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Contents

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On Iterative Maps

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Presented at the 2001 National Convention and awarded
"top four" status by the Awards Committee

Introduction

The function $f(x) = 2.8e^{0.002x}$ might be a model to determine the number of bacteria colonies living in a petri dish. The independent variable x represents time, and $f(x)$ is the population at time x (perhaps in hundreds, thousands or millions). This type of model is one with which we are all familiar. Another type of model we might look at is the iterative model. In this type of modeling, x would not represent time, but the initial population (or some other initial condition). To simulate ongoing time, the function is iterated several times, once for each unit of time desired. An easily imaginable example of this type of model is a population model. Take, for example,

$$f(x) = 3x$$

This rule tells us what effect one time interval has on the population. Suppose we start with a population of 200. One iteration of the function with$x = 200$ will tell us what happens to the population in one time interval (say, 1 month).

$$f(200) = 3(200) = 600$$

To find the population at 3 months, apply the function 3 times:

$$f(f(f(200))) = f(f(600)) = f(1800) = 5400$$

Iterative functions are the type of model that will be discussed here. Let's start with a little background information:

Iterative functions are maps. A *map* is a function whose range is a subset of its domain. The example map above, $f(x) = 3x$, has a domain consisting of the real numbers and a range consisting of the same. It makes sense that iterative functions must be maps. The number that 'comes out' of an iterative function must be able to 'go back into' the same function again. An $f(x)$ has to be able to be an x.

Orbits and Fixed Points of Iterative Functions

Every initial value plugged into $f(x)$ has an orbit. An *orbit* is defined as the set of points:

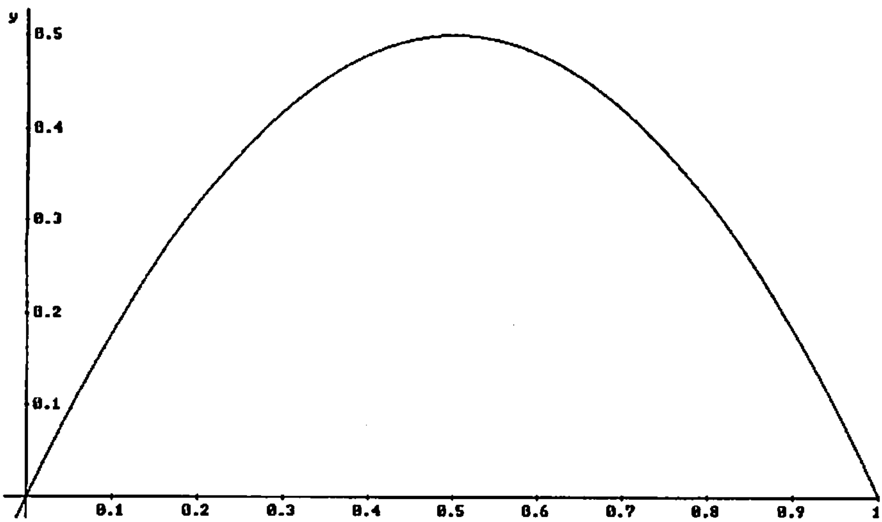
$$\{x, f(x), f^2(x), \dots\}.$$

The point x is called the initial point of the orbit.

Suppose we have a point x that we apply a function $f(x)$ to. That point x is called a *fixed point* if $f(x) = x$. Upon applying several iterations, we see that once we reach a state of $f(x) = x$, successive iterations will not budge $f(x)$. In this case, we can say that the orbit corresponding to the initial point x has only one element in it, x.

An example to illustrate the terms I have defined is the function

$$f(x) = 2x(1 - x) \tag{1}$$

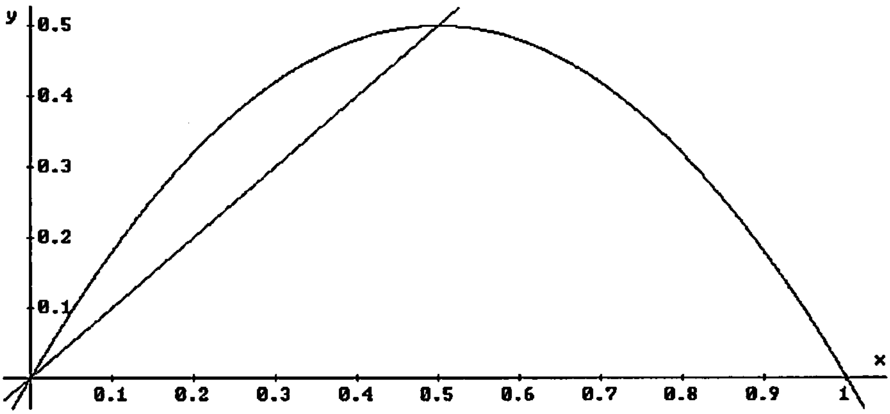


To find fixed points for this function, solve the equation $f(x) = x$ for x.

We have:

$$\begin{aligned} 2x(1-x) &= x \\ -2x^2 + x &= 0 \\ x &= 0 \ , \ 1/2 \end{aligned}$$

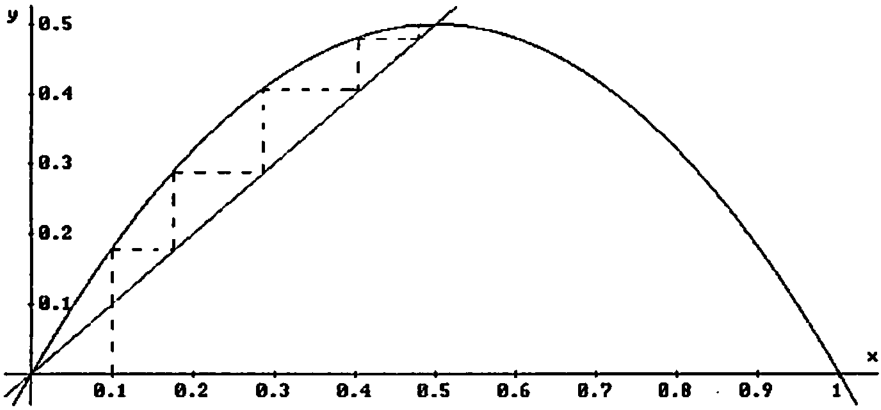
Another way to get an idea of where your fixed points are is to graph both the line $y = x$ and the function in question on the same set of axes. The fixed points will be the x-coordinates of the points where the two curves intersect.



The orbit of a point x under $f(x)$ can be graphically plotted relatively easily. We will do so using a tool called a cobweb plot. Follow these steps to produce a cobweb plot:

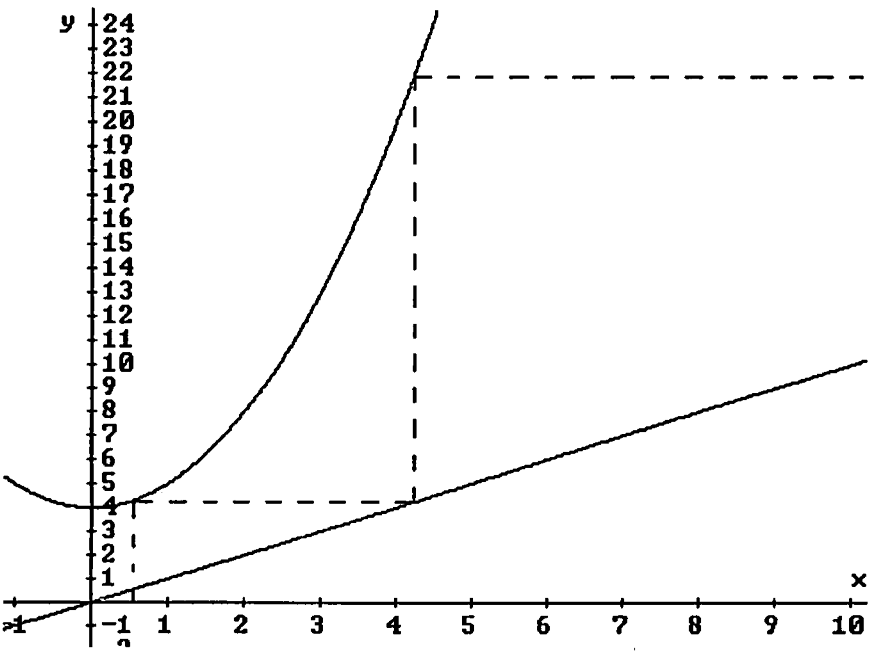
1. Graph the function $f(x)$ and the line $y = x$ on the same axes.
2. Pick an initial point.
3. From the x-axis, sketch a vertical dotted line from the initial point to the function curve.
4. From that point, sketch a horizontal line to the line $y = x$.
5. From that point, sketch a vertical line back to the function curve.
6. Repeat steps 4 and 5 to completion.

Not all functions have fixed points; some orbits are infinite in size. The function f is a case that has fixed points. Below is an example of a cobweb plot for (1) with initial condition $x = 0.1$.



Here is an example of a function that doesn't have any fixed points.

$$f(x) = x^2 + 4$$



Fixed points are an interesting occurrence in iterative modeling. All fixed points fall into two main categories: stable and unstable. A fixed point is considered stable if points near it are moved closer to it as the number of iterations increases. On the other hand, a fixed point is called unstable if points near it are moved farther away from it as the number of iterations increases. We will call unstable fixed points sources, and stable fixed points sinks. Take another look at (1). It appears that the fixed point $x = 0$ is a source, while $x = 1/2$ is a sink. Here is a theorem that will help decide whether a fixed point is a source or a sink:

Let f be a map on $\mathbb{R}$, and assume that p is a fixed point of f.

1. If $f'(p) < 1$, then p is a sink.
2. If $f'(p) > 1$, then p is a source.

(1, p.10)

In my studies, I have looked at four examples of iterative maps in depth. Their descriptions follow.

The Logistic Map

First, instead of just looking at function (1), let's look at the entire family of logistic functions:

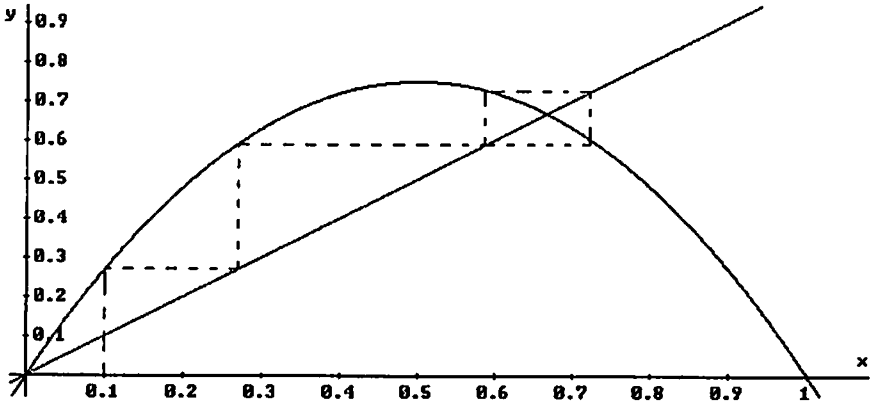
$$f(x) = ax(1 - x).$$

Function (1) is the particular member of the logistic map family of functions that has 2 as the value of a. What happens as we begin to change the value of a? In this case, if we vary a, it turns out that we begin to see some periodic points show up. A periodic point is a point p such that:

$$f^k(p) = p$$

In this environment, f^k represents the k^{th} iteration of f, not f to the k^{th} power. Fixed points are periodic points of period one. The orbit with initial point p (which has elements in it) is called a periodic orbit of period k. As an example, consider the function:

$$f(x) = 3x(1 - x) \tag{2}$$



With an initial point of 0.1, the function converges to a period two sink as iterations are applied.

Both (1) and (2) belong to this family of logistic maps. An interesting question may have come to you. How can we predict what happens to the size of the orbits as a is increased? To answer this question, I will use a tool called a *bifurcation* diagram. A bifurcation diagram can be created in several easy steps:

1. Decide what values of the constant a, and to what precision you wish to test over.

2. For each value of a, pick a random x to use for the initial condition. All x should be attracted to the same point, and this will illustrate that.

3. Begin to iterate the function, but ignore the first 100 (or so) iterations. This step allows the results to settle down into some type of orbit.

4. Plot the points (a, x), where $x \in \{x, f(x), f^2(x), \ldots\}$.

5. Repeat steps 1 to 4 until you have tested all values of that you wish to.

I wrote a program in C++ to create a bifurcation diagram for the logistic map. See Figure 1 for the result. Take a vertical slice of the graph (Fig. 1) at the value of a that you are interested in. The number of times it intersects the 'graph' will be the number of sinks in the largest orbit corresponding to that value of a. It turns out that, in this case, as a increases, so does the size of the orbit. As a approaches 4, the number of points in the orbit increases to a very large number!

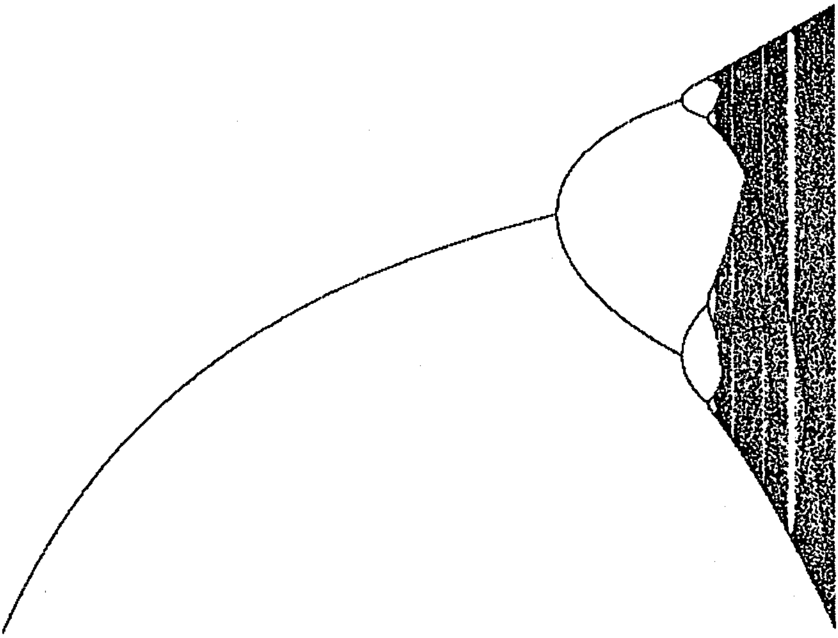


Figure 1: Bifurcation Diagram for the Logistic Map

The x-values lie on the vertical axis, and range from 0 *to* 1.
The a-values lie on the horizontal axis, and range from 1 *to* 4.

The Hénon Map

After exploring the logistic map, I ran across a version of the Hénon Map. It caught my eye because it was an iterative function in two dimensions. Here's the function:

$$f\left(x,y\right)=a-x^{2}+by,x) \tag{3}$$

The Hénon Map also displays some periodicity. A bifurcation diagram for the Hénon Map could be created by choosing either x or y to plot on the vertical axis, and choosing a or b to plot on the horizontal axis. To discover completely all possible periodicity, one must vary a and b separately and look at two bifurcation diagrams.

This is certainly a more difficult map to visualize than a one-dimensional map. See Figure 2 for some examples of orbits in this map. In each of the plots, $b = 0.4$ and a is varying. In plots a through f, $a = 0.9, 0.988, 1.0,$ $1.0293, 1.045$,and 1.2, respectively.

In a, b, and d, each cross represents a point in the orbit. In the others, the orbit points have become so many and so close together that the orbits appear to be in distinct pieces. Because these orbits are sinks, plots c, e, and f are called, respectively, four-, two- and one-piece *attractors*.

