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## A Message from the Editor

Wow! As I sit at my computer to write this message, it is hard to know what to say. Any project like The Pentagon evokes many emotions upon those involved. Feelings of frustration, of triumph, of gratitude, and even of love are among them. As I am sure all previous editors have done, I have poured something of myself into this journal. I know this journal now. I feel a great sense of pride (as well as relief) as I prepare to take this issue to the printer. My hope is that you will have the same pleasure in reading as I have had in editing.

You will probably notice a few relatively minor changes in The Pentagon, and as always I will remain open to comments on any changes made. Remember, this is your journal, and any suggestions you have for its improvement will be welcome. For the most part, however, the journal remains the same as it was under the magnificent leadership of Andrew Rockett, whose advice and encouragement shall not be forgotten by this editor.

There are so many individuals to thank, it will be impossible to name thern all; I offer my apologies in advance to those omitted below. I would like to thank, in no particular order, Andrew Rockett, former editor, without whose insights and guidance this effort would not have been possible; Sharon Kunoff, former business manager, for help in this transition period; Larry Scott, current business manager and chair of my department, who originally encouraged me to take on this project; Kenneth Wilke and Mary Elick, associate editors, for all their work and advice; the National Council of Kappa Mu Epsilon, for giving me their trust in handing me this opportunity; Arnold Hammel, current National President, whose incredible energy and enthusiasm have been inspiring; Harold Thomas (National President, 1989-1993), Helen Kriegsman (editor of The Pentagon, 1965-1971), J.D. Haggard (Problem Corner editor, 1957-1963), and others at Kansas Alpha at Pittsburg State University, for getting me involved in Kappa Mu Epsilon as an undergraduate; the many presenters of papers at the Biennial Convention in Durango, for all their hard work preparing the papers that make up the core of this journal; Colorado Gamma, for their hospitality during the convention; faculty advisors who prodded their students to complete their projects; and the many student members in Kappa Mu Epsilon at all levels, without whom this journal would cease to exist. A special "thank you" goes to my wife Martha and children Matthew, Patricia, and Tiffany, for their patience with me over the last few months.

Now, dear reader, I invite you to turn the page, prop your feet up, tune out the distractions of the day, and delve into an interesting article or problem. Enjoy!

# When Intuition Fails 

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Presented at the 1995 National Convention and awarded "top four" status by the Awards Committee.

While taking a course in probability and statistics, we encountered an interesting problem pertaining to counting basic events. The problem was first presented in a chapter exercise. "A group consists of four married couples. How many ways can the group be seated in a row if no man sits next to his wife?" The answer given in the book was 144, which the professor told us was incorrect. She, then, challenged us to find the correct solution. Several days later, the same problem was again discussed in the text, and the author gave another incorrect answer and also challenged the readers to discover the flaw in the incorrect solution. The two invitations to investigate this problem were enough to spark our curiosity.

Since this question deals with the order of the couples being seated, it is appropriate to use the concept of permutation to solve this problem. By definition, "a permutation of a set of objects is a mapping of the set onto itself." In a simpler form, a permutation is the arrangement of the elements of a set where the order of its elements is of importance. Using the generalized counting principle, the number of different permutations of a set of $n$ objects is $n!$. Because there is no restriction given to the seating of the first member of the group, there are eight different ways to seat the first individual. Now, the seated individual's mate cannot be included in the group left to be seated, therefore, leaving only six people. From this point, the remaining spaces fall into a simple permutation that is 6 !. The answer to this problem then is $8 \times 6$ !, which equals 5760 , obviously more than 144. The solution appears simple enough, but after close examination, the answer appeared to be incorrect because it excludes a term. The problem states that all eight individuals will be seated; however, $8 \times 6!$ contains only seven terms:

$$
A=\underline{8} \underline{6} \underline{5} \underline{3} \underline{2} \underline{?} \text {. }
$$

In essence, the solution $8 \times 6!$ omits an individual, which we felt was the only flaw in the intuitive approach presented by the author.

Since the direct way of approaching the problem did not work, we chose to search for the answer by using the complement ( $A^{\prime}$ ). The complement of "no man sits next to his wife" is "at least one man sits next to his wife." In order to satisfy all of the possibilities of "at least one man sits next to his wife," we had to consider all of the following: "only one man sits next to his wife," "only two men sit next to their respective wives," "only three men sit next to their respective wives," and "all four men sit next to their respective wives." Now, instead of considering each individual, which would give eight elements to work with, we consider each married couple as an entity of itself, thus producing only four elements ( $E$ ). The union of all four couples ( $E_{1} \cup E_{2} \cup E_{3} \cup E_{4}$ ) allowed us to consider all of the possibilities of the complement. As shown in Figure 1, the union of the four sets contains overlapping areas that must be considered when computing the probability of the complement. The following theorem allowed us to compute the probability of the complement:
Theorem. Let $A_{1}, A_{2}, \ldots, A_{n}$ be arbitrary events. Then

$$
\begin{aligned}
P\left(A_{1} \cap A_{2} \cap \cdots \cap A_{n}\right)= & \sum_{i=1}^{n} P\left(A_{i}\right)-\sum_{i<j} P\left(A_{i} \cap A_{j}\right) \\
& +\sum_{i<j<k} P\left(A_{i} \cap A_{j} \cap A_{k}\right) \\
& -\sum_{i<j<k<1} P\left(A_{i} \cap A_{j} \cap A_{k} \cap A_{l}\right)+\ldots \\
& +(-1)^{n+1} P\left(A_{1} \cap A_{2} \cap \cdots \cap A_{n}\right)
\end{aligned}
$$

The probability of the union of the four sets is

$$
\begin{aligned}
P\left(A^{\prime}\right)= & P\left(E_{1} \cup E_{2} \cup E_{3} \cup E_{4}\right) \\
= & P\left(E_{1}\right)+P\left(E_{2}\right)+P\left(E_{3}\right)+P\left(E_{4}\right)-P\left(E_{1} \cap E_{2}\right)-P\left(E_{1} \cap E_{3}\right) \\
& -P\left(E_{1} \cap E_{4}\right)-P\left(E_{2} \cap E_{3}\right)-P\left(E_{2} \cap E_{4}\right)-P\left(E_{3} \cap E_{4}\right) \\
& +P\left(E_{1} \cap E_{2} \cap E_{3}\right)+P\left(E_{1} \cap E_{2} \cap E_{4}\right)+P\left(E_{1} \cap E_{3} \cap E_{4}\right) \\
& +P\left(E_{2} \cap E_{3} \cap E_{4}\right)-P\left(E_{1} \cap E_{2} \cap E_{3} \cap E_{4}\right) .
\end{aligned}
$$

To compute $P\left(E_{i}\right)$, where $i=1,2,3,4$, couple $i$ is considered a unit, which can be seated in two ways, while the remaining three couples are considered as individuals, giving seven "people" to be seated; thus,

$$
P\left(E_{i}\right)=\frac{7!2!}{8!}, \quad i=1,2,3,4 .
$$



Figure 1. Four sets and their possible intersections.
In order to compute the probability of the two-way intersections, married couples $i$ and $j$ are considered as units, which can be seated in two ways. We are then seating six "people." Therefore,

$$
P\left(E_{i} \cap E_{j}\right)=\frac{6!(2!)^{2}}{8!}, \quad\left\{\begin{array}{l}
i=1,2,3 \\
j=i+1, \ldots, 4
\end{array}\right.
$$

By the same rational,

$$
P\left(E_{i} \cap E_{j} \cap E_{k}\right)=\frac{5!(2!)^{3}}{8!}, \quad\left\{\begin{array}{l}
i=1,2 \\
j=i+1, \ldots, 3 \\
k=j+1, \ldots, 4
\end{array}\right.
$$

and

$$
P\left(E_{1} \cap E_{2} \cap E_{3} \cap E_{4}\right)=\frac{4!(2!)^{4}}{8!}
$$

From this,

$$
P\left(A^{\prime}\right)=4 \cdot 2 \cdot \frac{7!}{8!}-6 \cdot 2^{2} \cdot \frac{6!}{8!}+4 \cdot 2^{3} \cdot \frac{5!}{8!}-2^{4} \cdot \frac{4!}{8!}
$$

When using the complement, the probability of "no man sitting next to his wife" equals $1-P\left(A^{\prime}\right)$. Therefore,

$$
P(A)=1-\frac{26496}{40320}=\frac{13824}{40320} .
$$

From this, we concluded that the number of ways to seat the group by the problem was 13,824 .

Since we had arrived at 13,824 seating arrangements, we were confident that it was the correct solution; however, we still needed to check its validity. Enumeration, which is the process of listing all of the possible outcomes, is the best way to positively find the correct number of seating arrangements. Since the process of listing all of the possible ways to seat the four couples would be too time consuming, the first step we took to complete the enumeration was to write a computer program in Fortran 77 that calculated each of the ways to seat the couples when the order of arrangement was of no concern. As the computer determined each unique seating arrangement, it kept track of the total, which turned out to be 40,320 or 8 !. From that point, we modified the existing program to calculate and list only the potential ways in which no man is seated next to his wife. The modified program produced 13,824 ways to seat the four couples. This answer, too, is clearly much more than the solution of 144 or the other solution of $8 \times 6!$ and verifies the complement solution.

Next we used tree factoring to try to determine a pattern in the enumerations. Six patterns resulted from the 13,824 arrangements. We can consider the seating of the four couples in two possibilities: either the third seat is occupied by the spouse of the first person or it is not.

If the first person's spouse is seated in the third seat, then the following three types of arrangements are observed: If the first person's spouse is seated in the third seat and the second person's spouse is seated in the fourth seat then

$$
A=8611 \underline{4} \underline{1} 1=384 .
$$

If the first person's spouse is seated in the third seat and the second person's spouse is seated in the fifth seat then

$$
A=8 \underline{6} 14 \underline{4} 1 \underline{1}=384 .
$$

If the first person's spouse is seated in the third seat and the second person's spouse is not seated in the fourth or fifth seat then

$$
A=\underline{8} \underline{6} \underline{4} \underline{2} \underline{2} \underline{2} \underline{1}=1536 .
$$

On the other hand, if the first person's spouse is not in the third seat then the first three seats are occupied by people from three different couples. Because of this, these three arrangements result: If the first three seats are occupied by three people from three different couples and the fourth seat is occupied by someone from the fourth couple then

$$
A=\underline{8} \underline{6} \underline{4} \underline{2} \underline{3} \underline{3} \underline{2} \underline{1}=6912 .
$$

If the first three seats are occupied by three people from three different couples and the fourth seat is occupied by the mate of the first or second person seated and the fifth seat is occupied by a member of the fourth couple then

$$
A=\underline{8} \underline{6} \underline{4} \underline{2} \underline{2} \underline{2} \underline{2} \underline{1}=3072 .
$$

If the first three seats are occupied by three people from three different couples and the fourth seat is occupied by the first or second person's mate and the fifth seat is occupied by someone other than from the fourth couple then

$$
A=\underline{8} \underline{6} \underline{4} \underline{2} \underline{2} \underline{1} \underline{1}=1536 .
$$

When combined, the two sets of patterns yield 13,824 arrangements. It is interesting to observe that the incorrect intuitive approach discussed earlier,

$$
A=\underline{8} \underline{6} \underline{5} \underline{4} \underline{2} \underline{1} ?
$$

contains a factor of 5 , while the six correct possibilities do not have a factor of 5 . Therefore, we discovered a single product would not give the solution as first thought.

After discovering the flaw in the author's intuitive approach, we successfully discovered and showed the correct answer through three different methods; therefore, we confidently submit 13,824 possibilities as the number of ways to seat a group of four married couples in a row where no man sits next to his wife.

Acknowledgements. We wish to express our gratitude to our advisor Dr. Gayle Kent. Her love for mathematics inspired us to find the correct answer to the problem. It was her enthusiasm that encouraged us to write this paper. We would also like to thank Dr. Albert Sheppard for his guidance in computer programming. His direction in the thought process of our program helped us to find one sure way to obtain an answer to our problem. We are truly grateful to both Dr. Kent and Dr. Sheppard for giving us the opportunity and knowledge to write and present this paper.

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# A History of Population Models 

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Presented at the 1995 National Convention

## Introduction

The study and description of interacting populations has permeated mathematical literature for the last century. We, as mathematicians and scientists, wish to have an accurate model of the events taking place in nature so that we can modify some of them for our benefit, while at the same time preserving the balance existing in nature. Some of the systems we want to evaluate include forest-pest dynamics and plant-herbivore interactions related to crop production. The benefits of being able to detail these interactions are obvious. Because of cases such as these, people have been trying to obtain a complete, mathematical description of plant-herbivore interactions, which fall under the broader class of predator-prey models.

Models involving two or more species are undeniably more complicated than those involving one species, and hence were developed after the one species models had been explored. In formulating these models, many important details were left out so that a solution could be found. In this, the computer age, one can formulate vastly more complicated models, and proceed to analyze them by numerically integrating the equations and plotting the trajectories. Thus, even if a closed form solution does not exist, we can obtain the behavior of the system via a graphical approach. When we apply this to a plant-herbivore model, we can then determine the long time behavior of the populations and what the approximate density of the populations will be.

In the present paper, we track population models from their humble beginnings as sequences, through selected developments of single-species models involving time derivatives, then close with multi-species models which exhibit periodic behavior.

## The First Models

The first account of mathematics being used as a descriptive tool for populations is when Leonardo of Pisa (Fibonacci) in 1202 used his famous sequence

$$
1,1,2,3,5,8,13,21, \ldots
$$

to answer a question about the growth of a rabbit population (see [3]). The sequence progresses using the recursion relationship

$$
\begin{gathered}
n_{k+2}=n_{k+1}+n_{k}, \quad k=1,2, \ldots \\
n_{1}=n_{2}=1,
\end{gathered}
$$

which is also referred to as a difference equation [1]. This model works for a short time frame, but in regards to the long-time behavior of a population, this model predicts an infinite population! Not a very physical outcome of a rabbit population. Many physical properties are left out of Fibonacci's population model, some of which include mortality, environmental carrying capacity, and predation, all of which are negative contributors to the rabbit population model.

The next advancement in the study of population dynamics involving single populations came from the "father of statistics," John Graunt of London. He compiled information regarding the population of London during the 1600 's, studied how the population changed during that period, and what the major influences were for population peaks and valleys. Table 1 is a sample of the data Graunt worked from.

Some of the questions posed and explored by Mr. Graunt [4] were:

- Was the population of London increasing or decreasing?
- What was the ratio of males to females?
- What was the age distribution of the population?
- How much time does it take to replenish the housing after a plague?

The answers to these questions were intended to provide insight to the ruling class of the time, and to be used for plotting a course for the growth and development of London. One could examine the tables and conclude that the population of London was growing at a steady (but small) rate, determine what percentage of the population was male or female, or evaluate the age classes of the population to see how many males were of a working age or how many females were of childbearing age.

One of the main points lacking from Graunt's method was the power of long-time prediction. His method was one of observation and reporting. It wasn't until much later (after the calculus had been developed) that a more precise population model could be derived which incorporated the aspect of long-time behavior and the power of prediction.

## The Diseases, and Cassalities this year being 1632.

ABortive and Stibom Gout ..... 4
Grief ..... 11
Affrighted Jaundies ..... 43
Aged ..... 628
Ague. ..... 43
Apoplex, and Meagrom ..... 17
Bit with a mad dog ..... 1
Bleeding. ..... 3
Bloody flux, scowring, and flux. ..... 348
Brused, Issues, sores, and ulcers. ..... 28
Burnt, and Scalded ..... 5
Burst, and Ruprure ..... 9
Cancer, and Wolf. ..... 10
Canker. ..... 1
Childbed ..... 171
Chrisomes, and Infants. ..... 2268
Cold, and Cough ..... 55
Colick, Stene, and Strangury ..... 56
Consumption ..... 1797
Convulsion. ..... 241
Cut of the Stone. ..... 5
Dead in the street, and starved ..... 6
Dropsie, and Swelling ..... 267
Drowned ..... 34
Executed, and prest to death ..... 18
Falling Sickness ..... 7
Fever. ..... 1108
Fistula ..... 13
Flocks, and small Pox ..... 531
French Pox ..... 12
Gangrene ..... 5
Jawraln ..... 8
Impostume ..... 74
Kild by several accidents. ..... 46
King's Evil. ..... 38
Lethargie. ..... 2
Livergrown. ..... 87
Lunatique. ..... 5
Made away themselves ..... 15
Measles ..... 80
Murchered ..... 7
Over-laid, and starved at nurse ..... 7
Palsie ..... 25
Piles ..... 1
Plague ..... 8
Planet. ..... 13
Pleurisie, and Spleen. ..... 36
Purples, and spotted Feaver ..... 38
Quinsie ..... 7
Rising of the Lights. ..... 98
Sciatica ..... I
Scurvey, and Itch ..... 9
Suddenly ..... 62
Surfet ..... 86
Swine Pox ..... 6
Teeth. ..... 470
Thrush, and Sore mouth ..... 40
Tympany ..... 13
Tissick. ..... 34
Vomiting ..... 1
Worms ..... 27
Increased in the Burials in the 122 Parishes, and at the Pest- house this year. ..... 993
Decreased of the Plague in the 122 Parishes, and at the Pest- house this year ..... 266

## Later Models

The first of the dynamic population models is due to Malthus, who in 1798 proposed the following as a single species population model:

$$
\begin{equation*}
\frac{d N}{d t}=r N \tag{1}
\end{equation*}
$$

This states that the rate of change of a population $N$ with respect to time is directly proportional to the current population [1]. The constant, $r$, is called the rate constant for population growth ( $r>0$ ) or population decay $(r<0)$. We know solutions for this equation, and they have the form

$$
\begin{equation*}
N(t)=N_{0} e^{r t} \tag{2}
\end{equation*}
$$

Equation (2) predicts exponential growth or exponential decay, depending upon the sign of $r$. Model (1) has the same fault as Fibonacci's. It predicts unlimited growth or unlimited decay.

