THE PENTAGON

A Mathematics Magazine for Students

Volume 62 Number 1 Fall 2002

Contents

Typeset in Scientific Word Printed in the United States of America.

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Papers written by undergraduate mathematics students for undergraduate mathematics students are solicited. Papers written by graduate students or faculty will be considered on a space-available basis. Submissions should be typewritten (double spaced with wide margins) on white paper, standard notation conventions should be respected any special symbols not typed should be carefully inserted by hand in black in. All illustrations must be submitted on separate sheets and drawn in black ink. Computer programs, although best represented by pseudocode in the main text, may be included as an appendix. Graphs, tables or other materials taken form copyrighted works MUST be accompanied by an appropriate release form the copyright holder permitting their further reproduction. Student authors should include the names and addresses of their faculty advisors. Final versions on 3.5 inch disk in "text only" (ASCII) format are appreciated. Contributors to The Problem Corner or Kappa Mu Epsilon News are invited to correspond directly with the appropriate Associate Editor.

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Statistical Analysis of Disease Rates in the Counties of Missouri

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Presented at the 2001 National Convention

Introduction

Science and mathematics are fundamentally tied together. Ask any chemist or physicist where they would be without mathematics. Some might say they wouldn't mind finding out, but as a general rule, the answer would be "somewhere back in the dark ages." Math and science go hand in hand. As I pursue a career in public health, I am interested in the way that math can be applied to biology. And I am fascinated by the idea that there may well be mathematicians on the research teams that lead us to the cures for AIDS and cancer. My presentation will be nothing of that magnitude, but my goal is to demonstrate one way that mathematics- and specifically statistical methods, have natural application to the fields of biology and medicine. I will investigate possible correlations between incidence of death due to various diseases, as well as possible correlations between environmental factors and death due to various diseases.

Nature of the Data

My study begins with some health data from my home state, Missouri. Missouri contains 114 counties, and the data is separated according to these. Three leading causes of death were selected: heart disease, lung cancer, and stroke. Obviously, more deaths will occur in a county where there are more people, so before making comparisons between counties, it is necessary to calculate a death rate. The death rate is computed by dividing the number of deaths in a county by the population of that county, and then multiplying by the standard population for which the rate will be expressed. For example, if Nowhere County, Missouri with population 20,000 had 500 deaths due to heart disease, then the death rate per 10,000 would be:

500 ,20,000 ′ 10,000 = 250

Likewise, difference in age distribution across the counties must be accounted for, since the probability of death is greater for older persons and the counties have varied proportions of older and younger persons. So an age-adjusted death rate is calculated, based on the standard of the 1940 census. The population of each county is divided into eleven age categories, and the death rate for each category is calculated. Call this the "age-specific death rate" (ASDR). Then each ASDR is multiplied by the proportion of the population that was in that age group in 1940, and these results are summed.

Age-adjusted death rates for heart disease, lung cancer, and stroke were available at the Missouri Department of Health website.

Also from the Department of Health, I obtained data on the percent of population below the 185 percent poverty level in each Missouri county. In figuring poverty level, a scale is created based on a standard minimal dollar amount necessary to provide food and housing for the members of a household. A household is below the 185 percent poverty level if their annual household income is less than 185 percent of this dollar amount.

The third type of data made use of comes from a study concerning children and lead poisoning. Any contact with lead is harmful, and lead poisoning is the result of even a small amount being swallowed or inhaled. Young children absorb lead more easily than adults do, putting them at a greater risk. So the Missouri Department of Health has been testing children for lead poisoning, and data on the percentage that tested positive was obtained. Since the sources of lead poisoning range from water and soil to food or painted surfaces, I reasoned that if a high percentage of children exhibit lead poisoning, then a high percentage of the population as a whole has probably been exposed.

Statistical Methods Used

The primary statistical tool used in the study is linear regression. Linear regression is used to show association between two random variables. The fitted regression line contains the expected y-value for every independent x-value. The line is determined by minimizing the sum of the squares of the deviations of the observed sample points from their expected values, known as the method of least squares. Scatter-plot diagrams are used to help visualize the regression. As the actual calculations become tedious, particularly with repetition, and since much statistical software is available, I have made use of a program called Windows Kwikstat, or WINKS, which is available through TexaSoft.

And finally, it should be noted that the statistical tools applied in this study show only a correlation between variables, and cannot predict a cause-effect relationship. For example, if a correlation is discovered between heart disease and stroke, the conclusion will be, "Missouri counties with a high incidence of heart disease are likely to have an elevated incidence of stroke," and not "A high incidence of heart disease causes an elevated incidence of stroke."

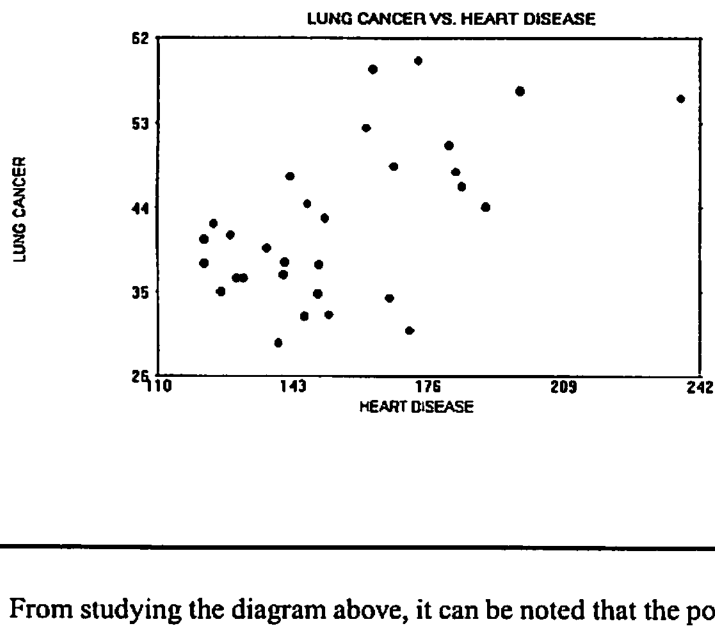
Without Further Ado. . .

Case Number One

COUNTY	HEART DISEASE	LUNG CANCER
Adair	149.3	38.0
Andrew	139.4	29.5
Atchison	150.2	30.2
Benton	162.6	58.7
Boone	121.4	38.1
Cape Girardeau	151.7	32.6
Carroll	166.6	34.4
Carter	198.7	56.4
Cole	129.2	36.5
Crawford	167.8	48.4
Dade	150.8	42.9
Dallas	123.7	42.3
Dent	184.4	46.3
Grundy	190.2	44.1
Holt	136.5	39.7
Howell	161.0	52.5
Johnson	171.6	30.9
Linn	149.0	34.8
Mississippi	237.3	55.6
Nodaway	125.5	35.1
Oregon	181.3	50.6
Perry	145.7	32.4
Pettis	142.3	47.3
Platte	121.4	40.6
Polk	140.8	38.2
Ste. Genevieve	130.8	36.5
Stoddard	182.9	47.8
Warren	140.7	36.9

I began with the question, "Is there a correlation between heart disease and lung cancer across the counties of Missouri?" As a set of thirty sample points is considered to be sufficient to create a linear model, I decided to zero in on the data from thirty counties. In order to randomly select these counties, all Missouri counties were alphabetized and numbered from one to 114. A random number generator was used to select thirty numbers between one and 114, and data from these thirty counties was entered into the WINKS program. The counties selected are shown on the previous page.

First a scatter plot was created, producing a graph where each county is represented by one sample point:



From studying the diagram above, it can be noted that the points do not seem to be scattered about randomly. That is, as the general rate of heart disease increases, the rate of lung cancer also seems to increase. Next, the linear regression is performed. A model of the form

$$Y = \beta_0 + \beta_1 X$$

will be generated, where Y is the approximate rate for lung cancer in a particular county when X is the rate for heart disease in that county. I will begin with the equation

$$Y_i = \beta_0 \beta_1 X_i + e_i$$

where the Y_i's are the rates for lung cancer for each county, and the X_i's the heart disease rates for each county. The β_0 represents the intercept for the linear model, and the β_1, the slope. The e_i represents the random error term, which will be minimized. Solving the equation for the error term yields

$$e_i = Y_i - \beta_0 - \beta_1 X_i.$$

Since the goal is to minimize the sum of squares, both sides are squared:

$$e_i^2 = (Y_i - \beta_0 - \beta_1 X_i)^2$$

So now the estimates for β_0 and $\beta_1$shall be obtained by minimizing the sum of the e_i^2 terms:

$$\Sigma_{i=1,n} (Y_i - \beta_0 - \beta_1 X_i)^2$$

Solving this equation, the estimates for β_0 and β_1 will be given by

$$\begin{aligned} \beta_0 &= Y_{Ave} - \beta_1 X_{Ave} \\ \beta_1 &= \frac{\Sigma (X_i - X_{Ave})(Y_i - Y_{Ave})}{\Sigma (X_i - X_{Ave})^2} \end{aligned}$$

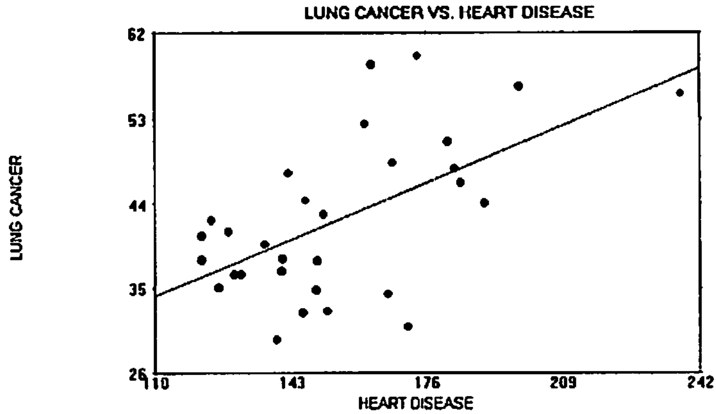
The average rate for heart disease (X_{Ave}) from the thirty counties is 155.8 and the average lung cancer rate (Y_{Ave}) is 42.0. Therefore, the following are obtained for intercept and slope, respectively:

$$\begin{aligned} \beta_0 &= 12.3 \\ \beta_1 &= .191 \end{aligned}$$

Thus the linear model achieved through this method is:

$$Y = 12.\bar{3} + .191X$$

Below is the same scatter plot seen before, with a graph of $Y = 12.3 + .191X$.



Next, a hypothesis test will be performed to determine the significance of the relationship observed. A null hypothesis (designated by H_0) and an alternative hypothesis (H_1) are formed, and a test statistic chosen to determine which hypothesis should be accepted. The test statistic yields a "$p - value$". In this case, a small $p - value$ supports our original, or null hypothesis. In the case of linear regression, if no linear association exists between the two sets of data, then the slope $\beta_1 = 0$. Thus we begin with the null and alternative hypotheses:

$$H_0 \ : \ \beta_1 = 0$$
$$H_1 \ : \ \beta_1 \neq 0$$

For the question of correlation between heart disease and lung cancer, the test statistic gives:

$$p \leq .001$$

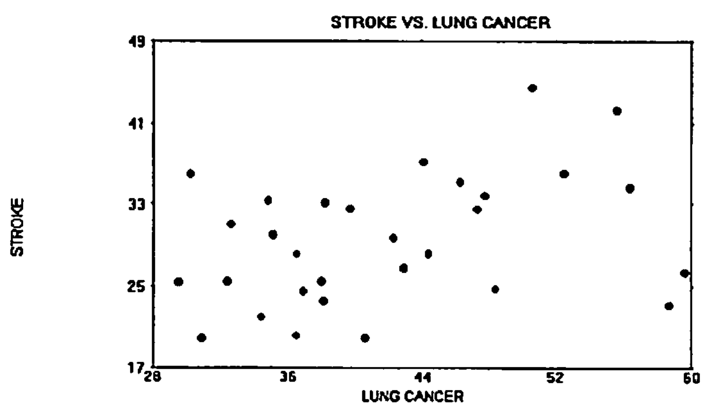
Thus, we conclude that the probability that the null hypothesis was accepted but not true is very small. So now an appropriate linear model has been designed, and the data from the remaining 84 counties can be used to test the model.

Case Number Two

A similar process is now used to determine a correlation between the rates of death due to lung cancer and the rates of death due to stroke across the counties of Missouri. Data on stroke for the 30 counties selected is as follows:

COUNTY	STROKE	COUNTY	STROKE
Adair	25.5	Holt	32.5
Andrew	25.4	Howell	36.0
Atchison	35.9	Johnson	19.9
Benton	23.2	Linn	33.3
Boone	23.6	Mississippi	42.3
Cape Girardeau	31.0	Nodaway	30.0
Carroll	22.0	Oregon	44.5
Carter	34.6	Perry	25.5
Cole	28.1	Pettis	32.5
Crawford	24.8	Platte	19.9
Dade	26.8	Polk	33.1
Dallas	29.7	Ste. Genevieve	20.2
Dent	35.1	Stoddard	33.8
Grundy	37.1	Warren	24.5
Hickory	28.2	Wayne	26.4

Data for lung cancer rates is the same as in the first comparison. It can be noted from a simple scatter plot that the correlation is not nearly so strong as in the HEART DISEASE VS. LUNG CANCER plot. That is, the sample points are not as tightly distributed about one clear line.



The hypothesis test described above was performed, and as expected, the p-value is higher: $p = 0.085$.

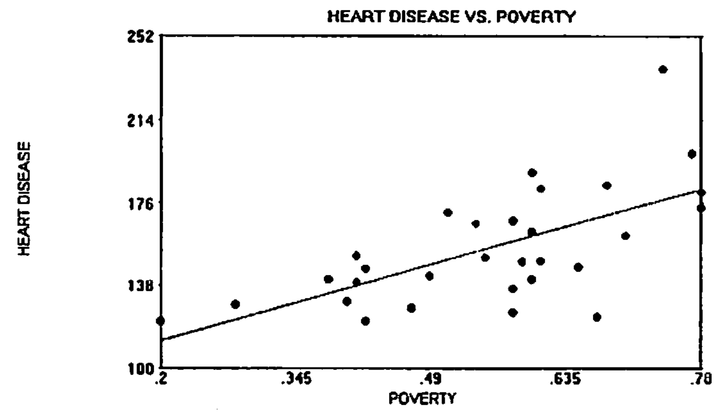
Thus the conclusion is made that there is less correlation between rates of lung cancer and stroke than between rates of heart disease and lung cancer.

Case Number Three

The study now turns to the data on poverty level. Similar procedures are exercised to answer the question, "Is there a correlation between rate of heart disease and poverty level in a Missouri county?" The data on poverty level is shown in this table:

COUNTY	POVERTY	COUNTY	POVERTY
Adair	0.61	Holt	0.58
Andrew	0.41	Howell	0.70
Atchison	0.55	Johnson	0.51
Benton	0.60	Linn	0.59
Boone	0.42	Mississippi	0.74
Cape Girardeau	0.41	Nodaway	0.58
Carroll	0.54	Oregon	0.78
Carter	0.77	Perry	0.42
Cole	0.28	Pettis	0.49
Crawford	0.58	Platte	0.20
Dade	0.55	Polk	0.60
Dallas	0.67	Ste. Genevieve	0.40
Dent	0.68	Stoddard	0.61
Grundy	0.60	Warren	0.38
Hickory	0.65	Wayne	0.78

A scatter plot of rate of heart disease versus percentage of poverty level is shown below.



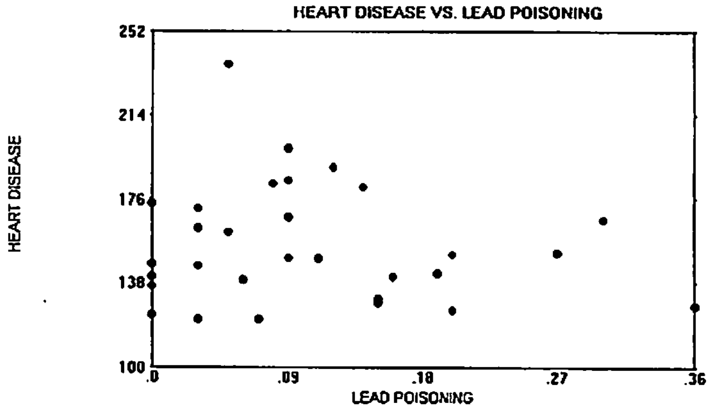
It seems that the data is closely distributed about the fitted regression line, and as suspected, the hypothesis test reveals that $p \leq .001$. Thus we conclude that the rate of heart disease and the level of poverty in a county are related.