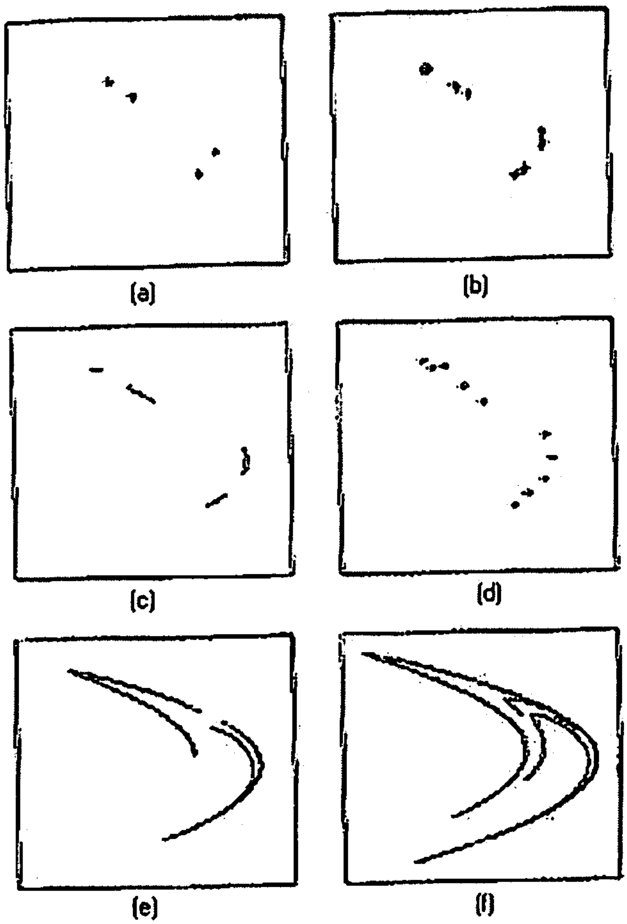


Figure 2: Some Sample Orbits for the Henon Map (1, p. 75)

Attempt at an Iterative Predator-Prey Model

The Hénon Map made me wonder about other two-dimensional iterative maps, so I tried to come up with an iterative, two-dimensional, population model that would model the populations of a group of rabbits and a group of foxes in an isolated environment. One of the design goals was to end up with a model that would produce similar results to some currently existing models that use differential equations. The function is as follows:

$$f(x, y) = \left(1.8xe^{-0.01y}, 0.8ye^{0.005x}\right) \tag{4}$$

Here, x represents the rabbit population, while y represents the fox population. While the model comes close to modeling the populations in the desired fashion, it has no periodicity, only fixed points. I rewrote the function as

$$f(x, y) = axe^{by}, cye^{dx}$$

I did this so that I could create bifurcation diagrams, and discover which values of a, b, c, d and gave me some periodicity in the orbits, but no reasonable values of these constants produced any kind of periodicity. The value of the fixed point does change, of course, depending on the values of a, b, c, and d. From this point on, I stuck with the a, b, c, and d values of (4).

I wrote another C++ program with the goal of observing the behavior of the model. The first program I wrote asked for initial conditions (i.e.: the initial number of rabbits and the initial number of foxes), and then output the results of iterating the function several times in the form of ordered pairs (x, y). Eventually, we arrive at a fixed point of 280 rabbits and 59 foxes. Looking at ordered pairs isn't very exciting however, so I wrote a third program to plot the points on the Cartesian Plane. One can really see what happens over time when one sees the graph. As the number of iterations is increased and more points are plotted, the points begin to spiral in toward the fixed point. See Figure 3.

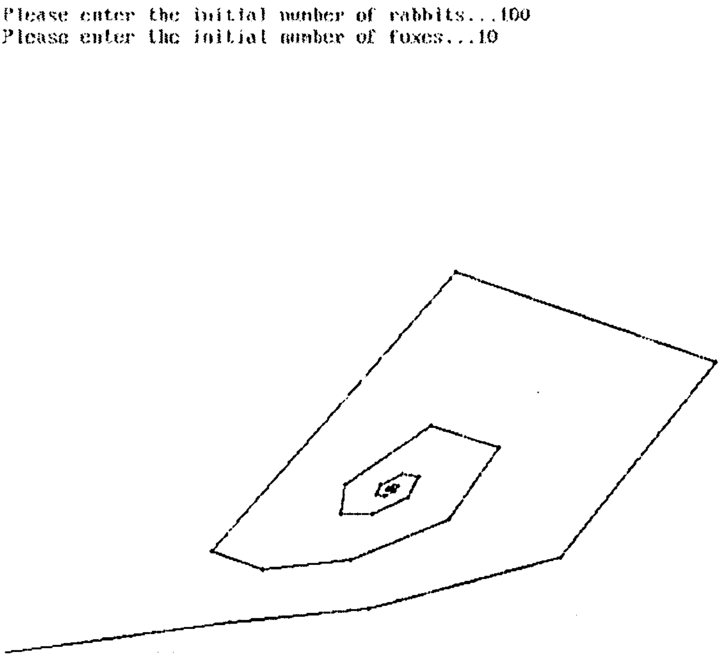


Figure 3: Orbit Plot for (4)

The x-axis represents rabbits, and ranges from 0 *to* 640, *while the y-axis represents foxes and ranges from* 0 *to* 240.

In Figure 3, the dots are the actual data points produced, while the lines are merely lines that connect the consecutive data points. The lines are not the actual curve that the points lie on, but the lines do help us see what the curve looks like. The initial values entered for the figure given were 100 rabbits and 10 foxes.

Revised Attempt at an Iterative Predator-Prey Model

The first attempt modeled the populations in a realistic manner, but it didn't possess any fixed points that were not of period one. I wanted a model that had a more cyclic feel, so I considered a new function, with the same final goals in mind. Here it is: $f(x, y)$

$$= \left(\frac{1.75\,(1+x)}{1+\log(1+0.0005y)}, 0.55\,(1+y)\,(1+\log(1+0.0002x))\right) \quad (5)$$

You could say that this map is on the other end of the spectrum. It does contain periodicity, but it seems a bit erratic to be a realistic model for modeling what it was intended to. The periodicity of this map, however, is different than the periodicity defined before. In the previous example, we had points in an orbit that were continually repeating themselves. In the case of (5), the points tend to follow the same curve over time, but don't land on the same points in that curve. Figure 4 shows a simple example of this for no relation in particular, while Figure 5 shows the same phenomena for (5).

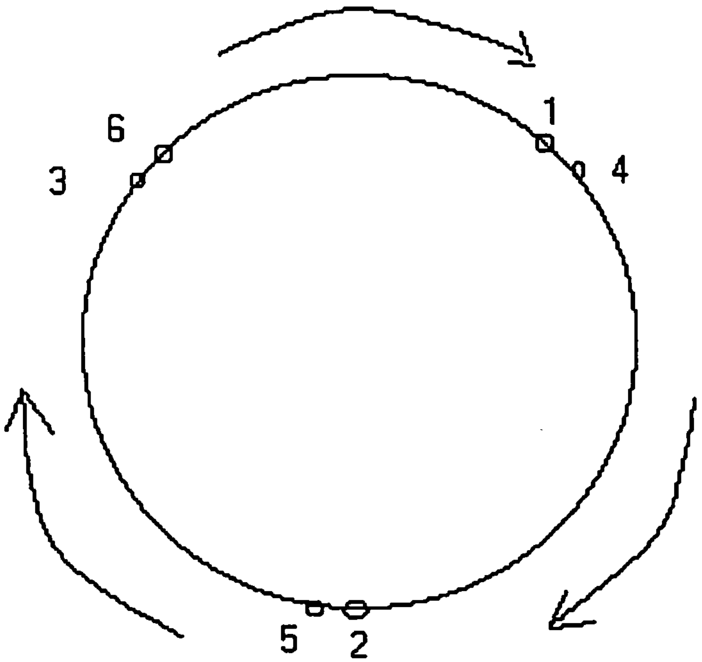


Figure 4: Example of interleaving points on a curve

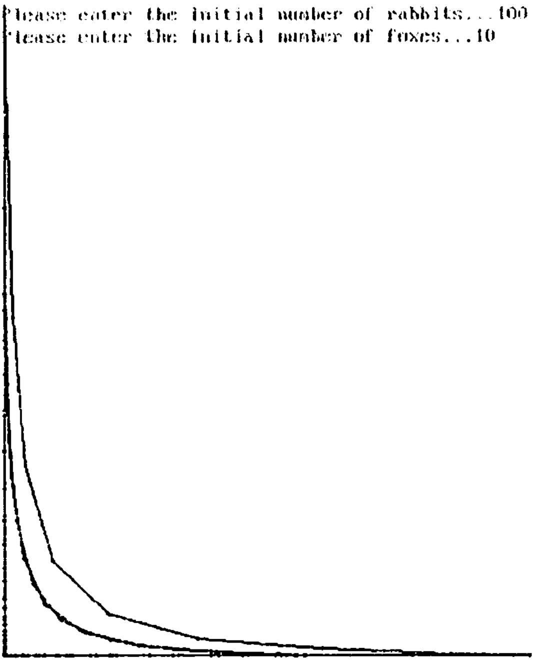


Figure 5: Orbit Plot for (5). Illustrates data point interleaving behavior

The x-axis again represents rabbits, and the y-axis represents foxes. The x-axis ranges from 0 to around 11 million, while the y-axis ranges from 0 to around 14 million.

I will refer to this behavior as data point <u>interleaving</u>. As in Figure 3, the dots represent actual data points, and the lines merely connect consecutive dots. The function (5) never allows the number of foxes or the number of rabbits to reach zero. While there appear to be points on the axes, this is because I had to zoom out quite a bit to fit the entire plot on the screen. The part of the graph that doesn't seem to mesh with the rest of it is the path of the data points the first time around. After the first "cycle", things seem to calm down into a regular cycle that the data point interleave themselves upon.

Conclusion

Iterative maps have been shown to possess many interesting behaviors. We have seen, in both one and two-dimensional maps, orbits, fixed points, periodic points, periodic orbits, and interleaving. Bifurcation diagrams and orbit plots have both been useful tools in analyzing these maps. Iterative maps may be only distant cousins to differential systems, but they are certainly no less interesting.

Acknowledgements. I would like to thank Dr. Charles Curtis, Associate Professor of Mathematics at Missouri Southern State College, for his advice and help in undertaking this project. It would not be what it is without him.

I would also like to thank Dr. Jack Oakes, Department Head of CIS at Missouri Southern State College, for teaching me to program in C++. He enabled me to learn and discover in a way not before possible.

References

1. Chaos, An Introduction to Dynamical Systems. Alligood, Kathleen T., et al. Springer-Verlag, New York-Inc. 1996.

Starting a KME Chapter

For complete information on starting a KME chapter, contact the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of the $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offering and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

Division of a Number by Multiple Divisors and the Underlying Geometry

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Introduction

By fourth grade the average American elementary school child has been introduced to the idea of long division. This mathematical concept is covered shortly after students learn the multiplication tables, and is usually mastered before the students leave grade school. In upper level college math courses the long division is revisited and studied in a more rigorous manner. The algorithm is studied and expanded for other situations at the graduate level and even the postgraduate level. In fact, the long division for numbers and polynomials is studied in a more thorough method at the university and doctoral level. Much advanced work has been done on the topic, especially concerning division of polynomials. At the postgraduate level, division algorithms have been developed for single variable polynomials and multivariable polynomials. This paper presents a manner of viewing the division algorithm differently than the convention used in most advanced texts.

In this paper we give a geometric representation of division. During the early period in a child's education, the student is unfamiliar with the Euclidean Space. This fact makes it impossible to use Euclidean geometry to interpret the result of a problem when a child is learning long division. Even more advanced texts do not interpret long division in such a manner, despite its many advantages. In this paper we begin by interpreting the standard division algorithm geometrically. Furthermore, we generalize grade school long division by considering division of an integer by several divisors.

Division of integers can easily be thought of as a shorthand method of subtraction. Thus, 61 divided by 7 to yield a quotient of 8 and a remainder of 5 is the same as subtracting 7 from 61, 8 times, until a remainder of 5 is obtained. Although long division can be looked upon as a shorthand method of subtraction, it is entirely based on the division algorithm. The following is a statement of the division algorithm. Let a, b be integers with

$b > 0$. Then there exist unique integers q and r such that $a = bq + r$ and $0 \leq r < b$ (Hungerford 1997).

This statement is found in many mathematics books, but as previously stated, standard texts do not give a geometric representation. Relating this statement of the division algorithm to the standard notion of long division, 'a' is the dividend, b is the divisor, q is the quotient, and r is the remainder. To study this in a geometrical fashion, we substitute x for q and y for r. This substitution yields the following equation: $a = bx + y$, which can also be written as $y = -bx + a$. Consider the division of 61 by 7. The corresponding algebraic equation is: $61 = 7x + y$ or $y = -7x + 61$. The graph of this equation yields a geometric interpretation. Also shown is the method that is used by grade school students to perform the division.

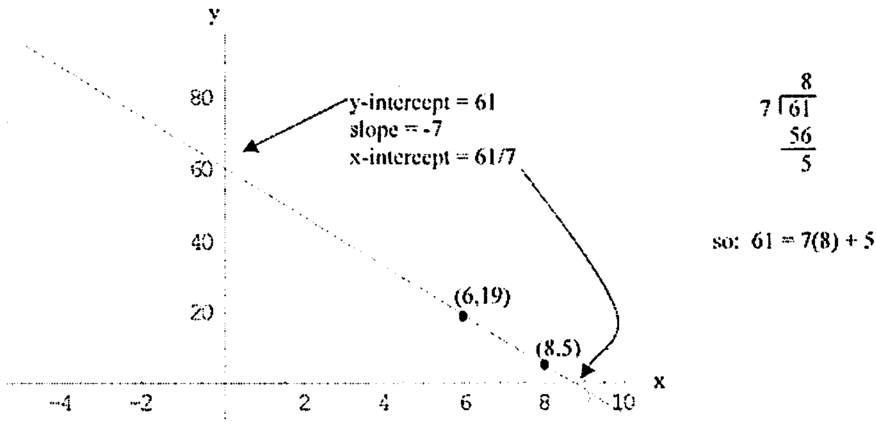

Figure 1

The x-intercept is approximately 8.71. The floor value of the x-intercept yields the integer 8, which is the quotient. The y-value at $x = 8$ is 5, which is the remainder. In general, dividing 'a' by a positive integer, b, gives rise to an equation $y = -bx + a$. The floor value, $[a/b]$, of the x-intercept is the quotient, and $-b[a/b] + a$ is the remainder. Applying the division algorithm to the previous example we know that there exist unique integers q and r such that $61 = 7q + r$. It is clear that $q = 8$ and $r = 5$. This means that the point with the Cartesian coordinates $(8, 5)$ is the only point on the line that fits the division algorithm. Although there are an infinite number of points on the line $y = -7x + 61$, points such as $(7, 12)$ and $(6, 19)$ are not representative of a proper division of 61 by 7. This follows from the uniqueness portion of the division algorithm. More specifically, the y values of all lattice points other than $(8, 5)$ are larger than 7. There are points on the line where $y < 7$, but in the division algorithm a, b, q, and r are restricted to integers so these points are also not representative of proper division.

It is clear that a geometric representation works well when a is positive, but what happens if the value for a is negative? Let $a = -61, b = 7$. So the equation that symbolizes this division geometrically is $-61 = 7x + y$. The following is the graph, accompanied by the longhand division.

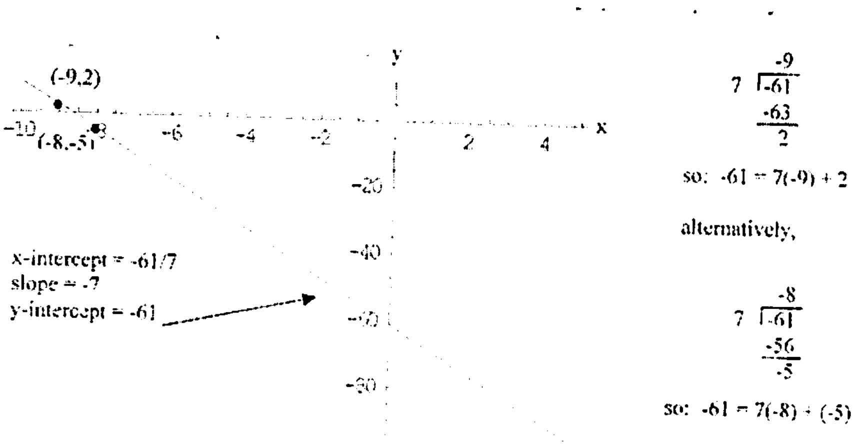

Figure 2

Arithmetically there appears to be two correct answers, but according to the division algorithm, r must be nonnegative and less than the divisor. The geometric representation clarifies the issue of a positive remainder. The x-intercept, approximately -8.71, is between -8 and -9. The floor value of this x-intercept is -9. The y-value at $x = -9$ is 2. Hence $-61 = 7(-9) + 2$ reflects the correct long division. The points at $(-8, -5)$ and $(-9, 2)$ are two of an infinite number of lattice points, but only $(9, 2)$ represent the correct division. Once again the geometric representation helps to shed light on the long division.

The standard division algorithm involves dividing a dividend by a single divisor. In this paper the standard division algorithm is generalized by introducing more than one divisor. Currently we are unaware of any text where long division of an integer is studied using multiple integral divisors. Basing this two-divisor algorithm on the original division algorithm the same limitations would be applied to b_1 and b_2 as were originally applied to b, and the limits for the remainder would remain constant. The following could be a proposed algorithm. Let a, b_1, b_2 be integers with $b_1, b_2 > 0$. Then there exist unique integers q_1, q_2, and r such that $a = b_1q_1 + b_2q_2 + r$

and $0 \leq r < min\{b_1, b_2\}$. In this two-divisor algorithm, a remains the dividend, b_1 and b_2 are the divisors, q_1 and q_2 are the quotients, and r is the remainder. At first glance, this algorithm looks suitable, but various situations must be considered.

