THE PENTAGON

A Mathematics Magazine for Students

Volume 54 Number 1 Fall 1994

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Two Interesting Approaches to Counting the Number of Spanning Trees of the Complete n-Partite Graph

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Presented at the 1993 National Convention and awarded THIRD PLACE by the Awards Committee.

The matrix tree theorem allows one to calculate the number of spanning trees of a graph. This theorem will be used to find a general formula for the number of spanning trees of a complete graph with n vertices and the complete n-partite graph. The formula for the complete graph is Cayley's formula for counting trees. These same formulas may also be obtained from a combinatorics point of view by giving the trees "names." Prüfer is the one credited with this approach.

The Matrix Tree Theorem.

Let us see what the matrix tree theorem states. "Let G be a connected labeled graph with adjacency matrix A. Then all cofactors of the matrix M are equal and their common value is the number of spanning trees of G" [Harary, 1969]. The matrix M is obtained from the negation of matrix A and by replacing the i-th diagonal by the degree of vertex v_i [Harary, 1969]. Let us consider a simple example of the matrix tree theorem. Define the labeled graph G to have four vertices and five edges as shown in Figure 1. The adjacency matrix A and the matrix M are

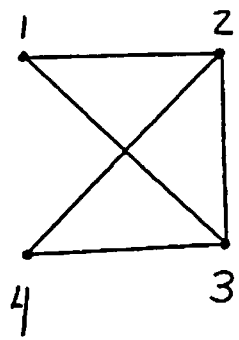

Figure 1.

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix} 2 & -1 & -1 & 0 \\ 1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}.$$

Now let us look at a cofactor of M, say the one found by crossing out the first row and the first column. The following matrix results

$$\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Taking the determinant of this new matrix yields the result of eight, which according to the theorem is the number of spanning trees of G. Let us verify this by examining the possible spanning trees of the labeled graph G (see Figure 2).

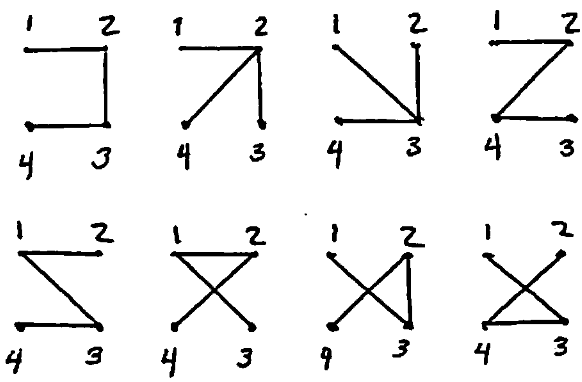

Figure 2.

These are the only possible spanning trees of G, and indeed there are eight of them.

The Complete Graph as a Special Case of the Matrix Tree Theorem.

Suppose the graph K_n is the complete graph with n vertices. Then the matrix A has 0's down the diagonal and 1's everywhere else. It follows that the matrix M has $(n-1)$'s down the diagonal and -1's everywhere else. Cross out the first row and the first column of the matrix M to obtain a submatrix C. By adding to the new first row of the matrix C all of the following rows, the resultant first row consists of just 1's. By adding this row to all of the rest of the rows, one at a time, a new matrix is obtained. This matrix is upper triangular. Also, the element in the 1,1 position has a value of 1; the rest of the elements on the diagonal, of which there are $n-2$, have the value n. So the value of this cofactor gives the value n^{n-2} as the result. By the matrix tree theorem, this result is the number of spanning trees of the graph K_n. Thus the number of spanning trees of the complete graph, i.e., the number of different labeled trees on n vertices, is n^{n-2}. This is Cayley's formula for counting labeled trees [Harary, 1967].

Once again, let us look at another example. Define the complete labeled graph K_4 to have four vertices as shown in Figure 3.

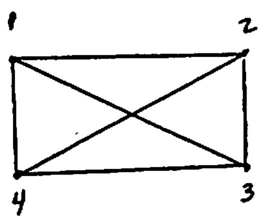

Figure 3.

The adjacency matrix A and the matrix M are

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}.$$

Crossing out the first row and the first column of the matrix M gives the matrix

$$C = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}.$$

Adding the second and third rows to the first row changes the matrix to

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

and then adding the first row to each of the second and third rows gives

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

Since the matrix is upper triangular, the determinant can be calculated by multiplying together the elements along the diagonal. This product is $4^2 = n^{n-2}$ and thus, by the matrix tree theorem, the complete labeled graph G has sixteen spanning trees. Let us verify that there are actually sixteen spanning trees by drawing them (see Figure 4).

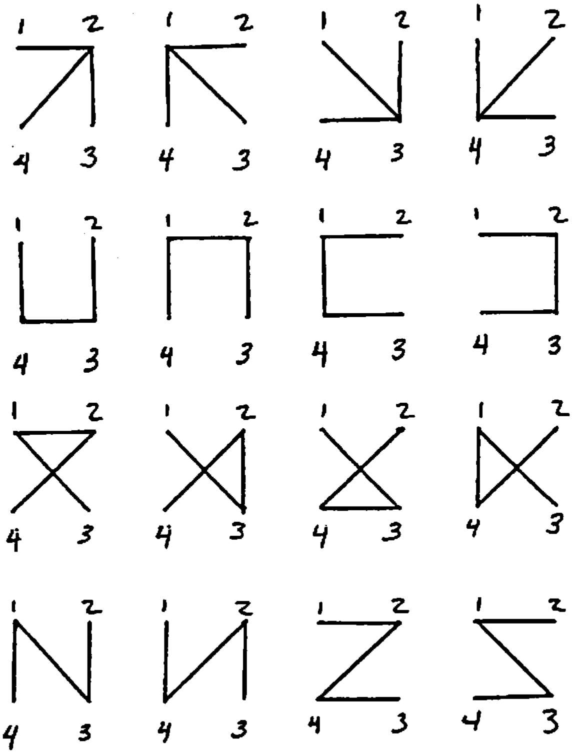

Figure 4.

These are all of the spanning trees of G, and indeed, there are sixteen of them.

The Complete Graph as Seen by Prüfer.

According to Prüfer [Harary, 1967], this formula, n^{n-2}, is without coincidence the same formula for finding the number of words using an n-letter alphabet that are $n-2$ letters in length. Thus each of the labeled spanning trees of the complete graph can be named uniquely, using the subscripts of the vertices of the complete labeled graph as the alphabet.

Suppose the complete graph G has n vertices labeled $v_1, v_2, \ldots, v_n$. For a given spanning tree T of K_n, the following is an algorithm for obtaining the "name" of T. The name of tree T is the set $(a_1, a_2, \ldots, a_{n-2})$ where a_i is a positive integer less than or equal to n. Also, when j does not equal k, a_j does not necessarily have to be different from a_k. Remove the endpoint v_i of the tree T whose subscript is the largest and remove the edge that connects v_i to v_j. Let $a_1 = j$. Repeat this process $n-3$ more times (that is until only two endpoints and the edge connecting them remain) to obtain a_2 through a_{n-2}. The result is the "name" $(a_1, a_2, \ldots, a_{n-2})$ [Harary, 1967].

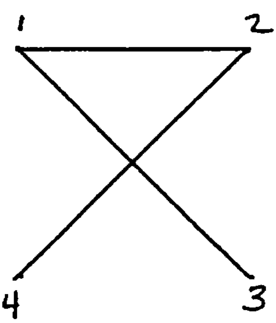

Figure 5.

Let us work an example. Take one of the spanning trees of the complete labeled graph G (with four vertices) and determine its name by the algorithm described above. For the tree T shown in Figure 5, remove the endpoint 4 and the edge connecting it to vertex 2. Since vertex 4 is the endpoint with the largest number, $a_1 = 2$. Thus we are left with the tree shown in Figure 6. Now the largest endpoint is 3. Remove it and the edge connecting it to vertex 1. Let $a_2 = 1$. Since all that now remains of tree T are two vertices, 1 and 2, and an edge connecting them, we stop. Thus the tree T has the name (2, 1). Note that no other tree has the name (2, 1). This can be verified by naming all of the sixteen spanning trees of the graph G.

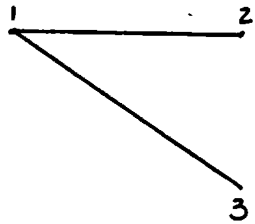

Figure 6.

It may be observed that the number of occurrences of the subscript of vertex v_i in the name of the tree is one less than the degree of that vertex. That is, if vertex v_i has degree d_i in a given tree, then the number i appears in the tree's name $d_i - 1$ times [Harary, 1967].

Let us examine the tree T (see Figure 5). The vertices 1 and 2 have degree two while the vertices 3 and 4 have degree one. Recall the name of the tree T is (2, 1). The vertices 1 and 2 each appear once — one less time than the degree of those vertices. The vertices 3 and 4 do not appear at all, which also is one less time than the degree of those vertices.

Using the above observation, the process can be reversed to obtain the spanning tree given its name. For example, let us look at the name (2, 1). Since the length of the name is two, we know there are two more vertices in the tree, the total thus being four. Let us write down this information and label the vertices (see Figure 7).

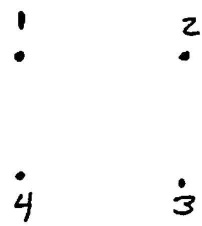

Figure 7.

Note that since vertices 1 and 2 each appear once in the name, they have degree two, and similarly, vertices 3 and 4 have degree one. Vertices 3 and 4 are the only endpoints. The largest endpoint must be connected to vertex 2, so an edge connects vertex 2 to 4 (see Figure 8).

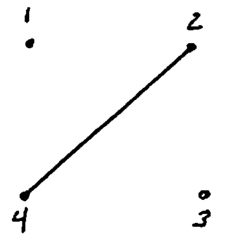

Figure 8.

The second largest endpoint must be connected to vertex 1, so an edge connects vertex 1 to 3 (see Figure 9).

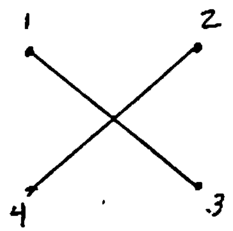

Figure 9.

The last edge to be drawn connects vertex 1 to vertex 2. This is known because these two vertices have degree two. So the tree shown in Figure 10 follows; note that it is the same tree with which we started in order to obtain the name (2, 1).

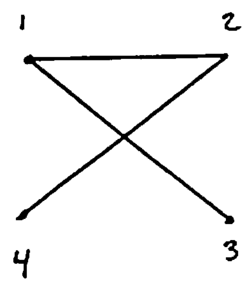

Figure 10.

A proof by induction will reveal that for each name of length $n-2$, there is a tree that corresponds to that name when the above process is followed. Since each tree has a unique name, it follows that the number of spanning trees of the complete graph is n^{n-2} [Harary, 1967].

The Complete n-Partite Graph.

A more general result exists for the n^{n-2} formula. This one is for complete n-partite graphs — graphs whose N vertices are grouped in clusters, varying from one to N in number, in such a way that any two members of a cluster are not joined together by an edge. The following is the formula [Oláh, 1968] for the number of spanning trees of a complete n-partite graph

$$N^{n-2} \times (N-L_1)^{L_1-1} \times (N-L_2)^{L_2-1} \times \cdots \times (N-L_n)^{L_n-1}$$

where N is the number of vertices, n is the number of clusters, and L_i is the number of vertices in cluster C_i found in the complete n-partite graph G. Note that if graph G were K_n, N would be equal to n and thus all but the first factor would have the exponent zero (and therefore equal one) and the previous formula of n^{n-2} would result. Thus n^{n-2} is a specific case of the complete n-partite graph formula.

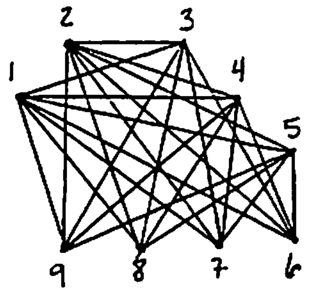

Figure 11.

Proof by the Matrix Tree Theorem.

Let us look at an example. Suppose we have the complete 3-partite graph shown in Figure 11. Its matrix M would be

$$\begin{bmatrix} 7 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 7 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 6 & 0 & 0 & -1 & -1 & -1 & -1 \\ -1 & -1 & 0 & 6 & 0 & -1 & -1 & -1 & -1 \\ -1 & -1 & 0 & 0 & 6 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 5 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & 0 & 5 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & 0 & 0 & 5 & 0 \\ -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 5 \end{bmatrix}$$

To find the number of spanning trees of this complete 3-partite graph, first remove the first row and first column. Then add all of the rows of the matrix to the last row to obtain

$$\begin{bmatrix} 7 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 6 & 0 & 0 & -1 & -1 & -1 & -1 \\ -1 & 0 & 6 & 0 & -1 & -1 & -1 & -1 \\ -1 & 0 & 0 & 6 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 5 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & 0 & 5 & 0 & 0 \\ -1 & -1 & -1 & -1 & 0 & 0 & 5 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Then add the last row to the first row, yielding

$$\begin{bmatrix} 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 6 & 0 & 0 & -1 & -1 & -1 & -1 \\ -1 & 0 & 6 & 0 & -1 & -1 & -1 & -1 \\ -1 & 0 & 0 & 6 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 5 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & 0 & 5 & 0 & 0 \\ -1 & -1 & -1 & -1 & 0 & 0 & 5 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Add one-seventh multiplied by the first row to all of the other rows in the matrix except for the last row. Then add the last row to all of the rows in the matrix except for the first row. The following matrix results

$$\begin{bmatrix} 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 7 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 6 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 6 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Using row operations similar to those just performed,

$$\begin{bmatrix} 7 & 1 & 1 \\ 1 & 7 & 1 \\ 1 & 1 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} 9 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

while changing the ones to zeros under the "mini matrix" in the last row.

Similarly,

$$\begin{bmatrix} 6 & 1 & 1 & 1 \\ 1 & 6 & 1 & 1 \\ 1 & 1 & 6 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Finally, the following matrix results:

$$\begin{bmatrix} 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Since the matrix is lower triangular, the determinant can be calculated by multiplying the diagonal entries together: $9 \cdot 7 \cdot 6^2 \cdot 5^3 \cdot 1$. This product can be rewritten as: $9^{3-2} \cdot (9-2)^{2-1} \cdot (9-3)^{3-1}(9-4)^{4-1}$, which is the application of the general formula for our complete 3-partite graph.

Let us look at the general case, the complete n-partite graph. Its formula can be easily derived from row operations on the matrix representation of the complete n-partite graph. The matrix representation of a complete n-partite graph is

$$\begin{bmatrix} \mathrm{B}_1 & -1 & \cdots & -1 \\ -1 & \mathrm{B}_2 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & \mathrm{B}_n \end{bmatrix}$$

where B_i is a mini matrix of size $L_i \times L_i$ and looks like

$$\begin{bmatrix} N-L_1 & 0 & \cdots & 0 \\ 0 & N-L_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & N-L_n \end{bmatrix}.$$

After repeated row operations (similar to those in the example above) to make the matrix lower triangular, we obtain the matrix

$$\begin{bmatrix} D_1 & 0 & \cdots & 0 & 0 \\ 0 & C_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & C_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & D_2 \end{bmatrix}$$

where D_1, C_i and D_2, respectively, are

$$\begin{bmatrix} N-L_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & N-L_1 \end{bmatrix}, \begin{bmatrix} N & 0 & \cdots & 0 \\ 0 & N-L_i & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & N-L_i \end{bmatrix}$$

and
$$\begin{bmatrix} N-L_n & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & N-L_n & 0 \\ 1 & 1 & \cdots & 1 \end{bmatrix}.$$

It can easily be seen that the matrix is now lower triangular. To compute a cofactor, cross out the first row and first column. The determinant of this cofactor may be computed by multiplying the diagonal elements together resulting in the complete n-partite graph formula for the number of spanning trees.

Proof using Oláh's Naming Method.

Similar to the names assigned to the spanning trees of the complete graph using Prüfer's method, Oláh discovered that names of the spanning trees of a complete n-partite graph may be uniquely assigned [Oláh, 1968]. However, the names of the complete n-partite graph take on a slightly different form. These names are broken down into $n+1$ segments of the form

$$(a_{0_1}, \ldots, a_{0_{n-2}})\,(a_{1_1}, \ldots, a_{1_{(L_1-1)}})\,(a_{n_1}, \ldots, a_{n_{(L_n-1)}})$$

where the entries are positive integers less than or equal to N. Furthermore, the name segment containing a_i corresponds to the cluster C_i, whereas the a_0 segment corresponds to an "overflow" name segment, as will be explained below.

The following is Oláh's algorithm [Oláh, 1968]. Remove the endpoint v_i whose subscript is the largest of the tree and remove the edge that connects v_i to v_j. Let j be the first letter in the name segment corresponding to cluster to which v_i belongs. Note that this segment will contain $L_i - 1$ entries. In the event that the name segment for a certain cluster becomes full, the subscript is entered into the "overflow" name segment. Repeat this process until only two endpoints and the edge connecting them remain. The result is the unique name of the spanning tree.

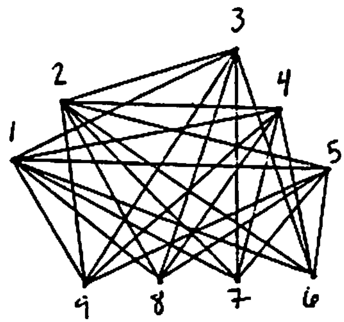

Figure 12.

Let us look at an example. Recall the complete 3-partite graph example used in the matrix tree theorem section (see Figure 12).Now, let us choose a spanning tree T (see Figure 13) of this graph where C_1 contains the vertices 1 and 2; C_2 contains the vertices 3, 4 and 5; and C_3 contains the vertices 6, 7, 8 and 9. The name will have the form "(_)(_)(_,_)(_,_,_)." Now, let us fill in the blanks.

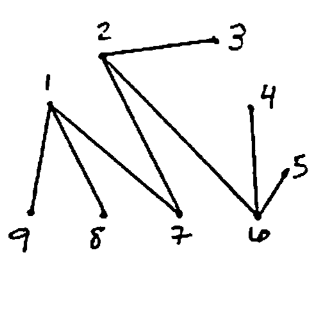

Figure 13.

