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How Many Is Enough?

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When the French arrived and began to settle Louisiana, they encountered a tribe of mound building Indians called Natchez. The Natchez were the largest and strongest tribe inhabiting the lower Mississippi River. The Natchez and French were hostile towards one another until they made what was to be a temporary peace. Temporary because in 1729 the Natchez attacked a French settlement and killed two hundred people. In retaliation, the French succeeded in killing the majority of the tribe; dispersing any survivors among other southeastern tribes.

What makes the Natchez especially interesting is their caste system; maintained by a set of marriage laws. The Natchez had a monarchy form of government. Their ruler was called the Great Sun. All members of the tribe belonged to an aristocracy or were commoners. The caste system was as follows:

Sun-the highest aristocrats (including the Great Sun)

Noble-the second level of aristocracy

Honored-the third level of aristocracy

Stinkard-the common people

The Natchez caste system was very strict; in whatever class one was born, one stayed. There was no way for an Honored to become Noble or a Sun. This caste system was maintained by their marriage laws. Interestingly, the system required that every marriage include at least one member of the Stinkard class. The class of any child resulting from the marriages is completely determined by the class of its parents. In Figure 1 we illustrate how the class of the child is determined.

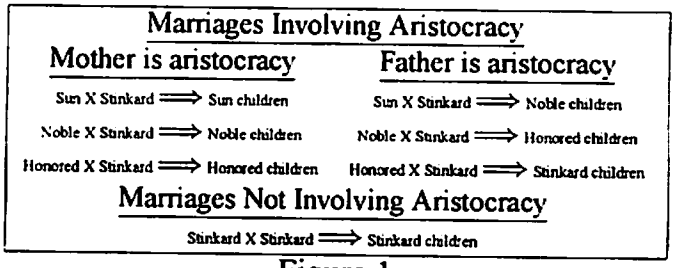

Figure 1

All Suns, including the Great Sun, had to marry from the Stinkard class. The children of female Suns married to Stinkards remained Suns themselves. However, the children of male Suns married to Stinkards were down-graded to being Nobles. Similar rules applied to all other Natchez of the aristocracy. The children of female Nobles or Honored married to Stinkards kept the same class as their mothers, while children of male Nobles or Honored were down-graded to Honored and Stinkard respectively. Any Stinkard not married to an aristocrat married a Stinkard. All children of such Stinkard X Stinkard marriages remained Stinkard.

These rules give rise to the main question we investigate in this paper. If the rules remain intact for multiple generations, will the caste system remain "stable"? By stable we mean that generation after generation there will always be enough members in each class in any given generation, so that the marriage rules can be applied for the next generation.

An interesting thought problem is to simply look at the marriage rules specified in Figure 1 and conjecture which, if any, of the classes will disappear first. This disappearance would lead to instability in the system.

In order to analyze this modeling problem we introduce some notation and impose several simplifying assumptions.

S_k = number of Suns in generation k

N_k = number of Nobles in generation k

H_k = number of Honored in generation k

C_k = number of Stinkards in generation k

Assumptions

I. Each class is comprised of an equal number of men and women.

II. Each member of the tribe gets married once and only once.

III. Every marriage produces 2 children; 1 boy and 1 girl.

IV. All initial generation values S_o, N_0, H_0, and C_0 are positive. Also $C0S0 + N0 + H0$ (In the initial generation there are a sufficient number of Stinkards to supply aristocracy with marriage spouses)

We will now develop recurrence relations for S_k, N_k, H_k, and C_k in terms of the previous generation numbers $S_{k-1}, N_{k-1}, H_{k-1}$,and C_{k-1}.

In Figure 2 we illustrate how the members in the $(k-1)^{st}$ generation of each class (left hand side) determine the number of members in the k^{th} generation of each class (right hand side). Following from left to right in Figure 2 we see how the k^{th} generation numbers are generated from the $(k-1)$ generation numbers.

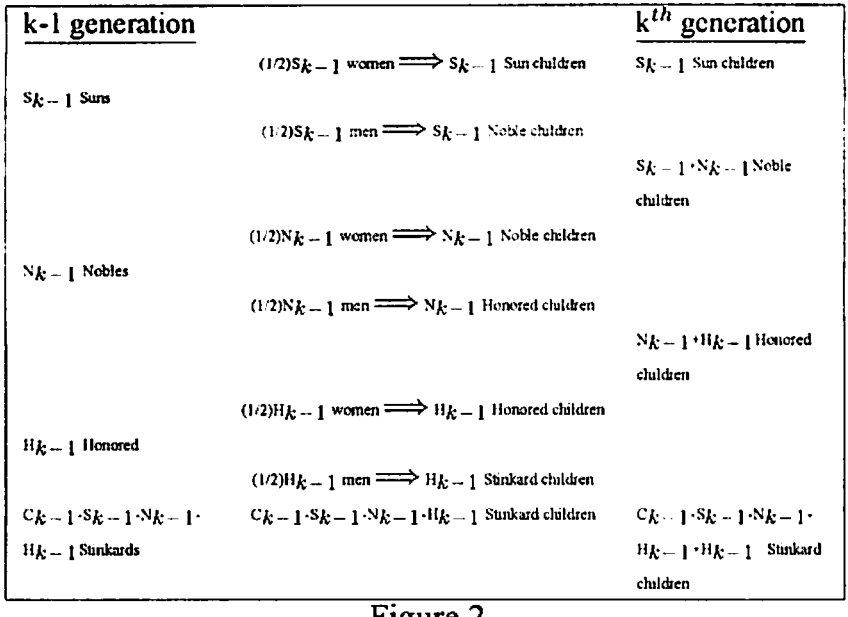

Figure 2

Note that although there are C_{k-1} Stinkards in the $(k-1)^{st}$ generation, S_{k-1} of them marry Suns, N_{k-1} of them marry Nobles, and H_{k-1} of them marry Honored, so there are only $C_{k-1} - S_{k-1} - N_{k-1} - H_{k-1}$ remaining to marry within the Stinkard class as the last line in Figure 2 depicts.

The right hand column in Figure 2 shows the number of members in each class in the k^{th} generation. We summarize the right hand column conclusions in the following recurrence relations.

$$S_k = S_{k-1} \tag{1}$$

$$N_k = N_{k-1} + S_{k-1} \tag{2}$$

$$H_k = H_{k-1} + N_{k-1} \tag{3}$$

$$C_k = C_{k-1} - S_{k-1} - N_{k-1} \tag{4}$$

The initial class populations S_0, N_0, H_0, and C_0 are all fixed constants

which satisfy Assumption IV.
Using (1) repeatedly we get the following sequence of equations.

$$S_k = S_{k-1} = S_{k-2} = S_{k-3} = ...S_2 = S_1 = S_0$$

$$\text{Thus } S_k = S_0 \tag{5}$$

Similarly we use (1) and (2) repeatedly.

$$N_k = N_{k-1} + S_0 = N_{k-2} + 2S_0 = N_{k-3} + 3S_0$$
$$= ...N_2 + (k-2)S_0 = N_1 + (k-1)S_0 = N_0 + kS_0$$

$$\text{Thus } N_k = N_0 + kS_0 \tag{6}$$

The development of the explicit formula for H_k is more complicated.

$$H_k = H_{k-1} + N_{k-1}; \text{ from (3)}$$

Replacing k by $k-1$ in (6) and substituting we get

$$H_k = H_{k-1} + N_0 + (k-1)S_0 \tag{7}$$

We now apply the iteration method repeatedly to (7).

$H_1 = H_0 + N_0$; substituting $k = 1$ in (7)
$H_2 = H_1 + N_0 + 1S_0$; substituting $k = 2$ in (7)
$= H_0 + 2N_0 + 1S_0$; substituting value of H_1 in previous line
$H_3 = H_2 + N_0 + 2S_0$; substituting $k = 3$ in (7)
$= H_0 + 3N_0 + (1+2)S_0$; substituting value of H_2 in previous line
$H_4 = H_3 + N_0 + 3S_0$; substituting $k = 4$ in (7)
$= H_0 + 4N_0 + (1+2+3)S_0$; substituting value of H_3 in previous line
$H_k = H_0 + kN_0 + (1+2+3+...+(k-2)+(k-1))S_0$; using the established pattern
$= H_0 + kN_0 + (1/2)k(k-1)S_0$; using a standard arithmetic progression formula.

We have thus developed an explicit formula for H_k. The explicit formula for the recurrence relation for C_k can be developed in a similar manner so we omit details.

$$C_k = C_0 - kN_0 - (1/2)k(k+1)S_0$$

We have now solved our recurrence relations (2), (3), (4), and (5). The explicit formulas we developed are repeated below.

$$S_k = S_0 \tag{8}$$

$$N_k = N_0 + kS_0 \tag{9}$$

$$H_k = H_0 + kN_0 + \frac{1}{2}k(k-1)S_0 \tag{10}$$

$$C_k = C_0 - kN_0 - \frac{1}{2}k(k+1)S_0 \tag{11}$$

Before proceeding we should note that in the development of these explicit formulas we used "pattern recognition" techniques that may lead to questionable results since one can never be completely certain that the pattern does not change in later iterations. Indeed, we can prove that our results are all valid using mathematical induction. We will prove that the recurrence relation

$$H_k = H_{k-1} + N_{k-1} \tag{12}$$

produces exactly the same results as the explicit solution

$$T_k = H_0 + kN_0 + \frac{1}{2}k(k-1)S_0 \tag{13}$$

Notice we are symbolizing the explicit formula T_k rather than H_k so that we can easily distinguish between the two formulas. With this notation we will use mathematical induction to prove

$$H_k = T_k \quad \text{for all} \quad k \in \{0, 1, 2, \ldots\} \tag{14}$$

From (13) with $k = 0$ we get $T_0 = H_0$ to establish the base case. We next assume that $H_n = T_n$ for some $n \in \{0, 1, 2, ...\}$.

$H_{n+1} = H_n + N_n$; by (12)
$= T_n + N_n$; by the induction hypothesis
$= H_0 + nN_0 + \frac{1}{2}n(n-1)S_0 + N_n$; by (13)
$= H_0 + nN_0 + \frac{1}{2}n(n-1)S_0 + N_0 + nS_0$ by (10)
$= H_0 + (n+1)N_0 + \frac{1}{2}S_0(n)(n+1)$; simplifying
$= T_{n+1}$ by (13).

By the principle of math induction we conclude that $T_n = H_n$ for all $n \in \{0, 1, 2, ...\}$. Of course, correctness of the explicit formulas for S_k, H_k, and C_k could be proven similarly.

We are now ready to argue that indeed the marriage rules do NOT permit a stable class system. In particular, we argue that the Stinkard popu-

lation will not be adequate to provide spouses for the aristocracy after a certain number of generations.

We will show that there necessarily exists a generation K for which

$$C_K < S_K + N_K + H_K$$

In other words in generation K there are not enough Stinkards to provide the aristocracy with marriage spouses.

We begin by letting

$$y(k) = S_k + N_k + H_k - C_k \tag{15}$$

Substituting explicit formula results from (8), (9), (10), and (11) we get

$$y(k) = S_0 + (N_0 + kS_0) + [H_0 + kN_0 + \frac{1}{2}k(k-1)S_0]$$

$$+[C_0 - kN_0 - \frac{1}{2}k(k+1)S_0]$$

Combining like terms we get

$$y(k) = S_0k^2 + 2N_0k + (S_0 + N_0 + H_0 - C_0)$$

Now if we temporarily assume that k is a real variable ($k \geq 0$) rather than an integer we note that $y(k)$ is a quadratic equation in k and therefore has a parabolic graph. The parabola opens upward since $S_0 > 0$ from Assumption IV. Also $(S_0 + N_0 + H_0 - C_0) \leq 0$ from Assumption IV. If $(S_0 + N_0 + H_0 - C_0) = 0$ then $y(k) > 0$ for all $k > 0$, (since $S_0 > 0$ and $N_0 > 0$ from Assumption IV). If instead,$(S_0 + N_0 + H_0 - C_0) < 0$ then the parabola $y(k)$ crosses the negative y axis and since it opens upward it necessarily crosses the k axis. From the quadratic formula the value at which it crosses is

$$x = \frac{-N_0 + \sqrt{(N_0)^2 - S_0(S_0 + N_0 + H_0 - C_0)}}{S_0}$$

If we let K be the first integer beyond x then clearly $y(K) > 0$ and substituting from (15) we get

$$S_K + N_K + H_K > C_K$$

We conclude that in the K^{th} generation there are not enough Stinkards to marry aristocracy. With our assumptions the caste system is NOT stable.

Our original modeling problem has been solved but that solution was, of course, based upon our modeling assumptions. Perhaps it would be possible to change our original assumptions in such a way that the caste sys-

tem would instead be stable. For example, suppose the Stinkard X Stinkard marriages are more prolific than those involving aristocracy. In particular, suppose that they produce twice as many children. In our original development of the four recurrence relations, this would change only C_k. The recurrence relations for S_k, N_k, and H_k would remain as in (8), (9), and (10) respectively. In Figure 2 the Stinkard X Stinkard marriages would produce $2(C_{k-1} - N_{k-1}S_{k-1} - H_{k-1})$ Stinkard children and adding those to the $H(k-1)$ produced by Honored marriages we get a new recurrence relation for C_k

$$C_k = 2C_{k-1} - 2S_{k-1} - 2N_{k-1} - H_{k-1} \tag{16}$$

The solution to the recurrence relation C_k is going to get rather complex. The solution is simplified somewhat if we replace k in (16) by $k+1$,

$$C_{k+1} = 2C_k - 2S_k - 2N_k - H_k$$

Substituting the values of S_k, N_k, and H_k from (8), (9), and (10) and simplifying we conclude

$$C_{k+1} = 2C_k - \frac{1}{2}S_0k^2 - [N_0 + \frac{3}{2}S_0]k - [H_0 + 2S_0 + 2N_0]$$

If we try to solve this recurrence relation using the iteration method the detail becomes rather complex. We will simplify the expression by letting

$$\begin{aligned} a &= -\tfrac{1}{2}S_0 \\ b &= -N_0 - \tfrac{3}{2}S_0 \\ c &= -H_0 - 2S_0 - 2N_0 \end{aligned} \tag{17}$$

Replacing these values in (17) we get the recurrence relation

$$C_{k+1} = 2C_k + ak^2 + bk + c \tag{18}$$

To solve this recurrence relation we substitute $k = 0, 1, 2, 3$, and 4 into the relation and simplify.

$C_1 = 2C_0 + c;$

$C_2 = 2^2C_0 + 2c + a1^2 + b1 + c;$

$C_3 = 2^3C_0 + 2^2c + 2a1^2 + 2b1 + 2c + a2^2 + b2 + c$

$\quad = 2^3C_0 + 2^2c + 2c + c + 2b1 + b2 + 2a1^2 + a2^2;$

$C_4 = 2^4C_0 + 2^3c + 2^2c + 2c + 2^2b1 + 2b2 + 2^2a1^2 + 2a2^2 + a3^2 + b3 + c$

$\quad = 2^4C_0 + 2^3c + 2^2c + 2c + c + 2^2b1 + 2b2 + b3 + 2^2a1^2 + 2a2^2 + a3^2;$

$C_5 = 2^5C_0 + 2^4c + 2^3c + 2^2c + 2c + c + 2^3b1 + 2^2b2 + 2b3 + b4$

$$
\begin{aligned}
&+2^3a1^2 + 2^2a2^2 + 2a3^2 + a4^2 \\
&= 2^5C_0 + c[2^4 + 2^3 + 2^2 + 2 + 1] + b[2^3(1) + 2^2(2) + 2^1(3) + 1(4)] \\
&+a[2^31^2 + 2^22^2 + (2)3^2 + (1)4^2].
\end{aligned}
$$

We have carried the process far enough to recognize a pattern.

$$
\begin{aligned}
C_k &= 2^kC_0 + c[2^{k-1} + 2^{k-2} + ... + 2^2 + 2 + 1] \\
&+b[2^{k-2}(1) + 2^{k-3}(2) + 2^{k-4}(3) + ... + 2(k-2) + 1(k-1)] \\
&+a[2^{k-2}1^2 + 2^{k-3}2^2 + ... + 2(k-2)^2 + 1(k-1)^2].
\end{aligned}
$$

Note that we have 3 series embedded in this expression. If we factor a 2^{k-1} out of the middle series and a 2^{k-2} out of the final series and simplify we get

$$
\begin{aligned}
C_k &= 2^kC_0 + c[2^{k-1} + 2^{k-2} + ... + 2^2 + 2 + 1] \\
&+\tfrac{1}{2}2^kb[\tfrac{1}{2} + \tfrac{2}{2} + \tfrac{3}{2^3} + \tfrac{4}{2^4} + ... + \tfrac{(k-1)}{2^{k-1}}] \\
&+2^k\tfrac{a}{2^2}[1^2 + \tfrac{2^2}{2^1} + \tfrac{3^2}{2^2} + \tfrac{4^2}{2^3} + ... + \tfrac{(k-1)^2}{2^{k-2}}]
\end{aligned}
\tag{19}
$$

We now seek explicit formulas for the three series that occur in C_k. To simplify we let

$$
\begin{aligned}
A &= 2^{k-1} + 2^{k-2} + ... + 2^2 + 2 + 1 \text{ (first series);} \\
B &= \tfrac{1}{2} + \tfrac{2}{2^2} + \tfrac{3}{2^3} + \tfrac{4}{2^4} + ... + \tfrac{k-1}{2^{k-1}} \text{ (middle series);} \\
C &= 1^2 + \tfrac{2^2}{2^1} + \tfrac{3^2}{2^2} + \tfrac{4^2}{2^3} + ... + \tfrac{(k-1)^2}{2^{k-2}} \text{ (last series)}
\end{aligned}
\tag{20}
$$

All three of these series can be evaluated by starting with the simple geometric series

$$
1 + x + x^2 + ... + x^{k-1} = \frac{x^k - 1}{x - 1} \tag{21}
$$

Plugging $x = 2$ into both sides of this equation yields a sum for series A.

$$
1 + 2 + 2^2 + 2^3 + ... + 2^{k-1} = \frac{2^k - 1}{2 - 1}) = 2^k - 1
$$

Differentiating equation (21) and then simplifying the right hand side yields

$$
\begin{aligned}
1 + 2x + 3x^2 + ... + (k-1)x^{k-2} &= (x-1)kx^{k-1} - \tfrac{x^k-1}{(x-1)^2} \\
&= \tfrac{(k-1)x^k - kx^{k-1}+1}{(x-1)^2}
\end{aligned}
$$

Multiplying both sides of (22) by x produces

$$x + 2x^2 + 3x^3 + ... + (k-1)x^{k-1} = \frac{(k-1)x^{k+1} - kx^k + x}{(x-1)^2} \qquad (22)$$

Substituting $x = \frac{1}{2}$ and doing more simplification yields a sum for series B.

$$\begin{aligned} &\tfrac{1}{2} + \tfrac{2}{2^2} + \tfrac{3}{2^3} + \tfrac{4}{2^4} + ... + \tfrac{k-1}{2^{k-1}} \\ &= 2^2[\tfrac{k-1}{2^{k+1}} - \tfrac{k}{2^k} + \tfrac{1}{2}] \\ &= \tfrac{2^2}{2^{k+1}}[(k-1) - 2k + 2^k] \\ &= (\tfrac{2^k - k - 1}{2^{k-1}}) \end{aligned}$$

