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For the addresses of the new editor & the business manager, see page 72.

Fibonacci and Other Polynomials: Pascal Comes Through Again

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Abstract

Fibonacci numbers, Lucas numbers, and Pascal's Triangle all make appearances when studying recursively defined polynomials of the form $P_n(x) = x \cdot p_{n-1}(x) + P_{n-2}(x)$. While Fibonacci numbers, Lucas numbers and Pascal's Triangle are all fascinating when studied alone, they come together in yet another way through these polynomials. This paper will examine the recursively defined polynomials and their connection to Fibonacci numbers, Lucas numbers, and Pascal's Triangle.

Preliminaries

Most of us are quite familiar with the Fibonacci sequence defined by the recurrence relation

$$F_n = F_{n-1} + F_{n-2} \qquad F_1 = 1 = F_2$$

which produces: $1, 1, 2, 3, 5, 8, 13, 21, \ldots$.

Perhaps slightly less well known are the Lucas Numbers. Given by the recurrence relation

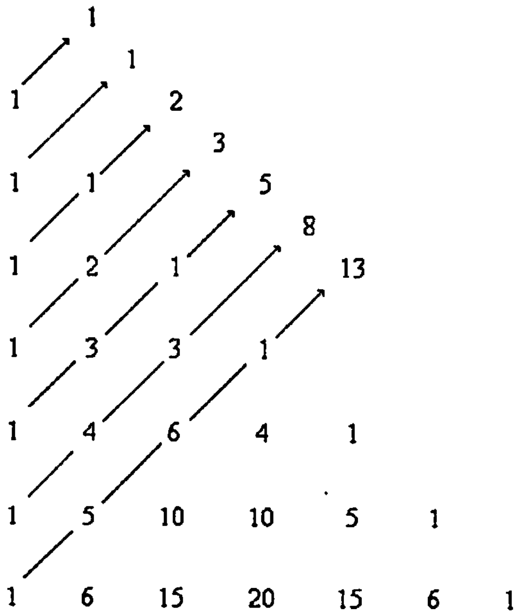
$$L_n = L_{n-1} + L_{n-2}, \quad L_1 = 2, L_2 = 1$$

producing the sequence: $2, 1, 3, 4, 7, 11, 18, 29, \ldots$.

We will be encountering Pascal's Triangle as well throughout the paper.

$$\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \\
 & & & & & & \vdots & & & & & &
\end{array}$$

Recall Pascal's Triangle is produced by summing the two numbers directly above the desired position. Now, if you left justify the triangle, and sum the diagonals you get Fibonacci numbers.



This, of course, is just a brief overview. It would be easy to spend more time discussing Fibonacci and Lucas numbers themselves. However, since this is not our objective, let us move on.

Fibonacci Polynomials

Fibonacci polynomials are defined by the recurrence relation;

$$f_n(x) = x \cdot f_{n-1}(x) + f_{n-2} \qquad f_1(x) = 1, f_2(x) = x$$

This produces a whole sequence of polynomials.

$$\begin{aligned}
f_1(x) &= 1 \\
f_2(x) &= x \\
f_3(x) &= x^2 + 1 \\
f_4(x) &= x^3 + 2x \\
f_5(x) &= x^4 + 3x^2 + 1 \\
f_6(x) &= x^5 + 4x^3 + 3x \\
f_7(x) &= x^6 + 5x^4 + 6x^2 + 1 \\
f_8(x) &= x^7 + 6x^5 + 10x^3 + 4x \\
f_9(x) &= x^8 + 7x^6 + 15x^4 + 10x^2 + 1 \\
f_{10}(x) &= x^9 + 8x^7 + 21x^5 + 20x^3 + 5x \\
&\vdots
\end{aligned}$$

From now on I will adopt the convention of using lowercase letters (f_n) to indicate the n^{th} polynomial, and upper case letters (F_n) to indicate the n^{th} number in a sequence.

Because of the definition of the polynomials we note that $f_n(1) = F_n$. Somewhat more surprising is the sequence that pops up if we evaluate the f_n at $x = 2$.

$$\begin{aligned}
f_1(2) &= 1 \\
f_2(2) &= 2 \\
f_3(2) &= 5 \\
f_4(2) &= 12 \\
f_5(2) &= 29 \\
f_6(2) &= 70 \\
&\vdots
\end{aligned}$$

This is known as the sequence of Pell numbers. An interesting site that is helpful in recognizing integer sequences is Sloan's Online Encyclopedia of Integer Sequences. The site is a searchable database of integer sequences with 79591 sequences catalogued.[1]

Going back to the polynomials themselves, if we arrange the coefficients of the f_n in a rectangular array with n on the vertical axis and powers of x on the horizontal axis, the triangle of coefficients form Pascal's Triangle.

$n \backslash x^j$	1	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
1	1												
2		1											
3	1		1										
4		2		1									
5	1		3		1								
6		3		4		1							
7	1		6		5		1						
8		4		10		6		1					
9	1		10		15		7		1				
10		5		20		21		8		1			
11	1		15		35		28		9		1		
12		6		35		56		36		10		1	
13	1		21		70		84		45		11		1

Hence,

$$f_n(x) = \sum_{k=0}^{\lfloor \frac{(n-1)}{2} \rfloor} \binom{n-k-1}{k} x^{n-2k-1}$$

which can be verified by an easy inductive proof. [2]

Lucas Polynomials

Lucas polynomials are defined by the recurrence relation;

$$l_n(x) = x \cdot l_{n-1}(x) + l_{n-2}(x), \qquad l_1(x) = 2, l_2(x) = x$$

This produces a sequence of polynomials similar to those obtained in section 2.

$$
\begin{aligned}
l_1(x) &= 2 \\
l_2(x) &= x \\
l_3(x) &= x^2 + 2 \\
l_4(x) &= x^3 + 3x \\
l_5(x) &= x^4 + 4x^2 + 2 \\
l_6(x) &= x^5 + 5x^3 + 5x \\
l_7(x) &= x^6 + 6x^4 + 9x^2 + 2 \\
l_8(x) &= x^7 + 7x^5 + 14x^3 + 7x \\
l_9(x) &= x^8 + 8x^6 + 20x^4 + 16x^2 + 2 \\
l_{10}(x) &= x^9 + 9x^7 + 27x^5 + 30x^3 + 9x
\end{aligned}
$$

Similar to the Fibonacci polynomials, if you evaluate the Lucas polynomials at $x = 1$, you get $l_n(1) = L_n$

Again, we find it useful to look at only the coefficients of the Lucas polynomials.

$n \backslash x^j$	1	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
1	2												
2		1											
3	2		1										
4		3		1									
5	2		4		1								
6		5		5		1							
7	2		9		6		1						
8		7		14		7		1					
9	2		16		20		8		1				
10		9		30		27		9		1			
11	2		25		50		35		10		1		
12		11		55		77		44		11		1	
13	2		36		105		112		54		12		1

Draw your attention again to the diagonals of this triangle. At first glance nothing seems to be especially exciting. However, the rows do exhibit the same structure as a Pascal triangle. For example, element $c_{n,j}$ corresponds to the coefficient of the x^j term in the n^{th} polynomial. Notice that $c_{n,j} + c_{n-1,j+1} = c_{n+1,j+1}$which is similar to Pascal's triangle. For example:

$c_{7,2} = 9$, $c_{6,3} = 5$, and $c_{8,3} = 14$

$$
\begin{array}{ccc}
 & & 5 \\
 & + & \\
9 & & \\
 & \backslash\backslash & \\
 & & 14
\end{array}
$$

$$
\begin{aligned}
c_{7,2} + c_{6,3} &= c_{8,3} \\
9 + 5 &= 14
\end{aligned}
$$

Look on the next page to see what happens if we rewrite the numbers.

In Fact...

Since there seems to be such similar structure between these two families of polynomials one might be inclined to look at polynomials of a general initial value:

$$p_n(x) = x \cdot p_{n-1}(x) + p_{n-2}(x), \qquad p_1(x) = N, p_2(x) = x$$

$$
\begin{aligned}
p_1(x) &= N \\
p_2(x) &= x \\
p_3(x) &= x^2 + N \\
p_4(x) &= x^3 + (N+1)x \\
p_5(x) &= x^4 + (N+2)x^2 + N \\
p_6(x) &= x^5 + (N+3)x^3 + (2N+1)x \\
p_7(x) &= x^6 + (N+4)x^4 + (3N+3)x^2 + N \\
p_8(x) &= x^7 + (N+5)x^5 + (4N+6)x^3 + (3N+1)x \\
p_9(x) &= x^8 + (N+6)x^6 + (5N+10)x^4 + (6N+4)x^2 + N \\
p_{10}(x) &= x^9 + (N+7)x^7 + (6N+15)x^5 + (10N+10)x^3 + (4N+1)x
\end{aligned}
$$

n \ x^j	1	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
1	1+1												
2		1											
3	1+1		1										
4		2+1		1									
5	1+1		3+1		1								
6		3+2		4+1		1							
7	1+1		6+3		5+1		1						
8		4+3		10+4		6+1		1					
9	1+1		10+6		15+5		7+1		1				
10		5+4		20+10		21+6		8+1		1			
11	1+1		15+10		35+15		28+7		9+1		1		
12		6+5		35+20		56+21		36+8		10+1		1	
13	1+1		21+15		70+35		84+28		45+9		11+1		1

As we have done before we will now look at the coefficients in a rectangular array. Again, we will see Pascal's triangle popping up, but this time it is even more surprising since we constructed our polynomials with a general initial value. The array shows two simultaneous Pascal triangles, explaining the repeated Pascal like structure in the two examples.

$n \backslash x^k$	1	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
1	$1N$												
2		1											
3	$1N$		1										
4		$1N+1$		1									
5	$1N$		$1N+2$		1								
6		$2N+1$		$1N+3$		1							
7	$1N$		$3N+3$		$1N+4$		1						
8		$3N+1$		$4N+6$		$1N+5$		1					
9	$1N$		$6N+4$		$5N+10$		$1N+6$		1				
10		$4N+1$		$10N+10$		$6N+15$		$1N+7$		1			
11	$1N$		$10N+5$		$15N+20$		$7N+21$		$1N+8$		1		
12		$5N+1$		$20N+15$		$21N+35$		$8N+28$		$1N+9$		1	
13	$1N$		$15N+6$		$35N+35$		$28N+56$		$9N+36$		$1N+10$		1

Hence we find explicit formulae for $p_{2n}(x)$ and $p_{2n-1}(x)$ given the initial values $p_1(x) = N$, $p_2(x) = x$.

$$\left.\begin{aligned} p_{2n}(x) &= \sum_{k=1}^{n} \left\{ \binom{n+k-2}{2k-1} N + \binom{n+k-2}{2k-2} \right\} x^{2k-1} \\ p_{2n-1}(x) &= \sum_{k=1}^{n} \left\{ \binom{n+k-3}{2k-2} N + \binom{n+k-3}{2k-3} \right\} x^{2(k-1)} \end{aligned}\right\} p_1(x) = N \text{ and}$$

$p_2(x) = x$

Proof: (by induction)

$$p_2(x) = \sum_{k=1}^{1} \left\{ \binom{1+k-2}{2k-1} N + \binom{1+k-2}{2k-2} \right\} x^{2k-1} = \left\{ \binom{0}{1} N + \binom{0}{0} \right\} x = x$$

$$\begin{aligned} p_3(x) &= \sum_{k=1}^{2} \left\{ \binom{2+k-3}{2k-2} N + \binom{2+k-3}{2k-3} \right\} x^{2(k-1)} \\ &= \left\{ \binom{0}{0} N + \binom{0}{-1} \right\} x^0 + \left\{ \binom{1}{2} N + \binom{1}{1} \right\} x^2 \\ &= x^2 + N \end{aligned}$$

Assume the formulae hold for all $p_t(x)$ with $2 < t \leq 2n-2$

$p_{2n-1}(x) = x \cdot p_{2n-2}(x) + p_{2n-3}(x)$

$$= \sum_{k=1}^{n-1} \left\{ \binom{n+k-3}{2k-1} N + \binom{n+k-3}{2k-2} \right\} x^{2k} + \sum_{k=1}^{n-1} \left\{ \binom{n+k-4}{2k-2} N + \binom{n+k-4}{2k-3} \right\} x^{2(k-1)}$$

Now by reindexing with $k = k+1$ into the second summand;

$$= \sum_{k=1}^{n-1} \left\{ \binom{n+k-3}{2k-1} N + \binom{n+k-3}{2k-2} \right\} x^{2k} + \sum_{k=1}^{n-2} \left\{ \binom{n+k-3}{2k} N + \binom{n+k-3}{2k-1} \right\} x^{2k}$$

$$= x^{2n-2} + N + \sum_{k=1}^{n-2} \left\{ \binom{n+k-2}{2k} N + \binom{n+k-2}{2k-1} \right\} x^{2k}$$

$$= \sum_{k=0}^{n-1} \left\{ \binom{n+k-2}{2k} N + \binom{n+k-2}{2k-1} \right\} x^{2k}$$

and after reindexing with $k = k-1$ we get.

$$p_{n-1} = \sum_{k=1}^{n} \left\{ \binom{n+k-3}{2k-2} N + \binom{n+k-3}{2k-3} \right\} x^{2(k-1)}$$

which is what we need. Now we will check the formula for the p_{2n}.

$p_{2n}(x) = x \cdot p_{2n-1}(x) + p_{2n-2}(x)$

$$= \sum_{k=1}^{n} \left\{ \binom{n+k-3}{2k-2} N + \binom{n+k-3}{2k-3} \right\} x^{2k-1} + \sum_{k=1}^{n-1} \left\{ \binom{n+k-3}{2k-1} N + \binom{n+k-3}{2k-2} \right\} x^{2k-1}$$

$$= x^{2k-1} + \sum_{k=1}^{n-1} \left\{ \binom{n+k-3}{2k-2} N + \binom{n+k-3}{2k-3} + \binom{n+k-3}{2k-1} N + \binom{n+k-3}{2k-2} \right\} x^{2k-1}$$

$$= x^{2k-1} + \sum_{k=1}^{n-1} \left\{ \binom{n+k-2}{2k-1} N + \binom{n+k-2}{2k-2} \right\} x^{2k-1}$$

$$p_{2n}(x) = \sum_{k=1}^{n} \left\{ \binom{n+k-2}{2k-1} N + \binom{n+k-2}{2k-2} \right\} x^{2k-1}$$

which completes the induction.

We have so far been taking initial values of the general polynomial sequence to be $p_1 = N$ and $p_2 = x$. What would happen if we took the initial values to be instead $p_1 = x$ and $p_2 = N$?

$$p_n(x) = x \cdot p_{n-1}(x) + p_{n-2}(x), \quad \text{with } p_1(x) = x, p_2(x) = N$$

$$\begin{aligned}
p_1(x) &= x \\
p_2(x) &= N \\
p_3(x) &= (N+1)x \\
p_4(x) &= (N+1)x^2 + N \\
p_5(x) &= (N+1)x^3 + (2N+1)x \\
p_6(x) &= (N+1)x^4 + (3N+2)x^2 + N \\
p_7(x) &= (N+1)x^5 + (4N+3)x^3 + (3N+1)x \\
p_8(x) &= (N+1)x^6 + (5N+4)x^4 + (6N+3)x^2 + N \\
p_9(x) &= (N+1)x^7 + (6N+5)x^5 + (10N+6)x^3 + (4N+1)x \\
p_{10}(x) &= (N+1)x^8 + (7N+6)x^6 + (5N+10)x^4 + (10N+4)x^2 + N
\end{aligned}$$

Again it is useful to look at the coefficients by themselves. You might be able to look at the sequence of polynomials above and have a good idea of what our array will look like (see the next page).

