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Where Do We Hunt for Ichabods?

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KS Delta

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Presented at the 2008 North Central Regional Convention and awarded "top four" status by the Awards Committee.

1. Introduction

Washburn University experienced an unexpected and disappointing decrease of 3.5% in enrollment in the fall of 2007 ([2]). As a mathematics capstone project we were asked to investigate trends in the numbers of graduating seniors in the geographic areas which have been the traditional source of the Washburn students, Ichabods, hopefully locating areas that are producing an increasing number of high school graduates. These trends could play a significant role in needed changes to Washburn's recruitment strategies. To conduct this investigation it was necessary to (1) determine if data detailing numbers of graduating seniors in various geographic locations was available, and if so (2) gather and format the data to support trend analysis of the data by geographic area, (3) determine whether time series forecasting or regression analysis was appropriate for the data, (4) use that method, to develop forecasting models of the data by geographic area, (5) determine and apply measures of effectiveness to the developed models to select the model which best represents the data trends for each geographic area, and (6) report trend results of the chosen models for each geographic area and make recommendations. The investigation revealed time series modeling was appropriate and led to the selection of "best" forecasting models for each geographic area. None of the selected models indicated a decline in source of potential students; however, some forecasted a positive linear trend of potential students which could be investigated by Washburn's recruiters.

2. Data Collection

Finding a reputable source of long term data to facilitate more accurate trend forecasting proved challenging, but after extensive searching, we found the data on the National Center for Education Statistics (NCES) website. This site is maintained by the United States Department of Education and is the primary body for collecting and analyzing data within the field of education. The NCES website contains a database with a vast array of data on public schools nationwide. The data is organized by state, year, county, and school district and then by the type of data collected. Using NCES website tools we were able to extract the numbers of students within a county that received a diploma at graduation for the years 1987 through 2005 for the counties in the geographic areas in which Washburn focuses its' recruitment strategies.

We partitioned the data in to seven geographic regions as shown in Figure 1, located on the web at http://www.kappamuepsilon.org/pentagon, with the state of Kansas divided into four regions, as well as regions in Nebraska, Missouri, and Oklahoma in which Washburn has focused its recruitment in the past. These regions will be the basis for how we separate the data to develop forecasting models on the data. We will refer to the Eastern counties in Kansas as "County Set 1", the four counties surrounding Washburn University as "County Set 2", the Southwestern Kansas counties as "County Set 3", the Northwestern Kansas counties as "County set 4", the Nebraska counties as "Counties in NE", the Oklahoma counties as "Counties in OK", and the Missouri Counties as "Counties in MO". The extracted data for each region appear in the column C of Appendices A-G located on the web at the address detailed earlier.

3. Time Series vs. Regression

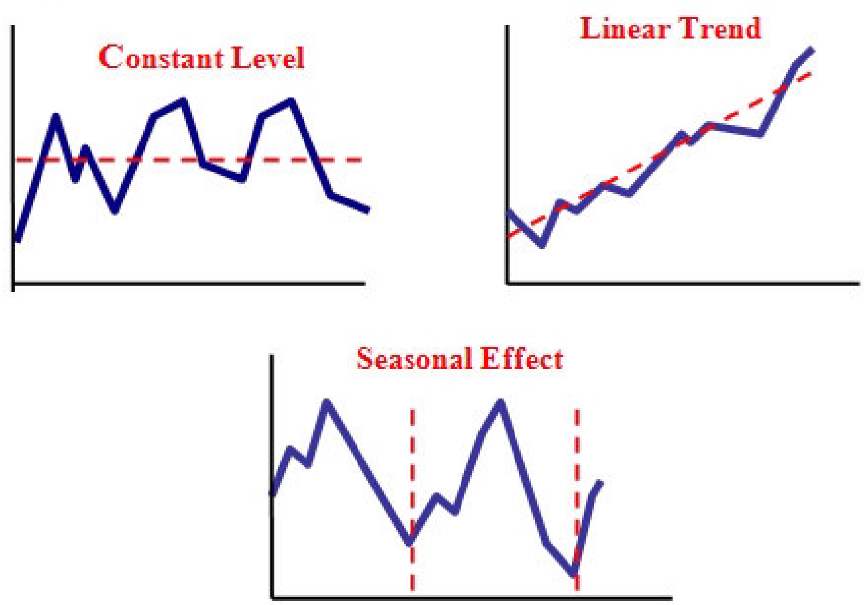
Before we could begin developing forecasting models, we needed to determine whether time series forecasting or regression analysis was appropriate for the data. As discussed in [5], regression analysis should be used if the data, in particular adjacent data values in the data sequence, are independent; however, we should chose time series forecasting methods if the data are not independent. If the data were independent then knowledge of the number of graduating seniors for the year 1987 for a particular region would not be useful in forecasting the number of graduates for 1988. Clearly that is not the case; in fact the value for 1987 should provide a reasonable forecast for 1988. Since our data are not independent we will use time series forecasting for developing our models.

Time Series. Time series forecasting consists of sequential data points that are equally spaced, in our case the spacing is yearly, and uses a model to forecast data based on known data, which restricts the forecast. The forecasts made with time series are based on previous values of data, such that previous years of graduation numbers play a role in the forecast. There were three basic patterns we considered in investigating the time series data: the constant level, linear trend and the seasonal effect. A constant level pattern shows random fluctuations superimposed on an otherwise constant value. With a linear trend, the data is steadily increasing or decreasing, such that a line can be fit to its increase in the form

$$X_t = A + B_t + e_t, \tag{1}$$

where X is the observed value, A is a constant (the expected value), B is the trend factor (slope) and e is the random error at time t. Seasonal effect superimposes systematic calendar related fluctuations on the underlying pattern and is often seen in service industries like stores and restaurants. We have shown these patterns in figure 2.

Figure 2:



We will not use the seasonal effect pattern, because graduation numbers occur on an annual basis and should not have any seasonal patterns, but we will be using the constant level and the linear trend time series patterns to predict the value, F_{t+1}, which is a function of the observed values of the time series. In the following trend models,

$$F_{t+1} = \text{forecast of the value of the time series at time } t+1, \text{ given the observed values } X_1 = x_1, X_2 = x_2, \ldots, X_t = x_t.$$

To use the linear trend model, we will compute the trend line to make a forecast, whereas, for a constant-level model we will consider several classic models: the last-value forecasting method, the averaging forecasting method, the moving-average forecasting method, the weighted-average forecasting method, and the exponential smoothing forecasting method [3].

4. Time Series Trend Models

Linear Trend Model. Working with linear trends requires the use of scatter plots, so we will refer to all scatter plots found in Appendix H, located on the web address on page 2. The red sloped line on each scatter plot indicates a linear trend line that can be fit to the data for a specific county set. Each line's equation is also shown on the scatter plot, and takes the form $y = mx + b$. To use the linear trend model there are three formulas that must be completed. First, we calculated the latest trend, L_{t+1}, of the time series at a time $t+1$, using the equation

$$L_{t+1} = \alpha \left(x_t - x_{t-1} \right) + (1 - \alpha) \left(F_t - F_{t-1} \right).$$

Here, L_{t+1} is the latest trend at the time $t+1$, which is based on the last two values x_t and x_{t-1} and the last two forecasts, F_t and F_{t-1}. The constants β and α are smoothing constants with $\beta > 0$, $\alpha < 1$, and $\alpha + \beta = 1$. The latest trend is the change in y where x is only changing one unit, because we are calculating yearly. Second, we calculated the latest trend factor, T_{t+1} using the equation

$$T_{t+1} = \beta L_{t+1} + (1 - \beta) T_t,$$

where T_{t+1} is the exponential smoothing estimate of the trend factor B (see equation (1)) at time $t+1$, given the observed values, $X_1 = x_1$, $X_2 = x_2$,..., $X_t = x_t$. The trend factor represents the most recent slope in the linear equation because it considers what the latest trend is based upon the actual changes between F_t and x_t. Lastly, the calculations of L_{t+1} and

T_{t+1} are used to calculate the forecast of the time series at $t+1$, F_{t+1}:

$$F_{t+1} = \alpha x_t + (1-\alpha) F_t + T_{t+1}.$$

To develop linear models, two initial assumptions must be made about values prior to any forecasting. The first assumption is the expected value of the time series, x_0, which is assumed to have remained unchanged prior to any forecasting. The second assumption is the specific value of the trend of the time series, T_1, prior to any forecasting. This linear trend method uses relatively complicated calculations, and because of this we used the Operational Analysis Software, *Crystal Ball*, which we found for download on the software's website. To see how this works on the data, we will show this process using county set 1, found in Appendix A, located on the web address on page 2, where $\alpha = 0.3$, $\beta = 0.3$, $x_1 = 9555$, $x_2 = 9893$, and $x_3 = 9655$. These constants were used based on typical values as discussed Lieberman and Hillier, but in the actual reports in Appendices A-G, α and β were found using the software *Crystal Ball* ([3]).

First, we must make the initial assumptions $x_0 = 9555$ and $T_1 = 0$, so that the first forecast,

$$F_1 = 9555 + 0 = 9555.$$

Then, we calculated L_2 and T_2 to find F_2:

$$L_2 = 0.3\,(9555 - 9555) + (1-0.3)\,(9555 - 9555) = 0$$

and

$$T_2 = 0.3 \cdot 0 + (1-0.3) \cdot 0 = 0,$$

and finally

$$F_2 = 0.3 \cdot 9555 + 0.7 \cdot 9555 + 0 = 9555,$$

so the forecast for 1988 is 9555. Next, we calculated the forecast for 1989, which is a little more exciting than the previous forecast.

$$L_3 = 0.3\,(9893 - 9555) + (1-0.3)\,(9555 - 9555) = 101.4$$

and

$$T_3 = 0.3 \cdot 101.4 + (1-0.3) \cdot 0 = 30.42,$$

so that

$$F_3 = 0.3 \cdot 9893 + 0.7 \cdot 9555 + 30.42 = 9686.82,$$

so the forecast for 1989 is approximately 9687. This process is repeated until there is no more data to process, and even after the data is done, the formulas for the linear trend method can be used to compute years ahead. For the data, we used *Crystal Ball* to forecast the next four years of graduation numbers. These can be seen in Reports 1-7 and in Appendices A-G, located on the web address on page 2.

Constant-Level Models.

Last-Value Forecasting Method.

The last-value forecasting method is one of the simplest types of forecasting available and can be represented with the equation

$$F_{t+1} = x_t.$$

In this equation the F_{t+1} is the forecast for a year, x_t represents the previous year's data, and the variable t represents time. To show how the last-value method works on the data, we will apply it to county set 2, which appears from first glance at the scatter plot in Appendix H, located on the web address on page 2, to have a fairly constant-level model because the slope is approximately zero. Let $x_1 = 2832$ and $x_2 = 2801$; thus

$$F_2 = x_1 = 2832 \text{ and } F_3 = x_2 = 2801.$$

This process is repeated using the previous year's graduation numbers as the forecast for the next year until no more data is left. The full set of forecasts can be seen in Appendix C located on the web address on page 2. This forecasting method is often imprecise because its variance may be very large since it is based on a sample size of one. However, according to Lieberman and Hillier, the last-value method may be (and often is) appropriate to use if "the underlying assumption about the constant-level model is 'shaky' and the process is changing so rapidly that anything before time t is almost irrelevant or misleading or the assumption that the random error e_t has constant variance is unreasonable and the variance at time t actually is much smaller than at previous times" ([3]). Therefore, when data is changing rapidly, the last value might be the only relevant data point to use when forecasting the next value ([3]).

Averaging Forecasting Method.

The average forecasting method is the other extreme because it uses all the data points in each county set and averages them to find F_{t+1}. This process is usually fairly accurate if the data is stable, and can be calculated from the equation

$$F_{t+1} = \sum_{i=1}^{t} \frac{x_i}{t}.$$

In this equation the F_{t+1} is the forecast for a year, x_i represents the summation of the previous year's graduation numbers, and the variable t represents the number of data points (or total of years that graduation numbers were summed for x_i). To show how the average forecasting method works on the data, we will apply it to county set 2, which appears to be a constant-level model. Let $x_1 = 2832$, $x_2 = 2801$, $x_3 = 2834$, and $x_4 = 2555$. Then

$$\begin{aligned} F_{1+1} &= F_2 = x_1 = 2832 \\ F_{2+1} &= F_3 = \frac{x_1 + x_2}{2} = \frac{2832 + 2801}{2} = 2816.5 \\ F_{3+1} &= F_4 = \frac{x_1 + x_2 + x_3}{3} = \frac{2832 + 2801 + 2834}{3} = 2822.33. \end{aligned}$$

This process repeats itself, and as each year passes another data number is added to the previous years and the t increases by one. The full set of forecasts can be seen in Appendix B, located on the web address on page 2. One disadvantage with the average forecasting method is that it assumes the data values will stay about the same over an extended amount of time, which is seldom the case. Therefore, when the data is current and stable, the average method is very helpful; however, it is unlikely to be accurate if data is very old and may no longer be relevant ([3]).

Moving-Average Forecasting Method.

