THE PENTAGON

A Mathematics Magazine for Students

Volume 62 Number 2 Spring 2003

Contents

Typeset in Scientific Word Printed in the United States of America.

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Chicken Pecking Orders

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Presented at the 2001 National Convention

Throughout the 1950s and the early 1960s, H.G. Landau, a mathematical sociologist, modeled the dominance behavior of animals. Using what he called dominance relations, he defined the hierarchy that exists in certain animal species. In 1980, Stephen Maurer at Swarthmore College built on Landau's foundation, looking at chicken dominance relations specifically. The relationship between chickens can be called a dominance relation since it is commonly recognized that between any two chickens in a barnyard, exactly one of the chickens will peck (dominate) the other. These dominance relations give rise to a barnyard hierarchy ranking sometimes called a pecking order. The number of other chickens in the barnyard that a given chicken pecks will be called the chicken's score. One of Landau's most clever theorems can be adapted to chickens with maximal score. If Hilda is a barnyard chicken with maximal score, and Gelda is any other chicken in the barnyard, then Landau proves that either Hilda pecks Gelda directly or instead pecks some other chicken which in turn pecks Gelda. An interesting theorem, indeed.

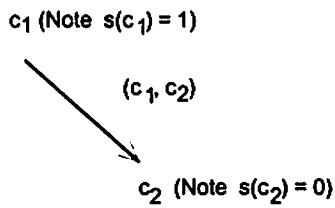
Maurer's paper focused on chickens of maximal score. He called them "kings". Perhaps "queens" would be a better name since most barnyard chickens are female. In the development which follows we instead focus our attention on the pecking orders. Our results will not depend significantly on Maurer's paper, nor on the barnyard behavior of Landau's chicken dominance. However, our terminology will indeed reflect this interesting application of graph theory.

We first introduce our terminology with its corresponding symbolism. A set $F = \{c_1, c_2, ..., c_n\}$ of chickens is an **n-flock** if and only if for any two different chickens c_i and c_j either c_i pecks c_j or c_j pecks c_i, but not both. In graph theory, this is an example of a **directed graph**, or **digraph**. The set F is called the **vertex set** of the digraph. The ordered pair (c_i, c_j) would represent a pecking relationship from chicken c_i to c_j. This ordered pair is called a **directed arc**. A directed graph in which for any two vertices a and b, there is either an arc (a, b) or an arc (b, a), but not both, is commonly called a **tournament**. Note that our definition of

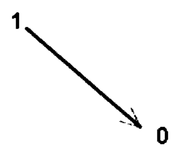
n-flock simply guarantees us that the digraph formed by the vertex set of chickens and the pecking relationships between them, does indeed form a tournament on n vertices as defined in graph theory.

A chicken's **score** $s(c_i)$ will be defined as the number of chickens in the barnyard pecked by chicken c_i. For the n-flock $\{c_1, c_2, ..., c_n\}$ of n chickens the sequence $s(c_1), s(c_2), ..., s(c_n)$ will be called the **score sequence** of the flock. A score sequence $P_1, P_2, ..., P_n$ will be called a **pecking order** if and only if $P_i \geq P_{i+1}$ for all $i = \{1, 2, ..., n-1\}$. Notice that a pecking order is simply a score sequence arranged in non-ascending order. In this paper we will focus entirely on these pecking orders. How many of them are there? How can we recursively count them? We begin by looking at special cases involving very small barnyards.

Example 1: $F = \{c_1, c_2\}$



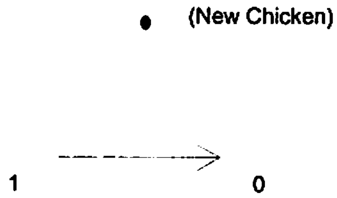
This digraph will be simplified by referring to the vertices of the digraph by the scores of the chickens, respectively. Note that the score sequence is 10, and that score sequence yields a pecking order of 10.



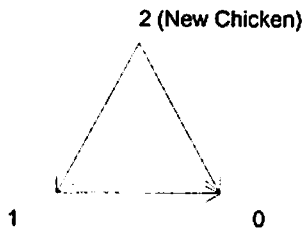
10 (For the sake of simplicity, the commas in the pecking order sequences are omitted.)

If instead c_2 pecks c_1, then the score sequence is 01, but the associated pecking order is 10 as well.

Example 2: Using a “recursive brute force” method, a 3-flock is investigated by considering all the possible score sequences generated by adding a third chicken to the single distinct pecking order, 10, for a 2-flock.

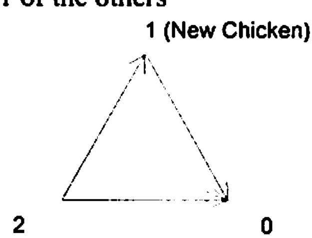


Case I: New chicken pecks the two others



210

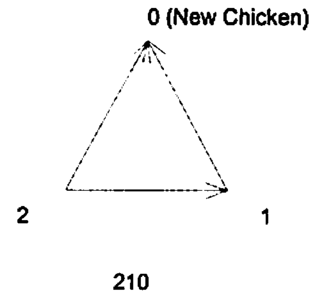
Case II: New chicken pecks only 1 of the others



111

210

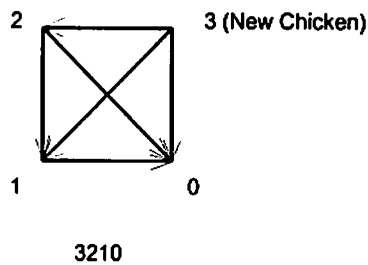
Case III: New chicken pecks none of the others



An examination of the resulting pecking orders shows that 210 and 111 are the only distinct ones.

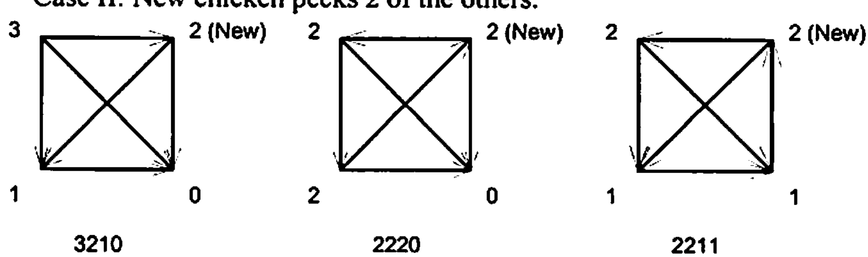
Example 3: We now investigate all possible score sequences obtained in a 4-flock when a fourth chicken is added to the two distinct pecking orders obtained in Example 2. We first consider the pecking order 210.

Case I: New chicken pecks the 3 other chickens.



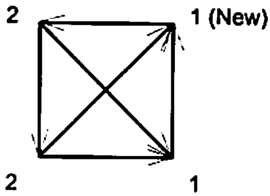
Note there are 3 choose 3, written as $C(3,3) = 1$, score sequence and one distinct new pecking order.

Case II: New chicken pecks 2 of the others.

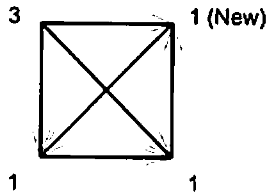


Note there are $C(3,2) = 3$ score sequences, but only two distinct pecking orders beyond Case 1; namely 2211 and 2220.

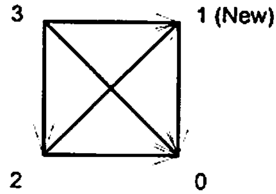
Case III: New chicken pecks 1 of the others.



2211



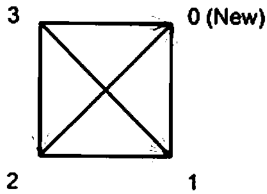
3111



3210

Note that there are $C(3,1) = 3$ score sequences with one new distinct pecking order, 3111.

Case IV: New chicken pecks none of the others.



3210

Note that there are $C(3,0) = 1$ score sequences, but not a new distinct pecking order.

In these 4 cases we constructed $C(3,3)+C(3,2)+C(3,1)+C(3,0) = 8$ score sequences and found only 4 distinct pecking orders: $3210, 3111, 2220, 2211$. If we perform the same process on the other distinct pecking order from 3 chickens, 111, we would again construct $C(3,3) + C(3,2) + C(3,1) + C(3,0)$ new score sequences, but would discern no new distinct pecking orders. Summarizing, for a 4-flock there are only 4 distinct pecking orders: $3210, 3111, 2220, 2211$.

The expression $C(3,3)+C(3,2)+C(3,1)+C(3,0)$ which surfaced in the previous example can quickly be evaluated by $(1+1)^3$ using the popular binomial expansion. Similarly, in a n-flock with 5 chickens the expression $(1+1)^4 = C(4,4)+C(4,3)+C(4,2)+C(4,1)+C(4,0) = 2^4 = 16$ can be used to count the number of new score sequences constructed when adding the fifth chicken. The 4 pecking orders $3210, 3111, 2220, 2211$ which we found in the case $n = 4$ give rise to $4 * 2^4 = 64$ new score sequences on 5 chickens. Of these 64 score sequences we found by inspection 9 distinct pecking orders. With hours of mechanical investigation we continued this brute force recursive approach to generate distinct pecking orders up to $n = 7$. Our results are summarized in Figure 1.

Size of n-flock	# of score sequences	Distinct pecking orders (by inspection)
2		1
3	$1 * 2^2 = 4$	2
4	$2 * 2^3 = 16$	4
5	$4 * 2^4 = 64$	9
6	$9 * 2^5 = 288$	22
7	$22 * 2^6 = 1408$	59
8	$59 * 2^7 = 7552$	???

Notice in the 7-flock we recursively generated 1408 score sequences and by inspection found that only 59 were distinct pecking orders. Looking forward to $n = 8$ we could foresee 7552 score sequences. It was clearly time to end this manual process.

With all this information it seemed reasonable to then construct a recursive formula which would predict the number of distinct pecking orders. This attempt failed. Our efforts then shifted attempting to design a process which would generate these distinct pecking orders for any n-flock without the duplications which severely complicated the previous process.

We will not try to detail the "sweat and tears" necessary to develop such a process. With minimal motivation we will state and prove a sequence of theorems that we developed which will ultimately drive the algorithm.

In Theorems 1-6 it will be understood that the sequence P_1 , $P_2, ..., P_n$ represents a pecking order on an n-flock $\{c_1, c_2, \ldots, c_n\}, n \geq 2$.

The first is a standard theorem borrowed from graph theory.

Theorem 1: $\sum_{i=1}^{n} P_i = \frac{n(n-1)}{2}$

Proof: Since there is exactly one pecking arc between any two chickens of an n-flock, there are n choices for the pecking chicken and $n-1$ choices for the pecked chicken. We multiply n by $n-1$ using the fundamental counting principle and divide by 2 since the pecking relation can go only one way.

The key to any pecking sequence is the first, and largest, entry. .

Theorem 2 $\frac{n-1}{2} \leq P_1 \leq n-1$

Proof: For a pecking order$P_1 \geq P_2 \geq ... \geq P_n$ it is clear that $nP_i \geq \sum_{i=1}^{n} p_i$. So, from Theorem 1, $nP_i \geq \frac{n(n-1)}{2}$.Dividing by n leads to $P_1 \geq \frac{n-1}{2}$. Also $P_1 \leq n-1$ since there are only $n-1$ other chickens in the n-flock.

Next, a lower bound for the P_k entries where $k\{2,3,...,n-1\}$ is established.

Theorem 3: $P_k \geq \frac{\frac{n(n-1)}{2} - \sum_{i=1}^{k-1} P_i}{n-k+1}$ for $k \in \{2,3,\ldots,n-1\}$

Proof: Consider only the tail-end of the pecking order $P_k \geq P_{k+1} \geq \ldots \geq P_n$ where $k \in \{2,3,\ldots,n-1\}$. By similar reasoning to that used in the previous theorem, $(n-k+1)P_k \geq \sum_{i=1}^{n} P_i - \sum_{i=1}^{k-1} P_i = \frac{n(n-1)}{2} - \sum_{i=1}^{k-1} P_i$. Dividing by $(n-k+1)$:

$$P_k \geq \frac{\frac{n(n-1)}{2} - \sum_{i=1}^{k-1} P_i}{n-k+1}$$

Before we switch our attention to the upper bounds of P_k, another theorem is needed to limit the sum of the first k entries of a pecking order.

Theorem 4: $\sum_{i=1}^{k} P_i \leq kn - \frac{k(k+1)}{2}$ for $k \in \{2,3,\ldots,n-1\}$

Proof: Consider again the pecking order $P_i \geq P_2 \geq \cdots \geq P_{k-1} \geq P_k \geq P_{k+1} \geq \cdots \geq P_n$. The sum of this pecking order is $\sum_{i=1}^{n} P_i = \sum_{i=1}^{k} P_i + \sum_{i=k+1}^{n} P_i = \frac{n(n-1)}{2}$. The tail-end sum $\sum_{i=k+1}^{n} P_i$ involves exactly $n-k$ chickens. If we disregard all pecking arcs to or from the remaining k chickens, we certainly have a "sub-n-flock of $n-k$ chickens. From Theorem 1 in this sub-n-flock along, there would be exactly $\frac{(n-k)(n-k-1)}{2}$ total pecking arcs. Of the pecking arcs we have disregarded, any arcs directed into the sub-n-flock do not contribute to the total score of the sub-n-flock. Any disregarded pecking arcs that are directed out of the sub-n-flock can only raise the total sum of the sub-n-flock. Accordingly, $\sum_{i=k+1}^{n} P_i \geq \frac{(n-k)(n-k-1)}{2}$. So, $\sum_{i=1}^{n} P_i + \sum_{i=k+1}^{n} P_i \geq \sum_{i=1}^{k} P_i + \frac{(n-k)(n-k-1)}{2}$. Then, $\frac{n(n-1)}{2} \geq \sum_{i=1}^{k} P_i + \frac{(n-k)(n-k-1)}{2}$, or, $\frac{n(n-1)}{2} - \frac{(n-k)(n-k-1)}{2} \geq \sum_{i=1}^{k} P_i$. Simplifying the left-hand side produces $kn - \frac{k(k+1)}{2} \geq \sum_{i=1}^{k} P_i$.

We can now easily construct a theorem which establishes the upper bounds for each P_k .

Theorem 5:

$$Pk \leq min\{P_{k-1}, kn - \frac{k(k+1)}{2} - \sum_{i=1}^{k-1} P_i\} for k \in, \{2,3,\ldots,n-1\}$$

Proof: From Theorem 4, $P_k \leq kn - \frac{k(k+1)}{2} - \sum_{i=1}^{k-1} P_i$. Also, for a score sequence to be a pecking order, $P_k \leq P_{k-1}$. the restriction on

k comes from the fact that the first entry P_1 is governed by Theorem 2. Once all of the other scores are known for a given pecking sequence, the last entry is the amount needed for the scores to add to the necessary total.

Theorem 6:

$$P_n = \frac{n\,(n-1)}{2} - \sum_{i=1}^{n} P_i$$

Proof: This theorem follows immediately from Theorem 1 if solved for P_n.

Theorems 2-6 can be combined into a single summary theorem.

