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A Proof of Admiral Peary's Expedition

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Presented at the 2003 National Convention and awarded "top four" status by the Awards Committee.

On April 6th, 1909, Admiral Robert E. Peary completed an expedition to the North Pole believing he indeed reached his destination. However, all the rest of the world had to go on was Peary's claim along with photographs of the expedition. As a result, skepticism developed over the next eighty years. Then in 1989 a group decided to launch a thorough investigation into Peary's claim. The Foundation for the Promotion of the Art of Navigation was looking for concrete, verifiable answers. In fact they determined, through a complex series of measurements involving vanishing points and shadow angles taken from photos from the expedition, that a set of equations did prove that Peary was indeed at the North Pole + / - 5 miles. In the fall of 2002, at the urging of my advisor who knew of my interest in both math and photography, I decided to undertake a detailed recreation of the original proof as an undergraduate research project.

In order to begin this proof, the initial investigation was begun on the internet. Simple inquiries into Peary's expedition gave little to work with and a more in-depth investigation involving more specific queries yielded even less. Convening back with my advisor, he told me that he first read about the proof in an old issue of *National Geographic* [1], so he recommended that I try and track the issue down. Upon locating the issue, I was happy to find a detailed 18 page article on the Foundations findings in regards to Peary's expedition complete with actual photographs from which their calculations and measurements were made. But where were the equations, the meat of the proof? Unfortunately, nowhere to be seen. Luckily, I found an e-mail address for the Foundation and contacted them accordingly. Yet, while I did receive a response from the Foundation, including one equation, it was still a fraction of what was needed. No direction was given at all as to how this one equation worked, not to mention how they even got it. So I was left with no choice but to again e-mail the Foundation in hopes that they could give me more.

In the mean time, my advisor thought we should perhaps go in another direction before tackling the equations, specifically, looking at how a camera even works in the first place. Upon researching this in the book *Basic*

Photography [2] much was found detailing the specifics of a camera, such as focal lengths, image distances, and the effects of tilt on the picture, etc. Most helpful to our understanding though was the lens maker formula that was provided: $\frac{1}{u} + \frac{1}{f} = \frac{1}{F}$ where u is the object distance, f is the focal length at infinity, F is the focal length for the object, and A is the distance from the object to the focal point at infinity. Expanding this equation, letting x be the thickness of the lens, then $A = u + x + f$ (see Figure 1).

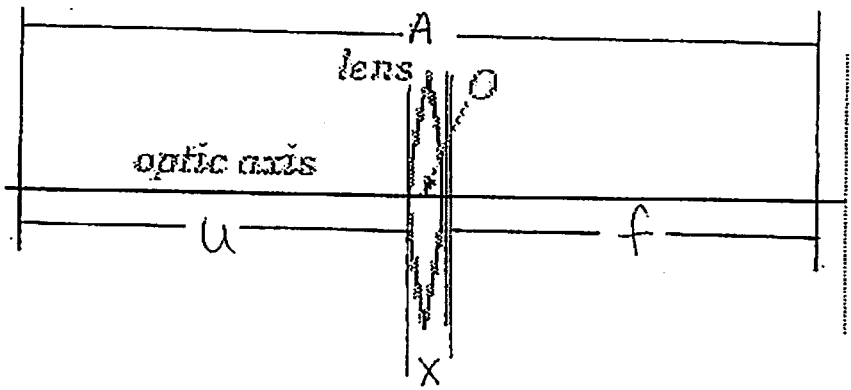

Figure 1

Solving for u, we get $u = A - x - f$. Thus the original equation could be rewritten as $\frac{1}{f} = \frac{1}{F} - \frac{1}{A-x-f}$ which turned out to be the one equation that the Foundation sent me.

Finally, the Foundation, specifically Terry Carraway, responded stating that they could indeed send me all the information via a fax. What was included in this fax were 6 equations, 5 new ones as well as the one previously sent. Essentially, the five equations were based on the structure of the photograph itself and measurements that were taken from these photos for computations. Since shadows of certain objects were the key, in order to make these measurements, certain procedures involved the analysis of these shadows. These procedures are listed below:

1. Draw diagonals on the photograph to determine the exact center. The center will correspond to the axis of the camera and a horizontal line through the center acts as a horizon.
2. Extend the line segments of the shadows under consideration to lines. All these lines will intersect at one point.
3. The point where the extensions of the shadows intersect will lie on the horizon.

4. The angle that the vertical plane of sun rays make with the cameras axis can be computed by the formula: $Tan(K) = \frac{L}{R}$ where L is the focal length of the camera and R is the horizontal distance between the center of the picture and the point of intersection of the extensions of the shadows.

An actual picture is shown in Figure 2

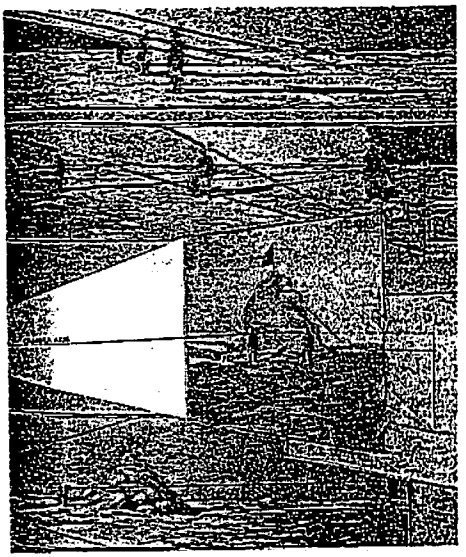

Figure 2

However, it is not obvious that the extension of the shadows would intersect in a single point and that they would intersect on the horizon of the picture. It was also no clear how you could use these measurements from the picture to gain any information about the actual angles that the sun rays made. This became the focus of our research. Why is this true?

We decided to construct a three dimensional model and to create a picture by projecting the images of the objects onto a plane. To do this projection, we placed the plane between the objects and a point that would correspond to the eye and drew lines from the eye to the objects, marking where the lines would intersect the plane as our image points. The objects located in the three dimensional coordinate system included three men, three rays of light that touch the top of the heads of each man, and the three shadows produced by the men. These nine objects were then projected onto the plane mentioned above for our picture. Through this model we hoped to answer three conjectures:

Conjecture 1. The extensions of the three shadows (in the plane of projection) will intersect in one point.

Conjecture 2. The intersection point of the point of Conjecture 1 will lie on the horizon (at the same height as that of they eye, located at $(4, 5, 3)$ or $z = 3$)

Conjecture 3. The angle that the vertical plane of sun rays makes with the plane of the picture is given by the equation $tan(r) = B/x$ where B is the distance of the eye to the picture and x is the horizontal distance between the eye and the point of intersection of the extensions of the shadows.

The model first consists of three men M_A, M_B, and M_C that are located in a three dimensional coordinate system with the coordinates:

$M_A = \{(x, y, z) : x = -4, y = 2, z = S, 0 \leq S \leq 6\}$
$M_B = \{(x, y, z) : x = -4, y = 4, z = S, 0 \leq S \leq 6\}$
$M_C = \{(x, y, z) : x = -4, y = 6, z = S, 0 \leq S \leq 6\}$.

Light rays (L_A, L_B,and L_C) from the sun are hitting the top of the heads of the three men producing shadows (S_A, S_B,and S_C), respectively) on the floor which will be the plane $z = 0$. The light rays will be parallel and are given the direction (a, b, c) where $a < 0, b > 0$, and $c < 0$. Since the light rays pass through the top of each man, we can express the equations of these light rays as:

$L_A = \{(x, y, z) : x = -4 + at, y = 2 + bt, z = 6 + ct, t\epsilon R\}$
$L_B = \{(x, y, z) : x = -4 + at, y = 4 + bt, z = 6 + ct, t\epsilon R\}$
$L_C = \{(x, y, z) : x = -4 + at, y = 6 + bt, z = 6 + ct, t\epsilon R\}$.

The eye is located at point $(4, 5, 3)$ and the plane the image will be projected on will be $x = 0$. The line segments of the shadows will then be projected onto the plane $x = 0$ to produce the picture. The image of each shadow is obtained by finding the intersection point of the projecting line (the line joining the eye to a point in the shadow) with the image plane $x = 0$. We will first determine the equation of the shadow of M_A projected onto $x = 0$.

To find S_A, the shadow of Man A, we need to find the line segment from the foot of Man A, $(-4, 2, 0)$, to where L_A intersects the $z = 0$ plane (the floor). This intersection will occur when $z = 0 = 6+ct$ and $t = -6/c$. Using this value of t, we obtain the end of the shadow of M_A (denoted by E_A) $= \{x = -4 - \frac{6a}{c}, y = 2 - \frac{6b}{c}, z = 0\}$

We can now find S_A by finding the line: $\{(x, y, z) : x = \alpha + \beta t, y = \gamma + \delta t, z = \mu + \upsilon t\}$ through the foot of Man A and E_A and by letting $t = 0$ be the first point and $t = 1$ for E_A.

By then restructuring t to $[0, 1]$, we have $S_A = \{(x, y, z) : x = \alpha + \beta t, y = \gamma + \delta t, z = \mu + \upsilon t, 0 \leq t \leq 1\}$

At $t = 0$, we find $-4 = \alpha, 2 = \gamma, \mu = 0$.

At $t = 1$, we get $4 - \frac{(6a)}{c} = -4 + \beta t, 2 - \frac{(6b)}{c} = 2 + \delta, 0 = \upsilon$. Solving these equations gives $\beta = \frac{-6a}{c}, \delta = \frac{-6b}{c}, \upsilon = 0$.

This results in $S_A = \{(x, y, z) : x = -4 - \frac{(6a)s}{c}, y = 2 - \frac{(6b)s}{c}, z = 0, 0 \leq S \leq 1\}$.

Now, to project the image of M_A, denoted I_A, onto the yz plane, we will need to find the equation of the lines, L_{PA}, from the eye, $E = (4, 5, 3)$, to points of M_A. We will use $t = 0$ for $(4, 5, 3)$ and $t = 1$ for $(-4, 2, S)$ where $0 \leq S \leq 1$: getting us the equation $L_{PA} = \{(x, y, z) : x = \alpha + \beta t,\ y = \gamma + \delta t,\ z = \mu + \upsilon t, t \varepsilon \mathrm{R}\}$. Using $t = 0$ gets us $4 = \alpha, 5 = \gamma, 3 = \mu$, and using $t = 1$ gets us $-4 = 4 + \beta$, $2 = 5 + \delta$, $S = 3 + \upsilon$. This results in $\beta = -8, \delta = -3, \upsilon = S - 3$. We now have, $L_{PA} = \{(x, y, z) : x = 4 - 8t, y = 5 - 3t, z = 3 + (S - 3)t, t \varepsilon \mathrm{R}\}$.

Next we need to find where our projection line hits the picture. This happens when $x = 0$ Setting $x = 0 = 4 - 8t$, we get $t = \frac{1}{2}$. The image point is then those points of L_{PA} where $t = \frac{1}{2}$ or $I_A = \{(x, y, z) : x = 0, y = 3\frac{1}{2}, z = 3 + \frac{S-3}{2}, 0 \leq S \leq 6\}$.

When we check this, we see that the top of Man A is where $S = 6$ and the top of M_A projects to $(0, 3\frac{1}{2}, 4\frac{1}{2})$ and the foot of the man $(S = 0)$ is projected to $(0, 3\frac{1}{2}, 1\frac{1}{2})$. Thus the man has shrunk from $6ft$ to $3ft$.

Now, to get our picture of the shadow, denoted by I_{SA}we will project S_A onto the $x = 0$ plane. To do this we first need to find the lines , L_{SA}, from the eye to points of the shadow. Using the same procedure as in finding I_A, and letting $t = 0$ for $(4, 5, 3)$ and $t = 1$ for $\{-4 - (6as/c), 2 - (6bs/c), 0\}$, we obtain $\alpha = 4, \gamma = 5, \mu = 3$ when $t = 0$.

At $t = 1$ the equations are $-4 - \frac{6as}{c} = 4 + \beta, 2 - \frac{6bs}{c} = 5 + \delta, 0 = 3 + \upsilon$. Solving these equations yields: $\beta = -8 - \frac{6as}{c}, \delta = -3 - \frac{6bs}{c}, \upsilon = -3$.

Thus L_{SA} is then given by $L_{SA} = \{(x, y, z)] x = 4 + (-8 - (6as/c))t, y = 5 + (-3 - (6bs/c))t, z = 3 - 3t, t \varepsilon \mathrm{R}\}$

To get the projection, we need to intersect L_{SA} with $x = 0$. Setting $x = 0$ gives $0 = 4 + \frac{(-8-(6as)t}{c}$

and solving for t gives $t = \frac{2c}{(4c+3as)}$

Thus I_{SA} is given by $I_{SA} = \{(x, y, z) : x = 4 + \frac{2(-8c-6as)}{(4c+3as)} = 0, y = 5 + \frac{2(-3c-6as)}{(4c+3as)}, z = 3 - \frac{6c}{(4c+3as)}, 0 \leq S \leq 1\}$.

The previous equation, I_{SA}, was for a line segment. In order to determine if the lines actually intersect we need the equations for the full lines. Using the form $z = my + Q$ and the points determined by using $S = 0$, and $S = 1$, we can calculate the equation of the line.

Letting $S = 0$ gives $y = 5 - \frac{3}{2} = \frac{7}{2}, z = 3 - \frac{3}{2} = \frac{3}{2}$ and letting $S = 1$

gives $y = 5 - \frac{(6c+12a)}{(4c+3a)}$, $z = 3 - \frac{6c}{(4c+3a)}$

We can now use the slope formula $m = \frac{Z_2 - Z_1}{Y_2 - Y_1}$.

Solving for m gives

$$\begin{aligned} & \frac{3 - \frac{(6c)}{(4c+3a)} - \frac{3}{2}}{5 - \frac{(6c+12b)}{(4c+3a)} - \frac{7}{2}} \\ = & \frac{\frac{3}{2} - \frac{(6c)}{(4c+3a)}}{\frac{3}{2} - \frac{(6c+12b)}{(4c+3a)}} \\ = & \frac{\frac{(12c+9a-12c)}{(4c+3a)}}{\frac{(12c+9a-12c-24b)}{(4c+3a)}} \\ = & \frac{9a}{(9a-24b)} \\ = & \frac{3a}{(3a-3b)} \end{aligned}$$

Then plugging m into the original equations results in $z = \frac{(3a)y}{(3a-8b)} + Q$.

Next we must solve for Q. Using one of our points in the above equation yields $\frac{3}{2} = \frac{(3a)}{(3a-8b)} \cdot \frac{7}{2} + Q$.

This equation can be solved for Q giving $\frac{(-6a-12b)}{(3a-8b)} = Q$.

Thus our extended line is given by $z = \frac{(3a)y}{(3a-8b)} + \frac{(-6a-12b)}{(3a-8b)}$

Now then, just as was done for Man A, the same will be done for Man B. That is we will find the equation of the line segment of the shadow image and convert it into a full line equation accordingly. The results of this analysis are listed below:

ManB = $\mathbf{M}_B = \{(x, y, z) : x = -4, y = 4, z = S, 0 \leq S \leq 6\}$

I_{SB} = image of shadow of Man B

$I_{SB} = \{(x, y, z) : x = 0, y = 5 - \frac{(2c+12b)}{(4c+3as)}, z = 3 - \frac{6c}{(4c+3as)}, 0 \leq S \leq 1\}$

Extension line for Shadow B

$z = \frac{(3a)y}{(a-8b)} + \frac{(-12a-12b)}{(a-8b)}$

Now then, just as was done for Man A & B, the same will be done for Man C. That is we will find the equation of the line segment of the shadow image and convert it into a full line equation accordingly. The results are:

Man C = $M_C = \{(x, y, z) : x = -4, y = 6, z = S, 0 \leq S \leq 6\}$

I_{SC} = image of shadow of Man C

$I_{SC} = \{(x, y, z,) : x = 0, y = 5 + \frac{(2c+12b)}{(4c+3as)}, z = 3 - \frac{6c}{(4c+3as)}, 0 \leq S \leq 1$

Extension line for Shadow C

$z = \frac{(3a)y}{(-a-8b)} + \frac{(18a+12b)}{(a+8b)}$

Now, in order to prove Conjectures 1 & 2 we must find an intersection between these three lines as well as show that they intersect at the horizon ($z = 3$) To do this, we will take two of the extended line equations, 1 & 2, and solve for y. Once we know our y value, we can then plug it back into one of the two equations and solve for z accordingly. Then we would solve equations 2 & 3 and hope for the same results. The three equations are:

$$z = \frac{(3a)y}{(3a-8b)} + \frac{(-6a-12b)}{(3a-8b)} \quad (1)$$

$$z = \frac{(3a)y}{(a-8b)} + \frac{(-12a-12b)}{(a-8b)} \quad (2)$$

$$z = \frac{(3a)y}{(-a-8b)} + \frac{(18a+12b)}{(a+8b)} \quad (3)$$

Setting the y's equal to each other from equations 1 and 2:

$$\frac{(3a)y}{(3a-8b)} + \frac{(-6a-12b)}{(3a-8b)} = \frac{(3a)y}{(a-8b)} + \frac{(-12a-12b)}{(a-8b)}$$

Solving for y yields

$$\left(\frac{(3a)}{(3a-8b)} - \frac{(3a)}{(a-8b)}\right) y = \frac{(6a+12b)}{(3a-8b)} - \frac{(12a+12b)}{(a-8b)}$$

$$\begin{aligned} y &= \frac{(6a^2 - 48ab + 12ab - 96b^2 - 36a^2 - 36ab + 96ab + 96b^2)}{(3a^2 - 24ab - 9a^2 + 24ab)} \\ &= \frac{(-30a^2 + 24ab)}{-6a^2} \\ &= \frac{(5a-4b)}{a} \end{aligned}$$

Setting the y's equal to each other from equations 1 and 3:

$$\frac{(3a)y}{(3a-8b)} + \frac{(-6a-12b)}{(3a-8b)} = \frac{(3a)y}{(-a-8b)} + \frac{(18a+12b)}{(a+8b)}$$

Solving for y yields

$$\left(\frac{(3a)}{(3a-8b)} + \frac{(3a)}{(a+8b)}\right) y = \frac{(6a+12b)}{(3a-8b)} + \frac{(18a+12b)}{(a+8b)}$$

$$\begin{aligned} y &= \frac{(6a^2 + 48ab + 12ab + 96b^2 + 54a^2 + 36ab - 144ab - 96b^2)}{(3a^2 + 24ab + 9a^2 - 24ab)} \\ &= \frac{(60a^2 - 48ab)}{12a^2} \\ &= \frac{(5a - 4b)}{a} \end{aligned}$$

Plugging the y value back into the first equations gives

$$\begin{aligned} z &= \frac{3ay - (6a + 12b)}{(3a - 8b)} \\ &= \frac{3a(5a - 4b)}{(3a - 8b)\,(a)} - \frac{(6a + 12b)}{(3a - 8b)} \\ &= \frac{9a - 24b}{(3a - 8b)} \\ &= 3 \end{aligned}$$

and into the second equation gives

$$\begin{aligned} z &= \frac{3ay - (12a + 12b)}{(a - 8b)} \\ &= \frac{3a(5a - 4b)}{(a - 8b)(a)} - \frac{(12a + 12b)}{(a - 8b)} \\ &= \frac{(15a - 12b - 12a - 12b)}{(a - 8b)} \\ &= \frac{3a - 24b}{(a - 8b)} \\ &= 3 \end{aligned}$$

and finally into the third equation gives

$$\begin{aligned} z &= \frac{-3a(5a - 4b)}{(a + 8b)} + \frac{(18a + 12b)}{(a + 8b)} \\ &= \frac{(3a + 24b)}{(a + 8b)} \\ &= 3 \end{aligned}$$

Thus since the three equations share a y & z value we know that the lines intersect, and the intersection point lies on the horizon. Thus conjecture 1 & 2 is true.