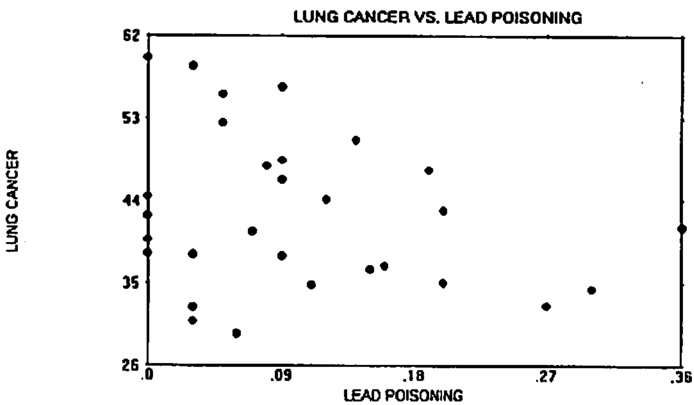
Case Number Four

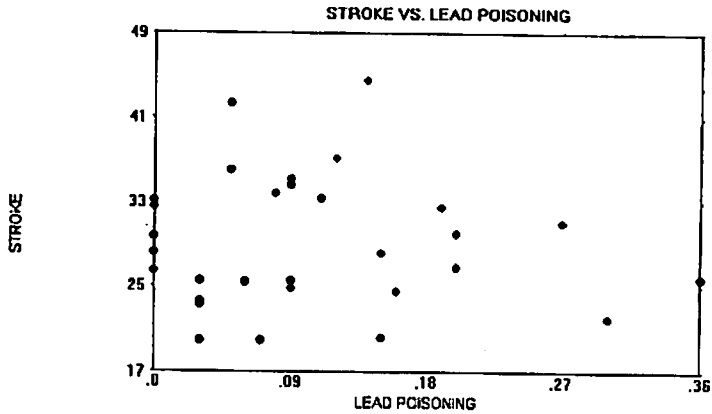
Now the percentages of children with lead poisoning across the counties are compared with the various disease rates. The percentages of children with lead poisoning for each county is listed below:

COUNTY	LEAD	COUNTY	LEAD
Adair	0.09	Holt	0.00
Andrew	0.06	Howell	0.05
Audrain	0.36	Johnson	0.03
Benton	0.03	Linn	0.11
Boone	0.03	Mississippi	0.05
Cape Girardeau	0.27	Nodaway	0.20
Carroll	0.30	Oregon	0.14
Carter	0.09	Perry	0.03
Cole	0.15	Pettis	0.19
Crawford	0.09	Platte	0.07
Dade	0.20	Polk	0.00
Dallas	0.00	Ste. Genevieve	0.15
Dent	0.09	Stoddard	0.08
Grundy	0.12	Warren	0.16
Hickory	0.00	Wayne	0.00

The following three graphs show the lead data plotted with heart disease, lung cancer, and stroke rates, respectively:


HEART DISEASE VS. LEAD POISONING
252
214
176
138
100
HEART DISEASE
.0
.09
.18
.27
.36
LEAD POISONING


LUNG CANCER VS. LEAD POISONING
62
53
44
35
26
LUNG CANCER
.0
.09
.18
.27
.36
LEAD POISONING



As is easily concluded from viewing the plots, there is little correlation between the lead data and that of any of the disease rates. This is confirmed by the p-values calculated: For heart disease versus lead poisoning, $p = .515$. For lung cancer versus lead poisoning, $p = .205$, and for stroke versus lead poisoning, $p = .708$.

Conclusion

And so in summary, six comparisons have been made, describing the levels of correlation between diseases and other factors. In four examples (lung cancer and stroke, heart disease and lead poisoning, lung cancer and lead poisoning, and stroke and lead poisoning), little or no correlation was found to be present. However, it was discovered that between heart disease and lung cancer as well as between heart disease and poverty, there is a significant amount of correlation. The linear models generated in these two cases can now be used to predict the rate of lung cancer in a Missouri county whenever the percent of poverty is known, and likewise to predict the rate of lung cancer whenever the rate of heart disease is known. And finally, the study itself provides a simplistic example of mathematics applied to a real-life question concerning public health.

Acknowledgements. I would like to thank my advisor Dr. Charles Curtis for his direction in working on this project. All the time and effort have been much appreciated! I would also like to thank Dr. John Messick, Dr. Vickie Roettger, Dr. Dennis Harmon, and Brandon Rekus for providing

me with helpful sources of information and useful feedback.

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Starting a KME Chapter

For complete information on starting a KME chapter, contact the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of the $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offering and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

A Look at the RSA Cryptosystem

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Introduction

With the world moving into a more technological era it is imperative that secure methods of transmitting information are developed. There are many applications for encryption methods in the business world alone. Suppose a company wishes to transfer sensitive data to an agent abroad, keeps confidential records of employees such as salaries and medical records or records on new developments that competitors would wish to see. The only way is to encrypt these files so that way if they are intercepted or the files are hacked all that the person would see is gibberish unless they possessed the decryption key. Not only in the business world but in our own day to day lives is this important. Files at our banks need to be kept secure or someone could easily transfer funds from our account into theirs [Meyer]. In the near future passports will be more like a drivers license with all of our information kept on a smart chip up to and including retinal patterns. Just think if this data were not secure, then anyone could fake the passport and travel into or out of our country illegally. If they happened to commit some criminal act "we" were the ones that made the trip, so it is entirely possible that an innocent bystander could find themselves in a terrible predicament just because our information was not secure. This may be a far fetched idea but the possibility is there. Many people see the world moving away from paper money into the realm of plastic entirely, and if that happens, protection of the data is paramount. In our personal lives it maybe that we are working on an open problem and we do not wish to have another person see our progress and perhaps steal our hard work; thus we need to encrypt the findings and keep them safe until we are ready to make the findings known to the public. Or maybe you simply wish to correspond with a friend over email and be assured that your writings are not being read by a third party. Considering the varying needs of encryption there are two types of encryption methods, private key, and public key crypto systems.

A private key system is just that, private. In this system only the sending and receiving people know how to encrypt/decrypt. There are many advantages to using this system. First and foremost is that any hacker trying to crack the files has nothing to work with. For example, is the encryption method just a shift in letters? Is it a complex mathematical formula? The numerous possibilities make breaking the code a long and difficult process. But if the data is being transmitted to the second party then there needs to be a way to deliver the key securely. If there is already a secure method of transmitting something then why would you need encryption? It could also be expensive if you handle things like the government does, at least in the movies, by locking the key in a bullet proof briefcase, handcuffing that to a trusted courier and surrounding him with body guards. Another solution is that if you are trying to get the key to an old friend you could reference something that presumably only the two of you would have knowledge of, such as a favorite childhood hide out or the name of a favorite waitress at the local pub. Barring a third party finding the key, this is an extremely secure method.

On the other hand, public key encryption makes known to the general public how to encrypt but keep the decryption key all to himself. This is a difficult task, since if you give away the method of encryption it only makes sense that you can "reverse" the process to decrypt. In many circumstances this is true, but one method of public key encryption is the RSA method, which will be discussed shortly.

In both systems there are several key components, plain text, cypher text, the cypher and the signature. The plain text just refers to the original data before encryption. Cypher text is the data after encryption. The cypher is the algorithm for encryption and the signature is just that, some form of signing the document so that you can be sure who it came from. Additionally Cryptography is the study of creating the cypher, cryptanalysis is the study of breaking the cypher and cryptology is the study of making and breaking ciphers.

History

If you would like to learn a little about the history of cryptology you should read David Kahn's The Codebreakers.

Basic RSA Algorithm

The RSA algorithm is fairly simple, the successful use of it however gets a bit tricky. The basic algorithm is as follows [Rosen]:

1. Find a number N that is the product of two primes p and q.
2. Find $\Phi(N) = (p-1)(q-1)$
3. Let e be relatively prime to $p-1$ and $q-1$ (an alternative to this is to let e be relatively prime to $\mathrm{lcm}(p-1, q-1)$)

4. Let $d * e = 1 \ mod \ \Phi(N)$
5. Let M be the plain text message and C be the encrypted message
6. Split the M up into blocks containing at most $log(N)$ digits
7. $M^e = C \ mod \ N$ is used to encrypt
8. $Cd = M \ mod \ N$ is used to decrypt

The pair (e, N) is the public key pair and is posted on a public domain where anyone can gain access to it. It is used to encrypt message sent to you. The pair (d, N) is the private key pair where only you have access to it and it is used to decrypt messages sent to you. If you wish to sign a message then all you need to do is encrypt the message with the receiver's public key pair, then take the last block and encrypt it with your private key. Once they receive your message they can decode the last block with your public key verifying that the message originated with you, since only you should have access to your private key, and then decoding the message with their private key.. Here is an example.

Suppose that Shawn sends the message "I really hope this presentation goes over well since my degree depends on this thesis!" to Bob.

Bob's key pair is
$N1 = 124576669624507270305958648624700264299341307748952$
97140356867
$e1 = 4543327881804725748506496889105708 31$
$d1 = 117039202705594294934338288813272073639586321896 0913$
3431847831

and Shawn's Key is
$N2 = 584972955201051875990568974351091486330703346508917$
$45783417664457394672248885340232640471097$
$d2 = 2465628746027770750402810778784416633540450628540039 0625$
$e2 = 278399743531815703777203646257888356293389226434386 9$
$45810178358169838444349767830259348 97345$

The message is first converted to a numeric text, in this example it is ASCII. Now the message looks like

I		r	e	a	l	l	y		h	o	p
073	032	114	101	097	108	108	121	032	104	111	112
e		t	h	i	s		p	r	e	s	e
101	032	161	104	105	110	032	112	114	101	115	101
n	t	a	t	i	o	n		g	o	e	s
110	116	097	116	105	111	110	032	103	111	101	115

	o	v	e	r		w	e	l	l		s
032	111	118	101	114	032	119	101	108	108	032	115
i	n	c	e		m	y		d	e	g	r
105	110	099	101	032	109	121	032	100	101	103	114
e	e		d	e	p	e	n	d	s		o
101	101	032	100	101	112	101	110	100	115	032	111
n		t	h	i	s		t	h	e	s	i
110	032	116	104	105	115	032	116	104	101	115	101

Since the message has more digits than the encryption modulus we need to separate it into blocks of at most $log(N)$ digits. Now it becomes

$M_1 = 073032114101097108108121032104111112101032161104105$
1100321121

$M_2 = 141011151011101160971161051111100321031111011150321$
1111810111

$M_3 = 403211910110810803211510511009910103210912103210010$
1103114101

$M_4 = 101032100101112101112100115032111110032116104105115$
0321161041

$M_5 = 01115101115$

When we encrypt this by raising each block to the $e1$ power and $mod\ N$, we get ...

$M_1^{e1}\ mod\ N_b = 884647694235234566809125397457636821548320028$
7352614312918258

$M_2^{e1}\ mod\ N_b = 122968746869899395129615199119903765646092681$
62582791668888424

$M_3^{e1}\ mod\ N_b = 116661369486076303865758572923021233946158902$
58554898110331337

$M_4^{e1}\ mod\ N_b = 296856735025942230092938685901880010838392615$
1007445018761172

$M_5^{e1}\ mod\ N_b = 599886666718425328717804036043929293123411417$
$881019208020 1023$

Now to sign it we take the last block and encrypt it with Shawn's private key. Thus the message is changed to:

M_1^{e1} *mod* N_b = 884647694235234566809125397457636821548320028735261431291825

M_2^{e1} *mod* N_b = 1229687468698993951296151991199037656460926816258279166888842

M_3^{e1} *mod* N_b = 1166613694860763038657585729230212339461589025855489811033133

M_4^{e1} *mod* N_b = 296856735025942230092938685901880010838392615100744501876117

$(M_5^{e1})^{e2}$ *mod* $N2$ = 1984044517579265109957684944011319109836035919413048814877184660776654535156096147953439869

Notice that we needed to re-block the message since the n used in the signature was larger that then encrypting n. There are two ways to avoid this. First a threshold value h is chosen for the system. Every member of the system has two pairs of keys, one for encryption and one for signatures such that $N_s < h < N_e$ where N_s is the signature key and N_e is the encryption key, then when you block the message, block it according the $Log\ (N_s)$ [Rivest].

Wen Bob receives the message he needs to decode it. So he takes the last block and decodes it using Shawn's public key, then takes the entire message and decodes it using his private key.

Now that this is done we just need to change them back into letters and we have sent a supposedly secure message. When changing the numerical message back into letters it is easiest to start from the end and work backwards since any leading zeros will probably be left off. Take for example a numerical message that starts out as 121324... If you work from left to right how do you know to split it up as 121, 324..... or 12, 132, 4..... By starting at the end of a message this will be avoided.

Strengths

There are several key factors that make the RSA desirable. One of these is the fact that you eliminate any need to send keys by expensive courier back and forth to the recipients. All you have to do is post the keys in a read only public file and everyone has access. Secondly and most importantly is the difficulty in factoring large numbers. If someone could factor N into its two primes they could very easily obtain the private key. One should note that, yes any number can be factored, but the length of time that is required is truly astounding in many cases when the number is chosen with care. Imagine for a moment it took 50 years to break the code. What good is a 50 year old secret really going to do anybody? By that time it is probably known anyway. So the security of the RSA algorithm from this aspect is very strong. Another good point is that key pairs can be generated rather quickly and easily. Lastly, as long as you choose the exponents carefully it takes an unreasonable amount of time to crack the cipher using these. Remember, who cares if the cypher can be broken 1000 years from now.

Attacks on N

Lets take a look at Fermat's factorization method which was developed in 1643. The algorithm for this method is [Rosen]:

Step 1: *Let* $x = \lceil \sqrt{n} \rceil + 1$

Step 2: *Let* $\Delta x = x^2 - n$

If Δx *is a perfect square then*

$n = \left(x - \sqrt{\Delta x}\right)\left(x + \sqrt{\Delta x}\right)$

If not let $x = x + 1$ and go to step 2.

Example:

Let $n = 221$

$x = \lceil \sqrt{221} \rceil = 15$

$\Delta x = 15^2 - 221 = 4$ which is a perfect square

$\therefore n = (15 - 2)(15 + 2) = 13 * 17$

With this method the number was factored in one step, where as division by small primes would have taken 5 steps (6 if you were crazy enough to start at 2!). For this method to be efficient the factors of N must be relatively close together, otherwise there will have to be too many iterations. So when choosing N one should be careful to find p and q so that they are far apart.

Another attack on N is to use the Pollard $p - 1$ method of factoring. This method makes use of Fermat's little theorem which states that if p is a prime and a is a positive integer with $p \nmid a$, then $a^{p-1} \equiv 1 \; mod \; p$. For the Pollard method let $a = 2$. The goal of this method is to find an integer k that is not too large such that $(p-1)|k!$. With this we can say that

$2^{k!} = 2^{(p-1)q} = (2^{p-1})^q \equiv 1^q = 1 \; mod \; p$

which implies that $p|(2^{k!} - 1)$. Let M be the least positive residue of $2^{k!} - 1 \; mod \; N$, so that $M = (2^{k!} - 1) - Nt$ for some integer t. Since $p|(2^{k!} - 1)$ and $p|N$, $p|M$ as well. Now all that is left is to find the gcd(M, N) which can be done fairly quickly using the Euclidean Algorithm. To factor a number using this the following algorithm works quickly[Rosen].

Step 1: *Let* $r_1 = 2$
Step 2: *Let* $r_2 = r^2 (\text{mod}\, N)$
Step 3: *Let* $r_3 = r^3 (\text{mod}\, N)$
⋮
Step k: *Let* $r_k = r^k (\text{mod}\, N)$

At each step compute $\gcd(r^k - 1, N)$, When this quantity is greater than one we have a divisor. It is possible that the divisor we get is N, but this does not happen as much as getting a proper divisor. This can be extended to bases other than 2 and this method is usually employed after division by small primes but before elliptic curve method, quadratic sieve and the other big guns. For this method to be efficient we must have the prime factors of N, p and q, such that $p-1$ and $q-1$ have relatively small prime divisors.

Example: $N = 3937 = 31 * 127$.
$30 = 2 * 3 * 5$ and $126 = 2 * 3^2 * 7$. Since there are no big primes this should factor fairly easily so let's try.
Let $r_1 = 2$
$r_2 = 2^2 = 4 \, mod\, 3937$ $\gcd(3, 3937) = 1$
$r_3 = 4^3 = 64 \, mod\, 3937$ $\gcd(63, 3937) = 1$
$r_4 = 64^4 = 16777216 = 1659 \, mod\, 3937$ $\gcd(1658, 3937) = 1$
$r_5 = 1659^5 = 12567009116212299 = 1272 \, mod\, 3937$
$\gcd(1271, 3937) = 31$
thus 31 is a divisor of 3937.

We were able to find the divisors after only four computations of the gcd, whereas with division by small primes it would take 10 divisions (or 11 if once again it was started at 2).

Continued fractions can be used to factor numbers as well. This method is based on the following theorem.

Theorem: Let n be a positive integer that is not a perfect square. Define, $\alpha_k = (P_k + \sqrt{n})\,|Q_k$, $a_k = [\alpha_k]$, $P_{k+1} = a_k Q_k - P_k$ and $Q_{k+1} = (n - P_{k+1}^2)/Q_k$, for $k = 0, 1, 2....$where

$\alpha_0 = \sqrt{n}$. Furthermore, let p_k/q_k denote the k^{th} convergent of simple continued fraction expansion of $\sqrt{n}$. Then

$$p_k^2 - nq_k^2 = (-1)^{k-1} Q_{k+1}$$

If it happens that k is odd and $Q_{k+1} = s^2$ and s is a positive integer then

$P_k^2 = s^2 \,(\text{mod}\, n)$ and we may be able to use this congruence of two squares modulo n 5 to find the factors of n. All that needs to be done is to carry out the algorithm for the continued fraction expansion of $\sqrt{n}$and look for perfect squares on the Q_k with even indices. These may lead to a non proper factor of n [Rosen].

