty, there is a fourth case in which the intersection contains elements from each set but is a subset of both sets. However, this is not possible for the triangles under consideration.

Case 1: Assume $T_b^q = T_r^c$. Then

$$(T_a^p \to T_b^q) \circ (T_r^c \to T_s^d) = (T_a^p \to T_b^q) \circ (T_b^q \to T_s^d) = (T_a^p \to T_s^d),$$

which is an element of R.

Case 2: Assume $T_b^q \subseteq T_r^c$. This implies that $T_b^q \cap T_r^c = T_b^q$. Thus, to evaluate the composition we will restrict the domain of $T_r^c \to T_s^d$ to the range of $T_a^p \to T_b^q$, namely T_b^q. Let $\rho\colon T_r^c \to T_s^d$. Then ρ is either a rescaling transformation, a translational symmetry, or a combination of both. In any case, the image of T_b^q under ρ is some triangle T_t^e. Thus, in the compositions we will replace T_r^c with T_b^q to obtain the following:

$$(T_a^p \to T_b^q) \circ (T_r^c \to T_s^d) = (T_a^p \to T_b^q) \circ (T_b^q \to T_t^e) = (T_a^p \to T_t^e),$$

which is an element of R.

Case 3: Assume $T_r^c \subseteq T_b^q$. This implies that $T_b^q \cap T_r^c = T_r^c$. Thus, we will restrict the range of $T_a^p \to T_b^q$ to the domain of $T_r^c \to T_s^d$, namely T_r^c. The pre-image of T_r^c is some triangle T_k^n equal to or strictly contained in T_a^p Hence, in the compositions we will replace T_b^q with T_r^c to obtain the following:

$$(T_a^p \to T_b^q) \circ (T_r^c \to T_s^d) = (T_k^n \to T_r^c) \circ (T_r^c \to T_s^d) = (T_k^n \to T_s^d)$$

where $T_k^n \to T_s^d$ is a transformation in R.

In any of the three above cases $(T_a^p \to T_b^q) \circ (T_r^c \to T_s^d)$ is an element of R. Since
$T_a^p \to T_b^q$ and $T_r^c \to T_s^d$ were fixed but arbitrarily chosen elements in R, the transformation that results from the composition of any two transformations in R is again in R. Therefore, the set of translational symmetries and rescaling transformations, R, of the Sierpinski triangle is closed. ■

In both [3] and [2], the claim is put forth that the set P of partial symmetries of the Sierpinski triangle forms a semigroup under the operation of composition .

Theorem 10 *The set of translational symmetries and rescaling transformations, R, of the Sierpinski triangle forms a subsemigroup of P.*

Proof. Since every translational symmetry and rescaling transformation is a partial symmetry, it follows that every element in R is contained in P. Thus, $R \subset P$. From our previous proof we know that R is closed. Thus, by definition, R is a subsemigroup. ■

As we discussed above in section 3, we find that R is also a semigroup.

Corollary 1 *The set R of the translational symmetries and rescaling transformation forms a semigroup under composition.*

5. Conclusion

From a group theory point of view an equilateral triangle and the Sierpinski triangle are indistinguishable. The study of the partial symmetries of an equilateral triangle and the Sierpinski triangle reveals that these structures are indeed quite different. The study of partial symmetries of the Sierpinski triangle allows for an analysis of every iterate of this structure's construction. Our study of the Sierpinski triangle has helped us gain some insight about the complex interior of this infinite structure. Nevertheless, we focused only on the set of translational symmetries and rescaling transformations, which is only one possible subset of the set of partial symmetries. The more we study the partial symmetries of the Sierpinski triangle, via semigroup theory, the more we can explore this fractal in its entirety.

From a group theory point of view an equilateral triangle and the Sierpinski triangle are indistinguishable. The study of the partial symmetries of an equilateral triangle and the Sierpinski triangle reveals that these structures are indeed quite different. The study of partial symmetries of the Sierpinski triangle allows for an analysis of every iterate of this structure's construction. Our study of the Sierpinski triangle has helped us gain some insight about the complex interior of this infinite structure. Nevertheless, we focused only on the set of translational symmetries and rescaling transformations, which is only one possible subset of the set of partial symmetries. The more we study the partial symmetries of the Sierpinski triangle, via semigroup theory, the more we can explore this fractal in its entirety.

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Exclusive Seating: An Application of the Inclusion-Exclusion Principle

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1. Introduction

The professor of a probability and statistics course posed this question to the class. "There are three couples, six people, seated in six chairs. In how many ways can those couples be seated so that no couple is sitting together?" I solved this problem by developing a formula for n couples and then solving it for $n = 3$ to get the answer of 240. This problem was explored by Joshua Weber in [1]. In his paper, Weber discusses two recursive formulas to find the number of seating arrangements for n couples; one formula is for n couples and another for n couples plus a singleton. In contrast, I provide a closed-form formula which gives an interesting application of the inclusion-exclusion principle for both cases along with a few other generalizations.

2. Development and Explanation of the Formula

To explain the process for finding the formula, consider n couples (that is, $2n$ individuals) being seated in a row with $2n$ chairs. Although we want to count the number of ways that no couple sits together, we start by counting how many ways to seat the individuals so that k of the couples are required to sit together. The k couples chosen to sit together can be chosen from the n couples in $\binom{n}{k} = \dfrac{n!}{k!\,(n-k)!}$ ways. Couples that are sitting

together can be considered single entities. For example, assume $n = 3$ and that couples are $A = \{a, \alpha\}$, $B = \{b, \beta\}$, and $C = \{c, \gamma\}$. If $k = 2$, we can select two couples to sit together in $\binom{3}{2} = 3$ ways. If couples A and B are chosen to sit together, then there are $2n - k$ entities to arrange in $(2n - k)!$ ways. In our example, there are $2 \cdot 3 - 2 = 4$ entities, A, B, c, and γ, to arrange in $(2 \cdot 3 - 2)! = 4! = 24$ ways. Couples sitting togethert can still be seated in 2 ways each. For example, couple A be be seated as $a\alpha$ or αa. So the k couples stting together can be seated in 2^k ways and still sit together. The product of these three terms

$$\binom{n}{k} 2^k \, (2n - k)!$$

attempts to count the number of ways in which k couples can sit together. However this term over counts the number of ways that k couples can sit together since the individuals that are not required to sit together ay end up being placed together unintentionally. In our example above, the seating arrangement (A, B, c, γ) gets counted for each of the couples $A\&B$, $A\&C$, $B\&C$, and $A\&B\&C$ sitting together. This over counting can be adjusted for by applying the inclusion-exclusion principle. The result takes the form

$$c\,(n) = \sum_{k=0}^{n} \left[(-1)^k \binom{n}{k} (2n - k)! 2^k \right],$$

from which the result 240 is acquired when there are three couples being seated. To demonstrate the principle, consider the expansion for $c(3)$:

$$\begin{aligned} c(3) &= \left[(-1)^0 \times \binom{3}{0} \times (2 \times 3 - 0)! \times 2^0 \right] \\ &\quad + \left[(-1)^1 \times \binom{3}{1} \times (2 \times 3 - 1)! \times 2^1 \right] \\ &\quad + \left[(-1)^2 \times \binom{3}{2} \times (2 \times 3 - 2)! \times 2^2 \right] \\ &\quad + \left[(-1)^3 \times \binom{3}{3} \times (2 \times 3 - 3)! \times 2^3 \right] \\ &= 720 + (-720) + 288 + (-48) \\ &= 240. \end{aligned}$$

This equation can be used for any n number of couples. The table below shows the results of the formula for $0 \leq n < 10$.

n	$c(n)$	n	$c(n)$
0	1	5	1,263,360
1	0	6	168,422,400
2	8	7	30,865,121,280
3	240	8	7,445,355,724,800
4	13,824	9	2,287,168,006,717,440

Table 1

3. Illustration of Process Using the Inclusion-Exclusion Principle

To aid in the understanding of this formula, consider the case for $n = 3$. Let the individuals of group 1 be represented by a and α, let group 2 be represented by b and β, and let group 3 be represented by c and γ. The table below demonstrates the possible combinations of seating arrangements for a in the first seat and α in the second seat.

a α b β c γ[3]	**a α** β b c γ	**a α** β c b γ	**a α** β c γ b
a α b β γ c	**a α** β b γ c	**a α** β γ b c	**a α** β γ c b
a α b c β γ	**a α** c b β γ	**a α** c β b γ[4]	**a α** c β γ b
a α b γ β c	**a α** γ b β c	**a α** γ β b c	**a α** γ β c b[5]
a α b c γ β	**a α** c b γ β	**a α** c γ b β	**a α** c γ β b
a α b γ c β	**a α** γ b c β	**a α** γ c b β	**a α** γ c β b

Table 2

But as mentioned above, if all the possibilities are shown in a like manner, it is easy to see that some possibilities are counted multiple times. After expression 1, the concept of double and triple counting is introduced. Arrangement 3 demonstrates triple counting; the arrangement is counted once for a and α sitting together, once for b and β sitting together, and once more for c and γ sitting together. Such multiple counting is compensated

for by the inclusion-exclusion principle. Arrangement 4 is an example of double counting in that it is counted for a and α and counted again for b and β; whereas, arrangement 5 is simply counted once for a and α. The inclusion-exclusion principle further demonstrates the idea of counting too much, then counting too little.

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| \\ &\quad - (|A \cap B| + |B \cap C| + |A \cap C|) \\ &\quad + |A \cap B \cap C| \end{aligned} \tag{5}$$

Equation 2 exhibits this concept. Figure 1, shown in the appendix, gives a visual illustration for three couples. The same idea can be considered for four couples where couple 1 is represented by a and α, couple 2 by b and β, couple 3 by c and γ, and couple 4 by d and δ. A table similar to that to table 2 can be generated to show couple arrangements.

$$|A \cup B \cup C \cup D|$$

$$= |A| + |B| + |C| + |D| \tag{6}$$

$$\begin{aligned} &- (|A \cap B| + |A \cap C| + |A \cap D| + |B \cap C| + |B \cap D| + |C \cap D|) \\ &+ |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| \\ &- |A \cap B \cap B \cap C \cap D| \end{aligned} \tag{7}$$

Equation 3 demonstrates the concept of the inclusion-exclusion principle for four couples. Figure 2 illustrates this case. The idea can be extended to the case of n couples.

4. Further Generalization and Explanations

To simply restate the formula, let $c(n)$ be the number of ways n couples can be seated so that no couples are sitting together. Then

$$c\left(n\right)=\sum_{k=0}^{n}\left[\left(-1\right)^{k}\binom{n}{k}\left(2n-k\right)!2^{k}\right]. \tag{8}$$

Now consider the concept of seating n equal sized groups of g individuals each. A group of size g will be considered to be sitting together when all of the people in the group are contiguous. There are two terms of the formula that need to be replaced and redefined. The first term to evaluate is 2^k. It was previously stated that this term was added to the formula to rearrange the individuals for each couple forced to sit together. From understanding that $2^k = (2!)^k$ for couples ($g = 2$), the replacement term of $(g!)^k$ can easily be derived. The second term to be replaced in the formula undergoes two rudimentary phases, each explained below. Looking at the term $(2n-k)!$, which is simplified from $(2n-2k+k)!$, it can be generalized by placing g into the term: $(gn-gk+k)!$ To further explain, a group that is defined to not be sitting together does not imply that no one in the group is sitting by another member of that same group. It simply means that at least one number from each group is not sitting with the group it sat with at first. The second consideration for this term is that of including s number of singletons not a part of any groups so that the term finally becomes $(gn-gk+k+s)!$. The formula is rewritten to let $c(n,g,s)$ be the number of ways that n groups of g along with s singletons can be seated such that none of the groups are together, thus

$$c\left(n,g,s\right)=\sum_{k=0}^{n}\left[\left(-1\right)^{k}\binom{n}{k}\left(gn-gk+k+s\right)!\left(g!\right)^{k}\right]. \tag{9}$$

Lastly, it must be noted that $n \geq 0$, $g \geq 2$ (the formula does not work for $g < 2$ but the answered can be obtained through logically reasoning), and $s \geq 0$. Also, this formula can only be used for couples being seated in a line as opposed to being seated round a table of arbitrary size or shape (the formula does not consider the possibility of the individuals being seated in the first position and the last position sitting together).

Appendix

$s = 0$		g		
		2	3	4
n	1	0	0	0
	2	8	504	35,712
	3	240	283,824	454,035,456
	4	13,824	400,028,544	20,337,266,294,784
	5	1,263,360	1,134,481,697,280	2,390,716,211,594,526,720

Table 3

$s = 1$		g		
		2	3	4
n	1	2	12	72
	2	48	3,816	331,776
	3	1,968	2,975,616	5,974,124,544
	4	140,160	5,343,200,640	347,454,074,880,000
	5	15,298,560	18,472,299,194,880	50,330,123,910,441,861,120

Table 4

$s = 2$		g		
		2	3	4
n	1	12	84	576
	2	336	32,544	3,400,704
	3	17,760	33,903,360	84,372,295,680
	4	1,543,680	76,432,170,240	6,278,472,568,995,840
	5	199,019,520	318,615,016,527,360	1,109,522,976,154,559,447,040

Table 5

Table 3 shows the results for when $n = 2, 3, 4$; $g = 1, 2, 3, 4, 5$; and $s = 0$. Tables 4 and 5 illustrate a comparison for the addition of singletons, 1 and 2 respectively.

Figure 1

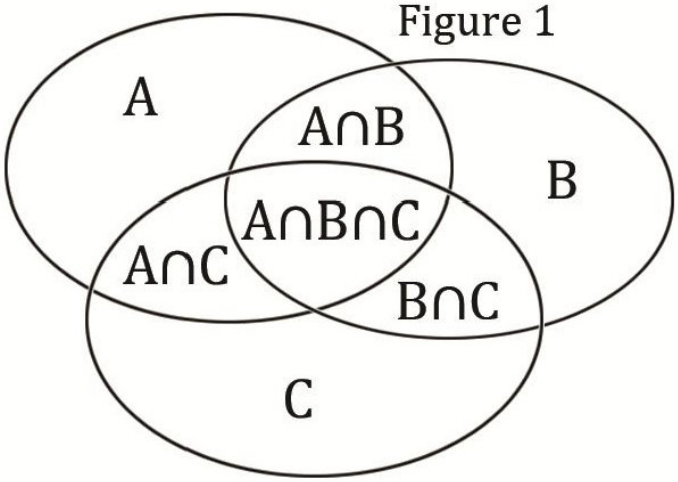
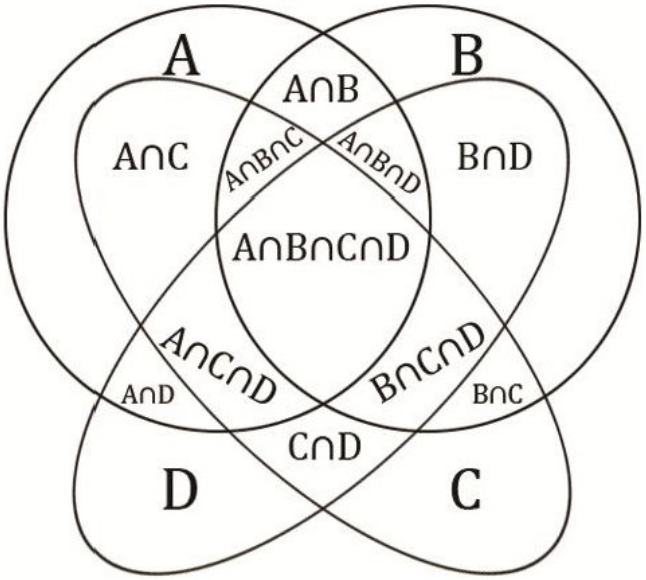


Figure 2



Reference

[1] J. Weber, Let's Be Seated, *The Pentagon* **56 No. 1**, (Fall 1996) 39-42.

Pentagon Editor Search

The current editor of *The Pentagon* would like to step down within approximately the next year, so Kappa Mu Epsilon seeks a new editor to begin summer of 2013. Interested individuals should contact the KME National President, Ron Wasserstein. His contact information is listed on page 2 of this issue. Below is a description of the job.

Pentagon Editor Job Description

KME's official journal, *The Pentagon*, is published twice annually. It is the editor's job to prepare *The Pentagon* for publication. It can be quite gratifying to be involved in the process of bringing the work of multiple authors together into a completed publication.

Each issue of *The Pentagon* consists of articles (typically 2-4), the Problem Corner, KME News, a list of active chapters, and a list of national officers. Certain issues contain additional features, such as reports of the national conventions. The Problem Corner Editor collects and assembles the material (problems and solutions) for the Problem Corner. The KME Historian collects and assembles the material for KME News.

The Pentagon Editor attends the national convention to encourage student paper presenters to submit their articles for publication. The Editor responds to inquiries about publishing in the Pentagon. The Editor maintains a list of reviewers (obtained by sending out to past reviewers, KME sponsors and Corresponding Secretaries, and others a request to serve as a reviewer). The Editor forwards submitted articles to appropriate reviewers, and guided by their reviews, determines which articles to publish. The Editor communicates with the Problem Corner Editor and the KME Historian to obtain their components of each issue. The Editor selects which articles will appear in a particular issue. The Editor typesets all of the components of the issue, ultimately preparing a single .pdf file for the issue, which is then forwarded to the Business Manager of *The Pentagon*.

The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before September 1, 2012. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2012 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)-622-3051)

NEW PROBLEMS 689-698

Problem 689. *Proposed by Tom Moore, Bridegewater State University, Bridgewater, MA.*

Prove that there are infinitely many triangular numbers which are the sum of three pentagonal numbers.

Problem 690. *Proposed by Tom Moore, Bridegewater State University, Bridgewater, MA.*

Show that the sequence $91, 8911, 889111, 88891111, ...$ (i.e., put one more 8 on the front and one more 1 on the end) consists solely of triangular numbers.

