THE PENTAGON

A Mathematics Magazine for Students

Volume 55 Number 2 Spring 1996

Contents

© 1996 by Kappa Mu Epsilon. All rights reserved. General permission is granted to *KME* members for noncommercial reproduction in limited quantities of individual articles, in whole or in part, provided complete reference is given as to the source.

Typeset in $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX Printed in the United States of America.

The Pentagon (ISSN 0031–4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Papers written by undergraduate mathematics students for undergraduate mathematics students are solicited. Papers written by graduate students or faculty will be considered on a space-available basis. Submissions should be typewritten (double spaced with wide margins) on white paper, standard notation conventions should be respected and any special symbols not typed should be carefully inserted by hand in black ink. All illustrations must be submitted on separate sheets and drawn in black ink. Computer programs, although best represented by pseudocode in the main text, may be included as an appendix. Graphs, tables or other materials taken from copyrighted works MUST be accompanied by an appropriate release from the copyright holder permitting further reproduction. Student authors should include the names and addresses of their faculty advisors. Final versions on 3.5 inch disk in "text only" (ASCII) format are appreciated. Contributors to *The Problem Corner* or *Kappa Mu Epsilon News* are invited to correspond directly with the appropriate Associate Editor.

Individual domestic subscriptions: $5.00 for two issues (one year) or $10.00 for four issues (two years); individual foreign subscriptions: $7.00 (US) for two issues (one year). Library rate: $10.00 (US) for two issues (one year) or $20.00 (US) for four issues (two years). Correspondence regarding subscriptions, changes of address or back copies should be addressed to the Business Manager. Copies lost because of failure to notify the Business Manager of changes of address cannot be replaced. Microform copies are available from University Microfilms, Inc., 300 North Zeeb Road, Ann Arbor, Michigan 48106–1346 USA.

EDITOR C. Bryan Dawson

Division of Mathematics and Computer Science
Emporia State University, Emporia, Kansas 66801
dawsonbr@esumail.emporia.edu

ASSOCIATE EDITORS

The Problem Corner Kenneth M. Wilke

Department of Mathematics
Washburn University of Topeka, Topeka, Kansas 66621
xxwilke@acc.wuacc.edu

Kappa Mu Epsilon News Mary S. Elick

Department of Mathematics
Missouri Southern State College, Joplin, Missouri 64801
elick@vm.mssc.edu

BUSINESS MANAGER Larry Scott

Division of Mathematics and Computer Science
Emporia State University, Emporia, Kansas 66801
scottlar@esumail.emporia.edu

Thank You, Referees!

The current and previous editors wish to thank the following individuals who refereed papers submitted to *The Pentagon* during the last two years.

V. Sagar Bakhshi
Virginia State University
Petersburg, Virginia

Duane Broline
Eastern Illinois University
Charleston, Illinois

Jeffrey Clark
Elon College
Elon College, North Carolina

Marvin Harrell
Emporia State University
Emporia, Kansas

James E. Lightner
Western Maryland College
Westminster, Maryland

Elizabeth G. Yanik
Emporia State University
Emporia, Kansas

We also wish to thank the many other individuals who volunteered to serve as referees but were not used during the past two years. Referee interest forms will again be sent by mail in the near future, so that interested faculty may volunteer. If you wish to volunteer as a referee, feel free to contact the editor (see page 2) for a referee interest form.

Back Issues

Is your journal collection complete? Copies of most back issues of *The Pentagon* are still available for $5.00 per copy. Please send inquiries to:

The Pentagon Business Manager
Division of Mathematics and Computer Science
Emporia State University
Emporia, KS 66801 USA

Fore!!!

Daniel Wessel, *student*

Kansas Delta

Washburn University
Topeka, KS 66621

Presented at the 1995 National Convention and awarded "top four" status by the Awards Committee.

In watching golf on Sunday afternoons I am amazed at the control that Greg Norman and other professionals have over the ball. On the contrary my ball hardly ever goes where I want it to go. I decided to investigate some of the underlying mathematics and physics which determine the path of the golf ball. Since I plan to teach high school mathematics, this undertaking might additionally serve as a source of problems and projects that I can use to stimulate interest in high school mathematics and physics students.

As an introduction to some of the basic principles involved, consider a program which I have entered on a TI-85 advanced scientific calculator. A copy of the complete program can be found in the Appendix. The following is an outline of the program's features:

1.) Prompts user for the distance to the flag.

2.) Asks user if a tree is wanted in the fairway. If yes, prompts for height and location of the tree, then displays the tree and flag on the screen.

3.) Prompts the user to choose a club from a menu (using irons only).

4.) Determines and displays the path of the ball (See Figure 1).

5.) Informs the user how far the ball landed from the flag.

In the program the golf ball path depends upon the golf club selected which in turn is used to determine the initial velocity and initial angle at which the ball leaves the ground. Several assumptions were made to simplify the model. They are listed below:

1.) The only force acting on the ball is due to gravity (the ball is in a vacuum).

2.) The initial angle at which the ball leaves the ground is the same as that of the loft angle of the clubhead (See Table 1).

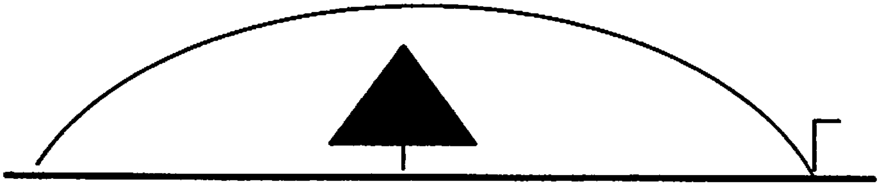


Figure 1

3.) During the swing the golf clubhead follows a circular path. The radius of that path is assumed to be the club length plus 15 inches.

4.) A golfer's swing speed (angular velocity) at the time of impact with the ball is the same for any club he uses.

5.) The golf ball's initial velocity as it leaves the clubhead is linearly related to the speed of the golf clubhead immediately before impact.

CLUB	LOFT
WOODS	
Driver	11°
IRONS	
#3	23°
#4	26°
#5	30°
#6	34°
#7	38°
#8	42°
#9	46°
PW	50°
From Golfsmith Clubhead and Component Catalog, 1994 p.42	

Table 1

Using these initial assumptions we can develop an equation which describes the path of the golf ball. In Figure 2 we picture the relationship between the position, velocity, and acceleration vectors along the golf ball path.

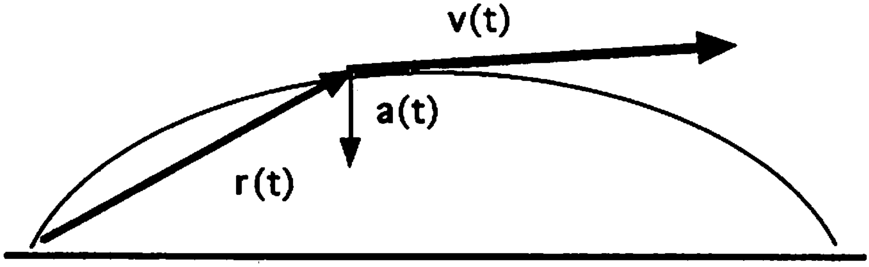


Figure 2

As in calculus,

$r(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ is the position vector at time t (seconds),
$v(t) = r'(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j}$ is the associated velocity vector, and
$a(t) = v'(t) = r''(t) = x''(t)\mathbf{i} + y''(t)\mathbf{j}$ is the associated acceleration vector,

where $\mathbf{i}$ is the unit vector in the positive horizontal direction, and $\mathbf{j}$ is the unit vector in the positive vertical direction.

In Assumption 1 we have assumed that gravity g is the only force acting on the golf ball and this exerts a force in a downward direction, so the components of the acceleration vector for any time t are necessarily given by

$$x''(t) = 0$$
$$y''(t) = -g.$$

Substituting the components back into the acceleration vector we get

$$a(t) = -g\mathbf{j}.$$

We picture the initial velocity vector $v(0)$ in Figure 3 where

$$v(0) = x'(0)\mathbf{i} + y'(0)\mathbf{j} \text{ and } v_0 = \sqrt{x'(0)^2 + y'(0)^2}.$$

In Figure 3, θ is the initial angle of the ball and v_0 is its initial velocity. Using some simple trigonometry we get

$$x'(0) = v_0 \cos\theta$$
$$y'(0) = v_0 \sin\theta.$$

Next, if the ball is assumed to be struck at the origin of our coordinate system,

$$x(0) = 0, \quad y(0) = 0.$$

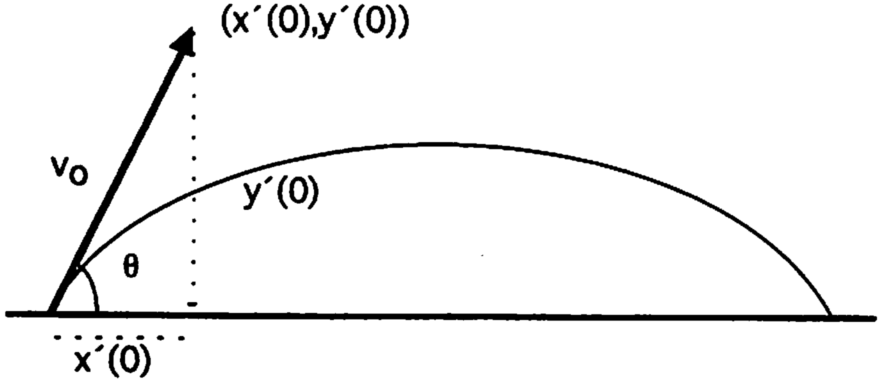


Figure 3

Summarizing and reorganizing the above conclusions, two differential equations with associated boundary conditions are generated:

$$\begin{aligned} x''(t) &= 0, \quad \text{with } x(0) = 0 \text{ and } x'(0) = v_0 \cos\theta; \\ y''(t) &= -g, \quad \text{with } y(0) = 0 \text{ and } y'(0) = v_0 \sin\theta. \end{aligned}$$

Equations of this nature are solved in most calculus texts, so the solutions are stated without the solution detail:

$$\begin{aligned} x(t) &= v_0 t \cos\theta, \\ y(t) &= -gt^2/2 + v_0 t \sin\theta. \end{aligned}$$

Eliminating the parameter t we get the quadratic equation

$$y = -gx^2/(2v_0^2 \cos^2\theta) + x \tan\theta.$$

This equation determines the parabolic path of the golf ball in my program. The constant g is taken as the standard 32 feet/second^2. Furthermore, the initial angle θ is taken as the loft angle of the selected golf club as stated in Assumption 2. The loft angle of the clubhead measures the angle of the slanted face of the clubhead. More specifically, it is the angle that the clubhead face makes with the shaft of the golf club. These angles (Table 1) are typical golf club lofts as found in the *Golfsmith Clubhead and Component* catalog. To keep the program uncluttered, the only choice for clubs are the irons (3–9 and the pitching wedge).

To determine the initial velocity v_0 in the program we first use Assumption 3 (the path of the clubhead is circular, with radius equal to the club length plus 15 inches). In *The Search for the Perfect Swing* by Cochran and Stobbs (page 206), the clubhead speed for the driver at the time of impact with the ball is typically about 162 feet/second (for a six ounce clubhead). Additionally, the driver club length is typically 43 inches. So the radius of a golf swing with the driver is 58 inches $(43+15)$. Now, using the trigonometric formula $v = r\omega$, we can convert the linear velocity v of 162 feet/second to an angular velocity ω of 33.52 radians/second, where r is the swing radius of 58 inches, which equals 4.83 feet.

Assumption 4 states that the golfer's angular velocity is the same for any club. Again using Assumption 3 we multiply the length of each club plus 15 inches by the constant angular velocity of 33.52 radians/second to determine the linear velocity of the clubhead at the time of impact with the ball. Golf club lengths with calculated clubhead speeds are given in the third column of Table 2.

CLUB	LENGTH (inches)	CLUB SPEED (ft/sec)	BALL SPEED (ft/sec)
WOODS			
Driver	43	162.0	214.0
IRONS			
#3	38.5	149.4	154.8
#4	38	148.0	148.2
#5	37.5	146.7	142.0
#6	37	145.3	135.4
#7	36.5	143.9	128.7
#8	36	142.5	122.1
#9	35.5	141.1	115.4
PW	35.5	141.1	115.4
Lengths from Golfsmith Catalog,1994 p. 42. Speeds calculated using model's assumptions.			

Table 2

In Assumption 5 we assume that the initial velocity v_b is linearly related to the clubhead speed v_c immediately before impact:

$$v_b = av_c + b.$$

In Cochran and Stobbs (page 206) we find that the expected initial velocity of the ball when hit with a driver with a clubhead speed of 162 feet/second is 214 feet/second. Unfortunately we could find no other data which predicts initial ball speeds for clubs other than the driver. We expect that the ball velocity leaving the pitching wedge would be somewhat less than the clubhead speed before impact. We estimate that the ball would leave the pitching wedge with a speed of 115 feet/second, which, with the clubhead speed of 141.1 feet/second from Table 2, will yield a reasonable traveling distance for the ball. Solving the system

$$\begin{aligned} v_b = av_c + b \text{ with } v_b &= 214 \text{ when } v_c = 162.0 \\ \text{and } v_b &= 115 \text{ when } v_c = 141.1 \end{aligned}$$

we get the linear equation coefficients $a = 4.74$ and $b = -553.37$. Using this equation we calculate the initial velocity of the ball as it leaves the clubhead for each of the irons in our golf bag. The results which are used in our program are stated in the fourth column of Table 2.

Using the model's assumptions, we have determined the initial velocity and initial angle for each club, and outlined the mathematics which determines the resulting path of the golf ball used in the program. We now investigate how closely these results mirror "golf course reality."

Using a typical loft angle for a driver, 11° (Table 1), and ball speed off the driver, about 214 feet/second (Table 2), and the projectile path developed in the program, a ball hit with the driver would go 178 yards. However, using empirically determined driving distances from Cochran and Stobbs (page 26) an average drive with this loft angle and initial velocity should go 218 yards. This result is rather surprising since we assumed our golf course resides in a vacuum, and therefore there was no air resistance drag which obviously would shorten the path of the ball even more.

Physicists sometimes assume that the drag is proportional to the velocity of the projectile but acting in the opposite direction (see Figure 4). We now investigate how the path of the ball is affected if this additional drag resistance is added to our model's assumptions.

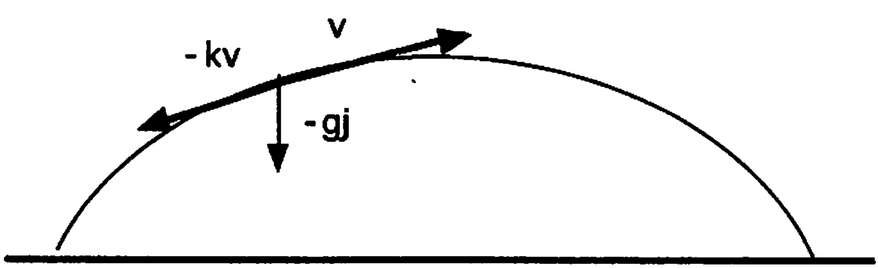


Figure 4

Recall that in the program discussion $a(t) = -g\mathbf{j}$ and $v(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j}$. When we also consider drag we get a more complicated set of equations:

$$a(t) = -kv(t) - g\mathbf{j} = -kx'(t)\mathbf{i} + [-ky'(t) - g]\mathbf{j}.$$

Also, recall that $a(t) = x''(t)\mathbf{i}+y''(t)\mathbf{j}$, so the resulting differential equations in this case are

$$x''(t) = -kx'(t), \text{ with } x'(0) = v_0 \cos\theta \text{ and } x(0) = 0$$
$$y''(t) = -ky'(t) - g, \text{ with } y'(0) = v_0 \sin\theta \text{ and } y(0) = 0.$$

The above boundary conditions are unchanged from our previous discussion without the additional drag assumption.

These differential equations are not solved in calculus; however they can be solved using beginning differential equations techniques. The solutions are

$$x(t) = (v_0 \cos\theta/k)[1 - e^{-kt}]$$
$$y(t) = 1/k[v_0 \sin\theta + (g/k)][1 - e^{-kt}] - (g/k)t.$$

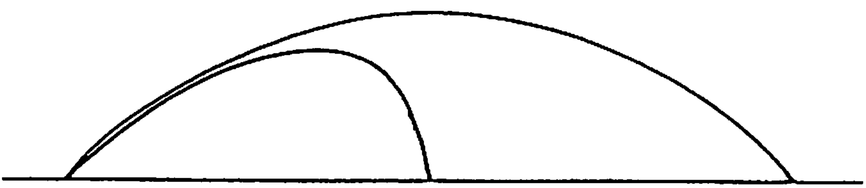


Figure 5

It is interesting to observe the shape of the drag curve compared to the parabolic non-drag curve (Figure 5); we used the TI-85 to graph both paths using a driver with an initial ball velocity of 214 feet/second (Table 2) and an initial angle of 11° (Table 1). In the equation involving drag, we used a value of .25 for the proportionality constant k. We have no scientific way of determining the most appropriate value of the constant of proportionality k. However, after trying several different values we found that a selection of .25 yields a graph which reasonably reflects our expectations of the drag curve.

Not surprisingly, adding the drag assumption produced a path which was shorter than the path in a vacuum. Consequently, there must be other real world factors involved. Indeed, another force that is acting on the ball is that caused by the spin of the golf ball. This force due to the spin of the golf ball is commonly called lift. The lift vector n is perpendicular to the velocity vector and counteracts the gravity vector (Figure 6). This upward lift results from air pressure due to the air moving more quickly above the ball than below. This is due to a spin of several hundred radians/second generated by the slanted edge of the clubhead at impact. The mathematics and physics involved here get rather complicated. We turn to *The Mathematics of Projectiles in Sport* by Neville de Mestre (page 147) for assistance. De Mestre determined that due to the shape, size, and characteristics of a golf ball, the drag is more accurately represented by a force which is proportional to the velocity squared (still acting in the opposite direction of the velocity).

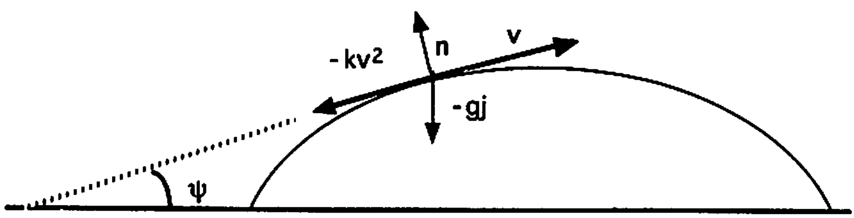


Figure 6

De Mestre uses these lift, drag, and gravity forces to determine the following system of differential equations:

$$\frac{d(v\cos\psi)}{dt} = \frac{-\rho A}{2m}v^2(C_D\cos\psi + C_L\sin\psi) \qquad \frac{dx}{dt} = v\cos\psi$$

$$\frac{d(v\sin\psi)}{dt} = \frac{-\rho A}{2m}v^2(C_D\sin\psi + C_L\cos\psi) - g \qquad \frac{dy}{dt} = v\sin\psi,$$

with initial conditions

$$x(0) = 0,\ y(0) = 0,\ v(0) = v_0,\ \text{and}\ \psi(0) = \theta,$$

and where

$v(t) =$ the ball's velocity vector

$\psi(t) =$ the angle the velocity vector makes with the horizontal (Figure 6)

$\rho =$ the density of air

$A =$ the cross-sectional area of the golf ball

$m =$ mass of the golf ball

$g =$ gravitational force

$C_D =$ drag coefficient (which varies with the velocity of the ball)

$C_L =$ lift coefficient (which varies with the amount of spin on the ball).