From this we obtain similar explicit formulae for the sequence when the initial values are $p_1 = x$ and $p_2 = N$.

$$\left.\begin{aligned}
p_{2n}(x) &= \sum_{k=1}^{n} \left\{ \binom{n+k-2}{2k-2} N + \binom{n+k-2}{2k-3} \right\} x^{2k-2} \\
p_{2n-1}(x) &= \sum_{k=1}^{n-1} \left\{ \binom{n+k-2}{2k-1} N + \binom{n+k-3}{2k-2} \right\} x^{2k-1}
\end{aligned}\right\} n \geq 2$$

$n \backslash x^j$	1	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8
1		1							
2	$1N$								
3		$1N+1$							
4	$1N$		$1N+1$						
5		$2N+1$		$1N+1$					
6	$1N$		$3N+2$		$1N+1$				
7		$3N+1$		$4N+3$		$1N+1$			
8	$1N$		$6N+3$		$5N+4$		$1N+1$		
9		$4N+1$		$10N+6$		$6N+5$		$1N+1$	
10	$1N$		$10N+4$		$15N+10$		$7N+6$		$1N+1$

Proof:

Take $p_1(x) = x$

$$p_2(x) = \sum_{k=1}^{1} \left\{ \binom{1+k-2}{2k-2} N + \binom{1+k-3}{2k-3} \right\} x^{2k-2} = N$$

$$p_3(x) = \sum_{k=1}^{1} \left\{ \binom{2+k-2}{2k-1} N + \binom{2+k-3}{2k-2} \right\} x^{2k-1} = (N+1)x$$

Assume the formulae hold for all $p_1(x)$ with $2 \le t \le 2n-2$.

$$p_{2n-1}(x) = x \cdot p_{2n-2}(x) + p_{2n-3}(x)$$

$$= \sum_{k=1}^{n-1} \left\{ \binom{n+k-3}{2k-2} N + \binom{n+k-4}{2k-3} \right\} x^{2k-1} + \sum_{k=1}^{n-2} \left\{ \binom{n+k-3}{2k-1} N + \binom{n+k-4}{2k-2} \right\} x^{2k-1}$$

$$= (N+1)x^{2n-3} + \sum_{k=1}^{n-2} \left\{ \binom{n+k-2}{2k-1} N + \binom{n+k-3}{2k-2} \right\} x^{2k-1}$$
$$= \sum_{k=1}^{n-1} \left\{ \binom{n+k-2}{2k-1} N + \binom{n+k-3}{2k-2} \right\} x^{2k-1}$$
which is what we needed. Now we want to affirm the formula for the even p.

$$p_{2n}(x) = x \cdot p_{2n-1}(x) + p_{2n-2}(x)$$
$$= \sum_{k=1}^{n-1} \left\{ \binom{n+k-2}{2k-1} N + \binom{n+k-3}{2k-2} \right\} x^{2k} + \sum_{k=1}^{n-1} \left\{ \binom{n+k-3}{2k-2} N + \binom{n+k-4}{2k-3} \right\} x^{2k-2}$$
$$= \sum_{k=1}^{n-1} \left\{ \binom{n+k-2}{2k-1} N + \binom{n+k-3}{2k-2} \right\} x^{2k} + \sum_{k=0}^{n-2} \left\{ \binom{n+k-2}{2k} N + \binom{n+k-3}{2k-1} \right\} x^{2k}$$
$$= (N+1)x^{2n-2} + \sum_{k=1}^{n-2} \left\{ \binom{n+k-1}{2k} N + \binom{n+k-2}{2k-1} \right\} x^{2k}$$
$$= (N+1)x^{2n-2} + N + \sum_{k=2}^{n-1} \left\{ \binom{n+k-2}{2k-2} N + \binom{n+k-3}{2k-3} \right\} x^{2k-2}$$
$$= \sum_{k=1}^{n-1} \left\{ \binom{n+k-2}{2k-2} N + \binom{n+k-3}{2k-3} \right\} x^{2k-2}$$
which completes the induction.

The next topic to be addressed is polynomials of the form:
$p_n(x) = x^m \cdot p_{n-1}(x) + p_{n-2}(x)$ with $p_1(x) = N$ and $p_2(x) = x$
This recurrence relation produces the sequence:

$$\begin{aligned}
p_1(x) &= N \\
p_2(x) &= x \\
p_3(x) &= x^m + N \\
p_4(x) &= x^{2m+1} + N \cdot x^m + x \\
p_5(x) &= x^{3m+1} + N \cdot x^{2m} + 2x^{m+1} + N \\
p_6(x) &= x^{4m+1} + N \cdot x^{3m} + 3x^{2m+1} + 2N \cdot x^m + x \\
p_7(x) &= x^{5m+1} + N \cdot x^{4m} + 4x^{3m+1} + 3N \cdot x^{2m} + 3x^{m+1} + N \\
p_8(x) &= x^{6m+1} + N \cdot x^{5m} + 5x^{4m+1} + 4N \cdot x^{3m} + 6x^{2m+1} \\
&\quad +3N \cdot x^m + x \\
p_9(x) &= x^{7m+1} + N \cdot x^{6m} + 6x^{5m+1} + 5N \cdot x^{4m} + 10x^{3m+1} \\
&\quad +6N \cdot x^{2m} + 4x^{m+1} + N \\
p_{10}(x) &= x^{8m+1} + N \cdot x^{7m} + 7x^{6m+1} + 6N \cdot x^{5m} + 15Nx^{4m+1} \\
&\quad +10N \cdot x^{3m} + 10x^{2m+1} + 4N \cdot x^m + x
\end{aligned}$$

Notice what happens if we take $y = x^m$

$$\begin{aligned} p_3(x) &= yx + N \\ &= f_2(y)x + f_1(y)N \\ &= f_2(x^m)x + f_1(x^m)N \end{aligned}$$

$$\begin{aligned} p_4(x) &= y^2x + N \cdot y + x \\ &= (y^2+1)x + N \cdot y \\ &= f_3(x^m)x + N \cdot f_2(x^m) \end{aligned}$$

where $f_3(x^m)$ and $f_4(x^m)$ are Fibonacci Polynomials evaluated at x^m.

In fact:

$$p_n(x) = x \cdot f_{n-1}(x^m) + f_{n-2}(x^m)N, \qquad n \geq 3$$

Proof:
Assume the formula holds for $n \geq 4$ then:

$$\begin{aligned} p_{n+1}(x) &= x^m \cdot p_n(x) + p_{n-1}(x)N \\ &= x^m\left(x \cdot f_{n-1}(x^m) + f_{n-2}(x^m)N\right) + x \cdot f_{n-2}(x^m) + f_{n-3}(x^m)N \\ &= x\left(x^m \cdot f_{n-1}(x^m) + f_{n-2}(x^m)\right) + \left(x^m f_{n-2}(x^m) + f_{n-3}(x^m)\right)N \\ &= x \cdot f_n(x^m) + f_{n-1}(x^m)N \end{aligned}$$

which is remarkable indeed!

Notice that by choosing $m = 1$ and $N = 1$ we produce the Fibonacci polynomials. Also, by choosing $m = 1$, and $N = 2$ we produce the Lucas polynomials. This formula provides an interesting way of looking at some of the polynomials discussed previously. It relates all of the polynomials of the form:

$$p_n(x) = x^m \cdot p_{n-1}(x) + p_{n-2}(x) \text{ with } p_1(x) = N \text{ and } p_2(x) = x$$

back to the Fibonacci polynomials.

In conclusion ...

We've looked at the recursive polynomials defined by $p_n(x) = x \cdot p_{n-1}(x) + p_{n-2}(x)$, and saw the connection that these polynomials have to Fibonacci numbers, Lucas numbers, and Pascal's Triangle.

Setting initial values of the p_n to $p_1 = 1$, and $p_2 = x$ produces what are called Fibonacci polynomials, which we denoted $f_n(x)$. We observed $f_n(1) = F_n$.

Similarly we saw that setting initial values of the p_n to $p_1 = 2$, and $p_2 = x$ produces what are called Lucas polynomials, which we denoted $l_n(x)$. We saw that $l_n(1) = L_n$.

We then saw that examining the polynomials produced by more general initial values result in coefficients that form two simultaneous Pascal Triangles.

Also, we considered the polynomials of the form:

$$\begin{aligned} p_n(x) &= x^m \cdot p_{n-1}(x) + p_{n-2}(x) \\ &\text{with initial values} \\ p_1(x) &= N \text{ and } p_2(x) = x \end{aligned}$$

and related them to the Fibonacci Polynomials in the following way:

$$p_n(x) = x \cdot f_{n-1}(x^m) + f_{n-2}(x^m)N$$

The next topic to be investigated is to examine what happens when we start looking at polynomials with various initial values (N, x and x, N) of the form:

$$p_n(x) = (x^2 + x + 1)p_{n-1}(x) + p_{n-2}(x)$$

or

$$p_n(x) = (x^3 + 3x + 1)p_{n-1}(x) + p_{n-2}(x)$$

Do they behave in a predictable way to those of the coefficients of the polynomials examined in this paper?

Acknowledgment. Dr. Boerkoel, who never passes up an opportunity to learn something new, continues to be an inspiration to me. His guidance and insightfulness were invaluable throughout the research and writing process of this paper.

References

1. Sloan's website is a searchable database of integer sequences with 79591 sequences catalogued. http://www.research.att.com/~njas/sequences/, January 20, 2003. Also, for a similar cataloguing in print see The Encyclopedia of Integer Sequences by N.J.A Sloane and S Plouffe.

2. For a more complete treatment of Fibonacci polynomials, and Lucas polynomials see Fibonacci and Lucas Numbers with Applications by T. Koshy.

Chaos

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Chaos is everywhere around us. In a chemistry class, all measurements are done to a certain number of significant digits, and the weather man is never exactly right with the forecast either. Nothing is totally predictable, which is contrary to what intuition says should happen. This predictable unpredictability is Chaos. This new science of chaos was too abstract for physicists but too experimental for mathematicians.[1]

Edward Lorenz was an early contributor to chaos, though he did not know it. Lorenz was a meteorologist at MIT. He developed a computer program that modeled weather, though not nearly as complex as the earth's atmosphere. One day, in his office, he decided to take another look at a pattern that his modeled weather had made. So he input the numbers back into the system as a starting point. The returned results were nowhere close to the results he expected because he input numbers with 3 significant digits, whereas the computer stored up to 6 significant digits. This minute change should not have made a large difference, but the results were nowhere close. Lorenz wrote a paper about his findings in which chaos was first modeled, yet they were buried deep in meteorological journals so no mathematicians read the article. In the paper, the term "Butterfly Effect" was born. The butterfly effect poses this question: "does the flap of a butterfly's wings in Brazil set off a tornado in Texas?" It is basically sensitive dependence on initial conditions. Lorenz's work was pioneering, yet mostly undiscovered until much later.[2]

Benoit Mandelbrot was the next big pioneer in the area of chaos. Mandelbrot did work in many different areas of math. He was working on economics at the time that he discovered chaos. He saw seemingly unrelated random events bound together by a pattern. Instead of finding a pattern in one area, he found a pattern that bound everything together; this pattern was the pattern of chaos. Mandelbrot also explored the flooding of the Nile River and also cotton prices; in all these, he saw the pattern

[1] Gleick: Chaos p. 38
[2] Ibid p. 9-32

of chaos. In his further studies, Mandelbrot posed the famous question: "how long is the coastline of Britain?" To answer this question, Mandelbrot turned to the idea of fractal dimension;t his gave rise to fractals - the beautiful pictures of chaos.[3]

After much study, Benoit Mandelbrot produced the most famous fractal of all. The set still bears his name, the Mandelbrot Set. The Mandelbrot Set is a beautiful picture, but it is formed using complex mathematics. First, the Mandelbrot Set is formed using a recursive formula.[4] Second, the Mandelbrot Set is a picture on the complex plane.[5]

The logistics equation is a recursive formula that exhibits chaos. Robert May, a biologist, was modeling populations in nature using the recursive equation $y = rx(1 - x)$, where the output, y, becomes the next input, x. r is the parameter for the equation and changes how the population grows and declines. After r reaches a certain number, the population does not settle down, like all biologists thought it should, but oscillates between 2 values, contrary to what biologists have seen in nature. r will eventually grow big enough that the population is chaotic, which biologists also had observed. May published his findings, but again, mathematicians did not find it until later.[6] (Appendix A)

The key to the logistics equation is that the logistics equation is non-linear. Non-linear equations are not studied in depth until graduate school. Both mathematics and science like linear equations, because they are simple and easy to understand. The logistics equation, uses squaring, which is a non-linear function. This non-linearity is what gave rise to the unexpected results that May received.

Mitchell Feigenbaum was doing research into non-linear equations at Los Alamos national laboratories when he came across the work by Robert May. Feigenbaum did more work with the logistics equation and found that the "splits" in oscillation were not random at all, but could be predicted. The ratio 4.6692016090 is the ratio of bifurcation and is also known as the Feigenbaum constant. These findings were later published in an article, not by Feigenbaum, called "Period Three Implies Chaos."[7]

Complex numbers are numbers outside the set of real numbers. They are formed using $\sqrt{-1}$. The complex numbers have usefulness in solving cubic equations and other equations. They are written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. Much of the work on chaos

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[3] Ibid p. 82-117

[4] Boston University p. 2

[5] Kaye: Chaos and Complexity p. 91

[6] Gleick: Chaos p. 69-77, 177

[7] Gleick: Chaos p. 166-181

was done on a computer. In fact, with out computers, chaos would still remain undiscovered. To a computer, $\sqrt{-1} = 0$. This poses a problem for doing work with complex numbers on a computer. The solution to this problem is to write the complex numbers as an ordered pair: (a, bi), stored in the computer as an array.

$a[1] = atof(anumber);$
$a[2] = atof(bnumber);$ //Stores complex number in array

The Mandelbrot Set's relationship to complex numbers is that the Mandelbrot Set is on the complex plane. It is done by iterating the squares of complex numbers plus a constant c.

$$z_0 = 0, \qquad z_1 = z_0{}^2 + c, \qquad z_n = z_{n-1}{}^2 + c.$$

If z_n diverges to infinity, the point is not in the Mandelbrot Set. If z_n does not diverge to infinity, but rather is contained, it is in the Mandelbrot Set. Just for one point, the math gets long and tedious, so it is obvious why computers were used in order to produce the Mandelbrot Set. The computer code uses the array that stored the complex number. Squaring the complex number (a, b) results in $(a^2 - b^2, 2ab)$

$$\begin{aligned} &b[0] = PlotArray[i-1][0] * PlotArray[i-1][0] \\ &\qquad -PlotArray[i-1][1] * PlotArray[i-1][1]; \\ &b[1] = 2 * PlotArray[i-1][0] * PlotArray[i-1][1]; \\ &PlotArray[i][0] = b[0] + a[1]; \\ &PlotArray[i][1] = b[1] + a[2]; \end{aligned}$$

PlotArray holds the array needed to plot the point on the screen. The Array b is a temporary array used for calculation purposes. The $[i-1]$ gives the recursive effect. This snippet of code is placed in a loop to give the number of points to plot.

Depending on the value of c, one can tell how many points z will oscillate between. Each bulb is associated with a number, the number of oscillating points. This biggest bulb is the period 1 bulb. Any point c picked in period one will converge to one point. The next biggest bulb (the one on the left) is the period 2 bulb and any value for c will oscillate between 2 points. The value for c does not have to change a great deal in order to change its bulb of residence. Even a very small change can greatly affect the outcome. (Appendix B) The number of bulbs is infinite.[8]

Julia Sets are very closely linked to the Mandelbrot Set. Julia Sets are formed in the same way as Mandelbrot Sets, but done by fixing c and iterating the starting location, z_0. If c is in the Mandelbrot Set, then the

[8] Devaney: Fractal Geometry of the Mandelbrot Set

Julia Set is connected; if not, the Julia Set is not connected and is called a Fatou dust. All Julia Sets are self similar. All the vertices in the Set are the same, just scaled down. The number of bulbs connected to each vertex tells which bulb of the Mandelbrot Set from which the Julia Set originates. The Julia Sets are closely linked to the Mandelbrot Sets.[9]

The Mandelbrot set is not self similar. The only similarity is that it is reflective about the real axis. Though it may appear to be self similar at first glance, it is not. Each of the bulbs is slightly different. Each bulb has a vertex at the end of it, and that vertex has the same number of antennae as the number of the bulb. The Fibonacci sequence is also hidden within the bulbs. Start with bulbs one and two. The biggest bulb between one and two is bulb three. The biggest between two and three is bulb five. This continues forever.[10]

The Mandelbrot Set is constructed from seemingly unrelated, almost random, numbers. Many other fractals are constructed in the same way. In the "chaos game," three vertices are chosen, and a random number, one through three, is generated. A point half-way to the chosen point is then plotted; this is repeated again and again. The fractal known as Serpinski's triangle emerges from the chaos of the pseudo-random points. This is another example of chaos.[11]

Chaos is all around us. Even the smallest changes can have a great impact in the future because chaos theory is based upon the sensitivity of initial conditions. The reason chaos theory was such a recent development is because of the necessity of computers to facilitate the discovery and the use of the non-linear equations used for calculations. Chaos will always be around us; we experience it every day.

Acknowledgements. I would like to thank Dr Troy Riggs for his assistance and guidance while doing the research.

Appendix A

Robert Mays Logistics Equations were modeled using the equation $y = rx(1-x)$. This produces a parabola. The recursive nature can be viewed as a series of line segments being reflected from the parabola, off the line $y = x$, back to the parabola. The graph that is produced will show convergence, oscillation, or chaos.

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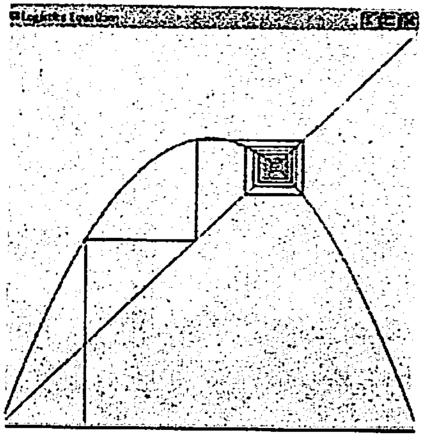
9 Boston University p. 6

10 Devaney: Fractal Geometry of the Mandelbrot Set

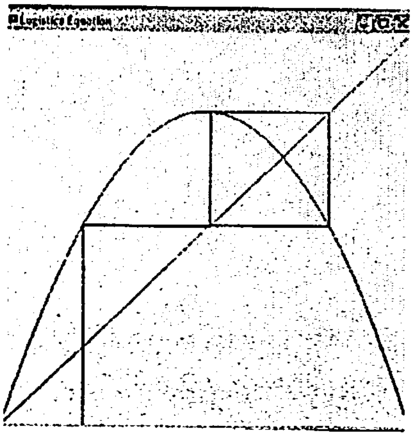
11 Boston University - Chaos in the Classroom



$r = 2.9$

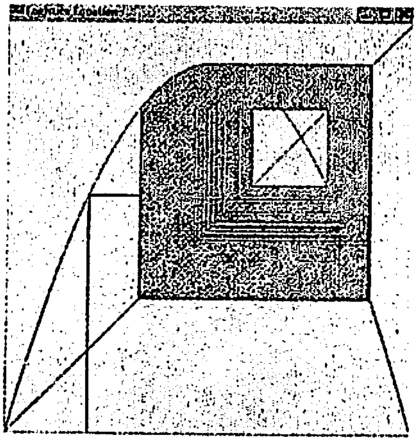
This is the logistic Equation with $r = 2.9$.