Problem 691. *Proposed by Tom Moore, Bridegewater State University, Bridgewater, MA.*

Which pentagonal numbers are the positive integer power of a single prime?

Problem 692. *Proposed by D. M. Batinetu-Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil Palade Secondary School, Buzau, Romania.*

Let

$$x_n = \sqrt[n]{\sqrt{2!} \cdot \sqrt[3]{3!} \cdots \sqrt[n]{n!}}.$$

Calculate

$$\lim_{n\to\infty} \left[\frac{(n+1)^2}{x_{n+1}} - \frac{n^2}{x_n} \right].$$

Problem 693. *Proposed by D. M. Batinetu-Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil Palade Secondary School, Buzau, Romania.*

Prove that if $x, y, m > 0$, then in any triangle ABC, the following holds

$$\frac{\tan^{2m+1}\frac{A}{2}}{\left(x\cot\frac{B}{2} + y\cot\frac{C}{2}\right)^m} + \frac{\tan^{2m+1}\frac{B}{2}}{\left(x\cot\frac{C}{2} + y\cot\frac{A}{2}\right)^m}$$
$$+ \quad \frac{\tan^{2m+1}\frac{C}{2}}{\left(x\cot\frac{A}{2} + y\cot\frac{B}{2}\right)^m} \geq \frac{(4R+r)\, r^m}{(x+y)^m\, s^{m+1}},$$

where R is the circumradius, r is the inradius, and s is the semiperimeter.

Problem 694. *Proposed by Jose Luis Diaz-Barrero, BARCELONA TECH, Barcelona, Spain.*

Let a, b, c be positive real numbers such that $a + b + c = 1$. Prove that

$$\frac{a^3}{a+b} + \frac{b^3}{b+c} + \frac{c^3}{c+a} \geq \frac{1}{6}.$$

Problem 695. *Proposed by Jose Luis Diaz-Barrero, BARCELONA TECH, Barcelona, Spain.*

Let $a > 1$ be a real number such that $\{a\} + \{1/a\} = 1$, where $\{x\}$ represents the fractional part of x. [Note that the golden ratio $\left(1+\sqrt{5}\right)/2$ has this property.] Compute

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\left\lfloor a^k \right\rfloor + \left\lfloor \frac{1}{a^k} \right\rfloor + \lfloor a \rfloor + \left\lfloor \frac{1}{a} \right\rfloor + 2 \right)^{-1},$$

where $\lfloor x \rfloor$ denotes the floor of x.

Problem 696. *Proposed by Ovidiu Furdui, Campia Turzii, Cluj, Romania*.

Let $\alpha > 4$ be a real number. Find in closed form the value of the sum

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\left\lfloor\sqrt{n+m}\right\rfloor^{\alpha}},$$

where $\lfloor x\rfloor$ denotes the floor of x.

Problem 697. *Proposed by Pedro H. O. Pantoja (student), Natal-RN, Brazil.*

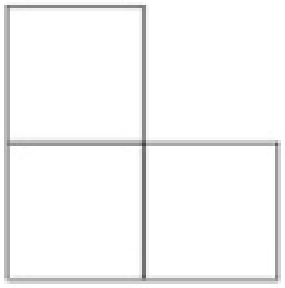
Let A and B be 2×2 matrices with real entries such that $\mathrm{Tr}\,(BA)=0$ and

$$\mathrm{Det}\left(A\cdot A^t+B^t\cdot B\right)+\mathrm{Det}\left(A\cdot A^t-B^t\cdot B\right)+\mathrm{Det}\left(2\,(AB)^2\right)=-1.$$

Compute $(AB)^{2012}$.

Problem 698. *Proposed by Mathew Cropper, Eastern Kentucky University, Richmond, KY.*

Start with an 8×8 array of squares. Remove the central 4×4 square. Note that 48 squares remain and 48 is divisible by 3. Determine the number of ways in which the 48 squares can be tiled with tiles made up of three squares in the shape of an L as indicated below.



SOLUTIONS TO PROBLEMS 669-678

Problem 669. *Proposed by Mohammad Azarian, University of Evansville, Evansville, IN.*

Show that

$$\int_{-1}^{1} \frac{1}{1+e^{2x}}\,dx = 1.$$

Solution *by the Cade Herron (student), TN Beta, East Tennessee State University, Johnson City, TN.*

Let $u = e^x$. Then $dx = \dfrac{du}{u}$. The definite integral becomes

$$\int_{-1}^{1} \frac{1}{1+e^{2x}}\,dx = \int_{e^{-1}}^{e} \frac{du}{u+u^3}.$$

By partial fraction decomposition, we have

$$\begin{aligned}
\int_{e^{-1}}^{e} \frac{du}{u+u^3} &= \int_{e^{-1}}^{e} \frac{du}{u} - \int_{e^{-1}}^{e} \frac{u\,du}{1+u^2} \\
&= \ln u\big|_{e^{-1}}^{e} - \frac{1}{2}\ln\left(1+u^2\right)\Big|_{e^{-1}}^{e} \\
&= 2 - \frac{1}{2}\left[\ln\left(1+e^2\right) - \ln\left(1+e^{-2}\right)\right] \\
&= 2 - \frac{1}{2}\ln\left(\frac{1+e^2}{1+e^{-2}}\right) \\
&= 2 - \frac{1}{2}\ln\left[\frac{e^2\left(1+e^2\right)}{1+e^2}\right] \\
&= 2 - \frac{1}{2}\ln e^2 \\
&= 1.
\end{aligned}$$

Also solved by Wen Shao and Rina Orimo (students), Northeastern State University, Tahlequah, OK; Tatsuya Eto (student), Northeastern State University, Tahlequah, OK; Joseph Garcia, Slippery Rock University, Slippery Rock, PA; Jose Luis Diaz-Barrero, Polytechnical University of Catalonia, Barcelona, Spain; Pedro H.O. Pantoja (student), Natal-RN, Brazil; and the proposer.

Problem 670. *Proposed by Mohammad Azarian, University of Evansville, Evansville, IN.*

Show that for any natural number n,

$$\sum_{k=0}^{n} \frac{1}{[k!\,(n-k)!]^2} < \left(\frac{2^n}{n!}\right)^2.$$

Solution *Jose Luis Diaz-Barrero, Polytechnical University of Catalonia, Barcelona, Spain.*

The inequality given is equivalent to

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \sum \frac{(n!)^2}{[k!\,(n-k)!]^2} < 2^{2n}.$$

We equate the coefficients of x^n on both sides of the identity

$$(1+x)^n\,(1+x)^n = (1+x)^{2n}.$$

Expanding both sides with the binomial theorem,

$$\left(\sum_{k=0}^{n} \binom{n}{k} x^k\right)\left(\sum_{k=0}^{n} \binom{n}{k} x^k\right) = \sum_{k=0}^{2n} \binom{2n}{k} x^k,$$

it follows immediately that

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

since $\binom{n}{k} = \binom{n}{n-k}$. So to complete the solution it will suffice to see that $\binom{2n}{n} < 2^{2n}$. This inequality is immediate because the left-hand side is the number of subsets of size n from a set A with $2n$ elements, and the right-hand side is the number of all subsets of A.

Also solved by Pedro H.O. Pantoja (student), Natal-RN, Brazil; and the proposer.

Problem 671. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let $ABCD$ be a trapezoid, with CD parallel to AB, side CD fixed, and AB a segment of constant length. Assume that $AD + BC$ is constant. Describe the locus of the point M at which sides AD and BC meet and the locus of the point O at which the diagonals of the trapezoid meet.

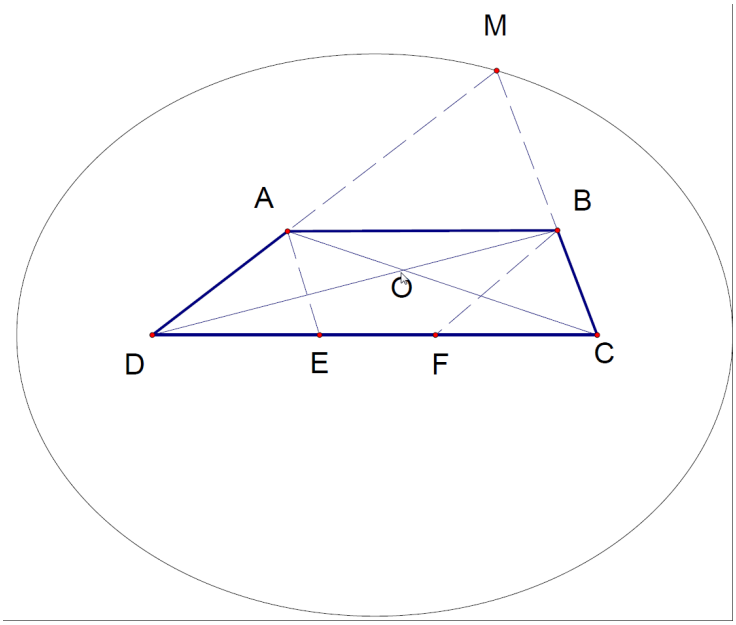
Solution *by the proposer.*

Draw through vertex A the parallel to side BC. It meets DC at point E. Since $\triangle MDC$ is similar to $\triangle ADE$, we have

$$\frac{MD}{AD} = \frac{DC}{DE} \Longrightarrow MD = \frac{AD \cdot DC}{DE} = \frac{AD \cdot DC}{DC - AB}.$$

Likewise, draw through vertex B the parallel side to side AD. It meets DC at point F. Since $\triangle MDC$ is similar to $\triangle CBF$, we have

$$\frac{MC}{BC} = \frac{DC}{CF} \Longrightarrow MC = \frac{BC \cdot DC}{CF} = \frac{BC \cdot DC}{DC - AB}.$$



Adding yields

$$MD + MC = \frac{DC\,(AD + BC)}{DC - AB} = K.$$

Therefore, M describes an ellipse with foci at D and C and major axis of length K. On the other hand, we have

$$BC + BF = BC + AD \text{ (constant)}.$$

So B describes an ellipse with foci at C and F. Now we consider the similar triangles AOB and DOC. We have $\dfrac{OB}{OD} = \dfrac{AB}{DC}$. Furthermore,

$$DB = DO + OB = OB\left(1 + \frac{CD}{AB}\right) = OB\left(\frac{AB + CD}{AB}\right),$$

from which it follows that

$$\frac{DO}{DB} = \frac{DC}{DC + AB} = k \text{ (a constant)}.$$

Since the ratio DO/DB is constant and D is a fixed point, the point O describes the transformation of the ellipse described by B by the homothety with center at D and coefficient of homothety $k = \dfrac{DC}{DC + AB}$.

Problem 672. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Find all positive solutions of the following system of equations:

$$\begin{cases} x_1 + x_2 + x_3 = x_4^2 \\ x_2 + x_3 + x_4 = x_5^2 \\ x_3 + x_4 + x_5 = x_6^2 \\ x_4 + x_5 + x_6 = x_1^2 \\ x_5 + x_6 + x_1 = x_2^2 \\ x_6 + x_1 + x_2 = x_3^2 \end{cases}.$$

Solution *by the proposer.*

Setting $x_7 = x_1$, $x_8 = x_2$, and $x_9 = x_3$, we can write the system in the more convenient form

$$x_k + x_{k+1} + x_{k+2} = x_{k+3}^2,\ k = 1, 2, ..., 6.$$

Let $x_i = \min\limits_{1 \le k \le 6} \{x_k\}$ and $x_j = \max\limits_{1 \le k \le 6} \{x_k\}$. Then $x_i^2 = x_{i-1} + x_{i-2} + x_{i-3} \ge 3x_i$, from which it follows that $x_i \ge 3$. On the other hand, $x_j^2 = x_{j-1} + x_{j-2} + x_{j-3} \le 3x_j$, from which it follows that $x_j \le 3$. Since we have $3 \le x_i \le x_j \le 3$, we get that $x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = 3$. Thus the only positive solution of the system is $(3, 3, 3, 3, 3, 3)$.

Problem 673. *Proposed by Pedro H.O. Pantoja (student), Natal-RN, Brazil.*

Let n, p, x be positive integers. Let $S_{n,p}$ be the set of all possible remainders on division of $x^{10\cdot 2^n}$ by p. Prove that there exists p so that $|S_{1,p}| = |S_{2,p}| = |S_{3,p}| = \cdots$, where $|S|$ represents the number of elements in S.

Solution *by the proposer.*

We will show that $p = 9$ satisfies the conditions of the problem. Looking first at fifth powers of $x \bmod 9$, we see that

x	0	1	2	3	4	5	6	7	8
$x^5 \bmod 9$	0	1	5	0	7	2	0	4	8

,

so the remainders are $0, 1, 2, 4, 5, 7, 8$. Now looking at squares of these $\bmod 9$, we see that

x	0	1	2	4	5	7	8
$x^2 \bmod 9$	0	1	4	7	7	4	1

.

Hence the reminders of $x^{10} \bmod 9$ are just $0, 1, 4, 7$. Looking at this subset of the above table, we see that the reminders of $x^{20} \bmod 9$ are just $0, 1, 4, 7$. Since this subset repeats with each squaring, we have $S_{n,9} = \{0, 1, 4, 7\}$ for all $n \geq 1$.

Problem 674. *Proposed by Ovidiu Furdui, Campia Turzii, Cluj, Romania.*

Let $\lfloor x \rfloor$ be the integer part of x and $\{x\}$ the fractional part of x. Find the value of

$$\int_0^1 x \left\{ \frac{1}{x} \right\} \left\lfloor \frac{1}{x} \right\rfloor \, dx.$$

Solution *by the proposer.*

Using the substituion $t = 1/x$, the integral becomes

$$\begin{aligned}
I &= \int_0^1 x \left\{ \frac{1}{x} \right\} \left\lfloor \frac{1}{x} \right\rfloor \, dx \\
&= \int_1^\infty \frac{\{t\} \cdot \lfloor t \rfloor}{t^3} \, dt \\
&= \sum_{k=1}^\infty \int_k^{k+1} \frac{(t-k)\, k}{t^3} \, dt \\
&= \sum_{k=1}^\infty \int_k^{k+1} \left(\frac{k}{t^2} - \frac{k^2}{t^3} \right) \, dt \\
&= \sum_{k=1}^\infty \left(k \left(\frac{-1}{k+1} + \frac{1}{k} \right) + \left(\frac{k^2-1}{2\,(k+1)^2} - \frac{1}{2} \right) + \left(\frac{1}{2\,(k+1)^2} \right) \right) \\
&= \sum_{k=1}^\infty \frac{1}{2\,(k+1)^2} \\
&= \frac{1}{2} \left(\frac{\pi^2}{6} - 1 \right).
\end{aligned}$$

Problem 675. *Proposed by Ovidiu Furdui, Campia Turzii, Cluj, Romania*.

Let k be a real number with $k > -1$. Calculate the double integral

$$\int_0^1 \int_0^1 \frac{x^{k+1}y^k}{x+y}\, dxdy.$$

Solution *by the proposer.*

Let I denote the value of the integral. By symmetry, we have

$$\int_0^1 \int_0^1 \frac{x^{k+1}y^k}{x+y}\, dxdy = \int_0^1 \int_0^1 \frac{y^{k+1}x^k}{x+y}\, dxdy.$$

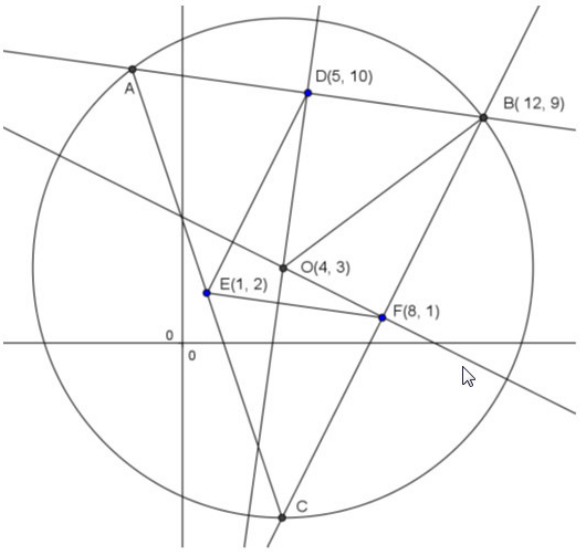
Thus

$$\begin{aligned} I &= \frac{1}{2}(I+I) \\ &= \frac{1}{2}\int_0^1 \int_0^1 \left(\frac{x^{k+1}y^k}{x+y} + \frac{y^{k+1}x^k}{x+y}\right) dxdy \\ &= \frac{1}{2}\int_0^1 \int_0^1 x^k y^k\, dxdy \\ &= \frac{1}{2(k+1)^2}. \end{aligned}$$

Problem 676. *Proposed by Ed Wilson, Eastern Kentucky University, Richmond, KY*.

Triangle ABC is circumscribed by a circle. The midpoints of the sides of the triangle are $(5, 10)$, $(1, 2)$, and $(8, 1)$. Determine the equation of the circle that circumscribes triangle ABC.

Solution *by the proposer.*



Let $D\,(5, 10)$ be the midpoint of AB, $E\,(1, 2)$ be the midpoint of AC, and $F\,(8, 1)$ be the midpoint of BC. Then DE is a midsegment of triangle ABC and is parallel to BC. The slope of BC equals the slope of DE. Thus, the slope of BC is 2. Since F is the midpoint of BC, the equation of the perpendicular bisector of BC is $y = -\frac{1}{2}x + 5$. Similarly, EF is a midsegment of triangle ABC and is parallel to AB. The slope of AB equals the slope of EF. The slope of AB is thus $-1/7$. Since D is the midpoint of AB, the equation of the perpendicular bisector of AB is $y = 7x - 25$. The center of the circle $O\,(4, 3)$ is the intersection of the perpendicular bisectors of the sides of triangle ABC. The equation of the line passing through BC (and F) is $y = 2x - 15$. The equation of the line passing through AB (and D) is $y = -\frac{1}{7}x + \frac{75}{7}$. The intersection of the lines is vertex $B\,(12, 9)$ of the triangle. The radius of the circle is $OB = 10$. Thus, the equation of the circle is $(x - 4)^2 + (y - 3)^2 = 100$.