The moving-average forecasting method is a combination of the averaging and last-value forecasting methods because it averages the data for the previous n years. The forecast is then calculated using the equation

$$F_{t+1} = \sum_{i=t-n+1}^{t} \frac{x_i}{n}.$$

For instance, in the data we used a moving-average method that takes into account the previous three years, so $n = 3$. We chose $n = 3$ because graduation numbers can have a high variability over an extended amount of time; however, a three year period allows for a small amount of change without the assumption that every value in the data set stays the same over the time span. Thus, the equation for this method looks like the following:

$$F_{t+1} = \frac{x_{t-2} + x_{t-1} + x_t}{3}.$$

In this equation, F_{t+1} is the forecast for a year, x_i represents the previous three years of data added together, and the variable t represents time. The moving-average method does not use very old data that might be irrelevant; instead it allows one to focus on the recent data to determine the forecast. To show how the moving-average forecasting method works on the data, we will apply it to county set 2, which appears to be a constant-level model. Let $x_1 = 2832$, $x_2 = 2801$, $x_3 = 2834$, $x_4 = 2555$, and $x_5 = 2555$. We had to start with F_4 because we need three years of previous data; thus

$$\begin{aligned} F_{3+1} &= F_4 = \frac{x_1 + x_2 + x_3}{3} = \frac{2832 + 2801 + 2834}{3} = 2822.33 \\ F_{4+1} &= F_5 = \frac{x_2 + x_3 + x_4}{3} = \frac{2801 + 2834 + 2555}{3} = 2730.00. \end{aligned}$$

Once again this process repeats itself until there are no more forecasts to be made. Each time there will only be three graduation numbers considered and the numbers will change with every new forecast, discarding the oldest data point and adding a new data point. The full set of forecasts can be seen in Appendix B, located on the web address on page 2. A disadvantage to this method is that x_{t-2} has the same weight placed on it as x_t, meaning that in the sample all three years have the same weight, so if all data values are not equally significant in determining the forecast, this method will be inappropriate. The last two methods, the weighted-average and exponential smoothing allow forecasting to emphasize more recent data in determining a forecast ([3]).

Weighted-Average Forecasting Method.

The weighted-average forecasting method is similar to the moving-average forecasting method, because it averages the data for the last n years as well. However, unlike the moving-average method, this method places a certain weight on each of the n years, so that the equation changes to

$$F_{t+1} = \sum_{i=t-n+1}^{t} \frac{\alpha_i x_i}{n}.$$

In this equation the F_{t+1} is the forecast for a year, x_i represents the previous years of data, n represents the number of previous years being summed, the variable t represents time, and the α_i represent weight constants. The constants share the relationship that $0 \leq \alpha_i \leq 1$, and the α_i's sum to one. If the constants are all equal this is just the moving-average, because every data value has the same weight. To show how the weighted moving-average forecasting method works on the data, we will apply it to county set 2, which appears to be a constant-level model, where $\alpha = 0.2$ and $\beta = 0.8$. Let $x_1 = 2832$, $x_2 = 2801$, $x_3 = 2834$, and $x_4 = 2555$, and let $n = 2$, so that we will be summing two previous years to obtain a forecast.

Thus,

$$\begin{aligned} F_{2+1} &= F_3 = (0.2)\, x_1 + (0.8)\, x_2 = (0.2)\,(2832) + (0.8)\,(2801) \\ &= 2807.50 \\ F_{3+1} &= F_4 = (0.2)\, x_2 + (0.8)\, x_3 = (0.2)\,(2801) + (0.8)\,(2834) \\ &= 2827.40. \end{aligned}$$

The weighted forecasting method is very similar to the moving average in that it discards older values as it progresses, however, with this method a greater weight is placed on the newer data and a lesser weight is placed on the older data. The full set of forecasts can be seen in Appendix B, located on the web address on page 2 ([3]).

Exponential Smoothing Forecasting Method.

The final forecasting method is the exponential smoothing forecasting method. This method is a weighted average of the previous forecast, F_t, and the previous observation, x_t. The equation for this method can be expressed in two different equations, one of which is the most used formula

$$F_{t+1} = \alpha x_t + (1-\alpha)\, F_t$$

and the other of which shows the recursive relationship between F_{t+1} and F_t:

$$F_{t+1} = \alpha x_t + \alpha\,(1-\alpha)\, x_{t-1} + \alpha\,(1-\alpha)^2\, x_{t-2} + \cdots.$$

In both of these equations F_{t+1} is the forecast for a year and α is a smoothing constant, $0 < \alpha < 1$, that can be determined randomly. The second equation shows that exponential smoothing places the most weight to x_t, the most recent data value, and smaller weights to earlier observations. For this project we will be using the first equation because it is in a simpler form even though it takes all previous observations into account without listing them all out. To use this method we had to select a value for α. Among statisticians there is no agreed upon method for determining an appropriate value for α. Typically α is chosen so as to minimize the measure of effectiveness, commonly mean absolute deviation (MAD) or mean squared error (MSE) as discussed in [4] and [5]. A small value for α causes the model to respond slowly to changes in data, while a large value for α causes to model to react quickly causing lots of variability. Hillier and Lieberman suggest that α should not exceed 0.3 and that a reasonable choice for α is 0.1. One important note on why Hillier and Lieberman suggest a lower α is because as α approaches one, the method becomes the last value method. This is important to realize because as α increases, all the weight is placed on the previous observation, x_t, and almost none is placed on the previous forecast, F_t. To show how the exponential smoothing forecasting method works on the data, we will apply it to county set 2, which appears to be a constant-level model, where $\alpha = 0.1$. Let $x_1 = 2832$, $x_2 = 2801$, $x_3 = 2834$, and $x_4 = 2555$. Thus,

$$\begin{aligned} F_{1+1} &= F_2 = (0.1)(2832) + (0.9)(2832) = 2832 \\ F_{2+1} &= F_3 = (0.1)(2801) + (0.9)(2832) = 2828.90 \\ F_{3+1} &= F_4 = (0.1)(2834) + (0.9)(2828.90) = 2829.41. \end{aligned}$$

This process continues until no more data is left to process, and can be seen in full in Appendix B, located on the web address on page 2. Exponential smoothing has two disadvantages, the first is it "lags behind a continuing trend," meaning if the constant-level model is incorrect the forecast will be several periods behind, and second is it is often difficult to choose a suitable smoothing constant ([3]).

5. Measures of Effectiveness

Now that we have discussed the various trend models we will be using in the calculations, we need to decide which measure of effectiveness to use in comparing the various forecasting models. Two measures of effectiveness frequently used in time series model comparisons are mean absolute deviation (MAD) and mean squared error (MSE). Both methods assess the error between the actual value and the forecasted value. The forecasting error, which is often called the residual and will be denoted as E_t, is the absolute value of the difference between the observed value, x_t, and the forecast for that year, F_t.

$$E_t = |x_t - F_t|.$$

The forecasting error is computed for each trend model that was used and is shown after each forecasting column, in the designated "Abs Dev" column. Using this forecasting error, we can calculate both MAD and MSE values. The MAD is the average of the errors, such that

$$MAD = \frac{\sum_{t=1}^{n} E_t}{n}.$$

The n designates the number of time periods (years) and t designates the particular time period. The calculated MAD can be found at the end of the "Abs Dev" column and is labeled "MAD." Next, to calculate the MSE, we created a new column, called the "Abs Dev Sq." column. This column is the square of the forecasting error, $(E_t)^2$. The MSE averages these squared forecasting errors, i.e.,

$$MSE = \frac{\sum_{t=1}^{n} E_t^2}{n}.$$

The MSE can be found at the end of the "Abs Dev Sq" column, and is labeled "MSE." The goal in evaluating MSE and the MAD is to minimize the value in the "Abs Dev" and "Abs Dev Sq." columns. To see how MAD and MSE work, we will work out a sample calculation using the last-value method for County Set 3, found in Appendix C, located on the web address on page 2. First, we calculated the forecasting error for each year by taking the absolute value of the difference between the actual graduation numbers of a given year and the forecast of a given year. This can be seen in column E of Appendix C. Next, we calculated the MAD based on the forecasting errors. Thus,

$$\begin{aligned} MAD &= (E_1 + E_2 + \cdots + E_{18})/18 \\ &= (125.0 + 18.0 + \cdots + 207.0)/18 = 247.44. \end{aligned}$$

To calculate the MSE we first squared each of the 18 original forecasting errors in column E. These squared forecasting errors are found in column F of Appendix C. Then, we calculated the average of these 18 squared errors, as show below, to determine the MSE:

$$\begin{aligned} MSE &= \left(E_1^2 + E_2^2 + \cdots + E_{18}^2\right)/18 \\ &= (15,625 + 324 + \cdots + 42849)/18 = 91,967.44 \end{aligned}$$

The MAD and MSE are both important; however, the mean absolute deviation is less affected by large forecasting errors than the MSE, which can be seen in the above example. Hence, because of variability we decided to use the MAD as the measure of effectiveness on the trend models ([3]).

6. Results

After calculating the MAD and MSE for each method for each county set, we examined the results and observed that the model with the lowest MAD in each county set also had the lowest MSE. Therefore, selecting the best method within each county set meant selecting the method with the lowest MAD value. The following table shows the MAD values for all methods performed on each county set, highlights the lowest MAD value, and identifies the "best" model, i.e., the one which produced the lowest MAD value, in the last column:

Cnty Set	Last Value	Avg	Mvg Avg	Wt Mvg Avg	Exp Sm (a) α =0.1 (b) α =0.2 (c) α =0.5	Linear Trend	Best Model
KS 1	282.7	795.5	466.3	309.3	(a) 702.0 (b) 614.0 (c) 407.2	Dbl Exp 242.8	**Linear Trend**
KS 2	143.3	265.3	189.8	151.2	(a) 253.8 (b) 230.8 (c) 168.4	Sgl Exp 152.2	**Last Value**
KS 3	247.4	520.0	301.3	252.7	(a) 488.4 (b) 411.3 (c) 278.4	Sgl Exp 246.0	**Linear Trend**
KS 4	143.2	279.7	234.6	158.0	(a) 263.7 (b) 242.8 (c) 189.7	Sgl Exp 144.5	**Last Value**
NE	335.8	433.8	314.6	321.0	(a) 391.6 (b) 361.6 (c) 294.4	Sgl Exp 492.6	**Exp. Sm. α=0.5**
OK	545.9	749.0	466.1	538.7	(a) 814.9 (b) 732.0 (c) 541.6	Sgl Exp 622.0	**Moving Average**
MO	403.8	830.7	580.2	426.3	(a) 813.2 (b) 714.9 (c) 437.0	Sgl Exp 400.7	**Linear Trend**

Before choosing a linear trend model to be the best model for the forecasting, we needed to confirm the time series data was in fact linear. As suggested in [1], we did this by conducting linear regression, not for forecasting purposes, but to determine statistically whether the data exhibited non-zero slope. Using, *Crystal Ball*, we conducted hypothesis testing with the null hypothesis stating the slope of the regression line was equal to zero. The resulting p-value was the probability of observing a sample statistic as extreme as the test statistic which is equal to the slope of the regression line divided by the standard error. We decided to reject the null hypothesis and conclude the data was linear if the p-value (significance level) was less than 0.01 (0.05 is a common choice [1]). All the statistics from the linear regressions are shown in Appendices A-G, located on the web address on page 2, with the p-value highlighted blue. As can be seen below, the p-values for all "best" linear trend models were well below 0.01.

County Set with Linear Trend Model	KS Set 1	KS Set 3	MO Set
p-value	0.0001	0.00002	0.0048

Thus we concluded these data sets were appropriate for linear trend model forecasting.

As a final test of the effectiveness of the "best" models, we used each model to forecast the number of graduates in each county set for 2005, the most recent year for which data was available, and we compared these forecasts with the actual number of graduates for 2005. The results appear in Table 1, located on the web address on page 2. The models performed well, with all but one having an error of less than 5 % and the other at 10%, which was considered acceptable. We then used the best models to forecast the number of graduates in each county set for 2006 and the results are shown in Table 2 located on the web address on page 2.

7. Recommendations

First, even though enrollment is declining, it is not because high school graduation numbers are decreasing. In fact, graduation numbers are remaining constant or increasing in all areas in which Washburn has traditionally focused its recruitment, as demonstrated by the choice of either a constant-level model or a linear trend model with positive slope for each geographic area. We would recommend Washburn recruiters investigate what factors could be leading to the decline in numbers of recruits. Second, in regions where a constant model is the best choice, since numbers of high school graduates are remaining relatively constant, we would recommend focusing on regions with a large number of forecasted graduates, in particular the Oklahoma county set. Third, in all regions showing a linear trend with positive slope, we would recommend Washburn consider focusing more resources on recruitment, because there are an increasing number of graduates available each year, and the forecasted numbers are large relative to other county sets.

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Almost Enough About Cardinal Numbers

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1. Introduction

To any given set one assigns its cardinal number, an entity which makes precise how many elements the set contains. Everybody knows the *finite* cardinal numbers: $0, 1, 2, \ldots$. As Halmos [3] shows, even without a general definition, one may work thoroughly with cardinal numbers, by repeatedly referring to sets they're related to (the definition Halmos gives in the end uses the theory of ordinal numbers). In this paper, we shall give a brief introduction to the world of cardinal numbers.

We systematically denote the cardinals numbers of sets A, B, by small letters a, b. We write $a = b$, or $A \sim B$, whenever there is a bijection from A onto B, and write $a \leq b$, or $A \precsim B$, whenever there is an injection from A into B. As the notation suggests, the relation $\leq$ is an *order*: it is reflexive, antisymmetric and transitive. Antisymmetry is not so easy to obtain, and Cantor himself got stuck trying (we refer to [2], pp. 218–22, for a brief account of the origins of set theory). In terms of sets, this all-important property is:

Theorem 1 If $A \precsim B$ and $B \precsim A$, then $A \sim B$.

Three ingenious little proofs are found in [2], pp. 248–9, [3], pp. 88–9, and [4], pp. 23–5. Boas [1], p. 21, gives references to further proofs. As one might expect, the order among cardinal numbers is *total*, that is, one always has either $a \leq b$ or $b \leq a$. This result too gave Cantor a tough time. But this really shouldn't surprise, since a proof calls for Zorn's lemma (see [3], p. 89), or some axiom to the same effect, and these were yet to be introduced.

2. Larger cardinal numbers

If $A \precsim B$, but not $B \sim A$, we write $A \prec B$, or $a < b$. One of Cantor's major findings was that, for any given set, a yet larger one can be obtained in a natural way:

Theorem 2 $A \prec \mathcal{P}(A)$, where $\mathcal{P}(A)$ is the set of subsets of A.