For conjecture 3, we will compute the angle that the vertical plane of sun rays makes with the picture plane by using the equation of the conjecture. This equation is $tangent(r) = \frac{B}{x}$ (see r Figure 3) where B is the distance from the eye to the picture and x is the horizontal distance between the eye and the point of intersection of the extensions of the shadows.

First off though, we must find the values of x and B. We know that the eye is at $(4, 5, 3)$, which gives $B = 4$. To find x, we must compute the horizontal distance between the eye and the point of intersection of the extension lines of the images of the shadows. We can do this by subtracting the y coordinates of these two points. This gives $x = \frac{5a-4b}{a} - \frac{5}{1} = \frac{5a-4b-5a}{a} = \frac{-4b}{a}$.

Then r is determined by the equation

$$tan(r) = \frac{B}{x} = \frac{4}{-4\frac{b}{a}} = \frac{-a}{b}$$

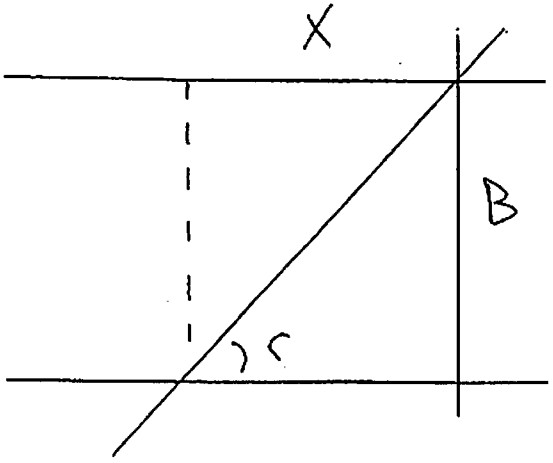

Figure 3

Now, to determine the validity of conjecture 3, we will go to the model and measure the angle that the shadow makes with the y axis. We will do this for Man A (see Figure 4). The foot of Shadow A is $(-4, 2, 0)$ and the end of the shadow is $(-4 - \frac{6a}{c}, 2 - \frac{6b}{c}, 0)$.

Using the definition of the tangent of an angle we find

$$tan(r) = \frac{height}{base} = \frac{-4 - (-4 - 6\frac{a}{c})}{2 - (6\frac{b}{c}) - 2} = \frac{6\frac{a}{c}}{-6\frac{b}{c}} = \frac{-a}{b}$$

Thus we have verified conjecture 3 and shown that is possible to use

information obtained from a picture to determine features of the real world that the pictures depict.

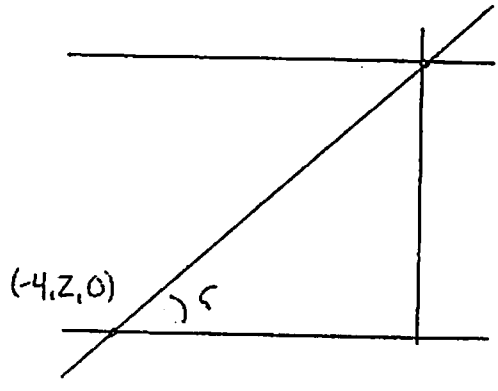

Figure 4

In their research, the foundation found two key angles. One is the angle we used in conjecture 3. The other being the vertical angle of the sun relative to the horizontal plane through the camera's (eye's) axis, which is what proved that Peary was actually there. In further research , we will investigate why this angle can be determined from the properties of the picture.

Acknowledgements. In closing, I would like to thank my advisor for his guidance and patience, as much was needed, as well as Terry Carraway of the Foundation for his help and information that was provided.

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Something Unexpected

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Presented at the 2003 National Convention

A standard problem in Calculus 1 is to investigate the convergence of Riemann Sums by calculating the sum for various increasing values of n and demonstrate how the left and right Riemann Sums converge to the definite integral. For the function $f(x)$ on the interval $[a, b]$ the Riemann Sums are defined as follows: The right Riemann Sum, $R_n(f(x), [a,b]) = \sum_{i-1}^{n} f\left(a + i\frac{(b-a)}{n}\right)\left(\frac{(b-a)}{n}\right)$ and the left Rieman Sum, $L_n(f(x), [a,b]) = \sum_{i=0}^{n-1} f\left(a + i\frac{(b-a)}{n}\right)\left(\frac{(b-a)}{n}\right)$. A typical numerical example might be for $f(x) = \sin(x)$ over$[0, \pi]$. The definite integral $\int_0^{\pi} \sin(x)dx = -\cos(x)]_0^{\pi} = 2$. Values for the Riemann Sums are given in the table:

	$L_n[0,\pi]$	$R_n[0,\pi]$
$n = 1$	0	0
$n = 2$	$\pi/2$	$\pi/2$
$n = 5$	1.93377	1.93377
$n = 10$	1.98352	1.98352
$n = 20$	1.99589	1.99589
$n = 100$	1.99984	1.99984
$\int_0^{\pi} \sin(x)dx$	2	2

The table gives numerical evidence that both Riemann Sums are converging to 2.

If we apply the same procedure to $f(x) = \cos^2(x)$over $[0, \pi]$ something unexpected happens. Using a TI-89 to calculate the corresponding

Riemann Sum table for we have:

	$L_n[0,\pi]$	$R_n[0,\pi]$
$n=1$	π	π
$n=2$	$\pi/2$	$\pi/2$
$n=5$	$\pi/2$	$\pi/2$
$n=10$	$\pi/2$	$\pi/2$
$n=20$	$\pi/2$	$\pi/2$
$n=100$	$\pi/2$	$\pi/2$
$\int_0^\pi \cos^2(x)dx$	$\pi/2$	$\pi/2$

The definite integral is $\int_0^\pi \cos^2(x)dx = \frac{\pi}{2}$. It appears that the Riemann Sums converges in one-step with $\frac{\pi}{2}$ for $n > 1$ for being the constant value. The numerical analysis leads to the following conjecture:

For $n \geq 1$, $L_n(\cos^2(x), [0,\pi]) = \frac{\pi}{2}$.

To prove this, the first thing we tried was breaking the proof into two cases: the first case being when n is an even integer and the second case being when n is an odd integer. In the first case, let $n = 2k$ for some integer k in the real numbers. Then one needs to show that the right and left hand limits equal $\frac{\pi}{2}$. What happens can be discovered by doing a few examples with even integers. For example let $n = 4$. The right Riemann Sum

is $R_4(\cos^2(x),[0,\pi]) = R_4\frac{(1+\cos(2x))}{2},[0,\pi]) = \left(\frac{\pi}{4}\right)\sum_{i=1}^{4}\frac{\left(1+\cos\left(\frac{2i\pi}{4}\right)\right)}{2}$ using the Double Angle Formula. Expanding, $R_4(\cos^2(x),[0,\pi])$

$$= \left(\frac{\pi}{4}\right)\left[\begin{array}{c}\left(\frac{1+\cos\left(\frac{2\cdot1\cdot\pi}{4}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{2\cdot2\cdot\pi}{4}\right)}{2}\right)\\ +\left(\frac{1+\cos\left(\frac{2\cdot3\cdot\pi}{4}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{2\cdot4\cdot\pi}{4}\right)}{2}\right)\end{array}\right]$$

$$= \left(\frac{\pi}{4}\right)\left[\left(\frac{1+\cos\left(\frac{2\pi}{4}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{4\pi}{4}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{6\pi}{4}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{8\pi}{4}\right)}{2}\right)\right].$$

Further reduction gives

$$\left(\frac{\pi}{4}\right)\left[\left(\frac{4}{2}\right)+\left[\left(\frac{\cos\left(\frac{\pi}{2}\right)}{2}\right)+\left(\frac{\cos(\pi)}{2}\right)+\left(\frac{\cos\left(\frac{3\pi}{2}\right)}{2}\right)+\left(\frac{\cos(2\pi)}{2}\right)\right]\right]$$

$$= \left(\frac{\pi}{4}\right)\left[\left(\frac{4}{2}\right)[0-\left(\frac{1}{2}\right)+0+\left(\frac{1}{2}\right)]\right] = \left(\frac{\pi}{4}\right)\left(\frac{4}{2}\right) = \frac{\pi}{2}.$$

Now one should begin by applying the definition of the right Riemann Sum to $n = 2k$ and one has

$$R_{2k}\left(\cos^2(x),[0,\pi]\right) = R_{2k}\left(\frac{(1+\cos(2x))}{2},[0,\pi]\right) = \left(\frac{\pi}{n}\right)\sum_{i=1}^{n}\frac{\left(1+\cos\left(\frac{2i\pi}{n}\right)\right)}{2}$$

(LarsonA19). So when $n = 2k$ the right Riemann Sum is $\left(\frac{\pi}{2k}\right)\sum_{i=1}^{2k}\frac{\left(1+\cos\left(\frac{2i\pi}{2k}\right)\right)}{2}$.

This reduces to $\left(\frac{\pi}{2k}\right)\sum_{i=1}^{2k}\frac{\left(1+\cos\left(\frac{i\pi}{k}\right)\right)}{2}$. Then one should begin by writing out the summation from $i=1$ to $2k$ giving one

$$\left(\frac{\pi}{2k}\right)\left[\left(\frac{1+\cos\left(\frac{\pi}{k}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{2\pi}{k}\right)}{2}\right)+\ldots+\left(\frac{1+\cos\left(\frac{(2k-1)\pi}{k}\right)}{2}\right)+\left(\frac{1+\cos(2\pi)}{2}\right)\right]$$

Next, factor out the one-halves out of each factor in the summation giving one

$$\left(\frac{\pi}{2k}\right)\left[\left(\frac{2k}{2}\right)+\left[\left(\frac{\cos\left(\frac{\pi}{k}\right)}{2}\right)+\left(\frac{\cos\left(\frac{2\pi}{k}\right)}{2}\right)+\ldots+\left(\frac{\cos\left(\frac{(2k-1)\pi}{k}\right)}{2}\right)+\left(\frac{\cos(2\pi)}{2}\right)\right]\right].$$

Then if one keeps reducing one is left with

$$\left(\frac{\pi}{2k}\right)\left[\left(\frac{2k}{2}\right)+\left[\left(\frac{\cos\left(\frac{\pi}{k}\right)}{2}\right)+\left(\frac{\cos\left(\frac{2\pi}{k}\right)}{2}\right)+\ldots+\left(\frac{\cos\left(\frac{(2k-1)\pi}{k}\right)}{2}\right)+\left(\frac{1}{2}\right)\right]\right].$$

We know that $\cos\left(\frac{i\pi}{k}\right)=-\cos\left(\frac{(i\pi)}{k}+\pi\right)$ thus follows that $\cos\left(\frac{i\pi}{k}\right)=-\cos\left(\frac{(i+k)\pi}{k}\right)$ (Larson A19). This step helps to show that all the cosines will cancel. If one breaks the summation into two parts from one to k and $k+1$ to $2k$,one can see the cancellations easier. One can see that when $i=1$ cosine is $\frac{\cos\left(\frac{\pi}{k}\right)}{2}$ and that will cancel with $\frac{\cos\left(\pi+\frac{\pi}{k}\right)}{2}$ when $i=k+1$. Thus follow that when $i=2$ cosine is $\frac{\cos\left(\frac{2\pi}{k}\right)}{2}$ and that will cancel with $\frac{\cos\left(\pi+\left(\frac{2\pi}{k}\right)\right)}{2}$ when $i=k+2$. This pattern follows throughout the problem with the last cancellation being when $i=k$ cosine is $\frac{\cos(\pi)}{2}$ which equals $\frac{-1}{2}$ will cancel with $\frac{\cos(2\pi)}{2}$ which equals $\frac{1}{2}$ when $i=2k$. When all the cancellations are done one is left with $\left(\frac{\pi}{2k}\right)\left(\frac{2k}{2}\right)$ and thus left with $\frac{\pi}{2}$. The same argument applies to

$$L_{2k}\left(\cos^2(x),[0,\pi]\right).L_{2k}\left(\cos^2(x),[0,\pi]\right)$$
$$=\left(\frac{\pi}{2k}\right)\sum_{i=0}^{2k}1+\cos^2\left(\frac{2i\pi}{2k}\right)$$
$$=\left(\frac{\pi}{2k}\right)\left[\left(\frac{1+\cos(0)}{2}\right)+\left(\frac{1+\cos\left(\frac{\pi}{k}\right)}{2}\right)+\ldots\left(\frac{1-\cos\left(\frac{2k-1}{k}\right)\pi}{2}\right)\right]$$

using the same argument as for , R_{2k}, $L_{2k}\left(\cos^2(x),[0,\pi]\right)$ reduces to $\left(\frac{\pi}{2k}\right)\left(\frac{2k}{2}\right)=\frac{\pi}{2}$ for $k\leq 1$.

For the odd case, let $n=2k+1$ for some integer k in the real numbers. One needs to show that the right and left Riemann Sums equal $\frac{\pi}{2}$. In doing a few examples with odd integers one will find that the term for term cancellation technique was not an option for odd values of n. For example let $n=5$. Thus the definition of the left Riemann Sum is $L_n\left(\cos^2(x),[0,\pi]\right).L_n\left(\frac{(1+\cos(2x))}{2},[0,\pi]\right)=\left(\frac{\pi}{n}\right)\sum_{i=0}^{n-1}\frac{(1+\cos(2i\pi))}{2}$. Then

substitute $n = 5$ one is left with

$$\left(\frac{\pi}{5}\right)\left[\left(\frac{1+\cos\left(\frac{2*0*\pi}{5}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{2*1*\pi}{5}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{2*2*\pi}{5}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{2*3*\pi}{5}\right)}{2}\right)+\left(\frac{1+\cos\frac{2*4*\pi}{5}}{2}\right)\right]$$

$$=\left(\frac{\pi}{5}\right)\left[\left(\frac{1+\cos\left(\frac{0\pi}{5}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{2\pi}{5}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{4\pi}{5}\right)}{2}\right)+\left(\frac{1+\cos\left(\frac{6\pi}{5}\right)}{2}\right)+\left(\frac{1+\cos\frac{8\pi}{5}}{2}\right)\right]$$

$$=\left(\frac{\pi}{5}\right)\left[\left(\frac{5}{2}\right)+\left[\left(\frac{\cos(0)}{2}\right)+\left(\frac{\cos\left(\frac{2\pi}{5}\right)}{2}\right)+\left(\frac{\cos\left(\frac{4\pi}{5}\right)}{2}\right)+\left(\frac{\cos\left(\frac{6\pi}{5}\right)}{2}\right)+\left(\frac{\cos\left(\frac{8\pi}{5}\right)}{2}\right)\right]\right]$$

$= \left(\frac{\pi}{5}\right)\left[\frac{5}{2}+\left[\frac{1}{2}.15408-.404508-.404508+.154508\right]\right] = \frac{\pi}{2}$. The sum is the expected $\frac{\pi}{2}$ but the pattern is not obvious as in the even case.

In looking for an alternative method we searched for a trigonometric identity for the summation of $cos^2(x)$. While searching the Internet I came across the Mathworld website which provided me with the following trigonometric identity

$\sum_{i=0}^{n} \cos^2(ix) = \left(\frac{1}{4}\right)(3+2n+\csc(x)\sin[x(1+2n)]$ (Mathworld). Before one uses this identity it would be a good idea to prove it since it appeared undocumented in the website. In proving this trigonometric identity the method of choice is Mathematical Induction (Epp 213).

Theorem: For all integers $n \geq 0$, $\sum_{i=0}^{n} \cos^2(ix) = \left(\frac{1}{4}\right)(3+2n+\csc(x)\sin[x(1+2n)]$.

Proof: The base case then is let $n = 0$, $\sum_{i=0}^{0} \cos^2(ix) = \cos^2(0x) = 1$.

