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Computing Homoclinic Bifurcations

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Motivation

Dynamical systems is a very large field of mathematics which can be broken down into two main categories: differential equations and iterated mappings. Both categories of dynamical systems have several possible areas of application, as well. For example, differential equations can be used to model drug flow in the blood, while iterated mappings can be use to model the path along which a bunny hops.

This research involves only iterated mappings but could be extended to differential equations rather easily. In particular, this research focuses on real, planar iterated mappings.

The phenomenon of interest that we wish to study is called the homoclinic tangle for reasons that will soon become evident. Homoclinic tangles occur near one or more saddle points and are the intersection of stable and unstable manifolds. *Manifolds*, or *separatrices*, are solutions going away from the saddle points that exhibit special properties and separate the general regions of various behavior [1]. Homoclinic tangencies occur when the stable and unstable manifolds are tangent and the unstable manifold of the saddle point becomes the stable manifold of the same saddle point [1]. Homoclinic tangencies separate where there is a homoclinic tangle and where no homoclinic tangle occurs. See Figure 1 (attached at the end of the paper) for an illustration of one such homoclinic tangle. The mapping shown in this figure is the Hénon mapping with $a = 1$ and $c = 0.5$. The Hénon mapping from $\Re^2 \longmapsto \Re^2$ is of the form:

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x^2 - ay + c \\ x \end{pmatrix}$$

Note that the Hénon mapping is named after Michel Hénon, an astrophysicist from Paris, who suggested in 1968 that this mapping be used to study changing orbits of asteroids or satellites [2].

In figure 1, the stable manifold is shown in red; the unstable manifold is shown in blue; and the saddle point is labelled in green.

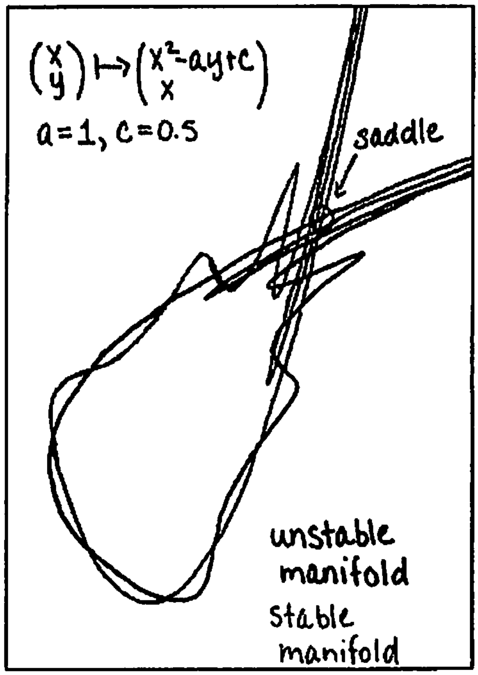

Figure 1

The Hénon mapping will be used for all further illustrations in this paper. In addition, the convention of using red for the stable manifold, blue for the unstable manifold, and green for the saddle point will also be used throughout the entirety of the paper.

The specific goal is to compute and visualize the homoclinic tangencies for planar iterated mappings from $\Re^2 \longmapsto \Re^2$. In order to accomplish this goal, we allow the system to bifurcate by changing the values of its parameters and then determine where the tangencies occur.

In order to understand the process of bifurcation, consider the following example. Consider again the Hénon mapping with $a = 0.3$ and $c = -1.2$. See Figure 2.

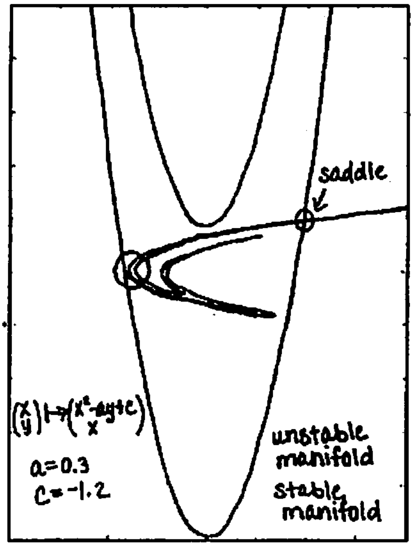

Figure 2

Notice how the stable and unstable manifolds are not quite touching in the circled region. However, when we allow the system to bifurcate and change the value of c to -1.5, we see that the manifolds are now overlapping in the same region. See Figure 3.

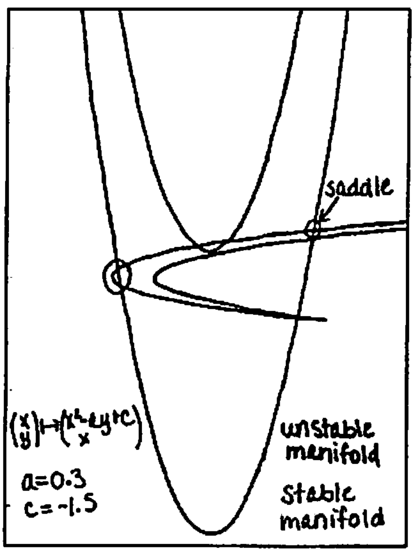

Figure 3

Thus, dynamical systems theory tells us that for some value of c between -1.2 and -1.5, the stable and unstable manifolds will be tangent. This is an example of a *homoclinic bifurcation*, and the tangency is called a *homoclinic tangency.* We will investigate the process of bifurcation in greater detail in Section Five.

The rest of this paper is organized as follows. In the remaining sections, we outline a method for computing and visualizing homoclinic bifurcations. In order to do so, we must discuss the theory and computational methods used in the calculation of saddle points, separatrices (or manifolds), tangencies, and bifurcations. At the end of this paper, we summarize our method and discuss several possibilities for further research.

Saddle Points

We begin our analysis of homoclinic bifurcations by first computing the saddle points for a given mapping. To that end, consider an iterated mapping from $\Re^2 \longmapsto \Re^2$ of the form

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x^2 - ay + c \\ x \end{pmatrix}.$$

We wish to find the *fixed saddle points* of the mapping. In order to do this, we first find the set of all fixed points for the mapping, since they obviously form a superset of the fixed saddle points. In order to find the fixed points, we simply solve the equations $x = f(x, y)$ and $y = g(x, y)$ using Newton's Method for nonlinear systems. This is a desirable algorithm to use for solving this system of equations since it guarantees super-convergence for most cases.

After finding the set of all fixed points for the mapping, we determine M, the matrix of partial derivatives for each fixed point. This is accomplished by solving the system of equations $\mathrm{M}x = \lambda x$. Here,

$$\mathrm{M} = \begin{bmatrix} f_x(x_0, y_0) & f_y(x_0, y_0) \\ g_x(x_0, y_0) & g_y(x_0), y_0) \end{bmatrix}.$$

Since M is a 2 by 2 matrix, this system can easily be solved. Many mathematical software packages have built-in subroutines to solve this particular system.

Let λ_1 and λ_2 denote the two eigenvalues of M for a given fixed point such that $|\lambda_1| > |\lambda_2|$. Then, (x_0, y_0) will be a saddle point of the mapping iff $|\lambda_1| > 1$ and $|\lambda_2| < 1$. As mentioned previously, saddle points locate the region of the homoclinic tangle.

Separatrices (Manifolds)

At a *saddle point*, there are two particular curves of interest called *separatrices* or the *stable and unstable manifolds.* These curves are defined in $\Re^2$ as follows:

- *Stable manifold*

$$\{x|f^n(x) \rightarrow (x_0, y_0)\ as\ n \rightarrow \infty\}$$

- *Unstable manifold*

$$\{x|f^{-n}(x) \rightarrow (x_0, y_0)\ as\ n \rightarrow \infty\}$$

Note that this definition can be changed to $\Re^n$ by simply changing the notation for the fixed point.

Thus, we see intuitively that the stable manifold is the set of all points such that when the mapping is iterated forward, the fixed saddle point is approached. Similarly, the unstable manifold is the set of all points such that when the mapping is iterated backward, the fixed saddle point is approached. This is illustrated in Figure 4.

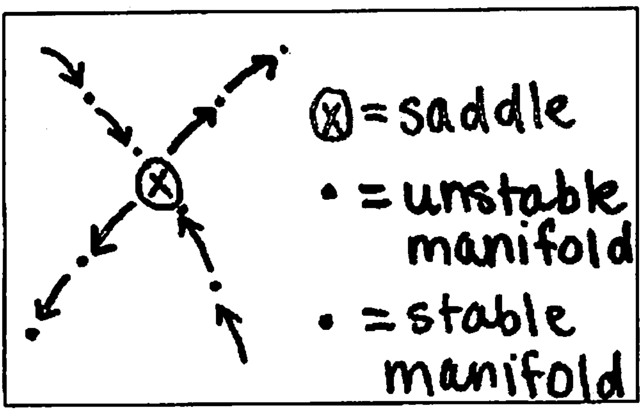

Figure 4

Separatrices are computed by first locating the saddle point. In order to compute the points on the unstable manifold, the mapping is iterated *forward* along the eigenvector with eigenvalue $|\lambda| > 1$. The points on the stable manifold are then computed by iterating the mapping *backward* along the eigenvector corresponding to the eigenvalue $|\lambda| < 1$.

Although this point will not be emphasized in the paper, it is important to add a "midpoint" between two consecutive points if the angle or distance between two consecutive points or vectors is too large. Otherwise, very odd, erratic behavior will be observed when interpolation is done between consecutive points on a manifold.

Once all the points on a manifold have been calculated, interpolation is done between all pairs of successive points. In order to do this, we used parametrized parabolas that are computed as follows:

1. Identify two successive points on a manifold between which to interpolate.
2. Identify their corresponding vectors which represent the direction of the next iterate. (Note that these are computed by applying the matrix of partial derivatives, M, to the original vector at the fixed point.)
3. Determine where the two vectors cross and label that point.
4. Compute and then draw in the arc of parabola that has as its endpoints the two points on the manifold and follows the two vectors.

See Figure 5 for an example of a parametrized parabola.

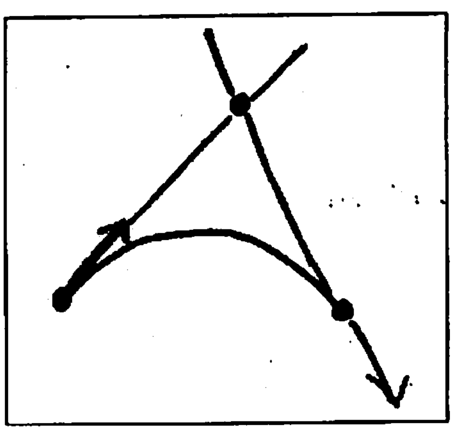

Figure 5

The blue dots in Figure 5 correspond to two successive points on the unstable manifold. The green vectors represent the direction of the next iterate at each point. The red lines are drawn to determine where the vectors cross. The black dot is the intersection of the two vectors. The blue arc of parabola is drawn by the interpolation scheme described above. In this manner, all successive points on a manifold are connected by interpolation.

Lastly, the triangles stemming from the two consecutive points on the manifold and the third point where the oriented line segments crossed are stored. This is an important data structure in the program.

Tangencies

At this point, we have computed the fixed saddle points and have calculated the stable and unstable manifolds. We have also interpolated between successive points on the two manifolds for visual aid in recognizing points on each manifold. The next step in computing homoclinic bifurcations is determine where the stable and unstable manifolds are *tangent*. Recall that this is important because tangencies determine where homoclinic tangles come into existence, and this is the phenomenon that we are interested in studying.

The specific goal then is to determine for each arc of parabola on the stable manifold and each arc of parabola on the unstable manifold if there exists a point on each parabola such that the manifolds will be *tangent* for some values of the *parameters*.

There are three possible cases for which the arcs of parabola on the stable and unstable manifolds will be tangent for some value of the parameters. All three cases are illustrated in Figure 6. The most obvious case is when the stable and unstable manifolds are tangent. This is illustrated by the diagram in the middle of Figure 6. The second case is when the stable and manifolds are separated by a positive distance for all points on the manifolds, but the tangent vectors at points on each manifold are parallel. This case is illustrated by the first diagram in Figure 6. The final case is when the two manifolds are overlapping but again the tangent vectors at points on each manifold are parallel. This is illustrated by the last diagram in Figure 6. Notice that in all three of these cases, the tangent lines and the line connecting them form an "H." For the purposes of this paper, this condition will be called the *H-condition.*

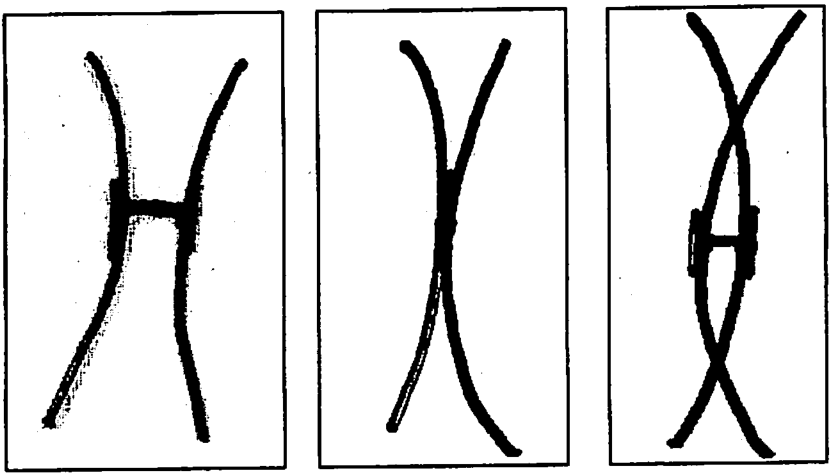

Figure 6

In order to determine where the *H-condition* occurs, we must first determine which system of equations must be satisfied for the set of parametric parabolas. To this end, let $\gamma(t)$ represent an arc of parabola on the unstable manifold and $\delta(s)$ represent an arc of parabola on the stable manifold be given by the following equations:

$$\begin{aligned} \gamma(t) &= \mathbf{a}_1 t^2 + \mathbf{a}_2 t + \mathbf{a}_3, 0 \le t \le 1 \\ \delta(s) &= \mathbf{b}_1 s^2 + \mathbf{b}_2 s + \mathbf{b}_3, 0 \le s \le 1 \end{aligned}$$

Note here that the $\mathbf{a}_i$ and the $\mathbf{b}_i$ are vectors.

