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Differentiating a^x: An Alternative Proof

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Presented at the 1991 National Convention and
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The problem of differentiating a^x is solved in calculus in the development of the natural logarithm and the exponential function, but although the solution is inventive, it is not very direct. Further, as Eidswick notes in [1], much of the rigor is lost in shuffle. Although it may not at first seem very natural or direct, we intend to differentiate a^x using sequences. In fact, many texts in elementary algebra informally define a^c for an irrational c as the limit of a sequence. For instance, 2^π is said to be the limit of the sequence 2^3, $2^{3.1}$, $2^{3.14}$, $2^{3.141}$, ... For our purposes we will suppose that a^c has been defined as such and that continuity and other properties of a^x have been established. We find a similar suggestion in [2], although Goffman proceeds to find a series representation of e^x, and he differentiates e^x by differentiating the power series. We, too, will rely on the nature of e^x to differentiate a^x; we will, however, proceed to this result in a manner that is more direct, and that is similar to the result in [1], but that is perhaps more motivated.

It is well known that the increasing sequence $(1+\frac{1}{n})^n$ converges to a value between 2 and 3, and this limit we call e. That is to say, $(1+\frac{1}{n})^n < e$ for all positive integers n. In particular, we note that $\dfrac{e^{1/n}-1}{1/n} > 1$. Some meditation upon this result will reveal the following: It is reasonable to expect that the sequence $\dfrac{e^{1/n}-1}{1/n}$ converges to 1. Now if e^x were differentiable at 0, then we would be assured that $\lim\limits_{x\to 0}\dfrac{e^x-1}{x} = 1$, since the sequence is simply the derivative "along a specific path" to 0. That is, if $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}$ exists, and if the above sequence converges to 1, then $\lim\limits_{x\to 0}\dfrac{e^x-1}{x} = 1$. We have, of course, speculated at length, but further investigation along these lines reveals another inequality involving e, namely, that $e \le \left(1+\frac{1}{n}+\frac{1}{n^2}\right)^n$. Thus, $e^{1/n} \le 1+\frac{1}{n}+\frac{1}{n^2}$, so that $n(e^{1/n}-1) \le 1+\frac{1}{n}$. Further, $(1+\frac{1}{n})^n < e$ implies that $1+\frac{1}{n} < e^{1/n}$, so that $1 < n(e^{1/n}-1)$. That is, $1 < n(e^{1/n}-1) \le 1+\frac{1}{n}$ for each n. Then, by the "squeeze" theorem, $n(e^{1/n}-1)$ converges to 1. This result is almost convincing, but we must still establish the existence of $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}$.

Let us consider this limit. If this limit exists then it is 1, and so for some $\delta > 0$, if $0 < x < \delta$ then it must be true that $\dfrac{e^x-1}{x} < \dfrac{n+1}{n}$. That is, we would have $e^x < (\frac{n+1}{n})x+1$ for each n. We will proceed to consider the converse to this result; namely, that if for each positive integer n there is a right neighborhood of 0 such that $e^x < (\frac{n+1}{n})x+1$, then $\lim\limits_{x\to 0}\dfrac{e^x-1}{x} = 1$. We present this as a major theorem that is preceded by several lemmas. We also will make use of the fact that $(1+\frac{1}{n})^n < e < (1+\frac{1}{n})^{n+1}$.

LEMMA 1. Let k be a positive integer and let $n \ge 2(k+1)$. Then $e^{1/(n-1)} < 1+\left(\frac{k+1}{k}\right)\frac{1}{n}$.

Proof. Now $e < (1+\frac{1}{n-2})^{n-1}$ for $n \geq 3$. Then, $e^{1/(n-1)} < 1+\frac{1}{n-2}$. Now $1+\frac{1}{n-2} \leq 1+\left(\frac{k+1}{k}\right)\frac{1}{n}$ if and only if $\frac{n}{n-2} \leq \frac{k+1}{k}$, which is true if and only if $1+\frac{2}{n-2} \leq 1+\frac{1}{k}$. But this is just $n \geq 2(k+1)$. Thus, $n \geq 2(k+1)$ implies that $e^{1/(n-1)} < 1+\left(\frac{k+1}{k}\right)\frac{1}{n}$.