Problem 677. *Proposed by the editor.*

Let $\{a_n\}$ be an integer sequence defined by

$a_1 = 0$, $a_2 = 2, a_3 = 5$, and $a_n = a_{n-1} + a_{n-2} - a_{n-3}$ for $n \geq 4$.

Find (with proof) a positive rational number Q so that $a_n \geq Qn - 3$ for all $n \geq 1$ and Q is the largest such rational number.

Solution *by Rex Edmonds (student), Slippery Rock University, Slippery Rock, PA.*

The sequence starts with $a_1 = 0$, $a_2 = 2$, $a_3 = 5$, $a_4 = 5 + 2 - 0 = 7$, $a_5 = 7 + 5 - 2 = 10$, $a_6 = 10 + 7 - 5 = 12$, $a_7 = 12 + 10 - 7 = 15$, $a_8 = 15 + 12 - 10 = 17$. We can inductively show that the terms have the form $\frac{5}{2}(n-1)$ when n is odd and $\frac{5}{2}(n-2) + 2$ when n is even. Then

$$\tfrac{5}{2}(n-1) = \tfrac{5}{2}n - \tfrac{5}{2} > \tfrac{5}{2}n - 3 = \tfrac{5}{2}(n-2) + 2.$$

Hence it follows that $a_n \geq \frac{5}{2}n - 3$. Therefore the largest Q so that $a_n \geq Q \cdot n - 3$ for all $n \geq 1$ is $Q = \frac{5}{2}$.

Also solved by Cade Herron (student), TN Beta, East Tennessee State University, Johnson City, TN; Moselle Christensen (student), Eastern Kentucky University, Richmond, KY; and the proposer.

Problem 678. *Proposed by the editor.*

An alphametic is a mathematical puzzle in which a set of words is written in the form of an ordinary addition sum and requires that the letters in the puzzle be replaced with decimal digits so that the result is a valid arithmetic sum. Solve the following alphametic in base 14:

		W	I	L	S	O	N
+	T	H	E	O	R	E	M
L	A	G	R	A	N	G	E

Solution *by Moselle Christensen, Eastern Kentucky Univ., Richmond, KY.*

0	1	2	3	4	5	6	7	8	9	10	11	12	13
A	L	I	N	M	G	S	E	H	-	R	W	O	T

		11	02	01	06	12	03
	13	08	07	12	10	07	04
1	00	05	10	00	03	05	07

Note that the values of W and H can be interchanged to provide a second solution.

Also solved by the proposer.

Report of the 38th Biennial Convention

Kappa Mu Epsilon
April 14-16, 2011
Harris-Stowe State University, Missouri Mu
St. Louis, Missouri

Thursday, April 14, 2011

On Thursday evening, April 14, a mixer was held in the Dr. Henry Givens Jr. Administration Building (HGA) Room 005. Participants were able to pick up registration packets at this mixer. At 8:00 p.m., the National Council, the Regional Directors and the Pentagon Editor and Business Manager held their biennial business meeting in HGA 208.

Friday, April 15, 2011

Friday's activities began with breakfast and continued registration in the Hallway outside Mini Auditorium, HGA 311. At 8:30 a.m. in the Mini Auditorium, HGA 311, the first general session began, with KME President Ron Wasserstein presiding. Dr. Henry Givens Jr., President of host Harris-Stowe State University, welcomed participants. Mark Hamner, KME Secretary, then called the roll. Twenty-five faculty members and 63 students were in attendance, for a total of 88 participants from 16 chapters.

The following ten new chapters installed during the 2009-2011 biennium were recognized:

Alabama Theta at Jacksonville State University, California Zeta at Simpson University, Florida Gamma at Southeastern University, Georgia Epsilon at Wesleyan College, Massachusetts Beta at Stonehill College, New Jersey Epsilon at New Jersey City University, New York Rho at Malloy College, North Carolina Eta at Johnson C. Smith College, North Carolina Zeta at Catawba College, and Rhode Island Alpha at Roger Williams College.

Rhonda McKee, KME President-Elect of Missouri Beta conducted the filing of delegates. There was no old business.

Introduction of new business began with a report from the nominating committee. Peter Skoner of Pennsylvania Mu, chair of the Nominating Committee, reported for the committee. Two officers were nominated to continue for another term in their current positions: Cynthia Woodburn of Kansas Alpha, Pittsburg State University as National Treasurer and Mark Hamner of Texas Gamma, Texas Woman's University as National Secretary. The nominees were introduced to the delegates and additional nominations were requested from the floor. There being none, nominations were

closed. A vote on the nominations was held during the business meeting on Saturday morning.

Revisions to the KME Constitution, to be voted upon at the Saturday morning business meeting, were discussed and announcement was made about the new KME Chapter Handbook.

Rhonda McKee, KME President-Elect presided over the student paper presentations, which began at 9 a.m. in the Mini Auditorium, HGA 311.

First Paper Session

- 1-1: The Role of Dynamic Representations in Student Development of Algebraic Concepts, by Marie Ermete, Michigan Beta, Central Michigan University
- 1-2: Creating Irreducible Divisor Graphs for Numerical Monoids, by James Gossell, Missouri Beta, University of Central Missouri
- 1-3: Dates, Times, and Students: A Graph Theoretical Solution to the Scheduler's Dilemma, by Andrew Lake, Kansas Delta, Washburn University

At 10:00 a.m. there was a 15-minute Refreshment Break in the hallway outside HGA 311.

Session #2 of the Student Presentations commenced at 10:15 a.m. in the Mini Auditorium, HGA 311.

Second Paper Session

- 2-1: Factorization in k-Furcus Semigroups, by Kaitlyn Cassity, Missouri Beta, University of Central Missouri
- 2-2: The Combinatorics of the Card Game 16, by Matthew Moreno, Texas Mu, Schreiner University
- 2-3: The Number π As a Limit, by Anthony Cammarano and Gabriela Rodrigues, New York Omicron, St. Joseph's College
- 2-4: Math Magic, by Sean Van Dyke, Kansas Delta, Washburn University

A group picture was taken in the Emerson Performance Center Theater at 11:30 a.m.

Lunch was held at 11:45 a.m. in HGA 315 followed by two meetings of the National KME Auditing Committee, Philip Lombardo, New York

Omicron, Chair (in HGA 218) and National KME Resolutions Committee, Pedro Muiño, Pennsylvania Mu, Chair (in HGA 220).

Session #3 of the Student Presentations commenced at 1:00 p.m. in the Mini Auditorium, HGA 311.

Third Paper Session

- 3-1: De Bruijn Sequences, by Daniel Demski, Michigan Beta, Central Michigan University
- 3-2: Exclusive Seating: An Application of the Inclusion-Exclusion Principle, by Kenneth Barker, Missouri Theta, Evangel University
- 3-3: Recursive Patterns and the Resistance of an Infinite Circuit, by Michael Phinney, Missouri Beta, University of Central Missouri
- 3-4: The Evolution of Cryptography, by Kimberly Thompson and Nick Wood, New York Rho, Molloy College

At 2:15 p.m. after the paper session, refreshments were served. At 2:30 p.m. in HGA 311, participants viewed Fun Math Videos.

Session #4 of the Student Presentations commenced at 2:45 p.m. in the Mini Auditorium, HGA 311.

Fourth Paper Session

- 4-1: The Hat Problem: A Mathematics Student's Perspective, by Michael Reb, Kansas Delta, Washburn University
- 4-2: Diophantine Monoids Defined by Second Order Recurrence Relations, by Trey Brock, Missouri Beta, University of Central Missouri
- 4-3: Bornologous Equivalencies, by Laura Stibich, Pennsylvania Mu, St. Francis University

Between 3:40 p.m. and 4:30 p.m., the KME Student Section met in HGA 311, while the Faculty Section met in HGA 315. The afternoon activities concluded at 4:30 p.m.

The convention banquet was held in the Clay Early Childhood Center at 7:00 p.m. Rhonda McKee, KME President-Elect served as emcee. KME President Ron Wasserstein presented the George R. Mach Distinguished Service Award to Pat Costello, Kentucky Alpha, and to Don Tosh, Missouri Theta for their outstanding service and dedication to KME. Following dinner, Steven G. Krantz of Washington University in St. Louis, gave the keynote address. His topic was A Matter of Gravity.

Saturday, April 16, 2011

Saturday's activities began between 7:30 a.m. and 8:15 a.m. with Registration and Breakfast in the hallway outside HGA 311.

Session #5 of the Student Presentations commenced at 8:30 a.m. in the Mini Auditorium, HGA 311.

Fifth Paper Session

- 5-1: It is Not Just How Far You Can See, by Yusuke Suita, Kansas Beta, Emporia State University
- 5-2: Diophantus: A Lesson Plan Module, by Jennifer Hayes, Missouri Beta, University of Central Missouri
- 5-3: Factor Graphs and the Summary-Product Algorithm, by Abram Demski, Michigan Beta, Central Michigan University
- 5-4: Application of Linear Algebra to Three-Dimensional Computer Graphics, by Jae Min Lee, Kansas Alpha, Pittsburg State University

At 9:45 a.m. a Refreshment Break was held in the hallway outside HGA 311. The Awards Committee met in HGA 315.

At 10:15 a.m., in the Mini Auditorium, HGA 311, the Second General Session began with President Ron Wasserstein presiding.

For the Continuation of New Business, KME constitutional revisions were discussed. During the discussion, a motion on Article XI to change the wording of "A simple majority of the chapters…" to "Two-thirds of the chapters …" was voted on. The motion was defeated by a vote of 12 opposed and 5 for the motion. Subsequently, all constitutional changes, as initially presented, were unanimously accepted by a vote of 18 for with none opposed.

The following national officers made reports: Charles Curtis, Editor, The Pentagon; Peter Skoner, Historian; Cynthia Woodburn, Treasurer; Mark Hamner, Secretary; Rhonda McKee, President-Elect; and Ron Wasserstein, President.

Following the national officer reports were reports from the Section Meetings, the Auditing Committee and the Resolutions Committee.

By unanimous acclamation, both officers were re-elected to serve another term in their current positions: Cynthia Woodburn of Kansas Alpha, Pittsburg State University as National Treasurer and Mark Hamner of Texas Gamma, Texas Woman's University as National Secretary. These officers were installed by President Ron Wasserstein.

The report of the Awards Committee and presentation of awards were made. The top four papers were: (in alphabetical order)

- James Gossell, Missouri Beta
- Andrew Lake, Kansas Delta
- Matthew Moreno, Texas Mu
- Kimberly Thompson & Nick Wood, New York Rho

These students were awarded a $100 check for their respective winning paper. In addition, $50 for travel was given to each chapter for each presenter. A new award called the "People's Choice Award" was presented to Jennifer Hayes of Missouri Beta. The "People's Choice Award" recipient received a Texas Instruments donated a graphing calculator. The winner of this new award was selected by submitted votes of the attending KME student members.

The convention concluded with the National Treasurer Cynthia Woodburn presenting checks for travel allowances to each chapter present.

Mark Hamner
National Secretary

Report of the National President

The first two years of my term as President of Kappa Mu Epsilon have been busy ones, and, thanks to the wonderful work of my colleagues on the National Council, productive ones.

Before I elaborate on the highlights, I take this moment to thank our hosts at Harris-Stowe University for hosting this convention. Special thanks to Ann Podleski for a job well done! I hope the Missouri Mu Chapter has found it rewarding to serve the society in this important way.

Highlights of our activities between the March 2009 convention in Philadelphia and this 2011 convention in St. Louis include:

- We installed ten new chapters (installing officer in parentheses).
  - Alabama Theta at Jacksonville State University, March 29, 2010 (Wasserstein)
  - California Zeta at Simpson University, April 4, 2009 (Don Tosh)
  - Florida Gamma at Southeastern University, March 31, 2010 (Wasserstein)
  - Georgia Epsilon at Wesleyan College, March 30, 2010 (Wasserstein)
  - Massachusetts Beta at Stonehill College, April 8, 2011 (Pete Skoner)
  - New Jersey Epsilon at New Jersey City University, February 22, 2010 (Wasserstein)
  - New York Rho at Malloy College, April 21, 2009 (Andy Rockett)
  - North Carolina Eta at Johnson C. Smith College, March 18, 2010 (Wasserstein)
  - North Carolina Zeta at Catawba College, September 17, 2009 (Wasserstein)
  - Rhode Island Alpha at Roger Williams College, November 13, 2009 (Pete Skoner)
- We developed and implemented a plan to regularly contact institutions without KME or PME chapters to encourage their participation in KME. Some positive results have occurred from this, but we are well aware that this will take regular effort over time to achieve maximum success.
- At the other end of the spectrum, we made contacts with chapters that have not been active to let them know they are approaching inactive status, and to try to encourage them to restart. This appears to have

been successful in several instances, but unfortunately several chapters have been moved to inactive status.

- The National Council met regularly (almost every month) by conference call. These regular meetings not only allowed us to get more business done more quickly, but also alleviated the need for a face to face meet between conventions, which reduced our expenses.
- We rewrote the KME Chapter Handbook, which was "completed" and announced to the chapters at this convention. "Completed" is in quotes because we recognize the need for continual updates to the handbook, which is available on our website.
- We changed our publication model for The Pentagon, moving from a printed subscription-based model to an online, open access model, which we believe will widen the accessibility and usefulness of The Pentagon.
- We reviewed the KME Constitution and recommended changes, which will be voted on at this convention.
- We named two outstanding volunteers as recipients of the George Mach Distinguished Service Award. (I have included their citations here, for the record.)
  - Pat Costello has faithfully served Kappa Mu Epsilon for many years. For almost 30 years, Pat has served as corresponding secretary for the KY Alpha chapter (Eastern Kentucky University), and served as host of the 1983 national convention. He also hosted a regional convention during his 12 years as regional director (southeast). His service then continued at the national level, where he served as president-elect of KME 1993-1997 and as president 1997-2001. In the fall of 2005 Pat returned to a prominent service role as editor of the Problem Corner for The Pentagon. In addition, Pat has been an unwavering supporter of his chapter, encouraging students to excellence in the pursuit of their educational goals. Only a few individuals serve an organization in so many ways so very well for so long. With gratitude, KME presented Pat Costello with the George R. Mach Distinguished Service Award for his outstanding contributions to the success and the well-being of Kappa Mu Epsilon.
  - Don Tosh has served as the corresponding secretary of Missouri Kappa (Drury University) for over 25 years. Don has served as a national officer for a dozen years, as Historian from 1997-2001, as President-Elect from 2001-2005, and as President from 2005-

> 2009. During his time as President he also took on the responsibility of Business Manager of the Pentagon, a capacity in which he still serves today. Don has been passionate about the Missouri Kappa chapter, encouraging student research and bringing students to national and regional conventions in a period that now spans four decades. Don has supported and continues to support KME with a great deal of time and sacrifice. With gratitude, KME presented Don Tosh with the George R. Mach Distinguished Service Award for his outstanding contributions to the success and the well-being of Kappa Mu Epsilon.

A list of bullet points is appropriate for a report such as this, but it belies the amount of effort made by the national officers, who serve as unpaid volunteers of this society. Indeed, one of the most rewarding aspects of serving as your President has been working with President-Elect Rhonda McKee, Treasurer Cynthia Woodburn, Secretary Mark Hamner, and Historian Pete Skoner on the National Council. They work hard, but do so cheerfully and professionally. We owe a special debt of gratitude for Rhonda's work in organizing this convention. It has also been a great privilege to serve with Chip Curtis, who does a wonderful job as Pentagon editor, and Don Tosh, past president, business manager of the Pentagon. Don is a good friend and tremendous source of encouragement. The Regional Directors (Kevin Charlwood, David Dempsey, Vince Dimiceli, Beth Mauch, and Pedro Muiño) contribute so much to KME, and I am deeply grateful for their service as well. Finally, thank you to all the faculty sponsors and corresponding secretaries who are the heart and soul of KME.

Of course, the most gratifying aspect of this work is the impact that KME has on the lives of thousands of mathematics students nationwide each year. This is what brings us all together, and is the reason why the hundreds of volunteers do what they do year in and year out.

This month marks the 80th anniversary of the founding of Kappa Mu Epsilon. We will withhold our celebration of the "80th birthday" until the 2013 convention, which will mark 80 years since our first KME Convention. We look forward to planning and carrying out an appropriate celebration.

Respectfully,
Ron Wasserstein
National President

Report of the National President-Elect

This is my first report as KME President-Elect. Having been the KME National Secretary for many years, I find learning about KME from another perspective very interesting and enjoyable. I have especially enjoyed the monthly conference calls, among the National Council, that have been made possible by President Ron Wasserstein. The Council has accomplished a lot in the past year, due in part to our frequent conference call meetings.

I will continue my predecessor's habit of reporting here primarily on conventions.

2011 National Convention

Kappa Mu Epsilon's 38th Biennial Convention is being held this weekend, April 14-16, in St. Louis. Our host chapter is Missouri Mu at Harris-Stowe State University. There are 16 chapters in attendance from nine states (Indiana, Kansas, Kentucky, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas). Eighteen students will present papers over the course of the next two days. Eighty-seven people are registered for the convention.

By way of comparison, 16 chapters from nine states (Georgia, Kansas, Maryland, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas) participated in 2009. Sixteen students presented papers Seventy-five people attended.

In 2007, 14 chapters from five states (Kansas, Missouri, New York, Oklahoma, and Tennessee) participated in 2007 in Springfield, Missouri.

In 2005 (Schreiner U., Kerrville, TX), there were 17 chapters from nine states (California, Kansas, Missouri, Michigan, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). There were 15 student presentations.

In 2003 (ORU, Tulsa, OK), there were 19 chapters from 9 states (Iowa, Kansas, Michigan, Missouri, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). Thirteen student papers were presented.