Then in order to determine when the H-condition is met, we must solve the following system of equations:

$$det(\gamma'(t), \delta'(s)) = 0 \tag{1}$$

$$(\gamma(t) - \delta(s)) \cdot \gamma'(t) = 0 \tag{2}$$

When these equations are solved simultaneously, a fifth-degree polynomial in s results. Because of its great length, the polynomial is left out of this paper. For simplicity's sake, we just call is $f(s)$. Then, in order for the H-condition to be satisfied, we wish to minimize $f(s)$. This is done using Sturm's Algorithm.

Sturm's Algorithm

Sturm's Algorithm is an algorithm that is very useful for classifying the roots of a polynomial with real coefficients. A description of the algorithm is included because of the interesting mathematical theory behind it. The algorithm's input is as follows:

Input:

- $f(x) = a_0 + a_1 x + \cdots + a_n x^n, a_i \epsilon \Re$
- $t \epsilon \Re$.

The output for Sturm's Algorithm is as follows:

Output:

- p, the number of real roots $> t$
- q, the number of real roots $< t$
- $2r$, the number of complex roots of f.

Sturm's Algorithm works by taking the polynomial $f(x)$ and the real number t as input. An associated polynomial $g(x)$ is then defined as follows:

$$g(x) = (x - t) f'(x)$$

Next, $L(f; x, y)$, a symmetric expression in x and y is defined as follows:

$$\begin{aligned} L(f; x, y) &= \frac{f(x)g(y) - f(y)g(x)}{x - y} \\ &= \sum_{h=0}^{n-1} \sum_{k=0}^{n-1} A_{hk} x^h y^k \end{aligned}$$

Because $L(f; x, y)$ is symmetric, it can be used to define the following quadratic form:

$$Q(f; u_0, u_1, \cdots u_{n-1}) = \sum_{h=0}^{n-1} \sum_{k=0}^{n-1} A_{hk} u_h u_k.$$

As with all quadratic forms, $Q(f; u_0, u_1, \cdots, u_{n-1})$ can be written in matrix form. Let M be the matrix of coefficients of Q. Then, we need to compute what is known as the signature of M.

The signature is simply equal to the two numbers that represent the number of positive and negative eigenvalues of a matrix. Thus,

$$\begin{aligned} \textbf{Signature} &= (number\ of\ \lambda > 0, number\ of\ \lambda < 0) \\ &= (p + r, q + r), \end{aligned}$$

where p, q, and r are as defined above.

For our purposes, we wish to minimize $f(s)$. Thus, we must run Sturm's Algorithm twice with $t = 0$ and $t = 1$ in order to determine when one real root between these two values has been obtained.

Bifurcation

After the arcs of parabola satisfying the H-condition have been selected via the above method, we must bifurcate the system in order to determine the exact values of the parameters for which the tangencies occur. We now give the definition of a bifurcation.

Bifurcations are changes in the structure of the curves of a nonlinear system as a parameter passes through a critical value (bifurcation point).

Figure 7 is used to illustrate the concept of a bifurcation. The diagram on the left in this figure is for two arcs of parabola, one on each manifold, and the value of a generic parameter a is set to a_0. Notice that in this picture, the two arcs of parabola are separated by a positive distance. In the second diagram, the value of the parameter has been changed from $a = a_0$ to $a = a_1$. Now, the two arcs of parabola have passed the tangency and are now overlapping. Because the structure of the curves has significantly changed, the system is said to have undergone a bifurcation.

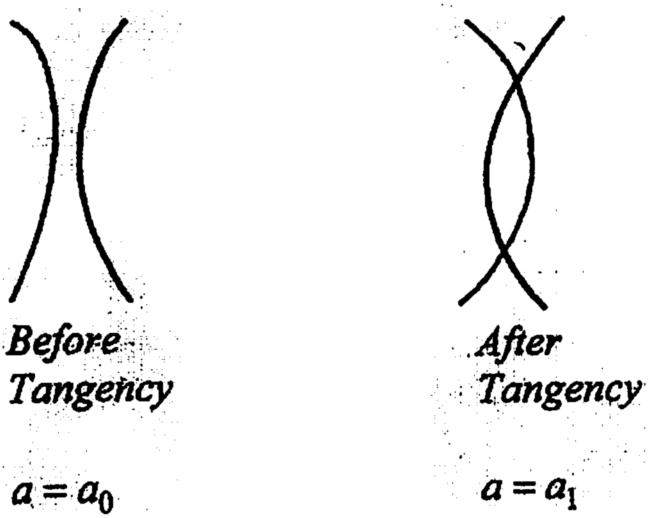

Figure 7

Recall that bifurcations are important to this research because we are looking for the exact value of the parameters such that the manifolds are tangent. In the example above, this would occur for some value of the parameter a between a_0 and a_1.

In order to compute the homoclinic tangencies, we follow the four-step procedure outlined below:

1. Locate the region for the tangency. (Note that this can be done by first determining where the triangles described earlier in the paper overlap.)
2. Determine two values of the parameter that correspond to the cases before and after the tangency.
3. Use Bisection to determine the value of the parameter corresponding to the minimum distance between the parabolas.
4. Trace out the curve in the parameter space where the tangencies occur.

An example of what such a curve in the parameter space might look like for the Hénon mapping is given in Figure 8. Notice that this curve looks very much like a phase diagram. We feel that understanding how these curves develop in the parameter space may help chemists understand phase transitions.

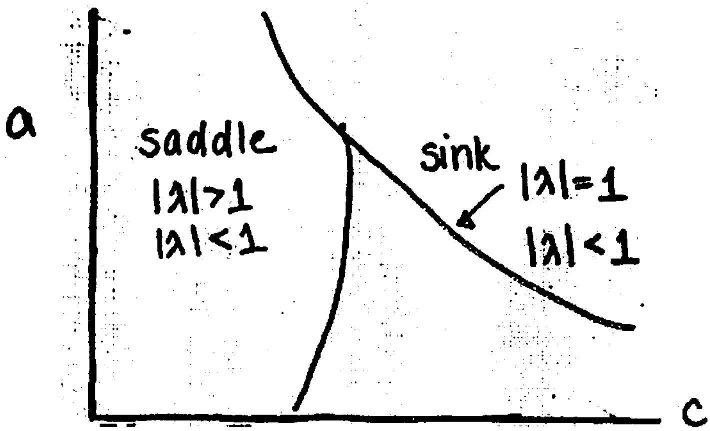

Figure 8

Conclusion

In summary, this implementation allows us to compute and visualize planar iterated mappings in $\Re^2$. This model could easily be extended to higher dimensions; however, this is not investigated in this paper.

Possibilities for Further Research

There are many other possibilities for further work. We list three such possibilities here.

1. How do such curves in the parameter space start and end?
2. Investigate mappings in C^2.
3. Visualize differential equations in $\Re^3$, particularly to determine the tangencies of stable and unstable surfaces.

Acknowledgment. This research was conducted at the 1998 Cornell University National Science Foundation's Research Experiences for Undergraduates in Mathematics Program under the direction of Professor John H. Hubbard.

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Starting a KME Chapter

For complete information on starting a KME chapter, contact the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately \$50) and the expenses of the installing officer. The individual membership fee to the national organization is \$20 per member and is paid just once, at that individual's initiation. Much of the \$20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offering and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

The Stable Marriage Problem

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In 1962, David Gale and Lloyd Shapley published a paper in which they introduced and solved The Stable Marriage Problem. Many years later, it was discovered that this algorithm had been in use since 1952 by The National Intern Matching Program to match graduating medical students (residents) with hospitals. The National Intern Matching Program is now call The National Resident Matching Program, and it still uses this algorithm.

There are two sets in The Stable Marriage Problem. M is the set of men and W is the set of women. The number of elements in M equals the number of elements in W. Both sets are finite. Associate to each man, $m \in M$, and ordered list of all women, $w \in W$, from most favorable to least favorable. Similarly associate to each woman, $w \in W$, an ordered list of all men, $m \in M$, ranked from most favorable to least favorable. In other words, each man ranks the women in a preference list from most favorable to least favorable and each woman ranks the men from most favorable to least favorable. There are no ties in these preference lists.

A matching is a $1-1$ correspondence between the men and the women. Under a matching f: $M \rightarrow W$, m is paired with $f(m)$ and w is paired with $f^{-1}(w)$. Let f: $M \rightarrow W$ be a matching, and let $m \in M$ and $w \in W$. If m prefers w to his mate and w prefers m to her mate, then (m, w) is called a blocking pair for f. A blocking pair will separate from their matches to better themselves. If f admits at least one blocking pair, then the matching f is called unstable. Otherwise f is called stable.

The following are two basic questions about the stable marriage problem:

1. Must there exist a stable matching f that maps M to W?

2. Is there an efficient algorithm for finding a stable matching (if one exists)?

The answer to both of these questions is yes. The efficient algorithm is called *The Gale-Shapley Algorithm* (or propose-dispose). By efficient, I mean it terminates in polynomial time, after n^2 iterations, where n is the number of men or women.

The Gale-Shapley Algorithm

Each man and woman starts unmatched. If a man, m, is unmatched, then he "proposes" to the most favorable woman, w, on his list to whom he has not already proposed. If that woman is unmatched, then she must accept. If that woman is matched to some other man, m', then she compares m and m' on her list. If she prefers m to m' then she "disposes" of m', who then becomes unmatched, and she accepts m. If she prefers m' to m, then she rejects the proposal of m and remains matched to m'. The above process continues, one proposal at a time, until everyone is matched. In this algorithm, order does not matter. This algorithm can be executed in two ways: one being where the men propose and the other being where the women propose. The following is an example using this algorithm where the men propose:

1	4123	1	4132
2	2314	2	1324
3	2431	3	1234
4	3142	4	4132
Men's Preferences		Women's Preferences	

M_1 proposes to W_4;she is unmatched, so she must accept.

M_2 proposes to W_2; she is unmatched, so she must accept.

M_3 proposes to W_3; she is already matched with M_2, so she compares M_2 to M_3. She prefers M_3, so she accepts M_3 and disposes M_2.

M_2 proposes to W_3; she is unmatched, so she must accept.

M_4 proposes to W_3; she is already matched with M_2, so she compares M_2 to M_4. She prefers M_2, so she rejects M_4 and stays with M_2.

M_4 proposes to W_1; she is unmatched, so she must accept.

Therefore, the matching is $f = \{(1,4), (2,3), (3,2), (4,1)\}$ It is stable because it has no blocking pairs.

Now I will prove the Gale-Shapley Algorithm always terminates with a stable matching.

Theorem: The Gale-Shapley Algorithm always terminates with a stable matching.

Proof: Define $f : M \to W$ by letting $f(m)$ equal the women with whom m is matched with at the end of the algorithm. We will show that no pair (m, w) can be a blocking pair for f. This means either m prefers his mate to w or w prefers her mate to m. If m prefers w to $f(m)$, then w must have either rejected or disposed m at some point earlier in the algorithm. Hence, w was either already matched with, or later was proposed to by and accepted, a man who is higher on her list than m. In either case, since a woman's situation can only improve during execution of the algorithm, we have w preferring her mate to m so that (m, w) cannot block f. Therefore, f is a stable matching.

I found many interesting facts about The Stable Marriage Problem. I will introduce some of them here. All possible executions of the Gale-Shapley Algorithm (with the men proposing) lead to the same stable matching. If $f : M \to W$ is a matching resulting from an execution of the Gale-Shapley algorithm (with the men proposing), then for each man, $f(m)$ is the best possible partner that m can have under any stable matching. It is surprising that, if each man is paired with his best stable partner, the result is a stable matching. It is not even clear that this should even give a matching, let along a stable one.

The matching that results from the Gale-Shapley Algorithm with the men proposing is called the man optimal stable matching. Denote the man optimal stable matching by $f_m : M \to W$. The matching that results from the Gale-Shapley Algorithm with the women proposing is called the woman optimal stable matching. Denote the woman optimal stable matching by $f_w : M \to W$.

Theorem: The man optimal stable matching is worst possible for the women because under f_m, each woman is paired with the worst possible partner she can have under any stable matching.

Proof: We are to show that for all women and all stable matchings $g : M \to W$ that w prefers her mate under g to her mate under f, the man optimal stable matching or she is indifferent between the two matchings. Assume for contradiction that w prefers her mate under f, the man optimal stable matching to her mate under g for some stable matching g and some woman w. Since g is stable, it has no blocking pairs. In particular $(f_{m^{-1}}(w), w)$ cannot block g. Now, w does prefer her mate under f_m over her partner under g, this is what our assumption says. It follows that $f_{m^{-1}}(w)$ prefers his partner under g over w, his partner under f_m. This contradicts the fact that $f_{m^{-1}}(w)$ has no stable partner he prefers to w. This shows that, under the man optimal stable matching, each woman is paired with the worst possible partner she can have under any stable matching.

Another interesting fact is that if $f_m = f_w$, then there exists only one stable matching.

Theorem: If $f_m = f_w$, then there exists only one stable matching.