The next advancement in population models came from Verhulst in 1838 and (independently) from Pearl and Reed in 1920 (see [6]). This model takes into account what is known as a "carrying capacity." The carrying capacity is an environmental limitation on population sizes and includes such factors as food supply, competition between species, availability of shelter, and other natural factors which affect population growth. The model has the form

$$
\begin{equation*}
\frac{d N}{d t}=r N\left(\frac{K-N}{K}\right) \tag{3}
\end{equation*}
$$

where $K$ is the carrying capacity, $N$ is the number of individuals in the population, and $r$ is as above. Integrating, we obtain the solution

$$
\begin{equation*}
N(t)=\frac{N_{0} K}{N_{0}+\left(K-N_{0}\right) e^{-r t}}, \tag{4}
\end{equation*}
$$

where $N_{0}$ is the initial ( $t=0$ ) population. Some typical trajectories are sketched in figure 1 for $K=1$ and different values of $N_{0}$.

From figure 1, we see that no matter what the population is, as time marches on, we increase or decrease to a population density of one (the horizontal asymptote). This model emulates what we observe in nature. A population maintains an equilibrium density, depending upon the available resources.

Observing the two previous models, we see a pattern emerging. This pattern involves the general form of a population model, and this form is

$$
\begin{equation*}
\frac{d N}{d t}=N f(N) \tag{5}
\end{equation*}
$$



Figure 1. Population trajectories.
which tells us the rate of change of a population is proportional to the current population. The proportionality function, $f(N)$, can be a constant as in (1) (direct proportionality), a linear function as in (3) (linearly proportional), or a more complicated function.

The question which naturally arises is one of interacting species. What happens when rabbits and foxes, deer and wolves, or insects and plants interact? We need another variable to describe the second population and more analysis techniques.

## Multi-Species Models

One of the fundamental examples of multi-species interactions is that of a predator and its prey. Following the reasoning presented above, we see that the general form of a two species model is

$$
\begin{align*}
& \frac{d x}{d t}=x f(x, y) \\
& \frac{d y}{d t}=y g(x, y), \tag{6}
\end{align*}
$$

where $x$ represents the prey population density and $y$ represents the predator population density.

For the analysis, we need some definitions:
phase plane: Solutions to (6) have the form $(x(t), y(t))$ and trace out curves in the $x-y$ plane. We call this the phase plane, and it describes (graphically) how the populations change with time.
nullclines: The paths in the phase plane defined by $\frac{d x}{d t}=0$ or $\frac{d y}{d t}=0$. These are paths in the phase plane where one of the populations is in equilibrium (not changing). With population models, we note that the axes are always nullclines.
equilibrium points: Points in the phase plane where the $x$ and $y$ nullclines intersect. These are points where both populations are in equilibrium. The action takes place around these points.
solution trajectories: Paths in the phase plane which are solutions to the set of differential equations describing the populations.

For a specific example, let $A, B, C$ and $D$ be positive constants and examine the following predator-prey model, first proposed by Volterra (see [5]):

$$
\begin{align*}
& \frac{d x}{d t}=x(A-C y)  \tag{7}\\
& \frac{d y}{d t}=y(D x-B) .
\end{align*}
$$

The individual terms are defined as follows: $A x$ is the prey growth term; $-C x y$ is the prey removal (due to predation) term; Dxy is the predator growth (due to prey consumption) term; and $-B y$ is the predator death term.
y, Predator


Figure 2. Nullclines and equilibrium points.

Using a standard approach to solving such a system, we divide the two equations obtaining

$$
\begin{equation*}
\frac{d x}{d y}=\frac{x(A-C y)}{y(D x-B)}, \tag{8}
\end{equation*}
$$

which can be separated and then integrated

$$
\begin{equation*}
\int \frac{A-C y}{y} d y=\int \frac{D x-B}{x} d x \tag{9}
\end{equation*}
$$

yielding the solution

$$
\begin{equation*}
A \ln |y|+B \ln |x|-C y-D x=K, \tag{10}
\end{equation*}
$$

where the constant $K$ depends upon the initial conditions, and is a parameter of the family of curves defined implicitly by equation (10). It can be shown that trajectories form a family of closed curves centered about the point $(B / D, A / C)$ in the phase plane. Now that we have the analytic solution to (7), we want to look at system (7) in the phase plane, and analyze what happens to the populations as time progresses. There are four nullclines given by $x=0, y=0, x=B / D$ and $y=A / C$. This results in two equilibrium points, $(0,0)$ and $(B / D, A / C)$. These are sketched in figure 2 (see previous page) for $B / D=3$ and $A / C=4$.

| Region | $\dot{X}$ |  | Region | $\dot{X}$ | $\dot{y}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x=0$ | no <br> change | negative | $y=4, x>3$ | no <br> change | positive |
| $y=0$ | positive | no <br> change | $x<3, y<4$ | positive | negative |
| $x=3, y<4$ | positive | no <br> change | $x>3, y<4$ | positive | positive |
| $x=3, y>4$ | negative | no <br> change | $x>3, y>4$ | negative | positive |
| $y=4, x<3$ | no <br> change | negative | $x<3, y>4$ | negative | negative |

## Table 2.

We are only concerned with the first quadrant, which is the biologically plausible region of the phase plane (nonnegative population values). Next, we examine how the populations are changing in each of the regions of

## Y. Predator



Figure 3. The direction field.
the phase plane defined by the nullclines. Taking a point in each of these regions and evaluating the right-hand side of our differential equations (7) gives us table 2.

Having the algebraic sign of the derivative enables us to determine the direction of movement of each population (increasing, decreasing, or remaining constant). These direction arrows are sketched in figure 3.

The arrows indicate the algebraic sign of the derivatives, which shows us the direction of the solution trajectories. The closed paths of (10) are now evident, and we note that these paths represent periodic fluctuations in the populations. A picture of system (7) is shown in figure 4 , for $A=4$, $C=D=1$ and $B=3$.

Periodic orbits are an important feature of a dynamical system, and for population models, they indicate co-existence with periodic fluctuations in both populations. In the previous example we had a family of periodic orbits. A special case arises when we have only one periodic orbit, and all solution trajectories tend toward it (see figure 5). This behavior is called a stable limit cycle, and it is found in population models as well as mechanical systems. Limit cycles play an important role in dynamical systems, and their study is a hot topic in current research.
Acknowledgements. I would like to acknowledge the constructive comments made to me by Dr. Richard Gibbs, my undergraduate advisor, and by Dr. Deborah Sulsky, my graduate advisor. Their time and patience has helped me in my endeavors with mathematics and in the mathematical community in which we all work.


Figure 4. System (7) in the phase plane.


Figure 5. A stable limit cycle.
Editor's note. The author prepared figures 4 and 5 using PhasePlane, The Dynamical Systems Tool [2].

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## Convention Presenters



Presenters of papers at the 30th Biennial Convention. Photograph courtesy of John Atkinson, MO Lambda.

See page 76 for the full convention photograph.

# Special Curves Connected 

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Presented at the 1995 National Convention

My original intention was to study Lissajous figures in order to relate them to a physics lab project. That proved to be an uninteresting geometric topic. My efforts were then expanded to planar curves in general. Most curves are the result of other curves or of points and curves "interacting." After looking at a number of curves and how they are formed, one awful question came up: "So what?" So what, indeed! As you will see, the answer to that question will connect some geometry to other parts of mathematics.

The first part of the research after the Lissajous figures was to study planar curves. First came the limaçon. The limaçon is the cycloid of two circles of the same radius, one rolling on the other. It can be formed by the parametric equations

$$
x=a \cos t+b \cos 2 t \quad y=a \sin t+b \sin 2 t .
$$

When $a=2 b$, the curve is a cardiod, and when $a=b$, it is a trisectrix. The trisectrix can be used to trisect an angle. Figure 1 shows the limaçon and its special cases (Lockwood [2, pp. 45-47, 50]).

The cissoid is more difficult to explain. The cissoid of Diocles is a special cissoid said to be with respect to a circle, a point on the circle, and a tangent line diametrically opposed to the point. This was used to solve the classic problem of doubling the cube. It is the result of the parametric equations

$$
x=\frac{2 a t^{2}}{1+t^{2}} \quad y=\frac{2 a t^{3}}{1+t^{2}} .
$$

Figure 2 (left side) shows the cissoid of Diocles with a circle of radius $a=\frac{1}{2}$ (Lockwood [2, pp. 130-132]).

A right strophoid can be formed by the equation

$$
y^{2}=\frac{x^{2}(1+2 a x)}{1-2 a x}
$$



Figure 1. Clockwise, from upper left: general limaçon ( $a=3$, $b=2$ ); cardioid ( $a=2, b=1$ ); trisectrix ( $a=b=1$ ).
where $a$ is a constant. Figure 2 (right side) shows a right strophoid with $a=1$ (Yates [3, p. 129]).


Figure 2. Cissoid of Diocles (left); right strophoid (right).
The lemniscate of Bernoulli is the cissoid of a circle of radius $a / 2$ with respect to a point $O$ that is a distance $a \sqrt{2} / 2$ from the center. It is formed using the polar equation

$$
r^{2}=a^{2} \cos (2 \theta)
$$



Figure 3. Lemniscate of Bernoulli.
Figure 3 shows a lemniscate with $a=1$.
These curves - the limaçon, cissoid, strophoid, and lemniscate - have something in common. They are all formed from conics under inversion in a circle. What is inversion in a circle, for those who are not familiar with it? When dealing with individual points, the inverse of a point $P$ with respect to a circle of radius $k$ and center $O$ is defined to be the point $P^{\prime}$ on ray $\overrightarrow{O P}$ such that $d(O P) d\left(O P^{\prime}\right)=k^{2}$ (see figure 4).


Figure 4. Inverse of a point.
Inversion is used with complex numbers and in non-Euclidean geometry models. Some interesting properties become evident. If a point $P$ is on the circle of inversion, for example, then the inverse is given by $P^{\prime}=P$. Also, if $P$ is inside the circle, then $P^{\prime}$ is outside, and vice versa. To construct an inverse point $P^{\prime}$ of a point $P$ inside a circle $\gamma$, let $\overline{T U}$ be the chord of $\gamma$ perpendicular to line $\stackrel{O P}{ }$ through $P$. Then $P^{\prime}$ is the point of intersection of line $\overleftrightarrow{O P}$ and the tangents to $\gamma$ at $T$ and $U$ (see figure 5).

If $P$ is outside of $\gamma$, then let $Q$ be the midpoint of $\overline{O P}$. Let $\sigma$ be the circle with center $Q$ and radius $d(O Q)=d(Q P)$. Then $\sigma$ cuts $\gamma$ in two


Figure 5. Construction of the inverse of a point inside the circle.


Figure 6. Construction of the inverse of a point outside the circle.
points $T$ and $U$, and $P^{\prime}$ is the intersection of lines $\overleftrightarrow{T U}$ and $\overleftrightarrow{O P}$ (Greenberg [1, pp. 243-244]). See figure 6.

When dealing with whole curves, therefore, the inverse of a curve is the locus of all inverse points of that curve. The inverse of a parabola, for example, with the center of inversion (that is, the center of the circle of inversion) at the vertex, turns out to be the cissoid of Diocles (figure 7).

The strophoid is the result of the inversion of a rectangular hyperbola with a vertex as the center of inversion (figure 8). Figure 9 shows how a rectangular hyperbola with the center of inversion at the center is inverted to a lemniscate.

Last, but not least, a conic with the center of inversion at a focus inverts


Figure 7. Parabola inverted to a cissoid of Diocles.


Figure 8. Hyperbola inverted to a right strophoid.
to a limaçon (figure 10). Note how a circle inverts to another circle. This should be no surprise since a circle is both a special conic and a special limaçon (Yates [3, pp. 129-130).

How are these inversions done? There are three methods (there may be more, but they are pretty elaborate.) First, construct several inverse points by hand to the given curve until they can be connected into some recognizable curve. This is long, tedious, inaccurate, and not really worth the time. The other methods involve simply transforming the equation of the given curve. Then the transformed equation can be put into a computer that plots graphs of equations. The second method is for an equation in cartesian form. To invert a given curve in a circle of radius $k$ centered at the origin, substitute for the $x$ value in the equation the value

$$
x=\frac{k^{2} x^{\prime}}{x^{\prime 2}+y^{\prime 2}}
$$



Figure 9. Hyperbola inverted to a lemniscate.


Figure 10. Conics inverted to limaçons. Clockwise, from top left: ellipse, hyperbola, parabola.
and substitute for $y$ the value

$$
y=\frac{k^{2} y^{\prime}}{x^{\prime 2}+y^{\prime 2}}
$$

For example, the equation for a parabola is

$$
y^{2}=h x
$$

(tilted to the side), where $h$ is a constant, and it inverts to

$$
y^{\prime 2}=\frac{h x^{\prime 3}}{1-h x^{\prime}}
$$

This is the general rule for all curves in such a situation, and it can be done in reverse in the same manner. The inversion is a little difficult to see at a glance of the original and transformed equations. The third method is more easily understood, and the way an inverse curve is formed becomes very recognizable. If a polar equation is given (as a function of $r$ and $\theta$ ), let $r_{1}$ be the radius for the original function. Then $r_{2}$ is the radius for the inverted function if

$$
r_{1} r_{2}=k^{2}
$$

$k$ again being the radius of the circle of inversion. For example, the polar equation for a conic is

$$
r_{1}=\frac{1}{a+b \cos t}
$$

For inversion in a circle of radius $k$, the transformed equation is

$$
r_{2}=k^{2}(a+b \cos t),
$$

or $k^{2}$ times its reciprocal, which is the equation for a limaçon in polar form. If the circle has radius $k=1$ (as in figures 7-10), the transformation is done by simply taking the reciprocal. This transformation applies to all such polar equations (Yates [3, p. 129]).

One can start making a few generalizations by just looking at the graphs. One observation would be that the tangents at the "cusp" of the limaçon in figure 10 (right side) are parallel to the asymptotes of the hyperbola. Inversion with respect to a circle has some applications. It can be applied to non-Euclidean geometry as a way to understand and work with the Poincaré model. It is also used with complex numbers, widely used in physics and computer science.
Acknowledgements. I would like first to thank Dr. Glenn Fenneman for his help in editing this paper. I would also like to thank Dr. Augie Waltmann, our corresponding Kappa Mu Epsilon secretary, for helping with the arrangements to present the paper at the national convention. Finally, many thanks go to Dr. Lynn Olson, my modern geometries professor, for helping me put this paper together. I never realized what fun rigorous proofs could be!

Editor's note. The author prepared figures 1-3 and 7-10 using Maple V (Waterloo Maple Software, Waterloo, Ontario) and figures 4-6 using The Geometer's Sketchpad (Key Curriculum Press, Berkeley, California).

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## Largest Prime

"In Euclid, there appears a proof of the statement that there is an infinite number of prime numbers. The greatest prime number known at the present time, however, is $2^{127}-1=170,141,183,460,469,231,731,687$, 303,715,884,105,727."
-The Pentagon, Vol. 1 Issue 1 Fall, 1941

In early 1994, Cray Research Inc. announced the discovery of the prime number $2^{859433}-1$, which has 258,716 digits. At 2322 digits per page, this number would take 112 pages of The Pentagon to print. That represents 1.4 times the current size of an entire issue.

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# A Study of Nineteenth and Twentieth Century Mathematics Textbooks 

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Presented at the 1995 National Convention

Dickens. Shakespeare. Hemingway. Whitman. Homer. Names such as these form the foundation of modern literature classes. All are masters of some form of writing style, ranging from the novel to the short poem to the play. Yet nowhere in this long list of great writers do you run across the names Colburn, Greenleaf, or Davies. Who are these men? They were among the many nineteenth century authors of mathematics textbooks that would shape the minds of numerous generations.

Why is it that textbooks are, in general, excluded from the royal treatment given to other period pieces? Are they, in fact, literature? One definition of literature is given as follows: "writings having excellence of form or expression and expressing ideas of permanent or universal interest." The condition of excellence is subjective at best, but there were undoubtedly a few select texts that were regarded as among the best. And what subject other than math could convey ideas of permanent or universal interest? So, it is by no stretch of the imagination that these texts are indeed literature.