Also $\left(\frac{1}{4}\right)(3+2*0+\csc(x)\sin[x1+2*0)] = \left(\frac{1}{4}\right)[3+0+\csc(x)\sin[x]]$. Then if one changes $csc(x)$ to $\frac{1}{\sin(x)}$ one is left with $\left(\frac{1}{4}\right)(3+\left(\frac{1}{\sin(x)}\right)\sin[x] = \left(\frac{1}{4}\right)4 = 1$ (Larson A19). Thus the base case is finished. Now one needs to move to the inductive step. Suppose $\sum_{i=0}^{k} \cos^2(ix) = \left(\frac{1}{4}\right)(3+2k+\csc(x)\sin[x(1+2k)])$ for some integer $k \geq 0$. The goal is to show that $\sum_{i=0}^{k+1} \cos^2(ix) = \left(\frac{1}{4}\right)(3+2(k+1)+\csc(x)\sin[x(1+2(k+1))])$. If one does some simplification one is left with $\left(\frac{1}{4}\right)(5+2k+\csc(x)\sin[3x+2kx])$. So $\sum_{i=0}^{k+1} \cos^2(ix) = \sum_{i=0}^{k} \cos^2(ix) + \cos^2((k+1)x)$. Thus $\sum_{i=0}^{k} \cos^2(ix) + \cos^2((k+1)x) = \left(\frac{1}{4}\right)(3+2k+\csc(x)\sin[x(1+2k)]) + \cos^2(kx+x)$. Then one should change the $csc(x)$ term into $\frac{1}{\sin(x)}$, combine the two sin

(x) terms, multiply the $cos^2(kx + x)$ by 4, and use the Double Angle Formula on the $cos^2(x)$ term such that $\cos^2(u) = \frac{(1+\cos(2u))}{2}$ with u being arbitrarily real numbers (Larson A19). So one has $\left(\frac{1}{4}\right)(3 + 2k + \left(\frac{\sin[x(1+2k)]}{\sin(x)}\right) + 4\left(\frac{(1+\cos(2(kx+x)))}{2}\right)$. If one does some simplification one is left with $\left(\frac{1}{4}\right)(3 + 2k + \left(\frac{\sin[x(1+2k)]}{\sin(x)}\right) + 2 + 2\cos(2(kx + x))$. So combine all the sines and cosines together by making a common denominator of $sin(x)$. Then one is left with $\left(\frac{1}{4}\right)(5 + 2k + \sin((2k + 1)x) + 2\sin(x)\left(\frac{\cos(2(kx+x))}{\sin(x)}\right)$. Then applying the Product-to-Sum Formula which is $\sin(u)\cos(v) = \left(\frac{1}{2}\right)[\sin(u + v) + \sin(u - v)]$ with u and v being arbitrarily real numbers, to the $2sin(x)cos(2(kx + x))$ term (Larson A19). We have $2sin(x)cos(2(kx + x)) = sin(2(kx + x) + x) - sin(2(kx + x) - x)$. Then after simplification we are left with $sin(2kx + 3x) - sin(2kx + x)$. Then replace $sin(2kx + 3x) - sin(2kx + x)$ term with the $2sin(x)cos(2(kx + x))$ term. So one is left with $\left(\frac{1}{4}\right)(5 + 2k + \frac{\sin((2kx)+\sin(2kx+3x)-\sin(2kx))}{\sin(x)}$. Then one can do some cancellation and is left with $\left(\frac{1}{4}\right)(5 + 2k + \left(\frac{\sin(2kx+3x)}{\sin(x)}\right)$. If one changes the $\frac{1}{\sin(x)}$ back into $csc(x)$ one is left with $\left(\frac{1}{4}\right)(5 + 2k + \csc(x)\sin[2kx + 3x])$ and thus for all integers $n \geq 0$, $\sum_{i=0}^{n} \cos^2(ix) = \left(\frac{1}{4}\right)(3 + 2n + \csc(x)\sin[x(1\_2n)]$ is true.

Therefore, one can continue the previous proof of showing that the left Riemann Sum of $cos^2(x)$ over $[0, \pi]$ is $\frac{\pi}{2}$. So we make a change in variables of the trigonometric identity to fit the left Riemann Sum. Thus, $\sum_{i=0}^{n-i} \cos^2(ix) + \cos^2(nx) = \left(\frac{1}{4}\right)(3 + 2n + \csc(x)\sin[x(1 + 2n)]) - \cos^2(nx)$ (Epp 185-186). Then one can apply the definition of the left Riemann Sum, which is $L_n\left(\cos^2(x), [0, \pi]\right) = \left(\frac{\pi}{n}\right)\sum_{i=0}^{n-1} \frac{\cos^2(i\pi)}{n}$ the trigonometric identity. Then one is left with $\left(\frac{\pi}{n}\right)\sum_{i=0}^{n-1} \frac{\cos^2(i\pi)}{n} = \left(\frac{\pi}{n}\right)\left[\left(\frac{1}{4}\right)(3 + 2n + \csc\left(\frac{\pi}{n}\right)\sin\left[\left(\frac{\pi}{n}\right)(1 + 2n)\right]) - \cos^2\left(\frac{n\pi}{n}\right)\right]$

After one does some simplification one is left with $\left(\frac{\pi}{n}\right)[\left(\frac{1}{4}\right)(3 + 2n + \csc\left(\frac{\pi}{n}\right)\sin\left[\left(\frac{\pi}{n}\right) + 2\pi\right]) - 1]$.

Since sine is a periodic function when one adds 2π to $\frac{\pi}{n}$ one will be back at $\frac{\pi}{n}$. Thus $sin[(\frac{\pi}{n}) + 2\pi)] = sin(\frac{\pi}{n})$. This fact helps to simplify the problem even more and the ability to write $csc(x)$ as $\frac{1}{\sin(x)}$. So one is left with $(\frac{\pi}{n})[(\frac{1}{4})(3 + 2n + \csc(\frac{\pi}{n})\sin[(\frac{\pi}{n})]) - 1]$

$= (\frac{\pi}{n})\left[(\frac{1}{4})(3 + 2n + (\frac{1}{\sin\frac{\pi}{n}})\sin\left[(\frac{\pi}{n})\right] - 1\right]$

$= (\frac{\pi}{n})[(\frac{1}{4})(3 + 2n + 1) - 1]$

$= (\frac{\pi}{n})[(\frac{1}{4})(4 + 2n) - 1]$

$= (\frac{\pi}{2})(\frac{n}{2})$

$= \frac{\pi}{2}$.

The same trigonometric identity also takes care of the right Riemann Sum for all $n > 1$, both even and odd.

In conclusion: For all $n > 1$,

$L_n(\cos^2(x), [0, \pi]) = R_n(\cos^2(x),)[0, \pi]) = \int_0^n \cos^2(x)dx = \frac{\pi}{2}$.

Three homework problems for those interested are:

1. Compare $L_n(\sin^2(x), [0, \pi])$, $R_n(\sin^2(x), [0, \pi])$,and $\int_0^n \sin^2(x)dx$.
2. Compare $L_n(\cos^4(x), [0, \pi])$, $R_n(\cos^4(x), [0, \pi])$,and $\int_0^n \cos^4(x)dx$.
3. Compare $L_n(\cos(x), [0, \pi])$, $R_n(\cos(x), [0, \pi])$,and $\int_0^n \cos(x)dx$.

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Reality is in the Eye of the Beholder

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For centuries philosophers have attempted to discover the true nature of reality through a process of purely rational thinking, while at the same time finding themselves unable to ignore their sensory observations of the external world. As much as we trust our senses to reveal to us information about our experiences, the dependability of such a subjective form of exposure must not remain unquestioned. Is it possible for humans to determine whether an objective reality exists beyond our realm of experience or knowledge? The answers to these questions may lie in the field of mathematics. Philosophers have long entertained many differing views on whether or not the study of mathematics can yield a deeper understanding of the universe. Indeed, many of the most esteemed philosophers have also been renowned mathematicians, as well, probably because they discovered that this intriguing world of numbers and logic just might hold the answers to that ultimate philosophical challenge: the definition of *truth*.

Our search for knowledge often begins with our senses. The ability to touch, taste, hear, see, and smell the things around us provides us with immediate and tangible confirmation of a world external to ourselves. In fact, most people are convinced that their observations are all but infallible; many have developed such a strong dependence on their vision that they would even refuse to believe something unless they have seen it for themselves. These people fail to realize, however, that their sense of sight is perhaps the most fallacious mode of perception we possess. Some of the most common illusions we experience on a regular basis include what appears to be the bending of objects when immersed in water (Kline 29)[1], a pair of railroad tracks meeting at a point on the horizon (27), or the enlargement of the sun as it sets (29) –no ne of which are the case. Each of our other senses can create similar illusions bearing a strong semblance of reality.

Some philosophers maintain not only that man can rely on his senses to acquire knowledge about reality, but that senses are the *only* means by which this is even possible. These philosophers – called *empiricists* – claim that man's experiences determine his capacity for knowledge, and

that there is no world external to his sensory perceptions – a theory which all but eliminates the possibility of an objective reality, such as the one that mathematics claims to support. Therefore, according to the empirical view, man cannot gain understanding about any fields of study that lie beyond the scope of his five senses, and thus remains ignorant of such topics as the far reaches of outer space and even the molecular structure of his own body. This standpoint clearly attempts to abolish the existence of mathematical truths on the basis that mathematics purports to give man access to a reality beyond his existence. George Berkeley, a noted empiricist, claimed that even matter itself did not exist and that "all our knowledge is in the mind" (12); one must assume that mathematics is no exception.

This sense-dependent perspective challenges one of the foundational teachings in philosophy, which is attributed to the ancient Greek thinker, Plato. His doctrines, now collectively called *Platonism*, assert that the material objects of this world are mere imitations of the true reality – a realm of Ideas upon which our entire universe has been based (Maddy 21). Ideas, including those of mathematics, are not only real, but they are "more real than all the phenomena of nature" (Møller 106) because they resemble more accurately the nature of the Forms, which are "eternal and immutable" (88). The knowledge we acquire through our senses is subordinate to these Forms because, unlike mathematics, it employs no element of reason (107). From this theory, Plato developed his concept of innate ideas: knowledge that exists in the mind of man prior to any experience ("Innate ideas"). Supporters of this assertion would therefore claim that man understands mathematical law not because he has created it, but because it is the discovery of a higher form of truth, which he has known all along.

Innate ideas are closely tied to the concept of intuition: man's ability to acquire knowledge through reason, to understand things he has not directly experienced. Our examinations and perceptions of the "external world" allow us to develop self-evident ideas for which no proof is necessary and to which no reasonable person would object. We develop such beliefs at an early age; in general, no child has to be *taught* that an object may appear to be different from a different perspective, or that it still exists even when one cannot see it (Maddy 70). We trust our intuition to allow us to make generalizations about the world based on our experiences, but this can lead us to develop false conclusions because we often generalize about things we cannot experience. For example, if a man only saw white swans, he may develop a belief that all swans are white, despite the fact that he has not visually perceived *every* possible swan (67). A mathematics professor would assert that "an example is not a proof," yet many people still

base their conceptions of truth upon intuitive beliefs. Nevertheless, *probable* truth is not *actual* truth, and neither does belief *influence* truth, which shows that intuition cannot provide a guaranteed basis for the theories of mathematics.

We have now encountered a conflict: Is mathematics "all in our minds" –s omething we have *invented* in order to describe the phenomena we experience – or is it the true reality – the natural laws of the universe that exist despite our ability to perceive them? The best response may be that mathematics is a combination of both views. We must rely on both our observations of the world and our ability to understand the world through our mental capacities. Sensory experience is not completely dependable, but it is the basis for most of our knowledge about the universe. Some of our best mathematical theories rely on truths we could not have known had we not experienced them in nature, such as the Law of Contradiction, which states that something cannot be both x *and* not-x. According to Morris Kline, "Sensations from the external world supply the raw material that the mind organizes" (18). However, there are many factors that seem to affect our existence, but which we cannot experience directly. To account for these influences we rely on our intuition, which Descartes referred to as "the conception of an attentive mind, so clear, so distinct, and so effortless that we cannot doubt what we have so conceived" (Platonic Realms). Unfortunately, one can easily see that our intuitive beliefs are often no more certain than our perceptual beliefs because intuitions are generally established by our experiences (Hacking 175). Thus, in order to attain a proper understanding of the universe, we must depend on both our perceptions and our reasoning. Paul Bernays once claimed that "the two tendencies... complement each other, and it would be doing oneself violence to renounce one or the other" (qtd. in Isaacson 134).

Although neither our sensory experiences nor our intuitions are completely infallible, together they create a solid foundation upon which we can base the truths of mathematics. Mathematicians rely upon several key factors, such as axiomatic truths, in order to legitimize their claims. Axioms are assumptions – defined as accepted general truths or principles – which serve as the intuitive basis for almost every mathematical theory. Most philosophers agree with Aristotle's point that one must begin the search for knowledge at some starting point, and why not begin at the point where everyone agrees? Besides, statements such as "all quantities are equal to themselves" are actually based on reason, and therefore indisputably hold philosophical – as well as mathematical – value (Kline 47). All mathematical figures and entities, such as *circle* and *parallel*, are based on such assumptions.

Another reason mathematical theories are so powerful is that their conclusions are drawn through deductive reasoning, a process that shows how a statement necessarily follows from the undeniable assumptions one began with. Deduction, as defined by Ian Hacking, "requires the intuition of initial propositions and consequential steps" (173). This method of proof is similar to Aristotle's form of syllogistic logic: If all men are mortal, and Socrates is a man, then one can reasonably conclude that Socrates is mortal (Møller 112). A mathematician has no use for any other types of proof, such as inductive or analogical, which to him are only mere conjectures (Kline 48). He bases his search for truth on his conviction that nothing less than one hundred percent certainty will do, and neither unconfirmed generalizations nor equivalences by analogy can provide him with that type of security.

However, the mathematician must eventually place his well-conceived theories under the scrutiny of "the final authority": his observations (77). No matter how many people agree with his theory, nor how much support he has accumulated to back up his hypotheses, a mathematician's conclusions are completely ineffectual if they do not correspond to his examination of the external world. Johannes Kepler, the famous mathematician who developed a proof for the heliocentric theory, often devised what appeared to be sufficient explanations of the movements of the heavenly bodies. But he often "sacrificed his most beloved mathematical hypotheses when he saw that they did not fit observational data" (75). Once mathematical conclusions are shown to be consistent with our observations, those conclusions should also yield "accurate predictions" of phenomena in the natural world (76), such as the occurrence of an eclipse or the reproductive growth patterns of animals.

The structural elements of the mathematical system – axioms, deductions, and observations – establish a firm basis for the claims of mathematicians, but many other factors lend additional support to the fact that mathematical laws are true descriptions of a reality external to our minds. The *realist* view of mathematical philosophy assumes that "mathematics is the scientific study of objectively existing mathematical entities" (Maddy 21), but many opponents of this perspective (aptly called *anti-realists*) would claim that mathematics is simply a construct of the human mind – an invention to serve our scientific purposes. Keith Devlin, author of *The Math Gene*, questions the logic of this reasoning by demonstrating the difference between *invention* and *discovery*. He claims that, "although mathematics may involve an element of creativity, it is most certainly an overall discovery" (141). This approach to mathematical reality supposes that the entire body of theorems and proofs already exist and that man just happens to

stumble upon them, as opposed to the view that man himself creates them. For example, Devlin points out that if a certain mathematician had failed to develop one of his theories, that would not have nullified the fundamental truth of the theory, and some other mathematician would likely have discovered it eventually. The fact that different men can come up with the same mathematical ideas without consultation –s uch as the simultaneous development of the calculus by Newton and Leibniz (Maddy 22) – also supports Devlin's distinction. Thus, the realist view of mathematics can be considered philosophically sound.

One of the most significant arguments in further defense of the realist perspective is that mathematics is astonishingly applicable and useful to our lives, especially in the development of scientific theories. Mathematics reveals errors in our sensory observations and allows us to understand phenomena we cannot possibly experience directly, such as the concept of infinity. To the amazement of many scientists, mathematical models accurately describe and predict the courses of objects and events in the universe (Hamming 10). And although a current mathematical theory may serve no immediately foreseeable purpose, the historical trend has been that science will ultimately find one. As Morris Kline remarks in his book *Mathematics and the Physical World*, the Greeks studied mathematical curves 1500 years before Kepler and Galileo discovered uses for them (472). The Quine-Putnam Indispensability Argument supports this view as well, stating that "we are committed to the existence of mathematical objects because they are indispensable to our best theory of the world and we accept that theory" (Maddy 30). And a final argument in favor of the reliability of mathematics is its overall simplicity, and the fact that the least complex mathematical theory tends to be the most accurate. The medieval philosopher William of Ockham developed a theory now known as Ockham's Razor, which states that, all things being equal, the simplest explanation is the most logical. This concept drove Kepler to discard the historical theories of the state of the universe and develop a straightforward, mathematically sound heliocentric theory, which we now accept as truth. Indeed, as Gudder once said, "The essence of mathematics is not to make simple things complicated, but to make complicated things simple" (*Platonic Realms*).

The study of mathematics is like any other philosophical subject: Man poses his questions about the nature of reality and attempts to reason out the answers based on his experiences and intuitions. The appeal of mathematical theories in the search for knowledge is that they are supported by a balanced framework of sensory perceptions as well as innate ideas. Mathematicians' final conclusions are based on reliable, self-evident truths, for-

mulated through a system deductive reasoning, and then further verified by observable data. One can further rely on the knowledge claimed by mathematics because it consistently proves to be an accurate assessment of the universe, and also provides a simple explanation for the phenomena we experience. As Galileo pointed out, "the book of nature is written in mathematics" (Møller 203), so it constitutes our best approach to understanding reality.

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Archimedes' Spiral

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Presented at the 2003 National Convention

We often hear that Archimedes is one of the greatest mathematicians of all times. Archimedes knew and used his limited tools 22 centuries ago to accomplish what I will talk about today. Archimedes lived in Syracuse and he applied mathematics to the so-called "real world." Often a mathematical concept is given the name of a mathematician who had nothing to do with it. *Archimedes' Spiral* however is properly named after Archimedes since he seems to be the first one to have defined it.

Archimedes defined (see Figure 1) the spiral as a straight line in the plane

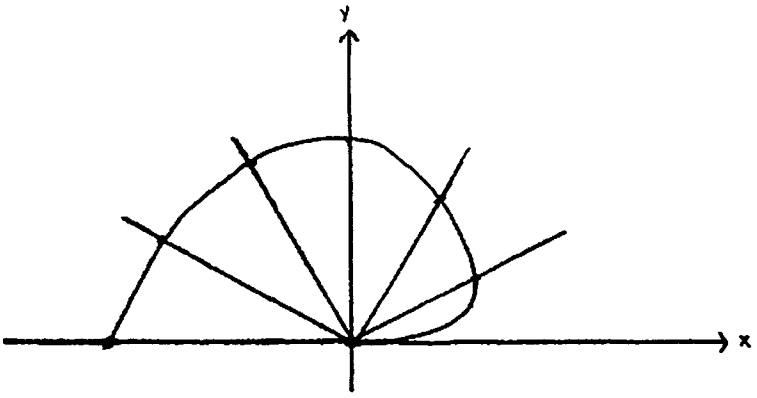

Figure 1: Archimedes' Spiral

that revolves at a constant rate about one of its ends (which remains fixed) and returns to its starting position. At the same time, a point moves at a constant rate along the moving line beginning at the fixed end, and it thus describes a spiral.

Most of us and those of you who had taken a geometry course have learned about the three problems of antiquity; one of them, if you recall, was that people in general and Greeks in particular could not trisect an arbitrary angle given a straight edge and a compass. But here comes Archimedes, the great Greek mathematician, who was able to trisect an angle using a straight edge and a compass provided he also had his spiral.

Using the above definition and the equation of the spiral in polar coordinates, which is $r = \theta$, we can see that there is a linear relation between the radius and the angle; in other words, as θ gets bigger, the radius gets larger. Therefore, since the angle is linear with the radius, divisions of the radius into n equal parts gives n equal angles on the spiral. So, let me try to show you how we can use the spiral as a geometric tool along with a straight edge and a compass to trisect an angle.

Using a straight edge and a compass, I can draw circles, lines, and also construct perpendicular lines. Take a line segment, place the compass on one edge and draw a circle of radius one, then draw another circle having the same radius, and where the two circles meet you can draw a perpendicular to the given line. Since I can draw perpendicular lines, I can also construct similar triangles where the ratio of the big triangle to the small one is of ratio 3:1 (see Figure 2).