An example similar to the single divisor instance, would be 61 divided by 7 and 4. In a general form, the equation would look like: $61 = 7q_1 + 4q_2 + r$. To change this into a geometric representation of the division problem, substitute x_1 and x_2 for q_1 and q_2 respectively, and also y for r. The resulting algebraic equation would be $61 = 7x_1 + 4x_2 + y$. The following is a graphical representation of this equation, along with one method of longhand division.

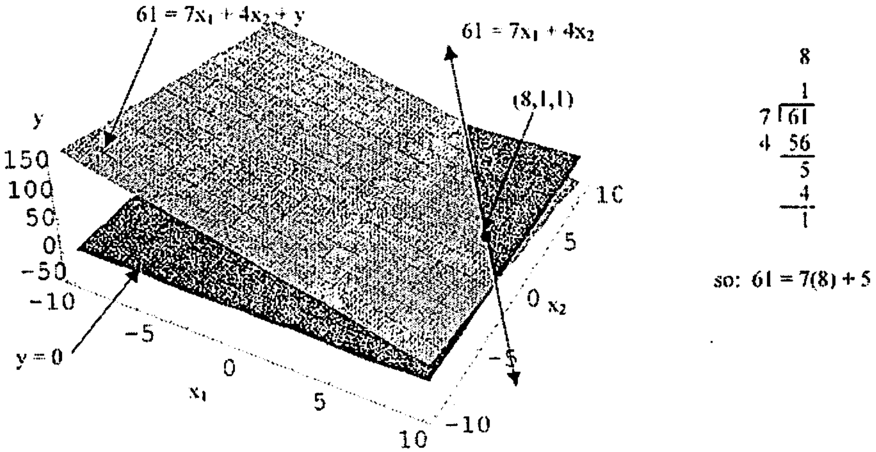

Figure 3

The first problem with this proposed two-divisor algorithm arises when the divisors are not taken into the dividend the maximum number of times. Two such examples are $61 = 7(7) + 4(3) + 0$ and $61 = 7(6) + 4(4) + 3$. These equations give rise to the ordered triples $(7, 3, 0)$ and $(8, 1, 1)$. These examples demonstrate that uniqueness of quotients and remainder need not hold in division with two divisors. The revised algorithm for division using two divisors is as follows: Let a, b_1, b_2 be integers with $b_1, b_2 > 0$. Then there exist integers q_1, q_2, and r such that $a = b_1q_1 + b_2q_2 + r$ and $0 \leq r < min\{b_1, b_2\}$.

For a moment, the algorithm itself will be studied without the grade school notion of division. This means that we must not be worried with the term division; instead concentrate on writing 'a' as a linear combination of b_1 and b_2 with an addition of a generalized remainder, r. The division algorithm is based on the equation $a = b_1q_1 + b_2q_2 + r$. Both b_1 and b_2 are assumed to be positive, but neither q_1 nor q_2 have a restriction and the generalized remainder may have no restrictions. Considering this, it is clear that the equation $a = b_1x_1 + b_2x_2 + y$ forms an infinite plane of lattice points. While the common notion of division is not being considered, what happens when the divisors are taken into the dividend zero times? When both quotients are zero, the point produced is always just $(0, 0, dividend)$, and in our illustration would be represented by the equation $61 = 7(0) + 4(0) + 61$.

An especially interesting question to ask when the standard notion of division is not taken into consideration is what happens when our divisors are both zero? When first learning division students learn that it is not possible to divide by zero. This impossibility is demonstrated by the ability to take 61 objects and put them into one group, two groups, and three groups demonstrating the division of 61 by 1, 2, and 3. The question is then posed, is it possible to take 61 objects and put them into zero groups, whatever that may mean? Without thinking in this way, we consider the plane $61 = 0x_1 + 0x_2 + y$. There is a graphic representation for a division problem with two divisors of zero; it is a plane at $y = dividend$. Without considering the typical concept of division the geometric representation becomes much broader and meaningful. No matter what the quotients, the generalized remainder is always equal to 61. The following is a graph of the aforementioned plane.

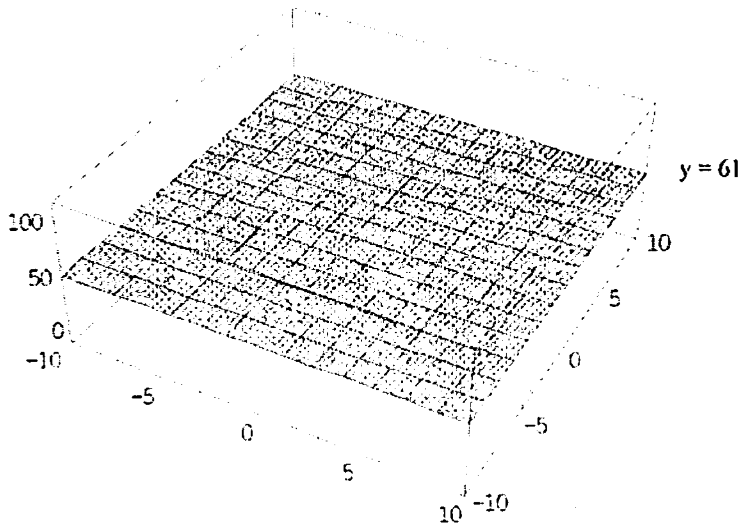

Figure 4

Intuitively, some specifically chosen pairs of the lattice points belonging to the plane $61 = 7x_1 + 4x_2 + y$, would define interesting lines. The most noteworthy lines lie on the intersections between the plane defined by the equation $61 = 7x_1 + 4x_2 + y$ and the planes parallel to the x_1, x_2 plane. The significance of these lines of intersection may be best exemplified by the intersection between the plane $61 = 7x_1 + 4x_2 + y$ and the x_1, x_2 plane itself, which is defined by the equation $y = 0$. This line contains all lattice points that have a y value of zero, which means that the remainder at these points is zero. As seen in Figure 3, the line at this intersection can be written as $61 = 7x_1 + 4x_2$. Hence, the lattice points contained in this line are all the Cartesian pairs that can be used to write 61 as a linear combination of 7 and 4. Two such points are represented by the equations $61 = 7(7) + 4(3) + 0$ and $61 = 7(3) + 4(10) + 0$. These two points define the previous line by the equation $x_2 - 3 = -\frac{7}{4}(x_1 - 7)$, i.e. $x_2 = -\frac{7}{4}x_1 + \frac{61}{4}$. Using this equation, it is possible to find an infinite number of solutions to the division that all have remainders of zero.

The ability to find an infinite number of points with the same remainder becomes even more useful when it can be generalized. Using this generalization, if we are given a specific division with one pair of quotients and a remainder, we will be able to produce an infinite number of quotient pairs with the same remainder. Referring back to the division algorithm, a, b_1, b_2, q_1, q_2, and r are all integers and $a = b_1q_1 + b_2q_2 + r$. The dividend, a, the divisors, b_1, b_2, and the remainder, r, should be held constant. The generalization begins with the basic concept that $b_1(+b_2) + b_2(-b_1) = 0$. This fact allows us to rewrite the original equation as follows: $a = b_1(q_1 + b_2) + b_2(q_2 - b_1) + r$. Now it is clear that we can multiply both b_1 and b_2 by the same integer and the equation will not be changed. The final generalization for any new pair of quotients while keeping the dividend, divisors, and remainder constant is as follows: $a = b_1(q_1 + mb_2) + b_2(q_2 - mb_1) + r$, where m is any integer. Referring back to Figure 3, we can find an infinite number of intersection points between the plane $61 = 7x_1 + 4x_2 + y$ and any plane parallel to $y = 0$, if we know one point along the line of intersection. Similarly, we could also find an infinite number of points along the intersection of the planes $61 = 7x_1 + 4x_2 + y$ and $y = m$, for any integer m.

Another result of interest is a generalization made when either q_1 or q_2 are held constant and the other is allowed to vary, along with the remainder. Using Figure 3 as a reference once again, we are now finding an infinite amount of intersection points between the plane $61 = 7x_1 + 4x_2 + y$ and any plane perpendicular to $y = 0$. This generality is more apparent than the last. Consider the following equations representing two points with the first quotient held constant at $q_1 = 3$: $61 = 7(3) + 4(9) + 4$ and $61 = 7(3) + 4(8) + 4$. It is obvious that each time q_2 decreases by one the remainder increases by b_2. It is also true that as q_2 increases by one, the remainder decreases by b_2. The generalization that can be made when a, b_1, b_2, and q_1 are held constant is $a = b_1q_1 + b_2(q_2 - m) + (r + mq_2)$, where m is any integer. A similar statement can be made when the second quotient is held constant rather than the first.

As with single divisor division, we check that these results are consistent when the dividend is negative. Consider a similar division of -61 divided by 7 and 4. The division can be written as the equation $-61 = 7q_1 + 4q_2+r$, and the corresponding geometric equation is $-61 = 7x_1+4x_2+y$. This equation can be satisfied in an infinite number of ways including the following: $-61 = 7(-8)+4(-2)+3, -61 = 7(-8)+4(-1)+(-1)$, and $-61 = 7(-7) + 4(-3) + 0$. The accompanying graph is a representation of the plane containing all the lattice points, and its intersection with the plane $y = 0$. It is evident that little changes when the dividend is negative.

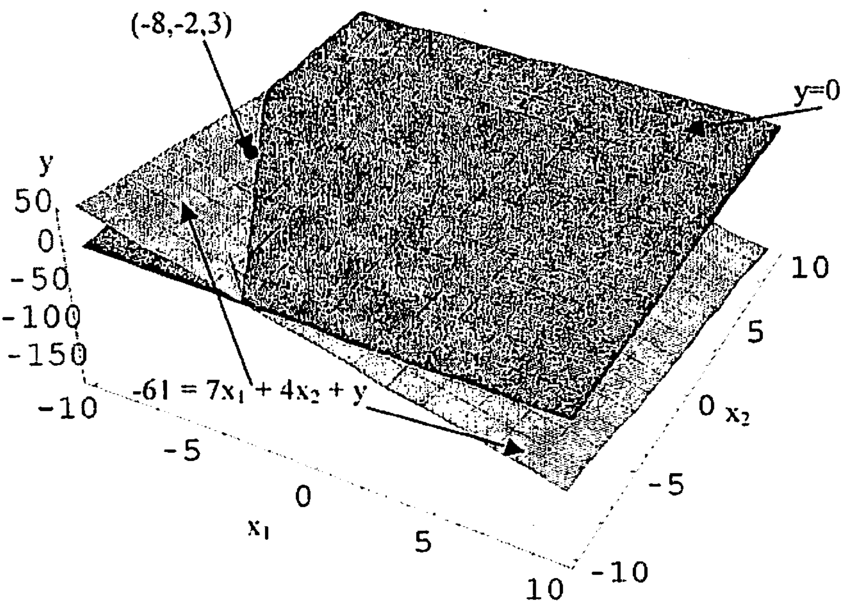

Figure 5

Continuing the generalization of division with multiple divisors, a dividend is now divided by three divisors. An example similar to the division used previously is 61 divided by $13, 7$, and 4. The equation that this division is based on would be $61 = 13q_1 + 7q_2 + 4q_3 + r$. There are an infinite number of ordered quadruples produced by this division. One such division is given in the equation $61 = 13(4) + 7(1) + 4(0) + 2$. When $q_1 = 0$, all lattice points contained in our two-divisor example are replicated in this set of ordered quadruples. The following are two examples of three-divisor division.

$$\begin{array}{r|r} & 4\\ & 1\\ & 0\\ \hline 13 & 61\\ 7 & \underline{52}\\ 4 & 9\\ & \underline{7}\\ & 2 \end{array} \qquad\qquad \begin{array}{r|r} & 0\\ & 8\\ & 1\\ \hline 13 & 61\\ 7 & \underline{56}\\ 4 & 5\\ & \underline{4}\\ & 1 \end{array}$$

There can be no geometric visualization to interpret three-divisor division, although there exist geometry in $\mathbb{R}_4$. Following the same substitution procedure as before we would produce the equation $61 = 13x1 + 7x2 + 4x3 + y$, which would be an equation of a hyperplane in $\mathbb{R}_4$. Since an actual representation of this geometry is impossible, we could find an alternative view of this division by writing the divisors and quotients as vectors. In out example, the vector representing the divisors would be in $\mathbb{R}_3$ and would be written as the following, $[13, 7, 4]$. Interestingly, we can show that this vector is orthogonal to the difference between any pair of vectors that represent quotients. Let $\cdot$ be the symbol for the dot product. If we have integers a, b_1, b_2 and b_3 with $b_1, b_2, b_3 > 0$, we know $a = b_1q_1 + b_2q_2 + b_3q_3 + r$ and $a = b_1q_1' + b_2q_2' + b_3q_3' + r$ for some integers $q_1, q_2, q_3, q_1', q_2', q_3'$, and r, with $0 \leq r < min\{b_1, b_2, b_3\}$, by a generalization of the division algorithm. By subtracting these equations we produce $0 = b_1(q_1 - q_1') + b_2(q_2 - q_2') + b_3(q_3 - q_3')$. This is the same as writing $[b_1, b_2, b_3] \cdot [q_1 - q_1', q_2 - q_2', q_3 - q_3']$. Hence, the vector $[b_1, b_2, b_3]$ is orthogonal to the vector representing the difference between the vectors $[q_1, q_2, q_3]$ and $[q_1', q_2', q_3']$.

Acknowledgments. A special debt of gratitude is owed to Dr. Kishor Shah who guided the direction of this project from start to finish. Another big thanks goes out to Dr. Les Reid, Dr. Richard Belshoff, Dr. John Kubicek, and the remainder of the mathematics faculty at Southwest Missouri State University. Without the help of these individuals this paper certainly would not have come to completion.

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Foci of Conic Sections

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Presented at the 2001 National Convention.

Introduction

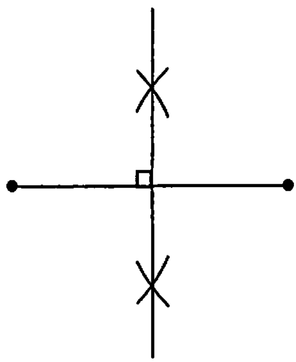
A problem in the Mathematics Magazine [1], posed by Kevin Ferland and Florian Luca, asks if one can construct the foci and major lines for any given conic section. This paper will show the construction for the foci of an ellipse, for the focus and directrix of a parabola, and for the foci and asymptotes of a hyperbola, as well as discuss tangent lines to these conic sections.

Basis Constructions:

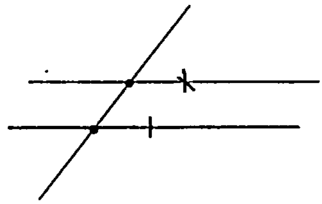
There are some basic constructions that we will be using:

Construction of:

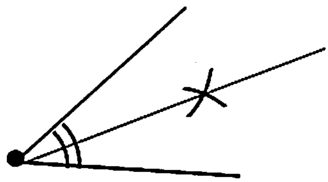
- Perpendicular Bisector (& Midpoint)



•Parallel lines



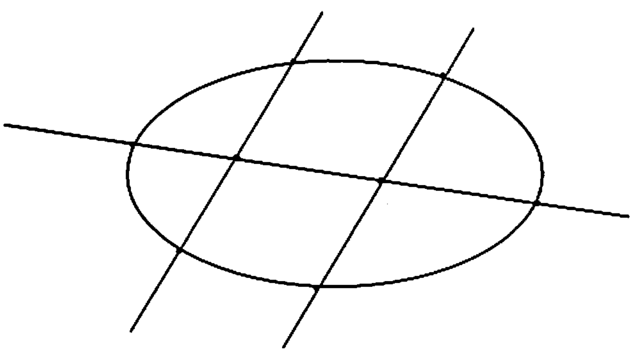
•Angle Bisector



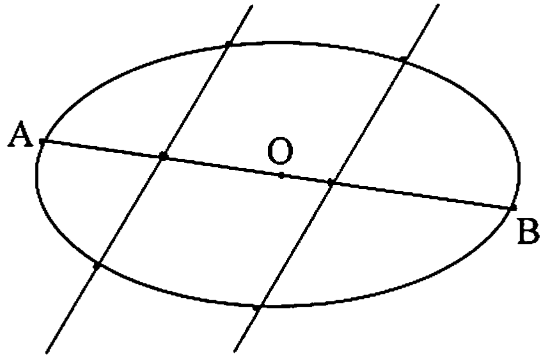
For more detailed explanations of these, see any basic geometry book, e.g. [2], [3].

Ellipse:

Given an ellipse, can the foci be constructed? Before we find the foci, it might be nice if we could find the center of the ellipse. This can be can be done by constructing two parallel lines intersecting the ellipse.