The ensuing question is quite simple: what could we hope to gain by studying mathematics textbooks? In the following pages, three primary areas will be brought forth: historical significance, social exposition, and the adaptation of mathematics and texts themselves. Of course, each of these areas contains numerous sub-groupings as well, which will be introduced and discussed when necessary and appropriate.

We have defined historical significance loosely as a collection of facts, numbers, and terms that could easily be found in a history textbook. Some of the most trivial of this material consists of mere price comparison. Far and away the most common type of goods mentioned in the texts (until just recently) were farm goods such as horses, cattle, and plows and house-
wares such as cloth, foodstuffs, and books. For example, from Mercantile Arithmetic (1806), a problem mentions that a horse cost $\$ 24$. By 1828, in Intellectual Arithmetic, that horse is worth $\$ 63$. This trend continues to 1884 when a horse was worth $\$ 140$ and to 1920 when a horse averaged around $\$ 170$. For reference, a horse today will easily stretch into the two thousand dollar plateau.

Similarly, we see salaries skyrocket from $\$ 300$ per year in 1818 (Federal Calculator) to approximately $\$ 1600$ in 1909 (Primary Arithmetic) to the current "astronomical" average of around twenty thousand dollars. It is interesting to note that in 1909 teachers were being paid around $\$ 1100$ per year, according to the texts. In other words, even eighty years ago teachers were being paid substantially less than the average laborer. There were countless other examples ranging from apples and oranges to hay and land (following the pattern of agriculture-related problems).

Many other types of numbers jumped out as we went through the texts. In 1920 the average beef consumption, according to School Arithmetic, was 194 pounds, in comparison to 64.1 pounds in America today. According to Elements of Numbers (1826), the President that year made $\$ 25,000$. It's hard to say exactly what the President makes today with all the benefits that are given them, but a conservative estimate puts the total well over $\$ 250,000$.

The reader can also see history in a more subtle manner by reading the questions closely. Frequently there are tables involving the states of the union which include well under the fifty we have now. The first mention of the automobile was in School Arithmetic in 1920. There is frequent mention in texts during or shortly after both world wars of boys practicing marching or of the size and capacity of ships and warplanes. In most texts, even as late as Intellectual Arithmetic in 1884, monetary values are often given in English currency. As far as money goes, in many of the texts found around the mid-nineteenth century such as Progressive Intellectual Arithmetic (1871), there are conversions given for the currencies of the different states (e.g. from New York dollars to Connecticut dollars).

It is also easy to ascertain the nature of things important and new to the people. Until early in the 1900 's, most texts discussed rural and agricultural settings. Many texts contained appendices that dealt with approximating the volume of grain bins, hay mows, cisterns, and the like. There are numerous chapters devoted to the topic of bartering, as in the Compendium of Arithmetic (1818).

Even words have come and gone from the American vocabulary. Words such as pantaloons, drovers, and steres are quite common in texts such as Normal Written Arithmetic (1863). Of course, words such as airplane, television, and car are absent. It is even possible to watch the slavery issue develop right before your eyes. In some texts during the 1830's and

1840's there is slight mention of slavery, which changes into mentionings of war preparation and weaponry during the 1860 's. However, as one would expect, there is no mention of any minorities, at least until A Higher Arithmetic in 1930 asks a question about an "Indian." After that, once again the minorities disappear.

As history merges more and more into social commentary, we begin to see the transition of "enlightenment" of our citizenry. The "political correctness" that both protects and hinders today is not evident in prior texts. There are countless questions found in texts that would most certainly not make production in today's books. For example, in the First Book of Arithmetic (1880) there is a table comparing and converting the different monetary values referred to in the Bible. There is also a reference to Christ in the Scholar's Guide to Arithmetic (1815) referring to calculating the age of other famous figures in history. It is highly doubtful that any text today would make mention of Biblical figures out of fear of offending someone of a different religion or faith. While this is a comment on the increased number of heritages in today's schools, it is also a comment on the fear that manufacturers have today of offending anyone.

This very same fear of offense is once again obvious when discussing gender roles in the home and workplace. In the previously mentioned School Arithmetic, there are a number of story problems involving tasks performed by various children. In all of these problems the reader encounters situations that would be called gender bias today. Only boys are mentioned when discussing farm work, or any kind of job for that matter. Girls are discussed strictly doing homebound jobs such as mending clothes, preparing meals, and buying groceries. When the occasional sport or recreational activity is mentioned, such as football and baseball in the late 1800 's, only boys are mentioned and described. Special activities for women are quilting bees and social gatherings.

There are a few other matters that come up in these texts that would also be "taboo" in today's books. There is frequent mention of ale, wine, or beer measure in books such as Arithmetick (1809) and American Tutor's Guide (1808). Common sense says that most elementary, or secondary for that matter, text books would not cover that material today.

There are a number of topics that today's books don't cover that were central to texts of the past. The most obvious when leafing through the older journals is the number of pages devoted to financial matters. In books such as National Arithmetic (1881), there are over twenty pages dedicated to material such as insurance, tare and tret( t ), commissions, and brokerages. Many books include sections strictly discussing stocks and bonds and various other investment opportunities such as annuities. This continues well into the nineteenth century until, suspiciously, approximately in the late 1920's, shortly before the great stock market crash. The texts also
go into simple and compound interest extensively. Once again, it may be worth reminding the reader that these books were designed for the elementary classroom. In general, however, the content of older texts is reasonably close to that of more modern examples. The basic arithmetic components are all there including addition, subtraction, multiplication, division, fractions, and ratios. Some of the key differences include symbology, difficulty of problems, number of problems, and some specific topics such as involution/evolution and permutations/combinations.

While most symbols remained constant throughout the last two hundred years, there are a few interesting exceptions. Most notable is the symbol used for division in the Federal Calculator. Instead of the traditional symbol resembling a dash placed through a colon, this text uses two dots on both top and bottom. In Elements of Numbers, the traditional fraction representation is replaced by a dash. Thus, three fourths would be represented as 3-4 instead of 3/4. Also in Elements, as well as in Arithmetick (1844), the decimal in decimal numbers is replaced by a comma. For example, as we would normally write 3.14159 , this number would be written 3,14159 instead. In addition, the text Compendium of Arithmetic uses no symbol for a decimal. Instead, a space is used (the above decimal would be written 3 14159).

One more difference in representing numbers can be found in both Elements of Numbers and Primary Arithmetic (1865), in which as our system advances million, ten million, hundred million, billion, etc., these texts count million, ten million, hundred million, thousand million, ten thousand million, hundred thousand million, billion, etc. In other words, million, billion, and so on have a six number period instead of the three number period as we know it (e.g. the number 123456789012 would be read one hundred twenty-three thousand four hundred fifty-six million, seven hundred eighty nine thousand and twelve).

The older texts also incorporate some material that is considered higher level now. These elementary texts (which range from first through eighth grade) often include sections on involution (computing the powers of numbers), evolution (extracting roots to the fifth degree), permutations and combinations, duodecimals, and arithmetic and geometric series, and some of these arithmetics even contained basic material on geometry, such as the Pythagorean Theorem and basic shapes. While this may indicate that older texts were more difficult, it is easier to see this increased level of ability when looking at the type of problems provided for practice.

By the third page after introducing arithmetic, students of the time were asked to add 472634926743 and 378276329833 . In the chapters on involution/evolution, children were asked to find the fifth root of 4182119424 (which happens to be 84). Upon introduction of multiplication, students were asked to multiply 8274937 and 2837495 . These types of problems are
not the exception but rather the rule. To make matters worse, the number of problems per section was greatly larger in past texts. In Intellectual Arithmetic (1828), one section of homework problems may run seven to eight pages and contain approximately one hundred twenty questions.

Even the type of problems is different. In the First Book of Arithmetic, there are whole sections of material dedicated to "slate and board" exercises as well as occasional oral activities, which differs from the texts of today which are strictly made up of paper and pencil replies. In Primary Arithmetic (1874), Intellectual Arithmetic (1828), and Progressive Intellectual Arithmetic, the whole book is a series of questions. There are no sections introducing material or explaining content; instead the text tries to lead the students into their own discovery by asking the right questions. Luckily for those students, many texts, such as Written Arithmetic (1866), give the answers, either written after the question or in the back of the text. This is another departure from modern arithmetic texts (Note: many secondary level mathematics texts do include at least some answers).

One final difference between questions of the past and those of the present deals with the goal of the question. The majority of questions asked in our current texts deal with some sort of contrived experience using apples, oranges, speeds of automobiles, or distances from place to place (the kind of problem we all grew up on). The goal for earlier questions was quite specifically to apply to real-life situations. Various texts had chapters entitled "On the Dairy," "At the Store," "On the Farm," and 'In the Bank" in which all questions were obviously designed to prepare the students for these situations after they left school. It is interesting to note that application type problems were prevalent in the older texts as we are once again moving in that direction.

That leads us to the next logical question: what of the textbooks of tomorrow? There are two basic possible trends that we can choose from: linear or cyclical. If the books follow the linear transformation, we can expect books with fewer and fewer application problems and increased theoretical questions. We can see the difficulty of material continue to decline until basic skills will begin to carry into the middle and upper grades. Mentioned in the questions we'll see increased "political correctness" until we have completely unbiased, generic people doing unbiased, generic tasks with unbiased, generic items. However, we should see better organization of material and students will be better able to use the texts. If, however, we find a cyclical pattern of events, we will see a return to stressing real-life problems. We will once again challenge students while still making the material clear and intuitive. Hopefully, we will also see less concern for proper wording of questions and increased importance placed on the pertinence of the questions.

Indeed, textbooks are literature by any definition and they should be
treated as such. There are numerous insights to be gained by studying these books in regard to what lies in the past and in what will lie ahead. From these preserved pieces of history we can study the history of our nation by learning what the children of that period were expected to learn. We can also see what was important to the people of the time as well as the new ideas that were constantly coming along. Finally, we can watch the slow change of the face of mathematics and how it is brought to students. Both the mathematical ideas and the writing of the texts themselves slowly adapt to the needs of the people. There is little argument then that the names of Colburn and Davies do indeed belong on the list of important authors. These were the men who would help shape the minds of a dozen generations by passing on the knowledge of mathematics. These were men who taught our own grandfathers and great-grandfathers. These were men who passed on a little piece of history. Let us learn from these texts again.

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## The Pentagon on Old Textbooks

The first issue of The Pentagon, dated Fall 1941, contains an article by E. R. Sleight of Albion College (MI Alpha) entitled "Robert Record's Whetstone of Witte." According to Sleight, this text appeared in 1556 as the second part to his 1540 publication Ground of Artes, which was the first widely used arithmetic text in the English language.

## Papers Anyone?

Students - do you have a mathematical interest that you would like to share with others? It's never too early to start working on papers for the regional conventions and the next national convention! When you write papers for regional conventions, please remember to submit them to The Pentagon for possible publication. Even if you are unable to attend a convention, you may still submit papers to this journal. See page 2 for details.

## KME On-Line

Yes, Kappa Mu Epsilon has hit the World Wide Web! The KME home page is physically located at Central Michigan University. Its URL is:
http://www.cmich.edu/kme.html
Information already available includes the organization and purposes of KME and membership information. Future plans for available information include the constitution of KME, the history of KME, information about national and regional officers, information about national and regional events, and chapters of KME by region.

Also, e-mail addresses for Pentagon personnel and KME National Officers are now listed on pages 2 and 77, respectively, of this publication.

# The Problem Corner 

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before July 1, 1996. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1996 issue of The Pentagon, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

## PROBLEMS 490-494

Problem 490. Proposed by Troy D. Van Aken, University of Evansville, Evansville, Illinois.

Suppose a person wants to cover a pickup bed that is four feet wide with a flexible plastic cover so that the cover rises one inch in the center (see figure below). If one assumes that the shape of the cover is circular, how large should the piece of plastic be cut? What if one assumes that the shape is parabolic?


Problem 491. Proposed jointly by Sammy and Jimmy Yu, special students at the University of South Dakota, Vermillion, South Dakota.

Evaluate the integral

$$
I=\int \frac{\sqrt{m-x^{n}}}{x^{1+n / 2}} d x
$$

Problem 492. Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.

The Fibonacci numbers are defined by $F_{0}=0, F_{1}=1$ and $F_{n+2}=$ $F_{n+1}+F_{n}$ for $n=0,1,2, \ldots$. The Lucas numbers are defined by $L_{0}=2$, $L_{1}=1$ and $L_{n+2}=L_{n+1}+L_{n}$ for $n=0,1,2, \ldots$. Show that

$$
\sum_{i=0}^{n} F_{i}^{2}=\left\{\begin{array}{l}
\frac{1}{5}\left(L_{2 n+1}+1\right) \text { if } n \text { is odd } \\
\frac{1}{5}\left(L_{2 n+1}-1\right) \text { if } n \text { is even } .
\end{array}\right.
$$

Problem 493. Proposed jointly by C. Bryan Dawson and Sam Snyder, Emporia State University, Emporia, Kansas.

Consider the decimal expansion of $\frac{\sin x}{x}$, where $x=10^{-k}$ for some nonnegative integer $k$ where $x$ is expressed in radians. Show that the first $6 k+3$ digits of the decimal expansion of $\frac{\sin x}{x}$ are given by $9_{2 k} 83_{2 k} 416_{2 k}$, where $r_{i}$ denotes $i$ repetitions of the digit $r$.

Problem 494. Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Let $x$ and $y$ be integers. Prove that if $3 x^{2}-2 y^{2}=1$, then $x^{2}-y^{2}$ is divisible by 40 .

Please help your editor by submitting problem proposals.

SOLUTIONS 472, 480-484

CORRECTION to Problem and Solution 475:
Condition (b) in the statement of the problem should have read

$$
\text { (b) } \sigma(n(n+2)) \phi(n(n+2))=\left(n^{2}+2 n+1\right)\left(n^{2}+2 n-3\right) \text {. }
$$

Also, the following portion of the reference to Prielipp's second solution should have read as follows:

A positive integer $\boldsymbol{m}$ is the product of two primes differing by 2 if and only if $\sigma(m) \phi(m)=(m-3)(m+1)$ (Page 56). This is problem 317 in the Canadian Mathematical Bulletin and was proposed by A.A. Mullin (Page 56) ... . These page numbers correspond to the page numbers of the solution published in the Spring 1995 (Volume 54, Number 2) issue of The Pentagon's Problem Corner. The Editor is indebted to Bob Prielipp's sharp eyes. The Editor apologizes for any inconvenience these typographical errors may have caused.

Editor's Comment. The names of Sammy and Jimmy Yu, special students, University of South Dakota, Vermillion, South Dakota were inadvertently omitted from the list of solvers of Problems 476 and 478 in the preceding column. The editor apologizes for this oversight.

Problem 472 (corrected). Proposed by Del Ebadi, Topeka West High School, Topeka, Kansas.

Given that the absolute value of the average of two real numbers is $4 \sqrt{6}$ and that the geometric mean of these numbers is $6 \sqrt{2}$, find the absolute value of the difference of the two numbers without finding the numbers themselves.

Solution by Laura Ramdarass and Trien Ngoc Vo (jointly), Austin Peay College, Clarksville, Tennessee.

Let the two numbers be $x$ and $y$. Then the absolute value of their average is

$$
\left|\frac{1}{2}(x+y)\right|=4 \sqrt{6}
$$

or

$$
\begin{equation*}
|x+y|=8 \sqrt{6} . \tag{1}
\end{equation*}
$$

The geometric mean of $x$ and $y$ is

$$
(x y)=6 \sqrt{2}
$$

or

$$
\begin{equation*}
x y=72 . \tag{2}
\end{equation*}
$$

Finally, from (1) and (2) we have

$$
|x-y|^{2}=|x+y|^{2}-4 x y=384-288=96
$$

or

$$
|x-y|=\sqrt{96}=4 \sqrt{6}
$$

since the absolute value is always positive.
Also solved by: The Alma College Problem Solving Group, Alma College, Alma, Michigan; Todd Bittinger, Shippensburg University, Shippensburg, Pennsylvania; Paul R. Cox, Rosary College, River Forest, Illinois; Clayton Dodge, University of Maine-Orono, Orono, Maine; Russell Euler, Northwest Missouri State University, Maryville, Missouri; J. Perez, Austin Community College, Austin, Texas; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; and J. Sriskandarajah, University of Wisconsin Center-Richland, Richland Center, Wisconsin.

Problem 480. Proposed jointly by Sammy and Jimmy Yu, special students at the University of South Dakota, Vermillion, South Dakota.