The largest endpoint is vertex 9, so remove it and the edge connecting it to vertex 1. Record the number 1 in the first blank of the last segment. The partial name is (_)(_)(_,_)(1,_,_) and the tree now is a shown in Figure 14.

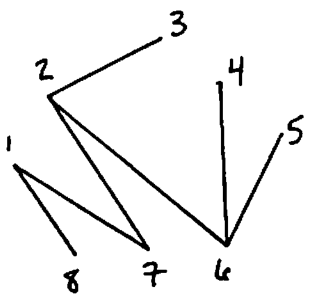

Figure 14.

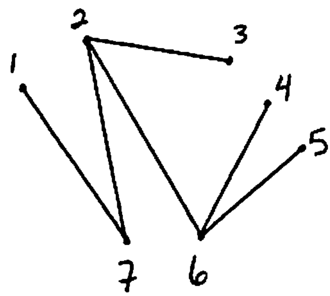

Figure 15.

Now the largest endpoint is vertex 8. Remove it and the edge connecting it to vertex 1. Record the number 1 in the blank next to the number 1. The tree now is as shown in Figure 15. The vertex 5 is the largest endpoint. Remove it and the edge connecting it to vertex 6. Record the number 6 in the first blank of the third segment of the name. The partial name is (_)(_)(6,_)(1,1,_) and the resulting tree is shown in Figure 16.

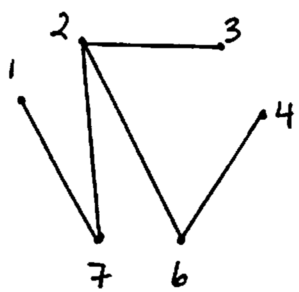

Figure 16.

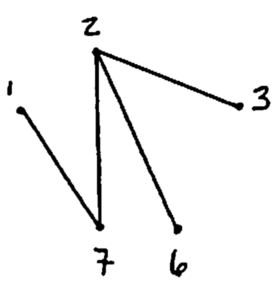

Figure 17.

Remove vertex 4 and the line connecting it to vertex 6. Enter the number 6 in the blank next to the current 6 in the name. Again, the partial name is now (_)(_)(6,6)(1,1,_) and the tree is as shown in Figure 17. Vertex 6 is the largest endpoint now. Remove it and the edge connecting it to vertex 2. Enter the number 2 into the last blank in the last segment in the name. The partial name becomes (_)(_)(6,6)(1,1,2) and the tree is shown in Figure 18. Now, vertex 3 is the largest endpoint. Remove it and the edge connecting it to vertex 2. Enter the number 2 into the blank in the first segment of the name. The vertex goes here this time because the segment associated with C_2 is full and thus goes into the "overflow" segment. The partial name (2)(_)(6,6)(1,1,2) and the tree is as shown in Figure 19. Finally, the vertex 2 is the largest endpoint. Remove it and the edge connecting it to vertex 7. Enter the number 7 into the blank in the second segment of the name. Since only the vertices 1 and 7 remain, of course they are connected by an edge and the complete name (2)(7)(6,6)(1,1,2) has been found.

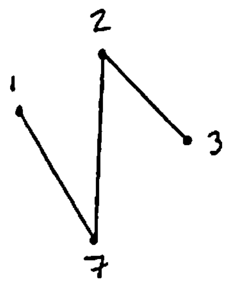

Figure 18.

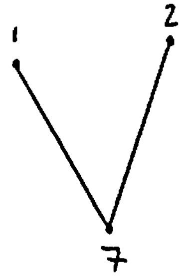

Figure 19.

Note that the number of subscripts of vertices appearing in the name is $N-2$, the same length names as seen in the complete labeled graph. In our example, tree T has nine vertices and the name has a total of seven vertices. Let us look at this in the general case.

Recall the general form of the name of a tree of a complete n-partite graph is

$$(a_{0_1}, \ldots, a_{0_{n-2}})\,(a_{1_1}, \ldots, a_{1_{(L_1-1)}})\,(a_{n_1}, \ldots, a_{n_{(L_n-1)}})$$

There are $n-2$ vertices in the first segment, $L_1 - 1$ vertices in the second, $L_2 - 1$ vertices in the third segment, and so on up to the last segment which has $L_n - 1$ vertices. Adding these numbers together yields

$$\begin{aligned}&(n-2)+(L_1-1)+(L_2-1)+\cdots+(L_n-1)\\ &= n-2 \;-\; n \;+\; L_1+L_2+\cdots+L_n\\ &= L_1+L_2+\cdots+L_n \;-\; 2 \;=\; N-2\end{aligned}$$

since $L_1 + L_2 + \cdots + L_n = N$. This sum of the vertices in all of the segments of the name is two less than the number of vertices in the complete n-partite graph.

Similarly to the complete graph, the number of occurrences of a vertex subscript corresponds to one less than the degree of that vertex. Again, let us look at our example tree T (see Figure 12). The vertices 3, 4, 5, 8 and 9 have degree one and do not appear in the tree's name. Similarly, vertex 7 appears once and has degree two,and vertices 1, 2 and 6 appear twice and have degree three. So the relationship of the number of occurrences of a vertex to the degree of that vertex still holds.

Using this observation, the process can be reversed and thus the spanning tree can be drawn from its name.

Now, given the name (2)(7)(6,6)(1,1,2), let us see if we can obtain the tree with which we started. We know that the tree is a 3-partite tree; C_1 has two vertices: 1 and 2; C_2 has three vertices: 3, 4, and 5; and C_3 has four vertices: 6, 7, 8 and 9. Also, we know that the following vertices are endpoints (because their names do not appear in the name): 3, 4, 5, 8 and 9.

To find the tree whose name is (2)(7)(6,6)(1,1,2), we must keep track of the endpoints. Our list of endpoints right now is 3, 4, 5, 8 and 9. We will start with vertex 9 since it is the largest. Vertex 9 is a member of C_3 so we connect vertex 9 to vertex 1 (since vertex 1 is the first vertex in the name segment belonging to C_3), remove 9 from our list of vertices and remove the 1 from the name. The following results: the name is now (2)(7)(6,6)(_,1,2), the list of endpoints is 3, 4, 5 and 8, and the tree is as shown in Figure 20.

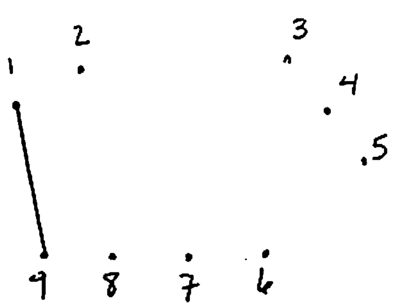

Figure 20.

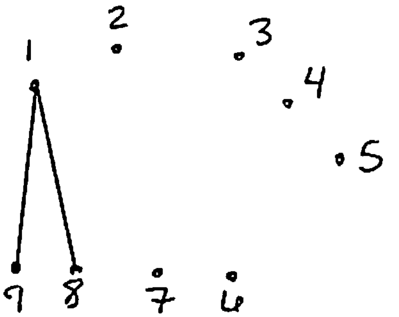

Figure 21.

Vertex 8 is an element of C_3 so we connect it to vertex 1. Remove 8 from the list of vertices and then remove 1 from the name and add it to the endpoint list (since it now is an endpoint in the part of the tree left to draw). The following results: the name is (2)(7)(6,6)(_,_,2), the list of endpoints is 1, 3, 4 and 5, and the tree is as shown in Figure 21. The largest endpoint is 5. Since it is an element of C_2, we connect it to vertex 6, remove vertex 5 from the endpoint list and remove vertex 6 from the name list. The following results: the name is (2)(7)(_,6)(_,_,2), the list of endpoints is 1, 3 and 4, and the tree is as shown in Figure 22. Vertex 4 is now the largest endpoint. It is also an element of C_2 and thus we connect it to vertex 6. Remove vertex 4 from the endpoint list. Remove vertex 6 from the name and place it in the endpoint list. The following results: the name is (2)(7)(_,_)(_,_,2), the list of endpoints is 1, 3 and 6, and the tree is now as shown in Figure 23.

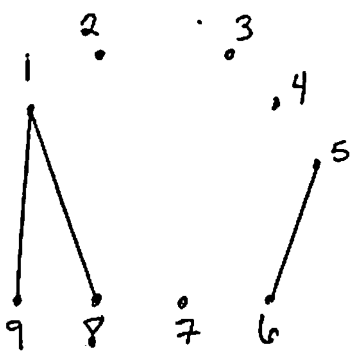

Figure 22.

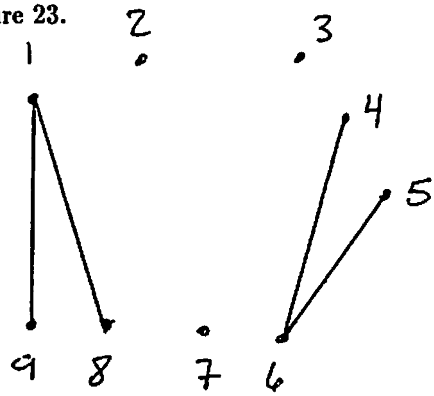

Figure 23.

Vertex 6 is the largest endpoint and is an element of C3. Connect vertex 6 to vertex 2, remove vertex 6 from the endpoint list, and remove vertex 2 from the name. The following results: the name is (2)(7)(_,_)(_,_,_), the list of endpoints is 1 and 3, and the tree is as shown in Figure 24. The largest endpoint is vertex 3 and is an element of C_2. Since there are no more vertices in the name segment corresponding to C_2, we connect vertex 3 to the vertex in the overflow segment, that is vertex 2. Remove vertex 3 from the endpoint list, remove vertex 2 from the name and add vertex 2 to the endpoint list. The following results: the name is (_)(7)(_,_)(_,_,_), the endpoint list is 1 and 2, and the tree is as shown in Figure 25. Again, vertex 2 is the largest endpoint and is an element of C_1. Connect vertex 2 to vertex 7, remove vertex 2 from the endpoint list, remove vertex 7 from the name and add it to the endpoint list. The following results: the name is (_)(_)(_,_)(_,_,_), the endpoint list is 1 and 7, and the tree is as shown in Figure 26.

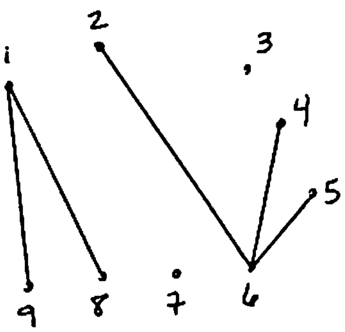

Figure 24.

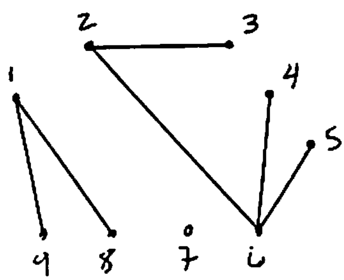

Figure 25.

Since there are no more vertices in the name, we connect the two vertices in the endpoint list, vertices 1 and 7. Thus the resulting tree is as shown in Figure 27. Notice that it is the same tree with which we started to obtain its name.

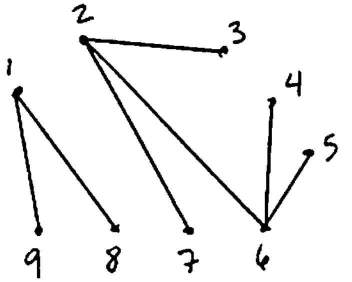

Figure 26.

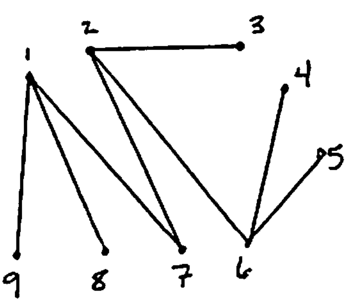

Figure 27.

For this example, we have shown that: given a tree, we can find its name that and given the name, we can find the tree. A proof by induction can prove that the general formula for the number of spanning trees of the complete labeled n-partite graph is correct.

As you have seen, there exist some interesting applications of the matrix tree theorem, resulting in a general formula for the number of spanning trees of a complete n-partite graph, along with the ability to uniquely define the names of the spanning trees.

Acknowledgement. Special thanks to Dr. Richard A. Gibbs for his help and patience in the preparation of this paper, which began as an independent study project during the Fall 1992 semester.

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Even the Least of These

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Presented at the 1993 National Convention.

In this paper I am going to describe a type of infinite sets of positive real numbers whose measure is zero. I will do this by describing the Cantor ternary set and its unique characteristics, then adapting it to any positive integer base expansion.

Measure theory is a branch of analysis. For our purpose we need to know that the measure of an interval $[a,b]$ is $b-a$. For example,

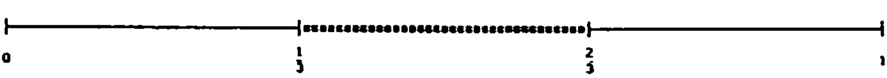
$$m([2,5]) = 5-2 = 3.$$

In fact,

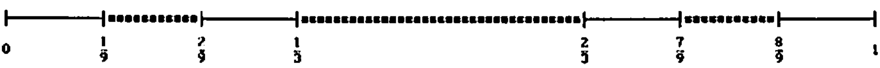
$$m([2,5)) = m([2,5)) = m((2,5]) = m((2,5)) = 3$$

since the measure of a single element set, and in fact any finite set, is zero.

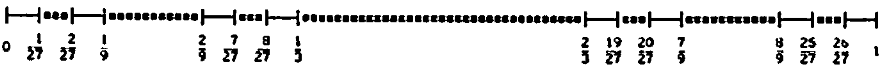
To begin we will first look at the Cantor ternary set, a subset S of the closed interval $[0,1]$. For simplicity it will be easier to first describe its complement C(S) relative to $[0,1]$. C(S) is the union of intervals constructed as follows: start with the middle third of $[0,1]$,



add to this the middle third of the two remaining subintervals,



add to this the middle third of the four remaining subintervals,



and so on, ad infinitum. From this definition it is easy to see that $m(\mathrm{C(S)})$ is a geometric series,

$$m(\mathrm{C(S)}) = 1/3 + 2(1/9) + 4(1/27) + 8(1/81) + \cdots$$

and can be simplified by factoring out 1/3,

$$= (1/3)\cdot\Big((2/3)^0 + (2/3)^1 + (2/3)^2 + (2/3)^3 + \cdots\Big)$$

$$= (1/3)\cdot\frac{1}{1-(2/3)} = 1\,.$$

The Cantor ternary set S is what is left after C(S) is removed from the closed interval $[0,1]$. So the measure of S is

$$m(\mathrm{S}) = m([0,1]) - m(\mathrm{C(S)}) = 1 - 1 = 0\,.$$

Although we have shown that set S in a sense takes up no "space" on $[0,1]$, it is not an empty set. Rather, it is an infinite set with as many elements as the original set $[0,1]$, which is uncountable. In fact we can prove the following theorem.

Theorem. The Cantor ternary set S has the same cardinality as $[0,1]$.

To show that two sets have the same number of elements, or cardinality, we have to establish a 1-to-1 correspondence between the two sets.

Proof. We can easily show that every number in $[0,1]$ has the following ternary, or base 3, expansion,

$$\frac{a_1}{3^1} + \frac{a_2}{3^2} + \frac{a_3}{3^3} + \cdots + \frac{a_n}{3^n} + \cdots$$

where $a_n = 0, 1$ or 2 for every positive integer n. As a base 3 "decimal," it is written in the following form:

$$.a_1 a_2 a_3 \ldots a_n \ldots {}_{(3)}$$

For example,

$$.101000_{(3)} = 1/3 + 0/9 + 1/27 = .370\overline{370}_{(10)}$$

and

$$.6_{(10)} = .1210\overline{1210}_{(3)}.$$

Now consider our definition of C(S) in its ternary expansion. Any number between $1/3$ and $2/3$ has ternary form

$$.1xxxx\ldots_{(3)}$$

where x is arbitrary. Any number between $1/9$ and $2/9$ has the form

$$.01xxx\ldots_{(3)}$$

and any number between $7/9$ and $8/9$ has the form

$$.21xxx\ldots_{(3)}.$$

Continuing in this fashion we see that every number in C(S) will have at least one 1 appearing in its expansion. Therefore S consists of only those numbers whose ternary expansion is composed entirely of 0's and 2's.

Now consider the interval $[0,1]$ in its binary expansion. We can see that every number would consist entirely of 0's and 1's by virtue of binary definition. For example,

$$.1011_{(2)} = 1/2 + 0/4 + 1/8 + 1/16 = .6875_{(10)}$$

and

$$.231_{(10)} = .00111011\ldots_{(2)}.$$

We define our 1-to-1 correspondence from S to $[0,1]$ as follows:

$$f(.a_1a_2a_3\ldots_{(3)}) = .b_1b_2b_3\ldots_{(2)}$$

where

$$b_i = \begin{cases} 0 & \text{if } a_i = 0 \\ 1 & \text{if } a_i = 2 \end{cases}.$$

For example,

$$f(.2022002\ldots_{(3)}) = .1011001\ldots_{(2)}.$$

Since any two distinct numbers in S will have distinct ternary expansions using only 0's and 2's, they will be mapped to distinct binary real numbers. This shows that f is 1-to-1.

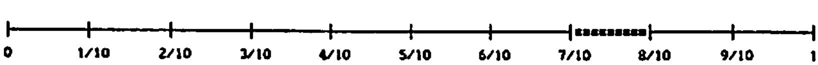
Also, since every number in $[0,1]$ has a binary expansion, we can find a ternary number in S that gets mapped to it. This shows that f is onto. Therefore, f is 1-to-1 and onto. So S is an (uncountably) infinite set of measure 0.

This is an important and well known example in analysis. Our extension of this result evolved from the rather harmless question we happened to ask ourselves, "How much of $[0,1]$ would be left if we left a digit out of the base 10 expansion of this set?" Before we give the answer stop and consider what you think might happen.

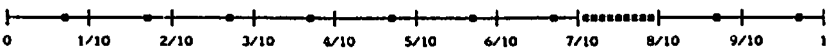
In fact the surprising result (at least to us) is the

Theorem. Leaving any digit out of the decimal expansion of $[0,1]$ leaves a set of measure 0.