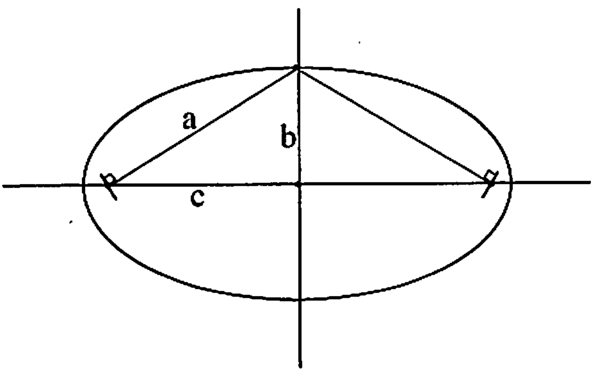
The line connecting the midpoints of the intersecting line segments goes through the origin.



To show this, let the line $y = mx+b$ intersect the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ at two distinct points (x_1, y_1) and (x_2, y_2) and M be the midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Then $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1 = \frac{x_2^2}{a^2} + \frac{y_2^2}{b^2}$ and $y_1 - mx_1 = b = y_2 - mx_2$, which gives us $\frac{x_1^2-x_2^2}{a^2} = \frac{y_2^2-y_1^2}{b^2}$ and $y_2 - y_1 = m(x_2 - x_1)$. Thus $\frac{(x_1x_2)(x_1-x_2)}{a^2} = \frac{(y_2+y_1)m(x_2-x_1)}{b^2}$, which yield $\frac{(x_1+x_2)}{a^2} = -\frac{(y_1+y_2)m}{b^2}$, and we see that the midpoint M lies on the line $y = -\frac{b^2}{a^2m}x$ through the origin, as do all midpoints of the parallel chords with slope m.

Hence the midpoint of line segment AB is the center O.

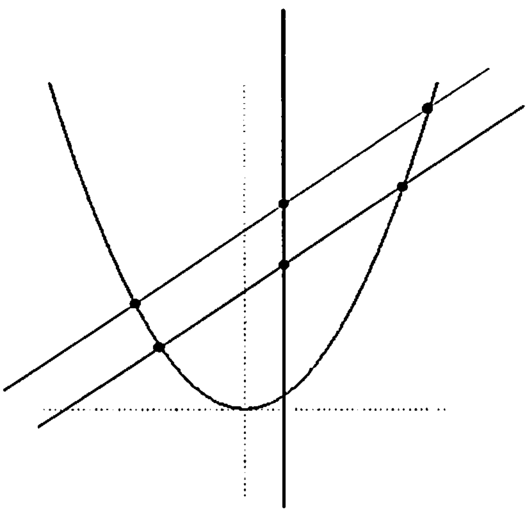
Once we know the center of the ellipse we can construct the major and minor axes using the symmetry of our ellipse. First construct a circle centered at O intersecting the ellipse. Then connect the points of intersection and find the midpoints. Connecting these midpoints creates the major and minor axes.



The foci are located at $(\pm c, 0)$, with $a^2 = b^2 + c^2$. So we construct a circle with the center at $(0, b)$ and the radius a to find the foci.

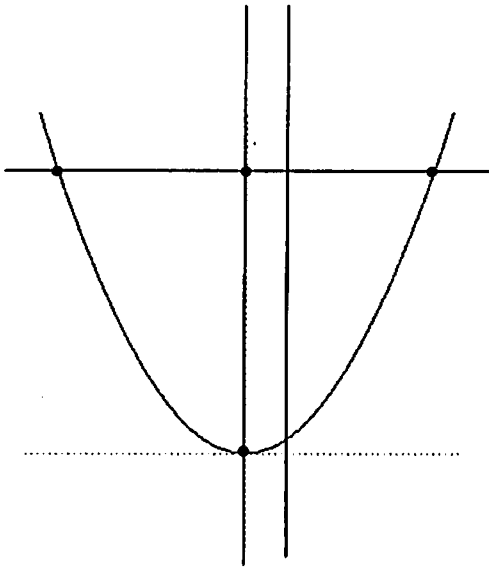
Parabola:

Can the focus and directrix of a given parabola be constructed? Before we can find the focus and directrix it would be helpful to find the vertex and axes of symmetry of the parabola. We can begin to do this by again considering two parallel lines that intersect the parabola. It turns out that the line connecting the midpoints is parallel to the axis of symmetry.

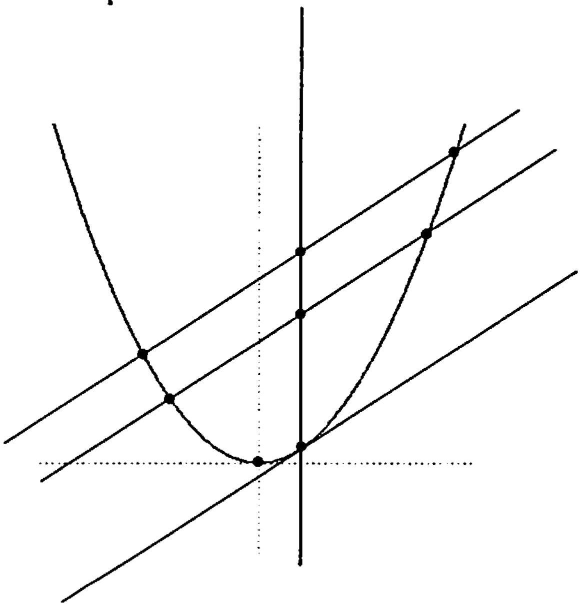


To see this, let the line $y = mx+b$ intersect the parabola $y = ax^2$ at two distinct points (x_1, y_1) and (x_2, y_2) and M be the midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Then $y_2 - y_1 = a\left(x_2^2 - x_1^2\right)$ and $y_2 - y_1 = m\left(x_2 - x_1\right)$. Hence we find that $a\left(x_2 + x_1\right)\left(x_2 - x_1\right) = m\left(x_2 - x_1\right)$ or $a\left(x_2 + x_1\right) = m$, which means that the midpoint M lies on the vertical line $x = \frac{m}{2a}$, as do all midpoints of the parallel chords with slope m.

Next we can construct a line perpendicular to the line connecting the midpoints. This line intersects the parabola at two points. Because of the symmetry of the parabola, the axis of symmetry is the perpendicular bisector of this chord. Where this line intersects the parabola is of course the vertex.

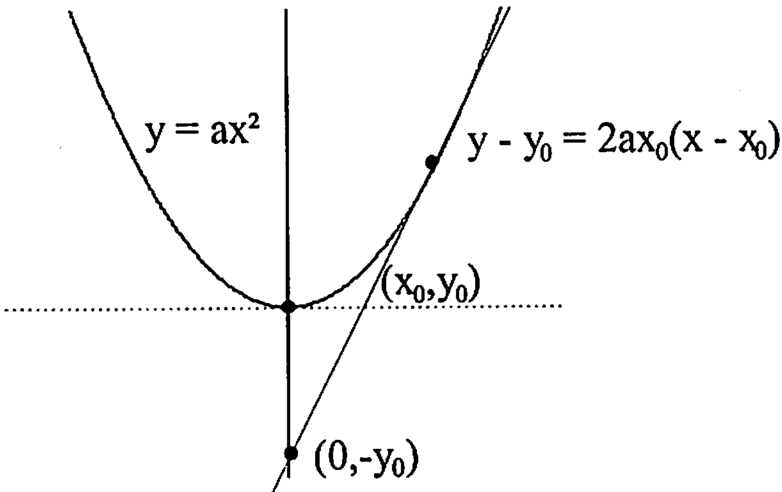


Next we construct a line tangent to the parabola, using for example the previous set of parallel lines. Where the line connecting the midpoints intersects with the parabola, we construct a line parallel to the others, this line is tangent to the parabola.



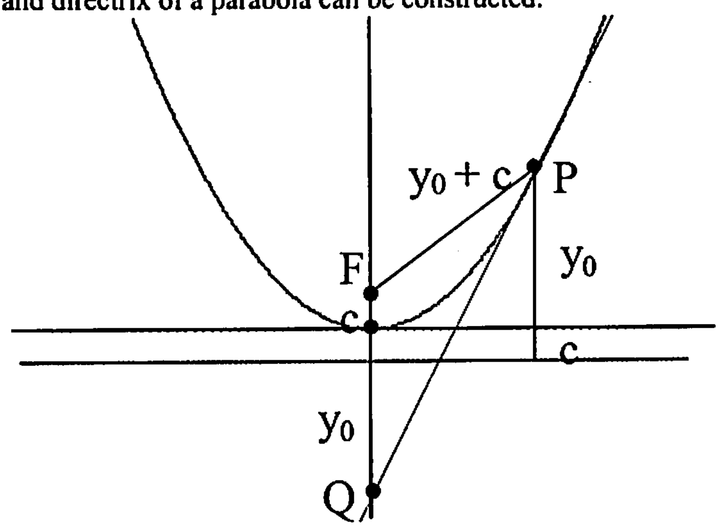
How can we use a tangent line to find the focus of the parabola? Note that the tangent line to $y = ax^2$ at $P = (x_0, y_0)$ is given by $y - y_0 = 2a$ $x_0(x - x_0)$. Note that the y-intercept Q of this line is $(0, -y_0)$. Hence if the focus $F - (0, c)$, then:

$|QF| = c + y_0 = distance(P, directrix) = |FP|$



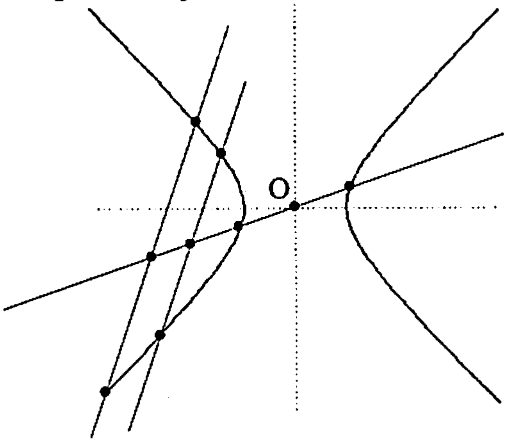
Thus F is the intersection of the major axis and the perpendicular bisector of QP.

From that it is easy to construct the directrix. We copy the length from the vertex to the focus along the major axis, and construct a line through that point perpendicular to the major axis. So we have shown how the focus and directrix of a parabola can be constructed.

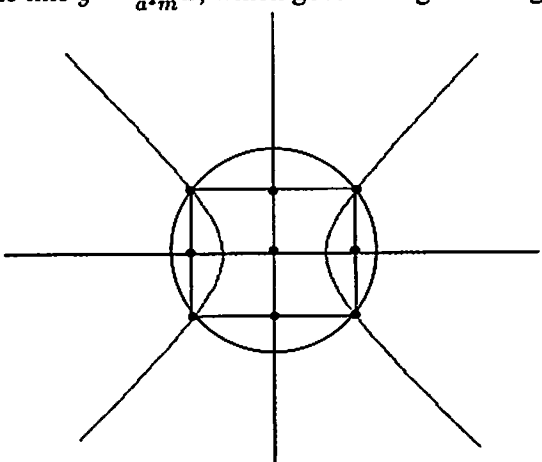


Hyperbola:

Given any hyperbola can the foci and asymptotes be constructed? To begin, we need to find the center. As in the case of the ellipse, if we construct parallel lines intersecting the hyperbola, and connect the midpoints, the connecting line goes through the center.

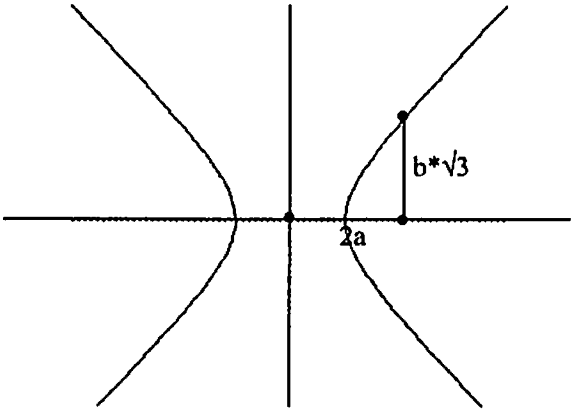


The proof is almost identical to the proof in the case of an ellipse. For an hyperbola, one finds that the midpoints of the parallel chords with slope m all lie on the line $y = \frac{b2}{a^2m}x$, which goes through the origin.



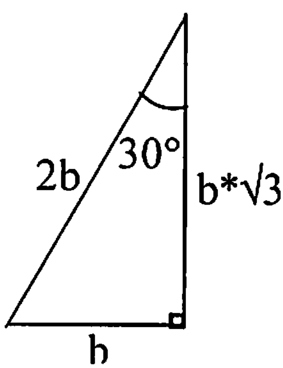
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

We can then construct the midpoint of the connecting line segment to determine the center. Then to find the major and minor axes, we construct a circle using the center as the midpoint. Then connect the points of intersection and find the midpoints. Connecting these midpoints creates the major and minor axes. Again we have used the symmetry of a hyperbola nicely.

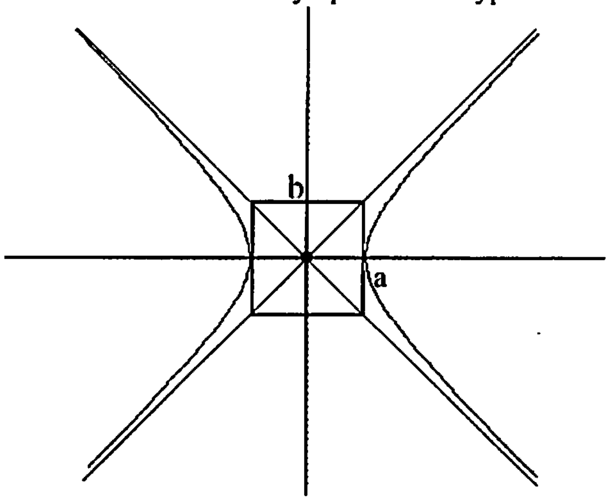


In order to construct the foci, we use the relationship $c^2 = a^2 + b^2$. Since we know a, if we can find b, we know c. If we let $x = 2a$, then $y = b\sqrt{3}$ (since $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$).

Then we construct a $30 - 60 - 90$ triangle with $b * \sqrt{3}$ as one of the legs. To do this we need to construct a $60°$ angle. This can be done by constructing an equilateral triangle. Then copy the angle to we get a $30 - 60 - 90$ triangle with the legs being b and $b * \sqrt{3}$, and the hypotenuse $2b$.

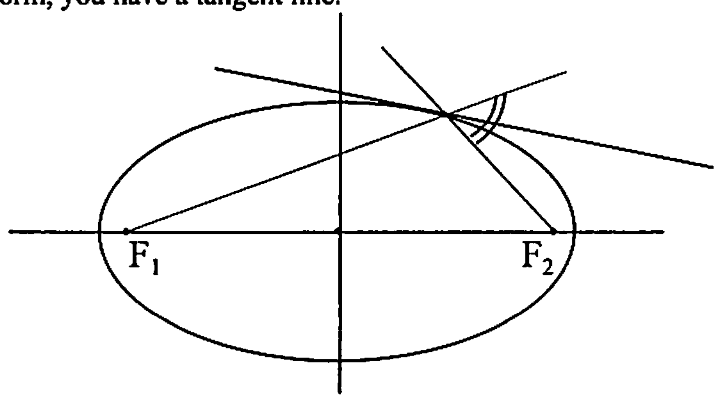


Having found b, we can construct the asymptotes with slopes $\pm b/a$, and find the foci. Then we know the hypotenuse with legs a and b is c from $c^2 = a^2 + b^2$. So if we draw a circle with center at the origin and a radius of c, where it intersects the major axes are the foci. We have shown how to construct the foci and the asymptotes of a hyperbola.

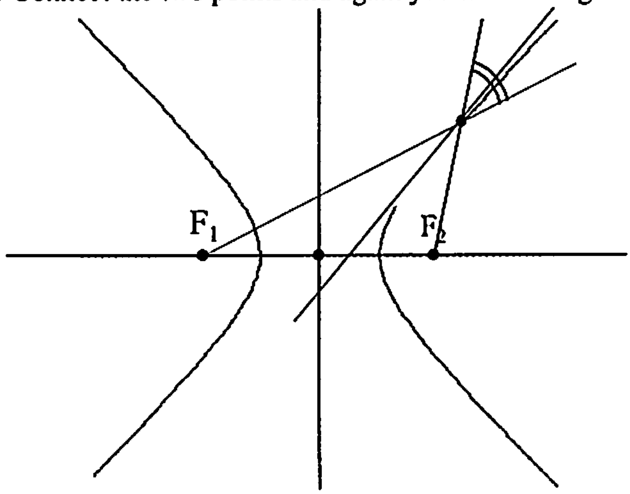


Tangents:

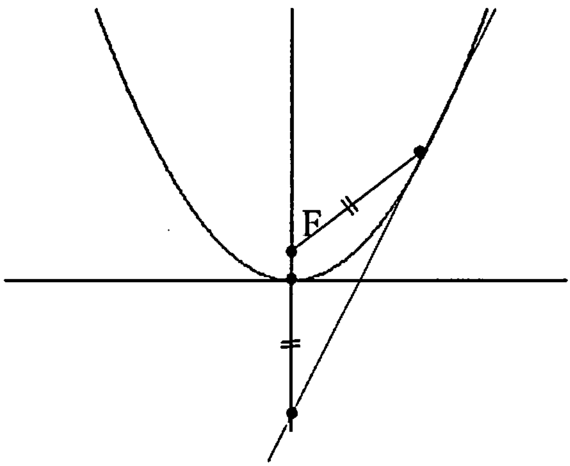
Once we have found the foci of the conic sections, we can easily construct tangent lines. For example, for an ellipse if you connect the two foci with any point on the ellipse and then take the angle bisector of the angle they form, you have a tangent line.