Let ( $a, b, c$ ) be a Pythagorean triple where $c^{2}=a^{2}+b^{2}$ and $T_{n}$ be the $n$th triangular number. Solve the Pythagorean equation

$$
\left(p T_{n}+c\right)^{2}=\left(p T_{n}+a\right)^{2}+(q b)^{2}
$$

for positive integers $p$ and $q$ in terms of $a, b, c$, and $n$.
Solution by Clayton Dodge, University of Maine-Orono, Orono, Maine.
Let $x=p T_{n}=\frac{1}{2} p n(n+1)$. Then we have $c^{2}=a^{2}+b^{2}$ and $(c+x)^{2}=$ $(a+x)^{2}+(q b)^{2}$. Multiplying out the second equation, subtracting the first equation and simplifying, we get

$$
2 c x=2 a x+\left(q^{2}-1\right) b^{2},
$$

or

$$
\begin{equation*}
2 x(c-a)=\left(q^{2}-1\right) b^{2} . \tag{1}
\end{equation*}
$$

Multiplying both sides of (1) by $c+a$ and then dividing by $b^{2}=c^{2}-a^{2}$ we get

$$
2 x=\left(q^{2}-1\right) b^{2}=p n(n+1)
$$

and finally

$$
\frac{p}{q^{2}-1}=\frac{c+a}{n(n+1)} .
$$

Now define $s$ by $p=s(c+a)$, so that

$$
q^{2}-1=s n(n+1)
$$

and

$$
q=s n^{2}+s n+1
$$

whose right side is a square for $s=0$ or $s=4$. Assuming that $c+a$ and $n(n+1)$ have no factors in common, then only these two values of $s$ always yield solutions. We then have the two solutions $p=0, q=1$ and $p=4(c+a), q=2 n+1$.

Also solved by: the proposers.
Problem 481. Proposed jointly by Sammy and Jimmy Yu, special students at the University of South Dakota, Vermillion, South Dakota.

Evaluate the integral

$$
I=\int \frac{m-x^{n}}{x^{1+n / 2}} d x
$$

Solution by The Alma College Problem Solving Group, Alma College, Alma, Michigan.

$$
\begin{aligned}
I & =\int \frac{m-x^{n}}{x^{1+n / 2}} d x \\
& =\int\left(\frac{m}{x^{1+n / 2}}-\frac{x^{n}}{x^{1+n / 2}}\right) d x \\
& =\int m x^{-(1+n / 2)} d x-\int x^{n-(1+n / 2)} d x \\
& =\frac{m x^{-n / 2}}{-n / 2}-\frac{x^{n / 2}}{n / 2}+C \\
& =\frac{-2\left(m+x^{n}\right)}{n x^{n / 2}}+C
\end{aligned}
$$

provided that $\boldsymbol{n} \neq 0$. If $\boldsymbol{n}=0$, then ( $I$ ) becomes

$$
\begin{aligned}
I & =\int\left(m x^{-1}-x^{-1}\right) d x \\
& =(m-1) \ln |x|+C .
\end{aligned}
$$

Also solved by: Clayton Dodge, University of Maine-Orono, Orono, Maine; Russell Euler, Northwest Missouri State University, Maryville, Missouri and the proposers. A partial solution was received from Leanne Majors, Holy Family College, Philadelphia, Pennsylvania.

Problem 482 CORRECTION. Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

The problem should have read:

$$
\text { Evaluate } I=\int_{0}^{\pi} \ln (\sin x) d x
$$

Solutions which involve complicated series were received from Clayton Dodge, University of Maine-Orono, Orono, Maine and Russell Euler, Northwest Missouri State University, Maryville, Missouri. This was caused by the inadvertent omission of the limits of integration, making the problem more difficult than was intended. The problem as corrected remains open. The editor apologizes to Clayton Dodge and Russell Euler for any inconvenience caused. One additional solution was received which was incorrect since the integration failed to consider the possibility of discontinuities occurring whenever $\sin x=0$.

Problem 483. Proposed by the editor.
Let $\left\{a_{i}\right\}$ for $i=1,2, \ldots, k$ be a sequence of positive integers such that $a_{1}+a_{2}+\cdots+a_{k}=200$. What values must be chosen for the $a_{i}$ in order to maximize the product of the $a_{i}$ ?

Solution by Paul R. Cox, Rosary College, River Forest, Illinois.
Let $N$ be a given integer, let $N \equiv m \bmod 3$ and let $p=(N-m) / 3$. We shall show that for any given integer $N$, the maximum product $P$ is given by $P=2^{a} 3^{b}$ where $b=p$ and $a=0$ if $m=0, b=p-1$ and $a=2$ if $m=1$ and $b=p$ and $a=1$ if $m=2$. We shall need the following

Lemma. For any fixed integer $k$ and any positive real number $N$, let $\left\{a_{i}\right\}$ denote a sequence of positive real numbers such that $N=a_{1}+a_{2}+\cdots+a_{k}$. Then the maximum product of the $a_{i}$ for $i=1,2, \ldots, k$ occurs when $a_{i}=$ $N / k$ for all values of $i=1,2, \ldots, k$.

Proof. By mathematical induction. For $k=1$, verification is obvious. For $k=2$, we have $N=a_{1}+a_{2}$. Then $a_{2}=N-a_{1}$. We want to maximize the product $a_{1} a_{2}=a_{1}\left(N-a_{1}\right)$. Treating $a_{1}$ as a variable, the usual calculus methods show that the maximum product occurs when $a_{1}=a_{2}=N / 2$.

For $k=n$, suppose that if for each real number $N=a_{1}+a_{2}+\cdots+a_{n}$, then the maximum product $P=a_{1} a_{2} \cdots a_{n}$ occurs when $a_{i}=N / n$ for $i=1,2, \ldots, n$. Then for $k=n+1$, suppose that $N=a_{1}+a_{2}+\cdots+a_{n+1}=$ $a_{n+1}+N$ where $N$ consists of the sum of the $n$ positive real numbers $a_{1}+$ $a_{2}+\cdots+a_{n}$. Then we need to maximize the product $P=a_{n+1}\left(a_{1} a_{2} \cdots a_{n}\right)$. But by the induction assumption the maximum product of the $a_{1}, a_{2}, \ldots, a_{n}$ is given by $N-c=((N-c) / n)^{n}$ where $c=a_{n+1}$. Then we need to maximize
the product

$$
P=c((N-c) / n)^{n}=n^{-n} c(N-c)^{n} .
$$

Then $d P / d c=n^{-n}(N-c)^{n-1}[(N-c)-n c]=0$ implies that either $c=N$ or $c=N /(n+1)$. Since $c=N$ is not allowable, we must have $c=N /(n+1)$ and the maximum product of the $k+1 a_{i}$ for $i=1,2, \ldots, n+1$ is given by $P=(N /(n+1))^{n+1}$ where each of the $a_{i}=N /(n+1)$. Since $d^{2} P / d c^{2}<0$ when $c=N /(n+1)$, this value of $c$ provides the maximal product of the $k+1 a_{i}$ for $i=1,2, \ldots, n+1$. This proves the Lemma.

Next we seek a value of $k$ which maximizes the product $(N / k)^{k}$. Using the usual first derivative techniques, we find that the desired value of $k$ is $k=N / e$, which is not an integer. Then since $2<e<3$, we must have each of the $a_{i}$ be either 2 or 3 for $i=1,2, \ldots, k$. Then since two 3 's produce a greater product than three 2's, the product consists of as many 3's as possible. Also two 2's provides a greater product than 3.1. Finally, by defining $m \equiv N \bmod 3$ and $p=(N-m) / 3$, we have the desired result: For any given integer $N=a_{1}+a_{2}+\cdots+a_{k}$, the maximum product $P=a_{1} a_{2} \cdots a_{n}$ is given by $P=2^{a} 3^{b}$ where $b=p$ and $a=0$ if $m=0$, $b=p-1$ and $a=2$ if $m=1$ and $b=p$ and $a=1$ if $m=2$. For $N=200$, the desired product is $2^{2} 3^{66}$.
Editor's comment. The calculus arguments can be obviated by the following observation: for $n>4$ one can replace $n$ in the sum with the integers 2 and $n-2$ which give a larger product. Repeated use of this observation shows that the desired product $P=2^{a} 3^{b}$ for some nonnegative integers $a$ and $b$. Then since $2^{3}<3^{2}, a \leq 2$, and the remainder of the desired result is easily established. This problem is very similar to a problem which appears in Mathematical Baffiers, by Angela Dunne, Dover Publications, 1980, pp. 119 and 132.

Also solved by: Clayton Dodge, University of Maine-Orono, Orono, Maine.

Problem 484. Proposed by the editor.
In the decimal system find all 10 digit palindromic numbers which are the product of two consecutive integers. A palindromic number has the same value regardless of whether it is read from right to left or vice versa.
Solution by Clayton Dodge, University of Maine-Orono, Orono, Maine.
By the lowly device of a BASIC computer program, listed below, we find the sole solution $2,704,884,072=52,008 \cdot 52,009$.
defdbl t
for $a=1$ to 9
for $b=0$ to 9

```
for c = 0 to 9
for d = 0 to 9
for e = 0 to 9
t = e*110000 + d*1001000 + c*10000100 + b*100000010
    + a*1000000001
s = int(sqr(n))
if t = s*(s + 1) then print t,s
next
next
next
nert
next.
end
```

By allowing the first digit to be zero, we find the five somewhat interesting cases ( $n$, number) where number denotes the palindromic number which is equal to $n(n+1)$ :

$$
\begin{aligned}
(n, \text { number })= & (25305,0626776260),(11110,1023443210), \\
& (6599,0043553400),(6324,0039999300) \\
& \text { and }(1000,0001001000) .
\end{aligned}
$$

Other cases involving fewer digits can be found easily.
Also solved by: Russell Euler and Jawak Sadek (jointly), Northwest Missouri State University, Maryville, Missouri.

Editor's comment. The joint solution by Russell Euler and Jawak Sadek used a multiple case analysis based upon several congruence conditions imposing conditions upon possible choices for $n$. Unfortunately, the most efficient solution utilizes a computer program after reaching the realization that the initial and final digits of the desired palindrome must be 2 or 6 .

## Math Joke

Did you know that at one time there was an American League baseball team made up entirely of mathematicians? Unfortunately, it disbanded shortly after it was formed, due to a ruling from the Commissioner. What was this ruling? He ruled that no matter how many games the mathematicians won, they could not be declared AL Champions. Why? Because they didn't even want to consider the possibility of an infinite Series!

# Kappa Mu Epsilon News 

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy KME events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, Missouri 64801.

## CHAPTER NEWS

## AL Gamma

University of Montevallo, Montevallo
Chapter President-Jamie Tallie
16 actives, 6 associates
Other 1995-96 chapter officers: Timo Langerwerf, vice president; Terra Cottingham, secretary; Kim Snider, treasurer; Larry Kurtz, corresponding secretary; Don Alexander, faculty sponsor.

## AR Alpha

Chapter President-Cindy Nicholsen
Arkansas State Univeraity, State University 14 actives, 7 associates
Other 1995-96 chapter officers: Odis Cook, treasurer; William Paulsen, corresponding secretary/faculty sponsor.

## CA Delta

California State Polytechnic University, Pomona
Chapter President-Sean Smith 10 actives, 1 associate
Other 1995-96 chapter officers: Steven Gurtin, vice president; Jennifer Baird, secretary; Maria Nuñéz, treasurer; Richard Robertson, corresponding secretary; Jim McKinney, faculty sponsor.

CO Gamma
Chapter President-Darren Gemoets
35 actives
On March 29, in conjunction with the Tenth Anniversary Observance of the installation of the chapter, CO Gamma initiated eight new members. Semester activities centered on preparing for and hosting the Biennnial Convention in April. The approximately 200 members from 35 chapters attending judged it a wonderfully successful convention! Other 1995-96 chapter officers: Tom Bruckner, vice president; Ben Moore, secretary; Stevan Scott, treasurer; Richard Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

CO Delta
Mesa State College, Grand Junction

Chapter President-Scott Davis 18 actives, 7 associates

At the fall meeting in September, keys and certificates were presented to those who had been initiated in April, 1994. The student members voted for $\$ 20.00$ annual local dues as a means of funding some chapter activities. They also decided to get chapter T-shirts as a means of increasing visibility on campus. Three students, along with the faculty sponsor and the corresponding secretary, had the priviledge of attending the Biennial Convention in Durango in April. All enjoyed the student papers and the opportunity to meet and share ideas with other KME chapters. Thirty-six members and guests were in attendance at the initiation banquet held on April 6, 1995. Seven new members were initiated at that time. Other 1995-96 chapter officers: Venus L. Martinez, vice president; Natisha R. Kimminau, secretary; Tammi I. Giroir, treasurer; Donna K. Hafner, corresponding secretary; Clifford C. Britton, faculty sponsor.

## FL Beta

Chapter President-Tammy Causey 11 actives, 8 associates Florida Southern College, Lakeland Other 1995-96 chapter officers: Shannon Tomarchio, vice president; Bradley Hof, secretary; Gayle S. Kent, corresponding secretary/faculty sponsor.

## GA Alpha

West Georgia College, Carrollton
On June 1, the Chapter hosted a reception for six new members following initiation. At the reception, several KME members were recognized as recipients of special awards or scholarships: Chris Flournoy received the Cooley Scholarship and the Boykin Scholarship; Amy Westbrook, the Whatley Scholarship; Chad Bean, the Boyd Award; and Helga Floodquist and Kristie Hannah received Crider Awards. Other 1995-96 chapter officers: Helga Floodquist, vice president; Amy Westbrook, secretary; Darron Robbins, treasurer; Thomas J. Sharp, corresponding secretary; Mark Faucette and Joe Sharp, faculty sponsors.

## IL Delta

College of St. Francis, Joliet
IL Delta held an induction ceremony on April 4. Two new inductees, accompanied by moderator Sister Virginia McGee, attended the Biennial Convention in Durango, CO, in late April. Although expenses exhausted chapter funds, the conventioneers thoroughly enjoyed the experience. After serving ten years as corresponding secretary for her chapter, Sister Virginia is retiring from this position and from teaching. Her dedication and efforts have touched many and are much appreciated. Other 1995-96 chapter
officers: Heather McNulty, vice president; Linda Wunder, secretary; John Salzer, treasurer. Corresponding secretary and faculty sponsor are to be named.

## IL Theta

Hlinois Benedictine College, Lisle
15 actives, 7 associates
Initiation was held in conjunction with the annual Mathematics and Computer Science Awards Banquet. Guest speaker, Dr. Jill Van Newenhiren from Lake Forest College, spoke on the topic "The Subtle Art of Voting." Student officers for $1995 \cdot 96$ will be elected in the fall. Mary Beth Dever is corresponding secretary/faculty sponsor.

## IN Delta

University of Evansville, Evansville
Chapter President-Steven Broad 63 actives, 25 associates
Other 1995-96 chapter officers: Glen Templeton, vice president; Carl Bergh, secretary; Troy D. VanAken, corresponding secretary; Mohammad Azarian, faculty sponsor.

## IA Alpha

Chapter President--Jack Dostal 34 actives
Students presenting papers at local KME meetings included Lisa Gaskell on "Fractals, the Geometry of Nature," Darci Lindeman on "Archimedes: The Sand Reckoner," and Saylar Craig on "I Think Knot." Mary Pittman addressed the spring initiation banquet on "An Introduction to Mayan Mathematics." Three new members were initiated: Nick Craig, Brad Klaes, and Matt Schafer. The highlight of the semester certainly was the KME National Convention held in Durango, CO. Eight students from Iowa Alpha attended, along with faculty members John E. Bruha and John S. Cross. Michelle Ruse's presentation, "An Introduction to Multiquadric Interpolation," was judged among the top four at the convention. She was presented with a check for $\$ 100.00$ and an HP calculator. The ten member Iowa Alpha delegation successfully braved two snowy crossings of Wolf Creek Pass under adverse weather conditions. A really great convention outing! Other 1995-96 chapter officers: Andrew Christianson, vice president; Jim Coons and Andy Schafer, secretary; Mary Pittman, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

## IA Gamma

Morningside College, Sioux City
Chapter President-Jason Shriver 9 actives, 4 associates
Other 1995-96 chapter officers: Jared Elwein, vice president; Heath Hopkins, secretary; Heather Schott, treasurer; Steve Nimmo, corresponding secretary/faculty sponsor.

IA Delta
Wartburg College, Waverly

Chapter President-Gretchen Roth 36 actives, 3 associates

On January 16, Professor Josef Breutzmann presented information about "News" on the internet. The program for the January meeting continued with the video of Van Beach's presentation of "Domes, Homes, and the Future," one of the papers presented at the 29th Biennial Convention. A video on Escher and his art was viewed at the March meeting. On March 17 and 18, the chapter co-hosted 45 high school students for the Explorations in Mathematical Sciences event. The initiation of 18 new members was held on March 18. Ron Stahlberg, a Wartburg graduate and Senior Systems Analyst at the University of Iowa Hospitals, was the initiation speaker. His topic was "Mathematics for Four Reasons." Adam Sanford presented his paper, "Special Curves Connected," at the 1995 Bi ennial Convention in April. The year's activities ended with a softball game and annual picnic on May 14. Other 1995-96 chapter officers: Adam Sanford, vice president; Lori Melaas, secretary; Amy Betz, treasurer; August Waltmann, corresponding secretary; Robin Pennington, faculty sponsor.