Proof: By mapping each $x \in A$ to the singleton containing it, we see that $A \precsim \mathcal{P}(A)$. We show that we do not have $A \sim \mathcal{P}(A)$ by proving that an arbitrary map $x \mapsto A_x$ from A into $\mathcal{P}(A)$ can't be onto. We define $B \subset A$ by

$$x \in B \Leftrightarrow x \notin A_x. \tag{2}$$

If $x \mapsto A_x$ were onto, then $B = A_{x_0}$, for some $x_0 \in A$. But then (2) would not hold for $x = x_0$, since both assumptions $x_0 \in B$ and $x_0 \notin B$ lead to a contradiction. In fact, if $x_0 \in B$, then $x_0 \in A_{x_0}$, since $B = A_{x_0}$, whereas (2) would imply that $x_0 \notin A_{x_0}$. The assumption $x_0 \notin B$ gives rise to a similar contradiction. Thus $x \mapsto A_x$ is not onto. ■

For any sets A, B, one may form the set A^B of maps from B into A. An important concept associated with a subset $X \subset A$ is its *characteristic map* χ_X, which, roughly speaking, attains 1 all over X, and 0 outside X. In other words, $\chi_X(x) = 1$ for $x \in X$ and $\chi_X(x) = 0$ for $x \in A - X$. We note that $\{0,1\}^A$ is the set of all characteristic maps of subsets $X \subset A$. If we assign to each element χ_X in $\{0,1\}^A$ the set X, it is easy to see that we get a bijection from $\{0,1\}^A$ onto $\mathcal{P}(A)$. Thus the Theorem 2 may be restated as $A \prec \{0,1\}^A$. In particular, $I \prec \{0,1\}^I$, where $I = [0,1)$. Since $\{0,1\}^I \subset I^I$, it follows that $I \prec I^I$, i.e., there are more maps from I into I than numbers in I. At the end of this note we learn the surprising fact that I^I has actually the same cardinal number as $\{0,1\}^I$, while it would appear to be far larger!

The set-theoretic notation for $\{0,1,2,\ldots\}$ is ω. We say that A is *countable* if $A \sim \omega$, which means that A may be written as a list. Theorem 2 implies that the set $\{0,1\}^\omega$ of all sequences of 0's and 1's is not countable. One could construe that as a proof that I is not countable, since each number in I can be written as a sequence of 0's and 1's (binary expansion). A technical detail prevents it: the number $.10111\ldots$, the rest being all 1's, can as well be written ending with 0's, as $.11000\ldots$. Thus, $\{0,1\}^\omega$ could indeed be larger than I.

A slight variant of the proof of Theorem 2, called Cantor's diagonal process, actually shows that I is not countable. We have to work with (at least) ternary expansion. To avoid ambiguity we use only expansions ending with 0's, rather than those ending with 2's. We take an arbitrary listing $i \mapsto .x_1^i x_2^i x_3^i \ldots$ of points in I. Mirroring the construction of the set B in the proof of Theorem 2, we let $x_i = 1$ if $x_i^i = 0$, and $x_i = 0$ otherwise. Then $.x_1 x_2 x_3 \ldots$ is a valid expansion, not ending with 2's, but is not listed. This shows that I is not countable.

We haven't made it clear whether actually $I \sim \{0,1\}^\omega$, but this is correct. The proof is a cute application of Theorem 1, and it suffices to produce an injection from I into $\{0,1\}^\omega$, and one from $\{0,1\}^\omega$ into I. For the first, we use binary expansions *not* ending with 1's in I, and map, say, $.010\ldots$ to $(0,1,0,\ldots)$. For the second, we use ternary expansion in I, and map, say, $(0,1,0,\ldots)$ to $.010\ldots$, which results in good expansions, not ending with 2's.

3. Cardinal arithmetics

Given cardinal numbers a, b, we select corresponding disjoint sets A, B. By definition, the sum $a+b$ and the product ab are the cardinal numbers of $A \cup B$ and $A \times B$, respectively (to obtain the product, A, B need not be disjoint). Such definitions extend naturally to an arbitrary family $\{a_i\}$ of cardinal numbers, but we do not need this extension.

Exponentiation is also simple: a^b is the cardinal number of A^B. The classical laws of exponents hold, and we are particularly interested in

$$a^{b+c} = a^b a^c, \qquad a^{bc} = (a^b)^c.$$

These are easy to obtain, and we outline the proof of the first. In terms of sets it says that $A^{B \cup C} \sim A^B \times A^C$, for disjoint B, C. But this is clear, since we can see a map on $B \cup C$ as the combination of a map on B and a map on C. Inequalities involving exponentiation also carry over to the universe of cardinal numbers. Of chief interest for us is the fact that if $a \le b$, then $a^c \le b^c$, of which we have already seen an example in $\{0,1\}^I \precsim I^I$.

One denotes by $\aleph_0$ the cardinal number of ω. We now perform little calculations with $\aleph_0$ and $2^{\aleph_0}$. To see that $2^{\aleph_0} + 2^{\aleph_0} = 2^{\aleph_0}$, we note that since $[0,2), [1,2) \sim I \sim \{0,1\}^\omega$, and $[0,2) = I \cup [1,2)$, the cardinal number of $[0,2)$ is both $2^{\aleph_0}$ and $2^{\aleph_0} + 2^{\aleph_0}$. To prove $\aleph_0 + \aleph_0 = \aleph_0$, we break ω into the sets ω_{e} and ω_{o} containing only even and odd numbers. We have that $\omega_{\mathrm{e}}, \omega_{\mathrm{o}} \sim \omega$, since $n \mapsto 2n$ is a bijection from ω onto ω_{e}, and $n \mapsto n+1$ is a bijection from ω_{e} onto ω_{o}. Thus the cardinal number of $\omega = \omega_e \cup \omega_o$ equals $\aleph_0 + \aleph_0$, as desired. This, in turn, enables us to compute

$$2^{\aleph_0} \cdot 2^{\aleph_0} = 2^{\aleph_0 + \aleph_0} = 2^{\aleph_0}. \tag{3}$$

Such an argument, for all its elegance, is somewhat mysterious: it conceals the very bijection from $\{0,1\}^\omega \times \{0,1\}^\omega$ onto $\{0,1\}^\omega$ it is to provide. However, it is a simple matter to spot it, if we are so inclined. We first observe that since (3) uses $\aleph_0 + \aleph_0 = \aleph_0$, we had better keep $\omega = \omega_e \cup \omega_o$ in mind. And since it uses the relation $a^{b+c} = a^b a^c$, it is wise to look again at its proof. We are thus led to combine a point in $\{0,1\}^{\omega_e}$ and one in $\{0,1\}^{\omega_o}$, say, the two lines

$$\begin{array}{cccccccccc} 0 & & 0 & & 1 & & 0 & & \cdots \\ \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow & \\ 1 & & 0 & & 1 & & 1 & & \cdots \end{array} \tag{4}$$

into the point in $\{0,1\}^\omega$ obtained by following the arrows. This readily translates into the bijection from $\{0,1\}^\omega \times \{0,1\}^\omega$ onto $\{0,1\}^\omega$ we're in search of.

An amazing corollary to computation (3) is the fact that $I \times I \sim I$, a discovery reported to have caused even agitation among Cantor's contemporaries.

By the way, it is easy to overlook the fact that we do *not* get a second proof that $I \times I \sim I$ by merely expressing the points in I in binary expansion, and using (4) to provide the desired bijection. The problem is that $.110101010\ldots$ must correspond to the pair consisting of $.1000\ldots$ and $.1111\ldots$, while, in order to avoid ambiguity, expansions ending with 1's are out.

But as it turns out, there is a clever way to make the argument work. It consists of attaching to each 0 the string of 1's preceding it, and treating each such cluster of digits as a single digit:

$$\begin{array}{cccccccc} .110 & & 10 & & 10 & & \cdots \\ \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow & \\ .10 & & 10 & & 10 & & \cdots \end{array}$$

which indicates that $.110101010\ldots$ now corresponds to the pair consisting of $.1101010\ldots$ and $.101010\ldots$.

We observe further that diagram (4) gives rise, if not to a bijection, at least to an injection from $I \times I$ into I. Since an injection the other way round is trivial to find, a third proof of $I \times I \sim I$ results by applying Theorem 1.

Also startling at Cantor's time, though now well-known, was his proof that $\aleph_0 \cdot \aleph_0 = \aleph_0$: one arranges $\omega \times \omega$ in the infinite array

$$\begin{array}{ccccccccccc}
(0,0) & & (0,1) & \rightarrow & (0,2) & & (0,3) & \rightarrow & (0,4) & (0,5) & \cdots \\
\downarrow & \nearrow & & \swarrow & & \nearrow & & \swarrow & & & \\
(1,0) & & (1,1) & & (1,2) & & (1,3) & & (1,4) & (1,5) & \cdots \\
& \swarrow & & \nearrow & & \swarrow & & & & & \\
(2,0) & & (2,1) & & (2,2) & & (2,3) & & (2,4) & (2,5) & \cdots \\
\downarrow & \nearrow & & & & & & & & & \\
(3,0) & & (3,1) & & (3,2) & & (3,3) & & (3,4) & (3,5) & \cdots \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots &
\end{array} \tag{5}$$

and then lists the points as suggested by the arrows.

Using Zorn's lemma one actually shows that $a + a = a \cdot a = a$ for all infinite a at once (see [3], pp. 96–8). In view of the copious evidence above to the diversity of techniques used in the specific cases $a = \aleph_0, 2^{\aleph_0}$, this is rather astonishing.

If a is infinite, induction then leads to $a^n = a$ for each $n \in \omega$. One is tempted to ask whether $a^{\aleph_0} = a$ should hold as well. If $a = \aleph_0$, it certainly doesn't, since $\aleph_0 < 2^{\aleph_0} \leq a^{\aleph_0}$. But if $a = 2^{\aleph_0}$, it does:

$$a^{\aleph_0} = (2^{\aleph_0})^{\aleph_0} = 2^{\aleph_0 \cdot \aleph_0} = 2^{\aleph_0} = a. \tag{6}$$

If we wish to identify the bijection from $(\{0,1\}^\omega)^\omega$ onto $\{0,1\}^\omega$ underlying (6), we should note that since $\aleph_0 \cdot \aleph_0 = \aleph_0$ was used, Cantor's array (5) must somehow be involved. Now, a point in $(\{0,1\}^\omega)^\omega$ is a sequence of sequences of 0's and 1's, that is, a "matrix" with an infinite number of lines, each of which is an infinite string of 0's and 1's. Hence, the map we're after assigns to a given element in $(\{0,1\}^\omega)^\omega$, say,

$$\begin{array}{ccccccccc}
1 & & 0 & \rightarrow & 0 & 1 & 1 & 1 & \cdots \\
\downarrow & \nearrow & & \swarrow & & & & & \\
0 & & 1 & & 0 & 1 & 0 & 0 & \cdots \\
& \swarrow & & & & & & & \\
1 & & 1 & & 0 & 1 & 0 & 1 & \cdots \\
\vdots & & \vdots & & \vdots & \vdots & \vdots & \vdots &
\end{array}$$

the element in $\{0,1\}^\omega$ obtained by following the arrows.

The fact that $a^{\aleph_0} = a$ for $a = 2^{\aleph_0}$ could give us the idea that this should hold of each $a \geq 2^{\aleph_0}$. Surprisingly, $a^{\aleph_0} = a$ does not hold either for an infinity of cardinal numbers a, which can be seen as "limiting" cardinal numbers. Such "limiting" occurs much as $\aleph_0$ limits the sequence $0, 1, 2, \ldots$. For all such special a, we have that $a < a^{\aleph_0}$ (see [4], p. 123).

It is but distressing that the behavior of a^b should display such anomalies, all the more so given that we never left the case $b = \aleph_0$. On the plus side, there is a general case where we easily find a^b: if b is infinite, and $2 \leq a \leq 2^b$, then

$$2^b \leq a^b \leq (2^b)^b = 2^{b \cdot b} = 2^b,$$

and it follows, by antisymmetry, that $a^b = 2^b$. In particular, by setting $a = b = 2^{\aleph_0}$, we establish our previous claim that $I^I \sim \{0,1\}^I$.

Acknowledgement. I want to thank the referee for a considerable improvement of the exposition.

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Problems should not be under consideration by other journals. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before January 1, 2011. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2011 issue of The Pentagon. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)-622-3051).

NEW PROBLEMS 659-668

Problem 659. *Proposed by Andrew Cusumano, Great Neck, NY.*

Find the value of the infinite series

$$\sum_{n=1}^{\infty} \frac{2n}{n^4+n^2+1}.$$

Problem 660. *Proposed by Hongbiao Zeng, Fort Hays State University, Hays, KS.*

Let n be a positive integer greater than 1. Let $f^{(0)}(x) = f(x) = (x-1)(x-2)\cdots(x-n)$. Let $f^{(i)}$ be the i^{th} derivative of $f(x)$. Let S_i denote the sum of all zeros of $f^{(i)}(x)$. Show that

$$\sum_{i=0}^{n-1} S_i = \frac{1}{n}\sum_{i=1}^{n} i^3.$$

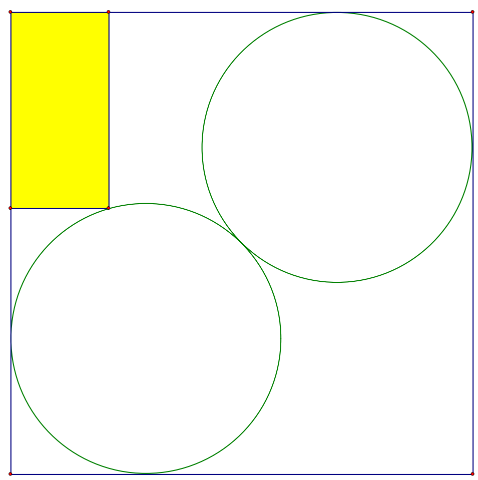
Problem 661. *Proposed by Hongbiao Zeng, Fort Hays State University, Hays, KS.*

Evaluate the double sum

$$\sum_{k=0}^{\infty}\sum_{i=1}^{\infty}(-1)^k\int_0^1 \frac{x^{2k}\left[1+x^{(2k+1)(i^2-1)}\right]}{\left(1+x^{2k+1}\right)^{i^2+1}}\,dx.$$

Problem 662. *Proposed by Ken Dutch, Eastern Kentucky University, Richmond, KY.*

In the following diagram, the shaded rectangle measures 2 cm by 4 cm. What is the radius of the circles in centimeters?