Proof: Let $g : M \to W$ be a stable matching. We have for each man that he prefers his partner under f_m to his partner under g, and he prefers his partner under g to his partner under f_w, or m is indifferent between all of them. But $f_m(m) = f_w(m)$ which implies $g(m) = f_m(m) = f_w(m)$. Therefore $g = f_m = f_w$. This shows that if $f_m = f_w$, then there exists only one stable matching.

There are also many questions about deceit in The Stable Marriage Problem. When this algorithm is executed from the male standpoint, no man acting along can falsify his preferences so that he does better then he would otherwise if he reported his true preferences. If any subset of men falsify their preferences, it is not possible for all of them to wind up with better partners than their male-optimal partners. However, when the algorithm is executed from the female standpoint, the men, by falsifying their preferences, can force the algorithm to produce the man optimal matching. Much research is still being done on The Stable Marriage Problem. Another approach to The Stable Marriage Problem was in the May 1998 issue of the Mathematics Monthly, Vol. 105, No. 5, pp. 430-445, by Michel Balinski and Guillaume Ratier.

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North Central Regional Convention of KME
April 7-8, 2000, in Atchison, KS

I will begin with a word about what this article is not. It is not an article about the advances made in mathematics during the twentieth century or about the state of mathematics at the end of the twentieth century. If such information is desired, a good article to read is one by Phillip A. Griffiths that appeared recently in The American Mathematical Monthly [18]. Professor Griffiths, having taught at institutions such as Harvard, Princeton, and UC Berkeley and having served as provost at Duke, is much more qualified to give such a perspective than I am. However, all professors were once students; Griffiths is no exception. As a student at Wake Forest College, he was awarded second place for his presentation at the 1959 National Convention of KME, and his paper was published in The Pentagon [17].

This article is about students and their projects, in particular those that have been published in The Pentagon, established in 1941 as the official journal of the Kappa Mu Epsilon national mathematics honor society. We will, therefore, be concerned only with the last six decades of the twentieth century. In particular, we wish to ask ourselves the following questions: How has student scholarship changed? How is student scholarship the same? What were student papers like 10, 25, or 50 years ago? We shall discuss this in six parts.

Part 1. Applications of Mathematics

Applications have always been of interest to students, although they seem to be a little more popular lately. The changes seen in this area have been in the type of applications that were of interest. What was interesting 50 years ago is not necessarily interesting now, and vice versa.

Consider, for instance, a 1947 paper by Thomas Selby [28], who had served as a captain in the field artillery. The paper was titled "Computation of Firing Data for Field Artillery." As we will do often, let's look at some quotes from the article to get some of its flavor:

> "The mathematics involved in the computation of firing data for the Field Artillery is very simple. We were told when we started the study of gunnery that all that was necessary to compute firing data was a knowledge of "grocery store" arithmetic. First is the understanding of the mil which is the unit of angular measurement used. The mil is defined as the amount of angle subtended by an arc one unit long at a distance of 1000 units from the vertex. the number 6,400 is used instead. The discrepancy is negligible when angles of not over 300 or 400 mils are used and can be ignored in larger angles when great accuracy is not essential."

Did you notice that issues of ease of computation were important? We shall return to this theme later as well. Selby did not ignore that issue, either as evidenced by the following:

> "Most people accustomed to five-place tables think the above computations are rather crude, but they should bear in mind that the method is designed to be used under adverse conditions where speed with some degree of accuracy is the important consideration. One artillery shell (105 mm.) covers an area of 15 yards in depth and 50 yards in width, making the first calculation accurate enough."

Cryptography was a popular application long before the modern RSA codes and other public key cryptosystems. One such article was "Modern Trends in Cryptography: The Fractionated Cipher," by S. H. Sesskin [30]. Sesskin's paper discussed the German Field Cipher of 1918 using the key word PENTAGON, one of many references in student projects over the years to KME or to its journal. I find the following comment interesting:

> "discussion will be limited strictly to paper and pencil ciphers, and, of course, will not include ciphers coming from the newer electronic devices. (Despite these new devices paper and pencil ciphers will be studied as long as there are spies and criminals who cannot have access to such devices, and as long as wars are fought in the field where such devices not only would prove cumbersome, but would require the maximum protection from capture.)"

Of course, hindsight is 20-20! Another of Sesskin's quotes, however, seems much more on the mark.

> "Practical ciphering has always been a compromise between space and time in an effort to obtain the maximum security in time at a minimum cost in words. And today it is more so than ever. In fact, today the balance is even more delicate, for the experts seek not so much an insoluble system, as one that will give security for a stated time."

Another timely application was discussed in Charles Trauth's paper "Motions of a Space Satellite" [36]. The article appeared in the Fall 1957 issue; however the presentation was awarded second prize at the 1956 National Convention, the year before Sputnik's October 4, 1957 launch. The figure below, from Trauth's article, shows the "preliminary schematic trajectory" of a rocket to launch a satellite.

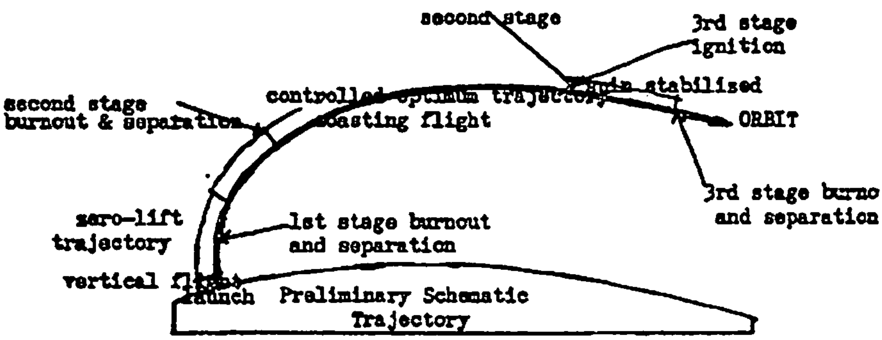


Figure from Fall 1957, p.24

In the 1960's, communication was an application of interest to at least one student. In a paper presented at the 1967 National Convention in Atchison, Kansas, April 7-8 (exactly 33 years before this regional convention, with the same dates and place!), Judy Kaldenberg discussed "Communication Networks Using Matrices" [21].

Finally, as an example of a more recent timely application, consider the paper of Michelle Biggers-Beach, whose presentation "The Orbit of Hale-Bopp" [3] was given at the 1997 National Convention during the time that the Hale-Bopp comet was visible in the sky. Her video presentation at that meeting was something that the authors of the previously mentioned papers could only dream of

Part 2. Computation

Another past interest was in methods of computing things that are now commonly done on a computer algebra system or calculator. These computations were carried out in many different ways; the variety of approaches may surprise you.

How many of today's students could extract a square root without a calculator? For that matter, how many of today's younger mathematics faculty could do so quickly? Just how much our perspectives have changed is illustrated by the following quote from an article by Judith Enos entitled "Methods of Extracting Square Roots" [14].

> "The average person knows but one or two ways of extracting square roots. The long division method is commonly taught in the junior high school, and if a person is lucky he is introduced to the logarithmic method of extracting square roots in the high school. Actually there are many distinctly different methods for finding square roots, and for each different method there are variations and generalizations."

Enos goes on to describe a dozen different methods!

Once upon a time, calculations fascinated many a person. That was true of Harvey Fiola, whose article "Integral Right Triangles of Equal Area" [15] appeared in the same issue as Enos' article. One example from his article is the three integral right triangles, i.e. Pythagorean triples, $(339252715200, 2066690884801, 2094350404801)$, $(4143735357600, 169202527102, 4147188470398)$,and $(966871583440, 725153687580, 1208589479300)$, all of which have the same area! The editor's note explains the situation further:

> "Adapted by the Editor from notes received from the author. The author is 19 years of age and works on his father's farm. He says, 'I have computed so many right triangles that I see them in the heavens.' "

Even today, most students learn how to graph simple functions by hand. But when it comes to curves such as

$$x^4 + x^3y + x^2y^2 + y^2x + xy + x + y = 0,$$

we now run to the computer. How would one carefully graph the above equation by hand, if one wanted more than just a rough sketch? Dale Schoenefeld spent an entire article answering that very question and graphing that equation in "The Use of the Analytical Triangle in Curve Tracing" [31].

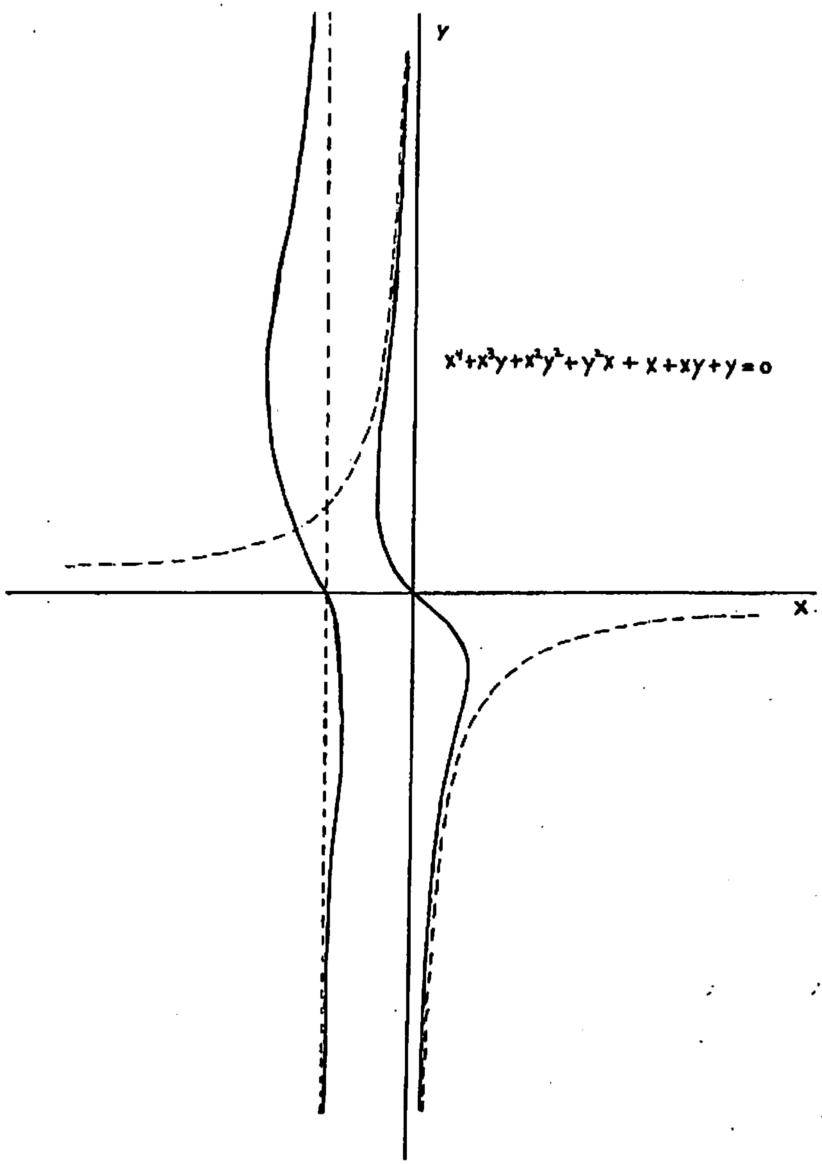


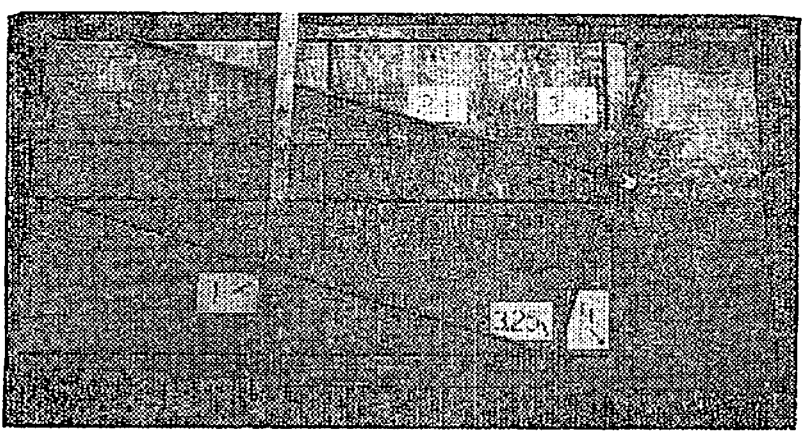
Figure 14 from Fall 1964, p.16

In "Nomography" [9], Eddie Dixon combined the two above ideas, using graphs to facilitate numerical computations. Nomograms were custom-created for specific computations; it is an art that is now mostly forgotten, as evidenced by my spell-checker failing to even recognize the word!

"Often in science courses a student is required to solve the same equation over and over again. Because of a similar situation occurring in industry, there developed a need for a rapid and simple method of solving these equations. The answer was nomography.

"A nomogram is an arrangement of two or more scales in such a manner that the value of an unknown variable may be determined by the use of a straight edge. The scales of the nomogram may be either straight or curved, uniform or nonuniform."

While many students worked on computations by hand, others worked on computation by mechanical devices. One of the most fascinating student efforts of this type appeared in the same issue as Dixon's nomography article: "Mechanical Solution of Cubic Equations" by John Couch [6]. Couch illustrated the solution of any cubic by means of the device pictured below (solution to $x^3 - 4x^2 + 3x - 2 = 0$). Of course, an exact solution could be found by hand using equations known for centuries; the idea, then, was not so much for practical use as it was for the pure fun of it. Couch's presentation won second place at the 1955 National Convention.



There were much more practical computing problems tackled, however. Consider "Solving a Differential Equation on a Differential Analyzer," by Joseph Weizenbaum [39], a paper awarded third place in 1953. Under the heading "Functional Description of an Analogue Machine," we find the following:

"An analogue computing machine performs its mathematical operations by measuring certain physical components or certain changes in certain components.