In 2001 (Washburn U., Topeka, KS), there were 20 chapters from 10 states (Colorado, Iowa, Kansas, Kentucky, Missouri, New York, Oklahoma, Ohio, Pennsylvania, and Tennessee)

The following chapters have participated in the last six conventions:

California Epsilon (2005)
Colorado Delta (2001)
Georgia Alpha (2009)
Indiana Delta (2011)
Iowa Alpha (2001, 2003)
Iowa Gamma (2001)
Kansas Alpha (2001, 2005, 2007, 2009, 2011)
Kansas Beta (2001, 2003, 2005, 2007, 2009, 2011)
Kansas Gamma (2001, 2003, 2007)
Kansas Delta (2001, 2003, 2005, 2007, 2009, 2011)
Kansas Epsilon (2001)
Kentucky Alpha (2001, 2011)
Maryland Beta (2009)
Maryland Epsilon (2009)
Michigan Beta (2003, 2005, 2009, 2011)
Missouri Alpha (2001, 2003, 2007)
Missouri Beta (2001, 2003, 2005, 2007, 2009, 2011)
Missouri Iota (2001, 2003, 2005, 2007, 2009, 2011)
Missouri Kappa (2001, 2003, 2005, 2007)
Missouri Mu (2011)
Missouri Theta (2001, 2003, 2005, 2007, 2009, 2011)
New York Eta (2001, 2003, 2005, 2007)
New York Lambda (2003, 2005)
New York Omicron (2005, 2009, 2011)
New York Rho (2011)
Ohio Alpha (2001)
Oklahoma Alpha (2003, 2007)
Oklahoma Gamma (2001, 2003, 2007)
Oklahoma Delta (2001, 2003, 2005, 2007, 2009, 2011)
Pennsylvania Lambda (2003, 2009)
Pennsylvania Mu (2005, 2009, 2011)
Pennsylvania Theta (2001)
Tennessee Gamma (2001, 2003, 2005, 2007)
Texas Gamma (2003, 2005, 2009, 2011)
Texas Mu (2003, 2005, 2009, 2011)

Thus, in the 2000's, 35 different chapters have participated. Six have participated in all six conventions. Two chapters are participating for the first time this year.

At the 2009 convention, the process for submitting papers was changed. Students were only responsible for having submitted by the deadline an abstract and a certifying letter from a faculty advisor. A full written paper is no longer required. This change was based on observation of the Pi Mu Epsilon conventions, and was made to facilitate more student paper submissions. The change seems to have helped increase the number of presentations, however, the number of submissions to The Pentagon may have decreased. The process will be reviewed again at the end of this convention.

This year the AMS and the ASA both contributed $500 each toward convention expenses. In addition Texas Instruments donated a graphing calculator, which will be given for the "People's Choice Award."

Ann Podleski of Missouri Mu at Harris-Stowe State University has done a wonderful job of making preparations for the convention and I am very grateful for dedication and hard work in making campus arrangements for us. In addition, she invited well-known mathematician and author Steven G. Krantz to be the Friday evening banquet speaker and we are so grateful that he accepted the invitation.

Regional Conventions

Two regional conventions were held in the spring of 2010. The North Central Regional Convention was hosted by my home chapter, Missouri Beta, at the University of Central Missouri. Ten student presentations were made. Eight chapters from Kansas and Missouri participated and 57 people attended.

The 2010 KME Northeast Regional Convention was held Saturday, April 10, 2010. The convention was hosted by New York Omicron at St. Joseph's College in Patchogue, NY.

Rhonda McKee
President-Elect

Report of the National Secretary

Kappa Mu Epsilon, National Mathematics Honor Society initiated 2,894 new members in 132 chapters during the 38th Biennium that ended April 14, 2011. This represents an increase of 450 new members compared to the last biennium. Twenty-Two active chapters did not report any initiates during the 38th biennium. The total membership of KME is 78,421.

As National Secretary, I receive all initiation reports from chapters, make a record of those reports, up-date mailing list information for corresponding secretaries and forward copies of the reports to other officers. At the beginning of each new biennium, I prepare a new KME brochure. During an academic year, I send out supplies to each chapter. The supplies include information brochures and membership cards. When a college or university petitions for a new chapter of KME, I send out a summary of the petition, prepared by the president, to each chapter and receive the chapter ballots.

Mark Hamner
National Secretary

Report of the National Historian

It has been a pleasure to serve as National Historian of Kappa Mu Epsilon National Mathematics Honor Society for two years now since I was elected at the 37th National Convention in Philadelphia. I have enjoyed serving with the many people who communicate with the office on a regular basis including national and regional officers, and corresponding secretaries. I also enjoy contributing in a small way to an organization of excellence for the most recent two of its 80 years of history.

The primary function of the national historian continues to be soliciting, collecting, maintaining and compiling records of chapter activities, installation of new chapters, and other society activities that have historical significance. Most of these records are gathered from individual chapters, who receive several electronic mail requests beginning in January and May of each year asking for a report of chapter activities from the previous semester. With the low cost and ease of group distribution messages, several requests are sent each semester. Each message sent typically results in additional reports being received. An additional and unexpected benefit is that when messages bounce back, it provides the opportunity to contact the chapter and learn if the corresponding secretary has changed; this helps maintain communication between the national society and the local chapters, and hopefully helps maintain the local leadership for each chapter. The work of the National Historian is impossible without the aid of the corresponding secretaries for each chapter. Thank you for all that you do in serving the students at your institution, your local Kappa Mu Epsilon chapter, and the national organization. I look forward to continue to work with you in the future.

During the past biennium, 77 of the active chapters responded at least once to the chapter news request. Special mention goes to the following 16 chapters for their cooperation in responding to all four inquiries: AL Zeta, IA Alpha, IL Zeta, KS Beta, KY Beta, MD Delta, MD Epsilon, MI Epsilon, MO Alpha, MO Theta, MO Nu, NC Zeta, PA Kappa, PA Mu, TX Mu and WI Gamma.

A special thank you also goes to the editor of The Pentagon, Chip Curtis of Missouri Iota chapter. He is very easy to work with, and accommodates the Chapter News section twice each year.

The national historian has also had the pleasure of serving as the installing officer for two new chapters. The Rhode Island Alpha chapter was installed at Roger Williams University in Bristol on November 13, 2009, and the Massachusetts Beta chapter was installed at Stonehill College in Easton last week on Friday, April 8, 2011. Helping the society grow and

involve more students and faculty in mathematics activities gives an opportunity to extend the mission of the honor society.

The national historian had the opportunity to represent Kappa Mu Epsilon at the board meeting of Pi Mu Epsilon at the MAA Mathfest, held in August, 2010 in Pittsburgh, PA, serve as a judge for student papers, and sit as a member of CUSAC (Committee on Undergraduate Student Activities and Chapters) of the MAA. It was an excellent opportunity to share and exchange ideas with the other mathematics honor society.

The national historian participated with the other national officers in revising the Chapter Handbook, in particular the section on Chapter Activities. The section provides examples of many service, academic, and fun activities that chapters can adopt. To look for even more ideas, spend some time reading chapter news reports in older editions of The Pentagon.

Since this year is the 80th Anniversary of the KME, the historian and other national officers updated the KME History and Information booklet from the 60th Anniversary Edition that was distributed at the 1991 national convention by then Historian Mary S. Elick. The booklet will be available on the national home page.

With the gradual transition of many society activities and processes away from paper and domestic mail to electronic communications, the society should investigate the establishment of an archive for historical documents and records. Preliminary discussions have begun at the National Council level and are expected to continue in the new biennium. Moving The Pentagon to an online format, and with all the hard work of Don Tosh and other national officers in making past issues of the journal available on the home page, is quite an accomplishment for the organization, and a step in the direction toward electronic record keeping and storage.

Thank you all for the opportunity to serve you and this outstanding organization for two years, and I look forward to continuing to work with you. This is especially true to the veteran national officers who welcomed me to the national council two years ago and have been all too accommodating in helping me learn the duties of the office. So, thank you to President Pythagoras Ron Wasserstein, President-Elect Euclid Rhonda McKee, Secretary Diophantus Mark Hamner, and Treasurer Newton Cynthia Woodburn. By the way, the legacy names are borrowed from the 80th Anniversary history.

Peter R. Skoner,
National Historian

Report of the National Treasurer

38th Biennium (March 11, 2009 – April 14, 2011)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows end-of-biennium assets of $70,366.56. The Cash Flow Report shows that we had an asset gain of $3,289.05 during the biennium. A National Council goal to maintain an asset base of at least $40,000 has been met.

BIENNIUM ASSET REPORT

Total Assets (March 10, 2009)		$67,002.51
Current Assets		
Kansas Teachers Community Credit Union		
Checking	38,110.76	
Share Account	2,255.80	
CD15229	10,000.00	
CD15261	10,000.00	
CD15288	10,000.00	
Total Current Assets		$70,366.56

BIENNIUM CASH FLOW REPORT

Receipts		
Initiation fees received	57,880.00	
Installation fees received	675.00	
Interest income	1,404.22	
Nat. Conv. Reg. Philly	4,585.00	
Gifts & misc. income	531.50	
Total Biennium Receipts		$65,075.72
Expenditures		
Association of College Honor Soc	849.00	
Administrative expenses	4,780.68	
National Convention expenses	31,990.61	
Regional Convention expenses	448.00	
Council Meetings travel	0.00	
Certificates, jewelry & shipping	9,631.04	
Installation expenses	176.05	
Overpayment returns	100.00	
Pentagon expenses	13,811.29	
Total Biennium Expenses		$61,786.67
Biennium Cash Flow		$3,289.05

The cash flow last biennium (07-09) was -$1,647.78. This biennium we initiated 450 more members and installed 7 more new chapters than last biennium. Pentagon expenses dropped by more than 50% since the last biennium and should drop even more with the move to electronic publication. There were no National Council Meeting expenses this biennium due to the move to having monthly conference calls as opposed to annual meetings. With lower interest rates, our interest income dropped. National convention expenses were much higher than in the past due to the venue. We also received a generous gift of $500 from the American Mathematical Society and another is on the way from the American Statistical Association to help with convention expenses for the 2011 national convention.

We have maintained our goal of maintaining assets of at least $40,000. The financial condition of Kappa Mu Epsilon is sound.

I want to offer sincere thanks to the dedicated, talented, hard-working professionals of the National Council with special thanks to Don Tosh for his efforts to lower Pentagon expenses. Without their help, support, and commitment to keep expenses low, especially in such tough economic times, the Treasurer's job would be much more difficult. In addition, a big thanks to the work of the corresponding secretaries who maintain such a vital role in Kappa Mu Epsilon.

Cynthia Woodburn
National Treasurer

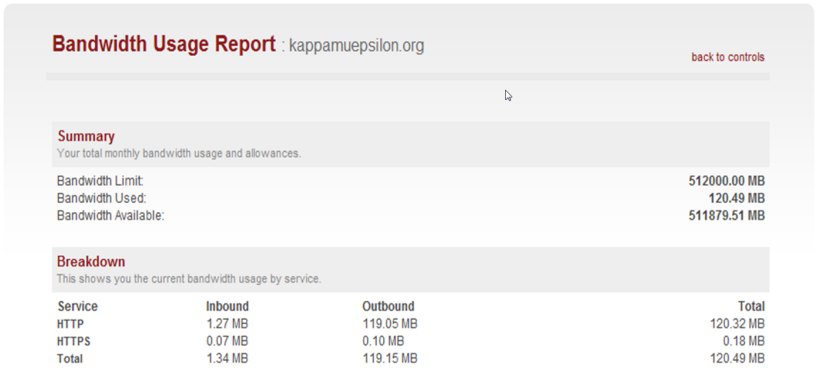
Report of the Webmaster

1. Architectural Improvements
   - Created the subdomain pentagon.kappamuepsilon.org
   - Split large webpages into smaller pieces where logical
     * Moved families of pages into subfolders
     * Removed anchor tags (CSS causing unexpected font style changes)
     * Moved The Pentagon to a new location to support the subdomain
   - Began moving common PHP and JavaScript functions to a shared repository
     * Reduces duplication
     * Improves maintainability
     * Improves site consistency
   - Increased usage of the ssh login
     * Faster updates to the website
     * Ability to generate simlinks (integrated pointer to a file)
   - Moved some dead code to archives
     * Will be removed when I can determine the code is indeed never accessed

2. Functional Improvements
   - Added a search feature to look up past issues of the Pentagon based on Table of Contents key word search
     * Not all links in results will match the PDF (links generated dynamically)
     * Search only available from data provided in the TOC
   - Added an auto generated password to account creation
   - Added a "recovery email" setting to quickly recover account details for users
   - Added an RSS feed
     * Updates on hold until announcements made at convention
     * Will generally duplicate data found on the home page (News)
   - Added a Links page

    * Updated Links page to remove dead links
  - Added a Mach Award page

3. Server Migration
  - Corrected email forwarding
  - Corrected database connection issues
  - Corrected "upload" email information and server configuration



Bandwidth Usage Report : kappamuepsilon.org

back to controls

Summary

Your total monthly bandwidth usage and allowances.

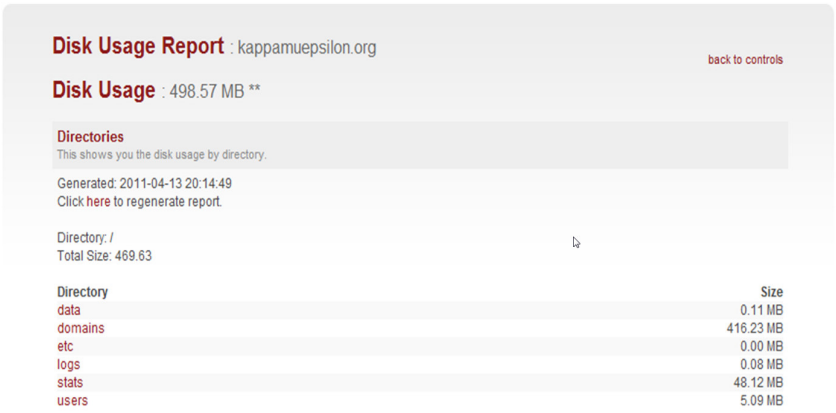
Bandwidth Limit:	**512000.00 MB**
Bandwidth Used:	**120.49 MB**
Bandwidth Available:	**511879.51 MB**

Breakdown

This shows you the current bandwidth usage by service.

Service	Inbound	Outbound	Total
HTTP	1.27 MB	119.05 MB	120.32 MB
HTTPS	0.07 MB	0.10 MB	0.18 MB
Total	1.34 MB	119.15 MB	120.49 MB

MediaTemple.net Bandwidth Usage Report



Disk Usage Report : kappamuepsilon.org

back to controls

Disk Usage : 498.57 MB **

Directories

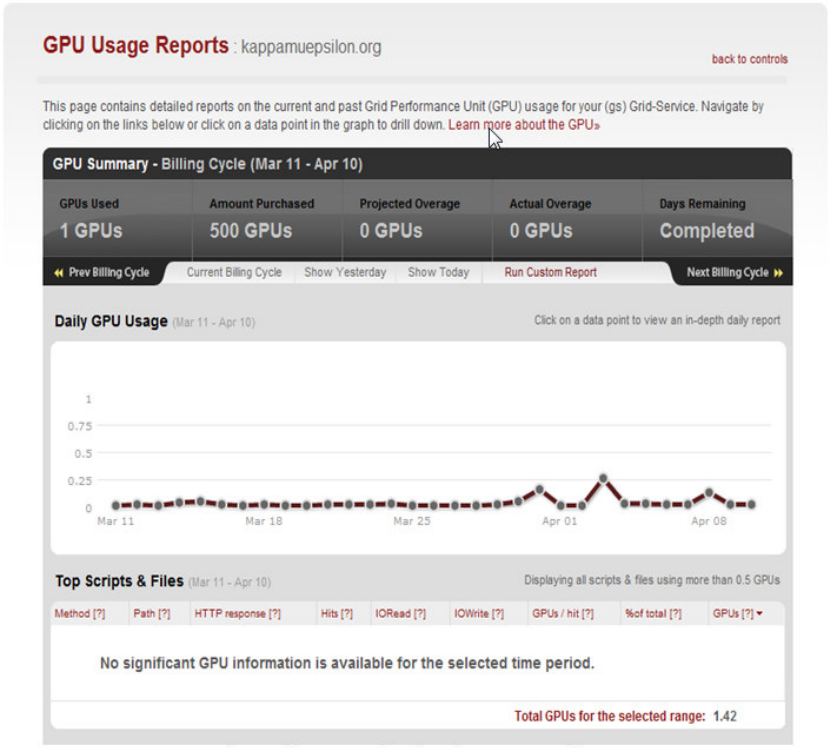
This shows you the disk usage by directory.

Generated: 2011-04-13 20:14:49
Click here to regenerate report.

