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An Introduction to Multiquadric Interpolation

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Introduction

We can find many mathematical problems in the world around us which do not have absolute answers, requiring us to find approximate solutions with as much accuracy as possible. For instance, if we try to construct a topographical map of a mountainous region, first we gather data by measuring some elevations and locations. In using the data to construct the map, we realize that because measuring every dip and valley would be impossible, the map must be constructed from a set of isolated points. The next step is either to guess the elevations between the data points, if there are enough points close enough together, or to estimate these elevations mathematically, using some form of an approximating method.

Since we would like to finish constructing the map by taking small regions around the known data points and finding approximating functions which, when graphed, will represent as precisely as possible the surface of the region, an entirely new problem arises. We now use these few, random data points to find an approximating surface by means of an interpolation method.

Interpolation Methods

The scenario above leads to an interpolation problem in $\mathbf{R}^3$. We expect that interpolation methods in $\mathbf{R}^3$ can be developed by generalizing interpolation methods appropriate for $\mathbf{R}^2$. Well-known and commonly used methods include Lagrange interpolatory polynomials, Taylor polynomials

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and cubic splines. We shall outline a general interpolation method appropriate for $\mathbf{R}^n$ called Multiquadric Interpolation and we shall compare Multiquadric Interpolation (MQ) with the classic methods by examining a specific problem in the plane.

Since we do not know the true elevation function, but only some point values of the function, the mathematical model is derived from the behavior of the approximating function. Thus, the accuracy of the approximation is an important factor in selecting a method to derive the estimated function from the given data.

Many interpolatory methods exist, such as the classical ones named above, and each has its strengths and weaknesses. Some of these strengths and weaknesses depend upon the function itself; therefore some functions are easily interpolated because they do not change much, while others are not so easily estimated because their behavior is unpredictable. The problem is to find interpolation methods which can predict such behavior even in difficult instances.

We offer an example of interpolation in $\mathbf{R}^2$ using the previously mentioned methods. We shall consider a known function, obtain data points by evaluating the function at specified abscissas and graph both the interpolating function and the true function. By considering the difference between these functions, we can measure the accuracy of our technique. We consider Runge's function $f(x) = \frac{1}{1+x^2}$ and seven specified data points.

Figure 1 shows the Lagrange approximation of $f(x)$, figure 2 shows the Taylor approximation of $f(x)$, and figure 3 shows the Cubic Spline approximation of $f(x)$. These figures were created on *MATLAB* using Neville's Algorithm for the Lagrange Polynomial, a simple program for the Taylor Polynomial and the Natural (Free) Cubic Spline algorithm found in Burden and Faires' *Numerical Analysis*, 5th edition [2]. As we can see in these figures some methods are more accurate than others, but we want more precision than these methods allow. Thus, we shall need to look for a more precise interpolation method.

Multiquadric Interpolation

These classical methods of interpolation use polynomials in the Cartesian plane to derive approximating functions, so it seems that we could use polynomials of degree one in the form of linear combinations calculated from the given ordered pairs for interpolation in the Cartesian plane. The following discussion leads to the formation of an interpolatory method called Multiquadric Interpolation, or MQ.

Suppose we are dealing with one independent variable and we are given the following data: $f(x_1)$, $f(x_2)$, $\ldots$, $f(x_n)$. The problem is to find an approximate $F(x)$ such that $F(x_i) = f(x_i)$ for $i = 1, 2, \ldots, n$ and $F(x)$ accurately describes the behavior of $f(x)$ between these points. Throughout

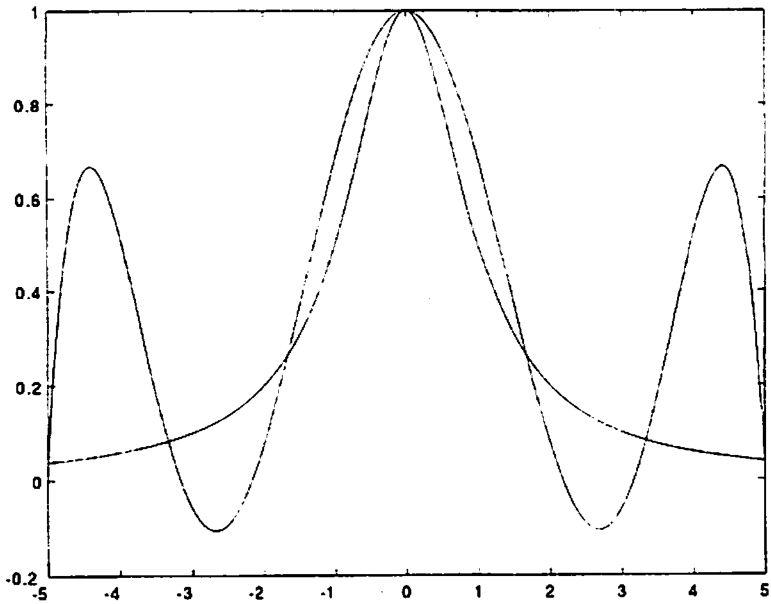


Figure 1. Lagrange approximation of $f(x)$.

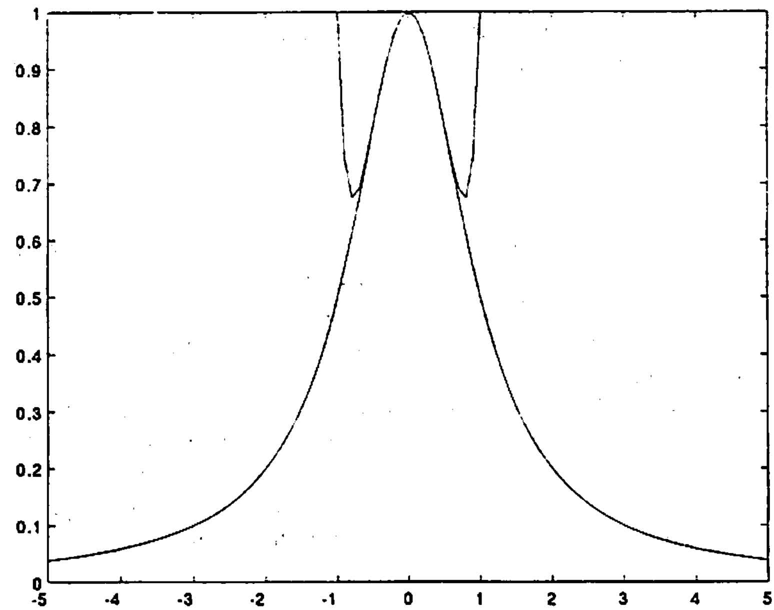


Figure 2. Taylor approximation of $f(x)$.

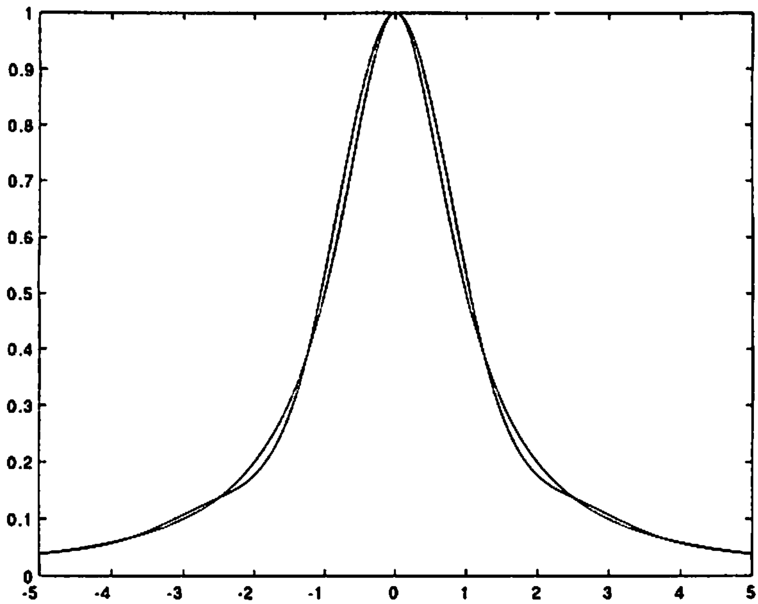


Figure 3. Cubic spline approximation of $f(x)$.

this report we shall use F to denote our interpolatory approximation to the function f with given data points. A reasonable function is the following:

$$F(x) = \sum_{i=1}^{n} c_i |x - x_i|,$$

where the c_i's are constants. These c_i's are calculated by solving the linear equation $A\mathbf{c} = \mathbf{b}$, where $\mathbf{c}$ is the unknown vector, $\mathbf{b}$ is the vector of length n with $f(x_1), f(x_2), \ldots, f(x_n)$ as its components, and $A = [a_{ij}]$ is an $n \times n$ matrix with $a_{ij} = |x_i - x_j|$. From this definition A is symmetric and has a principal diagonal of zeros.

In order to find the vector $\mathbf{c}$ of unknown coefficients, matrix A must be nonsingular; accordingly $A\mathbf{c} = \mathbf{b}$ will have a unique solution. Example 1 shows such an interpolation with one independent variable where the matrix is invertible; therefore $\mathbf{c}$ may be calculated uniquely. Figure 4 depicts graphically the interpolation method that finds a single approximating function.

Example 1. *Interpolate the function $f(x)$, given the following data: $f(1) = 4$, $f(2) = 3$, $f(4) = 7$.*

We have $n = 3$, $F(x) = \sum_{i=1}^{3} c_i |x - x_i|$, and $F(x_i) = f(x_i)$ for $i = 1, 2, 3$.

The equation $A\mathbf{c} = \mathbf{b}$ is

$$\begin{bmatrix} |1-1| & |1-2| & |1-4| \\ |2-1| & |2-2| & |2-4| \\ |4-1| & |4-2| & |4-4| \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 7 \end{bmatrix}.$$

We solve using Gaussian elimination. Starting by exchanging rows 1 and 2, we obtain

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 4 \\ 3 & 2 & 0 & 7 \end{array}\right].$$

We then subtract three times row 1 from row 3 and subtract two times row 2 from row 3 yielding

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & -12 & -10 \end{array}\right].$$

Backsolving results in $c_3 = 5/6$, $c_2 = 3/2$, and $c_1 = 4/3$. Hence

$$F(x) = \frac{4}{3}|x-1| + \frac{3}{2}|x-2| + \frac{5}{6}|x-4|.$$

Let $g(x) = (4/3)|x-1|$, $h(x) = (3/2)|x-2|$, and $k(x) = (5/6)|x-4|$. We graph each separate term to give a physical representation of the interpolation. Graphing $F(x)$ along with the terms illustrates that it is piecewise linear. We expect this result since the piecewise linear functions are a subspace of $C[1,4]$ and the addition of the lines, as shown in the graph, forms a line. The function is not differentiable at the data points since we only require that $F(x) = f(x)$ for the given data.

We can prove that in general the multiquadric matrix A will be nonsingular by developing an explicit formula for each of the c_i. We shall need three cases; first we develop a formula for c_i where $1 < i < n$, and then formulae for c_n and c_1 may be developed similarly. In each case the data is ordered so that $x_1 < x_2 < \cdots < x_n$ and $\mathbf{f}(x)$ is the known vector. Then

$$A = \begin{bmatrix} |x_1 - x_1| & |x_1 - x_2| & \dots & |x_1 - x_n| \\ |x_2 - x_1| & |x_2 - x_2| & \dots & |x_2 - x_n| \\ \vdots & \vdots & \ddots & \vdots \\ |x_n - x_1| & |x_n - x_2| & \dots & |x_n - x_n| \end{bmatrix}.$$

As a linear system the matrix equation $A\mathbf{c} = \mathbf{f}(x)$ becomes

$$\begin{cases} c_1|x_1 - x_1| + c_2|x_1 - x_2| + \cdots + c_n|x_1 - x_n| = f(x_1) \\ c_1|x_2 - x_1| + c_2|x_2 - x_2| + \cdots + c_n|x_2 - x_n| = f(x_2) \\ \qquad\qquad\qquad\vdots \\ c_1|x_n - x_1| + c_2|x_n - x_2| + \cdots + c_n|x_n - x_n| = f(x_n) \end{cases}.$$

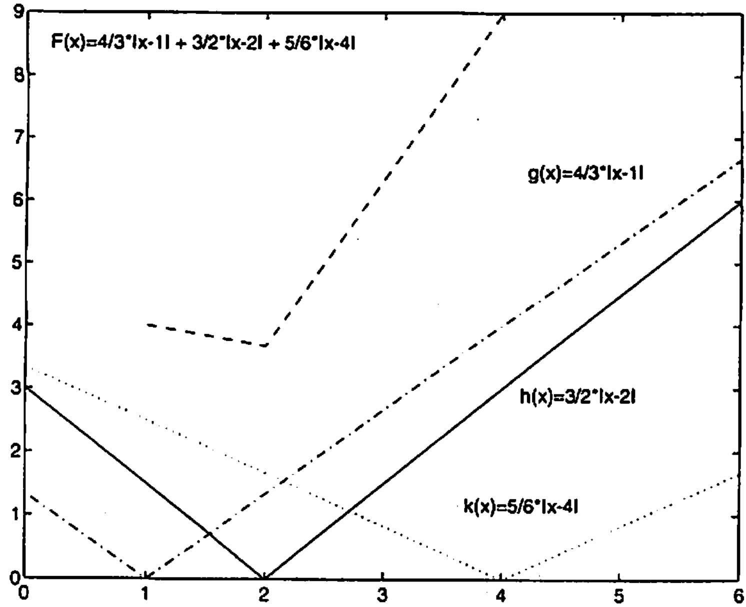


Figure 4. Pointwise addition to get linear piecewise approximation.

We shall now prove that the c_i have unique solutions for $1 < i < n$. We subtract the $(i-1)$st equation from the ith equation in $Ac = \mathbf{f}(x)$, giving us

$$c_1(|x_i - x_1| - |x_{i-1} - x_1|) + c_2(|x_i - x_2| - |x_{i-1} - x_2|) + \dots \\ + c_n(|x_i - x_n| - |x_{i-1} - x_n|) = f(x_i) - f(x_{i-1}).$$

Since $x_{i-1} < x_i$, the difference $|x_i - x_k| - |x_{i-1} - x_k|$ is greater than zero whenever $k < i$. If k is greater than or equal to i, the difference is negative. Thus the negative sign precedes the c_k whenever k is greater than or equal to i. Thus we get the following when we simplify:

$$c_1|x_i - x_{i-1}| + c_2|x_i - x_{i-1}| + \dots + c_{i-1}|x_i - x_{i-1}| - c_i|x_i - x_{i-1}| - \dots \\ - c_n|x_i - x_{i-1}| = f(x_i) - f(x_{i-1}).$$

We divide both sides by the common factor and obtain

$$(1) \qquad c_1 + c_2 + \dots + c_{i-1} - c_i - \dots - c_n = \frac{f(x_i) - f(x_{i-1})}{|x_i - x_{i-1}|}.$$

Similarly, we subtract the $(i+1)$st equation from the ith equation and obtain

$$(2) \qquad -c_1 - c_2 - \dots - c_i + c_{i+1} + \dots + c_n = \frac{f(x_i) - f(x_{i+1})}{|x_i - x_{i+1}|}.$$

We add (1) and (2) to get

$$-2c_i = \frac{f(x_i) - f(x_{i-1})}{|x_i - x_{i-1}|} + \frac{f(x_i) - f(x_{i+1})}{|x_i - x_{i+1}|}$$

and

$$c_i = -\frac{1}{2}\left(\frac{f(x_i) - f(x_{i-1})}{|x_i - x_{i-1}|} + \frac{f(x_i) - f(x_{i+1})}{|x_i - x_{i+1}|}\right).$$

Therefore each c_i is determined uniquely.

Similarly we can prove that c_1 and c_n are also determined uniquely. Since we obtain explicit formulae for each c_i, $A\mathbf{c} = \mathbf{f}$ has a unique solution, and thus A is nonsingular. It can be noted that in the interpolatory equation $F(x) = \sum_{i=1}^{n} c_i|x - x_i|$, the expression $|x - x_i|$ is the distance between x and x_i. Thus, we replace $|x - x_i|$ with $\sqrt{(x - x_i)^2}$. The definition of Euclidean distance in higher dimensional spaces can be represented similarly, only more variables are needed. Knowing this fact, we can now generalize the interpolation method to $\mathbf{R}^n$ by the following : Let x_i be any given data point in $\mathbf{R}^n$; then

$$F(x) = \sum_{i=1}^{n} c_i d(x, x_i),$$

where d is the Euclidean distance.

Beginning in three-dimensional space, the equation has two independent variables, the data is ordered triples, and the distance is

$$d((x, y), (x_i, y_i)) = \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

for $i = 1, 2, \ldots, n$. The approximating function is

$$F(x, y) = \sum_{i=1}^{n} c_i d((x, y), (x_i, y_i)).$$

So the $n \times n$ matrix A is

$$\begin{bmatrix} \sqrt{(x_1 - x_1)^2 + (y_1 - y_1)^2} & \cdots & \sqrt{(x_1 - x_n)^2 + (y_1 - y_n)^2} \\ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} & \cdots & \sqrt{(x_2 - x_n)^2 + (y_2 - y_n)^2} \\ \vdots & \ddots & \vdots \\ \sqrt{(x_n - x_1)^2 + (y_n - y_1)^2} & \cdots & \sqrt{(x_n - x_n)^2 + (y_n - y_n)^2} \end{bmatrix}.$$

In order to solve $A\mathbf{c} = \mathbf{f}(x, y)$, A must be nonsingular. The matrix A is nonsingular as Blumenthal knew in the 1930's when he obtained this result by use of Cayley-Menger Determinants [1]. It was not until some fifty years later, in the 1980's, that this fact was deemed important in approximation theory.

Now, A is invertible, so the unknown c_i's may be calculated. As in figure 4, which showed the addition of absolute value functions that yielded a piecewise linear approximating function, the interpolatory function may be visualized as a linear combination of frustrums of right circular cones which are added pointwise to form the approximating surface. We may note that if the frustrums are centered on the x-axis, the term $\sqrt{(x-x_i)^2+(y-y_i)^2}$ becomes $\sqrt{(x-x_i)^2+0}$. Hence, restricting the surface to the xz-plane reduces the problem to the previously studied case involving one independent variable. Thus, using distance in the interpolatory function is a proper generalization of the lower dimensional case.

If we consider the restriction to the xz-plane in the general case, where the frustrums do not lie on the x-axis, then a cone cut by a plane parallel to the axis of the cone, but not through the vertex, results in a hyperbola. Thus we may visualize the construction of the interpolatory function in the xz-plane as the pointwise addition of half-hyperbolas. It follows that any planar approximating function may be calculated using a linear combination of half-hyperbolas.

Now we generalize in $\mathbf{R}^3$ to construct the approximating surface using two independent variables. This can be visualized as in figure 5, which shows two cones with origins x_1 and x_2. A random x, y pair is selected, then the corresponding z values from the frustrum of each cone are added to obtain the z values of the approximating function, similar to the pointwise addition in example 1.

Here the resulting approximation will be a surface which may estimate the true surface. Since the equation involving absolute value was the first interpolation method that we studied, and examining its natural generalization in $\mathbf{R}^n$ resulted in a graph of half-hyperbolas, we now examine the equation of a hyperbola

$$y^2-(x-a)^2=r^2.$$

This can be written

$$y^2=(x-a)^2+r^2$$

or, finally,

$$y=\pm\sqrt{(x-a)^2+r^2}.$$

Since we want a function to approximate a surface, we shall only be concerned with the positive y values. Thus we may use $y=\sqrt{(x-a)^2+r^2}$ to form each term of our approximating function. We now substitute $\sqrt{(x-x_i)^2+r^2}$ for $\sqrt{(x-x_i)^2}$ in our previous development. The function becomes the following:

$$F(x)=\sum_{i=1}^{n}c_i\sqrt{(x-x_i)^2+r^2}. \tag{3}$$

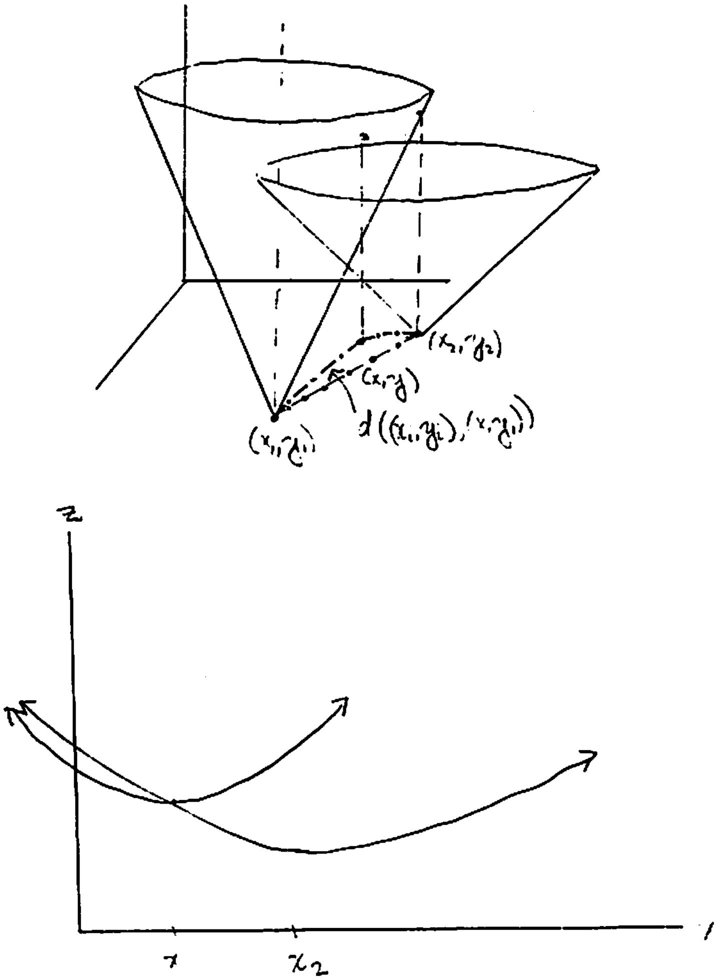


Figure 5. Interpolation in $\mathbf{R}^3$.