This is as opposed to a digital machine . The Wayne University differential analyzer - which is the original instrument built by Vannevar Bush of the Massachusetts Institute of Technology - measures rotations of shafts. It is entirely mechanical."

Although such devices were once rivals of electronic computers, we eventually see their mention in student papers totally disappear. The rise of the computer in student projects began in a Spring 1966 article entitled "Computer Application to Symmetric Double Integration by Hypercubes" by Jerry L. Lewis [23] with the following introduction:

"In this article we shall derive and apply to the digital computer a method for computing the numerical approximations to the double integral of a function of two variables over a two-dimensional hypercube."

Lewis' article contains the first student-generated code to appear in The Pentagon, written in FORTRAN. Six examples are given, the last of which is

$$\int_3^{11}\int_2^{8} e^y\,(1-x^2)\,dx dy,$$

for which an answer of -10270096.1000 is given. *Mathematica* gives an answer of -9696357.1009 instead; assuming *Mathematica*'s answer is accurate, Lewis' answer had an error of just under 6%. Not bad for one page of code in 1966!

Part 3. Computers

Besides being of interest in computations, computers themselves have been objects of interest. In 1958 a pair of articles about computers were published. One, by a faculty member, described digital computers. The other, "Electronic Analogue Computers" by Louis Kijewski [22], was by a student. Here are some interesting quotes from the introduction and conclusion of the article:

"An electronic analogue computer is a general-purpose problem-solving machine which is composed chiefly of electronic components but which may also include mechanical components. Variables of a problem are represented in the machine by voltages and mechanical displacements. the use of an analogue computer is particularly suitable for handling the more intricate problems involved in designing electronic brains for missiles and gun directors, where there may be ten parameters which will affect the speed of response to the target. the lower cost of the analogue, as compared to the cost of the digital, warrants its use in this field. Analogue computers are excellent tools which perform tedious calculations for man and leave him with extra time to do more creative work."

Analogue machines died a quick death in student use. Another interesting coincidence is that the first article in that same issue was an exposition of a classic problem that was eventually solved on the computer, namely the four color problem [26].

Eventually, focus shifted to applications of computer usage, and what can be done on a computer. We have already discussed that somewhat, but one more example is in order. The third-place paper at the 1981 convention was "Computer Graphics: Three Dimensional Representation of Spheres" by David Harris [19]. Harris writes:

> "At the beginning of the summer of 1979 I received a research participation award from the Clark Foundation. I modified a computer program that drew spheres on a high-resolution graphics terminal. The program ran on a PDP 11/45 computer coupled with a Genisco processor that controlled the graphics CRT. The primary use of the program was to draw molecules. illustrated with a series of prints reproduced from images on a CRT. The original slides were taken in a dark room with a camera mounted on a tripod at a distance of 18 [in] from the CRT display screen. The camera was set at a shutter speed of $\frac{1}{4}$ second and the F-stop was set at 4."

Wow! That's a lot different than clicking on the "print" icon! The original slides were in color, but *The Pentagon* printed them in black and white. Below is an example of what Harris had achieved.

Figure 8 from Fall 1981, p.9

How far we have come in just a couple of decades! Yet, consider the following quote from the conclusion of Harris' paper:

> "With the advent of faster computers, computer graphics is becoming better and easier."

Yes, that was better and easier at the time. A large number of such papers on computers followed, but have lately tapered off as computer science became its own field, with its own conferences and student groups.

Part 4. Ideas

Ideas have always driven mathematics. The same is therefore true of student research projects, and I suspect that fact will never change. Most of the ideas seem to come from geometry and precalculus mathematics.

Our first example of an idea comes from a paper by Thomas Potts entitled "Conic Sections with Circles as Focal Points" [25]. Potts' ideas was to use circles instead of points for the foci of the conics. An ellipse, along with its foci, is reproduced below from Pott's article.

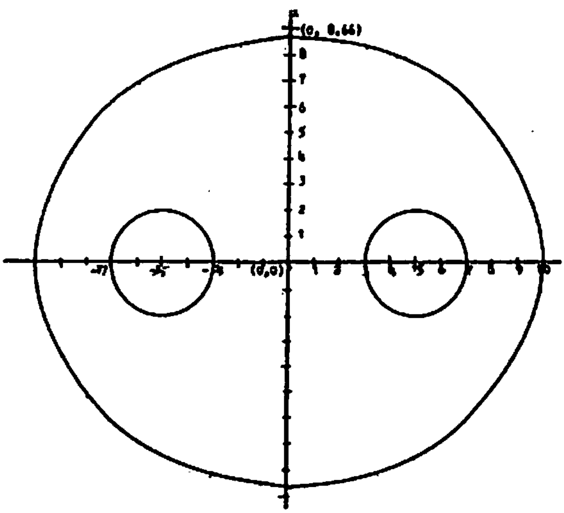


Figure 6 from Spring 1966, p.83

A similar idea was used previously by Morris Rosen in "Taxicab Geometry" [27]. Rosen's article looks at what the conic sections would be like if given their usual definitions but using the taxicab metric. Full of very interesting figures, his presentation won first place at the 1955 convention.

Another example of an idea driving a student project is "Pascal's Tetrahedron and the Trinomial Coefficients" by Janet Shorter and F. Max Stein [33]. Shorter participated in an Undergraduate Research Participation Program at Colorado State University under the direction of Stein. Stein and his students produced a good number of papers that appeared in The Pentagon, as well as other journals. This makes another point: most student research is, at least to some extent, directed by a faculty member. Although Stein preferred to be listed as co-author with his students, it has been more common for faculty members' contributions to be acknowledged in other ways, such as by a "thank-you" at the end of the article.

"Square Trigonometry" by William Georgou [16] was the 1971 convention winner. The idea of this paper was to use the unit square instead of the unit circle in defining the trigonometric functions. Georgou came upon the idea from a 1967 article in *The Mathematics Teacher* [2]. Again, I should digress momentarily to say that often the ideas undergraduate students use in research projects or papers are not their own original ideas, but ideas borrowed from others and expanded upon. That's not a bad trend in the opinion of this author, and it is a trend that I would expect to continue.

The functions in Georgou's article are named san, cus, tin, nas, suc, and nit, and an example of the graphs given by Georgou is below.

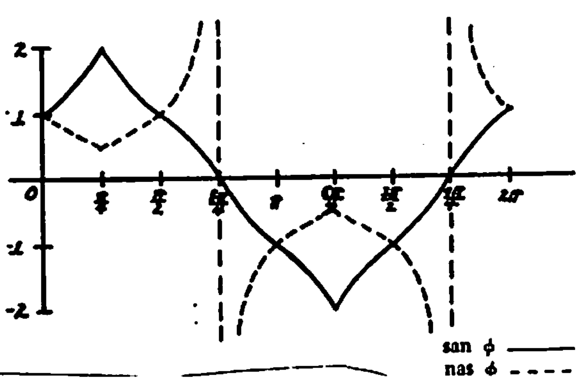
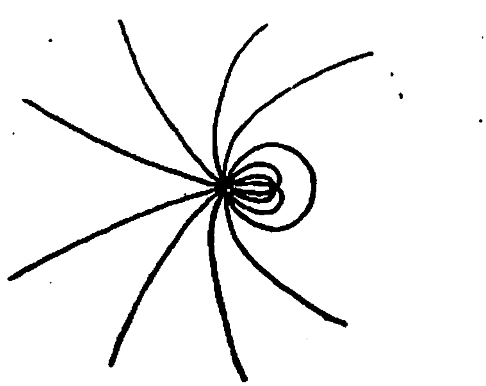


Figure 4 from Fall 1971, p.8

Myron Effing gave the winning presentation in the 1963 convention titled "Biangular Coordinates" [11]. Myron begins as follows:

> "The two most commonly used coordinate systems for locating points in a plane form a progression which leads to a third coordinate system. In the Cartesian systems, two distances are measured to locate a point, while in the polar coordinate system, a distance and an angle are measured. The third system is one in which two angles determine a point."

As with the previous four papers, the idea is simple yet interesting. Many questions within the reach of an undergraduate present themselves quite easily. The diagram showing the coordinates of a point is reproduced below, followed by a graph Effing calls an "arachnid" (spider).



(f) The Arachnid $\eta = \frac{4}{7}\xi$

Figure 1 from Fall 1963, p.3

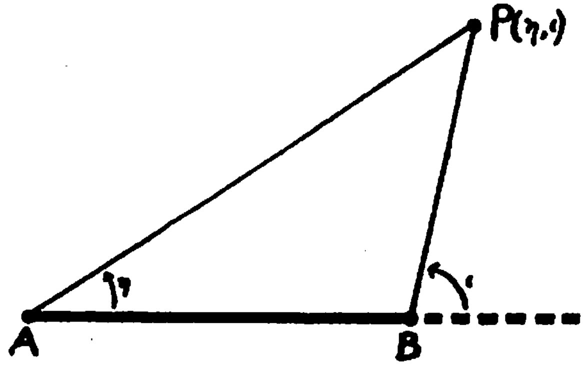


Figure 5(f) from Fall 1963, p.11

"The Four-Dimensional Cube" by Normal Sellers [29] discusses the tesseract, which is the extension to four dimensions of our two-dimensional visualization of a three-dimensional cube (confused?). Sellers' drawings are some of the best examples of hand-drawn figures I have seen, and the article is well worth a look just for those illustrations.

A final, more recent example of the same type is "Lengths of Generalized Rose Curves" by Ismat Hasan Shari [32]. Shari investigates graphs of the form $r(\theta) = \cos\left(\frac{m}{n}\theta\right)$, as opposed to those of the form $r(\theta) = \cos(m\theta)$.

Part 5. Trends

We have discussed several trends among student papers in the previous sections. What I would like to discuss here is the idea that mathematics, not unlike clothing, music, and entertainment, has its fashions as well. I have attempted to determine the "hot" topics of each decade of The Pentagon's existence. The following list isn't perfect, but it's my best shot:

1940's: history, geometry, and philosophy
1950's: cryptography
1960's: matrices, linear algebra and vector spaces
1970's: graph theory
1980's: games and game theory
1990's: fractals, chaos and dynamical systems

Notice that these are more in line with "popular" mathematics than with "serious" mathematical research, although not completely disjoint from the latter. An illustration of the trend of the 1990's and another example of a paper's reference to KME is the winning presentation from the 1991 convention, "What's the Fractal Dimension of KME?" by Mary Wilson [42]. The KME Fractal, reprinted below, appeared in her article.

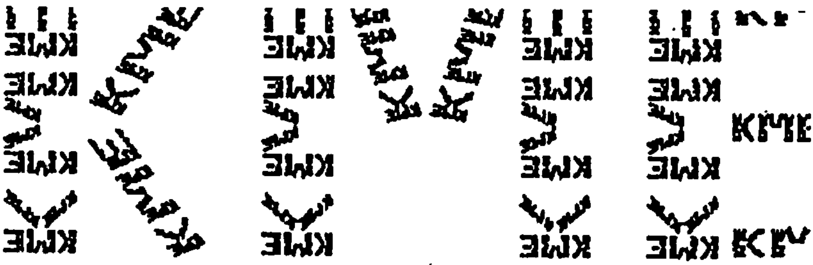


Figure 5 from Fall 1991, p.11

Another recent trend is that of the "catchy" title. While such titles somehow relate to their subject, in many it is not at all clear what the paper is about. Examples from the 1990's include "I Think Knot" [7], "Magical Minimal Mania" [1], "Even the Least of These" [41], "Let's Be Seated" [38], "When Intuition Fails" [5], "Choices, Choices, Choices" [4], "The Bobcat That Lived in a Polygon" [24], "As the Water Swirls" [20] (from 1988), "Fore!!!" [40], "I've Got a Secret" [37] and "Princess Diana, Paul Revere, and Group Theory???" [8]. Although a few older titles are this way, they are much fewer and farther between. The 1957 paper "Paradox Lost - Paradox Regained" by (Peggy) Steinbeck [34] has a nice literary sound to it, even though the title and author clash.

Part 6. Others

Of course, there have been many student articles that do not fit the types mentioned previously. Many are real "mathy," like "The Second Order Linear Differential Equation with Constant Coefficients and the Corresponding Ricatti Equations" [10]. Others were of mathematical exposition, e.g. "The Cantor Mapping" [13]; exposition is still a common theme of many of today's papers. Some were historical exposition (which is enjoying renewed interest) like "From Alice to Algebra" [12].

Our final example is a historical exposition written in the 1940's, an example of the hot topic of that decade as well. "The Newton-Leibniz Controversy" [35] was written by an alumnus (1942) of the host institution when it was still separate from the boy's college and called Mt. St. Scholastica College. Muriel Thomas was in attendance at the banquet [the banquet for which this address was given], more than half a century after her article was written, but unfortunately had to leave before the end of this presentation.

Conclusion

Kappa Mu Epsilon student scholarship will continue to see changes while remaining basically the same. "Hot topics" will come and go. Our interests will change and our applications of mathematics will change with them. We'll still see expository papers. We'll still see "ideas" as motivations for papers. But in the end, it will always come down to students with a curiosity about mathematics being led by faculty to discover the wonderful world of mathematical scholarship. As Solomon wrote, "There is nothing new under the sun" [Eccl. 1:9b].

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Entropy Properties of 2x2 Games

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Game theory has been examined and exploited in many ways. However, no attempt has been made to rate the value of specific games. That is, there is no way to explicitly describe how "interesting" a game is. This will be the subject of our discussion here. In order to accomplish this task, we will review some relevant concepts from game theory and borrow an idea from the seemingly unrelated field of information theory. After describing the manner in which we will judge games, the properties of this rating system will be examined, culminating in a theorem describing a function that serves as a lower bound for the value of the "quality" of any two-player game where at least one player has exactly two strategies and the other has at least two strategies (that is, any $2 \times n$ matrix where $n \geq 2$).