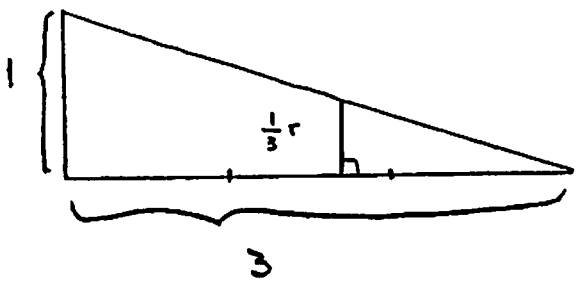

Figure 2: Constructing similar "3 to 1" triangles

If we have an angle θ and we also have in our possession Archimedes' spiral, we can go back to the spiral and mark that angle (see Figure 3). Then we take the radius r corresponding to θ and

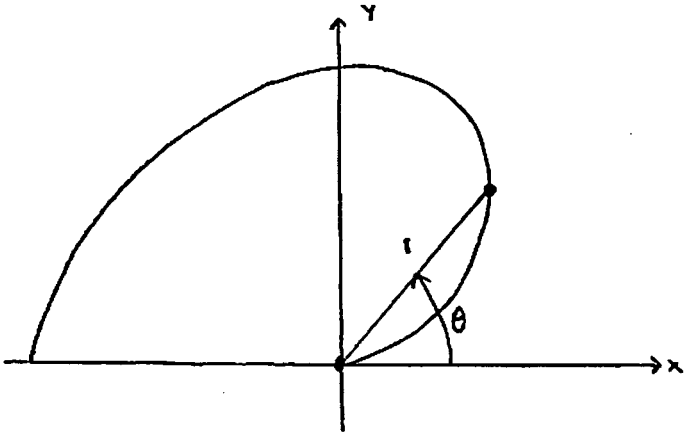

Figure 3: The angle θ and Archimedes' spiral

transpose it back to the similar triangles, and we can see that $r/3$ has been constructed. Now, we go back to the spiral, we draw a circle of radius $r/3$, and where this circle touches the spiral, that new angle is $\theta/3$ (see Figure 4).

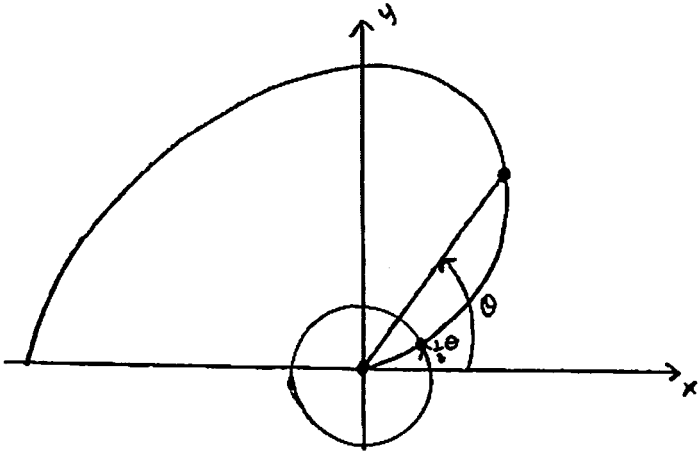

Figure 4: One-third r gives one-third θ

So, we were able to trisect an angle by using a straight edge, a compass and Archimedes' spiral.

Now, I would like to step forward and talk about the calculus of Archimedes' spiral. Archimedes' spiral is a curve and if we are given its parameters, we know that we can find the arc length of the curve and, further more, calculate its curvature.

To parameterize the curve we are better off working in rectangular coordinates (see Figure 5).

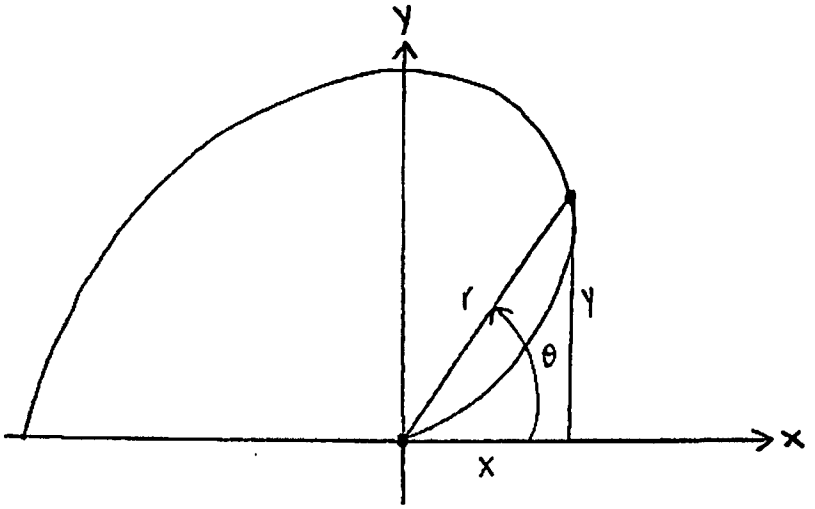

Figure 5: Changing to rectangular coordinates

So let us change from our polar equation $r = \theta$ for the spiral to the rectangular equation $r = \sqrt{x^2 + y^2}$, where $x = t\cos(t)$ and $y = t\sin(t)$. The arc length describes where the curve is in space and the equation for the arc length is given by

$$S = \int_a^b \sqrt{(x')^2 + (y')^2}\, dx\,,$$

where $x' = -t\sin(t) + \cos(t)$,

$$(x')^2 = t^2\sin^2(t) - 2t\sin(t)\cos(t) + \cos^2(t) \qquad (1)$$

and $y' = t\cos(t) + \sin(t)$,

$$(y')^2 = t^2\cos^2(t) + 2t\cos(t)\sin(t) + \sin^2(t) \qquad (2)$$

Adding (1) and (2) and combining terms using $\sin^2(t) + \cos^2(t) = 1$, we find that $S = \int_a^b \sqrt{t^2+1}\, dt$, which requires a trigonometric substitution to further solve. Letting $t = \tan(\theta)$, $dt = \sec^2(\theta)\, d\theta$ and $\sqrt{t^2+1} = \sqrt{\tan^2(\theta)+1} = \sec(\theta)$. Then $S = \int \sec(\theta)\cdot\sec^2(\theta)\, d\theta$ can be found using integration by parts to be

$$\frac{1}{2}\left(\sec(\theta)\tan(\theta) + \ln|\sec(\theta) + \tan(\theta)|\right)$$

so that

$$S = \frac{1}{2}\left(t\sqrt{t^2+1} + \ln\left|\sqrt{t^2+1} + t\right|\right)$$

An important use of the arc length is to find curvature. Curvature measures how sharply a curve bends. The curvature is given by the equation

$$\rho = \frac{(x')(y'') - (x'')(y')}{((x')^2 + (y')^2)^{3/2}}$$

where

$$\begin{aligned} x &= t\cos(t) \\ x' &= -t\sin(t) + \cos(t) \\ x'' &= -t\cos(t) - \sin(t) - \sin(t) = -t\cos(t) - 2\sin(t) \end{aligned}$$

and

$$\begin{aligned} y &= t\sin(t) \\ y' &= t\cos(t) + \sin(t) \\ y'' &= -t\sin(t) + \cos(t) + \cos(t) = -t\sin(t) + 2\cos(t) \end{aligned}$$

Then

$$\begin{aligned} (x')(y'') &= (-t\sin(t) + \cos(t))(-t\sin(t) + 2\cos(t)) \\ &= t^2\sin^2(t) - 2t\sin(t)\cos(t) - t\sin(t)\cos(t) + 2\cos^2(t) \\ &= t^2\sin^2(t) - 3t\sin(t)\cos(t) + 2\cos^2(t) \qquad (1) \end{aligned}$$

$$\begin{aligned} (x'')(y') &= (-t\cos(t) - 2\sin(t))(t\cos(t) + \sin(t)) \\ &= -t^2\cos^2(t) - t\sin(t)\cos(t) - 2t\sin(t)\cos(t) - 2\sin^2(t) \\ &= -t^2\cos^2(t) - 3t\sin(t)\cos(t) - 2\sin^2(t) \qquad (2) \end{aligned}$$

For the numerator of ρ, we subtract (2) from (1) and combine terms using $\sin^2(t) + \cos^2(t) = 1$ to get $t^2 + 2$. For the denominator, we use our arc-length calculation that $(x')^2 + (y')^2 = t^2 + 1$ to obtain $\left(t^2+1\right)^{3/2}$. Thus

$$\rho = \frac{t^2+2}{(t^2+1)^{3/2}},$$

showing us that the curvature of Archimedes' Spiral is positive and that as $t \to \infty$ the curvature $\rho \to 0$: The further out on the spiral, the more it is like a straight line.

An interesting physical application of Archimedes' spiral is found in sewing machines, in the part that spins the thread around the spool. The thread coming from the skein is kept taut and it is rolled around the spool, which turns and moves up and down in order to allow a uniform distribution of the thread. Now, the spiral comes in. To make sure that the thread is spun uniformly on all the parts of the spool, the oscillation movement needs to keep a constant speed. If the oscillation movement were faster in the center and slower towards the ends when the spool must change directions, the thread would not be distributed uniformly, but would tend to pile up at the spool extremities. Therefore, one needs a mechanism that makes the spool oscillate with a constant speed. This is made by regulating the oscillation spool through a profile made of two coupled spiral arcs (see Figure 6). The distance D, from the center of rotation to the tip of the pin

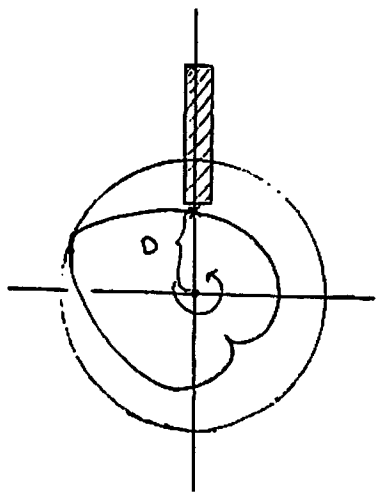

Figure 6: Converting circular motion to linear motion

resting on the spiral, is how the spinning motion is related to the vertical position of the point $(t\cos(t), t\sin(t))$. Using the distance formula, $D = \sqrt{(t\cos(t))^2 + (t\sin(t))^2} = t$, the rate of change of the vertical position of the pin is $dD/dt = 1$, showing that there is a constant change in position. If we take the second derivative, we get that the acceleration equals zero, and thus the tension is kept constant! And so we can certainly say that Archimedes' spiral can be used to convert uniform angular motion to uniform linear motion.

It is very interesting to learn how someone who lived so many centuries ago was able to accomplish so much using only the limited tools available to him. It is fascinating how we cannot trisect an angle using a straight edge and a compass alone, but this trisection becomes possible by using Archimedes' spiral. Since we are dealing with a curve, we can find its arc length and its curvature. Furthermore, we see how mathematical objects can have applications in physics.

Acknowledgements. This paper is an extension of the Mathematics Seminar ("MTH 90") presentation I prepared under the direction of Dr. Andrew M. Rockett and gave at the C.W. Post Campus of Long Island University in December 2002.

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KME Website

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The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before January 1, 2004. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 2004 issue of *The Pentagon*, with credit being given to the student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: ken.wilke@washburn.edu).

PROBLEMS 570-574

Problem 570. Proposed by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.

Let ABC be a triangle and H its orthocenter. Prove that $(AH + BH + CH)\sqrt{3} = AB + BC + CA$ if and only if at least one of the angles of triangle ABC is 60°.

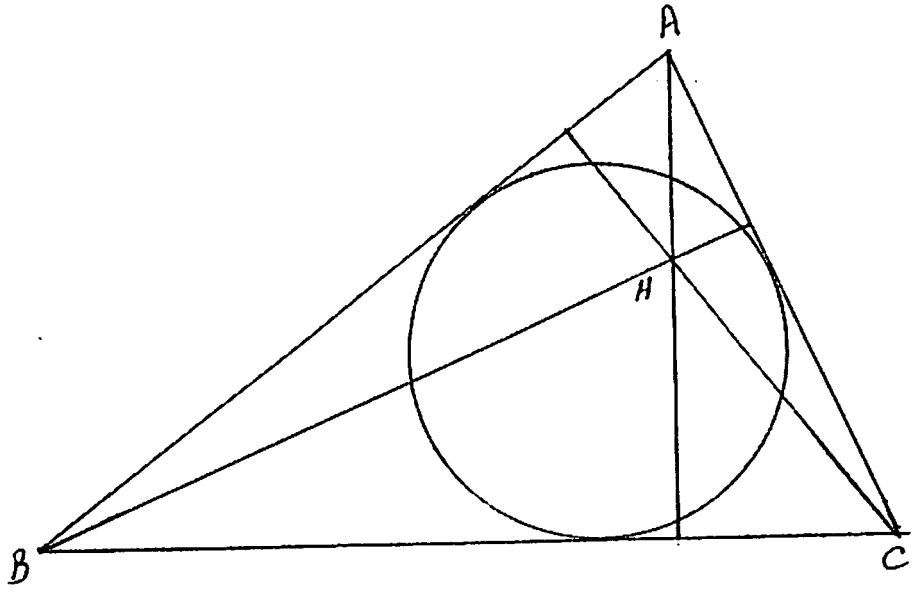

Figure for Problem 570

Problem 571. Proposed by James R. Bush, Waynesburg College, Waynesburg, Pennsylvania.

Find all values of a such that $x^2 - x + a$ divides $x^{15} + 27x^3 + 688x^2 - 1236x - 1092$.

Problem 572. Proposed by Jose Luis Diaz-Barrero, Universitat Politeccnica de Catalunya, Barcelona, Spain.

Prove that $\left(\sum_{k=1}^{n} \cosh x_k\right)^2 + \left(\sum_{k=1}^{n} \sinh x_k\right)^2 \geq n^2$
where $x_k \in \mathbb{R}, k = 1, 2, ..., n$.

Problem 573. Proposed by Pat Costello, Eastern Kentucky University, Richmond, Kentucky.

A pair of numbers is called amicable if the sum of the proper divisors of the first number is equal to the second number and vice versa. One way of constructing amicable pairs is to develop a "Thabit-Rule" which is just a set of values determines by a single input that need to be checked for primality. If all values turn out to be primes, they can be put together in some way to form an amicable pair.

Bill discovered the following "Thabit-Rule": if $p = 11 \cdot 2^n - 1, q = 2p - 2, r = 4p - 1$ and $s = (q + 1)(r + 1) - 1$ are all primes, then the numbers $2^{n+2}pqr$ and $2^{n+2}ps$ forms an amicable pair. He was so excited because he quickly calculated that for $n = 2$, $p = 43$ which is prime. His excitement waned when he computed $q = 85$ which is not prime. He was very disappointed when it turned out that not all four of the values, p, q, r, and s are prime. Find an amicable pair using Bill's "Thabit-Rule" or show that none exist.

Problem 574. Proposed by Thomas Chu, Austin, Texas.

Evaluate $(OA^2 - OB^2)(OB^2 - OC^2)(OC^2 - OA^2)$ in terms of sides a, b, and c of the triangle ABC, where OA, OB, and OC are the respective distances from the incenter O to the vertices of the triangle.

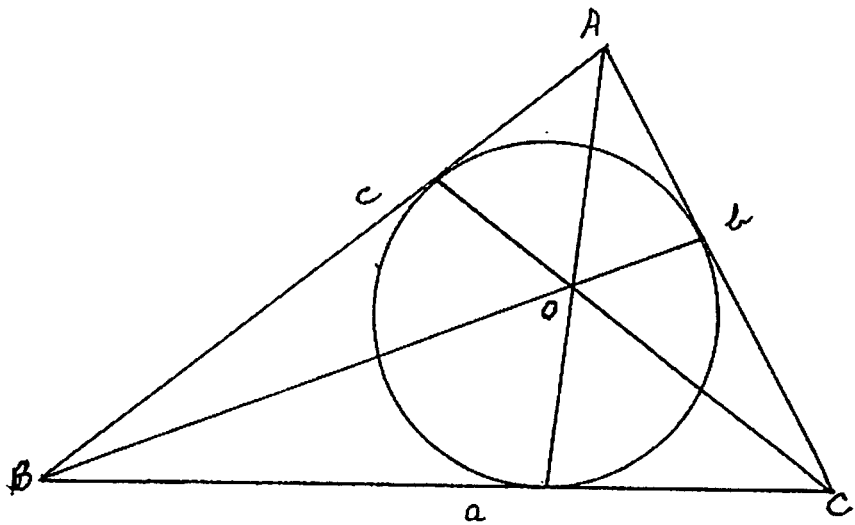

Figure for Problem 574

The editor wishes to acknowledge that Ian Martin, student, Waynesburg College, Waynesburg, Pennsylvania, was inadvertently omitted from the list of solvers for problem 555. Also Ryan Bates, student, Waynesburg College, Waynesburg, Pennsylvania was inadvertently omitted from the list of solvers for problem 556. The editor apologizes for any inconvenience this has caused.

Please help your editor by submitting problem proposals.

SOLUTIONS 560 – 564

Problem 560. Proposed by Jose Luis Diaz-Barrero, Universitat Politeccnica de Catalunya, Barcelona, Spain.

Let $\{a_n\}, n \in \mathbb{N}$ $(a_n > 0)$, be an arithmetic progression. Evaluate the sum $\sum_{n=1}^{\infty} \frac{1}{a_n a_{n+1} a_{n+2}}$.

Solution by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.

Let $a_{n+1} = a_n + r$ where $r \neq 0$. Then $a_{n+2} = a_{n+1} + r = a_n + 2r$.
Let $\frac{1}{a_n a_{n+1} a_{n+2}} = \frac{A}{a_n} + \frac{B}{a_{n+1}} + \frac{C}{a_{n+2}}$. Then
$1 = Aa_{n+1}a_{n+2} + Ba_n a_{n+2} + Ca_n a_{n+1} = A(a_n + r)(a_n + 2r) +$
$Ba_n(a_n + 2r) + Ca_n(a_n + r)$
$= a_n^2(A + B + C) + ra_n(3A + 2B + C) + 2r^2 A.$
Thus we have to solve the system of equations
$A + B + C = 0$, $r(3A + 2B + C) = 0$ and $2r^2 A = 1$.
Thus $A = \frac{1}{2r^2}$. Then $0 = A + B + C = 3A + 2B + C$ implies that
$2A + B = 0$
so that $B = \frac{-1}{r^2}$ and finally $C = -A - B = \frac{1}{2r^2}$.
Then $\frac{1}{a_n a_{n+1} a_{n+2}} = \frac{1}{2r^2}\left(\frac{1}{a_n} - \frac{2}{a_{n+1}} + \frac{1}{a_{n+2}}\right)$.
Let $Sn = \sum_{k=1}^{n} \frac{1}{a_k a_{k+1} a_{k+2}}$.
Then $Sn = \sum_{k=1}^{n} \frac{1}{a_k a_{k+1} a_{k+2}} = \frac{1}{2r^2} \sum_{k=1}^{n} \left(\frac{1}{a_n} - \frac{2}{a_{n+1}} + \frac{1}{a_{n+2}}\right)$
$= \frac{1}{2r^2}\left(\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_{n+1}} - \frac{1}{a_{n+2}}\right)$.
Finally $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{2r^2}\left(\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_{n+1}} - \frac{1}{a_{n+2}}\right)$
$= \frac{1}{2r^2}\left(\frac{1}{a_1} - \frac{1}{a_2}\right) = \frac{a_2 - a_1}{2r^2 a_1 a_2} = \frac{r}{2r^2 a_1 a_2} = \frac{1}{2ra_1 a_2}$
Also solved by the proposer.