Because of the variations in the drag and lift coefficients during the flight of the ball, de Mestre determines that these differential equations are best solved using numerical methods. However, de Mestre cites previous research (P.G. Tait) in which the lift coefficient C_L was found to be approximately proportional to the velocity ($C_L = C_L^* v$), where C_L^* is the constant of proportionality. Other research (Bearman and Harvey) argues that for golf speeds greater than 40 m/s, C_D could be considered constant. These two simplifying assumptions combined with some further assumptions concerning the angle $\psi(t)$ allow the stated differential equations to be replaced by a set of simplified differential equations which can be solved without numerical methods. De Mestre's solution is stated below:

$$x(t) = K_D \ln\left(1 + \frac{v_0 t}{K_D}\right)$$

$$y(t) = \left[\theta K_D - \frac{K_D^2}{v_0^2}(v_0 K_L - g) - \frac{K_D^2 g}{2v_0^2}\right] \ln\left(1 + \frac{v_0 t}{K_D}\right) + \frac{K_D(2v_0 K_L - g)}{2v_0} t - \frac{g}{4} t^2,$$

where $K_D = 2m/(\rho A C_D)$, $K_L = \rho A C_L^*/(2m)$, and the constants, other than C_L^*, are as earlier specified in the lift equation. Using the TI-85 we sketched the path of the golf ball in Figure 7 using the de Mestre solution and the corresponding path in a vacuum (short path).

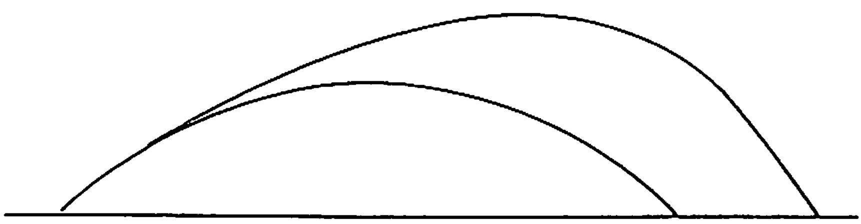


Figure 7

In this case we use metric values for the constants. In particular, $g = 9.8$ meter/second^2, $v_0 = 65.2$ meter/second, $\theta = .192$ radian, $A = .0014$ meter^2, $m = .046$ kg, $\rho = 1.3$ kg/meter^3, $C_D = 0.204$, and $C_L^* = 3.03$. Again we have no way of determining the values of C_D and C_L^* accurately, but the selected values result in a curve which appears close to what might be expected. Cochran and Stobbs (page 162) show a similar diagram without discussing the mathematical underpinnings of the curves. Notice that in the lift path, the ball's vertex is about two-thirds of the way to impact with the ground, and the ball appears to follow an approximately linear path in the early part of the flight.

We have investigated how lift and drag would effect the path of the golf ball. Our program's assumption that gravity was the only force acting on the golf ball did indeed significantly simplify the model. How about our other assumptions concerning the initial angle and initial velocity of the ball? How closely do these assumptions reflect reality?

Recall that we assumed that the initial angle of the golf ball as it leaves the ground is the same as that of the loft of the club (Assumption 2). In reality, several modifications are necessary. According to Cochran and Stobbs (page 151), there is a frictional force which decreases the initial angle significantly, especially for the higher-numbered irons. The decrease ranges from about 2° for the driver to 16° for the nine iron. (Cochran and Stobbs determined these range values using a constant club speed of 100 miles/hour.) In Figure 8, α is this angle of decrease. This angle results from the downward force as the ball slides along the clubhead. This action also is responsible for the initial spin on the ball which produces the lift.

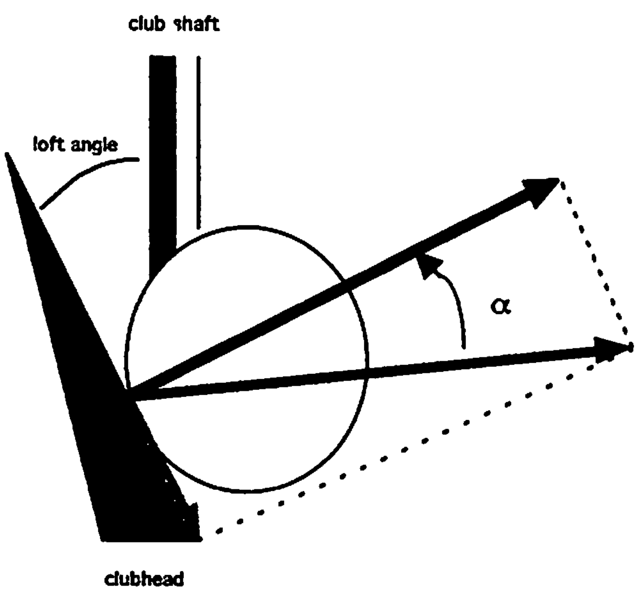


Figure 8

Although not mentioned in Cochran and Stobbs, there is another modification of the initial angle which must be considered. For the driver and lower-numbered irons the ball is typically placed forward from the middle of the golfer's stance. As a result the ball will be struck with a slightly upward motion which slightly increases the initial angle from that of the loft angle. This angle produces a counteracting increase in the initial angle of the ball. It ranges from 0° for the five iron to about 5° for a driver.

In Assumption 2 we assumed that the initial angle of the ball was equal to the loft angle. The modifications discussed above show that this is not very accurate. We have also shown that Assumption 1 concerning the medium in which the ball travels is not accurate. How about the remaining three assumptions, which all effect the initial velocity of the ball at impact with the clubhead?

In Assumption 5 it was stated that the initial velocity of the ball would be linearly related to the speed of the golf clubhead immediately before impact. As a result the length of the club completely determined the initial velocity of the ball. However, in a real-world golf situation there are many other factors that contribute to the initial velocity of the golf ball. One factor that needs to be considered involves the modified angle at which the ball leaves the club that was discussed a few paragraphs earlier. Cochran and Stobbs (page 152) argue that the greater the loft of the club face the more the initial velocity of the ball will be reduced. This is due to the frictional force that is occurring between the ball and the club face (Figure 8).

In addition to the loft, the club mass and the ball's elasticity also effect the initial velocity of the ball as it leaves the golf club. Cochran and Stobbs (page 229) give the following formula for the initial velocity, v_b:

$$v_b = v_c((1+e)/(1+(m/M))),$$

where

$v_c =$ velocity of the clubhead immediately before impact

$m =$ mass of the golf ball

$M =$ mass of the clubhead

$e =$ coefficient of restitution, which varies from .67 on a drive to .80 on a putt.

The coefficient of restitution depends upon the velocity of the clubhead at impact. It is a measure of the "bounce ratio" of the ball when it hits a surface (or in this case, the surface hits the ball). Cochran and Stobbs claim that this formula is reasonably accurate for a driver, but would not be very accurate for the higher-numbered irons. In Assumption 5 we stated that the initial velocity of the ball was linearly related to and therefore completely determined by the velocity of the clubhead at impact. In fact, it depends at least upon the masses of the ball and clubhead, and the coefficient of restitution of the ball. We conclude that Assumption 5 was also an over-simplification of reality.

We now turn our attention to Assumption 4 (a golfer's angular swing speed is the same for every club). The mass of the clubhead directly affects the velocity of the clubhead. Data in Table 3 shows that the weight of a golf clubhead slows the clubhead velocity. This is not surprising. A golfer could not swing a sledgehammer as fast as a golf club. We conclude that Assumption 4 is also an over-simplification of reality.

Clubhead Weight (ounces)	Clubhead Speed (ft/sec)
4	172
6	162
8	153
10	147
Cochran and Stobbs, p. 206	

Table 3

We began our investigation with a program which utilized a model containing five assumptions concerning the initial angle of the ball, the initial velocity of the ball, and the medium in which the ball travels. These initial assumptions seemed reasonable at the time, but we have argued that four of the five assumptions require significant modifications in order to better reflect golf course reality. The remaining assumption, involving a circular golf clubhead path, is also but a rough approximation to reality. However, perhaps surprisingly, this assumption may be the most realistic of the five. Video pictures in Cochran and Stobbs (page 25) show that the clubhead path on a typical swing is reasonably close to circular.

We have just begun to uncover the mysteries of the flight of the golf ball. In our investigation we have ignored something called the "yaw" of the golf ball. Yaw is the physicists' term for the sideways spin of the ball which causes hook and slice. We have also ignored the dimpling of the golf ball which effects both the lift and yaw of the ball. In addition, the flexibility of the golf club shaft effects the golf swing and the initial velocity of the ball at the time of impact. Furthermore, the internal material of the golf ball effects the elasticity of the ball which in turn effects the coefficient of restitution at impact. Additionally, the internal material of the ball also determines the weight. Cochran and Stobbs (page 176) imply that the standard weight for a golf ball is 1.63 ounces, but there is no corresponding standard size for a golf ball. They report that in 1968 the British used a ball of diameter 1.63 inches, while the Americans used a ball of diameter 1.68 inches. I am promoting an even bigger golf ball. I suggest a diameter of 1.75 inches. A ball with a diameter of 1.75 inches has two advantages over a smaller counterpart. First, the ball is easier to hit. Second, and more importantly, the ball will FLOAT.

Acknowledgements. I would like to thank Dr. A. Allan Riveland for all of his assistance in the preparation of this paper. Also I would like to thank Wayne Lanning, our golf expert, for his insight on the golf swing.

References

1. Bearman, P.W. and Harvey, J.K., "Golf Ball Aerodynamics," *Aeronaut Quarterly* 27 (1976), 112–122.
2. Cochran, Alastair and Stobbs, John, *The Search for the Perfect Swing,* J.B. Lippincott Company, Philadelphia, 1968.
3. de Mestre, Neville, *The Mathematics of Projectiles in Sport,* Cambridge University Press, Cambridge, 1990.
4. *Golfsmith Clubhead and Component Catalog,* 1994.
5. Halliday, David, Resnick, Robert, and Walker, Gearl, *Fundamentals of Physics,* fourth edition, Wiley, New York, 1993.
6. Jorgensen, Theodore P., *The Physics of Golf,* American Institute of Physics, New York, 1994.
7. Tait, P.G., "Some Points in the Physics of Golf (Part 1)," *Nature* 42 (1890), 420–423.
8. Tait, P.G., "Some Points in the Physics of Golf (Part 2)," *Nature* 44 (1891), 497–498.
9. Tait, P.G., "Some Points in the Physics of Golf (Part 3)," *Nature* 48 (1893), 202–205.

Appendix

```
PROGRAM:GOLF
:Lbl START
:Param
:FnOff    ' Clears all functions
:ClLCD    ' Clears view screen
:ClDrw    ' Clears all things drawn
:0->xMin
:.25->tStep
:-25->yMin
:100->yScl
:300->xScl
```

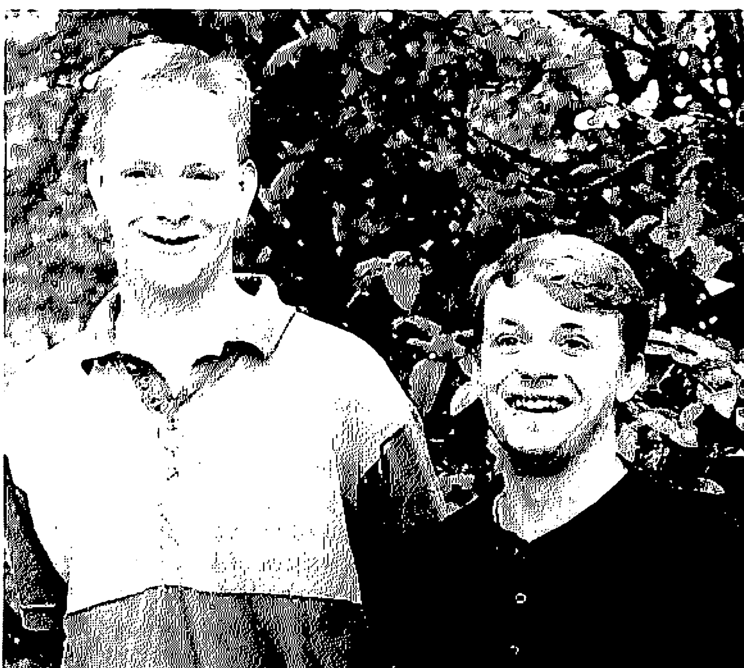
```
:Degree    'Sets degree MODE
:Disp "How far to the flag"
:Input " (in yards)?  ", F
:3F->F    ' Converts F to feet
:F+35->xMax
:(1/3)(F+35)->yMax
:Disp "Do you want a tree?"
:Menu(1,"Yes", A1,2,"No",A2)
:Lbl A1
:Disp "How far to the tree"
:Input " in yards)?", W
:3W->W    ' Converts W to feet
:Disp "How tall of tree?"
:Input " (in feet)?", H
:TREE    'Calls program TREE
:DispG    'Displays graph
:Pause
:Goto Club    'Goes to Lbl Club
:Lbl A2
:2->W
:0->H
:Goto Club
:Lbl Club
:{23,26,30,34,38,42,46,50}->Ang
:{154.8,148.2,142.0,135.4,128.7, 122.1,115.4,115.4}->Vel
:Disp "What club?"
:MENUS    'Call program MENUS
:If C==0
:Then
:Goto START
:End
:Ang(C-2)->A
:Vel(C-2)->V
:Line(F,0,F,20)    'Draws flag
:Line(F+9,20,F,20)
:poly({-16/((V cos A)^2), tan A, 0})
:max(Ans)->Z
:-16(W^2/(V cos A)^2) + W tan A
:Ans->B
:If B>H
:Then
:DrawF (x≤Z)(-16(x^2/(V cos A)^2)+x tan A)
:Pause
:Else
:DrawF (x≤Z)(x≤W)(-16(x^2/(V cos A)^2)+x tan A)
:If Z>W
:Then
:Line (W,B,.75W,0)
:Pause
:End
:Pause
:ClLCD
```

```
:ClDrw
:Disp "Try a new club"
:Disp " or new hole!"
:TREE
:Goto Club
:End
:int ((1/3)abs (Z-F))->S
:If S==0
:Then
:ClLCD
:Disp "Great Shot "
:Disp "You Hit The Pin!"
:Else
:ClLCD
:Disp "Your shot landed",S,"yards from the pin."
:END
:Pause
:Goto START
```

```
PROGRAM:MENUS
:Menu(1,"3iron",C1,2,"4iron",C2,3,"5iron",C3,4,"6iron",C4,5,"More",C5)
:Lbl C1
:3->C
:Goto E
:Lbl C2
:4->C
:Goto E
:Lbl C3
:5->C
:Goto E
:Lbl C4
:6->C
:Goto E
:Lbl C5
:Menu(1,"7iron",C6,2,"8iron",C7,3,"9iron",C8,4,"PW",C9,5,"New",D1)
:Lbl C6
:7->C
:Goto E
:Lbl C7
:8->C
:Goto E
:Lbl C8
:9->C
:Goto E
:Lbl C9
:10->C
:Goto E
:Lbl D1
:0->C
:Goto E
:Lbl E
:Return
```

```
Program:TREE
:Line(W,0,W,H)
:Line(W,H,W-20,10)
:Line(W-20,10,W+20,10)
:Line(W,H,W+20,10)
:Line(F,0,F,20)
:Line(F+9,20,F,20)
:Return
```

Convention Winners



Daniel Wessel (left) and Jeffrey Brown (right) presented two of the winning papers at the 30th Biennial Convention in Durango, Colorado (picture courtesy of Allan Riveland, Kansas Delta). Unfortunately, pictures of the other winners (Michelle Ruse, Tammy Causey and Dane Mooney) are currently unavailable. Anyone having pictures of the other winners should feel free to send them to the editor (see p. 2 for address).

The Bobcat That Lived in a Polygon

Phyllis Mahan, *student*

Kentucky Alpha

Eastern Kentucky University
Richmond, KY 40475

Presented at the 1995 National Convention

Introduction

It is a clear, cool March morning in eastern Kentucky. The sun is just beginning to come up and the grass is covered heavily with dew. A lone biologist, wearing layers of field clothes and tall rubber boots, hikes through an open field waving a Yagi antenna in front of her. One thought runs through her mind: Where are they now?

"They" are radio-collared bobcats and she goes out each day trying to locate them. She may never see them and may never be closer than 3 km to them. But with the beeping of the receiver, three or four fixes later, and the use of triangulation calculations, she will be able to determine approximately where they were. If she can locate the same bobcat a sufficient number of times over a period of six months or so, then she will be able to describe the size and shape of that bobcat's home range and compare it to that of other bobcats. She will also be able to determine if any bobcats are sharing a common area.

An animal's home range can be described as the area which is traversed by an individual animal in its normal activities of food gathering, mating, and rearing young. The most common method of describing an animal's home range is by drawing a minimum convex polygon around the animal's set of known location points. The study now turns to one of geometry.

Geometric objects such as points, line segments, and polygons have been studied since the beginning of mathematics. Geometric problems such as determining whether two line segments intersect or whether a point lies within a polygon are easily visualized and can often be solved by simply looking at a sheet of paper. However, generalizing a geometric problem so that a solution can be found for any possible set of points, line segments, and polygons requires non-trivial computer algorithms.

Computational geometry is the branch of computer science that studies algorithms for solving geometric problems. These algorithms have applications in many varied and interesting fields such as modern engineering, robotics, computer graphics, computer-aided design, very-large-system integration, and biology. Surprisingly, even though geometry has had a long history of useful applications, computational geometry has only recently begun to study these problems. It is not surprising that biologists use computational geometry as an aid in describing an animal's home range.

Background

Before discussing algorithms which describe an animal's home range, there are certain properties from geometry that can help us visualize the home-range polygon and better understand the algorithms that find this polygon.

The fundamental object in computational geometry is the point, which is an ordered pair of numbers such as coordinates in the Cartesian system. All other objects are described in terms of points. A line segment is a pair of points connected by a straight line segment. A polygon is an ordered sequence of points where successive points in the sequence are connected by straight line segments and the last point is connected to the first point to make a closed figure.

An animal's home range is described by a minimum convex polygon, known as the convex hull. The convex hull is the smallest convex polygon containing all points in the point set and where the vertices of the hull come directly from the point set. A convex polygon has the property that for any pair of points that belong to the polygon, the line segment connecting these points must itself lie entirely within the polygon (Fig. 1, top). A non-convex polygon has the property that there is a pair of points where the line segment connecting these points does not lie entirely within the polygon (Fig. 1, bottom). For example, triangles and regular octagons are convex, but arrow and star-shaped polygons are not.

A set of points can be classified with respect to a given polygon as exterior, interior, or boundary points (Fig. 2). An interior point is the center of a circle with sufficiently small radius which belongs entirely to the polygon. An exterior point is the center of a circle containing no point of the figure. A point is a boundary point if every circle drawn about the point always contains both interior and exterior points.

Questions

Algorithms used in finding the convex hull repeatedly ask three questions. Do two given line segments intersect? What is a simple closed path for the point set? Does a given point lie inside or outside a given polygon? These questions can often be answered by simply looking at a sheet

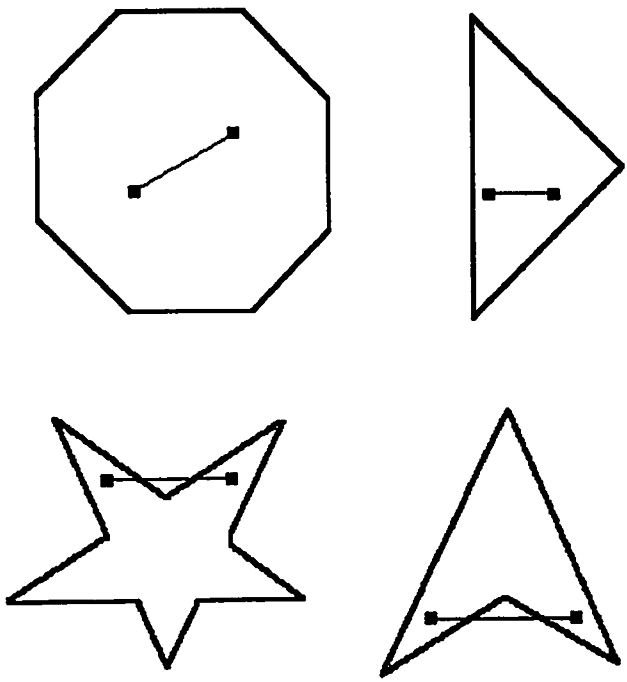


Figure 1. Top: convex polygons. Bottom: non-convex polygons.

of paper. But a computer cannot simply look at a sheet of paper. Hence, algorithms have been developed to instruct a computer to find solutions to these questions.