To solve the system $Ac = \mathbf{f}(x)$, we must first know that there is a unique solution. Micchelli proved the following result: Given any distinct points $x_1, \ldots, x_n$ in the plane, $(-1)^n \det \sqrt{1 + |x_i - x_j|^2} > 0$. This theorem says, in particular, that there is a unique surface $f(x) = c_1\sqrt{1 + |x - x_1|^2} + \cdots + c_n\sqrt{1 + |x - x_n|^2}$ which interpolates (data) $y_1, \ldots, y_n$ at $x_1, \ldots, x_n$ [4].

Thus, the function (3) will provide us with an interpolatory function for the known data. This approximating function, when graphed, will give an approximation to the true function as the pointwise addition of half-hyperbolas, given that the parameter r^2 is greater than zero.

We were able to move from one variable to two variables and even n variables using linear combinations of distance as the means of interpolation, so graphically we move from one to two to n variables using absolute values, hyperbolas, then hyperboloids of two sheets.

The equation of a hyperboloid of two sheets is the following:

$$w^2 - (x-a)^2 - (y-b)^2 = r^2.$$

Therefore,

$$w^2 = (x-a)^2 + (y-b)^2 + r^2,$$

and finally

$$w = \pm\sqrt{(x-a)^2 + (y-b)^2 + r^2}.$$

As before, we shall be concerned with the positive values, one sheet of a hyperboloid of two sheets: $w = \sqrt{(x-a)^2 + (y-b)^2 + r^2}$.

We substitute this for our distance function to form a new interpolatory function which has the following equation:

$$F(x,y) = \sum_{i=1}^{n} c_i \sqrt{(x-x_i)^2 + (y-y_i)^2 + r^2}.$$

This interpolatory function will provide an approximation for a function of two independent variables which may be represented graphically as a surface. Now the linear equation $A\mathbf{c} = \mathbf{f}(x,y)$ must have a unique solution so that $\mathbf{c}$ is a vector of constants and interpolation using $F(x,y)$ will be possible. Micchelli's theorem also guarantees the uniqueness of $\mathbf{c}$. Therefore, we have an interpolatory function. Micchelli's theorem says that interpolation is also possible for any finite number of independent variables:

$$F(x,y,\ldots,z) = \sum_{i=1}^{n} c_i \sqrt{(x-x_i)^2 + (y-y_i)^2 + \cdots + (z-z_i)^2 + r^2}.$$

We do not expect to be able to visualize the resulting hypersurface in any space greater than dimension three.

Accuracy Of Multiquadric Interpolation

As seen in figures 1–3, interpolation methods are not perfect. We hope to demonstrate that MQ proves to interpolate Runge's function far better than the classical methods, although not infallibly. Empirical studies of the veracity of interpolation methods have a long history in the mathematical literature. However, viable interpolatory schemes for scattered data were not systematically studied until the 1980's.

In 1982 Richard Franke realized a need to evaluate the accuracy as well as other factors of known interpolation methods of scattered data [3]. In his evaluation he rated these methods with letter grades A, ..., F on the basis of many characteristics that he considered important for analyzing the techniques. The method developed above, called Multiquadric Interpolation (MQ), received A's in Complexity, Accuracy and Visual and a B-/C- in time evaluation. These marks were overall superior to other methods considered. Still, this is only Franke's criteria for an A grade.

We find it necessary as well as worthwhile to test this newly found interpolatory method to discover its advantages and limitations. Taking a closer look at the MQ formula, we would like to find reasonable values for the parameter r^2.

Although a graph of an MQ approximation function may closely resemble that of the actual function, it does not necessarily mean it is the best or most precise interpolation, especially since we do not fully understand the unknown user-specified r^2 parameter in the equation.

Even if we find an r^2 value that yields an approximating function whose graph is close to that of the true function, it does not mean that we have found an optimal r^2. In fact, this is a current topic of mathematical research in the area of approximation theory, but there is currently no best algorithm for determining an r^2 parameter for all cases.

Optimizing r^2

Now we direct our examination of MQ to the search for an optimal parameter, r^2. Referring to Franke's work with interpolation, Audry Ellen Tarwater points out that Franke's evaluation clearly states that MQ is far better than all other methods evaluated, but "by optimizing r^2, the results obtained are significantly improved, indicating that MQ can be far better than previously expected" [5]. Possibilities for optimizing the r^2 parameter include finding a set numerical value from the given data, finding a variable r^2 such that r^2 is some function or optimizing r^2 with information other than the data points.

We can attempt to optimize r^2 as a scalar for Runge's function, which was exhibited in figures 1–3, by varying the r^2 value for different trials of approximating this function. In figures 6–13 (at the end of this paper) there are some examples of varying r^2 between zero and ten. This is a simple function in $\mathbf{R}^2$, so it is easy to substitute different values for r^2, find the L_1 error, and graph each approximation on the same graph with the true function, all in a reasonable amount of time and coding in *MATLAB*. Figure 6, with r^2 as ten, shows an undulating graph which resembles the Lagrange Polynomial method. We discover in figure 7 with r^2 as zero, that we do not the have smoothness of the original function. Now we can refer to Tarwater's investigation, which states that a larger r^2 increases the waves and a smaller r^2 decreases the smoothness of the graph [5].

We tried several values between zero and ten and found by a visual analysis, as well as numerical analysis of the errors, as seen in figures 7 and 8, that r^2 must be between zero and one. Figures 9 and 10 leave the optimal r^2 between 0.01 and 0.02. We further refined r^2 to 0.013 (figure 11) and 0.0133 (figure 12). But we see little difference in the final figures, so it seems we optimized the constant parameter as far as possible.

But this example oversimplifies the problem of parameter optimization, since all functions do not behave like Runge's function. Multiquadric approximations in $\mathbf{R}^3$ present an even bigger problem. First of all, the immense amount of computing time necessary to calculate c_i's and to graph it, as well as the vast possibilities of parameter values, make trial and error methods inappropriate for finding precise results in a reasonable amount of time.

Although we do not expect to find the optimal r^2 by means of trial and error, we can explore the behavior of a function in $\mathbf{R}^3$ and we might also gain some insight into the parameter using this method.

We investigated the parameter r^2 as it pertains to Franke's surface

$$\begin{aligned} f(x) =9\,&\left(.75\exp\left(-.25((x-3)^2+(y-3)^2)\right)\right.\\ &+.75\exp\left((-x/49)-(y/100)\right)\\ &+.5\exp\left(-.25((x-8)^2+(y-4)^2)\right)\\ &\left.-.2\exp\left(-1(x-5)^2-(y-8)^2\right)\right). \end{aligned}$$

On the 4-processor Cray Y-MP at the National Center for Supercomputing Applications, we ran a series of Fortran programs designed to solve the multiquadric matrix equation with various values for r^2. We not only varied r^2, but we also varied the number of data points used, which was anywhere from 20 up to 300 randomly selected data points. These programs yielded the coefficients of the vector c, which were then used to construct the multiquadric interpolatory function. We then generated a graphical representation of the surface within *MATLAB* running on a 4-processor Sun 670-MP. We found it convenient to dilate the domain uniformly so as to present the surface on the domain $[0,100]\times[0,100]$. A representative sample of the resulting surfaces are included in figures 13–19.

In the figures shown, 20 random data points were used and the surfaces were graphed using the dilated domain. Here we look at the L_1 error. In figure 13 the parameter is zero; we can see the decreased smoothness, not only due to the nature of MQ with small r^2 values, but also due to the small number of data points. The two peaks demonstrate this especially; both would appear to be cones. We had fewer data points on the lower peak, but we see that the higher peak appears somewhat conical.

We tried r^2 as 40 and found results we were likely to expect, as figure 14 shows; the peaks and valley are smooth, but the edges, which should be flat, are wavy. Figures 15 and 16 show the parameter as one and two, respectively, which helps us determine that the optimal r^2 must be between these values. Furthermore, we find as shown in figure 17 that the optimal parameter is near 1.3. The further refinements in figures 18 and 19 with the parameter equal to 1.33 and 1.339, show little difference by refinement of r^2 in the fourth significant digit. Again, we have gone as far as possible with this investigation of a constant parameter for this surface.

Conclusions

We found that varying r^2 gave us different graphs in $\mathbf{R}^2$ and different surfaces in $\mathbf{R}^3$. We consistently found that large values for r^2 resulted in poor interpolatory graphs and surfaces compared to the smaller r^2 values.

In constructing the Franke surface, not only did we change r^2, but we also varied the number of data points. We observe that the smaller the parameter value is, the better the approximated surface is; but we also see that the fewer the known data points we use, the less smooth surfaces constructed with a small r^2 value are. The construction of these surfaces involved fewer data points than the smooth surface interpolation.

Therefore we can conclude that not only does r^2 seem to be a small number in both of our cases, but the accuracy of the interpolation also depends upon the number of data points given. Furthermore, the location of the data points is important since some regions of the surface are predicted more accurately than others. We can see this specifically at the corners of the surface, which indicates that there are too few points, especially along the edges. Although our MQ approximations proved to be accurate when r^2 was small, they were not always efficient. The Franke surface construction required the use of a supercomputer for data point computation and *MATLAB* for generating a graphic representation. Although the supercomputer allows us to compute in a few minutes what takes over two hours on our usual computer, we do not always have access to such technology. So, MQ is not as efficient as it is accurate.

As Tarwater concludes in her study of the parameter r^2, "It has been shown that to optimize r^2, i.e. to minimize the error of the approximation, more factors are involved than the data locations" [5], we also conclude that information necessary to optimize MQ interpolation is missing. An interpolatory method may work well, but it cannot perfectly determine every function, so other techniques are still important. Also, analyzing the techniques for sensitivity, accuracy or other important traits cannot be overlooked. In conclusion, Multiquadric Interpolation seems to be one of the best interpolation methods available to us today.

Acknowledgements. I would like to thank the National Center for Supercomputing Applications for grant number TRA930381N which aided my research and data gathering. I cannot thank Dr. Timothy Hardy enough for all of his help, input, encouragement and patience. Thank you.

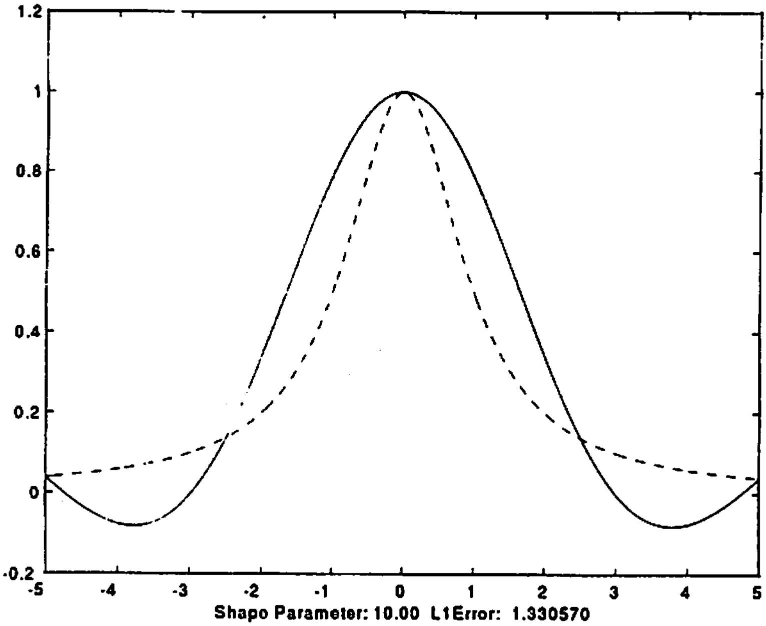
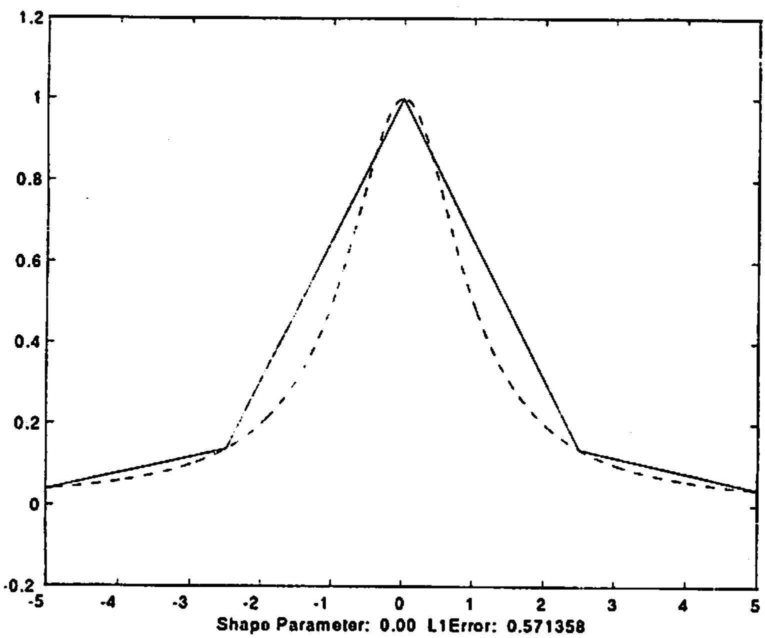


Figure 6.



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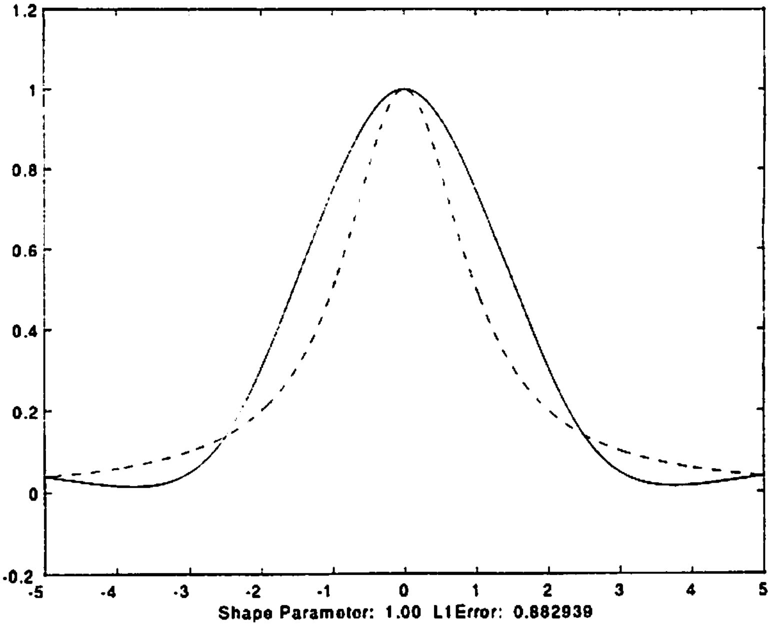

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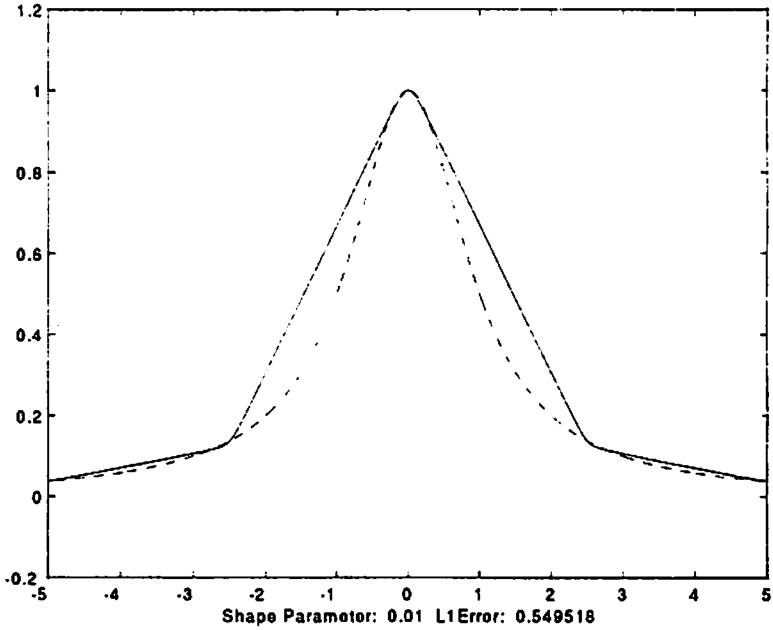

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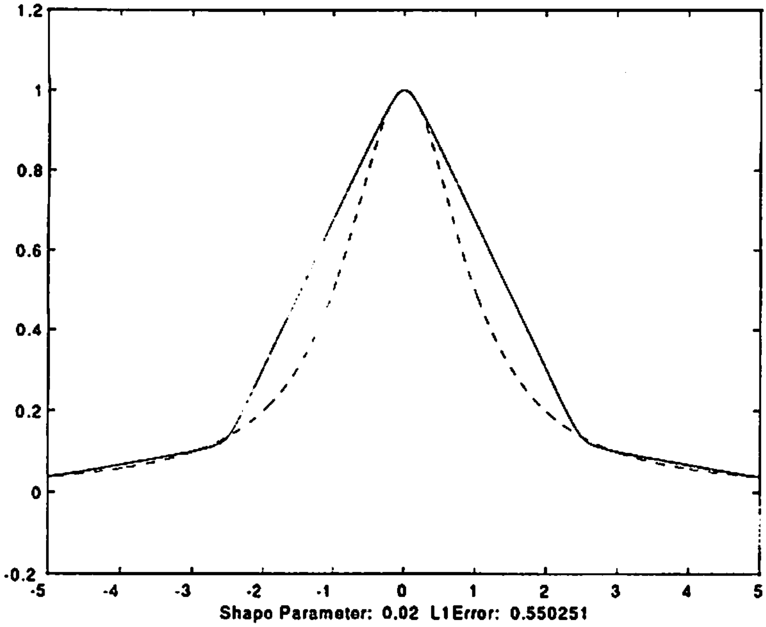

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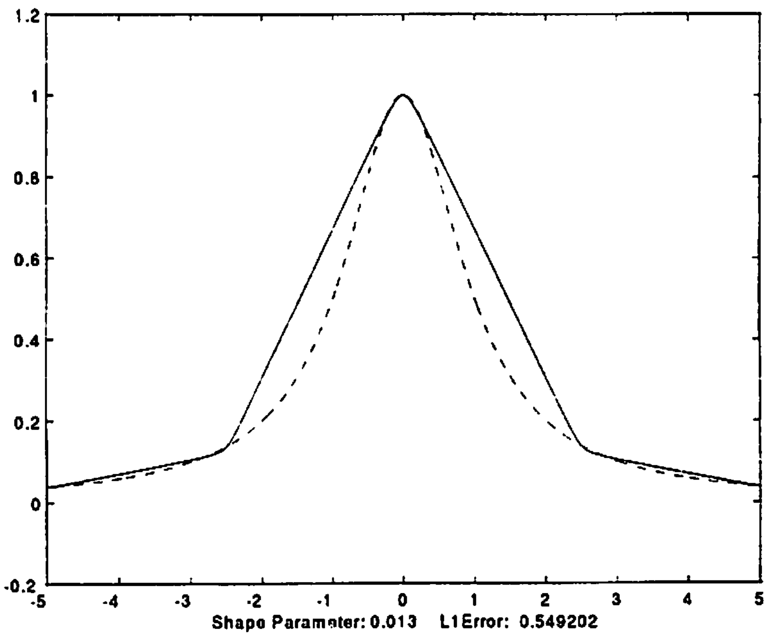

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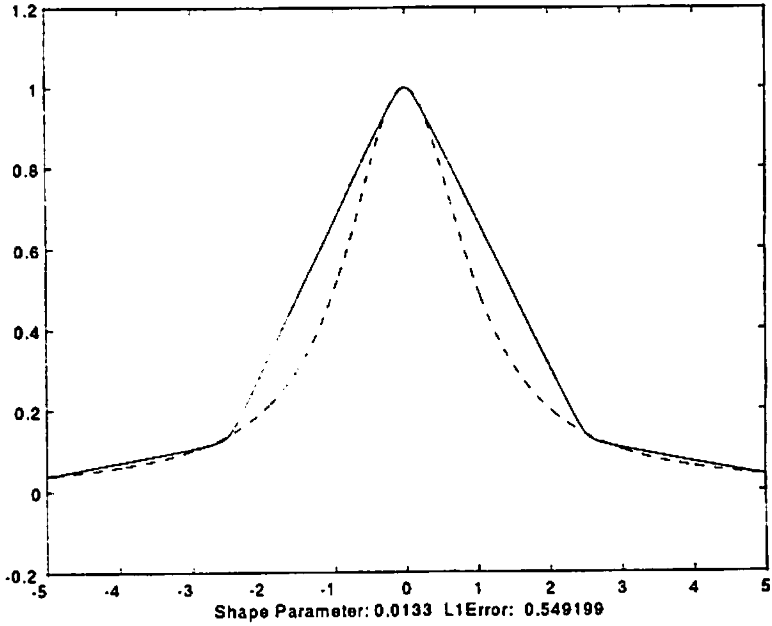

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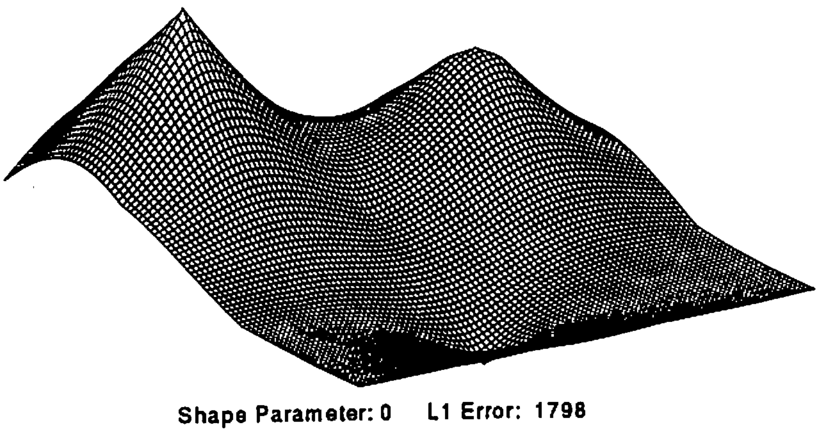

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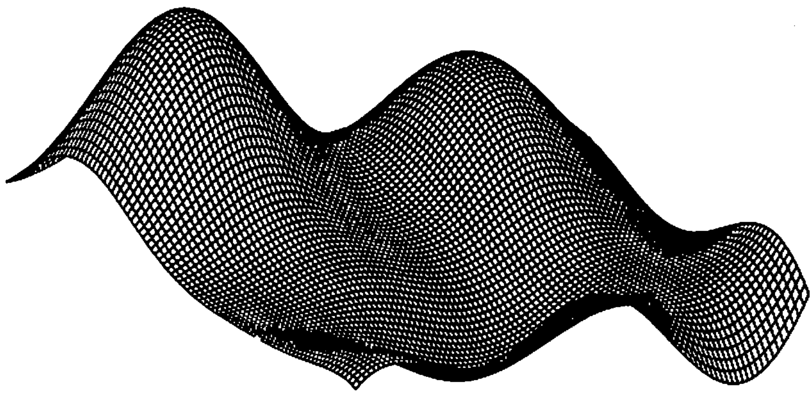

Figure 14.