Problem 561. Proposed by Jose Luis Diaz-Barrero, Universitat Politeccnica de Catalunya, Barcelona, Spain.

Determine all positive integers n such that $\frac{F_1}{\sqrt{F_2}+\sqrt{F_3}} + \frac{F_2}{\sqrt{F_3}+\sqrt{F_4}} + \frac{F_3}{\sqrt{F_4}+\sqrt{F_5}} + \cdots + \frac{F_{n-1}}{\sqrt{F_n}+\sqrt{F_{n+1}}} = n$
where F_n is the n^{th} Fibonacci number; i.e. $F_0 = 0$, $F_1 = 1$ and for $n > 2$, $F_n = F_{n-1} + F_{n-2}$.

Solution by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri.

Rationalizing the denominators and using the identity $\frac{F_i}{F_{i+2}-F_{i+1}} = -1$, gives $\sum_{i=1}^{n-1} \left(\sqrt{F_{i+2}} - \sqrt{F_{i+1}}\right) = n$ which is equivalent to $F_{i+1} - \sqrt{1} = n$. Thus $F_{n+1} = (n+1)^2$ which holds for only $n = 0$ or 11. Since n is a positive integer, $n = 11$. To complete the proof, we show that $F_{k+1} > (k+1)^2$ for all integers $k > 12$, $F_{k+1} = F_{13} = 233169 = 13^2$. Now assume that $F_{k+1} > (k+1)^2 + F_k$. But since $F_k > 2k+3$ whenever $k \geq 6$, we have $(k+1)^2 + F_k > k^2 + 2k + 1 + (2k+3) = (k+2)^2$. Hence $F_{k+2} > (k+2)^2$ for all integers $k > 12$ follows by mathematical induction. [It is easy to verify that for $2 \leq k \leq 11 F_k < k^2$. Ed.]

Also solved by the proposer.

Problem 562. Proposed by Pat Costello, Eastern Kentucky University, Richmond, Kentucky.

Show that $1 + 5^n + 5^{2n} + 5^{3n} + 5^{4n}$ is divisible by 11 whenever n, a positive integer, is not divisible by 5.

Solution by David Profili, McDaniel College, Westminster, Maryland.

The first few powers of 5 are: 1, 5, 25, 125, 625, and 3125. Reduced (*mod*11) these become respectively 1, 5, 3, 4, 9, and 1. Since $5^0 \equiv 1 (\text{mod } 11)$ it is clear that the pattern 1, 5, 3, 4, and 9 repeats itself; i.e., $n \equiv 1 (\text{mod } 5) \Rightarrow 5^n \equiv 5 (\text{mod } 11)$, $n \equiv 2 (\text{mod } 5) \Rightarrow 5^{2n} \equiv 3 (\text{mod } 11)$, $n \equiv 3 (\text{mod } 5) \Rightarrow 5^{3n} \equiv 4 (\text{mod } 11)$ and $n \equiv 4 (\text{mod } 5) \Rightarrow 5^{4n} \equiv 9 (\text{mod } 11)$. Note that the numbers 1, 5, 3, 4, and 9 for a cyclic group of order 5 under the operation of multiplication (mod 11). Since n is not divisible by 5, $n \equiv 1, 2, 3, 4 (\text{mod } 5)$ so that $5^n \equiv 5, 3, 4$, or 9 (mod 11). Rewriting the number $1 + 5^n + 5^{2n} + 5^{3n} + 5^{4n}$ as $1 + 5^n + (5^n)^2 + (5^n)^3 + (5^n)^4$ it is clear that each of these numbers in the sum reduce to exactly one of the numbers 1, 5, 3, 4, and 9 (although not necessarily in that order) so that the sum $1 + 5^n + 5^{2n} + 5^{3n} + 5^{4n} = 22 \equiv 0 (\text{mod } 11)$.

Solution by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri.

Since $5^5 = 3125 \equiv 1 (\text{mod } 11)$, $5^{5n} \equiv 1 (\text{mod } 11)$ for every positive integer n.

Thus $11 \mid (5^{5n}-1)$ which yields $11 \mid (5^n-1)(1+5^n+5^{2n}+5^{3n}+5^{4n})$. Assume that $11 \mid (5^n-1)$. Then since $5 \nmid n$, let $n = 5k+i$ where $i = 1, 2, 3$, or 4. Now $5^{5k+i}-1 = 11r$ for some integer r. Hence $5^i(3125^k-1) = 11r+1-5^i$. Since $11 \mid (3125^k-1)$, it follows that $11 \mid (1-5^i)$. But this leads to a contradiction for $i = 1, 2, 3, 4$. Hence $11 \mid (1+5^n+5^{2n}+5^{3n}+5^{4n})$ for all positive integers n that are not divisible by 5.

Also solved by Charles Ashbacher, Hiawatha, Iowa; Pat Costello, Eastern Kentucky University, Richmond, Kentucky;Je remiah Vedis, Waynesburg College, Waynesburg, Pennsylvania and the proposer.

Editor's Comment: In our second featured solution, the contradiction is established easily by trying to verify that $11 \mid (1-5^i)$ for $i = 1, 2, 3, 4$.

Problem 563. Proposed by the editor.

(a) Define $f(n)$ as the result of writing the positive integers in order; e.g. $f(1) = 1$, $f(3) = 123$, $f(10) = 12345678910$, and so on. What is the smallest integer n such that $f(n)$ is divisible by 241?

(b) Define $g(n)$ as the result of writing the positive integers in order;e .g. $g(1) = 1$, $g(3) = 321$, $g(10) = 10987654321$, and so on. What is the smallest integer n such that $f(n)$ is divisible by 241?

(This problem was suggested by Puzzle 188 from Carlos Rivera's website The Prime Puzzles & Problems Connection at www.primepuzzles.net and appears here with Mr.Rivera's kind permission.)

Solution by Pat Costello, Eastern Kentucky University, Richmond, Kentucky.

For part (a), test the values for $n = 1, 2, ..., 9$ and find that $f(n)$ is not divisible by 241. Then run the following UBASIC program.

```
10    S = 123456789
20    For I = 10 to 99
30    S = S*100 + I
40    If S@241 = 0 then print I: end
50    Next I
60    For I = 100 to 999
70    S = S*1000 + I
80    If S@241 = 0 then print I: end
90    Next I
100    End
```

And find that $n = 318$ is the smallest value of n where $f(n)$ is divisible by 241.

For part (b), test the values for $n = 1, 2, ..., 9$ and find that $g(n)$ is not divisible by 241. Then run the following UBASIC program.

```
10    S = 987654321
20    For I = 10 to 99
30    S = S*100 + I
40    If S@241 = 0 then print I: end
50    Next I
60    For I = 100 to 999
70    S = S*1000 + I
80    If S@241 = 0 then print I: end
90    Next I
100   End
```

(Note that additional loops can be added as necessary by following the pattern of lines 20 through 60 in either program).
And find that $n = 403$ is the smallest value of n where $g(n)$ is divisible by 241.

Also solved by Charles Ashbacher, Hiawatha, Iowa. One incorrect solution was received.

Problem 564. Proposed by the editor.

Find the smallest positive integer n such that $n/2$ is a perfect square, $n/3$ is a perfect cube, $n/5$ is a perfect fifth power and $n/7$ is a perfect seventh power.

Solution by Abby Fijas, Jamie Gulvin, Michael Johnston, and Christina Ruscio (jointly as members of the Honors Problem Solving Class), SUNY Fredonia, Fredonia, New York.

The solution must be of the form $n = 2^a 3^b 5^c 7^d$ since the smallest positive integer solution will have only the prim factors 2, 3, 5, and 7. Notice that $\frac{n}{2}$ is a perfect square so that 2 divides each of $a - 1, b, c$, and d. Similarly 3 divides each of $a, b - 1, c$, and d; 5 divides each of $a, b, c - 1$, and d and 7 divides each of a, b, c, and $d - 1$. Thus, in order to find the smallest positive value of n satisfying these criteria, a must be the least common multiple of 3, 5, and 7 which is one more than a multiple of 2; i.e. $a = 105$. Similarly b must be the least common multiple of 2, 5, and 7 which is one more than a multiple of 3; i.e. $b = 70$. Then c must be the least common multiple of 2, 3, and 7 which is one more than a multiple of 5; i.e., $c = 126$. Finally d must be the least common multiple of 2, 3, and 5 which is one more than a multiple of 7; i.e., $d = 120$.

Thus the answer is $n = 2^{105}3^{70}5^{126}7^{120}$.

Also solved by Pat Costello, Eastern Kentucky University, Richmond, Kentucky; Carl Libis, University of Rhode Island, Kingston, Rhode Island; James Perkins, Waynesburg College, Waynesburg, Pennsylvania; Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri and Aaron Scroggins, Northeastern State University, Tahlequah, Oklahoma.

Editor's Comment: All of the solutions received followed essentially the same idea. The featured solution was chosen for its crispness of presentation.

Starting a KME Chapter

For complete information on starting a KME chapter, contact the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of the $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offering and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

Kappa Mu Epsilon News

Edited by Connie Schrock, Historian
Updated Information as of September 2003

News of chapter activities and other noteworthy KME events should be sent to schrockc@emporia.edu or to

Connie Schrock, KME Historian
Mathematics Department
Emporia State University
1200 Commercial Street
Campus Box 4027
Emporia, KS 66801

Chapter News

AL Gamma
University of Montevallo

Other spring 2003 officers: Don Alexander, Corresponding Secretary.
New Initiates: Laura O'Dell

AL Zeta Chapter President – Scott Asher
Birmingham Southern 19 Actives, 12 New Members

Other spring 2003 officers: Isaac Dooley, Vice President; Ellen Segrest, Secretary/Treasurer; Mary Jane Turner, Corresponding Secretary.

Initiation for new members was held on May 12, 2003. There were 12 initiates. The ceremony was attended by faculty and members of KME. A reception was held in honor of the new members.

New Initiates: Ryan Bevis, Wesley Brown, Christa Calvert, Whitney Curvin, Sarah Gaskin, Emily Hunter, Meredith Kirkpatrick, Petra Kositzke, Wiley Truss, Mary Turner, Daniel Van Hauen, Lori Wijayasuriya.

CA Epsilon
California Baptist University 19 Actives

Other spring 2003 officers: Catherine Kong, Corresponding Secretary.

New Initiates: Jessica Aceves, Tawni Covington, Holly Curran, Amy Dutcher, Katy Howard, Derek Imai, Lisa Jackson, David Kelley, Elizabeth Matthews, Jessica Miller, Jeffrey Mulari, Sharon Olson, Michelle Pace, Brett Peterson, Kao Saechao, Philip Sanelle, F. Ryan Williams, Jim Buchholz, Elizabeth Morris.

CO Gamma
Fort Lewis College

Other spring 2003 officers: Deborah Berrier, Corresponding Secretary.

New Initiates: Lindsay Bush, Chris Colbert, Brad Gelljing, Aaron Jurkovich, Elisa Leonardo, Erin Pomeroy, Gretchen Tracy, Alex Turnipseed, Schuyler Engelenburg, Crystal Young.

CO Delta
Mesa State College

Other spring 2003 officers: Kim Schneider, Corresponding Secretary.

New Initiates: Diane Krieg, Jake Lewis, Randall Martens, Joel McBride, Mendy Osborn, Wayn Ward, Rebecca Watt, Peter Williams.

FL Beta Chapter President – Louis J. Rufo
Florida Southern College 14 Actives, 16 New Members

Other spring 2003 officers: Lindsay Stambaugh, Vice President; Houda A. Darwaiche, Secretary/Treasurer; Allen Wuertz, Corresponding Secretary.

GA Alpha Chapter President – Jessica Pritchett
State University of West Georgia 12 Actives, 15 New Members

Other spring 2003 officers: Chad Matthews, Vice President; Jessica Caldwell, Secretary; J.J. Wahl, Treasurer; Dr. Joe Sharp, Corresponding Secretary.

The Georgia Alpha Chapter of KME held its annual initiation meeting on April 16, 2003, and initiated 15 new members. This is the largest pledge class in the history of our chapter! After the initiation ceremony, we elected the Chapter Officers for 2003-2004 those are listed earlier on this report and attended a reception in honor of the 2003 initiates. At the reception, the names of the scholarship and award winners for 2003-2004 were announced (all of whom are members of KME): Bryan Stamps won the Burson Calculus Award, the Boyd Award, and one of the Marion Crider Awards; Chad Mathews won one of the Marion Crider Awards; Jessica Pritchett won one of the Marion Crider Awards;Danielle Ratzlaff won the Vachel Davis Whatley, Jr. Scholarship; Brian Bockelman won the Georgia Martin Scholarship; and Keisha Wilkey won the Chatty Pittman Scholarship.

New Initiates: Chikaodili Anyikire, Brian Bockelman, Jessica Caldwell, Clayton Culver, Erin Cook, Robert Graham Jr., Danielle Ratzlaff, Sharon Reed, Richard Reid, Craig Simpson, Rachel Trammell, Chris Williams, Marina Yevsikova, Florin Zidaru.

GA Delta Chapter President – Lauren Mobley
Berry College 30 Actives, 12 New Members

Other spring 2003 officers: Kevin Gammon, Vice President; Dana Roberts, Secretary; Melissa Mailloux, Treasurer; Ron Taylor, Corresponding Secretary.

New Initiates: Jennifer Dean, Anna Fincher, Kevin Gammon, Matthew Lewis, Melissa Mailloux, Lauren Mobley, Anna Morrison, Dana Roberts, Robyn Smith, David Squires, Michael VanSant, Jacqueline Black, Benjamin Chupp, Sarah Gowder, Kristin Johnson, William Leonard, Mathew Marsico, Heather Moore, Marju Purin, Dayna Rickett, Cary Sanders, Caio Soares, Matthew Wilson.

IA Alpha Chapter President – Sara Buchheim
University of Northern Iowa 38 Actives, 6 New Members

Other spring 2003 officers: Chad Tompkins, Vice President; Cindee Calton, Secretary; David Gisch, Treasurer; Mark Ecker, Corresponding Secretary

Student member David Gisch presented his paper "Isometries of Euclidean 3-Space Into Itself" at our first spring KME meeting on February 5, 2003 at Professor Mark Ecker's residence. Our second meeting was held on March 5, 2003 at Professor Ben Schafer's residence where Ben Wadsley presented his student paper on "Probabilistic Analysis of the NCAA Final Four". Faculty advisor, Mark Ecker, along with students Sara Buchheim and Ben Wadsley traveled to the KME National Convention at Oral Roberts University in Tulsa from March 27-29, 2003 where Sara presented her paper "Significant Predictors for UNI GPA's". Student member Jimmy Brito presented his paper, "Benford's Law: The First Digit Phenomenon" at our third meeting on April 9, 2003 at Professor Russ Campbell's residence. Student member James Mills addressed the spring initiation banquet with "Warning: Isometries are closer than they appear". Our banquet was held at La Fiesta restaurant on April 30, 2003 where six new members were initiated.

New Initiates: Michelle Boelman, Vanessa Kasch, Sara McCarty, Melissa Potter, Aaron Wilson, Matt Wood

IA Delta Chapter President – Wei (Peter) Yang
Wartburg College 51 Actives, 20 New Members

Other spring 2003 officers: Nicholas Wuertz, Vice President; Kristin Granchalek, Secretary; Mark Giesmann, Treasurer; Dr. Brian Birgen, Corresponding Secretary.

The Wartburg Chapter of KME held meetings on February 13, March 15, and May 20, 2003. The February meeting program consisted of upper classpersons sharing information about the upper division math courses that they experienced at Wartburg. The Chapter elected their officers for the next year and initiated 20 new members into the chapter at the March meeting. Some members had intended to attend the national meeting but later found that they had conflicts so nobody represented the chapter this year. A joint year-end picnic with the computer club and the physics club was the May meeting of the Iowa Delta Chapter of KME.

IL Beta

Western Illinois University

Other spring 2003 officers: Alan Bishop, Corresponding Secretary.

New Initiates: Amanda Cadieu, Emily Cilik, Mathew Johnson, John LeFager.

IL Zeta Chapter President – Theresa Meshes

Dominican University 16 Actives, 13 New Members

Other spring 2003 officers: Jennifer Soldat, Vice President; Melissa Sima, Secretary; James Mazorick, Treasurer, Sara Ziesler, Corresponding Secretary.

The Illinois Zeta Chapter of KME was involved in many activities this past spring including:

- Volunteer Peer Tutoring program: this spring we offered a record number of tutoring hours: 8 hours a week.
- Problem Solving Contest – Two problems were given out every two weeks throughout the academic year. Prizes were awarded for the best solutions. The contest is open to all undergraduate students, although the problems are distributed primarily in mathematics courses.
- Annual Induction Ceremony – The ceremony was held on April 6. This year's presentation included 2 alumnae speakers who are currently using their mathematical skills in their careers. This gives our new and current members an opportunity to learn about the uses of mathematics after graduating.
- On March 14th, Pi Day, we organized some celebrations. These included the movie A Beautiful Mind (cosponsored with Campus Activities), at which we gave out free pizza and drink; free pie and competitions from 11:30-1:30.

- This year the theme for Mathematics Awareness Month (April) was Mathematics and Art. We organized an exhibition of posters on this theme. This exhibition was in the Social Hall and Dining Hall and was on display for the whole of the month of April. The artists included Escher, Kandinksy, Brunelleschi, Charles Perry, Leonardo Da Vinci, and George Hart.
- This spring we began running Games Nights every other Wednesday. Students met and played board games from 10pm-12am. We provided free snacks, and participants gave us a $1 donation for charity.
- There was an annual end of the year party at a faculty member's house.

IN Delta
University of Evansville

Other spring 2003 officers: Joe Stickles, Jr., Corresponding Secretary.