Theorem 7: For a pecking order $P_1, P_2, ...P_n$ on an n-flock,$n2$,

a.$\frac{n-1}{2} \leq P_1 \leq n-1$

b.$\frac{\frac{n(n-1)}{2} - \sum_{i=1}^{k-1} P_i}{n-k+1} \leq P_k \leq \min\left\{P_{k-1}, kn - \frac{k(k+1)}{2} - \sum_{i=1}^{k-1} P_i\right\}$for $k \in \{2, 3, ..., n-1\}$

c.$P_n = \frac{n(n-1)}{2} - \sum_{i=1}^{n} P_i$

Note that (a) sets the upper and lower bounds for the first and maximal score in the pecking order. (b) similarly sets the range of acceptable values for the remaining terms of the pecking order except for the nth term. The nth is calculated exactly by (c).

Theorem 7 appeared to provide exactly the criteria needed to algorithmically construct pecking orders for each value of n. We have in Theorems 2-6 proven that these criteria are necessary conditions for a pecking order on an n-flock. Unfortunately, proving that these criteria were also sufficient conditions on an n-flock to guarantee a pecking order seemed to be a very difficult undertaking.

In Appendix A we show a tree diagram in which we used our results in Theorem 7 to manually construct all pecking orders on a 7-flock. Indeed, we obtained exactly the same 59 pecking orders that we found by the brute force recursive method discussed earlier in the paper.

To further substantiate the correctness and completeness of Theorem 7 we constructed a C program based on the criteria of the theorem. This program and the resulting output are provided in Appendix B. The same 59 pecking orders result paralleling our manual efforts. We also modified the C program to consider n-flocks with 8 and 9 chickens. We found 167 and 490 pecking orders respectively. Of course, manually verifying these results is unpractical.

At this point in our development we were reasonably convinced that Theorem 7 did indeed provide the exact criteria needed for the construction of all pecking orders and only pecking orders. As we have stated we had not yet proven that every sequence that satisfied the criteria in Theorem 7 was necessarily a pecking order. Maurer (1980), in the paper which motivated the development, had mentioned that Landau (1953) had found a characterization for score sequences. We expected that this result would be useful but kept our development independent as long as possible. As time ran short we turned to Landau. Interestingly, Landau's characterization of pecking orders was quite different than ours. However, Landau's powerful characterization helped us bridge the chasm we had failed to span. Using Landau's characterization theorem we will prove that the criteria in Theorem 7 will necessarily produce pecking orders. We will first state Landau's characterization theorem, then use this result to complete our development.

Theorem 8: (Landau) The n non-negative integers $S_1, S_2, ..., S_n$ is a score sequence if, and only if,

$$\sum_{i=1}^{n} S_i = \frac{n(n-1)}{2}, \text{and} \sum_{k} S_i \geq \frac{k(k-1)}{2}, \text{ for } k \in \{1, 2, ..., n-1\}$$

where $\sum_k S_i$ is the sum of any k of the S_i.

The hypotheses of the final theorem include only those parts of the Theorem 7 needed to reach Landau's characterization of pecking orders.

Theorem 9: For any positive integer $n, n \geq 2$, if $T_1, T_2, ..., T_n$ is a non-increasing sequence of nonnegative numbers for which

$$\begin{aligned} \text{i. } T_1 &\leq n-1 \\ \text{ii. } T_k &\leq kn - \frac{k(k+1)}{2} - \sum_{i=1}^{k-1} T_i \text{ for } k \in \{2, 3, ..., n-1\} \\ \text{iii. } T_n &= \frac{n(n-1)}{2} - \sum_{i=1}^{n-1} T_i, \text{ then} \end{aligned}$$

a. $\sum_{i=1}^{n} T_i = \frac{n(n-1)}{2}$

b. $\sum_k S \geq \frac{k(k-1)}{2}$ for any S of k members of sequence T_i where $k \in \{1, 2, ..., n-1\}$

c. The sequence $T_1, T_2, ..., T_n$ is a pecking order for an n-flock of n chickens.

Proof: (a)follows immediately from (iii) by adding the sum $\sum_{i=1}^{n-1}$ to both sides of the equation given in (iii). To prove (b) we let S be any set of k members of the nonincreasing sequence $T_1, T_2, \ldots, T_n$. Clearly

$$\sum_k S_k \geq T_{n-k+1} + T_{n-k+2} + \ldots + T_n \tag{1}$$

In a nonincreasing sequence, the sum of any k terms of the sequence must be greater than or equal to the sum of the last k terms of that sequence. By (ii) we have

$$T_k \leq kn - \frac{k(k+1)}{2} - \sum_{i=1}^{k-1} T_i \text{ for } k \in \{2, 3, \ldots, n\}$$

As a result

$$\sum_{i=1}^{k} T_i \leq kn - \frac{k(k+1)}{2} \text{ for } k \in \{2, 3, \ldots n\} \tag{2}$$

If we select k as $n-k$ then since $n-k \in \{2, 3, \ldots, n-1\}$ we have

$$\begin{aligned} 2 &\leq n-k \leq n-1 \\ 2-n &\leq -k \leq 1 \\ 1 &\leq k \leq n-2 \end{aligned}$$

and by replacing k by $n-k$ in (2)

$$\sum_{i=1}^{n-k} T_i \leq (n-k)\,n - \frac{(n-k)(n-k+1)}{2} \text{ for } k \in \{1, 2, \ldots, n-2\}.$$

This algebraically simplifies to

$$\sum_{i=1}^{n-k} T_i \leq \frac{n(n-1)}{2} - \frac{k(k-1)}{2}.$$

Using (a) we substitute in the above expression

$$\sum_{i=1}^{n-k} T_i \leq \sum_{i=1}^{n} T_i - \frac{k(k-1)}{2}$$

which simplifies to

$$\frac{k(k-1)}{2} \leq T_{n-k+1} + T_{n-k+2} + \ldots + T_n \text{ for } k \in \{1, 2, \ldots, n-2\}.$$

This inequality coupled with (1) gives us

$$\sum_k S_k \geq \frac{k(k-1)}{2} \text{ for } k \in \{1, 2, \ldots n-2\} \tag{3}$$

as desired. But the case in which $k = n-1$ remains unproven. In this special case (1) can be written :

$$\sum_k S_k \geq T_2 + T_3 + \ldots + T_n \tag{4}$$

By (b)

$$T_2 + T_3 + \ldots + T_n = \frac{n(n-1)}{2} - T_1. \tag{5}$$

By (i)

$$\begin{aligned} T_1 &\leq n-1 \\ -T_1 &\geq (n-1) \\ \frac{n(n-1)}{2} - T_1 &\geq \frac{n(n-1)}{2} - (n-1) \\ \frac{n(n-1)}{2} - T_1 &\geq \frac{(n-1)(n-2)}{2} \end{aligned} \tag{6}$$

But $k = n - 1$.
By (4), (5), and (6) we have

$$\sum_k S_k \geq \frac{(n-1)(n-2)}{2}.$$

But in this case $n - 1 = k$ so

$$\sum_k S_k \geq \frac{k(k-1)}{2} \text{ for } k = n-1.$$

Combining this with (3) we have

$$\sum_k S_k \geq \frac{k(k-1)}{2} \text{ for } k \in \{1, 2, 3, \ldots, n-1\}$$

and the proof of (b) is complete.
From (a), (b) and Landau's Theorem we can conclude that the sequence $T_1, T_2, \ldots, T_n$ is a score sequence. Since it is non-increasing, it is also a pecking order. Thus (c) is proven.

With the proof of Theorem 9 completed we can now, at last, conclude that the criteria stated in Theorem 7 are both necessary and sufficient for the construction of pecking orders of an n-flock of chickens.

Our original goal was to try to construct a formula, likely a recursive formula, to count distinct pecking orders. This was perhaps an unrealistic goal, but it led us naturally to a construction process for the pecking orders and the theorems which drive this process. Using our results it was not difficult to design a program which will not only count the pecking orders but also construct them. So, in a sense, we did accomplish our original goal- and more.

Acknowledgements. This paper was completed under the direction and guidance of Al Riveland of the Washburn University Mathematics and Statistics Department.

References

1. Landau, H.G., On Dominance Relations and the Structure of Animal Societies III: The Condition for a Score Structure, Bulletin of Mathematical Biophysics, 15 (1953), 143-148.
2. Maurer, Stephan B., The King Chicken Theorems, Mathematics Magazine, 53 (1980), 67-80.

Starting a KME Chapter

For complete information on starting a KME chapter, contact the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of the $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offering and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

Baseball: A Statistical Analysis

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The 2001 World Series between the Arizona Diamondbacks and the New York Yankees was one of the most exciting in Major League baseball history. Most of the games had excellent pitching, as well as great defense. However, the games were (and always are) decided by which team scored more runs. Which is more important to winning, offense or defense? In the following, this question, as well as many others, will be explored.

There will be two main parts to this paper. The first is to measure the importance of offense and defense on winning in the game of baseball. The second is to try to determine the greatest game ever pitched.

Part One: Importance of Offense and Defense

To begin the first section, an operational definition of offense and defense must be stated. Offense will be defined as the average on-base percentage of a team in a season, and the total runs scored by a team in a season. Defense will be defined as the total earned run average of all pitchers on a team in a season, and the average fielding percentage of a team in a season. The basic definition of on-base percentage is the percent chance in which a batter reaches base safely. The definition of runs scored is simply the number of runs a team scores in a season. The definition of earned run average is basically the average number of runs a pitcher is held accountable for in a nine innings, and the definition of fielding percentage is the percent chance that a fielder has to make a successful play.

The goals of the first part are as follows: 1. Determine if a team's offense or defense is significantly important to getting to a World Series, 2. Determine if a team's offense or defense is significantly important to winning a World Series, 3. Determine if runs, on-base percentage, earned run average, and fielding percentage are of the same importance to winning, and 4. If not, determine which are most and least important.

In doing the research, the winners and losers of the World Series will be looked at and ranked according to how they compared with the rest of the teams in the league that year in runs, on-base percentage, earned run average, and fielding percentage. The years that are to looked at are 1905-1960, because during that time period, there were two leagues with eight team each, so the number will be easier to work with. Since there are eight teams in each league, the winners and losers of the World Series will be ranked from one to eight, with the standard mean being 4.5, which is the average rank of all the teams in each year.

The mean of the ranks that were found for the winners were as follows:

On-base percentage = 2.35
Runs scored = 1.83
Earned run average = 1.79
Fielding percentage = 2.59

The mean of the ranks that were found for the losers were as follows:

On-base percentage = 2.21
Runs scored = 1.98
Earned run average = 1.92
Fielding percentage = 2.90

T-test: Comparing Means of Ranks to Standard Mean

To determine if these numbers were significantly lower than 4.5, a t-test was to be run on all of the means. For all the t-tests, the null hypothesis was that the mean of the ranks equaled 4.5, and the alternate hypothesis was that the mean of the ranks was less than 4.5. The significance level was chosen as 0.05, with $n = 56$, and critical values of 1.96 and -1.96. The t-values for the categories were as follows:

Winner t-values...

- ◊ On-base percentage: $t = -10.48$
- ◊ Runs scored: $t = -17.28$
- ◊ Earned run average: $t = -20.33$
- ◊ Fielding percentage: $t = -8.53$

Loser t-values...

- ◊ On-base percentage: $t = -11.61$
- ◊ Runs scored: $t = -15.48$
- ◊ Earned run average: $t = -14.62$
- ◊ Fielding percentage: $t = -6.64$

Since all of the t-values were less than -1.96, all the null hypotheses can be rejected. This means that the better a team's on-base percentage, runs scored, earned run average, and fielding percentage, the better chance a team has of getting to the World Series.

T-test: Comparing Means of Ranks of Winners to Losers

The next test to be run is a t-test for two means. In this test, the means of the ranks of the winners of the World Series will be tested against the means of the ranks of the losers to see if they are significantly different.

For these tests, the null hypotheses will be that the mean of the ranks of the winner is equal to the mean of the ranks of the loser. The alternative hypotheses will be that they are not equal to each other. Again, the significance level will be 0.05, with $n = 56$ and critical values of -1.96 and 1.96. The t-values were found as follows:

◊ On-base percentage: $t = 0.47$
◊ Runs scored: $t = -0.68$
◊ Earned run average: $t = -0.61$
◊ Fielding percentage: $t = -0.95$

Since all of the t-values are between -1.96 and 1.96, there is not sufficient evidence to reject the null hypotheses. This means that after a team gets to the World Series, their offense and defense is not significantly important to beating their opponent in the World Series.

Following this test, all of the means of the winners and losers were added up to find the means of the ranks of the winners and the losers together. It was found that:

◊ On-base percentage: 2.28
◊ Runs scored: 1.91
◊ Earned run average: 1.85
◊ Fielding percentage: 2.75

Analysis of Variance

In order to determine if any of these were more or less important than the others, an analysis of variance was run. For this test, the null hypothesis was that the mean of the ranks of runs equaled the mean of the ranks of on base percentage equaled the mean of the ranks of earned run average equaled the mean of the ranks of fielding percentage. The alternate hypothesis was that at least one was different. The significance level was again 0.05, with $n = 112$. The numerator degrees of freedom was the number of categories minus one, which equaled three, and the denominator degrees of freedom was the number of categories times $(n - 1)$, which was 444. So, after looking in a table, the critical value was 2.6049. After calculating the formula, it was found that $F = \frac{19.08}{2.0112} = 9.49$, and since that is greater than 2.6049, we reject the null hypothesis. This means that at least one of the categories (on-base percentage, runs, era, and fielding percentage) has a greater or lesser impact on winning in the game of baseball that the others.

Studentized Range

In order to determine which one(s) is (are) more or less important, the studentized range was used. For this test, if one mean of ranks minus another was greater than or equal to "w"(which will be calculated), then the two means are different. After calculations, w was found to be 0.486435. In looking at the means of the ranks and using subtraction, it is found that earned run average and runs scored are significantly more important to winning in the game of baseball than is fielding percentage, however, on-base percentage is not significantly more important than fielding percentage.

Part Two: Determine the Greatest Game Ever Pitched

The second part is to determine the greatest game ever pitched. In order to do this, we will look at the fourteen perfect games pitched in major league baseball history, find the probability of those games, and through that determine the greatest game ever pitched. A perfect game is defined as a game in which a pitcher allows no hits, no runs, and no opposing batter to reach first base. The fourteen perfect games pitched were by Cy Young, Addie Joss, Charlie Robertson, Don Larson, Jim Bunning, Sandy Koufax, Catfish Hunter, Len Barker, Mike Witt, Tom Browning, Dennis Martinez, Kenny Rogers, David Wells, and David Cone. After the on-base percentages of the batters faced were found, the probability of each perfect game was calculated as follows:

	On-base Percentages	
◊	Sandy Koufax:	0.0005423
◊	Len Barker:	0.0001592
◊	Cy Young:	0.0000933
◊	Tom Browning:	0.0000678
◊	Jim Bunning:	0.0000594
◊	Addie Joss:	0.0000531
◊	Mike Witt:	0.0000462
◊	David Wells:	0.0000398
◊	Catfish Hunter:	0.0000298
◊	Dennis Martinez:	0.0000245
◊	Kenny Rogers:	0.0000227
◊	David Cone:	0.0000187
◊	Don Larson:	0.0000089
◊	Charlie Robertson:	0.0000051

As you can see, Charlie Robertson seems to have the greatest game ever pitched. After further research, it was found that Don Larson pitched his perfect game in the 1956 World Series, while all the other pitchers pitched theirs during the regular season. Don Larson had much more pressure on his game, with many more fans watching, and a much higher "need to succeed". Therefore, although Charlie Robertson pitched the greatest game statistically, the "greatest game ever pitched" may be by Don Larson.