Differentiating equation (23) and simplifying the right hand side yields

$$\begin{aligned} &1 + 2^2x + 3^2x^2 + 4^2x^3 + ...(k-1)^2x^{k-2} \\ &= (x-1)[(k^2-1)x^k - k^2x^{k-1} + 1 \\ &\quad - \tfrac{(k-1)x^{k+1} - kx^k + x]2(x-1)}{(x-1)^4} \\ &= \tfrac{k^2x^{k+1} - x^{k+1} - k^2x^k + x - k^2x^k + x^k + k^2x^{k-1} - 1 - 2kx^{k+1} + 2x^{k+1} + 2^kx^k - 2x}{(x-1)^3} \\ &= \tfrac{(k^2 - 2k + 1)x^{k+1} + (2k - 2k^2 + 1)x^k + k^2x^{k-1} - x - 1}{(x-1)^3} \end{aligned}$$

Substituting $x = \frac{1}{2}$ and then simplifying the right hand side yields a sum for series C.

$$\begin{aligned} &1 + \tfrac{2^2}{2} + \tfrac{3^2}{2^2} + \tfrac{4^2}{2^3} + ... + \tfrac{(k-1)^2}{2^{k-2}} \\ &= -2^3\left(\tfrac{k^2 - 2k + 1}{2^{k+1}} + \tfrac{2k - 2k^2 + 1}{2^k} + \tfrac{k^2}{2^{k-1}} - \tfrac{3}{2}\right) \\ &= -\tfrac{2^3}{2^{k+1}}\left(k^2 - 2k + 1 + 2(2k - 2k^2 + 1) + 4k^2 - 3 * 2^k\right) \\ &= -\tfrac{1}{2^{k-2}}\left(k^2 + 2k + 3 - 3 * 2^k\right) \\ &= \tfrac{3*2^k - k^2 - 2k - 3}{2^{k-2}}. \end{aligned}$$

Now substituting these sums for A, B, and C, in (19) results in the following.

$$\begin{aligned} C_k &= 2^kC_0 + c(2^k - 1) + 2^{k-1}b[\tfrac{2^k - k - 1}{2^{k-1}}] + 2^{k-2}a[\tfrac{3*2^k - k^2 - 2k - 3}{2^{k-2}}] \\ &= 2^kC_0 + c(2^k - 1) + b(2^k - k - 1) + a(3 * 2^k - k^2 - 2k - 3) \\ &= (C_0 + c + b + 3a)2^k - ak^2 - (b + 2a)k + (c - b - 3a) \qquad (23) \end{aligned}$$

We must now return a, b,and c to their appropriate values as assigned in (17).

$$C_0 + c + b + 3a = C_0 - (H_0 + 2S_0 + 2N_0) + (-N_0 - \tfrac{3}{2}S_0) + 3(-\tfrac{S_0}{2})$$
$$= C_0 - H_0 - 5S_0 - 3N_0$$
$$b + 2a = -N_0 - \tfrac{3}{2}S_0 + 2(-\tfrac{S_0}{2}) = -N_0 - \tfrac{5}{2}S_0$$
$$-c - b - 3a = (H_0 + 2S_0 + 2N_0) + N_0 + \tfrac{3}{2}S_0 + \tfrac{3}{2}S_0 = H_0 + 5S_0 + 3N_0$$

Further substitution of these sums into (24) brings us to

$$C_k = (C_0 - H_0 - 5S_0 - 3N_0)2^k + \frac{1}{2}S_0k^2 + (N_0 + \frac{5}{2}S_0)k + (H_0 + 5S_0 + 3N_0)$$

We have arrived at our explicit formula for C_k given that the Stinkard X Stinkard marriages are twice as prolific. We return to the question of stability. Are there now enough Stinkards to provide spouses for aristocracy?

$$C_0 \geq H_0 + 5S_0 + 3N_0$$

For completeness we restate all assumptions of our modified model.
I'. Each class is comprised of an equal number of men and women.
II'. Each member of the tribe gets married once and only once.
III'. Every Stinkard X Stinkard marriage produces 4 children, 2 boys and 2 girls. All other marriages produce 2 children.
IV'. All initial generation $C_0 \geq H_0 + 5S_0 + 3N_0$.
The caste system for our model will be stable if

$$C_k \geq S_k + H_k + N_k \text{ for ALL } k \in \{0, 1, 2, ...\} \qquad (24)$$

Substituting the explicit formulas for C_k, S_k, H_k, and N_k from (8),(9),(10),(11) in (25) and simplifying we get

$$2^k(C_0 - H_0 - 5S_0 - 3N_0) + (2S_0)k + (4S_0 + 2N_0) \geq 0$$

The coefficient $C_0 - H_0 - 5S_0 - 3N_0$ is non-negative according to our modified Assumption (IV)'. The other coefficients $2S_0$ and $4S_0 + 2N_0$ are also positive so the above inequality is guaranteed.
We conclude that

$$C_k \geq S_k + H_k + N_k \text{ for all } k \in \{0, 1, 2, ...\}$$

so there are enough Stinkards to marry aristocracy in all generations. This caste system using our modified assumptions is indeed stable.

An interesting generalization to our model modification is to try to find the minimum "children ratio" for the Stinkard X Stinkard marriages which would yield stability for the caste systems. We know that for a ratio of 1 (2 children from 2 parents) the system is unstable. For a ratio of 2 (4 children from 2 parents) the system is stable. What is the minimum ratio between

1 and 2 for which stability occurs? We spent some time investigating this question.

In the modified recurrence relation for C_k we multiplied certain expressions by 2 since we had doubled the child production of the Stinkard X Stinkard marriages. Instead of multiplying by 2 it is possible to use an arbitrary multiplier, say t. Upon so doing we could, with some effort, obtain series with the same basic forms as A, B, and C in (20). The explicit formulas for these three series are much more complicated than those we obtained for the multiplier 2. The resulting explicit solution for C_k becomes extremely cumbersome and the general question of stability becomes unmanageable. It appeared that the best we could do was find a numeric approximation for the multiplier t using fixed numeric initial values of S_0, N_0, H_0, and C_0.

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by Suzanne Shontz

Motivation

Grade inflation has been a problem for numerous universities in recent years. Many believe that it started during the Vietnam War. Professors gave out higher grades in order to keeping students from flunking out and being drafted [5]. For example, in the 1960's, Stanford University graded according to a bell curve. This means that the majority of grades were *B*'s and *C*'s with fewer *A*'s, *D*'s, and *F*'s [2]. However, in 1970, Stanford University replaced the grade *F* with the mark *NC*, or no credit; if a student received an *NC*, it would not affect the student's gradepoint. Currently, the percentages of grades that Stanford hands out are over 50% *A*'s and 40% *B*'s [8]. At Columbia University, the situation is similar. In 1969, the university gave out only 7% *A*'s, while 25% of grades were *C*'s or below. In contrast, in 1993, Columbia awarded students with 26% *A* grades, and only 9% of the grades were *C*'s or lower [7]. The rapid transformation in grade distributions has had many adverse affects in our society.

Today's students are not prepared for careers once they get beyond school doors. They cannot read, write, or communicate effectively as compared to students in other nations, and grade inflation was a contributor to this problem. Grades are no longer a reflection of students' true talents and abilities [3]. Students' grades also affect faculty evaluations. This causes faculty to grade more leniently and students to take less-challenging courses [4]. Grade inflation encourages more competition, especially among students near the top of the class. Grades seem to follow the theory of supply and demand; if there is a large supply of *A* grades, then they will lose their value to future employers, admissions officers, and the students themselves [6]. Thus, it is important that grade inflation be examined in order to minimize these problems. A new method of evaluating students' performances in the classroom is essential for the success of our country in the future.

The rest of this paper is dedicated to finding a solution to the grade inflation problem at ABC College. In the next section, we state the problem and its assumptions and analyze the problem. In the third section we state and test a model. That is followed by the model's strengths and weaknesses. We draw our conclusions in the final section. Appendix A is a computer simulation of the rankings of seniors of a college of 600 students. Appendix B is the same simulation for a college of 10,000 students. Appendix C is a simulation for a college of 50 students without plus and minus grades given.

The Problem

The dean at A Better Class (ABC) College is concerned that the students' grades are so high that there is no obvious way to rank them according to class. The dean envisions a new method of grading where both the student's letter grade and the number of students receiving that particular grade for that section of the course are taken into account. These class ranks must be determined in order to decide which students are eligible for the scholarship at ABC College. It is our duty to answer the following questions: Is the dean's new way of grading workable if students receive plus and minus grades in their courses at ABC College? Is the dean's method of grading viable if students do not receive any plus or minus grades for their courses? Is there another possible method to determine a fair class ranking? Will a poor grade in one course affect the student's decile rank? If several students have one poor grade will this significantly affect the decile rankings?

Next, we state the assumptions under which we will be working. In the first case, we assume plus and minus grades are given at A Better Class (ABC) College, with the exception of $F+$ and $F-$ grades. In the second case, we assume that the only grades given are A, B, C, D, and F. A student will receive one of the above grades for every course taken at ABC College. That is, no courses may be taken pass/fail or audited. The courses can be divided into four difficulty levels: freshman (100 level), sophomore (200 level), junior (300 level), and senior (400 level). Each student takes a similar number of courses at each level. All of the courses at ABC College are three-credit-hour courses. We assume that the students at ABC College can be classified into four status levels: freshman, sophomore, junior, and senior, so that class ranks may be determined for each class.

We assume that the students at ABC College graduate at the end of four years in school. All students at ABC College are full-time students taking between 12 and 18 credit hours per semester.

A student receiving transfer credit from another college or university, Advanced Placement (AP) credit, or College-Level Examination Program

(CLEP) credit will receive credit, but this credit will not affect his/her ABC GPA. Finally, students applying for the scholarships must have taken classes at ABC College for two or more semesters.

Our main goal for a solution was to find a system that will place students in rank order by classroom achievement. Our subsidiary goal was to enhance the traditional GPA system. In order to reach these goals, we investigated several grading systems that are currently in use in universities in the United States.

The first grading system that we looked at is the one that exists at Dartmouth College. They use a system that is similar to the one that has been in use at McGill University in Canada for over fourteen years. At Dartmouth, they put two numbers on your transcript; the first number is the student's grade in the course, and the second number is the mean of the grades of all of the students in that course. There have been several variations of this system using either the median or the mode in place of the mean on the transcript [1]. Although this is a start, this method of grading does not address differences between instructors that teach the same course or the relative ability levels of the students taking the same course. Dartmouth also ranks their students by GPA, which will not benefit the problem at ABC College.

The next grading system we looked at was the system at Indiana University. Their transcripts show the student's grade in each particular course and the entire class' grade distribution [10]. We believe this system to be an improvement over the Dartmouth grading system because information about the grade distribution is given on the transcripts, making it easier to compare the students. However, it still does not give a fair ranking of the students, and it may be overwhelming to potential employers and admissions officers.

The final system that we looked at, and probably the most radical one, has recently been implemented at Duke University. They are using a grading system called the Achievement Index (AI) in conjunction with the GPA system. AI is a complicated system that seeks to rank students according to several criteria: the student's grade, the course difficulty, the class' distribution of grades, and the mean grade of the course [10]. This system has caused much controversy at Duke as of late.

Many students at Duke have expressed concerns about the purpose of the AI. Some of them think that it will cause more competition among the students. In addition, this competition will cause students to take easier classes. However, some students realize that the system currently in use (the GPA system) has its faults, and they recognize that there needs to be a change [9].

In contrast, there have been several people that advocate the use of the Achievement Index. One advantage of the number provided by the AI is that it is easy to understand, so employers and admissions officers should have no problem comprehending it. There are claims that if a student takes an easier course, the AI will reflect this decision by not rewarding him/her for it. Likewise, if a student takes a more challenging course, then the AI will reward that student. Also, if a student's letter grade were to decrease by one grade, such as from an *A-* to a *B+*, it will have little to no effect on his/her AI score [1].

In our opinion, the AI is the best model among these grading systems to help solve the problem of ranking the students at ABC College where grade inflation is prevalent. After examining the model used by Johnson in [4], we noticed that it uses a standard normal curve for scoring the students in each class, which is more representative of the actual population. It also takes into account many of the aspects of grade calculation that can exist at a college like ABC such as the grade earned by the student, the class' distribution of grades, and the average grade earned by the class. However, we think that the biggest problem with Johnson's AI model is that it is too complex. His model uses so many variables that it is hard to remember what they stand for, and it also requires the knowledge of what an inverse gamma distribution looks like and Bayesian statistics. We feel that an easier system can and should be devised so that one does not need a doctoral degree in statistics in order to comprehend it.

The Model

Our model is based upon the AI model discussed above. Our variation of the model, the Quality Performance Index (QPI), has simplified the complex structure of the AI model.

We define the following variables which will be used for ranking students. We use g_{ij} to denote the grade the ith student ($i = 1, 2, ..., N$,where N is the number of students at ABC College) received in the jth class ($j = 1, 2, ..., n$, where n is the number of courses the ith student is taking in a given semester) after his/her letter grade has been converted to a numerical score as follows: $F = 1$, $D- = 2$, ..., $A+ = 13$. We let M_j denote the average grade earned in a particular course. The measurement of central tendency can be defined to be the mean, the median, or the mode.

Next, z_{ij} denotes the z-score received by the ith student in the jth course that corresponds to the position of g_{ij} on the normal curve with mean M_j and standard deviation σ_j, where σ_j is the sample standard deviation of the jth class. Hence, z_{ij} is calculated as follows:

If $z_{ij} > 0$, the ith student is performing above average in the jth class. If $z_{ij} = 0$, the ith student is performing at the average level in the jth class.

If $z_{ij} < 0$, the ith student is performing below average in the jth class.

We use q_{ij} to denote the quality performance index of the ith student in the jth class. In the first semester in which this policy is implemented at ABC College, $q_{ij} = z_{ij}$. We will discuss what happens in subsequent semesters after defining QPI_i.

Finally, QPI_i denotes the quality performance index of the ith student at ABC College for a given semester. It is calculated as follows:

$$QPI_i = \frac{\sum_{j=1}^{n} q_{ij}}{n}.$$

In the semesters following the first semester, q_{ij} is given by

$$q_{ij} = z_{ij} + Q_j,$$

where

$$Q_j = \frac{\sum_{b \in K} QPI_b}{m-1},$$

where m is the number of students in class j and

$$K = \{k \mid \text{student } k \text{ is in class } j \text{ and } k \neq i\}.$$

This will take the ability of all of the students in the jth class into account.

The QPI of each student is used to rank the students in each class level. The person with the highest QPI score is the highest-ranking student. Similarly, the person with lowest QPI score is the lowest-ranking student. These class rankings are used to calculate the deciles to determine the students that qualify for the scholarship at ABC College.

In order to test our method, we further refine our assumptions as follows: The students at ABC College can take 12 hours, 15 hours, or 18 hours, in any given semester. Of all the students, 10% are taking 12 hours, 80% are taking 15 hours, and another 10% are taking 18 hours in any given semester. We distribute the four classes (freshmen, sophomore, junior, senior) among the four course levels as follows: the 100-level courses consist of two-thirds freshmen, one-sixth sophomores, one-twelfth juniors and one-twelfth seniors; the 200-level courses consist of one-fifth freshmen, three-fifths sophomores, one-tenth juniors and one-tenth seniors; the 300-level courses consist of one-fifth sophomores, three-fifths juniors and one-fifth seniors; and the 400-level courses consist of one-fourth juniors and three-fourth seniors. After a student has taken 30 credit hours, they are classified as a sophomore. After 60 credit hours, they are classified as a junior. After 90 credit hours, they are classified as a senior. Our computer simulation ensures that each student is promoted at the end of the school year to ensure graduation in four years.

Our model was tested on various-sized schools ranging from 100 to

9600 students. The computer simulation is shown in appendix A for a school with 600 students (168 of them being seniors) with plus and minus grades given. It shows that generally students with a high GPA rank in the top decile. However, the relative ranking of the students has changed within the deciles. For example, the student that would have had a GPA ranking of 1 has a QPI ranking of 5. This data suggests that the dean's proposal to take into account the information about the other students in the class seems to work well and is correlated with the GPA rankings. The computer simulation is shown in appendix B for a school with 10,000 students (2501 seniors) with plus and minus grades given. This data again suggests that the QPI and GPA are highly correlated and are in better correspondence at the larger school. The computer simulation shown in appendix C is for a school with 50 students (17 seniors). This data suggests that the dean's proposal does not work well for smaller schools without plus and minus grades given.

Strengths and Weaknesses of the Model

Our model is general enough to be applied to any college or university. We feel that it would work exceptionally well in larger universities for two reasons. First of all, the larger population is more likely to follow a normal distribution than a smaller one. Secondly, larger universities tend to have more of an open enrollment than smaller colleges. Therefore, there will be a wider variety of ability levels at larger universities. The QPI model is easy for the registrar to figure out. It will be simpler for professors to explain to students and in recommendation letters to future employers and admissions officers. The ranking is a better assessment of students in the classroom than the current system. It is not as affected by easy courses or hard professors as is the GPA system.

This model is flexible enough to be extended to other aspects of society. For example, the QPI model can be used to rank individual athletes, such as tennis players or golfers. This model can also be extended to other types of contests outside athletics, such as music competitions, dance contests, or science fairs.

On the other hand, the assumptions we made at the beginning of this paper are not necessarily true of all colleges and universities. However, they do reflect a measurement of central tendency of the colleges and universities in the United States and therefore are reasonable assumptions to have made. Our QPI model still does not account for differences across the disciplines, e.g., how do you compare an art student with a computer science major? It is virtually impossible to rank one above the other.

Conclusion

It is our recommendation to ABC College that they implement the QPI rating system to enhance the traditional GPA system. The QPI will rank the students more fairly than would a GPA because it is a relative scale that ranks to the mean. We believe that the GPA system should be kept because most people are more familiar with it, and, even with its faults, it is an absolute scale that reflects the quality of the students' work.

Appendix A
Comparison of QPI and GPA Scores for Seniors at a School of 600 Students with Plus and Minus Grades Given

QPI score	QPI rank	GPA score	GPA rank
14.0504	1	10.2683	20
13.8809	2	10.65	2
13.4337	3	10.525	6
12.588	4	10.4146	10
11.9484	5	10.7	1
11.1277	6	10.5	8
10.8383	7	10.575	4
10.6245	8	10.525	7
10.5766	9	10.4	11
0.723849	71	9.95	74
0.600638	72	9.54762	141
0.523648	73	10.025	54
0.136477	74	10.075	44
-0.129336	75	10.05	48
-0.397041	76	9.23256	158
-0.642972	77	9.95	73
-0.67389	78	9.73171	113
-0.738986	79	9.9	84
-0.772656	80	10.1	38
-.807698	81	9.8	95
-11.2055	163	8.95238	166
-11.3364	164	9.17073	160
-12.5402	165	9.14634	161
-13.1203	166	9.26829	157
-14.5066	167	9.375	154
-17.6519	168	8.69048	168

Appendix B
Comparison of QPI and GPA Scores for Seniors at a School of 10000 Students with Plus and Minus Grades Given

QPI score	QPI rank	GPA score	GPA rank
20.4401	1	10.95	4
19.0151	2	10.925	6
18.9455	3	11.025	3
0.0195565	1234	9.59524	1961
0.0120895	1235	9.61905	1917
0.00985858	1236	10.125	606
-0.00289452	1237	9.47619	2167
-0.00443649	1238	10.175	503
-0.00805843	1239	9.7561	1610
–0.0139558	1240	10.075	743
-19.1995	2499	9.1	2452
-19.5793	2500	9.05	2466
-20.1323	2501	8.95	2486

Appendix C
Comparison of QPI and GPA Scores for Seniors at a School of 50 Students with No Plus or Minus Grades Given

QPI score	QPI rank	GPA score	GPA rank
6.10667	1	10.0714	10
4.18896	2	10.425	2
3.7331	3	10.1707	8
3.19587	4	10.1707	7
2.84301	5	10.575	1
2.52736	6	10.2	6
2.08425	7	10.2	5
0.955701	8	10.35	4
0.720041	9	9.48837	16
-0.301998	10	10.05	11
-0.455409	11	10.425	3
-1.47585	12	10.125	9
-3.17285	13	9.73171	14
-3.23672	14	9.4186	17
-3.2646	15	9.975	12
-8.68956	16	9.6	15
-8.97137	17	9.825	13

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A mathematician confided
That a Möbius band is one-sided,
And you'll get quite a laugh
If you cut one in half
For it stays in one piece when divided.