New Initiates: Brian Alberding, Lyndsay Brown, Timothy DeBaillie, Christopher Elpers, Julie Elpers, Jeremy Funk, Lindsay Garrett, Samuel Glesting, David Haas, Eric Hahn, Brittney Harlan, Justin Higgs, Bradley Kempf, Elizabeth Krofcheck, Amy Lochmondy, Julie Lochmondy, Justin Martin, Eric Maurer, Casey Morford, Jesse Reiherzer, Eric Roberson, David Shelton, Karen Spindler, Valerie Stringer, Nathaniel Walizer, Marcus Wassmer, Rachel Wright.

KS Alpha
Pittsburg State University

Chapter President – Keith Smeltz
25 Actives, 18 New Members

Other spring 2003 officers: Libby Wonderly, Vice President; Jamie Fairbanks, Secretary; Meltem Tugut, Treasurer; Dr. Tim Flood, Corresponding Secretary.

The spring initiation was held May 6, 2003. Eighteen New Members were initiated. Following the initiation was the annual homemade ice-cream social hosted this year by Dr. Elwyn Davis and his wife Karen.

New Initiates: Rakshit Amba, Michael Brackett, Gates Brown, Kenneth Etzel, III, Jamie Fairbanks, Matthew Hill, Matthew Howard, Masahiro Itagaki, Loren Karleskint, Mamiko Nishida, William Pierce, Kunal Sood, Eric Sornson, Harsha Subramanya, Melton Tugut, Donavan Wheeler, Elizabeth Wonderly, Lindsey Carter.

KS Beta Chapter President – Melinda Born
Emporia State University

Other spring 2003 officers: Mindy Baker, Vice President; Leah Childers, Secretary/Treasurers; Connie S. Schrock, Corresponding Secretary.

The Kansas Beta Chapter of KME enjoyed an active semester in spring 2003. Four faculty and two student members attend the national convention at Oral Roberts University. Mindy Baker presented her paper, "Fibonacci Polynomials: Pascal Comes Through Again." Also, Leah Childers presented her paper, "Geometric Contortions: Exploring Elementary Functions on the Complex Plane." At our biannual induction ceremony Dr. G.P. Youvaraj spoke and the entire group enjoyed cookies and pizza.

KS Delta Chapter President – Zeb Kramer
Washburn University 30 Actives, 8 New Members

Other fall 2002 officers: Jeff Kingman, Vice President; Mary Noel, Secretary/Treasurer; Allan Riveland, Corresponding Secretary.

The Kansas Delta chapter of KME met for three luncheon meetings with the Washburn Math club during the semester. Speakers and/or mathematics presentations were part of the meetings. The chapter's annual KME Initiation Banquet was held on Feb. 17, 2003 with 11 new initiates. Students Jeff Kingman, Zeb Kramer, and Fred Hollingshead accompanied faculty AL Riveland and Kevin Charlwood to the National KME Convention at OK Delta in Tulsa, OK in March of 2003.

KS Gamma Chapter President – David Livingston
Benedictine College 15 Actives, 2 New Members

Other spring 2003 officers: Max Botta, Vice President; Andrea Archer, Secretary; Erin Stretton, Treasurer; Erin Stretton, Stu-Co Rep; Jo Ann Fellin, OSB, Corresponding Secretary.

On February 12, KS Gamma initiated two national members and ten associate members. The group enjoyed a spaghetti dinner following initiation. The spring semester featured many presentations. At the Faculty Colloquium in March, Sister Linda Herndon, OSB presented her doctoral research on "S. Mary Theresa Brentano: Innovator in the use of Magnetic Audio Tapes." In early April, two students talked about their directed research on Discovery Day at the college. Nikki Bruskerhoff did a power point presentation on "Fractals in nature" while Christina Hoverson demonstrated her work on "Exploring Geometry Using Sketchpad." Later in the month senior David Livingston presented his directed research on "chess, Linear Transformations, and Group Theory. In late April, Guest Speaker Art Janssen from Emporia State University gave a talk to a large gathering of math and economics students illustrating the relationship be-

tween these two fields of study. Three students and three faculty represented KS Gamma at the national convention in Tulsa. Co-recipients of Sister Helen Sullivan scholarships included Andrea Archer, Max Botta, Erin Stretton, Christina Hoverson, and Jon Baxa. They received certificates at the Honors Banquet on April 30. The grand finale for KS Gamma was its traditional cookout. This year it culminated Study Day on May 7th.

KY Alpha Chapter President – Shannon Hanner
Eastern Kentucky University 20 Actives

Other spring 2003 officers: Frank Donnelly, Vice President; DeAnna Shearer, Secretary; Kristen Barnard, Treasurer; Pat Costello, Corresponding Secretary.

At the January meeting, we elected a new Vice-President and discussed plans for the remainder of the year. Plans included designing and printing a T-shirt, participating in the Big Brothers/Big Sisters Bowl for Kid's Sake fund-raising event, and attending the national convention in Tulsa. The meeting at the beginning of March dealt with finalizing plans for attending the convention and initiation. Five students traveled by Enterprise van to the convention with Dr. C. The trip required an overnight stop in Evansville, IN on the way down and an overnight stop in Rolla, MO on the way back. A stop in St. Louis on the way back included going up the tram to the top of the Gateway Arch. Initiation of new members was held at the April meeting. Dr. Kirk Jones gave a talk entitled "Triangular Numbers." He explained how the talk was the result of trying to work with his daughter on a homework project for school. He showed several ways to get the formula for triangular numbers. He also showed many properties that these numbers have.

KY Beta Chapter President – Anthony Laschon
Cumberland College 27 Actives, 7 New Members

Other spring 2003 officers: Rose Olson, Vice President; John Nichols, Secretary; Vito Wagner, Treasurer, Jonathan Ramey, Corresponding Secretary.

On February 28, 2003, the Kentucky Beta chapter held an initiation and a joint banquet with Sigma Pi Sigma, physics honor society at the Atrium. Kappa Mu Epsilon inducted six new student members and one new faculty member at the banquet. presided over by outgoing President Anthony Laschon. As an additional feature, senior awards were given by the department at the banquet.

Jointly with the Mathematics and Physics Club, the Kentucky Beta Chapter hosted Dr. Bruce Kessler from W4estern Kentucky University on April 15. He spoke on "A Mathematical Glimpse of the Universe." On

April 16, members also assisted in hosting a regional high school math contest, held annually at Cumberland College. On May 2, the entire department, including the Math and Physics Club, Sigma Pi Sigma (Physics Honors Society), and the Kentucky Beta Chapter, held the annual spring picnic/cookout in Dr. Ramey's backyard.

LA Delta Chapter President – April Jeffcoat
University of Louisiana at Monroe 10 Actives

Other spring 2003 officers: Stephanie Hillhouse Welch, Vice President; Ashley Nero, Secretary; Sharee Davis, Treasurer; Jane Wampler, Corresponding Secretary.

The Louisiana Delta Chapter met twice during the spring semester for a pizza party and faculty "meet and greet". The second meeting included a talk by prominent woman mathematician Jane Curry who spoke about Women in Science. We also had an end of the semester party with ACM (the computer science honor club). We held our initiation this semester in which 16 new members were initiated. The University President and Provost were among our guests. Some of our members represented KME at several different events in the semester. One event was an "Ozzeball" tournament during ULM's Spring Fever week. Students along with faculty competed against other organizations in a volleyball type game played in two feet of mud. Although we didn't win the tournament, lots of fun was had by all.

MA Alpha Chapter President – Jonathan Katcher
Assumption College 12 Actives, 7 New Members

Other spring 2003 officers: Jennifer Genovevo, Vice President; Laura Beesley, Secretary; Rebecca Gladu, Treasurer; Charles Brusard, Corresponding Secretary.

Seven new members were initiated on May 1, 2003. Following the initiation ceremony, student members Laura Beesley, Scott Bouleter, and Jennifer Susel spoke on their research project "Constructing a bijective proof of the q-t-Catalan."

MD Alpha Chapter President – Carrie Kelley
College of Notre Dame of Maryland 8 Actives, 17 New Members

Other spring 2003 officers: Danielle Hughes, Vice President; Katrina Drew, Treasurer; Margaret Sullivan, Corresponding Secretary.

At our induction ceremony on April 27, 2003, a team of undergraduates from Johns Hopkins University presented their experiment and experience resulting from their participation in the NASA low-gravity experiment program. Their presentation was entitled "JHU STILL Mix Experiment" (Surface Tension Impelled Low-Gravity Liquid Mixing). They were with us

just two weeks after they had traveled to Texas to fly 18 "parabolas" in the low-gravity plane during which they conducted their surface tension experiments

New Initiates: Sarah Baxter, Jessica Brockmeyer, Meranda Byers, Rebecca Dodson, Katrina Drew, Swathi Gaddam, Danielle Hughes, Menew Karami, Amara Keldsen, Carrie Kelley, Naoko Kawamura, Melissa Martinez, Jin Yu Ou, Qiao Pan, Lauren Pecora, Laura Shirk, Zobia Shuaib.

MD Delta Chapter President –Brendon LaBuz
Frostburg State University 22 Actives

Other spring 2003 officers: Kandi Wertz, Vice President; Jacilyn Brant, Secretary; Crystal Beeman, Treasurer; Mark Hughes, Faculty Sponsor; Edward White, Corresponding Secretary.

The spring 2003 semester was a very active one for the Maryland Delta Chapter. We started the semester with an organizational meeting in February. March 14 brought our thirds annual celebration of Pi Day. Members contributed items for a very successful bake sale. The prize of a cherry pie was awarded to a student who won our contest by reciting 118 digits of Pi. The following week, Dr. Mark Hughes presented a talk on the brachistochrone problem. Our April meeting feature a talk by Dr. Gerald Wojnar that provided an introduction to category theory. During this meeting, we also held an election for next year's officers. Our officers-elect are Matthew Miller (President), Sherry Hartman (Vice President), Christopher Smoot (Secretary), and Dustin Robinson (Treasurer). As with the last year, our end-of-semester picnic had to be moved indoors due to the weather, but a good time was had by all. We wish all the best for our outgoing officers, all of whom will be graduating. In particular, outgoing President Brendon LaBuz will be continuing his mathematical studies as a graduate student at the University of Tennessee in Knoxville.

MO Alpha Chapter President – Robert Sorey
Southwest Missouri State University 45 Active Members, 10 New Members

Other spring 2003 officers: James Hayes, Vice President; Andrea Streff, Secretary; Melissa Schmidt, Treasurer;J ohn Kubicek, Corresponding Secretary.

The Missouri Alpha Chapter held monthly meetings and hosted the annual Mathematics Department Spring Banquet. The monthly meetings included presentations by two faculty, Dr. Jorge Rebaza and Dr. Kishor Shah, and presentations by two students, James Hayes and Rima Freeman.

MO Beta Chapter President – Elizabeth Jurshak
Central Missouri State University 20 Actives, 9 New Members

Other spring 2003 officers: Andrew Ray, Vice President; Brent Hoover, Secretary; Jacob Dubray, Treasurer; Andrew Nahlik, Historian; Rhonda McKee, Corresponding Secretary.

Missouri Beta chapter had a busy spring semester. In February, SRA Becky Jaimes, air traffic controller at Whiteman Air Force Base, spoke to the group about her job and how it is related to mathematics. Initiation was held in March. Six new full members and one associate member were initiated. Students Elizabeth Jurshak and Norman Elliott presented papers at the March meeting. They presented papers at the March meeting. They presented the same papers later that month at the national convention in Tulsa, OK. Elizabeth's paper was one of the top three papers at the convention. A total of eight students and two faculty members attended the convention. Pizza was served at the April meeting and elections for the 2003-2004 officers were held. Also, Jacob Dubray was announced as the winner of the Claude H. Brown Achievement Award for outstanding senior. An end-of-the-year picnic was held at Cave Hollow Park on May 1. Volleyball was the highlight of the evening.

MO Lambda Chapter President – Amy Lynn Kerling
Missouri Western State College 35 Actives, 15 New Members

Other spring 2003 officers: Gabe Wishnie, Vice President; Nicholas Limle, Secretary; James Blevins, Treasurer; Don Vestal, Corresponding Secretary.

Meetings were held monthly. In January we held elections for officers and initiated new members. In February we had our semester social at the home of Don Tosh. In late March, Don Tosh and three students – Kevin reed, Alex Rijo, and Joshua Wood – attended the national convention in Tulsa. In April, for our last meeting of the year, we tried a new format – a midweek evening pizza party followed by games. It was the best-attended meeting of the semester. We are going to continue with that format in the fall.

MO Zeta
University of Missouri, Rolla

Other spring 2003 officers: Roger Hering, Corresponding Secretary.

New Initiates: Philip Alt, Christine AuBuchon, Ed Bosanquet, Garrett Euler, Robert Fischer, Jennifer Feltcher, Stephen Kooi, Michelle Kuykendal, Seth Ledbetter, Adam Quigley, Shakila Ratnaraj, Sibila Ratnaraj, Benjamin Rhew, Steve Samuel, Erik Timpson, Nathan Tool, Dustin Trinkle, Lisa Wulff.

MS Alpha Chapter President – Shannon McVay
Mississippi University for Women 13 Actives, 4 New Members

Other spring 2003 officers: Henry Boateng, Vice President; Amy Ladner, Secretary; Sara Sheffield, Treasurer; Dr. Shaochen Yang, Corresponding Secretary.

The Mississippi Alpha chapter of KME had a busy spring semester including the first general meeting on January 30, a movie night watching a "Big Fat Greek Wedding" on February 4, Initiation on February 12, the next general meeting on February 26, an outing entitled "Little Tokyo" on March 24, a game night on April 8, and the final general meeting on April 16, 2003.

MS Epsilon Chapter President – Amy Pearson
Delta State University 13 Actives

Other spring 2003 officers: Becky Moore, Vice President; Beth Crisco, Secretary/Treasurer; Paula A. Norris, Corresponding Secretary.

NE Alpha Chapter President – Rochelle Swanson
Wayne State College 20 Actives, 5 New Members

Other spring 2003 officers: Dan Rasmussen, Vice President; Becky Cech, Secretary; Kristin Brudigam, Treasurer; Dr. John Fuelberth, Corresponding Secretary.

NE Beta
University of Nebraska, Kearney

Other spring 2003 officers: Katherine Kime, Corresponding Secretary.

New Initiates: Rodrigo Segura, Kevin Vogel, Matthew Culver, Sarah Dobbe, Richard Garrelts.

NE Delta Chapter President – Diana Faesser
Nebraska Wesleyan 19 Actives

Other spring 2003 officers: Angela Miller, Vice President; Val Stehlik, Secretary/Treasurer; Melissa Erdmann, Corresponding Secretary.

We were pleased to host Joe Gallian as a speaker in February. Twelve new members were inducted at a picnic in April.

New Initiates: Mary Arthur, Diana Faesser, Nicole Heath, Chad Katzberg, Kimberly Nathan, Kassandra Racek, Bradford Randazzo, Nickolas Scott, Valerie Stehlik, Kristin Pfabe, Taixi Xu, Melissa Erdmann.

NE Gamma
Nebraska Wesleyan University

Other spring 2003 officers: Melissa Erdmann, Corresponding Secretary.

New Initiates: Marcia Fiorelli, Jennifer Marie Rodin, Jason Sayers,

Trevor Schmidt, Stephanie Spahn, Larry Evans, Ashlie Hein, Gary Isom, Lucas Krakow, Nathan Lanik, Lori Pfingsten, Chris Seebohm, Mary Werner, Nathan Zobell, Scott Isom, Jennifer Warriner, Kayan Cary, Philip Cary, Angala Miller, Starla Tronstad, Josh Gathman, Rico Cobos, Kelly Kuzel.

NJ Beta

Montclair State University

Other spring 2003 officers: John G. Stevens, Corresponding Secretary.

New Initiates: Caroline Kamowski, Jessica Marini, Shweta Seshadri, David Stein.

NJ Gamma Chapter President – Katie Blackburn

Monmouth University 18 Actives

Other spring 2003 officers: Melissa McCormick, Vice President; Melissa Berfield, Secretary; Stephanie Beatty, Treasurer; Amanda Glynn, Historian; Judy Toubin, Corresponding Secretary.

The NJ Gamma Chapter of KME at Monmouth University was very successful in the Spring 2003 semester. The main focus of this semester was to organize this honor society and set precedents for induction procedures and community projects to be used for future years to come. This semester the national KME constitution was worked on and finalized. The Monmouth University constitution is still in progress. Over the summer months, the new officers of KME will be working to finish this before the Fall 2003 semester begins. The job tasks for each officer were also finalized this semester.

The KME induction ceremony for Spring 2003 semester occupied the majority of the honor society's time and energies. The Vice-President worked on a systematic process of narrowing the possible inductees who met all the standard requirements. Since this semester's induction members were larger than the previous year, a bigger room was needed. The officers spent time finding an appropriate-sized room and planned sufficient amounts of finger foods for the ceremony with MU's dining company. Also, the officers decided to make this ceremony more involved and prestigious than the previous year. Thus, hand-made invitations, programs, and table decorations were created. The Induction ceremony was held on Sunday, April 6, 2003 in Anacon Hall of Monmouth University. There was a guest speaker and family and friends were in attendance. This event was very successful.

Besides the Induction ceremony, the officers worked diligently on putting together community service projects for the KME members. This semester we collected over 5lbs of soda tabs to be donated to the Ronald McDonald House. We would like to continue this project in future years as well.

The officers discussed and brainstormed other community projects to be completed in the next semester.

Also, this semester, KME had a difficult time attempting to receive a budget from Monmouth University. Thus, the officers came up with a few projects as fund-raisers. One successful project was a candy fund-raiser in which 2 jars of candy were bought and for $1.00 people could guess how many were in each jar. The person who guessed the right amount of candy won that jar of candy. KME raised approximately $60.00. Other projects were discussed and recorded to use for next semester.

The officers felt it would be beneficial to have a few gatherings in which the new members had the opportunity to mingle and become acquainted with the current members. The money we raised was put towards holding a party for this occasion, which was held during the last month of the semester. Also, at this time, elections were held for the next year. The process of elections was also finalized.

Last but not least, the officers were encouraged by their faculty members to have educational forums held by various professors. Because of time constraints, only one was held this semester in which a faculty member presented on the topic of "Strings and Things." Next year, the officers are hoping to hold more of these mathematical sessions for the KME members as well as other interested students. Also, the officers discussed the idea of helping out at local schools with tutoring programs and/or holding mathematical contests.

NY Eta Chapter President – Amanda Masset
Niagara University 15 Actives

Other fall 2002 officers: Michelle Searles, Vice President; Marc Erickson, Secretary; Michael Bidzerkowny, Treasurer; Robert Bailey, Corresponding Secretary.