## KS Alpha

Chapter President-Shelly Younggren
Pittsburg State University, Pittsburg 72 actives
A pizza party and initiation were held in February for 22 new members. Following the initiation, plans were made for money raising activities to apply to expenses for attending the national convention in April. In this regard, a bake sale was held in February and a book sale in April. Both events were quite successful. The chapter hosted a guest speaker for the regular March meeting. Dr. David Surowski from Kansas State University presented an interesting program on "Games, Strategies, and Saddle Points." Six students and three faculty made the long drive in April to Durango, CO, to the National Convention. An enjoyable time was had by all! The chapter assisted the Mathematics Department faculty in administering and grading tests given at the annual Math Relays on April 25. Several members also worked on the Alumni Association's Annual Phon-a-thon. The final meeting for the semester was a social event held at the home of Professor Gary McGrath. Homemade ice cream and cake were served to those present. Officers for the coming year were elected at this meeting and the annual Robert M. Mendenhall awards for scholastic achievement were presented to Michelle Baier and Stephen Hilt. Other 1995-96 chapter officers: Bethany Schnackenberg, vice president; Shannon Wilkinson, secretary; Melissa Marsalis, treasurer; Harold Thomas, corresponding secretary; Cynthia Woodburn and Bobby Winters, faculty sponsors.

KS Beta
Emporia State University, Emporia

Chapter President--Kendra Dawson 35 actives, 11 associates

Other 1995-96 chapter officers: Ryan Karjala, vice president; Justin Elliott, secretary; Dustin Frank, treasurer; Connie Schrock, corresponding secretary; Larry Scott, faculty sponsor.

## KS Gamma

Benedictine College, Atchison
15 actives, 13 associates
Vice-president Gregory Boucher led KS Gamma during the spring semester as President Mary Kay Heideman was involved with student teaching. Initiated on January 26 were students Erik Kurtenbach, Eric Schultz, Bryan Speck, Seth Spurlock, and Dawn Weston. Honorary membership was conferred upon Sister Kathleen Brazzel, a charter member of KS Gamma who had never been formally initiated. The initiation, held in Schroll Center, concluded with a chili supper prepared by chefs Jodie Muhlbauer and Sister Jo Ann Fellin. As a fundraiser, the chapter sponsored a Valentine Dance on February 10. On March 20 a reception was held for Jim Ewbank, who retired from the College in May. Following the reception a lecture was given in Ewbank's honor by one of his former students, John Hutchinson. Hutchinson, currently Director of the National Institute for Aviation Research at Wichita State University, spoke on the topic "Head Injury Criteria." Gregory Boucher gave his convention presentation "Practical Fractical" on campus on April 10. Also attending the national convention in Durango were senior Michael Seebeck and faculty member Jo Ann Fellin, OSB, retiring National Treasurer of KME. KS Gamma honored the seniors at a dinner at Paolucci's in early May. The graduation valedictory was delivered by chapter member Sean Strasburg. Student officers for 1995-96 will be elected in the fall. Sister Jo Ann Fellin is corresponding secretary/faculty sponsor.

## KS Delta

Chapter President-Daniel Wessel 40 actives
Fourteen students were initiated at the annual initiation banquet held in March. The chapter met with the Washburn Math Club in early April to hear papers presented by Daniel Wessel and Jeff Brown. These papers were later presented at the National KME Convention in Durango, CO. Five students, three faculty members, and The Pentagon "Problem Corner" editor, Kenneth Wilke, all from Kansas Delta, attended the convention. Corresponding Secretary Dr. Al Riveland was elected to a four year term as National Treasurer. Semester activites ended with a KME/Math Club Picnic held the week before final exams. Other 1995-96 chapter officers: Kim Bell, vice president; Jim Stinson, secretary; Alex Alejandro, treasurer; Allan Riveland, corresponding secretary; Ron Wasserstein and

Gary Schmidt, faculty sponsors.

## KS Epsilon

Fort Hays State University, Fort Hays
29 actives, 6 associates
The chapter held monthly meetings and a spring banquet. Members also assisted with the library reorganization. Student officers for 1995-96 have not yet been elected; Charles Votaw and Mary Kay Schippers are corresponding secretary and faculty sponsor, respectively.

## KY Alpha

Eastern Kentucky University, Richmond
16 actives, 18 associates
The annual initiation ceremony, held in February, was followed by a party in the student center. Dr. Patrick Costello gave the initiation address entitled "Notable Numbers." In addition to selling computer disks, several other fundraisers were planned and carried out. A TI-82 calculator and two copies of the software package Derive were donated and sold. Pizza by the slice and other baked goods, many donated by faculty members, helped raise money for nine students, accompanied by Dr. Costello, to attend the National Convention in Durango, CO. It was the first plane flight for some of the students. Following convention activities, those attending enjoyed the trip to Mesa Verde National Park and the snow storm which accompanied it. Many souvenirs returned with the conventioneers, including a western style hat and Big Dog T-shirts. As a final semester activity, the organization observed Mathematics Awareness Day with a program of skits, music, poems, and a recital of interesting facts about $e$. Student officers for $1995-96$ will be elected in the fall. Pat Costello is corresponding secretary/faculty sponsor.

## KY Beta

Cumberland College, Williamsburg

Chapter President-Tessie Hale Black 35 actives, 14 associates

On March 3 the chapter initiated fourteen new student members at a banquet held at Cumberland Lodge. Graduating seniors and last year's inductees were also recognized at the banquet. The new chapter by-laws were presented and approved as well. On March 20, members assisted in hosting Dr. Tom Troland from the University of Kentucky. Following a spaghetti supper, Dr. Troland spoke on "The Tip of the Iceberg: Light and Dark Matter in the Universe." In April the chapter helped with the regional high school math contest held annually at the college. Other 199596 chapter officers: Eric Alan Thornsbury, vice president; Sherri Michelle McGeorge, secretary; William Patrick Giles, treasurer; Jonathan E. Ramey, corresponding secretary; John A. Hymo, faculty sponsor.

## MD Alpha

Chapter President-Regina Geiman
College of Notre Dame of Maryland, Baltimore
12 actives, 7 associates
A Career Night held on March 29 featured five college alumnae. Participating were Ann Shaughnessy Boland, '80, Vice-President, Information Systems for Value Behavioral Health; Shawne Fischer, '94, Information Systems, Bell Atlantic; Isma Hussain Gilani, '92, Software Development for Hubble Space Telescope, Computer Sciences Co.; Tracy Olert Starks, '84, Operations Research Analyst, Ballistic Research Lab, Aberdeen Proving Grounds; and Barbara Tipton, '66, Finance Industry Specialist, IBM. A presentation by Colleen Bergin, '94, also focused on career information. An Account Analyst with Alexander \& Alexander, Bergin spoke on "Mathematics in Business." The annual dinner induction meeting in May featured Dr. Richard Sahey, NASA Scientist, Goddard Space Center, who spoke on "Relativity and Cosmology." Other 1995-96 chapter officers: Shannon Spicer, vice president; Ana Casas, secretary; Jenny Dunning, treasurer; Sr. Marie A. Dowling, corresponding secretary; Joseph DiRienzi, faculty sponsor.

## MD Beta

Western Maryland College, Westminster

Chapter President-Kathy Gaston 18 actives, 4 associates

Following the induction of new members, MD Beta enjoyed an informal discussion of graduate school experiences by William Yankowsky, '93, who completed his master's degree in mathematics at UMBC in June. The chapter also sponsored a special Mathematics Careers Night featuring four Western Maryland College alumni. In early April Dr. Jay Anderson of Franklin and Marshall College visited the campus to talk on computers. In other activities, the organization sold soft pretzels and iced tea at the campus-wide Spring Fling, assisted in recruiting potential mathematics majors via a phonathon, and co-sponsored an end-of-year picnic for all mathematics majors. Two senior chapter officers, Robert Brown and Kari Dunn, presented honors papers and received honors in mathematics at commencement excercises in May. Other 1995-96 officers: Ivy Burklew, vice president; Leslie Huffer, secretary; Steven Eckstrom, treasurer; James Lightner, corresponding secretary/faculty sponsor.

MD Delta
Frostburg State University, Frostburg
Chapter President-Jesse Siehler 44 actives
On March 5, MD Delta inducted twenty new members. Following the initiation ceremonies, Professors John Jones and Kurtis Lemmert entertained the audience with a presentation on mathematical paradoxes. At the February meeting, Vice President Karl Streaker presented mathematical puzzles to the group; in late March, Dr. John Atkins of the West Virginia University Computer Science Department spoke on "Binary Trees Recon-
sidered." Other chapter officers for 1995-96: Dennis Moon, vice president; Amanda Sherman, secretary; Carla White, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

## MA Alpha

Assumption College, Worcester
13 actives
Eight new members were initiated on May 2. Following a dinner in honor of the new initiates, student David Brouillard presented a talk on "Public Key Encryption." Student officers for 1995-96 will be elected in the fall. Charles Brusard is corresponding secretary/faculty sponsor.

## MI Beta

Chapter President-Kristen Williams
Central Michigan University, Mount Pleasant
30 actives
Much of the semester was spent in preparation for the trip to Durango in April for the national convention. Thank you, Colorado Gamma, for a great location and a fantastic conference! Several CMU mathematics faculty spoke at meetings during the semester. Bill Miller talked on "Generalizing Pascal's Triangle," Douglas Nance addressed the group on "Why Study Mathematics?" and Richard Fleming spoke at spring initiation on "Some Outstanding Mathematicians." Near the end of the semester KME enjoyed a picnic with faculty from the Mathematics Department and members of the Actuarial Club, Gamma Iota Sigma. Other 1995-96 chapter officers: Chris Pesola, vice president; Curt Hanson, secretary; Tom Keller, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

MS Alpha
Mississippi University for Women, Columbus
Other 1995-96 chapter officers: Bethany J. Sims, vice president; Kathryn K. Flynn, secretary; Jamie C. Rohling, treasurer; Jean Ann Parra, corresponding secretary; Shaochen Yang, faculty sponsor.

MS Beta
Mississippi State University, Mississippi State

Chapter President-Charles Beyer
Other 1995-96 chapter officers: Michael Cox, vice president; Christin McCloskey, secretary; Anna Freibert, treasurer; Michael Pearson, corresponding secretary; Seth Oppenheimer, faculty sponsor.

## MS Delta

William Carey College, Hattiesburg
Members Ann Patterson and Tracie McLemore participated in the sixth annual Women in Science and Technology Conference in Oak Ridge, Tennessee. On April 25 the chapter held initiation ceremonies for thirteen new members, followed by election of new officers. Other 1995-96 chapter offi-
cers: Vickie Pickering, vice president; Lynn McShea, secretary; Joy Russell, treasurer; Charlotte McShea, corresponding secretary/faculty sponsor.

MS Epsilon
Delta State University, Cleveland

Chapter President-Danny Carpenter 20 actives
Spring activities for the newly installed MS Epsilon Chapter included initiation of one new member and the election of officers for the coming year. Other 1995-96 chapter officers: Renee Upton, vice president; Debra Joel, secretary; David James, treasurer; Paula Norris, corresponding secretary; Rose Strahan, faculty sponsor.

## MO Beta

Chapter President-Heather Scully Central Missouri State University, Warrensburg 25 actives, 10 associates

MO Beta Chapter sent six students and two sponsors to the national convention in Durango. All enjoyed the meetings and the snow. Semester programs included presentations on actuarial science and numerical tic-tactoe. Members assisted with the Missouri section MAA meeting held on the campus and also hosted a spring picnic. Other 1995-96 chapter officers: Ken Petzoldt, vice president; Lynn Graves, secretary; Chad Doza, treasurer; Rhonda McKee, corresponding secretary; Scotty Orr, Larry Dilley, and Phoebe Ho, faculty sponsors.

## MO Epsilon

Central Methodist College, Fayette
Chapter President-Beth Monnette
15 actives, 8 associates
Other 1995-96 chapter officers: Eric Kennedy, vice president; Sara Weiss, secretary; Lynn Klocke, treasurer; William D. McIntosh, corresponding secretary; Linda O. Lembke, faculty sponsor.

MO Eta
Northeast Missouri State University, Kirksville

Chapter President-Doug Cutler
25 actives, 5 associates

In February, the chapter hosted approximately 150 high school students at the Math Expo. The Expo featured both a contest and a speaker. Members participated, along with the Student MAA, ACM, and Actuary Club, in a faculty-student softball game. The faculty won again! Six students and three faculty attended the national convention at Fort Lewis College in April. Other 1995-96 chapter officers: Karen VanCleve, vice president; Amanda Nixon, secretary; Sarah Schwab, treasurer; Jennifer Griswold, historian; Mary Sue Beersman, corresponding secretary; Joe Hemmeter, faculty sponsor.

MO Theta
Evangel College, Springfield

Tosh, corresponding secretary/faculty sponsor.

## MO Iota

Missouri Southern State College, Joplin

Chapter President--Jolena Gilbert 33 actives
The chapter initiated 19 new student members in late March. A very successful book sale helped three students attend the National Convention in late April. The students were accompanied by two faculty and all had a wonderful time. A year-end lasagne party and volleyball game were held at the home of Mrs. Mary Elick. Student officers will be elected in the fall. Mary Elick and Charles Curtis are corresponding secretary and faculty sponsor, respectively.

## MO Lambda

Missouri Western State College, St. Joseph
Chapter President-Brian Bettis
46 actives, 15 associates
Eight new members were initiated into the chapter in February. Dr. Keith Brandt was the first presenter in the KME/Math Club sponsored "Faculty Lecture Series." The series was organized in response to expressed student desire to have faculty discuss mathematics not covered in the standard curriculum. Four students, along with Dr. and Mrs. Atkinson, traveled by van to the national convention in Durango. Two bake sales were held to raise money for the trip. A spring picnic was held at the home of graduating senior Valeri Jones. This year's event was in conjunction with both the Math Club and the Computer Science Club. Other 1995-96 chapter officers: Devon Kerns, vice president; Linda Meyer, secretary; Cindy Ready, treasurer; John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

NE Alpha
Wayne State College, Wayne

Chapter President-Rick Pongratz 30 actives, 11 associates
From funds earned from monitoring the computer lab, the club helped purchase a trash container to be put outside the building. Members helped plan and organize the annual building banquet. At the banquet the outstanding freshman mathematics award was presented to David Pease. For a social activity, a pizza party was held following the initiation ceremony. Several members were disappointed when the trip to Durango was cancelled due to weather. Other 1995-96 chapter officers: Trevor Rasmussen, vice president; Kathy Dalton, secretary/treasurer; Chris Wright, historian; John Fuelberth, corresponding secretary; Jim Paige, faculty sponsor.

NE Delta
Nebraska Wesleyan University, Lincoln

Other 1995-96 chapter officers: Charles McCutchen, vice president; Justin Rice, secretary; Justin Horst, treasurer; Gavin Larose, corresponding
secretary/faculty sponsor.
NH Alpha
Keene State College, Keene
Chapter President -Richard Elliott
37 actives
For Math Awareness Week, NH Alpha, along with Math Club, organized a poster display on coding theory and sponsored an encodingdeciphering contest open to the campus. Other spring activities included an initiation ceremony for ten new members in March and a year-end picnic in April. Other 1995-96 chapter officers: Lisa Smith, vice president; Sharon McCormick, secretary; Rodney Sleith, treasurer; Charles Riley, corresponding secretary; Ockle Johnson, faculty sponsor.

NY Alpha
Hofstra University, Hempstead
Chapter President-Aaron Riddle 15 actives, 15 associates
The chapter sponsored a seminar featuring former students who are now in graduate school. Other events included volleyball games, a movie night, and a social. Other 1995-96 chapter officers: Brandi York, vice president; Paul Ryan, secretary; Lisa Fontana, treasurer; Aileen Michaels, corresponding secretary/faculty sponsor.

## NY Eta

Niagara University, Niagara University
14 actives, 3 associates
Semester activities focused on a Career Day held April 21. Four alumni spoke to the undergraduates about the work they currently do and the job opportunities that exist in their particular fields. During the dinner which followed, students were able to ask more questions of the invited speakers. Student officers for 1995-96 have not yet been elected. Robert Bailey is corresponding secretary.

NC Gamma
Elon College, Elon College
The NC Gamma Chapter OR KME invited Witold Kosm 12 fom palachian State University to speak at the annual spring induction ceremony. The chapter also helped to sponsor a year end picnic. Other chapter officers for 1995-96: Sandy Crenshaw, vice president; Winn Crenshaw, secretary; Stefan Thompson, treasurer; David Nawrocki, corresponding secretary; Jeff Clark, faculty sponsor.

Semester activities centered on the trip to Durango for the National Convention. Fund raising efforts, begun in the fall, were brought to a close and the list of those planning to attend was finalized. The seven
students and three faculty members attending judged the convention a great success. The talks were well received and discussed at length during the long trip back to Ohio. The presentation of the George R. Mach Distinguished Service Award to Professor Emeritus Jim Smith was certainly the high point of the meeting for OH Zeta. Other 1995-96 chapter officers: Serena Wade, vice president; Tina Brown, secretary; Melissa Fruitig, treasurer; David L. Craft, corresponding secretary; Russell A. Smucker, faculty sponsor.