First, a few assumptions should be introduced. Throughout the text, the work "game" will be understood to mean a contest between two or more players where each has two or more strategies. Each player will choose one strategy with no prior knowledge of the other players' choices. These strategies, when considered in conjunction with each other, will result in a "payoff" for each player that can be described with a real value (it is convenient to think of the payoffs monetarily). The function that describes the relation between the various strategies and the resulting payoffs is commonly introduced visually as an n-dimensional array, where n is the number of players. So, for example, consider the array

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} 1,2 & -1,3 & 0,0 \\ 0,0 & 3,2 & 2,-1 \end{bmatrix} \end{array}$$

Here, the game is played by two players, P and Q. P's strategies correspond to the array's columns, and while Q's are the rows.

Notice that each entry in the array has two numbers. The first is P's payoff and the second Q's (in keeping with the money analogy, we will assume that each player is attempting to maximize their payoff). So, suppose that P had chosen the second strategy, while Q had chosen the first. Then, we see from the entry in the first row and second column that P gains -1 (or loses one dollar, if you like), while Q gains 3 (or receives three dollars). It should furthermore be noted that the scope of this paper is completely limited to two-person games (thus justifying the use of the word "game matrix") where each player's payoff is the negative of the other player's corresponding payoff (such a game is usually referred to as a "zero-sum game," since the sum of the values in every entry of the game matrix will be 0). The reason for this restriction will readily become apparent as the process of rating a game is introduced. Because we are only addressing zero-sum games, we can now streamline our notation a bit so that there is only one value in each entry of the matrix. This value will represent the amount the row player gains and the amount the column player loses. So, we can now write

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} 1,-1 & -1,1 & 0,0 \\ 0,0 & 3,-3 & 2,-2 \\ -2,2 & -2,2 & 1,-1 \end{bmatrix} \end{array}$$

as

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} -1 & 1 & 0 \\ 0 & -3 & -2 \\ 2 & 2 & -1 \end{bmatrix} \end{array}$$

Note that by rewriting our matrices in this fashion, the entries are the row player's payoff. (Here it is handy to think of P paying whatever payoff results to Q.)

One last subject from game theory must be addressed before we tackle the heart of our problem. It is a very simple task to create a game where, using only the tools we have thus far introduced, neither player could find a satisfying optimal strategy. Consider

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

Here, suppose that player P had a rational "best" choice that would optimize his payoff. Arbitrarily, let us assume it is the first column. But then Q, knowing P to be a rational sort, could anticipate this move and would undoubtedly choose the first row as his strategy. P, being rational and thus anticipating this thought process, would have instead chosen the second row, thus minimizing the payoff. But Q, anticipating *this*, would have chosen the second row, and so on. This is not a wholesome way to look at the game. So, we now introduce the idea of mixed strategies. That is, P will choose the first column with a probability $p_1$and the second with a probability p_2. Similarly, Q will choose the first row with probability $q_1$and the second row with probability q_2. We assume a large number of games have been played, and it is a simple task to calculate the expected payoff (as the sum of the product of each entry and the probability it will occur). It is this expected payoff that P must now minimize and Q must maximize, for obvious reasons. An invaluable theorem of game theory guarantees the existence of an equilibrium point for each player's mixed strategies. This describes the payoff for the optimal strategy of both players. Specifically, if one player plays the optimal strategy, the other loses all control of the game; the outcome is the same regardless of the second player's choice of strategies. Conversely, not playing one's optimal strategy gives the other player the possibility of worsening one's payoff. Calculating these optimal strategies is relatively uncomplicated. Viewing the probabilities of choosing each (both player's) strategy as free variables (but noting that if a player has n strategies, there will be only $n-1$ corresponding variables, as the various probabilities must sum to 1), find the multi-variable function describing the expected value (the sum of the product of each entry and the probability it will occur). Differentiating this function with respect to each variable and setting each result to 0 leads to a series of equations that can be solved, thus giving the optimal probabilities (this corresponds to maximizing or minimizing the value of a function as one learned to do in a first semester calculus course). For more information, refer to Owen's *Game Theory* [2](Chapter contains a proof of the Minimax Theorem described above) or Rapaport's *Two-Person Game Theory [3]*, for a bit more elementary approach.

We now turn our attention to information theory (or thermodynamics, if you prefer) in order to acquire our "interestingness" measure. In information theory, one refers to the *entropy* or *measure of uncertainty* of a finite set of probabilities corresponding to mutually exclusive events in order to describe how difficult it is to predict which one will occur. Explicitly, the entropy is defined as

$$H(p_1, p_2, ..., p_n) = \sum_{i=1}^{n} p_i \log_2 p_i,$$

where p_i is the probability of the i^{th} event occurring. Note that throughout the discussion, we adopt the common convention that $0 \log_2 0$ is defined to be 0. The properties of the uncertainty measure and the justification of the above formula as a suitable description should be available in any introductory information theory text, for those who are interested. Most of the author's reading came from the opening chapter of Ash's *Information Theory [1]*.

In order to apply this to a game (recalling that we have limited our attention to two-person, zero-sum games), define a game G with players P and Q and respective strategies $x_1, ..., x_m$ and $y_1, ..., y_n$. Calculate the optimal strategies of each player and let p_i be the probability with which P chooses x_i, defining q_i similarly. Then, we define the entropy associated with G as the arithmetic average of the entropy associated with each player's strategies, or

$$H(G) = \frac{H(p_1, ..., p_m) + H(q_1, ...q_n)}{2}$$

In this way, we can determine the "value" of the game. That is, those with small entropies (near 0), have fairly predictable outcomes, while those with large entropies (near 1) are about as unpredictable and random as possible. Accordingly, games with larger entropies are more "interesting."

Having found a way to describe the uncertainty inherent in a game, we can now examine the entropic properties of the game. We begin by noting that any game (recalling we have limited ourselves to those with which at least one player has exactly two strategies) can be "reduced," without changing the optimal strategies, to a $2x2$ matrix. The proof of this statement will not be given here, but it is not difficult to understand. In essence, it shows that the player with only two strategies can limit the game's payoff to linear combinations of a fixed number of values (actually, the number of values is equal to the number of strategies the other player can choose to play) in such a way that the other player can only suffer by ever considering the possibility of playing more than two strategies. In other words, the probability of choosing a strategy will be 0 for all but at most two strategies, or else the payoff will improve in favor of the player with only two strategies. Thus, when one player has exactly two strategies, the matrix can be reduced to a 2×2 matrix, and it is this situation that will occupy the remainder of the paper.

In order to describe our general game, arbitrarily fix four real values r, s, t, and u and order them in ascending order. So, without loss of generality, we have

$$r \leq s \leq t \leq u$$

These will be the payoffs in our game. Thus we must consider only 24(4!) matrices (that is, there are 24 possible ways to enter these payoffs into our game). However, it should be apparent that switching the order that the rows or columns (or both) appear will have absolutely no effect on the game itself or the game's entropy. Thus, we can always rearrange the game matrix so that the upper left entry has the greatest value, u. Now we need consider but six (3!) matrices. Of these six, four can immediately be eliminated by simple inspection, since the column player will have a dominant strategy in two, and the row player will have a dominant strategy in two others. In each case, the result is a trivial entropy of 0. (Note that these four matrices may not be bound by the inequality described near the end of the paper. However, having found that each has an entropy of exactly 0, there is no need to further explore the uncertainty of the game.) This process leaves us with but two matrices to be examined,

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} u & s \\ r & t \end{bmatrix} \end{array}$$

and

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} u & r \\ s & t \end{bmatrix} \end{array}$$

Now, for ease of computation and clarity, we introduce a scaling function S. Define S as

$$S\left(x\right) = \frac{x - r}{u - r}$$

applying S to each payoff in the above matrices gives

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix} \end{array}$$

and

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} 1 & 0 \\ a & b \end{bmatrix} \end{array}$$

where

$$\begin{aligned} a &= S\left(s\right) \\ b &= S\left(t\right) \end{aligned}$$

Notice,

$$0 \leq a \leq b \leq 1$$

In order to see that this does not alter the entropy of the game, it is a simple task to calculate the optimal strategies of the two new matrices, substitute in the definition of a and b, and note that this result matches the optimal strategies for the two old matrices. (The only time this scaling will not be possible is when all of the payoffs are exactly equal, but then we can assume each player picks one strategy at all times, since they are identical, giving an entropy of 0.)

Armed with our new matrices, it now becomes clear that they are equivalent to each other (an argument similar to the one that follows could have been applied before the scaling function S was introduced): Notice that our second matrix,

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} 1 & 0 \\ a & b \end{bmatrix} \end{array}$$

can be viewed as

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} -1 & -a \\ 0 & -b \end{bmatrix} \end{array}$$

(One way to think of this is to remember our money analogy. In this new matrix, Q is simply paying P instead of the other way around.) Applying S to each entry gives that the previous matrix is equivalent to

$$\begin{array}{cc} & Q \\ P & \begin{bmatrix} 0 & 1-a \\ 1 & 1-b \end{bmatrix} \end{array}$$

and thus

$$\begin{array}{cc} & Q \\ P & \begin{bmatrix} 1 & 1-b \\ 0 & 1-a \end{bmatrix} \end{array}$$

by swapping rows (recall this could be done as each player's strategy is still the same, they only appear in different orders). It is clear that

$$0 \leq 1-b \leq 1-a \leq 1$$

But these were the only limitations placed on our first matrix,

$$\begin{array}{cc} & P \\ Q & \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix} \end{array}$$

and so the analysis that follows is equally valid for either game. Henceforth, we will refer only to this last matrix.

Having found a succinct way to describe every game where at least one player has two strategies, we now turn our attention to increasing our understanding of this matrix. To accomplish this task, we will explicitly calculate $H(G)$, and find a pair of curves that always bound this entropy. In order to do so, we now introduce a few constants. We will denote the probability with which player P chooses the first column while utilizing the optimal strategy as p (so the second column will be chosen with a probability of $1-p$). Similarly, q will be the probability that Q chooses the first row. And finally, in anticipation of the final result, define

$$m = b - a$$

Simply following their definition, it is straightforward to show that

$$p = \frac{b-a}{1+b-a}$$

$$q = \frac{b}{1+b-a}$$

Hence,

$$H(p, 1-p) = \log_2(1+b-a) - \frac{b-a}{1+b-a}\log_2(b-a)$$

and

$$\begin{aligned} H(q, 1-q) &= \log_2(1+b-a) - \frac{b}{1+b-a}\log_2(b) \\ &\quad -\frac{1-a}{1+b-a}\log_2(1-a) \end{aligned}$$

and so

$$\begin{aligned} H(G) &= \log_2(1+b-a) \\ &\quad -\frac{(b-a)\log_2(b-a)+b\log_2(b)+(1-a)\log_2(1-a)}{2(1+b-a)} \end{aligned}$$

And, finally, it becomes time for our final result. Note

$$H(G) \leq 1$$

since $H(p, 1-p)$ and $H(q, 1-q)$ are, individually, always less than or equal to 1. (While we have not previously discussed this fact, it is a consequence of our definition of entropy.) However, finding a lower bound is not quite so straightforward. As it will turn out,

$$H\left(\frac{m}{m+1}, \frac{1}{m+1}\right) \leq H(G)$$

To see this, first note that

$$\begin{aligned} H\left(\frac{m}{m+1}, \frac{1}{m+1}\right) &= \log_2(m+1) - \frac{m}{m+1}\log_2(m) \\ &= \log_2(1+b-a) - \frac{b-a}{1+b-a}\log_2(b-a) \end{aligned}$$

So, the problem becomes to show that

$$\begin{aligned} &\log_2(1+b-a) - \frac{b-a}{1+b-a}\log_2(b-a) \\ \leq\ &\log_2(1+b-a) \\ &-\frac{(b-a)\log_2(b-a) + b\log_2(b) + (1-a)\log_2(1-a)}{2(1+b-a)} \end{aligned}$$

or, equivalently, that

$$0 \leq (b-a)\log_2(b-a) + b\log_2(b) - (1-a)\log_2(1-a)$$

(Here we just subtract the left side of the inequality from both sides and multiply by $2(1+b-a)$.) First note that if $b = a$, then the inequality does hold. So, let $a < b$ and define the function h as the right side of this new inequality:

$$h(a,b) = (b-a)\log_2(b-a) + b\log_2(b) - (1-a)\log_2(1-a).$$

Noting that

$$\frac{dh}{db} = \log_2\left(1 - \frac{a}{b}\right),$$

one realizes that $\frac{dh}{db} < 0$, since $a < b$. Thus it is clear that h is decreasing for a fixed a. But,

$$h(a,a) = -\log_2(a) - (1-a)\log_2(1-a) \geq 0$$
$$h(a,1) = 0$$

Thus, for any value of a between 0 and 1 and any b between a and 1,

$$h(a,b) \geq 0$$

as required.

To reiterate what we have found,

$$H\left(\frac{m}{m+1}, \frac{1}{m+1}\right) \leq H(G) \leq 1$$

for any non-trivial game G where one player has exactly two strategies.

Before we end, one is certainly entitled to examine this cryptic inequality a bit. The portion on the right is not mysterious. It simply reminds us that, as a direct consequence of our definition of a game's uncertainty, 1 describes the most uncertain game possible. The left inequality, however, is bit overpowering. To appreciate what this means, notice that we have found a lower bound for the average of two entropies (recall $H(G)$'s definition) in terms of the definition of entropy itself and the scaled entries of the game matrix, *not* the probabilities associated with the optimal strategies, as seems most natural. This undoubtedly deserves more than passing interest, however, for now, the matter must be postponed. I leave you with a picture of what we have been exploring.