LEMMA 2. Let k be a positive integer. Then there is a $\delta > 0$ such that if x is in $(0,\delta)$ then $e^x < \left(\frac{k+1}{k}\right)x+1$.

Proof. We let the positive integer k be given. Let $0 < \delta < \frac{1}{2(k+1)}$ and let x be in $(0,\delta)$. Let $n+1$ be the least integer greater than $1/x$; that is, $n+1 > \frac{1}{x} \geq n$. Now if it were the case that $n < 2(k+1)$, then $n+1 \leq 2(k+1)$. Then $\frac{1}{n+1} \geq \frac{1}{2(k+1)} > x$ so that $\frac{1}{x} > n+1$. So we must have $n \geq 2(k+1)$. Now by Lemma 1, $n+1 > n \geq 2(k+1)$ gives

$$e^{1/n} < 1+\left(\frac{k+1}{k}\right)\left(\frac{1}{n+1}\right).$$

Also, $\frac{1}{n+1} < x \leq \frac{1}{n}$ gives

$$1+\left(\frac{k+1}{k}\right)\left(\frac{1}{n+1}\right) < 1+\left(\frac{k+1}{k}\right)x.$$

Further, $e^{1/(n+1)} < e^x \leq e^{1/n}$. Thus,

$$e^x \leq e^{1/n} < 1+\left(\frac{k+1}{k}\right)\left(\frac{1}{n+1}\right) < 1+\left(\frac{k+1}{k}\right)x,$$

so that $e^x < 1+\left(\frac{k+1}{k}\right)x$.

LEMMA 3. Let k be a positive integer. Then $e^{1/(n+1)} > 1 + \left(\frac{k}{k+1}\right)\frac{1}{n}$ for $n \geq k$.

Proof. Now $\left(1 + \frac{1}{n+1}\right)^{n+1} < e$, so that $1 + \frac{1}{n+1} < e^{1/(n+1)}$. But $1 + \left(\frac{k}{k+1}\right)\frac{1}{n} \leq 1 + \frac{1}{n+1}$ if and only if $\frac{k}{k+1} \leq \frac{n}{n+1}$, which is true if and only if $n \geq k$. By choosing $n \geq k$, we proceed back to conclude that $e^{1/(n+1)} > 1 + \left(\frac{k}{k+1}\right)\frac{1}{n}$.

LEMMA 4. Let k be a positive integer. Then there exists $\delta > 0$ such that if x is in $(0, \delta)$ then $e^x > \left(\frac{k}{k+1}\right)x + 1$.

Proof: Let $0 < \delta < \frac{1}{k}$ and let $0 < x < \delta$. Let $n+1$ be the least integer greater than $\frac{1}{x}$. Then $\frac{1}{n+1} < x \leq \frac{1}{n}$. Now if it were the case that $n < k$ then $n+1 \leq k$. Then $\frac{1}{n+1} \geq \frac{1}{k}$, so that $x > \frac{1}{k}$. Thus we must have $n \geq k$. Then by Lemma 3,

$$e^{1/(n+1)} > 1 + \left(\frac{k}{k+1}\right)\frac{1}{n}.$$

Further, $e^{1/(n+1)} < e^x \leq e^{1/n}$. Then we have

$$\left(\frac{k}{k+1}\right)x + 1 \leq \left(\frac{k}{k+1}\right)\frac{1}{n} + 1 < e^{1/(n+1)}.$$

Then for the appropriately chosen δ, $0 < x < \delta$ implies that

$$\left(\frac{k}{k+1}\right)x + 1 < e^x.$$

THEOREM. $\lim_{x \to 0^+} \frac{e^x - 1}{x} = 1.$

Proof: Let $\epsilon > 0$ be given. Then for $\frac{\epsilon}{2} > 0$ there exists a positive integer K such that if $k \geq K$ is an integer we have $\left|\frac{k}{k+1} - 1\right| < \frac{\epsilon}{2}$ and $\left|\frac{k+1}{k} - 1\right| < \frac{\epsilon}{2}$. By Lemmas 2 and 4, there exist δ_1 and $\delta_2 > 0$ such that x in $(0, \delta_1)$ gives $e^x < \left(\frac{K+1}{K}\right)x + 1$ and x in $(0, \delta_2)$ gives $e^x > \left(\frac{K}{K+1}\right)x + 1$. Let $\delta = \min\{\delta_1, \delta_2\}$. Then for x in $(0, \delta)$,

$$\frac{K}{K+1} < \frac{e^x - 1}{x} < \frac{K+1}{K}.$$

Now we know that $\frac{K}{K+1} < 1 < \frac{K+1}{K}$. Then

$$\left|\frac{K+1}{K} - \frac{K}{K+1}\right| > \left|\frac{e^x - 1}{x} - 1\right|.$$