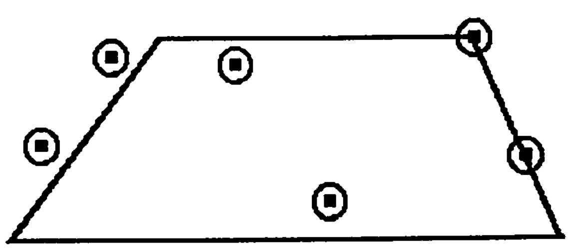


Figure 2. Exterior, interior, and boundary points.

Do two given line segments intersect? A straightforward way to solve this problem is to find the intersection point of the lines defined by the line segments and check whether this point falls between the endpoints of both the segments. Problems with this method arise when the two segments are nearly parallel to each other. The calculations are then sensitive to the precision of the division operator on the computer.

A more accurate method uses cross products and direction of turn. Let p_1, p_2, and p_3 be points where $p_1 = (x_1, y_1)$, $p_2 = (x_2, y_2)$, and $p_3 = (x_3, y_3)$. Let line 1 be the segment defined by the points p_1 and p_2, and let line 2 be the segment defined by the points p_2 and p_3. The direction of turn going from p_1 to p_2 to p_3 is a left turn if we travel counterclockwise along these points (Fig. 3, left) or a right turn if we travel clockwise (Fig. 3, right). The direction of turn can be determined by calculating the cross product of the vector made up of p_1 and p_2 with the vector made up of p_1 and p_3:

$$(p_2 - p_1) \times (p_3 - p_1) = (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1).$$

A left turn is made if the cross product is positive, and a right turn is made if the cross product is negative. A zero cross product indicates that the points are collinear.

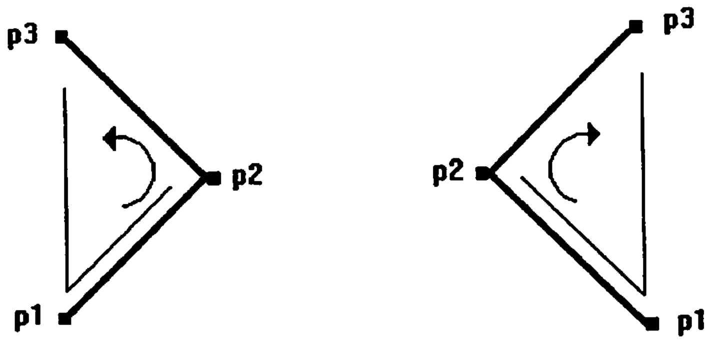


Figure 3. Left: left turn. Right: right turn.

The direction of turn is calculated four times to determine whether two line segments intersect — twice as we travel along one segment going to each of the other segment's endpoints, and twice more as we travel along the second segment going to each of the first segment's endpoints. If we make a right and a left turn as we travel along the first segment (Fig. 4, top left), and again a right and a left turn as we travel along the second segment (Fig. 4, top right), then we can conclude that the two segments intersect. If we make two right turns or two left turns as we travel along either segment then we can conclude that the two segments do not intersect (Fig. 4, bottom).

What is a simple closed path for a given point set? This question is also known as the travelling salesman problem. What is being asked is

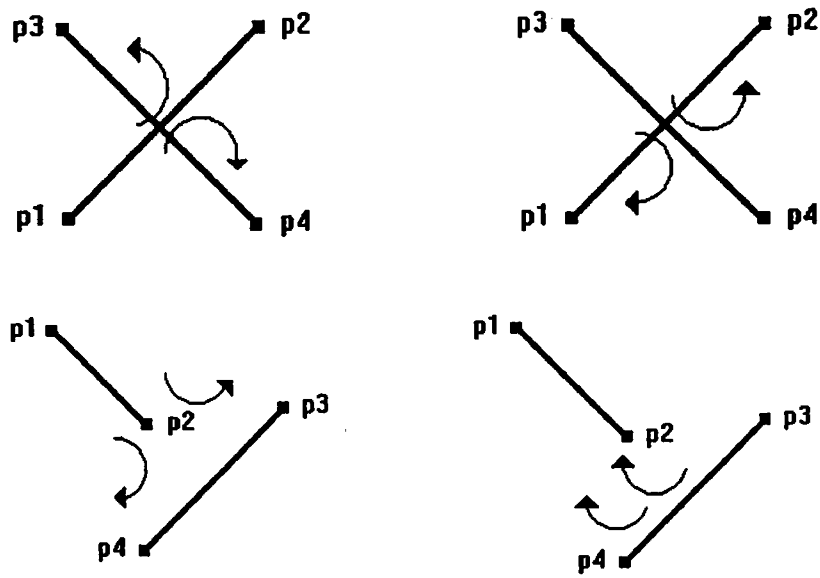


Figure 4. Top: intersecting lines. Bottom: non-intersecting lines.

what path can be followed through a set of given points so that each point is visited only once, the path never crosses itself, and we end at the point from which we started. For the travelling salesman problem we want this to be the shortest path. Thankfully, the shortest path is not necessary for our purposes since there is no algorithm available today which will find the shortest path for any given set of points.

We start by finding the anchor for the point set. The anchor is the lowest point in the set — the one with the least y-coordinate (Fig. 5, left). If there is more than one point with the least y, then among those points with the least y the anchor is the point with the least x-coordinate. Then we find the angle that each point makes as we travel from the point to the anchor and out in the positive horizontal direction from the anchor (Fig. 5, center). The points are then ordered by the angle they form from smallest to largest. Finally, adjacent points in the ordering are connected with straight line segments, and the last point is connected to the anchor. The resulting polygon, which is not necessarily convex, is a simple closed path for the point set (Fig. 5, right).

Does a given point lie inside or outside a given polygon? A straightforward way to solve this problem is to draw a long line segment from the point in any direction, long enough so that its endpoint is guaranteed to be outside the polygon, and count the number of times this test line crosses the polygon. If the number of hits is odd then the point must be inside the

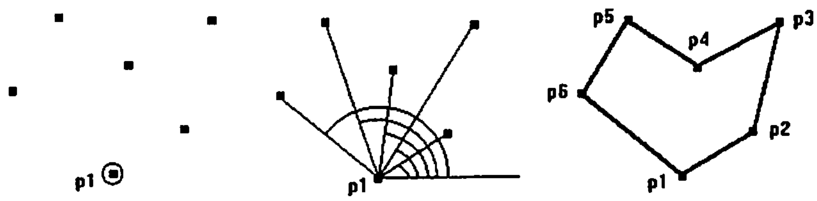


Figure 5. Construction of a simple closed path.

polygon. If the number of hits is even then it must be outside the polygon (Fig. 6, left).

Problems with this method arise when our test line hits a vertex point. How many hits do we count that as? If our point is outside and we count a vertex hit as 1 then we are saying the point is inside. So, let's say a vertex hit is 2. But then if our point is inside we would be saying it is outside. Another problem is encountered when we hit an entire line segment. How many hits should we say these are (Fig. 6, center)?

Luckily, there is a solution. We can look at the two line segments which share this vertex as an endpoint. If the two segments are on the same side of the test line then we will not increment the number of hits. If the two segments are on opposite sides of the test line then increment the number of hits by 1. When our test line encounters a line segment then we can look at the two segments which are attached to that segment's endpoints and follow this rule of same side or opposite sides (Fig. 6, right).

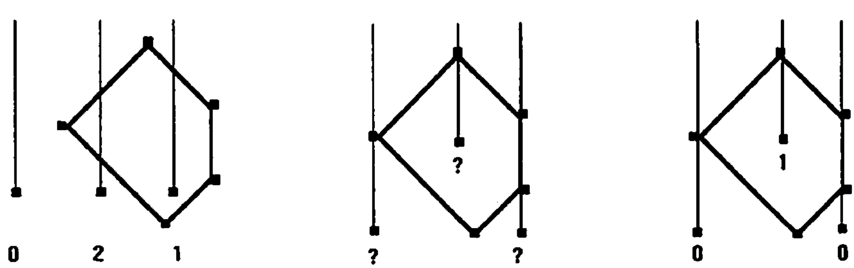


Figure 6. Is a point inside or outside a given polygon?

Convex Hulls

We are now well equipped to look at algorithms which determine the convex hull for a given point set. There are many algorithms available that find the convex hull. Two of the most popular are Jarvis' March and Graham's Scan.

Jarvis' March, also known as package wrapping, was created by R. A. Jarvis in 1973. This algorithm is easily visualized as wrapping a line around the set of points. We repeatedly anchor at an outermost point and sweep a line around until it hits another point. We start by finding the anchor for the set. This will be the first point on the hull. We anchor here and sweep a horizontal line, drawn in positive direction, upward until we hit a point in our set (Fig. 7, top left). This will be the second point on the hull. We then anchor at this point and sweep upward again until we hit the next point on the hull (Fig. 7, top right).

The computer isn't actually sweeping a line. At each anchor the angle made from every point in the set to that anchor is calculated. The point which forms the smallest angle with that anchor will be the point hit in the sweep.

When we reach the topmost point in our set we will then anchor and sweep a horizontal line, drawn in the negative direction, downward until we hit the next point on the hull (Fig. 7, bottom left). We continue to do this until our original anchor is hit (Fig. 7, bottom right). The package is now fully wrapped and we have our convex hull.

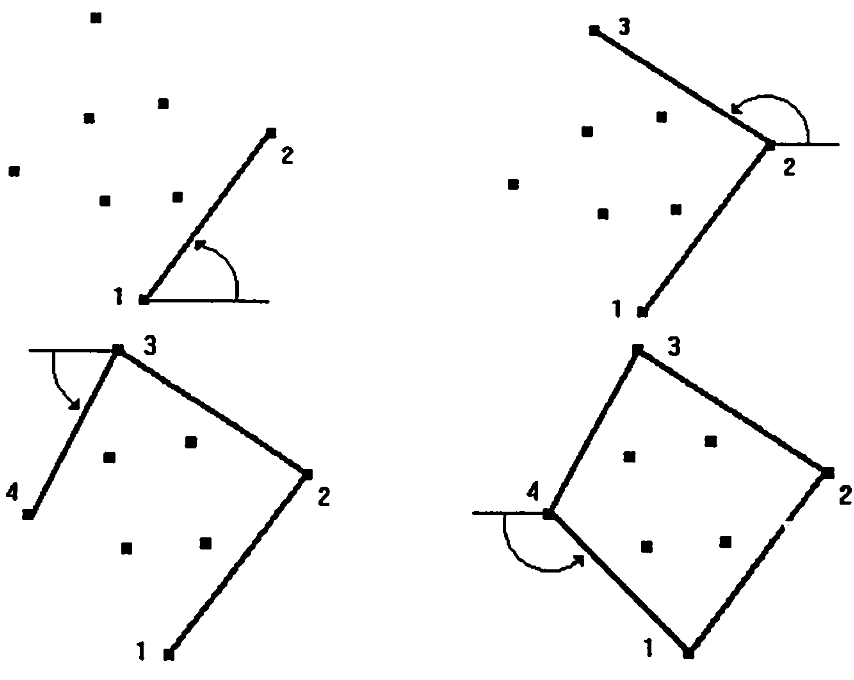


Figure 7. Jarvis' March.

This algorithm has time complexity of $O(nh)$, where n is the number of points in the set and h is the number of vertices on the hull. This time complexity can be explained by thinking that at each anchor (h) we calculate the angle from each point in the set (n). Hence, we have nh calculations. If many of the points from the point set lie on the hull this grows to $O(n^2)$.

Graham's Scan was created by R. L. Graham in 1972 and is a more efficient algorithm in most cases. We start by finding a simple closed path for our point set (Fig. 8, top row). Then the first three points in the path are included as candidates for the hull (Fig. 8, second row left). The next point on the path, p_4, is included as a candidate for the hull (Fig. 8, second row right). To determine if this point could be a hull point we calculate the direction of turn as we travel from p_2 to p_3 to p_4. If this is a left turn then we will leave p_4 as a candidate for the hull. Include the next point on the path, p_5, as a candidate for the hull. Calculate the direction of turn as we travel from p_3 to p_4 to p_5 (Fig. 8, third row left). If this is a right turn then we will remove the previous point, p_4, as a candidate and rename p_5 as p_4 (Fig. 8, third row right). Then calculate the direction of turn going from p_2 to p_3 to this new p_4. If it is a left turn then we can leave p_4 as a candidate. If it is a right turn we would backtrack again until a left turn is found.

Continue in this manner of including the next point on the path as a candidate, calculating the direction of turn for the last three candidates (Fig. 8, bottom left), and backtracking if necessary until a left turn is found. We continue until the first point in the set, the anchor, is included again. One final direction of turn is calculated (Fig. 8, bottom right), and our hull is now complete.

Graham's Scan has been shown to have a time complexity of $O(n \log n)$, where n is the number of points in the set. This time complexity can be explained by thinking back to the start of this algorithm. The first step was finding a simple closed path, which is basically sorting the points by the angle they form with the anchor. Most of the work in Graham's Scan is done here and many sorting algorithms have complexity $O(n \log n)$.

Conclusion

As we have seen, geometric problems are easily visualized and can often be solved by simply looking at a sheet of paper. However, these problems are more complicated to implement by computer. Study in computational geometry will undoubtedly continue as more people become intrigued by geometric problems that in most cases seem obvious at first glance. Scientists continue to look at problems with known solutions in hopes of finding a faster, more efficient algorithm.

With a set of known location points in hand and a good algorithm for finding the hull, our biologist can now more easily and accurately describe the home range of each bobcat she has followed tirelessly during the past

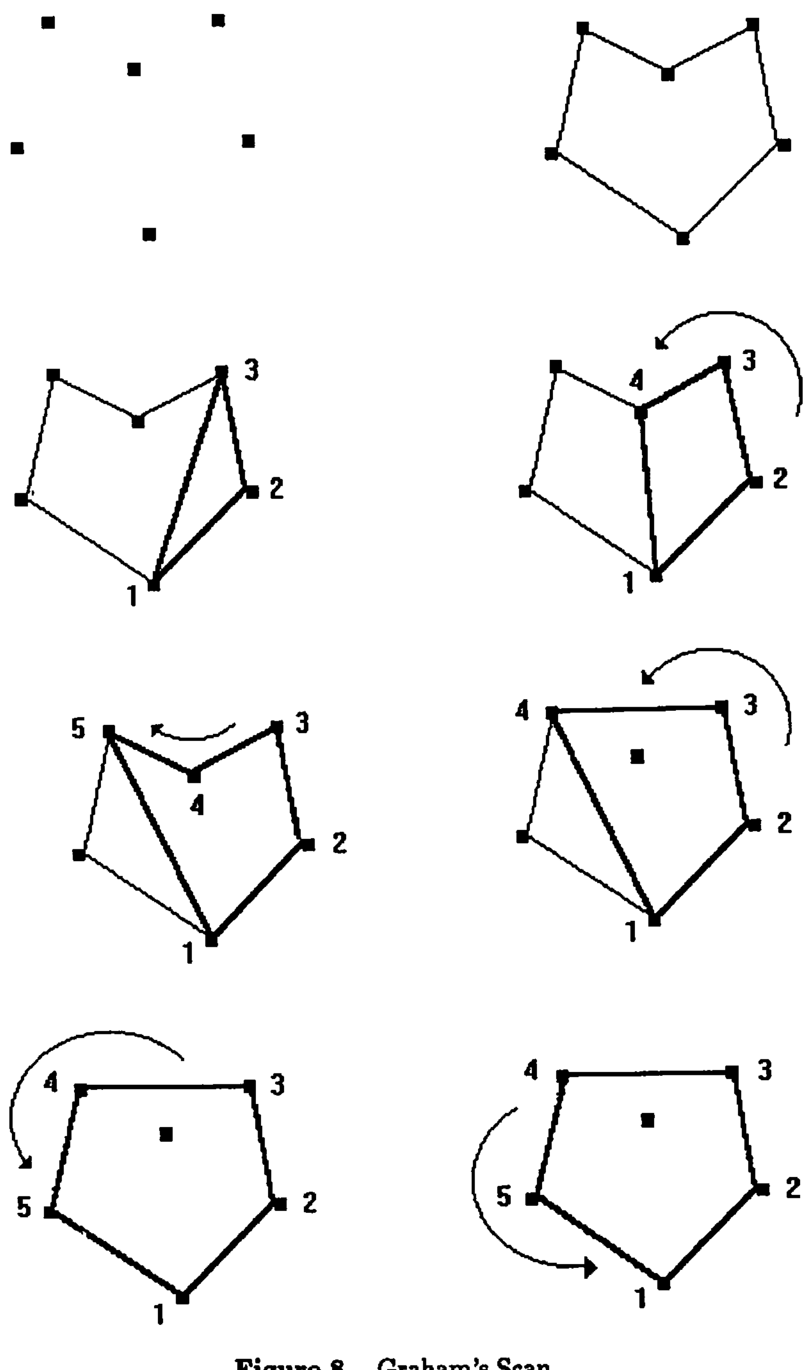


Figure 8. Graham's Scan.

months.

Acknowledgements. I would like to thank Dr. Pat Costello for encouraging me to write this paper; Dr. Don Greenwell for advising, reading, editing and providing criticism on this paper; and Dr. Bob Frederick for advising and explaining field and research biology methods.

KME Presidents

Four presidents of Kappa Mu Epsilon were able to attend the 30th Biennial Convention in Durango, Colorado. They are, from left to right, Arnold D. Hammel, Michigan Beta (Central Michigan University), 1993–present; George Mach, California Gamma (California Polytechnic State University), 1969–1973; James L. Smith, Ohio Zeta (Muskingum College), 1985–1989; and Harold L. Thomas, Kansas Alpha (Pittsburg State University), 1989–1993. Photograph courtesy of Arnold Hammel.

Playing Checkers with Mathematical Logic

Thomas A. Hileman, *student*

Ohio Alpha

Bowling Green State University
Bowling Green, OH 43403

Presented at the 1995 National Convention

Checkers, dating back over 4000 years, is the oldest continuous game in the history of the world. Egyptians kings took time off from building the pyramids to play it. The game of checkers is mentioned in Homer's *Odyssey* as well as in the writings of Plato. Throughout history, it has served as a source of entertainment and intellectual challenge for some very famous individuals. George Washington, Ben Franklin, Abraham Lincoln, Theodore Roosevelt, and Harry Houdini are included among these people. Edgar Allan Poe liked checkers more than any other educational pastime.

Although checkers has been played with many different variations, we chose to follow the rules of the American Checker Federation. The checkerboard has sixty-four alternating light and dark squares (see figure 1). For playing purposes, the board must be placed so that the bottom corner square on the left- hand side (known as the single corner) is dark. The dark squares are numbered from 1 to 32, starting from the upper left-hand corner. Each side starts with twelve pieces, one side being of light color and the other of dark color. In this article, we will refer to them as black and white even though they may be black and red or red and white. The twelve black pieces should be placed on the squares labeled 1 to 12 and the twelve white pieces on squares 21 to 32. The first move is always made by the player having the black pieces with each player alternating afterward.