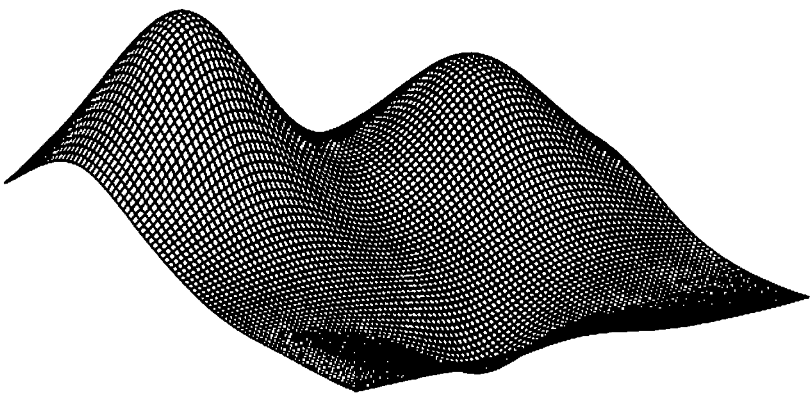

Figure 15.

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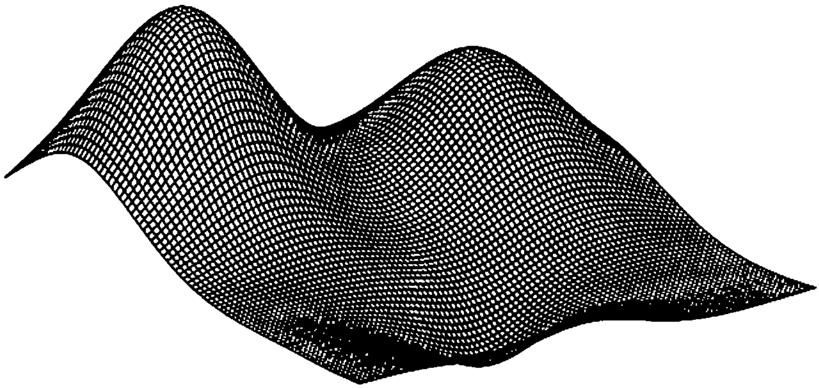

Figure 16.

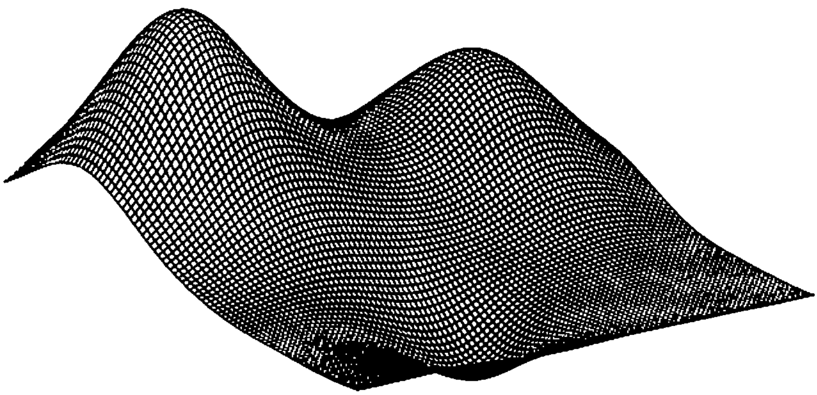

Figure 17.

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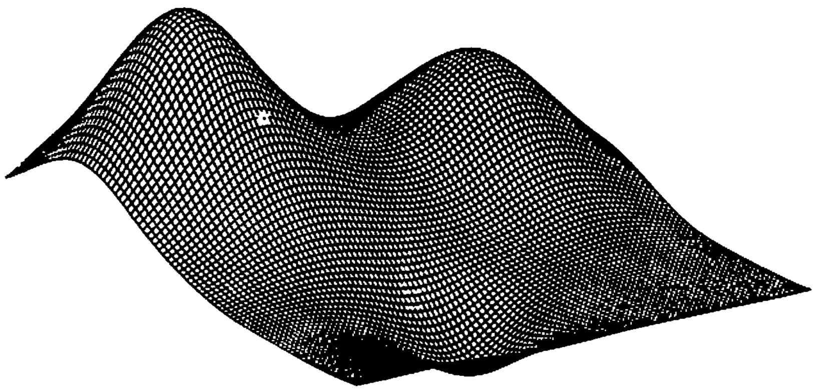

Figure 18.

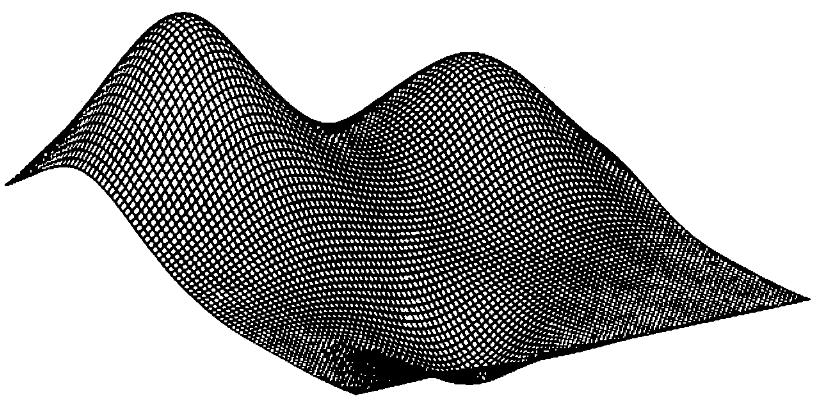

Figure 19.

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Palindromes

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Introduction

The mathematical puzzles that are learned when one is a child, the enigmas that face a young mind, are often those which haunt the older student. Why did this work? We wonder. Is it magic? Did I imagine it?

So, I began my search for a topic in a locale familiar to me since elementary school: books of mathematical puzzles. My original intent was to discuss a somewhat broader range of puzzles, such as the family of "magic squares," simple integer problems, and perhaps even some geometrical dilemmas. Curiosity overthrew intent, however, when I encountered one very simple-sounding problem, a problem that deals only with the counting integers, a problem that relies only on addition. It is, in strict point of fact, a parlor trick, more or less an intellectual amusement to fill the time. And yet, it has not, to my knowledge, been solved. Everyone can perform the simple calculations, and yet no one has explained it, no one has provided an answer to the question of "Why?". And so, this is something of a story — of how I came to narrow down the topic so very much, and what steps I have taken to increase the scope of my understanding about this puzzle.

Palindromes

Take a few moments to consider the following puzzle, haphazardly discovered amidst a sea of others in a book of mathematical curiosities: write down a positive integer, say 91; write the integer's digits in opposite order, and add the two integers; repeat this process, until the number reads the same, forwards and back. This is known as a palindromic number, a phrase more popularly associated with letters than with digits.

$$\begin{array}{r} 91 \\ +19 \\ =110 \\ +011 \\ =121 \end{array}$$

In the event that I have inadvertently chosen one of a select few counting integers that satisfies these conditions, I encourage the reader to take a few moments and try some other numbers, perhaps some of three and four digits, in order to rest assured that these statements hold true. A word of warning, however; even within the set of two-digit counting integers lie a few members that reach palindromes very slowly. For example, 89 and 98 each require twenty-four successive iterations of this formula in order to form a palindrome. Needless to say, it would be rather difficult to completely enumerate this process for even the two- and three-digit integers by hand. At this point, my investigation of this problem took on two very distinct and separate forms: the first, and perhaps the easier approach, which involves enumerating large sets of data using a computer program, and the second, more abstract method, which I will refer to as domain reduction. Neither was successful in providing "the" answer, but in my opinion, each approach has its merits and each has provided some interesting insights into the nature of the puzzle.

As mentioned above, and as perhaps the reader has discovered, one of the more difficult aspects of this problem is the enumeration of large sets of data. So, in the spirit of harmony with technology, I designed and implemented a computer program that does exactly that, as well as performing limited data analysis. Before discussing the results of this method, I would like to discuss the program itself, in order to assure the reader that all precautions to minimize error due to computer calculations have been taken. The difficulty with writing a computer program to handle this problem lies in the reversal of the digits to form the second number. This is by no means a standard operation, and would be very difficult to accomplish using the standard integer data type. In order to circumvent this, I have represented the numbers as a particular data structure referred to as a doubly-linked list. This has three major advantages, in terms of the problem: first, it simplifies the calculations involved, since the program can add the numbers in the manner that elementary school children are taught; second, it greatly reduces the computer error involved, since no member of the linked list will ever be greater than nineteen, and for most of the program execution time, all members are less than ten; and last, and perhaps most significant, the number can be reversed using nothing more complex than a simple while loop control structure. This abstract data structure is represented below as a series of nodes, which are linked through pointers to each other in a fashion that allows the list to be read forwards or backwards (see figure 1).

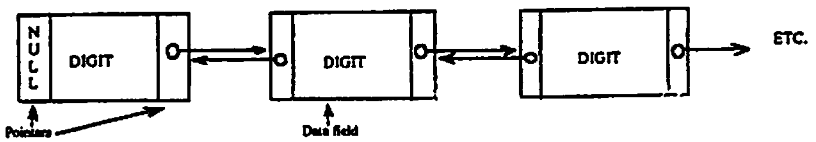

Figure 1. Data structure used for representing integers.

Each node contains a field called *digit*, which holds the corresponding digit for the number, and two pointer fields, one pointing forward in the list and the other pointing back. This method leaps one additional hurdle that, at first, I did not envision as a problem: the length of the integers. A standard integer data type can be no higher than $2^{15} - 1$, or 32,767; an unsigned integer data type is limited to numbers less than 2^{16}, or 65,536. Unfortunately, when a number is reiterated through this algorithm many times, the serviceability of the integer data type quickly moves from impractical to impossible. There does exist one major drawback to the linked list as a form of numeric representation: the limitation of computer memory. Each separate node, each digit, in the linked list requires more memory than the entire number would, if represented as an integer data type. It is therefore necessary to run this program on a computer with a reasonably large amount of memory.

Basically, the mechanics of the program follow exactly the steps of the algorithm as outlined in the problem, with the additional feature that a range of numbers may be entered. Each number, the resulting palindrome, and the number of iterations is written to a standard ASCII text file, using commas as delimiters in order to facilitate the use of spreadsheets in analyzing the data.

By this point, I am sure the reader would be interested to know the results of this little computer analysis. When I first designed and ran the program, as an additional feature I added a "safety valve," *just in case* (this is a standard practice to help avoid infinite loop situations). I wish to stress *just in case*, because I did not expect anything to go wrong with the program. To my total and complete surprise, this safety valve was reached a number of times, merely within the domain of 1 to 1,000. To assure the reader that, indeed, a reasonable safety valve was established, I wish to make note of a few figures. First, no number — other than these "safety violations" — exceeded 24 iterations for this domain; second, my initial safety valve was set at 2000 iterations. To illustrate a point, I at first believed that one of the ever-present bugs had infested the program, and I actually spent nearly four hours rearranging program code and nearly disabling the program altogether. I had of course fallen prey to a common fallacy — that is, what has been printed must be true! The problem stated very clearly, in print [1], that one could attempt this trick with any number. And so, I am now forced to admit that I may have misled the reader in some of my earlier statements, but that was intended to illuminate this point clearly: the problem being examined has perhaps been somewhat poorly explored. In fact, I do not know, and possibly cannot know, whether or not these numbers actually adhere to this system of palindromes; they may produce palindromes at a much higher number of iterations than the machine can test. I have examined them at iterations up to ten thousand, and they have continued to fail at each test.

In the interest of closer examination of these numbers, I began writing them to a separate text file during the execution of the program, calling the file "crazy.dat", as in 'These are driving me ... '. And, at first, it seemed as though a pattern of these numbers was forming: 196, 295, 394, ... It seemed as though I might have inadvertently stumbled on a clue to the puzzle after all. But as the pattern continued to evolve, I saw that I had actually expanded the complexity of the puzzle: 196, 295, 394, 493, 592, 689, 691, 788, 790, 879, 887, 978, 986 is the series of invalid numbers between 1 and 1,000. It *almost* forms a pattern. It *nearly* represents a very large piece of the puzzle. It *very clearly* is not a simple sequence! These same problems exist in every set of data I have generated; patterns are almost formed, sequences almost simple. Sets of data have been generated for the set 1 to 100,000. The appendix shows the output from the program for the data set 1 to 1,000 and the problematic integers between 1 and 10,000.

As a few more notes concerning this approach to analysis, I would like to mention some of the more interesting experiments I have performed on the sets of data. First, appended to the main body of the program is a function called "cmapit," which produces a color map of the data. This function first analyzes the data, and then draws lines of different color depending upon the number of iterations required to produce a palindrome. It produces a fairly striking effect, appearing more or less as a compressed wave, with a few extra odd lines. It demonstrates an idea, one that I still emphasize, about the data; the numbers, the inputs, cannot be considered as integer values, but instead should be viewed as collections of digits. As the sum of the digits gets higher, the color appears more intense, and vice versa.

Next, I used a standard spreadsheet to graph and analyze the data. Using parsing methods, it is fairly simple to read the data generated from the text files into the spreadsheet. Once again, the graphs demonstrate a

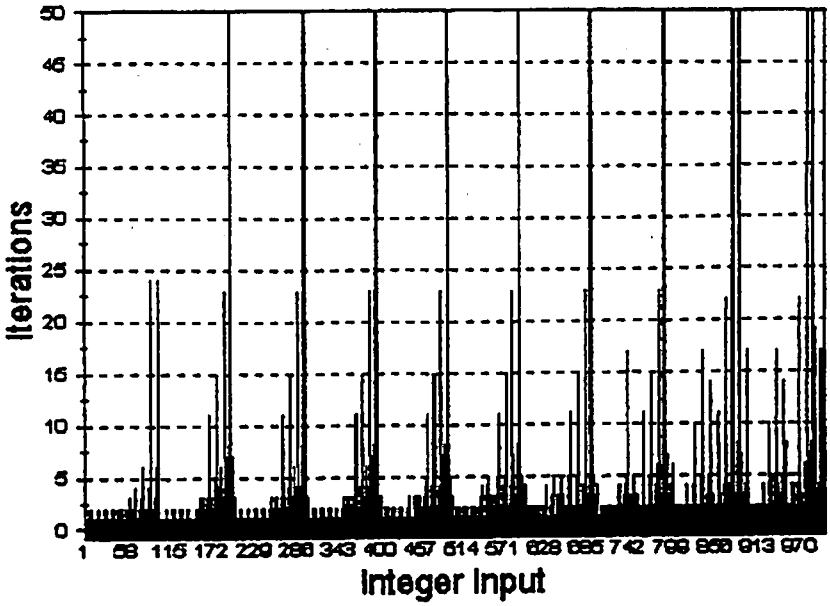

Figure 2. Iterations vs. integer input.

remarkable pattern. See figure 2.

The lines that shoot off of the top of the graph are the integers which do not seem to form palindromes. Note that no other integer reaches 25 iterations in this data set.

Next, I examined data sets formed under other number bases. The source code of the program can be easily altered to change the number base by simply changing the value of the variable base near the beginning of the code. A general pattern seems to be that number bases lower than ten possess many more integers that do not generate palindromes, while the other integers on average require more iterations to produce palindromes. Higher number bases seem to lean towards the other end of the spectrum, having less of each. This seems reasonable, since lower number bases carry more frequently than higher bases in the addition operator. This suggests that the palindrome is some sort of equilibrium point, just before which no digit has carried, or, if carried, then carried in a manner bordering on coincidence.

Although most of the data gathered from computer analysis did not provide any solid answers, the graphs and color maps led me to the next, and still developing, phase of the project. The graphs seem to indicate that it is more useful to consider the infinite set of counting integers as an infinite set of finite subsets of integers, i.e. to consider the counting integers to be divided into the one-, two-, three-, etc. digit number subsets. This seems reasonable, since each time the boundary of one of these subsets is crossed, the pattern of iterations abruptly and totally shifts. From this supposition came the idea of domain reduction.

The idea I have termed domain reduction represents in practice exactly what it appears to mean, that the domain of inputs can be reduced by a series of logical suppositions, until all possible cases have been enumerated. In theory, this sounds wonderful, but in practice it is a rather different story. Consider a two-digit number ab, such that a and b are each less than ten. Then the first iteration becomes

$$\begin{aligned} a&|b \\ +b&|a \\ =a+b&|a+b. \end{aligned}$$

Note that the mark "|" is used to separate the digits, and not to denote absolute value. Let $d = a + b$. Then d must be less than or equal to 18. If d is less than ten, then the number dd is a palindrome and iteration stops. Thus, we have established that all two-digit counting integers whose digits sum to be less than ten reach a palindrome in one iteration. Also, if d is 11, then iteration stops (i.e., $ab = 29, 92, 47, 74$, etc.). This limits us to a domain of $d \in \{10, 12, 13, 14, 15, 16, 17, 18\}$. In all of these cases, d would have to carry, and

$$d|d = 1|d-9|d-10.$$

The next step yields

$$\begin{aligned} 1|d-9&|d-10 \\ +d-10|d-9&|1 \\ =d-9|2d-18&|d-9. \end{aligned}$$

Now, if d = 10, 12 or 13, the process is complete at the second iteration, since no digit would carry. So this has already effectively removed all two-digit numbers such that $a + b < 14$ from the domain. With repeated applications of this method on the finite set of two-digit integers, we can enumerate all of the possibilities; see table 3 of the appendix for the results. This demonstrates that the iterations of the two-digit integers are dependent on a single variable, d = the sum of the two digits. However, from the outset the three-digit integers pose a much greater problem, since their iterations are dependent upon two variables, the middle digit b and the sum of the first and last digits d:

$$\begin{aligned} a|b&|c \\ +c|b&|a \\ =a+c|2b&|a+c. \end{aligned}$$

As the number of digits increase, the number of variables upon which the counting integers are dependent increases. For a counting integer with n digits, if n is even then the number of variables of dependence is $n/2$, and if n is odd, then the number of variables of reduction is $(n+1)/2$. This makes domain reduction theoretically possible, but practically, very unpalatable.