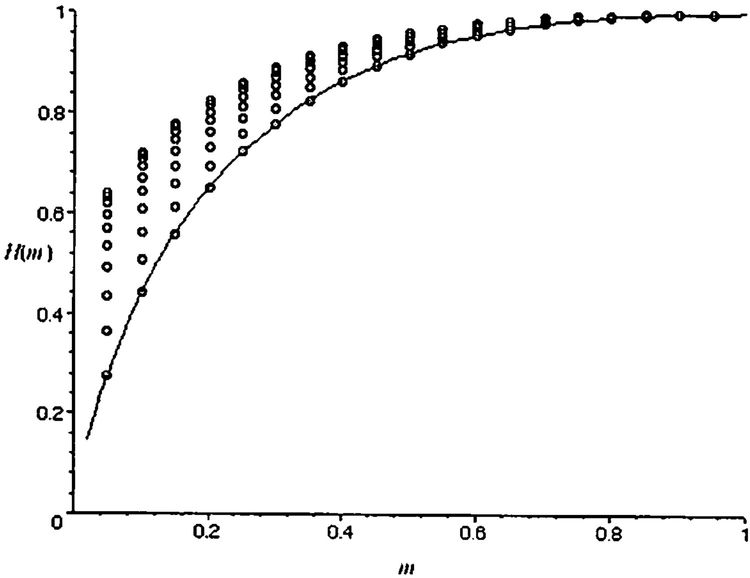

Figure 1

The horizontal axis is m, as defined above, and the vertical is the entropy of the general game we found above. For various values of m, ten values of a were chosen, thus uniquely defining a game. The circles correspond to these games. Along the bottom lies the graph of our lower bound. $H\left(\frac{m}{m+1}, \frac{1}{m+1}\right)$.

References

1. Ash, Robert B., Information Theory, New York (Dover Publications, Inc.), 1965.

2. Owen, Guillermo, Game Theory, Philadelphia (W. B. Saunders Company), 1968.

3. Rapaport, Anatol, Two-Person Game Theory, Ann Arbor (The University of Michigan Press), 1966.

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before January 1, 2002. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 2002 issue of *The Pentagon*, with credit being given to the student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

PROBLEMS 545-549

Problem 545. Proposed by the editor.

The sequences $A_k = 8*10^{2k}+4*10^k$, $B_k = 8*10^{2k}+4*10^k+1$ and $C_k = 4*10^k+1$ where k is a positive integer generate an infinite set of Heronian triangles in which the area of the triangle is 10^k times the perimeter of the triangle. These sequences allow automatic generation of an infinite list of these particular Heronian triangles through the use of a calculator or computer. Find another set of similar sequences which has the same property; i.e. the sequences generate an infinite set of Heronian triangles in which the area of the triangle is 10^k times the perimeter of the triangle and $B_k - A_k = 8$.

Problem 546. Proposed by Adrian C. Keister, Grove City College, Grove City, Pennsylvania.

Prove or disprove the following theorem:
Suppose a function f is three times differentiable on the interval (a, b). Suppose there exists a point c in (a, b) such that $f''(c) = 0$ but $f'''(c)$ is not equal to zero. Then c is an inflection point of f.

Problem 547. Proposed by the editor.

Evaluate the sum

$$\cos 9^\circ + \cos 49^\circ + \cos 89^\circ + \cos 129^\circ + \cos 169^\circ$$
$$+ \cos 209^\circ + \cos 249^\circ + \cos 289^\circ + \cos 329^\circ$$

A solution which does not use a calculator or computer is preferred.

Problem 548. Proposed by Jose Luiz Diaz, Universitat Politecnica de Catalunya, Terrassa, Spain.

Let n be a positive integer. Prove that

$$2F_{n+2} < \frac{F_n^2}{F_{n+2}} + \frac{F_{n+1}^2}{F_n} + \frac{F_{n+2}^2}{F_{n+1}}$$

where F_n is the n^{th} Fibonacci number. That is, $F_0 = 0$, $F_1 = 1$ and for $n \geq 2$, $F_n = F_{n-1} + F_{n-2}$.

Problem 549. Proposed by Bryan Dawson, Union University, Jackson, Tennessee.

A soft rain falls vertically at a speed of 12 miles per hour. You are in your car stopped at a stoplight. As the light turns green, you accelerate to 45 miles per hour and notice, of course, that more water now hits the windshield. If your windshield is inclined 60° from the horizontal, what is the ratio of the water hitting your windshield at 45 miles per hour compared to water hitting your windshield at rest ignoring any possible aerodynamic effects of the vehicle?

Please help your editor by submitting problem proposals.

SOLUTIONS 535–539

Problem 535. Proposed by the editor.

Determine the smallest value of $|18^p - 7^q|$ where p and q are positive integers and $||$ denotes absolute value.

Solution by Rozy Brar, California State University, Fresno, California.

Looking at possibilities for p and q, we notice that $18^p \equiv 18, 24, 32, 68$ and $72 \pmod{100}$ and $7^q \equiv 1, 7, 43$ and $49 \pmod{100}$. Considering possible differences, the smallest difference, in absolute value, between 18^p and 7^q is $11 = |18 - 7| = |32 - 43|$. Thus the smallest value of $|18^p - 7^q|$ is 11 when $p = q = 1$.

Problem 536. Proposed by the editor.

Let P(n) denote the product of the divisors of n (including 1 and n) Find, with proof, the smallest integer n such that $P(n) = n^8$ where:

(a) n is an integer;
(b) n is a perfect square; and
(c) n is a perfect cube.

Solution by Daniel Springer, California State University, Fresno, California.

Let the prime decomposition on n in canonical form be

$$n = \prod_{i=1}^{r} (p_i^{k_i}) \tag{1}$$

From (1), we use the number theoretical function $\tau(n)$ which denotes the number of positive divisors of the positive number n; i.e.

$$\tau(n) = \prod_{i=1}^{r} (k_i + 1) \tag{2}$$

where the prime decomposition of n in canonical form is given by (1).
By (2), the product of the positive divisors of n is given by

$$P(n) = n^{\left(\frac{\tau(n)}{2}\right)} \tag{3}$$

Then, by (3), $P(n) = n^8$ so that $\tau(n) = 16$. Thus

$$\tau(n) = \prod_{i=1}^{r} (k_i + 1) = 16 = 8 * 2 = 4 * 4 = 4 * 2 * 2 = 2 * 2 * 2 * 2$$

and the possible forms for n in part (a) are $p_1^{15}, p_1^7 p_2, p_1^3 p_2 p_3$, or $p_1 p_2 p_3 p_4$. Taking $p_1 = 2, p_2 = 3, p_3 = 5$ and $p_4 = 7$, and testing the possibilities shows that $n = 2^3 * 3 * 5 = 120$ is the smallest solution for part (a).

For part (b), since n is a perfect square, by (1), (2), and (3) we have

$$n = \prod_{i=1}^{r} \left(p_i^{2k_i}\right)$$

and

$$\tau(n) = \prod_{i=1}^{r} (2k_i + 1) = 16$$

which is impossible since $2k_i + 1$ is odd for all choices of i. Hence there are no solutions for part (b).

For part (c), since n is a perfect square, by (1), (2), and (3) we have

$$n = \prod_{i=1}^{r} \left(p_i^{3k_i}\right)$$

and

$$\tau(n) = \prod_{i=1}^{r} (3k_i + 1) = 16$$

so n must have the form p_1^{15} or $p_1^3 p_2^3$. Taking $p_1 = 2$ and $p_2 = 3$ and checking the possibilities, we find the smallest solution for part (c) to be $n = 2^3 * 3^3 = 216$.

Also solved by Clayton Dodge, University of Maine, Orono, Maine.

Editor's comment. Dodge correctly pointed out that $n = 1$ trivially satisfies all conditions of the problem and then found the same solutions as found by our featured solver. Sources for the results relied on by our featured solver are:

1. Oystein Ore, Number Theory and its History, McGraw-Hill Book Company, Inc., New York, 1949, p86. {Ore uses $v(n)$ instead of the more often seen $\tau(n)$.}
2. Oystein Ore, Number Theory and its History, McGraw-Hill Book Company, Inc., New York, 1949, p87. Theorem 5-2.

Problem 537. Proposed by the editor.

Let r, s, t, u and v be integers such that both their sum and the sum of their squares are divisible by an odd prime p. Prove that p also divides the quantity $r^5 + s^5 + t^5 + u^5 + v^5 - 5rstuv$.

Since no solutions have been received, this problem will remain open for another issue.

Problem 538. Proposed by the editor.

An eccentric gardener with a mathematical penchant has a group of gardens which have the following common properties: each garden has a triangular shape such that the area of the garden is twice the perimeter; each side is an integral number of feet; and in each garden two sides are consecutive integers. How many gardens does the eccentric gardener have and what are the dimensions of each garden?

Since no solutions have been received, this problem will remain open for another issue.

Problem 539. Proposed by the Albert White, St. Bonaventure University, St. Bonaventure, New York.

For points x, y, z, let $[x, y, z]$ denote the area of the triangle formed by the points x, y and z. Let a, b and c be the vertices of a right triangle. Find the point x such that $[a, b, x]^2 + [a, c, x]^2 + [b, c, x]^2$ is a minimum. For the purposes of this problem assume that all points lie in the same plane.

Solution by Carol Browning, Drury College, Springfield, Missouri.

Without loss of generality, we may situate the right triangle in the plane with the right angle c at the origin, the vertex a at the point $(A, 0)$ on the positive x-axis, and the vertex b at the point $(0, B)$ on the positive y-axis.

First notice that $[a, c, x]^2 = (\frac{AY}{2})^2$ and $[b, c, x] = (\frac{BX}{2})^2$. To find $[a, b, x]$, we consider the line from a to b as the base of the triangle formed by a, b, and x. Then the length of the base is $(A^2 + B^2)^{1/2}$. To find the height of that triangle, we drop a perpendicular from x to the line from a to b. The equation of the line from a to b is $y = (-\frac{B}{A})x + B$, and that line has slope $(-\frac{B}{A})$. The perpendicular then has slope $\frac{A}{B}$ and contains the point (X, Y), so it has the equation $y - Y = (\frac{A}{B})(x - X)$. Solving this system of two equations, we see that the perpendicular intersects the base at the point

$$\left(\frac{AB^2 + A^2X - ABY}{A2 + B2}, \frac{A^2B + B^2Y - ABX}{A^2 + B^2}\right)$$

After computing the distance from x to this intersection point, we find that

$[a,b,x]^2 = \frac{1}{4}[A^2B^2 - 2AB^2X - 2A^2BY + B^2X^2 + 2ABXY + A^2Y^2]$

Therefore, we want to minimize the function $f(X,Y) = \frac{1}{4}[A^2B^2 + 2A^2Y^2 + 2B^2X^2 - 2A^2BY - 2AB^2X + 2ABXY]$. It suffices to minimize $g(X,Y) = 4f(X,Y) = A^2B^2 + 2A^2Y^2 + 2B^2X^2 - 2A^2BY - 2AB^2X + 2ABXY$. The critical points for this function are the points where $G_x = 0$ and $G_y = 0$; that is, $4B^2X - 2AB^2 + 2ABY = 0$ and $4A^2Y - 2A^2B + 2ABX = 0$. Solving two equations in two unknowns, we find the only critical point to be when $X = \frac{A}{3}$ and $Y = \frac{B}{3}$.

The value of $G_{xx}G_{yy} - G_{xy}^2 = (4B^2)(4A^2) - (2AB)^2 = 12A^2B^2$, which is always positive. Since G_{xx} is always positive, the second partials test guarantees that the point $(\frac{A}{3}, \frac{B}{3})$ yields our desired minimum.

Therefore the point x that causes $[a,b,x]^2 + [a,c,x]^2 + [b,c,x]^2$ to be a minimum is found by moving from the right angle $\frac{1}{3}$ of the way to each of the other two vertices. In vector notation, if c is the point at which the right angle occurs, x is $\frac{1}{3}$ times the vector from c to a + $\frac{1}{3}$ times the vector from c to b.

Also solved by the proposer.

Editor's Comment. The proposer notes that the desired point is the triangle's center of mass.

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Kappa Mu Epsilon News

Edited by Don Tosh, Historian

News of chapter activities and other noteworthy KME events should be sent to Don Tosh, Historian, Kappa Mu Epsilon, Mathematics Department, Evangel College, 1111 N. Glenstone, Springfield, MO 65802, or to toshd@evangel.edu.

Chapter News

AL Gamma Chapter President—Chris Harmon
University of Montevallo, Montevallo 18 actives

This spring KME is sponsoring a field trip to the Marshal Space Flight Center in Huntsville, AL. Coffee mugs (bearing the derivative of the Cauchy integral formula) are being sold as a fundraiser. Other fall 2000 officers: Tommy Fitts, vice president; Jared Phillips, secretary/treasurer; Don Alexander, corresponding secretary; Michael Sterner, faculty sponsor.

AL Zeta Chapter President—Molly Gibson
Birmingham Southern College, Birmingham 15 actives

Other fall 2000 officers: Ansley Collins, vice president; Sarah Beth Coffey, secretary/treasurer; Mary Jane Turner, corresponding secretary; Jeff Barton, faculty sponsor.

AR Alpha Chapter President—Michael Mott
Arkansas State University, State University 7 actives, 3 associates

Other fall 2000 officers: Laura Firestone, secretary; Jacob Hamilton, treasurer; William Paulsen, corresponding secretary.

CO Delta Chapter President—Raymond W. Mitchell
Mesa State College, Grand Junction

Other fall 2000 officers: Jerri Fay, vice president; Richard Hasenauer, secretary; Abigail Fleming, treasurer; Donna Hafner, corresponding secretary; Kenneth Davis, faculty sponsor.