NY Iota
Pace University 20 Actives

Other spring 2003 officers: Geraldine Taiani, Corresponding Secretary.

NY Lambda Chapter President – Agathi Michael
C.W. Post Campus of Long Island University 16 Actives, 10 New Members

Other spring 2003 officers: Faith E. Signorile, Vice President; Sarah K. Corea, Secretary; Todd M. Tomeo, Treasurer; Dr. Andrew M. Rockett, Corresponding Secretary.

Nine students and one faculty member were initiated into the New York Lambda Chapter by the chapter officers during our annual banquet at the Greenvale Town House restaurant on the evening of March 23rd, bringing the Chapter membership to 235. Dr. Sharon Kunoff shared reminiscences

of her KME experiences as an undergraduate in the 1950's, the founding of our chapter and the first initiation in 1983, as well as trips with students to regional and national conventions and her years as business manager of The Pentagon while Dr. Andrew M. Rockett served as editor. Agathi Michael's after-dinner presentation on "Archimedes' Spiral" was the "dress rehearsal" for her presentation at the KME national convention held at Oral Roberts University in Tulsa, OK, March 27-29. Our evening concluded with the announcement of the 2002-2003 departmental awards: the Claire F. Adler Award and the Lena Sharney Memorial Award to Agathi Michael, the Joseph Panzeca Memorial Award to Daniel F. Swanson, and the Hubert B. Huntley Memorial Award to Michelle A. Rivera; the Dean Schmidt Graduate Scholarship Award to Michael J. Tiano; and the presentation by Dr. James V. Peters of MAA students memberships to Jill H. Kahan, Agathi Michael, Michelle Rivera, Faith E. Signorile, Daniel F. Swanson, and Michael J. Tiano.

NY Nu Chapter President – Alam Lamkin
Hartwick College 16 Actives, 6 New Members

Other spring 2003 officers: Jeremy Freeland, Vice President; Ezekiel Miller, Secretary; Jennifer Slade, Treasurer; Ron Brzenk, Corresponding Secretary.

OH Alpha
Bowling Green State University

Other spring 2003 officers: David Meel, Corresponding Secretary.

New Initiates: Sarah Bach, Veronica Burns, Heather Denlinger, Nathan Ellis, Carl Fayerwather, Seth Gilman, Todd Griffen, Katherine Grosswiler, Ken Hoskins, Clarissa Kinzel, Nathan Lowien, Dominic Marchi, Bryan McDaniel, Ross Muir, Jason Reissig, Jessica Schlaich, Carl Schlatter, Elizabeth Suchocki, Steven Walker, Tracy Wharton, Greg Wisniewski, Cameron Wright.

OH Epsilon
Marietta College

Other spring 2003 officers: John Michel, Corresponding Secretary.

New Initiates: Seth Avery, Danielle Cisler, Maria Cohenour, William Coniglio, Joshua Doak, Jennifer Foley, William Hohmann, Cody Lane, Holly McDaniel, Cheryl Pasquale, Derrick Pottmeyer, Trevor Schaffer, Emily Simms, Stephanie Snyder, Julianne Spaley, Joseph Starkey, Casey Trail, Lowell Warden III, Nicholas Kunze, Brian Stebbins, Brian Samol.

OH Eta
Ohio Northern University

Other spring 2003 officers: Donald Hunt, Corresponding Secretary.

New Initiates: Jeanine Arnold, Michelle Bash, Nathan Baxter, Elyse Cocco, Brad Crombie, Jarrett Dickey, Matthew Gerber, Julie Holda, William Humphries, IV, Michael Hylton, Amanda Kalal, Matthew Katschke, Jennifer Kirkham, Brandon Koester, Eleanor Kurtz, James Leemhuis, Christina Leidel, Kara Lewis, Cliton Luiso, Deborah McMechan, Jeffrey Meyer, Ashley Middelberg, Katey Nieset, Dustin Park, Katherine Peplowski, Ariane Petersen, Trevor Presgrave, Benjamkin Rowland, Justin Stiles, Jennifer Szippi, Matthew Tompary, Justin Vaught, Robert Withrow.

OH Gamma
Baldwin-Wallace College

Other spring 2003 officers: David Calvis, Corresponding Secretary.

New Initiates: Joseph Abraham, Heather Arnold, Tanya Biggs, Diana Bisesi, Jeremy Christman, Quinn Conley, Lauren Crandall, Julie Durkot, Khalid Farhood, Lisa Fox, Alex Furst, Allison Hegedus, Kerry Helm, Heidi Hist, Mary Kester, Erika Macho, Erin Mohar, Alison Plotts, Heather Ramsey, Ryan Ricklic, Jonathan Skuza, Kathryn Streiff, Plousia Vassilaras, Justin Yates.

OH Zeta
Muskingum College

Other spring 2003 officers: Richard Daquilla, Corresponding Secretary.

New Initiates: Gabriela Caruso, Cheryl Cole, Andrew Dean, Erin Klenk, Kurt Kontur, Albert Morell, Chad Parker, Richard Shawger, Jennifer Shay, Megan Strahota, Marquis Waller, Scott Ziegler.

OK Alpha Chapter President – Joe Gonzales
Northeastern State University 31 Actives, 7 New Members

Other spring 2003 officers: Stephanie Hilburn, Vice President; Katie McClure, Secretary; Amy Rose, Treasurer; Dr. Joan E. Bell, Corresponding Secretary.

Our spring initiation brought 16 new students into the Oklahoma Alpha Chapter. Three students (Joe Gonzales, Miri Whisnant, and Bryanne Weaver) and two faculty members (Dr. Joan E. Bell and Mr. Max Ellis) attended the 34th Biennial Convention of KME in Tulsa. Our spring speaker was Dr. Elwyn Davis, Mathematics Department Chair at Pittsburg State University. He did a hands-on demonstration of Riemann geometry using Lenart spheres. Close to 50 students/faculty attended the talk, which was followed by the annual spring ice cream social.

PA Alpha Chapter President – Sarah Plimpton
Westminster College 23 Actives, 8 New Members

Other spring 2003 officers: Heather Klink, Vice President; Danielle Zielinski, Secretary; Jessalyn Smith, Treasurer; Carolyn Cuff, Corresponding Secretary.

KME PA Alpha had an end of the year banquet honoring all graduating senior and initiating new members. The annual pizza party was held during finals.

New Initiates: Amy Putas, Nicole Potocnak, Christina Carlson, Bradley Patton, Thomas Spencer, Emily Wehr, Jollene Weister, Robert Wilson, Bradley Merklin, Lorrie Straka.

PA Delta
Marywood University

Other spring 2003 officers: Robert Ann Von Ahnen, Corresponding Secretary.

New Initiates: Sara Dzwonchyk, Eric Sponza.

PA Gamma Chapter President – Shannon Romeo
Waynesburg College 10 Actives, 11 New Members

Other spring 2003 officers: James Perkins, Vice President; Karen Montgomery, Secretary; Dr. James R. Bush, Corresponding Secretary.

The PA Gamma Chapter at Waynesburg College is planning a fall trip to the National Cryptologic Museum in Columbia, MD.

New Initiates: Kyla Deitz, Aaron LaBatte, Ryan Lawrence, James Lewis, Karen Montgomery, James Perkins, Zhanna Petrosyan, Shannon Romeo, Nathan Torboli, Jeremiah Vedis, Dr. Linda Moore.

PA Iota Chapter President – Lea Lantzy
Holy Family College 27 Actives, 3 New Members

Other spring 2003 officers: J. Russell Leidy, Vice President; Erica Condravy, Secretary; Matt Curran, Treasurer; Kimberly Presser, Corresponding Secretary.

This semester the club continued with their Adopt-a-Highway project from previous years.

The club started a new project under the leadership of Dr. James Hamblin called Sidewalk Math. This was tutoring for finals week held outside of the building using portable chalkboards. There were snacks and music and students were offered tutoring on any subject in mathematics. We hope that this remains a tradition here at Shippensburg University.

The students in the club continued to run a tutoring program for 3 evening each week to address the needs created by cutbacks in the university sponsoring tutoring. This was a very successful venture on their

part and much appreciated by the mathematics department here at Shippensburg University. They expanded the material covered in tutoring to include some of the courses that math majors take in their sophomore year here at Shippensburg.

The initiation ceremony/dinner was again a success. We have strengthened the responsibilities of members in our new constitution, so the club hopes to have more of an impact on the university and the department in the coming year.

New Initiates: Panagiota Asimakakis, Judith Canner, Elizabeth Corr.

PA Lambda Chapter President – Kate Wright
Bloomsburg University 30 Actives, 9 New Members

Other spring 2003 officers: Amy Tribendis, Vice President; Carlene Streit, Secretary; Steve Gentener, Treasurer; Elizabeth Mauch, Corresponding Secretary.

PA Mu
Saint Francis University

Other spring 2003 officers: Peter Skoner, Corresponding Secretary.

New Initiates: Ken Balough, Tracy Bender, Bridget Campbell, Shannon Decker, Haley Hawk, Antal Nemeth, David Staley, Lesley Wenzel, Tsegaselassie Workalehmahu, Tristan Cope.

PA Pi Chapter President – Davlyn Nauman
Slippery Rock University 10 Actives, 3 New Members

Other spring 2003 officers: Megan McKinney, Vice President; Justin Mashuda, Secretary/Treasurer; Elise Grabner, Corresponding Secretary.

PA Theta
Susquehanna University

Other spring 2003 officers: Carol Harrison, Corresponding Secretary.

New Initiates: Nile Abdel-Salam, Amanda Cherian, Alicia Ciotola, William Conklin, Michael Evans, Matthew Gaul, Suzanne Jacobson, Angela Long, Thomas Moore, Erica Nelson, Alison Pedley, Trevor Reeder, Jennifer Simpkins, Nicholas Woody, Stephen Zaharuk.

SD Alpha
Northern State University

Other spring 2003 officers: Raj Markanda, Corresponding Secretary.

New Initiates: Jon Tieszen, Gail Johnson, Jennifer Dolejsi, Dan Aspelund, Amber Mehaffey, Carin Rambow.

TN Alpha

Tennessee Tech University

Other spring 2003 officers: Michael Allen, Corresponding Secretary.

New Initiates: Jason Brunner, Katherine Fanning, Hunter Hughes, Charles Lanning III, Mary Kidd, Ritesh Motipera, Eric Pride, Chad Thompson, George Titsworth, Shane Walling, Natalie White.

TN Delta Chapter President – Elizabeth A. Weaver

Carson-Newman College 4 Actives, 4 New Members

Other spring 2003 officers: Marci Mitchell, Vice President; Chad Ramsey, Secretary; Houston Qualls, Treasurer; B.A. Starnes, Corresponding Secretary.

The Carson-Newman KME had a relatively quiet year. In the fall a social was held at the home of Dr. Starnes. A good time was had by all. The spring semester, however, featured nominal student involvement. At the end of the semester Dr. Starnes, Dr. Herring (dept. chairman) and Chad Ramsey worked on invited new members in. As a result, four new members were received. They will be inducted into the chapter next fall at a special ceremony.

TN Epsilon

Bethel College 3 Actives

Other spring 2003 officers: Russell Holder, Corresponding Secretary.

TN Gamma Chapter President –Breanne Oldham

Union University 18 Actives, 5 New Members

Other spring 2003 officers: Nikki Vassar, Vice President; Allen Smith, Secretary; Amanda Cary, Treasurer; Bryan Dawson, Corresponding Secretary.

Another semester, another tornado! Next year's KME students may have to estimate the probability of that happening in consecutive semesters. Our chapter's initiation banquet took place on April 7, 2003 at the Old Country Store. Carroll Griffin, an alumnus who was initiated into our chapter in the late 1960s, was the banquet speaker. We also held an end-of-the-year get-together along with our campus' chapter of ACM. At that meeting many awards were announced, including next year's Joe Tucker Scholarship recipient, Brian Taylor. The highlight of our semester, however, was the trip to the national convention in Tulsa. Two faculty and two students from our chapter attended;Rob ert Rickett presented his talk "Chaos". .

New Initiates: Sarah Campbell, Willie George, Ruth Hall, Dustin Rapp, Milan Zivkovic.

TX Alpha
Texas Tech University

Other spring 2003 officers: Anatoly Korchagin, Corresponding Secretary.

New Initiates: Sonya Sherrod, Britni Oaks, Jared Cole, Bryan McDaniel, Catherine Trogolo, Joshua Crabtree, Najib Nawas, Josh George, Matthew Wilson, Kenneth Jemmott, Jr., Michael Peterson, Christopher Bryant, Geffrey Jensen, Blake Baccigalopi, Colin Jennings, Matthew Vanderschaaf, Mitzi McNeese, Ying Chen, John Pinckney, Colin Dudley, John Garza, Stephanie Alley, Cassie Crews, Kenneth Kanipe, Jamie Millisor, Lina Alvarez, Andrew Ha, Joshua Brinkley, David Bua, Brenna Belew.

TX Eta Chapter President – Melissa Easley
Hardin-Simmons University 10 New Members

Other spring 2003 officers: Katie Wooten, Vice President; Kerra LeBlanc, Secretary/Treasurer; Frances Renfroe, Corresponding Secretary.

The 28th annual induction ceremony for the Texas Eta Chapter was held on March 27, 2003. There were ten new members: Rachel Dickerson, Joseph Fanning, Ty Lang, Kerra LeBlanc, Jason McCall, Amberlee Martin, Joshua Montgomery, Cynthia Pfau, Melissa Schaeffer, and Larry Smedley. With the induction of these members, membership in the local chapter stands at 233.

Leading the inducting ceremonies were President Melissa Easley and Vice President Katie Wooten. Following the induction ceremony, membership shingles and pins were presented to the 2002 inductees. Changing the format of our induction ceremony, KME then adjourned and the members, inductees, and chapter sponsors enjoyed pizza and cold drinks.

Newly elected chapter officers for the 2003-2004 year are: President Larry Smedley, Vice President Katie Wooten, and Secretary/Treasurer Kerra LeBlanc. Dr. Ed Hewett, Dr. James Ochoa, and Dr. Andrew Potter are chapter sponsors. Frances Renfroe is the corresponding secretary of the chapter.

New Initiates: Rachel Dickerson, Joseph Fanning, Ty Lang, Kerra LeBlanc Jay McCall, Amberlee Martin, Joshua Montgomery, Cynthia Pfau, Melissa Schaeffer, Larry Smedley.

TX Iota Chapter President – Anthony Joeris
McMurry University 20 Actives, 9 New Members

Other spring 2003 officers: Tracy Parker, Vice President; Katrina Brooks, Secretary; Jennifer Agirre, Treasurer; Kelly McCoun, Corresponding Secretary.

New Initiates: Jeremy Brown, Crisstie Crim, Kayla Gaskins-Dean, John Kniffen, Christopher Pratt, Kaci Rybolt, Diana Sheppard, Heather Sullivan, Dr. Kathryn Flores.

VA Gamma Chapter President – Ray Cain
Liberty University 183 Actives, 17 New Members

Other spring 2003 officers: Elizabeth Starkey, Vice President; Justin Hall, Secretary; Jackie Henry, Treasurer; Dr. Glyn K Wooldridge, Corresponding Secretary.

New Initiates: Mary Anna Brown, Raymond Cain, Amy Galvez, Steven Gregorin, Abigail Hagar, Justin Hall, Jacqueline Henry, Kelly Kalbach, Dana Kline, Danielle McNaney, William Meers, Joel Moylan, Benjamin Pittman, Melanie Pridgen, Megan Smith, Adrian Varon, Thuy Vu

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Are you an undergraduate student planning on attending graduate school?

Check out a free listing with the Registry of Undergraduate Researchers and Graduate Schools, sponsored by the Council on Undergraduate Research. The purpose of this registry is to facilitate match-making between undergraduates who have research experience with graduate schools seeking high quality students who are well prepared for research.

Any undergraduate may go to www.cur.org/ugreg/ to fill out a simple curriculum vitae form. There is no charge to you as a student. Student information records will be made available ONLY to bona fide Graduate Schools that contract with CUR for this service. Graduate School representatives may contact students to invite applications or visits to the campus and laboratory, or to share information about their research programs and financial support opportunities. .

If you are a student in your junior or senior year, REGISTER FOR FREE NOW! Juniors will be able to update their listing at the end of the summer and during their Senior year, to include any summer research experience or information about Senior Theses and test scores.

For additional information about the Registry, please visit http://www.cur.org/UGRegistryselect.html. Contact CUR at cur@cur.org or 202-783-4810 if you have any questions.

Report of the 34th National Biennial Convention

First Business Meeting was held on Friday, March 28, 2003. Rhonda McKee, national secretary, called the chapter roll. Seventy-eight students, 40 faculty sponsors and 1 guest were present, for a total of 119 participants. Twenty-two chapters were represented.

Charles Curtis moved that a vote be taken at this convention on a new chapter petition from California Baptist University. The motion was seconded and passed. The petition itself was then voted on and unanimously approved. The chapter at California Baptist University will be known as California Epsilon.

Pat Costello, chair of the nominating committee introduced the candidates for national treasurer and secretary. The candidates for treasurer were John Kubicek and Cynthia Woodburn. Only one candidate for secretary, Rhonda McKee, was nominated by the committee. Vitas for the candidates were distributed.

Final Business Meeting was held on Saturday, March 29, 2003. Each of the national officers gave a report. Al Riveland was presented a plaque in honor of his eight years of service as national treasurer, 1995-2003. Reports of the student sectional meeting and of the faculty sectional meeting were given. The auditing committee then gave their report. The committee recommended that KME accept the financial records and reports provided by treasurer Al Riveland. A motion was made, seconded and passed. The committee also recommended that Al Riveland be congratulated for excellent service to KME. A motion to that effect was also made, seconded and passed. The resolution committee presented their report.

Election of officers was next on the agenda. President Bob Bailey announced that other nominations for secretary would be accepted from the floor. After a brief silence, Jo Ann Fellin moved that Rhonda McKee be elected by acclamation. Connie Schrock seconded the motion, which passed. Secret ballots were distributed for the election of treasurer. After ballots were counted it was announced that John Kubicek was elected.

President Bob Bailey announced that he would accept invitations to host the 2005 national convention. No invitations were immediately extended, so President Bailey asked that chapters discuss the possibility with sponsors and faculty when they get back home.