According to Young [2], this still existed as an open problem as of 1992, a conclusion with which this research seems to concur most heartily! So, we ask the question "Was any progress made here?". Perhaps not. It seems, though, that some interesting methods have been, if not created, then at least applied in an intriguing manner. It may be possible as time goes on to develop an algorithm for domain reduction, which would make a great step towards a solution. And, in the meanwhile, I hope this little puzzle will continue to create a little of that sense of magic in the younger mathematicians in elementary classrooms, and maybe even a little in myself.

Acknowledgements. I would like to give great thanks to Dr. Ilene Morgan for the assistance and encouragement she provided.

Editor's Note. Anyone wishing to receive a copy of the program used by Mr. Brown may request one from the editor (address on page 2).

References

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Appendix

The figures on pages 30 and 31 are intended to demonstrate the pattern found in the palindrome generation program. In either case, the lines that shoot off of the top of the graphs indicate an integer without a generated palindrome.

Table 1 on pages 32–36 represents the data set 1–1000, with the enumerated palindromes and the number of iterations required to produce the palindrome. Please note that any integer on this table having a palindrome of 0 is an invalid integer, and is included in table 2. I have generated the complete data set of 1–100000, but it is too large to print. Table 2 on pages 36–37 lists the integers in the set 1–10000 which do not produce palindromes. It is included to show the emerging, and then crumbling, patterns in the invalid data set. Table 3 on page 38 displays the results of the domain reduction method of analyzing the set of two-digit counting integers.

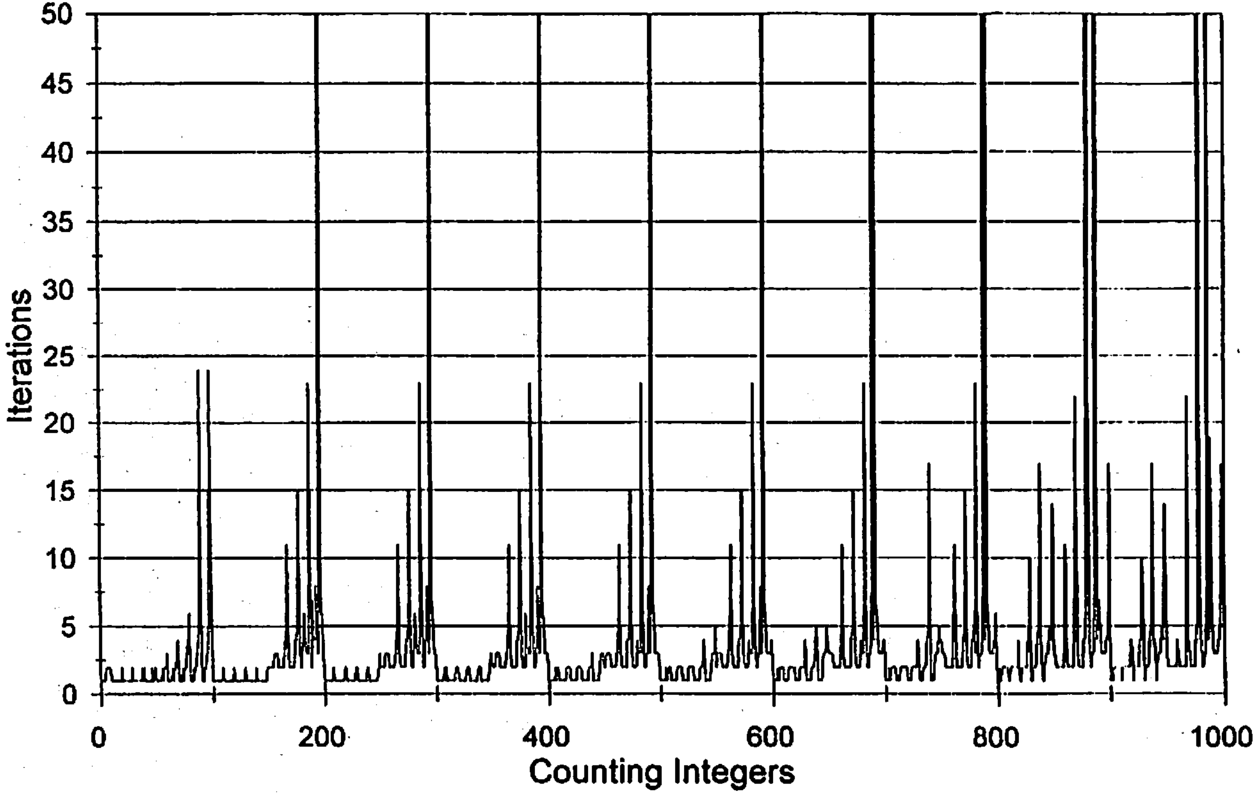

Figure 3. Iterations vs. counting integers

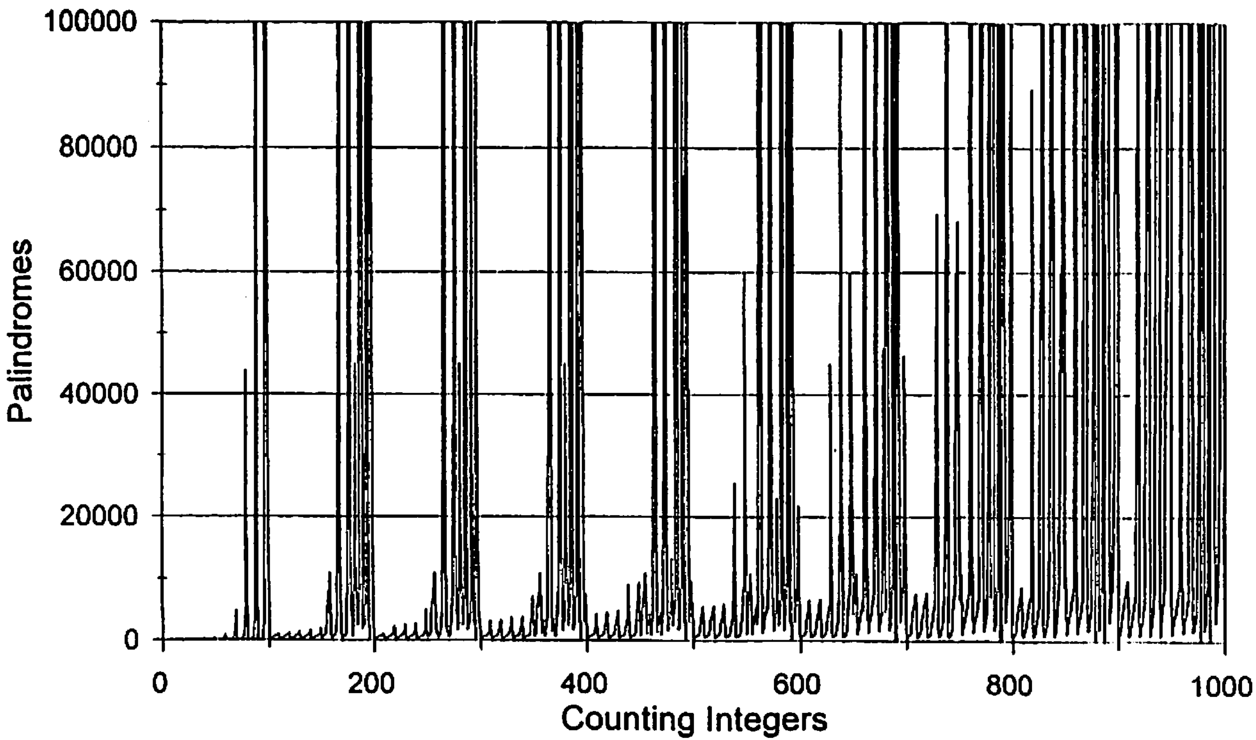

Figure 4. Palindromes vs. counting integers

Integer	Iterations	Palindromes
1	1	2
2	1	4
3	1	6
4	1	8
5	2	11
6	2	33
7	2	55
8	2	77
9	2	99
10	1	11
11	1	22
12	1	33
13	1	44
14	1	55
15	1	66
16	1	77
17	1	88
18	1	99
19	2	121
20	1	22
21	1	33
22	1	44
23	1	55
24	1	66
25	1	77
26	1	88
27	1	99
28	2	121
29	1	121
30	1	33
31	1	44
32	1	55
33	1	66
34	1	77
35	1	88
36	1	99
37	2	121
38	1	121
39	2	363
40	1	44
41	1	55
42	1	66
43	1	77
44	1	88
45	1	99
46	2	121
47	1	121
48	2	363
49	2	484
50	1	55
51	1	66
52	1	77
53	1	88
54	1	99
55	2	121
56	1	121
57	2	363
58	2	484
59	3	1111
60	1	66
61	1	77
62	1	88
63	1	99
64	2	121
65	1	121
66	2	363
67	2	484
68	3	1111
69	4	4884
70	1	77
71	1	88
72	1	99
73	2	121
74	1	121
75	2	363
76	2	484
77	3	1111
78	4	4884
79	6	44044
80	1	88
81	1	99
82	2	121
83	1	121
84	2	363
85	2	484
86	3	1111
87	4	4884
88	6	44044
89	24	8.8E+12
90	1	99
91	2	121
92	1	121
93	2	363
94	2	484
95	3	1111
96	4	4884
97	6	44044
98	24	8.8E+12
99	6	79497
100	1	101
101	1	202
102	1	303
103	1	404
104	1	505
105	1	606
106	1	707
107	1	808
108	1	909
109	2	1111
110	1	121
111	1	222
112	1	323
113	1	424
114	1	525
115	1	626
116	1	727
117	1	828
118	1	929
119	2	1331
120	1	141
121	1	242
122	1	343
123	1	444
124	1	545
125	1	646
126	1	747
127	1	848
128	1	949
129	2	1551
130	1	161
131	1	262
132	1	363
133	1	464
134	1	565
135	1	666
136	1	767
137	1	868
138	1	969
139	2	1771
140	1	181
141	1	282
142	1	383
143	1	484
144	1	585
145	1	686
146	1	787
147	1	888
148	1	989
149	2	1991
150	2	303
151	2	505
152	2	707
153	2	909
154	2	1111
155	3	4444
156	3	6666
157	3	8888
158	3	11011
159	2	1221
160	2	343
161	2	545
162	2	747
163	2	949
164	3	2662
165	3	4884
166	5	45254
167	11	8.9E+07
168	3	13431
169	2	1441
170	2	383
171	2	585
172	2	787
173	2	989
174	4	5115
175	4	9559
176	5	44044
177	15	8.8E+09
178	3	15851
179	2	1881
180	3	747
181	4	2662
182	6	45254
183	4	13431
184	3	2552
185	3	4774
186	3	6996
187	23	8.8E+12
188	7	233332
189	2	1881
190	7	45254
191	4	2552
192	4	6996
193	8	233332
194	3	2992
195	4	9339
196	501	0
197	7	881188
198	5	79497
199	3	3113
200	1	202
201	1	303
202	1	404
203	1	505
204	1	606
205	1	707
206	1	808
207	1	909
208	2	1111
209	1	1111
210	1	222
211	1	323
212	1	424
213	1	525
214	1	626
215	1	727
216	1	828
217	1	929
218	2	1331
219	2	2442
220	1	242
221	1	343
222	1	444
223	1	545
224	1	646
225	1	747
226	1	848
227	1	949
228	2	1551
229	2	2662
230	1	262
231	1	363
232	1	464
233	1	565
234	1	666
235	1	767
236	1	868
237	1	969
238	2	1771
239	2	2882
240	1	282

Table 1

241	1	383	321	1	444	401	1	505
242	1	484	322	1	545	402	1	606
243	1	585	323	1	646	403	1	707
244	1	686	324	1	747	404	1	808
245	1	787	325	1	848	405	1	909
246	1	888	326	1	949	406	2	1111
247	1	989	327	2	1551	407	1	1111
248	2	1991	328	2	2662	408	2	3333
249	3	5115	329	2	3773	409	2	4444
250	2	505	330	1	363	410	1	424
251	2	707	331	1	464	411	1	525
252	2	909	332	1	565	412	1	626
253	2	1111	333	1	666	413	1	727
254	3	4444	334	1	767	414	1	828
255	3	6666	335	1	868	415	1	929
256	3	8888	336	1	969	416	2	1331
257	3	11011	337	2	1771	417	2	2442
258	2	1221	338	2	2882	418	2	3553
259	2	2332	339	2	3993	419	2	4664
260	2	545	340	1	383	420	1	444
261	2	747	341	1	484	421	1	545
262	2	949	342	1	585	422	1	646
263	3	2662	343	1	686	423	1	747
264	3	4884	344	1	787	424	1	848
265	5	45254	345	1	888	425	1	949
266	11	8.9E+07	346	1	989	426	2	1551
267	3	13431	347	2	1991	427	2	2662
268	2	1441	348	3	5115	428	2	3773
269	2	2552	349	3	7337	429	2	4884
270	2	585	350	2	707	430	1	464
271	2	787	351	2	909	431	1	565
272	2	989	352	2	1111	432	1	666
273	4	5115	353	3	4444	433	1	767
274	4	9559	354	3	6666	434	1	868
275	5	44044	355	3	8888	435	1	969
276	15	8.8E+09	356	3	11011	436	2	1771
277	3	15851	357	2	1221	437	2	2882
278	2	1661	358	2	2332	438	2	3993
279	2	2772	359	2	3443	439	3	9119
280	4	2662	360	2	747	440	1	484
281	6	45254	361	2	949	441	1	585
282	4	13431	362	3	2662	442	1	686
283	3	2552	363	3	4884	443	1	787
284	3	4774	364	5	45254	444	1	888
285	3	6996	365	11	8.9E+07	445	1	989
286	23	8.8E+12	366	3	13431	446	2	1991
287	7	233332	367	2	1441	447	3	5115
288	2	1881	368	2	2552	448	3	7337
289	2	2992	369	2	3663	449	3	9559
290	4	2552	370	2	787	450	2	909
291	4	6996	371	2	989	451	2	1111
292	8	233332	372	4	5115	452	3	4444
293	3	2992	373	4	9559	453	3	6666
294	4	9339	374	5	44044	454	3	8888
295	501	0	375	15	8.8E+09	455	3	11011
296	7	881188	376	3	15851	456	2	1221
297	5	79497	377	2	1661	457	2	2332
298	3	3113	378	2	2772	458	2	3443
299	3	5335	379	2	3883	459	2	4554
300	1	303	380	6	45254	460	2	949
301	1	404	381	4	13431	461	3	2662
302	1	505	382	3	2552	462	3	4884
303	1	606	383	3	4774	463	5	45254
304	1	707	384	3	6996	464	11	8.9E+07
305	1	808	385	23	8.8E+12	465	3	13431
306	1	909	386	7	233332	466	2	1441
307	2	1111	387	2	1881	467	2	2552
308	1	1111	388	2	2992	468	2	3663
309	2	3333	389	3	7117	469	2	4774
310	1	323	390	4	6996	470	2	989
311	1	424	391	8	233332	471	4	5115
312	1	525	392	3	2992	472	4	9559
313	1	626	393	4	9339	473	5	44044
314	1	727	394	501	0	474	15	8.8E+09
315	1	828	395	7	881188	475	3	15851
316	1	929	396	5	79497	476	2	1661
317	2	1331	397	3	3113	477	2	2772
318	2	2442	398	3	5335	478	2	3883
319	2	3553	399	3	7557	479	2	4994
320	1	343	400	1	404	480	4	13431

Table 1 continued

481	3	2552	561	3	4884	641	1	787
482	3	4774	562	5	45254	642	1	888
483	3	6996	563	11	8.9E+07	643	1	989
484	23	8.8E+12	564	3	13431	644	2	1991
485	7	233332	565	2	1441	645	3	5115
486	2	1881	566	2	2552	646	3	7337
487	2	2992	567	2	3663	647	3	9559
488	3	7117	568	2	4774	648	5	59895
489	3	9339	569	2	5885	649	4	44044
490	6	233332	570	4	5115	650	3	4444
491	3	2992	571	4	9559	651	3	6666
492	4	9339	572	5	44044	652	3	8888
493	501	0	573	15	8.8E+09	653	3	11011
494	7	881188	574	3	15851	654	2	1221
495	5	79497	575	2	1661	655	2	2332
496	3	3113	576	2	2772	656	2	3443
497	3	5335	577	2	3883	657	2	4554
498	3	7557	578	2	4994	658	2	5665
499	3	9779	579	4	23232	659	2	6776
500	1	505	580	3	2552	660	3	4884
501	1	606	581	3	4774	661	5	45254
502	1	707	582	3	6996	662	11	8.9E+07
503	1	808	583	23	8.8E+12	663	3	13431
504	1	909	584	7	233332	664	2	1441
505	2	1111	585	2	1881	665	2	2552
506	1	1111	586	2	2992	666	2	3663
507	2	3333	587	3	7117	667	2	4774
508	2	4444	588	3	9339	668	2	5885
509	2	5555	589	6	1136311	669	2	6996
510	1	525	590	3	2992	670	4	9559
511	1	626	591	4	9339	671	5	44044
512	1	727	592	501	0	672	15	8.8E+09
513	1	828	593	7	881188	673	3	15851
514	1	929	594	5	79497	674	2	1661
515	2	1331	595	3	3113	675	2	2772
516	2	2442	596	3	5335	676	2	3883
517	2	3553	597	3	7557	677	2	4994
518	2	4664	598	3	9779	678	4	23232
519	2	5775	599	4	22022	679	4	47674
520	1	545	600	1	606	680	3	4774
521	1	646	601	1	707	681	3	6996
522	1	747	602	1	808	682	23	8.8E+12
523	1	848	603	1	909	683	7	233332
524	1	949	604	2	1111	684	2	1881
525	2	1551	605	1	1111	685	2	2992
526	2	2662	606	2	3333	686	3	7117
527	2	3773	607	2	4444	687	3	9339
528	2	4884	608	2	5555	688	6	1136311
529	2	5995	609	2	6666	689	501	0
530	1	565	610	1	626	690	4	9339
531	1	666	611	1	727	691	501	0
532	1	767	612	1	828	692	7	881188
533	1	868	613	1	929	693	5	79497
534	1	969	614	2	1331	694	3	3113
535	2	1771	615	2	2442	695	3	5335
536	2	2882	616	2	3553	696	3	7557
537	2	3993	617	2	4664	697	3	9779
538	3	9119	618	2	5775	698	4	22022
539	4	25652	619	2	6886	699	4	46464
540	1	585	620	1	646	700	1	707
541	1	686	621	1	747	701	1	808
542	1	787	622	1	848	702	1	909
543	1	888	623	1	949	703	2	1111
544	1	989	624	2	1551	704	1	1111
545	2	1991	625	2	2662	705	2	3333
546	3	5115	626	2	3773	706	2	4444
547	3	7337	627	2	4884	707	2	5555
548	3	9559	628	2	5995	708	2	6666
549	5	59895	629	4	45254	709	2	7777
550	2	1111	630	1	666	710	1	727
551	3	4444	631	1	767	711	1	828
552	3	6666	632	1	868	712	1	929
553	3	8888	633	1	969	713	2	1331
554	3	11011	634	2	1771	714	2	2442
555	2	1221	635	2	2882	715	2	3553
556	2	2332	636	2	3993	716	2	4664
557	2	3443	637	3	9119	717	2	5775
558	2	4554	638	4	25652	718	2	6886
559	2	5665	639	5	99099	719	2	7997
560	3	2662	640	1	686	720	1	747