Problem 663. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let $0 < a < b$ and $f : [a, b] \to \mathbb{R}$ be a continuous function. If $A(x)$ is a polynomial with real coefficients for which $A(a) < (b-a)^2 < A(b)$, show that there exist α, $\beta \in (a, b)$ such that

$$\int_a^b f(x)\,dx = f(\alpha)\sqrt{A(\beta)}.$$

Problem 664. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Find all triplets (a, b, c) of real numbers that satisfy the equations

$$a^6 = 5b^2 - 2, b^6 = 5c^2 - 2, c^6 = 5a^2 - 2.$$

Problem 665. *Proposed by Jason Gibson, Eastern Kentucky University, Richmond, KY.*

Let T be the set of integers greater than 1 whose prime divisors live in the set $\{2, 3, 5\}$. What is the sum of the reciprocals of the integers in the set T?

Problem 666. *Proposed by Ovidiu Furdui, Campia Turzii, 405100, Cluj, Romania.*

Let a be a positive real number. Find the value of

$$\lim_{n\to\infty} \sqrt[n]{\int_0^1 (1+ax^n)^n \, dx}.$$

Problem 667. *Proposed by Russell Euler, Northwest Missouri State University, Maryville, MO.*

For $n \geq 1$, find all Pythagorean triples (T_n, S_n, P_n) where T_n, S_n, and P_n are the n^{th} triangular, square, and pentagonal numbers, respectively.

Problem 668. *Proposed by the editor.*

Prove that the sum of the cubes of three consecutive positive integers can never equal the sum of the squares of two integers which are relatively prime.

SOLUTIONS TO PROBLEMS 641-648

Problem 641. *Proposed by Lisa Kay, Eastern Kentucky University, Richmond, KY.*

Suppose that there are five students enrolled in a chemistry class. They will have to complete five lab assignments. For each lab assignment, four of the students will work in two pairs while one student works independently. Each student will work independently for exactly one of the five labs. Each student will work with each of the other four students exactly once. How many different lab schedules are possible?

Solution *by the proposer.*

Label the five students A, B, C, D, and E. There are five ways to choose the student to work independently in Lab 1. Then there are three ways to divide the other four students into two pairs for Lab 1. Once the assignments have been made for Lab 1, there are four ways to choose the student to work independently in Lab 2. Then there are two ways to divide the other four students into two pairs for Lab 2. The independent student from Lab 1 cannot be paired with the student who was the Lab 1 partner of the independent student from Lab 2. (For example, if A were independent and B were paired with C in Lab 1, then A cannot be paired with C in Lab 2 because that would put D and E together a second time.) After the assignments have been made for Labs 1 and 2, there are three ways to choose the student to work independently in Lab 3. Then there is only one way to put the other four students into two pairs that will not cause duplications in the remaining labs. Once assignments have been made for Labs 1-3, there are two ways to choose the student to work independently in Lab 4. There is only one way to divide the other four students into two pairs for Lab 4. Once the assignments have been made for Labs 1-4, there is one way to choose the student to work independently in Lab 5. Then there is only one way to divide the other four students into two pairs for Lab 5. Hence, there are $5 \cdot 3 \cdot 4 \cdot 2 \cdot 3 \cdot 2 = 720$ to schedule the labs.

Also solved by James Bichler (student), Slippery Rock University, Slippery Rock, PA.

Problem 642. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let a, b, c be the lengths of the sides of a triangle ABC with heights h_a, h_b, and h_c, respectively. Prove that

$$\prod_{\text{cyclic}} \left(\frac{h_a}{h_b + h_c}\right)^{1/3} \leq \frac{1}{6}\left(\frac{a+b+c}{\sqrt[3]{abc}}\right).$$

Solution *by Michelle Zeng, Fort Hays State University, Hays, KS.*

The left side of the equation is

$$\begin{aligned} &\left(\frac{h_a}{h_b+h_c} \cdot \frac{h_b}{h_c+h_a} \cdot \frac{h_c}{h_a+h_b}\right)^{1/3} \\ = &\left(\frac{ah_a}{abch_b + abch_c} \cdot \frac{bh_b}{abch_c + abch_a} \cdot \frac{ch_c}{abch_a + abch_b}\right)^{1/3} (abc)^{2/3}. \end{aligned}$$

Notice that

$$ah_a = bh_b = ch_c = \text{ twice the area of triangle } ABC.$$

Hence, the above expression equals

$$\begin{aligned} &\left(\frac{1}{(ac+ab)(ab+bc)(bc+ac)}\right)^{1/3} (abc)^{2/3} \\ \leq &\left(\frac{1}{\left(2\sqrt{acab}\right)\left(2\sqrt{abbc}\right)\left(2\sqrt{bcac}\right)}\right)^{1/3} (abc)^{2/3} \\ = &\frac{1}{2} \\ \leq &\frac{1}{6}\left(\frac{a+b+c}{\sqrt[3]{abc}}\right). \end{aligned}$$

This last step results from $\sqrt[3]{abc} \leq \dfrac{a+b+c}{3}$, which implies that

$$\frac{1}{2} \leq \frac{1}{6}\left(\frac{a+b+c}{\sqrt[3]{abc}}\right).$$

Also solved by the proposer.

Problem 643. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

The equation $x^3 - 2x^2 - x + 1 = 0$ has three real roots $a > b > c$. Find the value of $ab^2 + bc^2 + ca^2$.

Solution *by Hongbiao Zeng, Fort Hays State University, Hays, KS.*

From the conditions, we know that

$$\begin{aligned} a+b+c &= 2 & (1)\\ ab+bc+ca &= -1 & (2)\\ abc &= -1. & (3) \end{aligned}$$

Multiplying (2) by a, b, and c, we have

$$a^2b + abc + ca^2 = -a,\ ab^2 + b^2c + abc = -b,\ abc + bc^2 + c^2a = -c,$$

respectively. Summing these, we get

$$\left(ab^2 + bc^2 + ca^2\right) + \left(a^2b + b^2c + c^2a\right) = -(a+b+c) - 3abc = 1. \quad (4)$$

From (1), we have $(a+b+c)^2 = 4$ and $(a+b+c)^3 = 8$. Therefore

$$a^2 + b^2 + c^2 = 4 - 2(ab + bc + ca) = 6$$

and

$$a^3 + b^3 + c^3 = 8 - 3\left(ab^2 + bc^2 + ca^2 + a^2b + b^2c + c^2a + 2abc\right) = 11.$$

We now evaluate $a^3b^3 + b^3c^3 + c^3a^3$. From (2), we have

$$(ab + bc + ca)^3 = -1.$$

Therefore,

$$\begin{aligned} & a^3b^3 + b^3c^3 + c^3a^3 \\ = \;& -1 - 3\left(ab^3c^2 + bc^3a^2 + ca^3b^2 + a^2b^3c + b^2c^3a + c^2a^3b + 2a^2b^2c^2\right) \\ = \;& -1 - 3\left(-b^2c - c^2a - a^2b - ab^2 - bc^2 - ca^2 + 2\right) \\ = \;& -4, \end{aligned}$$

so we have

$$\begin{aligned} & \left(ab^2 + bc^2 + ca^2\right)\left(a^2b + b^2c + c^2a\right) & (5)\\ = \;& a^3b^3 + b^3c^3 + c^3a^3 + ab^4c + bc^4a + ca^4b + 3a^2b^2c^2 \\ = \;& \left(a^3b^3 + b^3c^3 + c^3a^3\right) - \left(b^3 + c^3 + a^3\right) + 3a^2b^2c^2 \\ = \;& -4 - 11 + 3 = -12. \end{aligned}$$

From (4) and (5), we know that the sum of $\left(ab^2 + bc^2 + ca^2\right)$ and $\left(a^2b + b^2c + c^2a\right)$ is 1 and the product is -12. Therefore, one of them is 4 and the other is -3. From (1) and (3), we know that one and only one of a, b, c is negative. Since $a > b > c$, we know that $a > b > 0 > c$. Therefore, $a^2b + b^2c + c^2a > \left(b^3 + c^2b\right) + b^2c > 2b^2\,|c| + b^2c > 0$. So we must have $ab^2 + bc^2 + ca^2 = -3$.

Also solved by the proposer. Computer algebra solutions by students Chris Adkisson and Josh Sparks, Eastern Kentucky University, Richmond, KY.

Problem 644. *Proposed by Andrew Cusumano, Great Neck, NY.*

Find two primes whose reciprocals repeat after exactly 7 decimal places.

Solution *by Ed Wilson, Eastern Kentucky University, Richmond, KY.*

Let p be a prime. If the reciprocal of p repeats after 7 decimal places, we may assume that

$$\frac{1}{p} = .\overline{abcdefg}.$$

Multiplying by 10^7, we have $\frac{10000000}{p} = abcdefg.\overline{abcdefg}$. Subtracting $1/p$ and $.\overline{abcdefg}$ from the respective sides of the equation,

$$\frac{10000000 - 1}{p} = \frac{9999999}{p} = abcdefg.$$

Hence, p must be a prime factor of 9999999. The prime factorization of 9999999 is $3^2 \cdot 239 \times 4649$. The reciprocal of 3 repeats after exactly one decimal place. On the other hand,

$$\frac{1}{239} = .\overline{0041841} \text{ and } \frac{1}{4649} = .\overline{0002151}.$$

So the two primes whose reciprocals repeat after exactly 7 decimal places are 239 and 4649.

Also solved by Michelle Zeng, Fort Hays State University, Hays, KS; Chris Robinson (student), Centre College, Danville, KY; Donald Brewer (student), Eastern Kentucky University, Richmond, KY; Josh Sparks (student), Eastern Kentucky, Richmond, KY; and the proposer.

Problem 645. *Proposed by Ben Thurston, Florida Southern College, Lakeland, FL.*

What is the expected number of rolls of a fair die required to have all six faces come up at least once?

Solution *by Zachary Hopkins (student), Slippery Rock University, Slippery Rock, PA.*

Let x_i be the number of rolls required to get i different numbers, $i = 1, 2, ..., 6$. The expected number of rolls required to have all 6 faces come up at least once is

$$E\left(\sum_{i=1}^{6} x_i\right) = \sum_{i=1}^{6} E\left(x_i\right).$$

Each x_i is geometrically distributed with $P\left(x_i = 1\right) = 1$ and, for $2 \leq i \leq 6$, $P\left(x_i = y\right) = q^{y-1}p$, where $p = \left(7 - i\right)/6$ and $q = 1 - p$. The expected value of the geometric distribution is $E\left(x_i\right) = 1/p = 6/\left(7 - i\right)$. Thus $\sum_{i=1}^{6} E\left(x_i\right) = 14.7$.

Also solved by Sam Wysong (student), Manchester College, North Manchester, IN; and the proposer.

Problem 646. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, Illinois.*

Suppose that n is an odd integer. Show that

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots + \frac{1}{n-1} - \frac{1}{n} > \frac{1}{4} - \frac{1}{2\left(n+1\right)}.$$

Solution *by Josh Williams (student), Northwest Missouri State University, Maryville, MO.*

By direct computation, the inequality holds for $n = 3,\ 5,\ 7$. In what follows, we assume n is an odd integer greater than or equal to 9. It is well known that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \log 2$. By the alternating series error theorem, $0 < \left(-1\right)^n \left(\log 2 - S_n\right) < \frac{1}{n+1}$, where S_n is the n^{th} partial sum of the series. If n is odd, then $-\log 2 + 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots - \frac{1}{n-1} + \frac{1}{n} < \frac{1}{n+1}$, or equivalently,

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots + \frac{1}{n-1} - \frac{1}{n} > 1 - \log 2 - \frac{1}{n+1}. \qquad (1)$$

For $n \geq 9$, $\frac{1}{n+1} - \frac{1}{2(n+1)} = \frac{1}{2(n+1)} \leq 0.05$, so that

$$1 - \log 2 - \frac{1}{4} > \frac{1}{n+1} - \frac{1}{2(n+1)}. \tag{2}$$

The desired inequality now follows from (1) and (2).

[Editor's note: By grouping pairs, the left-hand side can be shown to be strictly greater than 0.25 when n is odd and ≥ 9.]

Also solved by Michelle Zeng, Fort Hays State University, Hays, KS; and the proposers.