## OH Eta

Ohio Northern University, Ada

Chapter President-Amy Gaiser 29 actives, 20 associates

OH Eta's activities for the semester included a guest speaker, Professor Chull Park of Miami University. The chapter assisted Professor David Reherer with ONU's celebration of Mathematics Awareness Week. An initiation ceremony for new members was held on Honors Day. The chapter also held a pizza party and elected new officers. Other 1995-96 chapter officers: Marlon Price, vice president; Angi Creason, secretary; Ken Fisher, treasurer; Tena Roepke, corresponding secretary; Harold L. Putt, faculty sponsor.

OK Alpha
Northeastern Oldahoma State Univeraity, Tahlequah
Chapter President-Carrie O'Leary
The chapter continues to hold joint activities with NSU's student chapter of MAA. The initiation of nine new members held in the banquet room of Roni's Pizza was well attended by both faculty and students. Members met several times to attempt to solve problems from The Pentagon and participated in "The Problem Solving Competition" sponsored by the MAA. In April the organization sponsored a talk by Jackie Sing, a recent NSU graduate and a member of KME. He spoke on "Life Before and During Graduate School." The chapter publicized National Mathematics Awareness Week on campus and celebrated with the annual pre-finals ice cream social. Students presented the annual "Mathematics Teacher of the Year" award to Dr. Darryl Linde. Other 1995-96 chapter officers: Jeana Wood, vice president; John Callaway, secretary; Peter Butz, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

## OK Gamma

Chapter President-Terry Price
Southwestern Oklahoma State University, Weatherford 20 actives, 10 associates
Other 1995-96 chapter officers: Kris Kessinger, vice president; Lori Ordway, secretary; Wayne Hayes, corresponding secretary; Radwan AlJarrah, faculty sponsor.

## PA Alpha

Weatminster College, New Wilmington

Chapter President-Karey Kustron
15 actives, 16 associates

Forty students attended a Careers Night sponsored by the chapter. Four alumni of Westminster discussed preparation for and experiences in their math and computer jobs. In observance of Math Awareness Week, PA Alpha sent a small packet of candy to each math major. An induction ceremony and luncheon was also held. Other 1995-96 chapter officers: Daniel Coffman, vice president; Laura Williams, secretary; Jason Thiel, treasurer; Jennifer Gatnarek, publicity; J. Miller Peck, corresponding secretary; Carolyn Cuff and Warren Hickman, faculty sponsors.

## PA Gamma

Waynesburg College, Waynesburg
Other 1995-96 chapter Zak, secretary; Jason Hoover, treasurer; A. B. Billings, corresponding secretary/faculty sponsor.

## PA Delta

Marywood College, Scranton

> Chapter President-Kim Fisher
> 5 actives

Several members attended the NCTM National Conference in Boston, Massachusetts, in April. Other 1995-96 chapter officers: Melissa Mang, vice president; Ann Conflitti, secretary/treasurer; Sr. Robert Ann von Ahnen, IHM, corresponding secretary/faculty sponsor.

## PA Eta

Grove City College, Grove City
Chapter President-Marcia Logan 37 actives
Initiation of new members and election of new officers were held on Thurday, March 9, 1995. Jack Schlossnagel, former Mathematics Department Chair, gave the initiation address. He reported on a poorly conceived statistical survey comparing the relative merits of bypass surgery and "medical treatment" in dealing with arterial blockage. The annual KME spring picnic was re-scheduled for next fall. The winner of the KME-selected Outstanding Freshman Mathematics Student was announced at the Parent's Day Award Ceremony on April 29. Three graduates, all former chapter members, returned to the college to discuss graduate school experiences and work experiences. The group included Julie Straub, Joe Magnotta, and Kristi Kowalski. Other 1995-96 chapter officers: Robin Jueschke, vice president; John Tynan, secretary; Kevin Porter, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Chapter President-Derek Smith Shippensburg University of Pennsylvania, Shippensburg 20 actives, 9 associates The chapter, together with Math Club, participated in the Adopt-a-

Highway program, spending one Saturday morning collecting highway litter. Three students attended the national convention in Durango. Chapter members are very proud of Christine Johnson, who presented a paper at the meeting. Upon returning from the convention, Jason Baker and Angela Foltz reported on their experiences and shared photos at the next chapter meeting. Possible ways to raise funds for future conventions was discussed. Jason and Angie sold T-shirts to help defray their convention costs. Spring initiation was held April 26, with nine new members coming aboard. The chapter looks forward to a great 1995-96. Other chapter officers for the coming year: Melissa Gladding, vice president; Todd Bittinger, secretary; Fred Mordai, treasurer; Becky Blair, historian; Michael Seyfried, corresponding secretary/faculty sponsor.

## PA Kappa

Holy Family College, Philadelphia

Chapter President-Leanne Majors
8 actives, 3 associates

During the semester, PA Kappa became more visible and viable both on campus and in the community. During Math Awareness Month, the chapter offered math puzzles and games with prizes for best solutions. They also hosted a grade school math competition and sponsored the math video "Stand and Deliver." As a community service, members coordinated a baby products drive in February for the residents of a local crisis pregnancy center. Leanne Majors spoke on "Magic Squares" at the spring initiation. Members formed a problem solving group, submitting solutions to problems posed in Math Horizons. Solutions submitted by Josh Wagner and Sarah Iskra were noted in the February, 1995, issue. Other 1995-96 chapter officers: Kimberly Doll, vice president; Nicholas Gross, secretary/treasurer; Sr. Marcella Louise Wallowicz, corresponding secretary/faculty sponsor.

## PA Mu

Saint Francis College, Loretto

Chapter President-Colleen Connors 36 actives, 7 associates
PA Mu participated in a variety of activities during the spring semester. The annual KME induction ceremony was held on February 15. The evening began with a mass celebrated by KME member Fr. Joseph Chancler, T.O.R. Following the mass, a dinner was held in the Maurice Stokes room for the 30 inductees, members, and guests. After the dinner, seven students were inducted into the chapter, bringing the total membership to 145 . Three faculty members and 13 students attended the KME national convention held in Durango. The trip was partially funded by Saint Francis College. Additionally, students and faculty sold candy and subs and held car washes to raise funds. The chapter participated in the Adopt-a-Highway program; approximately 15 people picked up litter from the highway near the college. The annual spring department picnic, following the road clean-up, was attended by 25 faculty and students. Other

1995-96 chapter officers: Jennifer Ropp, vice president; Richard Roth, secretary; Heather Barnick, treasurer; Peter B. Skoner, corresponding secretary; Adrian Baylock, faculty sponsor.

## SD Alpha

Chapter President-Aaron Greenwood
Northern State University, Aberdeen
13 actives
The chapter held monthly meetings to discuss business and organize fundraisers. Each semester chapter members put together a small handbook of algebra rules which they sell for $\$ 5.00$. As a second fundraiser, new this semester, memibers sold Hersheys, Kit Kats, Reeses, and Skittles. Both fundraisers were very successful. The chapter also prepared a Careers in Mathematics brochure. Other 1995-96 chapter officers: Scott Kortan, vice president; Misty Charron, secretary; Mike Munson, treasurer; Lu Zhang, corresponding secretary; Raj Markanda, faculty sponsor.

## TX Alpha

Texas Tech University, Lubbock
Other 1995-96 chapter officers: Tisha Blackburn, vice president; Yasser Khan, secretary; Chad McAllaster, treasurer; Edward Allen, corresponding secretary/faculty sponsor.

## TX Kappa

University of Mary Hardin-Baylor, Belton
The Texas Kappa Spring Symposium was held on the sixth of April. The guest speaker was Mr. Kendal Coen, an UMHB alumnus who is currently teaching at Ellison High School in Killeen, Texas. All those attending benefited from his dynamic talk on public school teaching. Other 1995-96 chapter officers: Lisa Hitt, vice president; Riki Perkins, secretary; Katharine Eversoll, treasurer; Peter H. Chen, corresponding secretary; Maxwell M. Hart, faculty sponsor.

VA Alpha
Virginia State University, Petersburg

The Fortieth Anniversary Banquet of VA Alpha Chapter was held on April 21, 1995, in Jones Dining Hall of Virginia State University. This was the site of the chapter installation ceremony and banquet in January, 1955. The Anniversary Banquet program included copies of photos from that event. The speaker for the occasion was Dr. Sadie Chavis Bragg, Acting Dean of Academic Affairs at the Borough of Manhattan Community College, the City University of New York, and formerly Associate Dean of Curriculum and Instruction and a Professor of Mathematics. A member of VA Alpha Chapter of Kappa Mu Epsilon, she is a consulting author of a middle school mathematics program and a co-author of Algebra $I$, both
published by Prentice-Hall, Inc. She is also co-author of Silver Burdett Mathematics K-8 Series, published by Silver Burdett and Ginn and Company. Two charter members were in attendance and seven new members were initiated. Donations were made to the Louise Stokes Hunter Scholarship Fund in the amount of $\$ 10,000$. Annual scholarships will be paid from the interest accrued beginning April 1996. Other 1995-96 chapter officers: Omar Khan, vice president; Jacqueline N. Payton, secretary; Emma B. Smith, treasurer; Joycelyn Harris, corresponding secretary; Azzala Owens, faculty sponsor.

## Excerpts from KME News Spring 1942

Chapter 1. OKLAHOMA ALPEA, Northeastern State College, Tahlequah, Oklahoma.
Oklahoma Alpha held its annual Founder's Day banquet on April 18, the exact date of the founding of Kappa Mu Epsilon. ...
Chapter 8. ILLINOIS ALPHA, Illinois State Normal University, Normal, Illinois.
... On April 3, a group of members presented a fifteen minute radio skit entitled "Mathematics for Victory" over WJBC, the local station. ...
Chapter 10. ALABAMA ALPHA, Athens College, Athens, Alabama.
... An initiation for associate members was held upon November 30th; the ceremony was followed by a breakfast. ...
Chapter 11. NEW MEXICO ALPHA, University of New Mexico, Albuquerque, New Mexico.
... Also, funds were solicited to assist Dr. Bernardo Baidaff in the publication of Boletin Matematico in Buenos Aires. Thanks to the assistance of several other chapters of Kappa Mu Epsilon, a total of eighteen dollars was raised. ...

Chapter 13. ALABAMA BETA, Alabama State Teachers College, Florence, Alabama.
Alabama Beta has been occupied with defense work. At the monthly meetings, defense stamps have been awarded as prizes in the mathematical contests. Instead of the annual banquet, Alabama Beta is using its banquet fund for the purchase of defense bonds.

Chapter 22. KANSAS GAMMA, Mount St. Scholastica College, Atchison, Kansas.
... November 13: ... "Mathematics and National Defense," with special emphasis on the role of the American college woman. ...

## Report on the 30th Biennial Convention

The Thirtieth Biennial Convention of Kappa Mu Epsilon was held April 20-22, 1995 at the Iron Horse Inn in Durango, Colorado with Colorado Gamma serving as the host chapter.

On Thursday evening, April 20, a registration/mixer was held at the Stagecoach Theatre of the Iron Horse Inn. The National Council and the Regional Directors met in the Wildflower Room of the Inn.

On Friday morning, April 21, registration continued at the Stagecoach Theatre of the Iron Horse Inn. The first general session commenced at 8:30 a.m. with Arnold Hammel of Michigan Beta, National President, presiding. Dr. Joel Jones, President of Fort Lewis College, gave an address of welcome and Patrick Costello of Kentucky Alpha, National President-Elect, responded for the Society. Ben Moore, President of Colorado Gamma, presented a greeting to the delegates.

A roll call of the chapters was made by Robert L. Bailey of New York Eta, National Secretary. Thirty-three chapters and nearly 200 members were in attendance. Travel vouchers were filed and delegate voting cards were issued. A number of special guests were introduced, including Dr . George Mach, CA ${ }^{\vdots}$ Gamma, and his wife. Dr. Mach is a former National Secretary and former National President of KME. Also introduced was Dr. James Smith, OH Zeta, former National President, as well as Ken Wilke, KS Delta, and his wife. Mr. Wilke is Problem Corner Editor of The Pentagon. The following new chapters installed during the 1993-95 biennium were recognized: Louisiana Gamma at Northwestern State University, installed March 24, 1993; Kentucky Beta at Cumberland College, installed May 3, 1993; Mississippi Epsilon at Delta State University, installed November 19, 1994. A moment of silence was observed in honor of Sr. Adrienne Eickman, WL Alpha, Region 2 Director, who passed away in January 1995. Victims of the Oklahoma bombing tragedy and their families were also remembered.

President Hammel announced that the Initiation/Installation Ceremony revision has been completed and that the new document will be sent to all chapters. As well, the petition form for potential new chapters has been revised and improved.

The Nominating Committee report was presented by Harold Thomas of Kansas Alpha, chair. The committee nominated Carol Harrison of Pennsylvania Theta and Waldemar Weber of Ohio Alpha to the office of Secretary, and John Fuelberth of Nebraska Alpha and Al Riveland of Kansas Delta to the office of Treasurer. The nominees were introduced to the delegates and additional nominations were requested from the floor. There being none,
nominations were closed.
Because of the number of papers of good quality which were submitted for this convention, President Hammel announced that parallel presentations would take place during the indicated " B " sessions which include graduate as well as unjudged papers. Papers being judged will be presented at the "A" sessions.

During the juice break, the Awards, Auditing and Resolutions Committees met at various locations at the Iron Horse Inn.

Delegates were introduced to Bryan Dawson and Larry Scott, both of KS Beta, who are assuming positions as Editor and Business Manager, respectively, of The Pentagon. Outgoing Editor Andrew Rockett (NY Lambda) and Business Manager Sharon Kunoff (NY Lambda) were warmly applauded for their excellent work on the journal during the last six years.

Patrick Costello of Kentucky Alpha, National President-Elect, presided during the presentation of the following Session A student papers in the Stagecoach Theatre, Iron Horse Inn:

1) Atmospheric Gambling: An Investigation of Rayleigh Scattering in the Atmosphere Using Monte Carlo Methods

JEFFREY BROWN, Kansas Delta Washburn University
2) Packing General Sets of Squares into a Rectangle CHARLES MILLER, Missouri Eta Northeast Missouri State University
3) An Introduction to Muhiquadric Interpolation
MICHELLE RUSE, Iowa Alpha University of Northern Iowa
4)

Special Curves Connected ADAM SANFORD, Iowa Delta

Wartburg College
5)

Another Look at Random Shuffes GRANT LATHROM, Missouri Alpha Southwest Missouri State University

Arnold Hammel, National President, presided during the presentation of the following Session B student papers in Buckskin Charlie's Restaurant, Iron Horse Inn:

HEATHER SCULLY, Missouri Beta Central Missouri State University
2)

A History of Population Models JOHNNY SNYDER, New Mexico Alpha University of New Mexico
3)

> Universal Chaos
> JENNIFER LYNCH, Missouri Zeta University of Missouri-Rolla

Following lunch, the convention reconvened at $1: 55 \mathrm{p} . \mathrm{m}$. in the Iron Horse Inn. Patrick Costello, National President-Elect, presided during the presentation of the following Session A student papers in the Stagecoach Theatre:
6)

> Playing Checkers with Mathematical Logic THOMAS HILEMAN, Ohio Alpha
Bowling Green State University
7)

> When Intuition Fails
> TAMMY CAUSEY \& DANE MOONEY, Florida Beta
> Florida Southern College
8)

> Practical Fractical GREGORY BOUCHER, Kansas Gamma Benedictine College

Arnold Hammel, National President, presided during the presentation of the following Session B student papers in Buckskin Charlie's Restaurant:
4)

The Bobcat That Lived in a Polygon PHYLLIS MAHAN, Kentucky Alpha Eastern Kentucky University
5)

The History of Mathematics rebecca wakely, Missouri Zeta University of Missouri-Rolla

DON GILBERT, JR., Ohio Zeta
Muskingum College

At 3:05 p.m., a student section met with Ben Moore, President of Colorado Gamma, presiding. A faculty section met with Arnold Hammel of Michigan Beta, National President, presiding.

At 4:05 p.m., the convention reconvened for the presentation of papers with Patrick Costello, National President-Elect, presiding. The following Session A papers were presented in the Stagecoach Theatre:
9)

> Heroism: The Norm Then and Now CHRISTINE JOHNSON, Pennsylvania Iota Shippensburg University

CATHERINE GOOD, Missouri Kappa
Drury College

> I Think Knot
> SAYLAR CRAIG, Iowa Alpha
> University of Northern Iowa

At 7:00 p.m., the convention banquet was held in the Stagecoach Theatre, Iron Horse Inn, with Ben Moore of Colorado Gamma as master of ceremonies. Patrick Costello of Kentucky Alpha, National President-Elect, proceeded to explain the criteria for the selection of the recipient of the George R. Mach Distinguished Service Award which is given each biennium to the person who has made major contributions to Kappa Mu Epsilon. The award originated in 1987 and prior winners have been Laura Green, Wilbur Waggoner, Fred Lott, Sr. Helen Sullivan and James Lightner. For this biennium, there are two recipients, James L. Smith and Carl Fronabarger, the citations for whom read as follows:

> CITATION FOR DR. JAMES L. SMITH
> TEE GEORGE R. MACH DISTINGUISHED SERVICE AWARD APRIL 21, 1995

As a faculty member at Westminster College, James Smith was initiated into the Pennsylvania Alpha Chapter of Kappa Mu Epsilon in 1957. That ceremony started a career of distinguished service to KME which continues to the present day. After a move to Muskingum College in New Concord, Ohio, Jim was instrumental in the establishment of the Ohio Zeta Chapter in May, 1969. He then served as the corresponding secretary through 1993 at which time he retired from teaching.