In conclusion, it was found that a team's offense and defense is significantly important to getting to a World Series, but not to beating their opponent in the World Series. Also, a team's earned run average and runs scored are significantly more important to winning than is fielding percentage, however, on-base percentage is not significantly more important than fielding percentage. Finally, although it is difficult to determine the greatest game ever pitched because of situational effects, it was found that the greatest game ever pitched statistically was Charlie Robertson.

Acknowledgements. I wish to acknowledge Dr. Bryan Dawson for his help with this paper.

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Solving problems is a practical art, like swimming or skiing, or playing the piano; you learn it only by imitation and practice.

\- G. Polya

Transformations of the Unit Circle

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Introduction

In this paper, we will explore different areas of mathematics through maps specific to each area. We will be dealing with functions on $\mathbb{R}^2$, with particular attention to the images of the unit circle. it is good to keep in mind that the points on the unit circle can be represented in various ways (as elements of $\mathbb{R}^2$ and as elements of $\mathbb{C}$). In each area, it will be clearly stated which notation will be used. Also in each area, there will be some background given including definitions and special properties necessary to understand the discussion. The areas explored will be Linear Algebra, Group Theory, continuous Functions, and Complex Variables.

Specifically, we will consider functions $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ (or $f : \mathbb{C} \longrightarrow \mathbb{C}$) whose restriction to the unit circle is some specified set. Also of interest will be those functions that are bijective. A *bijective function* $f : A \longrightarrow B$ is one that is

1. injective (one-to-one) - each element of B is mapped to by at most one element of A
2. surjective (onto) - every element B is mapped to by something in A.

Linear Algebra

An *invertible matrix* is one that has an inverse. A special property of invertible matrices is that the determinant is not equal to zero.

A *linear transformation* is a function $L : V \longrightarrow W$ assigning a unique vector $L(x) \in W$ to each $x \in V$ such that:

1. $L(x + y) = L(x) + L(y)$
2. $L(cs) = cL(x)$, for every x in V and every scalar $c \in \mathbb{R}$.

Linear transformations can be thought of as simple equations using matrix multiplication that involve a transformation matrix M and satisfy the preceding conditions. Here $L : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, so M is 2×2 and the elements of V and W are 2×1 vectors

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

where $x, y \in \mathbb{R}$. Our transformation equation is thus

$$\begin{bmatrix} u \\ v \end{bmatrix} = M \begin{bmatrix} x \\ y \end{bmatrix}$$

By dividing linear transformations into two categories by the types of transformation matrices used, singular (noninvertible) and invertible, we can see what two types of images are possible.

(a) Let's begin by looking at the image of the unit circle under a singular matrix

$$M = \begin{bmatrix} a & b \\ ka & kb \end{bmatrix}.$$

By multiplying the right side of the transformation equation, the following equations are obtained

$$\begin{aligned} u &= ax + by \\ v &= kax + kby. \end{aligned}$$

By solving the system, the image is simply the straight line

$$v = ku.$$

The next question to answer is whether the line is infinite or if it is restricted to a line segment under the restriction

$$x^2 + y^2 = 1.$$

To do this, first solve for y and substitute in the above equation.

$$u = ax + by = AX + b\sqrt{1 - x^2}$$

Next take the derivative and set equal to zero.

$$\begin{aligned} u' &= a + \frac{x}{\sqrt{1 - x^2}} b = 0 \\ \frac{a}{b} &= \frac{x}{\sqrt{1 - x^2}} \\ x &= \frac{a}{\sqrt{a^2 + b^2}} \end{aligned}$$

So by plugging this value back in the original equation, we obtain the maximum value

$$u = \sqrt{a^2 + b^2}$$

Similarly the minimum can be found using

$$u = ax - b\sqrt{1 - x^2}.$$

This leads to u being restricted to the interval

$$\left[-\frac{a^2 - b^2}{\sqrt{(a^2 + b^2)}}, \sqrt{a^2 + b^2} \right].$$

Thus the image is a line segment. So this function is definitely not bijective, since it is not surjective.

(b) Next we will examine the image using an invertible matrix M. In this case, however, it simplifies many calculations to first multiply both sides of the transformation equation by M^{-1}. so the equation now looks like

$$M^{-1}\begin{bmatrix}u\\v\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix},$$

where

$$M^{-1}=\begin{bmatrix}a & b\\c & d\end{bmatrix}.$$

By multiplying the left side

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}u\\v\end{bmatrix}=\begin{bmatrix}au+bv\\cu+dv\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

and substituting for x and y in this equation

$$x^2+y^2=1,$$

the following equation is obtained

$$\left(a^2+c^2\right)u^2+(ab+cd)\,uv+\left(b^2+d^2\right)v^2-1=0,$$

which is a conic whose general form is

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0.$$

Next we will use the discriminant to see what kind of conic this is.

$$B^2-4AC=-4\,(ad-cb)^2$$

Since $ad-cb$ is the determinant of an invertible matrix, the discriminant is not equal to zero and is most certainly always less than zero. and since $B\neq 0$ and $D=E=0$, this is a rotated ellipse at the origin. it is possible to show that all rotated ellipses centered at the origin can be obtained. This transformation equation is a bijection that takes the unit circle to ellipses centered at the origin.

Group Theory

A *group* is a set G paired with binary operation $*$ where G is closed under $*$ and the following conditions apply:

1. $*$ is associative
2. G contains an identity element e such that $e*x=x*e=x$
3. For every x in G, there exists an inverse $x\prime$in G where $x\prime*x-x*x\prime=e$.

A *group homomorphism* $f : G \to G\prime$ is a map if for all $a,b\in G$:

where

$$.f(ab) = f(a)f(b).$$

In this homomorphism, we will be referring to the points of the unit circle as ordered pairs $(1, \theta) \in \mathbb{C}$ and $\theta \in (-\pi, \pi]$. Define

$$x * y = (1, \theta)(1, \phi) = (1, \theta + \phi)$$

and $f : \mathbb{C}^* \to \mathbb{R}^+$.
Let

$$f(1,\theta) = \begin{cases} (2^q, 0) & \text{if } q = \frac{\theta}{2\pi\beta}, \text{ where } q \in \mathbb{Q} \text{ and} \\ & \beta \text{ is an irrational number between 0 and 1} \\ (1,0) & \text{otherwise} \end{cases}.$$

The bottom half of the unit circle is mapped to the interval $(0, 1)$ and the top half to $[1, \infty)$ where all points are rational powers of 2. So this homomorphism is not onto and therefore not bijective.

Complex Variables

So far we have seen that a circle can be sent to an ellipse, a line segment, and points on the real axis. So the next trial is to see if it can be sent to the entire real axis. This function w will differ from those previously mentioned in that it will go from $\mathbb{C}$ to $\mathbb{R}$. so in this setting we will think of the points of the unit circle as $a + bi$. the easiest way to start this process is to pick certain points on the circle and decide where they should be sent. Let's pick the following accordingly:

$$\begin{aligned} -i &\implies 0 \\ 1 &\implies 1. \end{aligned}$$

Now we start with linear fractional transformation

$$w(z) = \frac{az + b}{cz + d}.$$

Using these two conditions

$$w(z) = k\left(\frac{z + i}{z - i}\right).$$

So by assigning one more point

$$-1 \implies -1,$$

we obtain

$$w(z) = -i\left(\frac{z + i}{z - i}\right).$$

so our function breaks the circle at i and sends the right half to the positive real axis and the left half to the negative real axis. We have developed a bijective function that will send the unit circle to the entire real axis.

Now that the unit circle can be sent to the entire real axis, composition of functions can be used to send the unit circle to any curve of the form

$$y = f(x)$$

by letting

$$\begin{aligned} x &\Longrightarrow (x, f(x)) \\ (x, y) &\Longrightarrow (x, f(x) + y). \end{aligned}$$

So more specifically for a parabola,

$$\begin{aligned} x &\Longrightarrow (x, 4p(x-h)^2 + k) \\ (x, y) &\Longrightarrow (x, 4p(x-h)^2 + k + y). \end{aligned}$$

And also for a hyperbola,

$$\begin{aligned} x &\Longrightarrow \left(x, \frac{b}{a}\sqrt{x^2 + a^2}\right) \\ (x, y) &\Longrightarrow \left(x, \frac{b}{a}\sqrt{x^2 + a^2 + y}\right). \end{aligned}$$

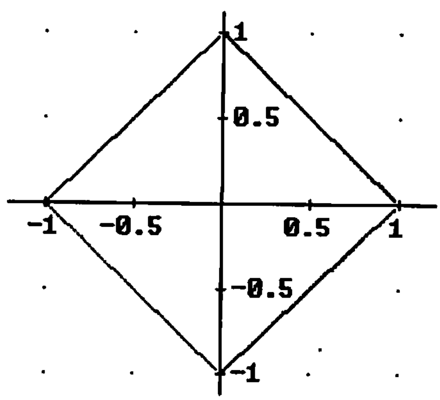
Note that to change the orientation of the curve would result by simply swapping the x and y coordinates in the image.

Continuous Functions

Next, let's set out to see if we can get a diamond from the unit circle. Let's start with the image of the diamond in $\mathbb{R}^2$ with vertices at $(0, 1)$, $(0 - 1)$, $(1, 0)$, and $(-1, 0)$. This equation will look like

$$|u| + |v| = 1$$

and the diamond itself would look like



We need to find a function $f(x,y) : \mathbb{R}^2 \Longrightarrow \mathbb{R}^2$ so that points on the unit circle are sent to points on the diamond. Start with $x^2 + y^2 = 1$. Let

$$x^2 = |u|$$

and

$$y^2 = |v|,$$

so

$$\begin{aligned} u &= \frac{x^3}{|x|} \\ v &= \frac{y^3}{|y|}. \end{aligned}$$

However, since 0 is a possible value for both x and y, we will have to define

$$f(x,y) = (u,v) \text{ where } \begin{aligned} u &= \begin{cases} \frac{x^3}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \\ v &= \begin{cases} \frac{y^3}{|y|} & \text{if } y \neq 0 \\ 0 & \text{if } y = 0 \end{cases} \end{aligned}$$

Conclusion

Reflecting upon what we have done, we see some of the differences between various branches of mathematics. We have seen Linear Algebra, Group Theory, Continuous Functions, and Complex Variables. By using the specific example of the unit circle, these differences have been amplified.

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At Point of Explosion

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Introduction

Elementary school students release balloons with their names and addresses attached with a note asking for a return letter so they will know how far their balloon traveled. Some of these balloons lose their helium while others are caught in trees and power lines preventing the balloons from gaining any significant distance. Other balloons seem to virtually disappear. Weather forecasters also release weather balloons into the atmosphere to measure information like atmospheric pressure and temperature. These balloons, normally filled with helium, rise high into the sky until they also disappear. What happens to these balloons if they gain enough altitude to be lost from sight? Many times, the balloons will lose their lift and come back to earth. However, if a balloon rose high enough into the atmosphere, the atmospheric pressure differential would cause the balloon to pop.

Background

While the authors were kicking some balloons around in a hallway, a discussion of how much pressure it would take to pop one of the balloons arose. Out of this lively and interactive discussion came an interesting question; how high would a balloon have to go up into the atmosphere before it would pop due to pressure differential?

Air pressure is the weight of the air pressing down on the earth. The higher a balloon goes into the air; the air pressure exerted on it is less. A similar example would be seen in a sea bubble rising from the ocean floor. The sea bubble would start out at a certain size and begin to rise to the surface. As it rose, the pressure would decrease allowing the bubble to get larger until it surfaced and popped. Like a sea bubble, a balloon expands as the pressure of the air inside the balloon pushes against the latex and outside air pressure. As the balloon rises into the atmosphere, the air pressure pushing in on the balloon decreases [3]. Soon, the only thing keeping the balloon from exploding is the latex. This is because there is less atmospheric pressure helping to keep the helium contained and thus holding the balloon's shape. The balloon's latex would eventually reach a point of fracture and the balloon will explode.

Stress, in this case, comes from pressure exerted on the walls of the balloon. Stress distribution towards the ends of an inflated balloon can be very complex, but towards the center the stress distribution is relatively simple. There are three types of stress that can act on a balloon: axial stress, hoop stress, and radial stress. Axial stress acts along the length of the balloon. Hoop stress acts along the diameter at the center of the balloon. Radial stress acts through the wall of the balloon and is usually negligible.

Axial stress = $\Pr/2t$ (tensile)

Hoop stress = $\Pr/t$ (tensile)

where P = pressure, r = radius of the balloon, t = thickness of the balloon wall

In the case of a long and skinny balloon, the hoop stress is always twice the axial stress. When the balloon is truly spherical, stress equals biaxial tension because both the axial and hoop stress will equal $\Pr/2t$; due to the spherical symmetry both axial and hoop directions are the same. The radial stress in the case of either of these balloons would be negligible. When stress starts acting on the walls, the latex starts stretching to the point where it can stretch no more, initiating cracks. Most often cracks run perpendicular to the maximum stress. This essentially means the cracks run axially. In the case of round balloons the tensile stress acts in all directions, tangential to the wall, making the direction of the cracks unpredictable. Once the balloon begins to crack at one point, it will send stress waves through the now increasingly stressed wall. The combined stress is enough to initiate additional cracks, which causes the popping of the balloon [2].

Model

To determine the height at which the balloon pops, a differential equation model was developed. In deriving the model, the following were assumed: no humidity, no difference due to the color of the balloons, no temperature impact, no wind, no abnormal atmospheric conditions, and all balloons were spheres.

$$\frac{dP}{dZ} = -kP\left(\frac{1}{h}\right)$$
$$\int \frac{1}{P}dP == -\frac{k}{h}\int dZ$$
$$\ln|P| = -\frac{k}{h}Z$$
$$P = e^{-\left(\frac{kZ}{h}\right)}$$
$$P = P_0 e^{-\left(\frac{Z}{h}\right)}$$

P_0 = pressure at sea level
h = scale height
P = pressure at height
Z = height

Experimental

The objective for the experiment was to determine the height at which sample balloons pop. To use the equation developed in the model, three variables and the constant, h, are used. Those variables are P_0, Z, and P. P_0 is given to be 1013 millibars (mbars) at sea level and Z is the unknown. Thus, a value for P needed to be found or determined before the equation could be used. To help determine P, an experiment was set up involving popping sample balloons with helium and recording the pressure when the balloons exploded. This was decided because an experimental value would be more specific for use with the sample balloons. The pressure found experimentally in the lab would be the pressure inside the balloon. The objective was to find a pressure outside of the balloon that would cause it to pop. To do that, assume

P = atmospheric pressure
P_i= initial pressure inside the balloon when released
P_b=experimentally found pressure differential when it pops.

Then $P_i - P = P_b$. Rearrange to $P = P_i - P_b$ to determine the atmospheric pressure when the balloons would pop.

Procedure

In keeping with the assumption of pressure and popping being independent of color, the procedure used balloons of different color. Using a helium gas tank, six balloons are expanded to the point of overcoming latex strength and thus popping. The pressures are recorded.

Data

Balloons are 7" round assorted colors made by the National Latex Products Company in Ashland, Ohio 44805.

Trial 1: 7.5 lbs/in^2
Trial 2: 9.0 lbs/in^2
Trial 3: 5.0 lbs/in^2
Trial 4: 5.2 lbs/in^2
Trial 5: 5.1 lbs/in^2
Trial 6: 7.5 lbs/in^2

Interpretation

The average pressure observed was 6.6 lbs/in^2(455 mbars) [4]. The value is only accurate to two significant figures. Using this value in $P = P_i - P_b$ yields the following result:

$$P_i - P = 455 mbars$$

It is known that P_i can be changed to whatever value desired. if $P_i =$ 1200 mbars then $P = 745$. If P_i 1100 mbars then $P = 645$. This shows the relationship between P_i and P. The lowest P_i that would still provide sufficient lift to the balloon so that it would rise would send the balloon to the greatest height before it exploded. In other words, the minimum P_i, where the balloon is just buoyant enough to lift, gives the maximum popping height [1].