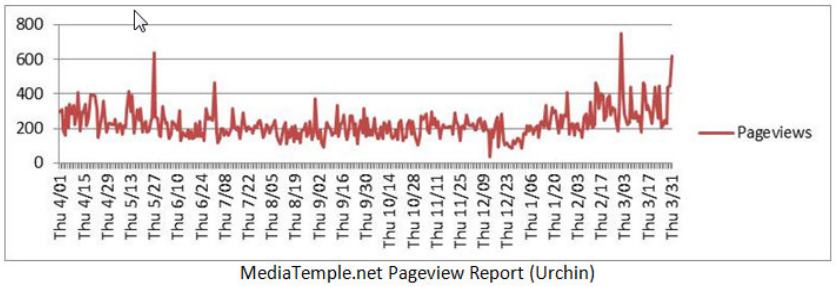
Directory: /
Total Size: 469.63

Directory	Size
data	0.11 MB
domains	416.23 MB
etc	0.00 MB
logs	0.08 MB
stats	48.12 MB
users	5.09 MB

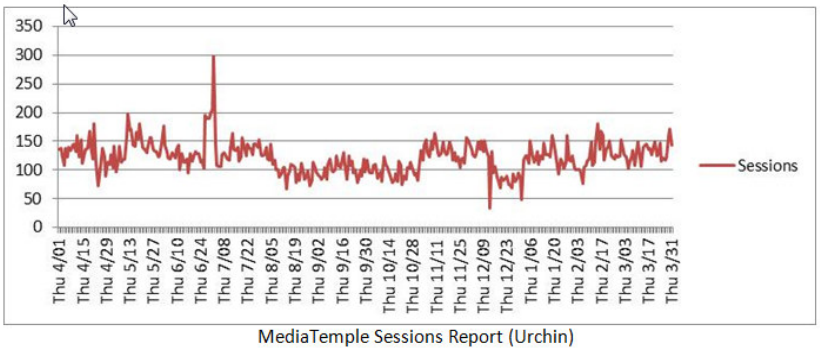
MediaTemple.net Disk Usage Report



MediaTemple.net GPU (Grid Performance Unit) Report



MediaTemple.net Pageview Report (Urchin)



MediaTemple Sessions Report (Urchin)

Report: Summary - Site113927
Date Range: 04/01/2010 - 04/13/2011

Total Sessions	46,205.00
Total Pageviews	88,704.00
Total Hits	140,479.00
Total Bytes Transferred	9.30 GB
Average Sessions Per Day	122.24
Average Pageviews Per Day	234.67
Average Hits Per Day	371.64
Average Bytes Transferred Per Day	25.20 MB
Average Pageviews Per Session	1.92
Average Hits Per Session	3.04
Average Bytes Per Session	211.14 KB
Average Length of Session	00:02:18

MediaTemple Traffic Summary Report (Urchin)

Report of the Pentagon Business Manager, April 2011

I took over as business manager of The Pentagon in December, 2006. My first issue was the Fall, 2006 issue (Volume 66, Number 1). This past biennium contains reports for issues from Spring, 2009 (Vol 68 Num 2) to Fall, 2010 (Vol 70 Num 1).

This past biennium has brought a momentous transformation to The Pentagon. After the printing and distribution of the Spring 2010 issue, the national council polled chapters about whether The Pentagon should go entirely electronic. The vote was affirmative, and as of this past fall, The Pentagon is now being provided electronically on our website, free of charge, to anyone with an interest in reading it. There were a small number of paid subscriptions, less than 100, that we decided to honor by printing and mailing a very limited number of hard copies. Printing and mailing a small number of issues is much more expensive per issue, but since there were so few issues involved, the overall effect has been a dramatic reduction is printing and mailing expenses. The effect can be seen in the receipt/expense table below. Costs have dropped from approximately $4500/issue to what will become $400/issue or less.

Along with the move to provide new issues on the KME website, I volunteered to scan old issues into electronic format if the executive would provide funding for a high speed scanner. They did and I did and as of now virtually the entire collection of past issues of The Pentagon has been scanned and is presently available online. I was going to make an appeal for copies of the few issues that I did not have but Chip Curtis, The Pentagon editor, was able to find the missing copies and the online collection should be complete by this summer. The online collection gives us a remarkable resource for examining the history of KME. I am working with the webmaster on an exhaustive keyword and author index that will allow easy access to article and author details.

Speaking of Chip Curtis, it has been a pleasure working with him and I look forward to helping him in the future, even as the role of business manager becomes less demanding and important than it has been in the past. I would also like to thank Kevin Reed, our webmaster, for the extra effort he has put forth in this process.

Don Tosh
Business Manager, The Pentagon

Pentagon receipts/Expenses 4/01/09 - 3/31/11

Beginning Balance 4/01/09	**$790.06**	**Ending balance 3/31/11**	**$1,249.00**
Receipts		**Expenses**	
Vol 68 Num 2 S09 printing	$2,784.55	Vol 68 Num 2 S09 printing	$2,784.55
Vol 68 Num 2 S09 Post/Hand	$1,318.33	Vol 68 Num 2 S09 Post/Hand	$1,318.33
Vol 69 Num 1 F09 printing	$3,194.12	Vol 69 Num 1 F09 printing	$3,194.12
Vol 69 Num 1 F09 post/hand	$1,368.76	Vol 69 Num 1 F09 post/hand	$1,368.76
Vol 69 Num 2 S10 printing	$2,915.09	Vol 69 Num 2 S10 printing	$2,915.09
Vol 69 Num 2 S10 post/hand	$1,497.79	Vol 69 Num 2 S10 post/hand	$1,497.79
Vol 70 Num 1 F10 printing	$348.39	Vol 70 Num 1 F10 printing	$348.39
Vol 70 Num 1 F10 post/hand	$0.00	Vol 70 Num 1 F10 post/hand	$138.79
Subscriptions	$867.50	Foreign/misc postage	$269.77
Total Receipts 4/01/09 - 3/31/11	**$14,294.53**	**Total expenses 4/01/09 - 3/31/11**	**$13,835.59**

Report of the Audit Committee

Audit Committee Members

- Philip Lombardo, New York Omicron, Chair
- Al Riveland, Kansas Delta
- David Creech, Michigan Beta
- Ryan Whaley, Kentucky Alpha
- Jessica Booth, Kansas Alpha

Audit Process

1. Prior to the national convention, Treasurer Cynthia Woodburn mailed biennium financial summary data to Philip Lombardo, the committee chair. At the convention, treasurer Woodburn provided the audit committee with biennium summary and detail documentation of receipts and payment transactions, as well as bank and savings account reconciliation documentation.
2. Prior to the national convention, Philip Lombardo contacted the Kansas Teachers Community Credit Union. He discovered that the account balances for the Kappa Mu Epsilon checking and savings accounts exactly match the totals from Treasurer Woodburn's biennium reports.
3. At the national convention, committee members spoke to President Ron Wasserstein and Secretary Mark Hamner to determine their impressions of the accuracy and completeness of the recording throughout the biennium. The committee examined the financial documentation provided by Treasurer Woodburn.

Recommendations

1. Information forwarded by the Treasurer to the committee chair prior to the national convention provides the opportunity for verification of assets in a careful manner and should be continued in the future.
2. The internal checks built into the regular financial processing between the Treasurer and the President and Secretary provide an important safeguard to the integrity of the office of the Treasurer and help avoid the necessity of an expensive external audit. These ongoing internal audit processes should be continued and updated by the National Council as needed.
3. Finally, the Audit Committee recommends the acceptance of the financial records and reports of Kappa Mu Epsilon for the 2009-2011 biennium as presented by the Treasurer Cynthia Woodburn.

Commendations

1. The committee commends Treasurer Woodburn for her exemplary maintenance, management and presentation of the financial records and for her generous donation of time as Treasurer for Kappa Mu Epsilon.
2. We further commend her for her valuable input to this Audit Committee and her helpful guidelines for the Audit Committee.
3. The committee commends President Ron Wasserstein, Secretary Mark Hamner and Treasurer Cynthia Woodburn for the manner in which they communicate and cooperate to maintain the internal checks which preserve the integrity of the office of Treasurer.
4. The committee commends the work of the previous audit committees and gratefully received reports provided by the Treasurer.

Report of the Resolutions Committee

The Resolutions Committee consisted of Amy Billups, student member from the Missouri Beta chapter; Seulki Kim, student member from the Kansas Beta chapter; Anna Lischke, student member from the Kansas Delta chapter; Rosemary Sherwood, student member from the Missouri Theta chapter; and Pedro Muíño, faculty member from the Pennsylvania Mu chapter, who acted as chair. The committee proposed the following resolutions.

Whereas Kappa Mu Epsilon has been holding its 38th Biennial Convention celebrating the 80th anniversary of Kappa Mu Epsilon on the beautiful and historic campus of the Harris-Stowe State University in the parabolic city of St. Louis and whereas the convention has been both a wonderful and instructive experience for all of us, be it resolved that we express appreciation:

$-e^{i\pi}$ To the host chapter, Missouri Mu; its student leaders, Cheryl Moonier, Tyra Scott and Rico Smith; its faculty sponsor, Dr. Ann Podleski; to the members of the Mathematics Department; to Dr. Lateef Adelani, College of Arts & Sciences Dean; to the university president, Dr Henry Givens Jr., for his inspirational welcome address; and to the administration of Harris-Stowe State University, for their Midwestern hospitality and efficient organization. They have been vital to the success of this national convention.

$\log(100)$ To each of the national officers of Kappa Mu Epsilon for the many hours of service they have contributed to the herculean tasks of preparing for and running the convention.

integer(π) To mathematician and author Dr Steven G. Krantz for his brilliant and riveting presentation on centroids as the banquet keynote speaker.

2^2 To the various committees (Nomination, Auditing, Awards, and, somehow recursively, Resolutions) who worked so diligently both before and during this convention to ensure its success.

$(2\phi - 1)^2$ To the twenty students who presented eighteen papers at this convention. We are all more educated after having attended those fantastic presentations.

$3!$ To Ron Wasserstein for his service as president-elect for four years and as president for two; and to Rhonda McKee, who has served as president-elect for two years. We wish them continuous success at the helm of this society.

$\sqrt{49}$To Cynthia Woodburn and Mark Hamner, who have completed their terms as national Treasurer and Secretary respectively and have graciously agreed to serve another cycle.

Respectfully submitted.
Pedro Muíño, chairperson, on behalf of the resolutions committee

Kappa Mu Epsilon News

Edited by Peter Skoner, Historian

Updated information as of October 2011

Send news of chapter activities and other noteworthy KME events to

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Saint Francis University
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Installation Report

Arkansas Alpha
Henderson State University

The Arkansas Beta Chapter of Kappa Mu Epsilon was installed at 3:00 P.M. on Monday, October 10, 2011, at a ceremony in the Newberry House on the campus of Henderson State University, in Arkadelphia, Arkansas. The meeting was conducted by faculty member Mr. David Gardner, who was initiated into the Arkansas Alpha chapter of KME as a student. KME President –Elect Rhonda McKee served as the Installing Officer. The eleven charter student members included Amy Benzi, Jones Carlson, Stefanie Davis, Kayla Hood, Constance Jones, Leonard Lawson, Devonta Morrison, Kayla Morrison, Kristina Stephens, Jeannie Toole, and James Edward Turner III, and the five charter faculty members included Debra Coventry, Carolyn Eoff, Duane Jackson, Michael Lloyd, and Holly Morado. One charter student member, Trae Warner, and one charter faculty member, Meredith Wright, could not attend. The first officers of Arkansas Beta, President James Turner, Vice President Kristina Stephens, Recording Secretary Stefanie Davis, Treasurer Jones Carlson, Corresponding Secretary Dr. Fred Worth, and Faculty Sponsor Dr. Carolyn Eoff, were installed during the ceremony. The initiates were welcomed by Mr. Bobby Jones, interim president of Henderson State University. About 30 people were in attendance. After the formal ceremonies, Rhonda McKee presented a talk entitled “Exploring the Fourth Dimension.”

Chapter News

AL Alpha – Athens State University

Chapter President – Lauren Atkins; 240 Current Members and 10 New Members

Other Spring 2011 Officers: Kayla Usery, Vice President; Dane Glover, Secretary; and Patricia Glaze, Cor. Sec. and Faculty Sponsor

The Alabama Alpha Chapter of KME at Athens State University will assist the Math and Computer Science (MACS) Club with their fundraiser at the Old Time Fiddler's Convention, held annually on the ASU campus. The money raised will go toward scholarships.

New Initiates - Lauren Renae Atkins, Shaun Bardell, Dane P. Glover, Treyla Shea Hardman, Brooke Allison McCain, Derek Newland, Elizabeth Pounders, Jennifer Ratley, Kayla Usery, and Jacob Walker.

AL Zeta – Birmingham-Southern College

Chapter President - Rebecca Terry; 40 Current Members and 19 New Members

Other Spring 2011 Officers: Amy Schumacher, Vice President; Stephanie Gossett, Secretary; and Bernadette Mullins, Treasurer and Cor. Sec.

What are Catalan numbers? And what do they have to do with the World Series? Dr. David Foreman of Samford University, KME Spring Guest Lecturer, helped us investigate these questions. A crowd of nearly 30 was surprised to learn that Catalan numbers can lend insight to how long we would expect the World Series to last.

New Initiates - Keenan Wayne Bailles, Arpan Bosmia, Wilson Stone Brummal, Caroline Merritt Crawford, Winston Murphree Crute, Yokeca Nicole Goff, Artem Mikhailovich Joukov, Danielle Marie Mayer, Callie Marie McGrath, Jennifer Auburn McKannan, Jacob Nicholas Plant Plagens, Huda Qureshi, Lauren Jean Reibe, Zachary James Richards, Virginia Anne Seale, Melanie Ann Short, Nino Christopher Cumba Yu Tiamco, Gaines Foster Yeilding, and Mengheng Zhou.

AL Eta – University of West Alabama

Corresponding Secretary - Hazel Truelove

New Initiates – Rita Bonner, Allison Harrod, Rebecca Murray, Kalynn Pruitt, and Zachary Long Riley.

AL Theta – Jacksonville State University

Chapter President – Brandy Greenleaf; 40 Current Members and 23 New Members

Other Spring 2011 Officers: Noel Overton, Vice President; Katie Henson, Secretary; Tyler Gable, Treasurer; and Dr. David Dempsy, Corresponding Secretary and Faculty Sponsor

The AL Theta Chapter has been holding regular monthly meetings throughout the school year. On March 1, 2011, the Alabama Theta chapter initiated 23 new student members. The afternoon ceremony was held on the 11th floor of Houston Cole Library. During the April meeting, new officers were elected and a graduation social event was planned for April 25. We hosted a graduation party and game night on April 25 at the home of Dr. Jan Case. Officers will be meeting in September to plan events for 2011-2012. Fall meeting times have been set for second Thursdays of the month.

CA Delta - Cal Poly Pomona

Corresponding Secretary - Patricia Hale

New Initiates - Karen Amagrande, Jose Ayala, Jerica Banares, Peter Banda, Brittany Camp, Israel Castro, Mayra Cervantes, Hannah Croft-Seidler, Cecilia Eriksson, Michael Grigsby, Arely Herrera, Jonathon Henry, Eric Ketchum, Hayk Kotelyan, Erica Puhawan, Eren Scott, Valerie Torpey, Jacqueline Trendt, Malakai Unland, Daisy Vasquez, and Stephanie Zajac.

CO Beta – Colorado School of Mines

Corresponding Secretary - Terry Bridgman

New Initiates – Oscar Aguilar, Harry Boyd, Colin Butler, David Chiavetta, Sara Clifton, Michael Firmin, Amy House-Thomas, Gregory Johnson, Sydney Liming, William McCollom, Connor McDonald, Tiffani Oney, Michael Paris, Jennifer Schrant, Erin Stephens, Steven Strong Sean Takahashi, and Eric Threet.

CO Delta – Mesa State College

Corresponding Secretary - Erik Packard

New Initiates – Caitlin Anderegg, Clinton Anderson, Brandon Bearden, Brittney Bristol, Lisa Driskell, Caitlin Heath, Michael Langston, Patricia Sanger, and Peter Schulze.

CT Beta - Eastern Connecticut State University

Corresponding Secretary - Christian Yankov; 14 New Members

Other Spring 2011 Officer: Mizan R. Khan, Treasurer

New Initiates - Colton Alexander, Maria Altieri, Daniel Arsenault, William Barrett, Adam Chase, Taylor Deguzis, Kristen Fitzsimons, Michael Ingram, Samantha LaPointe, Amy Laroux, Brian Murray, Antony Pizzoferrato, Joseph Saad, and Stephanie Smith.

FL Gamma – Southeastern University

Corresponding Secretary - Dr. Berhane T. Ghaim

New Initiates - William Erman, Camille Hines, Mackenzie Messenger, Cody Tessler, and Nathan Windhorst.

GA Alpha – University of West Georgia

Corresponding Secretary - Scott Sykes

New Initiates - Matthew Collins, Dorsey Gay, Tessa Gromoll, Keely Jones, Roger Lascorz, Esther Machado, and Patrick McAuliff.

GA Delta – Berry College

Corresponding Secretary - Ron Taylor

New Initiates - Suzanne Margaret Adkins, Anisa Ali, Calvin Blake Bannister, Kileen Arin Berry, Suzana Pierina Bowling, Allison Clare Byrd, Seth Jacob Camp, Melissa Ann Cook, Kenneth Terrell Henderson, Ashley Ann Holland, Christina Anna Le, Allison Danielle LeFeuvre, Justin Lee Lyle, Sheena Kirstin MacGowan, Jennifer Kay Massengale, Michael Stewart McCleod, Joshua Daniel Morris, Aaron Jacob Ostrander, Rachel Layne Stewart, Jonathan David Surls, Kari Michelle Theobald, Casey Earl Underhill, and Randoll Johnson Vick, Jr.

IA Alpha – University of Northern Iowa

Chapter Pres. – Jaime Zeigler; 35 Current Members and 9 New Members
Other Spring 2011 Officers: Tristam Nebelsick, Vice President; Kelsey Staudacher, Secretary; David Rygh, Treasurer; and Mark D. Ecker, Corresponding Secretary and Faculty Sponsor

Our first spring KME meeting was held on February 16, 2011 at Professor Mark Ecker's residence. Our second meeting was held on March 23, 2011 at Professor Syed Kirmani's residence where student member Jaime Zeigler presented her paper on "Variables Affecting Crime Rate." Student member David Rygh addressed the spring initiation banquet with "Individual Batting Factors Influencing a Team's Probability of Winning." Our banquet was held at Godfather's Pizza in Cedar Falls on April 27, 2011 where nine new members were initiated.

New Initiates – Michelle Burow, Adam Feller, Kara Gaul, Wesley Keene, Katherine Pearce, Logan Smith, Caitlin Stewart, Lisa Stoecken, and Adam Vaught.

IA Gamma – Morningside College

Corresponding Secretary - Eric Canning

New Initiates - Amy Elizabeth Baer, Fernando Franco Jr., Travis Lee Hanson, Emily Rose Lechtenberg, Stephanie Anne Norton, Rachel Justine Penning, Amanda May Von Ehwegen, and Jacob Mark Widman.