Example: $N = 1037$

k	a_k	p_k	P_k	Q_k
-2		0		
-1		1		
0	$\lfloor (0+\sqrt{1037})/1 \rfloor = 32$	32	0	1
1	$\lfloor (32+\sqrt{1037})/13 \rfloor = 4$	129	$32*1-0 = 32$	$(1537-32^2)/1 = 13$
2			$4*13-32 = 20$	$(1537-20^2)/13 = 49$

k	s	$\gcd(p_k+s, N)$	$\gcd(p_k-s, N)$
-2			
-1			
0			
1			
2	7	17	61

Thus $N = 1037 = 17*61$

Another method involves the Fibonacci sequence. Recall that the Fibonacci sequence is defined by:

$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$

It is easily shown that (see proof #1 in appendix)

$$\frac{F_{n+1}}{F_n} = 1 + \cfrac{1}{1+\cfrac{1}{\ddots \cfrac{1}{1+\frac{1}{n}}}} = 1 + \frac{1}{F_n/F_{n-1}} \qquad (1)$$

It is also true that (see proof #2 in appendix)

$$1 + \cfrac{1}{1+\cfrac{1}{1+ \ddots \cfrac{1}{1+\frac{1}{\alpha}}}} = \frac{\alpha F_{n+1}+F_n}{\alpha F_n + F_{n-1}} \qquad (2)$$

Factoring can either be done by checking $\gcd(N, Fn)$ or $\gcd(N, F_{n+1} * F_n^{-1})$. The second approach uses more than just factors of Fibonacci numbers which lends a small advantage to it. When factoring using this method, let $C = C^{-1} + 1$ with an initial C value of 1 and where C^{-1} indicates the inverse of C modulo N [Huber] (which is just the ratios of consecutive Fibonacci numbers from (1)) and compute $\gcd(N, C) = \gcd(N, F_{n+1} * F_n^{-1})$. (2) is used to find the number of recursions of $C = C^{-1} + 1$ that gets us back to the initial value of a. It is not set in stone that you have to use the Fibonacci numbers, but any two starting values will work. The Fibonacci sequence is used primarily because it has many divisors in it.

Example: Using $\gcd(F_n, N)$
$N = 1537$

$\gcd(N, 2) = 1$	$\gcd(N, 34) = 1$
$\gcd(N, 3) = 1$	$\gcd(N, 55) = 1$
$\gcd(N, 5) = 1$	$\gcd(N, 89) = 1$
$\gcd(N, 8) = 1$	$\gcd(N, 144) = 1$
$\gcd(N, 13) = 1$	$\gcd(N, 233) = 1$
$\gcd(N, 21) = 1$	$\gcd(N, 377) = 29$

Using $\gcd(N, F_{n+1} * F_n^{-1})$ it takes 13 iterations as well. In fact using the sequence starting at 1 and 3 it only takes 5 iterations. The method using $C = C^{-1} + 1$ takes on 12 iterations. Alternately, CFRAC has a k value of 53.

In order to defeat these types of attacks on N we must make sure that if $p|N$, then there is a large size u_p where Fu_p is the smallest Fibonacci number which contains p as a divisor.

Programs written in UBASIC are easily written for both the Continued Fractions method and the Fibonacci number method (see appendix for the versions used here). It appears as if the $C = C^{-1} + 1$ program often takes one less iteration than $\gcd(N, F_n)$ or $\gcd(N, F_{n+1} * F_n^{-1})$ and usually a lot fewer than the CFRAC. On the other hand the $\gcd(N, F_n)$ program takes much less time than any of the others because the are no inverse computations.

The last factorization method to be explored is the Pollard Rho method that was developed in 1974. The object is to choose integers $x_0, x_1 \ldots.., x_s$ so that they have distinct nonnegative residues modulo n, but where their least nonnegative residues modulo p are not distinct. Once we have found integers x_i and $x_j, 0 \leq i < j \leq s$, such that $x_i \equiv x_j \mod p$ but $x_i \neq x_j$ mod n, $\gcd(x_i - x_j, n)$ is a proper divisor of n. To find these integers we start with a seed value for x_0 and a polynomial function with integer coefficients of degree greater than 1 and compute the terms using

$$\chi_{k+1} = f(x_k) \bmod (n)$$

Once the sequence is generated compute $\gcd(x_{2i} - x_i, n)$ for $i \geq 1$. When choosing the polynomial be sure to find one that will produce a large number of integers x_i before they repeat. The polynomial $f(x) = x^2 + 1$ and an initial value of 2 is often used for this method.

Example: Let $N = 1037$

$x_0 = 2$
$x_1 = 5$ $\gcd(26 - 5, 1037) = 1$
$x_2 = 26$ $\gcd(159 - 26, 1037) = 1$
$x_3 = 677$ $\gcd(936 - 677, 1037) = 37$
$x_4 = 159$
$x_5 = 603$
$x_6 = 936$
Thus $N = 1037 = 37 * 29$

Choosing N to defeat this method is complicated enough that it is easier to pick N using the criteria put forth already. Just try to factor that number using this method. The same is also true of the elliptic curve method.

Attacks on d and e keys

Instead of trying to break the encryption by factoring N, we can attempt to find the d key directly. It is possible to try starting at 2 and working your way up, but unless the d key is extremely small this is very impractical! A total break, other than the previously mentioned method, of the system can occur when d is chosen too small.

Scenario 1:

Bob and Joe are attempting to send secret messages to each other about Shawn. Bob's public key is (154421,61445). Using this Joe encrypts the message "Meet me Sunday at the park."

The encrypted message reads

81638 104271 44321 142945 79131 41093 17780 109307 7535 36721 136865

Using just Bob's public key sly Shawn intercepts this message and manages to completely crack the system, enabling him to go eavesdrop on the not so careful duo.

How does Shawn crack the system? It is fairly simple actually. First we need a theorem.

Theorem:(M. Weiner) Let $N = pq$ with $q < p < 2q$. Let $d < N^{.25}/3$. Given (N, e) with $ed = 1 \ mod \ \Phi(N)$, one can effectively recover d [Boneh].

This method is based on the approximations using continued fractions. If one were to compute the $log\ N$ convergence of the continued fraction expansion of e/N one of these will have the d in the denominator. For devices when fast decryption is important, such as smart cards and the like, not being able to use small d keys is frustrating. However, using $\text{lcm}(p-1, q-1)$ as the d key seems to avoid this problem, since if the d is large like it needs to be, the lcm will be smaller allowing for faster decryption.

Here is what Shawn did in scenario 1.

$N = 154421$ and $e = 61445$

In the following table the column headings are the quotients generated by the Euclidean algorithm(see appendix) and the 1, 0, and 1 in the lower left corner are seed values. To find the value of the cells let k denote the column, h_k the column heading, a_k and b_k be the corresponding entries in that cell for the first row and second row respectively, then.

$a_k = h_k * a_{k-1} + a_{k-2}$ with $a_0 = 1$ and $a_{-1} = 0$ and $b_k = h_k * b_{k-1} + b_{k-2}$ with $b_0 = 0$ and $b_1 = 1$

	2	1	1	6	...	
1	2	3	5	33	...	154421
0	1	1	2	13	...	61445

The convergent fractions are simply each column written as a fraction.

After each fraction is found sly Shawn must test the encoded message using, in this case, the numerator as the d key. If he gets a sensible message back, he is done. Otherwise he must continue looking. For this particular case, the d key is found on the third convergent (the columns are treated as fractions). The way the box is set up here, the d is in the numerator since the continued fractions are converging to N/e instead of e/N. Bob's choice of the d key fits the necessary conditions perfectly. In order to make sure this does not happen easily we need d to be a fairly large number or we let d be chosen as the inverse of $e(mod\ \text{lcm}(p-1, q-1))$

Other Problems

In this section attacks on the other attacks on the system will be discussed.. The first attack to be addressed is what happens if N turns out to be a Carmichael number. A Carmichael number n is a number which has $B^n \equiv B(\ mod\ n)$ to every base B where $\gcd(B, n) = 1$, in other words it is pseudo-prime to every relatively prime base B.. Imagine the following scenario.

Scenario 2:

Suppose Bob chooses

$N = 7045248121 = 1010651 * 6971$

$e = 13$

$d = 4334911077$

Joe decides to send Bob the message "Shawn is a meanie". Converting this to ASCII and splitting into blocks of 9 gives Joe

083104061 119110032 105115032 097032109 101097110 105101
By using Bob's Public key Joe obtains
5672782078 3424431216 804255553 3229915751 4310093813
3734674217

But unknown to Bob and Joe Shawn is lurking in the shadows and intercepts the message. Looking up Bobs public key Shawn computes $modinv(e, N-1)$ and gets 6503305957. Now sly Shawn computes $modpow(message, 6503305957, N)$ and gets.....
083104061 119110032 105115032 097032109 101097110 105101
which is the original message.

Absolutely astounding!! Now why did this happen? If we look at Bob's choice for N we see that it is actually a product of three primes, $821*1231*6971$ and is a Carmichael number. In order to be a Carmichael number it must be the product of at least three primes, hence one of the reasons why we need to choose N to be the product of two primes. By choosing N in this manner we can avoid Carmichael numbers all together. Keep in mind that when dealing with the large numbers the primality tests are more than likely probabilistic and even if lady luck isn't present and the number is not prime, it is still a small probability that the number is actually Carmichael. In general, if $N = p_1p_2...p_k$, where p_j are distinct primes and satisfy $(p_j - 1)|(N-1)$ for all $1 \leq j \leq k$, then N is a Carmichael number [Rivest].

Scenario 3:

Bob wishes to send Joe a secret file so he looks up Joe's public key. Meanwhile the nefarious Shawn has secretly posted his own key under Joe's name. So old Bob sends the message, Shawn actually receives the message and since it is his key, voila! he decrypts Bob' secret message. Now Shawn is in a good mood this day so he uses Joe's real key and sends the message on to Joe, who can now decrypt it an also learn Bob's secret. Neither Bob nor Joe realizes that their communication has been compromised. Now imagine Shawn in a particularly foul mood, and instead of sending the original message onto Joe he alters the message first. Now Joe has received a message that is not correct. Shawn may even post a public key in Bob's name as well and pull the same trick. Now he can alter messages going both ways and if Joe and Bob only communicate through this channel they will never be the wiser.

This situation demonstrates the need for some way to validate the public keys. This method does not require the hacker to break the code at all so no matter how "secure" the crypto-system is, it can still be violated. There are effectively two ways to solve this problem. The first and easiest is to make sure the keys are posted by a trusted source as a read only file. Second is to send the keys to each other and have it signed by a trusted third party.

Scenario 4:

Bob has just gotten a promotion the encryption system manager for a large corporation. To make his job easier on himself he decides that he is going to use a common modulus system. Bob chooses an $N = p * q$ where p and q are large distinct primes. For each person in the system he chooses an e_p and d_p. Now everyone in the system has different keys but they use the same n. Unknown to Bob sly Shawn already works for the same company and now has even more chances to eavesdrop on Bob and Joe. From past experience Bob has learned how to choose the N very carefully and also how to choose the e and d keys. But somehow sly Shawn still breaks the code!

So what has Shawn done this time? The above is a common Modulus Setup [Costello] Here is Shawn's devious algorithm (actually it was developed by John DeLaurentis). Let E_b, D_b be Bob's key pairs and E_j, D_j be Joe's key pairs

Compute $A_0 = E_b D_b - 1$
$B_0 = \gcd(E_b D_b - 1, E_j)$
then compute $A_i = A_{i-1}/B_{i-1}$
$B_i = \gcd(A_i, E_j)$
until $B_i = 1$

Then he lets $A = A_i$ and uses the Euclidean algorithm to find a positive T so that
$TE_j = 1(mod\ A)$. Shawn now takes a block of the encrypted message, M, to Joe and computes
$M^T(mod\ N)$. The result is the numeric form for the original message and all he needs to do is change is back into letters. Unfortunately for Bob and Joe it seems that sly Shawn will always be one step ahead.

Scenario 5:

Bob is yet again attempting to send Joe an encrypted message. Joe's public key is
$N =$ 58497295520105187599056897435109148633070334650891745783417664457394672248885340232640471097
$e =$ 2014203474139153443

Trying to be sneaky he decides to send a test message to Joe so that if sly Shawn is still lurking in the shadows nothing important is compromised.

The message is

$$M = 58497295520105187599056897435109148633070334650891745783417664457394672248885340232640471096$$

When Bob performs the computations he notices something odd. $M^e \bmod N = M$ *!!!! He has just gotten back his own message!*

Which brings us to the next point. As [Blakely] points out, in the RSA system there are 9 messages which are unconcealable in any cryptosystem regardless of the choices for e and d. Thus we must be careful to avoid these messages. In fact there are some choices that will not encrypt anything at all! The moral of the story is always check to make sure you are actually encrypting your message.

Acknowledgements. I would like to thank Dr. Pat Costello for all of his help and support.

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Mathematical Discoveries of the Bernoulli Brothers

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Presented at the 2002 /Regional Convention

Many pairs of brothers have been famous, such as the Wright brothers and the Kennedy brothers. The Bernoulli family, a Swiss family, had its own pair of famous brothers – Jakob and Johann Bernoulli. These two men were not the only gifted members of their family. Eight Bernoullis were mathematically gifted – all within three generations (Simmons, 158). The mathematical genius started with the brothers Jakob (1654-1705) and Johann Bernoulli (1667-1748), sons of Nikolaus Bernoulli (1623-1708), and traveled through their descendants. These men did not originally study mathematics, however. Nikolaus wanted Jakob to become a Protestant pastor and Johann to become a doctor. Each obeyed his father and earned his degree in theology and medicine, respectively (Calinger, 418).

Jacob Bernoulli

But soon after they earned their degrees, they became interested in the "new math," what is called calculus today, from the papers of Gottlieb Wilhelm von Leibniz. Jakob and Johann Bernoulli started to write letters to Leibniz with their questions; they are now known as Leibniz's most important students (Simmons, 159). The friendship between these three men grew stronger over the years. They even defended Leibniz's calculus against Newton's calculus of fluxions (Calinger, 424).

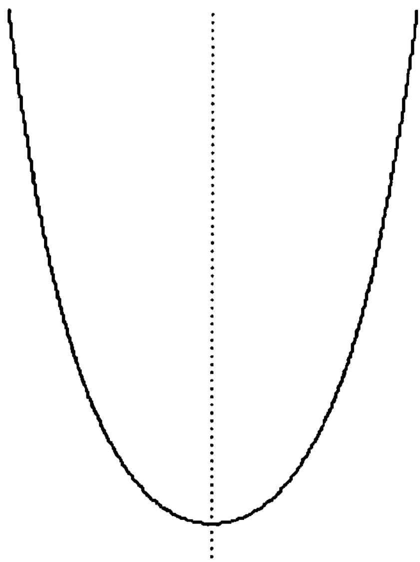
Johann Bernoulli

Jakob was the first person to use the word "integral" and later convinced Leibniz to change the name of the new math from *calculus summatorius* to *calculus integralis* (Calinger, 419). Jakob Bernoulli studied mathematics from other masters, not just Leibniz. He studied Descarte's *La Géometrie*, John Wallis's *Arithmetica Infinitorum*, and Isaac Barrow's *Lectiones Geometricae* (Calinger, 418). Jakob Bernoulli was not only interested in mathematics, but also in astronomy. He once said, "Against my father's will, I study the stars" (Calinger, 418). Johann Bernoulli studied physics, which deeply depends on mathematics. The Paris Academy awarded him its biennial prize three times – all for physics discoveries. He received the award in 1727 for his studies on the transmission of momentum, in 1730 for studies on the motion of planets in aphelion, and in 1734 for the inclination of planetary orbits toward the solar equator. In 1734, the Paris Academy awarded two first prizes – one to Johann Bernoulli, the other to his son, Daniel. Daniel Bernoulli read the letter from the Paris Academy first and was elated that both he and his father won. He waited for his father to get home from his lecturing job, but was surprised and shocked at his father's response. Guillen writes, "Johann was furious that the young man [Daniel] should now be considered his equal. He denounced the Academy for not distinguishing the master from the pupil and derided his son for not giving him proper credit" (67). This sense of competition and rivalry started with Jakob and Johann Bernoulli, then continued through the next generations. The Bernoulli brothers were especially competitive between themselves. Johann had the greater intuitive power and descriptive ability of the two; Jakob had a deeper intellect but took longer to solve problems (Calinger, 419). Since Jakob was the older brother but took longer to solve problems, he held some animosity towards his younger brother. When it comes to these brothers, "each became the other's fiercest competitor in mathematical matters" (Dunham, 191). These men held positions in universities in Europe. Jakob Bernoulli was the chair of the mathematics department at the University of Basel in Switzerland from 1687 until his death in 1705. Johann became a professor at Groningen University in 1697 then took over his brother's position at the University of Basel after his death (Simmons, 159). While at this university, Johann taught his greatest student, Leonard Euler (Eves, 357).

The Bernoulli family made many contributions to mathematics, some of the most famous being the catenary, the brachistochrone, and the proof of the divergence of the harmonic series.