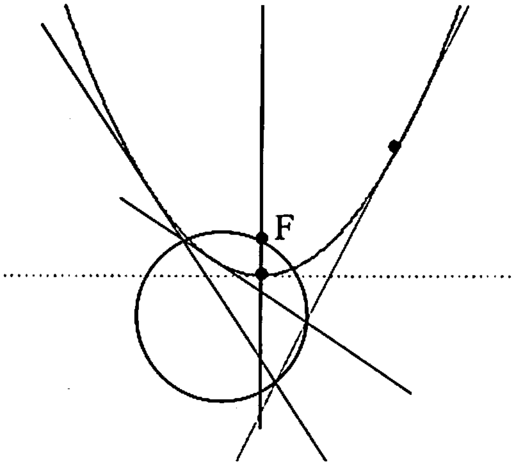


You do a similar thing for a hyperbola. For a parabola, take a point on the parabola, and construct a point, on the y-axis the same distance from the focus. Connect the two points and again you have a tangent line.



Another interesting fact about parabolas and tangent lines is that if you connect three tangent lines so they form a triangle, the circumscribed circle goes through the focus. Proofs for these interesting facts are left to the reader.





Conclusion

This paper shows that given any conic section it is possible to construct its foci and major axes. With these known we can then construct tangent lines rather easily.

Acknowledgments. I would like to thank Dr. Boerkoel for helping me throughout the entire process of writing this paper. You continue to inspire me.

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KME Website

The national *Kappa Mu Epsilon* homepage has moved. It is now housed at Eastern Kentucky University. The new URL is www.kme.eku.edu

Our thanks go to the folks at Eastern Kentucky University for hosting our website and Kirk Jones - the new webmaster. Also we want to thank Richard Lamb, Arnie Hammel and Carey Hammel for all the work they put into creating and maintaining our website up to now.

Quadratic Function and Irrational Numbers

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Besides the common examples of irrational numbers like $\pi, e, \sqrt{2}, \ln 2$, etc., there are some *artificial* irrational numbers as follows:

$$\alpha_1 = 0.101001000100001000001......,$$

$$\alpha_2 = 0.1011001110001111000011111.......$$

Let $x = 0.a_1a_2a_3...a_n...$ with $a_n \in \{0, 1, 2, ..., 9\}$ be an infinite decimal. Then a sufficient condition for x to be an irrational number is if x has the following two properties: (i) There is a digit a such that for any given positive integer k, there is a positive integer n_k such that $a_{n_k+1} = a_{n_k+2} = ... = a_{n_k+k} = a$. (ii) For any given positive integer k, there is a positive integer $m_k > k$ such that $a_{m_k} \neq a$.

The above two examples have these two properties. For number α_1, $a = 0$ and $a_{m_k} = 1$; for number α_2,we have $a = 0$ or 1 while $a_{m_k} = 1$ or 0 respectively.

Look at the following numbers:

$$\alpha_3 = 0.1234567891011121314151617181920......,$$

$$\alpha_4 = 0.135791113151719212325272931 3335......,$$

$$\alpha_5 = 0.2468101214161820222426283032 3436.......$$

They are all irrational numbers. Indeed, for α_3, α_4, since the number $10^n + 1$ will appear for any positive integer n, we have $a = 0$ and $a_{m_k} = 1$ for both α_3, α_4. As for α_5, the number $10^n + 2$ will appear, we have $a = 0$ and $a_{m_k} = 2$.

In this note we will prove the following number ξ_1 is irrational.

$$\xi_1 = 0.149162536496481100121144169196225.......$$

The number ξ_1 is formed by concatenation of squares of the sequence of natural numbers.

Let us consider $(10^n + 1)^2 = 10^{2n} + 2 \cdot 10^n + 1$. The first $n - 1$ digits of this number is 10...0 with $n - 2$ zeroes. Therefore, when $n > k + 2$, we can have k consecutive zeroes, i.e., $a = 0$. The first digit 1 of $(10^n + 1)^2$ can be taken as a_{m_k}.

To generalize this notion, consider

$$f(x) = Ax^2 + Bx + C$$

a quadratic function with positive interger coefficient. We can prove that the following number ξ_f is an irrational number.

$$\xi_f = 0.f(1)f(2)f(3)...f(n)....$$

If for instance $f(x) = 5x^2 + 23x + 7$, $f(1) = 35, f(2) = 73, f(3) = 121, f(4) = 179, f(5) = 247$, and we claim the following ξ_2 is an irrational number

$$\xi_2 = 0.3573121179247......$$

In the general case, since A, B, C are three given positive intergers, we may suppose that there is a fixed positive integer h such that $A < 10^h$, $B < 10^h$, $C < 10^h$. Let n be an arbitrarily large positive number . Consider

$$f(10^{hn}) = A \cdot 10^{2hn} + B \cdot 10^{hn} + C.$$

The first h digits of the positive integer $f(10^{hn})$ may not be all zeros. Since $B \cdot 10^{hn} + C < 10 \cdot 10^{hn} + 10^h < 10^{h(n+2)}$, the last $h(n+2)$ digits of $f(10^{hn})$ may not all be zeroes. But the middle $2hn - h - h(n+2) = h(n-3)$ digits are exactly all zeroes. Thus we may let $a = 0$ and a_{m_k} be the first digit of the positive integer A, then we can see that ξ_f is an irrational number.

The readers can generalize this result to prove that $\xi_f = 0.f(1)f(2)...f(n)...$ is an irrational number, where $f(x)$ is a polynomial with positive integer coefficients. So far we do not know whether these *artificial* irrational numbers are algebraic numbers. It is worth investigating how one can find algebraic irrational numbers by the help of polynomials with positive integer coefficients.

Acknowledgement. The authors are sincerely thankful to their advisor Dr. Jingcheng Tong and referees of this paper.

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The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before January 1, 2003. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 2003 issue of *The Pentagon*, with credit being given to the student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

Problems 555-559

Problem 555. Proposed by Maureen Cox and Albert White, St. Bonaventure University, St. Bonaventure, New York.

Define the sequence $\{a_n\}$ by $a_0 = 0, a_1 = 1, a_n = pa_{n-2} + (-1)^n$. If $-1 < p < 1$, does the series $\sum_{n=0}^{\infty} a_n$ converge?

Problem 556. Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Revolve the curve $y = x^{1/p}$ for $x, y \geq 0$ about the y axis where p is fixed and $p > 0$. Also revolve $y = 1/n$ where $n = 1, 2, 3...$ about the y axis. Let V_n denote the volume of each solid generated between $y = x^{1/p}$ and $y = 1/n$, for $n = 1, 2, 3, ...$ Find the conditions on p such that $\sum_{n=0}^{\infty} V_n$converges.

Problem 557. Proposed by Pat Costello, Eastern Kentucky University, Richmond, Kentucky.

From Pascal's Triangle we know that the maximum term in the sequence

$$\binom{3n}{0}, \binom{3n}{1}, \binom{3n}{2}, \ldots, \binom{3n}{3n}$$

occurs in the middle of the sequence. Find the maximum term in the sequence

$$\binom{3n}{0} * 2^{3n}, \binom{3n}{1} * 2^{3n-1}, \binom{3n}{2} * 2^{3n-2}, \ldots, \binom{3n}{3n} * 2^{3n-3n}$$

Problem 558. Proposed by Robert Rogers, SUNY College at Fredonia, Fredonia, New York.

Given a quintic polynomial $f(x)$ with exactly one inflection point at $x = 0$, one maximum at $x = -1$ and one minimum at $x = m$, what is the maximum value m can attain? [Note: For a cubic polynomial, $m = 1$.]

Problem 559. Proposed by the editor.

Let S_n and T_n denote perfect squares, each having exactly n digits and such that $S_n + R_n = T_n$ where $R_n = \frac{10^n - 1}{9}$, n is a positive integer > 1 and each digit of T_n equals 1 plus the corresponding digit of S_n. Furthermore the left most digit of S_n is not zero. For example, for $n = 4$ we have $S_n = 2025 = 45^2$ and $T_n = 56^2$ with $R_n = 1111$.

(a) Show that if n is an even integer, one can always find appropriate values for S_n and T_n.

(b) If n is an odd integer < 15 find all integers n and the corresponding values of S_n and T_n which satisfy the conditions of the problem.

Please help your editor by submitting problem proposals.

Solutions 545 – 549

Problem 545. Proposed by the editor.

The sequences $A_k = 8 * 10^{2k} + 4 * 10^k$, $B_k = 8 * 10^{2k} + 4 * 10^k + 1$ and $C_k = 4 * 10^k + 1$ where k is a positive integer generate an infinite set of Heronian triangles in which the area of the triangle is 10^k times the perimeter of the triangle. These sequences allow automatic generation of an infinite list of these particular Heronian triangles through the use of a calculator or computer. Find another set of similar sequences which has the same property; i.e. the sequences generate an infinite set of Heronian triangles in which the area of the triangle is 10^k times the perimeter of the triangle and $B_k - A_k = 8$.

Solution by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.

Note that since $A_k^2 + C_k^2 = B_k^2, A_k, C_k$ and B_k form a right triangle. Thus we seek another right triangle having sides A_1, C_1 and B_1 such that the right angle is $B_1, B_1 - A_1 = 8$ and Area $= 10^k P$ where P is the perimeter of triangle A_1, C_1 and B_1. The area and perimeter of triangle A_1, C_1 and B_1 are given by $\frac{A_1 C_1}{2}$ and $A_1 + C_1 + B_1$ respectively. Thus we must solve the equations

$$A_1^2 + C_1^2 = B_1^2 \quad (1)$$

$$B_1 = A_1 + 8 \quad (2)$$

$$\frac{A_1 C_1}{2} = 10^k (A_1 + C_1 + B_1) \quad (3)$$

Combining equations (1),(2) and (3) yields $2 \cdot 10^k (2A_1 + 8 + C_1) = C_1 A_1$ or

$$A_1 = 2 \cdot 10^k \left(\frac{C_1 + 8}{C_1 - 4 \cdot 10^k} \right) \quad (4)$$

Combining equations (1) and (2) yields

$$C_1^2 = 16 (A_1 + 4) \quad (5)$$

Combining equations (4) and (5) yields $2 \cdot 10^k \left(\frac{C_1 + 8}{C_1 - 4 \cdot 10^k} \right) = \frac{C_1^2 - 64}{16}$ which reduces to the quadratic equation

$$C_1^2 - 4 \cdot 10^k C_1 - 32 \cdot 10^k - 64 = 0 \quad (6)$$

The only positive root of equation (6) is $C_1 = 4 \cdot 10^k + 8$. Then $A_1 = 10^{2k} + 4 \cdot 10^k$ and $B_1 = 10^{2k} + 4 \cdot 10^k + 8$. The perimeter of this triangle is $20 \cdot 10^{2k} + 12 \cdot 10^k + 16$ and the area is $\frac{A_1 C_1}{2} = \frac{(10^{2k} + 4 \cdot 10^k)(4 \cdot 10^k + 8)}{2} = 2 \cdot 10^{3k} + 12 \cdot 10^{2k} + 16 \cdot 10^k = 10^k (A_1 + C_1 + B_1)$.

Problem 546. Proposed by Adrian C. Keister, Grove City College, Grove City, Pennsylvania.

Prove or disprove the following theorem:
Suppose a function f is three times differentiable on the interval (a, b).

Suppose there exists a point c in (a, b) such that $f''(c) = 0$ but $f'''(c)$ is not equal to zero. Then c is an inflection point of f.

Solution by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri.

We claim that $(c, f(c))$ has to be an inflection point. If not, then there exists an interval $I = (c-\epsilon, c+\epsilon) \subset (a, b)$ where $f''(x)$ does not change sign. Assume without loss of generality that $f''(x) \geq 0$ on I. Now since $f'''(c)$ exists,
$\lim_{x \to c^-} \frac{f''(x)-f''(c)}{x-c} = \lim_{x \to c^-} \frac{f''(x)}{x-c} = f'''(c) \leq 0$. Similarly
$\lim_{x \to c^+} \frac{f''(x)-f''(c)}{x-c} = \lim_{x \to c^+} \frac{f''(x)}{x-c} = f'''(c) \geq 0$.It follows that
$f'''(c) = 0$ in contradiction with the assumption of the problem. The claim follows.

Also solved by the proposer.

Problem 547. Proposed by the editor.

Evaluate the sum

$$\cos 9^\circ + \cos 49^\circ + \cos 89^\circ + \cos 129^\circ + \cos 169^\circ$$
$$+ \cos 209^\circ + \cos 249^\circ + \cos 289^\circ + \cos 329^\circ$$

A solution which does not use a calculator or computer is preferred.

Solution by Jimmy Kerl, Student, Northeastern State University, Tahlequah, Oklahoma.

Group the nine angles into three groups of three angles as follows: $\{9^\circ, 129^\circ, 249^\circ\}$, $\{49^\circ, 169^\circ, 289^\circ\}$ and $\{89^\circ, 209^\circ, 329^\circ\}$. Each group can be written in the form $\{X^\circ, X+120^\circ, X+240^\circ\}$ with $X = 9^\circ, 49^\circ$ and 89° respectively. Using the standard formula for $\cos(A+B)$ we have
$\cos X + \cos(X+120^\circ) + \cos(X+240^\circ)$
$= \{\cos X + \cos X \cos 120^\circ - \sin X \sin 120^\circ\}$
$\quad + \{\cos X \cos 240^\circ - \sin X \sin 240^\circ\}$
$= \cos X + \{(-\frac{1}{2}) \cos X - \frac{\sqrt{3}}{2} \sin X\} + \{-\frac{1}{2} \cos X + \frac{\sqrt{3}}{2} \sin X\} = 0.$
Therefore the sum $\cos X + \cos(X+120^\circ) + \cos(X+240^\circ) = 0$ for each group. Thus the total sum is also zero.

Solution by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.

Let

$$S = \sum_{k=0}^{8} \cos(9^\circ + 40k^\circ) \tag{1}$$

Multiplying both sides of this equation by $2\sin 20^\circ$ we have

$$2\sin 20^\circ \cos S = \sum_{k=0}^{8} (2\sin 20^\circ)\cos(9^\circ + 40k^\circ) \qquad (2)$$

Repeatedly using the identity

$$2\sin A\cos B = \sin(A-B) + \sin(A+B)$$

(2) becomes

$$2\sin 20^\circ S = \sum_{k=0}^{8}(\sin 20^\circ - (9^\circ + 40k^\circ)) + \sum_{k=0}^{8}(\sin 20^\circ + (9^\circ + 40k^\circ))$$

$$= (\sin 20^\circ - 9^\circ) + \sin(20^\circ + 320^\circ) = \sin 11^\circ + \sin 349^\circ \qquad (3)$$

Using the identity

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

in (3) shows that

$$\sin 11^\circ + \sin 349^\circ = 2\cos(-169^\circ)\sin(360^\circ) = 0$$

Hence $2\sin 20^\circ S = 0$ so $S = 0$.

Solution by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri.

Note that for any angle σ,

$$\sum_{k=0}^{8} e^{i(\theta+40^\circ)} = e^{i\theta}\sum_{k=0}^{8} e^{ik(40^\circ)} = e^{i\theta}\left(\frac{e^{0^\circ i} - e^{360^\circ i}}{1 - e^{40^\circ i}}\right) = 0 \qquad (1)$$

By equating the real and imaginary parts of (1) yields

$$\sum_{k=0}^{8}\cos(\theta + 40^\circ k) = 0 \qquad (2)$$

and

$$\sum_{k=0}^{8}\sin(\theta + 40^\circ k) = 0 \qquad (3)$$

respectively. The desired result follows by taking $\sigma = 9^\circ$ in (2).

Also solved by: Nada Bennett and Ericka Lund, students at California State University, Fresno, California and Albert White, St. Bonaventure University, St. Bonaventure, New York.