Application

Now that P is experimentally found, the differential equation model can be applied to the sample balloons. Thus the height at which the sample balloons pop can be related to the experimental P, 455 mbars at $P_i = 1200$ mbars and at $P_i = 1100$ mbars. The following table, relating height and pressure, was created from the differential equation model by inserting height values and solving for the pressure.

Height	Pressure (mbar)	Height	Pressure (mbar)
0	1013	18000	78.05440104
1000	878.5456472	19000	67.69432805
2000	761.9372701	20000	58.70933591
3000	660.8061919	21000	50.91691167
4000	573.098128	22000	44.15876716
5000	497.031457	23000	38.29762357
6000	431.0610297	24000	33.21442301
7000	373.8467834	25000	28.80590994
8000	324.2265196	26000	24.98253386
9000	281.1922978	27000	21.66663019
10000	243.8699598	28000	18.79084268
11000	211.5013737	29000	16.29675523
12000	183.4290339	30000	14.13370521
13000	159.0827042	31000	12.2577544
14000	137.9678356	32000	10.63079642
15000	119.6555197	33000	9.219782746
16000	103.7737769	34000	7.996051333
17000	90	35000	6.934744416

Table 1: Pressure vs. Height

Plotting this data yielded the following graph.

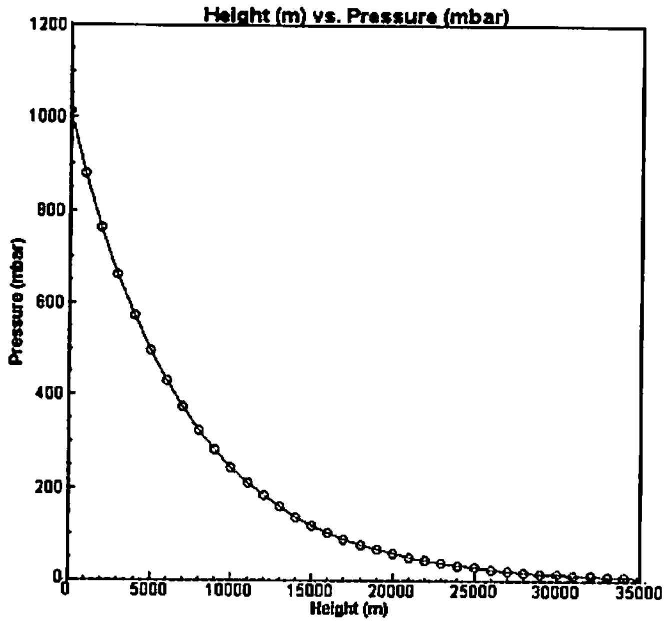

Figure 1: Graph of Table 1

Conclusion

When the children release balloons as mentioned in the introduction, the balloons that would gain the greatest distance would be the balloons that reached the greatest height. if the students release the sample balloons used in the experiment, the equation developed in the model and the experimentally found P would predict that the balloons would pop at a height of ˜2000 m or ˜6562 ft at $P_i = 1200$ or ˜3000 m or ˜9843 ft at $P_i = 1100$. Wind speed would affect the distance in that the harder the wind blows, the more horizontal distance the balloon would travel. The balloon pops due to a decreasing atmospheric pressure, which causes the balloons to expand and pop. Elasticity plays a role in determining when the balloon will pop and was included in the experimental findings of P. Elasticity along with temperature, humidity, wind, and abnormal atmospheric conditions were excluded in this model. This model simply took into account pressure and height. A possible future project could take into account these additional factors. One example would be that temperature would affect the pressure and the latex strength. The result would be that the colder the temperature, the more brittle the latex would become and cracking would be easier causing the balloon to pop sooner.

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Symmetry to Infinity

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Presented at the 2001 National Convention

Have you ever looked at the patterns in the tiles on the floor of your bathroom? Or perhaps you have noticed the patterns in your wallpaper or curtains. I bet it would surprise you to know that the patterns you notice in the tiles on the floor or the pattern in the wallpaper is strongly tied to mathematics.

Symmetry is key for having a pattern. Joyce defines symmetry as a transformation of the plane that moves the pattern so that it falls back on itself. The four symmetries that lie in a plane and that preserve distance are translations, rotations, reflections, and glide reflections. In the following paragraphs, we will explore these symmetries in more detail.

First we will look at translations. Translation symmetry means that you can slide the pattern so that it falls back upon itself. For any given translation we will restrict ourselves to sliding in only two different directions. Translations can also move different distances from the original spot.

Next we shall explore rotation symmetry. Rotation, as the word implies, rotates a shape around a point in the pattern. The possible angles of rotation happen to be 180^o, 120^o, 90^o, or 60^o, since these are the only ones that simultaneously divide 360^o and are angles in regular polygons. If a pattern is rotated 180^o, to get the pattern back to its original place we must rotate it twice. The first rotation brings the pattern to 180^o and the second to 360^o or 0^o. A 60^o angle needs more rotations to get it back to the original orientation. The first rotation brings the pattern to 60^o, second to 120^o, third to 180^o, fourth to 240^o, fifth to 300^o, and sixth to 360^o. Therefore we must rotate a 60^o angle 6 times to get it back to its original position. The order of rotation is the number of times needed to get a pattern back to its original position. An easier way to find the order of rotation than adding together all of the degrees, is to divide 360 by the degree of the angle. We will later denote the angles by their order of rotation. So, $180^o = 2$ degree rotation; $120^o = 3$ degree rotation; $90^o = 4$ degree rotation; and $60^o = 6$ degree rotation.

Reflection symmetries are similar to looking at mirrors. In reflections, a line is fixed on which you could place an imaginary mirror and the points on one side of the mirror are reflected, or exchanged with the points on the other side of the mirror. A common example of a reflection is a butterfly with the line of reflection down the middle.

The last kind of symmetry we will explore is called a glide reflection. It is a little more difficult than the other three to see and understand. First a reflection is applied along an axis, and then the translation occurs. When the finished glide reflection is accomplished, no rotations or reflections are symmetries. A common example is footprints because you can think of them reflecting and then gliding.

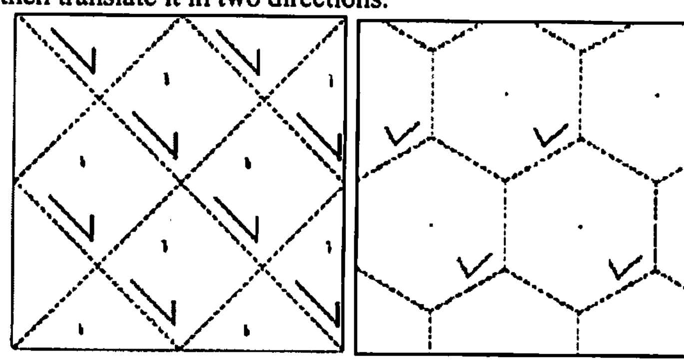
It would be believable that using combinations of these four symmetries would make an infinite amount of different patterns. Amazingly, there are essentially only 17 different groups of patterns. We will explore the term group later, but realize that there are only 17 different basic patterns. Using different colors and different shapes can make a pattern look different, but it is still the same group by its symmetries. We need to classify these groups, so we have names for each of the patterns and can group them together. One way to classify them is by their symmetries, and another way to classify them is by their lattices.

Lattices can be classified into five different categories. These five categories are square, hexagonal or triangular, rhombic, rectangular, and parallelogrammatic. For any point on the pattern, the lattice is the transformations of it by translation symmetries. Another way to think of lattices is the grid or the underlying shape of the pattern. Each lattice has certain symmetries that can be used. The parallelogrammatic lattice has translations and 180° rotations, but there are no reflections or glide-reflections. The rectangular lattice has translations, 180° rotations, and reflections. The rhombic lattice has translations, 180° rotations, reflections, and glide-reflections. The square lattice has translations, rotations of 90° and 180°, reflections, and glide reflections. The hexagonal lattice has translations, rotations of 60°, 120°, and 180° reflections, and glide reflections.

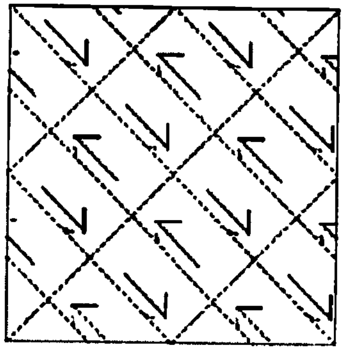
Now comes the fun part, the patterns! The patterns can be thought of as a game. This "game" can be played with seventeen sets of rules, but obviously not at the same time. The "board" of the game is a grid, which consists of squares, rectangles, parallelograms, rhombi, or triangles. Sound familiar? The board of the game is the lattice of the pattern. Some of the rule groups may use any one of the five boards while other rule groups use only one. This is apparent since not all symmetries work with all lattices. To understand the classification of the groups, we use different symbols to identify them. This notation is not found in all books. If the group's name has an x, it means that there exists a glide reflection in the pattern. If it has multiple x's, then there are as many glide reflections as the number of x's.

If an * appears, it means a reflection is in the pattern and like the x's, the number of * equals the number of reflections that are in the pattern. If it has a number it corresponds to the degree of rotation, as explained above. The numbers will be 2, 3, 4, or 6. If a number exists more than once then the number of that number corresponds to the number of rotations. Translations are in every single group so they are not denoted. As an aside all of the patterns go to infinity, meaning that they will extend to infinity in all directions, but they don't necessarily have a degree of freedom. A degree of freedom means how much the original shape can translate or better in how many directions it has freedom to translate. A picture of each of these groups is located after each explanation. These figures were found at Levy's website and were originally published in the 30th edition of the CRC Standard Mathematical Tables and Formulas. Now that we have that out of the way we can start the rule sets of the game.

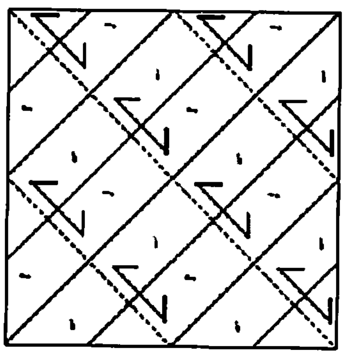
Rule Group #1 [p1]. This set of rules works on all of the grids. This is the easiest of the game and gives 2 degrees of freedom. First start with a shape, then translate it in two directions.



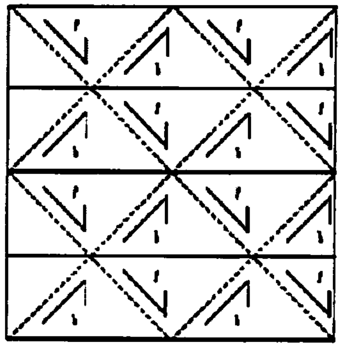
Rule Group #2 [pg (xx)]. This set of rules works only on the square and rectangle grid and only has one degree of freedom. The presence of two x's means that it has two glide reflections. So as the picture shows, first start with the shape, reflect it along its edge, and glide it either right or left. This is done twice.



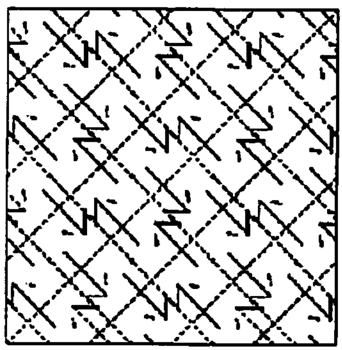
Rule Group #3 [pm (**)]. This set of rules works only on the square and rectangle grid and has only one degree of freedom. This has two *'s so it has two reflections. So as the picture shows, first start with the shape, reflect it on both sides, and then translate it up and down.



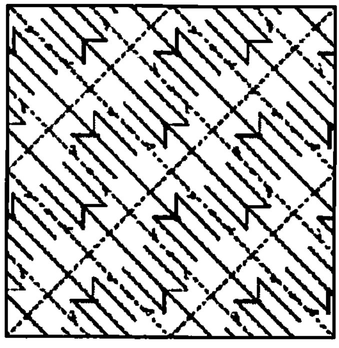
Rule Group #4 [cm (x*)]. This group has one degree of freedom and can be played on the rhombus, square, or triangular grid. Reflect the shape along its side and glide it in either direction. Then reflect the entire image to the next line.



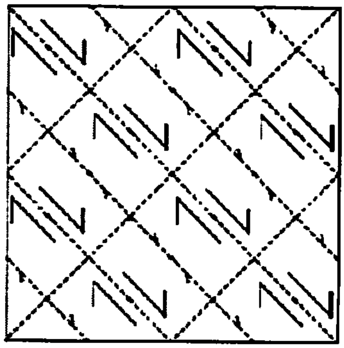
Rule Group #5 [pgg (22x)]. Only the square and rectangular grid works on this pattern and the degree of freedom is in only one direction. A glide reflection is done first on the shape then it is rotated 180° twice.



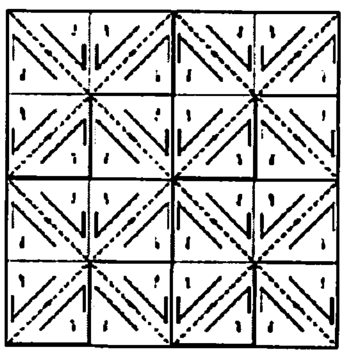
Rule Group #6 [pmg (22*)]. This only works on the square and rectangular grid and has only one degree of freedom. The figure has two 180° rotations and one reflection as well as translations.



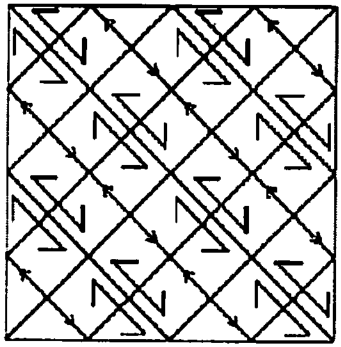
Rule Group #7 [p2 (2222)]. This uses all grids and has two degrees of freedom. It has four 180° rotations.



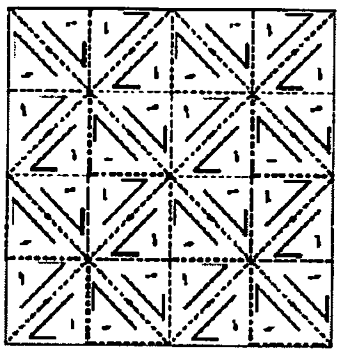
Rule Group #8 [cmm (2*22)]. This can be played on the rhombus, square, or triangular grid and has one degree of freedom. One 180° rotation is along the vertex while the other two are along the sides.



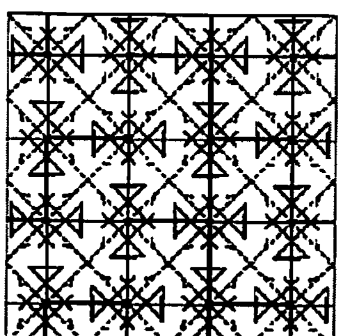
Rule Group #9 [pmm (*2222)]. This uses both the square and rectangular grids and has one degree of freedom. The four 180° rotations are along the sides and not the vertices.