In order to win the game, a player must either block the opponent's pieces or capture all of them. The player making the last move wins. At the beginning of the game all of the pieces (also called checkers) move like pawns; that is, they can move from one dark square to another either by stepping forward by one square or jumping an opponent's piece. Upon reaching an opponent's king row (the final row on the opposite side of the board), the checker is crowned by placing another checker on it and becomes a king. Crowning ends your move, and it is then your opponent's turn. This

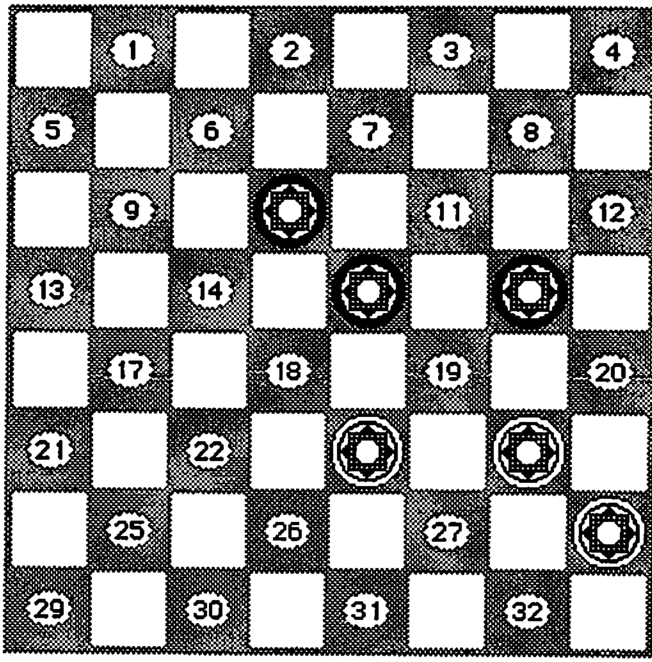


Figure 1. Checkerboard. Black's side at top, white's side at bottom.

is true even if you jumped into the king's row and have a possible jump out as well. Kings can move or jump either forward or backward one square at a time. This type of king is called a normal king. In our first version, we programmed the computer to play using flying kings, which can move the full length of the board. Since the moves of normal kings are a subset of the ones for flying kings, our strategy, originally developed for the flying kings, applies equally well to normal kings.

A checker must jump an adjacent piece of the opponent if the square beyond is vacant. When a piece is jumped, it is captured and removed from the board. Only one piece may be jumped at a time, but any number may be jumped in a series during one turn. If a jump is available, it must be taken; no other moves are allowed.

Antonio Torquemada, an author from Valencia, Spain, published the first book on checker methodology in 1547, and Pierre Mallet, a French mathematician, did the same for checker strategy in 1668. The first such book written in the English language was published by William Payne at London in 1756. From the viewpoint of master players, the game has advanced significantly since then in terms of knowledge accumulated. For instance, one American master player has compiled over seven hundred variations on a single opening. Many end games also have been solved and published. By way of comparison, checkers has advanced very little in terms of mathematical theory. Thus far, there is only the theory of the move (also called vantage), which is helpful in the end game but much too simplistic to be applied to an opening or mid game. Until computers arrived, checkers stood at a crossroads with no new theory, but only increasing data on specific end games, beginning gambits, and game records [1].

While working at International Business Machines, Arthur Samuel realized that checkers was ideally suited for computer programming and machine learning (now called artificial intelligence) because of its mathematical and logical nature. Beginning his research in 1952, he improved his checker program to the point where in the 1960's he was able to beat a master level player. However, he became frustrated under the limitations of the computing resources available to him at that time. Progress stood still until computer scientist Jonathan Schaeffer took an interest in checkers. Schaeffer, like many serious chess players, thought that checkers was childishly simple and not worthy of attention. They thought that for all practical purposes checkers had been solved and that chess was a more prestigious and intellectually challenging game to pursue. Thus, it was natural for early efforts to focus on the challenge of chess and ignore the game of checkers.

After a casual inquiry into the state of computerized checkers, Schaeffer began his quest to build the ultimate checker-playing machine, the Chinook, which currently ranks as the second best player in the world. Only Marion Tinsley, a mathematics professor from Florida State University, is better. Tinsley is widely regarded as the best player ever, losing only ten games in the last forty years while holding the world championship. So it was with much anticipation that a match of man against machine was to be played. Chinook first met Tinsley in the 1990 U.S. National Open and played a four game match, each ending in draws. In 1991, Tinsley visited Edmonton to play a friendly match against Chinook and won one game, drawing the remaining 13. Tinsley said this about Chinook: "He (Chinook) hasn't really developed much in the way of judgment and makes strange moves, but then gets down and fights like the very devil after getting into trouble. It's exciting to play him" [5]. At a forty match exhibition in London in 1993, Tinsley again prevailed over Chinook, with a record of 4–2–33. He defeated a machine that is able to consider three million different checker moves a minute and look 17 to 21 moves ahead. In its memory, Chinook has a record of every published game of Tinsley's, yet Tinsley comes out victorious. About the historic final game, the Sunday Times in London writes, "Amazingly the man used just half an hour thinking time for this historic game, while Chinook used an hour and half, during the course of which it saw no less than 270 million positions, but to no avail" [6].

While it is difficult to understand how Tinsley develops his strategy, Chinook's approach relies on a massive database to compare with the current board. In a sense, Chinook "works backward" to decide which of the moves is the best. However, the phrase "working backwards" is not what is intuitively expected [7]. Instead of drawing inferences from the goal of winning, Chinook considers preceding positions that would lead to it and, as a result, it considers many useless possibilities that could not be realized through forward play.

Chinook increases its database daily with the help of one hundred twenty computers which labor day and night to add even more moves to the massive database. According to Schaeffer, Chinook's prime advantage lies in the end game databases it can bring to bear in matches. Currently, Chinook can refer to all 2.3 billion positions attainable with six or fewer pieces, and the Alberta researchers are in the midst of calculating the 35 billion positions possible for seven pieces. "We believe that if we could reach the eight-piece database, which includes 400 billion positions, we would be virtually unbeatable," according to Schaeffer [5]. What this means is that currently with only six pieces left on the board Chinook already knows which moves lead to victory, defeat, or a drawn game. But all of this requires a massive hardware investment. The Chinook consists of a dedicated mainframe computer with gigabytes of data storage. It also requires a lengthy amount of time to consider moves because of the search time inherent in a database of such large size. Furthermore, Chinook has no real judgment or elegance of play and suffers from the lack of a coherent strategy.

My adviser and I wanted to program a microcomputer to play a good game of checkers using a simple set of logical rules. This purpose could best be accomplished through list processing, a concept suggested by Alonzo Church, a mathematical logician, in 1964. The first step was to get the program to follow checker rules. We chose to use Object Logo, because of its graphical user interface, list-processing capabilities, and object-oriented features [3]. Initially, our program played defensively by protecting threatened pieces and strengthening its position. We chose not to have the computer trade pieces to advance its position, but rather filled spaces from behind. This strategy led to a program that advanced very slowly and was trapped easily. A trap is a move where one side sacrifices a piece (or pieces) in exchange for pieces of the opponent. For example, when one piece is exchanged for two of your opponent's pieces it is called a two-for-one trap. To illustrate, black has pieces at 10, 15, and 16, with white having pieces at 23, 24, and 28, and with white's turn to move (see figure 1). A conservative player would move 24 to 20 and force the black piece either to move and be taken by 23 or to stay and be taken by the white piece at 20, producing a three-to-two piece advantage. However, the better choice for white would be to advance from 24 to 19 so that black is forced to jump from 15 to 24 and then white is left with a double jump, producing a two-to-one piece advantage. This changes the entire complexion of the game with white gaining a decisive advantage.

The improvements in our second version of the program resulted from the incorporation of forward and backward recursion, which allows us to have complete information (moves are described entirely) about moves and checker positions [3]. In the first version, the strategy was crippled, because we could not look past the second jump in a multiple jump move without recomputing each change of position. Our second version is more aggressive and sets traps without becoming trapped. We finally settled on four simple principles that were derived from analyzing masters play and that could be viewed as axioms for making recommendations in many situations [2] [4]. Rather than consulting a huge database, like Chinook, we used filters to shorten the time for picking moves. Each filter uses a relative hill-climbing function to evaluate possible moves while considering a given combination of our principles [7]. The filters were arranged in order of importance and programmed to pass through the subsequent ones after one filter produced a unique recommendation.

Our first filter checks for immediate victory or defeat. It first checks if there is a move that will win the game. If so, this move is performed. Next, it checks if there is a move that will lose the game. If there is such a move, the computer tries to avoid it. The second filter looks for traps to spring. We check for three types of general traps, which apply to any type of piece, and three types of traps for normal kings, as appropriate. The computer checks for traps that we can spring by comparing two-dimensional models to the board using a pattern recognition routine. The second filter also has a defensive aspect, since it detects potentially entrapping situations. If the program detects a trap about to be sprung on us in the next move, then the computer searches for a move to block or ruin our opponent's plans. The third filter looks to find traps that could be set in one move. The fourth and final filter looks ahead to a depth the programmer specifies and evaluates the board position of our pieces, the neighborhood support, and ability to get a king. A strategic position is determined by six positional arrays, which give priorities to some choices. For example, there are more opportunities for moves from the center of the board. Thus, the central squares, numbered 14, 15, 18, and 19, are important real estate for winning a checker game. Neighborhood support is a measure of the strength a piece gets from the proximity of its own pieces. Master players agree that the strongest formation in checkers is a triangle with its vertex facing an opponent. Furthermore, having checkers grouped together lends to greater strength when advancing pieces forward for an attack. Thus, neighborhood support is an important principle. The ability to get a king is also important, because a king allows for attacks from behind, where an opponent is weakest. Currently, our program is set to look six moves ahead when we are down in pieces and only four or two moves when we are ahead. Greater depths require more power, memory, and time.

The current version of our checker program plays a decent game. In terms of strengths, the current version is much more aggressive and can take advantage of opponents' errors. Furthermore, if it has a numerical advantage, then it forces its opponent into traps to further strengthen its position. The beginning game is much improved, too. It recognizes traps quickly and exploits them effectively in gaining a piece advantage or a positional advantage. The mid game also has a more aggressive strategy for obtaining kings. Even the end game is improved over the first version. It plays with greater conviction in defeating its opponent. However, the second version requires more time for calculations since it does more. At present, the time between moves can run up to five minutes with a fast computer, whereas in the first version the maximum time was around two minutes with a slower one.

Our program can be improved several ways. First, our filters could change their priorities from a game's beginning to the middle stage and, again, in the end game to allow strategy to be specialized for each phase [4]. We already implemented this idea somewhat minimally by increasing the search depth when there are fewer than seven pieces left. The search depth can be further increased when either side has two or fewer pieces without appreciatively increasing the wait time. We can also train our program to respond to particular situations in prescribed ways.

Although our program lacks the massive, efficient database that Chinook has, it plays a good game of checkers for its size (about 400 thousand bytes) and power (Apple Centris 650). An opponent wishing to defeat the computer must be able to force the computer into sophisticated traps that it has not been programmed to recognize. Otherwise, our program is excellent in detecting, setting up, and springing traps, as well as advancing pieces to produce both numerical and positional advantages. Although the computer efficiently advances pieces to the king's row to get them crowned, it is not aggressive enough in the end game to put its opponents away quickly. Sometimes it tends to advance pieces to the king's row rather than eliminate opponent's pieces. Finally, the time between moves should be reduced as much as possible. Although hardly a competitor for Tinsley or any other master, casual players can get hours and hours of enjoyment competing with the mathematical logic of our program.

Acknowledgements. I would like to thank my faculty advisor, Waldemar Weber, for his hard work and programming expertise in Object Logo. Without him, this project would never have been brought this far. For inspiration and support, I thank Renee Bannister, who proved not only to be a

worthy match in checkers but also in life. Finally, I would like to thank the Faculty Honors Committee of the Department of Mathematics and Statistics at Bowling Green State University for patience and encouragement.

Editor's Note. Anyone interested in a copy of the computer program (for Macintosh) described in this article may write Waldemar Weber, National Secretary of Kappa Mu Epsilon, Department of Mathematics and Statistics, Bowling Green State University, Bowling Green, OH 43403. Additionally, anyone interested in playing checkers may write Charles C. Walker, Secretary of the American Checker Federation, Post Office Drawer 365, Petal, MS 39465.

Marion F. Tinsley (1927–1995), whose checker play is described above, passed away after this article was written. For more information on Tinsley, see "Setting the Record Straight," *The Keystone Checker Review*, September 1995, pp. 939–940.

References

1. Goppelt, John W., "Mathematics and the Game of Checkers," *Journal of Recreational Mathematics* 4 (1971), 262–264.
2. Hopper, Millard, *Win at Checkers*, Dover Publications, New York, 1956; first edition appeared as *Checkers, All the Answers on the Art of Successful Checker Playing*, 1941.
3. *Object Logo for the Apple Macintosh and its Supplement for Object Logo 2.61*, Paradigm Software Inc., Cambridge, Massachusetts, 1990, 1992.
4. Pask, Richard, "Simple Strategy, the Opening and Early Midgame," *The Keystone Checker Review* 6 (1995), 835–838.
5. Peterson, Ivars, "The Checkers Challenge," *Science News* 140 No. 3 (1991), 40–41.
6. St. George, Donna, "Crown Him, He Can Beat a Computer, The Unsung Champion of Checkers," *The Philadelphia Inquirer*, April 11, 1993.
7. Wickelgren, Wayne A., *How to Solve Problems, Elements of a Theory of Problems and Problem Solving*, W. H. Freeman and Co., San Francisco, 1974.

KME WWW Information Request

Yes, there is a national Kappa Mu Epsilon World Wide Web home page! Its URL is:

http://www.cmich.edu/kme.html

Local chapters with WWW pages should send their URL's to our national president, Arnold Hammel (a.hammel@cmich.edu), so that links from the national page to local pages may eventually be added. A link from each local chapter's page to the national page would also be appreciated!

I Think Knot

Saylar Craig, *student*

Iowa Alpha

University of Northern Iowa
Cedar Falls, IA 50614

Presented at the 1995 National Convention

When we were children, a knot was either a major crisis in dressing ourselves or a good reason to climb a tree. The former is closer to the type that mathematicians study rigorously. Knot researchers have and are continuing to develop theories and classifications regarding their own kinds of knots. In this paper, I shall lead the reader through an investigation of these theories, their history, and some of the useful applications of the mathematics in other fields.

A knot, in useful knot theory, is an entanglement of a strand whose ends are ultimately joined together to form a "knotted loop." Then, we could say that a circle is a knot. Since it is the most trivial case of a knot, it has been nicknamed the "unknot." Other more interesting simple knots include the trefoil and the figure-eight knot (see figure 1).

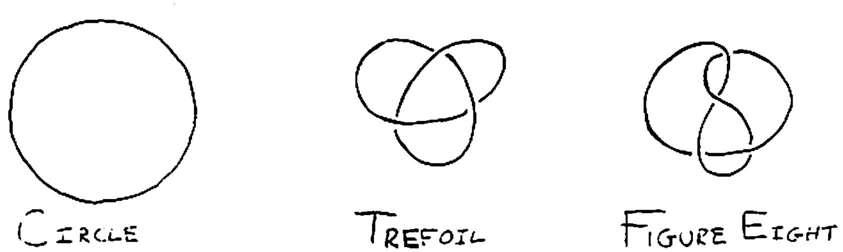


Figure 1

One of the simplest methods of generating new knots is to count the number of times it crosses itself, called simply the number of "crossings." The knots with up to 8 crossings (Epstein [1]) are in figure 2.

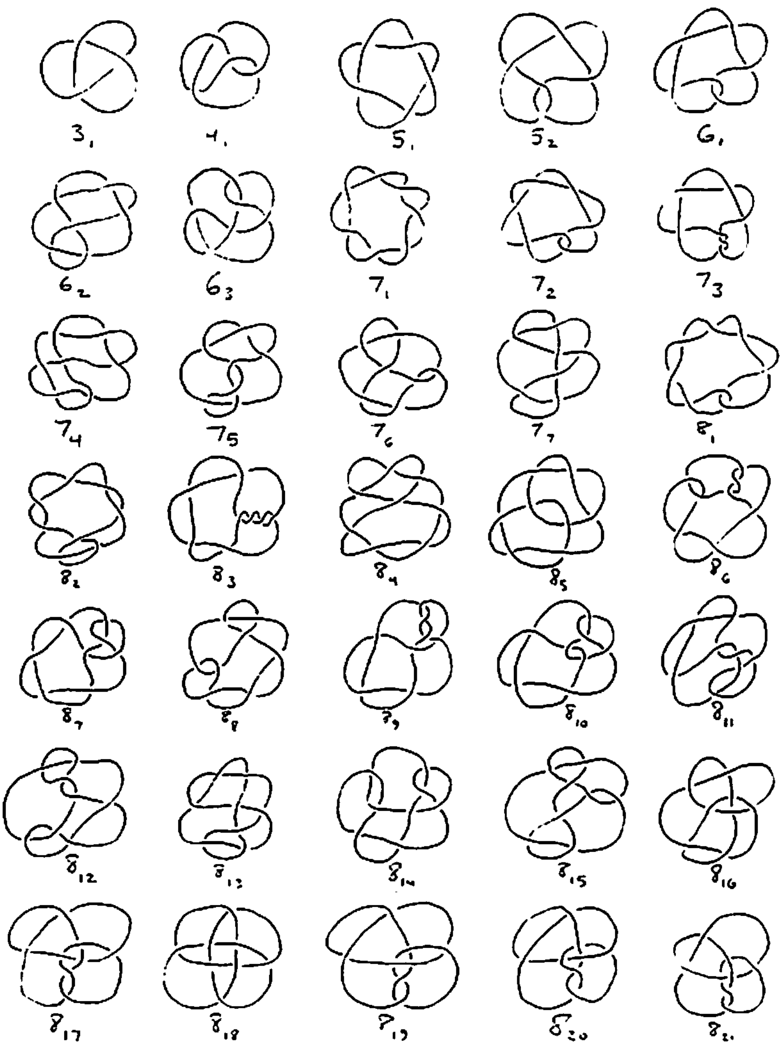


Figure 2

Understandably, the number of "different" knots we could possibly form is infinite. So, it would be nice to know what knots are actually the same (or equivalent), and which are completely different, and maybe even some sort of classification system that is more useful and more elaborate than just a counting of the crossings.

Even though knots were used elaborately in artwork all over the world as early as the 8th century, it wasn't until the mid-19th century that they were considered to have anything to do with mathematics. Karl Friedrich Gauss was the first to look at a knot as a mathematical unit. One of his students, Johann Listing, also studied them extensively. Later that century, when scientists all over the world were trying to understand the structure of the atom, Lord Kelvin theorized that atoms were vortex tubes of ether, and he and his colleague Peter Tait thought the tubes could be tangled in knots. So, they placed themselves on the path still being traveled today by knot theorists. By trial and error, they were able to classify knots of up to 11 crossings, which took years to develop. But, they eventually gave up, not knowing whether their lists were complete, and with no perfect way to tell whether two knots were actually equivalent (Watson [8]).

It was early in this century that we saw a better way to determine knot congruence. Kurt Reidemeister, in the 1920's, developed what are now known as the famous Reidemeister moves. He set down formal rules governing how one could deform a knot without actually changing its significant properties (Watson [8]). There are three Reidemeister moves (see figure 3).

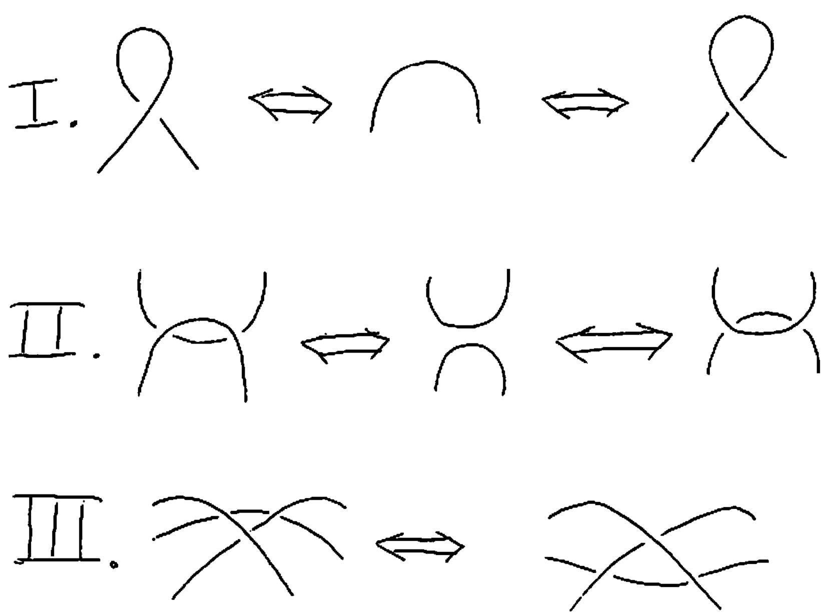


Figure 3

This, as simple as it may seem, opened up many doors for researchers. These, however, as we'll later see, are not enough to show equivalence of any two knots which are truly equivalent (Kauffman [3]). Later, in 1928, James Waddell Alexander came up with a very different way to classify knots. His idea was to gather information about a knot in such a way that a certain polynomial would be generated. This classification system actually goes pretty well up to about 9-crossing knots, where it starts to trip up (Watson [8]).