-Author Unknown

Big RAM

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Clustering technology is making a statement in the scientific and academic research communities. The Avalon cluster at Los Alamos National Labs ranks 113 on the Top500 list of the world's fastest computers, available online at

`http://www.top500.org/Top500.list.html`

Millions of dollars have been saved in scientific research labs where clusters such as Avalon have been built in place of parallel processing supercomputers, such as the SGI Origin 2000. This paper discusses some of the results of a study which sought to solve large mathematical problems on a cluster of 15 ordinary Intel processor based computers.

The problem chosen for this study is the problem of evaluating the determinant of a matrix using the method of row expansion. Implementing the method of row expansion is very computationally intensive and requires on the order of $n!$ multiplications for evaluating the determinant of an n by n matrix. A less computationally intensive alternative might involve the row reduction of a given matrix into an upper triangular form, at which point the determinant could be easily found by multiplying the diagonal entries of the matrix. The goal of the project, however, was to test the capabilities of the cluster with a problem that was large but easy to understand and straightforward to code. It was not this project's aim to simply solve determinants as quickly as possible; if it had been, other methods would have been used.

The resulting program is called detp (Determinants in Parallel). Running in parallel on the cluster, detp divides up the workload of the determinant computation by assigning portions of the problem to other machines in the cluster. For example, given a 10 by 10 matrix, the algorithm begins evaluating the determinant by taking the element in the first row, first column, raising it to a power of -1 (which is irrelevant at this point), and multiplying it by the determinant of the element's associated submatrix. This submatrix. will be a 9 by 9 matrix and will be passed on to another machine for determinant evaluation. Once the second machine is assigned the task of computing the determinant of a 9 by 9 matrix, it begins the

row expansion algorithm. Like the first machine, it delegates the tasks of solving the determinants of the associated 8 by 8 submatrices to other machines in the cluster. This continues until the problem has been divided into 2 by 2 matrices, at which point a determinant is quickly evaluated and returned.

In this study, a single computer is capable of evaluating the determinant of a matrix which is no larger than 10 by 10, but runs out of memory when evaluating determinants of any larger matrices. Since the order of the row expansion algorithm is $O(n!)$, incrementing the dimension of the matrix being solved increases the size of the problem from $(n-1)!$ to $n!$. Hence, solving an 11 by 11 matrix would require a machine to perform a number of operations no fewer than that which would be required in evaluating the determinants of eleven separate 10 by 10 matrices. The time required for evaluating the determinant of a 10 by 10 matrix on a single machine is only about 36 seconds. Yet, the memory required to solve the problem is very large. In the case of the 11 by 11 problem, all of the available memory is used up very quickly, leaving the machine unable to complete the evaluation.

From these findings, it was concluded that for this problem, memory is the most scarce resource. Recognizing this led to the development a mathematical method by which one could estimate the amount of memory that a given problem would require before running it on the cluster. Using this method, it would also be possible to determine the dimensions of the largest matrix that could be solved on the cluster, within the constraints of memory resources.

The following reasoning was used inductively to develop a formula for calculating the amount of memory required to evaluate the determinant of a matrix of a given dimension.

The row expansion algorithm necessitates the creation of submatrices during the computation of the determinant. The detp program creates one submatrix for each multiplication of the row expansion. For example, when solving a 4 by 4 matrix, expanding across the first row, detp generates 4 submatrices, each of size 3 by 3 for calculating the minor of each element in the given first row. The determinants of each of those four 3 by 3 matrices are calculated with a recursive call to the determinant solving algorithm, resulting in a total of twelve 2 by 2 matrices. In all, including the original 4 by 4 matrix, 17 matrices are formed. This yields the following formula which expresses the total number of data elements that must be stored throughout the computation of the determinant, in terms of the dimension, n, of the matrix being solved. (A data element, in the C programming language for example, would be one double variable or one int

variable, etc.. Matrices are stored as arrays of data elements.)

$$\sum_{k=0}^{n-2}(n-k)^2\frac{n!}{(n-k)!} \tag{1}$$

Each matrix is stored as an array of type double. Hence, each data element consumes eight bytes of memory. The total amount of RAM used throughout the computation of the determinant will be eight times the number of elements given by the above formula. This value will be called MEM.

We know that this formula is useful for predicting the value of MEM (actually $\frac{MEM}{8}$) by using inductive reasoning to prove its accuracy. The outline of the proof is as follows. Obviously, the number of data elements required to store a 2 by 2 matrix is 4. Evaluating the above summation with n=2 confirms this result. Now let $S(n-1)$ be the total number of data elements which must be stored in evaluating the determinant of a matrix with dimension n-1 by n-1. We know that for any higher dimension, n, the number of data elements which must be stored is equal to the number required by the original n by n matrix plus n times the amount required by a matrix of dimension n-1 by n-1. This relationship is expressed by the following recursive formula.

$$S(n) = n^2 + nS(n-1) \tag{2}$$

Now, if we replace $S(n-1)$ in this equation by the aforementioned summation evaluated for $n = n-1$ and reduce the result, it should look exactly like the original summation when all is finished. Applying the math, one can show that this is indeed the case.

The MEM value derived from the formula is really only good for heuristic purposes. It only tells us how much RAM will be used during the entire computation of the problem. It does not tell us the maximum amount of RAM needed at any given time, hereinafter referred to as MAX, a much more valuable piece of information. There is only one circumstance where this is not the case and the MEM value is equivalent to the MAX value. That circumstance occurs whenever the problem being solved is small enough that the cluster is actively processing nearly all of the problem's workload at any given time during computation, that is, the problem is small enough that the whole thing can be stored in RAM all at once. As the size of the problem grows, however, the MEM value will be much larger than the MAX value.

Knowing the MAX value would allow us to predict exactly how large a cluster would be required for evaluating a given large problem. This

would have some very effective applications, like budget planning when building new clusters for solving larger problems. To this end, an attempt has been made to add the necessary memory accounting code which would allow detp to determine the MAX value for each run of the program. The attempt failed when it was shown that the additional memory accounting code added so much overhead processing time and network bandwidth that it was self defeating. No further attempts at determining the MAX value were made, being beyond the scope of the research. For this study, the simple heuristic value of MEM is sufficient to show the relative magnitudes of problems of different sizes.

This study showed that a single machine could evaluate the determinant of a 10 by 10 matrix and that it failed to do so for an 11 by 11 matrix. It also showed that the cluster could evaluate the determinant of an 11 by 11 matrix and that it failed to do so for a 12 by 12 matrix. This begs the question of whether or not a reasonable addition of nodes to the cluster could provide enough resources for the cluster to solve the 12 by 12 problem. A computer lab nearby the one which contained the cluster used for this research contains 25 computers. The question raised was whether or not the addition of these 25 computers to the cluster would provide sufficient resources for solving the 12 by 12 problem. By using the formula to calculate the MEM value for the 12 by 12 problem, it became quite clear that even this more than doubling of the cluster's resources would still result in insufficient resources for solving the 12 by 12 problem.

The MEM value for an 11 by 11 matrix is approximately 1.3 GB and can be solved on the cluster of 15 machines which has a maximum memory capacity, including all physical RAM and hard disk swap space, of approximately 2.8 GB. The 12 by 12 matrix, on the other hand, has an MEM value of approximately 15.8 GB, far more than the capacity of the 15 node cluster. This is another good example of a time when it would be useful to know the MAX value of the problem. For instance, it may be that the MAX value is only about one half of the MEM value. Of course, even if that were the case, even half of 15.8 GB would still exceed the total capacity of the cluster. Adding the 25 workstations in the adjacent lab would boost the total memory capacity of the cluster to about 6.7 GB. Again, this is not even half of the MEM value of a 12 by 12 cluster. In reality, the MAX value is probably more than one half of the MEM value, placing the 12 by 12 problem even further outside the bounds of feasibility.

In conclusion, this paper has illustrated how memory is a limiting resource when using row expansion to evaluate the determinants of large matrices. It shows that even when clustering machines and offering vast memory resources, the nature of the determinant problem is such that it

would require unrealistic resources to solve.

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An Old Examination

"We are indebted to Professor Norman Anning for a copy of a Form III Algebra examination given in 1899 by the Education Department of Ontario, Canada. Professor Anning writes: "Form III equals eleventh grade...."

How many students of College Algebra can pass this examination?"

1. a. Solve $\frac{2}{3}(9-x)-\frac{1}{3}(13-x)-2=0$.

   b. Solve $\begin{aligned} 50x+51y&=152\\ 51x+52y&=155\end{aligned}$

2. a. Solve $12x^2-311x+1927=0$.

   b. Solve $\begin{aligned} 2x+3y-4z&=83\\ 8y-3z-6x&=54\\ 5z-y-3x&=-83\end{aligned}$

3. a. Divide $(x+y)^2-3(x-y)z+2z^2$ by $z-x-y$

   b. Prove the identity
   $(a+b+c)^2+a^2+b^2+c^2=(b+c)^2+(c+a)^2+(a+b)^2$

   c. Use the identity in (b) to find four numbers such that the sum of their squares shall be equal to the sum of the squares of 5, 4, and 3.

4. Express:

   a. x^6-y^6 in four factors,

   b. $(y-z)^3+(z-x)^3+(x-y)^3$ in three factors,

   c. $8b^2c^2+2c^2a^2+8a^2b^2-a^4-16b^4-c^4$ in four factors.

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Plaid or Pin-Striped: Which Gets You Closer to π?

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Mathematicians throughout the ages have been fascinated by the number π. A constantly reappearing quantity, it has been evaluated in many different fashions. Buffon's needle problem, dating from 1777, is the oldest problem dealing with geometrical probabilities, and was first considered by the French naturalist the Comte de Buffon. n needles, each one unit long, are randomly dropped upon a floor marked with vertical parallel lines, each one unit apart. Buffon approximates π by setting

$$\frac{1}{\pi} = \frac{x}{2n}$$

where x is the number of needles that cross a line.

We would like to consider Buffon's traditional model (case 1) and compare it to a related problem. The first case uses a set of 2n needles one unit in length dropped upon a floor marked with parallel vertical lines spaced one unit apart. We will count the number of needles that cross a line. The second case uses a set of n needles dropped upon a floor marked with two sets of parallel lines one unit apart, the first set vertical and second set horizontal. We will count the total number of lines that the needles cross; each needle may cross 0, 1, or 2 lines. We will estimate $\frac{1}{\pi}$, which is simpler computationally than estimating π. A common definition of a best estimate is one that has the smallest variance. The question then is whether the variance of the estimate of $\frac{1}{\pi}$ in the first case using $2n$ needles or the variance of the estimate of $\frac{1}{\pi}$ in the second case using n needles is smaller. Which is better, the pin-striped floor of case 1 or the plaid floor of case 2?

In the first case of a single set of vertical lines, Sheldon Ross[1] gives a good solution that goes something like the following: Let us consider one needle one unit in length and two parallel vertical lines spaced one unit apart [See Figure 1].

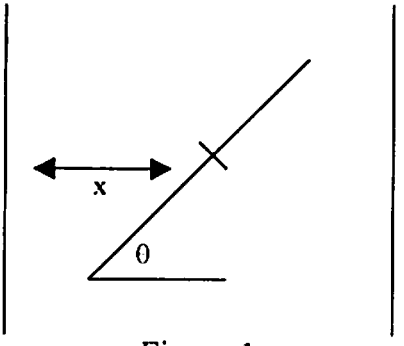

Figure 1

X is the horizontal distance between the center of the needle and the nearest line to the left. θ is the angle of elevation of the needle put in standard position. S is a discrete variable representing whether the needle crosses a line; S is equal to 1 if the needle crosses a line and S is equal to 0 if the needle does not cross a line. If the needle crosses the line to its left, x must be less than half the length of the side of the triangle adjacent to θ. If the needle crosses the line to its right, x must be greater than 1 minus half the length of the side of the triangle adjacent to θ [See Figure 2].

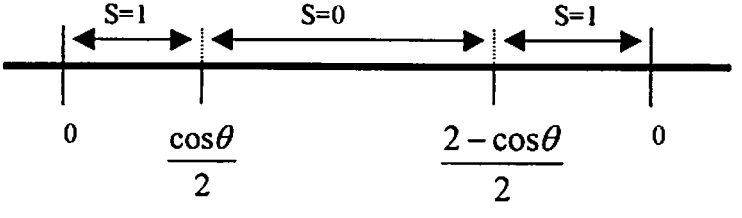

Figure 2

Then,

$$S = \begin{cases} 0 \text{ if } \frac{\cos\theta}{2} \le X \le 1 - \frac{\cos\theta}{2} \\ 1 \text{ if } 0 \le X \le \frac{\cos\theta}{2} \text{ or } 1 - \frac{\cos\theta}{2} \le X \le 1 \end{cases}$$

where X is a uniform continuous random variable between 0 and 1, and θ is a uniform continuous random variable between 0 and $\frac{\pi}{2}$. Their probability density functions are 1 and $\frac{2}{\pi}$ respectively, and since they are independent of each other, their joint probability density is the product $1 \times \frac{2}{\pi} = \frac{2}{\pi}$. We wish to find the probability that $S = 1$, which we write as $P(S = 1)$. We do this by integrating the joint density function of X and θ over the region where $S = 1$. $P(S = 0)$ is found the same way.

$$P(S = 1) = 2x \int_0^{\frac{\pi}{2}} \int_0^{\frac{\cos\theta}{2}} \frac{2}{\pi} dx d\theta = \frac{2}{\pi}$$

$$P(S=0)=\int_0^{\frac{\pi}{2}}\int_{\frac{\cos\theta}{2}}^{1-\frac{\cos\theta}{2}}\frac{2}{\pi}dxd\theta=\frac{\pi-2}{\pi}$$

Using a discrete probability table, we can find the expected value of S, the expected value of S squared, and the variance of S.

s	0	1
P_s	$\frac{\pi-2}{\pi}$	$\frac{2}{\pi}$

Expected (or mean) value of $S = E(S) = 0 \times \frac{\pi-2}{\pi} + 1 \times \frac{2}{\pi} = \frac{2}{\pi}$

$E(S^2) = 0^2 \times \frac{\pi-2}{\pi} + 1^2 \times \frac{2}{\pi} = \frac{2}{\pi}$

Variance of $S = V(S) = E(S^2) - E(S)^2 = \frac{2}{\pi} - \left(\frac{2}{\pi}\right)^2 = \frac{2\pi-4}{\pi^2}$

Averaging $2n$ such values for S and dividing by 2 will give an estimate for $\frac{1}{\pi}$ with variance

Now we will develop the variance of the estimate for the second case. In this case we will let X equal the horizontal distance between the center of the needle and the nearest line to the left, and we will let Y equal the vertical distance between the center of the needle and the nearest line below it. θ is still the angle of elevation of the needle put in standard position [See Figure 3].

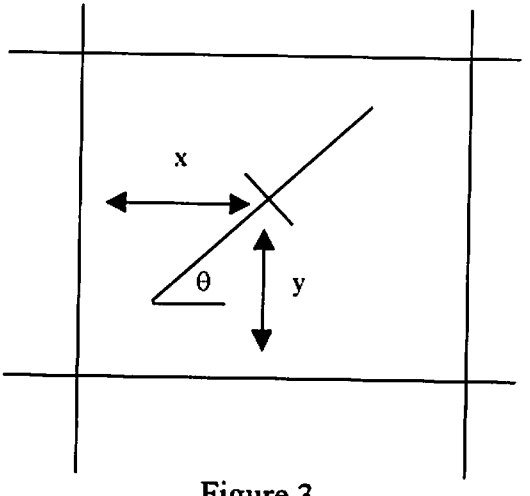

Figure 3

Since X has the same distribution as Y, the probability density functions of the three are as follows:

$$f(\theta)=\frac{2}{\pi},\qquad f(x)=1,\qquad \textit{and } f(y)=1$$

Again the variables are independent so the joint density is their product $\frac{2}{\pi}$. We will define the discrete variable T as follows; T is equal to 0 when the needle crosses no lines, T is equal to 1 when the needle crosses 1 line, and T is equal to 2 when the needle crosses 2 lines.

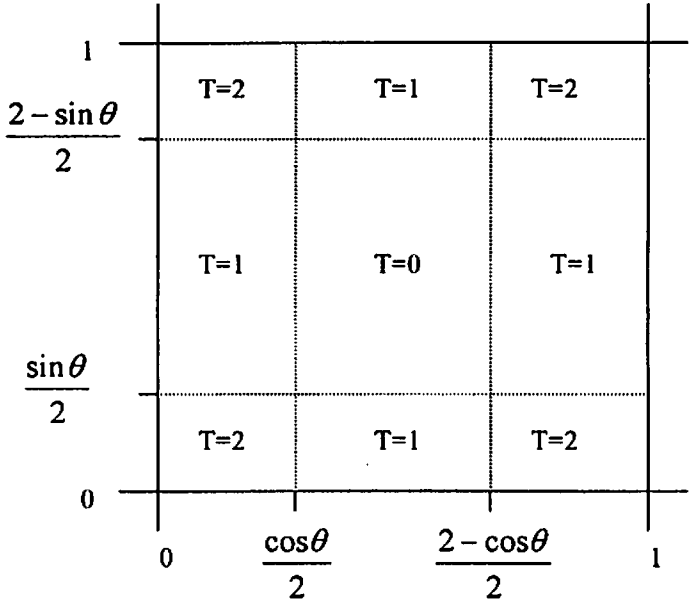

Figure 4

Integrating the joint probability density function over the regions indicated in Figure 4, we get:

$$P(T=2) = 4 \times \int_0^{\frac{\pi}{2}} \int_0^{\frac{\sin\theta}{2}} \int_0^{\frac{\cos\theta}{2}} \frac{2}{\pi} dx dy d\theta = \frac{1}{\pi}$$

$$P(T=1) = 2 \times \int_0^{\frac{\pi}{2}} \int_0^{\frac{\sin\theta}{2}} \int_{\frac{\cos\theta}{2}}^{1-\frac{\cos\theta}{2}} \frac{2}{\pi} dx dy d\theta + \int_0^{\frac{\pi}{2}} \int_{\frac{\sin\theta}{2}}^{1-\frac{\sin\theta}{2}} \int_0^{\frac{\cos\theta}{2}} \frac{2}{\pi} dx dy d\theta = \frac{2}{\pi}$$

$$P(T=0) = \int_0^{\frac{\pi}{2}} \int_{\frac{\sin\theta}{2}}^{1-\frac{\sin\theta}{2}} \int_{\frac{\cos\theta}{2}}^{1-\frac{\cos\theta}{2}} \frac{2}{\pi} dx dy d\theta = \frac{\pi-3}{\pi}$$

Creating a probability table for T, we can find the $E(T)$ and $V(T)$ as we did for S.

t	0	1	2
P_t	$\frac{\pi-3}{\pi}$	$\frac{2}{\pi}$	$\frac{1}{\pi}$

$$E(T) = 0 \times \tfrac{\pi-3}{\pi} + 1 \times \tfrac{2}{\pi} + 2 \times \tfrac{1}{\pi} = \tfrac{4}{\pi}$$

$$E(T^2) = 0^2 \times \tfrac{\pi-3}{\pi} + 1^2 \times \tfrac{2}{\pi} + 2^2 \times \tfrac{1}{\pi} = \tfrac{6}{\pi}$$

$V(T) = E(T^2) - E(T)^2 = \frac{6}{\pi} - \left(\frac{4}{\pi}\right)^2 = \frac{6\pi - 16}{\pi^2}$

Averaging n such values for T and dividing by 4 gives an estimate of $\frac{1}{\pi}$ with variance

This is over 37% smaller than the variance of the estimate based on one set of lines. In conclusion, the variance in case 2 is smaller than the variance in case 1. Because of its smaller variance, case 2 gives the better estimate. Astonishingly, it more accurately estimates $\frac{1}{\pi}$ using only half the number of needles used in case 1. Answering my previously stated question, the plaid floor in case 2 gives a more accurate and efficient estimate - no wonder the Scottish wear their kilts so proudly.