Editor's Comment: The other solutions involved the use of trigonometric identities in various ways. While this is a variation on an old problem, it is interesting to see the variety of methods used by our featured solvers. For another method of solution, see problem 213 in [1] in which one notices that the angles describe the vertices of a regular polygon of n sides and notice that the vector sum of the radii from the center to the vertices is zero. Note the relationship with complex numbers and the roots of the equation $x^n - 1 = 0$. This is a geometric interpretation of the solution by Russell Euler and Jawad Sadek. For another approach involving directed line segments, see problem 110 in [2].

1. Charles W. Trigg, Mathematical Quickies, Dover Publications, Inc., New York, 1985, pp. 58, 176-177

2. Ibid., pp 31, 126.

Problem 548. Proposed by Jose Luiz Diaz, Universitat Politecnica de Catalunya, Terrassa, Spain.

Let n be a positive integer. Prove that

$$2F_{n+2} < \frac{F_n^2}{F_{n+2}} + \frac{F_{n+1}^2}{F_n} + \frac{F_{n+2}^2}{F_{n+1}}$$

where F_n is the n^{th} Fibonacci number. That is, $F_0 = 0, F_1 = 1$ and for $n \geq 2, F_n = F_{n-1} + F_{n-2}$.

Solution by Carl Libis, Assumption College, Worcester, Massachusetts.

Using $F_{n+2} = F_{n+1} + F_n$ and simplify to obtain

$$\frac{F_n^2}{F_{n+2}} + \frac{F_{n+1}^2}{F_n} + \frac{F_{n+2}^2}{F_{n+1}} - 2F_{n+2}$$

$$= \frac{F_n^3 F_{n+1} + F_{n-1}^3 F_{n+2} + F_n F_{n+2}^3 - 2F_n F_{n+1} F_{n+2}^2}{F_n F_{n+1} F_{n+2}}$$

$$= \frac{F_n^3 F_{n+1} + F_{n+1}^3 (F_{n+1} + F_n) + F_n (F_{n+1} + F_n)^3 - 2F_n F_{n+1} (F_{n+1} + F_n)^2}{F_n F_{n+1} F_{n+2}}$$

$$= \frac{F_n^4 + 2F_n^3 F_{n+1} - F_n^2 F_{n+1}^2 + F_{n+1}^4}{F_n F_{n+1} F_{n+2}}$$

$$= \frac{2F_n^3 F_{n+1} + (F_n^2 - F_{n+1}^2)^2 + F_n^2 F_{n+1}^2}{F_n F_{n+1} F_{n+2}} > 0.$$

Thus, $\frac{F_n^2}{F_{n+2}} + \frac{F_{n+1}^2}{F_n} + \frac{F_{n+2}^2}{F_{n+1}} > 2F_{n+2}$.

Also solved by: Matthew Guilfoyle, Susquehanna University, Selgrove Pennsylvania and the proposer.

Problem 549. Proposed by Bryan Dawson, Union University, Jackson, Tennessee.

A soft rain falls vertically at a speed of 12 miles per hour. You are in your car stopped at a stoplight. As the light turns green, you accelerate to 45 miles per hour and notice, of course, that more water now hits the windshield. If your windshield is inclined 60 ° from the horizontal, what is the ratio of the water hitting your windshield at 45 miles per hour compared to water hitting your windshield at rest ignoring any possible aerodynamic effects of the vehicle?

Solution by Jennifer King and Scott Tobler (jointly), students, California State University, Fresno, California.

Let A denote the area of the windshield. The volume of water hitting the windshield at any one time can be found by multiplying the area of the windshield A by the velocity vector of the falling rain. If the car is at rest, the volume of rain hitting the windshield multiplied by the velocity vector of the rain with respect to the angle of the windshield; i.e. 60 °.

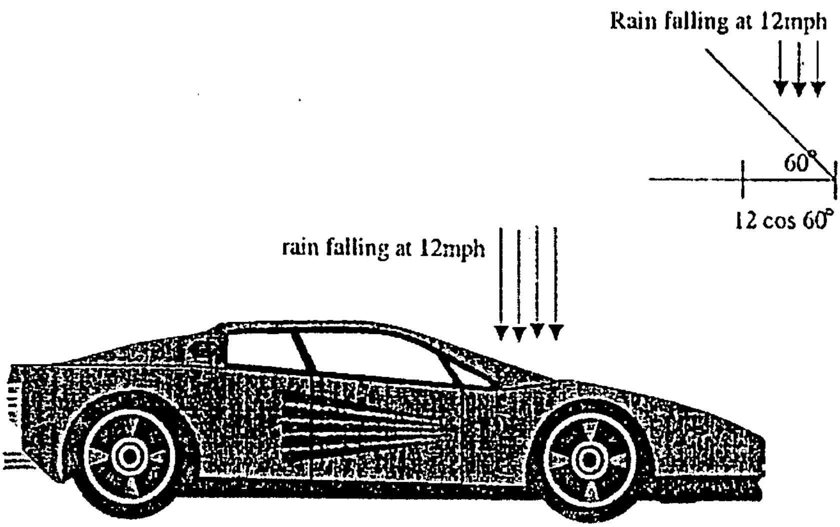

Figure 1

Thus the volume of water hitting the windshield when the car is at rest is $(12\cos 60°) \cdot A = 6A$.

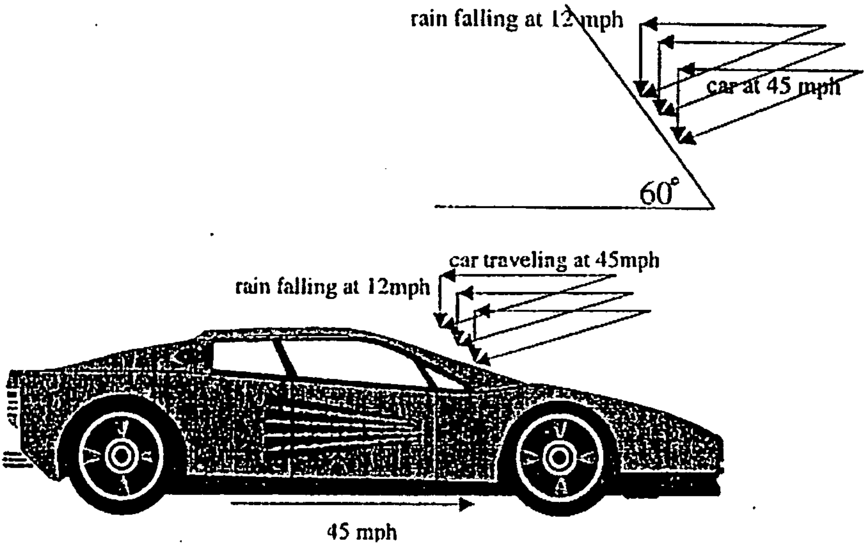

Figure 2

The velocity vector of the rain hitting the window when the car is moving at 45 mph is given by $12\cos 60^\circ + 45\sin 60^\circ$. Thus the volume of water hitting the windshield when the car is moving at 45 mph is given by $(12\cos 60^\circ + 45\sin 60^\circ)\cdot A = (6 + \frac{45}{2}\sqrt{3})\cdot A$. Hence the desired ratio is given by $(12\cos 60^\circ + 45\sin 60^\circ)\cdot \frac{A}{12\cos 60^\circ)\cdot A} = \frac{6+\frac{45}{2}\sqrt{3}}{6} \approx 7.495$.

Also solved by the proposer.

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Thank You, Referees!

The editor wishes to thank the following individuals who refereed papers submitted to *The Pentagon* during the last two years.

Gerald Bergum
South Dakota State University
Brookings, South Dakota

Duane M. Broline
Eastern Illinois University
Charleston, Illinois

Jeffrey Clark
Elon College
Elon College, North Carolina

Deborah Denvir
Marshall University
Huntington, West Virginia

Vincent Dimiceli
Oral Roberts University
Tulsa, Oklahoma

Timothy Flood
Pittsburg State University
Pittsburg, Kansas

Marc Goulet
University of Wisconsin - Eau Claire
Eau Claire, Wisconsin

Chenglie Hu
Fort Hays State University
Fort Hays, Kansas

Lyndell Kerley
East Tennessee State University
Johnson City, Tennessee

Larry Kurtz
University of Montevallo
Montevallo, Alabama

William R. Livingston
Missouri Southern State College
Joplin, Missouri

David Neel
Truman State University
Kirksville, Missouri

Rosalie Nichols
Fort Hays State University
Hays, Kansas

Michael Pearson
Mississippi State University
Mississippi State, Mississippi

Mohammad Riazi-Kermani
Fort Hays State University
Fort Hays, Kansas

Andrew Rockett
C.W. Post Campus of Long Island University
Brookville, New York

Thomas J. Sharp
State University of West Georgia
Carrollton, Georgia

Andrew Talmadge
Dillard University
New Orleans, Louisianna

Don Tosh
Evangel College
Springfield, Missouri

Lisa Townsley
Benedictine University
Lisle, Illinois

Also thanks to the many other individuals who volunteered to serve as referees but were not used during the past two years. Referee interest forms will again be sent by mail in the near future so that interested faculty may volunteer. If you wish to volunteer as a referee, feel free to contact the editor (see page 2) to receive a referee interest form.

Kappa Mu Epsilon News

Edited by Connie Schrock, Historian

News of chapter activities and other noteworthy KME events should be sent toschrockc@emporia.edu or to

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Chapter News

AL Beta
University of North Alabama, Florence 26 actives

AL Zeta Chapter President – Molly Gibson
Birmingham-Southern College 12 actives, 5 associates

Other fall 2001 officers: Auntara De, Vice President; Jennifer Turner, Secretary; Prof Mary Jane Turner, Corresponding Secretary.

The inititation was held December 5 for new members. Refreshments were served afterwards. Some members of the chapter will be presenting in Atlanta at the MAA/AMS Joint Meeting. There will be another initiation in the Spring.New Initiates – Barrett Patrick Cary, Isaac James Dooley, Yuliya A. (Julia) Kedrova, Oleathia, Elizabeth Leonard, Ellen Wright Segrest, Matthew Phillips Carter

CA Gamma Chapter President—Sheryl Cruz
Polytechnic State University, Obispo 14 actives, 5 associates

Other fall 2001 officers: Cassandra Fallscheer, Vice President; Natalie Schmitz, Treasurer; Jonathon Shapiro, Corresponding Secretary.

CO Beta Chapter President – Heather Barker
Colorado School of Mines 10 actives, 8 associates

Other fall 2001 officers: Kim Fleming, Vice President; Sean Campbell, Secretary; Aaron Schock, Treasurer; Ardel Boes, Corresponding Secretary.

CO Delta Chapter President—Ronald Mikluscak

Mesa State College, Grand Junction 25actives, 18 associates

Other fall 2001 officers: Thela Morales, Vice President; Wendy Serve, Secretary; Amber Sales, Treasurer; Kim Schneider, Corresponding Secretary.

In September, twenty members and guests attended a potluck dinner help at Lincoln Park where initiates from last year were given their certificates and pins. In November thirty-four members, initiates, and guests attended the initiation ceremony held in the Campbell College Center. Eighteen new members were initiated into the Colorado Delta Chapter. Former Corresponding Secretary Donna Hafner gave a brief talk about the history of Kappa Mu Epsilon. In December, eleven members met to have pizza before finals. New Initiates: Ana L. Aguilar, Jason Arthur, Paula Bauer, Eric Donato, Marlinda Fansher, Brian McCutchan, Tro Miller, Josh Nelson, Jamie Porta, Margo Rettig, Ambra Scarlett, Steve Smith, Nicole Taylor, Brian Willer, Julie Lay, Daniel Crumly, Warren MacEvoy, Anne Spalding.

GA Delta Chapter President – Cheryl Beierschmitt

Berry College 11 actives, 5 associates

Other fall 2001 officers: Jason Buczyna, Vice President; Ryan Fox, Secretary; Sarah Tweedie, Treasurer; Ron Taylor, Corresponding Secretary.

New Initiates – John Matthew Elmore, Melinda Page Graham, Madoka Hasegawa, Jessica Ann Vihon

IA Alpha Chapter President—Brad Rolling

University of Northern Iowa 36 actives, 7 associates

Other fall 2001 officers: Mark D. Ecker, Corresponding Secretary.

New Initaites – Lisa Gourley, Amanda Jamison, Matthew Lockner, Matt Nikkel, Elizabeth Robertson

IA Delta Chapter President—Matthew Trettin

Wartburg College 40 actives

Other fall 2001 officers: Kristina Blasber, Vice President; Jesse Oltrogge, Secretary; Seth Roberson, Treasurer; Dr. August Waltmann, Corresponding Secretary.

The October meeting was used to plan the chapter's homecoming activities and future meeting topics. Dr. Augie Waltmann shared a mathematical magic activity for which members were encouraged to discover the math behind the magic. Possible committee structures, bring a new KME prospect to a meeting, and a math game were enjoyed at the November meeting. In December the chapter had a pizza party, heard that a preparation for grad-school testing session is being arranged for the spring, and enjoyed a Christmas Greeting puzzle. Members were encouraged to submit a paper for the North Central Regional to be held April 26-27, 2002. Eric Leisge presented a brief form of his paper on Burnside's Theorem.

IL Delta
University of St.Francis — 12 associates

IL Eta — Chapter President—Jessica Holdsworth
Western Illinois University — 12 actives, 9 associates

Other fall 2001 officers: Eric Snodgrass, Vice President; Sandra Jazarb, Secretary; Hong Tran Treasurer; Alan Bishop, Corresponding Secretary

IL Theta — Chapter President—Steven Lafser
Benedictine University — 20 actives, 11 associates

The Math Club/KME met several times this semester. Featured events included a trip to Navy Pier to see the water garden, a T shirt contest (top 10 reasons to study math at BU) and watching the NOVA video on Fermat's Last Theorem. The KME induction was held at a tea honoring academic achievements in mathematics.

IL Zeta — Chapter President—Christine Pellini
Dominican University — 23 actives, 0 associates

Other fall 2001 officers: Theresa Meshes, Vice President; Rhonda Jurinak, Secretary; Michelle Blaszak, Treasurer; Sarah N. Ziesler, Corresponding Secretary.

Illinois Zeta ran a mathematical pproblem contest every two weeks, with prizes being awarded for the best solutions. We also organized a lunch for all members and ran a snack break during finals week.

IN Delta — Chapter President—Katherine Kratochvil
University of Evansville — 40 actives, 13 associates

Other fall 2001 officers: Kara Leonard, Vice President; Cathleen Morales, Secretary; Mohammad Azarian, Treasurer; Dr. Joe A. Stickles, Jr., Corresponding Secretary

KS Alpha Chapter President—Tim Pierce

Pittsburg State University 20 actives, 30 associates

This spring the Kansas Alpha Chapter was very active. We had one of the largest initiations in our history. We initiated 30 new members this spring. Our members also attended and presented at the Kansas Section meeting of the Mathematical Association of America and the KME National Convention. Many of our members were also recognized for outstanding scholarship at the math department's annual awards banquet. Dr. McGrath and his wife Fran hosted the annual KME Ice-Cream Social to close out the semester.

KS Beta Chapter President—Leah McBride

Emporia State University 14 actives, 5 associates

Other officers: Mindy Baker, Thad Davidson. The Kansas Beta chapter continued with the graphing calculator service project to the students in mathematics classes. Two sessions were held: one for beginners and another for more advanced users. Handouts with the directions were created and the sessions were lead by the student members. Two papers are underway for the regional conference. New Initiates: Amanda Anstine, Scott Keltner, Mindy Baker, Elizabeth Ohmie, Ton Boerkoel, Brian Hollenbeck, Amy Praiswater, Brad Tennant

KS Epsilon Chapter President—Charlotte Bigler

Fort Hays State University 17 actives, 4 associates

Other fall 2001 officers: Michael Breckenridge, Secretary; Jeff Sadler, Corresponding Secretary.

September – We held a softball party and barbecue (the faculty whipped up on the students). We also elected officers. In October, we had a Halloween party involving a video scavenger hunt. In November, held the initiation of four new KME members. Finally in December we had Christmas party including a visit by Santa Clause. New Initiates: Charlotte Bigler, Randi Gilber, Michael Brechenridge.

KS Gamma Chapter President—Angela Shomin

Benedictine College 9 actives, 11 associates

Other fall 2001 officers: David Livingston, Vice President; Brett Herbers, Secretary; Janeele Kroll, Treasurer; Erin Stretton, Stu-Go Rep; Jo Ann Fellin, OSB, Corresponding Secretary.

September was a time to tell others about Kansas Gamma. Janelle Kroll covered the desk for Club Night and provided games and information. Old and new members gathered at Schroll Center for the fall picnic on 3 September and were joined by the two new department faculty, Gelnn Adamson and Eric West. ON 15 October two students gave presetntations.