But
$$\left|\frac{K+1}{K} - \frac{K}{K+1}\right| = \frac{K+1}{K} - \frac{K}{K+1}$$

$$= \left|\frac{K+1}{K} - 1\right| + \left|\frac{K}{K+1} - 1\right| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

Thus, $\left|\frac{e^x - 1}{x} - 1\right| < \epsilon$ for x in $(0, \delta)$. That is, $\lim_{x \to 0^+} \frac{e^x - 1}{x} = 1$.

Now the left-hand limit may be found as follows:

$$\lim_{x \to 0^-} \frac{e^x - 1}{x} = \lim_{x \to 0^-} \frac{(e^x)(1 - e^{-x})}{x}$$

$$= \lim_{x \to 0^-} e^x \cdot \lim_{x \to 0^-} \frac{1 - e^{-x}}{x} = \lim_{y \to 0^-} \frac{e^y - 1}{y},$$

which equals 1. Taking these results together, we have $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$. We have previously noted that if we obtain this limit, then we may differentiate e^x; we restate the proof of this result for emphasis.

$$\lim_{x \to c} \frac{e^x - e^c}{x - c} = e^c \cdot \lim_{x \to c} \frac{e^{x-c} - 1}{x - c} = e^c \cdot \lim_{y \to 0} \frac{e^y - 1}{y},$$

which leaves e^c. Thus, the derivative of e^x at any point c is simply e^c.

The problem of differentiating a^x remains, but is now easily solved. Since e^x is a strictly increasing function (hence one to one), it has an inverse function, $\ln(x)$. The domain of $\ln(x)$ will be $\{x \in \mathbb{R} : x > 0\}$, so that $a^x = e^{x \ln(a)}$, where $a > 0$ and $a \neq 1$. Then,

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \lim_{x \to 0} \frac{e^{x \ln(a)} - 1}{x} = \lim_{y \to 0} \frac{e^y - 1}{y/\ln(a)}$$

$$= \ln(a) \cdot \lim_{y \to 0} \frac{e^y - 1}{y} = \ln(a).$$

Thus,

$$\lim_{x \to c} \frac{a^x - a^c}{x - c} = a^c \cdot \lim_{x \to 0} \frac{a^x - 1}{x} = a^c \cdot \ln(a).$$

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[2] Casper Goffman, *Introduction to Real Analysis*. New York: Harper and Row, 1966.

I've Got a Secret

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I've got a secret! First, step back in time for a moment to when you were a child. Most children at some point in time come in contact with the games of Scissors-Paper-Stone, Mastermind, and Battleship. As children, we tried to develop schemes of moves and actions to help us win at these games of strategy. We found our schemes by trial and error and thus we could never prove that they were the optimal strategies for the game. As time progressed, we found that mathematics could help us to find solutions to problems that apparently were completely without mathematics. This paper will focus on some techniques of mathematics which can aid us in determining optimal strategies for games of strategy. These techniques make up what mathematicians call the theory of games.

In this treatment of game theory, we will consider games played by two players. These two players could be people, nations, corporations, etc., and we will denote these players R and C. At this time we must make some assumptions concerning the game. First, we will assume that there are m possible moves for player R and n possible moves for player C. Also, for each move of R and C there is an associated payoff to each. We assume the game to be a "constant sum" game so that the sum of the payoffs to R and C is a constant throughout the game. This means we need only know the payoff to one player to find the payoff to the other.

We now note that our game can be described by a matrix $A = (a_{ij})$. Let the rows of the matrix denote the moves of R and the columns denote the moves of C. For $i = 1$ to m and $j = 1$ to n, let a_{ij} represent the payoff to R when R makes his i-th move and C makes his j-th move. Thus the matrix $A = (a_{ij})_{m \times n}$ is called the payoff matrix to R.