Problem 647. *Proposed by Panagiote Ligouras, Leonardo da Vinci High School, Noci, Italy.*

Let a, b, and c be the sides, and m_a, m_b, and m_c the medians of a triangle ABC. Prove or disprove that

$$m_a^2 m_b^2 + m_b^2 m_c^2 + m_c^2 m_a^2 \geq \frac{9}{4}\left(\frac{a^4 bc \cos A}{b^2+c^2} + \frac{ab^4 c \cos B}{c^2+a^2} + \frac{abc^4 \cos C}{a^2+b^2}\right).$$

Solution *by the proposer.*

We have

$$\frac{a^2b^2 + b^2c^2 + c^2a^2}{2} \geq \frac{a^4\left(b^2+c^2-a^2\right)}{b^2+c^2} + \frac{b^4\left(c^2+a^2-b^2\right)}{c^2+a^2} + \frac{c^4\left(a^2+b^2-c^2\right)}{a^2+b^2}. \tag{1}$$

In fact

$$\begin{aligned}\sum_{\text{cyclic}} \frac{b^2c^2}{2} &= \sum_{\text{cyclic}} \frac{a^2\left(b^2+c^2\right)}{4} = \sum_{\text{cyclic}} \frac{a^2}{b^2+c^2}\left[\frac{a^2 + \left(b^2+c^2-a^2\right)}{2}\right]^2 \\ &\geq \sum_{\text{cyclic}} \frac{a^2}{b^2+c^2} a^2\left(b^2+c^2-a^2\right) = \sum_{\text{cyclic}} \frac{a^4\left(b^2+c^2-a^2\right)}{b^2+c^2}.\end{aligned}$$

It is well known that

$$a^2+b^2 = 2m_c^2 + \frac{1}{2}c^2,\ c^2+b^2 = 2m_a^2 + \frac{1}{2}a^2,\ c^2+a^2 = 2m_b^2 + \frac{1}{2}b^2. \tag{2}$$

Using (2), we have

$$m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}\left(a^2+b^2+c^2\right)$$

and

$$\left(m_a^2+m_b^2+m_c^2\right)^2=\left[\frac{3}{4}\left(a^2+b^2+c^2\right)\right]^2. \tag{3}$$

Using (2), we have

$$4m_c^2=2a^2+2b^2-c^2,\, 4m_b^2=2a^2+2c^2-b^2,\, 4m_a^2=2c^2+2b^2-a^2.$$

Using these,

$$\begin{aligned}\left(4m_c^2\right)^2 &= \left(2a^2+2b^2-c^2\right)^2,\\ \left(4m_b^2\right)^2 &= \left(2a^2+2c^2-b^2\right)^2,\\ \left(4m_a^2\right)^2 &= \left(2c^2+2b^2-a^2\right)^2.\end{aligned}$$

Adding and simplifying gives

$$16\left(m_a^4+m_b^4+m_c^4\right)=9\left(a^4+b^4+c^4\right). \tag{4}$$

Subtracting (3) and (4) gives

$$16\left(m_a^2m_b^2+m_b^2m_c^2+m_c^2m_a^2\right)=9\left(a^2b^2+b^2c^2+c^2a^2\right).$$

Using (1) and (4), we have

$$\begin{aligned}&m_a^2m_b^2+m_b^2m_c^2+m_c^2m_a^2\\ \geq\ &\frac{9}{8}\cdot\left[\frac{a^4\left(b^2+c^2-a^2\right)}{b^2+c^2}+\frac{b^4\left(c^2+a^2-b^2\right)}{c^2+a^2}\right.\\ &\left.+\frac{c^4\left(a^2+b^2-c^2\right)}{a^2+b^2}\right].\end{aligned} \tag{5}$$

It is well known that

$$\cos A=\frac{b^2+c^2-a^2}{2bc},\ \cos B=\frac{c^2+a^2-b^2}{2ca},\ \cos C=\frac{a^2+b^2-c^2}{2ab}. \tag{6}$$

Using (5) and (6), we have

$$m_a^2m_b^2+m_b^2m_c^2+m_c^2m_a^2\geq\frac{9}{8}\left[\frac{2a^4bc\cos A}{b^2+c^2}+\frac{2ab^4c\cos B}{c^2+a^2}+\frac{2abc^4\cos C}{a^2+b^2}\right],$$

and finally

$$m_a^2m_b^2+m_b^2m_c^2+m_c^2m_a^2\geq\frac{9}{4}\left(\frac{a^4bc\cos A}{b^2+c^2}+\frac{ab^4c\cos B}{c^2+a^2}+\frac{abc^4\cos C}{a^2+b^2}\right).$$

Problem 648. *Proposed Ovidiu Furdui, University of Toledo, Toledo, OH.*

Let $k > 1$ be a real number. Find the value of $\int_0^1 \left\{ \frac{1}{\sqrt[k]{x}} \right\} dx$, where $\{a\} = a - \lfloor a \rfloor$ denotes the fractional part of a. [For example, $\{1.9\} = 0.9$.]

Solution *by Angel Plaza and Sergio Falcon, Universidad de Las Palmas de Gran Canaria, Las Palmas, Spain.*

By substituting $y = \dfrac{1}{\sqrt[k]{x}}$, we obtain

$$I = \int_0^1 \left\{ \frac{1}{\sqrt[k]{x}} \right\} dx = \int_1^\infty \{y\}\, k y^{-k-1}\, dy.$$

Note that, if $n < y < n+1$, then $\{y\} = y - n$. Therefore,

$$\begin{aligned}
I &= \sum_{n=1}^{\infty} k \int_n^{n+1} (y-n)\, y^{-k-1}\, dy \\
&= k \sum_{n=1}^{\infty} \int_n^{n+1} \left(y^{-k} - n y^{-k-1} \right) dy \\
&= k \sum_{n=1}^{\infty} \left[\frac{y^{-k+1}}{-k+1} + \frac{n y^{-k}}{k} \right]_n^{n+1} \\
&= \sum_{n=1}^{\infty} \left[n y^{-k} - \frac{k y^{-k+1}}{k-1} \right]_n^{n+1} \\
&= \frac{k}{k-1} + \sum_{n=1}^{\infty} \left[n y^{-k} \right]_n^{n+1} \\
&= \frac{k}{k-1} + \sum_{n=1}^{\infty} \left(\frac{n}{(n+1)^k} - \frac{n}{n^k} \right) \qquad (1) \\
&= \frac{k}{k-1} - \sum_{n=1}^{\infty} \frac{1}{n^k} \\
&= \frac{k}{k-1} - \zeta(k),
\end{aligned}$$

where $\zeta(k)$ is the Riemann zeta function which is finite since $k > 1$.

Also solved by Michelle Zeng, Fort Hays State University, Hays, KS; and the proposer. Chris Robinson (student), Centre College, Danville, KY was able to arrive at (1).

Announcement of the Thirty-Eighth Biennial Convention of Kappa Mu Epsilon

The Thirty-Eighth Biennial Convention of Kappa Mu Epsilon will be held April 14-16, 2011, on the campus of Harris Stowe State University, home of Missouri Mu Chapter, in St. Louis, Missouri. KME President-Elect Rhonda McKee(mckee@ucmo.edu) will send additional details to chapters in the fall. Each attending chapter will receive the usual travel expense ($.35/mile) reimbursement from the national office as described in Article VI, Section 2, of the Kappa Mu Epsilon Constitution.

A significant feature of our national convention is presentations by student members of Kappa Mu Epsilon. The mathematical topic selected by each student should be of interest to the author and of such scope that it can be given adequate treatment in a timed oral presentation. Senior projects and seminar presentations have been a popular way for faculty to get students to investigate suitable topics. Student talks to be presented at the convention will be chosen prior to the convention by the Selection Committee on the basis of the materials submitted. At the convention, the Awards Committee will judge the selected talks on both content and presentation, and the top four papers will receive a cash prize.

Who may submit a paper?

Any undergraduate student member of Kappa Mu Epsilon may submit an abstract for consideration as a talk at the national convention. Presentations may be coauthored.

Presentation Topics

Presentations at the convention should discuss material understandable by undergraduate students. The Selection Committee will favor abstracts that satisfy this criterion and which can be presented with reasonable completeness within the time allotted. Presentations may be original research by the student(s) or exposition of interesting but not widely known results. Presenters should always cite authors if presenting exposition of known results.

Presentation Time Limits

The time frame for presentations at the convention is 15 to 20 minutes.

How to Submit an Abstract

Students who wish to make a presentation at the national convention should submit an abstract of up to 500 words explaining the nature of the presentation and indicating the results. The abstract should be accompanied by a cover sheet and a letter from the student's project advisor. The cover sheet should include a statement from the corresponding secretary of the student's KME chapter, certifying the student's membership in KME. The letter from the project advisor should include a statement certifying that the student is doing the work specified in the abstract and the advisor's belief that the student will have a fully prepared presentation by the time of the convention.

Please send the cover sheet, abstract and advisor letter by electronic mail to:

Dr. Rhonda McKee, KME President-Elect
Department of Mathematics and Computer Science
University of Central Missouri
mckee@ucmo.edu

Selection of Papers for Presentation

A Selection Committee will review the abstracts submitted and will choose approximately fifteen papers to be judged for awards at the convention. The President-Elect will notify all authors of the status of their submissions after the Selection Committee has completed its deliberations.

Judging criteria include

- Choice and originality of topic
- Literature sources and references
- Depth, significance, and correctness of content
- Clarity and organization of materials
- Adherence to time constraints
- Effective use of graphs and/or visual aids
- Overall effect

Publication

A presenter who has not prepared a formal written paper by the time of the convention is encouraged to do so soon after the convention, so that the paper can be submitted for possible publication in *The Pentagon*. Unless published elsewhere, papers prepared from the prize-winning presentations will be published in *The Pentagon* after any necessary revisions have been completed. All other papers will be considered for publication. The editor of *The Pentagon* will contact each author during or soon after the convention to review his or her presentation and discuss requirements for publication.

To have a paper considered for publication, prepare it as a Microsoft Word document or .tex file, and include it as an attachment to an e-mail to the editor at curtis-c@mssu.edu. The electronic copy of the paper will be sent to a referee who will prepare an anonymous report. If the referee recommends publication and space is available, the paper will be published in one of the next several issues.

(Sample Cover Sheet)

Title of Presentation

Student's Name
Student's KME Chapter (State and Greek letter)
Student's College/University

I certify that (student's name) is a member of the (state and Greek letter) chapter of Kappa Mu Epsilon.

(corresponding secretary)

(date)

Thank You Referees!

The editor wishes to thank the following individuals who, during the last two years, refereed papers submitted to *The Pentagon*.

- John Behle, Harris-Stowe State University, St. Louis, Missouri
- Mark Bollman, Albion College, Albion, Michigan
- David Calvis, Baldwin Wallace College, Berea, Ohio
- Tim Flood, Pittsburg State University, Pittsburg, Kansas
- Berhane Ghaim, Southeastern University, Lakeland, Florida
- Chungsim Han, Baldwin Wallace College, Berea, Ohio
- Josh Brown Kramer, Illinois Wesleyan University, Bloomington, Illinois
- Jeremy Nadolski, Benedictine University, Lisle, Illinois
- Kimberly Presser, Shippensburg University, Shippensburg, Pennsylvania
- Andrew Rockett, C.W. Post Campus, Long Island University, Brookville, New York
- Evangelos Skoumbourdis, Liberty University, Lynchburg, Virginia
- Don Tosh, Evangel University, Springfield, Missouri
- Sister Marcella Louise Wallowicz, Holy Family University, Philadelphia, Pennsylvania

Also thanks to the many other individuals who volunteered to serve as referees but were not used during the past two years. If you wish to volunteer as a referee, feel free to contact the editor to receive a referee interest form.

Kappa Mu Epsilon News

Edited by Peter Skoner, Historian

Updated information as of March 2010

Send news of chapter activities and other noteworthy KME events to

Peter Skoner, KME Historian
Saint Francis University
117 Evergreen Drive
313 Scotus Hall
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or to
pskoner@francis.edu

Installation Report

Alabama Theta
Jacksonville State University

The Alabama Theta Chapter of Kappa Mu Epsilon was installed at 3:00 p.m. on Monday, March 29, 2010, at a ceremony in Houston Cole Library on the campus of Jacksonville State University, Jacksonville, Alabama. The meeting was conducted by Dr. David Dempsey. KME President Ron Wasserstein served as the Installing Officer. The charter members—Michelle Amosu, Korey Bentley, Timothy Daniel Bowden, Valarie Boyd, Eric Brown, Andres Camacho, Melissa Camp, David Chaney, Jason Cleveland, Tara Cook, Leah Dennis, Tyler Gable, Christina Rachael Ginn, Brandy Greenleaf, Katie Henson, Chad Horton, Jordyn Houser, Sunde Jones, Ashley Jordan, Oxana Katalevskaya, Cain Eric Kirk, Jayne Lampley, Anna Miller, Catrina Mize, Kristie Osborne, Lindsey Osborne, Jim Roebuck, Kristin Shirey, Jamie Shirley, Jessica Silvia, Page Trantham, and Amanda Webster—and faculty members—David Dempsey, Jeff Dodd, Amy Franklin, Fred Kelley, Sherry Kennedy, Rhonda Kilgo, Jaedeok Kim, Youngmi Kim, Martha Knight, Tom Leathrum, and Audria White—were initiated into the chapter. The first officers of Alabama Theta, President Brandy Greenleaf, Vice President Kristie Osborne, Recording Secretary Page Trantham, Treasurer Tyler Gable, and Corresponding Secretary/Faculty Sponsor David Dempsey were installed. The initiates were welcomed by JSU President William A. Meehan.

About 50 people were in attendance. After the formal ceremonies, Wasserstein presented a talk entitled "What Probability and Forrest Gump Teach Us About State Lotteries."

Florida Gamma
Southeastern University

The Florida Gamma Chapter of Kappa Mu Epsilon was installed at 4:pm on March 31, 2010, at a ceremony in Johnson Chapel on the campus of Southeastern University, Lakeland, Florida. The meeting was conducted by Dr. Berhane T. Ghaim. KME President Ron Wasserstein served as the Installing Officer. The charter members, Jabet Artis, Jeyan Bhakta, Timothy Burnham, Shaina Cannavaro, Lous Caponi, Brittany Hurst, Stephen Prosser, Meaghan Simms, Samatha Staples, and faculty members Shearen Fredere, Deborah Hazelbaker, Gary Kimball, TaeEun Kim, Todd Schraw, David Revell, were initiated into the chapter. The first officers of Florida Gamma, President Janet Artis, Vice President Meaghan Simms, Recording Secretary Samantha Staples, Treasurer Lous Caponi, Corresponding Secretary Dr. Berhane T. Ghaim, and Faculty Sponsor Dr. Berhane Ghaim were installed. The initiates were welcomed by Dr. Gordon Miller, Dean of the College of Arts and Sciences.

About 40 people were in attendance. After the formal ceremonies, Wasserstein presented a talk entitled "What Probability and Forrest Gump Teach Us About the Florida Lottery."