This shows convergence to one point, the point where the parabola intersects the line $y = x$.



$r = 3.2$

Here, $r = 3.2$. This clearly shows oscillation between 2 points.

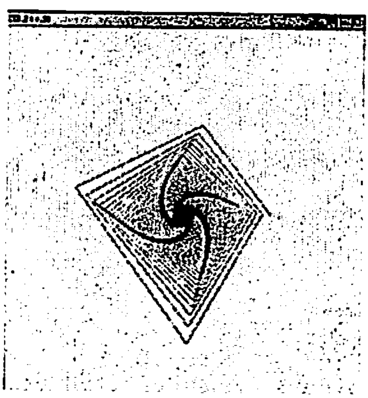


$r = 3.6$

Here, $r = 3.6$. Chaos has begun. There is no pattern.

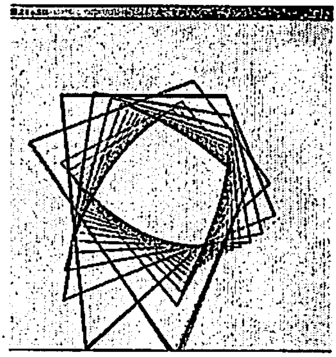
Appendix B

The program written shows a graph of the points produced when put through the algorithm for finding whether or not the point is in the Mandelbrot Set.



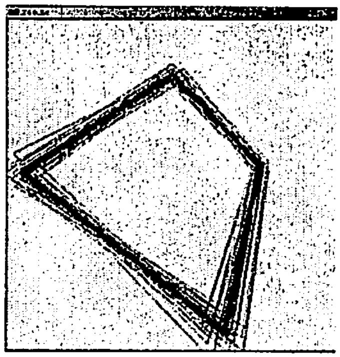
$c = 0.24 + .5i$

Converges to one point. In bulb one.



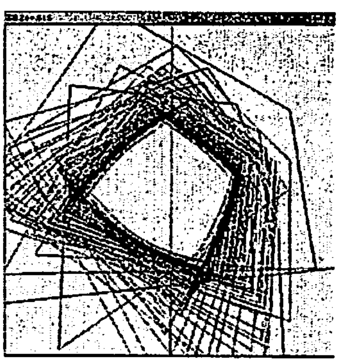
$c = 0.24 + 0.51i$

Oscillates among many points with so slight a change.



$c = 0.24 + 0.54i$

Clearly begins to oscillate among 4 points.



$c = 0.24 + 0.515i$

Chaos has ensued.

Even though the value of c changed very little, the change was great enough to change which bulb c existed in, and even whether it was in the Mandelbrot Set at all.

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Geometric Contortions of the Complex Plane: Exploring Elementary Complex Functions

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Introduction

In this paper we will begin by looking at some of the basic functions of the complex plane. We will discuss where specific regions are mapped, as well as where certain curves are mapped. Using the information from the discussed functions, we will map several interesting regions to the unit disk.

Complex Numbers

The complex number system C consists of the numbers $x + yi$, with $x, y \in \Re$, with an addition and a multiplication defined by:

1. $(x + yi) + (r + si) = (x + r) + (y + s)i$
2. $(x + yi) \cdot (r + si) = (xr - ys) + (yr + xs)i$

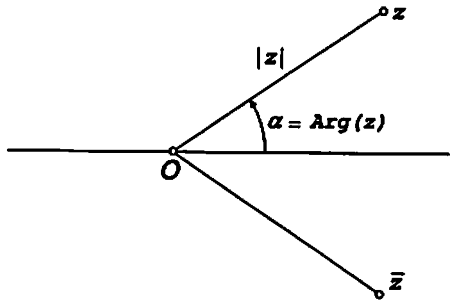
Properties of Complex Numbers

We will begin by stating some of the basic geometrical properties of the complex numbers.

Modulus and Argument of a Complex Number

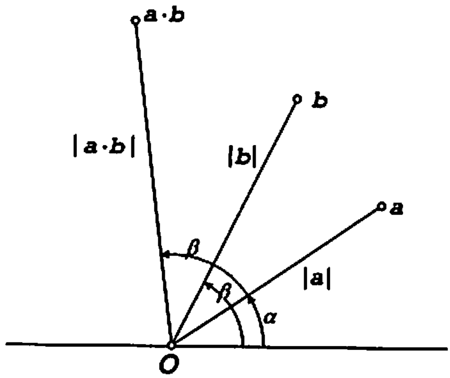
The modulus, or length of a complex number is defined as $|z| = \sqrt{x^2 + y^2}$, where $z = x + yi$.

If we define the complex conjugate of z by $\overline{z} = x - yi$, then $z\overline{z} = (x + yi)(x - yi) = x^2 + y^2 = |z|^2$. Note that $\overline{z}$ is the reflection of z in the real axis. Finally let the Argument of a Complex Number z be the angle made with the positive x-axis. Basically, the modulus and argument of a complex number represent the number in polar coordinates.



Multiplication

In complex multiplication moduli are multiplied and arguments added: the length of $|a \cdot b|$ is $|a||b|$, and the argument of $a \cdot b$ is $\alpha + \beta$. Hence we see that multiplication involves a rotation.



Euler's Formula

A very famous formula, due to Euler, is the following

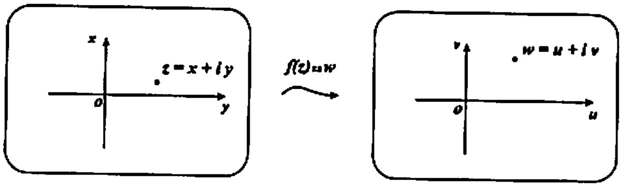
$$e^{i\theta} = \cos\theta + i\sin\theta$$

This allows us to describe complex multiplication rather elegantly:

$$ab = |a|e^{i\alpha} \cdot |b|e^{i\beta} = |a||b|e^{i(\alpha+\beta)}$$

Basic Functions

First we will define and analyze some basic functions using complex numbers. In order to do this we need to discuss some notation. In this paper the convention will be to map a complex number $z = x + iy$ to the complex number $w = u + vi$. Graphically $w = f(z)$ will be depicted with two planes: a z-plane with x,y-axis and a w-plane with u,v-axis.

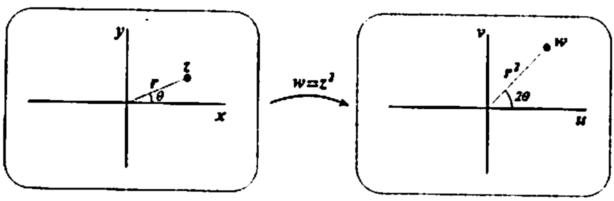


The Function z^2

Because of the multiplicative property of the complex numbers,

$$z^2 = (|z|e^{i\theta})^2 = |z|^2 e^{2\theta}$$

In a geometrical sense, that means the modulus of z is squared and the argument of z is doubled.



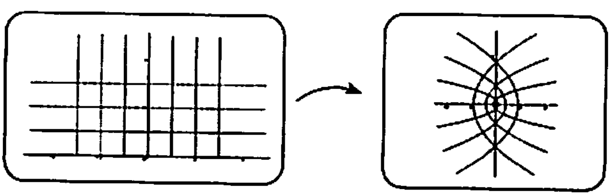
In order to get a better feel for this function we will see what happens to vertical and horizontal lines.

Consider a vertical line, $z = x_0 + yi$ with $x_0 \neq 0$.Then $z^2 = x_0^2 - y^2 + 2x_0yi$.
Hence $u = x_0^2 - y^2$ and $v = 2x_0y$, then

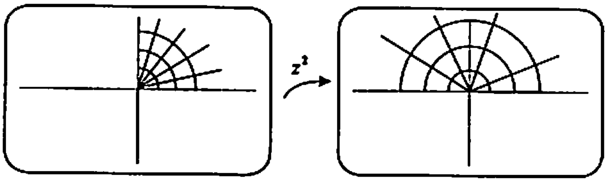
$$u = \frac{-v^2}{4x_0^2} + x_0^2$$

So vertical lines are mapped to parabolas. Similarly a horizontal line, $z = x + y_0 i$ with $y_0 \neq 0$ is mapped to the parabola $u = \frac{v^2}{4y_0^2} - y_0^2$ where $u = x^2 - y_0^2$ and $v = 2xy_0$.

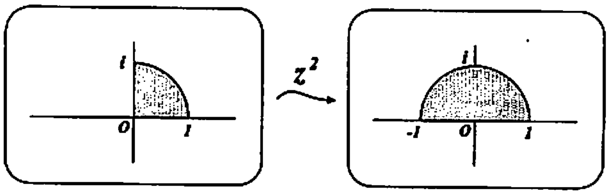
Here is a grid of horizontal and vertical lines transformed by z^2. Notice how the curves still intersect at a right angle.



Or looking at the circles and rays: $r_0 e^{i\theta}$ and $re^{i\theta_0}$



A useful region to look at is the first quadrant of the unit disk. It is mapped to the upper half of the unit disk. We will see applications of this later on.



The Function e^z

Let $e^z = e^{x+iy} = e^x(\cos y + i \sin y)$. Again we will look at vertical and horizontal lines. For example, a vertical line, $z = x_0 + yi$, is mapped by e^z as follows:

$$e^z = e^{x_0}(\cos y + i \sin y)$$

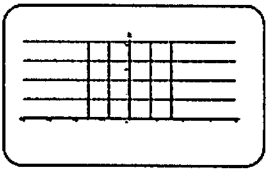
Since x_0 is fixed, with $u = e^{x_0} \cos y$ and $v = e^{x_0} \sin y$, we find

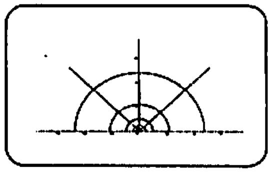
$$u^2 + v^2 = (e^{x_0})^2$$

In other words, the vertical line $z = x_0 + iy$ is mapped to a circle centered at the origin with a radius of e^{x_0}. A horizontal line, $z = x + y_0 i$ is mapped to

$$e^z = e^x e^{iy_0}$$

Clearly, $e^x > 0$, so this is a ray from the origin. e^z is periodic, because sine and cosine are periodic. An in interesting region to look at is the strip between $z = x + 0i$ and $z = x + \pi i$. As shown, it is mapped to the upper half plane. Vertical lines are mapped to circles and horizontal lines are mapped to rays as discussed earlier. Notice that the images of the lines also form right angles.





The Function sin (z)

The $\sin(z)$ can be defined as follows:

$$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}.$$

Hence

$$\begin{aligned}
\sin(z) &= \frac{e^{-y}e^{ix} - e^{y}e^{-ix}}{2i} \\
&= \frac{e^{-y}\cos x + e^{-y}i\sin x - e^{y}\cos x + e^{y}i\sin x}{2i} \\
&= \frac{e^{-y} + e^{y}}{2i} i\sin x + \frac{e^{-y} - e^{y}}{2i}\cos x \\
&= \cosh y \sin x + i\sinh y \cos x
\end{aligned}$$

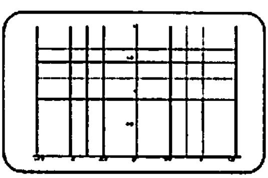
Looking at vertical lines: e.g. $z = x_0 + yi$ $\left(\text{e.g. } \frac{-\pi}{2} < x_0 < \frac{\pi}{2}\right)$. If $u = \cosh(y)\sin(x_0)$ and $v = \sinh(y)\cos(x_0)$ we see that

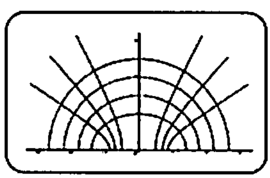
$$\left(\frac{u}{\sin x_0}\right)^2 - \left(\frac{v}{\cos x_0}\right)^2 = 1 \qquad (x_0 \neq 0)$$

So vertical lines go to hyperbolas. A horizontal line, $z = x + y_0 i$ $(y_0 \neq 0)$, with $u = \cosh(y_0)\sin(x)$ and $v = \sinh(y_0)\cos(x)$ is mapped to

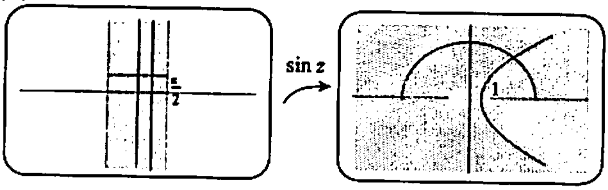
$$\left(\frac{u}{\cosh y_0}\right)^2 + \left(\frac{v}{\sinh y_0}\right)^2 = 1$$

Hence horizontal lines are mapped to ellipses. Here is a mapping of $\sin z$ for several horizontal and vertical lines (also in $\frac{-\pi}{2} < x < \frac{\pi}{2}$)





This region, is the strip given by $\left\{z \middle| \frac{-\pi}{2} < x < \frac{\pi}{2}\right\}$, is mapped. It is mapped 1-1 and onto to $C \backslash \left\{w = u + 0i : |u| \geq 1\right\}$. Hence we can talk about $\arcsin(z)$ (e.g. see (h)). That is, the strip is mapped to the entire plane except the real axis where $|x| > 1$. This region represents one period of $\sin(z)$.



The Function $\frac{1}{z}$

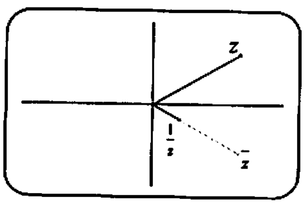
If you are familiar with inversions, this function might look familiar.

$$\frac{1}{z} = \frac{x}{x^2 + y^2} - \frac{y}{x^2 + y^2} i$$

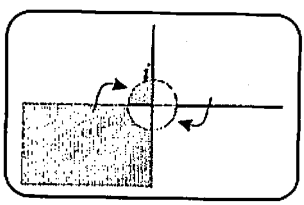
Another, sometimes more helpful way to look at $\frac{1}{z}$ is

$$\frac{1}{z} = \frac{\bar{z}}{|z|^2}$$

This can be visualized in the following way:

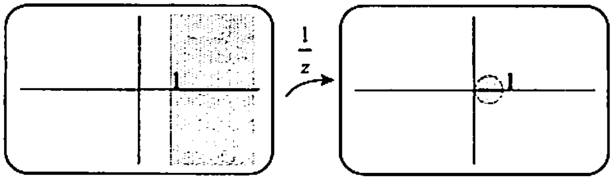


It is also interesting to note that regions outside the unit circle are reflected with respect to the real axis and then mapped inside the unit circle as shown.



Also it is good to note the well known fact that $\frac{1}{z}$ maps "circles" to "circles" (with "circles" here we mean both circles and lines).

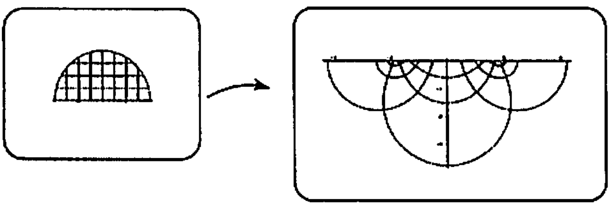
An interesting region to consider is $\{z = x + yi \mid x > 1\}$.



The Function $z + \frac{1}{z}$

$$z + \frac{1}{z} = z + \frac{\bar{z}}{|z|^2} = \frac{|z|^2 z + \bar{z}}{|z|^2} = \frac{x^3 + xy^2 + x}{x^2 + y^2} + i\frac{y^3 + yx^2 - y}{x^2 + y^2}$$

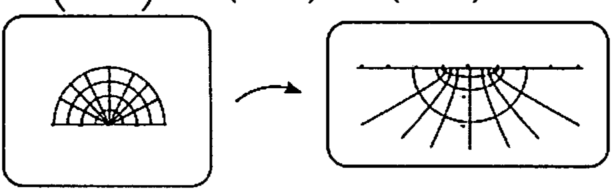
In order to get a better feel for this function, consider what happens to vertical and horizontal lines inside the upper half of the unit disk: (which is easy to do in Maple)



Another way to visualize this is looking at semi circles and rays:

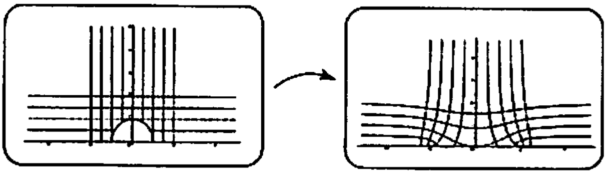
$$z + \frac{1}{z} = re^{i\theta} + \frac{1}{r}e^{-i\theta} = \left(r + \frac{1}{r}\right)\cos\theta + i\left(r - \frac{1}{r}\right)\sin\theta$$

So circles, $r_0 e^{i\theta}$ with $(r_0 > 0)$, go to ellipses $\left(\frac{u}{r_0 + \frac{1}{r_0}}\right)^2 + \left(\frac{v}{r_0 - \frac{1}{r_0}}\right)^2 = 1$ and rays, $re^{i\theta_0}$, go to hyperbolas $\left(\frac{u}{\cos\theta_0}\right)^2 - \left(\frac{v}{\sin\theta_0}\right)^2 = \left(r + \frac{1}{r}\right)^2 - \left(r - \frac{1}{r}\right)^2 = 4.$

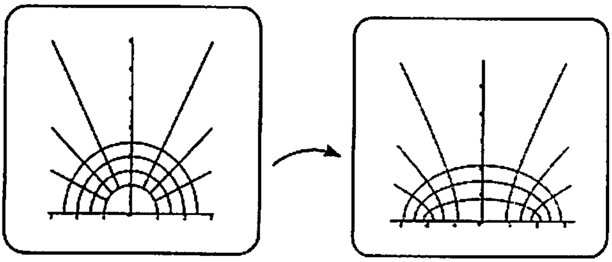


Hence $z + \dfrac{1}{z}$ maps the upper half of the unit disk to the bottom half-plane.