Proof. We will demonstrate this result for the digit 7. Let us now consider the closed interval $[0,1]$ in base 10 expansion. Numbers in $[0,1]$ of the form $.7xxxx\ldots_{(10)}$ where x is arbitrary will be between $7/10$ and $8/10$. This interval has measure 0.1 or $1/10$.



Numbers in the remaining 9 subintervals of the form $.x7xxx\ldots_{(10)}$ are indicated in the next figure and each has measure 0.01 or $1/100$.



Following the same pattern as for C(S), let T be the set of all numbers in $[0,1]$ whose decimal expansion contains no 7's. We can see that the measure of set C(T), all numbers in $[0,1]$ whose decimal expansion contains at least one 7, will be

$$m(\mathrm{C(T)}) = 1/10 + 9(1/100) + 81(1/1000) + \cdots$$

$$= (1/10)\cdot\Big((9/10)^0 + (9/10)^1 + (9/10)^2 + (9/10)^3 + \cdots\Big)$$

$$= (1/10)\cdot\frac{1}{1-(9/10)} = 1\,.$$

Set T is what is left after C(T) is removed from the closed interval $[0,1]$. So the measure of T is

$$m(\mathrm{T}) = m([0,1]) - m(\mathrm{C(T)}) = 1-1 = 0\,.$$

To prove that T is an (uncountably) infinite set we will find a 1-to-1 correspondence between set T and $[0,1]$. Consider set T in its decimal expansion and $[0,1]$ in a base 9 expansion. We define our 1-to-1 correspondence from T to $[0,1]$ as follows:

$$f(.a_1a_2a_3\ldots_{(10)}) = .b_1b_2b_3\ldots_{(9)}$$

where

$$b_i = \begin{cases} a_i & \text{if } a_i < 7 \\ a_i - 1 & \text{if } a_i > 7 \end{cases}.$$

For example,

$$f(.194868\ldots_{(10)}) = .184767\ldots_{(9)}\,.$$

Since any two distinct numbers in T will have distinct decimal expansions using the digits 0 to 9 except for 7, they will be mapped to distinct base 9 real numbers. This shows that f is 1-to-1.

Also, since every number in $[0,1]$ has a base 9 expansion, we can find a decimal number in T that gets mapped to it. This shows that f is onto. Therefore, f is 1-to-1 and onto. So T is an (uncountably) infinite set of measure 0.

We have demonstrated that leaving a single digit out of either base 3 or base 10 expansions of the closed interval $[0,1]$ leaves us with an (uncountably) infinite set of measure 0. Now consider the set $[0,1]$ where the numbers are written in base b "decimal" expansion, and let n be a digit between 0 and $b-1$ inclusive. Then the same type of argument allows us to conclude that omitting all numbers in $[0,1]$ whose expansion includes n will leave us with an uncountable set of measure 0. Indeed, we will have a lot of numbers left, but "most" of them will be deleted.

In the modern world of math and numerical analysis it is easy to regard a single number as insignificant, especially in comparison with infinite sets. We must be careful in doing so, as you have seen that leaving a single digit out of the decimal expansion of the uncountable set $[0,1]$ will leave us with an equivalent set of measure 0. We must remember that even the "least" of numbers are important.

Acknowledgement. Help on conceptual research from Dr. Don Tosh was much appreciated.

Abstracts of Papers Presented at the 1994 Region IV Convention

Edited by Mary Sue Beersman, Region IV Director

The 1994 *KME* Region IV Convention was held 22-23 April 1994 at Missouri Western State College in St. Joseph, Missouri. The Missouri Lambda Chapter hosted students and faculty from fifteen chapters. Even the weather cooperated for a beautiful weekend. Abstracts of the seven presentations are given below. First prize went to Kirk Drager from Kansas Delta.

Jeremy BENSON, *Forecasting: A Mathematical Approach.* Missouri Lambda (Missouri Western State College, St. Joseph).

The purpose of this paper is to show the double exponential smoothing forecast model and derive the formulas for the model. An empirical study of the model is also presented in the paper.

Kim DERRINGTON, *A Genetic Algorithm Applied to a Problem in Coding Theory.* Missouri Eta (Northeast Missouri State University, Kirksville).

Genetic algorithms are a relatively new optimization technique which are based on the mechanics of natural selection and natural genetics. A genetic algorithm combines the idea of survival of the fittest with a randomized information exchange. Error-correcting codes originated in response to problems in the reliable communication of digitally encoded information. A problem in the theory of error-correcting codes is determining the minimum distance of a code. The minimum distance d is an important concept in coding theory because from d we can determine how many errors a code can correct. This is usually a difficult problem to solve. I adapted an existing genetic algorithm to find the minimum weight vectors in a code and was able to show that genetic algorithms are a good technique to apply when searching for the minimum distance of a code.

Christopher A. DIX, *The Business of Calculus.* Iowa Alpha (University of Northern Iowa, Cedar Falls).

This paper is my way of showing that calculus will be valuable in my chosen career of business management. The paper includes two hypothetical situations that I might be involved in as a manager in which calculus would be very helpful in solving. The paper also includes corresponding tables and graphs which make the hypothetical situations more easily understood.

Kirk DRAGER, *Markov Chains and Ion Channel Conductance.* Kansas Delta (Washburn University of Topeka, Topeka).

This research utilized graph theoretical applications to model the kinetics of ion conducting protein found in nerve cells using data generated by a team of biochemists at the University of Kansas. The data show that over time electrical conductance measurements through the protein tend to cluster about four levels, which are presumed to represent different protein conformations. The first attempt to use a deterministic weighted digraph method was found inappropriate because an initial external input required to set the system in motion was absent. The model was changed to a probabilistic model know as a finite Markov chain where the weights on the digraph were replaced by probabilities. The Markov process includes results which allow calculation of the mean first passage matrix which calculates the average amount of time between observations of different states. Applying this to the ion channel problem indicates that the timing of these changes is within the range of known protein conformational state changes. Another goal of this project was to determine whether the conformational states identified were sufficient to describe the system. The Markov process can also be modified by redefining a state as the current state coupled with its previous state. This coupled analysis shows that for a given level of conductance, the probabilities for moving into other states differ depending on the preceding level. This suggests that further research could concentrate on this level of conductance.

Heather HOHNSTEIN, *The Mathematics of Future Value.* Nebraska Gamma (Chadron State College, Chadron).

The equation $x = (rP(1+r/12)^{12n})/(12(1+r/12)^{12n}-1)$ is a future value equation. It comes from a difference equation. Using the fact that the general difference equation is $y_{n+1} = Ay_n + B$, algebra and rearranging gives $y_n = A^n y_0 + (B/(A-1))(A^n - 1)$. Then, by deriving another equation for the interest I_{j+1} owed during the $(j+1)$-st month, another useful equation comes to life, $I_{j+1} = I_j - r/12(x - I_j) = (1 + r/12)I_j - (r/12)x$. Using the general form of the difference equation gives $y_n = (1 + r/12)^n (r/12)P - x((1 + r/12)^n - 1)$, but on a loan, at the end the loan will all be paid off and it has to be in terms of months, so doing some algebra and rearranging gives $0 = (1 + r/12)^{12n}P - 12x/r((1 + r/12)^{12n} - 1)$. Solving for x, $x = (rP(1 + r/12)^{12n})/(12(1 + r/12)^{12n} - 1)$.

Cynthia A. SCHWAB, *The Topological Equivalences of K and X.* Missouri Kappa (Drury College, Springfield).

In this paper I look at the topological qualities of letters graphed in $\mathbb{R}^2$. Specifically, I look at the letters X and K and show that they are topologically equivalent. To do so, I find a function α which maps from X to K and then show that α is a homeomorphism. In order to do this, I show that α is one-to-one, onto and bi-continuous. I next found an isotopy which changes the X into the K over time. I then used this isotopy in a computer program to show the X morphing into a K.

Amy WIEMERSLAGE, *The Mathematics of Just and Equal Temperament.* Iowa Alpha (University of Northern Iowa, Cedar Falls).

In this paper, two different ways of tuning fixed pitched instruments are presented - just and equal temperament. Though it may be practical to mathematically apply both concepts, it is musically and mechanically difficult to simultaneously attain the desired effects each offers.

Generalized p-Series With Applications

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Introduction.

It is well known that the p-series $\sum 1/n^p$ converges if $p > 1$ and diverges if $p \leq 1$, see for example [2, p. 500]. It is interesting to see this series and its test be generalized. A simple forward generalization of $1/n^p$ is $(a_n - a_{n-1})/a_n^p$ where $\{a_n\}$ is a sequence of positive real numbers. To see this, let $a_0 = 0$ and $a_n = n$ for $n = 1, 2, 3, \ldots$. If the sequence $\{a_n\}$ is also increasing, then the integral test applies to the series $\sum (a_n - a_{n-1})/a_n^p$ in the same way it applies to the p-series itself.

In Section 1, we will discuss the convergence and the divergence of a generalized p-series of this sort, namely, $\sum (a_n - a_{n-1})/a_n^p$, where $\{a_n\}$ is an increasing sequence of positive real numbers. Some error estimates and applications to integrals are also discussed. In Section 2, we will consider the more general p-series $\sum \mid a_n - a_{n-1} \mid / a_n^p$, where $\{a_n\}$ is a sequence of positive real numbers. The following is our formal definition of a generalized p-series.

Definition 1 For any sequence $\{a_n\}$ of positive real numbers, the series

$$\sum_{n=1}^{\infty} \frac{\mid a_n - a_{n-1} \mid}{a_n^p}$$

will be called the generalized p-series associated with $\{a_n\}$.

§1. The generalized p-series when $\{a_n\}$ is increasing.

In this section we will consider the generalized p-series when $\{a_n\}$ is an increasing sequence of positive real numbers. In this case $a_n - a_{n-1} > 0$ and the associated p-series will have the form $\sum (a_n - a_{n-1})/a_n^p$.

Theorem 1 Suppose $\{a_n\}$ is an increasing sequence of positive real numbers. Then (i) for $0 \le p \le 1$, the associated generalized p-series converges if and only if $\{a_n\}$ converges, and (ii) for $p > 1$, the associated generalized p-series converges.

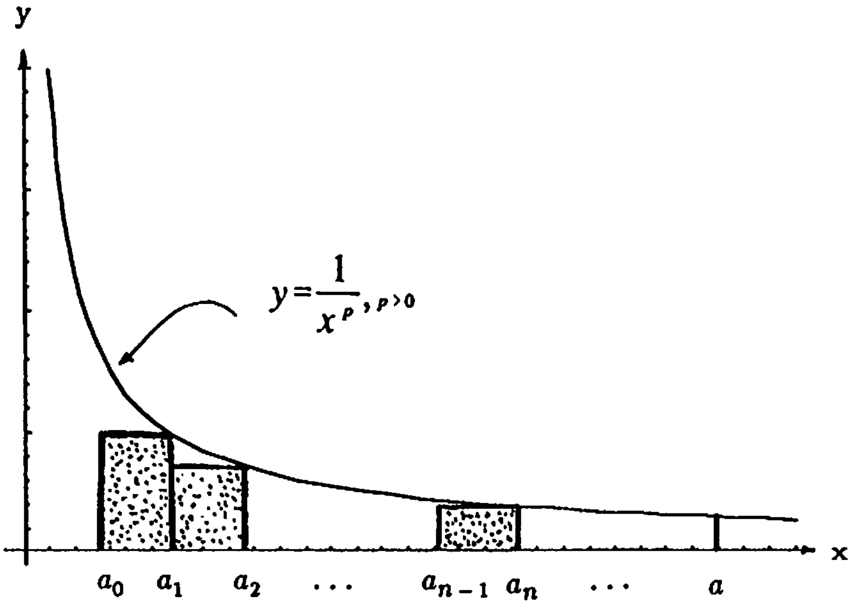

Figure 1.

Proof. (i) If $p = 0$, then the proof is trivial since the generalized p-series becomes a telescoping series. Now let $p > 0$. If $\lim_{n\to\infty} a_n = a < \infty$, then from Figure 1 it is easy to see that

$$\sum_{n=1}^{\infty} \frac{a_n - a_{n-1}}{a_n^p} < \int_{a_0}^{a} \frac{1}{x^p}\, dx < \infty .$$

If $\lim_{n\to\infty} a_n = \infty$, let m be any positive integer. Then for $k \ge m$ we have

$$\sum_{n=m}^{k} \frac{a_n - a_{n-1}}{a_n^p} \ge \frac{1}{a_k^p} \sum_{n=m}^{k} (a_n - a_{n-1})$$
$$= \frac{1}{a_k^p}(a_k - a_{m-1}) = \frac{1}{a_k^{p-1}} - \frac{a_{m-1}}{a_k^p} .$$

Since $\lim_{n\to\infty} a_n = \infty$,

$$\sum_{n=m}^{\infty} \frac{a_n - a_{n-1}}{a_n^p} \geq \lim_{k\to\infty}\left(\frac{1}{a_k^{p-1}} - \frac{a_{m-1}}{a_k^p}\right) = \begin{cases} 1 & \text{if } p = 1 \\ \infty & \text{if } p < 1 \end{cases}.$$

By the Cauchy criterion, see for example [1, p. 101], the last equality and the assumption that m is arbitrary, we conclude the divergence of $\sum (a_n - a_{n-1})/a_n^p$.

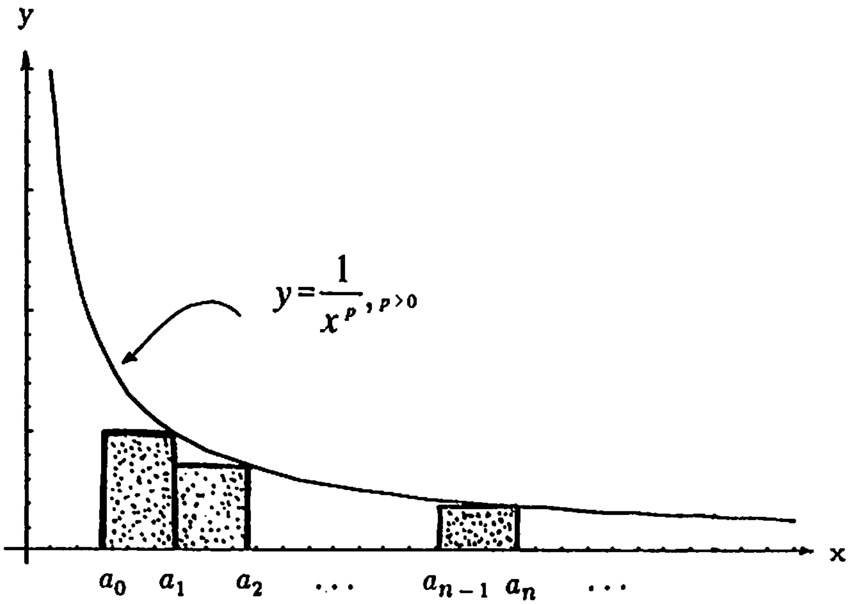

Figure 2.

(*ii*) From Figure 2 we can easily see that

$$\sum_{n=1}^{\infty} \frac{a_n - a_{n-1}}{a_n^p} < \int_{a_0}^{\infty} \frac{1}{x^p}\, dx < \infty .$$

Corollary 1 Suppose $\{b_n\}$ is an unbounded increasing sequence of positive real numbers. Then the series

$$\sum_{n=1}^{\infty} \frac{b_n^q - b_{n-1}^q}{b_n^r}$$

converges if $r > q > 0$ and diverges if $0 \leq r \leq q$.

Proof. Let $a_n = b_n^q$ and $p = r/q$. Then

$$\sum_{n=1}^{\infty} \frac{b_n^q - b_{n-1}^q}{b_n^r} = \sum_{n=1}^{\infty} \frac{a_n - a_{n-1}}{a_n^{r/q}} = \sum_{n=1}^{\infty} \frac{a_n - a_{n-1}}{a_n^p}$$

and the Corollary follows from Theorem 1,

Theorem 2 Suppose $\{a_n\}$ is an unbounded increasing sequence of positive real numbers such that $a_{n+1} - a_n \le a_n - a_{n-1}$ for $n = 1, 2, 3, \ldots$. Then

(i) the series $\sum_{n=1}^{\infty} \frac{a_n - a_{n-1}}{a_{n-1}^p}$ converges if $p > 1$ and diverges if $0 \le p \le 1$.

(ii)
$$\int_{a_r}^{a_{m+1}} \frac{1}{x^p}\,dx < \sum_{n=r}^{m} \frac{a_n - a_{n-1}}{a_n^p} < \int_{a_{r-1}}^{a_m} \frac{1}{x^p}\,dx$$

for $p > 0$ and for all positive integers r and m where $m \ge r$.

(iii)
$$\int_{a_r}^{\infty} \frac{1}{x^p}\,dx < \sum_{n=r}^{\infty} \frac{a_n - a_{n-1}}{a_n^p} < \int_{a_{r-1}}^{\infty} \frac{1}{x^p}\,dx$$

for $p > 1$ and for any positive integer r. In particular,

$$\int_{a_1}^{\infty} \frac{1}{x^p}\,dx < \sum_{n=1}^{\infty} \frac{a_n - a_{n-1}}{a_n^p} < \int_{a_0}^{\infty} \frac{1}{x^p}\,dx \qquad \text{for } p > 1.$$

(iv) if $S = \sum_{n=1}^{\infty} \frac{a_n - a_{n-1}}{a_n^p}$ and $S_r = \sum_{n=1}^{r} \frac{a_n - a_{n-1}}{a_n^p} + \int_{a_r}^{\infty} \frac{1}{x^p}\,dx$, then

$S \approx S_r$ with maximum error $E_r = S_r - S \le (a_{r+1} - a_r)/a_r^p$.