Acknowledgments. Thank you to Dr. Donald Tosh, Science and Technology Chairman, for his assistance with this paper.

References

1. Ross, Sheldon M., A First Course in Probability, 5th ed., Prentice Hall, Upper Saddle River, New Jersey, 1998.

Starting a KME Chapter

Complete information on starting a chapter of KME may be obtained from the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of the $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offering and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before July 1, 2000. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 2000 issue of *The Pentagon*, with credit being given to the student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 6621 (e-mail: xxwilke@acc.wuacc.edu).

PROBLEMS 530 - 534

Problem 530: Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Start at the origin, move to the right one unit, move up $\frac{1}{2}$ unit, move to the right $\frac{1}{4}$ unit, move up $\frac{1}{8}$ unit, etc. Connect the limiting point of the path from the origin to the origin by drawing a straight line connecting these two points. What is the area of the figure enclosed by connecting the origin and the limiting point of the original path?

Problem 531: Proposed by Russell Euler and Jawad Sadek jointly, Northwest Missouri State University, Maryville, Missouri.

A bridge in the form of a circular arc spans a river. At a distance of A feet measured horizontally from the shore, the bridge is B feet above the surface of the water. At the center of the bridge, the bridge is C feet above the surface of the water. Assuming that the bridge rests exactly on the shores, find the width of the river in terms of A, B, and C. For $B > C$, discuss the cases where there are zero, one, or two solutions.

Problem 532: Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Join consecutively the points

$$(1,0),(\frac{1}{2},(\frac{1}{2})^2),(\frac{1}{3},0),(\frac{1}{4},(\frac{1}{4})^2),...,(\frac{1}{2n},(\frac{1}{2n})^2),(\frac{1}{2n+1},0),...$$

with line segments, and include the point $(0,0)$ in the resulting graph. Use the x-axis as a base of the graph which should look like an infinite

series of triangles. Find the total area of the series of triangles. (This is a generalization of Pentagon problem 214).

Problem 533: Proposed by the editor.
Which of the following quantities is larger

$$(31415926535!)^2 \text{ or } 31415926535^{31415926535}?$$

Problem 534: Proposed by the editor.
The millennium is fast approaching. Whether it starts on January 1, 2000, as many people believe or January 1, 2001, as the purists argue, is not material to this problem. In honor of the millennium, now is a prime occasion to discover whether or not there is a prime p such that p! ends in exactly 2000 zeroes and whether there is a corresponding prime q such that q! ends in exactly 2001 zeroes?

Please help your editor by submitting problem proposals.

SOLUTIONS 520- 524

Problem 520: Proposed by the editor.
Let R_x denote the continued fraction

$$1/x + 1/x + 1/x + ...$$

Find all solutions in positive integers a and b, if any, of the equation

$$5R_a - 2R_b = 1.$$

Solution by Tom Elsner, Kettering University, Flint, Michigan.
As a continued fraction, R_x can be represented as the limit of the recursive sequence $R_x(n+1) = R_x + 1/x$ with $R_x(1) = 1/x$ which converges to a positive number for each positive x. Limit algebra implies that $R_x(R_x + x) = 1$ so that $R_x = \frac{-x+\sqrt{x^2+4}}{2}$ since the negative root is discarded. Then we can solve $5R_a - 2R_b = 1$ for b in terms of a. Our equation is

$$5(-a + \sqrt{a^2+4})/2 - (-b + \sqrt{b^2+4}) = 1 \qquad (1)$$

Solving (1) for b in terms of a we have

$$b = \frac{-(25a+10)\sqrt{a^2+4} + 44 + 10a + 25a^2}{2\left(5a + 2 - 5\sqrt{a^2+4}\right)} \qquad (2)$$

The roots of the numerator of equation (2) are $\frac{-24}{5}$ and $\frac{16}{15}$. The root of the denominator of equation (2) is $\frac{24}{5}$. These roots show that the sign

of b changes only twice in the positive domain. Then since a is a positive integer, only a = 2, 3, and 4 need to be checked because only for these values of a do the numerator and denominator of equation (2) have the same sign. The only solution in integers occurs when $a = 4$ and $b = 11$.

Also solved by: Bryan Fischer, Buffalo State College, Lackawana, New York. One incorrect solution was received.

Editor's comment: The following alternate approach avoids some of the tedious computations involved in our featured solution.

$5R_a - 2R_b = 1$ is equivalent to

$$2 + 5a - 2b = 5\sqrt{a^2 + 4} - 2\sqrt{b^2 + 4}. \qquad (*)$$

Since $x^2 + 4$ is the square of an integer only for the integer $x = 0$, the right side of (*) must be zero. Thus $25a^2 + 100 = 4b^2 + 16$ or $(2b + 5a)(2b - 5a) = 84$ where $(2b + 5a)$ and $(2b - 5a)$ have the same parity. Hence $(2b + 5a) = 42$ and $(2b - 5a) = 2$ or $(2b + 5a) = 14$ and $(2b - 5a) = 6$ since a and b are integers. Only the first pair yields integral values for a and b which are $a = 4$ and $b = 11$; these values satisfy equation (*). Finally we have

$$5R_a - 2R_b = 5\left(\frac{-4 + \sqrt{16 + 4}}{2}\right) + 2\left(\frac{11 - \sqrt{121 + 4}}{2}\right)$$

$$= -10 + \sqrt{5} + 11 - 5\sqrt{5} = 1.$$

Problem 521: Proposed by the editor.

Sam is delivering a large rectangular box of pizza in a dormitory. He must take the pizza down a hallway whose width is 40 inches. Unfortunately this hallway intersects another hallway whose width is 30 inches at right angles and the room which ordered the pizza lies down this second hallway. Assuming that he always carries the pizza box in a horizontal plane, what are the dimensions of the pizza box having maximum area which he can deliver? For the purposes of this problem, consider the pizza box as being a horizontal plate.

Solution by Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri.

Let A, B, C and D represent the vertices of the pizza box and let M be a corner where the hallways intersect. In order for the box to be turned from the 40" hallway into the 30" hallway, $MA < 40$" and $MD < 30$". If K represents the area of the triangle AMD, then $K = \frac{(AD)(AB)}{2} = \frac{(AM)(MD)(\sin AMD)}{2}$. It suffices to maximize K. Since $MA < 40$" and $MD < 30$", $K < \frac{(40)(30)}{2}$. So the maximum area of the box is 1200 square

inches and is achieved when $MA = 40$", $MD = 30$"and $\angle AMD = \frac{\pi}{2}$. As a consequence, $AD = 50$" and $AB = 24$". Of all the boxes that can be delivered, the box with dimensions 24" by 50" has both a maximum area and a maximum perimeter. There is a second solution where the box has dimensions 30" by 40" which has the same area. In this case the box fits snugly in the 40" hallway and when the 30" doorway is reached, the box fits that hallway snugly without turning.

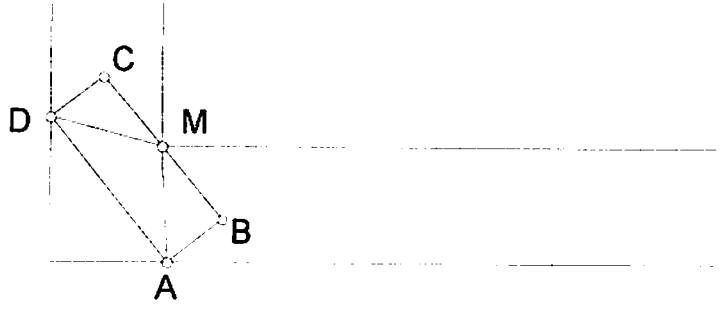

Figure for Solution to Problem 521

Editor's Comment: The solvers astutely pointed out that two solutions were possible even though your editor intended the 24" by 50" box to be the only solution. Is it possible that our solvers have had previous experience with both cases? Seriously though the methodology of the featured solution shows that for hallways of widths m and n, the dimensions of the largest pizza box is given by $\sqrt{m^2 + n^2}$ by $\frac{mn}{\sqrt{b^2+n^2}}$. (That is unless you prefer the "snug solution" where the box is m by n.)

Problem 522: Proposed by Bryan Dawson, Union University, Jackson, Tennessee.

A particular grade book allows 30 students to be listed on a page. Suppose a class consists of 33 students, with 30 names listed on one page and 3 on the next, and that the pages must be turned from one set of students to the other. If I record grades as I grade an assignment, in random order of students and turn the page only when necessary, what is the number of page turns needed, not counting turning to the correct page of the grade book for the first student? Assume that all students turned in the assignment.

Solution by: Brian Reich and Michelle Skurjanec (jointly), University of Wisconsin-River Falls, River Falls, Wisconsin.

Let a plus sign denote a student on the first page and a minus sign denote a student on the second page. Then the probability of one turn $P(1\ turn) = 2 \cdot (30!) \cdot (3!)/(33!)$. This case requires all three students from

the second page to appear as a block at the beginning (or end) of the list.

The probability of two turns $P(2\ turns) = 3 \cdot (30!) \cdot (3!)/(33!)$. This case has two subcases. The first subcase requires all three students from the second page to appear as a block but not occupying either the first or the last position in the list. There are $(29! \cdot 30! \cdot 3!)/33!$ listings of this type. The second subcase requires that all plus signs are in the middle of the listing and that each listing start with two consecutive minus signs and end with one minus sign or vice versa. There are a total of $(2 \cdot 30! \cdot 3!)/33!$ such listings. The probability of three turns $P(3\ turns) = 4 \cdot 29 \cdot (30!) \cdot (3!)/(33!)$. This case has essentially two subcases. The first subcase requires all three students from the second page to appear as a block but not occupying either the first or the last position in the list. There are $(29! \cdot 30! \cdot 3!)/33!$ listings of this type. The second subcase requires that all plus signs are in the middle of the listing and that each listing start with two consecutive minus signs and end with one minus sign or vice versa. There are a total of $(2 \cdot 30! \cdot 3!)/33!$ such listings.

The probability of three turns $P(3\ turns) = 4 \cdot 29 \cdot (30!) \cdot (3!)/(33!)$. This case has essentially two subcases. The first subcase requires the first student in the listing be a plus sign and the last two students be consecutive minus signs with the third minus sign student appearing somewhere in the middle of the listing between positions 2 and 30. Also from each of these listings a corresponding listing may be found by appropriately reversing the positions of the minus signs; i.e. putting two consecutive minus signs in a middle position and a single minus sign at the end of the list. There are $2 \cdot 29 \cdot 30! \cdot 3!$ such listings. The second subcase mirrors the first subcase except that here each listing begins with one (or two) students from the second page. These are mirror reflections of the listing of the first subcase and there are $2 \cdot 29 \cdot 30! \cdot 3!$ such listings.

The probability of four turns $P(4\ turns) = [30 \cdot 29 - 2 \cdot 29 + 29] \cdot (30!) \cdot (3!)/(33!)$. This case has two subcases. The first subcase requires that each listing begin and end with a plus sign with a pair of consecutive minus signs and a single minus sign occupying a separate interior space between spaces 2 and 31. There are $(30 \cdot 29 - 2 \cdot 29)30! \cdot 3!$ such listings. The second subcase requires that each listing begin (and end) with a single minus sign with the third minus sign occupying a space between spaces 3 and 30. There are $29 \cdot 30! \cdot 3!$ such listings.

The probability of five turns $P(5\ turns) = [30 \cdot 29 - 2 \cdot 29 + 29] \cdot (30!) \cdot (3!)/(33!)$. This case has two subcases. In the first subcase, the listing starts with a minus sign, end with a plus sign and the other two minus signs occupy separate interior spaces separated by at least one minus sign. The second subcase consists of the mirror reflections of the listings in the

first case so that all listings end with a minus sign and start with a plus sign and the other two minus signs occupy separate interior spaces separated by at least one minus sign. There are in all $2 \cdot [{}_{30}C_2 - {}_{29}C_1] \cdot 30! \cdot 3!$ such listings.

The probability of six turns $P(6\ turns) = [{}_{31}C_3 - (30 \cdot 29 - 2 \cdot 29) - 29] \cdot (30!) \cdot (3!)/(33!)$. In this case, each listing begins and ends with a plus sign with the each minus sign occupying an interior space and each minus sign is separated from the others by at least one plus sign. There are $[{}_{31}C_3 - (30 \cdot 29 - 2 \cdot 29) - 29] \cdot (30!) \cdot (3!)$ such listings. Now let X denote the number of page changes needed to complete the grading. Then

$$\begin{aligned} E(X) = 1 \cdot \{2 \cdot (30!) \cdot (3!)/(33!)\} + 2 \cdot \{3 \cdot (30!) \cdot (3!)/(33!)\} \\ +3 \cdot \{4 \cdot 29 \cdot (30!) \cdot (3!)/(33!)\} \\ +4 \cdot \{[30 \cdot 29 - 2 \cdot 29 + 29] \cdot (30!) \cdot (3!)/(33!)\} \\ +5 * \{[30 * 29 - 2 * 29 + 29] \cdot (30!) \cdot (3!)/(33!)\} \\ +6 \cdot \{[{}_{31}C_3 - (30 \cdot 29 - 2 \cdot 29) - 29] \cdot (30!) \cdot (3!)/(33!) \\ = \tfrac{29760}{5456} = 5 + \tfrac{5}{11}. \end{aligned}$$

Also solved by the proposer.

Editor's Comment: The following is another approach which yields the same solution.

Consider a string of 30 + signs where the plus sign denotes a student on the first page. Let a minus sign denote a student whose name appears on the second page. Then a possible listing of the student names might look like

$$++++-++++++++-++++++++++-++++++++++$$

There are a total of ${}_{33}C_3 = 5456$ possible listings. If one considers a string of 30 plus signs with spaces between them and spaces on each end available for the insertion of minus signs, there are 31 such spaces which may be used. Let s_j , where $1 < j < 6$, denote a listing with j sign changes.

First suppose that all three minus signs appear together as a block. There are ${}_{31}C_1 = 31$ such listings. If the block occupies an end space, we have a s_1 listing and if the block appears at an interior space we have a s_2 listing. There are ${}_2C_1 = 2s_1$ such s_1 listings and ${}_{29}C_1 = 29s_2$ such s_2 listings. Hence ${}_{31}C_1 = 31 = 2s_1 + 29s_2$.

Next suppose that all three minus signs appear in separate spaces. There are ${}_{31}C_3 = 4495$ such listings. If each of the end positions is filled with a minus sign, the third minus sign must fill an interior space and a s_4 listing results and there are ${}_{29}C_1 = 29$ such s_4 listings. If all three minus signs

occupy interior spaces, an s_6 listing results and there are ${}_{29}C_3 = 3654$ such s_6 listings. If one minus sign occupies and end space and the other two occupy interior spaces, an s_5 listing results and there are $2({}_{29}C_2) = 812$ such s_5 listings. Hence ${}_{31}C_3 = 4495 = 29s_4 + 812s_5 + 3654s_6$.

Finally suppose that the minus signs appear as a single minus sign and a pair of two consecutive minus signs. There are $2({}_{31}C_2) = 930$ such listings. If each block of minus signs occupies an end space, an s_2 listing results and there are $2({}_1C_1) = 2$ such s_2 listings. (The factor of 2 results from the choice of which end space the double minus sign occupies.) If each block of minus signs occupies an interior space, an s_4 listing results and there are $2({}_{29}C_2) = 812$ such s_4 listings. (The factor of 2 results from the choice of two possible choices of which interior space the double minus sign occupies.) If one block of minus signs occupies and end space and the other occupies an interior space, an s_3 listing results and there are $4({}_{29}C_1) = 116$ such s_3 listings. (The factor of 4 results from the choice of two end spaces and a choice of two possible choices of which end space the double minus sign occupies.) Hence ${}_{31}C_2 = 930 = 2s_2 + 116s_3 + 812s_4$.

Finally collecting all possible listings, we have

$${}_{33}C_3 = 5456 = 2s_1 + 31s_2 + 116s_3 + 841s_4 + 812s_5 + 3654s_6.$$

Then the desired expected value

$$E(X) = \tfrac{1}{5456}(1 \cdot 2 + 2 \cdot 31 + 3 \cdot 116 + 4 \cdot 841 + 5 \cdot 812 + 6 \cdot 3654)$$
$$= \tfrac{29760}{5456} = 5 + \tfrac{5}{11}$$ where X denotes the number of page turns required.

Problem 523: Proposed by the editor.

A customer stops in at a local convenience store and purchases four items. The customer was told that the cost was \$7.70. The clerk had inadvertently multiplied together the prices of the four individual items. The customer protested that the four prices should have been added together instead of being multiplied. The clerk said that was OK with him, but the result was still the same: exactly \$7.70. What were the prices of the four items?

Solution by: Charles Ashbacher, Hiawatha, Iowa.

Let a, b, c and d denote the respective prices in pennies. Then $a + b + c + d = 770$ and $abcd = 770000000$. A simple computer search reveals two solutions:

$1.00 + 1.00 + 2.20 + 3.50 = 7.70$ $\quad$ $1.00 * 1.00 * 2.20 * 3.50 = 7.70$

and

$1.00 + 1.25 + 1.60 + 3.85 = 7.70$ $\quad$ $1.00 * 1.25 * 1.60 * 3.85 = 7.70.$

Partial solutions were received from: Craig Davis, Kutztown University, Kutztown, Pennsylvania; Tom Elsner, Kettering University, Flint, Michi-

gan; Russ Euler and Jawad Sadek (jointly), Northwest Missouri University, Maryville, Missouri; and Brian M. Fischer, Buffalo State College, Lackawana, New York.

Editor's Comment: It is interesting that each of the partial solutions found the same solution with the prices \$1.00, \$1.00, \$2.20 and \$3.50. The assumption that two of the prices are \$1.00 happens to yield a solution, but not all of them.

Problem 524: Proposed by the editor. From the "Mathematical Scrapbook" section of the Pentagon, Spring 1943.

How much of the earth's surface would a man see if he were raised to the height of the radius above it?

Solution by: Protima Advanti, West Virginia Wesleyan College, Buckhannon, West Virginia. (Revised slightly by the Editor.)