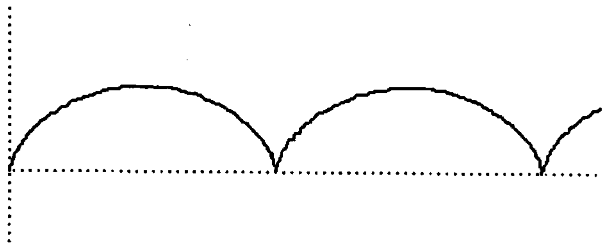
Jakob Bernoulli proposed the catenary problem in the May 1691 edition of *Acta Eridutorum*, a mathematical journal published by Leibniz. The problem stated, "And now let this problem be proposed: To find the curve assumed by a loose string hung freely from two fixed points" (Maor, 140). 'Catenary' comes from the Latin word *catena*, meaning chain (Maor, 140).

Catenary



Jakob Bernoulli gave this problem as a counter problem, or challenge, to Leibniz's isochronous curve – the curve along which a body affected only by gravity will fall with constant velocity (Burton, 435). Galileo guessed that this curve was a parabola, but was never able to prove it (Dunham, 192). Mathematicians accepted this until 1646 when Dutch scientist Christiaan Huygens proved that the shape of the catenary was not a parabola. He was just seventeen years old when he proved this, but he was unable to prove what the shape was (Maor, 140). However, like most mathematicians, Huygens kept this problem in the back of his head and continued to think about it; he did not give up on the catenary. Forty-five years later, at the age of sixty-two, he submitted a correct solution to this problem. Leibniz and Johann Bernoulli also provided correct solutions (Maor, 141). Leibniz solved the catenary in just twenty-four hours (Burton, 436). Jakob Bernoulli was unable to arrive at the correct solution because he focused on proving that the shape was a parabola (Dunham, 192). The fact that Johann solved the problem that Jakob proposed caused added tension and competition between these men. Jakob Bernoulli made up for it later when he solved general forms of the catenary that allowed for variations, such as an elastic chain or a chain of variable density (Burton, 436). The solution is part of the hyperbolic cosine function, $y = \frac{e^x+e^{-x}}{2}$. During the Seventeenth Century, the number e did not have a symbol. It was not known as its own function, but rather as the inverse of the logarithmic function (Maor, 142). The Italian Jesuit Vincenzo Riccati introduced the modern equation for the hyperbolic cosine in 1757; he was also part of a very mathematical family (Maor, 144). The catenary shape can be seen in many places in America today. One of the most famous is the Gateway Arch in St. Louis, Missouri, which is in the shape of an inverted catenary. A catenary can also be seen in suspension bridges, such as the Golden Gate Bridge in San Francisco, California. The shape of telephone wires and transmission lines is also a catenary (Burton, 436).

Johann Bernoulli proposed the brachistochrone problem in the June 1696 edition of *Acta Eruditorum*. He gave mathematicians seven months, until January of 1697, to provide a correct solution (Dunham, 199). The problem originally stated, "Suppose two nails are driven at random into a wall, and let the upper nail be connected to the lower by a wire in the shape of a smooth curve. What is the shape of the wire down which a bead will slide so as to pass from the upper nail to the lower in the least possible time?" (Simmons, 136). The problem assumed that there was no friction between the bead and wire and that the points were not directly on top of each other (Dunham, 200). Galileo believed that the solution was an arc of a circle, and his assumption had been accepted until the time of the Bernoullis (Burton, 436). By January of 1697, only Leibniz had submitted a correct solution. So, Johann expanded the time and reworded the problem to avoid any confusion. The new problem stated, "Among the infinitely many curves which join two given points...choose one such that, if the curve is replaced by a thin tube or groove, and a small sphere is placed in it and released, then this will pass from one point to the other in the shortest time" (Dunham, 199).



It is from this wording of the problem that we get the more common name for the brachistochrone, the 'shortest time' problem. Newton did not originally receive a copy of the brachistochrone problem. The Bernoullis sent Newton a copy of the reworded problem and he solved it within twelve hours of receiving it. He sent his solution in anonymously, but when Johann Bernoulli read it, he said, "The lion is known by his paw" (Burton, 373-74). The solution to the brachistochrone is the cycloid, more specifically, an inverted cycloid. The shape of the cycloid has been described as "the curve traced by a point on the rim of a wheel as it rolls along a horizontal surface"(Maor, 117). This solution was surprising to several mathematicians of that time, because so many mathematicians had investigated the cycloid that there seemed to be nothing more to discover about it (Burton, 436). The equation for the cycloid is a parametric equation defined by $x = t - sin(t)$ and $y = 1 - cos(t)$. Along with Leibniz and Newton, L'Hôpital and Jakob and Johann Bernoulli also provided correct solutions for the brachistochrone. Although these five men arrived at the same solution, some of them used different approaches to the problem. Johann used physics, relying on an analogous problem in optics: to find the curve described by a ray of light as it travels through successive layers of matter of increasing density. He applied Fermat's Principle to his solution (Maor, 117). Jakob Bernoulli took a more mathematical approach and he arrived at a differential equation. His solution led to the calculus of variations, which was later developed by Euler, Lagrange, and Legendre (Burton, 436).

The harmonic series is defined by $\{1/n\}$. The harmonic series is a series defined by the terms of the sequence $1, \frac{1}{2}, 1/3, 1/4, 1/5$, and so on. Johann Bernoulli suggested that this series diverged, or converged to infinity (Dunham, 195).

Even though each individual term is approaching zero, he hypothesized that the sum of the terms grows infinitely large. The series is defined as $1 + 1/2 + 1/3 + 1/4 + \ldots + 1/k + \ldots$. The limit of the sequence is zero, since each term grows closer to zero than the previous term, and the sequence converges. The limit of the series is the limit of the sequence of partial sums, which Bernoulli hypothesized to be infinity, which would mean that the series diverges. Dunham used the words "pathological counterexample" to describe the harmonic series. He defined pathological counterexample as, "a specific example that seems so counter-intuitive and bizarre as to warrant the label 'pathological'" (195-96). For example, one must sum up the first eighty-three terms to get the sum of the series past 5.00($1 + 1/2 + 1/3 + \ldots + 1/83 = 5.002068$). One must look at the first 227 terms for the sum to peak 6.00 at 6.00437. For the sum to reach barely over 20.00 one must sum the first quarter of a billion terms (Dunham, 196). How did Bernoulli ever think that this series would diverge? His proof rests on the convergence of a series that Leibniz had already proved. Bernoulli started his proof by defining $A = 1/2+1/3+1/4+\ldots+1/k+\ldots$, which is the harmonic series lacking the first term. Then he changed the numerators to $1, 2, 3, 4, \ldots$ so that $A = 1/2+2/6+3/12+4/20+\ldots$. If you reduce the fractions in this new A, you will get the original A; therefore, both of the series A are congruent. Next Bernoulli introduced $C = 1/2+1/6+1/12+1/20+1/30+\ldots = 1$, which is the series that Leibniz had already proved converged to one. He then defined $D = 1/6 + 1/12 + 1/20 + 1/30 + \ldots$ which is C minus its first term, 1/2. Therefore $D = C - 1/2 = 1 - 1/2 = 1/2$. Next he defined $E = 1/12 + 1/20 + 1/30 + \ldots$ which is D minus its first term, 1/6. Now $E = D - 1/6 = 1/2 - 1/6 = 1/3$. The next defined series was $F = 1/20 + 1/30 + \ldots$ which is E minus its first term, 1/12. Therefore $F = E - 1/12 = 1/3 - 1/12 = 1/4$. He similarly defined G as $1/30 + 1/42 + \ldots$, which is $F - 1/20 = 1/5$. Then he added the left sides of the equations so that $C + D + E + F + G + \ldots = 1/2 + (1/6 + 1/6) + (1/12 + 1/12 + 1/12) + (1/20 + 1/20 + 1/20) + \ldots$. Once he added these together he had $C + D + E + F + G + \ldots = 1/2 + 2/6 + 3/12 + 4/20 + \ldots$, which is A, previously defined, after changing the numerators. But if he added the right sides of the equations, he had $C+D+E+F+G+\ldots = 1+1/2+1/3+1/4$, which is the original A plus one. So Bernoulli showed that $C + D + E + F + G + \ldots = A$ and that it also equaled $1 + A$. He concluded then that $A = 1 + A$, or that "the whole equals the part" (Dunham, 196-98). Bernoulli thought that $1 + A$ was an infinite quantity, thus his proof was complete. Johann Bernoulli's proof was printed in his brother's *Tractatus de Seriebus Infinitus* (*Treatise on Infinite Series*) in 1689 (Dunham, 196). Modern mathematicians criticize Bernoulli's proof of the divergence of the harmonic series, because Johann Bernoulli treated the harmonic series as individual terms that could be manipulated at will. Today much more care is taken when working with series because they have been more rigorously defined. Another criticism is that Bernoulli proved divergence by proving that $A = 1 + A$. Today one would fix a number N and show that the series exceeds that number N. Proofs today also have to be much more rigorous than in Bernoulli's time. But in Bernoulli's defense, he wrote this proof approximately one hundred fifty years before a truly rigorous theory of series was developed. According the Burton, "One cannot deny the mathematical insight and cleverness of Bernoulli's argument" (199).

Jakob and Johann Bernoulli contributed much more to mathematics than just these three problems. Some of their other mathematical contributions include L'Hôpital's Rule, proving the convergence of the sum of the reciprocals of the squares, the book Ars Conjectandi, the Bernoulli Theorem (Law of Large Numbers), reflection and refraction, analytical trigonometry, and the early use of polar coordinates. But even this list is not exhaustive. Johann Bernoulli established a teacher/student relationship with Guillaume Francois Antoine Marquis de L'Hôpital. He sold his discoveries to L'Hôpital to be published in the first calculus text, entitled *Analyse des infiniment petits* (*Analysis of the Infinitely Small*) (Dunham, 191). This text contains what is now known as L'Hôpital's Rule, used when taking the limit of a function of an indeterminate form: either zero divided by zero or infinity divided by infinity. Jakob Bernoulli proved the convergence of the sums of the reciprocals, but he was unable to find what that sum was. However, his student Leonard Euler later proved that this series converges to pi squared divided by six, or approximately 1.64493 (Simmons, 159). Jakob Bernoulli wrote *Ars Conjectandi*, but his nephew published it eight years after his death, in 1713. It had four parts: (1) a reproduction of Huygen's *De Ratiociniis in Ludo Alegae*, along with Bernoulli's commentary; (2) standard results of permutations and combinations, in the same form as we use them today; (3) twenty-four games of chance from his day; (4) "Applications of the Previous Study to Civil, Moral, and Economic Problems" (Burton, 433-34). The fourth part was left incomplete because Bernoulli died before finishing it. But it is considered to be the most important part of his book. In it, Bernoulli discussed philosophical problems connected with probability theory, such as probability as a degree of certainty, moral versus mathematical expectation, etc. (Burton, 433-34)). The French mathematician Poisson later named the Bernoulli Theorem the Law of Large Numbers (Burton, 434). It states, "If p is the probability of an event, if k is the actual number of times the event occurs in n trials, if $\epsilon > 0$ is an arbitrary small number, and if P is the probability that the inequality $|k/n - p < \epsilon|$ is satisfied, then P increases to 1 as n grows without bound" (Simmons, 159). This theorem states that the more times a trial is run, the more likely you are to get the expected probability of the event occurring. For example, when tossing a coin, the probability of getting a head is $\frac{1}{2}$. If $n = 100$ and $k = 47$ then $|k/n - p| = |0.47 - 0.50| = 0.03$. If our $\epsilon = 0.01$, then we would have $P(k = 50)$, which is hard to do. However, if $n = 100,000$ and $k = 50,104$, then $|k/n - p| = |0.50104 - .5| = 0.00104$. If we use the same $\epsilon = 0.01$, then we would have $P(49,000 < k < 51,000)$ (Dr. Dawson, November 27, 2001). This theorem is the stepping-stone from insignificant problems with urns containing colored balls or dice tossing and card playing to valuable and scientifically justified applications of probability theory, such as mathematical statistics, demography, and the theory of random errors (Burton, 434-35). Johann Bernoulli used his physics knowledge to work with reflection and refraction of light. He also did mathematical work in analytical trigonometry (Eves, 354). Jakob Bernoulli introduced the use of polar coordinates in 1691. He was also one of the early students of mathematical probability (Eves, 286, 353).

All eight mathematicians in the Bernoulli family made a great impact on the mathematical world, especially these two brothers. Their mathematical skills were on the cutting edge of the 'new math' during the Seventeenth and Eighteenth Centuries. One should ask, "Where would we be mathematically without their work?" In reality, someone would have come along later and discovered what these men found, but it is fascinating to look at everything they were able to accomplish. Their competitiveness and desire to 'beat' each other to a correct solution probably helped propel their discoveries. The work of these two men should not be taken for granted. And their father did not even encourage them in mathematics; he actually discouraged all of their mathematical, astronomical, and physical endeavors. But Jakob and Johann Bernoulli had an inborn love for mathematics, which shone brightly in their lives, and to all future generations.

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What's Wrong Here?

$$\frac{n}{1-n} = n + n^2 + n^3 + \cdots \text{ and } \frac{n}{n-1} = 1 + \frac{1}{n} + \frac{1}{n^2} + \cdots,$$

so

$$\cdots + \frac{1}{n^2} + \frac{1}{n} + 1 + n + n^2 + \cdots = \frac{n}{n-1} + \frac{n}{1-n} = 0.$$

Does this puzzle you? If so you can feel comforted by the knowledge that the illustrious Euler fell into this trap, paradoxically in an essay in which he was urging great caution in use of divergent series.

The Pentagon, Fall 1961, p.48.

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before January 1, 2002. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 2002 issue of *The Pentagon*, with credit being given to the student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

PROBLEMS 559-564

Problem 559. Proposed by the editor. (Corrected)

Let S_n and T_n denote perfect squares, each having exactly n digits and such that $S_n + R_n = T_n$ where $R_n = \frac{10^n - 1}{9}$, n is a positive integer > 1 and each digit of T_n equals 1 plus the corresponding digit of S_n. Furthermore the left most digit of S_n is not zero. For example, for $n = 4$ we have $S_n = 2025 = 45^2$ and $T_n = 56^2$ with $R_n = 1111$.

(a) Show that if n is an even integer, one can always find appropriate values for S_n and T_n.

(b) If n is an odd integer < 15 find all integers n and the corresponding values of S_n and T_n which satisfy the conditions of the problem.

Problem 560. Proposed by Jose Luis Diaz-Barrero, Universitat Politeccnica de Catalunya, Barcelona, Spain.

Evaluate the sum $\sum_{n=1}^{\infty} \frac{1}{a_n a_{n+1} a_{n+2}}$.

Problem 561. Proposed by Jose Luis Diaz-Barrero, Universitat Politeccnica de Catalunya, Barcelona, Spain.

Determine all positive integers n such that $\frac{F_1}{\sqrt{F_2}+\sqrt{F_3}} + \frac{F_2}{\sqrt{F_3}+\sqrt{F_4}} + \frac{F_3}{\sqrt{F_4}+\sqrt{F_5}} + \cdots + \frac{F_{n-1}}{\sqrt{F_n}+\sqrt{F_{n+1}}} = n$
where F_n is the n^{th} Fibonacci number; i.e. $F_0 = 0$, $F_1 = 1$ and for $n > 2$, $F_n = F_{n-1} + F_{n-2}$.

Problem 562. Proposed by Pat Costello, Eastern Kentucky University, Richmond, Kentucky.

Show that $1 + 5^n + 5^{2n} + 5^{3n} + 5^{4n}$ is divisible by 11 whenever n, a positive integer, is not divisible by 5.

Problem 563. Proposed by the editor.

(a) Define $f(n)$ as the result of writing the positive integers in order; e.g. $f(1) = 1$, $f(3) = 123$, $f(10) = 12345678910$, and so on. What is the smallest integer n such that $f(n)$ is divisible by 241?

(b) Define $g(n)$ as the result of writing the positive integers in order; e.g. $g(1) = 1$, $g(3) = 321$, $g(10) = 10987654321$, and so on. What is the smallest integer n such that $f(n)$ is divisible by 241?

(This problem was suggested by Puzzle 188 from Carlos Rivera's website The Prime Puzzles & Problems Connection at www.primepuzzles.net and appears here with Mr.Rivera's kind permission.)

Problem 564. Proposed by the editor.

Find the smallest positive integer n such that $n/2$ is a perfect square, $n/3$ is a perfect cube, $n/5$ is a perfect fifth power and $n/7$ is a perfect seventh power.

Please help your editor by submitting problem proposals.

The editor wishes to acknowledge that Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan and Scott H. Brown, Auburn University, Montgomery, Alabama were inadvertently omitted from the list of solvers of Problem 548. The editor apologizes for any inconvenience this has caused.

SOLUTIONS 537, 538, 541 – 544

Problem 550. Proposed by Jose Luis Diaz, Universitat Politecnica de Catalunya, Terrassa. Spain.