At the local level Jim has been a source of inspiration and
guidance to an entire generation of Muskingum students. In the classroom he was an enthusiastic teacher who cared deeply for his students. These students have strong recollections of Dr. Smith's promotion of Kappa Mu Epsilon as a student organization. He believed that the strength of a chapter was developed through the involvement of all of its members. Ohio Zeta students could count on support from Jim in encouraging attendance at and providing transportation to regional and national conventions. Many of these students also received benefit from his guidance as they prepared and delivered student papers at these conventions. He helped coordinate the hosting of the Fall 1977 National Convention at Muskingum.

Jim Smith's national contributions started with his appointment as Regional Director of Region 2 in 1975 which continued until 1979. He was then elected President-Elect in 1981 and moved into the Presidency in 1985. He continued the strong tradition of leadership that this organization had attained through its rich heritage. Through the years Jim has always had a great deal of enthusiasm and affection for Kappa Mu Epsilon. His chapter letters made you want to get up and forge into action. A KME colleague once commented that Jim Smith was a person who you could call on, the job would get done and it would get done well.

In recognition of Jim Smith's outstanding dedication and service to Kappa Mu Epsilon, we take great pleasure in presenting to him the George R. Mach Distinguished Service Award.

## CITATION FOR DR. CARL FRONABARGER THE GEORGE R. MACH DISTINGUISHED SERVICE AWARD APRIL 21, 1995

On January 8, 1942, as a new faculty member at Southwest Missouri State College, Dr. Carl Fronabarger was initiated into the Missouri Alpha Chapter of Kappa Mu Epsilon. The 206th member of the Chapter, he was to serve Kappa Mu Epsilon in an exemplary manner for years to come. He was corresponding secretary of Missouri Alpha from 19421952 and continued as faculty sponsor until 1960. He was the General Chair for the Eighth National Convention held at Southwest Missouri State in 1951. From 1950-1952 he was an associate editor of The Pentagon in charge of "The Book Shelf," the book review section. In 1952 he became editor
of The Pentagon, a position he held until 1959. During this time he increased significantly the number of student papers being published in the journal. He served as National President from 1959 to 1963. At the local level, he supported many students in their mathematical studies and investigations and provided transportation to regional and national conventions so they could share their work as student papers. Even after his retirement from teaching in 1974 he remained interested in Kappa Mu Epsilon until his death in 1988. His commitment to the ideals and purposes of the organization did not go unnoticed by his students, two of whom have since served KME as national officers. Another is serving as a regional director and three continue serving as corresponding secretaries. In recognition of Dr. Fronabarger's outstanding dedication and service to Kappa Mu Epsilon, we take pleasure in presenting to his chapter, Missouri Alpha, the George R. Mach Distinguished Service Award.

The delegates of the convention were especially fortunate that Dr. Smith was in attendance at the banquet and so was able to receive this award in person. An award of $\$ 100$ will be given to Ohio Zeta, the chapter with which James Smith has been associated since 1969.

A representative of Missouri Alpha received the Mach Award on behalf of Dr. Fronabarger. An award of $\$ 100$ will be given to this chapter, with which Carl Fronabarger was associated from 1942 to 1988.

The keynote address was then given by Dr. Gustavus J. Simmons, Senior Fellow and Director for National Security Studies, Sandia National Laboratories (Ret.)

The convention reconvened on Saturday, April 22 at 8:15 a.m. when the group photograph was taken outside the Iron Horse Inn. At 8:45 a.m. Patrick Costello, National President-Elect, presided during the presentation of the following Session A papers in the Stagecoach Theatre:

## Fore!

DANIEL WESSEL, Kansas Delta
Washburn University

## Palindromes

CHRISTOPHER BROWN, Missouri Zeta
University of Missouri-Rolla
14) A Study of 19th and 20th Century Mathematics Textbooks DONOVAN DIEDE \& AARON GREENWOOD, South Dakota Alpha Northern State University

> Fractals: The Geometry of Nature LISA GASKELL, Iowa Alpha University of Northern Iowa

During the juice break, the Awards Committee met in the Wildflower Room at 10:15 a.m.

The second general session was held at 10:50 a.m. in the Stagecoach Theatre with Arnold Hammel, National President, presiding. The following national officers presented reports:

Business Manager, The Pentagon - Sharon Kunoff, New York Lambda (report was given by Andrew Rockett)<br>Editor, The Pentagon - Andrew Rockett, New York Lambda<br>National Historian - Mary Elick, Missouri Iota<br>National Treasurer - Sr. Jo Ann Fellin, Kansas Gamma<br>National Secretary - Robert L. Bailey, New York Eta<br>National President-Elect - Patrick Costello, Kentucky Alpha<br>National President - Arnold Hammel, Michigan Beta

Ben Moore of Colorado Gamma reported for the student section meeting, while Mary Sue Beersman reported for the faculty section meeting.

Peter Skoner, Chair of the Auditing Committee, reported that the National Treasurer's records were found to be accurate and in good order.

Ann Scheffing of Missouri Beta and Kristen Williams of Michigan Beta reported for the Resolutions Committee. The following resolutions were adopted:

Resolved: That the Thirtieth Biennial Convention of Kappa Mu Epsilon express its gratitude to Robert Bailey who has served 8 years as Secretary of Kappa Mu Epsilon and to Sister Jo Ann Fellin who has served 8 years as Treasurer of Kappa Mu Epsilon, both of whom have given so generously of their time and talent.

Resolved: That the Thirtieth Biennial Convention of Kappa Mu Epsilon express its gratitude to Andrew Rockett who has served 6 years as Editor of The Pentagon and to Sharon Kunoff who has served 6 years as Business Manager, both of whom have contributed their valuable time and expertise.

Resolved: That the Thirtieth Biennial Convention of Kappa Mu Epsilon express its appreciation:

1. To Dick Gibbs, Debbie Berrier, Ben Moore, and the members of Colorado Gamma for their work in the expeditious planning of this convention.
2. To Dr. Joel Jones, President of Fort Lewis College, for the gracious hospitality and the many services rendered the chapters and officers of the Convention.
3. To Dr. Gus Simmons for his entertaining banquet speech.
4. To President-Elect Patrick Costello and the Selection Committee who gave so unselfishly of their time in the paper selection process.
5. To all the students who prepared and presented papers at the Convention.
6. To the Auditing, Awards, and Nominating Committees for their diligent service.
7. To President Arnold Hammel, President-Elect Patrick Costello, Secretary Robert Bailey, Treasurer Sister Jo Ann Fellin, and Historian Mary Elick for the generous service preceding and during the Biennial Convention.
8. To all the Regional Directors, Faculty Sponsors, and Corresponding Secretaries whose leadership of local Kappa Mu Epsilon Chapters inspired the many student participants at the Thirtieth Biennial Convention.

President Arnold Hammel presided during the election of officers for 1995-1997. Ballots were cast and counted for the offices of National Secretary and National Treasurer. The winners are Waldemar Weber of Ohio Alpha, National Secretary and Al Riveland of Kansas Delta, National Treasurer.

An invitation to host the 1997 Biennial Convention was received from the chapters of Missouri Alpha, Theta and Kappa which will act as co-hosts for the convention. Delegates were reminded that additional invitations can be extended by any chapter to the National Council any time during the next few months.

Don Tosh of Missouri Theta, Chair of the Awards Committee, reported for the committee. Certificates of participation were presented to all 21 student speakers. In addition, the top four speakers each received their choice
of either an HP 48s or TI-82 calculator as well as an award of $\$ 100$. One calculator was also randomly awarded to one of the remaining seven presenters of judged papers. An additional calculator was randomly awarded to one of the six presenters of unjudged papers. The top four presenters (in alphabetical order) were announced as:

> Jeffrey Brown - Kansas Delta
> Tammy Causey \& Dane Mooney - Florida Beta
> Michelle Ruse - Iowa Alpha
> Daniel Wessel - Kansas Delta

The following officers were installed by Arnold Hammel, National President for the term 1995-97:

> National Secretary - Waldemar Weber, Ohio Alpha National Treasurer - Al Riveland, Kansas Delta

Arnold Hammel, National President, presented engraved plaques to the following individuals in appreciation of their service to Kappa Mu Epsilon:

Ken Wilke, Kansas Delta, Problem Corner Editor of The Pentagon
Andrew Rockett, New York Lambda, Editor of The Pentagon, 19891995

Sharon Kunoff, New York Lambda, Business Manager of The Pentagon, 1989-1995 (accepted by Andrew Rockett)

Sr. Jo Ann Fellin, Kansas Gamma, National Treasurer, 1987-1995
Robert L. Bailey, New York Eta, National Secretary, 1987-1995
Convention evaluation forms which had been distributed to the delegates earlier were collected by the host chapter. Copies of reports of the national officers and The Pentagon staff were made available to each chapter. Delegates were also reminded about the sightseeing tour of Mesa Verde which would take place immediately following the close of the convention.

Travel allowances were paid to the delegates by Sr. Jo Ann Fellin of Kansas Gamma, National Treasurer. The convention was adjourned at 12:15 p.m.

Robert L. Bailey

Report of the President
Since my installation as National President at the 29th Biennial Con-
vention, we have installed two new chapters, which brings the total number of active chapters to 116. I am pleased to report that none of our chapters were placed on inactive status during the past biennium. New chapters added are Kentucky Beta at Cumberland College (installed by Patrick Costello on May 3, 1993) and Mississippi Epsilon at Delta State (installed by Arnie Hammel, November 19, 1994). In addition, two other institutions have received petition forms. Also, eleven other colleges and universities have'indicated an interest in Kappa Mu Epsilon and received information about our organization. If you have friends and colleagues at schools that do not have a KME chapter, but are interested, have them contact me.

The National Council continues to support the regional structure of KME. Please refer to the report by President-Elect Patrick Costello for the summary of regional conventions held in 1994. With much gratitude we recognize the work and efforts of our Regional Directors. These people have served our Society well and deserve the thanks of each of us. We give our heartfelt sympathy to the family and Mount Mary College colleagues of Sister Adrienne Eickman. Sister Adrienne passed away to cancer in late December, 1994. She was a tireless worker for KME, her chapter, for her school, and for her students. We were blessed by her presence and she will be sorely missed. She had served as Region II Director since 1989. I would also like to especially cite Mary Sue Beersman, Region IV Director, and MO Eta (Northeast Missouri State University), and Raymond Terry, Region VI Director, and CA Gamma (California Polytechnic State University at San Luis Obispo), whose terms expire with this convention. The appointment of the next group of Regional Directors for Regions II, IV, and VI will be made shortly after this convention.

A special thanks is extended to each of the faculty who serve as corresponding secretaries and faculty sponsors with our active chapters. I want you to know how much your time and effort on behalf of your local members is appreciated. Keep up the good work. I know that many of you played an important role in assisting your students in the preparation of the excellent papers we have on the convention program this year. Furthermore, we all express our gratitude to each of the students who did the work, endured the stress and prevailed in submitting and presenting a paper at this convention. Without the student papers, the major focal point of the convention does not exist.

We are further indebted to all the individuals who did all the work necessary to bring this convention to fruition and to those who have served on the convention committees. A big thank you goes to the members of Colorado Gamma (under the capable direction of Dick Gibbs and Deborah Berrier) for their fine work. The cooperation of faculty and students in their willingness to serve on convention committees has been very impressive. Through the many phone calls I have had the neat opportunity to chat
with many of you during the last several months. This kind of response, both at convention time and throughout my tenure thus far, has made the privilege of being your President much easier.

I also want to recognize and applaud the outstanding efforts put forth by the members of the National Council in their respective areas of responsibility. We have special indebtedness to these very capable and conscientious individuals who so unselfishly give of their time and efforts in making our honor society the very best it can possibly be. It is a joy to work with them.

I next want to recognize the fantastic job which is being done by those who work with, manage, write for, and produce our journal, The Pentagon. We are most appreciative of the editorial leadership of Andy Rockett (NY Lambda) and the sound business management given to us by Sharon Kunoff (NY Lambda). We especially want to thank Andy and Sharon at this convention as they have requested to be replaced so as to resume other projects at their school. They have left The Pentagon in excellent condition and we are enthusiastic that a new team will soon be resuming these duties. This process is currently under discussion by the National Council. Also big thank yous to Associate Editors Mary Elick (MO Iota) and Kenneth Wilke (KS Delta). Mary edits the Chapter News section and Ken the Problem Corner. We are pleased that Ken is in attendance at this convention.

During the past biennium, I have represented Kappa Mu Epsilon at the Annual meeting of the Association of College Honor Societies (ACHS). It has been very helpful to meet with officers of the other 59 honor societies that are members of ACES. Hopefully this exchange of ideas and gathering of suggestions can be put to good use in possibly improving on the programs we currently have in place.

In summary, I can honestly say that I have thoroughly enjoyed the privilege of serving Kappa Mu Epsilon at the national level the past six years. I look forward to the next two years of my term as your National President. Best wishes to each of you as we continue to work for the improvement of this very special Honor Society - Kappa Mu Epsilon.

Arnold D. Hammel

## Report of the President-Elect

One responsibility of the President-Elect is to serve as coordinator of regional activities of the Society through the Regional Directors. During the Spring of 1994, there were five regional conventions held. They were:

Region I at Pennsylvania Lambda, Bloomsburg University, April 15-16, Carol Harrison, Regional Director.

Region II at Michigan Beta, Central Michigan University, March 18-19, Sister Adrienne Eickman, Regional Director.

Region III at Kentucky Alpha, Eastern Kentucky University, March 4, Pat Costello, Regional Director.

Region IV at Missouri Lambda, Missouri Western State College, April 22-23, Mary Sue Beersman, Regional Director.

Region $V$ at New Mexico Alpha, University of New Mexico, April 8-9, Richard Gibbs, Regional Director.

Programs at the regional conventions included student papers, guest talks, and good social times. We extend our sincere thanks to the host chapters, regional directors, and all who participated in this regional activity. We also appreciate the efforts of Raymond Terry, the Regional Director of Region VI, in attempting to have a regional convention in his region.

A second duty of the President-Elect is to coordinate nominations for the George R. Mach Service Award. In the Fall of 1994 a letter went out requesting nominations for the award. Those names received were added to the list of nominees carried over from previous years. I want to thank those Corresponding Secretaries and Faculty Sponsors who made nominations and the students and alumni who wrote letters in support of these nominations. At the National Council meeting in November a decision was made to present the award to two outstanding nominees at this convention. The recipients will be announced at the Saturday morning session.

It is another of the President-Elect's responsibilities to arrange for the presentation of student papers at the National Convention. I am pleased to report that twenty-three students, representing seventeen chapters and eleven states, submitted papers for this convention. Twenty-one papers were written by undergraduates and two by graduate students. The Paper Selection Committee read and ranked all twenty-three papers submitted. The committee indicated that it was difficult to rank the papers because they were all so good. The top fifteen undergraduate papers will be judged by the Awards Committee at the convention. On behalf of the Society, I want to extend special thanks to the members of the Paper Selection Committee who read and ranked the papers: Professor Rhonda McKee (Missouri Beta), Helmut Doll (Pennsylvania Lambda), and Mary Beth Dever (Illinois Theta). One difference this year can be noted in the convention schedule. Not just the papers that are to be judged are being presented. Twenty-one papers are being presented at this convention. The National Council made the decision to try to give everyone who submitted a paper the opportunity to present it at the convention. Consequently, there are two sessions of papers scheduled for Friday afternoon. Another change from recent conventions is that the rankings of the Paper Selection Committee
will be combined with the rankings of the Awards Committee to determine a final ranking of the judged papers. The top four judged papers will each receive $\$ 100$. On behalf of the Society, I want to express our sincere thanks to all twenty-three students who prepared and submitted papers. It is this work that makes for a truly successful convention.

Patrick J. Costello

## Report of the National Secretary

During the last biennium, three new chapters of Kappa Mu Epsilon were installed as listed below.

| Chapter | Institution | Installation Date |
| :--- | :---: | ---: |
| Louisiana Gamma | Northwestern State University | $3 / 24 / 93$ |
| Kentucky Beta | Cumberland College | $5 / 3 / 93$ |
| Mississippi Epsilon | Delta State University | $11 / 19 / 94$ |

During the last biennium 2,454 members were initiated. The 116 active chapters have a combined membership of 59,292 and the 29 inactive chapters have a combined membership of 6,390 , making the total membership of Kappa Mu Epsilon 65,682 at the end of the biennium.

As National Secretary, I maintain permanent files on all active and inactive chapters, including reports of all initiations. I order membership certificates and jewelry for all new members and I stock all supplies, including forms, invitations and jewelry. I assist corresponding secretaries in any way that I can and I take minutes of National Council meetings and Biennial Conventions.