In the figure let $RS = RQ = r$ denote the radius of the Earth. Let $VT = nr$ denote the height of the man above the Earth where n is a proportionality factor. Let $PR = x$: then $PV = r - x$ and $TR = (n+1)r$. Triangles TQR and STR are right triangles. Also denote angle PRQ by θ.

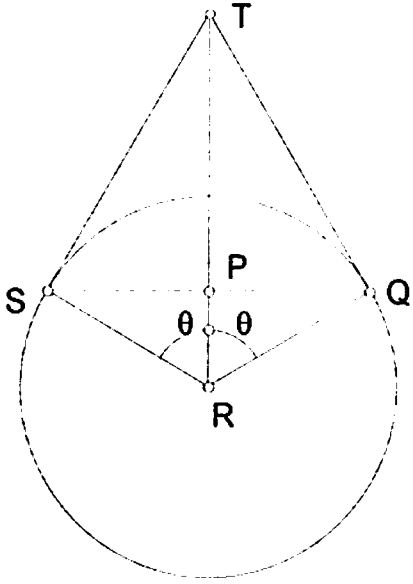

Figure for Solution to Problem 524

In triangle TQR, $\cos\theta = RQ/RT = \frac{1}{n+1}$. In triangle PQR, $\cos\theta = PR/RT = \frac{x}{r}$. Hence $x = \frac{r}{n+1}$. Then the height of the spherical cap is $r - x = \frac{rn}{n+1}$. Then since the surface area of the spherical cap is given by $2\pi rh$ where h is the height of the spherical cap, we find that the surface

area the man can see is given by $2\pi r(r-x) = 2\pi r^2 \frac{n}{n+1}$. Then since the surface area of the Earth is given by $4\pi r^2$, the man can see $\frac{n}{2(n+1)}$ per cent of the Earth's surface. In this problem, $n = 1$ so the man sees 25% of the Earth's surface. As the man moves farther away from the Earth, $n \longrightarrow \infty$ and since $\lim_{n\to\infty} \frac{n}{2(n+1)} = \frac{1}{2}$, the man can see $\frac{1}{2}$ of the Earth's surface at a maximum.

Also solved by: Tom Elsner, Kettering University, Flint, Michigan; Russ Euler and Jawad Sadek, jointly, Northwest Missouri State University, Maryville, Missouri; Brian M. Fischer, Buffalo State College, Lackawana, New York; and the Alma College Problem Solving Group, Alma, Michigan.

(An Old Examination *continued from page 27)*

5.a. Find a symmetrical rational integral homogeneous expression of two dimensions in x, y,and z, which is equal to 6 when $x = y = z = 1$; and which is equal to 11, when $x = y = 1$ and $z = 2$.

b. Simplify:
$\frac{a^3}{(a-b)(a-c)} + \frac{b^3}{(b-c)(b-a)} + \frac{c^3}{(c-a)(c-b)}$

6.a. Solve fully the quadratic equation $px^2 + qx + 4 = 0$.

b. Find an expression for the sum of the roots in (a) in terms of the coefficients.

c. Find the equation whose roots are the reciprocals of the roots of $5x^2 + 6x - 7 = 0$.

7.a. Find the square root of
$a^2 - 2a^{3/2}b^{-1/3} + 3ab^{-2/3} - 2a^{1/2}b^{-1} + b^{-4/3}$

b. Solve $\sqrt{x+3} + \sqrt{x} = 5$

8.a. Find two numbers, differing by 20, and such that five-eights of the less exceeds two-fifths of the greater by 1.

b. The sum of two numbers is 4, and the difference of their squares is also 4. Find the numbers.

c. A rectangular field contains 5 acres. If the field were 10 rods shorter and 4 rods broader the area would be the same. Find the length and breadth of the field.

-The Pentagon, Fall 1951 (pp.43-44)

Kappa Mu Epsilon News

Edited by Don Tosh, Historian

News of chapter activities and other noteworthy KME events should be sent to Don Tosh, Historian, Kappa Mu Epsilon, Mathematics Department, Evangel College, 1111 N. Glenstone, Springfield, MO 65802, or to toshd@evangel.edu.

INSTALLATION OF NEW CHAPTERS

North Carolina Delta

High Point University, High Point

The installation of the North Carolina Chapter of Kappa Mu Epsilon was held on March 24, 1999 in the Slane Student Center on the campus of High Point University. Donald Aplin, corresponding secretary of South Carolina Gamma at Winthrop University, Rock Hill, S.C., was the installing officer. Rob Harger of the Department of Mathematics, who took the primary responsibility to get a chapter of Kappa Mu Epsilon at High Point University, was the conductor for the ceremony. The founding group listed below includes fourteen students and five faculty.

Pennsylvania Pi

Slippery Rock University, Slippery Rock

The installation of the Pennsylvania Pi Chapter of Kappa Mu Epsilon was held on Monday, April 19, 1999, in the SWC Auditorium on the campus of Slippery Rock University. Dr. Peter R. Skoner, Great Lakes Regional Director, led the installation ceremony. Ms. Jennifer Aiello, President of the Math Club, arranged the ceremony and served as conductor. Fourteen students and four faculty constituted the founding group of the new chapter at Slippery Rock University. Those initiated included:

Students: Alexey Balykin, Travis Bargo, Carrie Birckbichler, Wesley Cross, Heather Gaston, Valerie Hensel, Crystal Hogue, Abby Hooks, Nishan Jayaratnam, Angela Miller, David Moskala, Todd Pavlik, Melissa Persing, and Andrew Wozniak.

Faculty: Dr. Michael E. Detlefsen, Dr. Elise M. Grabner, Dr. Gary C. Grabner, and Dr. Robert W. Vallin.

Also in attendance were Mathematics Department Chairman Dr. William Lindgren, three KME members from the Pennsylvania MU Chapter, and the family and friends of many of the inductees.

Jennifer Aiello provided the opening welcome. Dr. Lindgren offered the faculty address and challenged all present to a mathematics puzzle.

President Melissa Persing next introduced the officers of Pennsylvania Pi. Dr. Skoner then began the installation with an introduction to and a brief history of Kappa Mu Epsilon. Officers installed during the ceremony were Melissa Persing, President; Crystal Hogue, Vice President; Carrie Birckbichler, Recording Secretary; Nishan Jayaratnam, treasurer; Dr. Elise Grabner, corresponding secretary; and Dr. Robert Vallin, faculty sponsor. After the installation, Ms. Aiello offered concluding remarks. A reception was held following the ceremony.

Chapter News

AL Zeta Chapter President — Melanie Styers
Birmingham Southern College, Birmingham 20 actives

Two regular meetings were held during the semester. Other spring 1999 officers: Kelly O'Donnell, vice president; Elizabeth White, secretary/treasurer; Mary Jane Turner, corresponding secretary; Shirley Brannan, faculty sponsor.

AL Eta Chapter President — Justin Smith
University of West Alabama, Livingston 19 actives, 9 associates

Nine members were inducted and the chapter was represented at the National Convention in Lakeland. Other spring 1999 officers: James Zimlich, vice president; Jaime Shutt, secretary; Jason Overstreet, treasurer; Judy Massey, corresponding secretary/faculty sponsor.

CO Gamma Chapter President — Shannon Cummins
Fort Lewis College, Durango 15 actives, 6 associates

During the spring semester the chapter sponsored, along with the college Math/Computer Science Club, a showing of the movie "Pi". Pizza was served. There was also an initiation ceremony for 6 new members. Pizza was again served. Other spring 1999 officers: Kim Grote, vice president; Cindy Hilliker, secretary/treasurer; Richard Gibbs, corresponding secretary;Deborah Berrier, faculty sponsor.

CO Delta Chapter President — John Bright
Mesa State College, Grand Junction 17 actives

Twenty members and guests attended a picnic at Lincoln Park in April. Certificates and pins were presented to the November initiates, and new officers were elected for 1999-2000. Donna Hafner, who represented the chapter at the 32nd Biennial Convention, reported on convention activities and topics of discussion. Other spring 1999 officers: Amanda Widel, vice president; Sarah Kennedy, secretary; David Wing, treasurer; Donna Hafner, corresponding secretary; Kenneth Davis, faculty sponsor.

FL Beta Chapter President — Roxanne Back
Florida Southern College, Lakeland 37 actives, 10 associates

We hosted the National KME Convention. Other spring 1999 officers: Michelle Porter, vice president; Amanda Malone, secretary; Jeff Carlson, treasurer; Susan Rinker, corresponding secretary; Allen Wuertz, faculty sponsor.

GA Alpha Chapter President — Nancy Boyette
State University of West Georgia, Carrollton 18 actives, 8 associates

Our chapter held its Spring Social, which was very well attended by both students and faculty, on April 9 at one of the area seafood restaurants . On April 21, we held our annual Initiation Ceremony at which 8 new members were initiated. After the initiation, we elected new officers for 1999-2000. After the initiation, a reception was held in honor of the new initiates. At the reception the winners of this year's mathematics awards/scholarships were announced. The winners included: Boyd Award (Nancy Bryson), Burson Calculus Award (Amy Smith), Marion Crider Awards (James Henson, John Doughty, and Danny O'Rourke), Martin Scholarship (David Wilson), Whatley Scholarship (John Doughty), and the Cooley Scholarship (Chris Faucett). Other spring 1999 officers: Tatiana Mack, vice president; Amy Smith, secretary; Blake Smith, treasurer; Joe Sharp, corresponding secretary; Mark Faucette/Joe Sharp, faculty sponsors.

IL Zeta Chapter President — Karen Jarosz
Dominican University, River Forest 16 actives, 18 associates

Our spring induction ceremony was held on March 21, 1999. We were fortunate to have Dr. Ed Packel, Professor of Mathematics at Lake Forest College, as a guest speaker. His topic was Game Theory. On Sunday, April 25, a party for math majors and minors was held at a math professor's house. Other spring 1999 officers: Christa Lee, vice president; Anna Cantal, secretary; Heather Wasielewski, treasurer; Paul Coe, corresponding secretary/faculty sponsor.

IL Theta Chapter President — Dennis Wozniak
Benedictine University, Lisle 17 actives, 7 associates

The spring banquet featured the induction of 7 new members into KME. The guest speaker, Dr. Don Porzio of IMSA, spoke on "Making Mathematics Fun Again". KME chapter members worked with the Math Club to foster participation in a regional calculus competition. Over 30 students competed, with one team placing 5th overall. Other spring 1999 officers: Lisa Townsley Kulich, corresponding secretary/faculty sponsor.

IN Gamma Chapter President — Michelle Muhlenkamp
Anderson University, Anderson 7 actives

Other spring 1999 officers: Laura Jourdqan Walls, Stanley Stephens, corresponding secretary/faculty sponsor.

IA Alpha Chapter President — Suzanne Shontz
University of Northern Iowa, Cedar Falls 42 actives

One of our scheduled local meetings was cancelled because of a snow storm in progress. Student member Kamilla Guseynova presented her paper on "Fibonacci Numbers and their Applications" at the next meeting. Stephanie Timmerman addressed the Spring initiation banquet on "The Mathematics of Chemical Oscillations". The major event of the semester was the KME National Convention in Lakeland, FL. Six students, Allysen Edwards, Kamilla Guseynova, Beth Koch, Mary Noga, Suzanne Shontz and Gary Spieler, accompanied by faculty members John Cross and Mark Ecker flew to Tampa where they were joined by student Eric Clark of Western Illinois University. Three Iowa Alpha students presented papers to the National Convention: Mary Noga, Suzanne Shontz and Gary Spieler. Allysen Edwards and Kamilla Guseynova served on National KME committees at the convention. Suzanne Shontz's paper on "Computing Homoclinic Bifurcations" was selected as one of the top four papers at the convention and she received a check for $100 and the software package "Maple". Professor John Cross will retire from UNI this spring. For 33 of his 37 years of service he was the Faculty Sponsor and Corresponding Secretary for Iowa Alpha Chapter. During this period, exactly 400 new members were initiated into Iowa Alpha. We are most pleased that Dr. Mark Ecker will be the new faculty liaison. Other spring 1999 officers: Gary Spieler, vice president; Beth Koch, secretary; Mary Noga, treasurer; John Cross and Mark Ecker, corresponding secretary/faculty sponsor.

IA Delta Chapter President — Paul Seberger
Wartburg College, Waverly 58 actives

Seniors shared their Mathematics Capstone posters as the program for our January meeting. The February meeting program was a math puzzle, "How Is This True?" Some members traveled as a group to Iowa State University to meet with representatives of the Graduate School and the Engineering School one weekend in February. We initiated eighteen new members into our Chapter in March. Our Initiation Banquet speaker was Reid Koenig, Vice-President and Chief Operations Officer for CUNAA Mutual Life Insurance Company. Reid spoke about his career path in mathematics including his studies at Wartburg College through his current position. The April meeting consisted of a refreshment study-break

during the final exam week for our winter term. In May, we had a joint picnic with the Physics, Computer Science and Chemistry clubs at one of the city parks. Other spring 1999 officers: Robyn Brent, vice president; Janelle Young, secretary; Daniel Bock, treasurer; August Waltmann, corresponding secretary; Mariah Birgen, faculty sponsor.

KS Alpha Chapter President — Mandy Fritz
Pittsburg State University, Pittsburg 28 actives, 10 associates

The first meeting of the semester was held on February 3. Plans to attend the national convention were discussed along with the idea of developing a chapter T-shirt. A video about fractals was the program. The next meeting was on March 3. At this meeting the plans for the national convention were finalized and 10 new members were initiated. Jeremy Dill presented a program on "Digital Loops". This was the same presentation for which he won a prize at the MAA Summer Mathfest. After the meeting the member and initiates went out for pizza. In April, the main topic of the meeting was the trip to the national convention in Florida. For the program, Gayola Dodson gave a presentation about three famous female mathematicians. The plans for the chapter T-shirt were finalized and the shirt sold very well. The Kansas Alpha Chapter was represented by six students and the faculty sponsor at the national convention. The last activity of the school year was an ice cream social hosted by the corresponding secretary. Other spring 1999 officers: Catherine Ellis, vice president; Lisa Collier, secretary; Jeremy Dill, treasurer; Tim Flood, corresponding secretary; Yaping Liu, faculty sponsor.

KS Beta Chapter Presidents — Brian Albright & Casey Wilson
Emporia State University, Emporia 15 actives, 3 associates

Other spring 1999 officers: Aaric Davis, vice president; Katrina Penner, secretary/treasurer; Connie Schrock, corresponding secretary; Larry Scott, faculty sponsor.

KS Gamma Chapter President — Kevin Slattery
Benedictine College, Atchison 9 actives, 11 associates

On 22 March, KS Gamma initiated four new members-Lance Hoover, Clint Keading, Curtis Sander, and Matthew Wiggins. These four initiates were awarded Sister Helen Sullivan scholarships at the Honors Convocation on 24 March. That evening the College presented Dr. Vern Ostdiek its Distinguished Educator Award. Dr. Ostdiek, an active KS Gamma member who teaches half-time in mathematics and half-time in physics, will be on sabbatical during the fall semester. Sister Linda Herndon, OSB will take a leave from the faculty at the end of the spring semester to pursue doctoral study at the University of Wisconsin. The COMAP project

on "Curving Grade Inflation," completed last spring by three KS Gamma members, appeared on 14 April in the first issue of the College's Discovery Journal. President Kevin Slattery served on the Awards Committee and faculty sponsor Sister Jo Ann Fellin, OSB served as chair of the Audit Committee at the 32nd KME Biennial Convention in Lakeland, FL in late April. On 4 May, chapter members enjoyed a pizza party at the home of faculty member Richard Farrell. Other spring 1999 officers: Jo Ann Fellin, corresponding secretary/faculty sponsor.

KS Delta Chapter President — Laurie Payeur
Washburn University, Topeka 24 actives

On February 23, Kansas Delta had its annual spring initiation banquet. There were three initiates. In March, KME had a joint noon meeting with the Washburn Math club. In April, another joint meeting was held. At this meeting Laurie Payeur and Doug Appenfeller presented the papers that they would give at the National Biennial KME Convention. Seven students and three faculty attended that convention at Florida Southern College in Lakeland on April 22-24, 1999. Payeur and Appenfeller both were awarded Top-4 Honors for their presentations. Others attending were Stephanie Adelhardt, Justin Freeby, Milorad Felbapov, Elena Kanaeva, Stephanie Lambert, Donna LaLonde, Ron Wasserstein, and Al Riveland. In May, an afternoon picnic was held, jointly, with the Washburn Math Club. Other spring 1999 officers: Stephanie Lambert, vice president; Justin Freeby, secretary/treasurer; Allan Riveland, Corresponding Secretary; Ron Wasserstein and Donna LaLonde, faculty sponsors.

KS Epsilon Chapter President — Marian Riazi
Fort Hays State University, Hays 39 actives, 6 associates

Monthly meetings and a banquet were held. These included speakers on Optical Delusions and Animated Designs. Other spring 1999 officers: B. J. Peterson, vice president; Emily McDonald, secretary/treasurer; Chenglie Hu, corresponding secretary; Linda Kallam, faculty sponsor.

KS Zeta Chapter President — Thyrza Mucambe
Southwestern College, Winfield 11 actives, 5 associates

We had meetings every 2 weeks, and invited the whole school twice. The first time was only an introduction to the math lab and the facility, and also to inform students about the kind of assistance they can get from KME members. The second time was called Math Fun Day. We divided students into different teams and gave the same problems to each group. They also played Thangrameth and some other mathematical games. The prizes for Math Fun Day were coupons from different restaurants in town, which were donated by one of our new initiates. As well, some of the

members are helpers for our Bridges Conference in July. Other spring 1999 officers: Jeff Rahm, vice president; Carol Black, secretary; Mehri Arfaei, corresponding secretary; Reza Sarhangi, faculty Sponsor.

KY Alpha Chapter President — Brandy Smith
Eastern Kentucky University, Richmond 40 actives, 18 associates

The spring semester began with floppy disk sales (together with the ACM chapter) to students in the computer literacy class and the Mathematica class. A meeting in January made plans for attending the national convention. On February 23, eighteen students were initiated. Dr. Don Greenwell gave an interesting talk entitled "The Fifteen Puzzle." The fifteen puzzle is the numbers 1 through 15 scrambled in a 4x4 square. The object is to make one move at a time to get 1 through 15 back in order. Dr. Greenwell showed which scramblings can be undone and which cannot. (A computer program had been written that did scramblings but was frustrating because some could not be unscrambled.) April was declared Math Awareness Month and every day a list of several interesting facts about that day's number were placed all over the Wallace Building. For example, 20 is Niven, semiperfect, and tetrahedral. Nine students went with Dr. Costello to the national convention in Lakeland, Florida. Brandy Smith was one of the top four speakers with the talk "Triangle Tilings of the Hyperbolic Plane." After the convention banquet, several chapters joined our group in cosmic bowling. The lunch on Friday was next to the lake where an alligator was observed floating on the water. After the convention, the group drove to Clearwater Beach and spent an afternoon soaking up the sun. As Gordon Elmore commented, "It was a helluva trip!" Other spring 1999 officers: Charles Woolum, vice president; Amy Brewer, secretary; Shannon Purvis, treasurer; Pat Costello, corresponding secretary; Ray Tennant, faculty sponsor.