IA Delta - Wartburg College

Chapter President – Justin Denner; 34 Current Members and 19 New Members
Other Spring 2011 Officers: Daniel Mysnyk, Vice President; Andreanna Hoth, Secretary; Brian Birgen, Corresponding Secretary; and Dr. Terry Letsche, Faculty Sponsor

In March, nineteen new initiates were welcomed at our annual banquet and initiation ceremony. Our speaker was Michael Burk, a 1977 Wartburg Alum and math major, who is the Bishop for the southeastern Iowa Synod of the Evangelical Lutheran Church of America. In May, together with the Physics and Computer Science clubs, we hosted the departmental end of the year picnic. There were other social meetings throughout the year.

New Initiates - Justin Denner, Justin M. Heine, Andreana Gayle Hoth, Michael Chris Jorgensen, Jonathan E. Juett, Andrew Krehbiel, Anastasia J. Lundt, John Mallen, Erin Marshall, Nicholas Mercurio, Daniel Mysnyk, Allyssa K. Neighbor, Jamie Lynn Peterson, Stephanie Reiter, Derek Karl Wittenburg Schwanz, Neil Shields, Katherine M. Tjeerdsma, Zhijian Wang, and Adam Weber.

IL Zeta – Dominican University

Chapter President - Kim Plesnicar; 35 Current Members and 12 New Members

Other Spring 2011 Officers: Daniel Dziarkowski, Vice President; Eva Mehta, Secretary; Lisa Gullo, Treasurer; and Aliza Steurer, Cor. Sec.

KME operates as one student organization at Dominican University. KME/ Mathematics Club served free pie on Pi Day, organized the game "Who Wants To Be a Mathematician" during Dominican University's International Week, and they held the KME Initiation Ceremony on April 20, 2011.

New Initiates - Stephanie Ciura, Daniel Dziarkowski, Valerie Grano, Charles Harmata, Safia Jilani, Andrew Kurber Khadijah Muhammad, Jackelyn Nagel, John Pontikis, Dr. Sara Quinn, Jennifer Russ, and Samantha Spitzock.

IL Theta – Benedictine University

Corresponding Secretary - Dr. Thomas Wangler

New Initiates - Mirza Baig, Rikki D'Angelo, Reema Khatri, Vanessa Mensie, Natalia Poniatowska, Edward Price, and Sandra Tovalin-Schmidt.

IL Iota – Lewis University

Corresponding Secretary - Margaret M. Juraco

New Initiates - Kailey Allen, Ashley Cychosz, Elizabeth DeWaard, Alexandra Hahn, Feda Abu-Mallouh, Lauren McCarthy, Levi Moffett, Ron Jovi Ramirez, Megan Scara, Kelly Tillman, and Sara Tippy.

IN Alpha – Manchester College

Corresponding Secretary - Jim Brumbaugh-Smith

New Initiates - Melissa Bowman, Jessica Fauser, Alex Hokanson, Shayla Shirk, and Kameron Troxell.

IN Beta – Butler University

Chapter President – Sara Prusinski; 20 Current Members

Other Spring 2011 Officers: Ashley Drees, Vice President; Kristen Allen, Secretary; Eric Buenger, Treasurer; and Dr. Amos Carpenter, Corresponding Secretary and Faculty Sponsor

KME hosted two speakers Dr. Daniel Pritikin on Variations of the "Lights Out" Game on April 20, 2011 and Dr. William Fenton on April 25, 2011 on Some Mathematics in the Music. Both speakers were funded by the J. James Woods Lectures.

IN Delta - University of Evansville

Corresponding Secretary - Adam Salminen

New Initiates - Kathryn A. Buckner, Wolfe S. Greene, William C. Hoskins, Kenneth T. Kaufmann, Dylan E. Kleeman, Nicole E. Loehr, Desire L. Matouba, Kepra L. McBrayer, Sarah Beth McNeely, Deanna Michelle Miller, Samantha M. Pitt, Edgar F. Qualkenbush, Katelyn A. Spainhaur, Amber M. Thomas, and Allison A. Throm.

KS Alpha – Pittsburg State University

Chapter President – Vanessa Peach; 12 Current Members and 35 New Members

Other Spring 2011 Officers: Jordan Jameson, Vice President; Aisha Ford, Secretary; Wes Brook, Treasurer; Dr. Tim Flood, Corresponding Secretary; and Dr. Cynthia Woodburn, Faculty Advisor

The Kansas Alpha KME Chapter continued to have monthly noon meetings with pizza. In February, Dr. Cynthia Woodburn presented "Flatland meets Spaceland" and then students put together zonohedron sliceforms out of card stock. Vanessa Peach gave a talk entitled "Outstanding Mathematicians in Greece" at the March meeting. "Application of Linear Algebra to Three-Dimensional Computer Graphics" was presented by Jae Min Lee in April. The chapter celebrated Pi Day on March 14 by eating pizza pies and watching "The Great Pi/e Debate" video. Activities for the semester concluded with our annual KME Ice Cream Social in early May at the home of retired PSU mathematics professor Dr. Elwyn Davis.

KS Beta - Emporia State University

Chapter President – Yuying Tsao; 10 New Members

Other Spring 2011 Officers: Lezley Lawson, Vice President; Keely Grossnickle, Secretary; Yusuke Suita, Treasurer; and Dr. Connie Schrock, Corresponding Secretary and Faculty Sponsor

Spring semester KME attended the Biennial convention in St. Louis. One member, Yusuke Suita, presented a paper. We celebrated PI day with pie and many different activities. The end of the semester closed out with an all department picnic hosted by KME.

KS Delta – Washburn University

Chapter President – Kristen Beall; 26 Current Members and 15 New Members

Other Spring 2011 Officers: Markie Gallagher, Vice President; Lisa Wade, Secretary and Treasurer; Mike Mosier, Corresponding Secretary; and Dr. Kevin Charlwood, Faculty Sponsor

Kansas Delta held our annual initiation banquet on February 28, and initiated 15 new members. We took 5 students and 3 faculty to the National Convention in St. Louis and had a great time. Andrew Lake, Sean Van Dyke, and Michael Reb all presented papers, and Andrew Lake was recog-

nized with a top four award. We held our final meeting of the year in April, and elected new officers for the coming academic year.

New Initiates - Kristen Beal, William Beaton, Donovan Briggs, Markie Gallagher, Tyler Graves, Shuang Jiang, Laura Kinderknecht, Andrew Lake, Ty Lewis, Andrew Lichter, Stephen Littleton, Brandon Marshall, Garrett Mazachek, Seth M. Miller, Riley Piles, Gaspar Porta, Dustin Schnee, Joshua Thomason, Megan Thompson, Lisa A. Wade, Jennifer Wagner, and Haiji Wang.

KS Epsilon - Fort Hays State University

Corresponding Secretary - Jeffrey Sadler

New Initiates - Emma Dreiling, Paul Flesher, Linnea Gustafsson, Emily Lowry, Kaitlyn Paul, and Landon Taylor.

KY Alpha – Eastern Kentucky University

Corresponding Secretary - Patrick Costello

New Initiates - Lois M. Bailey, Moselle A. Christensen, Erik C. Cook, Allison C. Galassie, Amy D. Isaacs, Ethan E. Kilgore, Michael G. Mazzotta, Molly C. Mounce, Melissa H. Pierce, Laura L. Straub, A. Keeley Webb, and Ryan E. Whaley.

KY Beta – University of the Cumberlands

Chapter President – Amy Roberts; 36 Current Members and 9 New Members

Other Spring 2011 Officers: Megan Barrowman Brown, Vice President; Jerrid Neeley, Secretary; Clint Creekmore, Treasurer; Dr. Jonathan Ramey, Corresponding Secretary; and Dr. John Hymo, Faculty Sponsor

On February 18, 2011, the Kentucky Beta chapter held an initiation and a banquet at the Cumberland Inn. Kappa Mu Epsilon initiated eight new student members at the banquet, presided over by outgoing president, Amy Roberts. As an additional feature, senior awards were given by the department at the banquet. On April 28, the chapter initiated one additional student member at the Correll Science Complex. Jointly with the Mathematics and Physics Club, the Kentucky Beta Chapter hosted Dr. Carl Wagner from the University of Tennessee on April 11. He spoke about "Incompleteness, Undecidability, and Mathematics." On April 12, members also assisted in hosting a regional high school mathematics contest, held annually at University of the Cumberlands. On April 26, the entire department, including the Math and Physics Club, Sigma Pi Sigma (Physics Honors Society), and the Kentucky Beta Chapter, held a pizza and game night.

LA Gamma – Northwestern State University

Corresponding Secretary - Leigh Myers

New Initiates - Mary Hebert, Dustin McPhate, Michelle Simon, and Courtenay Stevens.

LA Delta - University of Louisiana at Monroe

Corresponding Secretary - Youssef Dib

New Initiates - Lindsey Alexander, Patrick Atkinson, Emily Bethea, Emily Cooke, Jade Doyle, Robert Esponge, Miranda Gagliano, Megan Guillot, Lauren Hall, Aaron Head, Christopher Maxwell, Badwi Mouawad, Thai Nguyen, Sandeep Pantha, Philip Petit, Jesse Pope, Kory Reeves, Ahmeed Sleem, and Blane Stroud.

MD Alpha – College of Notre Dame of Maryland

Chapter President – Rebecca Pettit; 22 Current Members and 14 New Members

Other Spring 2011 Officers: India Scott, Secretary; and Dr. Margaret Sullivan, Corresponding Secretary and Faculty Sponsor

Mr. Sun Jian, a Ph.D. candidate at The Johns Hopkins University, Baltimore, MD, spoke of his work with deep space pictures from the Hubble Space Telescope at our Initiation meeting March 20, 2011.

MD Beta – McDaniel College

Corresponding Secretary - Harry Rosenzweig

New Initiates - Sarah Holbrook, Brittany Nicholls, Jason Smith, and Taylor Thompson.

MD Delta – Frostburg State University

Chapter President – Joshua Wilson; 28 Current Members and 7 New Members

Other Spring 2011 Officers: Rachel Skipper, Vice President; Jesse Otto, Secretary; Kevin Loftus, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor

After an organizational meeting a few weeks into the spring semester, the Maryland Delta Chapter held its Initiation Ceremony on February 27, 2011. We were very pleased to welcome seven new members. The highlight of the day was a hands-on origami workshop conducted by our department chair, Dr. Marc Michael. Our new members were active participants in our annual Pi-Day Bake Sale which went off very successfully. The Chapter's next activity was to host an early April visit from FSU alumnus Dr. Brendon LaBuz. Dr. LaBuz is a former member of the Delta Chapter and is currently a professor of mathematics at Saint Francis University of the Pennsylvania Mu Chapter in Loretto, PA. He gave a well attended and very interesting presentation on topology as a topic for undergraduate research. Towards the end of April we had another meeting where faculty sponsor Dr. Mark Hughes gave a presentation on involutes, evolutes and Mathematica animations. This meeting also saw the election of new officers for the coming year. Congratulations to our new president Kevin

Loftus, vice president Justin Good and treasurer Marcus Carter. Jesse Otto will continue his good work as secretary. The Maryland Delta Chapter finished the term with an end of semester cookout. We had perfect weather (for a change!) for enjoying a picnic and getting some exercise with a soccer ball! We would like to end this summary by wishing the best of luck to our graduating president Josh Wilson who will soon be starting his career as a mathematics teacher and vice president Rachel Skipper who will start working towards her Ph.D. in Mathematics at SUNY–Binghamton this coming fall.

MD Epsilon – Stevenson University

Chapter President – Rebecca Hollins; 32 Current Members

Other Spring 2011 Officers: Megan Staudenmaier, Vice President; Diane Swale, Secretary; Rachel Buchanan, Treasurer; and Dr. Christopher E. Barat, Corresponding Secretary and Faculty Sponsor

In April, to celebrate Mathematics Awareness Month, the Chapter sponsored a talk on "Solving the Enigma: The History of the Cryptanalytic Bombe" by Jennifer Wilcox of the National Cryptologic Museum, Fort Meade, MD. The Chapter planted a gray birch on the Greenspring campus in the first week of May in memory of the deceased former chair of the Mathematics Department and Chapter member, Dr. Susan Slattery. An adjoining marker will be installed and dedicated at the beginning of the Fall 2011 term.

MI Alpha – Albion College

Corresponding Secretary - Mark Bollman

New Initiates - Abigail Carter, Chen Chen, Nicholas DeVinney, Jacob Engel, Cassandra Labadie, William Spencer, Hollis Williams, Marc Winter, and Jeremy Yu.

MI Delta – Hillsdale College

Chapter President – Kerry Frost; 52 Current Members and 6 New Members

Other Spring 2011 Officers: Juliana O'Neill, Vice President; Jonathan Gregg, Secretary; Meredith Langlois, Treasurer; and Dr. David Murphy, Corresponding Secretary and Faculty Sponsor

The goal of Geek Week is to promote honor societies on campus and to encourage their interaction. It is also a fundraising event for charity. The festivities began with Honorama on April 7, the annual bowling tournament for honoraries on campus. This year's KME team placed second overall, and our secretary, Jonathan Gregg, won the third place trophy for his individual score. The next event of Geek Week was the KME-sponsored Paper Airplane Challenge. Participants could enter a paper airplane constructed at our event into any of three competitions (duration of flight, target/precision flying, and loop-the-loop competitions) for a $1 donation.

Through the Honorama and Paper Airplane Challenge donations, KME raised $46.91 this year, all of which was contributed to Circle K (another campus organization that is the college branch of Kiwanis International). Our other event this semester was our Initiation Ceremony and Picnic, held on April 27.

New Initiates - James Michael Banovetz III, Celia Bigelow, Thomas Currey, Toby Alexander Flint, Jack Isaac Hummel, and Paul D. Schmitt.

MI Epsilon – Kettering University

Chapter President – Jessi Harden (A Section); 192 Current Members and 30 New Members

Other Spring Officers: Ryan McGuire (A Section), President Elect; Bryan Coburn (A Section), Vice President; Derek Hazard and Michael Steinert (A Section) and Shahnoor Amin (B Section), Secretaries; Kasey Simons (A Section), Treasurer; Boyan N. Dimitrov, Corresponding Secretary; and Ruben Hayrapetyan (Section A) and Ada Cheng (Section B), Faculty Sponsors

The Winter 2011 KME Initiation occurred on March 11 with 28 bright Kettering students and two new faculty (Dr. Matthew R. O'Toole and Dr. Daniela Szatmari-Voicu) joined the Michigan Epsilon Chapter. They were welcomed by the current Chapter student and faculty officers, and many parents of the newly initiated students. Dr. Hayrapetyan entertained the entire auditorium with some amazing attractive games from Mathematical Logic in the form of a quiz-competition (Prize Contest). The traditional Pizza/Movie was earlier on 5th Tuesday (February 8) at 12:20 with the movie "Isaac Newton. The Gravity of Genius," that refreshed the memories of all the attendees about most of the achievements of Newton in physics and mathematics, we learn and use today. Our Winter term students returned to campus in the Summer. Then the events continued with the movie "The Story of One," that was an amazing walk in the history how the human beings developed the numeric systems during the centuries, and how and where today's counting systems and symbolic notations are used. In addition was the second movie "Decoding Nazi Secrets," about the contributions of the UK (and International) mathematicians in the secret fights during the World War II. The attention of students was taken by the discoveries of Turing, by the first UK computers created then, by the fate of those mathematicians working in the group. The coming Fall Kettering University will host yet another High School Mathematics Olympiad, organized by the Mathematics Department. The Mathematics Olympiad at Kettering is a competition designed to identify and encourage students with interests and abilities in mathematics. Our goal is to develop the Olympiad into one of the most prestigious mathematical competitions in the region. The ex-

amination is designed for students in grades 9 through 12. However any student working towards a high school degree who is currently enrolled in a public school, private school or a home-school program can sit for the examination. The competition consists of six challenging problems and has a time limit of four hours. The problems range from "mind-benders" that require little mathematical skills to problems that require the knowledge of geometry, trigonometry and beginning calculus. No calculators are permitted for this competition.

New Initiates - Micah D. Anderson, Brad R. Bowns, John D. Califano, Kayla A. Cooley, Don J. Ebben, Kristi E. Grauf, Sarah N. Kulhanek, Joel D. Laber, Infane O. Lowe, Daniel D. Lynch, Graham J. MacMaster, Andrew T. Middleton, Kathryn L. Moody, Tammi L. Nowicki, Matthew R. O'Toole, Rachel L. Paulikonis, Benjamin B. Peterson, Nicholas E. Reding, Justin P. Reidling, Mariam K.Said, Cameron R. Schaefer, Tylor C. Schlink, Douglas J. Shuster, Ashley A. Siler, Victoria M. Sprague, Claire R. Swainson, Daniela Szatmari-Voicu, Andrea M. Thompson, and Rebecca R. Walt.

MO Alpha – Missouri State University

Chapter President – Christina Tharp; 33 Current Members and 4 New Members

Other Spring 2011 Officers: Brett Foster, Vice President; Ashley Lewis, Secretary; Lee Hicks, Treasurer; and Jorge Rebaza, Corresponding Secretary and Faculty Sponsor

KMS sponsored three seminars: Math Relay (teams of students), February 11, 2011; Speakers M. Weiss, A. Fraticelli, M. Ho, and D. Kline (Mathematics, MSU), March 24, 2011; and Speakers J. Groeble, J. Ellliot, and S. Pine (Mathematics, MSU), April 26, 2011.

New Initiates - Neil Grisham, Mariesa Ho, Cody Seidel, and Micah Weiss.