North Carolina Epsilon
Johnson C. Smith University

The North Carolina Eta Chapter of Kappa Mu Epsilon was installed at 1:30pm on Thursday, March 18, 2010, at a ceremony in the Technology Center on the campus of Johnson C. Smith University, Charlotte, North Carolina. The meeting was conducted by Quadashia Walker-Moss. KME President Ron Wasserstein served as the Installing Officer. The charter members, Niketa Jones, Catherine Few, Dawnite Gilmore, Andrew Milden, Chenea Wilson, Shawana Wilson, Lorren Baldwin, Maurice Scott, Quadasia Walker-Moss, R'Ameereah Alexander, Shimeca Bowman, Elliot Betrand, Marcia Higgins and faculty members Livinus Uko, Brian Hunt and Lakeshia Legette, were initiated into the chapter. The first officers of North Carolina Eta, President Niketa Jones, Vice President Maurice Scott, Recording Secretary Shimeca Bowman, Treasurer Quadashia Walker-Moss, Corresponding Secretary Lakeshia Leggette, and Faculty Sponsor Livinus Uko were installed.

About 25 people were in attendance. After the formal ceremonies, Wasserstein presented a talk entitled "What Probability and Forrest Gump Teach Us About the North Carolina Lottery."

New Jersey Epsilon
New Jersey City University; Jersey City, NJ

The New Jersey Epsilon Chapter of Kappa Mu Epsilon was installed at 3:00 P.M. on Monday, February 22, 2010, at a ceremony in the Student Union on the campus of New Jersey City University in Jersey City, New Jersey. The meeting was conducted by Karen Ivy. KME President Ron Wasserstein served as the Installing Officer. The charter members initiated into the chapter include students Abiodun Banner, Salma Bouazoui, Asmaa Bouayad, Juliette Cabrera, Phil Carrillo, Cody Chin, Azza Eltawil, Walter Fedzina, Tracy Goycochea, Teresa Kalinowski, Elizabeth Manfrede, Peter Morin, Maria Quiamboa, Edwin Rivera, Christopher William, and faculty members Deborah Bennett, Sandra Caravella, Zhixiong Chen, Yi Ding, Karen Ivy, Richard Riggs, Freda Robbins, and Beimnet Teclezghi. The first officers of New Jersey Epsilon were installed, including President Elizabeth Manfrede, Vice President Tracy Goycochea, Recording Secretary Phil Carrillo, Treasurer Cody Ching, Corresponding Secretary Beimnet Teclezghi, and Faculty Sponsor Yi Ding. The initiates were welcomed by Dr. Barbara Feldman, Dean of the College of Arts and Sciences. About 30 people were in attendance. After the formal ceremonies, Ron Wasserstein presented a talk entitled "What Probability and Forrest Gump Teach Us About the New Jersey Lottery."

Pictured are the student inductees, plus corresponding secretary Beimnet Teclezghi (back row, third from left), Dean of Arts and Sciences Barbara Feldman (second row, farthest to the right), and KME President Ron Wasserstein (second row, third from the left).

Rhode Island Alpha
Roger Williams University; Bristol, RI

The installation of the Rhode Island Alpha Chapter of Kappa Mu Epsilon was held in Room 200 of the Marine and Natural Sciences (MNS) Building on the campus of Roger Williams University on Friday, November 13, 2009, at 3:00 PM. KME National Historian Peter Skoner served as the installing officer. Dr. Annela Kelly, faculty sponsor and corresponding secretary, organized the event and participated in the ceremony; she was initiated previously into the society while a faculty member of the Louisiana Delta chapter at the University of Louisiana at Monroe. Dr. Skoner began the day before the combined installation and initiation ceremony by presenting a talk “And the Winner Is: Careers in Mathematics and Careers Using Mathematics.”

The faculty charter members of Rhode Island Alpha are Bruce Burdick, Earl Gladue, Ruth Koelle, Koray Özer, and Yajni Warnapala. The student charter members of Rhode Island Alpha are Joshua Boisclair, Jesse Farugella, Erin Gilliam, Katie Kanakos, Alyssa MacDonald, Barbara Mann, Andrew Richard, Danielle Rubenstein, and Raveena Siegel. Each initiate was invited to sign the Rhode Island Alpha Chapter Roll, and was presented with a membership card, KME brochure, a program announcing the charter initiates, and a KME jewelry pin. The officers of the Rhode Island Alpha chapter were also installed during the ceremony including Dr. Annela Kelly as faculty sponsor and corresponding secretary; Danielle Rubenstein as president, Raveena Siegel as vice president, Erin Gilliam as treasurer, Jesse Farugella as secretary, and Andrew Richard as webmaster. Each officer was charged with the responsibilities of the office, and each chose to accept those responsibilities. After Jesse Farugella described KME's crest, the organization was declared to be the Rhode Island Alpha Chapter of Kappa Mu Epsilon and the chapter's charter was presented to chapter president Danielle Rubenstein. Congratulations, fellowship, and delicious refreshments were enjoyed by all following the both the presentation and the installation/initiation ceremony. A total of about 30 people were in attendance.

Chapter News

AL Zeta – Birmingham Southern College

Chapter President – Xinyan Yan; 5 Current Members

Loree Killebrew, Vice President; Michael Graham, Secretary and Treasurer; Mary Jane Turner, Corresponding Secretary; Dr. Bernie Mullins, Faculty Sponsor.

The Fall Program featured Cameron Byrum, Ph.D. student at the University of Mississippi, who presented "Combinatorics on Partial Words."

CA Epsilon – California Baptist University

James Buchholz, Corresponding Secretary.

New initiates – Timothy Cahill, Brandi Cozzens-Ritz, Charlie Holderman, Katherine Kistler, Kristal McMillan, LeAnn Stephens, Jessica Worsham, Lisa Hernandez, Ali Mirazali, Brett Ackerman, David Brinkley, Rebecca Capata, Cassondra Conant, Kacy Craven, Tim Degeneffe, Joshua Ewell, Jonathon Fanning, Jessica Qwilt, Kamaljit Kaur, Nathan Sepulveda, Rebecca Whitley, Matt Ayers, Mayra Barajas, Anabel Flores, Teresa Gibbs. Meghan Hosfield, Benjamin Lee, Krystal Solomona, Brittany Walden.

CO Beta – Colorado School of Mines

Terry Bridgman, Corresponding Secretary

New Initiates - Kelly Andrews, Carl Blum, Brett Eagle, Sam Geldhof, Nicholas Gerstle, Cathryn Greene, Carl J. Hesse, Jeff Hilton, D. Ryan Hild, Sarah Hinnegan, Mohd Afiq Ishak, Emma Janisch, Courtney Kais, Satira Labib, Callen Liles, David Lipp, Daniel Moran, Marty Otzenberger, Daniel Pascua, Courtney Rohde, Tyler Rust, Tyler Scott, Quintin Sheridan, Brennan Sprinkle, Ashley Valdez, Brenden Villars, Nathan Welch, Scott Wiedemann, Kevin Alkema, Eric Eisinger, Ginger Gilfillan, Lemuel Godinez, Jennifer Gallardo, William Leavenworth, Marie-Claire LeLait, Blakelee Midyett, Samuel Schuessler.

CO Delta – Mesa State College

Erik Packard, Corresponding Secretary

New Initiates - Brendan Bridge, Jacob Cady, Heather Conder, Larissa Merrel, Ryan Monesmith, Anton Neff, Katherine Pearson, Donald Warbritton.

GA Beta – Georgia College and State University

Chapter President – Dr. Jason Huffman; 177 Current Members

Dr. Laurie Huffman, Corresponding Secretary;Dr. Jason Huffman, Faculty Sponsor.

IA Alpha – University of Northern Iowa

Chapter President – Reanna Collins; 12 Current Members, 3 New Members

Jaime Zeigler, Vice President; Kelsey Staudacher, Secretary; David Rygh, Treasurer; Mark D. Ecker, Corresponding Secretary.

Our first fall KME meeting was held on September 21, 2009 at Professor Jerry Ridenhour's house where student member David Rygh presented his paper entitled "NBA Draft Lottery Probabilities." The University of Northern Iowa Homecoming Coffee was held at Professor Suzanne Riehl's residence on October 17, 2009. Student member Jaime Zeigler presented her paper entitled "Which Variables Affect Crime Rates?" at our second meeting on November 9, 2009 at Professor Mark Ecker's home. Jim Stevenson addressed the fall initiation banquet with "Design for Six Sigma." Our fall banquet was held at the Pepper's Grill and Sports Pub in Cedar Falls on December 7, 2009 where three new members were initiated.

New Initiates – Nathan Kelly, Clint Meinecke, Anna Schumacher.

IL Beta – Eastern Illinois University

Nancy Van Cleave, Corresponding Secretary

New Initiates - Evan Bright, Katie Fehrenbacher, Brett Hafferkamp, Amy Helpingstine, David Hovorka, Steven Kilty, Timothy Russell, Ben Thompson, Dan Campbell, Paula Cler, Jon Harter, Emily Horigan, Robert Johnson, Adam Long, Bert Smallwood, Stephen Summers, Amy Vitzthum, Paula Wojcik, Stefan Aydt, Philip Boehl, Sarah Braun, Emily Hecht, Paul Ivnik, Garett Kerber, Joseph Leipert, Jared McClellan, Mollie Neff, Ashlee Sharp, Dominique Sims, Hannah Wilson.

IL Zeta – Dominican University

Chapter President – Natalie Waksmanski; 20 Current Members

Kim Plesnicar, Vice President; Michelle Nowak, Secretary; Nicole Marin, Treasurer; Aliza Steurer, Corresponding Secretary.

The Illinois Zeta chapter of KME and the Math Club at Dominican University operate together as one student organization. This fall the members of both worked hard to increase their activities around campus. They held many fun events, including a Sudoku competition, an origami event in which participants learned how to make polyhedra out of origami, and designed a new Math Club T-shirt. Each of these activities, as well as the organization's regular meetings, was well attended. In fact, the origami event was so popular that a second session will be held in the spring. The members look very forward to all of the spring 2010 activities.

IN Delta – University of Evansville

Chapter President – Mary Craighead

Carrie Schindler, Vice President; Annette DeWolf, Secretary; Dr. Talitha Washington, Webmaster; Dr. Adam Salminen, Corresponding Secretary; Dr. Mohamma Azarian, Faculty Sponsor.

During the fall of 2009, Indiana Delta offered free tutoring during finals week. In the last year, we introduced a new website which was created by Dr. Washington.

KS Beta – Emporia State University

Chapter President – Heather Julia; 25 Current Members, 4 New Members
Melissa Swager, Vice President; Richard Nelson, Treasurer; Yuchen Chen, Secretary; Dr. Connie Schrock, Corresponding Secretary

Last semester we held a Calculator Workshop in order to teach the basics for the TI Graphing Calculator for college algebra students, helped with math day, and had an ice cream social. We hosted a Jeopardy game and a math panel for the introduction to mathematics class. Towards the end of the semester we had initiations and an end of the semester party. We initiated two full members and two associate members.

KS Delta – Washburn University

Chapter President – Kyle Volle; 16 Current Members
Moriba DeCoteau, Vice President; Sean VanDyke, Secretary and Treasurer; Dr. Mike Mosier, Corresponding Secretary, Dr. Kevin Charlwood, Faculty Sponsor.

Two students, Sarah Butler and Christine Potter, prepared and presented papers at the Kappa Mu Epsilon National Convention held at the Doubletree Hotel in downtown Philadelphia March 26 – 28, 2009. Sarah's paper was titled, "Dancing with the Crops: Using Mathematical Modeling to Maximize Earnings for a Family Farm" and Christine's paper was titled, "The Peg Game." Christine won a "top four" award (not rank-ordered) for her presentation, out of 16 presentations given overall. Student Amanda McCullough also attended the convention, along with faculty members Dr. Donna LaLonde and Dr. Gaspar Porta. Dr. Porta served on the Resolutions Committee at the convention.

KS Gamma – Benedictine College

Chapter President – Matthew Weaver; 14 Current Members
Caitlin Kelly, Vice President; Christina Henning, Secretary and Treasurer; Dr. Eric West, Corresponding Secretary.

The highlight of the semester for our chapter was an end of the year Christmas party, which included watching math videos and a "geometrical gingerbread" contest.

KY Alpha – Eastern Kentucky University

Chapter President – Joshua Sparks; 15 Current Members
Brittney Walker, Vice President; Marc Jones, Secretary; Sarah Hale, Treasurer; Pat Costello, Corresponding Secretary.

The first meeting was held on September 18th. We had an election of officers and discussion of plans for the year. It was decided to continue viewing the series of talks by Arthur Benjamin entitled "Joy of Math." The October meeting was a viewing of the "Joy of 9." The December meeting was a Christmas party with a White Elephant gift exchange.

KY Beta – University of the Cumberlands

Chapter President – Lola Embree; 31 Current Members

Andrzej Lenard, Vice President; Bethany Quinn, Secretary; Cynthia Kaeser, Treasurer; Dr. Jonathan Ramey, Corresponding Secretary; Dr. John Hymo, Faculty Sponsor.

On September 8, the Kentucky Beta chapter helped to host an ice cream and volleyball party, along with Mathematics and Physics Club, Beta Beta Beta, and the Alchemist Club. Along with the Mathematics and Physics Club and Sigma Pi Sigma, the chapter had a picnic at Briar Creek Park on October 8. On December 11, the entire department, including the Kentucky Beta chapter, had a Christmas party with about 27 people in attendance.

New Initiates - Brian Bastin, Laura Bradford, Jessica Cox, Genna Cummins, Amanda Faulkner, Lauren Howe, Tina John, Cynthia Kaeser, Sarah Mullins, Bryan Parker, Justin Cody Philpot, Christopher Price, Joshua Ward, Jacob Carpenter, Clint Creekmore, Kyle Creekmore, Lola Embree, Kellie Hatter, Sarah Kamalian, Lucas Matlock, Bethany Quinn, Kelly Rogers, Brenda Woods.