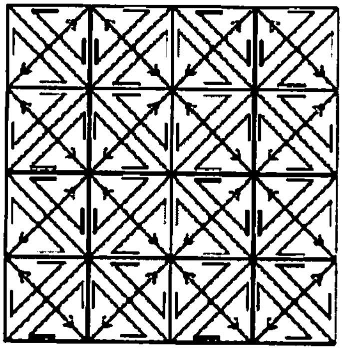
Rule Group #10 [p4 (442)]. This can only be played on the square grid and has no degrees of freedom. This rule set has two 90° rotations and a 180° rotation.



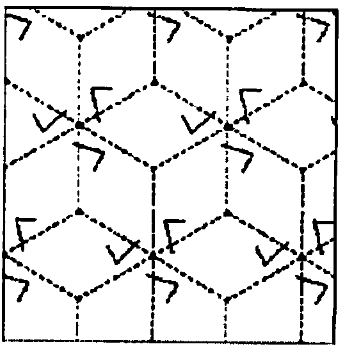
Rule Group #11 [p4g (4*2)]. This can also be played only on the square grid and has no degrees of freedom. This rule set has a 90° rotation on the vertex and a 180° rotation on the line.



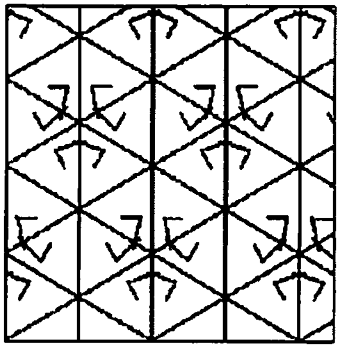
Rule Group #12 [p4m (*442)]. This like the last two can only be played on the square grid and has no degrees of freedom. This rule set has two 90° rotations along the sides and a 180° rotation along the side.



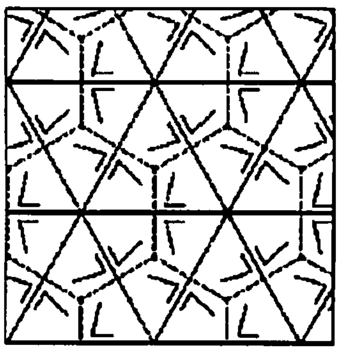
Rule Group #13 [p3 (333)]. This can only be played on the triangular grid and has no degrees of freedom. The three rotations occur all 120° and on the vertices.



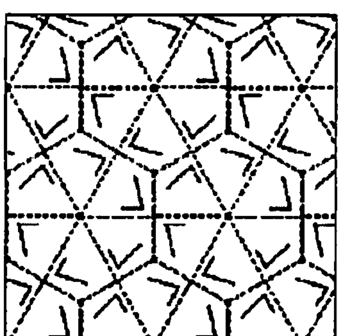
Rule Group #14 [p3m1 (3333)]. This can also only be played on the square grid and has no degrees of freedom. This rule set has four 120° rotations on the vertices.



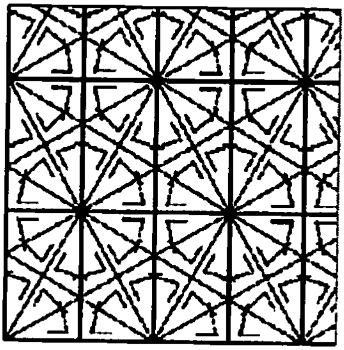
Rule Group #15 [p31m (3*3)]. This like the previous two can only be played on the triangular grid and has no degrees of freedom. One 120° rotation occurs on the vertex and the other 120° rotation occurs on the side.



Rule Group #16 [p6 (632)]. This again can only be played on the triangular grid and has no degrees of freedom. This rule set has one 60°, one 120°, and one 180° rotation all on vertices.

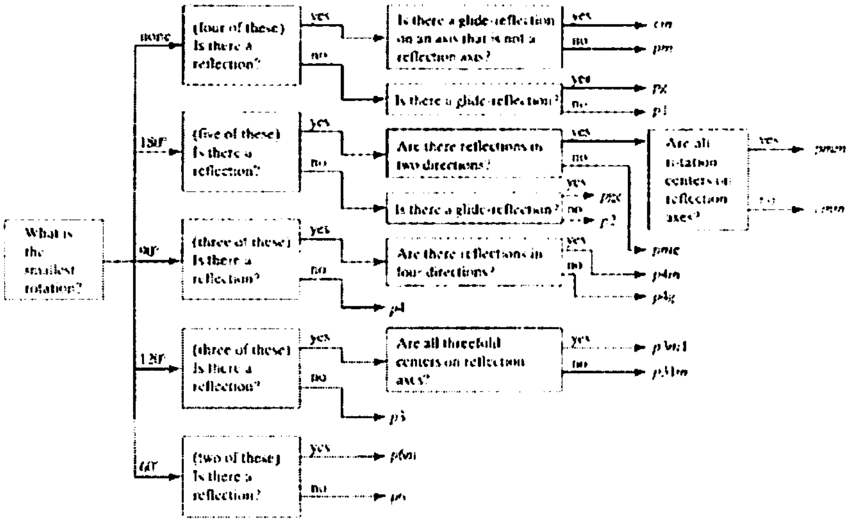


Rule Group #17 [p6m (*632)]. FINALLY!! This again can only be played on the triangular grid and has no degrees of freedom. This rule set has one 60°, one 120°, and one 180° rotation all on sides.



To more adequately understand these patterns, a simple explanation of groups is needed. Each of the above rule groups is a mathematical group. A mathematical group in general is a nonempty set with an operation that is closed, associative, has an identity element, and has inverses for every element. Typically, a group is put into a table to test the properties. Unfortunately, we cannot place these groups into tables because they are infinite. To test whether a group is closed, you see if the composition of two elements (symmetries) is also an element (symmetry). To test associativity, we have three elements (symmetries), A, B, and C (or translation, reflection, glide reflection) and if $(AB)C = A(BC)$ then the operation is associative. The operation must have an identity which when composed with another element (symmetry) gives back the element (symmetry). So if we call the identity I and have another element (symmetry) called A then if AI=A is true then I is the identity. In transformation symmetry, the identity (I) is the transformation that doesn't move. Lastly, to have a group every element (symmetry) needs an inverse. In symbols, this means $AA^{-1} = I$ where A is an element or symmetry and I is the identity. In symmetries, the inverse reverses what A did. So if A rotated 90° counterclockwise then A^{-1} would be a 90° clockwise rotation. A reflection's inverse is itself.

Distinguishing between the groups is a difficult task especially since different books and websites number the groups in different orders. The IUC (International Union of Crystallography) notation is the only notation that is uniform. Following is a flow chart, which can be useful for determining the IUC notation for a pattern. It came from Gallian's book.



The table below shows the 17 symmetry groups and how they are different. It came from Joyce.

Symmetry Group	IUC Notation	Lattice Type	Rotation Orders	Reflection Axes
1	p1	Parallelogrammatic	None	None
2	p2	Parallelogrammatic	2	None
3	pm	Rectangle	None	Parallel
4	pg	Rectangle	None	None
5	cm	Rhombus	None	Parallel
6	pmm	Rectangle	2	90°
7	pmg	Rectangle	2	Parallel
8	pgg	Rectangle	2	None
9	cmm	Rhombus	2	90°
10	p4	Square	4	None
11	p4m	Square	4+	45°
12	p4g	Square	4*	90°
13	p3	Hexagon	3	None
14	p31m	Hexagon	3*	60°
15	p3m1	Hexagon	3+	30°
16	p6	Hexagon	6	None
17	p6m	Hexagon	6	30°

+ = all rotation centers lie on reflection axes

* = not all rotation centers on reflection axes

These 17 groups are not only mathematically and artistically useful. They helped bring about the birth of modern mineralogy, the first model of DNA, and are useful in chemistry for determining lattices of atoms. After learning about these groups one can never look at wallpaper the same again.

Acknowledgements. I would like to take this moment to thank: Dr. Cynthia Woodburn for encouraging me to write this paper and for helping me with it along the way, and Dr. Ramón Figueroa, my advisor, for use of his technology and for encouragement. Without them I would not have a paper.

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The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before January 1, 2004. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 2004 issue of *The Pentagon*, with credit being given to the student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: ken.wilke@washburn.edu).

PROBLEMS 565-569

Problem 560. Proposed by Jose Luis Diaz-Barrero, Universitat Politeccnica de Catalunya, Barcelona, Spain. (Corrected)

Let $\{a_n\}_{n\in\mathbb{N}}$, $(a_n > 0)$ be an arithmetic progression. Evaluate the sum $\sum\limits_{n=1}^{\infty} \frac{1}{a_n a_{n+1} a_{n+2}}$.

Problem 565. Proposed by Jose Luis Diaz-Barrero, Universitat Politeccnica de Catalunya, Barcelona, Spain.

Let ABC be a non-degenerate triangle. Find the least upper bound of $\left(\frac{a+b}{c} - 1\right)\left(\frac{b+c}{a} - 1\right)\left(\frac{a+c}{b} - 1\right)$.

Problem 566. Proposed by Jose Luis Diaz-Barrero, Universitat Politeccnica de Catalunya, Barcelona, Spain.

Prove that $\left(\sum\limits_{k=1}^{n} \cosh x_k\right)^2 + \left(\sum\limits_{k=1}^{n} \sinh x_k\right)^2 \geq n^2$ where $k \in \mathbb{R}$, $k = 1, 2, ..., n$.

Problem 567. Proposed by Thomas Chu, Austin, Texas.

In triangle ABC let h_a, h_b, and h_c denote the altitudes from A, B, and C respectively. Let r denote the radius of the inscribed circle of triangle ABC. Prove that $\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r}$.

Problem 568. Proposed by Dan Buchnick, Ceasarea, Israel (Restated by the Editor).

Given any triangle ABC, let D, E, and F be points on BC, CA, and AB respectively, which do not coincide with any vertex of triangle ABC. Prove that the area of triangle DEF is greater than the area of at least one of the other triangles AFE, DBF, DEC except when the areas of all four triangles are equal.

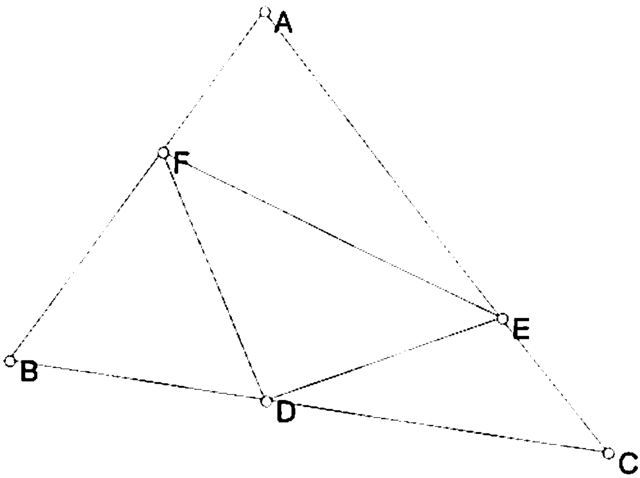


Figure 1

Problem 569. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Imagine a square surrounded by four semi-circles. Assume that the length of each side of the square is a. Construct a square that is tangent to each of the semi-circles. Now construct semi-circles on the side of each of the newly constructed square. Repeat these two steps ad infinitum. Let S denote the sum of the reciprocals of the areas of the squares and let C denote the sum of the reciprocals of the areas of the semi-circles. Prove that $\frac{S}{C} = \frac{\pi}{2}$.

Please help your editor by submitting problem proposals.

SOLUTIONS 555 – 558

Problem 555. Proposed by Maureen Cox and Albert White, St. Bonaventure University, St. Bonaventure, New York.

Define the sequence $\{a_n\}$ by $a_0 = 0, a_1 = 1, a_n = pa_{n-2} + (-1)^n$. If $-1 < p < 1$, does the series converge?

Solution by Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri.

Writing out the first few terms and noting the patterns, it follows by a simple induction argument that $a_{2n+1} = p^n - p^{n-1} - \cdots - p^2 - p - 1$ and $a_{2n+2} = p^n + p^{n-1} + \cdots + p^2 + p + 1$. Since $\lim_{n\to\infty} a_{2n} = \frac{1}{1-p}$, the general term does not go to zero and the series diverges.

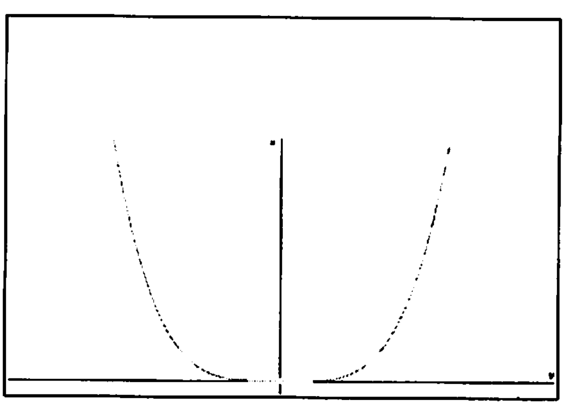
Also solved by: Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan; Carl Libis, University of Rhode Island, Kingston, Rhode Island and the proposer. One incorrect solution was received.

Problem 556. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York

Revolve the curve $y = x^{\frac{1}{p}}$ for $x, y \geq 0$ about the y axis where p is fixed and $p > 0$. Also revolve $y = \frac{1}{n}$ where $n = 1, 2, 3, \ldots$ about the y axis. Let V_n denote the volume of each solid generated between $y = x^{\frac{1}{p}}$ and $y = \frac{1}{n}$, for $n = 1, 2, 3, \ldots$. Find the conditions on p such that $\sum_{n=0}^{\infty} V_n$ converges.

Solution by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.

$y = x^{\frac{1}{p}}$ is equivalent to $x = y^p$. Consider the following figure



$x = y^p$

The volume V_n is given by

$$\pi \int_0^{\frac{1}{n}} y^{2p} dy = \left[\pi \cdot \frac{1}{2p+1} \cdot y^{2p+1}\right]_0^{\frac{1}{n}} = \left(\frac{\pi}{2p+1}\right)\left(\frac{1}{n^{2p+1}}\right)$$

Then $\sum\limits_{n\geq 1}^{\infty} V_n = \pi \cdot \frac{1}{2p+1} \cdot \sum\limits_{n\geq 1}^{\infty} \frac{1}{n^{2p+1}}$ which converges if and only if $2p+1 > 1$ which is equivalent to $p > 0$. Thus if $p > 0$ then $\sum\limits_{n\geq 1}^{\infty} V_n$ is a convergent series.

Also solved by the proposer.

Problem 557. Proposed by Pat Costello, Eastern Kentucky University, Richmond Kentucky.

From Pascal's Triangle we know that the maximum term in the sequence $\binom{3n}{0}, \binom{3n}{1}, \binom{3n}{2}, \ldots \binom{3n}{3n}$ occurs in the middle of the sequence. Find the maximum term in the sequence $\binom{3n}{0} \cdot 2^{3n}, \binom{3n}{1} \cdot 2^{3n-1}, \binom{3n}{2} \cdot 2^{3n-2}, \ldots \binom{3n}{3n} \cdot 2^{3n-3n}$.

Solution by Carl Libis, University of Rhode Island, Kingston, Rhode Island.