Table 1 continued

721	1	848	801	1	909	881	7	233332
722	1	949	802	2	1111	882	2	1881
723	2	1551	803	1	1111	883	2	2992
724	2	2662	804	2	3333	884	3	7117
725	2	3773	805	2	4444	885	3	9339
726	2	4884	806	2	5555	886	8	1136311
727	2	5995	807	2	6666	887	501	0
728	4	45254	808	2	7777	888	4	66066
729	4	69696	809	2	8888	889	6	881188
730	1	767	810	1	828	890	7	881188
731	1	868	811	1	929	891	5	79497
732	1	969	812	2	1331	892	3	3113
733	2	1771	813	2	2442	893	3	5335
734	2	2882	814	2	3553	894	3	7557
735	2	3993	815	2	4664	895	3	9779
736	3	9119	816	2	5775	896	4	22022
737	4	25652	817	2	6886	897	4	48484
738	5	99099	818	2	7997	898	6	449944
739	17	5.2E+09	819	4	89298	899	17	1.3E+11
740	1	787	820	1	848	900	1	909
741	1	888	821	1	949	901	2	1111
742	1	989	822	2	1551	902	1	1111
743	2	1991	823	2	2662	903	2	3333
744	3	5115	824	2	3773	904	2	4444
745	3	7337	825	2	4884	905	2	5555
746	3	9559	826	2	5995	906	2	6666
747	5	59895	827	4	45254	907	2	7777
748	4	44044	828	4	69696	908	2	8888
749	4	68486	829	10	8.9E+07	909	2	9999
750	3	6666	830	1	868	910	1	929
751	3	8888	831	1	969	911	2	1331
752	3	11011	832	2	1771	912	2	2442
753	2	1221	833	2	2882	913	2	3553
754	2	2332	834	2	3993	914	2	4664
755	2	3443	835	3	9119	915	2	5775
756	2	4554	836	4	25652	916	2	6886
757	2	5665	837	5	99099	917	2	7997
758	2	6776	838	17	5.2E+09	918	4	89298
759	2	7887	839	4	88088	919	4	125521
760	5	45254	840	1	888	920	1	949
761	11	8.9E+07	841	1	989	921	2	1551
762	3	13431	842	2	1991	922	2	2662
763	2	1441	843	3	5115	923	2	3773
764	2	2552	844	3	7337	924	2	4884
765	2	3663	845	3	9559	925	2	5995
766	2	4774	846	5	59895	926	4	45254
767	2	5885	847	4	44044	927	4	69696
768	2	6996	848	4	68486	928	10	8.9E+07
769	4	67276	849	14	8.8E+09	929	5	293392
770	5	44044	850	3	8888	930	1	969
771	15	8.8E+09	851	3	11011	931	2	1771
772	3	15851	852	2	1221	932	2	2882
773	2	1661	853	2	2332	933	2	3993
774	2	2772	854	2	3443	934	3	9119
775	2	3883	855	2	4554	935	4	25652
776	2	4994	856	2	5665	936	5	99099
777	4	23232	857	2	6776	937	17	5.2E+09
778	4	47674	858	2	7887	938	4	88088
779	6	475574	859	2	8998	939	4	168861
780	3	6996	860	11	8.9E+07	940	1	989
781	23	8.8E+12	861	3	13431	941	2	1991
782	7	233332	862	2	1441	942	3	5115
783	2	1881	863	2	2552	943	3	7337
784	2	2992	864	2	3663	944	3	9559
785	3	7117	865	2	4774	945	5	59895
786	3	9339	866	2	5885	946	4	44044
787	8	1136311	867	2	6996	947	4	68486
788	501	0	868	4	67276	948	14	8.8E+09
789	4	66066	869	22	8.8E+12	949	8	2322232
790	501	0	870	15	8.8E+09	950	3	11011
791	7	881188	871	3	15851	951	2	1221
792	5	79497	872	2	1661	952	2	2332
793	3	3113	873	2	2772	953	2	3443
794	3	5335	874	2	3883	954	2	4554
795	3	7557	875	2	4994	955	2	5665
796	3	9779	876	4	23232	956	2	6776
797	4	22022	877	4	47674	957	2	7887
798	4	48484	878	6	475574	958	2	8998
799	6	449944	879	501	0	959	4	112211
800	1	808	880	23	8.8E+12	960	3	13431

Table 1 continued

961	2	1441	976	4	47674	991	3	3113
962	2	2552	977	6	475574	992	3	5335
963	2	3663	978	501	0	993	3	7557
964	2	4774	979	4	155551	994	3	9779
965	2	5885	980	7	233332	995	4	22022
966	2	6996	981	2	1661	996	4	46464
967	4	67276	982	2	2992	997	6	449944
968	22	8.6E+12	983	3	7117	998	17	1.3E+11
969	4	138831	984	3	9339	999	16	8.9E+09
970	3	15851	985	6	1136311	1000	1	1001
971	2	1661	986	501	0			
972	2	2772	987	4	88088			
973	2	3883	988	6	881188			
974	2	4994	989	19	9E+10			
975	4	23232	990	5	79497			

Table 1 continued

Invalid Integer		
196	2584	4439
295	2586	4492
394	2674	4494
493	2676	4529
592	2764	4582
689	2766	4584
691	2854	4619
788	2856	4672
790	2944	4674
879	2946	4709
887	2996	4762
978	3493	4764
986	3495	4799
1495	3583	4852
1497	3585	4854
1585	3673	4889
1587	3675	4942
1675	3763	4944
1677	3765	4979
1765	3853	4994
1767	3855	5078
1855	3943	5168
1857	3945	5258
1945	3995	5348
1947	4079	5438
1997	4169	5491
2494	4259	5493
2496	4349	5528

Table 2

5581	7076	8345	9088
5583	7149	8349	9147
5618	7166	8359	9164
5671	7239	8418	9168
5673	7256	8435	9178
5708	7329	8439	9237
5761	7346	8449	9254
5763	7419	8490	9258
5798	7436	8508	9268
5851	7491	8525	9327
5853	7509	8529	9344
5888	7526	8539	9348
5941	7581	8580	9358
5943	7599	8598	9417
5978	7616	8615	9434
5993	7671	8619	9438
6077	7689	8629	9448
6167	7706	8670	9507
6257	7761	8688	9524
6347	7779	8705	9528
6437	7796	8709	9538
6490	7851	8719	9597
6492	7869	8760	9614
6527	7886	8778	9618
6580	7941	8795	9628
6582	7959	8799	9687
6617	7976	8809	9704
6670	7991	8850	9708
6672	8058	8868	9718
6707	8075	8885	9777
6760	8079	8889	9794
6762	8089	8899	9798
6797	8148	8940	9808
6850	8165	8958	9867
6852	8169	8975	9884
6887	8179	8979	9888
6940	8238	8989	9898
6942	8255	8990	9957
6977	8259	9057	9974
6992	8269	9074	9978
7059	8328	9078	9988
			9999

Table 2 continued

Table 3. Domain reduction

Sum of digits	Number of iterations	Integers in set								
1	1	10								
2	1	11	20							
3	1	12	21	30						
4	1	13	22	31	40					
5	1	14	23	32	41	50				
6	1	15	24	33	42	51	60			
7	1	16	25	34	43	52	61	70		
8	1	17	26	35	44	53	62	71	80	
9	1	18	27	36	45	54	63	72	81	90
10	2	19	28	37	46	55	64	73	82	91
11	1		29	38	47	56	65	74	83	92
12	2			39	48	57	66	75	84	93
13	2				49	58	67	76	85	94
14	3					59	68	77	86	95
15	4						69	78	87	96
16	6							79	88	97
17	24								89	98
18	6									99

Let's Be Seated

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Introduction

A common question that is used as an example in combinatorics is how many ways n couples can be seated in a row with $2n$ chairs so that no couple sits together. The usual example is for 3 couples, and the number of seating arrangements is 240. At the 1995 national KME convention in Colorado the problem was solved by Causey and Mooney [1] for 4 couples, and the answer is 13824. In this paper we generalize this result with a recursive formula for n couples. We also find a general formula for how many ways exactly one couple can sit together.

Results

First, we introduce some terminology.

Definition. *We define E_{2n} to be the number of arrangements of n couples ($2n$ individuals) so that no couple sits together.*

For example, $E_2 = 0$ since with one couple (two people), they must sit side by side. Also, $E_4 = 8$ since if the couples are $\{A, a\}$ and $\{B, b\}$ then the possible seating arrangements are

$$ABab \quad AbaB \quad aBAb \quad abAB \quad BAba \quad BabA \quad bABa \quad baBA.$$

The counting argument for this total is that there are 4 choices for the first position, 2 for the second position, and 1 each for positions three and four. The product of these, $4 \cdot 2 \cdot 1 \cdot 1 = 8$, gives the number of permutations with no couple sitting together. As mentioned above, $E_6 = 240$ and $E_8 = 13824$. We leave it to the reader to list all the possible seating combinations.

A second quantity is now defined.

Definition. *We define O_{2n+1} to be the number of arrangements of n couples and one loner ($2n+1$ individuals) so that no couple sits together.*

For example, if the couple is $\{A, a\}$ and the loner is B then the possible seating combinations are ABa and aBA, so $O_3 = 2$. In the case of a single loner, say A, the only seating arrangement is A and since no couple is sitting together we have $O_1 = 1$.

To list the arrangements for O_5, we assume the couples are $\{A, a\}$ and $\{B, b\}$ and the loner is E. Then the 48 possible seating combinations for O_5 are given in Table 1, with the counting subdivided according to where E is seated.

E in position	Seating Combinations							
1	EABab	EAbaB	EaBAb	EabAB	EBAba	EBabA	EbABa	EbaBA
2	AEBab	AEbaB	aEBAb	aEbAB	BEAba	BEabA	bEABa	bEaBA
3	ABEab	AbEaB	aBEAb	abEAB	BAEba	BaEbA	bAEBa	baEBA
	ABEba	AbEBa	aBEbA	abEBA	BAEab	BaEAb	bAEaB	baEAB
4	ABaEb	AbaEB	aBAEb	abAEB	BAbEa	BabEA	bABEa	baBEA
5	ABabE	AbaBE	aBAbE	abABE	BAbaE	BabAE	bABaE	baBAE

Table 1

Note that when E is located in a fixed position, we can then place the other individuals as if E was not present in E_4 ways, and there may be some extra combinations since E could be used to split a couple in a previously disallowed arrangement.

This leads to the following definition:

Definition. *We define $O_{2n+1,i}$ to be the number of arrangements of n couples and one loner ($2n+1$ individuals) so that no couple sits together given that the loner is sitting in position i.*

For example, from Table 1 in the case of O_5, we see that

$$O_{5,1} = O_{5,2} = O_{5,4} = O_{5,5} = 8 \text{ and } O_{5,3} = 16.$$

Note that

$$O_{2n+1} = \sum_{i=1}^{2n+1} O_{2n+1,i}. \tag{1}$$

For example, $O_5 = O_{5,1} + O_{5,2} + O_{5,3} + O_{5,4} + O_{5,5} = 48$.

To calculate $O_{2n+1,i}$ we reason as follows. When a loner is sitting in position i, you may seat the n couples in the E_{2n} ways that did not require the loner to be present. For example, in Table 1, see the first row of the arrangement for O_5 with E in position 3. Then there are extra seating arrangements that may be possible by placing a couple on each side of the loner. For example, see the second row of the arrangement for O_5 with E in position 3. This combines the loner with a couple who together may be considered a single entity (a loner) among the remaining $n-1$ couples. To see this, assume the loner E is in position 3. There are $2n$ ways of arranging a couple in positions 2 and 4, since there are $2n$ individuals left to choose from for position 2, but that individual's partner must occupy position 4. If the arrangement is $\_AEa\_\_\_\_\_$, then we see that AEa can be considered a single loner in position 2 among the remaining individuals, with the corresponding number of arrangements being $O_{2n-1,2}$.

This leads to the following formulas:

$$\begin{aligned}
O_{2n+1,1} &= E_{2n} \\
O_{2n+1,2} &= E_{2n} + 2nO_{2n-1,1} \\
O_{2n+1,3} &= E_{2n} + 2nO_{2n-1,2} \\
&\ \vdots \\
O_{2n+1,2n} &= E_{2n} + 2nO_{2n-1,2n-1} \\
O_{2n+1,2n+1} &= E_{2n}.
\end{aligned}$$

Adding these all together gives

$$\begin{aligned}
O_{2n+1} &= (2n+1)E_{2n} + 2n\sum_{i=1}^{2n-1} O_{2n-1,i} \\
&= (2n+1)E_{2n} + 2nO_{2n-1}.
\end{aligned} \tag{2}$$

We can find a recursive formula for E_{2n} as follows. There are $2n$ choices for the first position. Once that person (call him A) is chosen, then his partner becomes a loner among the remaining $2n-1$ individuals, except that the loner cannot sit in position 2. Thus the number of seating arrangements is

$$E_{2n} = 2n(O_{2n-1} - O_{2n-1,1}).$$

Since $O_{2n-1,1} = E_{2n-2}$, we arrive at the formula

$$E_{2n} = 2n(O_{2n-1} - E_{2n-2}). \tag{3}$$

Alternately using formulas (2) and (3) lets us calculate arbitrarily large values of E_{2n} and O_{2n+1}. For example, let us consider E_6. We know that $E_4 = 8$ and $O_5 = 48$. Therefore, we can find E_6 by

$$E_6 = 6(O_5 - E_4) = 6(48 - 8) = 240.$$

The first several values for these quantities are:

$$\begin{array}{rr} O_1 = 1 & E_2 = 0 \\ O_3 = 2 & E_4 = 8 \\ O_5 = 48 & E_6 = 240 \\ O_7 = 1968 & E_8 = 13824 \\ O_9 = 140160 & E_{10} = 1263360 \\ O_{11} = 15298560 & E_{12} = 168422400. \end{array}$$

We have verified the values up through E_{10} by means of two independent computer programs and we verified the values up through O_9 using combinatorial trees.

Finally, if we consider a couple sitting together as a loner among the other $n-1$ couples, the couple can be seated in 2 ways and the couple can be selected in n ways. The remaining couples can be seated in O_{2n-1} ways using this couple as a loner. So the number of ways of seating n couples so that exactly one couple sits together is $2 \cdot n \cdot O_{2n-1}$. For example, the number of ways of seating 3 couples so that exactly one couple sits together is $2 \cdot 3 \cdot O_5 = 288$.

We could not find a general formula for the number of ways exactly k of the n couples could sit together.

Acknowledgements. I would like to thank Allen Moore, a co-student at Evangel College, for his diligent work on the computer. This allowed a numerical check of the formula, and he also provided an extra mind to help figure out this problem. I would especially like to thank Dr. Don Tosh, chair of the Science and Technology department at Evangel College. Without his support, guidance, and encouragement this paper would not exist. I have been extremely fortunate to have studied under him. Thank you so much Dr. Tosh for helping me find my direction in both this problem and my life. Most of all I would like to thank God, who made this problem come to life and gave us the skills used in solving it.

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Oops!

The palindrome conjecture that is effectively debunked by Christopher Brown in his article on pages 23–38 of this issue was stated as fact, with no comment, reference, or accompanying proof, in the Fall 1943 — Spring 1944 issue (Vol. 3 Nos. 1 and 2) of *The Pentagon*, on page 37.

Symmetry Groups as Scientific Tools

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Presented at the 1996 Region IV Convention and awarded "top four" status by the Awards Committee.

Introduction

Group theory is a field first encountered by most mathematics undergraduates in their third or fourth year of studies. Symmetry groups are one of the earliest, and most intuitive, examples of groups encountered in a course in group theory. Thus, it is surprising that they are so powerful in analyzing very complex theories in the physical sciences. But this is exactly the case; in group theory, as in many fields of advanced mathematics, unexpected applications are the norm. Symmetry groups pop up in some of the most amazing places, and it is both interesting and instructive to look into a few of their applications.

A Quick Introduction to Group Theory

A group is a mathematical abstraction that can be used to describe many familiar items in mathematics, from integers to matrices to permutations. A group G is a set of elements (for convenience, let us call them a, b, c, and so on) with an operation (say, $\cdot$) such that the set and operation obey the following four rules:

1) **Closure:** Given any two elements a and b in G, $a \cdot b$ is also in G.
2) **Associativity:** Given any three elements a, b, and c in G, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
3) **Existence of the Identity:** There is some element, often denoted e, in G such that for any element a in G, $e \cdot a = a$. We say that e is the identity of G.
4) **Existence of the Inverse:** For any element a in G, there is some element, usually denoted a^{-1}, such that $a \cdot a^{-1} = e$. Then a^{-1} is called the inverse of a.

That's it! On such simple foundations are built the entire complex and magnificent field of modern group theory. To get a better handle on what groups are (and how wide-spread are their applications) consider the following example [5]:

> " ... imagine you are standing on a straight road that goes on forever in front and behind you. Stand stock still; this is the identity of a group. Walk forwards a little, then a little more. But you are now where you would have been had you just walked further in the first place. So moving along a straight line exhibits the closure property. Associativity can be demonstrated by walking different distances forwards and backwards in different sequences and noting that the end result is always the same. Finally, if you walk forwards a bit then backwards to where you started you have discovered the inverse."

For another example, using symmetries instead of motion, consider the dihedral groups. The dihedral group D_n is the group of symmetries of a planar n-gon. These groups arise often in studying groups and symmetry. Figure 1 shows all the elements in D_4. There are four clockwise rotations — through 0, 90, 180, and 270 degrees — and four reflections — one horizontal, one vertical, and two diagonal. Note that $R0$ is the identity element, for it does nothing. All four of the reflections and $R180$ are their own inverses. Also, $R90$ and $R270$ are inverses of each other. The interested reader should check the other two group properties, closure and associativity, on his or her own.

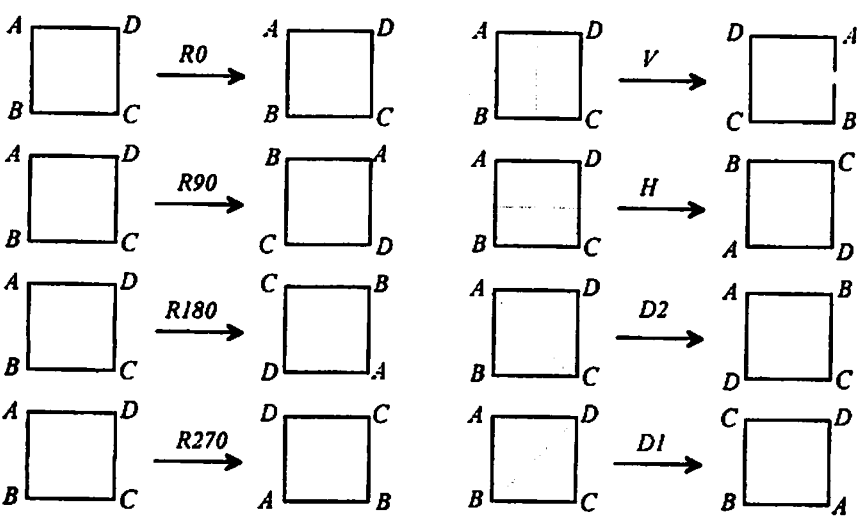

Figure 1. The dihedral group D_4

Symmetry in Physics

The first example above came from one of the earliest ties between group theory and physics, made by Sophus Lie. He articulated a link between group theory and motion (see [5], p. 48) in the form of several different continuous groups, now called, appropriately, Lie groups. Besides the already noted example, motion on a line, there are many other Lie groups that one is already familiar with; two of these are $O(n)$, the collection of transformations that preserve distance in n-dimensional space (including inversion, the transformation, often denoted P, that sends $(x_1, x_2, \ldots, x_n)$ to $(-x_1, -x_2, \ldots, -x_n)$) and $SO(n)$, the subgroup of $O(n)$ that only includes rotations. Two that are most likely unfamiliar to the reader are $U(n)$ and $SU(n)$, groups of distance-preserving transformations in an abstract space.

These last two groups are very important in the field of theoretical physics called gauge theory. Gauge theory deals with the fundamental interactions that comprise the four forces of nature. These interactions are governed by several different basic conservation laws, including the familiar law of conservation of energy. Since conservation laws can be thought of as symmetries in time, one might think that symmetry groups could help us understand these conservation laws. This is indeed the case, as Lie groups can be used to model science's basic conservation laws, e.g., formulating the conservation of angular momentum in terms of $SO(3)$, or describing the conservation of charge as a phase symmetry of the electron's wave equation (see [5], p. 48). This second example is very important to physicists, as local phase symmetry is the building block for the theories of the fundamental forces of nature: the electromagnetic force, the nuclear weak force, the nuclear strong force, and gravity. Quantum electrodynamics describes the electromagnetic force in terms of $U(1)$, a group with one generator; this generator corresponds directly to the photon (the generators of a group are its "building blocks;" using the generators and the operation of a group, one can enumerate all of the elements in a group). Similarly, the theory of weak interactions is based on $SU(2)$; this group has three generators that translate into the W^+, W^-, and Z particles. Strong nuclear forces are formulated in terms of a group with eight generators (gluons): $SU(3)$ (see [5], p. 50).

Indeed, this symmetry becomes even more important as scientists strive for the Holy Grail of theoretical physics: the Grand Unified Theory of Everything. If all of the fundamental forces are based on symmetry, then it may be possible to tie them all together in some unified field theory. Alas, this goal is as yet unattainable, as no acceptable theory of local phase symmetry has yet been advanced to describe gravity.

Symmetry in Chemistry

Another area of the physical sciences that uses group theory is spectroscopy, the study of the light given off by excited atoms and molecules. Fan Chung and Shlomo Sternberg explore this idea as it relates to a particular molecule, C_{60}, also known as the "buckyball," in their article "Mathematics and the Buckyball" [1].

Buckminsterfullerene is a highly symmetric carbon molecule that looks exactly like a soccer ball. Its rotational symmetry group is I (the symmetric rotations of an icosahedron). This group is isomorphic to A_5, the group of all permutations of 5 elements. What is being permuted in this case? The molecule C_{60} has 30 single bonds, each of which can be "connected" to a bond on the opposite side of the cage-like molecule. These connections define fifteen planes that bisect the buckyball at different angles. Among these different planes, there are five sets of mutually perpendicular planes that define a set of three coordinate axes. So we can see that each rotation of C_{60} will permute these five sets of axes ([1], p. 63). Now that we have all the rotations of buckminsterfullerene, we can get all of the symmetries by adding the inversion operation to I, getting a new, 120-element group: I_h.