KY Beta Chapter President — Candace Osborne Perry
Cumberland College, Williamsburg 50 actives

On March 26, the Kentucky Beta chapter held an initiation and banquet at the Atrium for 17 new student members, which brought the total membership to exactly 100. Members inducted last year and graduating seniors were also recognized during the banquet, presided over by outgoing president, Candace Perry. As a new feature, senior awards were given by the department at the banquet. On April 9, Kentucky Beta Chapter held a joint picnic with Beta Beta Beta (biology) and Sigma Pi Sigma (physics) honor societies. Jointly with the Mathematics and Physics Club, KY Beta hosted Dr. Carroll Wells from David Lipscomb University on April 15. He spoke on Euler. On April 16, members also assisted in hosting a regional

high school math contest, held annually at Cumberland College. On April 26, the chapter held a second initiation for six additional student members in the Gatliff Chapel. On May 3, the entire department, including the Math and Physics Club, Sigma Pi Sigma, and the Kentucky Beta Chapter, held the annual spring picnic at Briar Creek Park. Other spring 1999 officers: Christopher Stephens, vice president; Katherine Sizemore, secretary, Elizabeth Iles, treasurer; Jonathan Ramey, corresponding secretary; John Hymo, faculty sponsor.

MD Alpha Chapter President — Malini Hosakere
College of Notre Dame of Maryland, Baltimore 8 actives, 3 associates

Other spring 1999 officers: Kristen Balster, vice president; Christine Wooding, secretary; Vinima Kerof, treasurer; Sister Marie Augustine Dowling, corresponding secretary; Joseph Dirienzi, faculty sponsor.

MD Beta Chapter President — Jenny Addeo
Western Maryland College, Westminster 32 actives

Spring activities included the induction of 8 new members and a game night. For Career Night we had a dinner with talks about career opportunities by 3 alumni. We sponsored Senior Honors presentations by 2 members and we hosted the mathematics department spring picnic. As well we held elections of new officers for next year and provided tutoring service for lower level mathematics and computer science classes. Other spring 1999 officers: Tom Lapato, vice president; Christie Addeo, secretary; David Meckley, treasurer; Linda Eshleman, corresponding secretary; Harry Rosenzweig, faculty sponsor.

MD Delta Chapter President — Sean Carley
Frostburg State University, Frostburg 33 actives

On March 8 Maryland Delta inducted 12 new members. Professor Greg Latta of the Physics Department, who is a local observer for the National Weather Service, spoke at the induction on the topic of "Frostburg Weather Data". At a meeting later in the semester, the chapter discussed the possibility of organizing a used calculator exchange as a service and/or fundraising project. Other spring 1999 officers: Julie Robison, vice president; Katherine Taylor, secretary; Andrew Adam, treasurer; Edward White, corresponding secretary; John Jones, faculty sponsor.

MA Alpha Chapter President — Tammy Ives
Assumption College, Worcester 10 actives, 7 associates

Seven new members were initiated at a meeting on April 28, 1999. Following a dinner in honor of the new initiates, Professor Joseph Alfano, of the Assumption Faculty, spoke on "Counting Trees, Parking Cars, and

Other Uses for a Mathematics Degree." Other spring 1999 officers: Shelley Kijek, vice president; Cara Lambert, secretary; Charles Brusard, corresponding secretary/faculty sponsor.

MI Epsilon Chapter President — Suzanne Labadie
Kettering University, Flint 56 actives, 54 associates

B-Section activities included a showing of the movie "Mathematical Mystery Tour" and a pizza party where Kettering Professor Ilya Kudish spoke on "Back in the USSR". He provided insight into mathematics education in the Soviet Union and talked about his career and colleagues during the time that he taught there. "The Proof", a movie which chronicles the search for a proof of Fermat's Last Theorem, was also shown. Finally, we held our initiation ceremony and banquet during the spring. We initiated 54 new members, and followed the banquet with a talk on "Democracy is Never Fair: Why Every Voting Method is Inherently Flawed" by UM-Flint Professor Matt Wyneken. Other spring 1999 officers: Joel Austin, vice president; Robert Riley, secretary; Dan Holmquist, treasurer; Jo Smith, corresponding secretary; Brian McCartin, faculty sponsor.

MS Alpha Chapter President — Gordona Bauhan
Mississippi University for Women, Columbus 11 actives

Other spring 1999 officers: Julie Torrent, vice president; Jamie Rickert, secretary; Jacqueline Tharp, treasurer; Shaochen Yang, corresponding secretary; Beate Zimmer, faculty sponsor.

MS Gamma Chapter President — Jason Haight
University of Southern Mississippi, Hattiesburg 25 actives, 6 associates

Mississippi Gamma Chapter of KME celebrated its 50th anniversary on April 21, 1999. Six new members were initiated. Dr. Gaston Smith, a charter member, told about the history of the chapter. Pizza was served to 37 members and guests. Other spring 1999 officers: Paula Thigpen, vice president; Adrienne Davis, secretary; Alice Essary, treasurer/corresponding secretary; Bill Horner and Hosea Contreras, faculty sponsors.

MS Epsilon Chapter President — Amanda Seward
Delta State University, Cleveland 21 actives

The Mississippi Epsilon Chapter initiated seven new members on Sunday, March 21, 1999. Other spring 1999 officers: Ken Byars, vice president; Chad Huff, secretary/treasurer; Paula Norris, corresponding secretary; Rose Strahan, faculty sponsor.

MO Alpha Chapter President — Angie Horton
Southwest Missouri State University, Springfield 38 actives, 13 associates

Missouri Alpha met monthly during the Spring Semester. Each meeting included a presentation of a mathematics topic. Two presentations were made by faculty, Mr. Frank Gillespie and Dr. Yingcai Su. Student presentations were made by Sam Blisard and Michael Byrd. Three students and one faculty attended the National Convention at Florida Southern College where Sam Blisard and Michael Byrd presented papers, and Rachel Netzer served on the Nominations Committee. Other spring 1999 officers: Michael Byrd, vice president; Samual Blisard, secretary; Jessica McDonnell, treasurer; John Kubicek, corresponding secretary/faculty sponsor.

MO Beta Chapter President — Darin Tessier
Central Missouri State University, Warrensburg 25 actives, 8 associates

The Missouri Beta Chapter heard two talks this semester. In February, Dr. Cindy Ramey spoke on "Falling Bodies: A Motion Detector Project" and in March Dr. Kerry Henson spoke about the Y2K problem. Six students and two faculty attended the national convention in Lakeland. Melissa Elliott and Tracy Murphy tied for the Claude H. Brown-KME Mathematics Achievement Award for Outstanding Senior. Other spring 1999 officers: Becky Stafford, vice president; Andrew Feist, secretary; Warren Christensen, treasurer; Beth Usher, historian; Rhonda McKee, corresponding secretary; Larry Dilley, Phoebe Ho, Scotty Orr, faculty sponsors.

MO Gamma Chapter President — Jeremy Rush
William Jewel College, Liberty 11 actives, 4 associates

Our annual Spring initiation ceremony and banquet was held on March 23. Our speaker for the evening was Allison Cooper Hartnett, a 1998 graduate who is now working on a masters degree at UMKC in Kansas City. Other spring 1999 officers: Travis Wade, vice president; Joshua Stephenson, secretary; Joseph Mathis, treasurer/corresponding secretary/faculty sponsor.

MO Epsilon Chapter President — David Bates
Central Methodist College, Fayette 8 actives, 3 pledges

Other spring 1999 officers: Christina Miller, vice president; Sheryll Rector, secretary; William McIntosh, corresponding secretary; Linda Lembke and William McIntosh, faculty sponsors.

MO Eta Chapter President — Shawn Logan
Truman State University, Kirksville 12 actives, 6 associates

We held a volleyball night in January, Annual Math Expo in February, and the initiation and elections in April, with regular meetings every two

weeks. Other spring 1999 officers: Bryan Bichsel, vice president; Angela Kell, secretary; Chad Muse, treasurer; Mary Sue Beersman, corresponding secretary/faculty sponsor.

MO Theta Chapter President — Amanda Wachsmuth
Evangel University, Springfield 9 actives

We had monthly meetings and had an "all you can eat" semester social at a pizza buffet. Amanda Wachsmuth and Don Tosh attended the national convention in Lakeland, FL where Amanda presented a paper discussing the Buffon needle problem. Other spring 1999 officers: John Elliott, vice president; Don Tosh, corresponding secretary/faculty sponsor.

MO Iota Chapter President — JoAnna Scott
Missouri Southern State College, Joplin 29 actives

The highlight of the spring semester was undoubtedly the National Convention in Lakeland. Three of the four students attending experienced their first air travel. Member Mike Wilkerson presented his paper entitled "Big Ram." Other semester activities included regular monthly meetings, participation in the college phonathon, and the annual spring picnic at the home of Mrs. Elick. Other spring 1999 officers: Kristi Karber, vice president; Jill Hedger, secretary; Jonathon Shull, treasurer; Mary Elick, corresponding secretary; Charles Curtis, faculty sponsor.

MO Lambda Chapter President — Stephanie Tingler
Missouri Western State College, St. Joseph 34 actives

The Missouri Lambda Chapter initiated five new members on March 7. Dr. Keith Brandt was the speaker for the program. On April 29, student KME member Joe Roma gave a colloquium presentation on Zeros of Cubic Polynomials. The final event of the year was a cookout at Dr. Atkinson's. Election of officers for the next academic year was held. Other spring 1999 officers: David McCay, secretary; Shaun Piatt, treasurer, John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

NE Alpha Chapter President — Jeff Thoene
Wayne State College, Wayne 41 actives, 4 associates

Semester activities included a banquet, a speaker on Casino Games, and Combinatries. The outstanding Freshman Math Student was Matt Theiman. Other spring 1999 officers: Ross Volk, vice president; Brandi Hall, secretary; Brian Kesting, treasurer; John Fuelberth, corresponding secretary; Jim Paige, faculty sponsor.

NE Beta Chapter President — Kala Devi Ramalingam
University of Nebraska at Kearney, Kearney 17 actives, 7 associates

Other spring 1999 officers: Michael Sullivan, vice president; Tisha

Maas, secretary; Peter Okumah, treasurer; Stephen Bean, corresponding secretary; Richard Barlow, faculty sponsor.

NE Gamma Chapter President — Andy Boell
Chadron State College, Chadron 15 actives, 3 associates

A formal initiation was held on March 28 at the home of Dr. Ted Davis. Three members were inducted into our chapter. Other spring 1999 officers: Shaun Daugherty, vice president; Craig Bruner, Jr., secretary; Kendra Pedersen, treasurer; Robert Stack, corresponding secretary; Brent Rickenbach, faculty sponsor.

NE Delta Chapter President — Andres Garcia
Nebraska Wesleyan University, Lincoln 31 actives, 13 associates

Other spring 1999 officers: Timothy Anderson, vice president; Ann DiGiorgio, secretary/treasurer; Gavin LaRose, corresponding secretary/faculty sponsor.

NH Alpha Chapter President — Allan Barriere
Keene State College, Keene 19 actives, 6 associates

Other spring 1999 officers: Melissa Shepard, vice president; Laura Devoid, secretary; Travis Wakefield, treasurer; Vincent Ferlini, corresponding secretary; Ockle Johnson, faculty sponsor.

NM Alpha Chapter Presidents — Delores Gabaldon and Jennifer Gill
University of New Mexico, Albuquerque 90 actives, 14 associates

Other spring 1999 officers: Holly Dison, vice president; Merlin Decker, secretary/Web Master; Archie Gibson, treasurer/corresponding secretary/faculty sponsor.

NY Alpha Chapter President — William D'Angelo
Hofstra University, Hempstead 15 actives, 1 associate

Other spring 1999 officers: Andrea Genzale, vice president; Drew Batkin, secretary; Aileen Michaels, corresponding secretary/faculty sponsor.

NY Eta Chapter President — Brett Richner
Niagara University, Niagara University 20 actives, 18 associates

One of our regular meetings featured a former student, Nathan Williams, who presented a talk on factorials. Some fund-raising activities were undertaken this semester. In attendance at our April induction ceremony were family members, including parents and even grandparents in one instance. Other spring 1999 officers: Mike Simons, vice president; Kristen Grimm, secretary; Alan Hunt, treasurer; Robert Bailey, corresponding secretary; Wendy Duignan, faculty sponsor.

NY Iota Chapter President — Tanya Brown
Wagner College, Staten Island 8 actives, 6 associates

The two community service projects our chapter participated in were a success! During the spring semester, fifth graders from a local school came to visit Wagner College. They first saw a show in the Planetarium. After the show, the class met the members of Kappa Mu Epsilon. We played "Math Bingo" with the students. We all had a good time. We showed the youngsters through this game that math can be fun and interesting, as well as challenging. The fifth-graders certainly met the challenge! After the game we had some time to just talk to the students. They asked us many thought provoking questions about college life. The second major activity we participated in was the offering of our tutoring services to our peers at Wagner College for their upcoming mathematics finals. This activity was offered during the last three days of classes. We were pleased by the outcome of peers seeking a little extra help. Other spring 1999 officers: Roseanne Pianelli, vice president; Michael Rubinfeld, secretary; Catherine Bottiglia, treasurer; Stephen Sessions, corresponding secretary/faculty sponsor.

NY Kappa Chapter President — Maureen Ellison
Pace University, New York 15 actives, 6 associates

We held and induction dinner on April 28 at The Beekman. Other spring 1999 officers: Kyle Hill, vice president; Steven Bourgault, secretary; Andrea Liberatore, treasurer; Geraldine Taiani, corresponding secretary; Robert Cicenia, faculty sponsor.

NY Lambda Chapter President — Jill Kahan
C. W. Post Campus of Long Island University, Brookville 32 actives

Eleven new student members and one new faculty member were initiated by the chapter officers (with the assistance of Loriann Loraia, who was chapter president last year) during our annual banquet at the Greenvale Town House restaurant on the evening of April 11th, bringing the Chapter membership to 194. After dinner, Jill Kahan's presentation on " The Stable Marriage Problem" concluded with a lively demonstration of the Gale-Shapely algorithm using most of the newest members of New York Lambda. Our evening concluded with the announcements by Dr. Neo Cleopa of the departmental awards for 1998-99. The Lena Shamey Memorial Award went to Nicole Garofalo, the Joseph Panzeca Memorial Award went to Steven McKinnon, and the Claire F. Adler Award went to David Joseph and Jill Kahan. Alumni Concetta Vento Ahern, Colin Grimes and Anthony Weidner also attended. Steven McKinnon presented his paper on "Wavelets" at the 32nd Biennial Convention at Florida Southern Col-

lege on April 22-24. Nicole Garofalo spoke on "Continued Fractions and the Fibonacci Sequence" at the Metropolitan New York Section meeting of the MAA at Hofstra University on May 1. Other spring 1999 officers: Nicole Garofalo, vice president; Tanya Palacio, secretary; David Joseph, treasurer; Andrew Rockett, corresponding secretary; John Stevenson, faculty sponsor.

NY Mu

St. Thomas Aquinas College, Sparkill — 72 actives, 6 associates

Six members were initiated on April 19. Other spring 1999 officers: Joseph A. Keane, corresponding secretary/faculty sponsor.

OH Gamma — Chapter President — Anila Xhunga

Baldwin-Wallace College, Berea — 32 actives, 10 associates

Other spring 1999 officers: Duke Hutchings, vice president; Mary Guinn, secretary; Corina Moise, treasurer; David Calvis, corresponding secretary; David Calvis & Chungsim Han, faculty sponsors.

OH Zeta — Chapter President — Aaron Rose

Muskingum College, New Concord — 21 actives, 3 associates

Other spring 1999 officers: Anjanette Hadley, vice president; Jeff Shoemaker, secretary; Katrina Barr, treasurer; Richard Daquila, corresponding secretary; David Craft, faculty sponsor.

OK Alpha — Chapter President — Melinda Weigle

Northeastern State University , Tahlequah — 26 actives, 6 associates

The initiation of our six students was held in the banquet room of the Sirloin Stockade. We continue to have joint activities with NSU's student chapter of the MAA and participate in "The Problem Solving Competition" sponsored by the MAA. For the second year in a row, we designed and sold our "original" KME T-shirts. We sponsored a talk by Dr. Itrel Monroe, associate professor of mathematics from the University of Arkansas. The title of his presentation was "Electrostatics, Harmonic Motion, Geometry, and Brownian Motion." In April several hundred middle school and high school students came to an Education Fair held at NSU. Our KME group provided a display for the fair that promoted KME and the teaching of mathematics. We celebrated Math Awareness Month with our annual Ice Cream Social. Other spring 1999 officers: Tina Wolfe, vice president; Tera McGrew, secretary; Gregg Eddings, treasurer; Joan Bell, corresponding secretary/faculty sponsor.

OK Gamma — Chapter President — Kory Hicks

Southwestern Oklahoma State University, Weatherford — 30 actives

We had two students and two faculty attend the national convention at

Lakeland, FL. Other spring 1999 officers: Christy Koger, vice president; Shelly Davenport, secretary; Lindy Coley, treasurer; Wayne Hayes, corresponding secretary; Gerry East, faculty sponsor.

PA Epsilon
Kutztown University, Kutztown — 10 actives, 12 associates

Spring 1999 officer: Cherry Mauk, corresponding secretary.

PA Iota — Chapter President — Don Miller
Shippensburg University, Shippensburg — 24 actives, 8 associates

Other spring 1999 officers: Tom Ruffner, vice president; Jaymie Kenny, secretary; Mike Seyfried, treasurer/corresponding secretary; Cheryl Olsen, faculty sponsor.

PA Kappa — Chapter Presidents — Linda Bruce & Lindsay Janka
Holy Family College, Philadelphia — 6 actives, 2 associates

During the Spring semester, two new members were welcomed into the Chapter. The spring induction ceremony was held on Friday, March 19, in conjunction with the biology honor society. The keynote address was given by Dr. Paul Hieble from Smith Klein Beecham. The topic of Dr. Hieble's lecture was Development of Drugs for Prostatic Pharmacology vs. Surgery in the Treatment of Prostatic Hyperplasia, which was accessible and applicable to both the future biologists and future statisticians in the audience. During Math Awareness Month, the members of the PA Kappa sponsored their fifth annual grade school mathematics competition on Saturday, April 24. Six local elementary schools participated. The three top schools received commemorative plaques. All entrants received certificates of participation. Other spring 1999 officers: Brian Minster, secretary/treasurer; Sr. Marcella Louise Wallowicz, corresponding secretary/faculty sponsor.

PA Mu — Chapter President — Troy Mohney
Saint Francis College, Loretto — 18 actives, 8 associates

The induction ceremony was held on Thursday, February 12, in the Stokes Room at Saint Francis College. A formal dinner preceded the actual initiation ceremony for the eight new student members. The members conducted several fund raising activities to help pay travel to the national convention. Two students and two faculty attended the convention at Florida Southern College. Other spring 1999 officers: Tracy Paxon, vice president; Rebecca Espenlaub, secretary; Kourosh Barati-Sedeh, treasurer; Pete Skoner, corresponding secretary; Amy Miko, faculty sponsor.

PA Omicron Chapter President — Christy Lynch
University of Pittsburgh at Johnstown, Johnstown 17 actives, 13 associates

Meetings: We had a general organization meeting in February at which we made plans for inductions. Semester activities included an Induction Ceremony for 13 inductees which was held April 8. We honored three retiring math faculty. Other spring 1999 officers: Marie Hufford, vice president; Erin Hepinger, secretary; Michelle Vincenzini, treasurer; Nina Girard, corresponding secretary/faculty sponsor.