MO Beta – University of Central Missouri

Chapter President – Andrew Stallman; 25 Current Members and 9 New Members

Other Spring 2011 Officers: Trey Brock, Vice President; Christina Koehler, Secretary; Cynthia Craft, Treasurer; Jason Merten, Historian; Rhonda McKee, Corresponding Secretary; and Steve Shattuck and Dale Bachman, Faculty Sponsors

New Initiates - Justin Adkins, Jade Biesemeyer, Amy Billups, Valenica Brancato, Tia Burkhardt, John Crooker, Codey Davis, Cody Elpers, Jennifer Granicke, Zachary Foster, Joel Jeffries, Sara Kennedy, David Lewis, Kevin Loeffler, Annie Lowe, Michael Phinney, Christopher Purcell, Emilee Rice, Jacob Roach, Dustin Smith, Hannah Williams, and Alexandra Wolf.

MO Gamma – William Jewell College

Corresponding Secretary - Erin Martin

New Initiates - Ashutosh Dahal, Bikesh Dahal, Aayush Regmi, and Brett Whisler.

MO Epsilon – Central Methodist University

Corresponding Secretary - Linda O. Lembke

New Initiates - Megan Davidson, Jacob Heppner, Jacob Kleyh, Lisa Macon, Eric Millam, Tyler B. Padgett, Amer Elizabeth Pinson, Amber Strubberg, Stephanie Sullivant, Cody Wallingford, and Ashton Julianna Zimmerman.

MO Zeta – Missouri University of Science and Technology

Corresponding Secretary - Dr. Vy K. Le

New Initiates - Margaret Adams, Joel Allen, Kenneth Bassler III, James Benetatos, Peter Bergamini, Patrick Bradford, Andrew Braun, James Bridges, Daniel Bunselmeyer, Amy Cady, Cailie Carlile, Shaun Catlett, Brian Charles, Min Kyung Cho, Jessica Chowning, Nicholas Cooley, Andrew Christian, Bill Cushman, Harshil Desai, Kevin Dillon, Timothy Doonan, Gabriel Ellis, Brandon Ensor, Logan Ewigman, Allyson Finch, Ryan Fischer, Ryan Foshage, Tyler Gach, Cory Gilliam, Nolan Goth, Yishak Habtemichael, David Hengst, Chris Hillebrenner, Michael Hillstrom, Matthew Holmes, Ryan Holzum, Andrew Jabrani, Joshua Jensby, Austin Johnson, Jordan Kellerman, Tyler Knewtson, Robert Kreutzer III, Brandon Lahmann, Erik Larince, Jordan C. Lee, William Marchetto, Matthew Mashek, Ryan Mathes, Jackson Meyer, Megan Meyer, Kristen Mills, Robert Nevett, Victoria Prokopf, Cory Reed, Thomas R. Rehmeier, Lainey Ross, Melanie Rupprecht, Frank Sauer, William Selby, Jacqueline Soderstrom, Jason Stumfoll, Dale Summers, Tyler Thompson, Luther Triplett, David Uhlman, Nishant Uniyal, Brian Van Booven, Jacob Voss, Douglas Weidman, Ryan Weldon, Logan Wesley, John P. Winder, Caroline Wright, Charles Wright, and Andrew Yost.

MO Eta – Truman State University

Corresponding Secretary - Jason Miller

New Initiates - Katelyn N. Beike, Brianna K. Beitling, Michelle R. Berryman, Katie S. Boevers, Kendall O. Brown, Natalie E. Clark, Brie Daniels, Erik D. Dauster, Molly M. Dieckman, Melissa L. Greene, Alexander M. Kaizer, Rebecca L. Martin, Colleen S. McNamee, Angela N. Page, Samantha C. Ressler, Lauren E. Snyder, and Sarah L. Weinhold.

MO Theta – Evangel University

Chapter Pres. – Katie Strand; 15 Current Members and 7 New Members Other Spring 2011 Officers: Elizabeth Baumeister, Vice President; and Don Tosh, Corresponding Secretary and Faculty Sponsor

Meetings were held monthly. In January we initiated 7 new members. In February, an alumnus who is an actuarial fellow, Aaron Nieuwsma, gave an excellent presentation on actuarial science. In April Dr. Tosh and six students attended the national convention in St. Louis. One of our members, Ken Barker, presented a paper at the national convention. Also in April we had our end-of-year ice cream social at the home of Don Tosh.

MO Iota – Missouri Southern State University

Corresponding Secretary - Charles Curtis

Other Spring 2011 Officers: Rich Laird and Grant Lathrom, Faculty Sponsors

New Initiates - Nicole Green, Adebayo Orunpekun, Jared Smith, and Peter Thompson.

MO Kappa – Drury University

Corresponding Secretary - Dr. Bob Robertson

New Initiates - Rosalia Alcoser, Amy Briggs, Sarah Clayton, Michael Czajka, Amanda Gingras, Cynthia Lombardo, Matthew McCoy, Jennifer Nash, Kieran Ojakangas, and Sayan Patra.

MO Lambda – Missouri Western State University

Chapter President – Jeffrey Stevens

Other Spring 2011 Officers: Danielle Wagner, Vice President; Lucinda McDaniel, Secretary; Steven Brown, Treasurer; and Dr. Steve Klassen, Corresponding Secretary

At Missouri Western State University, fourteen new members were initiated into the Missouri Lambda chapter. The Spring initiation ceremony showcased our newly renovated, state-of-the-art planetarium and featured a "Wonders of the Universe" presentation hosted by Jerry Wilkerson.

New Initiates - Steven Brown, Dr. Brian Bucklein, Ashley Callen, Rashell Cogan, Brandin Erickson, Brandon Harrington, Emily Hickman, DooHyun Jo, Lucinda McDaniel, Morgan Russell, Darrin Smith, Jacqueline Stevens, Jeffrey Stevens, and Danielle Wagner.

MO Nu – Columbia College

Chapter President - Tomas Horvath; 12 Current Members and 8 New Members

Other Spring 2011 Officers: Kyle Christian, Vice President; Carolyn Summers, Secretary; Ran Kim, Treasurer; and Dr. Kenny Felts, Corresponding Secretary and Faculty Sponsor

MS Alpha – Mississippi University for Women

Chapter President – Chelsea Pugh; 14 Current Members and 4 New Members

Other Spring 2011 Officers: Meagan Vaughan, Vice President; Menuka Ban, Secretary; Dr. Joshua Hanes, Corresponding Secretary and Faculty Sponsor

During the fall semester, we held three chapter meetings. In November 2010, our chapter participated in Operation Christmas Child and donated four full shoeboxes. In the spring semester, we had three chapter meetings, initiated four members, and created a chapter t-shirt. In April 2011, our chapter participated in Columbus Multiple Sclerosis Walk, raising $1,520 from KME members, friends, and family.

New Initiates - Menuka Ban, Leota Cornelius, Chelsea Pugh, and Meagan Vaughan.

MS Epsilon – Delta State University

Corresponding Secretary - Paula Norris

New Initiates - Israel Kipapa, John Patrick Roach, and Jessica Szot.

NC Epsilon – North Carolina Wesleyan College

Chapter President – Andrew Webb; 24 Current Members and 7 New Members

Other Spring 2011 Officers: Jessica Willis, Vice President; Daniel Moore, Secretary; Elizabeth Uzzell, Treasurer; and Bill Yankosky, Corresponding Secretary and Faculty Sponsor

On Monday, March 21, 2011 the North Carolina Epsilon chapter of KME held its fourth annual initiation ceremony. During the ceremony seven students were initiated. This year's initiates were Holly Deaver, Trevour Huber, Jena McFadden, Brittany Nichols, Linh Nguyen, Tyler Olkowski, and Deanna Petersen. Following the ceremony a program was held in which four mathematics major alumni spoke about the different things they are doing now. The alumni in attendance were Josh Allen ('08), Brittany (Wright) Jernigan ('09), Laura (Hilbert) Faucette ('01), and Ben Lilley ('10). All are doing slightly different things and it was wonderful to hear them share their experiences since graduation and reminisce about their time at NCWC. The entire evening was great and everyone really appreciated the opportunity to talk with the alumni who took the time to come back and participate.

NC Zeta – Catawba College

Chapter President – Cynthia Cook; 21 Current Members and 5 New Members

Other Spring 2011 Officers: Spencer Ashley, Vice President; Zachary Owen, Secretary; Bridgett Hendersen, Treasurer; and Doug Brown, Corresponding Secretary and Faculty Sponsor

The NC Zeta Chapter initiated five new members in a ceremony held February 3, 2011. After the ceremony, a talk was given by Doug Brown on the sometimes unsavory consequences of the Axiom of Choise. The Chapter also sponsored a Pi Day race involving a run of 3.14 kilometers followed by a pie-eating contest.

NC Eta – Johnson C. Smith University

Chapter President – Niketa Jones; 25 Current Members and 11 New Members

Other Spring 2011 Officers: Maurice Scott, Vice President; Shimeca Bowman, Secretary; Quadashia Walker-Moss, Treasurer; Dr. Lakeshia Legette, Corresponding Secretary; and Dr. Brian Hunt, Faculty Sponsor

NE Beta – University of Nebraska Kearney

Chapter President – Valerie Sis; 15 Current Members and 7 New Members
Other Spring 2011 Officers: Kandi Young, Vice President; Brian Flannery, Secretary; Kali Anderson, Treasurer; and Dr. Katherine Kime, Corresponding Secretary and Faculty Sponsor

This spring, members of KME held a Math Fun Day at a local middle school. Work continued on the electric data base of members, etc. At initiation this spring, we had three parents attending. Dr. Richard Barlow, long-time sponsor of KMW now retired, provided pizza and attended with his sister Roberta Barlow. A large group photo was taken.

New Initiates - April Christman, Tiffani Doss, Chen Hou, Joshua Kershaw, Brittany Spiehs, Zach Springer, and Alicia Titus.

NE Delta – Nebraska Wesleyan University

Chapter President – Brent McKain; 19 Current Members and 7 New Members
Other Spring 2011 Officers: Macklin Warrington, Vice President; Abigail Raasch, Secretary and Treasurer; and Melissa Erdmann, Corresponding Secretary and Faculty Sponsor

On a lovely spring evening in Lincoln, Nebraska, we initiated seven new members into KME. Afterwards, we enjoyed grilled meats and other treats outside at a joint mathematics/physics picnic. Another highlight of the spring was a Pi Day Fun Run of 3.14 miles. Each participant received an entire pie.

NJ Beta – Montclair State University

Corresponding Secretary - John G. Stevens

New Initiate - Daniel J. Traum.

NJ Delta – Centenary College

Chapter President – Kim Kupper
Other Spring 2011 Officers: Ashley Burger, Vice President; Carissa Utter, Secretary; Brandon Iuzzolin, Treasurer; and Kathy Turrisi, Corresponding Secretary and Faculty Sponsor

Initiation ceremony was held on Sunday, May 1. Four new members were initiated into the New Jersey Delta Chapter of KME. The students are excited to be members and look forward to the coming year.

New Initiates - Pete DeMary, Judith A. Miller, Sara Jayne McCollum, and Devon Pierce Paffendorf.

NY Eta – Niagara University

Corresponding Secretary - Maritza M. Branker

New Initiates - Veronica L. Campanella, Cameron J. Fitch, Sarah M. Kojm, Renee Landrigan, Megan M. Marcin, Megan E. McGahan, Margaret A. Schuler, Lukas J. Shumway, and Christopher R. Wirth.

NY Iota – Wagner College

Corresponding Secretary - Dr. Zohreh Shahvar

New Initiates - Brandi Adduce, Nicholas Apicella, Kelsey Brenner, Lirie Cekalniku, Jo Anna Ciavarella, Ryan Cornell, John DeMartino, Kelly Dennis, Gia DeStefanis, Krey Keller, Joseph Kreitzer, Robert MacDonald, Teuta Marsic, Medije Mashkulli, Carley Nicoletti, Anthony Rafetto, Christina Schroeder, Samantha Schueschner, Sophie Tripp, and Melanie Valencia.

NY Kappa – Pace University

Corresponding Secretary - Lisa Fastenberg

New Initiates - Lauren Banko, David Bennett, Robert Hamilton, Qinghui Ji, Samantha Kwek, Sichen Liu, Melissa Meireles, Pooja Pasawala, and Marcelo Zimmler.

NY Lambda – C.W. Post Campus of Long Island University

Chapter President – Jennifer Hanly; 26 Current Members and 12 New Members

Other Spring 2011 Officers: Janine Neyssen, Vice President; Elyse Capozza, Secretary; Daniel Barone, Treasurer; Dr. Andrew M. Rockett, Corresponding Secretary; and Dr. Mahmoud Zeinalian, Faculty Sponsor

Twelve students were initiated by the chapter officers during our annual banquet at Greenvale Town House restaurant on the evening of April 3, 2011, bringing the chapter membership to 337. After the initiation, Debra Prevete spoke on "Can a regular polygon have lattice point vertices?" and Dr. Katherine Hill-Miller, Dean of the College of Liberal Arts and Sciences, recognized the recipients of the departmental awards for 2010-2011: the Clair F. Adler Award to Alyson Lamberti; the Lena Sharney Memorial Award to Janine Neyssen and Nicole Vasheo; the Joseph Panzeca Memorial Award to Debra Prevete and Andrea Wapnick; and the Hubert B. Huntley Memorial Award to Amanda Mazza.

New Initiates – Michael Cohen, Thomas Fallon, Brittany Greene, Christine Koenigsmann, Jamianne Kruse, Matthew Lucas, Amanda Mazza, Gladney Nose, Mei Qiao, Christopher Salvato, Paul Samber, and Paige Wehren.

NY Nu – Hartwick College

Chapter President – Ashley Hunt

Other Spring 2011 Officers – Nicole Besancon, Vice President; Julie Kessler, Secretary; Rebecca Lounsbury, Treasurer; and Ron Brzenk, Corresponding Secretary and Faculty Support

New Initiates - Diana Acker, Rachele Anderson, Nicole Besancon, Florence A. Christoph, Nicholas Clair, Katharine Holmes, Isaac Hughes, Ashley Hunt, Jaclyn Patterson, Catherine Weigel.

NY Omicron – St. Joseph's College

Chapter President – Melissa A. Bernstein; 40 Current Members and 22 New Members

Other Spring 2011 Officers: Charles C. Essig, Vice President; Maggie Kumpas, Secretary; Gabriela Rodrigues, Treasurer; Elana Reiser, Corresponding Secretary; and Dr. Donna Marie Pirich, Faculty Sponsor

A group of students attended the KME National Convention and two of our students presented. An initiation ceremony was held this semester. We have continued to staff our mathematics clinic, which offers free tutoring to local high schoolers.

NY Rho - Molloy College

Chapter President – Kimberly Thompson; 51 Current Members and 25 New Members

Other Spring 2011 Officers: Cara Rudolfsky, Vice President; Lara Sehne, Secretary; Brigid Damm, Treasurer; Manyiu Tse, Corresponding Secretary; and Deborah Upton, Faculty Sponsor

Kimberly Thompson and Nick Wood presented a paper of interest on Cryptography at the KME National Convention. We continued to hold "Calculus Corner" free walk-in tutoring sessions for those that need help in Calculus as well as other levels of mathematics. We created "MathHead Colloquium;" the mission is to motivate mathematics and mathematics education majors to be more involved in the research aspects of mathematics. We defined a mathematics head to be a person of mathematics moving in the speed of education. We partnered with the Math and Computer Science Club and created "In the Know Series." The mission is to see how mathematics can be applied outside of the classroom.

OH Gamma - Baldwin-Wallace College

Corresponding Secretary and Faculty Sponsor - David Calvis; 46 Current Members and 15 New Members

Ohio Gamma received 15 new initiates into membership on Friday, April 29, 2011. There were a total of about 40 students, faculty, family members, and guests in attendance.

New Initiates - Paige Boughton, Cassandra Clancy, Katherine Crossen, Erik Drost, Melissa Fannon, Kenneth Janosko, Ha Lai, Joseph Marosan, John Muryn, Tuan Nguyen, Larry Pearson, Brigitte Petrash, Melissa Small, Kristin Stark, and Kristen Szutkowski.

OH Epsilon – Marietta College

Chapter President – Lauren Litts; 40 Current Members and 23 New Members

Other Spring 2011 Officer: John Tynan, Corresponding Secretary and Faculty Sponsor

New Initiates - Matthew Boothe, Marena Current, Stephanie Davis, Alisha Dunn, Derek

Cavender, Craig Enos, Aleisha Guiler, Matthew Harper, James Houck, Jarvis Huck, Scott Kimmey, Cally Lee, Randall Ludolph, Muzi Luo, William McCulloch, Michael Paulus, Kaitlin Rinaldo, Cody Smith, Tyler Stathers, Yuan Tao, Cameron Tope, Mark Torres, and Xinyang Wu.

OH Zeta - Muskingum University

Corresponding Secretary - Richard Daquila

New Initiates - Kristyn Heagen, Daniel Heflin, William Lake, and Wade Litt.

OH Eta – Ohio Northern University

Corresponding Secretary - Donald Hunt

New Initiates - Samantha K. Bouhall, Tim Chaffin, Ryan Christman, Hannah E. DePriest, Zachary Lewis Dunn, Leah Felice Easterling, David Kauffman, Brian Klingenberg, Jennifer L. Krauss, Sarah Massella, Nicholas Saunders, J.C. Schroeder, Joanna L. Snyder, Sarah Thompson, Matthew Zirkle, and Caitlin Zook.

OK Alpha – Northeastern State University

Chapter President – Toni Slagle; 69 Current Members and 12 New Members

Other Spring 2011 Officers: Seth Vansell, Vice President; Jacob Curley, Secretary; Jonathan Moyer, Treasurer; and Dr. Joan E. Bell, Corresponding Secretary and Faculty Sponsor

Our spring initiation brought 12 new members into our chapter. Members worked on problems in The Pentagon and submitted one solution to the editor. OK Alpha members (Toni Slagle, Wen Shao, Leah Imboden, Abraham (Rho) Middleton) and future member (Ryan Berkley) attended the 73rd annual Oklahoma-Arkansas section meeting of the Mathematics Association of America. Also attending were NSU mathematics faculty Dr. Darryl Linde, Dr. Joan E. Bell, Dr. John Diamantopoulos, Dr. Mark Buckles, and Mr. Dan Sisk. The students competed in the 3rd annual student competition which included both a team competition in "Jeopardy" format, as well as an individual integration bee. Wen Shao tied for third place in the integration bee. We sponsored the annual KME ice cream social for mathematics majors and faculty. While we enjoyed ice cream sundaes, students and faculty told their favorite mathematics jokes.