To relate all of this to spectroscopy, we must appeal to group representation theory. A group representation is a mapping of every element in a group to a set of matrices such that if element r is mapped to a matrix A and element s is mapped to matrix B, then rs is mapped to AB. The simplest example of this is with a group of rotations: a rotation in 3-space through an angle α about the z-axis can be represented by the matrix

$$\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Any rotation in I can be represented by a 3×3 matrix in a similar manner. But this representation is not unique, nor is it necessarily in terms of 3×3 matrices, since for any given representation it may be possible to break it down into smaller portions. A matrix which can be broken down is said to be reducible, while one that cannot is called irreducible. This is important because "the structure of a representation is determined by its irreducible components and their multiplicity" ([1], p. 67).

Issai Schur, one of the founders of group representation theory, proved a lemma that helps us determine all of the irreducible representations of a group. The lemma is this: two irreducible representations have either nothing in common or everything in common (i.e., they are identical). The representations of a group can be denoted by the dimension of its matrices in boldface, with a prime mark used when a group has more than one distinct irreducible representation of a given dimension. For the icosahedral group I there are 5 different irreducible representations: **1**, **3**, **3′**, **4**, **5**. To incorporate the inversion operator and thus characterize I_h, we need only note that the representation of P is $-1 \cdot E$, where E is the identity matrix. Thus, any matrix representing a rotation can be turned into a rotational inversion by multiplying it by $-E$ ([1], p. 68).

Now we have all of the tools we need to describe the spectroscopy of C_{60}. Every molecule will absorb and emit light at certain characteristic frequencies. These frequencies correspond to the vibrational energies of the molecule. Based on its structure, one can determine that buckminsterfullerene has 46 distinct vibrational energies in the one-photon state, the state that describes the infrared emission spectrum of molecules. Thus, we would expect it to emit light at 46 different frequencies based on structural considerations alone. However, experiments show that there are only 4 lines ([1], p. 64).

The reason for this is what scientists call "selection rules" that determine that certain emissions are forbidden. These selection rules come straight from group theory. Emissions are determined by quantum transitions between vibrational states, and these transitions can be described with matrix multiplication and representations of the states in matrix terms. As a consequence of Schur's lemma, the only allowable transitions will be those that correspond to the 3^- representations of the vibrational state. Thus, the multiplicity of 3^- in the space of the vibrational states will give us the number of infrared lines. This multiplicity is 4, exactly the number of observed spectral lines ([1], p. 69).

The Jewel in the Crown: Mathematical Crystallography

So we now see that one way to use group theory in chemistry is to explore the basic structure of molecules and their energy characteristics. However, this is a fairly recent application of group theory. Chemists have been using groups in a different region of science for over 150 years: the field of crystallography.

The first big breakthrough in crystallography occurred when R-J Haüy dropped a friend's crystal which he was studying. When the crystal broke into identical shapes, Haüy proposed the model of a crystal that we use today: a crystal is made up of identical "building blocks" (atoms, lattice points, polyhedra, or whatever) whose shape is particular to the type of crystal ([2], p. 4).

This model restricted the shapes that these building blocks could have to be those with 2-, 3-, 4-, or 6-fold symmetries, for no other shapes can fill space ([2], p. 18). To describe these symmetries, crystallographers turn to point groups, those symmetry groups whose transformations leave at least one point fixed. Marjorie Senechal very cleverly linked these point groups to the already discussed group $O(3)$, the symmetries of a sphere ([2], p. 32). Thus, to determine the groups of rotational symmetry, one need only find the finite subgroups of $SO(3)$, a task that is not as daunting as it first appears. Using an important result in group theory — the orbit-stabilizer theorem — and the so-called crystallographic restriction (symmetries of order 5 and greater than 6 are not allowed), Marjorie Senechal cleverly classifies all of these subgroups ([2], pp. 34-36). The rotational symmetries are limited to the groups Z_n (the group of integers under addition modulo n), D_n (the nth dihedral group; see figure 1), T (the rotational symmetries of the tetrahedron), and O (the rotational symmetries of the octahedron and the cube) where n can take on the values 2, 3, 4, and 6 ([3], p. 33). To extend these groups to include all of the point groups, we merely need to combine these rotational groups with the inversion operator by taking their internal direct product (a special group theoretic operation, the details of which are not as important as the result). By doing this, we can see that there are 32 crystallographic point groups.

But crystals extend indefinitely, so we need to include some more symmetry operations to classify all of the 3-dimensional symmetries of crystals: glide reflection, screw rotation, and rotatory inversion. A glide reflection is a reflection followed by a translation; a screw rotation is a rotation followed by a translation; and a rotatory inversion is a rotation followed by an inversion. It should be clear that these operations result from composing translation with the primary point group operations, reflection and transformation. Similarly, the so-called space groups, those groups which characterize the three-dimensional symmetries of crystal lattices, can be formed by combining translations with our previously classified point groups. Carrying out this classification yields 230 space groups.

The most amazing thing about these classifications is that they took place decades before the existence of experimental methods for testing the symmetries! Thus, using symmetry, group theory was able to see where science could not. A much more dramatic example of this is the history of the discovery of quasicrystals (crystalline solids with fivefold symmetry), which can be found in the very engaging article by van Baeyer [4].

Conclusion

These examples should demonstrate very clearly that group theory is a very powerful tool for scientific inquiry. Many students, myself included, get frustrated with the lofty heights of mathematical abstraction in which group theory resides. These illustrations serve to "ground" the topics and highlight the efficacy and surprising beauty of symmetry and its close cousin, group theory.

Acknowledgements. I would like to thank Dr. Lynn Olson for providing the impetus to write this paper in his abstract algebra class, and then for suggesting that I present it. I would also like to thank Dr. Glenn Fenneman for his valuable aid in revising the paper and Dr. Augie Waltmann for helping with arrangements to present at the regional convention.

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KME WWW Update!

The national Kappa Mu Epsilon home page has been expanded and updated (thank you, Carey Hammel!). Information is now available on almost any KME topic of interest, including meetings, links to local chapter home pages, history, and much, much more! The URL is:

http://www.cmich.edu/kme.html

All local chapters with home pages should send their URL's to Arnold Hammel (a.hammel@cmich.edu) if they have not already done so. It is hard to provide a link to a page that no one knows about!

Plans are also underway for a home page for *The Pentagon*. Anyone with suggestions may send them to the editor (address on p. 2).

KME Quiz

Test your knowledge of Kappa Mu Epsilon! The answers to many of these questions can be found in this issue or past issues of *The Pentagon*. As an incentive, the first entry received from each state to correctly answer all of the questions below will receive a one-year extension of their subscription to *The Pentagon*! The names of winners will appear in the Fall 1997 issue. The deadline for submissions is July 1, 1997. Send entries to the editor (address on page 2). Have fun!

1. Name the only state with active Kappa, Mu, and Epsilon chapters of KME.

2. Name the states with active chapters with names of exactly two of Kappa, Mu, and Epsilon.

3. Which state had the first Delta chapter?

4. Which state has held the record for having the most chapters for the longest period of time?

5. Which state with five or more chapters has gone the longest without an installation of a new chapter?

6. Name the first letter of the Greek alphabet for which more active chapters of KME are named than are named for the preceeding letter.

7. What is the greatest number of currently active chapters that happened to be installed in the same year?

8. Fifty years ago, was "University" or "Teacher's College" more commonly in the name of an institution with a KME chapter?

9. What chapter is credited with writing "The Math Student Blues," a song which appeared in *The Pentagon* some time in the 1940's?

10. Name the only institution with an active chapter of KME whose name does not include either "University" or "College."

11. Name the only city with three active chapters of KME.

12. Which chapter has initiated the most total members into KME?

13. Which active chapter is located the furthest north?

14. Which active chapter is located the furthest east?

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before July 1, 1997. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1997 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

PROBLEMS 500–504

Problem 500. Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

The nth triangular number is given by $t_n = n(n+1)/2$ where n is a positive integer. Prove that there are an infinite number of triangular numbers which can be expressed as a sum of two distinct triangular numbers.

Problem 501. Proposed by Charles Ashbacher, Cedar Rapids, Iowa.

Given any integer $n > 1$, the Smarandache function $S(n)$ is the smallest integer m such that n divides $m!$. The formula for the determinant of a 2×2 matrix is well known. Prove that there exists an infinite set of 4-tuples (a_1, a_2, a_3, a_4) such that:

a) all a_i are composite and greater than 2;

b) $a_i \neq a_j$ for $i \neq j$; and

c) we have

$$\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix} = \begin{vmatrix} S(a_1) & S(a_2) \\ S(a_3) & S(a_4) \end{vmatrix}.$$

Problem 502. Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

Prove that the following procedure always yields a Pythagorean triangle. (1) Add together either the reciprocals of two consecutive even integers or the reciprocals of two consecutive odd integers. (2) In the reduced form of the resulting fraction, the numerator and denominator are the legs of a Pythagorean triangle and the hypotenuse is 1+ the denominator if the denominator is even and the hypotenuse is 2+ the denominator if the denominator is odd.

Problem 503. Proposed by C. Bryan Dawson, Emporia State University, Emporia, Kansas.

Using only a compass and an unmarked straightedge, construct the orthocenter, circumcenter, centroid, and the nine-point circle of an arbitrary triangle using the compass six or fewer times. The drawing of the nine-point circle is included as one of the uses of the compass.

Problem 504. Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

If A, B, and C are the angles of a triangle, prove that

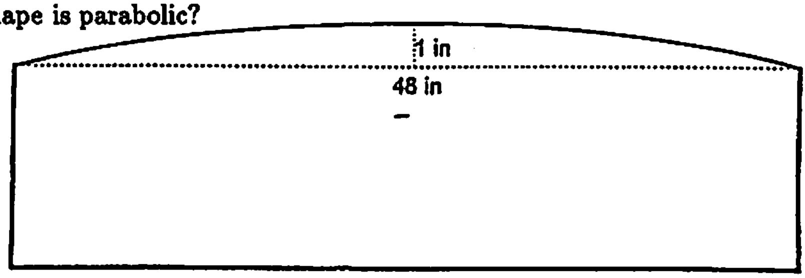
$$2\cos A\cos B\cos C = 1 - \cos 2A - \cos 2B - \cos 2C.$$

Please help your editor by submitting problem proposals.

SOLUTIONS 490–494

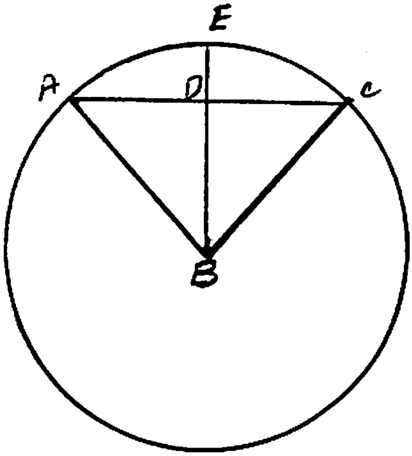
Problem 490. Proposed by Troy D. Van Aken, University of Evansville, Evansville, Illinois.

Suppose a person wants to cover a pickup bed that is four feet wide with a flexible plastic cover so that the cover rises one inch in the center (see figure below). If one assumes that the shape of the cover is circular, how large should the piece of plastic be cut? What if one assumes that the shape is parabolic?



Solution by Ali Ghorashi, University of Southern Louisiana, Lafayette, Louisiana (revised by the editor).

Solution to part (a). Assume that the shape of the cover is circular. Then we have the following figure:

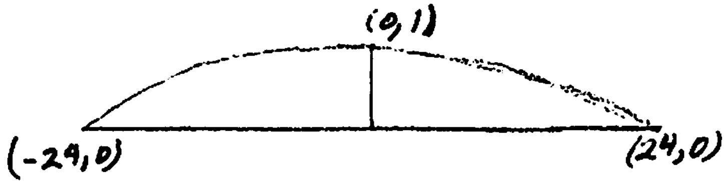


Let BE be a radius which is perpendicular to chord AC. Let it cut the circular cover at E and chord AC at D. Then since $BA = BE = BC$, triangles ABD and BCD are congruent and $AD = DC$. Then since $DE = 1$, $BD = BC - 1$. By the Pythagorean Theorem, $(BC-1)^2 + CD^2 = BC^2$, and since $CD = 24$, we have

$$BC = (1/2)(CD^2 + 1) = 577/2 = 288.5.$$

But angle $DBC = \sin^{-1}(CD/BC) = \sin^{-1}(24/288.5)$. Then the length of the arc AEC is $2 \cdot BC \cdot \sin^{-1}(24/288.5) = 48.05553628$ inches.

Solution to part (b). Assume that the shape of the cover is parabolic. Then we have the following figure:



Given the fact that the parabola passes through the three points $(0,1)$, $(-24,0)$ and $(24,0)$ and that the general equation for a parabola is given by $ax^2 + bx + c = y$, one can easily show that $a = -1/576$, $b = 0$ and $c = 1$

so that the equation for the parabolic cover is

$$y = -(x/24)^2 + 1.$$

Then since $y' = -(x/288)$, the standard arc length formula gives

$$I = \int_{-24}^{24} \sqrt{1 + (x/288)^2}\, dx = 48.05549783 \text{ inches.}$$

Also solved by: Clayton W. Dodge, University of Maine—Orono, Orono, Maine. One incorrect solution was received.

Problem 491. Proposed jointly by Sammy and Jimmy Yu, special students at the University of South Dakota, Vermillion, South Dakota.

Evaluate the integral

$$I = \int \frac{\sqrt{m - x^n}}{x^{1+n/2}}\, dx.$$

Solution by Martha Degen, Alma College, Alma, Michigan.

Consider the case $n = 0$. Then the integral I becomes

$$I = \int \frac{\sqrt{m-1}}{x}\, dx = \sqrt{m-1}\ln|x| + C.$$

When $n \neq 0$, let $u = \sqrt{m - x^n}$ and $dv = x^{-(1+n/2)}\, dx$ and integrate by parts. Then $du = 1/2(m - x^n)^{-1/2}\left(-nx^{n-1}\right)\, dx$ and $v = -2x^{-n/2}/n$. Then I becomes

$$\begin{aligned} I &= \frac{-2\sqrt{m - x^n}}{n\sqrt{x^n}} - \int \frac{x^{-1+n/2}}{\sqrt{m - x^n}}\, dx \\ &= \frac{-2\sqrt{m - x^n}}{n\sqrt{x^n}} - \frac{2}{n}\int \frac{dw}{\sqrt{m - w^2}} \end{aligned}$$

where $w = x^{n/2}$ and $dw = (n/2)x^{n/2-1}\, dx$, so that $x^{n/2-1}\, dx = (2/n)dw$. Now I becomes

$$I = \frac{-2\sqrt{m - x^n}}{n\sqrt{x^n}} - \frac{2}{n}\sin^{-1}\left(w/\sqrt{m}\right) + C$$

or

$$I = \frac{-2\sqrt{m - x^n}}{n\sqrt{x^n}} - \frac{2}{n}\sin^{-1}\left(x^{n/2}/\sqrt{m}\right) + C.$$

Also solved by: Tamara Adams and Timothy Sipka (jointly), Alma College, Alma, Michigan; Clayton W. Dodge, University of Maine—Orono, Orono, Maine; Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin and the proposers. One incorrect solution was received.

Problem 492. Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.

The Fibonacci numbers are defined by $F_0 = 0$, $F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for $n = 0, 1, 2, \ldots$. The Lucas numbers are defined by $L_0 = 2$, $L_1 = 1$ and $L_{n+2} = L_{n+1} + L_n$ for $n = 0, 1, 2, \ldots$. Show that

$$\sum_{i=0}^{n} F_i^2 = \begin{cases} \frac{1}{5}(L_{2n+1} + 1) \text{ if } n \text{ is odd} \\ \frac{1}{5}(L_{2n+1} - 1) \text{ if } n \text{ is even.} \end{cases}$$

Solution by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

First we establish the following lemmas. Let $\alpha = (1 + \sqrt{5})/2$ and $\beta = (1 - \sqrt{5})/2$. Then $\alpha + \beta = 1$ and $\alpha\beta = -1$.

Lemma 1. $5F_i^2 = L_i^2 - 4(-1)^i$.

Proof. Since $F_i = (\alpha^i - \beta^i)/\sqrt{5}$ and $L_i = \alpha^i + \beta^i$, we have $5F_i^2 = (\alpha^i - \beta^i)^2 = (\alpha^i + \beta^i)^2 - 4(\alpha\beta)^i = (\alpha^i + \beta^i)^2 - 4(-1)^i$.

Lemma 2. $\sum_{i=1}^{n} L_i^2 = L_n L_{n+1} - 2$.

Proof. Note $\sum_{i=1}^{1} L_i^2 = L_1^2 = 1^2 = 1$ and $L_1 L_2 - 2 = 3 - 2 = 1$. Thus the desired result holds for $n = 1$. Next assume that

$$\sum_{i=1}^{k} L_i^2 = L_k L_{k+1} - 2,$$

where k is an arbitrary fixed positive integer. Then

$$\begin{aligned} \sum_{i=1}^{k+1} L_i^2 &= \sum_{i=1}^{k} L_i^2 + L_{k+1}^2 = L_k L_{k+1} - 2 + L_{k+1}^2 \\ &= L_{k+1}(L_k + L_{k+1}) - 2 = L_{k+1} L_{k+2} - 2. \end{aligned}$$

Thus

$$\sum_{i=1}^{n} L_i^2 = L_n L_{n+1} - 2$$

for each positive integer n by mathematical induction.

Lemma 3. $L_n L_{n+1} = L_{2n+1} + (-1)^n$.

Proof. Since $\alpha + \beta = 1$, we have $L_n L_{n+1} = (\alpha^n + \beta^n)(\alpha^{n+1} + \beta^{n+1}) = (\alpha^{2n+1} + \beta^{2n+1}) + (\alpha\beta)^n(\alpha+\beta) = L_{2n+1} + (-1)^n$.

Proof of main result. By Lemma 1, Lemma 2, and Lemma 3,

$$\begin{aligned}\sum_{i=0}^{n} F_i^2 = \sum_{i=1}^{n} F_i^2 &= \frac{1}{5}\sum_{i=1}^{n} L_i^2 - \frac{4}{5}\sum_{i=1}^{n}(-1)^n \\ &= \frac{1}{5}(L_n L_{n+1} - 2) - \frac{4}{5}\sum_{i=1}^{n}(-1)^n \\ &= \frac{1}{5}(L_{2n+1} + A),\end{aligned}$$

where $A = ((-1)^n - 2 - 4\sum_{i=1}^{n}(-1)^n)$. Now if n is odd, $A = -1 - 2 - 4(-1) = 1$. If n is even, $A = 1 - 2 - 4(0) = -1$. Therefore,

$$\sum_{i=0}^{n} F_i^2 = \begin{cases} \frac{1}{5}(L_{2n+1} + 1) \text{ if } n \text{ is odd} \\ \frac{1}{5}(L_{2n+1} - 1) \text{ if } n \text{ is even.} \end{cases}$$

Also solved by: Clayton Dodge, University of Maine—Orono, Orono, Maine; Carl Libis, University of Southwestern Louisiana, Lafayette, Louisiana and the proposer.

Problem 493. Proposed jointly by C. Bryan Dawson and Sam Snyder, Emporia State University, Emporia, Kansas.

Consider the decimal expansion of $\frac{\sin x}{x}$, where $x = 10^{-k}$ for some nonnegative integer k where x is expressed in radians. Show that the first $6k+3$ digits of the decimal expansion of $\frac{\sin x}{x}$ are given by $9_{2k}83_{2k}416_{2k}$, where r_i denotes i repetitions of the digit r.

Solution by Clayton Dodge, University of Maine—Orono, Orono, Maine.

Divide each term of the Maclaurin series for $\sin x$ by x to obtain

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots.$$

Setting $x = 10^{-k}$ we get

$$\frac{\sin x}{x} = 1 - \frac{10^{-2k}}{6} + \frac{10^{-4k}}{120} - \frac{10^{-6k}}{5040} + \cdots$$

$$
\begin{aligned}
&= 1 - (.166'6')10^{-2k} + (.00833'3')10^{-4k} \\
&\quad - (.0001984'126984')10^{-6k} + \cdots \\
&= 1 - (1 - .833'3')10^{-2k} + (.000833'3')10^{-4k} \\
&\quad - (.0001984'126984')10^{-6k} + \cdots \\
&= (1 - 10^{-2k}) + (.833'3')10^{-2k} + (.00833'3')10^{-4k} \\
&\quad - (.0001984'126984')10^{-6k} + \cdots ,
\end{aligned}
$$

which is the desired form since $1 - 10^{-2k} = .9_{2k}$ and $(.833'3')10^{-2k} = .0_{2k}833'3'$. Also $.0_{4k}33'3' + (.00833'3')10^{-4k} = (.34166'6')10^{-4k}$, placing exactly $2k$ threes between the 8 and the 4. In a similar manner one can show that the next term leaves [exactly] $2k$ sixes before altering the digits. In this solution the notation $'abcd\ldots'$ denotes an infinitely repeated block of digits.

Also solved by: the proposers.

Problem 494. Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

Let x and y be integers. Prove that if $3x^2 - 2y^2 = 1$, then $x^2 - y^2$ is divisible by 40.

Solution by Clayton Dodge, University of Maine—Orono, Orono, Maine.