GA Alpha Chapter President—Karen Jones
State University of West Georgia, Carrollton 20 actives

At our first meeting on October 18, membership certificates and pins were distributed to the new initiates and a Fall Social was planned at a local Mexican restaurant for later in the semester. For the fourteenth consecutive year, Georgia Alpha sponsored its Food and Clothing Drive for the Needy with the proceeds being delivered to the Salvation Army at the end of the semester. Our Fall Social was well attended and everyone had a very nice time. Other fall 2000 officers: Christin Phillips, vice president; Kaitlin Lewis, secretary; Daemon Whittenburg, treasurer; Joe Sharp, corresponding secretary; Mark Faucette and Joe Sharp, faculty sponsors.

GA Gamma Chapter President—Tony McCullers
Piedmont College, Demorest 4 actives

Other fall 2000 officers: Heather Knight, vice president; Amie Mills, secretary; Tony McCullers, treasurer; Shahryar Heydari, corresponding secretary/faculty sponsor.

IL Zeta Chapter President—Janine Smuda
Dominican University, River Forest 28 actives

Other fall 2000 officers: Nora Hussar, vice president; Christine Pellini, secretary; Amanda Schreckenberg, treasurer; Sarah Ziesler, corresponding secretary/faculty sponsor.

IL Theta Chapter President—Aero McReynolds
Benedictine University, Lisle 13 actives

President McReynolds gave a talk on her summer actuarial internship. A representative of the Casualty Actuary Society also gave a talk on careers in actuarial science. Other fall 2000 officers: Mariam Ahmed, secretary; Lisa Townsley Kulich, corresponding secretary; Tracii Friedman, faculty sponsor.

IN Beta Chapter President—J. Nicholas Deibel
Butler University, Indianapolis

Other fall 2000 officers: Rebecca Wahl, corresponding secretary/faculty sponsor.

IN Gamma Chapter President—Courtney Taylor
Anderson University, Anderson 8 actives, 8 associates

Other fall 2000 officers: Matthew Cherry, vice president; Stanley Stephens, corresponding secretary/faculty sponsor.

IA Alpha Chapter President—Allysen Edwards

University of Northern Iowa, Cedar Falls 31 actives

Student member Brad Rolling presented his paper "Mayan Mathematics" at our first fall KME meeting in September at Professor Syed Kirmani's residence. Our homecoming coffee was held at Mr. Carl Wehner's residence on September 30. Our second meeting was held on October 25 at Professor Greg Dotseth's residence. Michelle Day presented her student paper on "Emee Netter" at our November meeting held at Professor Russ Campbell's residence. Student member LaNel Carey addressed the spring initiation banquet with "Perfect Numbers". Our banquet was held at Beck's restaurant on December 4, where six new members were initiated. Other fall 2000 officers: Brad Rolling, vice president; Teresa Grothus, secretary; Barbara Meyers, treasurer; Mark Ecker, corresponding secretary/faculty sponsor.

IA Gamma Chapter President—A. Kruger

Morningside College, Sioux City 18 actives

Four of our members tutored TAG students. Other fall 2000 officers: Mary Curry, vice president; Michelle Harvey, secretary; Kyle Kolander, treasurer; Doug Swan, corresponding secretary/faculty sponsor.

IA Delta Chapter President—Janelle Young

Wartburg College, Waverly 38 actives, 3 associates

Our September meeting consisted of planning the annual KME Roy's Egg-Cheese booth for Homecoming and planning other activities during the year. Our program was senior Molly Mason reporting about her summer internship with the Federal Reserve Board in Washington, D.C., where she helped prepare currency exchange rate reports. In October the Chapter learned that, because of donated supplies and good sales, the Roy's Egg-Cheese booth made approximately $225 this year at Homecoming. Volleyball and other activities with the physics club, Psi Phi, were also discussed. Chrissy Johnson, a 1999 graduate of Wartburg, spoke about her career path in mathematics since her graduation. She found her business electives very helpful for her work applying her mathematics major.

Other fall 2000 officers: Dawn Brandau, vice president; Matthew Reuer, secretary; Angela Helland, treasurer; August Waltmann, corresponding secretary; Mariah Birgen, faculty sponsor.

KS Beta Chapter President—Katrina Penner

Emporia State University, Emporia 22 actives, 2 associates

Other fall 2000 officers: Leah McBride, vice president; Melinda Born, secretary; Thad Davidson, treasurer; Connie Schrock, corresponding secretary; Larry Scott, faculty sponsor.

KS Gamma Chapter President—Angela Shomin

Benedictine College, Atchison 7 actives, 15 associates

At a 12 September meeting two KS Gamma members talked about their summer intern experiences - Brett Herbers with Xerox Corporation and Angela Shomin with Intel. On 20 September KS Gamma students gathered in Schroll Center for a Pizza Party to welcome new students attracted to KME through the Club Fair held on 13 September. In November two KS Gamma members joined the faculty in welcoming prospective students at the "Destination BC Open House." Junior Angela Shomin talked about her interest in Math/CS and about her experience as part of a COMAP team last year. Sophomore April Bailey spoke of her motivation to pursue mathematics and be certified to teach on the secondary level. Each shared family and demographic background as well. A large crowd of students came to Marywood, home of moderator Sister Jo Ann Fellin, for the traditional Wassail party on 4 December. Other fall 2000 officers: Janelle Kroll, vice president; Jo Ann Fellin, corresponding secretary/faculty sponsor.

KS Delta Chapter President—Mark Allen Smith

Washburn University, Topeka 26 actives

The Kansas Delta Chapter of KME met with the Washburn University math club, Mathematica, on three different occasions during the semester. At each there was a lunch and at the last two we had a speaker. Other fall 2000 officers: Sue Ann Hamon, vice president; Melissa Mikkelsen, secretary/treasurer; Allan Riveland, corresponding secretary; Ron Wasserstein and Donna LaLonde, faculty sponsors.

KS Epsilon Chapter President—Wendy Scott

Fort Hays State University, Hays 21 actives, 30 associates

Other fall 2000 officers: Lora Clark, vice president; Adam North, secretary; Chenglie Hu, corresponding secretary; Lanee Young, faculty sponsor.

KY Alpha Chapter President—Katy Fritz

Eastern Kentucky University, Richmond 38 actives

The semester began with floppy disk sales (together with the ACM chapter) to students in the computer literacy class and the Mathematica class. At the September meeting, we had the election of Treasurer and discussed plans for the year. It was decided to contribute to the campus collection for Habitat for Humanity. The October meeting was held on the 30th and was advertised as a Ghostly Meeting. At homecoming, John Leachman (a KME member) was elected King. At the November meeting, Dr. Matthew Cropper gave a talk entitled "Stable Marriages and Surprising Relations." He picked 4 males and 4 females from the audience and made them rank the members of the opposite sex and matched them up. In December we had our White Elephant Gift Exchange at the Christmas party at J. Patrick's. The big gag gift was a pair of women's underwear that was big enough for both Alison and Katy to fit in. As a service project at Christmas time, members collected toys for children ages 3-5, put them in six stockings, and took them to the Salvation Army. Other fall 2000 officers: Alison Marshall, vice president; Jon Fulkerson, secretary; Melanie Puckett, treasurer; Pat Costello, corresponding secretary.

KY Beta Chapter President—Eric Ritchie

Cumberland College, Williamsburg 29 actives

On September 5, 2000, the Kentucky Beta chapter officers helped to host an ice cream party for the freshmen math and physics majors. Along with the Mathematics and Physics Club and Sigma Pi Sigma, the chapter had a picnic at Briar Creek Park on September 19. Several members of the chapter traveled to Marshall Space Flight Center in Huntsville, Alabama, on November 4. On November 16, the Kentucky Beta Chapter had an informal dinner get-together at the steakhouse in Corbin. On December 8, the entire department, including the Math and Physics Club, the Kentucky Beta chapter, and

Sigma Pi Sigma had a Christmas party with almost 60 people in attendance. Other fall 2000 officers: Amanda Kidd, vice president; Kevin Floyd, secretary; Mark Vernon, treasurer; Jonathan Ramey, corresponding secretary; John Hymo, faculty sponsor.

MD Beta Chapter President—Paul Ostazeski
Western Maryland College, Westminster 21 actives

We held a picnic for all mathematics majors at the beginning of the fall semester. Dennis Lucey, one of our student members, gave a talk about the research he did this past summer on Braid Theory at Louisiana State University. The movie The Matrix was shown at a pizza party. An ongoing activity was providing free tutoring services for students taking mathematics and computer science courses. Other fall 2000 officers: Amy Bittinger, vice president; Teresa Needer, secretary; Susan Miller, treasurer; Linda Eshleman, corresponding secretary; Harry Rosenzweig, faculty sponsor.

MD Delta Chapter President—Erin Resh
Frostburg State University, Frostburg 32 actives

Maryland Delta Chapter opened the semester with an organizational meeting in September. It was agreed to pursue the possibility of setting up a volunteer KME tutoring service for courses at the calculus level and above. After much work on the part of our vice president, Angela Myers, this service was instituted during the last few weeks of the semester, and is expected to continue in the spring. The semester was capped off by a talk from exchange professor Diarmuid O'Driscoll on the beauty and customs of Ireland, his native country. Other fall 2000 officers: Angela Myers, vice president; Sabrina Ritchie, secretary; Carrie Snyder, treasurer; Edward White, corresponding secretary; John Jones, faculty sponsor.

MI Epsilon Chapter Presidents—Drew Spooner & Justin McCurdy
Kettering University, Flint 74 actives

A-section: During the Summer Term of 2000 the Math Noon-Time Movie "The Man Who Loved Numbers" was shown on the second week of the term. This is the remarkable story of the self-taught Indian clerk, Srinivas Ramanujan, who burst on the mathematical scene earlier this century. The ripples from his work are still felt today. New A-section officers were elected, and two pizza parties were held. Professor Brian McCartin presented his exciting talk entitled "Musical Geometry" wherein some applications of simple geometry are applied to understand classical music. He did it with a finest manner playing some musical examples for the public on his own synth, and it was fun for all, intelligible to everyone, even with no knowledge of music theory. The membership certificates and pins for new members were distributed. Professor Jo Smith was presented with a recognition plaque in appreciation for her contributions and hard work to KME as the Faculty Corresponding Secretary. Professor Boyan Dimitrov stepped forward to volunteer as Faculty Corresponding Secretary for the chapter. B-Section: "The Man Who Loved Numbers" was shown for a second time for the students of the fall term. New B-section officers were elected, and two pizza parties were held. Professor Brian McCartin repeated his presentation on the "Musical Geometry" and played more musical examples for the public. Everyone had fun again. Membership Certificates and pins for new members were distributed. The greatest most news come when our faculty sponsor, Professor Brian McCartin, received "The Kettering University Year of 2000 Outstanding Researcher Award" in recognition and appreciation for his more than 80 published papers and contributions in the field of Computational Mathematics, Wave Propagation and Human Biology, ranging from the design of stealth vehicles to stock option pricing, from the analysis of human genetics to the geometry of music. We are especially proud of his 44 papers in about 10 years since joining Kettering's faculty, and of 8 publications that he has co-authored with Kettering undergraduates. Congratulations, Brian! For more information, please see http://www.kettering.edu/acad/scimath/appmath/award.html or http://www.kettering.edu/~kme/. Other fall 2000 officers: Heather Palmer & Stephen Cmar, vice presidents; Lindsay Allor & Jill Tebbe, secretaries; Ben Myers & Stuart Sherry, treasurers; Boyan Dimitrov, corresponding secretary; Brian McCartin, faculty sponsor.

MS Alpha Chapter President—Chris Sansing

Mississippi University for Women, Columbus 12 actives, 2 associates

In September we held a monthly meeting at which we changed the constitution. In October we held a monthly meeting and also had an initiation for two new members. At the November meeting Dr. LeRoy Wenstrom from the Mississippi School for Mathematics and Science gave a talk "0=1 or How to Read Theorems More

Carefully". Other fall 2000 officers: Mindy Hill, vice president; Jennifer Kimble, secretary; Kent Smith, treasurer; Shaochen Yang, corresponding secretary; Beate Zimmer, faculty sponsor.

MS Epsilon Chapter President—Ashley Burleson
Delta State University, Cleveland 15 actives

Other fall 2000 officers: L'Kenna Whitehead, vice president; Natalie Pavlich Hanks, secretary/treasurer; Paula Norris, corresponding secretary; Rose Strahan, faculty sponsor.

MO Alpha Chapter President—Sheri Puestow
Southwest Missouri State University, Springfield 30 actives, 6 associates

MO Alpha chapter hosted the departmental fall picnic and had monthly meetings. Featured speakers at the meeting included a representative from the Career Planning and Placement Office and two faculty members. Other fall 2000 officers: Jeff Rice, vice president; Erin Stewart, secretary; Jason Grout, treasurer; John Kubicek, corresponding secretary/faculty sponsor.

MO Gamma Chapter President—Laura Cline
William Jewell College, Liberty 16 actives

Other fall 2000 officers: Shane Price, vice president; Joel Campbell, secretary; Truett Mathis, treasurer/corresponding secretary/faculty sponsor.

MO Epsilon Chapter President—Sarah Moulder
Central Methodist College, Fayette 10 actives

Other fall 2000 officers: Amy Ketchum, vice president; Beth Kurtz, secretary/treasurer; William McIntosh, corresponding secretary; Linda Lembke & William McIntosh, faculty sponsors.

MO Theta Chapter President—David Bush
Evangel University, Springfield 5 actives, 2 associates

Meetings were held monthly. A social was held at the home of Don Tosh. Other fall 2000 officers: John Gale, vice president; Don Tosh, corresponding secretary/faculty sponsor.