We also assume that both players are conservative in that:

(1) Player R will always choose the move that maximizes his minimum earnings regardless of the move C makes.

(2) Player C will always choose the move that minimizes the maximum earnings of R regardless of the move that R makes.

For an example, suppose A is the payoff matrix:

$$
\begin{array}{ccl}
 & \begin{matrix}\text{row}\\ \text{minimum}\end{matrix} & \\
\begin{bmatrix} 5 & 1 & -1 & 2 \\ 3 & 4 & 1 & 2 \\ -2 & 2 & 1 & 0 \end{bmatrix} & \begin{matrix} -1 \\ 1 \\ -2 \end{matrix} & \left.\vphantom{\begin{matrix}1\\1\\1\end{matrix}}\right\} \begin{matrix}\text{R's minimum earnings}\\ \text{per each move of C}\end{matrix} \\
\begin{matrix} 5 & 4 & 1 & 2 \end{matrix} & & \\
\underbrace{\text{column maximum}}_{\begin{matrix}\text{R's maximum earnings}\\ \text{per each move of C}\end{matrix}} & &
\end{array}
$$

Thus, given their conservative strategies, R would choose move 2 and C would choose move 3. Notice that R could make more if he chose another move but that would be dependent on the move C chose.

Because of the software we eventually use to solve these games, we focus on C's problem. Let q_i denote the relative frequency with which C makes his i-th move and p_j the relative frequency with which R makes his j-th move. We can also think of q_i as the probability that C makes move i on a given playing of the game. Then the vectors

$$P = (p_1, p_2, \ldots, p_m) \text{ and } Q = (q_1, q_2, \ldots, q_n)$$

are called strategies for R and C respectively. Note that

$$q_1 + q_2 + \cdots + q_n = 1 \text{ and } p_1 + p_2 + \cdots + p_m = 1.$$

If R uses strategy P and C uses strategy Q, then PAQ^T is called the value of the game. Hence if PAQ^T is greater than zero then R has the advantage, but if it is less than zero, C will have the advantage. When the value of the game is zero, we say the game is fair. If the game is played many times, R will eventually win an amount equal to PAQ^T.

Now suppose C uses strategy $Q = (q_1, q_2, \ldots, q_n)$ and R makes his first move. Then the payoff to R will be

$$a_{11}q_1 + a_{12}q_2 + \cdots + a_{1n}q_n.$$

Similarly if R makes his second move, his payoff will be

$$a_{21}q_1 + a_{22}q_2 + \cdots + a_{2n}q_n.$$

If R makes his last move the payoff to R is

$$a_{m1}q_1 + a_{m2}q_2 + \cdots + a_{mn}q_n.$$

Let V be the maximum of these sums. Thus C's problem becomes to choose $q_1, q_2, \ldots, q_n$ such that V is a minimum where:

$$\begin{array}{ccccccccccc}
a_{11}q_1 & + & a_{12}q_2 & + & \cdots & + & a_{1n}q_n & \leq & V \\
a_{21}q_1 & + & a_{22}q_2 & + & \cdots & + & a_{2n}q_n & \leq & V \\
 & \ddots & & & & & & & \\
a_{m1}q_1 & + & a_{m2}q_2 & + & \cdots & + & a_{mn}q_n & \leq & V \\
q_1 & + & q_2 & + & \cdots & + & q_n & = & 1
\end{array}$$

$$\text{and } q_i \geq 0 \text{ for all } i \leq n.$$

Observe that if a given scalar is added to each entry in the payoff matrix A, the strategies of R and C would not change. Thus we are free to assume that A has only positive entries and hence V is also positive. Letting

$$x_1 = q_1/V,\ x_2 = q_2/V,\ \ldots,\ x_n = q_n/V,$$

we have

$$Z = x_1 + x_2 + \cdots + x_n = q_1/V + q_2/V + \cdots + q_n/V$$
$$= (q_1 + q_2 + \cdots + q_n)/V = 1/V.$$

C's problem now becomes:

$$\text{maximize } Z = x_1 + x_2 + \cdots + x_n \text{ where}$$

$$\begin{array}{ccccccccc}
a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & \leq & 1 \\
a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & \leq & 1 \\
 & \ddots & & & & & & & \\
a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & \leq & 1
\end{array}$$

$$\text{and } x_i \geq 0 \text{ for all } i \leq n.$$

Problems of this type are known as linear programming problems.