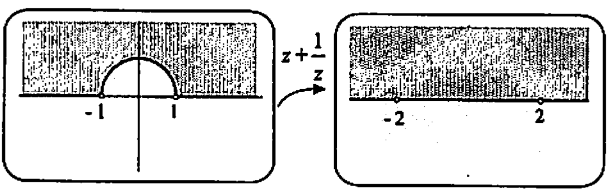
Now consider the upper half-plane without the unit disk. Looking at vertical and horizontal lines we see: (using Maple)



or using rays and circles:



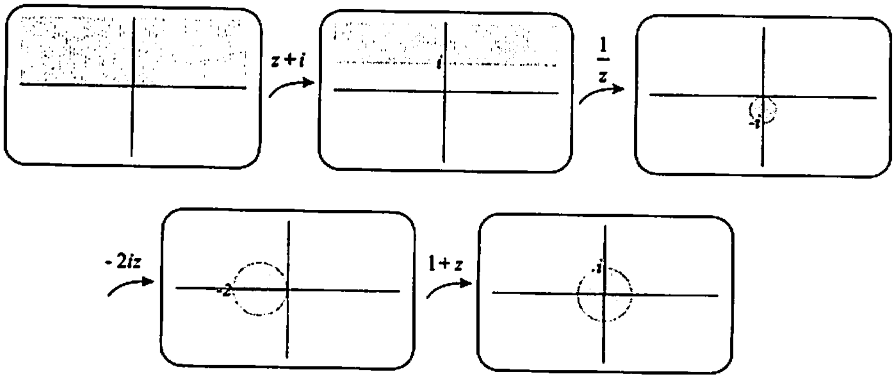
Both cases illustrate that this region is mapped to the entire upper half-plane. Combining these, means $z+\frac{1}{z}$ maps the upper half-plane $(Im(z) > 0)$ to the entire plane without $\{z \in \Re \mid |z| \geq 2\}$.



The Function $\dfrac{z-i}{z+i}$

This function can be understood more easily if we think about it as a composition of several functions already discussed. Since an equivalent form of this is:

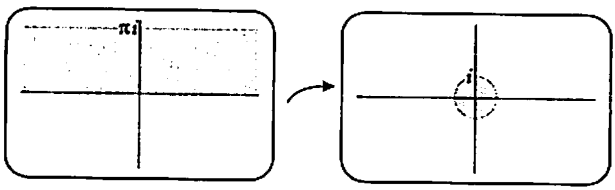
$$\frac{z-i}{z+i} = 1 - 2i\frac{1}{z+i}$$



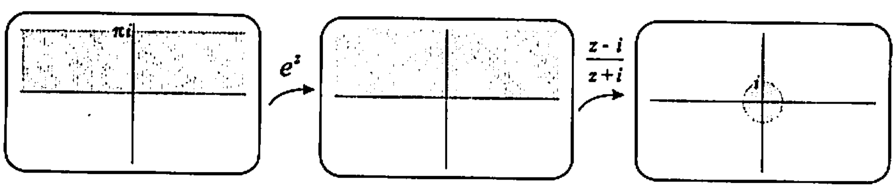
Regions

The functions we have just discussed will help us with our next task. We can now map many regions to e.g. the unit disk $\{z \mid |z| < 1\}$. We will be mapping open regions to the open unit disk bijectively (1-1 and onto). In all the pictures the boundary lines are not included.

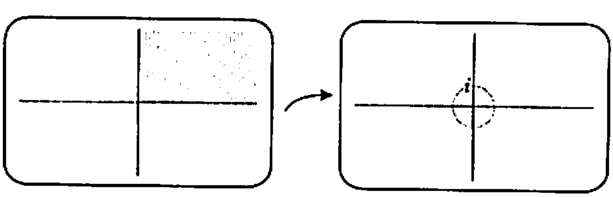
a) Map the strip $\{z = x + yi \mid 0 < y < \pi\}$ onto the unit disk.



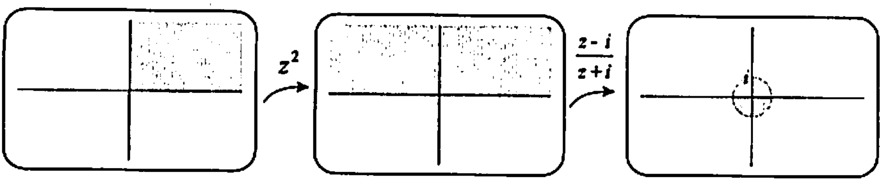
This could be done as follows:



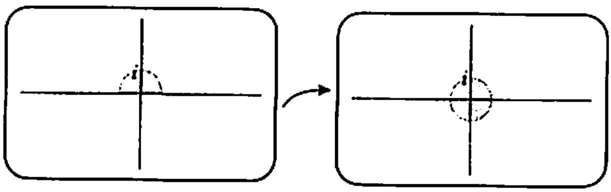
b) Another interesting region we want to map onto the unit disk is the first quadrant, $\{z \mid Re(z) > 0,\ Im(z) > 0\}$.



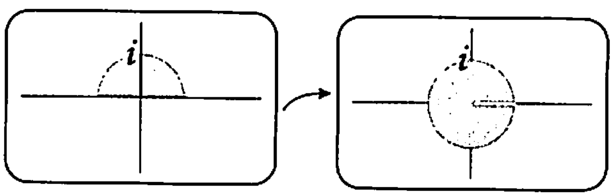
which can be done as follows:



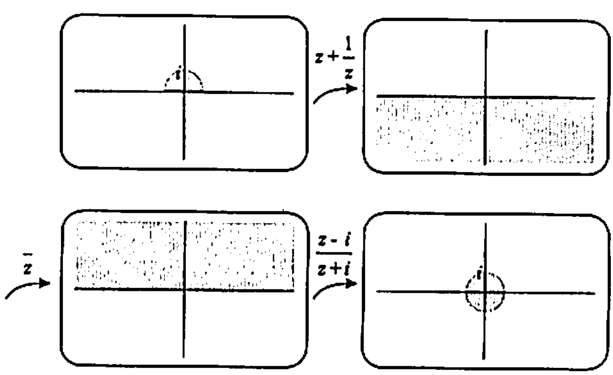
c) Next consider $\{z \mid Re(z) > 0, |z| < 1\}$ onto the unit disk.



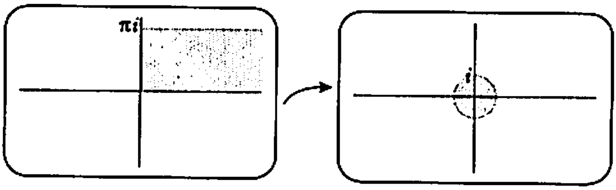
While this may seem to be a trivial example, it is not as simple as it first appears because we want the entire unit circle. For example, z^2 would not work because we would miss points on the real axis.



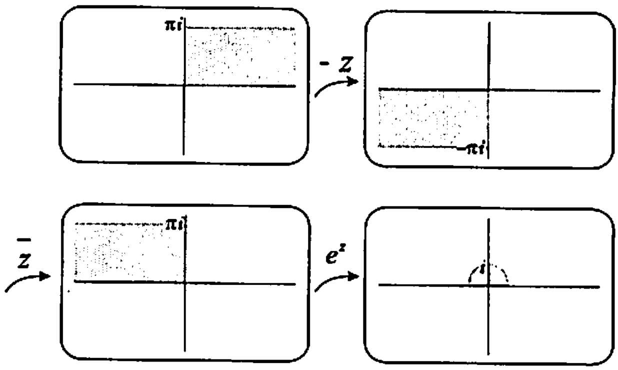
Instead



d) Next we want $\{z \mid Re(z) > 0,\ 0 < Im(z) < \pi\}$ to be mapped onto the unit disk.

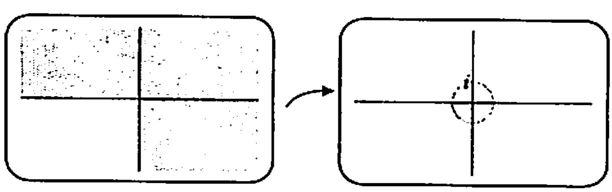


This region can be mapped to the upper half of the unit disk as follows:

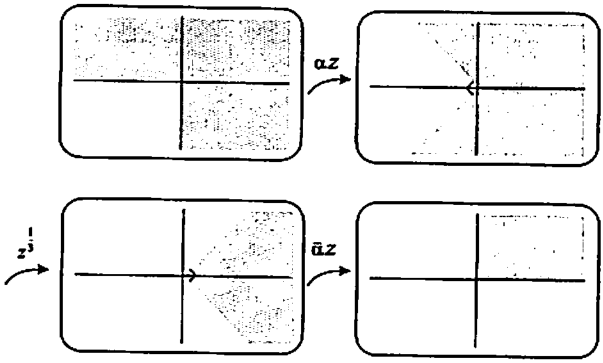


From (c) we see this can be mapped to the unit disk.

e) Our next region is C without Quadrant 3, $\{z \mid \frac{-\pi}{2} < Arg(z) < \pi\}$, mapped onto the unit disk.

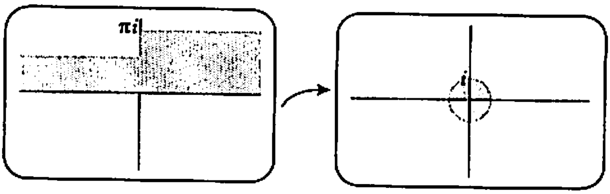


This can be mapped to the first quadrant as follows:

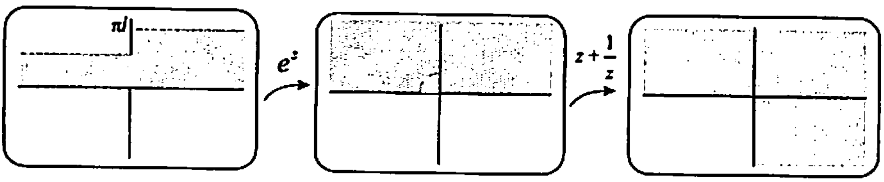


where $\alpha = e^{\frac{-\pi}{4}i}$. Now we are at the region in (c) that we have already mapped to the unit disk.

f) Consider the following mapping of $\{z \mid 0 < Im(z) < \frac{\pi}{2}\} \cup \{z \mid Re(z) > 0,\ \frac{\pi}{2} \leq Im(z) < \pi\}$ onto the unit disk.

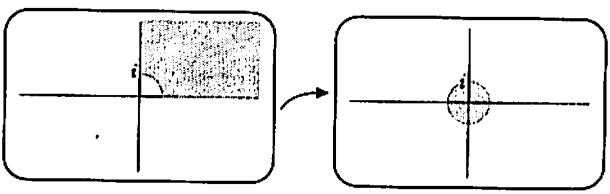


This can be accomplished by

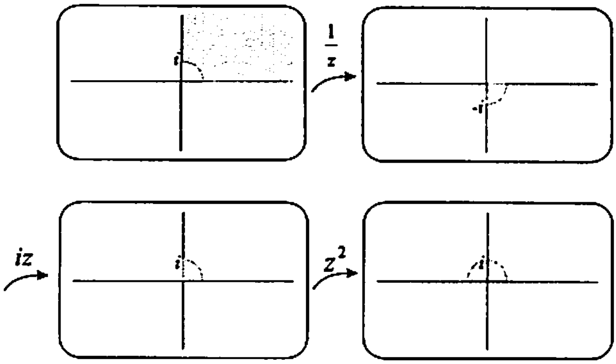


which can be mapped to the unit disk as in (e).

g) Our next region is the first quadrant without the unit disk, $\{z \mid |z| > 1,\ 0 < Arg(z) < \frac{\pi}{2}\}$, mapped onto the unit disk.

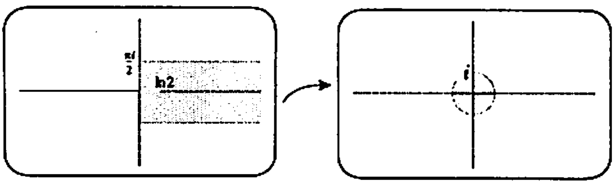


Consider

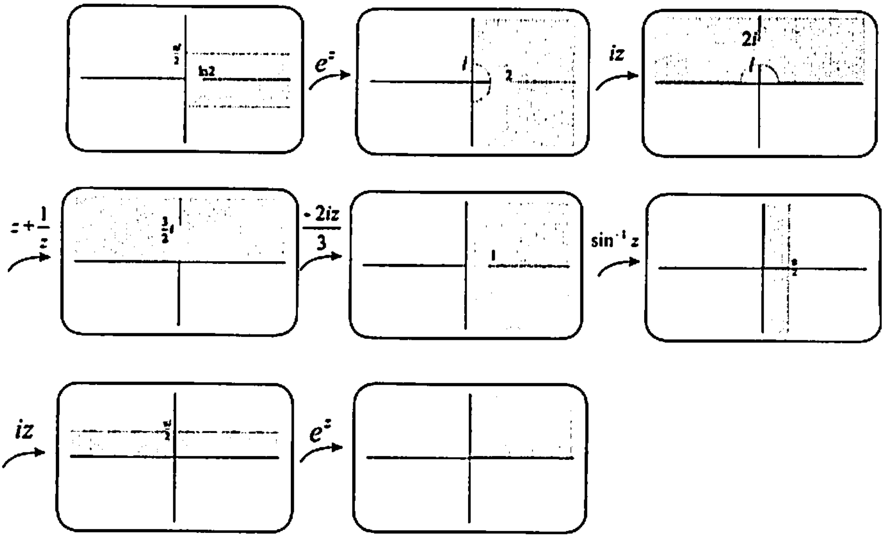


And since we mapped the upper half of the unit circle to the entire unit disk in (c), we are done.

h)An interesting region to consider is $\left\{z \mid \frac{-\pi}{2} < Im(z) < \frac{\pi}{2},\ Re(z) > 0\right\} \backslash \left\{z \in \Re \mid |z| \geq \ln 2\right\}$ onto the unit disk. In other words part of a strip with a ray removed mapped onto the unit disk.

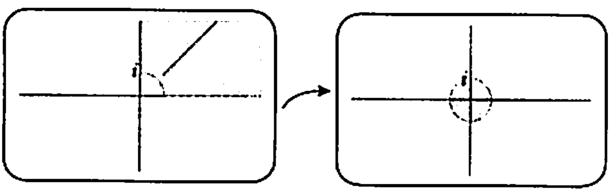


To map this to the unit disk is a bit harder to do then with the region in (d).

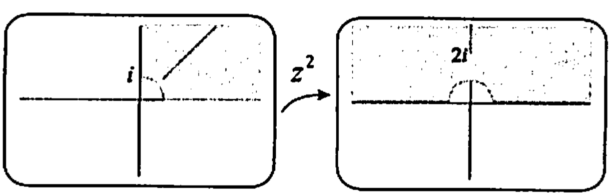


Now we are at the region discussed in (b), so we can easily complete the mapping.

i) A related region is the region of (g) minus the ray $\{z = t+ti \mid t > 1\}$.

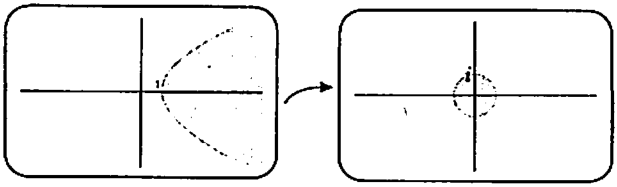


If we map $z \longmapsto z^2$,

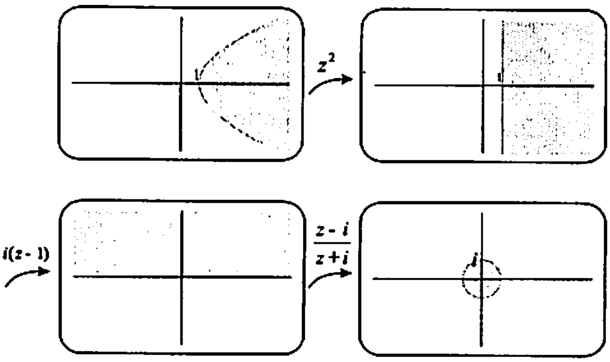


then we have a similar region to the one that was discussed in the previous example. Hence this can indeed be mapped to the unit circle.

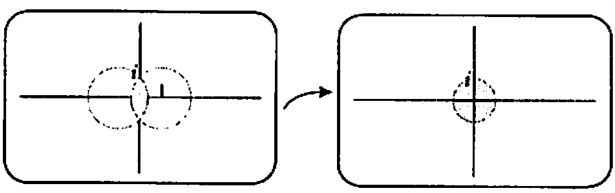
j) Consider the following mapping of $\{z \mid x > \sqrt{y^2+1}\}$ to the unit disk.