TN Beta Chapter President — Shannon Gosnell
East Tennessee State University, Johnson City 17 actives, 9 associates

Other spring 1999 officers: Ken Proffitt, vice president; Susan Hosler, secretary; Justin Hyder, treasurer; Lyndell Kerley, corresponding secretary/faculty sponsor.

TN Gamma Chapter President — Lori Davis
Union University, Jackson 15 actives, 6 associates

During the 1998-99 academic year the TN Gamma Chapter of KME sponsored talks by Dr. Bryan Dawson of Union University (and Editor of the Pentagon) and by Dr. Dave Gibson of Murray State University. The annual KME initiation banquet was held on April 15 in the Taylor Room of the Casey Jones Old Country Store in Jackson, TN. Twenty-two faculty and students gathered to initiate six new members. New student officers were elected to serve during the upcoming academic year. Dr. Bryan Dawson agreed to serve as Corresponding Secretary for the upcoming year and Dr. Matt Lunsford will continue to serve as the Faculty Sponsor. Julie Jones, a 1994 graduate of Union and KME member, was the invited speaker for the event. Dr. Bryan Dawson and student member Jennifer Middleton attended the KME National Convention in Lakeland, FL. Jennifer presented her senior seminar paper entitled "Nature: A Continual Portrait of Mathematics". The Chapter ended the year with a joint social event with the local ACM student chapter. Both organizations met May 6, 1999 on the campus for a cookout. Seniors in both organizations were recognized. Other spring 1999 officers: Mandy Davidson, vice president; Lindsey Crain, secretary; Cathie Scarbrough, treasurer; Don Richard, corresponding secretary; Matt Lunsford, faculty sponsor.

TN Delta Chapter President — Robert Johnson
Carson-Newman College, Jefferson City 12 actives, 8 associates

Semester activities included an Initiation Banquet and picnic. The topic for the Math/Philosophy Forum was "Is There Proof of God?" Three of our new members, Jeffrey Daniel, Nicholas Stepp and Benjamin Weaver, participated in the 15th annual Mathematical Contest in Modeling on Febru-

ary 5-8 and have received Honorable Mention. Other spring 1999 officers: Melissa Holland, vice president; Sarah Montgomery, secretary; Brian Renninger, treasurer; Catherine Kong, corresponding secretary/faculty sponsor.

TN Epsilon — Chapter President — Jennifer Dowdy
Bethel College, McKenzie — 10 actives, 1 associates

In addition to monthly meetings, the chapter gathered for special movie nights and participated with the Gamma Beta Phi honor society in campus events. Other spring 1999 officers: Jonathan Lankford, vice president; Christina Hill, secretary; Jamie Wiggleton, treasurer; Russell Holder, corresponding secretary; David Lankford, faculty sponsor.

TX Alpha — Chapter President — Rachel Dowlen
Texas Tech University, Lubbock — 15 associates

Other spring 1999 officers: Douglas Burkes, vice president; Dana Hernes, secretary; John Granier, treasurer; Victor Shubov, corresponding secretary/faculty sponsor.

TX Eta — Chapter President — Micah Lindstrom
Hardin-Simmons University, Abilene — 19 actives, 4 associates

TX Eta, in conjunction with the Big Country Council of Teachers of Mathematics and Science, sponsored a Math-Science UIL contest on November 7. On February 6 our chapter hosted the Math Count contest sponsored by the Abilene Chapter of Professional Engineering Society. The 25th annual induction banquet for the Texas Eta chapter was held March 6. There were four new members inducted, bringing the total membership to 197. Leading the induction ceremonies were Wendy James, Gareth Jenkins, Sarah McCraw, and Micah Lindstrom. Following the induction Mrs. Kathy Hale, Secondary Math Specialist for Region XIV Education Service Center, made a presentation on fractals. Other spring 1999 officers: Sarah McCraw, vice president; Jenifer Leonard, secretary; James Martin, treasurer; Frances Renfroe, corresponding secretary; Edwin Hewett, Andrew Potter and James Ochoa, faculty sponsors.

TX Kappa — Chapter President — Mary Bruton
University of Mary Hardin-Baylor, Belton — 15 actives, 5 associates

Other spring 1999 officers: Alicia Kuehl, vice president; Belinda Smith, secretary; Peter Chen, corresponding secretary; Maxwell Hart, faculty sponsor.

VA Gamma — Chapter President — Sarah Lemon
Liberty University, Lynchburg — 32 actives, 8 associates

Most of our programs were career oriented, with former students speak-

ing about their experiences in graduate school, actuarial careers, and teaching. Faculty members spoke on several occasions about their work. All the members experienced a profitable year. Other spring 1999 officers: Sean Burtt, vice president; Bobbi Heim, secretary; Curtis Hartman, treasurer; Glyn Wooldridge, corresponding secretary; Sandra Rumore, faculty sponsor.

WI Gamma Chapter President — Jeff Clay
University of Wisconsin-Eau Claire, Eau Claire 13 actives

We have had 13 initiates during the biennium. Officers were elected in May. Other spring 1999 officers: Elizabeth Whitney, vice president; Julie Loasching, secretary; Dion Meyer, treasurer; Marc Goulet, corresponding secretary.

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Report of the 32nd *National Biennial Convention*

The 32nd National Biennial Convention of the *Kappa Mu Epsilon*, National Mathematics Honor Society, was hosted by the Florida Beta Chapter at Florida Southern College, Lakeland, Florida, April 22-24, 1999. Meeting in buildings designed by Frank Lloyd Wright (1869-1959), the Convention certainly promised to be attractive, and when combined with the invitation of good old-fashioned southern hospitality, it suddenly became irresistible. In 1513, Juan Ponce de León (1460-1521), a Spanish explorer, discovered and named the Florida peninsula, which he initially thought to be an island of the Bahamas. The name was reverently appropriate, since springtime contained the "Season of Flowers," which he observed as part of Easter [*Pascua florida*], and the luxuriant vegetation that he found growing there would impress anyone still seeking the legendary Fountain of Youth. Though no one ever found it, everyone attending the Convention was youthful enough. Indeed, many students and some faculty, representing twenty-seven chapters from sixteen states, participated in the roll call. The installation of ten new chapters during the preceding biennium and the approval of several more for the current biennium also reminded us that, as an honor society, *Kappa Mu Epsilon* continues to grow by promoting excellence in mathematics.

After the first general session was called to order by Patrick Costello, our National President, we received a genial welcome from Thomas Reuschling, the President of Florida Southern College, and provided an appropriate response through Robert Bailey, our National President-Elect. Roxanne Back, Student President of the Florida Beta Chapter, then extended her greetings, and we began to attend to the official business of the organization as provided by the constitution. This session included (a) the identification of student delegates for the participating chapters, (b) the recognition of new chapters together with the formal reception of petitions from qualified institutions for additional ones, and (c) a review of old business with the introduction of new business, as well as (d) a report of the Nominating Committee, chaired by Arnold Hammel of Michigan Beta, Central Michigan University, with the introduction of candidates for regional and national positions. Arnie had served *Kappa Mu Epsilon* as a former President. He also helps to maintain the national web site. Genuine interest was shown in the announcement that the Pennsylvania Pi Chapter was just installed at Slippery Rock University on Monday, earlier in the week of April 18-24, 1999.

The Convention then recessed to begin hearing the presentation of student papers. Members may submit their papers for competition through

the National President-Elect, who chairs the Selection (or Program) Committee, which judges them for content, while an independent Awards Committee judges them for presentation. Their combined rankings determine the winners. Nineteen papers were submitted, and four winners* emerged from fifteen presentations this year. Student presentations of the morning included

Computing Homoclinic Bifurcations
*Suzanne Shontz, Iowa Alpha
University of Northern Iowa

Bigger, Better Powerball
Mary Noga, Iowa Alpha
University of Northern Iowa

An Investigation of the Graph of a Cubic Equation to Determine the Integer Solutions to a Modified Cubic Equation in Three Variables
Sam Blisard, Missouri Alpha
Southwest Missouri State University

Graphical Encryption Technique
Andrew Boell, Nebraska Gamma
Chadron State College

They were followed by a group picture on the stage of the auditorium and a picnic lunch along the shore of Lake Hollingsworth. Student presentations of the early afternoon included

Postmodernism in Mathematics, the Loss of Certainty as Seen in a Discussion of Kurt Gödel's Incompleteness and Consistency Theorems
Robert J. Johnson, Tennessee Delta
Carson-Newman College

Non-periodic Tiling and Quasicrystals
Michael Byrd, Missouri Alpha
Southwest Missouri State University

How Many is Enough?
*Laurie Paycur, Kansas Delta
Washburn University

Mathematics in the Natural World
Jennifer Ann Middleton, Tennessee Gamma
Union University

Faculty and Student Sectional Meetings provided an interesting recess as well as important feedback on important questions, such as (1) How

should the organization respond to rising costs for providing services? This perennial question exists at both local and national levels, since quantity discounts offer special savings, and (2) What ideas can be suggested for encouraging more students to join? According to Solomon A. Garfunkel and Gail S. Young, "The sky is falling," *Notices of the American Mathematical Society* 45(1998), pages 256-257, national enrollment in upper division courses of mathematics fell about 30% over the ten year period from 1985 to 1995. In partial response to the first question, the National Council will explore ways to continue providing current services at reduced costs, and in partial response to the second one, local chapters are encouraged to develop and maintain their own web sites, which can be linked to the national home page at

`http://www.cst.cmich.edu/org/kme_nat`

the following chapters already have their own web sites and would be pleased to assist others

upon request (universal resource locators are subject to change without notice)

Alabama Zeta, Birmingham-Southern College
`http://www.bsc.edu/science/math/kme.htm`

Arkansas Alpha, Arkansas State University
`http://www.csm.astate.edu/students/kme/index.html`

Colorado Beta, Colorado School of Mines
`http://magma.Mines.EDU/Stu_life/organ/kme/kme.html`

Connecticut Beta, Eastern Connecticut State University
`http://www.ecsu.ctstateu.edu/depts/matcs/nhs.html#kappa`

Illinois Eta, Western Illinois University
`http://www.wiu.edu/users/mikme/`

Indiana Alpha, Manchester College
`http://www.manchester.edu/department/mathcptrsci/kme.htm`

Iowa Alpha, University of Northern Iowa
`http://www.math.uni.edu/KME/KME.html`

Kansas Gamma, Benedictine College
`http://www.benedictine.edu/math-cs/kme.html`

Kentucky Alpha, Eastern Kentucky University
`http://eagle.eku.edu/faculty/pjcostello/kme`

Maryland Beta, Western Maryland College
http://www.wmc.car.md.us/HTMLpages/Academics/Math/KME.html

Mississippi Gamma, University of Southern Mississippi
http://www.math.usm.edu/organizations_html/kme.html

Missouri Alpha, Southwest Missouri State University
http://studentorganizations.smsu.edu/KME

Missouri Zeta, University of Missouri at Rolla
http://www.umr.edu/~kme

New Mexico Alpha, University of New Mexico
http://www.math.unm.edu/~kme

New York Eta, Niagara University
http://www.niagara.edu/math/kme.html

New York Lambda, C. W. Post Campus of Long Island University
http://www.cwpost.liunet.edu/cwis/cwp/clas/math/kme.htm

New York Xi, Buffalo State College
http://math.buffalostate.edu/~kme/index.html

Ohio Alpha, Bowling Green State University
http://www.bgsu.edu/departments/math/kme

Pennsylvania Xi, Cedar Crest College
http://www.cedarcrest.edu/academic/mat/saa.htm

Evidently, initiatives, such as these, help *Kappa Mu Epsilon* to grow and become more effective both locally and nationally. Student presentations of the late afternoon included

Matrix Multiplication Using Strassen's Algorithm
Gary Spieler, Iowa Alpha
University of Northern Iowa

Triangle Tilings on the Hyperbolic Plane
*Brandy M. Smith, Kentucky Alpha
Eastern Kentucky University

While the first day of the Convention ended with an informal mixer, the second day ended with a more formal banquet and keynote address by Jonathon Stadler of Coastal Carolina University. He was initiated as an undergraduate student by the Ohio Alpha Chapter at Bowling Green State University and recently completed his doctorate degree at Ohio State

University. Jonathon showed us some interesting patterns that he found among the permutations, which describe patterns used by jugglers. For a popular introduction to this subject, see Peter J. Beek and Arthur Lewbel, "The mathematics of juggling" under "The science of juggling," *Scientific American* 273:5 (Nov 1995), pages 92-97

`http://www.juggling.org/papers/science-1`

Door prizes from some of our sponsors were distributed at the end of Friday's banquet (I won a fashionable "Explore Mathematics" fishing hat, donated by Waterloo Software). Saturday morning came early. Student presentations that followed a continental breakfast in the auditorium breezeway included

Plaid or Pin-Stiped:
Which Gets You Closer to Pi?
Amanda Wachsmuth, Missouri Theta
Evangel University

In the Spirit of Klein,
Making Connections between the Möbius and Lorentz Groups
Shawn Logan, Missouri Eta
Truman State University

Wavelets
Steven G. McKinnon, New York Lambda
C. W. Post Campus of Long Island University

Going Up?
*Douglas Appenfeller, Kansas Delta
Washburn University

Big RAM
Michael L. Wilkerson, Missouri Iota
Missouri Southern State College

The second general session, which was called to order by our National President to bring this Convention to a triumphant conclusion, continued the business of the first general session that started the preceding day. After receiving the following reports of the National Officers and Pentagon Staff, the delegates accepted the Report of the Auditing Committee, chaired by Jo Ann Fellin of Kansas Gamma, Benedictine College, by accepting the financial records for the 1997-99 Biennium as presented by National Treasurer, A. Allan Riveland, and congratulating him on his excellent service over the past four years. Since the incoming resources are monitored by the National Secretary and the outgoing expenses are monitored by the National President, there seem to be adequate safeguards for maintaining the

financial integrity of the Society. The Audit Committee did not see a need for any further, independent, external review. Jo Ann had served *Kappa Mu Epsilon* as a former Treasurer.

The delegates also accepted the Report of the Resolutions Committee, chaired by Andrew Rockett of New York Lambda, C. W. Post Campus of Long Island University. Indeed, "whereas Florida Southern College and the surrounding community of Lakeland have provided this Convention with gracious hospitality and perfect weather, be it resolved

1. That this Thirty-Second Biennial National Convention express its gratitude to the Florida Beta Chapter for the thorough arrangements they have planned and carried out so successfully, and
2. That this Convention recognize and thank Thomas Reuschling, President of Florida Southern College, as well as Tonya Brown, Troy Renoll, Heather Tommarchio, William Albrecht, Gayle Kent, and Allen Wuertz, together with all the other members of Florida Beta, who devoted countless hours to ensure the success of this meeting.

"Whereas the success of any undertaking is directly proportional to the dedication and ability of its leaders, be it further resolved

1. That this Thirty-Second Biennial National Convention express its gratitude to (a) Mary Sue Beersman, for her many years of productive service as the Region 4 (North Central) Director, (b) Rhonda McKee, for her willingness to undertake the responsibility of leading this region into the next biennium, (c) C. Bryan Dawson, for his skilled organization and promotion of *The Pentagon* over the past four years, and (d) Steve Nimmo, for his willingness to assume the editorship of our Society's journal,
2. That this Convention express thanks to the National Officers, *The Pentagon* Business Manager, and the continuing Regional Directors for their devotion and effort not only during the past biennium but also for the one to come, and
3. That this Convention acknowledge the participation of the students and faculty, who served on the Auditing, Awards, Local Arrangements, Nominating, Paper Selection, and Resolutions Committees, which is so essential for the success of this meeting.

"Finally, whereas the primary purpose of *Kappa Mu Epsilon* is to encourage participation in mathematics and the development of a deeper understanding of its beauty, be it further resolved

1. That the students, who prepared, submitted, and then presented their papers, be given special commendation by this Thirty-Second Biennial National Convention for their enthusiasm and dedication,
2. That this Convention express thanks to Jonathon Stadler for sharing his insights, knowledge, and juggling skills, during his address at the Friday night banquet, and
3. That this Convention recognize the contributions of AMC Theaters, Barnes and Noble Campus Bookstore, Cypress Gardens, Jeff's Discount Printing, the Lakeland Chamber of Commerce, Texas Instruments, and Waterloo Maple, to the success of this meeting."

It may be obvious from these stirring resolutions that Andrew had served *Kappa Mu Epsilon* as a former Editor of *The Pentagon*.

Under new business, the delegates voted to accept the petitions of Piedmont College for the Georgia Gamma Charter and Trinity University for the Texas Lambda Charter. There were no dissenting votes in either case. The delegates also recognized C. Bryan Dawson of Tennessee Gamma, Union University, with a certificate of appreciation as the outgoing Editor of *The Pentagon* and voted to accept the appointment of Steve Nimmo of Iowa Gamma, Morningside College, as his replacement. Mary Sue Beersman of Missouri Eta, Truman State University, also received a certificate of appreciation for her service as the Director of the North Central Region, and Rhonda L. McKee of Missouri Beta, Central Missouri State University, was appointed as her replacement. A. Allan Riveland of Kansas Delta, Washburn University, and Waldemar Weber of Ohio Alpha, Bowling Green State University, were elected to succeed themselves for a second term as the National Treasurer and Secretary by acclamation.

The Kansas Delta Chapter of Washburn University offered to host the Thirty-Third Biennial National Convention of the *Kappa Mu Epsilon*, National Mathematics Honor Society (this invitation was subsequently reviewed and accepted by the National Council during a special meeting, November 13-14, 1999). Meanwhile, the Thirty-Second Biennial National Convention, so charmingly hosted by the Florida Beta Chapter of Florida Southern College adjourned after receiving the Report of the Awards Committee, chaired by Donna Hafner of Colorado Delta, Mesa State College. The winners of the paper competition, listed in order of presentation, were Suzanne Shontz of Iowa Alpha, Laurie Payeur of Kansas Delta, Brandy M. Smith of Kentucky Alpha, and Douglas Appenfeller of Kansas Delta, as already noted. Each winner received a cash prize of $100 as well as a hand-held calculator, furnished by Texas Instruments, and each presenter

received a certificate of appreciation as well as a two-year extension of their subscription to *The Pentagon*. Indeed, impressed by the vitality and growth of the Society, everyone, who attended this Convention, returned home with precious memories.

Report of the President

It has been an exciting and hectic first two years as President. During this time, we installed ten new chapters. These chapters were Pennsylvania Omicron at the University of Pittsburgh-Johnstown, Michigan Delta at Hillsdale College, Michigan Epsilon at Kettering University, Kansas Zeta at Southwestern College, Tennessee Epsilon at Bethel College, Georgia Beta at Georgia College and State University, Missouri Mu at Harris-Stowe State College, Alabama Eta at the University of West Alabama, New York Xi at Buffalo State College, and North Carolina Delta at High Point University. Since Pennsylvania Pi has already been installed at Slippery Rock University during the current biennium, the current number of active chapters has increased to 127.

In addition, two petitions have recently been approved by the National Council. They are to be acted on at this convention. The petitions are from Piedmont College in Demorest, Georgia, and Trinity University in San Antonio, Texas. I have also corresponded with sixteen other colleges and universities that have indicated interest in establishing a chapter of *Kappa Mu Epsilon* (including Pacific University, Eastern Michigan, Stockton College, Caltech, Bennett College, and the University of Missouri-St. Louis). Several of these have petitions. If you have friends and colleagues at schools that do not have a KME chapter, but are interested, have them contact me.