LA Gamma – Northwestern State University

Chapter President – Jacob Matherne; 15 Current Members; 5 New Members

Brittney Domangue, Vice President; Allison Hardy, Secretary; Philip Adams, Treasurer; Leigh Ann Myers, Corresponding Secretary; Lisa Galminas, Faculty Sponsor.

The Louisiana Gamma Chapter of Kappa Mu Epsilon sponsored a toy drive for children in the LSU Health Sciences Center Hospital during the 2009 Christmas holiday season. We collected lots of toys and delivered them to the children (ages 0-18) in the hospital. The toy drive was a great success and a very rewarding experience for all of us.

MD Alpha – College of Notre Dame

Chapter President – Miku White; 13 Current Members; 7 New Members

Jill Iracki, Vice President; Amanda Rapczak, Secretary; Jessica Loveless, Treasurer; Margaret Sullivan, Corresponding Secretary.

Our Induction on 15 November 2009 included an entertaining and informative presentation on “How Natural is Nature?” by Christopher Wells, a PhD candidate in Physics at The Johns Hopkins University and an adjunct faculty member at the College.

MD Beta – McDaniel College

Chapter President – Fenghao Wang; 8 Current Members; 2 New Members
John Maddrey, Vice President; Greg Allen, Secretary; Ben Sapp, Treasurer; Dr. Harry Rosenzweig, Corresponding Secretary; Spencer Hamblen, Faculty Sponsor.

This past fall, the Maryland Beta chapter of KME ran tutoring sessions for four hours per week to help students in any of the lower level mathematics courses, up to Calculus II. They also sponsored a very successful picnic, in the first week of September, and a talk, on The Logic of Secrets, at the induction ceremony in December.

MD Delta – Frostburg State University

Chapter President – Kelly Seaton; 20 Current Members
Joshua Wilson, Vice President; Joseph Bascelli, Secretary; Bradley Phillips, Treasurer; Mark Hughes, Corresponding Secretary.

This past fall semester the members of the Maryland Delta chapter engaged in a number of activities. In mid-September, we organized a "meet and greet" for faculty and students in the departments of mathematics, physics and engineering. There was a good turnout and a fun time was had by all. As in the past few years, KME students set up displays and manned the tables for the annual Majors Fair at Frostburg State University providing information for those interested n majoring in mathematics or joining KME. In November, KME sponsored a lecture by Dr. Frank Barnet of FSU's Department of Mathematics on parallel computing using clusters of video game consoles. At a fundraiser just before Christmas, we were able to sell about 50 Pi blankets.

MD Epsilon – Stevenson University

Chapter President – Jennifer Kurek; 26 Current Members, 15 New Members
Brittany Miller, Vice President; Justin Bobo, Secretary; Matthew Bramble, Treasurer; Dr. Christopher E. Barat, Corresponding Secretary.

On September 15, 2009, 15 new members (13 students and two faculty) were initiated into the Chapter. The guest speaker, Dr. Ron Wasserstein, spoke on "What Probability and Forrest Gump Teach Us about Lotteries." For the second straight year, the Fall Raffle was a considerable success, raising about $600 for Chapter activities. In Spring 2010, the Chapter will sponsor a speaker to honor Mathematics Awareness Month.

MI Delta – Hillsdale College

Chapter President – Jennifer Falck; 23 Current Members; 12 New Members

Jonathan Gregg, Vice President; Benjamin Wood, Secretary; David Murphy, Corresponding Secretary.

After being essentially inactive for the 2008-2009 academic year, we started Fall 2009 with a new faculty sponsor and many excited students. The semester's activities began with Fall Initiation on September 30 at which 12 new members were initiated, including one faculty member and 11 students. Once new initiates joined our chapter, we held elections for the officers for the year. We planned two events in October. The first was to watch "Hard Problems", the documentary about the International Mathematical Olympiad, and the second was a Euchre night.

New Initiates – Katherine Christiansen, Jessica Basitan, Jennifer Falck, Brooke Knight, James Joseph, William Kanitz, Benjamin Wood, Eric Schweller, Paul Mueller, David Charles Murphy, Rachel M. Bradford, Nathan D. English, Jonathan Gregg, Patience Henry, Catherine Nass, Juliana Rose O'Neill, David Racke, Catherine Rook, Heather Lynn Shiner, Joshua Trojniak, Jayme Louise Weber.

MI Epsilon – Kettering University

Chapter President – Jeff Nolen (A Section) and Matthew Sornig (B Section); 198 Current Members

Phillip Besoiu (A Section) and Starla Walters (B Section), Vice Presidents; Kathleen Moufore (A Section) and Shahnoor Amin (B Section), Secretaries; Jessi Harden (A Section) Treasurer; Boyan Dimitrov, Corresponding Secretary; Ruben Hayrapetyan (Section A, Winter and Summer terms), and Ada Cheng (Section B, Spring and Fall terms), Faculty Sponsors.

Kettering University has an active KME Society life. Two and a half years ago the former Science and Mathematics Department split into 4 separate departments. Their heads were chosen to be the Chairs of the respective programs of the SM Department. Later these were elected/appointed as permanent Department Heads as follows: Department of Mathematics (Dr Leszek Gawarecki); Department of Physics (Dr Bahram Roughani); Department of Chemistry and Biochemistry (Dr. Stacy Seeley); Computer Science Department (Dr. John Geske). The former Department Head, Dr David Green retired in January 2009, and was elected as Professor Emeritus in September 2009.

The Summer and Fall 2009 there was no new KME Initiation. However, the traditional Pizza/Movie Parties continued with the movie "Time Travelers" where most of the bravest physicists illustrated their views on the feasibility to travel in the time and in constructing time machines. On August 20, the movie was "Infinite Secrets." Exciting news for our entire Kettering Society came in the Fall, that our colleague, Professor Brian McCartin was selected for the 2010 very prestigious Chauvenet Prize recipient for Mathematical Expository Writing “e-The master of All,” given by the Mathematical Association of America (MAA). Professor McCartin is the founder and the first faculty sponsor of the Kettering University section of Kappa Mu Epsilon. On the afternoon of January 14th, the Mathematical Association of America did award him the 2010 Chauvenet Prize for Mathematical Exposition at their Annual Meeting in San Francisco. Due to health problems, Professor McCartin was not able to attend the presentation. Instead, the Mathematics Department and Kappa Mu Epsilon sponsored his Chauvenet Prize Presentation titled "A Mathematical Org-e" in the Crib-a-Thon at 12:20 on First Thursday of the Winter Term of 2010. CONGRATULATIONS, BRIAN! WE ARE PROUD OF YOU AS A COLLEAGUE, AS AN HONORABLE MAN AND SUCCESSFUL MATHEMATICIAN!

Kettering University financially supports the Mathematics Olympiads, organized by a group of enthusiastic faculty. The Mathematics Olympiad at Kettering is a competition designed to identify and encourage students with interests and abilities in mathematics. This was the 9th Olympiad held at Kettering. Our goal is to develop the Olympiad into one of the most prestigious mathematical competitions in the region. The examination is designed for students in grades 9 through 12. However any student working towards a high school degree who is currently enrolled in a public school, private school or a home-school program can sit for the examination. The competition consists of four to six challenging problems and has a time limit of four hours. The problems range from "mind-benders" that require little mathematical skills to problems that require the knowledge of geometry, trigonometry and beginning calculus. No calculators are permitted for this competition. The last Math Olympiad was held on November 21, 2009 with about 80 participants.

MO Alpha – Missouri State University

Chapter President – Christina Tharp; 38 Current Members

Jacob Swett, Vice President; Ashley Lewis, Secretary; Brett Foster, Treasurer; Jorge Rebaza, Corresponding Secretary.

The KME Annual Picnic was held on September 15, 2009. Seminars were held on the following dates with the following speakers: 09/24/09 Ken Vollmar (Computer Science), MSU; 10/22/09 Sam Lynch (Mathematics), MSU; and 11/19/09 Nick McCurry (Mathematics), MSU.

New Initiates - Brandi Bowers, Cheryl Ginnings, Zach Jones, Casandra Lewis, Danny Liggett, Shelby Vicat, Sarah Vitali, Laurie Washington.

MO Beta – University of Central Missouri

Chapter President – Jennifer Hayes;30 Current Members, 4 New Members

Andrew Stallmann, Vice President; Phat Hoang, Secretary; Cynthia Craft, Treasurer; Rhonda McKee, Corresponding Secretary; Rhonda McKee, Steve Shattuck and Dale Bachman, Faculty Sponsors.

The Missouri Beta chapter was quite active in the fall, with monthly programs and an end of the semester social. We are preparing to host the North Central Regional Convention this spring.

New Initiates – Curtis Burns, Lauren Cox, Matthew McDole, Samantha Platt.

MO Gamma – William Jewell College

Chapter President – Jennifer McKnight; 7 Current Members

John Spiegel, Vice President; Emma Farris, Secretary and Treasurer; Dr. Neil Nicholson, Corresponding Secretary.

On December 5, 2009, eight students (including 4 current KME members) became William Jewell College's inaugural William Lowell Putnam Competition participants.

MO Zeta – Missouri University of Science and Technology

Dr. Vy K. Le, Corresponding Secretary.

New Initiates - Katie Gredell, Jill Hecht, Travis Hemsath, Gina Hentschke, Thomas Herbst, Benjamin Huffman, Nicholas LaBarge, Michael Laskowsky, Brian Latal, Zach Marsden, Alexandra Rasband, Stephany Rich, Thomas Russell Jr., Ross Shafer, David Wehner, Samuel Wehner, Alexander White, Edward Belanger IV, Travis Brenneke, Patrick Hill, Michael Laurentius, Rebecca E. Pooker, Ryan Rader.

MO Theta – Evangel University

Chapter President – Jennifer Brocker; 12 Current Members

Rachel Hughes, Vice President; Don Tosh, Corresponding Secretary.

Meetings were held monthly. In September we had our first meeting at the home of Don Tosh. In November most members were able to attend the Math Conference held at Missouri State University.

MO Nu – Columbia College

Chapter President – Magda Pride; 9 Current Members, 4 New Members
Tomas Horvath, Vice President; Becca Kunce, Secretary; Chris Hawkins, Treasurer; Dr. Kenny Felts, Corresponding Secretary.

MS Alpha – Mississippi University for Women

Dr. Shaochen Yang, Corresponding Secretary; 6 Current Members
Dr. Joshua Hanes, Faculty Sponsor.

NC Epsilon – North Carolina Wesleyan College

Bill Yankosky, Corresponding Secretary.

New Initiates – Matthew Dougherty, Twyla Harrison-Hendricks, Samantha Kaye House, Melita Michelle Lewis, Denver Nixon, Cassandra Slater, Susan Elizabeth Uzzell, John Williamson.

NC Zeta – Catawba College

Chapter President – Scott Campbell; 12 Current Members; 12 New Members
John Hoehman, Vice President; Zachary Owen, Secretary; Cody Ashby, Treasurer; Doug Brown, Corresponding Secretary.

The NC Zeta Chapter was initiated during the Fall Semester 2009, as reported in the last issue of the Pentagon. The Chapter, in conjunction with the Catawba College Math Club, sponsored a talk on mathematics and poetry by Dr. Patrick Bahls of UNC-Asheville. The two organizations also hosted a bowling night for students and faculty. Several members also attended the NC state dinner of the MAA, hosted by UNC-Asheville.

NE Alpha – Wayne State College

Chapter President – April Groteluschen; 1 Current Member
Emily Gardner, Vice President; Amy Doerr, Secretary; Deena Bignell, Treasurer; Dr. Jennifer Langdon, Corresponding Secretary.

NE Beta – University of Nebraska Kearney

Chapter President – Abigail Om; 9 Current Members; 3 New Members
Valerie Sis, Vice President; Tierra Webb, Secretary; Wes Sanders, Treasurer; Dr. Katherine Kime, Corresponding Secretary.

A new bulletin board, with a color block design, was created. The current President, Abigail Om, proposed several designs and one was chosen last spring. Abby, who is also an art major, painted the KME crest and the surrounding ribbon which has the KME motto in Greek. Wes Sanders and Valerie Sis also participated in making the board.

New Initiates – Kali Anderson, Tomoo Kasuya, Kandi Young.

NE Delta – Nebraska Wesleyan University

Chapter President – Thao Nguyen; 15 Current Members

Brent McKain, Vice President; Melissa Erdmann, Corresponding Secretary.

In the autumn we were pleased to hear two KME members, Ana Burgers and Brent McKain, give talks about their summer REU experiences. We assisted with a mathematics outreach event, Math Night, at a local elementary school. In December we enjoyed a holiday party complete with a math-related gift exchange and mathematical versions of Christmas carols.

NJ Delta – Centenary College

8 Current Members

Linda Ritchie, Treasurer; Kathy Turrisi, Corresponding Secretary.

At the April 2009 Induction ceremony, eight new members were initiated: Math merit awards were also handed out to graduating seniors. Fall 2009 Community service Praxis 1 tutoring offered to Education Dept. KME students offer Praxis 1 tutoring campus wide to assist students. This free tutoring has prepared students to pass their Praxis 1 exam. KME students also assisted/participated in a stem cell drive to bring awareness to the community of the importance of stem cells. Stem cells are important to give another person a chance at life and Prof Turrisi's husband, David, is in need of a life saving stem cell transplant. KME, along with Centenary College, NJ increased the National Bone Marrow Bank by 100 people by offering people the simple kit to swab their cheeks to become a donor by being added to the National Bone Marrow bank. As always, opportunities are posted on Centenary College Blackboard Organizational tab that KME students can take advantage of during the year. These items have included but are not limited to internships, tutoring, and conferences.

New initiates: Michael L. Ahrens, Kristen A. Wirasnik, Penny J. Zitomer, Lauren Culbert, Ismael Garcia III, Sean Hutchinson, Andrew Pancoast, and Christine Merriman.

NY Nu – Hartwick College

Chapter President – Melanie Hart

Dan Parisian, Vice President; Kaitlyn King, Secretary; Matthew Shoudy, Treasurer; Ron Brzenk, Corresponding Secretary.