Linear programming originated largely during World War II as a method for specifying routes that would minimize travel distance for the limited shipping facilities available to the Allies and for determining the best method of allocating scarce labor, machine tools, and plant facilities to produce war goods. The general form of a linear programming problem is as follows:

$$\text{maximize } Z = Z(X) = c_1x_1 + c_2x_2 + \cdots + c_nx_n \text{ subject to}$$

$$\begin{array}{ccccccccc}
a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & \leq & b_1 \\
a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & \leq & b_2 \\
 & \ddots & & & & & & & \\
a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & \leq & b_m
\end{array}$$

$$\text{and } x_i \geq 0 \text{ for all } i \leq n.$$

Letting $C = (c_1, c_2, \ldots, c_n)$, $X = (x_1, x_2, \ldots, x_n)$, $B = (b_1, b_2, \ldots, b_m)$ and

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & \ddots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

we can abbreviate our system as:

$$\text{maximize } Z = Z(X) = CX \text{ where } AX^{\mathrm{T}} \leq B^{\mathrm{T}} \text{ and } X \geq 0.$$

A point $X = (x_1, x_2, \ldots, x_n)$ is a feasible point for the linear programming problem if X satisfies each of the inequalities including $x_i \geq 0$ for all i. The set of all feasible points is called the feasible region. A feasible point Y is said to be the optimal solution to the linear programming problem if no other feasible point gives Z a larger value than does Y. That is, $Z(Y) \geq Z(X)$ for any X in the feasible region. An extreme point, also called a corner point, of the feasible region is a point P that is not an interior point of a line segment joining two other points of the feasible region.

For any linear programming problem the following can easily be established:

(1) The feasible region is convex; that is, any line segment joining two points in the region lies entirely in that region.

(2) The feasible region is a closed set; that is, the region contains all of its boundary points.

If in addition the feasible region is bounded, then there will be an optimal solution (a continuous function on a compact set always attains a maximum value). We now demonstrate the key point in solving linear programming problems, namely that the optimum solution must occur at an extreme point of the feasible region.

Suppose P_1, P_2, ... , P_k are the corner points of the feasible region and Q is an interior point (see Figure 1). Examining Z at each of the points P_1, P_2, ... , P_k, let us assume that Z attains its maximum value over these points at P_1. Now assume that the line through P_1 and Q strikes the boundary at point R where R lies between P_i and P_{i+1}. Thus $R = tP_i + (1-t)P_{i+1}$ for some real number t where $0 \leq t \leq 1$. The value of Z at R becomes

$$\begin{aligned} Z(R) = C \cdot R &= C \cdot (tP_i + (1-t)P_{i+1}) \\ &= t(C \cdot P_i) + (1-t)(C \cdot P_{i+1}) \\ &= tZ(P_i) + (1-t)Z(P_{i+1}). \end{aligned}$$

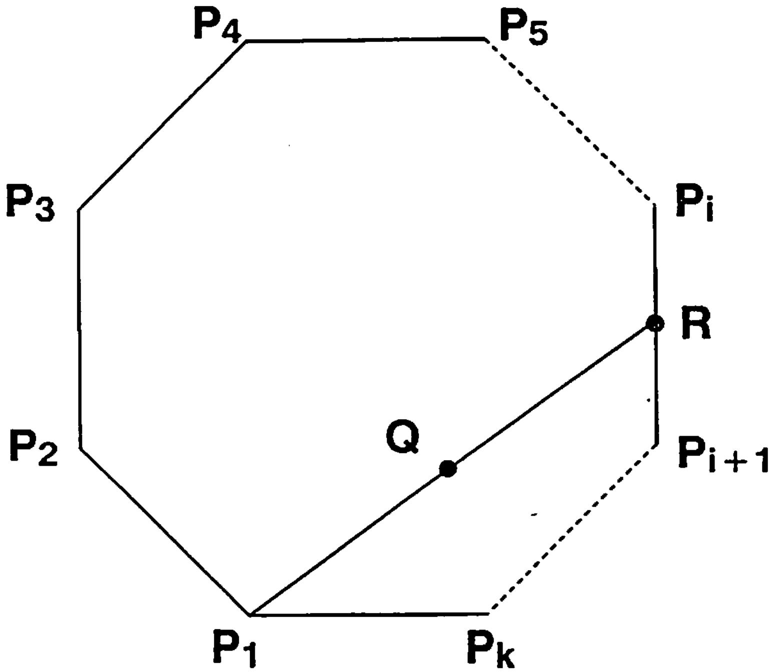

Figure 1.