If a, b, and c are distinct nonzero complex numbers, determine

$$\frac{bc(a+1)}{(a-b)(a-c)}+\frac{ac(b+1)}{(b-a)(b-c)}+\frac{ab(c+1)}{(c-a)(c-b)}.$$

Solution by Jimmy Kerl, student, Northeastern State University, Tahlequah, Oklahoma.

$$\frac{bc(a+1)}{(a-b)(a-c)}+\frac{ac(b+1)}{(b-a)(b-c)}+\frac{ab(c+1)}{(c-a)(c-b)}=$$

$$\frac{bc(a+1)(b-c)+ac(b+1)(c-a)+ab(c+1)(a-b)}{(a-b)(a-c)(b-c)}=$$

$$\frac{ab^2c+b^2c-abc^2-bc^2+abc^2+ac^2-a^2bc-a^2c+a^2bc+a^2b-ab^2c-ab^2}{(a-b)(a-c)(b-c)}=$$

$$\frac{b^2c-bc^2+ac^2-a^2c+a^2b-ab^2}{(a-b)(a-c)(b-c)}=$$

$$\frac{b^2c-bc^2+ac^2-a^2c+a^2b-ab^2}{b^2c-bc^2+ac^2-a^2c+a^2b-ab^2}=1.$$

Also solved by: Karen Crider, Student, Waynesburg College, Waynesburg, Pennsylvania; Albert White, St. Bonaventure University, St. Bonaventure, New York; Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri; SUNY Fredonia Math Club Student Group, University of New York at Fredonia, Fredonia, New York; and the proposer.

Problem 551. Proposed by Carol Browning, Greg Eastman, and John House (jointly).

The divisibility test for 3 is simple: a natural number is divisible by 3 if and only if the sum of its digits is divisible by 3. The same test works for 9. Notice that the test seems to work for 3 in base 4 but fails for 3 in base 5. Give conditions on divisor d and base b (with $d < b$) for which d divides n if and only if d divides the sum of the digits of n expressed in base b.

Solution by Tara Coraizi and Jennifer Tharp (jointly), Students, Waynesburg College, Waynesburg, Pennsylvania.

Let $n = a_0 + a_1 b + a_2 b^2 + \cdots + a_k b^k$ be the representation of n in base b. Then

$$n = \sum_{j=1}^{k} a_j b^j.$$

Let $S = a_0 + a_1 b + a_2 b^2 + \cdots + a_k b^k$ be the sum of the digits of n in base b. Then

$$S = \sum_{j=1}^{k} a_j.$$

We claim that for any positive integer $d < b$ such that $b \equiv 1 \pmod{d}$, $d \mid n$ if and only if $S \mid n$. Then $b \equiv 1 \pmod{d} \iff b^j \equiv 1 \pmod{d}$ for $j = 1, 2, \ldots, k \iff a_j b^j \equiv a_j \pmod{d}$ for $j = 1, 2, \ldots, k$

$$\iff \sum_{j=1}^{k} a_j b^j = \sum_{j=1}^{k} a_j \pmod{d}$$

$$\iff n \equiv S \pmod{d}$$

This last statement establishes that $d \mid n$ if and only if $S \mid n$.

Also solved by: Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri; SUNY Fredonia Math Club, University of New York at Fredonia, Fredonia, New York; and the proposer.

Problem 552. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Determine if each of the following two series $\sum\limits_{j=1}^{\infty} \frac{1}{x_j}$ are convergent or divergent. If a series is convergent, find the sum of the series.

(a) Define $x_1 = 1$, $x_n = \sum\limits_{j=1}^{n-1} j x_j$, $j \geq 2$.Find $\sum\limits_{j=1}^{\infty} \frac{1}{x_j}$.

(b) Define $x_1 = 1, x_n = \sum_{j=1}^{n-1} \frac{1}{j} x_j, j \geq 2$.Find $\sum_{j=1}^{\infty} \frac{1}{x_j}$.

Solution by Doug Joseph, Pittsburg State university, Pittsburgh, Kansas.
Solution to part (a):

$$\sum_{j=1}^{\infty} \frac{1}{x_j} = 1 + 1 + \frac{1}{(1+2(1))} + \frac{1}{(1+2(1)+3(3))} + \frac{1}{5(1+2(1)+3(3))} + \cdots$$

$$= 1 + 1 + \frac{1}{3} + \frac{1}{12} + \left[\frac{1}{5(12)} + \frac{1}{6(5)(12)} + \cdots\right]$$

$$= 1 + 1 + \frac{1}{3} + \frac{1}{12} + \frac{1}{12}\left[\frac{1}{5} + \frac{1}{6(5)} + \frac{1}{7(6)(5)} + \cdots\right]$$

$$= 1 + 1 + \frac{1}{3} + \frac{1}{12} + \frac{4!}{12}\left[\sum_{k=5}^{\infty} \frac{1}{k!}\right] \qquad (1)$$

Applying the ratio test we have

$$\lim_{n\to\infty}\left|\frac{\frac{1}{(n+1)!}}{\frac{1}{n!}}\right| = \lim_{n\to\infty}\left|\frac{n!}{(n+1)!}\right| = \lim_{n\to\infty}\left|\frac{1}{n+1}\right| = 0.$$

Thus the series converges by the ratio test.
Furthermore, (1) becomes

$$1 + 1 + \frac{1}{3} + \frac{1}{12} + \frac{4!}{12}\left[\sum_{n=0}^{\infty} \frac{1}{n!} - \left(1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}\right)\right]$$
$$= \frac{29}{12} + 2e - \frac{65}{12} = 2e - 3.$$

Solution to part (b):

$$\sum_{j=1}^{\infty} \frac{1}{x_j} = 1 + 1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3} + \frac{2}{7} + \cdots$$
$$= 1 + \sum_{n=2}^{\infty} \frac{2}{n} = 1 + 2\sum_{n=2}^{\infty} \frac{1}{n} = 2\sum_{n=2}^{\infty} \frac{1}{n} - 1 \qquad (2)$$

then since $\sum_{n=1}^{\infty} \frac{1}{n}$ is a divergent series, (2) also diverges.

Also solved by: Pat Costello, Eastern Kentucky University, Richmond, Kentucky; Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri; Carl Libis, Assumption College, Worcester, Massachusetts and the proposer.

Problem 553. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Al spotted a squirrel in the family room. He had Samantha stand in the doorway so that the squirrel would not go into the kitchen. Then Al opened the back door so that the squirrel would run outside. Using the handle of a broom, he poked under the entertainment center until the squirrel ran out.

The squirrel ran a series of straight lines from (0,0) to (15,-9) to (19,-5) to (23,-5) where the back door is located. When the squirrel was located at (15,-9), Samantha abandoned her post and ran in the opposite direction along the path $y = -10 - (x - 16)^2$. Samantha started at (16, -10) and stopped at (21,-35). Assuming that the squirrel ran at a constant speed of 110 feet per minute while Samantha ran at a constant speed of 150 feet per minute, does the squirrel reach the back door before Samantha reaches her stopping point?

Solution by Jessica Roth, Sarah del Monte and Greg Richir (jointly), students, SUNY College at Fredonia, Fredonia, New York.

(Revised by the editor) Since Samantha does not start running until the squirrel reaches the point (15,-9) the distance from (0,0) covered by the squirrel is immaterial. The remaining distance to the back door covered by the squirrel is $4 + 4\sqrt{2}$ ft. Then the time, x minutes, taken by the squirrel to reach the back door is found from the ratio $\frac{110\,\text{ft}}{1\,\text{min}} = \frac{4+4\sqrt{2}\,\text{ft}}{x\,\text{min}}$. So $x = 0.0878$ minutes. This means that the squirrel took 0.0878 minutes to reach the back door. The distance Samantha ran is given by the arc length integral $\int_b^a \sqrt{1 + f\,'(x)^2}dx$ where $f(x) = -(x - 16)^2 - 10$, $f\,'(x) = -2(x - 16)$, $a = 21$, and $b = 16$ since Samantha started at $(16, -10)$ and stopped at $(21, -35)$. Thus Samantha's distances equals

$$\int_{16}^{21} \sqrt{1 + f\,'(x)^2}dx \qquad (1)$$

Letting $u = 32 - x$, then $du = -2xdx$ in (1) we have,

$$\frac{1}{2}\int_{-10}^{0} \sqrt{1 + u^2}du \qquad (2)$$

Now taking $u = \tan\theta, du = \sec^2\theta d\theta$ (2) becomes

$$\frac{1}{2}\int_{u=-10}^{u=0} \sqrt{\sec^2\theta}\sec^2\theta d\theta$$

$$\begin{aligned}
&= \frac{1}{2}\int_{u=-10}^{u=0} \sec\theta\sec^2\theta d\theta \\
&= \frac{1}{2}\int_{u=-10}^{u=0} \left(1+\tan^2\theta\right)\sec\theta d\theta \\
&= \frac{1}{2}\int_{u=-10}^{u=0} \sec\theta d\theta + \frac{1}{2}\int_{u=-10}^{u=0} \sec\theta\tan^2\theta d\theta \qquad (3)
\end{aligned}$$

By taking $w = \tan\theta, dw = \sec^2\theta, dv = \tan\theta\sec\theta$, $v = \sec\theta$,and using integration by parts, we get

$$\begin{aligned}
\frac{1}{2}\int_{u=-10}^{u=0} \sec^3\theta d\theta &= \frac{1}{4}\left[\ln\left|\sec\theta+\tan\theta\right| + \tan\theta\sec\theta\right] \\
&= \frac{1}{4}\left[\ln\left|-u+\sqrt{1+u^2}\right| + u\sqrt{1+u^2}\right]_{u=-10}^{u=0} \\
&= \frac{-1}{4}\left[\ln\left|-10+\sqrt{101}\right|\right] + \frac{5}{2}\sqrt{101}
\end{aligned}$$

Then calculating the length of time Samantha took to reach her stopping point, we find she took $\frac{\frac{-1}{4}\left[\ln\left|-10+\sqrt{101}\right|\right]+\frac{5}{2}\sqrt{101}}{150} \approx 0.1725$ min. Thus the squirrel reached the back door before Samantha stopped running.

Also solved by Russ Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri and the proposer.

Problem 554. Proposed by the editor.

Let f be defined by the relation $f = 2 + \sqrt{44g^2+1}$ where g is a positive integer. Show that if f is an integer f is a perfect square.

Solution by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri.

f is an integer if and only if $\sqrt{44g^2+1}$ is an integer. Write $\sqrt{44g^2+1} = a$, where a is an integer, and square both sides. We get $44g^2 = a^2 - 1 = (a-1)(a+1)$. Since $a-1$ and $a+1$ are of the same parity, they must be both even. We may write $11g^2 = \frac{a-1}{2}\cdot\frac{a+1}{2} = b(b+1)$ where b is an integer. Since 11 is prime, $11 \mid b$ or $11 \mid (b+1)$ and $g^2 = \frac{b(b+1)}{11}$. In general, if two relatively prime numbers have their product a perfect square, then each one of them must be a perfect square too (it is straight forward to see this is true if we write the prime factorization of each number). We have two cases.

Case1. If $11 \mid b$, then $\frac{b}{11} = c^2$ and $b+1 = d^2$ for some integers c and d. Write $\sqrt{44g^2+1} = \sqrt{44\frac{b(b+1)}{11}+1} = \sqrt{4(d^2-1)d^2+1} = \sqrt{(2d^2-1)^2} = 2d = 2d^2-1$. It follows that $f = 2 + 2\sqrt{44g^2+1} = 2 + 2(2d^2-1) = 2 + 4d^2 - 2 = 4d^2$; a perfect square.

Case 2. If $11 \mid (b+1)$, then $\frac{b+1}{11} = c^2$ and $b = d^2$ for some integers c and d. This implies $d^2 + 1 = 11c^2$. It follows that c and d are not of the same parity. Therefore, if $d = 2n$ and $c = 2m+1$ for some integers m and n, then by substitution we get $4n^2+1 = 11(2m+1)^2 = 44m^2+44m+11$. It follows that $4n^2 = 44m^2 + 44m + 10$. Divide both sides by 2 and get $2n^2 = 22m^2 + 22m + 5$. This is a contradiction as the two sides of the equality are not of the same parity. The case when d is odd and c is even leads to a similar contradiction. It follows that Case 1 is always true which leads to the desired result.

Editor's Comment: For a similar problem see, [1] which cites a problem from the J. Kurshak Competition, 1969.

My own solution is as follows.
Let $f = \sqrt{44g^2+1}$ where f is a positive integer. Then

$$f^2 - 44g^2 = 1 \qquad (1)$$

which is a Pell equation. Since $44 = 4 \cdot 11$ the solutions of equation (1) are found among the solutions of the Pell equation

$$x^2 - 11y^2 = 1 \qquad (2)$$

All solutions of equation (2) are given by the relation

$$x_k + y_k\sqrt{11} = \left(x_1 + y_1\sqrt{11}\right)^k = \left(10 + 3\sqrt{11}\right)^k \qquad (3)$$

where k is a positive integer. Since $10^2 - 11 \cdot 3^2 - 1$ is the smallest solution of equation (2), clearly each solution of equation (1) corresponds to a solution of equation (2) in which y_k is even. $\left(10+3\sqrt{11}\right) = 199 + 60\sqrt{11} = x_2 + y_2\sqrt{11}$ shows that solutions where y_k is even do exist. Hence all solutions of equation (2) have the form

$$
\begin{aligned}
x_{2k} + y_{2k}\sqrt{11} &= \left(x_1 + y_1\sqrt{11}\right)^{2k} \\
&= \left(x_k + y_k\sqrt{11}\right)^2 \\
&= \left(x_k^2 + 11y_k^2\right) + 2x_k y_k\sqrt{11} \qquad (4)
\end{aligned}
$$

But since $x_k^2 - 11y_k^2 = 1$, we have $x_{2k} = 2x_k^2 - 1$. Then since $y_{2k} = 2x_k y_k$ we have $g = \frac{y_{2k}}{2}$. Thus

$$
\begin{aligned}
f &= \sqrt{44g^2 + 1} \\
&= \sqrt{44\left(\frac{y_{2k}}{2}\right)^2 + 1} \\
&= \sqrt{44\left(x_k y_k\right)^2 + 1} \\
&= \sqrt{11\left(2x_k y_k\right)^2 + 1} \\
&= 2x_k^2 - 1 = x_{2k}
\end{aligned}
$$

Hence $2 + 2\sqrt{44g^2 + 1} = 2 + 2\left(2x_k^2 - 1\right) = 4x_k^2$ which is a perfect square.

1. Andreescu and Gelca, Mathematical Olympiad Challenges, Birkhauser, 2000, Problem 3 on p.71, solution on p.217.

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Call For Papers for the Thirty-Fourth Biennial Convention of Kappa Mu Epsilon

The Thirty-Fourth Biennial Convention of Kappa Mu Epsilon will be hosted by the Oklahoma Delta chapter at Oral Roberts University in Tulsa, Oklahoma. The convention will take place March 27-29, 2003.

A significant feature of our national convention will be the presentation of papers by student members of Kappa Mu Epsilon. The mathematical topic selected by each student should be of interest to the author and of such scope that it can be given adequate treatment in a timed oral presentation. Senior projects and seminar presentations have been a popular way for faculty to get students to investigate suitable topics. The deadline for receiving papers is February 1, 2003. Information about paper submission and presentation is available at www.kme.eku.edu/announce.html.

Are you an undergraduate student planning on attending graduate school?

Check out a free listing with the Registry of Undergraduate Researchers and Graduate Schools, sponsored by the Council on Undergraduate Research. The purpose of this registry is to facilitate matchmaking between undergraduates who have research experience with graduate schools seeking high quality students who are well prepared for research.

Any undergraduate may go to www.cur.org/ugreg/ to fill out a simple curriculum vitae form. There is no charge to you as a student. Student information records will be made available ONLY to bona fide Graduate Schools that contract with CUR for this service. Graduate School representatives may contact students to invite applications or visits to the campus and laboratory, or to share information about their research programs and financial support opportunities. .

If you are a student in your junior or senior year, REGISTER FOR FREE NOW! Juniors will be able to update their listing at the end of the summer and during their Senior year, to include any summer research experience or information about Senior Theses and test scores.

For additional information about the Registry, please visit http://www.cur.org/UGRegistryselect.html. Contact CUR at cur@cur.org or 202-783-4810 if you have any questions.

Reports of the Regional Conventions

Report of the North Central Regional Convention

The KME North Central Regional Convention was held April 26-27, 2002 at Central Missouri State University in Warrensburg, Missouri. sixty-two people attended from eleven different chapters in four states. The host chapter was Missouri Beta. Eight student papers were presented, with awards going to the top three papers. The award winners were Mindy Baker from the Kansas Beta Chapter, Leah McBride from the Kansas Beta Chapter, and Joseph Bohanon from Missouri Theta Chapter. Dr. Curtis Cooper of Central Missouri State University gave an after-lunch talk titled "Mersenne Primes and GIMPS."

Report of the Southeastern Regional Convention

The KME Southeastern regional was hosted by the Mississippi Beta chapter of KME (Mississippi State University) on March 22-23, 2002, on the campus of MSU. We had thirty-one students and faculty in attendance, as well as three visitors from Southern Farm Bureau Life Insurance Co.

The meeting started with a reception on Friday night, and on Saturday began the formal meeting at approximately 8:30am.

There were five student talks, as well as a special lecture by Pat Costello. We had a group problem solving competition, with students from different schools teamed together for about forty-five minutes to see how well they could collaborate on short notice. After lunch on Saturday, we had a career presentation from our visitors. The meeting concluded at about 2:30pm.

We awarded a $100.00 prize to Caroline Ellis (TN Gamma) for her paper/presentation, Mathematical Discoveries of the Bernoulli Brothers. The best problem solving team was awarded a $50.00 prize ($10.00 per team member.)