Also, some knots are "chiral." This means that they have "handedness," or that the mirror image of a knot is not equivalent to the original knot. The trefoil knot can be considered to be either left- or right-handed, and the two cannot be deformed into each other using legal Reidemeister moves (see figure 4). Another problem with the Alexander polynomial is that it does not distinguish between two knots that are mirror images of each other, but not equivalent using Reidemeister moves. The most significant contribution of Alexander was the proposition that knot theory and algebra were somehow linked (Watson [8]).

Figure 4

British mathematician John Horton Conway invented a very simple but effective invariant in knots. It is called the skein relation, and involves oriented knots (where strand(s) have a given direction). It defined three types of relationships at crossing regions (Jones [2]; see figure 5). Then, the skein relation states that

$$(1/t)V_{L_+} - tV_{L_-} = \frac{t-1}{\sqrt{t}}V_{L_0},$$

where t is the invariant of the knot. This turns out to be quite useful in discovering basic differences between knots. This idea of an invariant was expanded on greatly by future knot theorists.

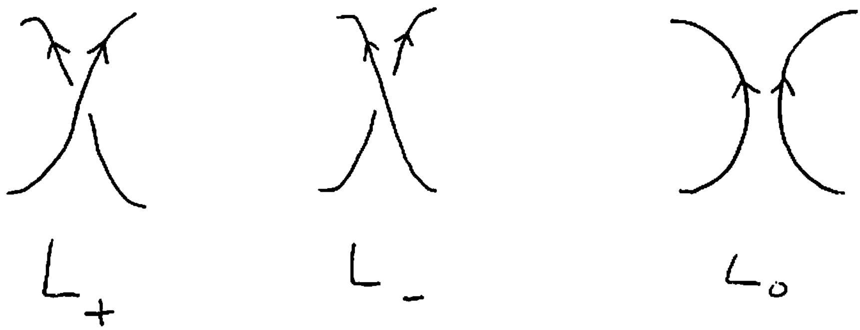


Figure 5

Other simple values used in many different theories about knots are the twist, writhe, and linking numbers of a given knot. We use a crossing number similar to the skein (Kauffman [3]; see figure 6).

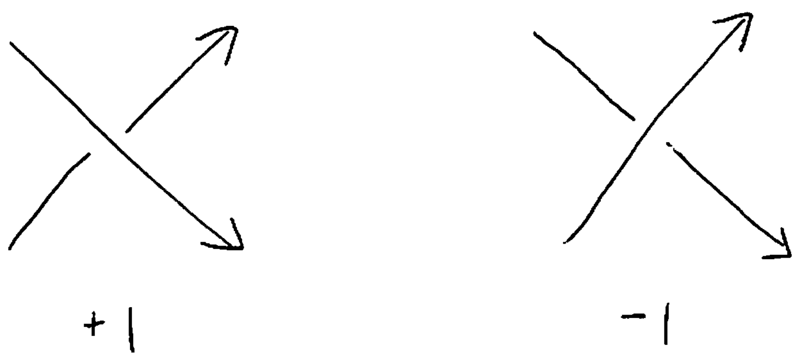


Figure 6

The linking number is then half of the sum of these crossing values of all of the different links of the knot configuration. It follows that if two knots are in fact linked, the linking number will be nonzero. If they are not linked, the linking number will be zero. We cannot say, however, that two links are not linked if they cannot be disentangled (Kauffman [3]). See figure 7. The writhe number is the sum of the crossing values obtained from the crossings within a single-link knot.

Consider a knot along with a parallel strand which follows the same crossing orders as the original knot. Now, we can compute the linking numbers again (Kauffman [3]; see figure 8).

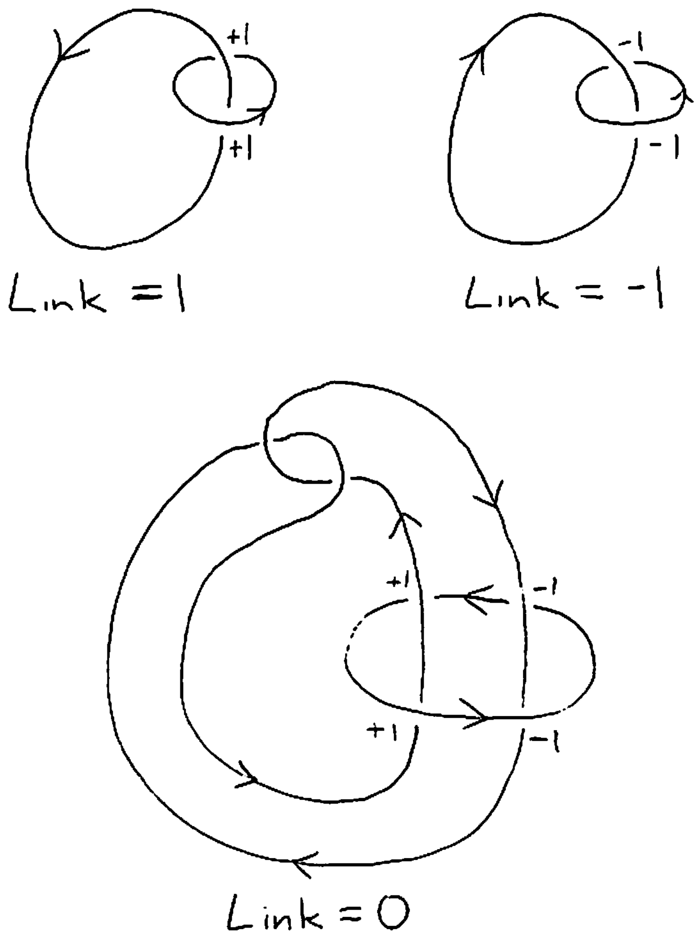


Figure 7

Now, what if we were able to twist the parallel strands one full positive revolution, and then compute the new linking number for the entire configuration? After this development, we define the knot's twisting number to be 1. It is easily proven that the linking number is always the sum of the writhe number (of the generating 1-link knot) and the twist number (Kauffman [3]; see figure 9). These numbers show up all over the place in many other approaches to knot theory.

The next significant work in the study of knots was done by Vaughan Jones in the early 1980's. He eventually devised another polynomial to classify knots. His work involves the use of braids, formed by the four

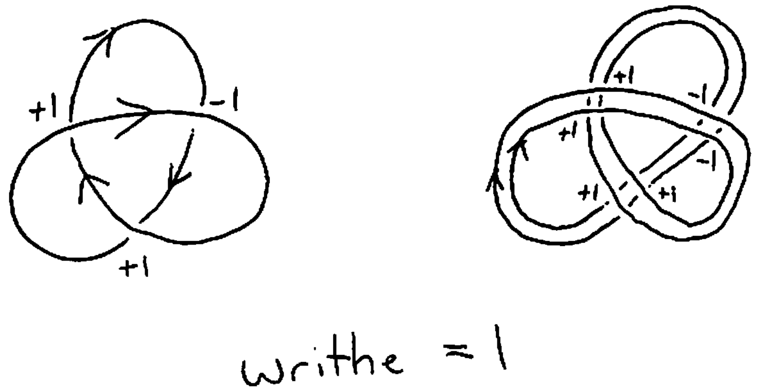


Figure 8

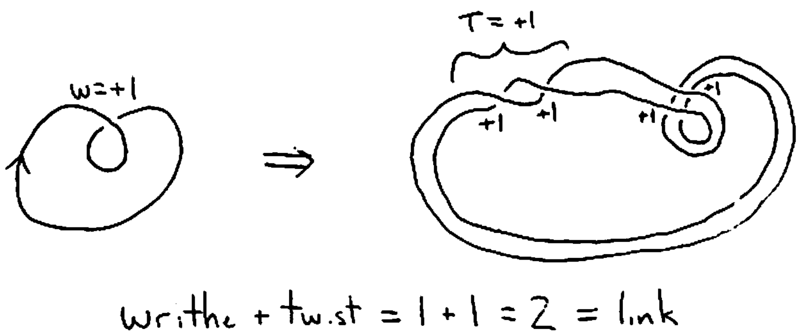


Figure 9

braid generators (see figure 10). He showed that any knot could be formed by ordering these braid generators (top to bottom), then connecting the loose ends at the bottom with the loose ends at the top. But, there are cases when more than one braid algorithm can be used to produce a given knot.

Braid generators can be manipulated, as an "operation," much like the way integers are manipulated in modern algebra, to form "groups" of knots. The Jones polynomial is actually calculated through a series of rules about how to cut up a knot or link into its simplest components. The algorithm records a factor for each part of the knot (Watson [8]).

To understand the Jones polynomial more thoroughly, it makes sense to first consider the bracket polynomial for knots. This method uses two knot components (much like the i and j components of vectors), paired with variables A and B in the following axiom (see figure 11):

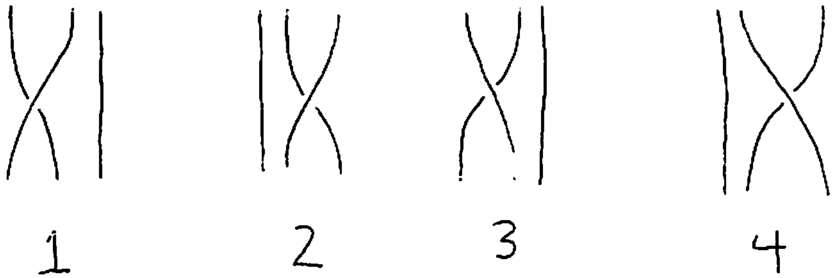


Figure 10

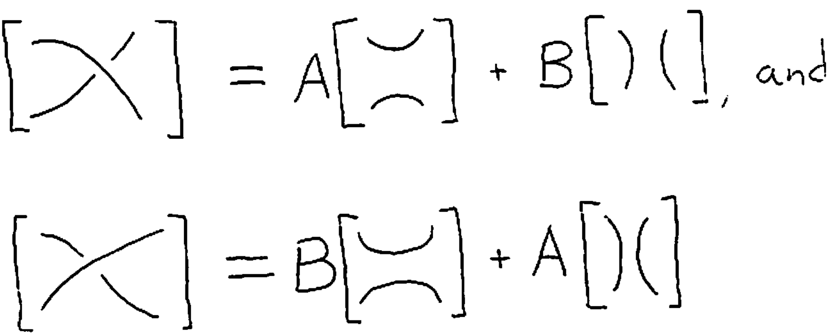


Figure 11

These crossings represent those present in unoriented knots, that is, knots with no direction put on the strand(s). An unoriented crossing yields two of each kind of region between strands. If we splice "across the A-way," we get the diagram in the A bracket, and the "B-way" gives us its diagram. This is true in both equations of the axiom. An example of how this is used to break up knots is in figure 12.

The other axiom in bracket theory states that we can rename any number n of disjoint simple closed curves as d^n. See, for example, figure 13.

The polynomial we obtain for any knot K is denoted $[K]$, and has component parts in A, B, and d. This, of course, may require a few Reidemeister moves and such to get started, especially if using braid generators to come up with new candidates for evaluation. This was the basis that the next pair of knot theorists needed for the discovery of the decade.

In May 1984, Vaughan Jones was meeting with Joan S. Birman, who had been studying the topological side of knot and braid theories for quite some time. Jones had vast experience in Von Neumann algebras, whose

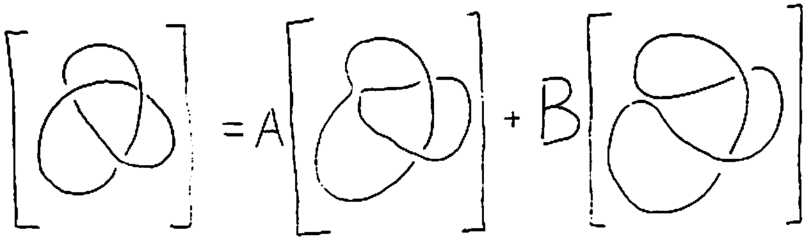


Figure 12

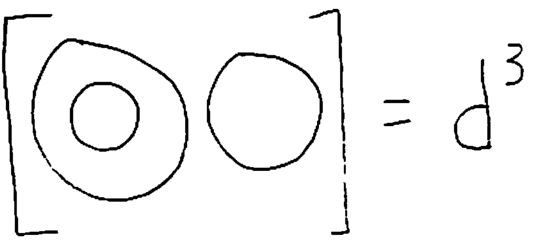


Figure 13

equations and properties happened to bear a strong resemblance to expressions found in knot theory and topological braid relations, thought Jones during their meetings. Using this combination, in concert with what he had learned about the Alexander polynomial and the skein relation, Jones invented a new invariant polynomial method for distinguishing knots. He knew that it would also be useful in statistical mechanics (which will be discussed later in the paper). After the word traveled, many other knot theorists went to work on another invariant: a two-variable polynomial composed of aspects of the Jones and Alexander polynomials, called HOMFLY (an acronym using the names of its inventors). Until 1990, it was the best one around (Jones [2]).

Although the Jones polynomial proved not to be the ultimate in knot invariants, it made an astonishing impact on the theory of knots. It does a better job of distinguishing knots than the Alexander method, since it picks up differences the Alexander polynomial misses, such as handedness; but the Jones polynomial is hard to compute for complex knots. So, Tait could have devised his tables in an exponentially shorter length of time. Subsequently, Jones earned the Fields Medal in 1990 for his work (Watson [8]).

The next popular method for dealing with knots was to first consider the space surrounding the knot, or the knot's complement. Intuitively, we can think of a bowl of jelly with a knot tube removed. This complement is considered to be a space, much like a topological or metric space. The 3-dimensional spaces, conceptually, make the most sense, although this is not a requirement of complement spaces (Epstein [1]). Jones and Birman sometimes considered knot compliments existing in continual dimensional space, meaning that it would have x dimensions, where x is any real number, including irrationals (Kolata [4]).

Then, in this complement space, the order of symmetry is considered one of the classification elements. Another way to form these knot complements is to consider an n-dimensional space, where n is the number of links in the knot. Researchers then bend the axes (with respect to hyperbolic and/or elliptic geometry) into the shape of the knot, then examine the resulting space. If you do this with the Borromean rings, you get a rhombic dodecahedron (Epstein [1]; see figure 14).

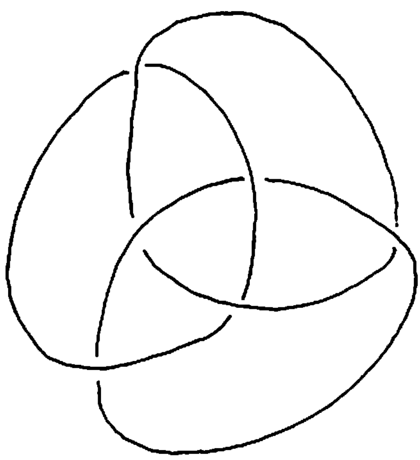


Figure 14. Borromean rings.

The limitation of this method is that is doesn't distinguish all knots that are different. Some different knots will generate the same space, but these are quite complex knots. So, we have yet another useful method (if conceivable by the involved persons) of knot classification.

Other methods of dealing with this problem have been explored in great detail, but sometimes to no avail. Bracket methods have been expanded and explored, but don't do as much as the Jones polynomial. The method of alternating knots and links involves the shading (in the diagram) of the different regions set apart in the knot; this method works only in knots where, if one travels along the strand, one encounters crossings in an alternating fashion: under, over, under, over, under, etc. Then, either all regions adjacent to A crossings (from bracket methods) or B crossings can be shaded without losing internal consistency. Counting the number of shaded regions yields the degree of the knot (Kauffman [3]). But, as mentioned earlier, it does not apply to all knots.

Braid generators can easily be related to algebraic structures, as mentioned earlier, and in the work of Birman and Jones. But, like the alternating knots and links, not all knots can be created with braid generators. And, distinguishability remains a problem (Kauffman [3]).

The Tutte polynomial is computed by determining how many loops and isthmus (loop with the strand running back through it) the knot has (but not at this oversimplified level). It is a recursive definition, and uses more variables than most other polynomials. Consequently, the ordering system is more in a tree format.

Countless mathematicians have tried to expand the HOMFLY polynomial, or combine other methods, much like the way HOMFLY was devised (Kauffman [3]). But, most have made little progress as a blanket method for all researchers.

In the field of topology, where knot theory is most often classified, all knots are considered to be homeomorphic to each other, since the topological definition of a knot states that a subset K of 3-space is a knot if there is a homeomorphism that maps the unit circle onto K. In other words, a knot is homeomorphic to any "closed curve" in topology. Sometimes it is useful to think of knot theory as a branch of 3-dimensional topology (Patty [5]).

As we investigate further, it is quite simple to show that knot equivalence (through Reidemeister moves) is indeed an equivalence relation. If a knot contains a finite number of crossings (or, considered to be the union of a finite number of line segments), it is defined to be a polygonal knot, and these segments are called its edges. The endpoints (intuitively, the "turning points") are the vertices of the knot. If a knot is equivalent to a polygonal knot, it is defined to be tame; if not, it is wild. Obviously, tame and wild knots will never be equivalent (Patty [5]).

Like most other methods of dealing with knots, topologists choose to consider the 3-dimensional knots using their 2-dimensional "shadows," but with visible order given to the types of crossings. The point in 2-space that the crossing gets "mapped down to" is called a double point. If the position of the knot is such that more than one "layer" gets mapped down to one point, this point is called a triple point, quadruple point, or "n-tuple point," as a general multiple point. Define a knot in regular position to have a finite number of multiple points which are all double points, where none of these double points is at a vertex of the knot. In each double point, the point whose z-image is larger is called the over-crossing; the other, the under-crossing (Patty [5]).

Here's where the real topology kicks in: any polygonal knot can be rotated into regular position. Also, the Reidemeister moves can be explained topologically using the notions of homotopy and isotopy (Patty [5]). Now, we are starting to get another classification system, although it's almost too rigorous to be useful in some cases.

Deeper into the topology, we use triangulation of a 3-manifold to form a topological tetrahedron and/or other 3-dimensional (Euclidean) solids which have corresponding knots, using their edges and vertices as in the above "knot" definitions. In this mode, it is quite easy to tell whether or not a knot will be chiral, by trying to put an orientation along the edges of the corresponding polyhedra (Patty [5]).

There are definitely those aspects of knot theory that are generally agreed upon (i.e., Reidemeister "equivalence" rules), and those parts of knot theory that are surrounded by conflict (polynomials, generators, etc.). However, we cannot discard knot theory in the face of uncomfortable conflict within mathematics or the shadow of internal inconsistencies, since, as we shall soon see, knot theory has practical applications that are useful and in high demand in other fields.

Even though knot theory seems to be too abstract for intuition at times, much less reality manifestation, there are cases where it has shown up outside conceptual mathematics. For example, the method of computing knot invariants that involves cutting and re-tying of the knot (bracket method) is a lot like the way that enzymes break up strands of protein, or even DNA molecules, and then recombine them. So, molecular biology is very interested in these recombination moves of knot theory. Previously, biologists did what early mathematicians did. They took photographs with the electron microscopes, then got out the string and tried to make their own discoveries about the knots. That is, until they discovered that the mathematicians were already working on it. The math was especially helpful in determining if and how many algorithm(s) of Reidemeister moves and/or orders of recombinations existed. Then, they could track what actually happened to the molecule when exposed to the enzyme — step by step (Peterson [6]).

Then, with this data, they were able to predict what other types of strands and configurations could and do exist. In many cases, they have found their predictions existing in the natural world. But, they believe that these problems are sufficiently solved; any knot that will show up in this type of biology work will be easily classifiable in the system they use (Peterson [6]).

In addition to Lord Kelvin's theories of knots and their relationship to the study of physics, scientists now believe that there is a connection between the theory of knots and the interactions of elementary particles. The crossings in a (schematic) diagram of a knot are used to represent the different types of interactions. The specific knot is a sort of summation of all of the different types of interactions the two particles (each having its own link) can have (Peterson [6]).