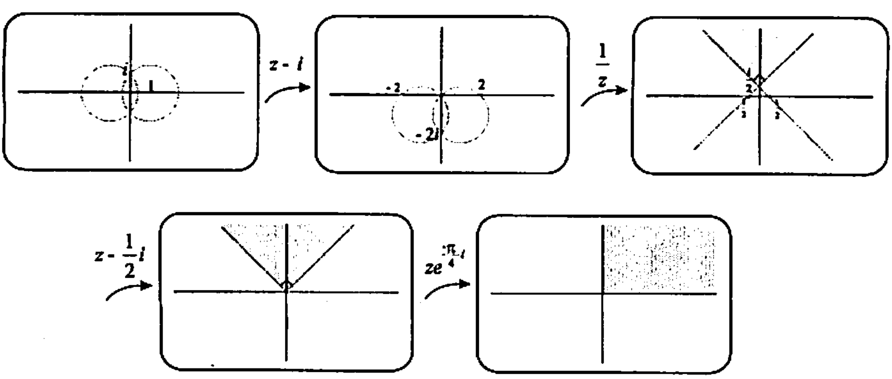
This can be done as follows:



k) Next consider the intersection of two disks, $z = \sqrt{2}\, e^{i\theta} + 1$ and $z = \sqrt{2}\, e^{i\theta} - 1$, mapped onto the unit disk.

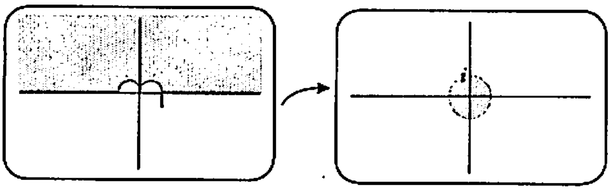


We can map this region in the following manner:

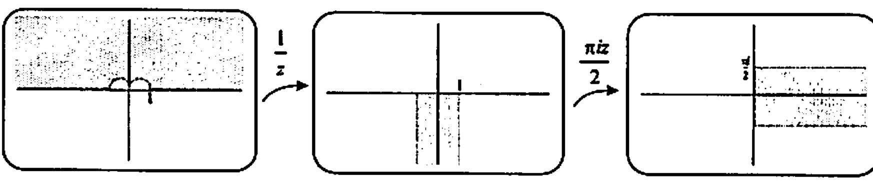


Now that we are at the region discussed in (b), so we can easily finish our mapping to the unit disk.

l) The next region to map is $\{z \mid Im(z) > 0\} \backslash \{z \mid |z - \frac{1}{2}| \leq \frac{1}{2} \; or \; |z + \frac{1}{2}| \leq \frac{1}{2}\}$, a half plane with two half disks scooped out.

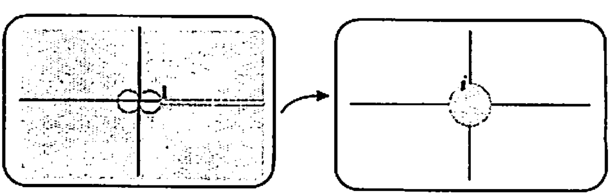


This map can be done as follows: (recall that $\frac{1}{z}$ maps "circles" to "circles")

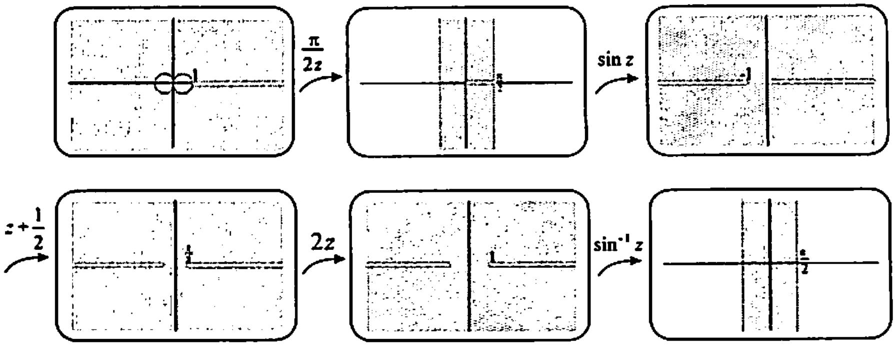


We have already mapped similar strips to the unit disk. This strip will need to be scaled and shifted, but essentially the mapping is as in (d).

m) Our last region to consider is the complement of $\left(\{z \mid |z - \frac{1}{2}| \leq \frac{1}{2} \; or \; |z + \frac{1}{2}| \leq \frac{1}{2}\} \cup \{z \in \Re \mid x > 1\}\right)$ mapped onto the unit disk.



It can be mapped by:



Now we are at a strip which is similar to the one mapped in (a).

Riemann's Mapping Theorem

In this paper I have looked at some elementary functions and their geometric behavior. I have also looked at many regions which I mapped to the unit disk, using these elementary functions. Riemann's Mapping Theorem says much more.

Riemann's Mapping Theorem: Every simply connected open region, R, $R \neq C$, can be mapped conformly to the unit disk (uniquely if we specify e.g. $f(z_0) = 0$, and $f'(z_0) > 0$).

I have just begun my study of Complex Analysis (and not yet covered conformal mappings). The material in this paper has helped me get a better feel for complex functions and understand their structure and behavior. All maps studied have been bijectives (in fact, even conformally) of open regions to the open unit disk.

Acknowledgments. I would like to thank Dr. Boerkoel for his guidance and help throughout this paper. Under his guidance I discovered the information in this paper. Without his help, this paper would not be. I would also like to thank Dr. Simpson. His help with the computer and graphics was invaluable.

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The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before January 1, 2005. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 2005 issue of *The Pentagon*, with credit being given to the student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: ken.wilke@washburn.edu).

PROBLEMS 570-574

Problem 571. Proposed by James R. Bush, Waynesburg, College, Waynesburg, Pennsylvania (Corrected).

Find all values of a such that $x^2 - x + a$ divides

$$x^{15} + 27x^3 + 688x^2 - 1352x - 1092$$

Problem 572. Proposed by Jose Luis Diaz-Barrero, Universitat Politeccnica de Catalunya, Barcelona, Spain (Corrected). [The editor inadvertently used the proposer's problem 566 in the previous issue instead of this problem which was intended to be used as Problem 572. The editor apologizes for the mistake.]

Let z_1, z_2 and z_3 be nonzero complex numbers such that

$$z_1^3 + z_2^3 + z_3^3 = 0$$

Show that

$$\frac{z_1^3 + z_2^3 + z_3^3}{z_1^3 z_2^3 z_3^3}$$

is an integer and determine its value.

Problem 575. Proposed by Pat Costello, Eastern Kentucky University, Richmond, Kentucky.

Given a positive integer m, then take the sum of its digits to obtain a different number, take the sum of the digits of this new number to obtain yet another number and so on until the remaining number has only one digit. We call the one digit number the digital root of m. Suppose that we have a recursive sequence defined by

$$\begin{aligned} x_1 &= 3 \\ x_2 &= 6 \\ x_3 &= 9 \\ x_n &= x_{n-1} + x_{n-2} + x_{n-3} \end{aligned}$$

and for all integers $n \geq 4$. Show that the digital root of x_n is always 3.

Problem 576. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Find the maximum of the function $y = f(x)$ on the interval $[e, \pi]$ given that $\ln f(x) = \frac{xf'(x)}{f(x)}$ and $f(\pi) = e$.

Problem 577. Proposed by Thomas Chu, Austin, Texas.

In triangle ABC let O denote the center of the inscribed circle. Prove that

$$OA^2 + OB^2 + OC^2 = \frac{a^2(b+c) + b^2(c+a) + c^2(a+b) - 3abc}{a+b+c}$$

where a, b and c denote the sides of triangle ABC and OA, OB and OC denote the distances from the incenter O to the respective vertices of the triangle.

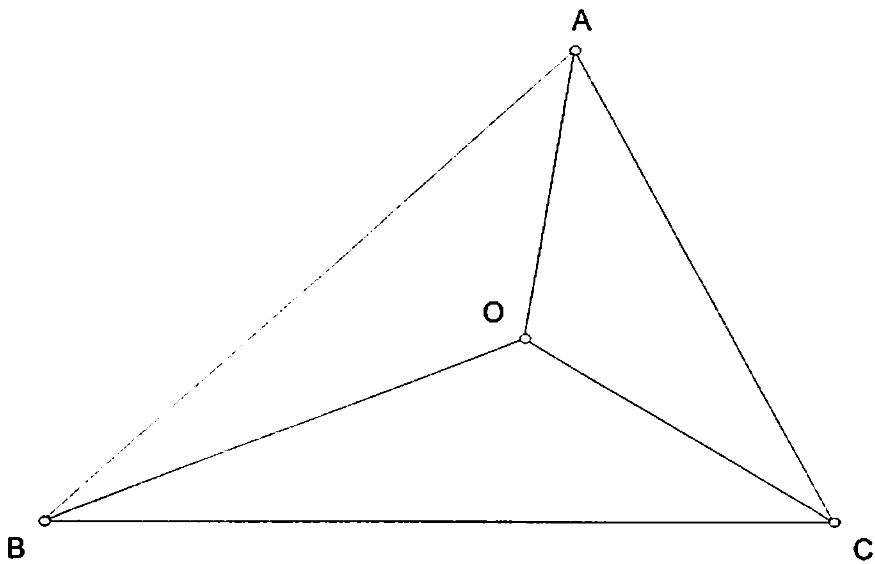

Figure for Problem 577

Problem 578. Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.

Let triangle ABC be a triangle such that $\sin A, \sin B$ and $\sin C$ are in arithmetic progression. Prove that $\tan\left(\frac{A}{2}\right)\tan\left(\frac{C}{2}\right)=\frac{1}{3}$.

Problem 579. Proposed by M. Khoshnevisan, Griffith University, Gold Coast, Queensland, Australia.

A Generalized Smarandache Palindrome (GSP) is a concatenated number of the form $a_1a_2...a_na_n...a_2a_1$ or $a_1a_2....a_{n-1}a_{n-1}...a_2a_1$ where $a_1, a_2, ..., a_n$ are positive integers of various numbers of digits which are not palindromic numbers. Find the number of GSP of four digits which are not palindromic numbers.

The editor wishes to acknowledge receipt of late solutions for problem 560 from Nicholas a. Bernini, student, Slippery Rock University, Slippery Rock, Pennsylvania and Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri.

Please help your editor by submitting problem proposals.

SOLUTIONS 565 – 569

Problem 565. Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.

Let ABC be a non-degenerate triangle. Find the least upper bound of $\left(\frac{a+b}{c}-1\right)\left(\frac{b+c}{a}-1\right)\left(\frac{a+c}{b}-1\right)$.

Solution by Melissa Masek, student, Northwest Missouri State University, Maryville, Missouri.

We shall show that $\left(\frac{a+b}{c}-1\right)\left(\frac{b+c}{a}-1\right)\left(\frac{a+c}{b}-1\right) \leq 1$. The least upper bound of the given expression is 1. First, if $a=b=c$, then

$$\left(\frac{a+b}{c}-1\right)\left(\frac{b+c}{a}-1\right)\left(\frac{a+c}{b}-1\right) \leq 1$$

is equivalent to

$$(a+b-c)(b+c-a)(c+a-b) \leq abc$$

To show this inequality, notice that

$$\sqrt{(a+c-b)(a-c+b)} = \sqrt{a^2-(b-c)^2} \leq \sqrt{a^2} = a$$

Then

$$\sqrt{(b+a-c)(b-a+c)} = \sqrt{b^2-(c-a)^2} \leq \sqrt{b^2} = b$$

and

$$\sqrt{(c+b-a)(c-b+a)} = \sqrt{c^2-(a-b)^2} \le \sqrt{c^2} = c$$

where the fact that a, b and c are positive used in conjunction with the triangle inequalities, guarantee that $a+b-c$, $b+c-a$ and $c+a-b$ are positive numbers. It follows by multiplication that

$$\sqrt{(a+b-c)^2(b+c-a)^2(c+a-b)^2} \le abc$$

Thus since $a+b-c$, $b+c-a$ and $c+a-b$ are all positive numbers, we get

$$(a+b-c)(b+c-a)(c+a-b) \le abc$$

and the desired result follows.

Also solved by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan and the proposer.

Problem 566. Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.

Prove that $\left(\sum\limits_{k=1}^{n} \cosh x_k\right)^2 + \left(\sum\limits_{k=1}^{n} \sinh x_k\right)^2 \ge n^2$ where $k \in \mathbb{R}$.

Solution by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.

We shall prove the result by mathematical induction. For $n=1$ $\cosh^2 x_1 + \sinh^2 x_1 = \left(\frac{e_1^x+e_1^{-x}}{2}\right)^2 + \left(\frac{e_1^x-e_1^{-x}}{2}\right)^2 = \frac{e_1^{2x}+e_1^{-2x}}{2} \ge 1$ since $e^t + e^{-t} \ge 2$ for all real numbers t. Now assume that the desired result holds for all integers n. Then for $n+1$ we have

$$\left(\sum_{k=1}^{n+1} \cosh x_k\right)^2 + \left(\sum_{k\ge 1}^{n+1} \sinh x_k\right)^2$$

$$= \left(\sum_{k=1}^{n} \cosh x_k\right)^2 + (2\cosh x_{n+1})\left(\sum_{k=1}^{n} \cosh x_1\right) + \cosh^2 x_{n+1}$$

$$+ \left(\sum_{k=1}^{n} \sinh x_k\right)^2 + (2\sinh x_{n+1})\left(\sum_{k=1}^{n} \sinh x_1\right) + \sinh^2 x_{n+1}$$

$$= \left(\sum_{k=1}^{n} \cosh x_k\right)^2 + \left(\sum_{k=1}^{n} \sinh x_k\right)^2 + (\cosh^2 x_{n+1} + \sinh^2 x_{n+1}) +$$

$$2 * \sum_{k=1}^{n} (\cosh x_{n+1} \cosh x_n + \sinh x_{n+1} \sinh x_n)$$

$$\ge n^2 + 1 + 2\sum_{k=1}^{n} \cosh(x_{n+1} + x_k) \ge n^2 + 1 + 2n = (n+1)^2$$

since $\cosh u = \frac{e^u + e^{-u}}{2} \geq 1$ for all real numbers u, $\cosh^2 u + \sinh^2 u \geq 1$ and $\cosh a \cosh b + \sinh a \sinh b = \cosh(a+b)$.

Also solved by Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri; Carl Libis, University of Rhode Island, Kingston, Rhode Island; Stefan Sillau, Colorado School of Mines, Golden, Colorado and the proposer.

Editor's Comment. Euler and Sadek point out that this problem is a special case of the more general result

$$\left(\sum_{k=1}^{n} \cosh x_k\right)^{2m} + \left(\sum_{k=1}^{n} \sinh x_k\right)^{2m} \geq \left(\sum_{k=1}^{n} \cosh x_k\right)^{2m}$$
$$\geq \left(\sum_{k=1}^{n} 1\right)^{2m} \geq n^{2m} \text{ for any positive integers } m.$$

Problem 567. Proposed by Thomas Chu, Austin, Texas.

In triangle ABC let h_a, h_b, and h_c denote the altitudes from A, B, and C respectively. Let r denote the radius of the inscribed circle of triangle ABC. Prove that $\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r}$.

Solution by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.

Let $[ABC]$ denote the area of triangle ABC and I denote its incenter. Since $[ABC] = [AIB] + [BIC] + [CIA] = \frac{ar}{2} + \frac{br}{2} + \frac{cr}{2} = sr$ where $s = \frac{a+b+c}{2}$, then $\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{a}{2}[ABC] + \frac{b}{2}[ABC] + \frac{c}{2}[ABC] = \frac{a+b+c}{2}[ABC] = \frac{s}{rs} = \frac{1}{r}$.

Also solved by Scott H. Brown, Auburn University, Montgomery, Alabama;Ru ssell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri; Messiah College Problem Solving Group, Messiah College, Grantham, Pennsylvania; Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan and the proposer.

Problem 568. Proposed by Dan Buchnick, Ceasarea, Israel (Restated by the Editor).

Given any triangle ABC, let D, E, and F be points on BC, CA, and AB respectively, which do not coincide with any vertex of triangle ABC. Prove that the area of triangle DEF is greater than the area of at least one of the other triangles AFE, DBF, DEC except when the areas of all four triangles are equal.

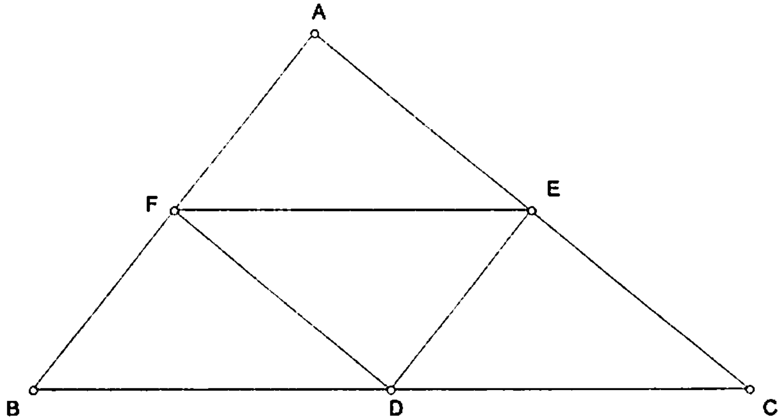

Figure for Problem 568

Solution by Clayton W. Dodge, University of Maine, Orono, Maine.

Let A', B', and C' be the midpoints of sides BC, CA, and AB respectively. Let $[ABC]$ denote the area of triangle ABC.

Case 1. If both E and F are midpoints, then $[DEF] = \frac{[ABC]}{4}$, no matter where D lies on side BC, since one can look at triangle DEF as having base EF, which is parallel to BC, so the altitude from D is constant. Without loss of generality, suppose D is closer to B than to C. Using BD as base of triangle BDF, then $BD < BA'$, so $[BDF] < [DEF] = \frac{[ABC]}{4}$.