Kappa Mu Epsilon News

Edited by Connie Schrock, Historian

News of chapter activities and other noteworthy KME events should be sent to schrockc@emporia.edu or to

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Campus Box 4027
Emporia, KS 66801

Chapter News

AL Alpha

Athens State University, Athens

Other spring 2002 officers: Jennie Legge, Corresponding Secretary.

AL Eta

University of West Alabama, Livingston

Other spring 2002 officers: Judy Massey, Corresponding Secretary.

AL Gamma

Chapter President–Cari Cook

University of Montevallo, Montevallo — 17 Actives, 9 New Members

Other spring 2002 officers: Amanda McCormick, Vice President; Hollie Smith, Secretary; Laura Moore, Treasurer; Don Alexander, Corresponding Secretary.

The chapter met to hold an initiation ceremony on March 5, 2002 to induct 9 new members. Graduate Tommy Fitts is currently working for NSA at Fort Mead near Washington D.C. Jessica Batting is teaching at Pinson Valley High School and planning to go to graduate school in the fall. Graduating senior Jared Phillips will attend Emory University in the fall, to begin work on his Ph.D. in mathematics.

AL Zeta

Chapter President–Scott Asher

Birmingham Southern College, Birmingham — 16 Actives, 6 New Members

Other spring 2002 officers: Isaac Dooley, Vice President; Ellen Segrest, Secretary/Treasurer; Mary Jane Turner, Corresponding Secretary.

CO Beta

Colorado School of Mines, Golden — 23 New Members

Other spring 2002 officers: Ardel Boes, Corresponding Secretary

CO Delta Chapter President–Ron Mikluscak

Mesa State College, Grand Junction 22 Actives

Other spring 2002 officers: Troy Miller, Vice President; Wendy Serve, Secretary; Paula Bauer, Treasurer; Kim Schneider, Corresponding Secretary.

We had an organizational pot luck in the spring and elected officers for the next academic year.

CO Gamma

Fort Lewis College, Durango

Other spring 2002 officers: Deborah Berrier, Corresponding Secretary.

GA Alpha Chapter President–Jessica Pritchett

State University of West Georgia, Carrollton 25 Actives, 6 New Members

Other spring 2002 officers: Beth Gibbs, Vice President; Bryan Stamps, Secretary; J.J. Wahl, Treasurer; Dr. Joe Sharp, Corresponding Secretary.

On April 22, 2002, the Georgia Alpha Chapter of KME held its 28th annual Initiation Meeting at which 6 new members of KME were initiated bringing the total membership of the chapter to 257. After the initiation, new officers were elected for 2002-2003. The new officers are listed above. A reception followed in honor of the 2002 initiates and at the reception the names of the winners of the mathematics scholarships and awards for 2002-2003 were announced. The Burson Calculus Award went to Jessica Pritchett, The Boyd Award in Mathematics went to Bryan Stamps, the three Marion Crider Award winners were Beth Gibbs, Chad Mathews, and Bryan Stamps. The Georgia Martin Scholarship went to J.J. Wahl, the Whatley Scholarship winner was Danielle Ratzlaff, and the winner of the first ever Dr. Chatty Pittman Memorial Scholarship was Jessica Pritchett.

GA Beta

Georgia College & State University, Milledgeville

Other spring 2002 officers: Michael Marion, Corresponding Secretary.

IA Alpha Chapter President–Chad Florke

University of Northern Iowa. Cedar Falls 34 Actives, 6 New Members

Other spring 2002 officers: Jessica Hagberg, Vice President; Rebecca Sherfield, Secretary; Scott Hirschman, Treasurer; Mark Ecker, Corresponding Secretary

Out first spring KME meeting was held at Professor Mark Ecker's residence on February 13, 2002. The second meeting was held on March 12, 2002 at Professor Cathy Miller's residence where Carl Gross presented his student paper "Omar Khayyam: Islamic Scholar of the Middle Ages." Student member, Scott Hirschman present his paper "The Cantor Set" at our third meeting on April 17, 2002 at Professor Syed Kirmani's residence. Faculty advisor Mark Ecker and KME president Chad Florke traveled to the KME regional convention at Central Missouri State University on April 26-27, 2002 where Chad presented his paper "NBA Draft Lottery Probabilities." Student member Elizabeth Robertson addressed our spring initiation banquet with "The Platonic Solids." The banquet was held at Bonanza restaurant on Mary 1, 2002 were six new members were initiated.

IA Delta Chapter President–Jesse Oltrogge

Wartburg College, Waverly 41 Actives, 30 New Members

Other spring 2002 officers: Matthew Townsley, Vice President; Alanson Ridpath, Secretary; Derek Riley, Treasurer; Dr. August Waltmann, Corresponding Secretary.

The January, February, and March meetings involved planning future events including encouraging attendance at the Regional Meeting of KME, voting about admittance of a new KME Chapter, and planning the initiation banquet, which was held on March 23. We initiated thirty new members into the Chapter, the largest group of new members for several years. The traditional year-end picnic was held on May 20.

IA Gamma Chapter President–Mike Husmann

Morningside College, Sioux City 16 Actives, 2 New Members

Other spring 2002 officers: David Dziurawiec, Vice President; Justin Lewis, Secretary; Fawn Woiwood, Treasurer; Eric Canning, Corresponding Secretary.

Several members worked the TAG (Talented and Gifted) students at local schools.

IL Delta

University of St. Francis, Joliet

Other spring 2002 officers: Richard Kloser, Corresponding Secretary.

IL Eta Chapter President–Jessica Holdworth

Western Illinois University, Macomb 13 Actives, 8 New Members

Other spring 2002 officers: Eric Snodgrass, Vice President; Sandra Jarzab, Secretary; Hong Tran, Treasurer; Alan Bishop, Corresponding Secretary.

IL Theta Chapter President–Mariam Ahmed

Benedictine University, Lisle 10 Actives, 8 New Members

Other spring 2002 officers: Afroze Mirza, Vice President; Colleen Powers, Secretary; Jon Rink, Treasurer; Manu Kaur, Corresponding Secretary.

As we had several mathematicians join the math department this year, the induction included 4 math faculty members as well as 4 undergraduate students. In February, the students invited a speaker from Lucent Technologies to discuss her work on telephone switch problem solving. The spirit week activities on campus featured a LEGO building contest sponsored by the KME /Math Club.

IL Zeta Chapter President–Christine Pellini

Dominican University, 23 Actives, 4 New Members

Other spring 2002 officers: Rhonda Jurinak, Vice President; Theresa Meshes, Secretary; Michelle Blaszak, Treasurer; Sarah N. Ziesler, Corresponding Secretary.

Induction ceremony was held on April 8th with speakers Karen Jarosz and Angie Pascente, both alumnae and past KME presidents. Other activities were: members provided 5/6 hours tutoring per week as a service to the university; trips were organized to see A Beautiful Mind and Proof, the Pulitzer Prize winning Broadway play; a group of members went to hear Underwood Dudley speak at the annual ACCA (Associated Colleges in the Chicago Area) mathematics symposium; a picnic was organized in finals weeks; we ran a bi-weekly problem contest; we cosponsored " The Weakest Link, Dominican Style". This year we also won the "Outstanding Honors Society" award.

IN Delta Chapter President–Ellen Miles

University of Evansville, Evansville 15 Actives, 16 New Members

Other spring 2002 officers: Megan Baumgart, Vice President; Katherine Achim, Secretary; Dr. Joe A. Stickles, Jr., Corresponding Secretary.

IN Gamma

Anderson University, Anderson

Other spring 2002 officers: Stanley L. Stephens, Corresponding Secretary.

KS Alpha

Pittsburg State University, Pittsburg

Other spring 2002 officers: Tim Flood, Corresponding Secretary.

KS Beta Chapter President–Leah McBride

Emporia State University, Emporia 20 Actives, 1 New Member

Other spring 2002 officers: Melinda Born, Vice President; Mindy Baker, Secretary; Thad Davidson, Treasurer; Connie S. Schrock, Corresponding Secretary.

The Kansas Beta chapter has been active this semester. Student and faculty members attended the Kansas Section Mathematical Association of America Meeting at Ottawa University. Leah McBride presented her paper "Complex Numbers in Plane Geometry." Mindy Baker presented a paper entitled, "Mathematical Games: Variations on Nim." Allison Fairburn presented her research project, "Continued Fractions," and received first place. Also students and faculty members attended the Regional KME convention at Central Missouri State University. Leah McBride and Mindy Baker both placed in the top three for the presentation of their papers at this convention. Our organization was also well represented at a university wide research and creativity day where students exhibited the projects that they had been working on this spring.

KS Delta Chapter President–eb Kramer
Washburn University, Topeka 30 Actives, 8 New Members

Other spring 2002 officers: Jeff Kingman, Vice President; Mary Noel, Secretary/Treasurer; Allan Riveland, Corresponding Secretary.

Once a month during the semester, KME met for a noon luncheon with the Math Club. Presentations were given by Dr. Ken Ohm at one meeting and by Robert O'Loughlin at another. The Spring initiation banquet was held on February 21 with eight initiates. A Washburn and KME alumnus Hal Dick was the invited speaker. Students Mary Noel, Jeff Kingman, Ezra Rush, and Robert O'Loughlin with faculty Al Riveland and Kevin Charlwood attended the regional conference. Robert O'Loughlin presented a paper, "Song of Banacio: Tonal Excursions in a Familiar Sequence" at the conference.

KS Epsilon Chapter President–Charlotte Bigler
Fort Hays State University, Hays 20 Actives, 5 New Members

Other spring 2002 officers: Michael Breckenridge, Secretary/Treasurer; Jeff Sadler, Corresponding Secretary.

The Kansas Epsilon Chapter of KME continued to be active during the spring of 2002, holding monthly meetings in conjunction with the FHSU Math Club. These meetings occurred monthly, frequently with guest speakers and delicious snacks. The chapter was scheduled to take part in a service project this spring, but the project was cancelled due to inclement weather. The annual KME banquet was held on April19th at the FHSU student union, catered by union food service. The banquet proceedings included delicious Mexican food, the initiation of three new members, and a delightful talk by a FHSU/KME alumnus, Crystal Holdren-Vaceura, detailing her work involving mathematics at an insurance company in Nebraska.

KS Gamma Chapter President–Angela Shomin
Benedictine College, Atchison 11 Actives, 9 New Members

Other spring 2002 officers: David Livingston, Vice President; Brett

Herbers, Secretary; Janelle Kroll, Treasurer; Jerin Stretton, Stu-Go Rep; Jo Ann Fellin, OSB, Corresponding Secretary.

On February 4th, Kansas Gamma initiated faculty member Eric West along with three students. Dr. West successfully defended his dissertation and will participate in spring commencement at the University of Kansas. Sister Linda Herndon, OSB completed her Ph.D. work and will graduate from the University of Wisconsin at Madison. Having completed a two-year leave and then a sabbatical this academic year, she will return to Benedictine in the fall. Several students gave presentations during the semester. Angela Shomin spoke about GAP as a teaching tool. Janelle Kroll, Brett Herbers, and Eric Brownson gave presentations for Discovery Day. Alumnus Kevin Slattery returned to campus on April 12th to share a picnic honoring the seniors at the home of retired faculty member Richard Farrell. On April 22nd Dr Al Riveland from Washburn spoke to the group about research projects that he has directed during his teaching career. Eric Brownson and Angela Shomin were honored at the Honors' Banquet for completing their math comprehensive exams with distinction. Sister Helen Sullivan Scholarships were presented that evening to Andrea Archer, Max Bottam and Erin Stretton. Max Botta, Rachel Small, and David Livingston attended the regional meeting in Warrensburg. David served on the awards committee. Also attending the regional meeting were faculty members, Sister Jo Ann Fellin, OSB, Eric West, and Glenn Adamson.

KY Alpha

Eastern Kentucky University, Richmond

Other spring 2002 officers: Pat Costello, Corresponding Secretary.

KY Beta Chapter President–Michael Hesson

Cumberland College, Williamsburg 26 Actives, 10 New Members

Other spring 2002 officers: Rebecca Faw, Vice President; Mark Vernon, Secretary; Kirill Yakovlev, Treasurer, Jonathan Ramey, Corresponding Secretary.

On February 22, 2002, the Kentucky Beta chapter held an initiation and banquet at the Atrium. Kappa Mu Epsilon inducted ten new student members at the banquet, presided over by outgoing president, Michael Hesson. As an additional feature senior awards were given by the department at the banquet.

Jointly with the Mathematics and Physics Club, the Kentucky Beta Chapter hosted Dr. Carroll Wells from David Lipscomb University on April 11. He spoke on "Math Class Goes to the Movies: A Look at How Hollywood has Solved Freshman Math Problems." On April 12, members also assisted in hosting a regional high school math contest, held annually at Cumberland College. On April 22, the entire department, including the Math and Physics Club, Sigma Pi Sigma (Physics Honors Society), and the Kentucky Beta Chapter, Held the annual spring picnic at Briar Creek Park.

LA Delta Chapter President–April Jeffcoat
University of Louisiana, Monroe 28 Actives

Other spring 2002 officers: Stephanie Hillhouse, Vice President; Rhonda Antley, Secretary/Treasurer; Maribeth Olberding, Corresponding Secretary.

The Louisiana Delta Chapter met the third Friday of each month for a seminar/speaker symposium. We had Dr. Annela Kelly talk to us on game theory and Dr. Paul Fisher spoke to us on astronomy and math. We also continue to attend many functions on campus, i.e. campus clean ups, spring fever week, etc. We have a 2nd annual end of the year Luau planned to celebrate the end of the year along with those KME seniors who will be leaving ULM. The officers meet at least two weeks prior to any function to talk about what needs to be done. We plan to form at least two teams to participate in the "Reclaiming our Campus" day on April 27th. This is a day where organizations get together to help paint and fix up the interior of campus buildings.

LA Gamma Chapter President–Robert Hawkins
Northwestern State University, Natchitoches 20 Actives, 13 New Members

Other spring 2002 officers: Alyson Schexnayder, Vice President; Stewert Thompson, Secretary; Courtney Gillan, Treasurer; Leigh Ann Myers, Corresponding Secretary.

Held an Annual Initiation Ceremony and Banquet on April 24, 2002.

MA Alpha Chapter President–Renee Salvas
Assumption College, Worcester 2 Actives, 17 New Members

Other spring 2002 officers: Jennifer Susel, Vice President; Jonathan Katcher, Secretary; Charles Brusard, Corresponding Secretary.

The Massachusetts Alpha Chapter initiated seventeen new members on May 7th. Assumption students Jennifer Susel and David Katcher spoke on "An Analysis of the Mathematical Game DOIN." (DOIN is a strategy game invented by Mr. Katcher.)

MD Beta Chapter President–Jason Pyle
McDaniel College, Westminster 20 Actives, 5 New Members

Other spring 2002 officers: Andrew Ewing, Vice President; Dana Miller, Secretary; Susan Miller, Treasurer; Linda Eshleman, Corresponding Secretary.

Activities included: Induction of new members, Career Night, Phonathon to encourage applicants to enroll at the college and major in Mathematics, speech by former alumnus on his Ph.D. work in Applied Physics, spring picnic for math majors.

MD Delta Chapter President–Angela Myers

Frostburg State University, Frostburg 33 Actives, 7 New Members

Other spring 2002 officers: Timothy Sloane, Vice President; Sherri Raley, Secretary; Michael Shaffer, Treasurer; Mark Hughes, Faculty Sponsor; Edward White, Corresponding Secretary.

Maryland Delta Chapter enjoyed another active semester under the guidance of its energetic officers. In March Dr. Gerard Wojnar gave two informative talks, one on "Lesser-Known Insights on Polynomials" and one on rational functions. The centerpiece of the semester was the Pi Day celebration on March 14th. Chapter members decorated the halls around the math classrooms with balloons, streamers, and pi poster. A 300 ft. chain of linked rings representing the digits of pi, prepared by our secretary, Sherri Ralye, was hung from the ceiling. A bake sale was help, and pi stickers were given out. ("Pi Day 2002", "Got Pi?", "Pi is for Lovers", "3.14 is Never Enough" were some of the choices) The winner of the pi digit contest recited 126 digits of I, and received as a prize, of course, a pie. In April Dr. Mark Hughes gave a two-part lecture on Euler, entitled "Euler's Derangements and Other Amusements." An end-of-semester picnic was saved by from the rain. New officers for the fall are Brendon La Buz, President; Kandi Wertz, Vice President; Jacilynn Brant, Secretary; Kesha Frisby, Treasurer.

MI Epsilon Chapter President–Justin McCurdy

Kettering University, Flint 132 Actives, 38 New Members

Winter 2002 officers: Paul Young, President; Mary Palbykin, Vice President; Aaron Micyus, Treasurer; Nicole Primeau, Secretary.

Other spring 2002 officers: Leo Bush, Vice President; Scott Mulligan, Treasurer; Rebecca Barthlow, Secretary; Boyan Dimitrov, Corresponding Secretary.

During the Winter Term the KME Applied Math Noon-Time Movie took place. The movie "Sir Isaac Newton: The Gravity of Genius" was presented. The students could see lots of mathematical relevance in the works of Newton as well as most of his physics, astronomy, and other achievements. Pizza and pop was served. The Initiation Ceremony for the new members was on February 18th. 36 new, top quality Kettering students became members of the Chapter, as well as Ada Chend, a new Math faculty member. Our keynote speaker, Prof. Marc Lipman (from Oakland University), gave an artistic, exciting talk on "Doomsday Calender—When's Yours?"