Proof. (i) This part follows from Theorem 1 and the two inequalities

$$\sum_{n=2}^{\infty} \frac{a_n - a_{n-1}}{a_{n-1}^p} \le \sum_{n=1}^{\infty} \frac{a_{n-1} - a_{n-2}}{a_{n-1}^p} \qquad \text{for } p > 1,$$

$$\sum_{n=1}^{\infty} \frac{a_n - a_{n-1}}{a_{n-1}^p} \ge \sum_{n=1}^{\infty} \frac{a_n - a_{n-1}}{a_n^p} \qquad \text{for } 0 \le p \le 1.$$

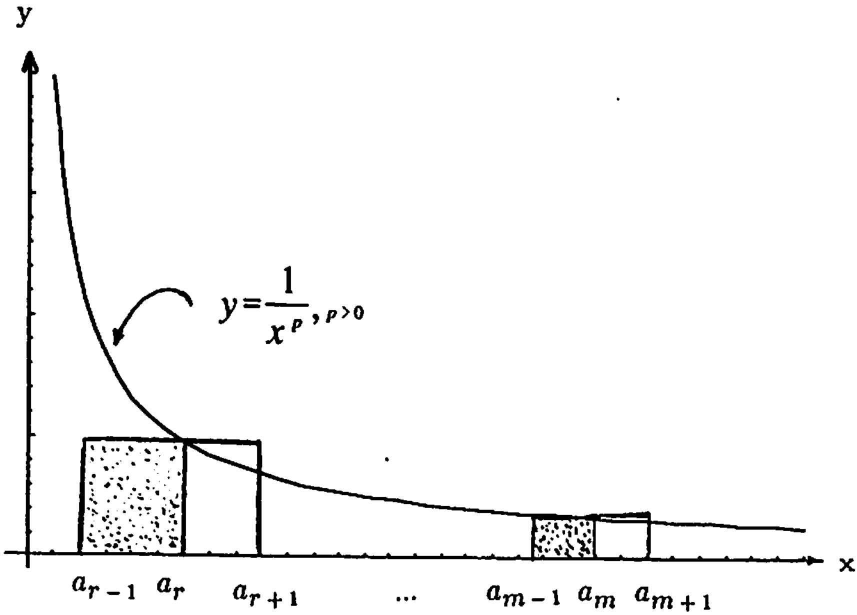

Figure 3.

(*ii*) Since $a_{n+1} - a_n \leq a_n - a_{n-1}$ for $n = 1, 2, 3, \ldots$, then for $p > 0$ and for all positive integers r and m where $m \geq r$, we have

$$\sum_{n=r}^{m} \frac{a_n - a_{n-1}}{a_n^p} \geq \sum_{n=r}^{m} \frac{a_{n+1} - a_n}{a_n^p} .$$

From Figure 3, it can easily be seen that

$$\int_{a_r}^{a_{m+1}} \frac{1}{x^p}\, dx < \sum_{n=r}^{m} \frac{a_{n+1} - a_n}{a_n^p} \quad \text{and} \quad \sum_{n=r}^{m} \frac{a_n - a_{n-1}}{a_n^p} < \int_{a_{r-1}}^{a_m} \frac{1}{x^p}\, dx$$

and the desired inequality follows from the last three inequalities.

(*iii*) This part follows from (*ii*) by taking the limit as $m \to \infty$.

(*iv*) From (*ii*) we have

$$\int_{a_{r+1}}^{\infty} \frac{1}{x^p}\, dx < \sum_{n=r+1}^{\infty} \frac{a_n - a_{n-1}}{a_n^p} < \int_{a_r}^{\infty} \frac{1}{x^p}\, dx$$

and, if we add

$$\sum_{n=1}^{r} \frac{a_n - a_{n-1}}{a_n^p}$$

to each side, we obtain

$$\sum_{n=1}^{r} \frac{a_n - a_{n-1}}{a_n^p} + \int_{a_{r+1}}^{\infty} \frac{1}{x^p}\, dx < S < S_r .$$

Since

$$\sum_{n=1}^{r} \frac{a_n - a_{n-1}}{a_n^p} + \int_{a_{r+1}}^{\infty} \frac{1}{x^p}\, dx = S_r - \int_{a_r}^{a_{r+1}} \frac{1}{x^p}\, dx ,$$

$$S_r - \int_{a_r}^{a_{r+1}} \frac{1}{x^p}\, dx < S < S_r .$$

Thus,

$$0 < S_r - S \le \int_{a_r}^{a_{r+1}} \frac{1}{x^p}\, dx .$$

But

$$\int_{a_r}^{a_{r+1}} \frac{1}{x^p}\, dx \le \frac{a_{r+1} - a_r}{a_r^p} ,$$

hence $0 < S_r - S \le (a_{r+1} - a_r)/a_r^p$ and this completes the proof.

Theorem 3 Let $f(x)$ be a positive, continuous and increasing function for $x \ge 0$. If $f(x-1)/f(x)$ is an increasing function for $x \ge 1$, then

$$\int_{1}^{\infty} \frac{f(x)^q - f(x-1)^q}{f(x)^r}\, dx$$

converges if $r > q > 0$ and diverges if $0 \le r \le q$.

Proof. Let

$$g(x) = \frac{f(x)^q - f(x-1)^q}{f(x)^r} \qquad \text{for } x \ge 1.$$

Then

$$g(x) = \frac{1}{f(x)^{r-q}} \left(1 - \left(\frac{f(x-1)}{f(x)} \right)^q \right).$$

Since $f(x)$ and $f(x-1)/f(x)$ are increasing, it is clear that $g(x)$ is a product of two decreasing positive functions, so it is decreasing. It is obvious that $g(x)$ is positive and continuous. If we let $b_n = f(n)$, then by

Corollary 1 the series $\sum(b_n^q - b_{n-1}^q)/b_n^r$ converges if $r > q > 0$ and diverges if $0 \le r \le q$. The theorem now follows by the integral test.

The following lemma identifies a class of increasing functions for which $f(x-1)/f(x)$ is also increasing.

Lemma 1 Let $f(x)$ be a positive, differentiable and increasing function for $x \ge 0$. If $f''(x) < 0$ (i.e., the graph of $f(x)$ is concave down), then $f(x-1)/f(x)$ is an increasing function.

Proof. Let $g(x) = f(x-1)/f(x)$. We show that $g'(x) > 0$. Since

$$g'(x) = \frac{f'(x-1)f(x) - f'(x)f(x-1)}{f(x)^2},$$

it is enough to show that $f'(x-1)f(x) - f'(x)f(x-1) > 0$. By assumption $f(x)$ is increasing, so $f(x) > f(x-1)$ and $f'(x-1) > 0$. Thus

$$\begin{aligned} f'(x-1)f(x) - f'(x)f(x-1) &> f'(x-1)f(x-1) - f'(x)f(x-1) \\ &= f(x-1)\cdot\left(f'(x-1) - f'(x)\right). \end{aligned}$$

But $f''(x) < 0$ implies that $f'(x)$ is decreasing, which implies that $f'(x-1) - f'(x) > 0$. Therefore $g'(x) > 0$.

Corollary 2 Let $f(x)$ be a positive, differentiable and increasing function for $x \ge 0$. If $f''(x) < 0$ (i.e., the graph of $f(x)$ is concave down), then the integral

$$\int_1^\infty \frac{f(x)^q - f(x-1)^q}{f(x)^r}\, dx$$

converges if $r > q > 0$ and diverges if $0 \le r \le q$.

Proof. By Lemma 2 the function $f(x-1)/f(x)$ is increasing and so the result follows from Theorem 3.

Now we give some theoretical and numerical applications of the theorems and corollaries we proved above.

Example 1 The first application of the generalized p-series is the well known p-series itself. Letting $a_n = n$ so $\sum 1/n^p = \sum(a_n - a_{n-1})/a_n^p$, we may use Theorem 1.

Example 2 The convergence or divergence of the following series can be determined by using the generalized p-series tests developed in this section. Other tests may also be used, but some may require more time and effort. (i) The series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{(n+1)^{p/2}}$$

converges for $p > 1$ and diverges for $0 \le p \le 1$. Let $a_n = \sqrt{n+1}$ and use Theorem 1. (ii) The series

$$\sum_{n=1}^{\infty} \frac{\ln\left(\frac{n+1}{n}\right)}{\left(\ln(n+1)\right)^p}$$

converges for $p > 1$, and diverges for $0 \le p \le 1$. Let $a_n = \ln(n+1)$ and use Theorem 1. (iii) The series

$$\sum_{n=1}^{\infty} \frac{\tan \dfrac{(n+1)\pi}{2(n+2)} - \tan \dfrac{n\pi}{2(n+1)}}{\left(\tan \dfrac{(n+1)\pi}{2(n+2)}\right)^p}$$

converges for $p > 1$ and diverges for $0 \le p \le 1$. Let $a_n = \tan\ ((n+1)\pi)/(2(n+2))$ and use Theorem 1. (iv) The series

$$\sum_{n=1}^{\infty} \frac{n^q - (n-1)^q}{n^r}$$

converges if $0 < q < r$ and diverges if $0 \le r \le q$. Let $b_n = n$ and use Corollary 1.

Example 3 The integral

$$\int_1^{\infty} \frac{x^q - (x-1)^q}{x^r}\, dx$$

converges if $0 < q < r$ and diverges if $0 \le r \le q$. Let $f(x) = x$ and use Theorem 3.

Example 4 In this example we will apply Theorem 2 to find some numerical estimates on two of the series of Example 2. (i) Let $a_n = \sqrt{n+1}$ and use Theorem 2 to get

$$1.07003 \le \sum_{n=1}^{15} \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}} \le 1.38629\ ,$$

$$0.46457 \le \sum_{n=1}^{15} \frac{\sqrt{n+1}-\sqrt{n}}{n+1} \le 0.75\ ,$$

$$0.7071 \le \sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n+1} \le 1$$

and

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n+1} \approx \sum_{n=1}^{62} \frac{\sqrt{n+1}-\sqrt{n}}{n+1} + \int_{\sqrt{63}}^{\infty} \frac{1}{x^2}\,dx$$

$$\approx 0.73437 + 0.12599 \approx 0.86036\ ,$$

with the maximum error less than 0.001. (*ii*) Let $a_n = \ln(n+2)$ and use Theorem 2 again to get

$$0.78054 \le \sum_{n=1}^{8} \frac{\ln\left(\frac{n+2}{n+1}\right)}{\ln(n+2)} \le 1.20055\ ,$$

$$0.49321 \le \sum_{n=1}^{8} \frac{\ln\left(\frac{n+2}{n+1}\right)}{\left(\ln(n+2)\right)^2} \le 1.0084\ ,$$

$$0.91024 \le \sum_{n=1}^{\infty} \frac{\ln\left(\frac{n+2}{n+1}\right)}{\left(\ln(n+2)\right)^2} \le 1.4427$$

and

$$\sum_{n=1}^{\infty} \frac{\ln\left(\frac{n+2}{n+1}\right)}{\left(\ln(n+2)\right)^2} \approx \sum_{n=1}^{58} \frac{\ln\left(\frac{n+2}{n+1}\right)}{\left(\ln(n+2)\right)^2} + \int_{\ln 60}^{\infty} \frac{1}{x^2}\,dx$$

$$\approx 0.93078 + 0.24424 \approx 1.17502\ ,$$

with the maximum error less than 0.001.

§2. The generalized p-series when $\{a_n\}$ is not increasing.

In this section we will consider the generalized p-series when $\{a_n\}$ is a non-increasing sequence of positive real numbers.

Definition A sequence $\{a_n\}$ of positive real numbers is called a sequence of uniformly bounded successive change if there exists a constant $M > 0$ such that $|a_n - a_{n-1}| \leq M$ for all $n = 1, 2, 3, \ldots$.

Theorem 4 Let $\{a_n\}$ be a sequence of positive real numbers. Then (i) If $\{a_n\}$ has a uniformly bounded successive change and satisfies the condition $|a_n - a_m| \geq S$ for some $S > 0$ and all $m \neq n$, then its associated generalized p-series $\sum |a_n - a_{n-1}|/a_n^p$ converges for $p > 1$. (ii) if $\lim_{n\to\infty} a_n = \infty$, then its associated generalized p-series $\sum |a_n - a_{n-1}|/a_n^p$ diverges for $p \leq 1$.

Before we prove this theorem, the following lemma is in order.

Lemma 2 Suppose $\{a_n\}$ is a sequence of positive real numbers. Then (i)if $\{a_n\}$ has a uniformly bounded successive change, then the series $\sum 1/a_n^p$ diverges for $p \leq 1$. (ii) if $|a_n - a_m| \geq S$ for some $S > 0$ and all $m \neq n$, then the series $\sum 1/a_n^p$ converges for $p > 1$.

Proof. (i) If $\{a_n\}$ has an accumulation point, say a, then $a \geq 0$ and there are infinitely many a_i's such that $a_i < a + 1$. Thus

$$\sum_{n=1}^{\infty} \frac{1}{a_n^p} \geq \sum_{n=1}^{\infty} \frac{1}{(a+1)^p} = \infty,$$

so the series is divergent. If $\{a_n\}$ has no accumulation points, then define the sequence $\{b_n\}$ inductively as $b_1 = \min\{a_k : k = 1, 2, 3, \ldots\}$ and $b_n = \min\left\{\{a_k : k = 1, 2, 3, \ldots\} - \{b_k : k = 1, 2, 3, \ldots, n-1\}\right\}$ for $n \geq 2$. Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{a_n^p} \geq \sum_{n=1}^{\infty} \frac{1}{b_n^p}.$$

To complete the proof, we will show that $\sum 1/b_n^p$ diverges. Since $\{a_n\}$ is a sequence of uniformly bounded successive change, there exists a constant $M > 0$ such that $|a_n - a_{n-1}| \leq M$ for all n. We will show that $\{b_n\}$ is also of uniformly bounded successive change. In fact, $b_n - b_{n-1} \leq M$ for all n. To see this, suppose $b_m - b_{m-1} > M$ for some m. Let $A = \{a_i : a_i \leq b_{m-1}\}$. Since $\{a_n\}$ has no accumulation points, A is finite. Therefore there is at least one $a_i \in A$ such that $a_{i+1} \notin A$. From the definition of $\{b_n\}$ it follows that $a_{i+1} \geq b_m$. Thus we have $a_{i+1} - a_i \geq b_m - b_{m-1} > M$, which is a contradiction. Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{b_n^p} = \sum_{n=1}^{\infty} \frac{1}{b_n - b_{n-1}} \cdot \frac{b_n - b_{n-1}}{b_n^p} \geq \frac{1}{M} \sum_{n=1}^{\infty} \frac{b_n - b_{n-1}}{b_n^p}.$$

By Theorem 1, $\sum (b_n - b_{n-1})/b_n^p$ diverges. Hence, $\sum 1/b_n^p$ diverges.

(*ii*) Let $\{b_n\}$ be as defined in part (*i*) above. Since $|a_n - a_m| \geq S$, $b_n - b_{n-1} \geq S$. Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{a_n^p} = \sum_{n=1}^{\infty} \frac{1}{b_n^p} = \sum_{n=1}^{\infty} \frac{1}{b_n - b_{n-1}} \cdot \frac{b_n - b_{n-1}}{b_n^p} \leq \frac{1}{S} \sum_{n=1}^{\infty} \frac{b_n - b_{n-1}}{b_n^p}.$$

By Theorem 1, $\sum (b_n - b_{n-1})/b_n^p$ converges. Hence, $\sum 1/b_n^p$ converges.

Remark If $\{a_n\}$ is not a sequence of uniformly bounded successive change, then Lemma 2(*i*) is not true. To see this let $a_n = n^2$. So $a_n - a_{n-1} = 2n - 1 \to \infty$ as $n \to \infty$, and $\sum 1/a_n = \sum 1/n^2 < \infty$. Also, the condition $|a_n - a_m| \geq S$ for some $S > 0$ and all $m \neq n$ can not be dropped from Lemma 2(*i*) even if $\lim_{n\to\infty} a_n = \infty$. For this, let $\{a_n\}$ be the sequence:

$$\begin{gathered} 2,\ 2+\tfrac{1}{2}, \\ 3, 3+\tfrac{1}{3}, 3+\tfrac{2}{3}, \\ 4, 4+\tfrac{1}{4}, 4+\tfrac{2}{4}, 4+\tfrac{3}{4}, \\ \ldots, \\ n, n+\tfrac{1}{n}, n+\tfrac{2}{n}, n+\tfrac{3}{n}, \ldots, n+\tfrac{n-1}{n}, \\ n+1, \ldots \end{gathered}$$

Then $\sum 1/a_n^2 \geq \sum n/(n+1)^2 = \infty$.

Proof of Theorem 4. (*i*) Since $\{a_n\}$ is a sequence of uniformly bounded successive change, there exists a constant $M > 0$ such that $|a_n - a_{n-1}| \leq M$ for all $n = 1, 2, 3, \ldots$. Therefore,

$$\sum_{n=1}^{\infty} \frac{|a_n - a_{n-1}|}{a_n^p} \leq M \sum_{n=1}^{\infty} \frac{1}{a_n^p}.$$

Since $|a_n - a_m| \geq S$, Lemma 2(*ii*) implies the convergence of $\sum 1/a_n^p$ and therefore $\sum |a_n - a_{n-1}|/a_n^p$ converges.

(*ii*) The proof of this part is a slight modification of the proof of the second part of Theorem 1. Let m be a positive integer. For $k \geq m$, let $A_k = \{a_m, \ldots, a_k\}$, $m_k = \min A_k$ and $M_k = \max A_k$. It is not hard to see that then

$$\begin{aligned} \sum_{n=m}^{k} \frac{|a_n - a_{n-1}|}{a_n^p} &\geq \frac{1}{M_k^p} \sum_{n=m}^{k} |a_n - a_{n-1}| \\ &\geq \frac{1}{M_k^p}(M_k - m_k) = M_k^{1-p} - \frac{m_k}{M_k^p}, \end{aligned}$$

and, since $\lim_{k\to\infty} M_k = \lim_{k\to\infty} a_k = \infty$, it follows that

$$\sum_{n=m}^{\infty} \frac{|a_n - a_{n-1}|}{a_n^p} \geq \lim_{k\to\infty}\left(M_k^{1-p} - \frac{m_k}{M_k^p}\right) = \begin{cases} 1 & \text{if } p = 1 \\ \infty & \text{if } p < 1 \end{cases}.$$

Since this limit holds for any m, the series $\sum |a_n - a_{n-1}|/a_n^p$ diverges by the Cauchy criterion.