The sequence $\binom{3n}{i} \cdot 2^{3n-i}, i = 0, 1, 2, \ldots, 3n$ increases and then decreases. The maximum term occurs for the first i for which the ratio $\frac{\binom{3n}{i} \cdot 2^{3n-i}}{\binom{3n}{i+1} \cdot 2^{3n-i-1}}$ is less than 1. We have

$$1 > \frac{\binom{3n}{i} \cdot 2^{3n-i}}{\binom{3n}{i+1} \cdot 2^{3n-i-1}} = \frac{\frac{2 \cdot (3n)!}{i!(3n-i)!}}{\frac{(3n)!}{(i+1)!(3n-i-1)!}} = \frac{2(i+1)}{3n-i}$$

$$\Longleftrightarrow 3n - i > 2i + 2$$

$$\Longleftrightarrow n - \frac{2}{3} > i$$

Thus the ratio is first less than 1 for the value $i = n$. Thus the maximum term is $\binom{3n}{n} \cdot 2^{2n}$.

Also solved by: Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan and the proposer.

Problem 558. Proposed by Robert Rogers, SUNY College at Fredonia, Fredonia, New York.

Given a quintic polynomial $f(x)$ with exactly one inflection point at

$x = 0$, one maximum at $x = 1$, and one minimum at $x = m$, what is the maximum value m can attain? [Note: For a cubic polynomial, $m = 1$.]

This problem duplicates problem 540 which was originally published in the Fall 2000 issue (Vol. 60, No. 1, p.50). The problem has remained open without a solution being published previously.

Solution by Robert Rogers, SUNY College at Fredonia, Fredonia, New York.

The maximum value for m is $2+\sqrt{3}$. Without loss of generality, assume $m > 1$. Let $f'(x) = k(x+1)(x-m)\left(x^2+sx+t\right)$ where $s^2 - 4t \leq 0$ and k is constant. Expanding and taking $k = 1$ for sake of convenience, we have $f'(x) = x^4+(1+s-m)x^3+(t+(1-m)s-m)x^2+((1-m)t-ms)x-mt$. Since $f''(0) = 0$, we have $(1-m)t-ms = 0$ so that $t = \frac{ms}{1-m}$. This yields the following formula for $f''(x)$:

$$\begin{aligned} f''(x) &= 4x^3+3(1+s-m)x^2+2(t+(1-m)s-m)x \qquad (1) \\ &= x\left[4x^2+3(1+s-m)x+2\left(\frac{ms}{1-m}\right)+(1-m)s-m\right] \end{aligned}$$

Since the only inflection point occurs at $x = 0$, the quadratic factor in (1) can have at most one real root. Thus

$$3(1+s-m)x+2\left(\frac{ms}{1-m}\right)+(1-m)s-m$$

$$(3(1+s-m))^2-4\cdot4\cdot2\cdot\left(\frac{ms}{1-m}+(1-m)s\right) \leq 0 \qquad (2)$$

Simplifying we obtain,

$$9s^2+2\left(\frac{7m^2+2m+7}{m-1}\right)+9m^2+14m+9 \leq 0$$

Setting the left side of (2) equal to a and noting that $a \leq 0$, we solve for s obtaining

$$s = \frac{7m^2+2n+7 \pm \sqrt{9a(m-1)^2-32(m^4-2m^3-6m^2-2m+1)}}{9(1-m)}$$

Since s must be a real number,

$$9a(m-1)^2-32\left(m^4-2m^3-6m^2-2m+1\right) \geq 0$$

so that

$$a \geq \frac{32(m+1)^2\left(m^2-4m+1\right)}{9(m-1)^2}$$

But since $a \le 0$, this can happen only if $m^2 - 4m + 1 = (m-2)^2 - 3 \le 0$. The maximum value of m for which this occurs is $m = 2 + \sqrt{3}$. Substituting $m = 2 + \sqrt{3}$ and the corresponding values of s and t back into the original derivative, one obtains,

$$f'(x) = k\left(x^4 - 8\left(\frac{\sqrt{3}+1}{3}\right)x^3 + \left(4\sqrt{3}+8\right)x^2\right) - \frac{20\sqrt{3}}{3} - \frac{35}{3}$$

and so

$$f(x) = k\left(\frac{x^5}{5} - 2\left(\frac{\sqrt{3}+1}{3}\right)x^4 + \left(\frac{4\sqrt{3}+8}{3}\right)x^3 - \left(\frac{20\sqrt{3}+35}{3}\right)x\right)$$

Problem 559. Proposed by the editor. (Correoted)

Let S_n and T_n denote perfect squares, each having exactly n digits and such that $S_n + R_n = T_n$ where $R_n = \frac{10^n - 1}{9}$, n is a positive integer > 1 and each digit of T_n equals 1 plus the corresponding digit of S_n. Furthermore the left most digit of S_n is not zero. For example, for $n = 4$ we have $S_n = 2025 = 45^2$ and $T_n = 56^2$ with $R_n = 1111$.

(a) Show that if n is an even integer, one can always find appropriate values for S_n and T_n.

(b) If n is an odd integer ≤ 15 find all integers n and the corresponding values of S_n and T_n which satisfy the conditions of the problem.

Solution by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan. Part (a) only.

Let $n = 2k$. Then $R_{2k} = 10^{2k} - 1 = \frac{10^k - 1}{9}\left(10^k + 1\right)$. Taking $S_n = S_{2k} = a^2$ and $T_n = T_{2k} = b^2$, we have $b^2 - a^2 = R_{2k} = 10^{2k} - 1 = \frac{10^k - 1}{9}\left(10^k + 1\right)$. Taking $b + a = 10^k + 1$ and $b - a = \frac{10^k - 1}{9}$ and solving for a and b yields $b = 55\ldots56$ where there are $k-1$ fives followed by a six and $a = 44\ldots45$ where there are $k-1$ fours followed by a five. Thus $T_{2k}^2 = (55\ldots56)^2$ and $S_{2k}^2 = (44\ldots45)^2$. Both T_{2k}^2 and S_{2k}^2 each have exactly $2k$ digits and the digits of T_{2k}^2 exceeds the corresponding digit of S_{2k}^2 by 1.

Also solved by Charles Ashbacher, Hiawatha, Iowa.

Editor's Comment: My own solution for part (a) parallels our featured solution except that I expressed everything in terms of R_k; e.g., $b + a = 10^k + 1 = 9R_k + 2$ and $b - a = \frac{10^k - 1}{9} = R_k$. Thus $b = 5R_k + 1$ and $a = 4R_k + 1$ which agrees with Furdui's result. Also $T_{2k}^2 - S_{2k}^2 = \left(25R_k^2 + 10R_k + 1\right) - \left(16R_k^2 + 8R_k + 1\right) = 9R_k^2 + 2R_k = R_k\left(9R_k + 2\right) = R_k\left(10^k + 1\right) = R_{2k}$. A numerical set of solutions for part (b) was received from Charles Ashbacher, Hiawatha, Iowa without any listing of the program used. Here is my solution for part (b).

Since T_{2k+1}^2 and S_{2k+1}^2 each have exactly $2k+1$ digits, we must have $10^{2k} < S_{2k+1}^2 < T_{2k+1}^2 < 10^{2k+1}$ so that $10^k < S_{2k+1} < T_{2k+1} < 10^k\sqrt{10}$. Also since $R_{2k+1} \equiv 11 \pmod{100}$, an examination of the 22 possible two digit ending for squares, shows that $S_{2k+1} \equiv 5 \pmod{10}$. Using the UBASIC program below, the following possible choices for S_{2k+1} and T_{2k+1} were found:

$2k+1$	S_{2k+1}	T_{2k+1}	S_{2k+1}^2	T_{2k+1}^2
3	17	20	289	400
5	115	156	13225	24336
7	2205	2444	4862025	5973136
11	245795	267444	60415182025	71526293136
15	17663395	20569556	311995522926025	423106634037136

Of these, only the solutions for $2k+1 = 5, 7$, and 11 are acceptable as satisfying the conditions of the problem. The UBASIC program follows.

10 Input n (an odd integer)
20 $k = (n-1)\backslash 2 : B = int(10\hat{\ }k * sqrt(10) : R_n = (10\hat{\ }n - 1)\backslash 9$
30 For $j = 10\hat{\ }k + 5$ to B step 10
40 $T = j\hat{\ }2 + R_n$
50 If $sqrt(T) = isqrt(T)$ then 60 else 70
60 Print $n, j, isqrt(T), j^2, T^2$
70 Next j
80 End

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Kappa Mu Epsilon News

Edited by Connie Schrock, Historian
Updated Information as of December 2002

News of chapter activities and other noteworthy KME events should be sent to schrockc@emporia.edu or to

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Mathematics Department
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Campus Box 4027
Emporia, KS 66801

Chapter News

AL Alpha
Athens State University

Other fall 2002 officers: Jennie Legge, Corresponding Secretary.

FL Beta Chapter President – Teiauna Stockman
Florida Southern College 14 Actives

Other fall 2002 officers: Adele Douglin, Vice President; Julie Everett, Secretary/Treasurer; Allen Wuertz, Corresponding Secretary.

GA Alpha Chapter President – Jessica Pritchett
State University of West Georgia 25 Actives, 7 New Members

Other fall 2002 officers: Beth Gibbs, Vice President; Bryan Stamps, Secretary; J.J. Wahl, Treasurer; Dr. Joe Sharp, Corresponding Secretary.

During the fall semester of 2002, the Georgia Alpha Chapter of KME conducted its annual food and clothing drive for the needy with the proceeds being donated to the Salvation Army. We also had our Fall Social at a local restaurant with a total of 19 people attending. A fine time was had by all.

IA Alpha Chapter President – Elizabeth Robertson
University of Northern Iowa 30 Actives

Other fall 2002 officers: Sara Buchheim, Vice President; Sara Hirschman, Secretary; Scott Hirschman, Treasurer; Mark Ecker, Corresponding Secretary

Student member Sara Buchheim presented her paper "Significant Predictors for UNI GPA's"at our first fall KME meeting on September 23, 2002 at Professor Mark Ecker's residence. The University of Northern Iowa Homecoming Coffee was held at Professor (emeritus) Carl Wehner's residence on October 21, 2002 at Professor Syed Kirmani's residence where Sara Hirschmann presented her student paper on "Hyperbolic Tesselations." Student member Chad Thompkins presented his paper, "The Bernoulli Brothers" at out hired meeting on November 18, 2002 at Professor Doug Mupasiri's residence. Student member Marie Calkins addressed the fall initiation banquet with "17 Wallpaper Group Quilt." Our banquet was held at The Brown Bottle restaurant on December 9, 2002 where seven new members were initiated.

New Initiates: Jimmy Brito, Cindee Calton, David Gisch, Adam Hoefer, James Mills, Marshall Stuart, Ben Wadsley.

IA Beta

Drake University

Other fall 2002 officers: Lawrence Naylor, Corresponding Secretary.

IA Delta Chapter President – Jesse Oltrogge

Wartburg College 53 Active Members

Other fall 2002 officers: Matthew Townsley, Vice President; Alanson Ridpath, Secretary; Derek Riley, Treasurer; Dr. August Waltmann, Corresponding Secretary.

The October meeting began with a scavenger hunt. The business meeting was used planning the Homecoming fund-raiser and other events. The Roy's Place egg-cheese sandwich sales were a big success with alumni at Homecoming on October 19. On December 12, the Chapter had a pizza party as the Christmas season event.

IN Beta

Butler University

Other fall 2002 officers: Amos Carpenter, Corresponding Secretary.

New Initiates: Christine M. Berkesch, Krystal A. Briney, Brittany A. Brown, Jalene A. Campbell, Melanie L. Cole, Clint R. Garrett, Jennifer M. Henry, Jocelyn M. Ivey, Jennifer E. Legge, Lucia Quevedo, Jennifer E. Thompson, Rebecca G. Wahl, Amy E. Zapp.

IL Theta Chapter President – Colleen Powers

Benedictine University 7 Active Members

Other fall 2002 officers: Jonathan Rink, Vice President; Samee Haq, Secretary; Erica Andrews, Treasurer; Manmohan Kaur, Corresponding Secretary.

KS Beta Chapter President – Melinda Born
Emporia State University 21 Actives, 5 New Members

Other fall 2002 officers: Thad Davis, Vice President/Treasurer; Allison Fairburn, Secretary; Thad Davidson, Treasurer; Connie S. Schrock, Corresponding Secretary.

KS Delta Chapter President – Zeb Kramer
Washburn University 30 Actives, 8 New Members

Other fall 2002 officers: Jeff Kingman, Vice President; Mary Noel, Secretary/Treasurer; Allan Riveland, Corresponding Secretary.

The Kansas Delta chapter of KME met for three meetings with the Washburn Math Club. All were luncheon meetings with good food and speakers and/or mathematics presentations.

KS Epsilon Chapter President –Charlotte Bigler
Fort Hays State University 20 Actives, 5 New Members

Other fall 2002 officers: Jeff Sadler, Corresponding Secretary.

New Initiates: Sara M. Mead, Gregory D. Hanna, Christine Hindman. Danielle Augustine.

KS Gamma Chapter President – David Livingston
Benedictine College 11 Actives, 9 New Members

Other fall 2002 officers: Max Botta, Vice President; Andrea Archer, Secretary; Erin Stretton, Treasurer; Erin Stretton, Stu-Co Rep; Jo Ann Fellin, OSB, Corresponding Secretary.

Sister Linda Herndon rejoined the faculty this fall after a three-year leave during which time she completed the Ph.D. degree at the University of Wisconsin. This brought the number of faculty in the department to 4.5 again. Kansas Gamma had its first gathering at a cookout at Schroll Center on 20 September. In early September Andrea Archer and Matt Reel participated in the "Successful Student Seminar" session for beginning freshmen facilitated by Sister Jo Ann Fellin. They were joined by Christina Hoverson for the department sharing at the Open House for prospective students held on 9 November. Glenn Adamson, faculty moderator, spoke on "Catalan Numbers" at the 18 November meeting in Westernman Hall. The group gathered at Marywood for Christmas Wassail on 10 December hosted by Sister Jo Ann Fellin, OSB. Emeritus faculty member Richard Farrell joined in the festivities.

KY Alpha Chapter President – Shannon Hanner

Eastern Kentucky University 20Actives

Other fall 2002 officers: Frank Donnelly, Vice President; DeAnna Shearer, Secretary; Kristen Barnard, Treasurer; Pat Costello, Corresponding Secretary.

At the September meeting, we had the election of officers and discussed plans for the year. The annual picnic was a joint picnic with the Stat Club. It was held at Million Park on a hot Sunday afternoon. Attendance was very good. At the October meeting, Dr. Robert Nelson gave a talk entitled "Modular Origami Polyhedra." Dr. Bob showed us how to make an octahedron using six square gyroscope modules. He also showed hot to make an origami penguin. At the November meeting, Jason Davis gave a presentation of his senior Honors thesis. His thesis was on "Sequences and Society." Jason showed how sequences like the Fibonacci sequence occur in nature. In December we had our Christmas party. The start of the party was a matching game where objects laid out on a table needed to be matched with a phrase on a sheet of paper. At the White Elephant Gift Exchange, several people got huge chocolate bars.

KY Beta Chapter President – Anthony Laschon

Cumberland College 27 Actives

Other fall 2002 officers: Rose Olson, Vice President; John Nichols, Secretary; Vito Wagner, Treasurer, Jonathan Ramey, Corresponding Secretary.

On September 10, the Kentucky Beta chapter officers helped to host an ice cream party for the freshmen math and physics majors. Along with the Mathematics and Physics Club and Sigma Pi Sigma, the chapter had a picnic at Briar Creek Park on October 8. On December 13, the entire department, including the Math and Physics Club, the Kentucky beta chapter, and Sigma Pi Sigma had a Christmas party with about 35 people in attendance.