Robert L. Bailey

> Financial Report of the National Treasurer 1993-95 Biennium (March 23, 1993 through March 20, 1995)

Assets at the beginning of biennium
\$55,780. 57
RECEIPTS

| Receipts from Chapters |  |
| :--- | ---: |
| Initiates (2454) | 36810.00 |
| Jewelry (special one-time offer for bulk supply) | 6189.00 |
| Jewelry |  |
| Supplies | 221.05 |

Interest ..... 3684.77
Miscellaneous ..... 1038.42
Certificate/Crest replacements ..... 90.00
Overpays-underpays ..... 45.00
Insuff funds reimbursement ..... 180.00
Credit Memo ..... 100.00
New Chapter Fees ..... 618.82
Capital Gain ..... 4.60
Total Receipts during biennium ..... 43968.74
Receipts plus Assets
EXPENDITURES
Jewelry (automatic issue) ..... 14930.55
Jewelry for resale (special offer for bulk supply) ..... 1056.01
Printing (other than for certificates) ..... 1681.21
Pentagon ..... 13700.00
Conventions/meetings ..... 19375.02
1993 Biennial Convention ..... 13628.38
1994 Regional meetings ..... 1389.19
National Council Meeting 1993 ..... 1571.83
National Council Meeting 1994 ..... 1713.76ACHS meeting attendance1071.86
ACHS duesMiscellaneous
547.96
Supplies for resale
263.67
Supplies not for resale
303.87
Telephone
1346.26
Postage
618.82
Chapter installations
195.00
Insufficient fund debit memos
75.00
Overpay refunds
Total ExpendituresAssets at end of bienniumExchange National Bank \#34689616936.47
World Savings CD \#62539739

99,749.31
activities. In an effort to accomplish this latter task, requests are sent out twice a year soliciting news of your chapters. After your responses are compiled and edited, they are then published in the Chapter News section of The Pentagon. It is worth remembering that when you respond to the chapter news request and have your chapter activities published in the $K M E$ journal, you have one more source of record should your own files be lost or destroyed.

During the past biennium 84 chapters have responded at least once to this request. The following chapters are to be commended for responding to each of the four requests: CO Gamma, GA Alpha, IA Alpha, IA Gamma, IA Delta, KS Alpha, KS Gamma, KS Delta, KS Epsilon, KY Alpha, MD Beta, MD Delta, MI Beta, MS Alpha, MO Beta, MO Epsilon, MO Eta, MO Iota, MO Lambda, NE Alpha, NY Eta, NY Kappa, OK Alpha, PA Beta, PA Gamma, PA Delta, PA Eta, PA Iota, PA Kappa, PA Mu, TX Alpha, TX Eta, and TX Kappa.

I have come to appreciate the work of many during the time I have served as your National Historian. The other National Officers with whom I have had the privilege of serving have all been top notch. I am grateful to Andy Rockett for the cooperation and assistance he has provided during the six years we have worked together. He and his work will be long remembered. And finally, I am most appreciative of the faculty who serve as corresponding secretaries and faculty sponsors. Many of the same names show up on Chapter News responses year after year as you work faithfully with KME and your current students. As a corresponding secretary myself, I know that is not always easy. A big thanks to all of you. I have enjoyed serving as your National Historian.

Mary S. Elick

## Report of the Editor of The Pentagon

Volumes 53 and 54 of The Pentagon featured ten student papers, five faculty papers, and the abstracts of seven student papers presented at the 1994 Region IV convention. "The Problem Corner" and "Kappa Mu Epsilon News" sections appeared in every issue and thus continued to form the core of our journal.

Unsolicited manuscripts were refereed by faculty volunteers. The efforts of sixteen such individuals were acknowledged in the Spring 1994 issue. They have been a great help to me and I am sure that the authors also found their constructive comments most useful.

The continued effort and prompt attention to detail by associate editors Kenneth M. Wilke and Mary S. Elick greatly simplified my task, as did the
support and assistance of Sharon Kunoff and the KME national officers.
I thank the National Council for allowing me to serve Kappa Mu Epsilon as the editor of your journal for these last six years. I am grateful to have had the opportunity to work with so many student authors and their faculty advisors. I shall treasure the many friendships formed in shared pursuit of our mutual desire to appreciate the beauty of mathematics.

Andrew M. Rockett

## Report of the Business Manager of The Pentagon

I am sorry that I could not be here to present my final report as Business Manager of The Pentagon in person. I have been enjoying a sabbatical this year which has given me the opportunity to travel and I have taken full advantage of it. That and the pressure of the holidays made it impossible for me to be with you today. Even though I will no longer serve in the capacity of Business Manager I hope to be with you for the next biennial in 1997.

The past six years have been good ones for The Pentagon. We service over 3000 subscribers both in this country, Europe, Asia, and Africa. Most of our chapter secretaries are making sure that the addresses we get for new subscribers are permanent addresses and not dorm rooms, so that the new member receives his or her subscription for the full 2 year term and is also in receipt of the renewal notice at the end of the two year subscription. A few chapters enter their math department as a permanent address for those students who cannot give a more permanent address. The chapter advisor can usually locate those students and make sure that The Pentagon is received. I would like to suggest that all chapters consider this solution for students who cannot provide a permanent address or Post Office Box.

I will conclude my tenure as Business Manager with the Spring Pentagon, Volume 54-2. All correspondence after June 1, 1995 should be sent to Larry Scott in Emporia, Kansas. I want to wish him well. I hope you will cooperate with him as fully as you did with me. I imagine he will have his own innovations which will make the business of The Pentagon better. Larry's address will appear in the Spring Pentagon as well as on the renewal notices to go out in June.

My job as Business Manager was made simpler and more pleasant by the fine working relation I have had with The Pentagon Editor Andrew Rockett. His help was invaluable. I also want to thank the wonderful officers and board members of Kappa Mu Epsilon that I have had the privilege of working with for the past six years. This is a mathematical fraternity in the truest sense and I want to thank you all for allowing me
to have this unique experience.
On a personal note, I would like to encourage all student delegates to read their Pentagons, to send in solutions to the Problem Corner and to send in articles if they have something to share. My first experience with The Pentagon was in the 1950's when Professor Frank Hawthorne of Hofstra University encouraged me to send in a solution to The Pentagon. Frank Hawthorne, a former Pentagon officer, died recently, but his influence, as you can see, lives on. We your officers and faculty advisors hope that we can influence you in the same way. Good luck to you all!

Sharon Kunoff



James L. Smith (right) receives the George R. Mach Distinguished Service Award from President-Elect Patrick Costello. Photograph courtesy of John Atkinson, MO Lambda.

## 31st Biennial Convention - Springfield!

The National Council has accepted a proposal from Missouri Alpha at Southwest Missouri State University, Missouri Theta at Evangel College, and Missouri Kappa at Drury College, all located in Springfield, to host the next Biennial Convention in 1997. Start making plans to be there now!


# Kappa Mu Epsilon National Officers 

Arnold D. Hammel<br>President<br>Department of Mathematics<br>Central Michigan University, Mt. Pleasant, Michigan 48859<br>a.hammel@cmich.edu<br>Patrick J. Costello<br>President-Elect<br>Department of Mathematics, Statistics and Computer Science<br>Eastern Kentucky University, Richmond, Kentucky 40475<br>matcostello@acs.eku.edu<br>Waldemar Weber<br>Secretary<br>Department of Mathematics and Statistics<br>Bowling Green State University, Bowling Green, Ohio 43403<br>kme-nsec@mailserver.bgsu.edu

A. Allan Riveland

Treasurer
Department of Mathematics and Statistics
Washburn University, Topeka, Kansas 66621
zzrive@acc.wuacc.edu

Mary S. Elick
Historian

> Department of Mathematics
> Missouri Southern State College, Joplin, Missouri 64801 elick@vm.mssc.edu

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, The Pentagon, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

# Active Chapters of Kappa Mu Epsilon 

## Listed by date of installation.

| Chapter | Location Ins |
| :---: | :---: |
| OK Alpha | Northeastern Oklahoma State University, Tahlequah |
| IA Alpha | University of Northern Iowa, Cedar Falls |
| KS Alpha | Pittsburg State University, Pittsburg |
| MO Alpha | Southwest Missouri State University, Springfield |
| MS Alpha | Mississippi University for Women, Columbus |
| MS Beta | Mississippi State University, Mississippi State College |
| NE Alpha | Wayne State College, Wayne |
| KS Beta | Emporia State University, Emporia |
| NM Alpha | University of New Mexico, Albuquerque |
| IL Beta | Eastern Ilinois University, Charleston |
| AL Beta | University of North Alabama, Florence |
| AL Gamma | University of Montevallo, Montevallo |
| OH Alpha | Bowling Green State Unjversity, Bowling Green |
| MI Alpha | Albion College, Albion |
| MO Beta | Central Missouri State University, Warrensburg |
| TX Alpha | Texas Tech University, Lubbock |
| TX Beta | Southern Methodist University, Dallas |
| KS Gamma | Benedictine College, Atchison |
| LA Beta | Drake University, Des Moines |
| TN Alpha | Tennessee Technological University, Cookeville |
| NY Alpha | Hofstra University, Hempstead |
| MI Beta | Central Michigan University, Mount Pleasant |
| NJ Beta | Montclair State College, Upper Montclair |
| IL Delta | College of St. Francis, Joliet |
| KS Delta | Washburn University, Topelca |
| MO Gamma | William Jewell College, Liberty |
| TX Gamma | Texas Woman's University, Denton |
| WI Alpha | Mount Mary College, Milwaukee |
| OH Gamma | Baldwin-Wallace College, Berea |
| CO Alpha | Colorado State University, Fort Collins |
| MO Epsilon | Central Methodist College, Fayette |
| MS Gamma | University of Southern Mississippi, Hattiesburg |
| IN Alpha | Manchester College, North Manchester |
| PA Alpha | Weatminster College, New Wilmington |
| IN Beta | Butler University, Indianapolis |
| KS Epsilon | Fort Hays State University, Hays |
| PA Beta | LaSalle University, Philadelphia |
| VA Alpha | Virginia State University, Petersburg |
| IN Gamma | Anderson University, Anderson |
| CA Gamma | California Polytechnic State University, San Luis Obispo |
| TN Beta | East Tennessee State University, Johnson City |
| PA Gamma | Waynesburg College, Waynesburg |
| VA Beta | Radford University, Radford |
| NE Beta | Kearney State College, Kearney |

18 April 1931
27 May 1931
30 Jan 1932
20 May 1932
30 May 1932
14 Dec 1932
17 Jan 1933
12 May 1934
28 March 1935
11 April 1935
20 May 1935
24 April 1937
24 April 1937
29 May 1937
10 June 1938
10 May 1940
15 May 1940
26 May 1940
27 May 1940
5 June 1941
4 April 1942
25 April 1942
21 April 1944
21 May 1945
29 March 1947
7 May 1947
7 May 1947
11 May 1947
\& June 1947
16 May 1948
18 May 1949
21 May 1949
16 May 1950
17 May 1950
16 May 1952
6 Dec 1952
19 May 1953
29 Jan 1955
5 April 1957
23 May 1958
22 May 1959
23 May 1959
12 Nov 1959
11 Dec 1959

| IN Delta | University of Evansville, Evansville | 27 May 1960 |
| :---: | :---: | :---: |
| OH Epsilon | Marietta College, Marietta | 29 Oct 1960 |
| MO Zeta | University of Missouri-Rolla, Rolla | 19 May 1961 |
| NE Gamma | Chadron State College, Chadron | 19 May 1982 |
| MD Alpha | College of Notre Dame of Maryland, Baltimore | 22 May 1963 |
| IL Epsilon | North Park College, Chicago | 22 May 1963 |
| OK Beta | University of Tulsa, Tulsa | 3 May 1964 |
| CA Delta | California State Polytechnic University, Pomona | 5 Nov 1964 |
| PA Delta | Marywood College, Scranton | 8 Nov 1964 |
| PA Epsilon | Kutztown University of Pennsylvania, Kutztown | 3 April 1965 |
| AL Epsilon | Huntingdon College, Montgomery | 15 April 1965 |
| PA Zeta | Indiana University of Pennsylvania, Indiana | 6 May 1965 |
| AR Alpha | Arkansas State University, State University | 21 May 1965 |
| TN Gamma | Union University, Jackson | 24 May 1965 |
| WI Beta | University of Wisconsin-River Falls, River Falls | 25 May 1965 |
| IA Gamma | Morningside College, Sioux City | 25 May 1965 |
| MD Beta | Western Maryland College, Westminster | 30 May 1965 |
| IL Zeta | Rosary College, River Forest | 26 Feb 1967 |
| SC Beta | South Carolina State College, Orangeburg | 6 May 1967 |
| PA Eta | Grove City College, Grove City | 13 May 1967 |
| NY Eta | Niagara University, Niagara University | 18 May 1968 |
| MA Alpha | Assumption College, Worcester | 19 Nov 1968 |
| MO Eta | Northeast Missouri State University, Kirksville | 7 Dec 1968 |
| IL Eta | Western Illinois University, Macomb | 9 May 1969 |
| OH Zeta | Muskingum College, New Concord | 17 May 1969 |
| PA Theta | Susquehanna University, Selinggrove | 26 May 1969 |
| PA Iota | Shippensburg University of Pennsylvania, Shippensburg | 1 Nov 1969 |
| MS Delta | William Carey College, Hattiesburg | 17 Dec 1970 |
| MO Theta | Evangel College, Springfield | 12 Jan 1971 |
| PA Kappa | Holy Family College, Philadelphia | 23 Jan 1971 |
| CO Beta | Colorado School of Mines, Golden | 4 March 1971 |
| KY Alpha | Eastern Kentucky University, Richmond | 27 March 1971 |
| TN Delta | Carson-Newman College, Jefferson City | 15 May 1971 |
| NY Iota | Wagner College, Staten Island | 19 May 1971 |
| SC Gamma | Winthrop University, Rock Hill | 3 Nov 1972 |
| IA Delta | Wartburg College, Waverly | 6 April 1973 |
| PA Lambda | Bloomsburg University of Pennsylvania, Bloomsburg | 17 Oct 1973 |
| OK Gamma | Southwestern Oklahoma State University, Weatherford | 1 May 1973 |
| NY Kappa | Pace University, New York | 24 April 1974 |
| TX Eta | Hardin-Simmons University, Abilene | 3 May 1975 |
| MO Iota | Missouri Southern State College, Joplin | 8 May 1975 |
| GA Alpha | West Georgia College, Carrollton | 21 May 1975 |
| WV Alpha | Bethany College, Bethany | 21 May 1975 |
| FL Beta | Florida Southern College, Lakeland | 31 Oct 1976 |
| WI Gamma | University of Wisconsin-Eau Claire, Eau Claire | 4 Feb 1978 |
| MD Delta | Frostburg State University, Frostburg | 17 Sept 1978 |
| IL Theta | Illinois Benedictine College, Lisle | 18 May 1979 |
| PA Mu | St. Francis College, Loretto | 14 Sept 1979 |
| AL Zeta | Birmingham-Southern College, Birmingham | 18 Feb 1981 |
| CT Beta | Eastern Connecticut State University, Willimantic | 2 May 1981 |
| NY Lambda | C.W. Post Campus of Long Ialand University, Brookville | 2 May 1983 |
| MO Kappa | Drury College, Springfield | 30 Nov 1984 |

CO Gamma
NE Delta
TX Iota
PA Nu
VA Gamma
NY Mu
OH Eta
OK Delta
CO Delta
NC Gamma
PA Xj
MO Lambda
TX Kappa
SC Delta
SD Alpha
NY Nu
NH Alpha
LA Gamma
KY Beta
MS Epsilon

Fort Lewis College, Durango<br>Nebraska Wesleyan University, Lincoln<br>McMurry College, Abilene<br>Ursinus College, Collegeville<br>Liberty University, Lynchburg<br>St. Thomas Aquinas College, Sparkill<br>Ohio Northern University, Ada<br>Oral Roberts University, Tulsa<br>Mesa State College, Grand Junction<br>Elon College, Elon College<br>Cedar Crest College, Allentown<br>Missouri Western State College, St. Joseph<br>University of Mary Hardin-Baylor, Belton<br>Erskine College, Due West<br>Northern State University, Aberdeen<br>Hartwick College, Oneonta<br>Keene State College, Keene<br>Northwestern State University, Natchitoches<br>Cumberland College, Williamsburg<br>Delta State University, Cleveland

29 March 1985
18 April 1986
25 April 1987
28 April 1987
30 April 1987
14 May 1987
15 Dec 1987
10 April 1990
27 April 1990
3 May 1990
30 Oct 1990
10 Feb 1991
21 Feb 1991
28 April 1991
3 May 1992
14 May 1992
16 Feb 1993
24 March 1993
3 May 1993
19 Nov 1994

## Starting a KME Chapter

Complete information on starting a chapter of KME may be obtained from Arnold Hammel, National President (see address on p. 77). Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; student members must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately $\$ 50$ ) and the expenses of the installing officer. The individual membership fee to the national organization is $\$ 15$ per member and is paid just once, at that individual's initiation. Much of this $\$ 15$ is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offerings and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