Since $0 \le t \le 1$, we see that $Z(R)$ lies between $Z(P_i)$ and $Z(P_{i+1})$, both of which are less than or equal to $Z(P_1)$ and thus we conclude $Z(R) \le Z(P_1)$. Using this same argument, since Q falls between P_1 and R, the value of Z at Q is less than or equal to its value at P_1. Hence the maximum value of Z over the entire feasible region occurred at P_1, an extreme point.

We now want to solve the "Scissors-Paper-Stone" game. If we let 1 denote a win and -1 denote a loss, an appropriate payoff matrix for R would be

$$
\begin{array}{cc}
 & \overbrace{\begin{array}{ccc}\text{Sc} & \text{P} & \text{St}\end{array}}^{\text{C}} \\
\text{R}\left\{\begin{array}{c}\text{Sc} \\ \text{P} \\ \text{St}\end{array}\right. & \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}
\end{array}
$$

As indicated earlier, we are free to add 2 to each entry in this matrix and thus have

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

Thus C's problem becomes to maximize

$$\text{maximize } Z = x_1 + x_2 + x_3 \text{ where}$$

$$\begin{array}{rcrcrcl} 2x_1 & + & 3x_2 & + & x_n & \leq & 1 \\ x_1 & + & 2x_2 & + & 3x_n & \leq & 1 \\ 3x_1 & + & x_2 & + & 2x_n & \leq & 1 \end{array}$$

$$\text{and } x_i \geq 0 \text{ for all } i \leq 3.$$

Figure 2 shows this system graphically and the feasible region is the region on and inside the "box" with corners P_1 to P_8. The coordinates of these eight points are given in Table 1.

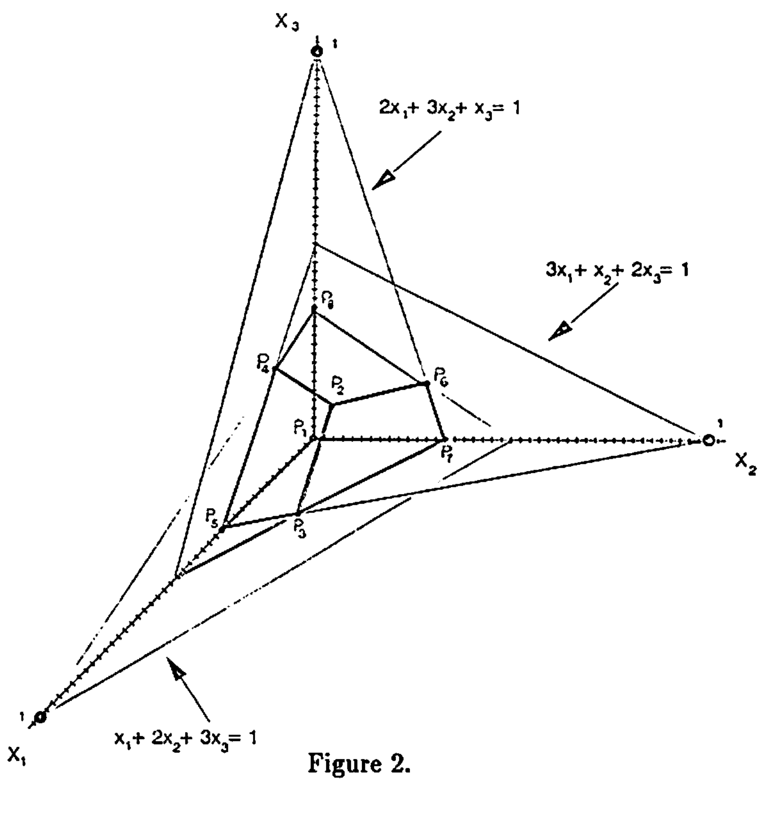

Figure 2.