LA Delta Chapter President – April Jeffcoat

University of Louisiana at Monroe 10 Actives

Other fall 2002 officers: Stephanie Hillhouse Welch, Vice President; Ashley Nero, Secretary; Sharee Davis, Treasurer; Jane Wampler, Corresponding Secretary.

The Louisiana Delta Chapter met twice during the fall semester for a pizza party and a faculty "meet and greet." The second meeting included a student math competition with prizes awarded to those members who finished the problem first. We also had a holiday party with ACM (the computer science honor club) at the end of the semester. Some of our members represented KME at several different events in the semester. One event was the University Mile, which is a walk in which organizations on campus participate in to show their support for the university. The other event ways Reclaiming Our Campus, which was a day that volunteers paint and fix up the buildings on campus.

MD Alpha

College of Notre Dame of Maryland

Other fall 2002 officers: Margaret Sullivan, Corresponding Secretary.

New Initiates: Melissa P. Hildt, Christine M. Rybarczyk, Flavia Sasaki, Rebecka A. Sullens.

MD Beta Chapter President – Andy Ewing

McDaniel College 19 Actives, 3 New Members

Other fall 2002 officers: Dave Profilli, Vice President; Chris Drupieski, Secretary; Matt Demos, Treasurer; Linda Eshleman, Corresponding Secretary.

In October, chapter members had a dinner meeting at a local restaurant. Induction of new members was held and Dr. James Lightner, part National president of KME and a emeritus professor of McDaniel College (formerly Western Maryland College) gave an informative talk on the history of KME.

New Initiates: Sarah E. Vannoy, Sarah Elizabeth Voskuhl, Scott, Zentz.

MD Delta Chapter President –Brendon LaBuz

Frostburg State University 22 Actives

Other fall 2002 officers: Kandi Wertz, Vice President; Jacilyn Brant, Secretary; Crystal Beeman, Treasurer; Mark Hughes, Faculty Sponsor; Edward White, Corresponding Secretary.

Maryland Delta chapter enjoyed a fall picnic in October, and held two monthly meetings to lay plans for future events.

MI Beta Chapter President –

Central Michigan State University 20 Actives, 8 Faculty

Other fall 2002 officers: Arnold Hammel, Corresponding Secretary.

New Initiates: Nicholaus T. Bauer, Mindy S. Bradford, Shawna L. Dattolo, Jeffrey L. Ginn, Phillip E. Grebur, Philip J. Hackney, Justin M. Hardy, Kristin M. Huhn, Michael T. Johnson, Jennifer J. Kruk, Erin R. Militzer. Casey C. Mungall, Gbenga O. Olumnolade, Stephen A. Orzel, Alicia D. Pressey, Kendall R. Root, Robert V. Spencer, Andrea L. Tompkins, Bradley M. Wahr, Cheryl M. Waineo, Sarah D. Widajewski, Eric D. Wiesenauer, Jessica L. Wolschlager.

MI Delta Chapter President – Ana Pavasovic
Hillsdale College 20 Actives, 8 Faculty

Other fall 2002 officers: Erin De Pree, Vice President; Kaitie Nelson, Secretary; Dr. John H. Reinoehl, Corresponding Secretary.

The Michigan Delta Chapter of KME offers a free tutoring program for student in the mathematics classes. They also hosted a speech by a visitor from the Michigan Tech graduate program.

New Initiates: Lara D. Beever, Todd T. Iannantuono, Alan D. O'Keefe, Sarah E. Roache, John G. Schweitzer Jr., Amy N. Watson, Crystal L. Cornett, Coral A. Shaw

MI Epsilon Chapter President –Lynette Fulk (S) / Rebecca Barthlow (F)
Kettering University 160 Actives, 32 Faculty

Other summer 2002 officers: Gayle Ridenour, Vice President; Kathleen Monfore, Secretary; George Hamilton, Treasurer; Other fall 2002 officers: Julie Xiong, Vice President; Jamie Taylor, Treasurer; Justin Via, Secretary; Boyan Dimitrov, Corresponding Secretary.

Summer 2002: (A section) – During the Summer Term of 2002 the KME Applied Math Noon-Time Movie took place on 2nd Thursday July 25. There was a Pizza Party and the Movie "Some Mathematics of Baseball". Lots of mathematical themes are involved there. Membership pins for the new members (pledged in Winter) were distributed.

At a second gathering professor Ruben Harapetyan presented us a talk on "The Miracle of Evariste Galois". He provided some biographical details of this great Mathematician and also presented some of his momentous achievements in Mathematics.

Fall 2002: (B section) – For the Fall Term of 2002 the Second week Math Noon Time Movie "Some Mathematics of Baseball" was repeatedly presented, and pizza & pop was served again. The new members got their membership pins.

There was a Second Pizza Party on 8th Tuesday (November 26) noon. The speaker this time was the incoming B-Section KME Faculty Sponsor Processor Ada Cheng. Her enthusiastic talk "Is it Math or is it Magic?" was a challenging opportunity for listeners to find how some series of tricky questions may magically lead to the right numerical answer. New membership certificates were also distributed.

Office: http://www.kettering.edu/acad/scimath/appmath/
MI Epsilon Chapter: http://www.kettering.edu/~kme/

MO Alpha Chapter President – Natalie Merriman
Southwest Missouri State University 32 Active Members, 10 New Members

Other fall 2002 officers: James Hayes, Vice President; Andrea Streff, Secretary; Melissa Schmidt, Treasurer; John Kubicek, Corresponding Secretary.

For Fall 2002 the Missouri Alpha Chapter of KME hosted a departmental picnic and held monthly meetings. Featured speakers at the meetings included two faculty, Larry Campbell and Robert Thurman, and three students, Melissa Schmidt, Andrea Streff, and Anne Webber.

New Initiates: Brigitte Carr, Alison Cornell, Rima Freeman, Dan Billingsley, Michael Edwards, Jennifer Pope, Chris William Pratte, Annette Richmond, Aimee Engler, Marissa Wolfe.

MO Beta Chapter President – Elizabeth Jurshak
Central Missouri State University 20 Actives, 9 New Members

Other fall 2002 officers: Andrew Ray, Vice President; Brent Hoover, Secretary; Jacob Dubray, Treasurer; Andrew Nahlik, Historian; Rhonda McKee, Corresponding Secretary.

Missouri Beta chapter met each month for the fall 2002 semester. At the September meeting, we watched Donald in Math Magic Land and viewed some math-related web sites. Later in Sept., we took a field trip to hear artist Dick Termes speak about his "Termespheres." He illustrated some great relationships between math and art. At the October meeting, we initiated nine new members and Dr. So gave a presentation on Taxi Cab Geometry. In November, Jason Dehn and Andrew Nahlik were presented the "Top Freshman Award", and Dr. Sundberg led an activity titles "Searching for Pythagoras." In December, the students challenged the faculty to a volleyball game. We ended in a tie – 2 games to 2! We then went to Dr. McKees house for food and games.

MO Gamma Chapter President – Josh Bebout
William Jewell College 12 Active Members

Other fall 2002 officers: Christine Deatherage, Vice President; Stephanie Murdock, Secretary; Joseph T. Mathis, Treasurer & Corresponding Secretary.

MO Kappa Chapter President – Jonathan McCrary
Drury University 18 Actives

Other fall 2002 officers: Stacy Dare, Vice President; Carrie Wright, Secretary; Abbey Parsley, Treasurer; Charles Allen, Corresponding Secretary.

The first activity of the semester was a pizza party held at Dr. Allen's house. The winner of the annual Math Contest was Partick Meuhlan for the Calculus II and above Division. Prize money was awarded to the winners at a pizza party held for all contestants. A sub-sandwich luncheon was held for the reports of undergraduate research projects (potential KME papers) by Stephen Dickey and Tracy Go0ering and Craig Johnson. The club finished the semester with an ice-cream social. The Math Club has also been running a tutoring service for both the day school and the continuing education division as a money-making program.

MO Lambda Chapter President – Firas Al-Takrouri
Missouri Western State College 20 Actives, 10 New Members

Other fall 2002 officers: Yevgeniy Kondratenko, Vice President; Trevor Huseman, Secretary; Kurt Czerwien, Treasurer; Don Vestal, Corresponding Secretary.

MO Theta Chapter President – James Beyer
Evangel University 6 Actives

Other fall 2002 officers: Eric Block, Vice President; Don Tosh, Corresponding Secretary.

Meetings were held monthly and the fall social was held at the home of Don Tosh.

MO Zeta
University of Missouri-Rolla

Other fall 2002 officers: Roger Hering, Corresponding Secretary.

New initiates: Bernard J. Fendler, Travis J. Hebner, Jeffrey K. Houghton, Laura K. Stirnemann, Elliot L. Strathman, Jason R. Trent, Melinda S. Wormington, Travis F. Yates.

MS Alpha Chapter President – Shannon McVay
Mississippi University for Women

Other fall 2002 officers: Henry Boateng, Vice President; Lailah Bruce Valentine, Secretary; Sara Sheffield, Treasurer; Shaochen Yang, Corresponding Secretary.

During the month of September, the Mississippi Alpha chapter of KME held their monthly meeting. October contained initiation, a movie night (A Beautiful Mind), and a game night.

MS Beta
Mississippi State University

Other fall 2002 officers: Vivien Miller, Corresponding Secretary.

MS Epsilon Chapter President – Amy Pearson
Delta State University 13 Actives

Other fall 2002 officers: Becky Moore, Vice President; Jason Umfress, Secretary/Treasurer; Paula A. Norris, Corresponding Secretary.

MS Gamma

The University of Southern Mississippi

Other fall 2002 officers: Jose Contreras, Corresponding Secretary.

New Initiates: Amanda Mixon, Tracy Hardwell, Amanda Meadows, Jessica Evans, Jason Baxter.

NE Beta Chapter President – Jill Delka

University of Nebraska at Kearney 12 Actives, 3 New Members

Other fall 2002 officers: Tom Mezger, Vice President; Stephanie Becker, Secretary; Jay Powell, Treasurer; Dr. Katherine A. Kime, Corresponding Secretary.

Our chapter cosponsored, with the Math department, a "Math Open House" in Oct. 2002 to inform campus members about our programs, careers in mathematics, etc. Our president, Jill Delka, made a proposal to Pepsi for drinks (a campus opportunity) and Pepsi did provide us with 4 cases of soda. About 20 people attended. "Chickenfoot", a dominoes game, which builds trees of dominoes, was played. Brochures were available, copies of career-related websites were displayed, and there were exhibits of various manipulatives used in teaching future elementary school teachers.

We had an initiation of four new members. Two seniors completed their courses at UNK and are student teaching in Spring 2003. As one of these was the president, we now have a new president, Tom Mezger, for Spring 2003.

New Initiates: Emilee Gusso, Kenda Olson, Nick Svehla, Nathaniel Watt.

NJ Gamma Chapter President – Katie Blackburn

Monmouth University 18 Actives

Other fall 2002 officers: Melissa McCormick, Vice President; Melissa Berfield, Secretary; Stephanie Beatty, Treasurer; Amanda Glynn, Historian; Judy Toubin, Corresponding Secretary.

Since we are a relatively new chapter, we are still trying to get organized.

NM Alpha Chapter President – Joshua Lee Fisher

University of New Mexico 30 Actives, 11 New Members

Other fall 2002 officers: Aaron Cabral, Vice President; Robert Seletsky, Secretary; Galvin Mendel-Gleason, Treasurer; Dr. Terry A. Loring, Corresponding Secretary.

New Initiates: Amber L. Ambercrombie, Mallika Das, Haley Elizabeth Bell, Lisa Glendenning, Rachel Lacy, Frank Maldonado, Chris McKelvey, Anah Mourant, Eric Nelson, Jaclyn Saavedra, Wenbin Luo.

NY Alpha
Hofstra University

Other fall 2002 officers: Aileen Michaels, Corresponding Secretary.

New Initiates: Ronald Giarraffa, William J. Harney, Andrew Lazowski, Elizabeth D. Russell.

NY Eta Chapter President – Amanda Masset
Niagara University 15 Actives

Other fall 2002 officers: Michelle Searles, Vice President; Marc Erickson, Secretary; Michael Bidzerkowny, Treasurer; Robert Bailey, Corresponding Secretary.

The NY Eta Chapter had several special programs, which elicited a good deal of interest from students and faculty alike:

Origami and Math Magic, presented by Dr. Bill Price of Niagara University, September 19. The New York State Assessment Module, presented by Dr. Sue McMillen of Buffalo State Mathematics Department, Buffalo, NY, October 23. Careers for Math Majors, presented by Christopher Lagrow of Niagara University, October 29. A demonstration of Maple 8.0 software, presented by Kevin Boon of Waterloo Software, Kitchener, ON, Canada, December 2.

NY Iota
Wagner College

Other fall 2002 officers: Zohreh Shahvar, Corresponding Secretary.

New Initiates: Deborah Raffone, Robert Liberto, Denise Franzakis, Nina Domdeth, Adam Pantaleo, Thomas Sheeran Jr., Michael J. Brillante, Hiromi Fujii, Danielle Guiga, YanHang Z. Hensley, Angela Schiavone, Graziella Stasolla, Nao Tanaka, Linh N. Do, Thomas De Mayo, Laura Bruzzese, Maryanne Gaul, Michael Aciemo, Stephanie Whitehead, John Lanza III, Frank Marelli, Jessic Violet Klett.

NY Mu
St. Thomas Aquinas College

Other fall 2002 officers: Dr. Joseph A Keane, Corresponding Secretary.

OH Eta Chapter President – Derek Heckler
Ohio Northern University 29 Actives, 16 New Members

Other fall 2002 officers: Matt Suchan, Vice President; Casey Leichty, Secretary; Julia Gould, Treasurer; Donald Hunt, Corresponding Secretary.

OK Alpha Chapter President – Stephanie Hilburn
Northeastern State University 31 Actives, 7 New Members

Other fall 2002 officers: Joe Gonzales, Vice President; Katy McClure, Secretary; Amy Rose, Treasurer; Dr. Joan E. Bell, Corresponding Secretary.

Our fall initiation brought 6 new students into our chapter. Dr. mark Buckles, Assistant Professor of Mathematics, was also initiated. Retired KME faculty members Dr. A.C. Nunley, Dr. Herbert Monks, and Dr. David Fitzgerald, as well as many other faculty, came to support the new initiates.

We participated in several NSU events. We were pleased to sponsor OK Alpha member Amy Rose, as one of the five finalists for the NSU Homecoming this year. We sponsored the "KME Pumpkin Patch" at NSU's annual Halloween Carnival. The children fished for pumpkins with meter stick fishing poles. Our chapter participated in the Redmen Ralley – a high school recruitment effort for NSU. We passed out packets containing information about KME, the math department at NSU and a math puzzle.

We were honored to host our fall speaker Dr. Floyd Coppedge. He is the current Oklahoma Secretary of Education and also is a member of our Oklahoma Alpha chapter since 1959. He shared a wealth of experience in mathematics and education with us.

Our annual mathematics book sale brought in over $100. We have worked on problems from the Pentagon and other journals at several of our meetings. The Christmas party held before finals week was enjoyed by many. A logic game provided the entertainment this year, with the students coming out on top.

OK Delta
Oral Roberts University

Other fall 2002 officers: Vincent Dimiceli, Corresponding Secretary.

New Initiates: Joshua Bimat, Mark Cleveland, Yanina Levchenko.