First we solve the given Diophantine equation

$$3x^2 - 2y^2 = 1. \tag{1}$$

By trial with the aid of a calculator or computer program, one finds that the first three solutions in positive integers of equation (1) are $(x, y) = (1, 1)$, $(9, 11)$ and $(89, 109)$ and that these are the only solutions in positive integers with $x < 100$. Assuming that $x_{n+1} = ax_n + by_n$ for some integers a and b, one determines from the three solutions listed above that these formulas produce $(x_4, y_4) = (881, 1079)$ and $(x_5, y_5) = (8721, 10681)$ as the next two solutions which can be easily verified. By taking $(x_1, y_1) = (1, 1)$ and $3x_{n+1}^2 - 2y_{n+1}^2 = 1$ for some pair of positive integers, and letting

$$x_{n+1} = 5x_n + 4y_n \tag{2}$$

and

$$y_{n+1} = 6x_n + 5y_n, \tag{3}$$

we have

$$
\begin{aligned}
3x_{n+1}^2 - 2y_{n+1}^2 &= 3(5x_n + 4y_n)^2 - 2(6x_n + 5y_n)^2 \\
&= 3x_n^2 - 2y_n^2 = 1.
\end{aligned}
$$

Hence by mathematical induction all integer pairs (x_n, y_n) where x_n and y_n are as defined in (2) and (3) respectively provide solutions of equation (1).

For sake of completeness, one can solve the recursion relations (2) and (3) to obtain

$$x_n = 5x_{n+1} - 4y_{n+1} \tag{4}$$

and

$$y_n = -6x_{n+1} + 5y_{n+1}. \tag{5}$$

It now follows that if there is any other solution in positive integers to equation (1), then one can apply (4) and (5), repeatedly if necessary, to obtain smaller solutions of equation (1). Eventually one would obtain a solution to equation (1) with $x < 100$. But since there are no solutions of equation (1) with $x < 100$ other than those already found above, the recursion formulas (2) and (3) give all solutions to equation (1) since $(x_1, y_1) = (1, 1)$.

Observe that in all solutions of equation (1), both x and y are odd. Hence both $x + y$ and $x - y$ are even. Thus it suffices to show that either $x + y$ or $x - y$ is divisible by 20. Note that $x_1 - y_1 = 0$ and is divisible by 20.

First suppose that $x_n + y_n = 20u$ for some integer u. Then by (2) and (3), $x_{n+1} - y_{n+1} = (5x_n + 4y_n) - (6x_n + 5y_n) = -(x_n + y_n) = -20u$. Next suppose that $x_n - y_n = 20v$ for some integer v. Then similarly $x_{n+1} + y_{n+1} = (5x_n + 4y_n) + (6x_n + 5y_n) = 11x_n + 9y_n = 11x_n + 9y_n + (20v - x_n + y_n) = 20\left((x_n + y_n)/2 + v\right)$. This shows that if either $x_n - y_n$ or $x_n + y_n$ is divisible by 20, then either $x_{n+1} + y_{n+1}$ or $x_{n+1} - y_{n+1}$ is likewise divisible by 20. This completes the proof.

Also solved by: the proposer.

Algebra Christmas Carol

To the tune of *Jingle Bells* (verse and chorus)

Dashing through group theory, in a one-semester course.
Over fields we go, laughing 'till we're hoarse!
We study commutative rings, making spirits bright.
Oh what fun it is to sing an Algebra song tonight!

Oh, Algebra! Algebra! Algebra all the way!
Oh what fun it is to take in a one-semester course!
Algebra! Algebra! Algebra all the way!
Oh what fun it is to take in a one-semester course!

—the editor.

Report of the Region IV Convention

Prepared by Mary Sue Beersman, MO Eta, Region IV Director

The 1996 Region IV convention of Kappa Mu Epsilon was held April 26–27, 1996 at Washburn University in Topeka, Kansas with Kansas Delta serving as the host chapter. Approximately eighty-five members attended representing fourteen chapters.

On Friday evening a registration/Casino Night was held at the Memorial Union. A lot of colorful money changed hands (it was pink, green, and blue!).

On Saturday morning, after welcomes by Dr. Aaron Stucker, Chair of the Department of Mathematics and Statistics at Washburn University, and by Daniel Wessel, President of Kansas Delta, two separate sessions of student presentations began. The two sessions were needed since thirteen papers were to be presented. Presiders at the sessions were Dr. Billy Milner and Dr. Ken Ohm. In Session A the following student papers were presented:

Progression of Chaos Theory
Mary Kay Vaske, SD Alpha, Northern State University

Let's Be Seated
Joshua James Weber, MO Theta, Evangel College

Real Division Algebras and Dickson's Construction
Heather Golliher, IA Alpha, University of Northern Iowa

Investigations of Biological Computers and Graph Theory
Kimberly Bell, KS Delta, Washburn University

Pinochle Probability
Crystal Vacura, KS Epsilon, Fort Hays State University

Higher Order Niven Numbers
Lyle Bertz, MO Beta, Central Missouri State University

Coding Theory
Sherry Brennon, KS Alpha, Pittsburg State University

The following student presentations were given in Session B:

Symmetry Groups as Scientific Tools
Andy Miller, IA Delta, Wartburg College

Better Understanding the Simplex Method
Mark Garton, MO Kappa, Drury College

Analyzing Atonal Music
Carmen Witten, KS Epsilon, Fort Hays State University

Tinkering with the Quaternion
Dawn M. Weston, KS Gamma, Benedictine College

Molecules and Their Symmetries: Determining the Hybridization of a Central Atom Using Point Groups
Suzanne Shontz, IA Alpha, University of Northern Iowa

Unusual Methods of Representing Triangles
Richard Williamson, Mo Iota, Missouri Southern State College

After lunch in the Memorial Union Dr. Ron Wasserstein gave an interesting and entertaining talk entitled "What a Coincidence" and the awards ceremony was held. The top two presenters (in alphabetical order) in each session were announced as Lyle Bertz, Missouri Beta and Joshua James Weber, Missouri Theta for Session A and Andy Miller, Iowa Delta and Carmen Witten, Kansas Epsilon for Session B. In addition to chapters represented by student presentations, MO Eta, MO Lambda and KS Beta were present.

Report of the Region V Convention

Prepared by Donna K. Hafner, CO Delta

The Region V Convention was held April 19–20, 1996, in Grand Junction, Colorado, hosted by CO Delta at Mesa State College. Being at the farthest northwest point in the region, we were joined by only two other chapters — Colorado Gamma (Fort Lewis College) and New Mexico Alpha (University of New Mexico). Convention activities were planned and coordinated by Clifford Britton, CO Delta Faculty Sponsor, and Donna K.

Hafner, Corresponding Secretary.

This convention was held jointly with the Rocky Mountain and Intermountain Section Meetings of The Mathematical Association of America. KME participants were invited to attend three significant MAA addresses as well as many of the MAA papers.

Dr. Gustavus Simmons, Rothschild Professor of Mathematics at Cambridge and Retired Director of National Security Studies at Sandia National Laboratories, opened the meetings on Friday with "Secrets & Geometry." His address was both delightful and educational with some surprising results! Following the presentation of numerous MAA papers, Dr. Fred Adler, Professor of Mathematical Biology, University of Utah, addressed the group with "Equalization and Optimization by Colonies of Foraging Ants," a problem illustrating some basic principles of ecology and calculus. A social hour followed by a joint KME/MAA banquet was held on Friday evening. Dr. Kenneth Ross, the President of MAA, University of Oregon, presented the banquet address.

KME members gathered for a Saturday breakfast and then attended the colorful and intriguing address "Patterns, Symmetry, & Chaos" by Dr. Martin Golubitsky, Cullen Distinguished Professor of Mathematics, University of Houston. The following KME student papers were then presented.

Functions of Bounded Variation
Darren Gemoets, CO Gamma, Fort Lewis College

Some Occurrences of the Fibonacci Sequence
Natisha Kimminau, CO Delta, Mesa State College

Stability Considerations for Numerical Methods
Johnny Snyder, NM Alpha, University of New Mexico

Each student presenting a paper received a Certificate of Recognition, a cash award, and a two-year extension of his/her subscription to *The Pentagon*. Fruit, cheese, cookies, and beverages were served during the awards and brief general information session prior to adjournment.

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Kappa Mu Epsilon News

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy KME events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, Missouri 64801.

CHAPTER NEWS

AL Zeta Chapter President — Scott A. Matthews

Birmingham-Southern College, Birmingham 31 actives

Other chapter officers for 1996–97: James M. Corder, vice president; Mary Jane Turner, corresponding secretary; Raju Sriram, faculty sponsor. Secretary and treasurer are to be elected later.

CO Beta Chapter President — Michael Colagrosso

Colorado School of Mines, Golden 15 actives, 7 associates

Tracy Gardner, chapter member and past president, wrote regular articles for the student newspaper, each of which included an interesting math problem. Several members volunteered time to assist with the "Math Counts" competition. Members also provided free tutoring at the local schools for students K–12. Bi-weekly problem sessions were held featuring various Putnam competition problems. T-shirts bearing the slogan "KME because there are only three kinds of people, those who can count and those who can't" were designed and sold. Other chapter officers for 1996–97: Tiffany Maier, vice president; Dawn Pribyl, secretary; Grant Erdmann, treasurer; Ardel Boes, corresponding secretary/faculty sponsor.

CO Gamma

Fort Lewis College, Durango 25 actives

Ten new members were inducted at an Initiation Pizza Party held in March. Chapter President Darren Gemoets presented a paper at the Region V Convention at Mesa State College in April. Student officers for 1996–97 will be elected in the fall. Faculty officers: Richard A. Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

CO Delta

Chapter President — Natisha R. Kimminau

Mesa State College, Grand Junction

24 actives, 7 associates

CO Delta held its seventh annual initiation banquet and ceremony at the Holiday Inn on March 27, 1996. Thirty-nine members, initiates, and guests were in attendance. Six students and one faculty member were initiated. KME pins and certificates were presented to the newly initiated members at a meeting in early May. On April 19–20, the chapter hosted the 1996 KME Region V Convention. Nineteen members registered, including members from NM Alpha and CO Gamma. The meeting was held jointly with the Rocky Mountain and Intermountain Sections of the MAA. Other chapter officers for 1996–97: Deborah J. McCurley, vice president; Robin L. O'Connor, secretary; Tassie S. Medlin, treasurer; Donna K. Hafner, corresponding secretary; Kenneth Davis, faculty sponsor.

GA Alpha

Chapter President — Tonja Davis

West Georgia College, Carrollton

20 actives, 7 associates

On May 29, 1996, the GA Alpha Chapter held its annual initiation meeting at which time seven new pledges were inducted into KME. At the reception following the initiation, the names of the mathematics scholarship and award winners for 1996–97 were announced. The winners, all KME members, are as follows: Crider Awards — Stephanie Parker and Kristy Williams; Burson Award — Mark Thomas; Cooley Scholarship and Whatley Scholarship — Tonja Davis. Other chapter officers for 1996–97: Stephanie Parker, vice president; Michael Jumper, secretary; Kristy Williams, treasurer; Joe Sharp, corresponding secretary/faculty sponsor; Mark Faucette, faculty sponsor.

IL Beta

Chapter President — Sarah Schuette

Eastern Illinois University, Charleston

Meetings were held throughout the semester. February activites included the ICTM Math Contest and the ICTM Conference. Chapter initiation of new members was held in April, as was the KME Honors Banquet. The KME picnic, held jointly with Math Club, took place April 14. Other chapter officers for 1996–97: Lisa Stranz and Justin Large, vice presidents; Susan Schmid, secretary; Carrie Webb, treasurer; Lloyd Koontz, corresponding secretary/faculty sponsor.

IL Delta

Chapter President — Mike Mravle

College of St. Francis, Joliet

25 actives, 11 associates

Other chapter officers for 1996–97: Heather McNulty, vice president; Toni Dactilidis, secretary; Meg McAleer, treasurer; Rick Kloser, corresponding secretary/faculty sponsor.

IL Theta Chapter President — Jennifer Larson

Benedictine University, Lisle 16 actives, 7 associates

Chapter members, in conjunction with Math Club, sponsored an alumni career night. Featured speakers were recent math graduates who discussed their current career activities. The spring induction ceremony was held in conjunction with the annual Math/Computer Science Awards Banquet. Other chapter officers for 1996–97: Geoffrey Pacana, vice president; Donna Snaidauf, secretary; Mary Beth Dever, corresponding secretary/faculty sponsor.

IN Alpha Chapter President — Nikki Erdelyi

Manchester College, North Manchester 11 actives, 4 associates

The 1996 banquet of the Manchester College Department of Mathematical Sciences and the IN Alpha Chapter of KME was held on Thursday, May 2, at the Ponderosa Restaurant in Warsaw. Three graduating seniors, Craig Strong, Joseph Vairo, and Rebekah Ousley, were honored at the event, and four new members were inducted into KME. The after-dinner address was given by Curt Beery, a past president of the chapter and a 1992 alumnus of the college. His topic was "Mathematics and My Career: From MC to MMA." Curt is currently employed as a programmer analyst at Mennonite Mutual Aid in Elkhart. Other chapter officers for 1996–97: Sidath Senadheera, vice president; Jennifer Bowman, secretary; Ron Whybrew, treasurer; Stan Beery, corresponding secretary; Andrew Rich, faculty sponsor.

IN Gamma Chapter President — Mary K. Stevens

Anderson University, Anderson 10 actives, 6 associates

Other chapter officers for 1996–97: Rhonda J. Merrill, vice president; Jeffrey R. Smith, secretary/treasurer; Stanley L. Stephens, corresponding secretary/faculty sponsor.

IA Alpha Chapter President — Matthew D. Schafer

University of Northern Iowa, Cedar Falls 31 actives

Students presenting papers at local KME meetings included Heather Golliher, who spoke on "Real Division Algebras and Dickson's Construction," and Sid Bos, whose topic was "The Assignment Problem: Decision vs Search." Two new members were initiated at the spring initiation banquet. Amber Grotjohn gave the banquet address. Her topic was "Wind Power." A highlight of the spring semester was the KME Region IV Convention in Topeka, KS. Heather Golliher, Andy Miller from Wartburg College in Waverly, Andy Schafer and Suzanne Shontz, along with UNI faculty John E. Bruha and John S. Cross, made the round trip in a brand new Dodge rental van. Heather presented her paper, "Real Division Algebras," and

Suzanne presented a paper entitled "Molecules and their Symmetries" to the convention. KME members assisted with the telethon, Science and Technology Day, and Mathematics Awareness Week. Other 1996–97 chapter officers: Mary E. Pittman, vice president; Suzanne M. Shontz, secretary; Amber Grotjohn, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

IA Gamma Chapter President — Heather Schott

Morningside College, Sioux City 8 actives, 3 associates

Other 1996–97 chapter officers: James Nicolaisen, vice president; Jared Ellwein, secretary; Heather Kelly, treasurer; Douglas Swan, corresponding secretary/faculty sponsor.

IA Delta Chapter President — Joy Trachte

Wartburg College, Waverly 54 actives, 2 associates

The first meeting of the new year was a party held at the home of faculty member Dr. Robin Pennington. Members brought board and card games; faculty members provided a variety of edible goodies. On February 21 KME met with other interested students and faculty to suggest renovation needs and plans for Becker Hall of Science. Lists of ideas and needs developed through the end of March were then shared with the college administration. On March 15–16, the chapter co-hosted thirty-six high school participants in the 1996 Explorations in Mathematical Sciences event. At the annual initiation banquet, held March 16, seventeen new members were welcomed into the chapter. Daniel Nettleton, former chapter president and Ph.D. candidate at the University of Iowa, gave the banquet address. He spoke about applications of statistics and how mathematics is used in statistical work. Members met with the computer science and physics clubs in early May to celebrate the year with a picnic and volleyball competition. Other 1996–97 chapter officers: Shilah Lybeck, vice president; Richard Kloster, secretary; Christopher Judson, treasurer; August Waltmann, corresponding secretary; Lynn Olson, faculty sponsor.

KS Alpha Chapter President — Kathleen Denney

Pittsburg State University, Pittsburg 63 actives

Spring Semester activities began with a pizza party and initiation in February for eight new members. Following the initiation, guest speaker Dr. David Surowski from Kansas State University presented an interesting talk on "The Fundamental Theorem of College Algebra." The regular March meeting program was given by Sherry Brennon. She gave a trial run of her paper that was subsequently presented at the Regional Convention in April. Her topic was "Coding Theory." Two additional new members who were unable to attend the February initiation were initiated at this meeting. The April meeting featured a presentation by Dr. Elwyn Davis, Mathematics Department Chairman, on "Analytical Hierarchical Processes and Decision Making." Six students and one faculty member attended the Region IV Convention at Washburn in late April. The chapter assisted the Mathematics Department faculty in administering and grading tests given at the annual Math Relays on April 23. Several members also worked on the Alumni Association's Annual Phon-a-thon. The final meeting for the semester was the traditional homemade ice cream and cake social held at Professor Gary McGrath's home. Officers for the coming school year were also elected. The annual Robert M. Mendenhall awards for scholastic achievement were presented to Natalia Ivanova, Bethany Schnackenberg, and Dan White. Dr. Harold Thomas, KS Alpha corresponding secretary for the past 29 years, informed the group that this was his final meeting as corresponding secretary. Dr. Thomas is looking forward to phased retirement beginning with the '96-'97 fall semester. His role as corresponding secretary will be ably filled by Dr. Cynthia Woodburn. Other 1996–97 chapter officers: Matthew Jackson, vice president; Heather Seybold, secretary/treasurer; Bobby Winters, faculty sponsor.

KS Beta — Chapter President — Brenda Sloop

Emporia State University, Emporia — 33 actives, 11 associates

Other chapter officers for 1996–97: Andy Applegarth, vice president; Ruth Dale, secretary; Shannon Decker, treasurer; Connie S. Schrock, corresponding secretary; Larry Scott, faculty sponsor.

KS Gamma — Chapter President — Erik J. Kurtenbach

Benedictine College, Atchison — 15 actives, 16 associates

Charter member Mary Margaret Downs Intfen and her husband George were honored by the college with the school's highest honor on February 17 at the Scholarship Dinner and Auction. On March 3 Jeff Blanchard, Chad Eddins, and Dan Kenney were initiated into KS Gamma and twelve new associate members were received. Initiation at Marywood was followed by a chili supper. Three students presented their research at the March 27 Discovery Day on campus. Dawn Weston spoke on "Tinkering with the Quaternion," Seth Spurlock talked about "Exploring $Z_5[i]$," and Jimmy Wang demonstrated his "Personal Financial Manager." The Math Ed students demonstrated their Home Pages as part of the day's activites and KS Gamma sponsored Richard Delaware of UMKC as a guest speaker. On April 24 the group gathered for dinner at Paolucci's and presented key rings to the graduates. Named as Sister Helen Sullivan Scholarship recipients at the April 25 Honors Convocation were: Jeff Blanchard, Chad Eddins, Christie Engelbert, Dan Kenney, Erik Kurtenbach, Seth Spurlock, and Dawn Weston. President Dawn Weston presented her research paper at the Regional Meeting at Washburn. Erik Kurtenbach served as a judge for the other section of papers. A final speaker for the year was co-sponsored by KS Gamma and the Physics Club. Faculty sponsor Jo Ann Fellin, OSB, will be on sabbatical during the 1996–97 year. Other chapter officers for the coming year: Chad W. Eddins, vice president; Dawn M. Weston, secretary; Christine M. Engelbert, treasurer; Eric Schultz, StuGo Representative; Seth Spurlock, *Exponent* Editor; Linda Herndon, OSB, corresponding secretary/faculty sponsor.

KS Delta Chapter President — Mandy Chester

Washburn University, Topeka

Much of the spring activity focused on hosting the Region IV KME Convention. The Convention was attended by approximately 85 people from 14 chapters. KS Delta member Kim Bell was one of the 13 student paper presenters. Chapter president Dan Wessel presided over several of the activities and faculty sponsor Dr. Ron Wasserstein gave the luncheon address. Other spring activities included an April induction of four new members and a joint picnic in May with the Washburn Mathematics Club. Other 1996–97 chapter officers: Kevin Hennessy, vice president; Jim Stinson, secretary/treasurer; Allan Riveland, corresponding secretary; Donna LaLonde and Ron Wasserstein, faculty sponsors.

KY Alpha Chapter President — Lynne Brosius

Eastern Kentucky University, Richmond 16 actives, 18 associates

Once again, the semester began with the sale of floppy discs as a chapter fund-raising activity. The agenda of the February meeting, the first meeting of the semester, included discussion on hosting and/or attending a regional convention. It was decided not to host a regional convention this year. The March initiation ceremony for new members featured a talk by Dr. Margaret Yoder entitled "Braid Equivalences," and was followed by a party in the student center. In April, Laura Melius from the Career Development and Placement Office gave a presentation regarding the software system Résumé Expert that is available to students and alumni who are job seeking. New officers for the coming year were elected at the May meeting. Other 1996–97 officers: Kevin Zachary, vice president; Heather Sadler, secretary; Elizabeth Barrett, treasurer; Patrick Costello, corresponding secretary/faculty sponsor.