Angela Shomin talked anbout her experience at Spring where she completed an internship last summer. Janelle Kroll gave her directed research talk on "Priomitive Pythagorean Triples." Four students – Angela Sshomin, Brett Herbers, Matt Reel and Erin Stretton – participated in a sharing session for prospective students and their parents on 3 November at the BC Open House. ON 10 November four members – David Livingston, Erin Stretton, Andrea Archer, and Martha Buckles – attended a career day at the University of Kansas with faculty member Eric West. Sister Jo Ann Fellin, OSB also attended and participated as a panelist on the topic of "College Teaching as a Profession." On 26 November the chapter co-sponsored with the Physics department a speaker from Astral Point Communications. Dr. Dan Moriarty spoke about his work with "Optical Networking, Metro Transport." The traditional Wassail party was enjoyed on 3 December at the home of Sister Jo Ann.

MI Epsilon (Section A) Chapter President—Drew Spooner
Kettering University 74 actives, 30 associates

During the Winter Term of 2001 the KME Applied Math Noon-Time Movie took place on the 2nd Thursday. The Movie "The Video Math Festival" was selected from a collection of juried mathematical videos presented at the 1998 International Congress of Mathematicians in Berlin. The videos are winners of a world-wide competition, in which the international jury evaluated their mathematical relevance, technical quality, and artistic imagination. The mathematical themes include visualizations of classical mathematical ideas of Archimedes, Eratosthenes, Pythagorean, and Fibonacci, as well as application of modern numerical methods to real world simulations. Pizza and pop was served.

The Initiation Ceremony for the new members of our Michigan Epsilon Chapter was on Friday, March 9, at the University Cafeteria. 30 new, top quality Kettering students become members of the chapter. Our Keynote Speaker, Prof. Charles MacCluer of Michigan State University, a highly regarded Applied Mathematician, spoke about "The Industrial Mathematics." It was an intriguing excursion through the examples and areas of application in his newly published book. There were many family and faculty members coming to join us in honoring our new members.

MI Epsilon (Section B) Chapter President—Justin McCurdy
Kettering University 128 actives, 32 associates

For the Spring Term of 2001 the Second week Math Noon-Time Movie "The Video Math Festival" was repeatedly presented, and pizza & pop was served again.

The Initiation Ceremony for the new members was held on June 1, at the University Cafeteria, and 32 new, top quality Kettering students became members of the Chapter. The Keynote Speaker at this event was Dr. Ronald Mossier who recently retired from Daimler Chrysler after a long and productive career there as an Industrial Mathematician. His talk entitled "Rating Baseball Relief Pitchers" was extremely artistic, interesting, and was extremely well received.

On June 22 Kettering will graduate its next class of future leaders. Nine of those graduates will be honored by receiving the President's Medal. Of those nine students, precisely six of them are members of our Chapter of Kappa Mu Epsilon. I am confident that our new initiates will carry on this tradition of excellence.

Yet another initiate came to Kettering: The newly hired Prof. Ruben Hayrapetyan and Prof. Joe Salacuse enthusiastically started the Kettering Mathematics Olympiad series for 9-12 Grade high school students. The first year competition was held on Saturday, April 28. This is a great way to advertise the Mathematics Program at Kettering. The University Presidency promised to provide some "prize money" for the top competitors on a regular basis every year.

MO Alpha Chapter President—Jack McCush

Southwest Missouri State University 42 actives

Other fall 2001 officers: Nichole Gray, Vice President; Abby Cossiboom, Secretary; Josie Madl, Treasurer; Dr. John Kubicek.

During the fall 2001 semester the Missouri Alpha chapter of KME held montly meetings and hosted the annual Mathematics Department picnic. Featured speakers at the meetings included a student lead panel discussion and presentations by two faculty members.

MO Epsilon Chapter President—Trent Kraemer

Central Methodist College 9 actives, 0 associates

Other fall 2001 officers: Cassie Laffoon, Vice President; Kendall Clark Secretary; Kendall Clark, Treasurer; William McIntosh, Corresponding Secretary

MO Iota Chapter President—Dondi Mitchell, Ted Walker

Missouri Southern State College 10 actives, 0 associates

Other fall 2001 officers: Chip Curtis, Corresponding Secretary

We held monthly meetings. We also worked at the concession stands at our home football games as a fundraiser.

MO Mu Chapter President—James Hammond

Harris-Stowe State College 15 actives, 6 associates

Our KME chapter sponsored a series of weekly meetings, called Math Thursdays, which involved a variety of math related activities. These included graphing calculator workshops, presentations by the math history class, presentations by in-service teachers on classroom math activities for middle school and high school students, review sessions for the PRAXIS (licensure exam for teachers), and opportunities to interact with math faculty and students at Harris-Stowe State College. Some of our KME members attended the Second Annual Missouri Pre-Service Mathematics Teacher Conference where they attended workshops presented by Missouri mathematics teachers and had the chance to network with other Missouri pre-service mathematics teachers.

MO Zeta Chapter President—Ryan Hatfield

University of Missouri-Rolla 5 actives, 5 associates

Projects included a "Bridging the Gap" activity for Girl Scouts and completion of a chapter banner. Also, at one of our meetings we were pleased to have former astronaut Col. Tom Akers give a talk and slide show. He discussed his work on the space shuttle and the ways that mathematics is used in astronautics.

MS Alpha Chapter President—Mindy Hill

Mississippi University for Women 11 actives, 1 associates

Other fall 2001 officers: Jennifer Kimble, Vice President; Lailah BruceSecretary; Fransisca Lahagu, Treasurer; Dr. Shaochen Yang, Corresponding Secretary

09/24/01 – Monthly Meeting; 10/12/01 – Initiation, MAA Video – Careers in Mathematics; 10/29/01 – Octoberfest – Pumpkin Contest.

MS Beta

Mississippi State University

New Initiates: Rosemary Harrell, Sean Lestrade, Jonathan Boggess, Seth harbin, Trisha Gilbreath, Greg Henry, Chang-Ho Lee, Neil Williams.

MS Gamma

Other fall 2001 officers: Jose N. Contreras, Corresponding Secretary.

New Initiates: Ben Mitcham, Amy Bishop.

NC Gamma Chapter President—Sharon Blatt

Elon University 25 actives, 16 associates

Other spring 2001 officers: Kathleen Iwancio, Vice President; Judyth Richardson, Secretary; Katie Park, Treasurer; James Beuerle, Corresponding Secretary.

NE Beta Chapter President—Scott Barber
University of Nebraska at Kearney 7 actives, 5 associates

Other fall 2001 officers: Jeny Rutar, Vice President; Lisa Beckenhauer, Secretary; Tom Mezger, Treasurer

NE Delta Chapter President—Michael Vech
Nebraska Wesleyan University unknown # of actives, 4 associates

Other fall 2001 officers: Sarah Barnes, Vice President; Brain Danforth, Secretary/Treasurer; Melissa Erdmann, Corresponding Secretary.

NY Eta Chapter President—Ryanne Fullerton
Niagra University 18 actives, 15 associates

Other fall 2001 officers: Maryann Blouin, Vice President; Tim Paluch, Secretary/Treasurer; Robert Bailey, Corresponding Secretary.

Other than our annual initiation ceremony, we had a movie night that featured "Good Will Hunting." Plans were made for fall activities, including two computer/calculator demonstration meetings.

NY Iota Chapter President – Donna Chan
Wagner College 21 actives, 13 associates

Other fall 2001 officers: Peter Herbst, Vice President; Christina Prundl, Secretary; Jason Nadal, Treasurer; Zohreh Shahuar, Corrsponding Secretary.

NY Kappa Chapter President—Janine Pryor
Pace University 12 actives, 5 associates

Other fall 2001 officers: Dina Taiani, Corresponding Secretary.

New Initiates: William Batiste, Maria deConti, Angelo Motta, Thomas Rizzolo, Grigor Sargsyan.

NY Lambda Chapter President—Andrea Lorusso
C.W. Post Campus of Long Island University 21 actives

Other fall 2001 officers: Kira Adel, Vice President; Lisa Cook, Secretary; Heidi Campbell, Treasurer; Dr. Andrew M. Rockett, Corresponding Secretary.

NY Mu
St Thomas Aquinas College 80 actives, 5 associates

Other fall 2001 officers: Joseph A. Keane, Corresponding Secretary
New Initiates: Melissa Arneaud, Jennifer Lo Prete, Deidre O'Shea, Dennis Ryan, Kerrianne Schwartz

NY Nu Chapter President—Catherine Paolucci
Hartwick College 15 actives, 6 associates

Other fall 2001 officers: Adam Parsells, Vice President; Jaclyn Raffo, Secretary; Barrett Snedaker, Treasurer; Ron Brzenk, Corresponding Secretary. New Initiates: Leigh Voigt, Jeffery Jackson, Jennifer Repetto Kimberly Zern, Kelly Meyer, Douglas Weidner, Lisa Gillon, Bradley Molnar

OH Eta Chapter President—Stacey Stillion

Ohio Northern University 31 actives, 18 associates

Other fall 2001 officers: Reid Moore, Vice President; Starli Klobetanz, Secretary; Sarah Miller, Treasurer; Donald Hunt, Corresponding Secretary

OH Gamma Chapter President—Jeff Smith

Baldwin-Wallace College 11 actives, 23 associates

Other spring 2001 officers: Corina Moise, Vice President; Marianne Fedor, Secretary; Jason Popovic, Treasurer; David Calvis, Corresponding Secretary

OK Alpha Chapter President—Aaron Lee

Northeastern State University 35 actives, 5 associates

Other fall 2001 officers: Emily Walker, Vice President; Lara Lancaster, Secretary; Erin Renfrow, Treasurer; Dr. Joan E. Bell, Corresponding Secretary.

Our fall initiation brought 8 new students into our chapter. Dr. Casey Mann, Assistant Professor of Mathematics, was also initiated. We sponsored the "KME Pumpkin Patch" at NSU's annual Halloween Carnival. The children fished for plastic pumpkins with meter stick fishing poles. We met to work on problems in the Pentagon and the MAA journals. Our chapter participated in a high school recruitment effort for NSU. We passed out packets containing information about KME, the Mathematics Department at NS, and a math puzzle. The annual book sale was held in November. The Christmas party in December was a RIOT! We played the game "mafia" – with the sponsor Dr. Bell "done in" first.

OK Gamma

Other fall 2001 officers: Gerry East, Corresponding Secretary.

New Initiates: Summer Al-Jarrah, Jarred Blount, Kevin Brown, Adam Carr, Kyran Jones, Molly Priest, Susanna Rogers.

PA Alpha Chapter President –Brian Sullivan

Westminster College 23 actives, 12 associates

Other fall 2001 officers: Ann Ullmann, Vice President; Emily Henry, Secretary; Sarah Plimpton, Treasurer; Carolyn Cuff, Corresponding Secretary.

PA Gamma

Other fall 2001 officers: Dr. James R. Bush, Corresponding Secretary.

New Initiates : Jeremy Diehl, Angela McEwen, Sara Eagleston, james R. Hinz, Amy Snyder, Tiffany leyda, Rebecca Stevens, Kevin Blostic, Robin Simmons, Thomas Morris, Lloyd Kennedy, Timothy Baily

PA Kappa Chapter President – Joseph Coll

Holy Family College 7 actives, 7 associates

Other fall 2001 officers: Angela Cardamone, Vice President; Erica Bucci, Secretary; Melissa Murphy, Treasurer; Dr. Anne E. Edlin, Corresponding Secretary.

PA Lambda

Other fall 2001 officers: Elizabeth Mauch, Corresponding Secretary.New Initites: Jennifer Bettine, Angela Kelly, Aaron Pence, Steve Wilson, Heather Yanchunas

PA NU

Other fall 2001 officers : Jeff Nelson, Corresponding Secretary.

New Initiates : Leigh Voigt, Jeffery Jacson, Jennifer Reptetto, Kimberly Zern, Kelly Meyer, Douglas Weidner, Lisa Gillon, Bradley Molnar

PA Pi Chapter President—James Oakley

Slippery Rock University of Pennsylvania 8 actives, 5 associates

Other fall 2001 officers: Leah Schilling, Vice President; David Czapor, Secretary; Gary Grabner, Treasurer; Elise Grabner, Corresponding Secretary.

SC Gamma Chapter President—Anna Ulrey

Winthrop University 7 actives, 11 associates

Other spring 2001 officers: Laura Taylor, Vice President; Angel Rushton, Secretary; Mathew Foth, Treasurer; Frank Pullano, Corresponding Secretary

SD Alpha Chapter President – Ben Bouza

Northern State University 14 actives, 6 associates

Other fall 2001 officers: Scott Heyne, Vice President; Josh Bragg, Secretary; Amy Gurney, Treasurer; Dr. Raj Markanda, Corresponding Secretary.

We celebrated 2001 Mathematics Week in April with a pizza party and a lecture on Fractals by Dr. Raj Markanda.

TN Gamma Chapter President—Patricia Rush

Union University 20 actives, 4 associates

Other fall 2001 officers: Caroline Ellis, Vice President; Breanne Oldham, Secretary; Amanda Cary, Treasurer; Bryan Dawson, Corresponding Secretary.

This semester's events for our chapter began with a fall social in September. In our October meeting, Amanda Cary shared with us about her experiences s a summer intern at a Tyson Chicken plant. Where she put her statistics classes to good use. At the end of November, several of our members gave senior project presentations. Our final fall event was a Christmas social (potluck and white elephant gift exchange_ at the home of Dr. Lunsford. We also continued our tradition of sponsoring a needy child for the annual Carl Perkins Christmas Program.

TX Alpha Chapter President—Jenifer Blasingame
Texas Tech University 14 actives, 14 associates

Other fall 2001 officers: Paul Brock, Vice President; April Glenn, Secretary; Matt Grisham, Treasurer; Dr. Anatoly B. Korchagin, Corresponding Secretary.

TX Eta Chapter President—Crystal Cookey
Hardin-Simmons University 9 actives, 11 associates

The 26th annual induction ceremony for the Texas Eta Chapter at Hardin-Simmons University in Abilene, Texas, was held March 22, 2001. There were eleven new members: Emily Barrow, Melissa Easley, Kurt Evans, Amber Holloway, Wen Kuo, Shannon McLaughlin, Andy Nelson, Michael Piland, Stephanie Stephenson, Kane Swetnam, and Andrea Wooley. With the induction of these members, membership in the local chapter stands at 213.

Leading the induction ceremonies were President Crystal Cooksey, Vice-President Brooke Motheral, and Secretary-Treasurer Katie Smith. Following the induction ceremony, membership shingles and pins were presented to the 2000 inductees. New officers and club pictures were taken. KME then adjourned, and the members, inductees, and chapter sponsors enjoyed pizza and cold drinks.

Crystal Cooksey was reelected President and Brooke Motheral was re-elected Vice-President for the 2001-2002 year. In addition Amber Holloway was elected secretary-treasurer for the 2001-2002 year. Dr. Ed Hewett, Dr. James Ochoa, and Dr. Andrew Potter are chapter sponsors. Frances Renfroe is the corresponding secretary of the chapter.

TX GAMMA

New Initiates: Dayna Ford, Amisha Whitby, Carol Wagner, Diana Dickey, Donna Maxwell, Linda Fuqua, AadaleneWright, Whitney Hoobws, Ebony McGee, Feba Varghese, You Ting Wu, Tracy Chandler Stegmar, Jysenia M. Gwin

TX Lambda

Trinity University

Other fall 2001 officers: Diane Saphire, Corresponding Secretary.

TX Mu Chapter President—Leigh Anne Owens

Schreiner University 15 actives

Other fall 2001 officers: Geneva Conner, Vice President; Jeremy Gutierrez, Secretary/Treasurer; William Silva, Corresponding Secretary

We had three meetings this fall. Texas Mu is organizing a regional meeting at Schreiner University on April 5/6. 2002.

VA Beta

Other fall 2001 officers : Stephen Corwin, Corresponding Secretary.

New Initiates : Kristin L. Bellando, Christopher A. Clark, Andrea K. Crigger, John C. Desmedt II, Timothy S. Dutton,Brandon M. Monahan, Dianna L, Musick, Amanda Swetnam, Christin M. Talbot

WV Alpha Chapter President—Adam Fletcher

Bethany College 35 actives, 35 associates

Other fall 2001 officers: Joshua Barker, Vice President; Jennifer Spencer, Secretary; Kyle Hostetler, Treasurer; Dr. Mary Ellen Komorowski, Corresponding Secretary.

The Alpha chapter of KME at Bethany is planning to host the ECC Math Competition in the Spring 2002. Students from 10-15 colleges will spend a day in competition on a Team-structured exam.

Plans for the Spring initiation are being formulated. Officers will oversee the election of eligible candidates by reviewing the information received from the Registrar's Office.

Members of KME are meeting regularly with the Math and Computer Science club. Together they have spent time tutoring area children at the Wheeling Public Library, conducted a Computer Gaming Competition for area high school students and have reorganized the student Advanced Computer Lab which was relocated to Richardson Hallo of Science lower level.