Seventeen students from ten chapters presented fourteen papers throughout the two days. The authors were invited to submit their papers for publication in *The Pentagon*, which is our official journal. Listed in order of

presentation, they were:

"*Portfolio Theory: A Constrained Optimization Problem*"
Yana Levchenko and Susan Wurster, Oklahoma Delta
Oral Roberts University

"*A Mathematical Model of Airport Efficiency*"
Tracy Goering, Missouri Kappa
Drury University

"*Geometric Contortions of the Comples Plane: Exploring Elementary Complex Functions*"
Leah Childers, Kansas Beta
Emporia State University

"*Archimedes Spiral*"
Agathi Michael, New York Lambda
C.W. Post Campus of Long Island University

"*An Investigation of the Set of ANS Numbers*"
Norman Elliot, Missouri Beta
Central Missouri State University

"*Significant Predictors for UNI GPA*"
Sara Buchheim, Iowa Alpha
University of Northern Iowa

"*Laplace Transformations: A Model of Column Chromatography*"
Abigail Wade and Susan Wurster, OK Delta
Oral Roberts University

"*Reality is in the Eye of the Beholder*"
Tiffany Judkins, TX Mu
Schreiner University

"*A Proof of Admiral Peary's Expedition*"
Stephen Dickey, Missouri Kappa
Drury University

"*Something Unexpected*"
April Egli, PA Lambda
Bloomsburg University

"*Fibonacci and Other Polynomials: Pascal Comes Through Again*"
Mindy Baker, Kansas Beta
Emporia State University

"Analyzing a Spirograph Shape with an outer Ellipse and Inner Rotating Circular Wheel"
Elizabeth Kay Jurshak, MO Beta
Central Missouri State University

"Chaos"
Robert Rickett, TN Gamma
Union University

"A Program Algorithm for Boolean Expression Simplification with Karnaugh Maps"
Joel Votaw and Franklin Daniel Reed, OK Delta
Oral Roberts University

All student paper presenters were called up to the stage and were presented with KME mugs. Don Tosh and the awards committee then announced the award-winning paper presenters. The four award winners were Mindy Baker of Kansas Beta chapter, Elizabeth Jurshak of Missouri Beta chapter, Stephen Dickey of Missouri Kappa and Tracy Goering of Missouri Kappa. Each award winner was presented with a TI 2000 Voyager calculator and a $100 check.

The newly elected officers, Rhonda McKee and John Kubicek, were then officially installed by President Bob Bailey.

Written reports of all the officers, along with travel allowance checks, were distributed to each chapter and then the meeting adjourned.

Report of the National President

My first two years as President have been a rather exciting learning experience. I have been ably assisted in this effort by discussions with former Presidents and by the rest of the national officers. Within this past biennium, one new chapter has been installed. It is New Jersey Gamma at Monmouth University (installed by John Stevens, April 21, 2002). Two chapters, which had been deactivated, have been reactivated. They are Texas Gamma at Texas Women's College and Alabama Alpha at Athens University. The latter school was originally installed as Athens College in March 1942. The current number of active chapters now stands at 131. Please take the opportunity to congratulate these three chapters as well as other chapters you interact with in your region. It is encouraging to me that we can add to the list of active chapters not only with new chapters, but also by those, which, for one reason or another, have previously been deactivated.

Recently, two petitions have been received. They are from California Baptist University (Riverside, CA) and Thiel College (Greenville, PA).

The National Council has approved of the first petition, so we will be able to act on this petition at this convention. Your chapter should have already received a summary sheet about California Baptist University to consider prior to this convention. The petition from Thiel College arrived only a few days ago, so it will be considered at a later date.

I have corresponded with several other colleges and universities that are interested in starting a new chapter. They include University of Mobile, Whittier College, Cal State Fullerton, Dominican College, Framingham State College, Columbia University, Norfolk State University, College of DuPage, University of Colorado at Colorado Springs, Pepperdine College, University of Texas at Dallas, Lycoming College, Marymount University, Alcorn State University, St. Augustine's College, Lee University, Jacksonville State University, and Paul Quinn College. Petitions have been sent to several of these. Many of these requests have come from individuals who have found our website and have studied the information contained therein. I am happy to announce that we now have a new "official" position in our organization. It is that of web master, and assuming that position is Kirk Jones, KY Alpha. We are appreciative of his efforts to update and improve the site. We are still in the process of trying to get our own URL address.

The National Council continues its support of the regional structure. The report by Don Tosh details the activity of the various regions. We are grateful for the hard work that our Regional Directors do to keep the activities of the region running smoothly. I would like to especially thank Carol Harrison, who will be retiring soon and leaving her position as the New England Director. Peter Skoner is the Great Lakes Director and has agreed to another term in office. Thanks, too, to the continuing Directors of the Southeast Region (C. Bryan Dawson), the North Central Region (Cynthia Woodburn), and the South Central Region (Gerry East).

Many thanks go to each of the faculty members who serve as Corresponding Secretaries and Faculty Sponsors of our active chapters. These individuals are the lifelines between the national, regional and local organizations, and our chapters cannot exist without these key persons in place. Their encouragement is often realized in those students who attend a convention, such as this one, and present papers. All of us want to thank each of the presenters who spent countless hours in researching, writing, editing and practicing the talks that we were privileged to hear.

The main purpose of the convention would cease to exist without these dedicated students. We are also indebted to those faculty and students who agreed to serve on the various committees, even though we didn't always give them much time to make a decision! My sincere thanks go

to the members of the Oklahoma Delta chapter, who made all the local arrangements and took care of all the behind-the-scenes details that always crop up, usually at the last minute.

During the past biennium, I was privileged to represent Kappa Mu Epsilon at one meeting of the Association of College Honor Societies. It was very helpful to me to talk with other individuals who are involved with various honor societies. Many of the sessions I attended were specifically geared to improve not only the national organizations, but also the regional and local ones as well.

I would like to recognize the outstanding services being performed by those individuals who produce our official journal, The Pentagon. This work requires it lot of dedication on the part of the editor, Steve Nimmo, as well as the business manger, Larry Scott. These two are ably assisted by the various editors, contributors and reviewers that take part in the journal's activities. Thanks to these individuals, we have a quality periodical that reflects well on the society.

One of the goals that the National Council has been striving for during the last few years has been a revision of our national constitution. Last fall, the proposal for the revised document was circulated to each chapter, which then had a chance to approve or disapprove of the revisions in accordance to Article VIII.3 of the (now old) constitution. As a result, I am happy to report that the revised constitution is now in effect. When copies are printed up, your chapter will be receiving a copy. This result could not have been realized without the assistance of an accumulation of suggestions and ideas from previous National Councils over the years.

Earlier, I mentioned the help I have received from other members of the National Council. I would now like to be more specific in my gratitude to those individuals. Al Riveland, our National Treasurer, has done a commendable job of providing periodic financial reports, taking care of checks, filling out our income tax forms and keeping an eye out on the bottom line. Al's term is up this year, so we will be electing a new treasurer at this convention. We wish Al well in his "retirement." Our national Secretary, Rhonda McKee, is diligent in maintaining records of initiations as well as keeping a current log of information on each chapter's corresponding secretary. In addition, she supplies initiation materials to each chapter on a regular basis. Since Rhonda is actually finishing out the term of our former Secretary, Waldemar Weber, the position of National Secretary has come open and will be filled at this convention. Connie Schrock, our National Historian, keeps track of the news for individual chapters and serves as an editor for The Pentagon in this capacity. Our National President-Elect, Don Tosh, handled the collection, selection, and schedul-

ing of papers at this convention in addition to the scheduling and publicity for regional conferences last year. In addition, Don cuts down on costs by producing our certificates and also acts as a calligrapher when needed for certificates and crests. I have enjoyed serving as National President, even during the hectic times, and look forward to serving my remaining two years. It is my hope that we can continue working together to improve our organization.

Bob Bailey
National President

Report of the President Elect

The president elect is responsible for working with regional directors in the coordination of regional activities and conventions. There were three regional conventions planned for the spring of 2002. In the Southeastern Region (Region 3, Regional Director Bryan Dawson) MS Beta (Corresponding Secretary Michael Pearson) at Mississippi State University in Mississippi State hosted a convention on March 22-23. In the North Central Region (Region 4, Regional Director Cynthia Woodburn) MO Beta (Corresponding Secretary Rhonda McKee) at Central Missouri State University in Warrensburg hosted a convention on April 26-27. In the South Central Region (Region 5, Regional Director Gerry East) TX Mu (Corresponding Secretary Bill Sliva) at Schreiner University in Kerrville planned a convention for April 5-6 but unfortunately, there was no response to Texas Mu's invitations to attend the convention and we reluctantly decided to cancel that convention. That was unfortunate since that was a gracious offer from a new chapter to host the convention, and they put a lot of effort into preparing for the convention. It is nice to have them present at this national convention and they even have one of their students presenting a paper. I was able to attend the two regional conventions, and both were well attended and efficiently run. The papers were good and the host chapters and regional directors are to be congratulated on a job well done. I would like to encourage chapters to consider hosting regional conventions for next year. We already have the North Central regional convention decided for next year, but I am hoping that we will have conventions in several other regions as well.

The President-Elect also arranges the presentation of student papers. We are trying a new grading system this year that we hope will simplify the judging of papers. As in past years, the four top papers will receive $100 each from the National Council as well as other prizes.

The president-elect is also responsible for accepting nominations for the George R. Mach Distinguished Service Award. This award was estab-

lished by the National Council in 1987 and is named in honor of George Mach in recognition and appreciation for his 21 years of exemplary service as a member of the National Council. It is to be awarded each biennium to a person who has made major contributions to the Society. We were pleased to award this honor to Mary Elick this year, and I would like to encourage you or your chapter to send me nominations for people that you believe have served KME in a distinguished way.

Don Tosh
President-Elect

Report of the National Secretary

Kappa Mu Epsilon, National Mathematics Honor Society initiated 2,400 in 113 chapters during the 34th biennium that ended March 1, 2003. That brings the total membership of KME to 67,589 members in 13 active and 31 inactive chapters.

As National Secretary, I receive all initiation reports from chapters, make a record of those reports, update mailing list information for corresponding secretaries, and forward copies of the reports to other officers. In the fall of each year, I send out supplies to each chapter. The supplies include information brochures, membership cards, a hard copy of the initiation report form and one or two copies of the brochure "A Matter of Honor." I also take minutes of all business meetings of the organization and all meetings of the national council. The minutes of the November meeting of the national council are included in the packet of information given to each chapter at this convention.

In the past two years, I have created a MS Access database with chapter information. This makes updating information and creating reports much faster and easier than before. I can then also share the database information with other officers electronically. The initiation reports are also now transmitted electronically to the other officers. This makes processing of the reports faster and saves on postage.

I would like to express my appreciation to the other national officers for their hard work and cooperative spirit. It has been a joy to work with them over the past two years.

Rhonda McKee
KME National Secretary

Report of the National Treasurer

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows end-of-biennium assets of $48,982.24. The Cash Flow Report shows that we had an asset gain of $11,057.28 during the biennium. A National Council goal to maintain an asset base of at least $30,000 has been met.

BIENNIUM ASSET REPORT

Total Assets (March 16, 2001)		$37,924.96
Current Assets		
US Bank	17,036.05	
Educational Credit Union	31.946.19	
Total Current Assets (March 1, 2003)		$48,982.24

BIENNIUM CASHFLOW REPORT

Receipts		
Initiation fees received	47,950.00	
Installation fees received	310.00	
Interest Income	2,908.04	
Inventory Income	56.00	
Total Biennium Receipts		$51,224.04
Expenditures		
Assoc. College Honor Soc	1,387.07	
Administrative expenses	3,546.15	
National Convention Expenses	5,722.90	
Regional Convention Expenses	890.00	
Council meeting travel	2,157.80	
Certificates, jewelry, ship	14,505 .60	
Installation expenses	200.00	
Inventory expenses	9.97	
Pentagon expenses	11,544.00	
Miscellaneous expenses	203.27	
Total Biennium Expenses		$40,166.76
Biennium Cash Flow		**+$11,057.28**

Last biennium (99-01) the cash flow was a negative $6,440.57. The major reasons for this significant cash flow increase of over $17,000 are as follows:

1. we initiated 311 more members this biennium over the last,

2. initiation expenses were reduced significantly by the certificate processing as well as the shipping of both pins and certificates out of Evangel University,
3. national convention expenses were low due to the centrally located site and low attendance,
4. Pentagon expenses were reduced.

When I initially took this office, I inherited financial records that were in remarkably good order, thanks to the well-organized and detailed work of my predecessor, JoAnn Fellin. I will leave this office satisfied that our financial record keeping has been made easier and more efficient with the introduction of computer and e-mail based processing. I am also satisfied that overall, KME remains in very sound financial condition with no immediate need to further raise initiation fees.

My sincere thanks to all the National Officers with whom I have worked throughout my eight years in office. They are a very dedicated group of talented, hard-working professionals. Special thanks go to Arnie Hammel, Pat Costello, Bob Bailey, Waldemar Weber, and Rhonda McKee. Without the continuous help and cooperation of the President and Secretary, the Treasurer's job would be more difficult and considerably less enjoyable. Thanks also to the work and cooperation of the corresponding secretaries who keep the KME organization alive and vibrant.

A. Allan Riveland
National Treasurer

Report of The Pentagon Editor

Fourteen papers comprise volumes 61 and 62 of The Pentagon. All fourteen papers are student papers students with eighteen student authors. Seventeen of the eighteen student papers were presented at national or regional Kappa Mu Epsilon conventions. The Problem Corner, Kappa Mu Epsilon News, Convention Reports, and the newly incorporated List of Recent Initiates make up a large portion of the journal and are essential to its success.

Manuscripts received by The Pentagon other than those presented at our conventions are still refereed by faculty volunteers. The efforts of twenty such referees were acknowledged in the Spring 2002 issue, and almost that many have already been utilized since that time. Since the Spring 2002 issue, fifty-three referees have volunteered their services to The Pentagon. The generous offers of your time are greatly appreciated. Nearly all referees are from schools with KME chapters. These individuals

have been a great help to the editor.

The efforts of the associate editors Connie Schrock and Kenneth M. Wilke are greatly appreciated as well as Don Tosh who is responsible for the List of Recent Initiates. The Pentagon would not be the quality journal that many look forward to receiving without all of their hard work. Our business manager, Larry Scott, continues to keep the journal running smoothly. The members of the national council have also been very helpful with answering questions, editing convention reports, forwarding papers to the editor, and dealing with many additional details of running this journal.

Always continue to keep those manuscripts coming!

Steven D. Nimmo
Editor of The Pentagon

Report of the Pentagon Business Manager

It is a pleasure to make my fourth Business Manager's report at this 34th Biennial Convention. As many of you know, the Business Manager's primary responsibility is to maintain a current list of subscribers, to oversee our mailings, and to assist the Editor in managing The Pentagon.

All new initiates receive a two-year subscription to The Pentagon and are encouraged to continue their subscriptions for a modest fee of $5.00 per year. The library rate is $10.00 per year and international subscriptions are $7.00 per year. Issues are mailed in December and May of each academic year. Our mailing list includes subscribers in this country, South America, Asia, Africa, and Europe. During the past biennium, we have serviced approximately 3000 subscribers per issue. Roughly 500 renewal notices are mailed to subscribers each semester. Please watch the expiration date for your subscription and renew early.

Postal regulations for bulk mailings require that each address be validated at least once per year and that each address contain either a street address or post office box number. Please check your address and make corrections if necessary. Postal service penalties for returning undeliverable issues have varied greatly over the past Biennium. Please use a permanent address and make sure that your address is valid. Address corrections may be made by sending an email to scottlar@emporia.edu.

Complementary copies of The Pentagon are sent to the library of each college or university with an active chapter of Kappa MU Epsilon. Anyone contributing an article for an issue will receive two free copies., Speakers at the 34th Biennial Convention will have their subscriptions extended for two years.

I am appreciative of the support and assistance given by the National Council. I would like to thank Steven D. Nimmo, Editor of The Pentagon;

Robert Bailey, KME President; A. Allan Riveland, KME Treasurer; and Rhonda McKee, KME Secretary. Their cooperation and assistance have made things work smoothly. I gratefully acknowledge the assistance of my secretary, Teresa Rios.

Larry Scott
Business Manager

The Pentagon Financial Report

Balance on Hand April 5, 2001			**$649.70**
Receipts:			
Subscriptions and Royalties		2,889.57	
KME Grant		500.00	
Total Receipts		3,389.57	
Expenditures:			
ESU Postage & Supplies (6/1/01 to 6/30/02)			212.47
ESU Postage & Supplies(7/1/02 to 3/21/0,3)			210.30
ESU Postage	18.36		
Bulk Rate Permit	125.00		
Morningside Postage & Supplies Vol 60 #2			727.89
Morningside Postage & Supplies Vol. 61 #1			764.56
Morningside Postage & Supplies Vol. 61 #2			665.32
Morningside Postage & Supplies Vol. 62 #1			757.46
Copyright (Vol. 60 #1, Vol 60 #2)			60.00
Total Expenditures	3,541.36		
Balance on Hand March 21, 2003			**$497.91**

Report of the Audit Committee

The Audit Committee consisted of Jo Ann Fellin, 0SB, KS Gamma, faculty, Chair, Andrew Rockett, NY Lambda, faculty, Joe Gonzales, OK Alpha, student, Kevin Reed, MO Theta, student, and Ben Wadsley, IA Alpha, student.

The national treasurer, A. Allan Riveland, provided complete information to the committee in a well-organized format to expedite the review

process. Information provided to the chair prior to the meeting made verification of assets easily completed.

Information reviewed

Asset and Cash Flow Reports for the 2001-2003 biennium
Pay Order transactions and interview with the National President
Receipt transactions and interview with the National Secretary
Verification of checking balance with the USBank of Topeka
Verification of savings account balance with the Educational Credit Union

Comments

1. The committee finds the internal checks to be important safeguards for the National Treasurer. The committee sees no reason to recommend a professional audit at this time.
2. The committee favorably notes that the asset reserves are at the level recommended by the National Council.
3. The committee chair found that the information provided to the chair prior to the meeting facilitated considerably the verification of assets and believes that this procedure should be continued in the future.

Commendations

1. The committee commends Professor Riveland for his excellent maintenance and presentation of the financial records and for his dedication and generous donation of time through eight years of service as Treasurer for Kappa Mu Epsilon.
2. The committee commends the national President, Secretary, and Treasurer for the manner in which they communicate and cooperate to maintain the internal checks, which preserve the integrity of the office of the Treasurer.
3. The committee commends the work of the previous audit committees and gratefully received reports provided by the treasurer.

Recommendations

1. The Audit Committee recommends the acceptance of the financial records and reports of Kappa Mu Epsilon for the 2001-2003 biennium as presented by the National Treasurer, A. Allan Riveland.
2. The Audit Committee recommends that A. Allan Riveland be congratulated on his excellent service to Kappa Mu Epsilon as Treasurer over the past eight years.

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

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Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959

IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981

NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February, 2001
GA Delta	Berry College, Mount Berry	21 April, 2001
TX Mu	Schreiner University, Kerrville	28 April, 2001
CA Epsilon	California Baptist University, Riverside	21 April, 2003

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