New initiates: Roderick Bledsoe, Kalin Bradshaw, Blane Burge, Molly Erwin, Tatsuya Eto, Jonathan Garcia, Leah Imboden, Abraham Middleton, Taylor Pride, Joshua Qualls, Tandy Roberts, and Wen Shao.

OK Gamma - Southwestern Oklahoma State University

Corresponding Secretary - Tom McNamara

New Initiates - Holly Ausmus, Brenna Cary, Rachelle Dougherty, Samantha Driskill, Kimes Gilbert, Rebecca Hawking, Zach Hayes, Kandi Huereca, Cal Humphreys, Ty Allen Mays Jr., Johnathan Walker, and Jeffery Vest.

OK Delta – Oral Roberts University

Chapter President – Lori Fielding; 203 Current Members and 12 New Members

Other Spring 2011 Officers: Daniel Holman, Vice Pres.; Jessica Shearer, Secretary and Treasurer; Dr. Vincent Dimiceli, Corresponding Secretary and Faculty Sponsor

New Initiates - David Adams, Joanna Brannberg, Aaron Beavers, David Bulger, Chad Carroll, Darryl Charron, Lori Fielding, Christina Halsmer, Daniel Holman, Maddie Sue Judd, Robert Kreis, Michael Mangum, Sean McDonough, Jesse Patsolic, Joseph Peterson, Wendy Phillips, Grant Shaida, Jessica Shearer, Melissa Slinkard, Jacob Sweat, Derek Taylor, Mio Wakai, Andrew Walter, Joshua Weed, Michael Wilson, and Ashley Wood.

OK Epsilon – Oklahoma Christian University

Chapter President – Jacob Clark; 20 Current Members and 11 New Members

Other Spring 2011 Officers: Cady Block, Vice President; Jordan Courtemanche, Secretary and Treasurer; Dr. Ray Hamlett, Corresponding Secretary and Faculty Sponsor; and Craig Johnson, Faculty Sponsor

PA Delta - Marywood University

Corresponding Secretary - Thomas Kent

New Initiates - Lauren Bianchetti, Brenda Dixon, Bradley Fenstermaker, Corey Foote, Amanda George, Michael Kuniega, Stephanie Lavelle, Ariel Miano, Christopher Ratchford, Kenneth Rushinski, Geri Smith, Kaitlyn Thompson, Lauren Thorne, Amberly Warner, and Sarah Yeust.

PA Epsilon - Kutztown University

Corresponding Secretary - R.S. Schaeffer

New Initiates - Alicia Bromwell, Korey Castronuovo, Kristopher Cornista, Jordan Diehl, Adam Fenstermaker, Brooke Geist, Lauren Hart, Amanda Hutton, Kristen Huy, Tabitha Hylton, John Jablonski, Sean Judge, Kira Knechtle, Alison Koser, Stephanie Kurtz, Megan Pulse, Maria Rodriguez, Sara Romano, Luis Salazar Jr., Joy Schwenk, Nicole Sciandra, Alyson Serafin, Doreen Smeck, Jordan Smith, Joseph Stempo, Kelly Wagner, Clinton Watton, Patrick Wiltrout, and Krystin Wright.

PA Iota – Shippensburg University

Chapter President – Josh Ide; 13 Current Members

Other Spring 2011 Officers: Rebecca Rotz, Vice President; Matthew Simeone, Secretary; Gianni Assi, Treasurer; and Dr. Paul Taylor, Corresponding Secretary and Faculty Sponsor

New Initiates - Gianni Assi, Joshua Ide, Rebecca Rotz, and Matthew Simeone.

PA Kappa – Holy Family University

Chapter President – Michael Browning; 8 Current Members and 3 New Members

Other Spring 2011 Officers: Jacqueline Gallelli, Vice President; Alyssia Overline, Secretary; Michelle Kustra and Catherine Blumenstock, Treasurers; and Sister Marcella Louise Wallowicz CSFN, Corresponding Secretary and Faculty Sponsor

In March, the chapter hosted a CSFN grade school mathematics competition. One hundred eighty students from grade schools affiliated with the Sisters of the Holy Family of Nazareth (CSFN) in Pennsylvania, New York and Puerto Rico participated. A top prize of $125.00 (in recognition of the Sisters' 125th anniversary of ministry in the United States) was to be awarded. Two students from St. Adalbert School in Elmhurst, NY, tied for first place. In lieu of a tie breaker, two $125 prizes were awarded. The third place finisher, also from St. Adalbert School received $25. At the School of Arts and Sciences Honor Society Initiation Evening on March 25, 2011, Emily Anick, Angela Hand and Gidget Montelibano were initiated into the PA Kappa Chapter of KME.

PA Mu – Saint Francis University

Chapter President – Michelle Wetzel; 57 Current Members and 19 New Members

Other Spring 2011 Officers: Katie Dacanay, Vice President; Colin Trout, Secretary; Laura Stibich, Treasurer; Peter Skoner, Corresponding Secretary; and Katherine Remillard, Faculty Sponsor

The Pennsylvania Mu Chapter held initiation ceremonies on Thursday, February 10, 2011 in the Christian Hall Conference Room. The evening began with a prayer from chapter Chaplain Fr. Joseph Chancler, continued with dinner, followed by a talk "Pi is a Harsh Mistress: A Tale of Irrational Passion," by Professor Dr. Pedro Muíño, and concluded with the initiation ceremony for the nineteen new members including Jenna Bailey, Marissa Basile, Quy Cao, Dane-Marie Greaves, Addison Fox, Courtney Francis, Theodore Jagielski, Maura Jones, Sean Kane, Ryan Knee, Dr. Ying Li, Adam Mengel, Lucas Mignogna, Brittany Miller, Julie Moore, Amber Shaikh, Jessica Ulishney, Matt Warfel, and Mara Weinzierl. KME members Quy Cao, Teddy Jagielski, Laura Stibich, and Colleen Stock along with Professor Brendon LaBuz, attended the Allegheny Mountain Section of the Mathematical Association of America Conference at Clarion University on April 8-9, 2011. The students participated in problem solving competitions and attended student and faculty talk sessions.

Two faculty members and three students from Pennsylvania Mu attended the 38th Biennial National Convention on April 14-16, 2011 at Harris-

Stowe State University in St. Louis, Missouri. Laura Stibich presented her paper from her summer research with Dr. Brendon LaBuz, Assistant Professor of Mathematics, entitled "Bornologous Equivalencies." Colin Trout and Matt Skoner served as the convention delegates. The Pennsylvania Mu chapter served at Try-Math-A-Lot, a mathematics competition for sixth and seventh grade students from area schools, held on Wednesday, May 4, 2011 on the campus of the University of Pittsburgh at Johnstown, and attended by 304 students from 21 schools. Saint Francis University representatives conducted mathematics quiz bowl competitions in four different rooms during three different time periods. Attending and serving as quiz bowl judges were Mr. Michael Blaisdell, Dr. Katherine Remillard, and Dr. Peter Skoner. Serving as moderators, scorers, and timekeepers for the quiz bowl were KME students Marissa Basile, Alyshia Lacey, Brittany Miller, Matt Skoner, and Laura Stibich.

PA Pi – Slippery Rock University

Chapter President - Jessica Hoffman; 15 Current Members and 12 New Members

Other Spring 2011 Officers: Ashley Dublin, Vice President; Grace Evans, Secretary; Rex Edmonds, Treasurer; Elise Grabner, Corresponding Secretary and Faculty Sponsor

All of our new inductees presented papers at the Allegheny Mountain section of the MAA spring meeting at Clarion University in April.

New Initiates - Mark Beckwith, Andrew Brown, Ethan Corle, Ashley Dublin, Amanda Eplett, Grace Evans, Joseph Garcia, Amanda Goodrick, Rachel Henderson, Kaila Kramer, Kristen Leya, and Kristy Snyder.

PA Rho – Thiel College

Chapter President – Matthew Lowry

Other Spring 2011 Officers: Jacob Shaffer, Vice President; Marion Edwards, Recording Secretary; Alex Johnson, Treasurer; and Max Shellenbarger, Corresponding Secretary

On Sunday, Mary 27, 2011, four new members were initiated into the Pennsylvania Rho Chapter of KME at Thiel College in Greenville, PA. Professor Max Shellenbarger, Corresponding Secretary, and David Wierzchowski, current President, served as the Masters of Ceremony. Dr. Troy VanAken, guest speaker, gave a presentation about computer code and security. The members were then initiated and the new officers installed.

New Initiates - Marion Edwards, Matthew James Lowry, Samantha Mallon, and Ning Zhang.

PA Theta – Susquehanna University

Corresponding Secretary - Lisa Orloff Clark

New Initiates - Samantha Berheimer, Geoffrey Lamb, Amy Palmer, Katelin Peropat, Ethan Sentz, and Megan Zingaretti.

PA Xi – Cedar Crest College

Corresponding Secretary - Patrick Ratchford

New Initiates - Emily Eshleman, Kayla Hager, Molly McQuilken, Samantha Olier, Aarti Ramdaney, and Jillian Webberson.

RI Alpha – Roger Williams University

Corresponding Secretary - Dr. Ruth Koelle

New Initiates - Nicky Bierniarz, Christina Fontana, Andrew Mitchell, Susan Mitchell, and Sara Spellman.

TN Beta – East Tennessee State University

Chapter President – Jessie Deering; 766 Current Members and 13 New Members

Other Spring 2011 Officers: Tony Rodriguez, Vice Pres.; Jessica Lunsford, Secretary; Terrance McDermott, Treasurer; Robert Gardner, Corresponding Secretary and Faculty Sponsor, and Robert Beeler, Faculty Sponsor

The Tennessee Beta chapter of Kappa Mu Epsilon met on April 25, 2011 to elect new officers. The formal initiation ceremony was held on April 28, 2011 at the ETSU Department of Mathematics and Statistics' annual honors banquet. The banquet was held at the Johnson City Cuban restaurant "Bodega" and included a presentation by guest speaker Dr. Trachette L. Jackson, faculty member at the University of Michigan. Dr. Jackson's presentation was in the general category of quantitative biology. The new initiates are: Kelechukwu Alu (graduate student), Josh Brooks (graduate student), Whitney Forbes, Paul Hoilman, Halie Jones, Martha Liendo, Jessica Holt, Terrance McDermott, Tony Rodriguez, Brandon Sexton, Brennan Trent, Adam White, and Rouying Wang. Some additional happy news from the TN Beta chapter is that member Paul Hoilman was a 2011, 19th round draft choice by Major League Baseball team the Chicago Cubs!

New Initiates - Kelechukwu Alu, Josh Brooks, Whitney Forbes, Paul Hoilman, Halie Jones, Martha Liendo, Jessica Lunsford, Terrance McDermott, Tony Rodriguez, Brandon Sexton, Brennan Trent, Adam White, and Ruoying Wang.

TN Gamma – Union University

Chapter President – Rebecca Eaton; 37 Current Members and 10 New Members

Other Spring 2011 Officers: Emilie Huffman, Vice President; Kim Lukens, Secretary and Treasurer; Seth Kincaid, Historian and Webmaster; Michelle Nielsen, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor

On April 25, 2011, the Tennessee Gamma Chapter of KME held the an-

nual Spring initiation banquet at the Old Country Store in Jackson, TN. Approximately 20-30 KME members attended the banquet, 10 of which were initiated as new KME members. Dr. Brian Taylor, an alumnus of Union University, gave a talk on his work at St. Jude Children's Research Hospital.

New Initiates - Jonathon Gwaltney, Matthew Johnson, Michael Lam, Jessica Lee, Katherine Long, Molly Mitchell, Michelle Nielsen, Mary Ellen Poe, Katherine Shelnutt, and Ryan Spencer.

TN Delta - Carson-Newman College

Chapter President – Terry Rogers; 27 Current Members and 9 New Members

Other Spring 2011 Officers: Lata Kodali, Vice President; and Kenneth Massey, Corresponding Secretary and Treasurer

The KME student membership was down this year, but we had a healthy group of nine new initiates this spring. Unfortunately, our picnic at the dam was rained out. Instead we met on the balcony of the business building and enjoyed food cooked by our officers.

New Initiates - Abbey Atchley, Allison Denny, Corbin Hedges, Rachel Hodge, Ashley Hutton, Rachel Logemann, Michael Lugo, Amy McLaughlin, and Christopher Yeary.

TN Epsilon – Bethel University

Corresponding Secretary - Mr. Russell Holder; 7 Current Members

Other Spring 2011 Officer: Mr. David Lankford, Faculty Sponsor

TX Alpha – Texas Tech University

Corresponding Secretary - Magdalena Toda

New Initiates - Kaitlin Blocker, Erin Fitzgerald, Rebecca Lelko, Chelsea Martin, Shauna Melton, Christine Rice, and Ashley Ross.

TX Gamma – Texas Woman's University

Corresponding Secretary - Dr. Mark Hamner

New Initiate – Jennifer L. Kirk.

TX Kappa – University of Mary Hardin-Baylor

Chapter President – Christi D'Herde; 14 Current Members and 6 New Members

Other Spring 2011 Officers: Ashley Lawson, Vice President; Helen Wong, Secretary; Trevor Ash, Treasurer; Peter H. Chen, Corresponding Secretary; and Maxwell Hart, Faculty Sponsor

New Initiates - Melissa Donham, Blake Heller, Kolton Keith, Katelyn McKinzey, Evan Mullins, and Jeremy Sapp.

TX Lambda – Trinity University

Corresponding Secretary - Diane Saphire

New Initiates - Jaskirat Batra, Alexander Butler, Elana Edwards, Marlies Hager, Rachael Heineman, Selman Kaldiroglu, Leigh Logsdon, Maxwell Robinson, Daniel Villamizar, Ian White, and Katelin Whittaker.

TX Mu – Schreiner University

Chapter Pres. – Audra Burnap; 24 Current Members and 9 New Members
Other Spring 2011 Officers: Denise Begley, Vice President; Caitlin Gayle, Secretary; Matthew Moreno, Treasurer; William M. Sliva, Corresponding Secretary and Faculty Sponsor

Matthew Moreno presented his ongoing research each at the national conference in April. He was one of the prize recipients.

VA Alpha – Virginia State University

Corresponding Secretary - Azzala Owens

New Initiates - Jeriel Champion, LaShona A. McClean, Lynette Obiero, Christine Saywack, and Shelly J. Simons.

VA Gamma – Liberty University

Corresponding Secretary - Dr. Tim Van Voorhis

New Initiates - Jee-Yeon Byeon, Joy Distler, Robert Feldges, Jesse Keyton, Jin Li, Rachel Morgan, Kevin Ng, Ashley Richtarik, Sarah Seaman, Timothy Shafer, Lindsey Stevenson, Timothy Vernon, and Jacob Yacovelli.

VA Delta – Marymount University

Chapter President – Hannah Korbach; 31 Current Members
Other Spring 2011 Officers: Matthew Villemarette and Eric Kamta, Vice President, Secretary, and Treasurer; William Heuett, Corresponding Secretary; and Elsa Schaefer, Faculty Sponsor

WI Gamma – University of Wisconsin-Eau Claire

Chapter President – Joshua Frinak; 60 Current Members and 15 New Members
Other Spring 2011 Officers: Kaisey Garrigan, Vice President; Kristina Bleess, Secretary; Bret Meier, Treasurer; and Dr. Simei Tong, Corresponding Secretary and Faculty Sponsor

Wisconsin Gamma has actively involved in the annual Math Retreat Day. In addition to the over 50 student/faculty research talks, we sponsored two new events: Mathematician Talent Show and Math Competition. They went very well and will continue these efforts in next spring.

WV Alpha - Bethany College

Corresponding Secretary - Dr. Mary Ellen Komorowski

New Initiates - Joseph A. Douglas II, James Z. Klingensmith, Bryce Wesley Patterson, Olivia M. Pavlic, David G. Pivik, Emily Rebecca Sechrest, Alexander Clark Stubbs, and John Richard Tritschler.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February 2001
GA Delta	Berry College, Mount Berry	21 April 2001
TX Mu	Schreiner University, Kerrville	28 April 2001
NJ Gamma	Monmouth University	21 April 2002
CA Epsilon	California Baptist University, Riverside	21 April 2003
PA Rho	Thiel College, Greenville	13 February 2004
VA Delta	Marymount University, Arlington	26 March 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 February 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 March 2005
SC Epsilon	Francis Marion University, Florence	18 March 2005
PA Sigma	Lycoming College, Williamsport	1 April 2005
MO Nu	Columbia College, Columbia	29 April 2005
MD Epsilon	Stevenson University, Stevenson	3 December 2005
NJ Delta	Centenary College, Hackettstown	1 December 2006
NY Pi	Mount Saint Mary College, Newburgh	20 March 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 April 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 October 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 March 2008

CA Zeta	Simpson University, Redding	4 April 2009
NY Rho	Molloy College, Rockville Center	21 April, 2009
NC Zeta	Catawba College, Salisbury	17 September, 2009
RI Alpha	Roger Williams University, Bristol	13 November, 2009
NJ Epsilon	New Jersey City University, Jersey City	22 February, 2010
NC Epsilon	Johnson C. Smith University, Charlotte	18 March, 2010
AL Theta	Jacksonville State University, Jacksonville	29 March, 2010
GA Epsilon	Wesleyan College, Macon	30 March, 2010
FL Gamma	Southeastern University, Lakeland	31 March, 2010
MA Beta	Stonehill College, Easton	8 April, 2011
AR Alpha	Henderson State University, Arkadelphia	10 October, 2011