Remarks (i) The condition $|a_n - a_m| \geq S$ for some $S > 0$ and all $m \neq n$ can not be dropped from Theorem4(i). Let $\{a_n\}$ be the sequence:

$$\begin{array}{cl} 3, 2, & (3,2 \text{ is repeated 1 time}) \\ 5,4,5,4, & (5,4 \text{ is repeated 2 times}) \\ 7,6,7,6,7,6, & (7,6 \text{ is repeated 3 times}) \\ \ldots, & \\ 2n+1, 2n, 2n+1, 2n, \ldots, 2n+1, 2n, & (2n+1, 2n \text{ is repeated } n \text{ times}) \\ \ldots & \end{array}$$

From the pattern of repetition in the terms of the sequence $\{a_n\}$, we observe that

$$\sum_{n=1}^{\infty} \frac{|a_n - a_{n-1}|}{a_n^2} > \sum_{n=1}^{\infty} \frac{n}{(2n)^2} = \sum_{n=1}^{\infty} \frac{1}{4n}$$

and $\sum 1/4n$ diverges. Notice that this sequence can be modified by adding (subtracting) small numbers to (from) the repeated terms to obtain a sequence with distinct terms which has a divergent associated p-series.

(ii) If the sequence $\{a_n\}$ is not a sequence of uniformly bounded successive change, then Theorem 4(i) is not true. Let $\{a_n\}$ be the sequence

$$a_n = \begin{cases} e^n & \text{if } n \text{ is even} \\ n & \text{if } n \text{ is odd} \end{cases}.$$

Then $\{a_n\}$ is not a sequence of uniformly bounded successive change and

$$\sum_{n=2}^{\infty} \frac{|a_n - a_{n-1}|}{a_n^2} > \sum_{n \text{ odd}, n \geq 3}^{\infty} \frac{|a_n - a_{n-1}|}{a_n^2}$$

$$= \sum_{n \text{ odd}, n \geq 3}^{\infty} \frac{e^{n-1} - n}{n^2} = \infty.$$

(iii) If $\lim_{n\to\infty} a_n \neq \infty$, then Theorem 4($ii$) is not true. Let $\{a_n\}$ be the sequence $a_1 = 1$, and for $n \geq 2$,

$$a_n = \begin{cases} 2-\dfrac{1}{n^2} & \text{if } n \text{ is even} \\ 2+\dfrac{1}{n^2} & \text{if } n \text{ is odd} \end{cases}.$$

Then

$$\sum_{n=2}^{\infty}\frac{|a_n-a_{n-1}|}{a_n} = \sum_{n=1}^{\infty}\frac{|a_{2n}-a_{2n-1}|}{a_{2n}} + \sum_{n=1}^{\infty}\frac{|a_{2n+1}-a_{2n}|}{a_{2n+1}}$$

$$= \sum_{n=1}^{\infty}\frac{\dfrac{1}{(2n)^2}+\dfrac{1}{(2n-1)^2}}{2-\dfrac{1}{(2n)^2}} + \sum_{n=1}^{\infty}\frac{\dfrac{1}{(2n+1)^2}+\dfrac{1}{(2n)^2}}{2+\dfrac{1}{(2n+1)^2}}$$

$$\leq \sum_{n=1}^{\infty}\frac{4}{(2n-1)^2} < \infty.$$

Corollary 3 Let $\{b_n\}$ be a sequence of positive real numbers, and let q and r be positive real numbers. Then the series

$$\sum_{n=1}^{\infty}\frac{|b_n^q-b_{n-1}^q|}{b_n^r}$$

(i) converges if $\{b_n\}$ satisfies the conditions of Theorem 4(i) and $r>q>0$ and (ii) diverges if $\{b_n\}$ satisfies the conditions of Theorem 4(ii) and $0<r\leq q$.

Proof. Let $a_n = b_n^q$ and $p = r/q$. Then

$$\sum_{n=1}^{\infty}\frac{|b_n^q-b_{n-1}^q|}{b_n^r} = \sum_{n=1}^{\infty}\frac{|a_n-a_{n-1}|}{a_n^{r/q}} = \sum_{n=1}^{\infty}\frac{|a_n-a_{n-1}|}{a_n^p}$$

and the Corollary follows from Theorem 4.

Corollary 4 Let $\{b_n\}$ be a sequence of positive real numbers and let q and r be positive real numbers. Then the series

$$\sum_{n=1}^{\infty}\frac{|b_n-b_{n-1}|^q}{b_n^r}$$

(i) converges if $\{b_n\}$ satisfies the conditions of Theorem 4(i) and $r>q\geq 1$ and (ii) diverges if $\{b_n\}$ satisfies the conditions of Theorem 4(ii) and $r\leq q\leq 1$.

Before we prove this corollary, the following lemma is needed.

Lemma 3 For any real numbers a and b with $a \geq b > 0$, (i) if $p \geq 1$, then $(a-b)^p \leq a^p - b^p$ and (ii) if $p \leq 1$, then $(a-b)^p \geq a^p - b^p$.

Proof. The case $p = 1$ is trivial. For $p \neq 1$, let $f(x) = (x-b)^p - x^p + b^p$. It is easy to see that

$$f'(x) = p(x-b)^{p-1}\left(1 - \left(\frac{x}{x-b}\right)^{p-1}\right)$$

So if $p > 1$, then $f'(x) < 0$ for all $x > b$ and if $p < 1$, then $f'(x) > 0$ for all $x > b$.

Proof of Corollary 4. (i) Suppose $q \geq 1$. Then by Lemma 3, $|b_n - b_{n-1}|^q \leq |b_n^q - b_{n-1}^q|$. Therefore,

$$\sum_{n=1}^{\infty} \frac{|b_n - b_{n-1}|^q}{b_n^r} \leq \sum_{n=1}^{\infty} \frac{|b_n^q - b_{n-1}^q|}{b_n^r}.$$

By Corollary 3(i), the series $\sum |b_n^q - b_{n-1}^q|/b_n^r$ converges, and so $\sum |b_n - b_{n-1}|^q/b_n^r$ converges.

(ii) Suppose $q \leq 1$. Then by Lemma 3, $|b_n - b_{n-1}|^q \geq |b_n^q - b_{n-1}^q|$. Therefore,

$$\sum_{n=1}^{\infty} \frac{|b_n - b_{n-1}|^q}{b_n^r} \geq \sum_{n=1}^{\infty} \frac{|b_n^q - b_{n-1}^q|}{b_n^r}.$$

By Corollary 3(ii), the series $\sum |b_n^q - b_{n-1}^q|/b_n^r$ diverges, and so $\sum |b_n - b_{n-1}|^q/b_n^r$ diverges.

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1. Bartle, R. G. and Sherbert, D. R., *Introduction to Real Analysis, 2nd ed.* John Wiley & Sons Inc., 1992.
2. Fraleigh, J. B., *Calculus with Analytic Geometry, 3rd ed.* Addison-Wesley, 1990.

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1995. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1995 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROBLEMS 472 (corrected) and 480-484.

Problem 472 (corrected). Proposed by Del Ebadi, Topeka West High School, Topeka, Kansas. Given that the absolute value of the average of two real numbers is $4\sqrt{6}$ and that the geometric mean of the two numbers is $6\sqrt{2}$, find the absolute value of the difference of the two numbers without finding the numbers themselves.

Problem 480. Proposed jointly by Sammy and Jimmy Yu, special students at the University of South Dakota, Vermillion, South Dakota. Let (a, b, c) be a Pythagorean triple where $c^2 = a^2 + b^2$ and T_n be the nth triangular number. Solve the Pythagorean equation

$$(pT_n + c)^2 = (pT_n + a)^2 + (qb)^2$$

for positive integers p and q in terms of a, b, c and n.

Problem 481. Proposed jointly by Sammy and Jimmy Yu, special students at the University of South Dakota, Vermillion, South Dakota.

Evaluate the integral

$$I = \int \frac{m - x^n}{x^{1+n/2}}\, dx$$

Problem 482. Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Evaluate the integral

$$I = \int \ln(\sin x)\, dx$$

Problem 483. Proposed by the Editor. Let $\{a_i\}$ for $i = 1, 2, \ldots, k$ be a sequence of positive integers such that $a_1 + a_2 + \cdots + a_k = 200$. What values must be chosen for the a_i in order to maximize the product of the a_i?

Problem 484. Proposed by the Editor. In the decimal system find all 10 digit palindromic numbers which are the product of two consecutive integers. A palindromic number has the same value regardless of whether it is read from right to left or vice versa.

Please help your editor by submitting problem proposals.

SOLUTIONS 460, 464, 470, 471, 473 and 474.

Problem 460. Proposed by the Editor. The natural numbers 281 and 1926 have the property that

$$1926^2 + 5 \equiv 0 \pmod{281}$$

and

$$281^2 + 5 \equiv 0 \pmod{1926}.$$

Prove that there are an infinite number of pairs of natural numbers with this property and find an infinite family of solutions.

Solution by Lamarr Widmer, Messiah College, Grantham, Pennsylvania.

For each positive integer k, we define

$$a_k = \left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)^{k-1} + \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^{k-1}.$$

Then we have $a_1 = 2$, $a_2 = 3$ and $a_k = 3a_{k-1} - a_{k-2}$ from which we can conclude, by mathematical induction, that $\{a_k\}$ is a sequence of positive integers. Note that

$$\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)^{-1} = \left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right).$$

We can easily verify that for all $k \geq 2$, $a_k^2 + 5 = a_{k-1}a_{k+1}$ and so both a_{k-1} and a_{k+1} are divisors $a_k^2 + 5$. Thus for any positive integer n, we have

$$a_n^2 + 5 \equiv 0 \pmod{a_{n+1}}$$

and

$$a_{n+1}^2 + 5 \equiv 0 \pmod{a_n}$$

so that our sequence $\{a_k\}$ furnishes the desired infinite family of solutions. The first ten terms of this sequence are 2, 3, 7, 18, 47, 123, 322, 843, 2207 and 5578.

Another infinite family of solutions is furnished by consecutive terms of the sequence $\{b_k\}$ where

$$b_k = \left(\frac{1}{2}-\frac{\sqrt{5}}{6}\right)\left(\frac{7}{2}+\frac{3\sqrt{5}}{2}\right)^{k-1} + \left(\frac{1}{2}-\frac{\sqrt{5}}{6}\right)\left(\frac{7}{2}-\frac{3\sqrt{5}}{2}\right)^{k-1}.$$

This sequence satisfies the recurrence relation $b_k = 7b_{k-1} - b_{k-2}$ and its first ten terms are 1, 1, 6, 41, 282, 1926, 13201, 90481, 620166, and 4250681. This family generates the solution given in the statement of the problem.

A third infinite family of solutions is furnished by consecutive terms of the sequence $\{c_k\}$ where

$$c_k = \left(\frac{3}{2}-\frac{13\sqrt{21}}{42}\right)\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)^{k-1} + \left(\frac{3}{2}-\frac{13\sqrt{21}}{42}\right)\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)^{k-1}.$$

This sequence satisfies the recurrence relation $c_k = 5c_{k-1} - c_{k-2}$ and its first ten terms are 1, 2, 9, 43, 206, 987, 4729, 22658, 108561, and 520147.

A fourth infinite family of solutions is furnished by consecutive terms of the sequence $\{d_k\}$ where

$$d_k = \left(1-\frac{4\sqrt{21}}{21}\right)\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)^{k-1} + \left(1-\frac{4\sqrt{21}}{21}\right)\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)^{k-1}.$$

This sequence satisfies the recurrence relation $d_k = 5d_{k-1} - d_{k-2}$ and its first ten terms are 1, 3, 14, 67, 321, 1538, 7369, 35307, 169166, and 810523.

Solution by the proposer.

Since $1926^2+5 = 281 \cdot 13201$ and $281^2+5 = 1926 \cdot 41$, the numbers 41, 281, 1926 and 13201 form a sequence of four numbers containing three consecutive pairs of integers which satisfy the relation

$$x_{n+2}x_n = x_{n+1}^2 + 5 . \tag{1}$$

Without loss of generality, we can assume that $x_n < x_{n+1}$. Then $x_{n+2} = x_{n+1} + 5/x_n > x_n + 5/x_n$ whenever $x_{n+1} > 2$. By replacing n by $n+1$ in (1) we have

$$x_{n+3}x_{n+1} = x_{n+2}^2 + 5 . \tag{2}$$

Subtracting (1) from (2) and combining we obtain

$$x_{n+3}x_{n+1} - x_{n+2}x_n = (x_{n+2} + x_{n+1})(x_{n+2} - x_{n+1}) \tag{3}$$

Hence
$$x_{n+1}(x_{n+3} + x_{n+1}) = x_{n+2}(x_{n+2} + x_n) . \tag{4}$$

Suppose that $(x_{n+1}, x_n) = d$. Then by (1) either $d = 1$ or $d = 5$. Now $d = 5$ implies that 5 divides x_{n+2}. But then 25 does not divide both sides of (1). Hence $d = 1$. Since $x_n < x_{n+1}$ we also have $x_{n+1} < x_{n+2}$. Thus the $\{x_n\}$ form an infinite sequence for $n = 0, 1, 2, \ldots$.

Now by (4) and $(x_{n+2}, x_{n+1}) = 1$, x_{n+1} divides $x_{n+2} + x_n$ or

$$x_{n+2} \equiv -x_n \pmod{x_{n+1}} . \tag{5}$$

Relation (5) allows one to generate easily an infinite sequence of x_n which satisfy the conditions of the problem. It also allows one to "descend" to find generators of sequences satisfying the conditions of the problem:

(1, 1)	leads to	1, 1, 6, 41, 281, 1926, 13201, ...
(1, 2)	leads to	1, 2, 9, 43, 206, ...
(1, 3)	leads to	1, 3, 14, 67, 321, ...
(1, 6)	leads to	1, 6, 41, 281, ...
(2, 1)	leads to	2, 1, 3, 14, 67, 321, ...
(3, 1)	leads to	3, 1, 2, 9, 43, ...
(2, 3)	leads to	2, 3, 7, 18, 47, 123, ...
(3, 2)	leads to	3, 2, 3, 7, 18, ...
(6, 1)	leads to	6, 1, 1, 6, 41, 281, ...

Thus it appears that there are four basic sequences of integers which satisfy relation (1).

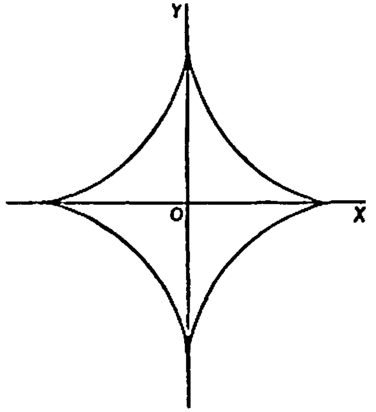

Figure 1.

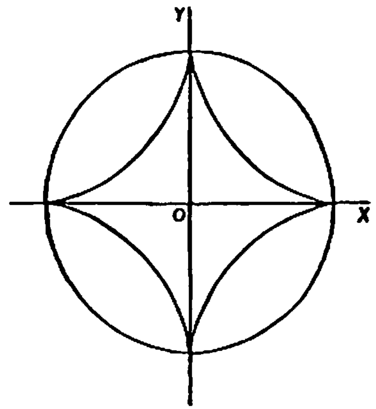

Figure 2.

Problem 464. Proposed by Mary Elick, Missouri Southern State College, Joplin, Missouri. Let C denote the curve given by the equation

$$x^{2/3} + y^{2/3} = 1$$

as shown in Figure 1. Suppose that the curve C is made from a flexible material which is attached to the coordinate axes at the points (0,1), (1,0), (0,-1) and (-1,0) and which may be moved without changing the length of C. Let C_p denote the "companion curve" for C which is formed by inflating C with air until it "pops outward" as shown in Figure 2. (a) Find the equation of the companion curve C_p. (b) Find the derivative, if it exists, at the point (0,1) on the curve C_p. (c) Given that the curve C is the circle described by the equation $x^2 + y^2 = 1$, what is the equation of the "popped in" companion curve C_p resulting from "deflating " curve C appropriately?

Solution by Patrick J. Costello, Eastern Kentucky University, Richmond, Kentucky.

The idea of "popping out" and "popping in" is reflecting the curve across the line segments connecting the points $(0,1)$, $(1,0)$, $(0,-1)$ and $(-1,0)$. Since both of the curves mentioned in the problem are symmetric across the line $y = x$, this reflection can be accomplished pointwise by reflecting points through the midpoint of the line segments. For example, in the first quadrant each point is reflected through the point $(1/2, 1/2)$; i.e. the point (x, y) goes into the point $(1 - x, 1 - y)$ as shown in Figure 3.

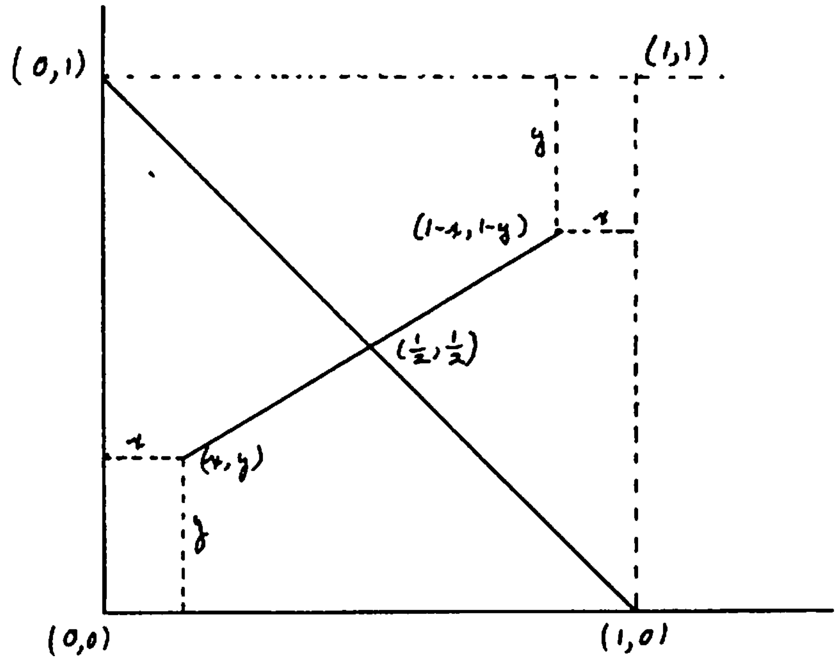

Figure 3.