MO Iota Chapter Presidents—Dondi Mitchell & Ted Walker
Missouri Southern State College, Joplin 15 actives

Meetings were held monthly. We worked at the concession stands at the home football games as a fund-raiser. We invited a representative from the Mathematics and Statistics Department at University of

Missouri - Rolla and a representative from the Mathematics Department at the University of Missouri - Columbia to give mathematics talks and visit with our majors about graduate school. We finished the fall with a Christmas party. Other fall 2000 officers: Luke Krudwig, vice president; Heather Vannaman, secretary; Jeremy Goins, treasurer; Chip Curtis, corresponding secretary; Rich Laird, faculty sponsor.

MO Kappa Chapter President—Kasey Boggs
Drury University, Springfield 18 actives, 6 associates

There was a high turn out for the first social event of the semester held at Dr. Carol Browning's house. The winner of the annual math contest this year was David Shade for the Calculus II and above division and Jeff Clark for the Calculus I and below division. Prize money was awarded to the winners at a pizza party held for all contestants. The chapter participated in the Annual Exploration in Mathematics and the Physical Sciences, which is a recruitment workshop designed for high school students. Sub sandwiches were served to the chapter at the preliminary undergraduate research talks given by Michael Victorine and David Crabtree. The Math Club has also been running a tutoring service for both the day school and the continuing education division (Drury Evening College). The semester ended with a Christmas party at Mr. Greg Eastman's house. Other fall 2000 officers: David Crabtree, vice president; Karen Sandreczki, secretary; Jay Howell, treasurer; Charles Allen, corresponding secretary; Al Letarte, faculty sponsor.

MO Lambda Chapter President—Shane Taylor
Missouri Western State College, St. Joseph 10 actives

Other fall 2000 officers: Josh Albin, vice president; Robert Horton, secretary/treasurer; Donald Vestal, Jr., corresponding secretary/faculty sponsor.

MO Mu Chapter President—James Hammond
Harris-Stowe State College, St. Louis 15 actives, 5 associates

Chapter president James Hammond ran an information session about Kappa Mu Epsilon at the Freshmen and New Student Orientation at the beginning of the semester. He and vice president Bruce Green organized a math club meeting which was open to all interested students. Activities at the meeting included experimentation with Mobius strips and a golf game involving geometry and trigonometry. Other fall 2000 officers: Bruce Green, vice president; Jack Behle, corresponding secretary; Ann Podleski, faculty sponsor.

NE Beta Chapter President—Brenna Knott
University of Nebraska at Kearney, Kearney 20 actives, 1 associate

Other fall 2000 officers: Jenny Gier, vice president; Jeny Rutar, secretary; Scott Barber, treasurer; Stephen Bean, corresponding secretary; Richard Barlow, faculty sponsor.

NE Delta Chapter President—Chad Parker
Nebraska Wesleyan University, Lincoln 22 actives

Other fall 2000 officers: David Sovey, vice president; Thor Esbensen, secretary/treasurer; Kristin Pfabe, corresponding secretary/faculty sponsor.

NM Alpha Chapter President—Tony Malerich
University of New Mexico, Albuquerque 110 actives, 13 associates

Our chapter activities are listed at: http://math.unm.edu/~kme. Other fall 2000 officers: William Tierney, vice president; Paloma Wells, secretary/treasurer; Archie Gibson, corresponding secretary/faculty sponsor.

NY Eta Chapter President—Chris Laden
Niagara University, Niagara University 20 actives

Other fall 2000 officers: Courtney Fitzgerald, vice president; Amanda Everts, secretary/treasurer; Robert Bailey, corresponding secretary; Eduard Tsekanovskii, faculty sponsor.

NY Kappa Chapter President—Janine Pryor
Pace University, New York 15 actives

Other fall 2000 officers: Geraldine Taiani, corresponding secretary; Robert Cicenia, faculty sponsor.

NY Lambda Chapter President—Stephanie Calzetta
C.W. Post Campus of Long Island University, Brookville 25 actives

Other fall 2000 officers: Renee des Etages, vice president; Elizabeth Keating, secretary; Andrea Lorusso, treasurer; Andrew Rockett, corresponding secretary; John Stevenson, faculty sponsor.

NY Nu Chapter President—Catherine Paolucci
Hartwick College, Oneonta 12 actives

Other fall 2000 officers: Meghan Hickey, vice president; Theresa Spano, secretary; Barrett Snedaker, treasurer; Ronald Brzenk, corresponding secretary/faculty sponsor.

OH Gamma Chapter President—Jeff Smith
Baldwin-Wallace College, Berea 11 actives

The chapter sponsored a talk by Dr. Gordon Wade from the Department of Mathematics and Statistics at BGSU. Other fall 2000 officers: Conna Moise, vice president; Marianne Fedor, secretary; Jason Papovic, treasurer; David Calvis, corresponding secretary/faculty sponsor.

OH Zeta Chapter President—Kelly Fonner
Muskingum College, New Concord 15 actives, 6 associates

Other fall 2000 officers: Jarrod Dalton, vice president; Tonia Baker, secretary; Dave Morgan, treasurer; Richard Daquila, corresponding secretary/faculty sponsor.

OK Alpha Chapter President—Aaron Lee
Northeastern State University, Tahlequah 35 actives, 1 associate

Our Fall initiation ceremonies brought 13 new students into our chapter. Dr.Wendell Wyatt, assistant professor of mathematics, also became a member. Although not a member of KME, the Dean of our College of Math, Science, and Nursing, Dr. Douglas Harrington, has been very supportive of the Oklahoma Alpha Chapter for many years. The students voted to pay Dr. Harrington's membership and surprise him at the Fall initiation. We sponsored a booth at NSU's free annual Halloween Carnival. Our "KME Pumpkin Patch" was a hit among the small children. They fished for pumpkins with meter stick fishing poles. KME participated in a high school student orientation and recruitment at Northeastern State University. We passed out packets containing information about KME, the Mathematics Department at NSU, and the math puzzle "Cherry in the Limeade Glass." We sponsored two speakers this semester. Dr. Wendell Wyatt, NSU, presented a workshop on "The Geometer's Sketchpad." Mr. David O'Neil, mathematics teacher at Tahlequah High School, gave a presentation on "National Board Certification." The annual book sale was held in November. The Christmas party held before finals week was filled with pizza, Christmas treats, and games, and again the students beat the faculty! Other fall 2000 officers: Miranda Hale, vice president; Gabriela Veith, secretary; Shannon Hilburn, treasurer; Joan Bell, corresponding secretary/faculty sponsor.

OK Gamma Chapter President—Jamie Smith

Southwestern Oklahoma State University, Weatherford 27 actives, 8 associates

We held a meeting every three weeks during the fall. Other fall 2000 officers: Kristy Koger, vice president; Julie Williams, secretary; Alana Schimmer, treasurer; Wayne Hayes, corresponding secretary; Gerry East, faculty sponsor.

OK Delta Chapter President—Jason Buendorf

Oral Roberts University, Tulsa 39 actives

Other fall 2000 officers: Molly VanGorp, vice president; Ivan Anderson, secretary/treasurer; Dorothy Radin, corresponding secretary; Vincent Dimiceli, faculty sponsor.

PA Alpha Chapter President—Melissa Fye

Westminster College, New Wilmington 12 actives

We took a trip to Carnegie Science Center, sponsored a movie night, and sponsored Career Night. Other fall 2000 officers: Gabriella Orr, vice president; Lee Stefanis, secretary; Erin Clohessy, treasurer, Warren Hickman, corresponding secretary, Carolyn Cuff, faculty sponsor.

PA Delta Chapter President—Susan Carlo

Marywood University, Scranton 3 actives

Other fall 2000 officers: Susan Kulikowski, secretary/treasurer; Sr. Robert Ann Von Ahnen, corresponding secretary/faculty sponsor.

PA Mu Chapter President—Geri Cooper

Saint Francis University, Loretto 18 actives

PA Mu co-sponsored the Seventh Annual Science/Mathematics Day on November 17, where nearly 400 high school students and teachers from 25 high schools came to campus for a variety of presentations, a science bowl competition, and several other challenging activities. Several members also participated in a week-long summer mathematics camp for middle school girls, where 30 girls participated in enrichment activities and applications of mathematics. KME members participated as group leaders, mentors, presenters, and role models. Other fall 2000 officers: Derek Warner, vice president; Jamie Krusinsky, secretary; Matt Bollinger, treasurer; Peter Skoner, corresponding secretary; Amy Miko, faculty sponsor.

PA Omicron Chapter President—Andrew Stumpf

University of Pittsburgh at Johnstown, Johnstown 33 actives

There were no programs, meetings, or activities solely sponsored by KME chapter; however, several activities were held in conjunction with the UPJ Math Club on campus. These included: a welcome back pizza party with video showing of "Good Will Hunting", volunteering as registration helpers for the Math on Sat./Science on Sat. series of workshops for teachers, selling NCTM "I Love Math" products to teachers, participating in UPJ Homecoming Window Painting contest, a bus trip to Carnegie Science Center in Pittsburgh, and a Holiday/end of term pizza party with faculty. Other fall 2000 officers: Todd McDowell, vice president; Christopher Wain, secretary; Chad Long, treasurer; Nina Girard, corresponding secretary/faculty sponsor.

PA Pi Chapter President—Crystal Hogue

Slippery Rock University, Slippery Rock 8 actives

Other fall 2000 officers: Carrie Birkbichler, vice president; David Czapor, secretary; Carrie Bisher, treasurer; Elise Grabner, corresponding secretary; Gary Grabner, faculty sponsor.

SC Gamma Chapter President—Sheri Alderman

Winthrop University, Rock Hill 10 actives

Dr. Don Bentley, Pomona College, presented a guest lecture entitled "Chance Answers to Questions About the Dead Sea Scrolls" on October 17. This KME sponsored lecture was open to the entire Winthrop University population and general public. Approximately 30 people were in attendance. Other fall 2000 officers: Allen Plyler, vice president; Andrew Dean, secretary; Andrew Lanier, treasurer; Frank Pullano, corresponding secretary; Jim Bentley, faculty sponsor.

TN Gamma Chapter President—Andy Nichols

Union University, Jackson 15 actives, 4 associates

Our semester activities began with a back-to-school pizza party on September 25. Melissa Culpepper, a recent graduate, spoke on November 8 about her experiences as a first-year high school teacher. On December 1, three of our officers presented their senior seminar projects. Jamie Mosley presented "Blaise Pascal: Proving God?? A Mathematical Interpretation of Pascal's Wager." Andy Nichols presented "In Defense of Euclid." Nicki McDowell presented "The Traveling Salesman of the Brooklyn Subway." That evening a KME Christmas party was held at the home of Dr. Dawson, where Mrs. Dawson led the activity of making icosahedral Christmas decorations out of Christmas cards. We also continued our tradition of sponsoring a needy child for the annual Carl Perkins Christmas Program. Other fall 2000 officers: Jamie Mosley, vice president; Nicki McDowell, secretary; Patricia Rush, treasurer; Bryan Dawson, corresponding secretary; Matt Lunsford, faculty sponsor.

TN Epsilon Chapter President—Jennifer Dowdy

Bethel College, McKenzie 5 actives

Other fall 2000 officers: Belinda Thompson, vice president; Christina Hill, secretary/treasurer; Russell Holder, corresponding secretary; David Lankford, faculty sponsor.

TX Iota Chapter President—Geoffery Whitley

McMurry University, Abilene 18 actives

Jennifer Hebert, a Mathematics Major at McMurry, participated in the Summer Undergraduate Mathematical Sciences Research Institute at Miami University in the summer of 2000 and she wrote a paper entitled, "The Traveling Saleswoman Meets Sperner." Go Jennifer! Other fall 2000 officers: Silvia Reyes, vice president; Jennifer Hebert, secretary; Justin Snyder, treasurer; Kelly McCoun, corresponding secretary/faculty sponsor.

TX Kappa Chapter President—Robin Stokes

University of Mary Hardin-Baylor, Belton 15 actives

Other fall 2000 officers: Rachel Goad, vice president; Lynett Kaluza, secretary; Peter Chen, corresponding secretary; Maxwell Hart, faculty sponsor.

WV Alpha Chapter President—Amanda Stafford

Bethany College, Bethany 15 actives, 9 associates

Other fall 2000 officers: Adam Fletcher, vice president; Joshua Barker, secretary; Jonathan Gaffney, treasurer; Mary Ellen Komorowsk corresponding secretary/faculty sponsor.

WI Gamma Chapter President—Terry Svihovec

University of Wisconsin, Eau Claire 10 active, 16 associates

Other fall 2000 officers: Jessica Kiciak, vice president; Brea Burmeister, secretary; Jared Balkman, treasurer; Marc Goulet, corresponding secretary/faculty sponsor.

Do you feel that because of your interest in mathematics you must excel at the bridge table or on the chess board? If you have had feeling of inadequacy on these scores be sure to read the following from the writings of the celebrated and versatile Henri Poincare.

"...According to this, the special aptitude for mathematics would be due only to a very sure memory or to a prodigious force of attention. it would be a power like that of the whist-player who remembers the cards played; or, to go up a step, like that of the chess-player who can visualize a great number of combinations and keep them in his memory. Every good mathematician ought to be a good chess-player and conversely; likewise he should be a good computer. Of course that sometimes happens; thus Gauss was at the same time a geometer of genius and a very precocious and accurate computer.

But there are exceptions; or rather I err; I cannot call them exceptions being more than the rule. Gauss it is, on the contrary, who was an exception. As for myself, I must confess, I am absolutely incapable of even adding without mistakes. In the same way I should be but a poor chess-player; I would perceive that by a certain play I should expose myself to a certain danger; I would pass in review several other plays, rejecting them for other reasons, and then finally I should make the move first examined, having meantime forgotten the danger I had foreseen."

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Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959

PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975

FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000