Topics of importance in statistical mechanics include the behavior of molecules in a condensing vapor and the lining up of electron spins when a material becomes magnetized. Knot theorist Louis Kauffman says, "In a physical situation, you often have a summation over a lot of different interactions that can happen, and the [knot] invariants seem to be ... averages over all these different possibilities." Some mathematicians have even been leaning on the physicists for the equations they use to try to come up with new invariants. In any case, both fields are finding out that these physical theories have a lot to do with the mathematics, and vice versa. A problem in either one could be converted into a problem for the other to deal with, and perhaps more easily (Peterson [6]).

Some physicists are now theorizing that the fundamental nature of reality is composed much like that of a very complex coat of chain mail. Abhay Ashtekar and Lee Smolin at Syracuse University are investigating the behavior of the sub-sub-microscopic fabric of space and time. If their theories are correct, it is all best understood as a densely woven skein of loops, coils, and braids. A lot of this theory relates directly to the big bang theory of the creation of the universe. However, they pride themselves in using data based on the "solid ground" of Einsteinian relativity and gravity theories and quantum mechanics, which are arguably hard to grasp [7]. If they are correct, the demand for progress in knot theory will be staggering, and the possibilities endless.

After a rough survey of knot theory, it is intended that the reader should be able to explain the basic concepts and the motivations behind some of the related methodologies of knot theory. Also, it is important to consider the various applications that have turned up for knot theorists. Even though the theory of knots is in its developing stages, it has come a long way, manifesting itself as a definite necessity in the world of mathematics.

Acknowledgements. I would like to thank Dr. Maura Mast for her time and advice regarding my research and paper. I would also like to thank our faculty advisor, John Cross, for sparking my interest in KME, and "taking care of all of us" here at the Iowa Alpha Chapter at the University of Northern Iowa.

References

1. Epstein, David and Gunn, Charlie, *Not Knot* (video and supplement), A. K.

Peters, Minneapolis, 1991.

2. Jones, Vaughan F. R., "Knot Theory and Statistical Mechanics," *Scientific American* **263** (Nov. 1990), 98–103.
3. Kauffman, Louis H., "New Invariants in the Theory of Knots," *American Mathematical Monthly* **95** (March 1988), 195–242.
4. Kolata, Gina, "Solving Knotty Problems in Math and Biology (work of V. Jones)," *Science* **231** (Mar. 28, 1986), 1506–1508.
5. Patty, C. Wayne, *Foundations of Topology*, PWS-Kent Pub. Co., Boston, 1993 (pp. 274–279).
6. Peterson, Ivars, "Unknotting a Tangled Tale," *Science News* **133** (May 21, 1988), 328–330.
7. Waldrop, M. Mitchell, "Viewing the Universe as a Coat of Chain Mail," *Science* **250** (Dec. 14, 1990), 1510–1511.
8. Watson, Andrew, "Twists, Tangles, and Topology," *New Scientist* **132** (Oct. 5, 1993), 42–46.

Top Tens

Top ten chapters in numbers of student papers appearing in *The Pentagon*, including this issue:

1.	Kansas Gamma	23
2.	Iowa Alpha	20
T3.	Kansas Beta	16
T3.	Missouri Alpha	16
5.	Kansas Delta	13
6.	Pennsylvania Lambda	10
T7.	Kansas Alpha	8
T7.	Michigan Alpha	8
T7.	Missouri Beta	8
T7.	New York Alpha	8

Top ten states in number of student papers (from schools with KME chapters only) appearing in *The Pentagon*:

1.	Kansas	64
2.	Missouri	41
3.	Iowa	28
4.	Pennsylvania	22
T5.	New York	13
T5.	Michigan	13
T7.	Illinois	11
T7.	Ohio	11
9.	California	7
T10.	Colorado, Texas, Wisconsin	6

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before January 1, 1997. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 1997 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

PROBLEMS 495–499

Problem 495. Proposed jointly by Sammy and Jimmy Yu, students at the University of South Dakota, Vermillion, South Dakota.

Evaluate

$$\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{b-c}{2}\right)\cos\left(\frac{c-a}{2}\right)$$

if $\sin a + \sin b + \sin c = \cos a + \cos b + \cos c = 0$.

Problem 496. Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

Find the smallest positive integer that can be increased by 50% by moving the digit on the extreme right to the extreme left.

Problem 497. Proposed by Charles Ashbacher, Cedar Rapids, Iowa.

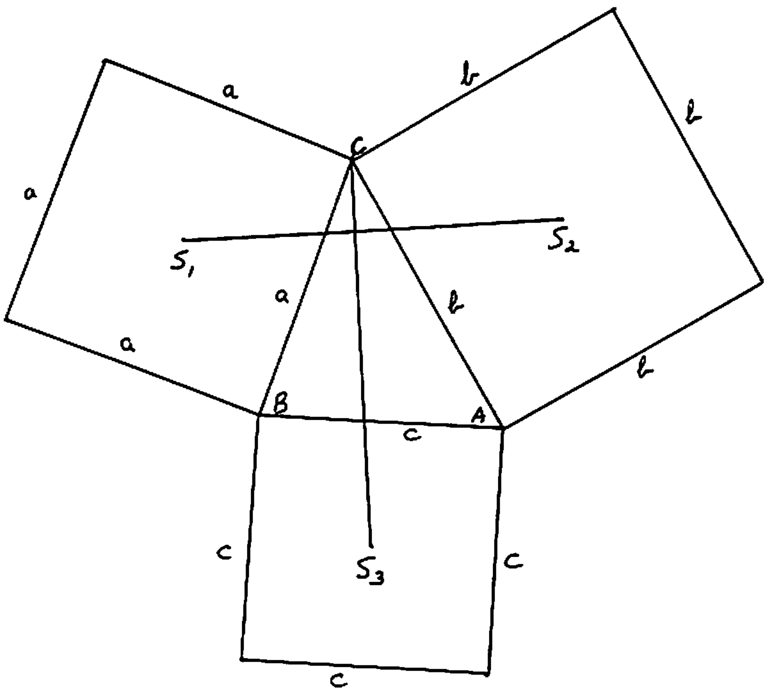
The Smarandache function $S(n)$ is defined in the following way: $S(n) = m$ is the smallest integer such that n evenly divides $m!$. The Euler phi function $\phi(n)$ is defined by letting $\phi(n)$ be the number of positive integers less than or equal to n that are relatively prime to n. Prove the following:

(a) The equation $S(\phi(n)) = n$ has no solution.

(b) The equation $n - S(\phi(n)) = 1$ has an infinite number of solutions.

Problem 498. Proposed by Oscar R. Casteneda, Southwest High School, San Antonio, Texas.

Let ABC be an arbitrary triangle with sides of lengths a, b and c. Contruct squares facing outward on each of the sides of the triangle. Prove that the length of the line segment S_1S_2 connecting the centers of two adjacent squares equals the length of the line segment CS_3 connecting the center of the third square with the common point of the other two squares. Also prove that these two line segments are perpendicular.



Problem 499. Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.

The Fibonacci numbers are defined by $F_0 = 0$, $F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for $n = 0, 1, 2, \ldots$. Evaluate the following expression for all integers $n \geq 1$:

$$F_{2n+1}F_{2n-1} - {F_{2n}}^2 - {F_{n+1}}^2{F_{n-1}}^2 + 2F_{n+1}{F_n}^2F_{n-1} - {F_n}^4.$$

Please help your editor by submitting problem proposals.

SOLUTIONS 477, 485–489

Problem 477. Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

Let n be an integer ≥ 2. Express

$$\sum_{k=2}^{n} \binom{n}{k-2}\binom{n}{k}$$

as a binomial coefficient and prove that your equality is correct.

Solution by Carl Libis, University of Southwestern Louisiana, Lafayette, Louisiana.

More generally we shall show that

$$\sum_{k=j}^{n} \binom{n}{k-j}\binom{n}{k} = \binom{2n}{n-j}.$$

Repeated use of the relation $\binom{m}{k} = \binom{m-1}{k}\binom{m-1}{k-1}$ and the fact that $\binom{m}{k} = 0$ for $k > m$ yields the following proof.

$$\begin{aligned}
\binom{2n}{n-j} &= \binom{2n-0}{n-j-0} = \binom{2n-1}{n-j-0} + \binom{2n-1}{n-j-1} \\
&= \binom{2n-2}{n-j-0} + 2\binom{2n-2}{n-j-1} + \binom{2n-2}{n-j-2} \\
&= \binom{2n-3}{n-j} + 3\binom{2n-3}{n-j-1} + 3\binom{2n-3}{n-j-2} + \binom{2n-3}{n-j-3} \\
&= \sum_{k=0}^{3} \binom{3}{k}\binom{2n-3}{n-j-k} \\
&= \cdots = \sum_{k=0}^{n} \binom{n}{k}\binom{2n-n}{n-j-k} = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-j-k} \\
&= \sum_{k=0}^{n} \binom{n}{k}\binom{n}{j+k} = \sum_{k=j}^{n+j} \binom{n}{n-k}\binom{n}{k} = \sum_{k=j}^{n} \binom{n}{k-j}\binom{n}{k}.
\end{aligned}$$

In particular for $j = 2$ we have

$$\sum_{k=2}^{n} \binom{n}{k-2}\binom{n}{k} = \binom{2n}{n-2}.$$

Also solved by: David Bayne, Missouri Western State College, St. Joseph, Missouri; Clayton Dodge, University of Maine—Orono, Orono, Maine; Russell Euler, Northwest Missouri State University, Maryville, Missouri and the proposer.

Problem 485. Proposed by Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin.

Find the sum of the following infinite series.

$$\sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{m\binom{2m}{m}}$$

Solution by Russell Euler, Northwest Missouri State University, Maryville, Missouri.

It is well known [1] that if $x \leq 1$ then

$$\text{(1)} \qquad \sum_{m=0}^{\infty} \frac{(-1)^m 2^{2m}(m!)^2 x^{2m+1}}{(2m+1)!} = \frac{\ln\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}.$$

In particular, if $x = .5$ then (1) becomes

$$\sum_{m=0}^{\infty} \frac{(-1)^m (m!)^2}{2(2m+1)!} = \frac{\ln(.5+\sqrt{1.25})}{\sqrt{1.25}}$$

and so we have

$$\text{(2)} \qquad \sum_{m=1}^{\infty} \frac{(-1)^{m-1}[(m-1)!]^2}{2(2m-1)!} = \frac{2\ln\left((1+\sqrt{5})/2\right)}{\sqrt{5}}.$$

Note that

$$\text{(3)} \qquad \frac{[(m-1)!]^2}{2(2m-1)!} = \frac{(m!)^2}{m(2m)(2m-1)!} = \frac{1}{m\binom{2m}{m}}.$$

Substituting (3) into (2) shows that the sum of the given series is

$$(2/\sqrt{5})\ln((1+\sqrt{5})/2).$$

Also solved by: the proposer. One incorrect solution was received.

[1] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series and Products*, fifth edition, Academic Press, 1994, p. 53.

Problem 486. Proposed by T. Yau, Pima Community College, Tucson, Arizona.

Consider the Smarandache function $S(n)$ which is defined as the smallest integer such that $S(n)!$ is divisible by n. Find $\max\{S(n)/n\}$ over all positive composite integers $n \neq 4$.

Solution by Troy VanAken, University of Evansville, Evansville, Illinois.

We shall show that the maximum of $S(n)/n$ over all positive composite integers $n \neq 4$ is $2/3$. When n is even, n divides $(n/2)!$ for $n > 4$. Hence $S(n)/n \leq (n/2)/n = 1/2$. When n is odd we distinguish two cases.

Case (a). Suppose that $n = p^2$ for some odd prime p. Then $S(n) = 2p$ so $S(n)/n = 2/p \leq 2/3$ with equality for $p = 3$.

Case (b). Suppose that $n = pm$ where p is the smallest prime divisor of n, m is odd and positive and $m > p$. Then $S(n) \leq m$ and thus $S(n)/n \leq m/n = 1/p < 2/3$. This completes the proof.

Also solved by: Charles Ashbacher, Des Moines, Iowa; Clayton Dodge, University of Maine—Orono, Orono, Maine; Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin and the proposer.

Editor's comment. Bob Prielipp notes that this same problem appeared as problem 4528 in the May 1995 issue of *School Science and Mathematics*.

Problem 487. Proposed by the editor.

Suppose that the sides of triangle ABC are all integers. If the measure of angle A is four times the measure of angle B, find the smallest possible integer lengths for the sides of triangle ABC.

Solution by Sammy and Jimmy Yu, jointly, students at the University of South Dakota, Vermillion, South Dakota.

By the Law of Cosines, since a, b and c are integers, $2\cos B = (a^2 + c^2 - b^2)/ac$ is a rational number. Let $2\cos B = m/n$ where m and n are both positive integers and $(m,n) = 1$. Now applying the Law of Sines, we have

$$\begin{aligned} a/b &= \sin A/\sin B = \sin 4B/\sin B \\ &= 2(2\sin B\cos B)(2\cos^2 B - 1) \\ &= (2\cos B)^3 - 2(2\cos B) \\ &= (m^3 n - 2mn^2)/n^3. \qquad (1) \end{aligned}$$

Also

$$\begin{aligned} c/b &= \sin C/\sin B = \sin(\pi - 5B)/\sin B = \sin 5B/\sin B \\ &= (\sin 4B\cos B + \cos 4B\sin B)/\sin B \end{aligned}$$

$$= \left((2\cos B)^3 - 2(2\cos B)\right)\cos B + 2(2\cos^2 b - 1)^2 - 1$$

$$(2) \qquad = (2\cos B)^4 - 3(2\cos B)^2 + 1 = (m^4 - 3m^2n^2 + n^4)/n^4.$$

Combining (1) and (2) we have

$$(3) \qquad a : b : c = m^3n - 2mn^3 : n^4 : m^4 - 3m^2n^2 + n^4.$$

Then since $(m, n) = 1$, the highest common factor above is 1 and we can take $a = m^3n - 2mn^3$, $b = n^4$, and $c = m^4 - 3m^2n^2 + n^4$. Next, since $A + B + C = \pi = 5B + C$, then $0 < 5B < \pi$ or $0 < B < \pi/5$. Hence

$$(4) \qquad \cos(\pi/5) < m/2n < \cos 0 \text{ or } 1.618 < m/n < 2.$$

The smallest integers m and n such that $(m, n) = 1$ which also satisfy (4) are $m = 5$ and $n = 3$, which result in the triangle ABC having the smallest integral sides a, b and c with $a = 105$, $b = 81$ and $c = 31$.

Also solved by: Clayton Dodge, University of Maine—Orono, Orono, Maine; David Bayne, Missouri Western State College, St. Joseph, Missouri; and Jackie Roehl, student, Austin Peay University, Clarksville, Tennessee. One incorrect solution was received.

Problem 488. Proposed by the editor.

Prove or disprove that

$$\sqrt{5} + \sqrt{26 + 2\sqrt{17}} = \sqrt{13 + 2\sqrt{38}} + \sqrt{18 - 2\sqrt{38} + 2\sqrt{65 - 10\sqrt{38}}}.$$

Solution I by Clayton W. Dodge, University of Maine—Orono, Orono, Maine.

Since we have

$$\left(\sqrt{5} + \sqrt{13 - 2\sqrt{38}}\right)^2 = 5 + 2\sqrt{5}\sqrt{13 - 2\sqrt{38}} + 13 - 2\sqrt{38}$$

$$(1) \qquad = 18 - 2\sqrt{38} + 2\sqrt{65 - 10\sqrt{38}},$$

then

$$(2) \qquad \sqrt{18 - 2\sqrt{38} + 2\sqrt{65 - 10\sqrt{38}}} = \sqrt{5} + \sqrt{13 - 2\sqrt{38}}.$$

Next we see that

$$(3) \qquad 2\sqrt{17} = 2\sqrt{169 - 152}$$

and

$$(4) \qquad 26+2\sqrt{17}=13+2\sqrt{38}+2\sqrt{169-4\cdot 38}+13-2\sqrt{38}.$$

Taking square roots of both sides of (4) we have

$$\sqrt{26+2\sqrt{17}}=\sqrt{13+2\sqrt{38}}+\sqrt{13-2\sqrt{38}}$$

or

$$\sqrt{5}+\sqrt{26+2\sqrt{17}}=\sqrt{13+2\sqrt{38}}+\sqrt{5}+\sqrt{13-2\sqrt{38}},$$

and finally using (2) we have

$$\sqrt{5}+\sqrt{26+2\sqrt{17}}=\sqrt{13+2\sqrt{38}}+\sqrt{18-2\sqrt{38}+2\sqrt{65-10\sqrt{38}}}.$$

Solution II by Sammy and Jimmy Yu, jointly, students at the University of South Dakota, Vermillion, South Dakota.

Let

$$(1) \qquad p\pm 2\sqrt{q}=\left(\sqrt{\alpha}\pm\sqrt{\beta}\right)^2=(\alpha+\beta)\pm 2\sqrt{\alpha\beta}$$

where p, q,α and β are all positive and $\alpha\geq\beta$. Then $\alpha+\beta=p$ and $\alpha\beta=q$ so that α and β are the roots of the equation $x^2-px+q=0$ where $\alpha\geq\beta$. Now the given equality can be shown as follows:

$$\begin{aligned}
\text{RHS} &= \sqrt{13+2\sqrt{38}}+\sqrt{18-2\sqrt{38}+2\sqrt{65-10\sqrt{38}}}\\
&= \sqrt{13+2\sqrt{38}}+\sqrt{5+(13-2\sqrt{38})+2\sqrt{5}\sqrt{13-2\sqrt{38}}}\\
(2) \qquad &= \sqrt{13+2\sqrt{38}}+\sqrt{5}+\sqrt{13-2\sqrt{38}}.
\end{aligned}$$

Now consider $\sqrt{13+2\sqrt{38}}$. By taking $p=13$ and $q=38$ in (1) and taking α and β as the roots of the equation $x^2-13x+38=0$, $\alpha=(13+\sqrt{17})/2$ and $\beta=(13-\sqrt{17})/2$. Hence

$$\sqrt{13+2\sqrt{38}}=\sqrt{(13+\sqrt{17})/2}+\sqrt{(13-\sqrt{17})/2}$$

and

$$\sqrt{13-2\sqrt{38}}=\sqrt{(13+\sqrt{17})/2}-\sqrt{(13-\sqrt{17})/2}.$$

Thus

$$\begin{aligned}
&\sqrt{13+2\sqrt{38}}+\sqrt{5}+\sqrt{13-2\sqrt{38}} \\
&\quad=\sqrt{(13+\sqrt{17})/2}+\sqrt{(13-\sqrt{17})/2}+\sqrt{5} \\
&\qquad+\sqrt{(13+\sqrt{17})/2}-\sqrt{(13-\sqrt{17})/2} \\
&\quad=2\sqrt{(13+\sqrt{17})/2}+\sqrt{5} \\
&\quad=\sqrt{5}+\sqrt{26+2\sqrt{17}}=\text{ LHS},
\end{aligned}$$

which completes the proof.

Also solved by: David Bayne, Missouri Western State College, St. Joseph, Missouri; Robyn M. Carley, student, Austin Peay University, Clarksville, Tennessee; Russell Euler, Northwest Missouri State University, Maryville, Missouri; Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin; and Jackie Roehl, student, Austin Peay University, Clarksville, Tennessee. One incomplete solution and one incorrect solution were also received.

Editor's comment. Most solutions were similar to Dodge's solution. The other solution was given to show a more general approach. For a related problem, see problem 169, *Crux Mathematicorum* (Eureka) Vol. II, 1976, pp. 233–234.

Problem 489. Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.

The Pell numbers P_n and their associated numbers Q_n satisfy the relations $P_{n+2}=2P_{n+1}+P_n$, $P_0=0$, $P_1=1$, and $Q_{n+2}=2Q_{n+1}+Q_n$, $Q_0=1$, $Q_1=1$. Show that (a) $P_{n+1}=(Q_n+Q_{n+1})/2$ and (b) $Q_{n+1}=P_n+P_{n+1}$.

Solution by Scott H. Brown, Auburn University, Auburn, Alabama.