With this kind of pointwise reflection, the part of the curve C in the first quadrant which is below the line $y = x$ is reflected to the "popped out" part of C_p which is above the line $y = x$ and vice versa. However, the symmetry of the curve C across the line $y = x$ .means the two separate reflected parts form the desired curve C_p. In the second quadrant, you reflect a point through the point (-1/2, 1/2). Here the point (x, y) goes into the point $(-1 - x, 1 - y)$. Proceed similarly for the third and fourth quadrants.

(a) The companion curve C_p of $x^{2/3} + y^{2/3} = 1$ is given by the following four pieces:

$$(1-x)^{2/3} + (1-y)^{2/3} = 1 \text{ when } 0 \leq x \leq 1,\ 0 \leq y \leq 1$$
$$(-1-x)^{2/3} + (1-y)^{2/3} = 1 \text{ when } -1 \leq x \leq 0,\ 0 \leq y \leq 1$$
$$(1-x)^{2/3} + (-1-y)^{2/3} = 1 \text{ when } 0 \leq x \leq 1,\ -1 \leq y \leq 0$$
$$(-1-x)^{2/3} + (-1-y)^{2/3} = 1 \text{ when } -1 \leq x \leq 0,\ -1 \leq y \leq 0$$

Note that the latter two curves simply are the reflections of the first two across the x-axis. In fact, the four pieces are quarters of the original curve centered at the points (1, 1), (-1, 1), (1,-1) and (-1,-1).

(b) Using implicit differentiation, the derivative of the first piece above at the point (x, y) is given by

$$y' = -\left(\frac{1-y}{1-x}\right)^{1/3}$$

Evaluating this expression at the point $(0,1)$ yields a value of 0 for the derivative. This is valid since the expression for y' is valid for any $x = 1$. Also, a similar check of the corresponding portion of the curve in the second quadrant yields the same answer at the point $(0,1)$ so at the point $(0,1)$, $y' = 0$.

The transformations $x' = 1 - x$ and $x' = -1 - x$ invert to identical transformations $x' = 1 - x$ and $x' = -1 - x$. Hence the "deflating" operation uses the same substitutions in the same quadrants as the "inflating" operation.

(c) The companion curve C_p of $x^2 + y^2 = 1$ is given by the following four pieces:

$$\begin{array}{l} (1-x)^2 + (1-y)^2 = 1 \text{ when } 0 \le x \le 1,\ 0 \le y \le 1 \\ (-1-x)^2 + (1-y)^2 = 1 \text{ when } -1 \le x \le 0,\ 0 \le y \le 1 \\ (1-x)^2 + (-1-y)^2 = 1 \text{ when } 0 \le x \le 1,\ -1 \le y \le 0 \\ (-1-x)^2 + (-1-y)^2 = 1 \text{ when } -1 \le x \le 0,\ -1 \le y \le 0 \end{array}$$

Note that these are simply the quarters of the unit circle centered at the points $(1,1)$, $(-1,1)$, $(1,-1)$ and $(-1,-1)$.

Also solved by Huashan Chen, Northwest Missouri State University, Maryville, Missouri (parts a and c); Russell Euler, Northwest Missouri State University, Maryville, Missouri; and the proposer.

Editor's Comment. Russell Euler notes that the equations defining C_p in part (a) can be combined into the form $(|x| - 1)^{2/3} + (|y| - 1)^{2/3} = 1$. Similarly the equations defining C_p in part (c) can be combined into the form $(|x| - 1)^2 + (|y| - 1)^2 = 1$.

Problem 470. Proposed by J. Sriskandarajah, University of Wisconsin Center-Richland, Richland Center, Wisconsin. Find all possible integers a and b such that $a^b + b^a = 100$.

Solution by Jimmy and Sammy Yu, jointly, special students, University of South Dakota, Vermillion, South Dakota.

Without loss of generality, we can assume that $a \leq b$. Then from the statement of the problem, $1 \leq a \leq b \leq 99$ so that $a^a \leq a^b \leq b^b$ and $a^a \leq b^a \leq b^b$. Adding these two inequalities, we have $2a^a \leq a^b + b^a \leq 2b^b$ or $a^a \leq 50 \leq b^b$. Now since a and b are positive integers and $3^3 \leq 50 \leq 4^4$, we must have $1 \leq a \leq 3$ and $4 \leq b \leq 99$. Also the given equation implies that a and b have the same parity.

If $a = 1$, then $b = 99$. If $a = 2$, then let $b = 2m$. Then $2^{2m} + (2m)^2 = 100$ or $2^{2m-2} + m^2 = 25$. Hence m is odd, $m < 5$ and $m \geq 2$ since $b \geq 4$. Thus $m = 3$ and $b = 6$. If $a = 3$, then b is odd. Since $4 \leq b \leq 99$, $b \geq 5$. Now $b^3 = 100 - 3^b \leq 100 - 3^5 = -143$ establishes that there is no solution when $a = 3$. Hence the only solutions to the given equation in integers a and b are $(a,b) = (1,99)$, $(2,6)$, $(6,2)$ and $(99,1)$.

Also solved by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; Brain Tobben, Southwest Missouri State, Springfield, Missouri; and the proposer.

Problem 471. Proposed by J. Sriskandarajah, University of Wisconsin Center-Richland, Richland Center, Wisconsin. Find all positive integers a, b and c such that $a^3 + b^3 + c^3 = abc$ where abc denotes a three digit number formed by concatenating a, b and c in the order shown and not the product abc.

Composite of solutions by Charles Ashbacher, Cedar Rapids, Iowa; Christopher Brenner, Shippensburg University, Shippensburg, Missouri; Ryan Faith, Rosary College, River Forest, Illinois; John P. Hughes, Frostburg State University, Frostburg, Maryland; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; Jimmy and Sammy Yu, special students, University of South Dakota, Vermillion, South Dakota; and the proposer.

Each solver submitted a computer program which led to the following four solutions: $1^3 + 5^3 + 3^3 = 153$, $3^3 + 7^3 + 0^3 = 370$, $3^3 + 7^3 + 1^3 = 371$ and $4^3 + 0^3 + 7^3 = 407$. Then, since only positive integers may be used, the only solutions are $1^3 + 5^3 + 3^3 = 153$ and $3^3 + 7^3 + 1^3 = 371$.

Bob Prielipp noted that this problem is related to Problem E1810 in *The American Mathematical Monthly* 75 (March 1968), page 264 which shows that for each positive integer n, if $C(n)$ is the sum of the cubes of the decimal digits of n, then iterating C eventually leads to one of the following repeating cycles: 1, 55-250-133, 136-244, 153, 160-217-352, 370, 371, 407 and 919-1459.

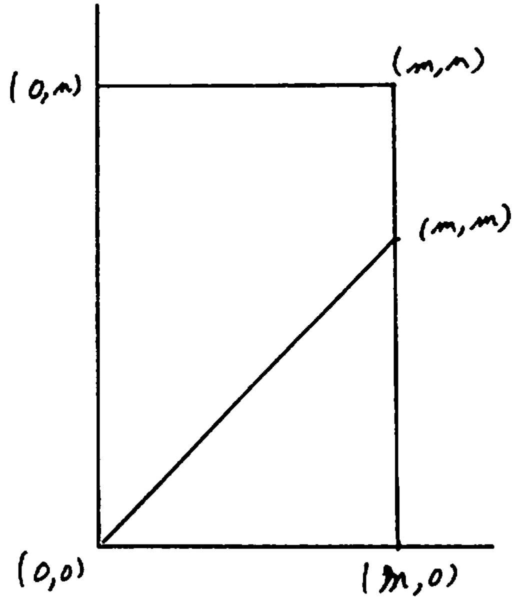

Figure 4.

Problem 473. Proposed by Del Ebadi, Topeka West High School, Topeka, Kansas. Two numbers a and b are selected randomly; a is chosen from the interval $[0,2]$ and b is chosen from the interval $[0,3]$. Find the probability that $a > b$. What is the probability if 2 and 3 are replaced by m and n, respectively?

Solution by Charles Ashbacher, Cedar Rapids, Iowa.

Consider Figure 4 (above) in which the rectangle has its lower left corner at the origin and the other corners at the points indicated. Let m and n be real numbers such that $m < n$. Draw the line connecting the origin with the point (m,m).

The rectangular region has area mn corresponding to all possible outcomes. The set of all possible outcomes in which $m > n$ is the set of points lying in the region below the line formed by the points $(0,0)$ and (m,m). The area of this region is $m^2/2$. Thus the required probability is $(m^2/2)/(mn) = m/2n$. For the values $m = 2$ and $n = 3$, the probability becomes $1/3$.

Also solved by Jimmy and Sammy Yu, jointly, special students, University of South Dakota, Vermillion, South Dakota; and Russell Euler, Northwest Missouri State University, Maryville, Missouri.

Problem 474. Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Evaluate

$$\mathrm{I} = \int \frac{1}{1-\sin(x)}\,dx$$

by converting the integrand to the form $\mathrm{D}_x h(x)$.

Solution by Russell Euler, Northwest Missouri State University, Maryville, Missouri.

$$\frac{1}{1-\sin(x)} = \frac{1}{\sin^2(x/2)+\cos^2(x/2)-2\sin(x/2)\cos(x/2)}$$

$$= \frac{1}{\left(\sin(x/2)-\cos(x/2)\right)^2} = \frac{1}{\cos^2(x/2)(\tan(x/2)-1)^2}$$

$$= \frac{\sec^2(x/2)}{(\tan(x/2)-1)^2} = \frac{d}{dx}\left(\frac{-2}{\tan(x/2)-1}\right),$$

so that $\mathrm{I} = -2/(\tan(x/2)-1)+C$ for any constant C.

Also solved by J. Sriskandarajah, University of Wisconsin-Richland Center, Richland Center, Wisconsin; Jimmy and Sammy Yu, jointly, special students, University of South Dakota, Vermillion, South Dakota; and the proposer.

Editor's Comment. Other answers which are equivalent to the given solution found by the solvers include $\cos(x)/(1+\sin(x))+C$, $(1+\sin(x))/\cos(x)+C$ and $\tan(x)+\sec(x)+C$.

Kappa Mu Epsilon News

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy *KME* events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, Missouri 64801.

CHAPTER NEWS

AL Beta

University of North Alabama, Florence — 26 actives

Student officers for 1994-95 have not yet been determined. Eddy Brackin is corresponding secretary; Pat Roden is faculty sponsor.

AL Gamma

Chapter President - Bryan Garreit

University of Montevallo, Montevallo — 15 actives, 3 associates

Chapter activities included initiation ceremonies, a retirement party for Professors Buttle and Cardine, and a presentation by Dr. Kwin concerning his research. Other 1994-95 chapter officers: Jamie Tauie, vice president; Melissa Ellison, secretary; Aleksis Langerwarf, treasurer; Larry Kurtz, corresponding secretary; Don Alexander, faculty sponsor.

AR Alpha

Arkansas State University, State University — 10 actives, 5 associates

Officers for 1994-95 have not yet been elected. William Paulsen is corresponding secretary/faculty sponsor.

CA Delta

Chapter President - Gillian Robbins

California State Polytechnic University, Pomona — 25 actives, 5 associates

CA Delta held bake sales to raise money. Members also sold t-shirts bearing the *KME* logo. The chapter sponsored pizza parties each quarter and a picnic in May for Math Department faculty, staff, and students. Plans were also made for a summer beach party and a trip to Disneyland. Other officers for 1994-95: Deborah Garcia, vice president; Jennifer Baird, secretary; Mirela Ciocan, treasurer; Richard Robertson, corresponding secretary; Jim McKinney, faculty sponsor.

CO Gamma Chapter President - Ben Moore

Fort Lewis College, Durango 27 actives

CO Gamma held two meetings and an initiation ceremony. Four chapter members attended the Regional Convention at the University of New Mexico; chapter president, Susan Clinkenbeard, took first prize in the undergraduate category. Plans to host the 1995 Biennial Convention are also underway. Members are looking forward to welcoming everyone to Durango in April of 1995. Other 1994-95 officers: Jody Davis, vice president; Faith Ward, secretary; Noreen Frost, treasurer; Richard A. Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

CO Delta Chapter President - Scott B. Davis

Mesa State College, Grand Junction 9 associates

The CO Delta Chapter held its Initiation Banquet on April 29, 1994, with 46 members, initiates, spouses, and guests in attendance. Other officers for 1994-95: Joy E. Rayside, vice president; April R. Galyean, secretary; William J. Haworth, treasurer; Donna Hafner, corresponding secretary; Clifford C. Britton, faculty sponsor.

CT Beta Chapter President - Noreen Gorka

Eastern Connecticut State University, Willimantic 15 actives, 9 associates

Other officers for 1994-95: Sharon Ficarra, vice president; Jason Brong, secretary/treasurer; Stephen Kenton, corresponding secretary/ faculty sponsor.

GA Alpha Chapter President - Polly Quertermus

West Georgia College, Carrollton 25 actives, 7 associates

The Georgia Alpha Chapter met on June 2, 1994, to initiate seven new members and elect officers for 1994-95. At the reception which followed, it was announced that four *KME* members had won academic scholarships for next year. The recipients and their awards are: Chad Bean, Crider Award; Helga Floodquist, Whatley Scholarship; Chris Flournoy, Boykin Scholarship and Crider Award; and Kristie Hannah, Cooley Scholarship. Other officers for 1994-95: Chad Bean, vice president; Chris Flournoy, secretary; Kristie Hannah, treasurer; Thomas J. Sharp, corresponding secretary/faculty sponsor.

IL Beta Chapter President - Steve Trepachko

Eastern Illinois University, Charleston 42 actives, 15 associates

February activities included a Bowling Night and a presentation by Dr. Broline on Maple. In March, members attended the ICTM Conference held in University Union and heard a talk by Dr. Coulton. Another March event was the ICTM Math Contest. *KME* Initiation and the Honors Banquet were held in April. A spring picnic closed out a successful semester. Other officers for 1994-95: Brittany Zupan, vice president; Jenny Wilhelmsen, secretary; Curtis Price, treasurer; Lloyd L. Kontz, corresponding secretary/faculty sponsor; Paul Tougaw, faculty sponsor.

IL Delta Chapter President - Anthony Mravle
College of St. Francis, Joliet 18 actives, 11 associates

A taffy apple sale helped raise money for members to attend the Regional Convention at Central Michigan University in April. Other officers for 1994-95: Leah Boeckmann, vice president; Janice Hinkleman, secretary; Phillip Graf, treasurer; Sister Virginia McGee, corresponding secretary/faculty sponsor.

IL Eta Chapter President - Brian D. Dalpaiz
Western Illinois University, Macomb 5 actives, 3 associates

Other 1994-95 officers: Andrea P. Stoltz, vice president; Richard T. Jensen, secretary/treasurer; Larrry J. Morley, corresponding secretary/ faculty sponsor.

IN Gamma Chapter President - Julia Frasure
Anderson University, Anderson 18 actives, 6 associates

Other officers for 1994-95: Matthew Weippert, vice president; Nathan Porter, secretary/treasurer; Stanley L. Stephens, corresponding secretary/ faculty sponsor.

IA Alpha Chapter President - Lisa Gaskell
University of Northern Iowa, Cedar Falls 38 actives

Students presenting papers during the spring semester at IA Alpha *KME* meetings included John Hamman on "Dynamics of Geometry," Amy Wiemerslage on "Just and Equal Temperament," and Andrew Christianson on "Utilizing Mathematica to Calculate the Magnetic Susceptibility of $(CuIn)_{1-x}Mn_{2x}Te_2$ with $x = .005$." There was strong interest in the *KME* Region IV Convention held April 22-23, 1994, in St. Joseph, MO. Chapter members Christopher Dix and Amy Wiemerslage presented papers at the convention. Amy's presentation was awarded third place. Seven students attended the convention along with faculty John E. Bruha and John S. Cross. Nine new student members were initiated into Iowa Alpha Chapter this spring. Michelle Ruse addressed the initiation banquet on the topic, "An Introduction to Multiquadric Interpolation." Other 1994-95 officers: Dan Gruman, vice president; Jack Dostal, secretary; Chris Dix, treasurer; John S. Cross, corresponding sponsor/faculty sponsor.

IA Beta Chapter President - Laura Peterson

Drake University, Des Moines 13 actives

On April 15 members met to view a video on the proof of Fermat's Last Theorem. In early May the chapter sponsored a colloquium on "Pianos and Drums, Mathematically Speaking," by Ruth Gornet of Texas Tech University. The chapter also heard a presentation by Cliff Hall and Lee Vettleson concerning their work on the Math Modeling Contest. Other 1994-95 officers: Becky Marjerison, vice president; Jeff Sass, secretary/treasurer; H. K. Krishnapriyan, corresponding secretary; Lawrence W. Naylor, faculty sponsor.

IA Gamma Chapter President - Dean Stevens

Morningside College, Sioux City 12 actives, 3 associates

IA Gamma organized a mathematics tutoring program this past semester at a local high school; three chapter members tutored one hour each week. A car wash held to raise money was so successful that members plan to make it an annual event. Other 1994-95 officers: Cara Kern, vice president; Jason Shriver, secretary; Denise Anderson, treasurer; Steve Nimmo, corresponding secretary/faculty sponsor.

IA Delta Chapter President - Wendy Ahrendsen

Wartburg College, Waverly 46 actives, 5 associates

The January meeting began with a pizza party, followed by a presentation by Vice President Becky Hertenstein on the "Geometry of Soap Bubbles." Soap solution, gumdrops, and toothpicks were provided for members to try their own geometric soap bubbles. The program for the February meeting was presented by Secretary Wendy Ahrendsen who spoke on the graphic works of M. C. Escher. On March 18 and the morning of March 19, the chapter co-hosted, with the Mathematics and Computer Science Department, 43 high school participants in the first annual "Explorations in Mathematical Sciences." The initiation banquet for 17 new members was held the afternoon of March 19. Tim Alpers from IBM spoke on the topic "Technology and Change: Prepare For It." The *KME* picnic, followed by a softball game, was held in May. Other 1994-95 officers: Kelly Berkeland, vice president; Gretchen Roth, secretary; Adam Sanford, treasurer; August Waltmann, corresponding secretary; Lynn Olson, faculty sponsor.