	Extreme Point (P)			Value of Z
P	X_1	X_2	X_3	Z
P_1	0	0	0	0
P_2	7/42	7/42	7/42	1/2
P_3	12/42	6/42	0	3/7
P_4	6/42	0	12/42	3/7
P_5	14/42	0	0	1/3
P_6	0	12/42	6/42	3/7
P_7	0	14/42	0	1/3
P_8	0	0	14/42	1/3

Table 1.

Examining Z at each extreme point P_i (where $1 \leq i \leq 8$), we find the maximum value of Z to be 1/2 occurring when $x_1 = x_2 = x_3 = 7/42$. Since $V = 1/Z$, we then have $V = 2$ and $q_1 = x_1V = (7/42)(2) = 1/3$, $q_2 = x_2V = (7/42)(2) = 1/3$ and $q_3 = x_3V = (7/42)(2) = 1/3$. Therefore we conclude that the optimal strategy for C is (1/3, 1/3, 1/3). This indicates C should play each of his 3 moves 1/3 of the time each.

Because of the complexity of the other two games we are considering, we need another method for solving linear programming problems. To our knowledge, this is the first mathematical treatment of these two games. The method we will use is called the "simplex method." The simplex method was developed in the summer of 1947 by Dr. George B. Dantzig. Dr. Dantzig and a group of his colleagues were asked by the Air Force to derive an inter-industry model. This model was to be used to efficiently coordinate the energies of whole countries in the event of another world war. The development of large scale electronic computers made it possible to work with such large linear programming problems.

We are now ready to solve Mastermind. Recall that the object of Mastermind is for the code breaker C (in as few guesses as possible) to find the order of colored pegs that the code maker R has chosen. The code maker will score each guess of the code breaker with pegs of his own. He will give the code breaker a white peg for each correct color in the correct position and a black peg for each correct color in the wrong position.

To begin with, we will allow three colors for code making and two holes for the guesses. The three colors are denoted c_1, c_2 and c_3. Thus both code maker and code breaker will have nine possible moves for the initial guess and code. The possible moves for R and C are:

Move	R	C
1	c_1c_1	c_1c_1
2	c_1c_2	c_1c_2
3	c_1c_3	c_1c_3
4	c_2c_1	c_2c_1
5	c_2c_2	c_2c_2
6	c_2c_3	c_2c_3
7	c_3c_1	c_3c_1
8	c_3c_2	c_3c_2
9	c_3c_3	c_3c_3

This means we will have a 9×9 matrix. We set up our scoring for the code maker as follows: two white pegs = -4, one white peg = -2, one black peg = -1, two black pegs = -2 and two empty holes = 4. Thus an appropriate matrix could be

$$\begin{array}{cc} & \mathrm{C} \\ \mathrm{R} & \begin{bmatrix} -4 & -2 & -2 & -2 & 4 & 4 & -2 & 4 & 4 \\ -2 & -4 & -2 & -2 & -2 & -1 & -1 & -2 & 4 \\ -2 & -2 & -4 & -1 & 4 & -2 & -2 & -1 & -2 \\ -2 & -2 & -1 & -4 & -2 & -2 & -2 & -1 & 4 \\ 4 & -2 & 4 & -2 & -4 & -2 & 4 & -2 & 4 \\ 4 & -1 & -2 & -2 & -2 & -4 & -1 & -2 & -2 \\ -2 & -1 & -2 & -2 & 4 & -1 & -4 & -2 & -2 \\ 4 & -2 & -1 & -1 & -2 & -2 & -2 & -4 & -2 \\ 4 & 4 & -2 & 4 & -2 & -2 & -2 & -2 & -4 \end{bmatrix} \end{array}$$

Adding five to each entry and using the resulting matrix on a software package that performs the simplex method, we find that the code maker should choose moves 1, 5 and 9, 1/3 of the time each and the code breaker should choose moves 2, 3 and 8, 1/3 of the time each. Using a different software package, we could have as easily gotten different moves for the code breaker; for example, moves 2, 6 and 7 could have been chosen. The simplex method does allow ties of the optimal solution to occur at different points. Thus the code maker should choose codes of one color only and the code breaker should initially guess different colors.