The Pell numbers, which satisfy the recurrence relation

$$P_{n+2}=2P_{n+1}+P_n,\ P_0=0,\ P_1=1, \tag{1}$$

have the Binet form

$$P_n=(\alpha^n-\beta^n)/\sqrt{8} \tag{2}$$

for n a nonnegative integer. The associated numbers Q_n in this problem, which satisfy the recurrence relation

$$Q_{n+2}=2Q_{n+1}+Q_n,\ Q_0=1,\ Q_1=1, \tag{3}$$

have the Binet form

$$Q_n = (\alpha^n + \beta^n)/2 \tag{4}$$

for n a nonnegative integer. Note that $\alpha = 1 + \sqrt{2}$ and $\beta = 1 - \sqrt{2}$ satisfy both recurrence relations (1) and (3). To prove part (a) we have

$$\begin{aligned} P_{n+1} &= (\alpha^{n+1} - \beta^{n+1})/\sqrt{8} = \left(\alpha^n(1+\sqrt{2}) - \beta^n(1-\sqrt{2})\right)/\sqrt{8} \\ &= \left(\alpha^n(\sqrt{2}+2)/2 + \beta^n(2-\sqrt{2})/2\right)/2 \end{aligned} \tag{5}$$

and

$$\begin{aligned} (Q_n + Q_{n+1})/2 &= \left((\alpha^n + \beta^n)/2 + (\alpha^{n+1} + \beta^{n+1})/2\right)/2 \\ &= \left(\alpha^n((1+\alpha)/2) + \beta^n((1+\beta)/2)\right)/2 \\ &= \left(\alpha^n(\sqrt{2}+2)/2 + \beta^n(2-\sqrt{2})/2\right)/2 \\ &= P_{n+1} \quad \text{(by (5))}. \end{aligned}$$

To prove part (b) we have

$$\begin{aligned} Q_{n+1} &= (\alpha^{n+1} + \beta^{n+1})/2 = \left(\alpha^n(1+\sqrt{2}) + \beta^n(1-\sqrt{2})\right)/2 \\ &= \left(\alpha^n(\sqrt{2}+2) + \beta^n(\sqrt{2}-2)\right)/\sqrt{8} \\ &= (\alpha^n(\alpha+1) + \beta^n(-\beta-1))/\sqrt{8} \\ &= (\alpha^n - \beta^n)/\sqrt{8} + (\alpha^{n+1} - \beta^{n+1})/\sqrt{8} \\ &= P_n + P_{n+1}. \end{aligned}$$

Also solved by: Charles Ashbacher, Cedar Rapids, Iowa; Oscar Robert Casteneda, Southwest High School, San Antonio, Texas; Paul R. Coe, Rosary College, River Forest, Illinois; Clayton Dodge, University of Maine—Orono, Orono, Maine; Carl Libis, University of Southwestern Louisiana, Lafayette, Louisiana; Todd Mateer, Grove City College, Grove City, Pennsylvania; Bob Prielipp, University of Wisconsin—Oshkosh, Oshkosh, Wisconsin; Jackie Roehl, student, Austin Peay University, Clarksville, Tennessee; Troy VanAken, University of Evansville, Evansville, Illinois; and the proposer.

"Those who have solved have triumphed."

Kappa Mu Epsilon News

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy *KME* events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, Missouri 64801.

CHAPTER NEWS

AL Beta — Chapter President — Miranda Williams

University of North Alabama, Florence — 26 actives, 10 associates

Other 1995–96 chapter officers: Tamra May, vice president; Caacie Brown, secretary; Eddy J. Brackin, corresponding secretary; Patricia Roden, faculty sponsor.

AL Gamma — Chapter President — Jamie Tallie

University of Montevallo, Montevallo — 5 associates

Other 1995–96 chapter officers: Timo Langerwerf, vice president; Kim Snider, secretary; Terra Cottingham, treasurer; Larry Kurtz, corresponding secretary; Don Alexander, faculty sponsor.

AR Alpha — Chapter President — Cindy Nicholson

Arkansas State University, State University — 27 actives, 10 associates

Other 1995–96 chapter officers: Donna Shepherd, secretary; Odis Cook, treasurer; William Paulsen, corresponding secretary/faculty sponsor.

CA Delta — Chapter President — Sean Smith

California State Polytechnic University, Pomona — 10 actives, 3 associates

Activities of CA Delta included weekly meetings, problem solving sessions, and planning sessions. Other 1995–96 chapter officers: Steven Guertin, vice president; Jennifer Baird, secretary; Maria Muñoz, treasurer; Richard Robertson, corresponding secretary; Jim McKinney, faculty sponsor.

CO Gamma — Chapter President — Daren Gemoets

Fort Lewis College, Durango — 25 actives

Two meetings were held during the fall semester. On November 15, three new members were initiated. Pizza and soft drinks were served. Plans are underway to attend the Region V Convention at Mesa State College in April of 1996. Other 1995–96 chapter officers: Tom Bruckner, vice president; Ben Moore, secretary; Stevan Scott, treasurer; Richard Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

CO Delta Chapter President — Scott B. Davis
Mesa State College, Grand Junction 14 actives

CO Delta chapter began the 1995–96 academic year with a September picnic in Hawthorne Park. Keys and certificates were presented to those initiated last April; the members who attended the Thirtieth Biennial Convention in Durango reported to the group; and plans for the academic year were discussed. In October, the decision was made to host the 1996 Region V Convention. This meeting will be held jointly with the Rocky Mountain and Intermountain Sections of the MAA on April 19–20, 1996, on the Mesa State College campus. Other 1995–96 chapter officers: Venus L. Martinez, vice president; Natisha R. Kimminau, secretary; Tammi I. Giroir, treasurer; Donna K. Hafner, corresponding secretary; Clifford C. Britton, faculty sponsor.

CT Beta Chapter President — Kerry Foust
Eastern Connecticut State University, Willimantic 12 actives

Other 1995–96 chapter officers: Margaret Weaver, vice president; Laura Dawley, secretary; Terri Boshka, treasurer; Mizan Khan, corresponding secretary/faculty sponsor.

FL Beta Chapter President — Tammy Causey
Florida Southern College, Lakeland 18 actives

Other 1995–96 chapter officers: Shannon Tomarchio, vice president; Bradley Hof, secretary; Gayle S. Kent, corresponding secretary/faculty sponsor.

GA Alpha Chapter President — Chris Flournoy
West Georgia College, Carrollton 25 actives

The Georgia Alpha chapter of KME once again sponsored a food and clothing drive for the needy. The Fall Social, held in November at a local restaurant, was enjoyed by all who attended. Other 1995–96 chapter officers: Helga Floodquist, vice president; Amy Westbrook, secretary; Darron Robbins, treasurer; Joe Sharp, corresponding secretary/faculty sponsor; Mark Faucette, faculty sponsor.

IL Beta Chapter President — Andrew Gherna
Eastern Illinois University, Charleston 37 actives

In September, at the first meeting of the year, members heard a talk by Dr. Gregory Galperin entitled "Geometric Kaleidoscope." In addition to holding regular meetings, members enjoyed a Math Club/KME picnic in the fall and a Christmas party in December. Other 1995–96 chapter officers: Lisa Stranz, vice president; Amanda Fejedelem/Sheila Simmons, secretaries; Sarah Schuette/Jennifer Feig, treasurers; Lloyd Koontz, corresponding secretary; Lloyd Koontz/Patrick Coulton, faculty sponsors.

IL Delta Chapter President — Mike Mravle
College of St. Francis, Joliet 10 actives

Other 1995–96 chapter officers: Heather McNulty, vice president; Linda Wunder, secretary; John Salzer, treasurer; Rick Kloser, corresponding secretary/faculty sponsor.

IN Delta Chapter President — Steven Broad
University of Evansville, Evansville 84 actives, 24 associates

Other 1995–96 chapter officers: Carl Bergh, vice president; Glen Templeton, secretary; Troy D. VanAken, corresponding secretary; Mohammad Azarian, faculty sponsor.

IA Alpha Chapter President — Jack Dostal
University of Northern Iowa, Cedar Falls 34 actives

Students presenting papers at Iowa Alpha chapter meetings included Julie Rullan on "A Brief Overview of Transfinite Numbers," Jim Coons on "An Introduction to Carmichael Numbers," and Matt Schafer on "Probabilistic Combinations in the Game of Blackjack." Brad Klaes gave the banquet address for the fall initiation of six new members. His topic was "Econometric Analysis of Land Values in Iowa." The annual KME Homecoming Coffee was held on October 7, 1995, at the home of Professors Emeritus Carl and Wanda Wehner. KME members assisted with the Fall Phonathon for the Mathematics Department and helped with the Mathematics-Science Symposium in November. Other 1995–96 chapter officers: Andrew Christianson, vice president; Jim Coons/Andy Schafer, secretaries; Mary Pittman, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

IA Gamma Chapter President — Jason Shriver
Morningside College, Sioux City 8 actives

Other 1995–96 chapter officers: Jared Elwein, vice president; Heath Hopkins, secretary; Heather Schott, treasurer; Steve Nimmo, corresponding secretary/faculty sponsor.

IA Delta Chapter President — Gretchen Roth
Wartburg College, Waverly 39 actives, 2 associates

The first meeting of the year was held on September 20. Adam Sanford reported on the April, 1995 KME National Meeting in Durango, Colorado, and presented "Special Curves Connected," the paper he had given at that meeting. Charles Leohr, Operations/Network Manager for the Wartburg Computer Center, presented "Surf the NET" at the October 18 meeting. On November 29, members helped decorate a Christmas tree for the Festival of Trees and enjoyed refreshments at the Christmas party. Other 1995–96 chapter officers: Adam Sanford, vice president; Lori Melaas, secretary; Amy Betz, treasurer; August Waltmann, corresponding secretary; Robin Pennington, faculty sponsor.

KS Alpha — Chapter President — Shelly Milledge

Pittsburg State University, Pittsburg — 60 actives, 4 associates

The chapter held monthly meetings in October, November, and December. Fall initiation was held at the October meeting. Four new members were initiated at that time. The meeting was preceded by a pizza party. The chapter hosted a guest speaker for the November meeting. Dr. Ilene H. Morgan of the University of Missouri—Rolla presented an interesting program on "Weird Dice and Polynomials." In December, a special pre-final exam and pre-Christmas social event was held at the home of Dr. Harold Thomas, Kansas Alpha corresponding secretary. The group viewed one of the award-winning paper presentations given at the Durango, Colorado, national convention. They also enjoyed several culinary delights prepared by faculty members or spouses. Other 1995–96 chapter officers: Bethany Schnackenberg, vice president; Shannon Wilkinson, secretary; Melissa Marsalis, treasurer; Harold L. Thomas, corresponding secretary; Cynthia Woodburn/Bobby Winters, faculty sponsors.

KS Beta — Chapter President — Kendra Dawson

Emporia State University, Emporia — 33 actives, 11 associates

Other 1995–96 chapter officers: Ryan Karjala, vice president; Dustin Frank, secretary; Justin Elliott, treasurer; Connie S. Schrock, corresponding secretary; Larry Scott, faculty sponsor.

KS Gamma — Chapter President — Dawn Weston

Benedictine College, Atchison — 14 actives, 11 associates

Early in the fall semester, chapter members met at lunch in the cafeteria to elect officers and formulate plans. A "hogie night" followed soon after on September 24 in Schroll Center. The chapter sponsored two coffee nights at the Roost as a fund-raising effort. Dawn Weston became Professor Raven for the problems which appear in the campus newspaper each issue. Initiation of Christie Engelbert was held on October 30. Following the initiation, math education students demonstrated their "home pages" which they had prepared for a course for teachers. On December 5, Kansas Gamma sponsored student presentations. Jimmy Wang and Gregory Boucher demonstrated a software application which they developed as a CS Seminar project. Gregory also gave the fractal talk which he presented at the national convention last spring. Retired professor Jim Ewbank hosted the chapter for the KME wassail at his home on December 8. Other 1995–96 chapter officers: Seth Spurlock, vice president; Christie Engelbert, associate vice president; Bryan Speck, secretary/treasurer; Eric Schultz, Stugo Rep; Jo Ann Fellin, OSB, corresponding secretary/faculty sponsor.

KS Delta Chapter President — Daniel Wessel
Washburn University, Topeka 37 actives

Kansas Delta chapter joined with the Washburn Mathematics Club for two events during the fall semester. A mathematics picnic was held in September and in November, Jeff Brown, a recent graduate of the department, discussed the field of actuarial science. Also in November, many KME students assisted the department with its annual Math Day. On this day a competitive examination was given to 500 area high school students. Other 1995–96 chapter officers: Kim Bell, vice president; Jim Stinson, secretary; Alex Alejandro, treasurer; Allan Riveland, corresponding secretary; Gary Schmidt, faculty sponsor.

KS Epsilon Chapter President — Crystal Holdren-Vacura
Fort Hays State University, Fort Hays 26 actives, 1 associate

Other 1995–96 chapter officers: Amy Kresin, vice president; Jerrod Hofaker, secretary/treasurer; Ellen Veed, corresponding secretary; Mary Kay Schippers, faculty sponsor.

KY Alpha Chapter President — Crystal Colwell
Eastern Kentucky University, Richmond 25 actives

The semester began with a fund raiser, the sale of floppy disks to students in the computer literacy class and the *Mathematica* class. Officers were elected the first meeting of the year and were installed following the election. In September, new faculty member Ray Tennant gave a lively talk on "Non-Euclidean Tessellations and the Incredible Hyperbolic Shrinking Fish." In mid-October a picnic for faculty and students was held at Million Park. The event featured volleyball, softball, and good food. The November meeting included viewing a videotape of one of the talks from the 1995 National KME Convention. The last activity of the semester was the Christmas party with the traditional white elephant gift exchange. Rules were changed slightly so that a gift could only be stolen three times. There were a lot of good gifts brought this year; one male Chinese grad student ended up with earrings. Other 1995–96 chapter officers: Eva Richardson, vice president; Susan Mattingly, secretary; Rachel Scott, treasurer; Pat Costello, corresponding secretary.

KY Beta — Chapter President — Tessie Hale Black

Cumberland College, Williamsburg — 20 actives

On September 4, 1995, chapter officers helped host an ice cream party for the freshmen math and physics majors. Along with the Mathematics and Physics Club, the chapter held a picnic at Briar Creek Park on September 11. Several members of the chapter traveled to Marshall Space Flight Center in Huntsville, Alabama, on November 11. On the last day of classes, December 12, the entire department, including the Math and Physics Club and the Kentucky Beta chapter, had a Christmas party with over 55 people in attendance. Other 1995–96 chapter officers: Eric Alan Thornsbury, vice president; Sherri Michelle McGeorge, secretary; William Patrick Giles, treasurer; Jonathan E. Ramey, corresponding secretary; John A. Hymo, faculty sponsor.

MD Alpha — Chapter President — Regina Geiman

College of Notre Dame of Maryland, Baltimore — 11 actives, 7 associates

During the fall semester several activities were held. These included an interdepartmental volleyball game, a fund-raising activity, and a presentation by Amy Poling, '96, who spoke concerning her research while an intern at Los Alamos Research Lab in New Mexico. Other 1995–96 chapter officers: Shannon Spicer, vice president; Jenny Dunning, secretary; Ana Casas, treasurer; Sr. Maria A. Dowling, corresponding secretary; Joseph DiRienzi, faculty sponsor.

MD Beta — Chapter President — Kathy Gaston

Western Maryland College, Westminster — 18 actives

Other 1995–96 chapter officers: Ivy Burklew, vice president; Leslie Huffer, secretary; Steve Eckstrom, treasurer; Toni Smith, historian; James Lightner, corresponding secretary/faculty sponsor.

MD Delta — Chapter President — Jesse Siehler

Frostburg State University, Frostburg — 33 actives

Maryland Delta chapter met twice during the fall semester, not counting a rained-out picnic in September. In October the group viewed a video dealing with the life of the Indian mathematician Ramanujan. In November, KME President Jesse Siehler gave a presentation on "The Brouwer Fixed Point Theorem." Other 1995–96 chapter officers: Dennis Moon, vice president; Amanda Sherman, secretary; Carla White, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

MI Beta Chapter President — Kristen Williams

Central Michigan University, Mount Pleasant 18 actives

The MI Beta chapter is proud that President Kristen Williams had her project paper, "Buffon's Needle Problem," published in the Fall 1995 Michigan Council of Teachers of Mathematics journal, *Mathematics in Michigan*. KME, along with the Central Michigan University Mathematics Department and the Actuarial Club, Gamma Iota Sigma, hosted a Homecoming Alumni picnic in late October. Other 1995–96 chapter officers: Chris Pesola, vice president; Curt Hanson, secretary; Tom Keller, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

MS Alpha Chapter President — Jon V. Rost

Mississippi University for Women, Columbus 13 actives, 2 associates

In addition to regular meetings and a November initiation, Mississippi Alpha sponsored a Problem Solving Contest, posting a new problem each month. The first person to solve the problem was given a certificate. Other 1995–96 chapter officers: Bethany J. Sims, vice president; Katheryn Kelly Flynn, secretary; Jamie C. Rohling, treasurer; Jean Ann Parra, corresponding secretary; Shaochen Yang, faculty sponsor.

MS Delta Chapter President — Tracie McLemore

William Carey College, Hattiesburg 18 actives

Other 1995–96 chapter officers: Vickie Pickering, vice president; Lynn McShea, secretary; Joy Russell, treasurer; Charlotte McShea, corresponding secretary/faculty sponsor.

MS Epsilon Chapter President — Danny Carpenter

Delta State University, Cleveland 13 actives

Other 1995–96 chapter officers: Renee Upton, vice president; Debra Joel, secretary; David James, treasurer; Paula Norris, corresponding secretary; Rose Strahan, faculty sponsor.

MO Alpha Chapter President — Matthew Brom

Southwest Missouri State University, Springfield 15 actives, 4 associates

Missouri Alpha, in conjunction with the Student Chapter of MAA, sponsored two social events: a fall picnic and an end of the semester pizza party. Both were open to members, prospective members, faculty and staff. Regular monthly meetings of KME were conducted with programs presented by both faculty and students. One student presented a paper he had written as a special project for honors. Other 1995–96 chapter officers: Catherine Montgomery, vice president; Brian Spicer, secretary; Jason Plumhoff, treasurer; Ed Huffman, corresponding secretary/faculty sponsor.

MO Beta
Chapter President — Heather Scully

Central Missouri State University, Warrensburg — 25 actives, 10 associates

Missouri Beta Chapter organized an intramural volleyball team during the fall semester (and actually won some of their games!!). Eli Bowman and Tammy Hutto were named last year's top freshmen. Joy Birchler received the ACM-KME Merit Scholarship. Programs for the semester included the topics "Taxicab Geometry" and "Raindrops and Rainbows." A committee was formed to design a KME sweatshirt. The new sweatshirts have the KME crest on the back and the letters KME on the front. Other activities for the semester included a book sale, tutoring in the Math Clinic, and a Christmas party. Other 1995–96 chapter officers: Ken Petzoldt, vice president; Lynn Graves, secretary; Chad Doza, treasurer; Joy Birchler, historian; Rhonda McKee, corresponding secretary; Larry Dilley, Phoebe Ho, and Scotty Orr, faculty sponsors.

MO Gamma
Chapter President — LeAnn Lotz

William Jewell College, Liberty — 14 actives

In the fall, a special program presentation was given by Dr. Leo Reid of the Mathematics Department at Southwest Missouri State University, Springfield. Other 1995–96 chapter officers: Stephanie Pauls, vice president; Kay Brock, secretary; Joseph T. Mathis, treasurer/corresponding secretary/faculty sponsor.

MO Epsilon
Chapter President — Beth Monnette

Central Methodist College, Fayette — 13 actives

Other 1995–96 chapter officers: Eric Kennedy, vice president; Sara Weiss, secretary; Lynn Klocke, treasurer; William D. McIntosh, corresponding secretary; Linda O. Lembke, faculty sponsor.

MO Eta
Chapter President — Doug Cutler

Northeast Missouri State University, Kirksville — 24 actives, 4 associates

Missouri Eta sponsored a spades tournament for the entire Math Division and organized a faculty/student softball game. Other 1995–96 chapter officers: Karen Van Cleave, vice president; Amanda Nixon, secretary; Sarah Schwab, treasurer; Mary Sue Beersman, corresponding secretary; Joe Hemmeter, faculty sponsor.