KS Alpha Chapter President - Andrew Buchholz

Pittsburg State University, Pittsburg 83 actives

The Spring Semester activities started with a pizza party and initiation in February for 13 new members. The chapter hosted a guest speaker at the March meeting; Dr. David Surowski from Kansas State

University presented the topic "Knights and Knaves." Several members worked on the Alumni Association's Annual Phon-a-thon. Their efforts were rewarded with 4th place prize for the most money raised by a student organization. Three students and two faculty attended the Region 4 convention held in St. Joseph, MO, in April. The chapter also assisted the Math Department faculty in administering and grading tests given at the annual Math Relays on April 26. The final meeting for the semester was a social event held at Professor Gary McGrath's home. Homemade ice cream and cake were served to those present. Officers for the 1994-95 school year were elected. The annual Robert M. Mendenhall awards for scholastic achievement were presented to Michelle Hudiburg, Yiwen Mao, Christopher Murphy, and Mitch Richling. Other 1994-95 officers: Bethany Schnackenberg, vice president; Zoeann Michel, secretary; Sherry Brennon, treasurer; Harold L. Thomas, corresponding secretary; Bobby Winters, faculty sponsor.

KS Beta Chapter President - Jason Henry
Emporia State University, Emporia 24 actives, 11 associates

Other 1994-95 officers: Stacey Walker, vice president; Sherry Drummond, secretary; Michelle Martling, treasurer; Connie S. Schrock, corresponding secretary; Larry Scott, faculty sponsor.

KS Gamma Chapter President - Mary Kay Heideman
Benedictine College, Atchison 18 actives, 14 associates

President Michael McGuire continued to meet with KS Gamma members regularly to plan activities. Initiated on March 15 were students Angela Behrnes, Gregory Boucher, Gerard Pineda, Michael Rhoden, Sean Strasburg, and faculty member Ann Petrus, CDP. Initiation, held in Schroll Center, was followed by a spaghetti dinner. On April 13 the group enjoyed dinner at Paolucci's in honor of the seniors and Sister Ann Petrus, who served as KS Gamma moderator this year. At the Honors Convocation this spring, the following recipients of Sister Helen Sullivan Scholarship awards were presented certificates: Mary Kay Heideman, Jodie Muhlbauer, Gerard Pineda, Sean Strasburg, and Michael Rhoden. Attending the regional meeting in St. Joseph, MO, on April 23 were nine students and two faculty members. Other 1994-95 officers: Greg Boucher, vice president; Jodie Muhlbauer, secretary; Gerard Pineda, treasurer; Jo Ann Fellin, OSB, corresponding secretary/faculty sponsor.

KS Delta Chapter President - Jeffrey Brown
Washburn University, Topeka 43 actives

In March, seventeen initiates were inducted into KS Delta at the annual spring initiation dinner. The chapter met in May to elect officers for the 1994-95 school year. In addition to these two meetings, the organization joined with the local Mathematics Club for several meetings. Programs for these meetings consisted of presentations by faculty or students on various mathematical topics. Other 1994-95 officers: Vincent Davis, vice president; Daniel Wessel, secretary; Karen Richard, treasurer; Allan Riveland, corresponding secretary; Gary Schmidt and Ron Wasserstein, faculty sponsors.

KS Epsilon

Fort Hays State University, Fort Hays — 29 actives, 3 associates

Spring activities consisted of monthly meetings and a spring banquet. Student officers for 1994-95 have not yet been elected. Charles Votaw is corresponding secretary and Mary Kay Schippers is faculty sponsor.

KY Alpha

Chapter President - Paula Christian

Eastern Kentucky University, Richmond — 20 actives. 19 associates

The first meeting of the semester, postponed due to bad weather, centered on plans for hosting a regional convention in conjunction with the department symposium in early March. The convention turned out to be a real success; some conventioneers even got to learn line dancing at the end of the day! The annual initiation ceremonies for new members featured a talk by Dr. Robert Buskirk entitled "Topological Curve Theory," and was followed by a party in the student center. The MAA videotape on Paul Erdös provided the program for the April meeting. At the last meeting, new officers were elected and installed. Two fund raisers were held during the semester. The first, in February, consisted of a two-night seminar on factoring for the students in the developmental math courses and carried a $2 charge per session. As a second fund-raiser, the chapter offered, toward the end of the semester, to buy and sell the TI-85 graphing calculators that are now being required of students in the College Algebra course. Not only did this serve as a fund-raiser, but it also allowed *KME* members to purchase calculators at a bargain price. Other 1994-95 officers: John Ward, vice president; Andrea Warren, secretary; Andrea McCreary, treasurer; Pat Costello, corresponding secretary/faculty sponsor.

KY Beta

Chapter President - Misti Honeycutt

Cumberland College, Williamsburg — 35 actives

On February 15, 1994, KY Beta co-hosted a visiting lecturer, Dr. Wells from Western Kentucky University. Following a spaghetti supper, Dr. Wells gave a topology talk entitled "Twisting and Turning." The next day, he spoke on origami and The Konigsberg Bridge problem. Initiation for 11 students and one faculty member was held in April.

Members also assisted in hosting a regional high school math contest, held annually at Cumberland College. Other 1994-95 officers: John Douglas Lovin, Jr., vice president; Eric Alan Thornsbury, secretary; Donald R. Poynter, Jr., treasurer; Jonathan E. Ramey, corresponding secretary; John A. Hymo, faculty sponsor.

MD Alpha — Chapter President - Christina Marsalek

College of Notre Dame of Maryland, Baltimore — 13 actives, 9 associates

MD Alpha spring semester activities included monthly meetings, a reception for guest lecturer, Dr. Joel E. Cohen, on March 22, and the annual induction dinner meeting on May 9. Dr. Cohen, Professor of Populations from Rockefeller University, spoke on "Models and How they Relate to the Real World." The induction address, "The EXXON Valdez Oil Spill," was given by Peter Olsen, USCG, Commander Coast Guard Reserve. Four members were inducted and nine students were received into temporary membership. Other 1994-95 officers: Emelia Tracey, vice president; Shannon Spicer, secretary; Donna Zajaczkwoska, treasurer; Sister Marie A. Dowling, corresponding secretary; Joseph DiRienzi, faculty sponsor.

MD Beta — Chapter President - Robert Brown

Western Maryland College, Westminster — 20 actives, 6 associates

As a scholarship fund raiser, MD Beta sponsored a booth at the College's Spring Fling. An induction was held in April. Other 1994-95 officers: Emily Snyder, vice president; Kari Dunn, secretary; Kathy Gaston, treasurer; James Lightner, corresponding secretary/faculty sponsor.

MD Delta — Chapter President - John Hughes

Frostburg State University, Frostburg — 44 actives

The February 20 induction of 15 new members featured a presentation by Dr. Mark Hughes entitled "Episodes from the History of Mathematics." In March, the chapter enjoyed a computer graphics demonstration by *KME* president John Hughes and fellow Computer Science Club members, Jeff Blank and Aaron Ward. Dr. Edward White gave a talk in April on "Stereograms." Other officers for 1994-95: Kileen Baker, vice president; Karl Streaker, secretary; Melissa Thomas, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

MA Alpha Chapter President - Michelle Paradise

Assumption College, Worcester 14 actives

Eight new members were initiated on March 28, 1994. Following a dinner in honor of the new members, Professor William Katcher of Assumption's Computer Science Faculty spoke on "Financial Simulation." Other 1994-95 officers: Wu-Ling, secretary; Charles Brussard, corresponding secretary/faculty sponsor.

MI Beta Chapter President - Nicole Zakrajsek

Central Michigan University, Mount Pleasant 32 actives

Meetings were held every two weeks during the winter semester. Much of the early part of the semester was spent in preparation for hosting the Region II Regional Convention which was held on March 18-19, 1994. The chapter enjoyed meeting guests from the various visiting chapters. The organization, along with the departmental actuarial club, co-hosted a spring picnic for members and faculty. Other 1994-95 officers: Christine Riggs, vice president; Tara Kelly, secretary; Rich Lamb, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

MS Alpha Chapter President - Nancy Piper

Mississippi University for Women, Columbus 16 actives, 4 associates

Other 1994-95 officers: Laura Pendergest, vice president; Jill Whites, secretary; Mary-Margaret Wolff, treasurer; Jean Parra, corresponding secretary; Shavchen Yang, faculty sponsor.

MS Gamma Chapter President -Joon Lee

University of Southern Mississippi, Hattiesburg 20 actives, 7 associates

Other 1994-95 officers: Pavor Cubranic, vice president; Kathy Jones, secretary; Alice W. Essary, treasurer/corresponding secretary; Barry Piazza, faculty sponsor.

MO Alpha Chapter President - Grant Lathrom

Southwest Missouri State University, Springfield 50 actives, 11 associates

During the spring semester, Missouri Alpha continued its cooperative efforts with the student chapter of the Mathematical Association of America. The two organizations held several joint meetings and celebrated the end of the semester and academic year with the annual spring *KME*/MAA Banquet, May 5, 1994. Other 1994-95 officers: Matthew Brom, vice president; Catherine Montgomery, secretary; Kaycia Riepe, treasurer; Ed Huffman, corresponding secretary/faculty sponsor.

MO Beta Chapter President - Steven Shattuck

Central Missouri State University, Warrensburg 18 actives, 5 associates

MO Beta chapter initiated 16 full members and 6 associate members in March. The annual Klingenberg Lecture was given by Kirk Monsees, a 1992 graduate of CMSU, who is employed with an actuarial consulting firm. Four students and two sponsors attended the regional meeting in St. Joseph. Both faculty and students enjoyed "ice-blocking" at the annual end-of-semester picnic. Other 1994-95 officers: Ann Scheffing, vice president; Mindy Eder, secretary; Chad Doza, treasurer; Grace Lee, Historian; Rhonda McKee, corresponding secretary; Scotty Orr, Larry Dilley, and Phoebe Ho, faculty sponsors.

MO Epsilon Chapter President - Heather Warren

Central Methodist College, Fayette 12 actives, 9 associates

At the initiation banquet on April 20, Visiting Professor Galina Piatnitskaia, from Northwest Polytechnic Institute in St. Petersburg, Russia, spoke on mathematics education in Russia. Other 1994-95 officers: Jason Graves, vice president; Audrey Heidekrueger, secretary; Mitu Bajpayer, treasurer; William D. McIntosh, corresponding secretary; Linda O. Lembke, faculty sponsor.

MO Eta Chapter President - Chad Tatro

Northeast Missouri State University, Kirksville 24 actives, 6 associates

Other 1994-95 officers: Sarah Schwab, vice president; Josh Aldrich, secretary; Tanya Walter, treasurer; Mary Sue Beersman, corresponding secretary; Jay Belanger and Joe Hemmeter, faculty sponsors.

MO Theta Chapter President - Kelly Godzwa

Evangel College, Springfield 12 actives, 6 associates

Other 1994-95 officers: Don Tosh, corresponding secretary/faculty sponsor.

MO Iota

Missouri Southern State College, Joplin 27 actives

Initiation of seventeen new members was held in March. In early April, chapter members assisted with the MO section meeting of the Mathematical Association of America held on the campus of MSSC. In particular, members hosted a reception for the MAA student members who attended, and assisted with preparations for a Panel Discussion on Careers. Members of the panel were MSSC alumni, many of them former *KME* members. Chapter member Tom Wofford presented a talk at the MAA section meeting entitled "Galileo, Parabolas, and Projectiles." He subsequently presented his paper again at the Undergraduate Research

Conference in Kalamazoo, MI. In late April, a delegation of eight attended the Region IV Conference in St. Joseph, MO. A meeting will be held in late summer to elect officers for next year. Mary Elick is corresponding secretary; Chip Curtis is faculty sponsor.

MO Lambda Chapter President - Dawn Powell

Missouri Western State College, St. Joseph 40 actives

The MO Lambda chapter was busy spring semester planning for and hosting the Region IV Convention. Ninety-four students and faculty attended the April 22-23 event. Ira Papick of University of Missouri at Columbia was the luncheon speaker. Other semester activities included the initiation of 12 students in February, a food sale in February, and the spring picnic and election of officers in May. Other 1994-95 officers: Tracy Schemmer, vice president; Ryoko Tamoto, secretary; Henry Trammell, treasurer; John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

NE Alpha Chapter President - Michelle Roberts

Wayne State College, Wayne 25 actives

As a money-making activity, club members monitored the Math-Science building during the evenings. Each year the chapter administers a competitive examination to identify the Outstanding Freshman. This year's winner is Brian Steinhoff, whose name will be engraved on a permanent plaque. Brian will also have his national *KME* initiation fees paid and will receive a one year's honorary membership in the local chapter. Four members, accompanied by faculty member Fred Webber, attended the Region IV convention at Missouri Western State College in St. Joseph in April. Also in April, club members made posters to promote National Mathematics Week and members Leslie Iwai and Heather Phinney presented their senior honors papers. Other 1994-95 officers: Darin Brumbaugh, vice president; Robert Schultz, secretary/treasurer; Todd Koehler, historian; Fred Webber, corresponding secretary; Jim Paige and John Fuelberth, faculty sponsor.

NE Gamma Chapter President - Amy Wiese

Chadron State College, Chadron 12 associates

Other 1994-95 officers: Todd Ziettlow, vice president; Corby Dayhoff, secretary; Ken Schultz, treasurer; James Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

NE Delta Chapter President - Patti Hultine

Nebraska Wesleyan University, Lincoln 28 actives, 11 associates

Other 1994-95 officers: Patricia Wahle, vice president; Randal Steele, secretary; Charles McCutchen, treasurer; Muriel Skoug, corresponding secretary/faculty sponsor.

NH Alpha Chapter Presidents - Tracey Thibeault and Suzanne Letendre

Keene State College, Keene 29 actives, 11 associates

The induction ceremony for eleven new members was followed by a problem solving contest. The chapter sponsored a campus wide contest during Math Awareness Week in April. Other spring activities included the design and sale of T-shirts as a fund raiser and an end-of-the-year picnic. Other 1994-95 officers: Tammy Spearrin, vice president; Tina Haggett, secretary; Shayne Noyes, treasurer; Charles Riley, corresponding secretary; Ockle Johnson, faculty sponsor.

NM Alpha Chapter President - Paul Frazier

University of New Mexico, Albuquerque 40 actives, 17 associates

Other 1994-95 officers: Theron Kissinger, vice president; Beth Stevens, secretary; Shuke Miao, treasurer; Richard Metzler, corresponding secretary/faculty sponsor.

NY Eta

Niagara University, Niagara University 16 actives, 9 associates

Chapter activities centered on two projects during the spring semester. The first of these, the video project, involved having copies made of the top four talks from the 1993 Biennial Convention and mailing these out to interested *KME* chapters nationwide. The second was a Career Day featuring several alumni who spoke to undergraduates about experiences in the work world. Discussions among students and alumni continued informally during the dinner which followed. Chapter officers for 1994-95 will be elected in the fall. Robert L. Bailey is corresponding secretary.

NY Kappa Chapter President - Dawn Tomlinson

Pace University, New York 20 actives, 7 associates

Other 1994-95 officers: Andrea Marchese, vice president; Terese Lester, secretary; Geraldine Taiani, corresponding secretary; Blanche Abramov and John W. Kennedy, faculty sponsors.

NY Lambda Chapter President - Suzanne Hecker

C. W. Post Campus of Long Island University, Brookville 28 actives

Following the initiation of 14 new members at the annual banquet at the Roslyn Café on April 11, Colin R. Grimes spoke on "A Game Theory Analysis of Morra." The following weekend, he repeated his talk at the *KME* Region I Convention held at Bloomsburg University; student members Concetta Vento and Khaled Amleh also attended, as did Dr. Kunoff. Other chapter officers: Khaled Amleh, vice president; Concetta Vento, secretary/treasurer; Andrew M. Rockett, corresponding secretary; Sharon Kunoff, faculty sponsor.

NY Nu Chapter President - Cafer Barutcuzade

Hartwick College, Oneonta 17 actives, 15 associates

On April 21, 1994, the NY Nu Chapter inducted fifteen new members, bringing the total membership of the two-year old chapter to 46. The new inductees included fourteen students and one faculty member. The faculty member inducted was Dr. Ann Hibner Koblitz of Hartwick College's History Department. Dr. Koblitz is actively involved in researching the history of science and mathematics and in the status of women as professionals in science and mathematics. She is the author of the widely acclaimed book *A Convergence of Lives: Sofia Kovalevskaia: Scientist, Writer, Revolutionary*; with Michael Fellows and Neal Koblitz, she also co-authored the recent article "Cultural Aspects of Mathematics Education Reform" which appeared in the January, 1994, issue of *Notices of the American Mathematical Society*. Following the induction ceremony, Dr. Koblitz gave an informative, provocative, and entertaining talk on "Gender and Mathematics" in which she surveyed some of the accomplishments of women in mathematics, highlighted the early acceptance of women into the mathematical community, and cited numerous instances of inequities that still exist for women in mathematics. Her talk was followed by the annual induction banquet at a local restaurant. Seven members of the local chapter graduated from Hartwick on May 29, 1994, and proudly wore their honor cords of silver and rose pink. Other 1994-95 officers: Marina Y. Mikhailova, vice president; Marisa Faith Stumpf, secretary; Lucianne Lowell, treasurer; Gary E. Stevens, corresponding secretary/faculty sponsor.

NC Gamma Chapter President - Amy Hill

Elon College, Elon College 33 actives, 6 associates

During the spring semester, NC Gamma sponsored two guest speakers, Bill Love of the University of North Carolina at Greensboro, and Richard Carmichael of Wake Forest University, who spoke at the annual *KME* induction ceremony. The chapter also organized a trip to the MAA sectional meeting in Morristown, TN, in April and held a year end cookout for graduating seniors and new chapter inductees. Other 1994-95 officers: Beth Campbell, vice president; Shelia Dove, secretary; Meredith Webster, treasurer; David Nawrocki, corresponding secretary; Jeffrey Clark, faculty sponsor.

OH Alpha Chapter President - Alisha Reesh
Bowling Green State University, Bowling Green 37 actives, 12 associates

On April 12, 1994, OH Alpha held a banquet initiation for twelve new members. Program for the event was given by Dr. Curtis Bennet of the Bowling Green State University mathematics faculty. His topic was "The William Lowell Putnam Mathematical Competition." Other 1994-95 officers: Leah Walden, vice president for programming; Leah Breckstein, vice president for initiation; Kevin Kundert, secretary/treasurer; Waldemar Weber, corresponding secretary; Stephen McCleary, faculty sponsor.