PA Alpha Chapter President – Sarah Plimpton
Westminster College 23 Actives, 8 New Members

Other fall 2002 officers: Heather Klink, Vice President; Danielle Zielinski, Secretary; Jessalyn Smith, Treasurer; Christopher Medjesky, Publicity; Carolyn Cuff, Corresponding Secretary.

PA Lambda
Bloomsburg University

Other fall 2002 officers: Elizabeth Mauch, Corresponding Secretary.

New Initiates: Abram Campbell, Andrew M. Clark, Michael Clark, Teresa M. Druker, Steve Gentner, Dan. C. Glamp, Megan W. Hoopes, David S. Matweecha, Ryan Melnychuk, Katie Miller, Mary Louise Schleppy, Amy Tribendis, Shelly Woodmansee, Nicole M. Zeisler.

PA Zeta Chapter President – Daniel J. Galiffa
Indiana University of Pennsylvania 14 Actives, 14 New Members

Other fall 2002 officers: Kristin Cousins, Vice President; Kenda Johns, Secretary; Anna Baughman, Treasurer; Daniel A. Burkett, Corresponding Secretary.

PA Eta
Grove City College

Other fall 2002 officers: Marvin C. Henry, Corresponding Secretary.

New Initiates: Emily Fechko, Frederick J. Jenny, Ian Luke Kane, Jack D. Kendall, Coleen McManamy, Julie Milinovich, Keith Palmisano, Karen Sundberg, Andrew Wirt.

PA Iota Chapter President – Shannon Wickline
Holy Family College 20 Actives, 16 New Members

Other fall 2002 officers: Jackie Miller, Vice President; Lea Lantzy, Secretary; Carolyn Dahl, Treasurer; Kimberly Presser, Corresponding Secretary.

This semester the club ran 2 fund-raisers: selling candles and selling candy. Both fund-raisers were very successful. The students in the club volunteered to start a tutoring program in the evenings to address the needs created by cutbacks in the university sponsored tutoring. This was a very successful venture on their part and much appreciated by the mathematics department here at Shippensburg University. The students began doing some outreach to students in the department, which were accrediting our outstanding initiation class to this year. The initiation ceremony/dinner was a rousing success. Our current membership has not almost doubled and we were able to gauge at the party a renewed enthusiasm among the students.

New Initiates: Fallon Baskerville, Tim Berkheimer, Kathleen Bray, Heather Carling, Erica Condravy, Matt Curran, Jennifer Doherty, Pat Galuska J. Vince Lang, J. Russell Leidy, Kevin Schieler, Jennifer Stein, Joni Weller, Heather Wildasin, Ji Young Choi, James Hanbein.

PA Pi Chapter President – Leah Shilling
Slippery Rock University 12 Actives

Other fall 2002 officers: David Czapor, Vice President; Davlyn Nauman, Secretary; Elise Grabner, Corresponding Secretary.

PA Omicron Chapter President – Rachelle Bouchat
University of Pittsburgh at Johnstown 21 New Members

Other fall 2002 officers: Matthew Williams, Vice President; Maura Partyka, Secretary; Michelle Shoub, Treasurer; Dr. Nina Girard, Corresponding Secretary.

All Activities were held in conjunction with the UPJ Math Club, which included: a bus trip to the Carnegie Science Center in Pittsburgh, PA, welcome back bowling party, student movie night, various service projects; and end of the term faculty/student get-together.

SC Gamma Chapter President – Laura Taylor
Winthrop University 15 Actives, 2 New Members

Other fall 2002 officers: Sarah Knight, Vice President; Angel Rushton, Secretary; Angel Rushton, Treasurer; Frank Pullano, Corresponding Secretary.

New Initiates: Christine Marie Jones, Vanny Tang Yib, Rudy Wiegand.

SC Delta
Erskine College

Other fall 2002 officers: Ann F. Bowe, Corresponding Secretary.

New Initiates: Kokou Abalo, Amber Gremmer, Jessica Tepper.

TN Alpha
Tennessee Tech University

Other fall 2002 officers: Michael R. Allen, Corresponding Secretary.

New Initiates: David Butz, Logan Cardwell, Paul Schuer, Michael H. Robinson, Jefferey D. Bailey, Mark E. Livengood, Jr., Jeff Husband, Patrick Currier, Crystal Childers, Nathaniel G. Johnson, Alicia Page, Rachel Roberts, Armando M. Rodriguez, R. Patrick Vernon.

TN Delta Chapter President – Elizabeth A. Weaver
Carson-Newman College 11 Actives

Other fall 2002 officers: Marci Mitchell, Vice President; Chad Ramsey, Secretary; Houston Qualls, Treasurer; B.A. Starnes, Corresponding Secretary.

The Tennessee Delta Chapter had a quiet fall semester. We conducted one meeting to determine the time and place for the fall get-together. This event was held on 21 Nov 02 at the home of Dr. Starnes in New Market. A good time was had by all. Approximately 24 people attended.

TN Gamma Chapter President –Breanne Oldham
Union University 19 Actives, 6 New Members

Other fall 2002 officers: Nikki Vassar, Vice President; Allen Smith, Secretary; Amanda Cary, Treasurer; Bryan Dawson, Corresponding Secretary.

Our fall activities began with a Pizza Social on September 23. Our seniors gave seminar presentations in late November; it is hoped that some of these will be presented at the convention in Tulsa. Our final fall event was a Christmas social, including a "Dirty Santa" gift exchange, at the home of Dr. Lunsford. We also continued our tradition of sponsoring a needy child for Christmas.

TX Gamma Chapter President – Dayna Ford
Texas Woman's University

Other fall 2002 officers: Dr. Mark Hamner, Corresponding Secretary.

New Initiates: Alison Harrell, Aziel Wilson, Chui Wa Wong, Cody Wilson, Deborah Sparkman, Heather McClendon, Janet Sipes, Jenny Tarin, Lacy Pemberton, Lucinda Migot, Maria Arvisu, Meagan Pollock, Tillie Bradley, Heather Jungman, Yolanda Butler, Charlotte Bell, Kim Campbell, Rhonda Shaw, Kate Amorella.

TX Kappa Chapter President – Jill Klentzman
University of Mary Hardin-Baylor 10 Actives, 6 New Members

Other fall 2002 officers: William Rogers, Vice President; Kesi Perkins, Secretary; Peter H. Chen, Corresponding Secretary.

TX Mu Chapter President – Tiffany Judkins
Schreiner University 9 Active Members

Other fall 2002 officers: Kelly McCullough, Vice President; Shelley Stark, Secretary; Shannon Solis, Treasurer; William Sliva, Corresponding Secretary.

VA Alpha Chapter President – Andrew Wynn
Virginia State University 17 Actives, 20 New Members

Other fall 2002 officers: Kia Garner, Vice President; Shanna Scott, Secretary; Dr. Emma B. Smith, Treasurer; V.S. Bakhshi, Corresponding Secretary.

Three meetings were held during the fall semester. They tutored student in the Mathematics Lab at Virginia State University. They also assisted the students in the Student Support Center at VSU.

WV Alpha Chapter President – Jennifer Spencer
Bethany College 20 Actives

Other fall 2002 officers: Jason Fogg, Vice President; David Smithbauer; Steve Wood, Treasurer; Mary Ellen Komorowski, Corresponding Secretary.

The Alpha Chapter of KME has been assisting the Mathematics and Computer Science Club at Bethany College with the Allison Library Project. They serve as volunteer librarians. The members also act as tutors, graders, and office helpers in the Mathematics and Computer Science Department. Officers are planning a schedule of tutoring activities at the Wheeling Public Library and at local elementary and secondary schools. Induction of new members will take place in March, 2003.

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The Mathematical Scrapbook

The Integraph

The Mathematical Scrapbook was a regular feature of The Pentagon up until the Fall of 1982. This article originally appeared in the Fall 1972 issue of The Pentagon, Vol. 32, No. 1, pg 50-52. At that time, Richard Lee Barlow of Kearney State College was the editor for the Scrapbook.

Geometric constructions of irrational numbers have interested many a student of mathematics from ancient times to the present. One of the more fascinating irrational numbers to construct is π. It is impossible to construct π with only a straight edge and compass. In fact, not even a curve of higher order defined by an integral algebraic equation, for which π is the ordinate corresponding to a rational value of the abscissa, has been found to exist. The geometric construction of π requires the use of a transcendental curve which can be constructed using a transcendental apparatus such as the **integraph** which traces the curve by continuous motion.

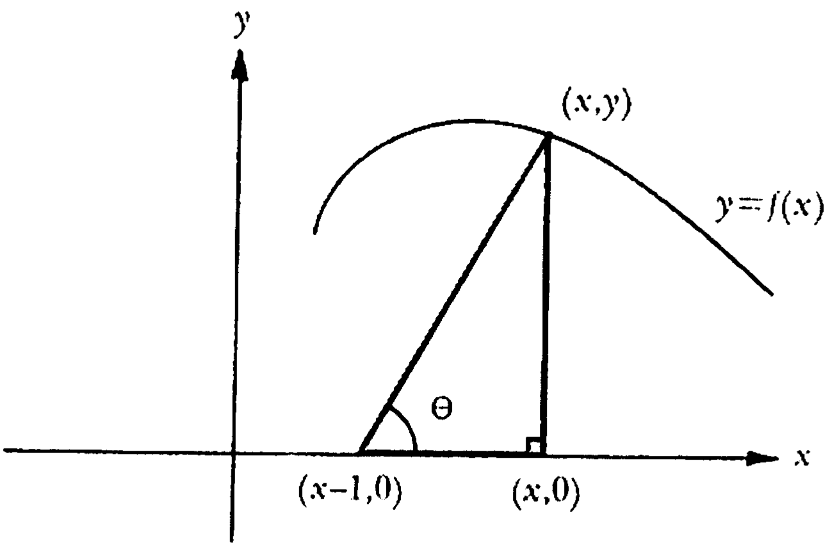

Figure 1

The integraph was invented by the Russian engineer Abdank-Abakanowicz and constructed by Coradi of Zürich. It enables one to trace the curve of the integral $Y = F(x) = \int f(x)dx$ given the differential curve $y = f(x)$. The integraph is so constructed that, when the guiding point of the linkwork of the integraph follows the differential curve, the tracing point will trace the integral curve.

Consider any point (x, y) of the differential curve $y = f(x)$ and construct the auxiliary triangle having for its vertices the points (x, y), $(x, 0)$, and $(x - 1, 0)$ as shown in Figure 1 on the previous page.

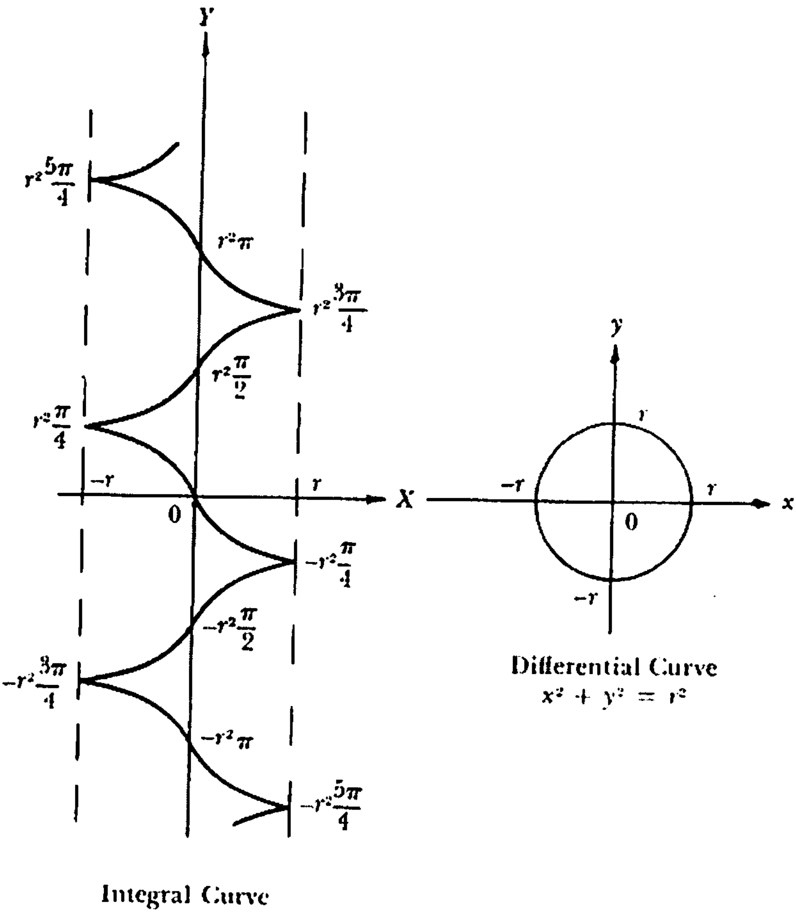

Figure 2

Note that the resulting triangle is a right triangle whose hypotenuse forms angle θ with respect to the x-axis and that $\tan\theta - y$. Therefore, the hypotenuse of the triangle is parallel to the tangent to the integral curve $Y = f(x)$ at the point (X, Y) corresponding to the point (x, y) on $y = f(x)$. The integraph is thus constructed so that the tracing point shall move parallel to the variable direction of the hypotenuse of the triangle, while the guiding point follows the differential curve $y = f(x)$. This is accomplished by connecting the tracing point of the integral curve with a sharp-edged roller whose plane is vertical and which moves so as to be always parallel to the hypotenuse of the auxiliary triangle. A weight is used to press the roller firmly upon the paper so that its point of contact can advance only in the plane of the roller. The integraph can be used to approximate definite integrals which will allow us to construct π as follows.

Let the differential curve $y = f(x)$ be the circle $x^2 + y^2 = r^2$. Hence $y = \sqrt{r^2 - x^2} = f(x)$. The integral curve $Y = F(x) = \int \sqrt{r^2 - x^2}dx$, by the use of trigonometric substitution $x = r\sin\phi$, becomes $Y = \frac{r^2}{2}\sin^{-1}\frac{x}{r}$ $\frac{x}{2}\sqrt{r^2 - x^2}$.

The integral curve thus consists of a series of congruent branches, the Y-intercepts of which have ordinates $0, \pm\frac{r^2\pi}{2}, \pm r^2\pi, \pm\frac{r^2 3\pi}{2}, \ldots$. The lines $x = \pm r$ intersect the curve $Y = F(x)$ at ordinates $\pm r^2\frac{\pi}{4} \pm r^2\frac{3\pi}{4} \pm r^2\frac{5\pi}{4}, \ldots$, as shown in Figure 2 on the previous page.
By letting $r = 1$, the ordinates of these intersections will construct the irrational number π and its multiples. The integraph thus allows us to trace the curve efficiently and with unusual sharpness. Using an integraph, can you construct 3π?

KME Website

The national KME website can be found at

`http://kme.eku.edu/`

Below is a partial list of items that are available on the site:

- How to start a KME chapter
- Information on KME conventions
- The cumulative subject index of *The Pentagon*

You can get a web page template from the Kentucky Alpha chapter. Its URL is

`http://math.eku.edu/PJCostello/kme/`

When you design a chapter homepage, please remember to make it clear that your page is for your chapter, and not for the national organization. Also, please include a link to the national homepage and submit your local chapter webpage's URL to the national webmaster. By doing so, other chapters can explore activities of your chapter and borrow some great ideas from you!

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Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983

MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February, 2001
GA Delta	Berry College, Mount Berry	21 April, 2001
TX Mu	Schreiner University, Kerrville	28 April, 2001

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