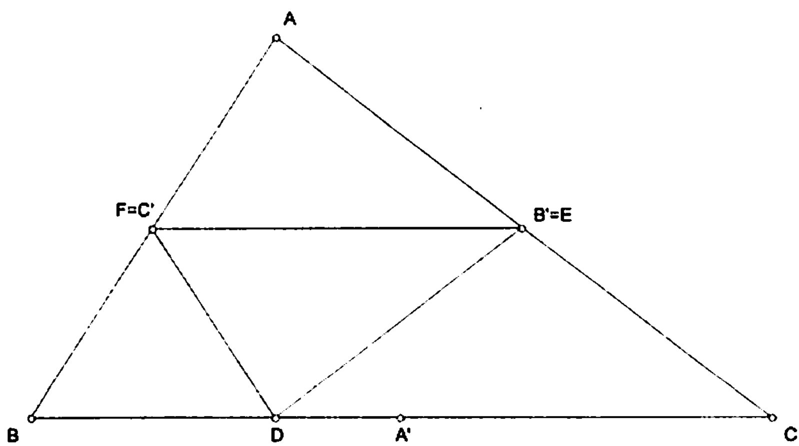

Figure for Case 1

Case 2. If at least two of D, E, and F are not midpoints, and some two of them are halfway or less from one vertex. Without loss of generality, say E lies between A and B' and F lies at C' or between A and C'. [The case where both E and F are at B' and C' has already been handled.] If $F = C'$, then the altitude from A to side EF in triangle AEF is equal to the distance from B to line EF and hence is less than the altitude from D to EF. If F is not equal to C', then the distance from B to EF is less than the distance from A to EF and the inequality is sharpened. In either case, by considering EF as the base for both triangles AEF and DEF, we have that $[AEF] < [DEF]$.

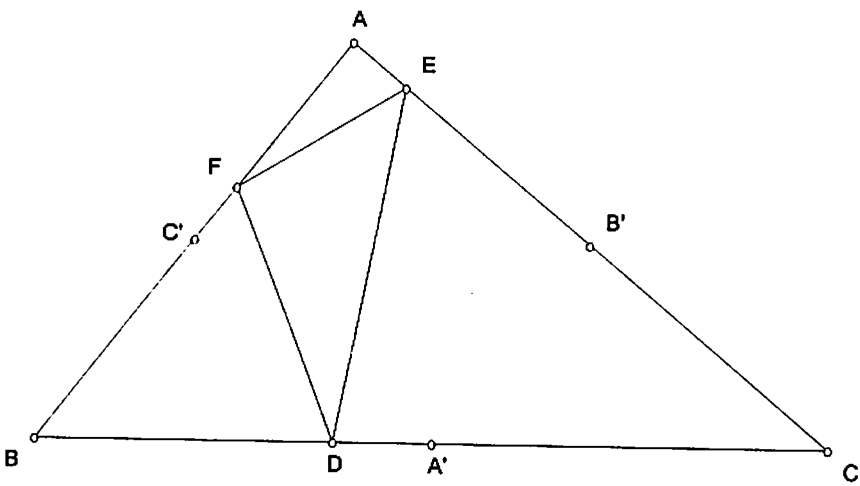

Figure for Case 2

Case 3. Suppose now that D lies between B and A', E lies between C and B', and F lies between A and C'. In the following string of equality and inequalities, we leave the base of each triangle fixed and move the third vertex to alter the altitude to that base. We have $[A'B'C'] = [A'EC'] < [DEC'] < [DEF]$. The string of inequalities involves changing one vertex at a time. So think of the fixed side in each change as being the base of the triangle and that its third vertex, altering the height to that base. In the first equality the base is parallel to the side of triangle ABC on which the small triangle's third vertex lies, so the altitude to that base is unchanged. In the other cases, the third vertex moves to increase the altitude. In the first equality, the base $A'C'$ is parallel to the side of triangle ABC on which the small triangle lies, so the altitude to that base is unchanged. In the other cases, the third vertex moves to increase the altitude.

Thus $[DEF] > \frac{[ABC]}{4}$ and hence at least one of the other three triangles must be smaller than $\frac{[ABC]}{4}$. The theorem follows.

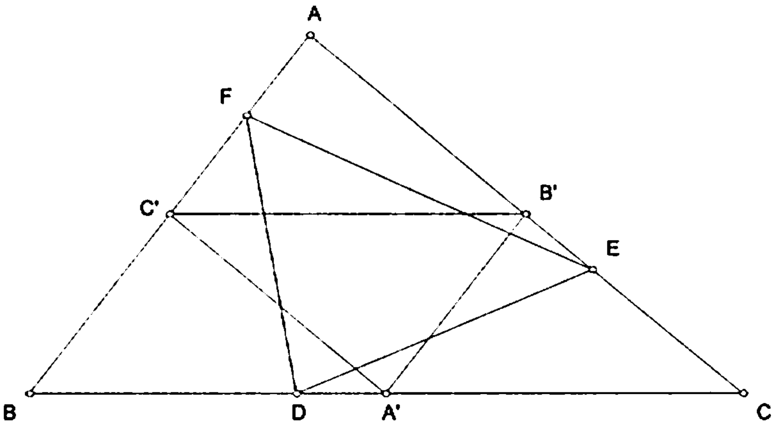

Figure for Case 3

Also solved by Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri, and the proposer.

Editor's Comment. Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan, notes that this problem appeared in the Pi Mu Epsilon Journal in 1989. The solution given there is couched in terms of lengthy analysis using inequalities. There the proposer attributes this problem to a result published by Paul Erdos and Hans Debrunner in 1958. Ironically, the editor of the Pi Mu Epsilon Journal at the time was none other than our featured solver whose solution is quite pretty and succinct! Euler and Sadek's solution derives the desired result as a special case of another geometric inequality.

Problem 569. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Imagine a square surrounded by four semicircles. Assume that the length of each side of the square is a. Construct a square that is tangent to each of the semicircles. Now construct semicircles on the side of each of the newly constructed square. Repeat these two steps ad infinitum. Let S denote the sum of the reciprocals of the areas of the squares and let C denote the sum of the reciprocals of the areas of the semicircles. Prove that $\frac{S}{C} = \frac{\pi}{2}$.

Solution by Peter Schallot, student, Slippery Rock University, Slippery Rock, Pennsylvania.

If a square with side a is surrounded with four semicircles, which in turn are surrounded by another square constructed tangent to each of the semicircles, and this process is repeated indefinitely, the result will look as follows:

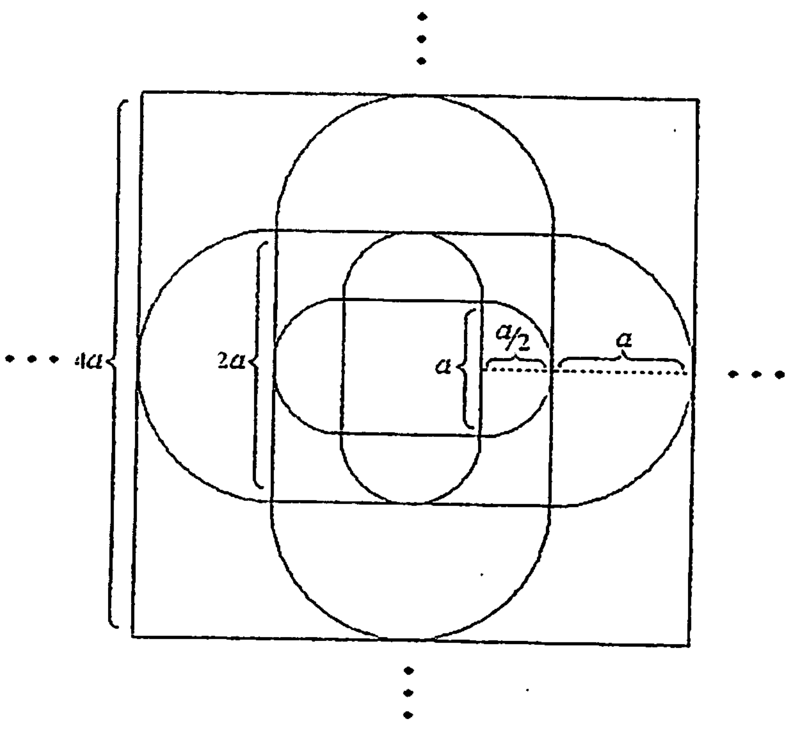

Figure for Problem 569

The sum of the areas of the squares and circles are, respectively, $\sum^{\infty}(2^{n-1}a)^2$ and $\sum^{\infty} 2\pi(2^{n-2}a)^2$. Thus if S and C are the sums of the reciprocals of the areas of the squares and circles respectively, we have $S = \sum^{\infty} \frac{1}{(2^{n-1}a)^2} = \frac{4}{3a^2}$ and $C = \sum^{\infty} \frac{1}{(2\pi)(2^{n-2}a)^2} = \frac{8}{3a^2\pi}$. It follows that $\frac{S}{C} = \frac{\pi}{2}$.

Also solved by Charles Ashbacher, Hiawatha, Iowa; Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri; Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan; Melissa Masek, student, Northwest Missouri State University, Maryville, Missouri; Elizabeth Sickler, student, Messiah College, Grantham, Pennsylvania; Kathleen Trimmer, student, Waynesburg College, Waynesburg, Pennsylvania, and the proposer.

Starting a KME Chapter

For complete information on starting a KME chapter, contact the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of the $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offering and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

Thank You Referees!

The editor wishes to thank the following individuals who refereed papers submitted to *The Pentagon* during the last two years.

Kevin Anderson
Missouri Western State College
St. Joseph, Missouri

Y. Balas
University of Southern Mississippi
Hattiesburg, Mississippi

John Behle
Harris Stone State College
St. Louis, Missouri

James R. Bush
Waynesburg College
Waynesburg Pennsylvania

Bryan Dawson
Union Universit
Jackson, Tennessee

Vicent J. Ferlini
Keene State College
Keene, New Hampshire

Marc Goulet
University of Wisconsin - Eau Claire
Eau Claire, Wisconsin

Yaping Liu
Pittsburg State University
Pittsburg, Kansas

William R. Livingston
Missouri Southern State College
Joplin, Missouri

Phoebe McLaughlin
Central Missouri State University
Warrensburg, Missouri

Joshua Moon
Liberty University
Lynchburg, Virginia

Jeff Poet
Missouri Western State College
St. Joseph, Missouri

Evangelos Skoumbourdis
Liberty University
Lynchburg, Virginia

Ron Taylor
Berry College
Mount Berry, Georgia

Glyn K. Wooldridge
Liberty University
Lynchburg, Virginia

Also thanks to the many other individuals who volunteered to serve as referees but were not used during the past two years. Referee interest forms will again be sent by mail in the near future so that interested faculty may volunteer. If you wish to volunteer as a referee, feel free to contact the editor (see page 2) or the editor-elect (see page 72) to receive a referee interest form.

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Kappa Mu Epsilon News

Edited by Connie Schrock, Historian
Updated Information as of January 2004

News of chapter activities and other noteworthy KME events should be sent to schrockc@emporia.edu or to

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Mathematics Department
Emporia State University
1200 Commercial Street
Campus Box 4027
Emporia, KS 66801

Chapter News

AL Alpha Chapter President – Mark Christopher
Athens State University 35 Actives, 11 New Members

Other Fall 2003 officers: Hubert Gonder, Vice President; Sabrina Balch, Secretary; Dottie Fuller, Treasurer/Corresponding Secretary.

Alabama Alpha Chapter of KME at Athens State has several activities planned for the fall semester. All activities are joint efforts with our Math and Computer Science Club (MACS).

A "welcome back to school" cook-out/meeting was held on September 14, 2003. Twelve members attended. We had a great picnic lunch under shade trees outside Waters Hall. MACS officers were elected and KME and MACS members made plans for our role in the 35th annual Fiddler's Convention for the weekend on Oct. 3-4, 2003. A nationally recognized convention, Fiddler's is a huge money-raising event for Athens State annually. KME members will sell Bar-B-Q sandwiches, chips, and drink for the two-day event and our goal is to raise $1000 from the effort. The money will be used to sponsor the math modeling and simulation competition again this year, to send two students to the ACTM Fall Forum in November and to send several students to the MAA regional conference this year. We look forward to a weekend of fun, fellowship, and productivity!

A lunch meeting was planned for the 2nd Tuesday of November. Holiday projects will be planned at this meeting.

AL Zeta
Birmingham Southern 14 Actives

Fall 2003 officers: Isaac Dooley, Vice President;Ell en Segrest, Secretary/Treasurer; Mary Jane Turner, Corresponding Secretary.

CA Epsilon Chapter President – Jeffrey R. Mulari

California Baptist University 13 Actives

Other Fall 2003 officers: Derek Imai, Vice President; Tawni L Covington, Secretary; Holly Curran, Treasurer; Catherine Kong, Corresponding Secretary.

KME and the Math Club at CBU sponsored Math Fun Day at Jefferson Elementary School, Riverside, California on October 21,2003. There were 20 CBU students who participated. They created their own math activities for more than 120 K-6 children according to the themes provided by Brandon Davis, HEARTS (after school program) director.

GA Alpha Chapter President – Jessica Pritchett

State University of West Georgia 25 Actives

Other Fall 2003 officers: Chad Matthews, Vice President; Jessica Caldwell, Secretary; J.J. Wahl, Treasurer; Dr. Joe Sharp, Corresponding Secretary.

The Georgia Alpha conducted its annual Food and Clothing Drive for the needy from November 15 – December 15 with the proceeds being given to The Salvation Army. We had our Fall Social at a local Mexican restaurant on November 22 with several students, guests, and faculty in attendance. A fine time was had by all!

CO Beta

Colorado School of Mines

New Initiates: Dylan Jones, Xiao Liang, Steve Politis, Luis Tenorio

CT Beta

Eastern Connecticut State University

New Initiates –Kathlee n Burdelski, Richard Freeman, Gregory Hebb, Suzanne Lester, Stephen Maslanka, Ryan Ouillette, Kerri Pion, Danielle Polchinski

IA Alpha Chapter President – Cindee Calton

University of Northern Iowa New Members: 8 students, 22 Faculty Members

Other Fall 2003 officers: Ben Wadsley, Vice President; Melissa Potter, Secretary; Michelle Boelman, Treasurer; Mark Ecker, Corresponding Secretary

Student member Melissa Potter presented her paper "Hyperbolic Tessellations"at our first fall KME meeting on Sept. 17, 2003 at Professor Mark Ecker's residence. The University of Northern Iowa Homecoming Coffee was held at Professor (emeritus) Carl Wehner's residence on October 11, 2003. Our second meeting was held on October 15, 2003 at Professor Jerry Ridenhour's and our third meeting was held on November 12, 2003 at Professor Min Lee's residence. Student member Matthew Wood addressed the fall initiation banquet with "Cubo – A Dice Game with a Difference." Our banquet was held at Greestreets restaurant in the Holiday Inn on December 10, 2003 where eight new members were initiated.

New Initiates: Martha Aragon, Donald Daws, Nicki Gannon, Paul Grammens, Kyle Lockie, Lindsey Lullman, Nicholas Sly, Michael Tetzloff

IA Delta Chapter President – Wei (Peter) Yang

Wartburg College 41 Actives

Other Fall 2003 officers: Nicholas Wuertz, Vice President; Kristin Granchalek, Secretary; Mark Giesmann, Treasurer; Dr. Brian Birgen, Corresponding Secretary.

This year we have established a biweekly card night, when members and non-members can gather to play card games. Mathematical games like "Set" are played as well as more standard games like "500" and "Euchre." This has helped to increase interest in the club and increase involvement. The club will be designing and selling Wartburg KME t-shirts this year. At the Wartburg Homecoming Renaissance Fair, our club successfully ran our traditional fund-raiser by selling egg-cheeses. We also helped sponsor a number of students who attended the Annual Regional Workshop in the Mathematical Sciences in Lincoln, Nebraska. We created a web site for the local club with links from the Wartburg and Math Department homepages.

IN Beta Chapter President – Christine Berkesch

Butler University 16 Actives

Other Fall 2003 officers: Jennifer Thompson, Vice President; Jennifer Legge, Secretary; Clint Garrett, Treasurer; Amos Carpenter, Corresponding Secretary.

The Beta chapter of KME had two speakers for the Fall 2003 semester. The first talk was "Graphs of Essentially Equivalent Lattice Paths" by Dr. Rick Gillman, Valparaiso University, on October 21, 2003. The second speaker was Dr. Richard Varga, Kent State University. Dr.Varga gave two talks – the first "Gerschgorin and His Circles I" on October 28, 2003 and the second "Gerschgorin and His Circles II" on October 30, 2003.

IN Gamma

Anderson University

New Initiate: Aaron Alexander

IL Zeta Chapter President – Jen Soldat

Dominican University 30 Actives

Other Fall 2003 officers: Merrit DeBartolo, Vice President; Kathrina Parkhill, Secretary; Maria Guzman, Treasurer, Marion Weedermann, Corresponding Secretary.

The Illinois Zeta Chapter of KME was involved in many activities this past spring including:

- – This semester, the Zeta Chapter hosted a table at Dominican University's Activities Fair to bring about awareness of mathematics on campus, to advertise tutoring, and to recruit potential KME members.
  - We sponsored our yearly visit to various math classes to inform students of our presence here on campus.
  - The members of KME volunteered for a total of eight hours per week of math tutoring in our Academic Resource Center.
  - We hosted monthly KME problem contests, open to all students. The winners received a small prize.
  - The biggest event we hosted was an evening event with Arthur Benjamin, Mathemagician, from Harvey Mudd College, CA. We invited Dr. Benjamin to come to campus so he could combine mathematics and magic to mystify and amaze an audience of about 60 people from the Dominican University and surrounding communities.
  - We also had two "picnics" in our Social Hall to gather all members of KME, where we had lunch and played math-related games.
  - At the end of the semester, we started our clothing sale, which is to be completed early in the spring 2004 semester.