OH Eta Chapter President - Jeana Fox
Ohio Northern University, Ada 34 associates

Chapter activities included initiation of new members on May 4 and a meeting to elect officers on May 16. Other 1994-95 officers: Lori Burgett, vice president; Andy Bushong, secretary; Rick Mokros, treasurer; Tena Roepke, corresponding secretary; Harold Putt, faculty sponsor.

OK Alpha Chapter President - Ryan Swank
Northeastern Oklahoma State University, Tahlequah 32 actives, 1 associate

OK Alpha continues to hold joint activities with NSU's student chapter of MAA. In February the chapter provided refreshments following a talk by Dr. Jerry P. King, Lehigh University mathematics professor. The title of his presentation was "Elegance: Mathematics as Art." The initiation of six members into the chapter was held in the banquet room of Roni's Pizza. The organization participated in "The Problem Solving Competition," sponsored by the MAA, and observed National Mathematics Awareness Week with a viewing of the video "Fermat's Last Theorem." A pre-finals ice cream social was also held. Other 1994-95 officers: Allison Selby, vice president; Jennifer Beals, secretary; Rod Bell, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

OK Delta Chapter President - D. Brock Leach
Oral Roberts University, Tulsa 16 actives

Other 1994-95 officers: Jonathan Forster, vice president; Scott Fowler, secretary; Mark Szewczul, treasurer; Debbie Oltman, corresponding secretary; Susan Carr, faculty sponsor.

PA Alpha Chapter President - Melissa Napoleon
Westminster College, New Wilmington 21 actives, 8 associates

Other 1994-95 officers: Kara Sheets, vice president; Sue Shaffer, secretary; Karin Speer, treasurer; Miller Peck, corresponding secretary; Carolyn Cuff and Warren Hickman, faculty sponsors.

PA Beta Chapter President - Anne Hofmann
La Salle University, Philadelphia 10 actives, 5 associates

Other 1994-95 officers: Molly McAvoy, vice president; Jen Bostak, secretary; Janet Munyan, treasurer; Hugh N. Albright, corresponding secretary; Carl P. McCarty, faculty sponsor.

PA Gamma Chapter President - Gwen Nicklow
Waynesburg College, Waynesburg 10 actives, 8 associates

Other 1994-95 officers: Laura Marquis, vice president; Crystal Thomas, secretary; Paul Gacek, treasurer; A. B. Billings, corresponding secretary/faculty sponsor.

PA Delta Chapter President - Ann Conflitti
Marywood College, Scranton 5 actives, 6 associates

PA Delta held initiation ceremonies on May 10, 1994. Other 1994-95 officers: Abigail Brace, vice president; Kim Fisher, secretary; Melissa Mang, treasurer; Sr. Robert Ann von Ahnen, corresponding secretary/faculty sponsor.

PA Epsilon Chapter President - Sheri Smucker
Kutztown University of Pennsylvania, Kutztown 12 actives, 7 associates

Other 1994-95 officers: Michelle Wiley, vice president; Brandi Thiele, secretary; Karen Biesecker, treasurer; Cherry C. Mauk, corresponding secretary; Randy Schaeffer, faculty sponsor.

PA Eta Chapter President - Kristin Gieringer
Grove City College, Grove City 27 actives

The *KME* selected Outstanding Freshman Mathematics Student was announced at the Parent's Day Award Ceremony on April 30. Other chapter activities included the initiation of new members and the election of next year's officers. The *KME* Spring Picnic was canceled due to inclement weather. Other 1994-95 officers: Claudine Desjardins, vice president; Danielle Miller, secretary; Bryan Weet, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

PA Iota Chapter President - Jason Baker
Shippensburg University of Pennsylvania, Shippensburg 20 actives, 6 associates

During the spring semester, PA Iota members, in conjunction with Math Club members, sponsored two talks and a very successful candy sale. The chapter was also involved in resuming the publication of a newsletter containing bio's of new faculty, information about *KME*, and information about careers in mathematics. Other 1994-95 officers: Angela Foltz, vice president; Todd Bittinger, secretary; Jenna Hoopengardner, historian; Fred Mordai, treasurer; Michael D. Seyfried, corresponding secretary/faculty sponsor.

PA Kappa Chapter Presidents - Sarah Iskra and Joshua Wagner
Holy Family College, Philadelphia 8 actives, 5 associates

Five new members were inducted into PA Kappa chapter on March 28, 1994. Following the initiation ceremonies, conducted by chapter member Daniel Lubicky, Dr. Sandra Fillebrowne of St. Joseph's University presented a media-enhanced lecture on "Chaos and Dynamic Systems." A pizza party concluded the events of the evening. Members enrolled in Modern Geometry wrote research papers on projective geometry and prepared computer graphics presentations on fractals and chaos. Other 1994-95 officers: Leanne Majors, secretary/treasurer; Sr. Marcella Louise Wallowicz, corresponding secretary/faculty sponsor.

SC Beta Chapter President - Tekye Evans
South Carolina State University, Orangeburg 24 actives, 10 associates

Other 1994-95 officers: John Harlen, vice president; Aronda Fraizer, secretary; Bridget Nimmons, treasurer; Cynthia T. Davis, corresponding secretary/faculty sponsor.

SC Gamma Chapter President - Tiffany Allen
Winthrop University, Rock Hill 15 actives, 3 associates

Other 1994-95 officers: Candace Rogers, vice president; Jamie Pittman, secretary; Ronald Knox, treasurer; Donald Aplin, corresponding secretary; Jim Bentley, faculty sponsor.

SD Alpha

Chapter President - Scott Kortan

Northern State University, Aberdeen 17 actives, 5 associates

Other 1994-95 officers: Shannon Catron, vice president; Sheryl Woelber, secretary; Ryan Darling, treasurer; Lu Zhang, corresponding secretary; Raj Markanda, faculty sponsor.

TN Beta

East Tennessee State University, Johnson City 18 actives, 13 associates

Thirteen new members were initiated on March 25, 1994. The ceremony was conducted by the chapter officers, Marvin Johnson, Jennifer Shupe, and Michele Cooke. Program for the event was given by Dr. Jeff Knisley who spoke on unsolved problems in mathematics. Dr. George Poole, the chair of the mathematics department, then presented the award for outstanding graduating senior in mathematics to Marvin Johnson. Officers for 1994-95 have not yet been determined. Lyndell Kerley is corresponding secretary.

TN Delta

Chapter President - Brenda Bleavins

Carson-Newman College, Jefferson City 16 actives, 5 associates

TN Delta held an initiation banquet at the Little Dutch Restaurant in March and a picnic at Panther Creek Park in April. Also in April, members assisted the department in hosting the annual meeting of the southeastern region of Mathematical Association of America. Other 1994-95 officers: Jay Mutterspaugh, vice president; Amy Smith, secretary/treasurer; Verner Hansen, corresponding secretary; Carey R. Herring, faculty sponsor.

TX Alpha

Chapter President - Curt Bourne

Texas Tech University, Lubbock 60 actives, 30 associates

Other 1994-95 officers: Nora Chang, vice president; Wes Kirk, secretary; Chuck Steed, treasurer; Edward J. Allen, corresponding secretary/faculty sponsor.

TX Eta

Chapter President - Ann Meuret

Hardin-Simmons University, Abilene 14 actives, 13 associates

Thirteen members were inducted at TX Eta's twentieth annual induction banquet on March 26, bringing the total membership in the chapter to 158. Amy Garrison, a graduating senior at H-SU, addressed the chapter on her work in industry the previous summer. Leading the induction ceremonies were Kristen Hieronymus, Robyn Eads, and Amy

Garrison. Other officers for 1994-95: Jeremy Fitch, vice president; Robyn Eads, secretary; Carmen Turner, treasurer; Frances Renfroe, corresponding secretary; Charles Robinson, Ed Hewett, and Dan Dawson, faculty sponsors.

TX Kappa Chapter President - James Davidson
University of Mary Hardin-Baylor, Belton 15 actives, 10 associates

TX Kappa held its Spring Symposium on April 14, 1994. The topic was "Awards and Scholarships for Mathematics Majors." As a result of the discussions and others to follow, there will soon be several new awards and scholarships which will benefit campus mathematics students. The chapter joined the 1994 University of Mary Hardin-Baylor Homecoming Parade on April 15 with an entry entitled "Trajectory." A special award was received for originality. All proceeds from the Spring Bake Sale held April 20 benefited the Children's Miracle Network. Other 1994-95 officers: Eric Madsen, vice president; Mary Cook, secretary; Rachel Mc Wha, treasurer; Peter Chen, corresponding secretary; Maxwell Hart, faculty sponsor.

VA Alpha Chapter President - Romon Williams
Virginia State University, Petersburg 36 actives

The annual initiation and banquet of Virginia Alpha chapter was held on Friday, April 22, 1994, at the Holiday Inn in Petersburg, VA. The banquet address was given by Dr. Christine Darden, a member of Virginia Alpha and an aerospace engineer at Langley Research Center, Hampton, VA. Awards presented at the banquet included the Louise Stokes Hunter Award in honor of the founder of VA Alpha chapter. Other 1994-95 officers: Carol Elias, vice president; Ann Flynn, secretary; M. H. Tewari, treasurer; Emma B. Smith, corresponding secretary; V. S. Bashshi, faculty sponsor.

WI Alpha Chapter President - Erin Hein
Mount Mary College, Milwaukee 5 actives, 5 associates

Graduates Silvia Navarro and Karen Hauck presented senior mathematics papers at the last spring meeting in May. Other 1994-95 officers: Shima Tsujimoto, vice president/secretary; Erin Hein, treasurer; Sister Adrienne Eickman, corresponding secretary/faculty sponsor.

1994 Regional Conventions

Edited by Patrick J. Costello, President-Elect

There were five *KME* Regional Conventions held during the Spring of 1994. The following reports were prepared from materials submitted by the host schools or the Regional Directors.

Report of the 1994 Region I Convention

The Region 1 Convention was held April 15-16 at Bloomsburg University in Bloomsburg, Pennsylvania. There were six chapters represented: Pennsylvania Zeta, Indiana University of Pennsylvania; Pennsylvania Theta, Susquehanna University; Pennsylvania Lambda, Bloomsburg University; Pennsylvania Mu, St. Francis College; New York Eta, Niagara University; and New York Lambda, C.W. Post Campus of Long Island University.

The meeting began with a dinner Friday night. Dr. John Riley from Bloomsburg University gave an interesting lecture on "Chaotic After-Dinner Thoughts." Afterwards there was a social gathering.

Saturday morning there were seven student talks. The best speakers were presented with awards donated by Texas Instruments and Future Graph. The talks were:

Genetic Algorithms
Badri Ramaswami (Susquehanna University)

The Postage Stamp Problem
Jim Rodenhaver (Bloomsburg University)

Primality Testing
Patricia George (St. Francis College)

The Mandelbrot Set
Joe Gallagher (Bloomsburg University)

Knots and DNA
Sharon Kane (Bloomsburg University)

A Game Theory Analysis of Morra
Colin Grimes (C.W. Post Campus of Long Island University)

The Decimal Expansion of $1/(b-1)^2$ *in Base b*
Lauree Attinger (Bloomsburg University)

Report of the 1994 Region II Convention

The Region II Convention was held March 18-19 at Central Michigan University in Mt. Pleasant, Michigan. There were three chapters represented: Illinois Delta, College of St. Francis; Michigan Beta, Central Michigan University; and Wisconsin Alpha, Mount Mary College. Thirty-five students and faculty were present. Convention activities were ably coordinated by Erica Hall, President; Sara Meese, Vice President; and Arnold Hammel, Faculty Advisor.

The meeting opened Friday evening with registration and a banquet at the CMU University Center. Participants enjoyed a fine dinner and a delightful address by Dr. Thomas Miles of Central Michigan University. Dr. Miles' talk entitled "Kappa, Mu, and Epsilon" looked at some topics in mathematics and physics which use the symbols κ, μ and ϵ for notation and applications. He also related these three Greek letters in an interesting manner to what membership and participation in an organization like *KME* really involves. The topping on the ice cream dessert at the banquet was maple syrup that was processed at the Shepherd Sugar Bush of which advisor, Arnold Hammel, is a volunteer sapper. After the banquet participants enjoyed bowling, wallyball, and other activities at the CMU Student Activity Center.

Saturday morning there were three student talks. An MAA book was given to each speaker. The talks were:

A Simple Mathematical Concept Application to New Material Development
Tom Fobear (Central Michigan University)

Educational System of Great Britain
Jane Peterson (Central Michigan University)

An Implementation of Large Precision Integer Arithmetic
Scott Radtke (Central Michigan University)

After the talks students and faculty met separately to discuss issues of interest. A report of these sessions was made at the closing session.

Report of the 1994 Region III Convention

The Region III Convention was held Friday, March 4 at Eastern Kentucky University in Richmond, Kentucky. There were three chapters represented: Kentucky Alpha, Eastern Kentucky University; South Carolina Gamma, Winthrop University; and Tennessee Alpha, Tennessee Technological University. Over 85 students and faculty were present. Convention activities were ably coordinated by Paula Christian, President; Tony Baugh, Vice President; Andrea Warren, Secretary; and Pat Costello, Faculty Advisor.

The convention was held in conjunction with the department's tenth annual Symposium in the Mathematical Sciences. The meeting opened early with a welcome by Dr. Mary Fleming, chair of the department, and Dr. Patrick Costello, Faculty Advisor of Kentucky Alpha. In all there were thirteen student talks, one faculty talk, and two invited talks. Dr. J. Jaromczyk from the computer science department at the University of Kentucky gave an interesting talk entitled "One Circle, Two Circles, ... What is Computational Geometry?" Dr. R. Kryscio from the statistics department at the University of Kentucky gave an informative talk on "Statistical Aspects of Clinical Research."

There were four *KME* student talks scheduled. Each speaker was presented with an EKU mug. The talks were:

On the Arithmetic of Polynomials Over Finite Fields
Leanne Link (Tennessee Tech University)

Lowess Approximation
Amy Deal (Winthrop University)

Have You Ever Heard of Catalan Numbers?
Paula Christian (Eastern Kentucky University)

Smith Numbers: An Infinite Subset of the Natural Numbers
Kathy Lewis (Eastern Kentucky University)

During a morning break the *KME* students and faculty met and introduced themselves. After lunch a tour of campus was provided. The day ended with some instructions and practice at line dancing.

Report of the 1994 Region IV Convention

The Region IV Convention was held April 22-23 at Missouri Western State College in St. Joseph, Missouri. There were fifteen chapters represented: Iowa Alpha, University of Northern Iowa; Kansas Alpha, Pittsburg State University; Kansas Beta, Emporia State University; Kansas Gamma, Benedictine College; Kansas Delta, Washburn University; Kansas Epsilon, Fort Hays State University; Missouri Beta, Central Missouri State University; Missouri Eta, Northeast Missouri State University; Missouri Theta, Evangel College; Missouri Iota, Missouri Southern State College; Missouri Kappa, Drury College; Missouri Lambda, Missouri Western State College; Nebraska Alpha, Wayne State College; Nebraska Beta, Kearney State College; Nebraska Gamma, Chadron State College; and South Dakota Alpha, Northern State University. There were 104 students and faculty were present. Convention activities were ably coordinated by Tracy Schemmer, President; Lee Napravnik, Vice President; Dawn Powell, Secretary; Denise Fuller, Treasurer; Jerry Wilkerson, Advisor; and John Atkinson, Corresponding Secretary.

The meeting opened Friday evening with registration. Afterwards there was a social gathering in the Nelle Blum Student Union.

Saturday morning started with a welcome by Dr. Martin Johnson, Dean of Liberal Arts and Sciences. Between two sessions of student presentations there were separate student and faculty discussion sessions. Participants enjoyed a fine luncheon and a delightful address by Dr. Ira Papick of the University of Missouri, Columbia. Dr. Papick's talk was entitled "Mathematics: The Terrible Beauty." After the luncheon there was a presentation of awards.

There were eight student talks. The top three speakers were presented with monetary prizes. The talks were:

The Mathematics of Future Values
Heather Hohnstein (Chadron State College)

Forecasting: A Mathematical Approach
Jeremy Benson (Missouri Western State College)

The Mathematics of Just and Equal Temperament
Amy Wiemerslage (University of Northern Iowa)

Spirographics for the Lazy
Stan Yoder (Evangel College)

The Business of Calculus
Christopher Dix (University of Northern Iowa)

Markov Chains and Ion Channel Conductance
Kirk Drager (Washburn University)

A Genetic Algorithm Applied to a Problem in Coding Theory
Kim Derrington (Northeast Missouri State University)

The Topological Equivalence of K and X
Cynthia Schwab (Drury College)

Report of the 1994 Region V Convention

The Region V Convention was held April 8-9 at the University of New Mexico in Albuquerque, New Mexico. There were four chapters represented: Colorado Gamma, Fort Lewis College; Colorado Delta, Mesa State College; Oklahoma Gamma, Southwestern Oklahoma State University; and New Mexico Alpha, University of New Mexico. Over 22 students and faculty were present. Convention activities were ably coordinated by Terry Lynn Vigil, President; Theron Kissinger, Treasurer; and Dick Metzler, Corresponding Secretary.

The meeting opened Friday evening with registration and a welcome. Afterwards there was a reception that included a game of Math Pictionary at the blackboard.

Saturday morning started with a mixer in the Math Library. After the student presentations, participants enjoyed a fancy catered luncheon in the historic Bobo Room on campus. After the luncheon there was a presentation of awards.

There were five student talks. All speakers were presented with monetary prizes. The talks were:

A Review of the Konigsberg Bridge Problem
William Grover (University of New Mexico)

An Introduction to Dynamical Systems and Their Analysis
Johnny Snyder (University of New Mexico)

Generalized p-Series with Applications
Lori Johnson-Williams (Southwestern Oklahoma State University)

Integer Sequences: Curious and Otherwise
Susan Clinkenbeard (Fort Lewis College)

The Irrationality of e
Steven Elliott (University of New Mexico)

Kappa Mu Epsilon National Officers

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation.

Chapter	Location	Installation Date
OK Alpha	Northeastern Oklahoma State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State College	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State College, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950

IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	Kearney State College, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri - Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin - River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Northeast Missouri State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel College, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971

KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	West Georgia College, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin - Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Illinois Benedictine College, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry College, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993