KY Beta Chapter President — Timothy D. Wilson

Cumberland College, Williamsburg 27 actives

On February 27 the KY Beta Chapter initiated seven student members and one faculty member. Outgoing president Tessie Black presided over the initiation ceremony, banquet, and related activies. Last year's inductees, as well as graduating seniors, were also recognized at the event. In April, members assisted in hosting the regional high school math contest held annually at Cumberland College. The entire department, including the Math and Physics Club and the KY Beta Chapter, enjoyed a picnic at Briar Creek Park in early May. On the first day of finals, a pizza party was held immediately following the calculus exams. Other 1996–97 chapter officers: Story A. Robbins, vice president; Melynda K. Hazelwood, treasurer; Jonathan E. Ramey, corresponding secretary; John A. Hymo, faculty sponsor.

MD Alpha — Chapter President — Shannon Spicer

College of Notre Dame of Maryland, Baltimore — 11 actives, 7 associates

During the spring semester, chapter members heard a presentation by Ana Casas, who reported on her engineering internship with Baltimore Gas and Electric Company. In March, Dr. Horace Russell, Associate Dean, College of Engineering of University of Maryland at College Park, spoke on the Dual Degree Program. Several sports events were held, including a volleyball game with the Loyola Physics Club. In May, nine students were inducted and two students were received into temporary membership at the annual induction dinner. John McGowan, Special Program Manager, AT&T, presented the program for the event. His topic was "Trends in Communications Technologies." Other 1996–97 chapter officers: Rachel Keffer and Jolanta Krywonis, co-vice presidents; Carolyn Pointek, secretary; Marie Morrow, treasurer; Sister Marie Augustine Dowling, corresponding secretary; Margaret Sullivan, faculty sponsor.

MD Beta — Chapter President — Leslie Huffer

Western Maryland College, Westminster — 26 actives

A highlight of spring events was a Mathematics Careers Night, held in early April, featuring various alumni of the college. Speakers for the dinner meeting included Carey Noll Emmons, '79, of NASA-Goddard Space Flight Center, Greenbelt, MD; Dan Holoski, '94, of Radical Solutions, Reisterstown, MD; and Sue Shermer Seevers, '71, Department of Defense (NSA), Fort Meade, MD. Other 1996–97 chapter officers: Toni Smith, vice president; Julie Brown, secretary; Lori Mowen, treasurer; James Lightner, corresponding secretary/faculty sponsor.

MD Delta — Chapter President — Joseph Palardy

Frostburg State University, Frostburg — 43 actives

In February the group enjoyed a dialog with Chinese mathematics professor Dr. Sun. The March meeting, a get-acquainted session for new members-to-be, featured mathematical recreations. This was followed a few days later by the induction itself which welcomed 16 new members into the organization. Chapter president Jesse Siehler provided a talk and demonstration on the mathematics of juggling for the occasion. The April program was given by one of Frostburg State's COMAP modeling teams: Leo Cyr, Jesse Siehler, and Jeff Wolfe. They previewed the presentation of their COMAP problem solution, which they later delivered at the MD-DC-VA sectional meeting of the MAA. Other 1996–97 chapter officers: Heidi Femi, vice president; Carla White, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

MA Alpha

Assumption College, Worcester 9 actives, 3 associates

Three new members were initiated the first of May. Following a dinner in honor of the new initiates, Professor Charles Brusard spoke on "The Gibbs Phenomenon in Fourier Series." Student officers for 1996–97 are to be elected in the fall. Charles Brusard is corresponding secretary/faculty sponsor.

MI Beta Chapter President — Carrie Rickabaugh

Central Michigan University, Mount Pleasant 45 actives

Winter 1996 meetings of MI Beta included an open house for prospective members, activities at the Student Activity Center, a Careers in Mathematics Seminar, and winter initiation. Senior students were the speakers at the Careers in Mathematics meeting. Guest speaker at the winter initiation was CMU Mathematics Professor Kirsten Fleming, who gave an interesting presentation on "Cryptography from Julius Ceasar to Sneakers." Her encoded messages, when decoded, were mathematical quotes. Chapter President Kristen Williams presented the program at one of the regular meetings; her topic was "Mathematics in Art." Kristen is also working on the local KME Homepage for the World Wide Web. It should be available for browsing sometime during the summer months. Discussions at the 1995 KME National Convention led Kristen and MI Beta member Rich Lamb to begin developing a homepage for the national KME. This was started in the fall of 1995 but student teaching and graduation delayed completion. Current plans are for Arnie Hammel to assist his son, Carey, during summer 1996 in efforts to expand the page and get it "out there" so individual chapters can link to it and the whole world can discover what the great organization of Kappa Mu Epsilon is all about!! The spring concluded with a picnic co-hosted by the Mathematics Department, KME, the Statistics Club, and the Actuarial Club. Other 1996–97 chapter officers: Kevin Zajac, vice president; Norma Reynolds, secretary; Debbie Sink, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

MS Alpha Chapter President — Karen Chandler

Mississippi University for Women, Columbus 12 actives

Other chapter officers for 1996–97: Amee Jo Miles, vice president/treasurer; Karen Chandler, secretary; Jean Ann Parra, corresponding secretary; Shaochen Yang, faculty sponsor.

MS Beta Chapter President — Jii Khoo

Mississippi State University, Mississippi State 20 actives, 7 associates

Other chapter officers for 1996–97: Brandon Butler, vice president; Christin McCloskey, secretary; Steve Wilson, treasurer; Michael Pearson, corresponding secretary/faculty sponsor.

MS Gamma Chapter President — Chuck Fleming

University of Southern Mississippi, Hattiesburg 20 actives

Other chapter officers for 1996–97: Mary Bassinger, vice president; Leigh Lynn, secretary; Alice W. Essary, treasurer/corresponding secretary; Barry Piazza, faculty sponsor.

MS Epsilon Chapter President — Robert East

Delta State University, Cleveland 17 actives

Other chapter officers for 1996–97: Kim Grimes, vice president; Alex Roehm, secretary/treasurer; Paula Norris, corresponding secretary; Rose Strahan, faculty sponsor.

MO Alpha Chapter President — Catherine Montgomery

Southwest Missouri State University, Springfield 12 actives, 4 associates

During the spring semester the chapter held two initiations, the second to accommodate two students who did not learn about the first initiation due to communication problems. Regular monthly meetings were held. A highlight of the semester was the Annual KME Banquet which has grown to include all faculty, staff, graduating senior math majors, math scholarship recipients, freshmen math awardees, and retired faculty. This year the banquet was attended by about 90 people and was considered a great success. Other 1996–97 chapter officers: Lisa Burger, vice president; Jennifer Mulder, secretary; Miriam Ligon, treasurer; Ed Huffman, corresponding secretary/faculty sponsor.

MO Beta Chapter President — Lynn Graves

Central Missouri State University, Warrensburg 25 actives, 6 associates

The February initiation of new members featured speaker Srikant Radhakrishnan whose topic was properties of Pascal's Triangle. In March, volunteers helped with the annual Math Relays. CMSU graduate Jeff Quibell was the speaker for the Klingenbery Lecture, also held in March.

Activities of the April meeting included election of new officers and a hands-on demonstration of the new TI-92 calculator. Nine students and two faculty attended the Region IV KME Convention in Topeka, KS. MO Beta now has a home page on the internet. You can visit them at http://153.91.1.102/~kme/kme.html. Their page also has a link to the national home page. Other 1996–97 chapter officers: Cassie Young, vice president; Carla Brown, secretary; Barbara Hart, treasurer; Joy Birchler, historian; Rhonda McKee, corresponding secretary; Larry Dilley, Phoebe Ho, and Scotty Orr, faculty sponsors.

MO Gamma — Chapter President — Amy Fifer

William Jewell College, Liberty — 13 actives, 5 associates

Other 1996–97 chapter officers: Lori Cantrall, vice president; Allison Cooper, secretary; Joseph T. Mathis, treasurer/corresponding secretary/faculty sponsor.

MO Epsilon — Chapter President — Gary Smith

Central Methodist College, Fayette — 13 actives, 5 associates

Other 1996–97 chapter officers: Michele Niemczyk, vice president; Victoria Vahle, secretary; William D. McIntosh, corresponding secretary/faculty sponsor; Linda O. Lembke, faculty sponsor.

MO Eta — Chapter President — Amanda Nixon

Truman State University, Kirksville — 20 actives, 4 associates

Other 1996–97 chapter officers: Kristen Moffitt, vice president; Laurel Berner, secretary; Jenny Griswold and Karen VanCleave, treasurers; Doug Cutler, historian; Mary Sue Beersman, corresponding secretary; Joseph Hemmeter, faculty sponsor.

MO Iota — Chapter President — Jolena Gilbert

Missouri Southern State College, Joplin — 20 actives, 6 associates

The organization met for regular monthly meetings and twice monthly for problem solving sessions. In February, several members car-pooled on a field trip to the MPSI Company in Tulsa, OK, where a former chapter president, Robyn Housman Caruthers, works as Manager of Modeling Research. Robyn and her husband, who works in demographics, gave members a tour of the plant and discussed the operation of MPSI. Initiation for nine new members was held in late March. As usual, many parents and friends attended the initiation ceremonies and the banquet which followed. Several students participated in the first Missouri MAA Collegiate Mathematics Competition at Southeast Missouri State University in Cape Girardeau. A delegation of eight attended the Region IV Convention at Washburn University in Topeka, KS, in late April. Richard Williamson presented a paper at the convention concerning unusual transformations of triangles. He also presented the paper at the Missouri Academy of Science Meeting at Drury College in Springfield, MO. At the Washburn Convention, Chapter President Jolena Gilbert served on the paper judging committee. Also in April, member Angela Selleck Long presented a talk to the Mo-Kan Council of Teachers of Mathematics entitled "Braid Theory." A party at the new home of Dr. and Mrs. Pat Cassens in early May closed out a successful semester. Student officers for 1996–97 will be elected in the fall. Faculty officers: Mary Elick, corresponding secretary; Chip Curtis, faculty sponsor.

MO Lambda Chapter President — Tanya Griffin

Missouri Western State College, St. Joseph

Professor Steve Klassen was the guest speaker for the March 3 initiation of five new members into the MO Lambda Chapter of Kappa Mu Epsilon. Four student members and one sponsor attended the Region IV Convention in Topeka, KS. MO Lambda President Brian Bettis served on the paper judging team. In observance of Mathematics Awareness Week, chapter members helped sponsor a series of three guest speakers. At the spring picnic Tanya Griffin was honored as the winner of the Riemann Award for outstanding accomplishment and promise in the field of mathematics. She was also elected president at that time. Other 1996–97 chapter officers: Cindy Ready, vice president; Stacey Cabill, treasurer; John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

NE Alpha Chapter President — Rick Pongratz

Wayne State College, Wayne 36 actives, 14 associates

Fourteen new members were initiated into NE Alpha Chapter at the spring semester initiation. The club selected Igor Proscurshim the Outstanding Freshman Student based on exam performance. KME members helped the Mathematics Department with the annual high school mathematics contest. In addition, the club was give the responsibility of overseeing the balloting for the selection of the Outstanding Wayne State College Math/Science Professor of the Year. Other 1996–97 chapter officers: Rustin Slaughter, vice president; Becky Proskocil, secretary/treasurer; John Fuelberth, corresponding secretary; James Paige, faculty sponsor.

NE Beta Chapter President — Jerrid Freeman

University of Nebraska—Kearney, Kearney 10 actives, 10 associates

Other 1996–97 chapter officers: Kim Flessner, vice president; Jeremy Suing, secretary; Justin Falor, treasurer; Charles Pickens, corresponding secretary; Peggy Miller, faculty sponsor.

NE Delta Chapter President — Justin Rice
Nebraska Wesleyan University, Lincoln 16 actives, 6 associates

Other 1996–97 chapter officers: Dusten Olds, vice president; Christin Cordes and J. P. Johnson, secretary/treasurer; Gavin Larose, corresponding secretary/faculty sponsor.

NM Alpha Chapter President — Brian Sanchez
University of New Mexico, Albuquerque 80 actives, 10 associates

Other 1996–97 chapter officers: Larry Montaño, vice president; Chris Blackwood, secretary; Johnny Snyder, treasurer; Archie G. Gibson, corresponding secretary/faculty sponsor.

NY Alpha Chapter President — Aaron Riddle
Hofstra University, Hempstead 15 actives, 11 associates

Chapter activities included a talk on careers in mathematics and an end-of-term picnic and barbeque. Other 1996–97 chapter officers: Brandi York, vice president; Paul Ryan, secretary; Lisa Fontana, treasurer; Aileen Michaels, corresponding secretary/faculty sponsor.

NY Eta Chapter President — Stacey Lauricella
Niagara University, Niagara University 18 actives, 11 associates

Other 1996–97 chapter officers: Jennifer Egan, vice president; Amy Maar, secretary; Lara Brown, treasurer; Robert L. Bailey, corresponding secretary; Kenneth Bernard, faculty sponsor.

NY Kappa Chapter President — Jennifer Smith
Pace University, New York 20 actives, 8 associates

Other 1996–97 chapter officers: Julia Chan, vice president; Angela Stone, secretary; Melahu Aynalem, treasurer; Geraldine Taiani, corresponding secretary; Blanche Abramov and John Kennedy, faculty sponsors.

NY Lambda Chapter President — Joseph D. Sprague
C. W. Post Campus-Long Island University, Brookville 32 actives

Twelve new members were initiated during the annual induction banquet held March 25 at the Greenvale Town House Restaurant. Dr. Debra V. Curtis, member #31 and now Assistant Professor of Mathematics at Bloomfield College in New Jersey, described in her talk, "From C.W. Post to Professor: Paths and Pitfalls," some of her experiences since graduating. The evening concluded with the announcements by Dr. Maithili Schmidt-Raghavan, Dean of the College, of the departmental awards. Other 1996–97 chapter officers: Joseph Glorioso, vice president; Justine D. Bello, secretary; Colin R. Grimes, treasurer; Andrew M. Rockett, corresponding secretary; Sharon Kunoff, faculty sponsor.

NC Gamma

Chapter President — Tiffany Scobey

Elon College, Elon College — 28 actives, 12 associates

NC Gamma has begun a monthly math contest through the student newspaper. The chapter also sponsored guest speakers Bill Love from the University of North Carolina at Greensboro and Theresa Early of Appalachian State University. In addition members helped sponsor a year-end picnic for graduating mathematics and computer science students. Other 1996–97 chapter officers: Kristin Miller, vice president; Amy Markijohn, secretary; Todd Williard, treasurer; David Nawrocki, corresponding secretary; James Allis, faculty sponsor.

OK Alpha

Chapter President — Carrie O'Leary

Northeastern Oklahoma State University, Tahlequah — 44 actives, 4 associates

OK Alpha continued to enjoy joint activities with NSU's student chapter of MAA. Members met for problem sessions featuring problems from *The Pentagon* and *The American Mathematical Monthly* and participated in "The Problem Solving Competition" sponsored by the MAA. The initiation of five students was well attended by both faculty and students. John Callaway has set up a database that includes information about the chapter's membership. He is also working on a local World Wide Web site. In March, the chapter sponsored a talk by Joe Michalcik, Actuary and Vice President of Blue Cross/Blue Shield of Oklahoma. At the annual "pre-finals" ice cream social, members of OK Alpha and the MAA student chapter gave their annual "Mathematics Teacher of the Year" award to Dr. Joan E. Bell. They also presented her with yellow roses and a plaque in appreciation and recognition of meritorious service in her ten year sponsorship of Kappa Mu Epsilon. Other 1996–97 chapter officers: Laura Cole, vice president; Lisa Eidson, secretary; Peter Butz, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

PA Alpha

Chapter President — Laura Williams

Westminster College, New Wilmington — 18 actives, 3 associates

Other 1996–97 chapter officers: Laurel Scaff, vice president; Heather Carson, secretary; Jill Schuller, treasurer; Jen Gatnerak, publicity; J. Miller Peck, corresponding secretary; Carolyn Cuff and Warren Hickman, faculty sponsors.

PA Gamma

Chapter President — Erin L. Korns

Waynesburg College, Waynesburg — 17 actives, 9 associates

Other 1996–97 chapter officers: Etta M. Nethken, vice president; Linda K. Smitley, secretary; Amanda J. Beisel, treasurer; Anthony Billings, corresponding secretary/faculty sponsor.

PA Eta

Chapter President — Ronna Matich

Grove City College, Grove City 33 actives

In late March PA Eta held an initiation ceremony for new members and elected new officers. The winner of the KME-selected Outstanding Freshman Mathematics Student was announced at a Parents' Day Award Ceremony on April 29. Chapter members had the opportunity on April 17 to hear Dr. Barbara Faires from Westminster College speak on "Tilings of the Plane." The group also held a fund-raising book sale. Other 1996–97 chapter officers: Suzette Cramer, vice president; Lori Young, secretary; Eric Blum, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

PA Iota

Chapter President — Rebecca Shubert

Shippensburg University of Pennsylvania, Shippensburg 23 actives, 10 associates

On April 21, initiation was held for 10 new members at a local restaurant. Also in April, together with the Math Club, the organization took part in the Adopt-a-Highway Program, cleaning up a section of a local roadway. Three faculty members and eight students enjoyed attendance at a regional meeting of the EPADEL Section of the MAA. Other 1996–97 chapter officers: Mary Wenrich, vice president; Cindy Hefty, secretary; Vicki Shanahan, historian; Michael D. Seyfried, treasurer/corresponding secretary; Gene Fiorini, faculty sponsor.

PA Kappa

Chapter President — Nicholas Gross

Holy Family College, Philadelphia 6 actives, 3 associates

On March 29 the PA Kappa Chapter held its annual initiation ceremony, inducting three new members. At this time, the chapter also celebrated its 25th anniversary on campus and honored Sister M. Grace Kuzawa, CSFN, Ph.D., for her 25 years of service to the chapter and her 40 years of service as a faculty member at Holy Family College. Many chapter alumni were present to offer testimonials, recognizing Sister's contribution to their mathematical development. During April, Mathematics Awareness Month, numerous activities were held to promote math awareness on campus. Among these, "Math Problems for Prizes" were offered daily during the last week of the month. On April 27 the chapter sponsored its second annual grade school mathematics competition. Eight local schools participated in arithmetic, problem solving, basic algebra, basic geometry, and math puzzle competitions. The top three schools received plaques and each participating student received a certificate. Other 1996–97 chapter officers: Thomas Feldmann, vice president; Lisa Esposito, secretary; Cheryll Stone-Schwendiman, treasurer; Sr. Marcella Wallowicz, corresponding secretary/faculty sponsor.

TN Delta — Chapter President — Deron C. Walraven

Carson-Newman College, Jefferson City — 12 actives, 4 associates

The chapter held an induction banquet in late March and observed Mathematics Awareness Week April 21–27. Other 1996–97 chapter officers: Michael D. Kelley, vice president; Jana L. Taylor, secretary/treasurer; Catherine Kong, corresponding secretary/faculty sponsor.

TX Eta — Chapter President — Sylvia Cantu

Hardin-Simmons University, Abilene — 15 actives, 9 associates

In conjunction with the Big Country Council of Teachers of Mathematics, TX Eta Chapter sponsored a Math-Science UIL Contest in November. In February the group hosted the Math Count Contest sponsored by the Abilene Chapter of Professional Engineering Society. The 22nd annual induction banquet for the chapter was held March 2. Nine new members were inducted, bringing the membership in the local chapter to 175. Induction ceremonies were under the direction of President Jeremy Fitch, Vice President John Lally, and Secretary Alexandria Hadlock. Program for the event was given by Dr. Edwin Hewett, Head of the Mathematics Department, who made a presentation on "Interesting Numbers." Other 1996–97 chapter officers: Christina Fischer, vice president; Stephanie Helbert, secretary; Phillip Hubbard, treasurer; Frances Renfroe, corresponding secretary; Edwin Hewett and Dan Dawson, faculty sponsors.

TX Kappa — Chapter President — David Hogan

University of Mary Hardin-Baylor, Belton — 12 actives, 2 associates

TX Kappa held its annual Spring Symposium on April 11. Discussion followed a presentation by Dr. William Harding who shared his ideas on how to teach basic concepts in calculus to children. Other 1996–97 chapter officers: Carrier Tucker, vice president; Kristi Davis, secretary; Andrea Hankins, treasurer; Peter H. Chen, corresponding secretary; Maxwell Hart, faculty sponsor.

WI Gamma — Chapter President — Steve Wall

University of Wisconsin—Eau Claire, Eau Claire — 20 actives, 12 associates

Other 1996–97 chapter officers: Kady Hickman, vice president; Kendra Zillmer, secretary; Jeremy Eppler, treasurer; Marc Goulet, corresponding secretary/faculty sponsor.

Trivia

Can you find all the chapters listed above whose schools have changed names since their last appearance in KME News? The number of such universities is positive.

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Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation.

Chapter	Location	Installation Date
OK Alpha	Northeastern Oklahoma State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959

IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel College, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	West Georgia College, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984

CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994

Starting a KME Chapter

Complete information on starting a chapter of KME may be obtained from Arnold Hammel, National President (see address on p. 77). Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; student members must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of this $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offerings and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.