KS Alpha Chapter President – Keith Smeltz
Pittsburg State University 15 Actives, 12 New Members

Other Fall 2003 officers: Libby Wonderly, Vice President;Ja mie Fairbanks, Secretary; Meltem Tugut, Treasurer; Dr. Tim Flood, Corresponding Secretary.

This fall, we began having noon meetings one Tuesday a month. The chapter provided pizza and soft drinks for the meetings. This format has seemed to work well. We had two meetings this fall as follows:

- – September 9, KME President Keith Smeltz presented "A Guide to Finding Rules for Testing Divisibility by Any Number."
  - October 14, KME member Gates Brown gave a presentation on Bernoulli.

The fall initiation was held November 11. A recent graduate who is an Operations Research Systems Analyst at the TRADOC Analysis Center (a division of the US Army) presented "Let's Play War." Dr. Flood hosted the annual KME Christmas party on December 8.New Initiates – Amy Baldwin, Ashley Bates, Brent Cameron, Joshua Gier, Pei-Ting Hsu, Brett Palmer, Noah Smith, Chandra Spence, Matt Weber, Justin Weimer, Melissa Beyer, Kristen Bell, Tony Brown, Brent Riedy, Amber Schmidt

KS Beta Chapter President – Melinda Born
Emporia State University 24 Actives, 5 New Members

Other Fall 2003 officers: Leah Childers, Vice President; Chris Dobbs, Secretary; Raegen Dyro, Treasurer; Connie S. Schrock, Corresponding Secretary.

KS Beta chapter of KME sponsored a calculator help session designed to help College Algebra students to become familiar with his or her calculator. We also hosted the North Central Regional Convention last spring.

KS Delta Chapter President – Zeb Kramer
Washburn University 30 Actives, 8 New Members

Other Fall 2003 officers: Jan Misak, Vice President; Jeff Kingman, Secretary/Treasurer; Allan Riveland, Corresponding Secretary.

The Kansas Delta chapter of KME met for three luncheon meetings with the Washburn Math Club during the semester. Speakers and/or mathematics presentations were part of the meetings.

KS Gamma Chapter President – Erin Stretton
Benedictine College 10 Actives, 1 New Member

Other Fall 2003 officers: Massimo Botta, Vice President; Christina Hoverson, Secretary; Andrea Archer, Treasurer; Linda Herndon, OSB, Corresponding Secretary.

Kansas Gamma started the Fall 2003 semester with a cookout September 25 at the home of Richard Farrell, emeritus professor. Initiation of one transfer student, Rosemary Kasten, was held on October 29. We concluded the activities of the fall semester with the traditional wassail party of December 7 hosted by faculty member Sister Jo Ann Fellin, OSB.

KY Beta Chapter President – Matthew Rasure
Cumberland College 25 Actives

Other Fall 2003 officers: Stephanie Isaacs, Vice President; Justin Williams, Secretary; Vito Wagner, Treasurer, Jonathan Ramey, Corresponding Secretary.

On September 18, the Kentucky Beta chapter officers helped to host an ice cream party for the freshmen math and physics majors. Along with the Mathematics and Physics Club and Sigma Pi Sigma, the chapter had a picnic at Briar Creek Park on October 2. On December 11, the entire department, including the Math and Physics Club, the Kentucky Beta chapter, and Sigma Pi Sigma had a Christmas party with about 30 people in attendance.

LA Delta Chapter President – Aaptha Murthy

University of Louisiana at Monroe 26 Actives

Other Fall 2003 officers: Katie Roussy, Vice President;Ap ril Jeffcoat, Secretary; Sharee Davis, Treasurer; Serpil Saydam, Corresponding Secretary.

The Lousiana Delta Chapter of KME met four times during the fall semester. We started the semester with a pizza social and later we held elections for officers. In October, we had a picnic. The third meeting included a panel discussion about graduate school. Our panel members were Dr. Andrew Hetzel and Dr. Mario Christou. We also had an end of semester party with ACM (the computer science student organization).Some of our members represented KME at several different events. One event was the "University Mile" race during ULM's homecoming week.

LA Gamma Chapter President – Anne Vakarietis

Northwestern State University of Louisiana 22 Actives

Other Fall 2003 officers: Jackie Adair, Vice President; Karen Yurk, Secretary; Shelley Koone, Treasurer; Leigh Ann Myers, Corresponding Secretary.

Student members of Kappa Mu Epsilon volunteered to tutor freshman mathematics students at NSU during finals exam week.

MD Beta Chapter President – Chris Drupieski

McDaniel College 24 Actives, 0 New Members

Other Fall 2003 officers: Laura Albaugh, Vice President; Sarah Vannoy, Secretary; Matt Demos, Treasurer; Linda Eshleman, Corresponding Secretary.

Guest Speaker, Fred Butler, an alumnus of the Chapter who is finishing up his PH.D. at the University of Pennsylvania, gave a talk on his thesis topic Rook Theory during one of our meetings. At another meeting, we had a guest speaker Dr. Charles Toll from the National Security Agency who gave a talk title "Polynomials over Finite Fields."

Our group also sponsored the Senior Honors presentation of our treasurer Matt Demos whose topic was "The Development of Solutions to Quadratic Equations." Besides programs and meetings the Maryland Beta chapter found themselves busy with "Pizza, Problems, and Ping-Pong" nights –a time for math majors to solve problems, eat pizza, and then play ping-pong. The chapter had two fund-raisers: a ping-pong tournament and cookie sales at Homecoming.

MD Delta Chapter President –Matthew Miller
Frostburg State University 22 Actives

Other Fall 2003 officers: Chris Smoot, Vice President; Sherry Hartman, Secretary; Dustin Robinson, Treasurer; Mark Hughes, Corresponding Secretary.

The Maryland Delta Chapter held its Fall 2003 organizational meeting in September. The meeting was followed by a book giveaway for KME members. Students were able to choose from the hundreds of volumes left to the Mathematics Department by long time faculty member Dr. Richard Weimer upon his retirement. Our meeting in October was devoted to planning a fund-raising activity and it was decided that coupons for a local deli would be sold. It turned out to be a very successful operation and $200 was raised by semester's end. Our final meeting for the semester was held in November, during which a lecture entitled, "Gauss, the Lemniscate and the Arithmetic-Geometric Mean" was presented by Dr. Mark Hughes.

MI Delta Chapter President – Coral Shaw
Hillsdale College 17 Actives, 3 Faculty

Other Fall 2003 officers: Dan Englert, Vice President; Craig Fraser, Secretary; Dr. John H. Reinoehl, Corresponding Secretary.

The Delta Chapter of Michigan offers a free tutoring program for mathematics students.

MI Epsilon Chapter President –Lynette Fulk (A) / Rebecca Barthlow (B)
Kettering University 189 Actives, 32 Faculty (2 Sections)

Other **Section A** (Fall 03/Summer 04) officers: Gayle Ridenour, Vice President; Kathleen Monfore, Secretary; George Hamilton, Treasurer.

Other **Section B** (Fall 03/Summer 04) officers: Julie Xiong, Vice President; Jamie Taylor, Treasurer; Justin Via, Secretary;

Corresponding Secretary: Boyan Dimitrov, Corresponding Secretary.

Summer 2003 (A Section):

There was a pizza party/movie was held on the 3rd Tuesday (July 29) at 12:20 in one of the largest classrooms of Kettering University. The movie was "Giants of Applied Mathematics" (Catheleen Morawetz). It was curiosity, fun, enjoyment, and lots of news from the frontier of mathematics for our students, which vast majority is in engineering field. Pins and membership cards were distributed. Another pizza party was held on Thursday September 11 at 12:20 and the presentation of the movie "Carl Frederich Gauss, Titan of Science" also took place.

Fall 2003 (B Section):

Rather than the traditional pizza party/movie over the lunch hour, we tried something new this term. There was a pizza party/movie on Thursday (Nov. 6) from 6-8 p.m. and the movie the KME students and many others enjoyed was "The Beautiful Mind."

For more information, please see our website at:

http://www.kettering.edu/acad/scimath/appmath/

MI Epsilon Chapter can also be found at: http://www.kettering.edu/~kme/

MO Alpha Chapter President – Jennifer Pope

Southwest Missouri State University 37 Active Members, 7 New Members

Other Fall 2003 officers: Shawn Poindexter, Vice President; April Williams, Secretary; Michael Sallee, Treasurer; John Kubicek, Corresponding Secretary.

For the Fall 2003 semester the MO Alpha of KME hosted the fall departmental picnic and held three monthly meetings. The meeting speakers were two faculty members, Dr. Kishor Shah and Dr. Cameron Wickham, and two student speakers, Shawn Poindexter and Robert Sorey.

MO Epsilon Chapter President – Temitop Ogunmola

Central Methodist College 8 Actives

Other Fall 2003 officers: Jennifer Kirchner, Vice President; Laura-Ann Linvall, Secretary; Linda O. Lembke, Corresponding Secretary.

We initiated three new members last May and elected new officers. Long-time corresponding secretary Dr. William McIntosh retired in May 2002 and Dr. Lembke took over his job. Dr. Jerry Priddy joined the math faculty in August 2002 and become one of the faculty sponsors of the Missouri Epsilon chapter of KME at that time.

New Initiates: Heidi Dailey, Nicole Roberts, Andi Skinner

MO Gamma Chapter President – Josh Bebout
William Jewell College 12 Active Members

Other Fall 2003 officers: Christine Deatherage, Vice President; Stephanie Murdock, Secretary; Joseph T. Mathis, Treasurer & Corresponding Secretary.

Two chapter meetings were held in the Fall 2003. The first was a talk by Prof. Mathis on Fibonacci numbers and Symmetric Groups, S_n. The second was in conjunction with the Physics Department student sections of SPS and Sigma Pi Sigma to hear a speaker talk about and display his collection of old cameras and the history of photography.

MO Kappa Chapter President – Stephen Dickey
Drury University 22 Actives, 12 New Members

Other Fall 2003 officers: Tracy Goering, Vice President; Erin Sly, Secretary; Amy Van Fossen, Treasurer;Cha rles Allen, Corresponding Secretary.

The first activity of the semester was a pizza party held at Dr. Allen's house. The winner of the annual Math Contest was Cody Pace for the Calculus II and above division and Adam Scott for the Calculus I and below division. Prize money was awarded to the winners at a pizza party held for all contestants. A sub-sandwich luncheon was held for the reports of undergraduate research projects (potential KME papers) by Jeff Clark and Heidi Hulsizer. The Math Club has also been running a tutoring service for both the day school and the continuing education division as a money-making project. The also made some "Math Helpers" and sold them to the lower division math classes.

MO Lambda Chapter President – Amy Lynn Kerling
Missouri Western State College 25 Actives

Other Fall 2003 officers: Gabe Wishnie, Vice President; Nicholas Limle, Secretary; James Blevins, Treasurer; Don Vestal, Corresponding Secretary.

MO Theta Chapter President – Kevin Reed
Evangel University 9 Actives, 2 New Members

Other Fall 2003 officers: Rijo Alex, Vice President; Don Tosh, Corresponding Secretary.

Meetings were held monthly. We changed to an evening format, which increased the student turnout. The free pizza also helped. Although initiations are usually held in the spring, we did initiate two new students in the November meeting.

MO Zeta

University of Missouri-Rolla

New initiates: Stephen Bridert, Greg Cartee, Mike Cress, Parul Hogrebe, Tim Ivancic, Nicole McBride, Sandeep Pedam, Chris Potter, Geoffrey Reedy, Karen Schindler, Alexis Sientins, Ryan Thorton

MS Alpha Chapter President – Shannon McVay

Mississippi University for Women 13 Actives, 4 New Members

Other Fall 2003 officers: Henry Boateng, Vice President; Amy Ladner, Secretary; Sarah Sheffield, Treasurer; Dr. Shaochen Yang, Corresponding Secretary.

The Mississippi Alpha chapter of KME had a busy fall semester including the first general meeting on September 11, Initiation on September 23, and the next monthly meeting on October 21.

MS Epsilon Chapter President – Laura Wallace

Delta State University 12 Actives

Other Fall 2003 officers: Amy Rowe, Vice President; Frank Rice, Secretary/Treasurer; Paula A. Norris, Corresponding Secretary.

NE Beta Chapter President – Stephanie Becker

University of Nebraska at Kearney 13 Actives, 2 New Members

Other Fall 2003 officers: Brandon Hauff, Vice President; Sarah Wall, Secretary; Jay Powell, Treasurer; Dr. Katherine Kime, Corresponding Secretary.

This semester, our chapter hosted Math Fun Night at the student union, open to all students on campus. We played "Set," "Chicken Foot" (dominoes), and other games. We had information on math classes and a display of many manipulatives and geometric objects. We also showed a short movie on the teaching of R.L. Moore. The chapter received $185 in funding from University Program and Facility Fees (UPFF), which was used for food. We also received two cases of Pepsi (Pepsi Sponsoring Support).

New Initiates: Eric Watson, Heather Schroeder

NE Delta Chapter President – Diana Faesser

Nebraska Wesleyan 19 Actives

Other Fall 2003 officers: Angela Miller, Vice President; Val Stehlik, Secretary/Treasurer; Melissa Erdmann, Corresponding Secretary.

NH Alpha

Keene State College

New Initiates: Lynn French, Lisa Hultgren, Allysha Lane, Robert Luz, Luke Mitchell, Dennis Muhonen, Erik Parkkonen, Abbey Pelkey, Joshua Pelton, David Rossall, Leah Ryan, Kathy Souza, Robert Tiebout, Matthew Tirrell, Jon Winn, Hugh Moore, Joseph Brady, Andrea Drake, Gregory DuBois, Megan Humphreys, Michael Melillo, Chelsea Wilcox, Josephine Witkowski, Jacqueline Boldman, Karen Stanish, Danny Franklin, Christopher Jackson

NJ Gamma Chapter President – Stephanie Beatty and Melissa McCormick

Monmouth University 36 Actives

Other Fall 2003 officers: Amanda Glynn, Vice President; Christina Colanero, Secretary; Paul Zoccali, Treasurer; Melissa Berfield, Historian; Jessica Gregory and Kristin Romans, Junior Liaisons; Judy Toubin, Corresponding Secretary.

The chapter began its annual lecture series with a presentation by Prof. Louis Penge. We would like to thank Prof. Penge for his effort and the faculty for their support. The soda tab collection for the Ronald McDonald House Charities continued in the fall with great success. To raise money for spring induction, the officers participated in a fund-raising activity.

NY Eta Chapter President – Michelle Searles

Niagara University 30 Actives, 22 New Members

Other Fall 2003 officers: Matthew Nethercott, Vice President; Megan Zdrojewski, Secretary; Michael Bidzerkowny, Treasurer; Robert Bailey, Corresponding Secretary.

We had an October induction this year. In November, a career evening was presented by Christopher LaGrow, a career counselor at Niagara University. This provided an opportunity for students in mathematics or mathematics education to explore possibilities for future jobs.

NY Kappa Chapter President –

Pace University 20 Actives

Other Fall 2003 officers: Geraldine Taiani, Corresponding Secretary.

No students applied for membership last year. We will have an induction in the spring.

OK Alpha Chapter President – Miri Whisnant
Northeastern State University 42 Actives, 11 New Members

Other Fall 2003 officers: Carrie Hoffman, Vice President; Nick Jones, Secretary/ Treasurer; Dr. Joan E. Bell, Corresponding Secretary.

Our fall initiation brought 11 new students into our chapter. Six NSU faculty members came to support the new members. We participated in several NSU events this semester. We sponsored the KME Pumpkin Patch at NSU's annual Halloween Carnival. The children fished for pumpkins with meter stick fishing poles. OK Alpha donated two hams for the needy at Thanksgiving. We continue to have joint activities with NSU's student chapter of the MAA and participate in "The Problem Solving Competition" promoted by the MAA. Our annual fall book sale brought in $116. Our fall speaker was from the National Imagery Mapping Agency. The theme for our 5th KME T-shirt this year was Top 5 Math Jokes. Almost all 50 shirts have been sold! At one of our meetings, Dr. Bell surprised everyone with her collection of Tavern Puzzles. The Physics Club joined us and everyone had a great time trying to solve the puzzles. The Christmas party was pizza, Christmas treats and the girls again the guys in Taboo. We also made baskets of treats to send to the math students at our other campus in Broken Arrow.

PA Alpha Chapter President – Heather Klink
Westminster College 18 Actives

Other Fall 2003 officers: Danielle Zielinski, Vice President, Bradley Patton, Secretary; Nicole Potocnak, Treasurer; Carolyn Cuff, Corresponding Secretary.

PA Alpha had an ice cream social for new students in October and served pizza and donuts during finals week.

PA Nu
Ursinus College

New Initiates: Daniel Augelli, Chad Hogg, Katie McLaughlin, Kacie Meyer, John Quinn, Seth Ratjski, Jeff Struble, Catherine Welsh, Dharmesh Sheth, Kristin Galie, Michael Hughes

PA Iota Chapter President – Kristina Wile
Shippensburg University 27 Actives, 3 New Members

Other Fall 2003 officers: Christa Friewald, Vice President; Lauren Frazier, Secretary; Judith Canner, Treasurer; Kimberly Presser, Corresponding Secretary.

This fall the KME chapter again sponsored and directed a math tutoring center two nights a week in the math department. This service was invaluable to the department and will hopefully be continued in the spring semester. The group also sponsored student visits to the Gettysburg Career Conference in October and some social activities in the end of the semester (including a Hershey Bears hockey game). Two fund-raisers were held jointly with the Math Club: selling of Math Department T-shirts and mugs.