MO Iota
Chapter President — Jolena Gilbert

Missouri Southern State College, Joplin — 15 actives, 10 associates

The first meeting of the semester was organizational in nature and included the election of new officers. Chapter members again worked concession stands at the home football games as a fund-raising activity. All who worked three or more games were eligible for a $50.00 drawing; An Pham was the winner. A Problem Solving Group was formed under the direction of Dr. Chip Curtis and student Richard Williamson. In October, a careers talk was given by MSSC graduate, Phillip Brown. Preliminary plans were made for a spring service project. A Christmas party and traditional white elephant gift exchange at the home of Mrs. Mary Elick closed out semester activities. Other 1995–96 chapter officers: April Dickens/An Pham, vice presidents; Jennifer Schumaker, secretary; Vicki Nelson, treasurer; Mary Elick, corresponding secretary; Chip Curtis, faculty sponsor.

MO Kappa — Chapter President — Pat Roper

Drury College, Springfield — 11 actives, 4 associates

The first activity of the semester was a pizza and movie rush party for the potential KME members held at the house of the new department chair of mathematics, Dr. Carol Collins. The winners of the annual math contest this year were Aaron Wilson, Calculus II and above division, and Dena Wisner, Calculus I and below division. Prize money was awarded to the winners at a pizza party held for all the contestants. A bonfire was held at Dr. Allen's house. A luncheon was held at which Kate Good and Mike West gave math talks for their senior assessment. The math club has also been running a tutoring service for both the day school and the Continuing Education Division, Drury Evening College, as a money-making project. The semester ended with a Christmas party at the home of Dr. Ted Nickles. Other 1995–96 chapter officers: Mark Garton, vice president; Michelle Biggers, secretary/treasurer; Charles Allen, corresponding secretary; Don Moss, faculty sponsor.

MO Lambda — Chapter President — Brian Bettis

Missouri Western State College, St. Joseph — 43 actives, 12 associates

Fall initiation of six new members was held on October 22. Social activities of the semester included a "Welcome Back" picnic in September and a Thanksgiving Potluck Dinner in mid November. Two KME members gave presentations in early December. Three additional meetings were held during the semester for organization and planning. Other 1995–96 chapter officers: Devon Kerns, vice president; Linda Meyer, secretary; Cindy Ready, treasurer; John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

NE Alpha — Chapter President — Rick Pongratz

Wayne State College, Wayne — 20 actives

The Nebraska Alpha chapter had two fund-raising activities during the fall 1995 semester. The first of these, selling coupons for Godfather pizzas, netted the organization $180. The second activity involved serving as security escorts from 10 p.m. to midnight for students going to and from the library during finals week. The escorting service was funded by Wayne State College student senate and the club earned $50. Other 1995–96 chapter officers: Trevor Rasmussen, vice president; Kathy Dalton, secretary/treasurer; John D. Fuelberth, corresponding secretary; James Paige, faculty sponsor.

NE Gamma Chapter President — Tricia Taylor

Chadron State College, Chadron 25 actives, 3 associates

Other 1995–96 chapter officers: J.J. Fernandez, vice president; Kacy Carpenter, secretary; Ken Schultz, treasurer; James Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

NE Delta Chapter President — Adam Newman

Nebraska Wesleyan University, Lincoln 17 actives

Other 1995–96 chapter officers: Charles McCutchen, vice president; Justin Rice, secretary; Justin Horst, treasurer; Gavin Larose, corresponding secretary; Bill McClung/Gavin Larose, faculty sponsors.

NH Alpha Chapter President — Richard Elliott

Keene State College, Keene 25 actives

Chapter activities included a fall picnic and participation in Keene's autumn pumpkin festival. Other 1995–96 chapter officers: Lisa Smith, vice president; Sharon McCormick, secretary; Rodney Sleith, treasurer; Charles Riley, corresponding secretary; Ockle Johnson, faculty sponsor.

NM Alpha Chapter President — Brian Sanchez

University of New Mexico, Albuquerque 76 actives, 12 associates

Other 1995–96 chapter officers: Larry Montaño, vice president; Chris Blackwood, secretary; John Snyder, treasurer; Archie G. Gibson, corresponding secretary/faculty sponsor.

NY Alpha Chapter President — Aaron Riddle

Hofstra University, Hempstead 21 actives, 4 associates

Chapter members heard a presentation given by Aaron Riddle concerning his experiences as an actuarial intern. Social activities included three student/faculty volleyball games, a dinner to honor new inductees, and a holiday party. Other 1995–96 chapter officers: Brandi York, vice president; Paul Ryan, secretary; Lisa Fontana, treasurer; Aileen Michaels, corresponding secretary/faculty sponsor.

NY Eta Chapter President — Ken Krawczyk

Niagara University, Niagara University 12 actives

Other 1995–96 chapter officers: Emily Hurlbert, vice president; Rebecca Bauer, secretary/treasurer; Robert Bailey, corresponding secretary;

Kenneth Bernard, faculty sponsor.

NY Kappa Chapter President — Jennifer Smith
Pace University, New York 20 actives, 6 associates

Other 1995–96 chapter officers: Teresa Lester, vice president; Geraldine Taiani, corresponding secretary; Blanche Abramov, faculty sponsor.

NY Lambda Chapter President — Joseph D. Sprague
C. W. Post Campus—Long Island University, Brookville 20 actives

Other 1995–96 chapter officers: Joseph Glorioso, vice president; Justine D. Bello, secretary; Colin R. Grimes, treasurer; Andrew M. Rockett, corresponding secretary; Sharon Kunoff, faculty sponsor.

NY Nu Chapter President — Clifford A. Baxter, IV
Hartwick College, Oneonta 22 actives

Other 1995–96 chapter officers: Karen A. Martin, vice president; John Painter, secretary; Jennifer L. Sutphen, treasurer; Gary E. Stevens, corresponding secretary/faculty sponsor.

OH Eta Chapter President — Amy Gaiser
Ohio Northern University, Ada 40 actives

Other 1995–96 chapter officers: Marlon Price, vice president; Angi Creason, secretary; Ken Fisher, treasurer; Tena Roepke, corresponding secretary; Harold Putt, faculty sponsor.

OK Alpha Chapter President — Carrie O'Leary
Northeastern Oklahoma State University, Tahlequah 28 actives, 7 associates

Fall initiation ceremonies for eleven students, held in the banquet room of a local restaurant, were attended by over thirty faculty and students. The chapter continues to sponsor a monthly math contest. In other activities, members viewed the video "There is Life After Math," and added $97 to their treasury via the annual book sale. The Christmas "pizza party" was again a success! The game of the evening was "Win, Lose, or Draw." John Callaway is working on setting up a database that will contain information about past and current chapter membership. Other 1995–96 chapter officers: Jeana Wood, vice president; John Callaway, secretary; Peter Butz, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

OK Gamma Chapter President — Terry Price
Southwestern Oklahoma State University, Weatherford 20 actives

One faculty and eight students traveled together to Oklahoma City for an evening of fun; they visited "Brick Town" and the Brick Town Spook House. Other 1995–96 chapter officers: Tracy Sipe, vice president;

Kristin Biddy, secretary/treasurer; Wayne Hayes, corresponding secretary; Rochelle Beatty, faculty sponsor.

PA Alpha — Chapter President — Karey Kustron

Westminster College, New Wilmington — 15 actives

Other 1995–96 chapter officers: Daniel Coffman, vice president; Laura Williams, secretary; Jason Thiel, treasurer; Miller Peck, corresponding secretary; Warren Hickman/Carolyn Cuff, faculty sponsors.

PA Gamma — Chapter President — Jacob Trombetta

Waynesburg College, Waynesburg — 14 actives, 3 associates

Other 1995–96 chapter officers: Frank Luzar, vice president; Leslie Zak, secretary; Jason Hoover, treasurer; A.B. Billings, corresponding secretary/faculty sponsor.

PA Delta — Chapter President — Kim Fisher

Marywood College, Scranton — 4 actives

Other 1995–96 chapter officers: Melissa Mang, vice president; Ann Conflitti, secretary/treasurer; Sr. Robert Ann von Ahnen, IHM, corresponding secretary/faculty sponsor.

PA Iota — Chapter President — Derek Smith

Shippensburg University of Pennsylvania, Shippensburg — 19 actives, 3 associates

Fall initiation was held at the home of Dr. Fred Nordai, Assistant Department Chair. Dr. and Mrs. Nordai were gracious hosts, entertaining KME officers, new initiates, and several faculty members in what was a delightful evening. Plans are underway for more activities in the spring. Other 1995–96 chapter officers: Melissa Gladding, vice president; Todd Bittinger, secretary; Rebecca Shubert, historian; Fred Nordai, treasurer; Michael Seyfried, corresponding secretary/faculty sponsor.

PA Kappa — Chapter President — Leanne Majors

Holy Family College, Philadelphia — 5 actives, 3 associates

Society members hosted a "Cabaret in the Commons" on October 29. Plans for a high school math competition were formulated; however, due to minimal response, the event was postponed until Spring 1996. Members met to make plans for the chapter's 25th Anniversary which occurs on January 23, 1996. The celebration will take place on March 29 in conjunction with the annual installation of officers and induction of new members. All Holy Family KME alumni will be invited. Sr. Grace Kuzawa, CSFN, one of the founding chapter members, will be honored for her dedication to the PA Kappa Chapter and for her 40 years of service in the Math Department at Holy Family College. Another founding chapter member, Mr. Louis Hoelzle, Math Chairman at Bucks County Community College and adjunct faculty member at Holy Family, will also be recognized for 25 years of membership in KME. Other 1995-96 chapter officers: Kimberly Doll, vice president; Nicholas Gross, secretary/treasurer; Sr. Marcella Wallowicz, corresponding secretary/faculty sponsor.

PA Mu Chapter President — Colleen Connors

Saint Francis College, Loretto 23 actives

The highlight of the fall semester was the poster session for senior seminar projects. Posters, available on Thursday, December 7, 1995, were viewed by faculty and students from across campus. The topics included "The Analysis of a Continuing Education Student Survey," "The Space Shuttle Challenger," "Technology in the Mathematics Classroom," "Driving Record," "Ordinary and Symmetric Derivatives," "The Analysis of Conclusions Drawn by the Authors of the Bell Curve," "A Look at Two Popular Compression/Decompression Algorithms," "Projectile Motion," and "The Tacoma Narrows Disaster." The Department of Chemistry, Mathematics, and Physical Science has created a newsletter entitled *CHEMAPS* (the CHEmistry, MAthematics and Physical Science newsletter). The first issue was published in the fall of '95. This newsletter gives an update of SFC students, Department News, The KME Honor Society, Department Awards, and any special announcements. A KME update is also given in the Department Home Page on the Internet system. Other 1995-96 chapter officers: Jennifer Ropp, vice president; Richard Roth, secretary; Heather Barnick, treasurer; Peter Skoner, corresponding secretary; Adrian Baylock, faculty sponsor.

TN Delta Chapter President — Lori George

Carson-Newman College, Jefferson City 14 actives

Chapter activities included a fall picnic and also a Christmas party. Other 1995-96 chapter officers: Alexander J. Mutterspaugh, vice president; John Tarwater, secretary; Amy S. Smith, treasurer; Catherine Kong, corresponding secretary/faculty sponsor.

TX Iota Chapter President — David Gregory Warden

McMurry University, Abilene

Other 1995-96 chapter officers: Kory D. Okerstrom, vice president; Karen Chronister, secretary; Dianne Dulin, corresponding secretary; Bill J. Dulin, faculty sponsor.

TX Kappa Chapter President — Mary Cook

University of Mary Hardin-Baylor, Belton 6 actives, 1 associate

Other 1995-96 chapter officers: Lisa Hitt, vice president; Riki Perkins, secretary; Katharine Eversoll, treasurer; Peter H. Chen, corresponding secretary; Maxwell M. Hart, faculty sponsor.

VA Alpha Chapter President — Debra Marks

Virginia State University, Petersburg

The Virginia Alpha chapter of Kappa Mu Epsilon, in conjunction with the Walter E. Johnson Mathematics Club and the Student Chapter of the Mathematics Association of America presented an "Afternoon of Papers" on November 1, 1995. Papers were presented by Dr. Christopher Barat, professor of mathematics at Virginia State University and Mr. Romon Williams, graduate student and former vice-president of Kappa Mu Epsilon, Virginia Alpha chapter. Dr. Barat presented a paper entitled "The History of Mathematics," highlighting the course that he is presently teaching on that subject. Mr. Williams presented a paper entitled "Laser Doppler Anemometry Measurements of a Pulsatile Flow Within a Pediatric Left Ventricular Assist Device," which described the research that he performed this past summer at Pennsylvania State University. These presentations were open for faculty, students, and the general public. Other 1995–96 chapter officers: Omar Khan, vice president; Barbara Montgomery, secretary; Emma B. Smith, treasurer; Joycelyn F. Josey-Harris, corresponding secretary; Azzala Owens, faculty sponsor.

WI Beta Chapter President — Kathleen Freese

University of Wisconsin—River Falls, River Falls 15 actives, 12 associates

The chapter held meetings every other Monday at 3:20 p.m. Additional activities included a picnic, a bake sale, and a senior send-off party. Other 1995–96 chapter officers: Stacie O'Connor, vice president; Catherine Bernhardt, secretary; Debra Robinson, treasurer; Robert Coffman, corresponding secretary.

WI Gamma Chapter President — Steve Wall

University of Wisconsin—Eau Claire, Eau Claire 20 actives, 10 associates

Other 1995–96 chapter officers: Kady Hickman, vice president; Brenda Bychinski, secretary; Mike Lockwood, treasurer; Marc Goulet, corresponding secretary/faculty sponsor.

Top Ten

Most current consecutive appearances in KME News: 1. Iowa Alpha, 49; 2. Kansas Alpha, 47; 3. Missouri Epsilon, 37; 4. Nebraska Alpha, 35; 5. Maryland Delta, 26; 6. Pennsylvania Kappa, 23; 7. Oklahoma Alpha, 20; 8. Iowa Delta, 18; T9. Michigan Beta and Missouri Iota, 17.

Announcement of the Thirty-First Biennial Convention of Kappa Mu Epsilon

The Thirty-First Biennial Convention of Kappa Mu Epsilon will be hosted by the three chapters Missouri Alpha, Missouri Theta, and Missouri Kappa in Springfield, Missouri. The convention will take place April 3–5, 1997. Each attending chapter will receive the usual travel expense reimbursement from the national funds as described in Article VI, Section 2, of the Kappa Mu Epsilon Constitution.

A significant feature of our national convention will be the presentation of papers by student members of Kappa Mu Epsilon. The mathematical topic selected by each student speaker should be of interest to the author and of such scope that it can be given adequate treatment in a timed oral presentation. Student talks to be judged at the convention will be chosen prior to the convention by the Selection Committee on the basis of submitted written papers. At the convention, the Awards Committee will judge the selected talks on both content and presentation. The rankings of both the Selection and Awards Committees will determine the top four papers.

Who may submit a paper?

Any undergraduate or graduate student member of Kappa Mu Epsilon may submit a paper for consideration as a talk at the national convention. A paper may be co-authored. If selected for presentation at the convention, the paper must be presented by one (or more) of the authors.

Presentation topics

Papers submitted for presentation at the convention should discuss material understandable by undergraduates who have completed only differential and integral calculus. The Selection Committee will favor papers that satisfy this criterion and which can be presented with reasonable completeness within the time allotted. Papers may be original research by the student(s) or exposition of interesting but not widely known results.

Presentation time limits

Papers presented at the convention should take between 15 minutes and 25 minutes. Papers should be designed with these time limits in mind.

How to prepare a paper

The paper should be written in the standard form of a term paper. It should be written much as it will be presented. A long paper (such as an honors thesis) must not be submitted with the idea that it will be shortened at presentation time. Appropriate references and a bibliography are expected. Any special visual aids that the host chapter will need to provide (such as a computer and overhead projection system) should be clearly indicated at the end of the paper.

The first page of the paper must be a "cover sheet" giving the following information: (1) title, (2) author or authors (these names should not appear elsewhere in the paper), (3) student status (undergraduate or graduate), (4) permanent and school addresses and phone numbers, (5) name of the local KME chapter and school, (6) signed statement giving approval for consideration of the paper for publication in *The Pentagon* (or a statement about submission for publication elsewhere) and (7) a signed statement of the chapter's Corresponding Secretary attesting to the author's membership in Kappa Mu Epsilon.

How to submit a paper

Five copies of the paper, with a description of any charts, models, or other visual aids that will be used during the presentation, must be submitted. The cover sheet need only be attached to one of the five copies. The five copies of the paper are due by February 4, 1997. They should be sent to:

Dr. Patrick Costello, KME President-Elect
Dept. of Math, Stat, CSC
Eastern Kentucky University
Richmond, KY 40475–3133

Selection of papers for presentation

A Selection Committee will review all papers submitted by undergraduate students and will choose approximately fifteen papers for presentation and judging at the convention. Graduate students and undergraduate students whose papers are not selected for judging will be offered the opportunity to present their papers at a parallel session of talks during the convention. The President-Elect will notify all authors of the status of their papers after the Selection Committee has completed its deliberations.

Criteria used by the Selection and Awards Committees

Each paper will be judged on (1) topic originality, (2) appropriateness to the meeting and audience, (3) organization, (4) depth and significance of the content, and (5) understanding of the material. Each presentation will be judged on (1) style of presentation, (2) maintenance of interest, (3) use of audio-visual materials (if applicable), (4) enthusiasm for the topic, (5) overall effect, and (6) adherence to the time limits.

Prizes

All authors of papers presented at the convention will be given two-year extensions of their subscription to *The Pentagon*. Authors of the four best papers presented by undergraduates, as decided by the Selection and Awards Committees, will each receive a cash prize of $100.

Publication

All papers submitted to the convention are generally considered as submitted for publication in *The Pentagon*. Unless published elsewhere, prize-winning papers will be published in *The Pentagon* after any necessary revisions have been completed (see page 2 of *The Pentagon* for further information). All other papers will be considered for publication. The Editor of *The Pentagon* will schedule a brief meeting with each author during the convention to review their manuscript.

Has Your Subscription Expired?

Your *Pentagon* subscription expires with the volume and number that appears in the upper right corner of your address label (see back cover). Since this issue is Volume 55 Number 2, if the code 55-2 appears on your label then THIS IS YOUR LAST ISSUE!

To renew, please send your check — just $10 for four more issues (domestic individuals only; see page 2 for rates for libraries and foreign subsciptions) — together with your name and address and a copy of your old address label to:

The Pentagon Business Manager
Division of Mathematics and Computer Science
Emporia State University
Emporia, KS 66801 USA

Please renew promptly to avoid gaps in your journal collection.

Kappa Mu Epsilon National Officers

Arnold D. Hammel *President*

Department of Mathematics
Central Michigan University, Mt. Pleasant, Michigan 48859
a.hammel@cmich.edu

Patrick J. Costello *President-Elect*

Department of Mathematics, Statistics and Computer Science
Eastern Kentucky University, Richmond, Kentucky 40475
matcostello@acs.eku.edu

Waldemar Weber *Secretary*

Department of Mathematics and Statistics
Bowling Green State University, Bowling Green, Ohio 43403
kme-nsec@mailserver.bgsu.edu

A. Allan Riveland *Treasurer*

Department of Mathematics and Statistics
Washburn University, Topeka, Kansas 66621
zzrive@acc.wuacc.edu

Mary S. Elick *Historian*

Department of Mathematics
Missouri Southern State College, Joplin, Missouri 64801
elick@vm.mssc.edu

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation.

Chapter	Location	Installation Date
OK Alpha	Northeastern Oklahoma State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959

IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Northeast Missouri State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel College, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	West Georgia College, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Illinois Benedictine College, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984

CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994

Starting a KME Chapter

Complete information on starting a chapter of KME may be obtained from Arnold Hammel, National President (see address on p. 77). Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; student members must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $50) and the expenses of the installing officer. The individual membership fee to the national organization is $20 per member and is paid just once, at that individual's initiation. Much of this $20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offerings and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.