For the Spring Term, the Math Noon-Time Movie, "Sir Isaac Newton: The Gravity of Genius" was repeatedly presented. Pizza and pop was also served. The Initiation Ceremony was on May 17th. 38 new members were initiated. Also two new faculty members, the visiting professor Dr. Vladimir Rykov and professor Dr. Peter Stanchev became members. The Keynote Speaker was Professor Eddie Cheng (from Oakland University). His talk entitle "Sometimes You Need To Be Greedy"in troduced the new members ant the guests into the rational use of the common sense (greediness) in some optimization problems related to economic situations. Unsolved problems were offered for a $1,000,000 award.

MO Alpha

Southwest Missouri State University, Springfield

Other spring 2002 officers: John Kubicek, Corresponding Secretary.

MO Beta Chapter President–Beth Kuenzel

Central Missouri State University, Warrensburg

Other spring 2002 officers: Erica Dinesen, Vice President; Sarah Hymes, Secretary/Treasurer; Rhonda McKee, Corresponding Secretary.

The Missouri Beta Chapter spent quite a bit of time this spring preparing for the North Central Regional Convention, which was held on our campus on April 26-27. We enjoyed the conference very much and would recommend the experience to everyone. The Claude H. Brown Senior Mathematics Achievement Award went to Beth Kuenzel.

MO Epsilon Chapter President–Trent Kraemer

Central Methodist College,Fayette 9 Actives, 4 New Members

Other spring 2002 officers: Cassie Laffoon, Vice President; Kendal Clark, Secretary/Treasurer, Linda Lembke, Corresponding Secretary.

MO Gamma

William Jewell College, Liberty

Other spring 2002 officers: Joseph T. Mathis, Corresponding Secretary.

MO Iota

Missouri Southern State College, Joplin 6 New Members

Chip Curtis, Corresponding Secretary.

We had monthly meetings, held initiation on April 1st, participated in the state math contest April 11/12 at Truman State University in Kirksville, participated in the KME regional convention April 26/27 in Warrensburg, and had an end of the semester picnic.

MO Kappa
Drury University, Springfield
Other spring 2002 officers: Charles Allen, Corresponding Secretary.

MO Lambda Chapter President–Firas Al-Takrouri
Missouri Western State College, St. Joseph 20 Actives, 10 New Members
Other spring 2002 officers: Yevgeniy Kondratenko, Vice President; Trevor Huseman, Secretary; Kurt Czerwien, Treasurer; Don Vestal, Corresponding Secretary.

MO Theta Chapter President–James Beyer
Evangel University, Springfield 5 Actives, 4 New Members
Other spring 2002 officers: Eric Block, Vice President; Don Tosh, Corresponding Secretary.

Meetings were held monthly, and the semester social was held in February at the home of Don Tosh. Dr. Tosh and four students – James Beyer, Joseph Bohanon, Andrew Hauck, and Jason Monroe – attended the regional convention April 26-27 at Warrensburg. Joey presented a paper entitle "The Planarity of Hasse Diagrams" and won an award as one of the top three presenters.

MO Zeta
University of Missouri-Rolla, Rolla
Other spring 2002 officers: Roger Hering, Corresponding Secretary.

MS Alpha Chapter President–Jennifer Kimble
Mississippi University for Women, Columbus 12 Actives, 2 New Members
Other spring 2002 officers: Shannon McVay, Vice President; Lailah Valentine, Secretary; Fransisca Lahagu, Treasurer; Shaochen Yang, Corresponding Secretary.

In February we had a meeting, watched the movie A Beautiful Mind, and held Initiations. During March we had a meeting and attended the KME regional conference at Mississippi State University. In April Mrs. Claudia R. Carter from Mississippi School for Mathematics and Science gave a talk entitled "Rolling Wheels, The Parametrics of Cycloids." During the talk we investigated with a simple computer program the effects of the modifying various constants in cycloid-type equations.

MS Beta Chapter President–Robin Terrell
Mississippi State University, Mississippi State
Other spring 2002 officers: Robin Bridges, Vice President; Greg Henry, Secretary/Treasurer; Michael Pearson, Corresponding Secretary.

MS Beta hosted the Southeastern Regional Convention. New officers for next year were elected: Sean Lestrade, President; Tricia Gilbreath, Vice

President; Francis Hamilton, Secretary/Treasurer, Prof. Tuncay Aktosun, Corresponding Secretary.

MS Epsilon Chapter President–Ashley Burleson Pullen
Delta State University, Cleveland 11 Actives, 5 New Members

Other spring 2002 officers: Amy Denise Pearson, Vice President; Jason Umfress, Secretary/Treasurer; Paula A. Norris, Corresponding Secretary.

NE Alpha Chapter President–Joshua Suing
Wayne State College, Wayne 27 Actives, 11 New Members

Other spring 2002 officers: Brian Pribnow, Vice President; Christina Ruttne, Secretary; Jeff Manka, Treasurer; John Fuelberth, Corresponding Secretary.

NE Beta Chapter President–Scott Barber
University of Nebraska - Kearney, Kearney 12 Actives, 3 New Members

Other spring 2002 officers: Jenny Rutar, Vice President; Lisa Beckenhauer, Secretary; Tom Mezger, Treasurer; Katherine Kime, Corresponding Secretary.

In spring 2002, we had monthly meetings and the initiation of 3 new members in March. In April we had an event, "Donald in Mathmagicland" in which the classic film was shown to a group of about 50 undergraduate students, with discussion of Mathematics (tutoring) services and pizza. Also, a KME t-shirt was designed by several members and was manufactured and made available to all members. The shirts were neon green with purple cosine rose on the front and "I am a denominator. I make a difference" in purple on the back. The semester finished with a picnic and election of next year's officers.

NE Delta Chapter President–Derek Brandt
Nebraska Wesleyan University, Lincoln 13 Actives, 6 New Members

Other spring 2002 officers: Rhea May, Vice President; Joshua Snyder, Secretary/Treasurer; Melissa Erdmann, Corresponding Secretary.

NH Alpha Chapter President–Craig Sheil
Keene State College, Keene 34 Actives, 16 New Members

Other spring 2002 officers: Peter Perrinez, Vice President; Ingrid Ayer, Secretary; David Caplette, Treasurer; Vincent J. Ferlini, Corresponding Secretary.

NH Alpha sponsored trips to MAA Northeast Region Conference at Bridgewater State College (16-17th of November 2001) and Hudson River Undergraduate Math Conference at Hamilton College (27th April 2002). Also, NH Alpha sponsored four student math talks at the Keene State College Academic Excellence Conference (6th April 2002).

NJ Beta

Montclair State University, Upper Montclair

Other spring 2002 officers: John Stevens, Corresponding Secretary.

NJ Gamma Chapter President–Felicita Ramos

Monmouth University, West Long Branch 27 Actives, 27 New Members

Other spring 2002 officers: Katie J. Blackburn, Vice President; Candice Ann Smith, Secretary; Erin Yourman & Franci Laska, Co-Treasurers; Judy Toubin, Corresponding Secretary.

The chapter had one meeting and Installation Ceremony on April 21, 2002. They also held a workshop on Islamic Art presented by Dr. Lynn Bodner. The members are in continuous contact via email.

NY Alpha

Hofstra University, Hempstead

Other spring 2002 officers: Aileen Michaels, Corresponding Secretary.

NY Eta Chapter President–Ryanne Fullerton

Niagara University,Niagara University 20 Actives, 22 New Members

Other spring 2002 officers: Maryann Blouin, Vice President; Tim Paluch, Secretary/Treasurer; Robert Bailey, Corresponding Secretary.

Two of our members accompanied Dr. Richard Cramer-Benjamin to a workshop on technology in late November in Albany, NY. Dr. Cramer-Benjamin also conducted evening workshops on the classroom use of TI calculators and the Geometer's Sketchpad. In addition, the Mathematics Department and our chapter sponsored a one-day workshop on the Geometer's Sketchpad for area teachers and students on April 15.

NY Kappa Chapter President–Maria De Conti

Pace University, New York 25 Actives, 10 New Members

Other spring 2002 officers: Geraldine Taiani, Corresponding Secretary.

NY Lambda Chapter President–Andrea M. Lorusso

C.W. Post Campus of Long Island University, Brookville 21 Actives, 12 New Members

Other spring 2002 officers: Kira A. Adel, Vice President; Lisa M. Cook, Secretary; Heidi A. Campbell, Treasurer; Dr. Andrew M. Rockett, Corresponding Secretary.

Twelve students were initiated into the New York Lambda Chapter by the chapter officers during our annual banquet at the Greenvale Town House restaurant on the evening of April 17th, bringing the Chapter membership to 225. Lisa M. Cook's after-dinner presentation on "Hot Hands" in Basketball: Myth or Math? was based on her honors thesis prepared under the direction of Dr. Geoffrey C. Berresford. The evening concluded with the announcement of Dean Paul Sherwin of the 2001-2002 departmental awards: the Claire F. Adler Award to Andrew Lorusso, the Lena Sharney Memorial Award to Lisa M. Cook, the Joseph Panzeca Memorial Award to Justina M. Raynor, and the Hubert B. Huntley Memorial Award to Agathi Michael; and the presentation by Dr. James V. Peters of MAA students memberships to Kira A. Adel, Heidi A. Campbell, Lisa M. Cook, Andrea M. Lorusso, Agathi Michael, and Justina M. Raynor.

NY Mu

St Thomas Aquinas College,Sparkill — 82 Actives, 5 New Members

Other spring 2002 officers: Dr. Joseph A Keane, Corresponding Secretary.

NY Nu — Chapter President–Adam Parsells

Hartwick College, Oneonta — 12 Actives, 5 New Members

Other spring 2002 officers: Leslie Quattrini, Vice President; Andreas Tsolakis, Secretary; Vicki Chan, Treasurer; Ron Brzenk, Corresponding Secretary.

OH Eta — Chapter President–Stacey Stillion

Ohio Northern University, Ada — 39 Actives, 16 New Members

Other spring 2002 officers: Reid Moore, Vice President; Starli Klobetanz, Secretary; Sarah Miller, Treasurer; Donald Hunt, Corresponding Secretary.

OH Epsilon

Marietta College, Marietta

Other spring 2002 officers: John R. Michel, Corresponding Secretary.

OH Gamma — Chapter President–Jason Popovic

Baldwin-Wallace College, Berea — 25 Actives, 17 New Members

Other spring 2002 officers: Marianne Fedor, Vice President; Mark Skor, Secretary; Valerie Cubon, Treasurer; David Calvis, Corresponding Secretary.

OK Alpha — Chapter President–Emily Walker

Northeastern State University, Tahlequah — 38 Actives, 8 New Members

Other spring 2002 officers: Lara Lancaster, Secretary; Erin Renfrow, Treasurer; Dr. Joan E. Bell, Corresponding Secretary.

The spring initiation of 8 new members was held at The Grand China Restaurant. Dr. Julie Sawyer was the chairman of our "Math Fair at Oaks" project in April. To spread the excitement of math, we put on a math fair for 50 5th and 6th graders at Oaks Mission School. The students did math posters to compete for graphing calculators, math games, and our KME math T-shirts. Judges were the interim dean of our college, Dr. Craig

Clifford, the chair of our department, Dr. Darryl Linde, and Dr. Wendell Wyatt. All the students did hands-on math activities in four stations: making circles/manipulative geometry with Dr. Wendell Wyatt, magic squares with Dr. Joan E. Bell, networks with Sam Snow, and, geometry art with Stephanie Hilburn and Joe Gonzales. Now for the 5th year in a row, our chapter designed and sold 70 KME T-shirts. We combined our annual spring ice cream social with a presentation by our State Representative Jim Wilson on "Schizophrenic Mathematics." It was standing room only with 120 in attendance.

PA Alpha Chapter President–Brian Sullivan
Westminster College, New Wilmington 23 Actives, 8 New Members

Other spring 2002 officers: Jeannie Ullman, Vice President; Emily Henry, Secretary; Sarah Plimpton, Treasurer; Carolyn Cuff, Corresponding Secretary.

PA Beta
La Salle University, Philadelphia

Other spring 2002 officers: Faculty Sponsor and Corresponding Secretary: Jonathan C. Knappenberger.

PA Epsilon
Kutztown University, Kutztown

Other spring 2002 officers: Randy Schaeffer, Corresponding Secretary

PA Eta
Grove City College, Grove City

Other spring 2002 officers: Marvin C. Henry, Corresponding Secretary.

PA Gamma
Waynesburg College, Waynesburg

Other spring 2002 officers: James Bush, Corresponding Secretary.

PA Kappa Chapter President–Joseph Coll
Holy Family College, Philadelphia 6 Actives, 7 New Members

Other spring 2002 officers: Angela Cardamone, Vice President; Erica Bucci, Secretary; Melissa Murphy, Treasurer; Dr. Anne E. Edlin, Corresponding Secretary.

PA Lambda Chapter President–Rachel Keller
Bloomsburg University, Bloomsburg 25 Actives, 15 New Members

Other spring 2002 officers: Brian Bankis, Vice President; Heather Yanchunas, Secretary; Kristen Detweiler, Treasurer; Elizabeth Mauch, Corresponding Secretary.

PA Mu Chapter President–Geri Cooper

Saint Francis University, Loretto 10 New Members

Other spring 2002 officers: Derek Warner, Vice President; Matt Bollinger Secretary; Adam Tebbe, Treasurer; Pete Skoner, Corresponding Secretary.

Ten Saint Francis University students were inducted into the Pennsylvania Mu Chapter of the Kappa Mu Epsilon National Mathematics Honor Society on Tuesday, March 12. A reception preceded the initiation ceremony for the 10 new student members: Thomas Anderson, senior computer science; Amy Croskey, sophomore chemistry; Matthew Farabaugh, junior mathematics; Jelena Petrovic, sophomore chemistry; Andrew Farrari, sophomore engineering; Joseph Bopp, junior chemistry; Rebecca Dombrowski, freshman mathematics secondary education; Susan Horten, junior mathematics secondary education; Chad O'Brien, junior mathematics secondary education; Lawrence Quinn, sophomore computer science.

The ceremony was led by KME officers Gerri Cooper, Jamie Krusinsky, Adam Tebbe, and Derek Warner, faculty advisor Ms. Amy Miko, and corresponding secretary Dr. Peter Skoner

The chapter, along with the mathematics department, sponsored the annual Pi Day celebration on March 14, at 1:59 in the afternoon by offering a piece of pie to students, faculty, or staff who stopped by the department office to honor the famous irrational number.

The chapter also hosted with the Laurel Highlands Mathematics Alliance, a Math Day for sixty-eight local high school students from eight area high schools on March 8. Each student had the opportunity during the day to attend two presentations and participate in mathematical contests.

All students had the chance to compete in a mathematics quiz game, "Who Wants to Look Like a Millionaire (in a New T-shirt)?" in the afternoon. The finalists were asked up to six questions of increasing difficulty until they answered one wrong. Students who answered at least six questions correctly were awarded a mathematics t-shirt. T-shirts were also given to students who found the answer to several logic puzzles.

PA Pi Chapter President–James Oakley

Slippery Rock University, 8 Actives, 2 New Members

Other spring 2002 officers: Leah Schilling, Vice President; Dave Czapor, Secretary; Gary Grabner, Treasurer; Elise Grabner, Corresponding Secretary.

PA Xi Chapter President–Jennifer Weinberg

Cedar Crest College, Allentown 0 Actives, 2 New Members

Other spring 2002 officers: Marie Wilde, Faculty Sponsor.

SC Gamma Chapter President–Anna Ulrey

Winthrop University, Rock Hill 18 Actives, 2 New Members

Other spring 2002 officers: Laura Taylor, Vice President; Angel Rushton, Secretary; Nathan Foth, Treasurer; Frank Pullano, Corresponding Secretary.

This year KME hosted two social events, one at the start of each semester. The pizza party at the start of the Fall 2001 semester provided freshman mathematics majors an opportunity to meet the upperclassmen and ask questions about what it means to major in mathematics. At the start of the Spring 2002 semester, KME hosted an ice-cream social for all mathematics majors. Also, the members of KME designed and ordered t-shirts that they now proudly wear on the campus!

SD Alpha Chapter President–Ben Bouza

Northern State University, Aberdeen 10 Actives, 5 New Members

Other spring 2002 officers: Scott Heyne, Vice President; Josh Bragg, Secretary; Amy Gurney, Treasurer; Dr. Raj Markanda, Corresponding Secretary.

TN Delta Chapter President–Kryshelle R. Smith

Carson-Newman College, Jefferson City 17 Actives, 2 New Members

Other spring 2002 officers: Robyn A. Peabody, Vice President; Hannah L. Baugher, Secretary; A. Houston Qualls, Treasurer; Dr. Catherine Kong, Corresponding Secretary.

The Spring KME banquet was held on April 4, 2002, a Career Seminar was on April 5, 2002, and on April 30, 2002 was the Spring KME picnic.

TN Epsilon

Bethel College, McKenzie 3 Actives

Other spring 2002 officers: Russell Holder, Corresponding Secretary.