NY Omicron – St. Joseph's College

Chapter President – Nicole Hatzispirou; 166 Current Members; 19 New Members

Julie-Anne Henken, Vice President; Melissa A. Bernstein, Secretary; Charles C. Essig, Treasurer; Elana Reiser, Corresponding Secretary.

The New York Omicron Chapter has volunteered to tutor local high school students on Saturdays. We are also planning for a 2010 initiation.

OK Alpha – Northeastern State University

Chapter President – Toni Slagle (Fall 2009), Caleb Knowlton (Spring 2009); 68 Current Members; 11 New Members

Seth Vansell, Vice President; Katherine Thompson, Secretary; Chris Palmer, Treasurer.

Aaron Labounty, Vice President; Callie Wilson, Secretary; Stephanie Emerson, Treasurer; Dr. Joan E. Bell, Corresponding Secretary.

Our spring initiation brought 10 new members into our chapter. During our February meeting we solved several problems from The Pentagon and submitted the solutions for publication. Dr. Diamantopoulos invited several mathematicians to speak at NSU. Dr. Cynthia Woodburn, Pittsburg State University, spoke on the topic "Do U Sudoku?" Dr. Elwyn Davis, Pittsburg State University, spoke on "Adventures on the Sphere and the E6B Flight Computer." OK Alpha chapter members (Evan Linde, Brandon Childress, Jooyong Kim, Katherine Thompson, Toni Slagle, and Lori Bottger) attended the 71st annual Oklahoma-Arkansas section meeting of the Mathematics Association of America and participated in the student competitions. At our annual ice cream social at the end of the semester we watched "The Great Pi versus e Debate." Our fall initiation brought 11 new members into our chapter. We sponsored several speakers this semester. Dr. Michael Schillaci, Asst. Prof. of Physics, NSU, spoke on "Making an MRI of a Pencil." Dr. Giovanni Petris, Dept. of Mathematical Sciences, University of Arkansas, spoke on "A Random Walk to the Top," and also visited with students about their graduate program in mathematics. KME also played a role in the presentation by Underwood Dudley of "Angle Trisectors." Our chapter designed and sold 48 math T-shirts this fall. The design resembled the popular "I 'heart' New York" shirts. We also helped with the NSU Student Foundation phonathon. We ended the semester with a Christmas party for KME members, math majors and faculty. Dr. Darryl Linde, Math Dept Chair, served his famous homemade pizza.

New initiates: David Duncan, Chelsea L. Hayes, Kaci D. Hood, Jooyong Kim, Joseph J. Kok, Charity J. Martinez, Chris G. Palmer, Parker J. Richey, Alexander J. Risenhoover, and Seth M. Vansell, Cassie A. Blankenship, Rebekah J. Burnett, Colin M. Clinton, Rachel M. Jones, Tabitha M. Lawrence, Jonathan W. Moyer, Christopher J. Nettles, Katie M. Ozburn, Avrahm J. Thorne, Anthony C. Wellman, Christina D. Williams.

OK Epsilon – Oklahoma Christian University

Ray Hamlett, Corresponding Secretary.

New Initiates – Cady Block, Julia Bryson, Micah Chandler, Jacob Clark, Luke Heithold, Adam Huse, Rachel Lewis, Wellars Muhoza, Breanna Reams, Drew Sandlin, Jannea Turner.

PA Iota – Shippensburg University

Chapter President – Jessica Armstrong; 13 Current Members; 5 New Members

Kelly Toppin, Vice President; Lauren Robinson, Secretary; Drew Snyder, Treasurer; Paul Taylor, Corresponding Secretary; Ji Young Choi, Faculty Sponsor.

PA Kappa – Holy Family University

Chapter President – Michael Browning; 6 Current Members

Jacqueline Galelli, Vice President; Sabrina Luczyszyn, Secretary; Colleen Siemers, Treasurer; Sister Marcella Louise Wallowicz CSFN, Corresponding Secretary.

In collaboration with the Math Club, the honor society members hosted the annual Evening of Mathematical Suspense on October 29, 2009. The Halloween-themed event is in the form of a Math Murder Mystery/Dinner Theatre in which the participants solve math problems in order to obtain the clues to solve the murder mystery. Many students and their families participated, enjoying pizza and other refreshments. Various items with the university logo were awarded as prizes.

In December, 2009 the honor society members performed several math songs during the university's annual Christmas celebration. The group delighted the audience with their special renditions of The Twelve Weeks of Calculus, Fill the Board With Differentials and Santa's Got Statistics on You.

PA Lambda – Bloomsburg University

Elizabeth Mauch, Corresponding Secretary.

New initiates – Keri Williams, Emily Barkanic, Nicholas Boccella, Michael Brabander, Ashley Callguire, Emily Claypotch, Diana Gray, Kyle Huber, Thomas Kaszmetskie Jr., Carrie Mensch, Cullen Mentzell, Jenna Mordan, Wayne Nilsen, Amy Schultz, Lindsay Shemansky, Carly Smith, Eric Subach, Trisha Temple.

PA Mu – Saint Francis University

Chapter President – Darci Jones; 37 Current Members

Michelle Wetzel, Vice President; Rachel Capizzi, Secretary; Aaron Osysko, Treasurer; Peter Skoner, Corresponding Secretary; Katherine Remillard,, Faculty Sponsor.

Several KME faculty and student members participated in the Sixteenth Annual Saint Francis University Science Day held Tuesday, November 24, 2009. Some KME members served as session moderators for faculty making presentations. Others served as moderators, judges, scorekeepers, and timers for the Science Bowl. A total of 435 high school students from 26 area high schools attended.

PA Pi – Slippery Rock University

Elise Grabner, Corresponding Secretary.

New Initiates - Nathan Leidwinger, Matthew F. Belella, Jesse P. Waldorf, Djordje Petkovic, Samantha G. Corvino, Ryan Smail.

PA Rho – Thiel College

Chapter President – David Wierzchowski; 17 Current Members

Krista Wissenbach, Vice President; Adam Troup, Secretary; Devin Todd, Treasurer; Max Shellenbarger, Corresponding Secretary; Dr. Jie Wu, Faculty Sponsor.

The Pennsylvania Rho chapter held meeting on a monthly basis due fall term of 2009. The members participate in the Homecoming Parade in October. We are had a fund raiser which benefited the Greenville Good Sheppard Center. $100.00 was donated. The chapter also had t-shirts printed for each member. No new members were added during fall term as we only select new members in the spring.

PA Sigma – Lycoming College

Chapter President – Steven Brown

Jake Crosetto, Vice President; Meg Bittle, Secretary; Christopher Dahlheimer, Treasurer; Santu de Silva, Corresponding Secretary.

SC Epsilon – Francis Marion University

Damon Scott, Corresponding Secretary.

New Initiates - Renata Cumbee, Alex W. Lee, Mallory Proctor, David J. Propst, Shereena Lashele Thompson, Jeremiah Emerson Williams, Joseph S. Conner, Staci E. Poston, Dylan Cook.

TN Delta – Carson-Newman College

Chapter President – Gretchen Hill; 308 Current Members

Luke Morton, Vice President; Andrew Hansen, Secretary; Kenneth Massey, Treasurer/Corresponding Secretary.

Being too cold for an outdoors picnic, the Carson-Newman KME held a banquet at Smokey's Barbeque. Students, faculty, retirees, family, and friends were invited. The group was amused with some entertaining stories about certain department members in their younger days. Afterward, the students held a bonfire at VP Morton's estate.

TN Epsilon – Bethel University

Russell Holder, Corresponding Secretary; David Lankford, Faculty Sponsor; 7 Current Members

TN Gamma – Union University

Chapter President – Will Sipes

Rebecca Eaton, Vice President; Jacob White, Secretary; Emily Huffman, Treasurer; Dwayne Jennings, Corresponding Secretary; Matt Lunsford, Faculty Sponsor.

KME hosted a back-to-school cookout on September 28, 2009 for all KME members, mathematics majors, and minors.

TX Gamma – Texas Woman's University

Chapter President - Danielle Silva; 25 Current Members, 10 New Members

Brent Bradley, Vice President; Abbi Martinez, Secretary; Vincent Gil, Treasurer; Dr. Mark Hamner, Corresponding Secretary.

New initiates – Mary Lou Rodriguez, Margarita Perez, Vincent Gil, Denise J. Murphy, Megan B. Endress, Brent Bradley, Abbi Martinez, Guadalupe Yesenia Nieto, Eric Bengfort, Megan Warren.

TX Iota – McMurry University

Dr. Kelly McCoun, Corresponding Secretary

New Initiates – Kaitlyn Barnhill, Stormye Calhoun, Justin Eddy, Matthew Montross, Heather Whitehead.

TX Kappa – University of Mary Hardin-Baylor

Chapter President – Kellie Thomas; 12 Current Members

Mattie Billington, Vice President; Christi D'Herde, Secretary; Dr. Peter H. Chen, Corresponding Secretary; Maxwell Hart, Faculty Sponsor.

TX Mu – Schreiner University

Chapter President – Leigh Ann Brown; 14 Current Members

Denise Begley, Vice President; Stephen Franklin, Secretary; Matthew Moreno, Treasurer; William M. Sliva, Corresponding Secretary; Clinton Coles and William Sliva, Faculty Sponsors.

Texas Mu had three lunch meetings during the fall semester. The September meeting was our typical business meeting. The October meeting we played the "16 game." (Four cards are dealt from a deck that has no face cards. The value of the cards are as shown with the ace being a one. The object of the game to be the first to combing the four cards to produce 16 using any combination of addition, subtraction, multiplication, division and exponents.) The November meeting, William Sliva gave a PowerPoint presentation on Hubble Telescope discoveries.

WI Gamma – University of Wisconsin-Eau Claire

Chapter President – Caitlyn Hellenbrand; 76 Current Members, 24 New Members

Chelsey Drohman, Vice President; Alyssa Frey, Secretary; Amaris Lieske, Treasurer; Simei Tong, Corresponding Secretary.

Four students from UW-Eau Claire received travel grants from MAA to present research results at the National Joint Mathematics Meetings in San Francisco in January, 2010. Senior Daniel Wackwitz (faculty mentor Michael Penkava) presented his poster, "Classification of Real Z_2-Graded Associative Algebras" and his poster received outstanding poster award. This is the third year in a row that Penkava's student received an poster award there. Wackwitz also gave a presentation titled "Moduli Spaces of Low Dimensional Associative Algebras and Their Deformations" at an AMS special session. Senior Kaitlyn Hellenbrand (faculty mentor Colleen Duffy) presented her poster, "Polynomial Equations over Matrices." Junior Mark Bauer and sophomore Hong Yang (faculty mentor Simei Tong) presented a joint poster, "Optimizing the Evacuation of Hospitals Phase II" at the same conference. The UW-Eau Claire team consisting of Kathy Du, Mei Pak, Mark Bauer, Matt Fjerstad, Kevin Kropp, and James Hollman (Faculty mentor Kris Presler) took second place at Actuarial Case Competition sponsored by Travelers in the fall of 2009. Six teams participated in this event (University of Minnesota, University of St. Thomas, Bentley College, Bryant University, University of Connecticut, and UW-Eau Claire). The team from the University of Minnesota was first, UW-Eau Claire was second, and Bryant University was third.

New Initiates - Jacob Brickner, Marcie Childs, Jiaying Du, Josh Frinak, Tyler Genovese, Adrienne Graese, Zachary Hines, Kendra Keon, Ganna Kotenko, Katherine A. Kuehn, Promise Lohse, Sean Murray, Caitlin Olig, Alice Oswalt, Austen Ott, Rachel Otte, Shawn Peters, Paul Stroik, Andrew Joseph Swoboda, Daniel Wackwitz, Zachary Westphal, Tristan Jay Newman Williams, Hong Yang, Xiaoqin Zhou, Andrew Anhalt, Andrew Bartlein, Ryan Davis, Melanie Griesbach, Mitchell J. Hahn, Trevor Hamann, Emily Hiebl, Wai Ling Ho, Junfeng Hou, Shu Hui Loh, Tim Kleinschmidt, Ari David Kramer-Morning, Melissa Larrabee, Sara Mattes, Sara Nelson, Charles Mark Rowe, Rebekah Sippert, William (BJ) Stokes, Nicholas Sullivan, Noah Williams.

WV Beta – Wheeling Jesuit University

Theodore Erickson, Corresponding Secretary

New Initiates – Amanda Smider, Brandi Sroka, Brandon Thurber, Justin Pastorius, DoBin Choi, Kyle Kepreos.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February 2001
GA Delta	Berry College, Mount Berry	21 April 2001
TX Mu	Schreiner University, Kerrville	28 April 2001
NJ Gamma	Monmouth University	21 April 2002
CA Epsilon	California Baptist University, Riverside	21 April 2003
PA Rho	Thiel College, Greenville	13 February 2004
VA Delta	Marymount University, Arlington	26 March 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 February 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 March 2005
SC Epsilon	Francis Marion University, Florence	18 March 2005
PA Sigma	Lycoming College, Williamsport	1 April 2005
MO Nu	Columbia College, Columbia	29 April 2005
MD Epsilon	Stevenson University, Stevenson	3 December 2005
NJ Delta	Centenary College, Hackettstown	1 December 2006
NY Pi	Mount Saint Mary College, Newburgh	20 March 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 April 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 October 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 March 2008

CA Zeta	Simpson University, Redding	4 April 2009
NY Rho	Molloy College, Rockville Center	21 April, 2009
NC Zeta	Catawba College, Salisbury	17 September, 2009
RI Alpha	Roger Williams University, Bristol	13 November, 2009
NJ Epsilon	New Jersey City University, Jersey City	22 February, 2010
NC Epsilon	Johnson C. Smith University, Charlotte	18 March, 2010
AL Theta	Jacksonville State University, Jacksonville	29 March, 2010
FL Gamma	Southeastern University, Lakeland	31 March, 2010