PA Pi Chapter President – Davlyn Nauman
Slippery Rock University 10 Actives

Other Fall 2003 officers: Megan McKinney, Vice President; Justin Mashuda, Secretary/Treasurer; Elise Grabner, Corresponding Secretary.

SD Alpha Chapter President – Amber Mehaffey
Winthrop University 14 Actives, 6 New Members

Other Fall 2003 officers: Gail Johnson, Vice President; Jennifer Dolejsi, Secretary; Jon Tieszen, Treasurer; Dr. Raj Markanda, Corresponding Secretary.

TN Epsilon
Bethel College 3 Actives

Fall 2003 officers: Russell Holder, Corresponding Secretary.

TN Gamma Chapter President –Nikki Vassar
Union University 16 Actives, 4 Associates

Other Fall 2003 officers: Allen Smith, Vice President; Willie George, Secretary;Br ian Taylor, Treasurer; Bryan Dawson, Corresponding Secretary.

The TN Gamma chapter began its fall activities with the traditional pizza party as an opportunity to recruit associate members. Member Brian Taylor spoke during the October meeting about his summer experience as an intern at St. Jude Children's Research Hospital and some of the mathematics involved in MRI imaging. The chapter's November activity was part of our university's "Day of Remembrance," where faculty, staff, and students volunteered in carious places around the community as a way of saying "Thanks!" for the community's help after last year's tornado damage on campus. Dr. Lunsford took a team of seven students to the Downtown Christian Academy, a school for disadvantaged children, where they led in several activities, some of which were mathematical.

The December meeting was held at the home of Dr. Lunsford and included a meal and a "Dirty Santa" gift exchange. The chapter also continued another tradition by co-sponsoring a needy child at the annual Carl Perkins Christmas dinner.

TX Kappa Chapter President – Jill Klentzman
University of Mary Hardin-Baylor 10 Actives, 0 New Members

Other Fall 2003 officers: Rona Greene, Vice President; Amanda Simmons, Secretary; Peter H. Chen, Corresponding Secretary.

TX Mu Chapter President – Kelly McCullough
Schreiner University 7 Active Members

Other Fall 2003 officers: Afton Sands, Vice President; Shelley Stark, Secretary/Treasurer; William Sliva, Corresponding Secretary.

Subscription Renewals and Change of Address

Your *Pentagon* subscription expires with the volume and number that appears in the upper right corner of your address label (see back cover). Since this issue is volume 63 Number 2, if the code 63-2 appears on your label then this is your last issue!

Early renewals save us the cost of mailing out renewal notices. To renew, please send your check - just $10 for four more issues (domestic individuals only; see page 2 for rates for libraries and foreign subscriptions) - together with your name and address and a copy of your old address label to:

Rich Laird, *The Pentagon* Business manager
Department of Mathematics
Missouri Southern State University
3950 E. Newman Road
Joplin, MO 64804 USA

Please renew promptly to avoid gaps in your journal collection.

If you have a change of address, please don't forget *The Pentagon*! Send your change of address to the business manager at the above address. Failure to do so may cost KME additional postage fees!

Announcement of the Thirty-Fifth Biennial Convention of Kappa Mu Epsilon

The Thirty-Fifth Biennial Convention of Kappa Mu Epsilon will be hosted by the Texas Mu chapter at Schreiner University in Kerrville, Texas. The convention will take place April 14-16, 2005. Each attending chapter will receive the usual travel expense ($.30/mile) reimbursement from the national office as described in Article VI, Section 2, of the Kappa Mu Epsilon Constitution.

A significant feature of our national convention will be the presentation of papers by student members of Kappa Mu Epsilon. The mathematical topic selected by each student should be of interest to the author and of such scope that it can be given adequate treatment in a timed oral presentation. Senior projects and seminar presentations have been a popular way for faculty to get students to investigate suitable topics. Student talks to be judged at the convention will be chosen prior to the convention by the Selection Committee on the basis of submitted written papers. At the convention, the Awards Committee will judge the selected talks on both content and presentation. The rankings of both the Selection and Awards Committees will determine the top four papers.

Who may submit a paper?

Any undergraduate or graduate student member of Kappa Mu Epsilon may submit a paper for consideration as a talk at the national convention. A paper may be co-authored. If a paper is selected for presentation at the convention, the paper must be presented by one or more of its authors.

Presentation topics:

Papers submitted for presentation at the convention should discuss material understandable by undergraduates who have completed only calculus courses. The Selection Committee will favor papers that satisfy this criterion and which can be presented with reasonable completeness within the time allotted. Papers may be original research by the student(s) or exposition of interesting but not widely known results.

Presentation time limits:

Papers presented at the convention should take between 15 minutes and 25 minutes. Papers should be designed with these limits in mind.

How to prepare a paper:

The paper should be written in the standard form of a term paper. It should be written much as it will be presented. A long paper (such as an honors thesis) must not be submitted with the idea that it will be shortened at presentation time. Appropriate references and a bibliography are expected. Any special visual aids that the host chapter will need to provide (such as a computer, overhead projection system, etc.) should be clearly indicated at the end of the paper.

The first page of the paper must be a "cover sheet" giving the following information:

- – Title
  - – Name of the author or authors
    - * these names should not appear elsewhere in the paper
  - – Student status
    - * undergraduate or graduate
  - – Author's permanent and school addresses and phone numbers
  - – Name of the local KME chapter and school
  - – Signed statement giving approval for consideration of the paper for publication in *The Pentagon*
    - * or a statement about submission for publication elsewhere
  - – Signed statement of the chapter's Corresponding Secretary attesting to the author's membership in Kappa Mu Epsilon

How to submit a paper:

Five copies of the paper, with a description of any charts, models, or other visual aids that will be used during the presentation, must be submitted. The cover sheet need only be attached to one of the five copies. The five copies of the paper are due by February 1, 2005. They should be sent to:

Dr. Don Tosh, KME President-Elect
Department of Science and Technology
Evangel University
1111 N. Glenstone
Springfield, MO 65802

Selection of papers for presentation:

A Selection Committee will review all papers submitted by undergraduate students and will choose approximately fifteen papers for presentation and judging at the convention. Graduate students and undergraduate students whose papers are not selected for judging may be offered the opportunity to present their papers at a parallel session of talks during the convention. The President-Elect will notify all authors of the status of their papers after the Selection Committee has completed its deliberations.

Criteria used by the Selection and Awards Committees:

Judging criteria used by both the Selection Committee and Awards Committee will include:

- – Choice and originality of topic
  – Literature sources and references
  – Depth, significance, and correctness of content
  – Clarity and organization of materials
  – Overall effect.

In addition to the above criteria, the Awards Committee will judge the oral presentation of the paper on:

- – Adherence to the time constraints
  – Effective use of graphs and/or visual aids

The rubric used for judging is available from the President-Elect.

Prizes:

All authors of papers presented at the convention will be given two-year extensions of their subscription to *The Pentagon*. Authors of the four best papers presented by undergraduates, as decided by the Selection and Awards Committees, will each receive a cash prize.

Publication:

All papers submitted to the convention are generally considered as submitted for publication in *The Pentagon*. Unless published elsewhere, prize-winning papers will be published in *The Pentagon* after any necessary revisions have been completed (see page 2 of *The Pentagon* for further information). All other papers will be considered for publication. The Editor of *The Pentagon* will schedule a brief meeting with each author during the convention to review his or her manuscript.

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New Editor & Business Manager

Charles "Chip" Curtis of Missouri Southern State University (Missouri Iota) has been named the next editor and Rich Laird has been name the next business manager of *The Pentagon* . Any correspondence for the editor arriving after June 1, 2004 should be sent to:

Charles Curtis
Department of Mathematics
Missouri Southern State University
3950 E. Newman Road
Joplin, MO 64804
email: curtis-c@mssu.edu

Dr. Curtis will be the tenth editor of this journal. Previous editors were C.V. Newsom (NM Alpha) 1941-1943; Harold D. Larsen (NM Alpha and MI Alpha) 1943-1953; Carl V. Fronabarger (MO Alpha) 1953-1959; Fred W. Lott, Jr. (IA Alpha) 1959-1965; Helen Kriegsman (KS Alpha) 1965-1971; James K. Bidwell (MI Beta) 1971-1979; Kent Harris (IL Eta) 1979-1989; Andrew M. Rockett (NY Lambda) 1989-1995; C. Bryan Dawson (KS Beta and TN Gamma) 1995-1999; and Steven D. Nimmo (IA Gamma) 1999-2004.

Correspondence for the business manager arriving after June 1, 2004 should be sent to:

Rich Laird
Department of Mathematics
Missouri Southern State University
3950 E. Newman Road
Joplin, MO 64804
email: laird-r@mssu.edu

Dr. Laird will be the tenth business manager of this journal. Previous business managers were C.B. Barker (NM Alpha) 1941-1943; Frank C. Gentry (NM Alpha) 1943-1945; Harold D. Larsen (NM Alpha & MI Alpha) 1945-1953; Dana Sudborough (MI Beta) 1953-1957; Wilbur J. Waggoner (MI Beta) 1957-1977; Douglas W. Nance (MI Beta) 1977-1985; Gerald L. White (IL Eta) 1985-1989; Sharon Kunoff (NY Lambda) 1989-1995; Larry Scott (KS Beta) 1995-2004.

Extracting Cube Roots

Editor's Note: As a high school student, I was fortunate enough to have a wonderful geometry teacher who took an interest in me and helped to foster my love for mathematics. In class, we had just gone over the extraction of square roots and I wondered if a similar technique existed for higher roots. She loaned me one of her own very old worn mathematics textbooks that contained a detailed explanation of extracting cube roots. The book could very well have been the same book as Richard Lee Barlow's great-grandfather used in high school. The following article is reprinted from a portion of the Spring 1981 issue of The Pentagon (pp. 115 – 122).

To find the cube root of a number, one must first determine the number of figures in the cube root. Ray's New Practical Arithmetic, published in 1877 by Van Antwerp, Bragg & Company, uses the following approach.

"The cube root of 1 is 1, and the cube root of 1000 is 10; between 1 and 1000 are all numbers consisting of one, two, or three figures, and between 1 and 10 are all numbers consisting of one figure; therefore, when a number consists of one, two, or three figures, its cube root consists of one figure." Similarly, the text states that numbers consisting of four, five or six figures have cube roots of two figures; numbers consisting of seven, eight, or nine figures have cube roots of three figures; etc. Therefore, the following rules are stated.

1. If a number is pointed off into periods of three figures each, the number of periods will be the same as the number of figures in the cube roots.
2. The cube of the units will be found in the first period, the cube of the tens in the second period, the cube of the hundreds in the third period, etc.

The following examples illustrate this procedure.

1. The point off of 876453921 into periods of three figures each is $87\dot{6}45\dot{3}92\dot{1}$.
2. Similarly, the point off of 37683.5624 is $3\dot{7}68\dot{3}.56\dot{2}40\dot{0}$.

As further explanation, the text gave the following: "Rule – Place a point over the order units, and then over every third order from units to the left and to the right. Remark 1 – The first period on the left of the integral part of the number will often contain but one or two figures. Remark 2 – When the first period on the right of the decimal part contains but one or two figures, ciphers must be annexed to complete the period."The cube root extraction process is illustrated as follows:

Example 1 – Extract the cube root of 13824.

Solution Steps.

1. Point off 13824 into periods of three figures each as $1\dot{3}82\dot{4}$.

$$\begin{array}{l} 1\dot{3}82\dot{4}\ (2 \\ \underline{8} \\ 5824 \end{array}$$

2. The largest cube in 13 is 8 which has as its cube root 2. Place the root 2 on the right, and subtract the cube 8 from 13 getting the remainder of 5. Now bring down the next period of 3 digits, namely 824.

3. Square the root 2 and multiply if by 300; i.e., $22 * 300 = 1200$. This number is the trial divisor. Find how many times 1200 is contained in 5824. The result is 4. Place 4 in the root to the right of 2.

$$\begin{array}{rr|l} & & 1\dot{3}82\dot{4}\ (24 \\ & & \underline{8} \\ 2 * 2 * 300 = & 1200 & 5824 \\ 2 * 4 * 30 = & 240 & \\ 4 * 4 = & \underline{16} & \\ & 1456 & \\ 1456 * 4 = & & \underline{5824} \\ & & 0 \end{array}$$

4. Multiply 2 by 4 and by 30. Next square 4. Then add these products of 240 and 16 to 1200, getting the sum of 1456 which is called the complete divisor. Multiply 1456 by 4, and subtract the product 5824 from 5824. The remainder is 0. Therefore, 13824 is a perfect cube having a cube root of 24.

As a second example, the following is presented:

<u>Example 2</u> – Extract the cube root of 413.5147

Solution:

$$
\begin{array}{r|l}
\multicolumn{1}{r}{} & 41\dot{3}.51\dot{4}70\dot{0}\ (7.45+ \\
\multicolumn{1}{r}{} & \underline{343} \\
7 * 7 * 300 = 14700 & 70\,514 \\
7 * 4 * 30 = 840 & \\
4 * 4 = \underline{16} & \\
15556 & \underline{62\,224} \\
74 * 74 * 300 = 1642800 & 8\,290700 \\
74 * 5 * 30 = 11100 & \\
5 * 5 = \underline{25} & \\
1653925 & \underline{8\,269625} \\
 & 21075
\end{array}
$$

Hence, $\sqrt[3]{413.5147} = 7.45$.

To justify the above approach in determining the cube root of a number, the text offered the following geometric explanation, which is quoted directly:

> *After finding that the cube root of the given number will contain two places of figures (tens and units), and that the figure in the tens' place is 2, form a cube, A, Fig. 1, 20 (2 tens) inches long, 20 in. wide, and 20 in. high; this cube will contain $20 \times 20 \times 20 = 8000$ cu. in.; take this sum from the whole number of cubes, and 5824 cu. in. are left, which correspond to the number 5824 in the numerical operation.*

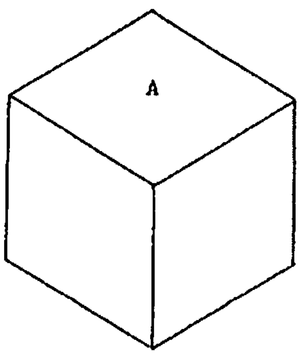

Figure 1

It is obvious that to increase the figure A, and at the same time preserve it a cube, the length, breadth, and height must each receive an equal addition. Then, since each side is 20 in. long, square 20, which gives $20 \times 20 = 400$, *for the number of square inches in each face of the cube; and since an addition is to be made to three sides, multiply the 400 by 3, which gives 1200 for the number of square inches called the trial divisor, because, by means of it, the thickness of the additions is determined.*

By examining Fig. 2 it will be seen that, after increasing each of the three sides equally, there will be required 3 oblong solids, C, C, C, of the same lengths each of the sides, and whose thickness and height are each the same as the additional thickness; and also a cube, D, whose length, breadth, and height are each the same as the additional thickness. Hence, the solid contents of the first three rectangular solids, the three oblong solids, and the small cube, must together be equal to the remainder (5824).

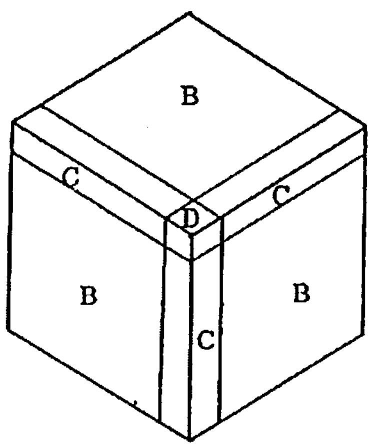

Figure 2

*Now find the thickness of additions. It will always be something less than the number of times the trial divisor (*1200*) is contained in the dividend (*5824*). By trial, we find* 1200 *is contained* 4 *times in* 5824*; proceed to find the contents of the different solids. The solid contents of* *the first three additions, B, B, B are found by multiplying the number of sq. in. in the face by the thickness; there are* 400 *sq. in. in the face of each, and* $400 \times 3 = 1200$ *sq. in. in one face of the three; then, multiplying by* 4 *(the thickness) gives* 4800 *cu. in. for their contents. The solid contents of the three oblong solids, C, C, C, are found by multiplying the number of sq. in. in the face by the thickness; now there are* $20 \times 4 = 80$ *sq. in. in one face of each, and* $80 \times 3 = 240$ *sq. in. in one face of the three; then multiplying by* 4 *(the thickness), gives* 960 *cu. in. for their contents. Lastly, find the contents of the small cubs, D, by multiplying together its length, breadth, and thickness; this gives* $4 \times 4 \times 4 = 64$ *cu. in.*

If the solid contents of the several additions be added together, as in the margin, their sum, 5824 *cu. in. will be the number of small cubes remaining after forming the first cube, A. Hence, when* 13824 *cu. in. are arranged in the form of a cube, each side is* 24 *in.; that is, the cube root of* 13824 *is* 24.

$$\begin{array}{rl}
\multicolumn{2}{l}{ADDITIONS} \\
B \times B \times B = 4800 & cu.in. \\
C \times C \times C = 960 & cu.in. \\
D = 64 & cu.in. \\
Sum = 5824 &
\end{array}$$

Can you determine the cube root of 1029.6213504 using this extracting process?

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959

IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981

NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February, 2001
GA Delta	Berry College, Mount Berry	21 April, 2001
TX Mu	Schreiner University, Kerrville	28 April, 2001
NJ Gamma	Monmouth University	21 April, 2002
CA Epsilon	California Baptist University, Riverside	21 April, 2003

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