TN Gamma Chapter President–Patricia Rush

Union University, Jackson 19 Actives, 6 New Members

Other spring 2002 officers: Caroline Ellis, Vice President; Breanne Oldham, Secretary; Amanda Cary, Treasurer; Bryan Dawson, Corresponding Secretary.

Activities: Spring Initiation Banquet on April 11. Christy Moore ('94), who currently teaches high school mathematics at the University School of Jackson, was the guest speaker. Had end of the year celebration for graduating seniors.

TX Alpha Chapter President–Christopher D. Wreh II
Texas Tech University, Lubbock 14 Actives, 18 New Members

Other spring 2002 officers: Bradly Ames, Vice President; Katie Paske, Secretary; Rachelle McNeely, Treasurer; Dr. Anatoly Korchagin, Corresponding Secretary.

TX Iota Chapter President–Anthony Joeris
McMurry University, Abilene 30 Actives, 11 New Members

Other spring 2002 officers: Josiah Burroughs, Vice President; Ronald Briscoe, Secretary; Denis Conner, Treasurer; Dr. Kelly L. McCoun, Corresponding Secretary.

The Texas Iota Chapter at McMurry University held its Spring 2002 KME Initiation banquet at Enrique's Mexican Food Restaurant this year. The Texas Iota Chapter inducted 9 new student members and 2 new faculty members this year. The invited speaker for the evening was Mr. J. Paul Holcomb, a McMurry alumnus, who retired from Rockwell Collins in 2000.

TX Gamma Chapter President–Dayna Ford
Texas Woman's University, Denton 13 Actives, 13 New Members

Other spring 2002 officers: Whitney Hobbs, Vice President; Ebony McGee, Secretary; Chui Wa Wong, Treasurer; Dr. Mark Hamner, Corresponding Secretary.

TX Kappa Chapter President–Rachel Goad
University of Mary Hardin-Baylor, Belton 10 Actives, 6 New Members

Other spring 2002 officers: Lynett Kaluza, Vice President; Peter H. Chen, Corresponding Secretary.

TX Mu
Schreiner University, Kerrville

Other spring 2002 officers: William Sliva, Corresponding Secretary.

VA Alpha Chapter President–Joi Young
Virginia State University, Petersburg 17 Actives, 20 New Members

Other spring 2002 officers: Adina Holmes, Vice President; Kia Garner, Secretary; Dr. Emma B. Smith, Treasurer; V.S. Bakhshi, Corresponding Secretary.

VA Beta Chapter President–Brandon Monahan

Radford University, Radford 13 Actives, 3 New Members

Other spring 2002 officers: Andrea Crigger, Vice President; Stephen Corwin, Secretary; David Albig, Treasurer, Stephen Corwin, Corresponding Secretary.

VA Gamma Chapter President–Andrew Felker

Liberty University, Lynchburg 15 Actives, 7 New Members

Other spring 2002 officers: Seth Dimond, Vice President; Hope Holloway, Secretary; Ronald Forch, Treasurer; Dr. Glyn K. Wooldridge.

WV Alpha

Bethany College, Bethany

Other spring 2002 officers: Mary Ellen Komorowski, Corresponding Secretary.

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Kappa Mu Epsilon

List of Recent Initiates

(as of October 2002)

Alabama Gamma University of Montevallo, Montevallo

Daniel D. McMillan, Christopher D. Nelson, Amy DePriest, Cari Cook, Dave Shanklin, Laura Moore, Amanda McCormick, Cory Holcomb, Hollie Smith

Alabama Epsilon Huntingdon College, Montgomery

Audrey Davie, Tonya Blankenship, Shelby Wills, Emilia Anna Lusnia, James Houston Kervin II, Radhika Iye, Anna M. Chetovkhina

California Delta CA State Polytechnic University, Pomona

Timothy L. Cobler, William F. Edwards, Jean I. Garcia, Lily Ho, Brian N. Li, Jason P. Matthews, Bartlomiej Podlesny, Marco A. Sanchez, Nerissa V. Soriano, Ryan S. Sousley, Emily N. Thompson

Colorado Beta Colorado School of Mines, Golden

Ricky Chan, Judy Chen, Kristina Coleman, Elizabeth Chorney, Scott Danford, Brandon DeHammer, Michael Deneff, Jr., Rachel Fallen, Anthony Febbraro, Andrew Paul Getz, Levi Hendrickson, Matthew W. Koenig, Jie Qing Li, Korinne M. McCoy, Kara Michelle Namanny, An Ngo, Suzanne Pearson, Brian Scroggs, Stefan Sillau, Zachary Smith, Pajau Vangay, Terri D. Wagner, Angela Yoshino

Georgia Alpha State University of West Ga., Carrollton

Ndidi. W. Ananaba, Chad Mathews, Juan Orphee, Bryan E. Stamps, John A. Wahl, Jr., Keisha R. Wilkey

Georgia Beta Georgia College & State University, Milledgeville

Carlos E. Aguilera, Martha Allen, Ryan A. Bell, Rima Chaudhuri, Denise M. Deloach, Ryan S. Kitson, Marko Krstulovic, Michael Marion, James W. Mozley, Luke D. Niday, Edward Zac Ray, Karyn L. Reeves, Janet M. Shiver, Emily Smith, Latha Swamy, Tahirih Varner, Gita C. Williams

Illinois Delta University of St. Francis, Joliet

Diedra A. Duncan, Christopher M. Farkas, Adam P. Haldorson, Jamie M. Kaffel, Jason R. Mizell, Jonathan D. Paul, Julie A. Pohlman, Erik N. Volkening

Illinois Zeta Dominican University, River Forest

Amanda Lange, Jennifer Soldat, James Wazorick, Conrad Zadlo

Illinois Eta Western Illinois University, Macomb

Amber L. Bricker, David J. Carpenter, Andrea L. Gillette, James C. Gumbart III, Stephanie M. Quick, Brian T. Sullivan, Eva M. Zellman, Julie K. Koch

Illinois Theta Benedictine University, Lisle

Nicholas Ciotola, Timothy D. Comar, Katharine Curtis, Manmohan Kaur, Matthew Koch, Warren Luckner, Tom Wangler, Brian Nelson

Indiana Alpha Manchester College, North Manchester

Ihab Abu-Zayda, Michael Caldwell, Sandra Haist, Chad Heck, Andrew Hodges, Laura Koczan, Jonathan May, Steven McCune, Becky Moening, Erin Reep, Mary Swartzentruber, Kara Wolheter

Indiana Delta University of Evansville, Evansville

Katherine Achim, Jeremy W. Aldridge, Nicholas L. Armstrong, Megan Therese Baumgart, Brian L. Cooper, Michael M. Fiedler, Colleen K. Gordon, Matthew W. Hartman, Marie H. Hoffman, Timothy J. Kiegel, Ellen Miles, Rex E. Riester, Jason A. Samples, David Scott Siebert, Amanda B. Singer, Li Situ

Iowa Alpha University of Northern Iowa, Cedar Falls

Sara Buchheim, Marie Calkins, Emily Doyle, Sara Hirschman, Jennifer McFate, Chad Tompkins

Iowa Gamma Morningside College, Sioux City

Justin Lewis, Fawn Woiwood

Kansas Alpha Pittsburg State University, Pittsburg

Koevi Agbemehin, Brendan Burke, William Cox, Ashley Gourley, Chris Holland, Erica Hoover, Tai-Hsiang Huang, Andrea Huser, Kelli Langan, Jisel Riachi, Keith Smeltz, Ryan Soendker, Rebecca Spencer, Adam Sponsel, Angela Steele, Andrew Super

Kansas Epsilon Fort Hays State University, Hays

Michelle Bui, Hongbiao Zeng, Nathan R. Jones, Teresa Detweiler, Lindsey Bailey

Massachusetts Alpha Assumption College, Worcester

Kellie Brown, Emelia Chabot, Wendy M. Cook, Kerri Corbett, Carolyn J. Cormier, Jonathan M. Katcher, Nicole Gallant, Michelle L. Gaudette, Diana M. Gross, Kristen M. Malone, Caitlin Marcotte, Eileen A. Mawn, Daniel McNeil, Jessica A. SaferRenee D. Salvas, Meghan W. Sullivan, Jennifer M. Susel

Missouri Epsilon Central Methodist College, Fayette
Jeremy Wayne Hagerman, Jennifer Kirchner, Laura-Ann Lindvall, Temitope Olaniyi Ogunmola

Missouri Zeta University of Missouri-Rolla, Rolla
Claire Elsea, Blake Gray, Callin Hahn, Brian Heckman, Andrew Rolf, Christina Welch

Missouri Iota Missouri Southern State College, Joplin
Ryan Burpo, Janice Espinosa, Daina Marshall, Justin Moll, Curtis Montgomery, Martin Stahl

Nebraska Delta Nebraska Wesleyan University, Lincoln
John Beltz, Jason Damme, Rhea May, Angela Miller, Joshua Snyder, Mason Ullrich

New Jersey Beta Montclair State University, Upper Montclair
Matthew G. Orlando

New Jersey Gamma Monmouth University, West Long Branch
Stephanie M. Beatty, Melissa Ann Berfield, Katie J. Blackburn, Andrew L. Bowler, Christina L. Colanero, John T. Dzwieleski, Amanda K. Glynn, Wendy Jo Horowitz, Gina M. Imbro, Franci R. Laska, Jennifer Lee, Scott J. Mark, J.D. Martino, Melissa McCormick, Felicita Rosa Ramos, Lisa M. Ricciardelli, Michelle I. Sikorski, Candice Ann Smith, Jonathan Daniel Thompson, Erin Yourman, B. Lynn Bodher, Bonnie Gold, Richard A. Kuntz, Biyue Liu, Thomas J. Smith, Judy Toubin, Margaret A. Wilson

New York Eta Niagara University, Niagara
Michael R. Bidzerkowny, Patricia Buscaglia, Ashley Churder, Allison Coomey, Valerie L. Devine, Marc C. Erickson, Melissa Giblin, Nicole R. Grosso, Alissa M. Guggi, Andre L. Maillet, Milena Matijevic, Samantha Meyer, Joseph G. Nyiri, Jennifer O'Laughlin, Sara L. Parsons, Eric James Renne, Jenifer Reynolds, Nancy A. Sansevere, Michelle Marie Searles, Rachel Sherwood, Tracy A. TaylorJustin W. Telech

New York Iota Wagner College, Staten Island
Monica Capobianco, Joanna Kolodzigski, Carolyn Palinkas, Angelo Pappalardo, Diana Senese, Neil Woltjen, Adrian Ionercu

New York Lambda C.W. Post Campus of Long Island University, Brookville
Lorelle J. Berger, Jennifer Bucalo, Sarah K. Corea, Shehzia Faruque, Eric A. Girolamo, Elina E. Kafkalia, Janis T. Mazza, Christine M. Metz,

Agathi Michael, Catherine G. Noto, Justina M. Raynor, Lawrence Schiralli

New York Nu Hartwick College, Oneonta

Vicki Oi Yan Chan, Jolene Curry, David Long, Zia Mahmood, Andrei Zaslavsky

North Carolina Gamma Elon University, Elon

Adam Benjamin, Daisy Cunningham, Emily Dreyer, Richard Dutton, Jennifer L. Hornback, Courtney S. Johnson, Whitney A. Lesch, Chaska Mendoza, April Schexnayder, Nathan Shown, Jennifer Sweeney, Jennifer Walker

Ohio Alpha Bowling Green State University, Bowling Green

Jennifer Barton, Ryan Bohn, Darren M. Bolinger, Remi M. Bowers, Adam Combs, Melanie Dunn, Marcy E. Gertsen, Shannon Hall, Jodi Harbal, Ashley Jones, Meredith Krebs, Kristin L. McDonald, Christine M. Oedy, Ryan Rahrig, Melissa Ryther, Nathan VanNuys, Douglas M. Dunkle, Amanda Bell, Rochelle L. Wolber

Ohio Gamma Baldwin-Wallace College, Berea

Russel E. Book, Timothy Fijalkovich, Jeffrey C. Fox, Edna G. Fromer, Erin J. Hahl, Alex C. Johnson, Lisa Ann Kaltenbach, Thomas N. Kovacevich, Jessica Lawson, Christopher Luzniak, Kevin O'Malley, Elizabeth Ptaszek, May RaffayAmanda Roble, Theodore Ryan, George Trimble, John Wadas

Ohio Eta Ohio Northern University, Ada

Shiloh A. Archer, Rebecca Lynn Busch, Tracy L. Egut, Ashley Lynette Franz, Amy E. Hayden, Peter Ryan Jankovsky, Lori Michelle Levering, Miranda N. Shaw, Kristin K. Stechschulte, Matthew Thomas Suchan, Thomas W. Turney, Matthew David Valerio, Jason Yagielo, Heather L. Buehler, Eric J. Coleman, Casey A. Leichty

Oklahoma Delta Oral Roberts University, Tulsa

Treena T. Balds, Michael Butcher, Jonathan Carlson, Jennie Crouch, Lynn Daugherty, Jeremy Jones, Shibu Matthew, Kathleen Ndackson, Leonard Osahor, Reid Philips, Franklin Daniel Reed, Joshua Rodriguez, Abigail E. Wade, Susan E. Wurster

Pennsylvania Alpha Westminster College, New Wilmington

Colleen H. Conwell, Alan D. Higby, Heather E. Klink, Christopher A. Medjesky, Michael G. Piddington, Jessalyn A. Smith, Jessica A. Young, Danielle D. Zielinski

Pennsylvania Gamma Waynesburg College, Waynesburg
Ryan Bates, Cardell Butler, Tara Corazzi, Elliot Eubanks, Susan Hamilton, James Janicki, Christopher KisnerBret McKinney, Jennifer Tharp

Pennsylvania Theta Susquehanna University, Selinsgrove
Gretchen Anderson, Joshua S. Bachman, Shannon M. Barnett, Teresa J. Bixby, Michael E. Bowman, Amanda J. Geiser, Tara J. Heydenreich, Jennifer S. Knaub, Robert J. Logan, Matthew S. Nagy, Lindsay A. Shaffer, Nathan D. Trick, Jack E. Watt, Timothy P. Weston

Pennsylvania Nu Ursinus College, Collegeville
Natalie M. MacConnell, Nour Z. Moghrabi, Katherine E. Northrup, John J. Grebe, Stephanie L. Zimmer, Jessica A. Braun, Frank G. Romascavage, Chaitanya Desai, Christopher L. Lorenzo, Anh Thuy Hua, Bryan J. Brook, David H. Parker

Pennsylvania Pi Slippery Rock University, Slippery Rock
Megan McKinney, Davlyn Nauman, Robert E. Buck, J. Lyn Miller,

South Dakota Alpha Northern State University, Aberdeen
Merissa Spellman

Texas Alpha Texas Tech University, Lubbock, TX
Brady L. Ames, Jacquelyn L. Moore, Katherine D. Paske, Rachelle McNeely, Jordan C. Frank, Joseph A. Comingore, Luis D. Marrufo, Christopher D. Wreh, II, Harold A. Buell, Jr., Clinton M. Hattaway, Corey Holliman, Graciela Perez Ruiz, Danielle M. Costilla, Samuel H. Thomas, Thomas J. Cronick, Sean J. Anderson, Nichole A. Hester, Drew N. Hirt

Texas Iota McMurry University, Abilene
Jennifer Sogand Agirre, Garret James Bryl, Katrina Jane Hale, Ryan Lee Henderson, Alyson Coleen Henry, Vickie Lynn McElroy, Mindy Michelle Mitchell, Tracy Janelle Parker, Kay Leigh Younggren, Dr. Hans Goeckner, Dr. Robert Rittenhouse

Texas Kappa University of Mary Hardin Baylor, Belton
Rachel Ach, Joseph Daugherty, Jill Klentzman, Rachel Perkins, William Rogers, Lori Wynkoop

Virginia Beta Radford University, Radford
Candace J. Gilliam, Rosanna M. Gould, Rhonda D. VanDyke

Virginia Gamma Liberty University, Lynchburg

Mihaela R. Bojor, Christopher David Cooper, Andrew Stephen Felker, Cori Elizabeth Murphy, Elizabeth Rachel Starkey, David R. Dinsmore, Kathy D. Spradlin

West Virginia Alpha Bethany College, Bethany

Nathan Cole, Melissa Gibson, Brian J. Laick, Anthony Lamia, Danielle M. Pazzabon, Theodor Richardson, Melanie Sands, David Smithbauer, David B. Weibe, Brendan Welsh, Jonathan P. Wilkins, Christopher P. Wilson

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KME Website

The national KME website can be found at

`http://kme.eku.edu/`

This is a new URL, so please update your bookmarks and links from chapter pages! Below is a partial list of items that are available on the site:

- How to start a KME chapter
- Information on KME conventions
- The cumulative subject index of *The Pentagon*

You can get a web page template from the Kentucky Alpha chapter. Its URL is

`http://math.eku.edu/PJCostello/kme/`

When you design a chapter homepage, please remember to make it clear that your page is for your chapter, and not for the national organization. Also, please include a link to the national homepage and submit your local chapter webpage's URL to the national webmaster. By doing so, other chapters can explore activities of your chapter and borrow some great ideas from you!

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Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983

MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February, 2001
GA Delta	Berry College, Mount Berry	21 April, 2001
TX Mu	Schreiner University, Kerrville	28 April, 2001

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