The National Council continues to support the regional structure of KME. Please refer to the report by President-Elect Bob Bailey for the report of regional conventions. With much gratitude, we recognize the work and efforts of our Regional Directors. These people have served our Society well and deserve the thanks of each of us. I would especially like to cite Mary Sue Beersman, North Central Region Director, whose term expires with this convention. Since 1991, Mary Sue has been diligently looking after KME interests in this region. Thanks Mary Sue, for a job well done.

A special thanks is extended to each of the faculty who serve as corresponding secretaries and faculty sponsors with our active chapters. I know many of you played an important role in assisting your students in the preparation of the excellent papers we have on the convention program this year. Furthermore, we all express our gratitude to each of the students

who did the research, the writing, the editing, and the practicing of the presentations we will hear. Without the student papers, the major focal point of the convention does not exist. We are also indebted to all the individuals who did the work in preparation for this convention. Everything seems to be in place for a great convention. I also want to thank all of those who agreed to serve on convention committees. Without exception, everyone who was asked to serve on a committee willingly agreed to do so. This has made the privilege of being your President much easier. To all of your at Florida Beta, and to each committee member, please accept our most sincere thanks for jobs well done.

During the past biennium, I have represented *Kappa Mu Epsilon* at one annual meeting of the Association of College Honor Societies (ACHS). It has been very helpful to meet with officers of other honor societies that are members of ACHS for the purpose of exchanging ideas and acquiring suggestions as to how we can possibly improve on the programs we currently have in place. Some of these ideas will be implemented in the near future. I have also submitted two annual reports to ACHS giving specific details regarding activities of our Society during each of the past two academic years.

I want to recognize the fantastic job which is being done by those who work with, manage, write for, and produce our journal, *The Pentagon*. We are most appreciative of the editorial leadership of Bryan Dawson and the sound business management given by Larry Scott. It was a sad day when I received Bryan's request to step down as Editor. I know he will continue top play an integral part in KME for many years to come at his new school in his new role. The Council has approved the appointment of Steve Nimmo from Morningside College as the new Editor. While his chapter was not planning to attend, Steve agreed to come to the convention to learn the ropes from Bryan. Please welcome him and give him any suggestions you might have for the continued excellence of our journal.

I also want to recognize and applaud the outstanding and sometimes gargantuan efforts put forth by the other members of the National Council in their respective areas of responsibility. Al Riveland has automated our financial reports so that even a nonfinancial person like I can easily see what we have and where things are going. Waldemar Weber has made the Society look much more professional with our letterhead paper and envelopes and new initiation report forms. Don Tosh has brought the Chapter News section of *The Pentagon* into the computer age with the electronic submission of chapter reports. Bob Bailey has handled the publicity, collection, selection, and scheduling of papers at this convention and the publicity and scheduling of regional conventions last year.

I can honestly say that I have thoroughly enjoyed the privilege of serving *Kappa Mu Epsilon* as President the past two years. I look forward to the next two years of working with all the capable and conscientious individuals that make up this Society. Best wishes to each of you as we continue to work for the improvement of KME.

Pat Costello

Report of the President-Elect

The president-elect is responsible for serving as coordinator of regional activities of the society by working with regional directors. During the sporing of 1998, there were four regional conventions held. They were in:

Combined Regions 1 (New England) and 2 (Great lakes) at Pennsylvania Iota, Shippensburg University, April 17-18, Carol Harrison, Director of Region 1, and Peter Skoner, Director of Region 2;

Region 3 (Southeastern) at Kentucky Alpha, Eastern Kentucky University, February 27, Gayle Kent, Regional Director.

Region 4 (North Central) at Missouri Gamma, William Jewell College, April 3-4, Mary Sue Beersman, Regional Director;

Region 5 (South Central) at Colorado Gamma, Fort Lewis College, February 27-28, Donna Hafner, Regional Director.

Programs at the regional conventions included student papers, guest speakers, and various social events. Regions 1 and 2 were able to take advantage of the fact that the Eastern Pennsylvania and Delaware Section of the Mathematical Association of America (MAA) was scheduled for the same weekend. This allowed the students to hear some of the MAA talks as well as make contact with students and faculty from several schools that do not necessarily have KME chapters. Region 3 was able to integrate its activities with the 14th Annual Symposium in Mathematics, Statistics and Computer Science. These meetings could be a trend in enriching our regional conventions by interacting with other groups that have meetings at the same location and time. A second meeting for Region 4 was scheduled for April 3-4 but had to be canceled. Our sincere thanks is extended to the four host chapters, their regional directors, and all those who participated in these important regional activities. We also appreciate the efforts of all the remaining regional directors in attempting to have regional conventions in their regions.

A second duty of the president-elect is to coordinate nominations for the George R. Mach Service Award. In the spring of 1998 a letter went out to all chapters requesting nominations for the award. Although the award will not be presented at this convention, perhaps there will be nominations

for the next one.

Another of the president-elect's duties is to publish announcements of the national convention, In addition to an announcement that appeared in the Spring 1998 issue of *The Pentagon*, announcements of this meeting have appeared in the AMS Notices, the MAA Focus magazine, and the MAA web site.

One of the most important responsibilities of the president-elect is to organize the presentation of student papers at the national convention. I am pleased to report that nineteen undergraduate students, representing twelve chapters and eight states, submitted papers for this convention. The Paper Selection Committee read and ranked the papers submitted. The committee experienced some difficulty ranking the papers because of their high overall quality. The Awards Committee will be judging papers presented at the convention. On behalf of the society, I want to extend special thanks to the members of the Paper Selection Committee who read and ranked the papers: Professors Richard Gibbs (Fort Lewis College), Chenglie Hu (Fort Hays State University) and Glyn Wooldridge (Liberty University). The rankings of the Paper Selection Committee will be combined with the rankings of the Awards Committee to determine a final ranking of the judged papers. The top four judged papers will each receive $100. The nineteen papers ranked by the paper Selection Committee have been placed on the program for presentation at the convention. On behalf of the society, I want to express our sincere thanks to all students who prepared and submitted papers. It is this work that makes for a truly successful convention.

Robert L. Bailey

Financial Report of the National Treasurer

1997-1999 Biennium (March 11, 1997-March 31, 1999)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows end-of-biennium assets of $44,365.53. The Cash Flow Report shows that we had an asset gain of $9,771.41 during the biennium.

A National Council goal to maintain an asset base of at least $30,000 has been met.

Biennium Asset Report

Total Assets (March 11, 1997) $34,594.12	
Current Assets	
Mercantile Bank	18,202.16
Educational Credit Union	

Savings Accounts	3,163.37
Certificates of Deposit	23,000.00
Total Current Assets (March 31, 1999)	$44,365.53
Biennium Gain	$9,771.41

Biennium Cash Flow Report

Receipts		
Initiation Fees received	45,120.00	
Installation Fees received	1,049.50	
Interest Income	2,864.78	
Inventory Income	357.57	
Gifts received	5,171.99	
Overpayments received	125.00	
Total Biennium Receipts		$54,688.84
Expenditures		
Assoc. of College Honor Societies	826.88	
Administrative expenses	4,659.80	
National Convention expenses	7,749.49	
Regional Convention expenses	1,207.61	
Council Meeting travel	1,750.93	
Certificates, jewelry, shipping	16,026.96	
Installation expenses	650.21	
Inventory expenses	14.15	
Overpayments returned	120.00	
Pentagon expenses	11,664.01	
Miscellaneous expenses	247.39	
Total Biennium Expenses		$44,917.43
Biennium Cash Flow		+9,771.41

During the previous biennium (95-97), the cash flow was a negative $10,861.82. The net difference in the two biennia is over $19,000. More than $5,000 of that amount can be attributed to a gift to KME by the Dorothy Horn Estate. Close to $6,000 came from reduced expenses for the National Convention held in Springfield, Missouri. Pentagon expenses were trimmed by over $1,000. The remaining portion of the difference is due to an increase in the initiation fees collected this biennium. The number of new initiates increased by over 200. Additionally, the initiation fee was $20 during the whole current biennium. It was $15 for the first nine months of the previous biennium.

Al Riveland

Report of the National Secretary

The *Kappa Mu Epsilon*, National Mathematics Honor Society, initiated 2,261 new members in 127 active chapters with a cumulative total of 56,918 members during the biennium that closed on March 31, 1999. Additionally, there are 28 inactive chapters with another total of 6,159 members for a grand total of 63,077 members, since the founding sixty-eight years ago on April 18, 1931, at Northeastern Oklahoma State Teachers College, Tahlequah, Oklahoma. The rate of increase accelerated approximately 10% with the installation of ten new chapters during the last biennium.

Pennsylvania Omicron, installed by Peter Skoner on April 10, 1997
University of Pittsburgh at Johnstown, Johnstown

Michigan Delta, installed by Arnie Hammel on April 30, 1997
Hillsdale College, Hillsdale

Michigan Epsilon, installed by Arnie Hammel on March 28, 1998
Kettering University, Flint

Kansas Zeta, installed by Bryan Dawson on April 14, 1998
Southwestern College, Winfield

Tennessee Epsilon, installed by Matt Lunsford on April 16, 1998
Bethel College, McKenzie

Georgia Beta, installed by Joe Sharp on April 25, 1998
Georgia College and State University, Milledgeville

Missouri Mu, installed by Don Tosh on April 25, 1998
Harris Stowe State College, St. Louis

Alabama Eta, installed by Eddy Joe Brackin on May 4, 1998
University of West Alabama, Livingston

New York Xi, installed by Robert Bailey on May 12, 1998
Buffalo State College, Buffalo

North Carolina Delta, installed by Donald Aplin on March 24, 1999
High Point University, High Point

Also, during the current biennium, we welcome
Pennsylvania Pi, installed by Peter Skoner on April 19, 1999
Slippery Rock University, Slippery Rock

As national secretary, I maintain permanent records of all initiates in local chapters. I also assist the corresponding secretaries, regional directors, and national officers, in any way that I can. For example, I order official key pin insignia and membership certificates from our jewelers, the J. O.

Pollack Company of Chicago, Illinois, and my project to create and maintain an electronic file of our membership continues. Another responsibility of my office involves the preparation of the minutes of the meetings of the National Council and Biennial Conventions.

Waldemar Weber

Report of the National Historian

The files of the national historian are being maintained and continually updated with reports received from chapters concerning their activities. I would like to thank the previous historian, Mary Elick, for presenting me with files that were systematic and well maintained. I must admit that when I became historian two years ago I was unaware of the magnitude of the job.

Twice a year a request for chapter news is sent to each chapter. These reports along with installation reports are edited and forwarded to the editor of *The Pentagon* for inclusion in the section on Chapter News. The unedited original reports are placed in the chapter folder and become a permanent record of each chapter's activities. I encourage each chapter to take advantage of this opportunity to record officers and activities.

During the last two years, I began to use electronic mail as well as surface mail for reporting chapter activities. I intend to continue sending both written and electronic requests, and I am happy to deal with either kind of report. I print electronic copies of all electronic responses for inclusion in your files. In the last reporting cycle, over 60% of the responses were electronic. If you are not receiving a form through the electronic mail, it is probably because I do not have a correct address for your corresponding secretary.

In this biennium 80 chapters reported at least once. To be commended are the 28 chapters which reported all four times: Alabama Gamma, Alabama Zeta, Colorado Delta, Georgia Alpha, Iowa Alpha, Iowa Delta, Kansas Alpha, Kansas Beta, Kansas Gamma, Kansas Delta, Kentucky Alpha, Maryland Beta, Maryland Delta, Mississippi Epsilon, Missouri Beta, Missouri Eta, Missouri Theta, Missouri Lambda, Nebraska Alpha, New Mexico Alpha, New York Alpha, New York Eta, Ohio Gamma, Oklahoma Alpha, Pennsylvania Iota, Pennsylvania Kappa, Tennessee Gamma, and Tennessee Delta. Additionally, reports were received about the installations of 9 new chapters: Alabama Eta, Georgia Beta, Kansas Zeta, Michigan Delta, Michigan Epsilon, Missouri Mu, New York Xi, Pennsylvania Omicron, and Tennessee Epsilon.

I would like to thank you for your active participation in maintaining the history and tradition of KME. I would especially like to thank the edi-

tor of *The Pentagon*, Brian Dawson, for the pleasant working relationship we have had. I look forward to working with the new editor, Steve Nimmo, as well. Finally, I would especially like to thank the corresponding secretaries whose extra efforts keep the lines of communication open. Their dedication is one of the main reasons for the continuing progress of KME.

Don Tosh

Report of the Editor of The Pentagon

Twenty-three papers comprise volumes 57 and 58 of The Pentagon. Of these, twenty are student papers, one is a joint student-faculty paper, and two are faculty papers. Fifteen of the student papers were presented at national or regional Kappa Mu Epsilon conventions; one of the faculty papers was a banquet address from the previous convention, and the other faculty paper grew out of the same banquet address. *The Problem Corner*, *Kappa Mu Epsilon News* and convention reports continue to make up a large portion of the journal and are essential to its success.

Manuscripts received by *The Pentagon* other than those presented at our conventions are still refereed by faculty volunteers. The efforts of twenty such referees were acknowledged in the Spring 1998 issue, and more than twenty referees have already been utilized since that time. Nearly all referees are from schools with KME chapters. These individuals have been a great help to the editor.

The cumulative subject index for *The Pentagon* was completed during this biennium. The index is available at the national KME web site. The index is arranged by generic class title or subject in order to be of greatest use to students and faculty. The wealth of interesting information in past volumes of the journal is readily apparent when one browses the index.

The efforts of associate editors Don Tosh and Kenneth M. Wilke have been greatly appreciated. Their diligent hard work has helped make *The Pentagon* a journal that many look forward to receiving. Larry Scott, business manager, has been of greater help to me than he will ever know. The members of the national council have also been very helpful in editing convention reports, forwarding papers to the editor, and dealing with many additional details of running this journal.

I trust that our organization will give the incoming editor, Steve Nimmo, the same high level of support that it has granted to me. Most of all, keep those manuscripts coming!

C. Bryan Dawson

Report of the Business Manager of The Pentagon

It is a pleasure to make my first Business Manager's report at this 32nd Biennial Convention. As many of you know, the Business Manager's primary responsibility is to maintain a current list of subscribers, to oversee our mailings, and to assist the Editor in managing *The Pentagon*.

All new initiates receive a two-year subscription to *The Pentagon* and are encouraged to continue their subscriptions for a modest fee of $5.00 per year. The library rate is $10.00 per year and international subscriptions are $7.00 per year. Issues are mailed in December and May of each academic year. Our mailing list includes subscribers in this country, South America, Asia, Africa, and Europe. During the past biennium, we have serviced more than 2500 subscribers per issue. Approximately 500 renewal notices are mailed each semester. Please watch the expiration date for your subscription and renew early.

Postal regulations for bulk mailings require that each address be validated at least once per year and that each address contain either a street address or Post Office Box number. Please check your address and make corrections if necessary. We typically pay more than $50 in penalties for forwarding issues and returning undeliverable issues. Please use a permanent address and make sure that your address is valid.

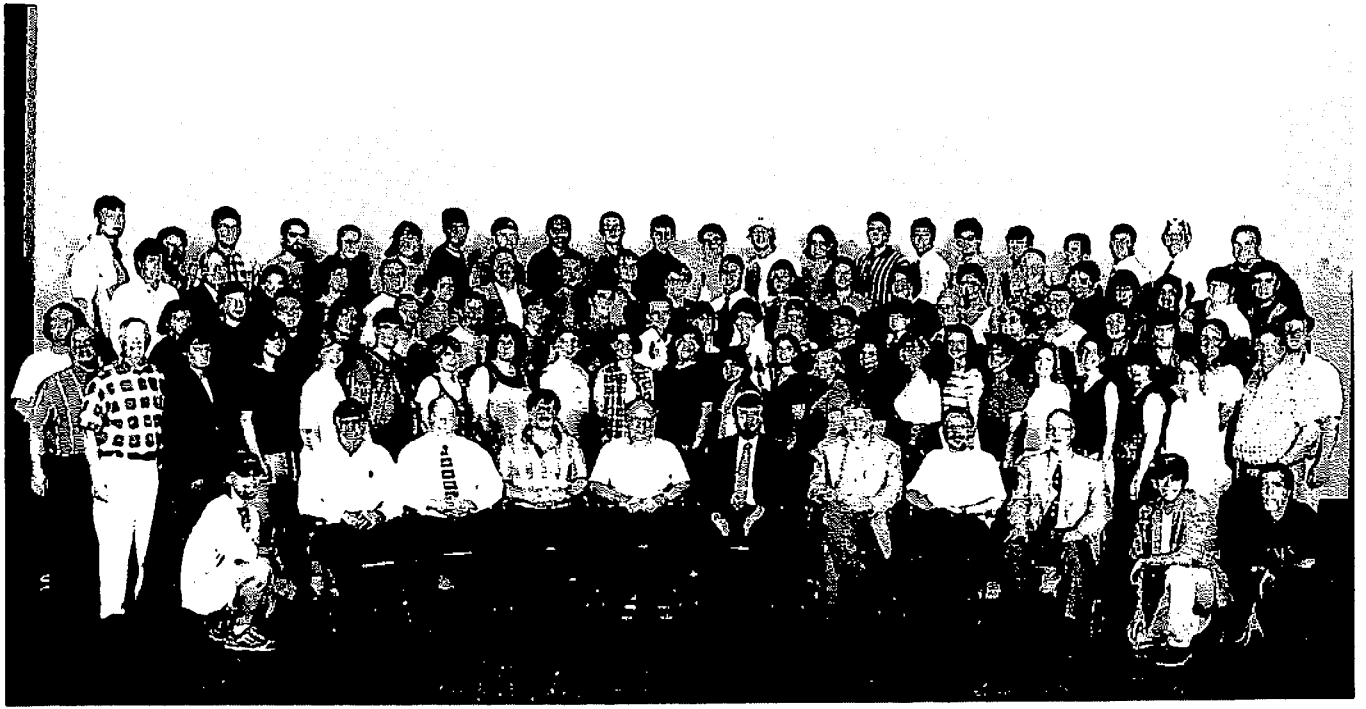
Complementary copies of *The Pentagon* are sent to the library of each college or university with an active chapter of *Kappa Mu Epsilon*. Anyone contributing an article for an issue will receive two free copies. Speakers at the 32nd Biennial Convention will have their subscriptions extended for two years.

I am appreciative of the support and assistance given by the National Council. I would like to thank C. Bryan Dawson, Editor of *The Pentagon*; Pat Costello, KME President; A. Allan Riveland, KME Treasurer; and Waldemar Weber, KME Secretary. Their cooperation and assistance have made things move smoothly. I gratefully acknowledge the assistance of my secretary, Sharon Brown.

Larry Scott

I don't know what I may seem to the world, but, as to myself, I seem to have been only as a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

-I. Newton



Thirty-Second Biennial Convention of *Kappa Mu Epsilon*, April 22-24,1999, Lakeland, Florida, hosted by Florida Beta.

National officers are seated in the front row. From left to right: A. Allan Riveland, C. Bryan Dawson, Steve Nimmo, Waldemar Weber, Patrick J. Costello, Larry Scott, Don Tosh, and Robert Bailey

Kappa Mu Epsilon National Officers

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Robert Bailey *President-Elect*
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Don Tosh *Historian*
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Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983

MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College	12 May 1998
NY Xi	Buffalo State College, Buffalo	May 12, 1998
NC Delta	High Point University, High Point	March 24, 1999
PA Pi	Slippery Rock University, Slippery Rock	April 19, 1999

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