The James Alllison Mathematics Library was established in honor of the late professor of Mathematics who passed away unexpectedly on Sept 7, 2000. President of KME, Adam Fletcher, spearheaded the establishment of the Library. Students may enter the library by obtaining a key from the officers of the Math/Computer Science Club.

Kappa Mu Epsilon

List of Recent Initiates
(as of May 2002)

Alabama Alpha Athens State University, Athens
Tony Anders, Kalyn Benson, Holly Camp, Dr. Neal Fentress, Ms. Dottie Fuller, Patsy Glaze, Robbie Griffin, Jessica Hulgan, Carol Jones, James Joy, Ms. Jennie Legge, Dr. Leigh Lunsford, Arthur Morris, Michelle Rosson, Emily Satterfield, Linda Schmidt, Lisa T. Warden, Susan Wilbanks, Alodie W. Brown, Chadrick B. Hastings, Lindsey McCaghren, Ricky Miller, Daniel Spears, Joseph Swinford

Alabama Eta University of West Alabama, Livingston
Tammy Bonner, Kim Giles, Randall L. Harrison, Jacob A. Ivey, McKiva James, Thomas J. Ratkovich

Colorado Gamma Fort Lewis College, Durango
Melanie Crawford, John Elliott, Ron Estler, Tim Hawkins, Malcolm E. Jewell III, James Lapp, Denise LeCrone, Carl Lienert, Lin Martin, Jacquelyn Rapp, Tim Rousset, Kenny Smith, Ryan Smith, Amanda Speegle, Rene Stein, Jake Wills

Indiana Gamma Anderson University, Anderson
Erin M. Daniels, Nathan T. Mercer, Denver W. Whittington

Iowa Delta Wartburg College, Waverly
Tate Behning, Joshua Christensen, Ellen Dane, Devansh Dhutia, Kyle Fencl, Bethany Franzman, Derek Gibson, Talitha Goldammer, Michelle Graaf, Kristin Haase, Amy Hammond, Amanda Hofer, Angela Holthaus, Rachel Husbyn, Aaron Jensen, Benjamin Kalkwarf, Christopher Kistler, Caleb Klein, Ryan Melin, Eric Muhle, Allison Oliver, Audra Ramsey, Jason Rasmussen, Derek Riley, Nathan Scheibe, Jonathan Schmidt, Matthew Stoffel, Matthew Townsley, Margaret Wrage, Wei Yang

Kansas Beta Emporia State University, Emporia
Amanda Anstine, Scott Keltner, Mindy Baker, Elizabeth Ohmie, Ton Boerkoel, Brian Hollenbeck, Amy Praiswater, Brad Tennant

Kansas Delta Washburn University, Topeka
Carolyn Cole, Catherine Ann Dowd, Jeffrey A. Kingman, Michael C. Mosier, Mary Elizabeth Noel, Ezra Livingstone Rush, Justin S. Stuewe

Kansas Gamma Benedictine College, Atchison
Andrea N. Archer, Massimo Botta, Erin E. Stretton, Eric M. West.

Kentucky Alpha Eastern Kentucky University, Richmond
Kristen M. Barnard, Philip D. Boggs, Justin T. Bowling, Kenneth D. Clark, Jason M. Davis, Amber R. Fulkerson, Eugena L. Gabbard, Shannon R. Hanner, Rexena Napier, Jong Sue Oh, Wesley D. Penn, Logan J. Sams, Jeremy S. Steele, Kristina M. Wallace

Maryland Beta Western Maryland College, Westminster
Laura E. Albaugh, Matthew P. Demos, Christopher Martin Drupieski, James M. Lipchock, Erin E. Shockley

Maryland Delta Frostburg State University, Frostburg
Gregory Barnhart, Crystal Beeman, Jacilynn Brant, Joshua Broadwater, Christopher Connelly, Michael Millione, Justin Rephann

Michigan Epsilon Kettering University, Flint
John W. Aarssen, Michael A. Biancalana, Leah L. Birchmeier, Kenneth D. Blecker, Mark L. Boehmer, William O. Calihan, Ada Cheng, Anthony Ciricola, Thomas L. Clark, Tyler A. Clarke, David M. DePaul, Christopher E. de Nijs, Justin A. Dodds, Michael R. Ekoniak III, Eric A. Ernst, Aaron M. Galbraith, Gerald J. Hysko, Carson M. Ingels, Ryan S. Jones, Ryan A. Kuhlman, Adam Y. Kurmally, Leo J. Leddy, Gordon A. Mausolf, Jessica L. Miller, Anthony F. Murphy, James R. Pettit, Thomas M. Redding, Michael A. Rekeny, Gayle L. Ridenour, Kelly L. Rose, Chad D. Slick, Sanja Sljivar, Jonathan M. Taylor, Scott D. Wauthier, LaRitta D. Webb, Steven A. Wessels, Daniel J. Wiant, Matthew A. Wright

Mississippi Alpha Mississippi University for Women, Columbus
Henry A. Boateng, Fransisca Asalina Lahagu, Shannon M. McVay

Mississippi Epsilon Delta State University, Cleveland
Cheryl Elizabeth Crisco, Larry Guise, Becky Waynelle Moore, Brandon Dale Sweet, Brad Alan Wallace

Missouri Alpha Southwest Missouri State University, Springfield
Timothy D. Brown, Jr., Amy Henken, Andrea Hurmence, Bridget Rowe, Melissa Schmidt, Andrea M. Streff, Angela Whytlaw

Missouri Beta Central Missouri State University, Warrensburg
Kassandra Barnett, Duane Bennett, Matt Detrick, Chris Dunnaway, Emily Goode, Brent Hoover, Bryon Howlett, Richard Lienou, Seth McNew, Andrew Nahlik, Kamen Nikolov, , Ross Weinberg

Missouri Gamma William Jewell College, Liberty

Cara Bartow, Christine Deatherage, Brooke Hildreth, Cassie Klein, Yuri Loktionov, Courtney Meyer, Aaron Sheffield, Erica Voeks

Missouri Theta Evangel University, Springfield

James E. Beyer, Jared M. Farnsworth, Joshua D. Groves, Andrew Lloyd Hauck

Missouri Kappa Drury University, Springfield

Jeff Clark, Lisa Dunham, Nicole Goldkamp, Ian Hendry, Jungmi Kim, Jeremy McMillan, Derek Mease, Heidi Miller, Dr. Robert Robertson, Mike Schluckebier, Young Soo Song, Virginie Stanley

Missouri Lambda Missouri Western State College, St. Joseph

Kevin L. Anderson, Emmy Bucher, Kurt M. Czerwien, Julie A. Hertel, Trevor Huseman, Thejaswi Karumanchi, Ludwig Nelson, Michael B. Ottinger, Jianping Su, Amy Wayman

Nebraska Alpha Wayne State College, Wayne

Kristin Brudigam, Janet Coco, Kimberly J. Daake, Jeremy Holtz, Kyle Mather, Leslie McFarland, Nicholas Muir, Dan Negus, Daniel Rasmussen, Laura Ann Sather, Rochelle Swanson

Nebraska Beta University of Nebraska at Kearney, Kearney

Stefanie Becker, Brandon Hauff, Jay Powell

New York Alpha Hofstra University, Hempstead

Tiffany Baron, Juan David Bolivar, Lauren Butler, Keith Comito, Scott Michael Davis, Enrico Fabbri, Richard Koenig, Karilyn Machen, Michael McGrath, Allison Powers, David Raikowski, Hector Rivera

Ohio Epsilon Marietta College, Marietta

Robert L. Behnke, Emilee C. Connors, Eric C. Davis, Kristi A. Long, Matthew D. Lupardus, Heath E. Lynch, Levi P. Maurer, Alison R. Ruff, Webo Wu, Brynn M. Moss, William E. Veigel

Oklahoma Alpha Northeastern State University, Tahlequah

John Diamantopoulos, Joseph F. Gonzales, Nicole M. Irick, Emily G. Kemp, James S. Kerl, Chi Kuen Ng, Abir Taha, Melanie D. Thomas

Pennsylvania Eta Grove City College, Grove City
Travis Barham, Jonathan Chambers, Michelle Hicks, Sandy Klemm, Ethan Smith

Pennsylvania Kappa Holy Family College, Philadelphia
Bradley T. Schneider, Danielle Bennett, Sarah Keeling, Lauren Gaughan, Jennifer Huetger, Scott Gabriele, Joseph Dollard

Pennsylvania Lambda Bloomsburg University, Bloomsburg
Saad Amin, Jason Bailey, Kelly Barchik, Lisa Bieryla, Rich Dempsey, Brock S. Devlin, Stacey L. DeWaelsche, April M. Egli, Basil Moskva, Michael Moskva, Janine DiSalvatore, Matt Keys, Maria Occhiato, Darrell Wolfe, Jill Yucins

Pennsylvania Mu Saint Francis University, Loretto
Thomas Andersen, Joseph Bopp, Amy Croskey, Rebecca Dombrowski, Matthew Farabaugh, Andrew Ferrari, Susan Horten, Chad O'Brien, Jelena Petrovic, Lawrence Quinn

South Carolina Gamma Winthrop University, Rock Hill
Lauren Renel Huntsberger, Sean Gibson Rae

South Dakota Alpha Northern State University, Aberdeen
Amy Bertsch, Julie Czmowski, Shane Haggerty, Shaun Haggerty, Dustin Hjelmeland

Tennessee Gamma Union University, Jackson
Nathan Creel, Ashley Merrick, Kelly Meyer, Allen Smith, Brian A. Taylor, Jennifer Nicole Vassar

Tennessee Delta Carson-Newman College, Jefferson City
Marci L. Mitchell, Chad A. Ramsey

Texas Mu Schreiner University, Kerrville
Colleen Marie Adcox, Teresa Joanne Gonzales, Kelly Kathryn McCullough, Shelley Rebecca Stark, Anelise Williamson

Announcement of the Thirty-Fourth Biennial Convention of Kappa Mu Epsilon

The Thirty-Fourth Biennial Convention of Kappa Mu Epsilon will be hosted by the Oklahoma Delta chapter at Oral Roberts University in Tulsa, Oklahoma. The convention will take place March 27-29, 2003. Each attending chapter will receive travel expense reimbursement from the national office as described in Article VI, Section 2, of the Kappa Mu Epsilon constitution. Mileage was raised from \$.25 to \$.30 per mile this past year by the national executive.

A significant feature of our national convention will be the presentation of papers by student members of Kappa Mu Epsilon. The mathematical topic selected by each student should be of interest to the author and of such scope that it can be given adequate treatment in a timed oral presentation. Senior projects and seminar presentations have been a popular way for faculty to get students to investigate suitable topics. Student talks to be judged at the convention will be chosen prior to the convention by the selection committee on the basis of submitted written papers. At the convention, the Awards Committee will judge the selected talks on both content and presentation. The rankings of both the Selection and Awards Committees will determine the top four papers.

Who may submit a paper?

Any undergraduate or graduate student member of Kappa Mu Epsilon may submit a paper for consideration as a talk at the national convention. A paper may be co-authored. If the paper is selected for presentation at the convention, it must be presented by one or more of its authors.

Presentation topics

Papers submitted for presentation at the convention should discuss material understandable by undergraduates who have completed only calculus courses. The Selection Committee will favor papers that satisfy this criterion and which can be presented with reasonable completeness within the time allotted. Papers may be original research by the student(s) or expositions of interesting but not widely known results.

Presentation time limits

Papers presented at the convention should take between 15 and 25 minutes. Papers should be designed with these limits in mind.

How to prepare a paper

The paper should be written in the standard form of a term paper. It should be written much as it will be presented. A long paper (such as an honors thesis) must not be submitted with the idea that it will be shortened at presentation time. Appropriate references and a bibliography are expected. Any special visual aids that the host chapter will need to provide (such as a computer and overhead projection system) should be clearly indicated at the end of the paper.

The first page of the paper must be a "cover sheet" giving the following information: (1) title, (2) the name of the author or authors (these names should not appear elsewhere in the paper), (3) the student's status (undergraduate or graduate), (4) the author's permanent and school addresses and phone numbers, (5) the name of the local KME chapter and school, (6) a signed statement giving approval for consideration of the paper for publication in The Pentagon (or a statement about submission for publication elsewhere) and (7) a signed statement by the chapter's Corresponding Secretary attesting to the author's membership in Kappa Mu Epsilon.

How to submit a paper

Five copies of the paper, with a description of any charts, models, or other visual aids that will be used during the presentation, must be submitted. The cover sheet need only be attached to one of the five copies. The five copies of the paper are due by February 1, 2003. They should be sent to:

Dr. Don Tosh, KME President-Elect
Department of Science and Technology
Evangel University
1111 N. Glenstone
Springfield MO 65802

Selection of papers for presentation

A Selection Committee will review all papers submitted by undergraduate students and will choose approximately fifteen papers for presentation and judging at the convention. Graduate students and undergraduate students whose papers are not selected for judging will be offered the opportunity to present their papers at a parallel session of talks during the convention. The President-Elect will notify all authors of the status of their papers after the Selection Committee has completed its deliberations.

Criteria used by the Selection and Awards Committees

Judging criteria used by both the Selection Committee and Awards Committee will include: (1) originality of topic and appropriateness for student audience, (2) clarity and organization of materials, (3) depth, significance, and correctness of content, and (4) credit given to literature sources and faculty directors as appropriate. In addition to the above criteria, the Awards Committee will judge the oral presentation of the paper on: (1) overall style and effectiveness of presentation, (2) presenter's apparent understanding of content, (3) effective use of technology and/or appropriate audio-visual aids, and (4) adherence to the time constraints imposed.

Prizes

All authors of papers presented at the convention will be given two-year extensions of their subscription to The Pentagon. Authors of the four best papers presented by undergraduates, as decided by the Selection and Awards Committees, will each receive a cash prize.

Publication

All papers submitted to the convention are generally considered as submitted for publication in *The Pentagon*. Unless published elsewhere, prize-winning papers will be published in *The Pentagon* after any necessary revisions have been completed (see page 2 of *The Pentagon* for further information). All other papers will be considered for publication. The Editor of *The Pentagon* will schedule a brief meeting with each author during the convention to review his or her manuscript.

Graduate School Registry

Are you an undergraduate student planning on attending graduate school? Check out a free listing with the Undergraduate Researchers' Graduate School Registry, sponsored by the Council on Undergraduate Research! The purpose of this registry is to facilitate matchmaking between undergraduates with research experience and a desire to pursue an advanced degree, and graduate schools seeking high quality students who are well prepared for research.

Any undergraduate may go to www.cur.org/ugreg/ to fill out a simple curriculum vitae form. There is no charge to you as a student. Student information records will be made available ONLY to bona fide Graduate Schools that contract with CUR for this service. Graduate School representatives may contact students to invite applications or visits to the campus and laboratory, or to share information about their research programs and financial support opportunities. .

If you are a student in your junior or senior year, **REGISTER FOR FREE NOW!** Juniors will be able to update their listing at the end of the summer and during their Senior year, to include any summer research experience or information about Senior Theses and test scores.

For additional information about the Registry, please visit http://www.cur.org/UGRegistryselect.html. Contact CUR at cur@cur.org or 202-783-4810 if you have any questions.

From the Pages of...

The Fall 1968 issue of *The Pentagon* (p.37) reported the installation of the New York - Eta Chapter (Niagara University). The faculty sponsor and corresponding secretary is none other than our esteemed national president, Robert L. Bailey. Might this be the first time Bob Bailey is mentioned in *The Pentagon*? And we have not seen the last.

Also in the same issue on page 48, Kenneth M. Wilke (and Don Page) noticed the solution to problem 208 was incomplete. So Ken has been working problems from *The Problem Corner* for at least 34 years and 351 problems! That's a lot of problems.

Kappa Mu Epsilon National Officers

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Back Issues

Is your journal collection complete? Copies of most back issues of *The Pentagon* are still available for $5.00 per copy. Please send inquiries to:

The Pentagon Business manager
Division of Mathematics and Computer Science
Emporia State University
Emporia, KS 66801 USA

Do You Have Your Seal Yet!

Would you like to get a copy of the seal of Kappa Mu Epsilon? It is available for free along with other logos and graphics files, information, links, etc. from the national KME Web Site! As you construct your own web pages, let people know you are connected to KME. You might also add a link to some portion of the national website, such as the general information about KME or to the cumulative subject index of *The Pentagon*. The URL for the national homepage is:

`www.kme.eku.edu/`

When you design a chapter homepage, please remember to make it clear that your page is for your chapter, and not for the national organization. Also, please include a link to the national homepage and submit your local chapter webpage's URL to the national webmaster. By doing so, other chapters can explore activities of your chapter and borrow some great ideas from you!

You can get a web page template from the Kentucky Alpha chapter. The URL is

`eagle.eku.edu/faculty/pjcostello/kme/`

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983

MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February, 2001
GA Delta	Berry College, Mount Berry	21 April, 2001
TX Mu	Schreiner University, Kerrville	28 April, 2001

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