The last game we want to solve is Battleship. Due to the complexities of the full scale version of Battleship we deal with one ship, taking up 3 slots, on a 5×5 board. Thus with these boundaries, we find thirty moves possible for the target, C, and twenty-five moves for the shooter, R. The twenty-five moves for the shooter are simply the twenty-five slots on the board and Figure 3 shows three of the thirty possible moves for the target. We set up scoring for the shooter so that his payoff is 1 for a hit and -1 for a miss. Hence our matrix will be 25×30 and consist of 1's and -1's.

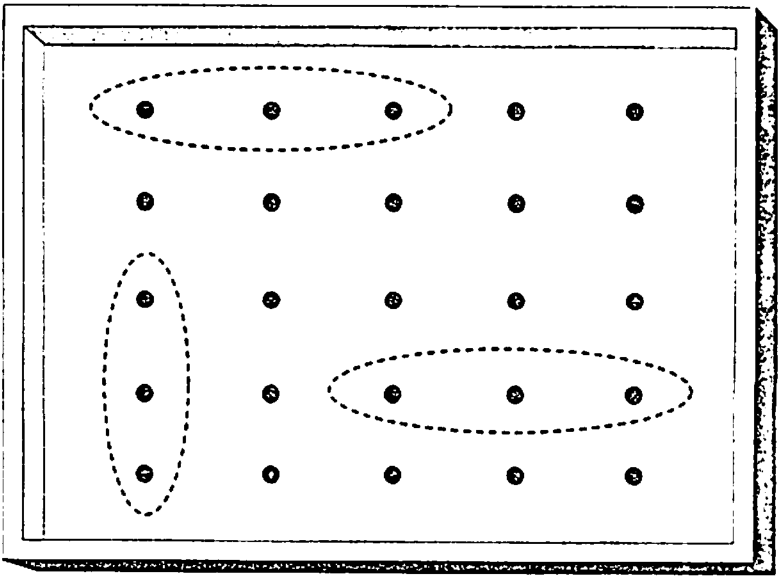

Figure 3.

Due to the limitations on memory in personal computers, this problem was solved using the Alabama Cray Supercomputer. The program to solve the problem was written in FORTRAN and makes use of the IMSL libraries on the supercomputer which contain FORTRAN subroutines that perform the simplex method. The resulting optimal positions for the shooter and target are shown in Figure 4 and Figure 5. In Figure 4 we see the eight optimal positions for the target. Each of these positions should be chosen 1/8-th of the time each. So now you know my secret.

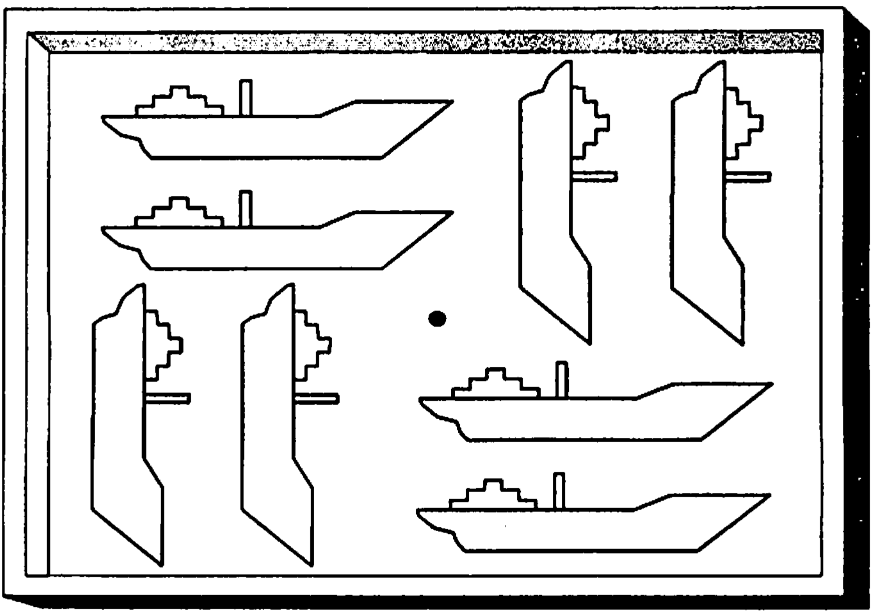

Figure 4.

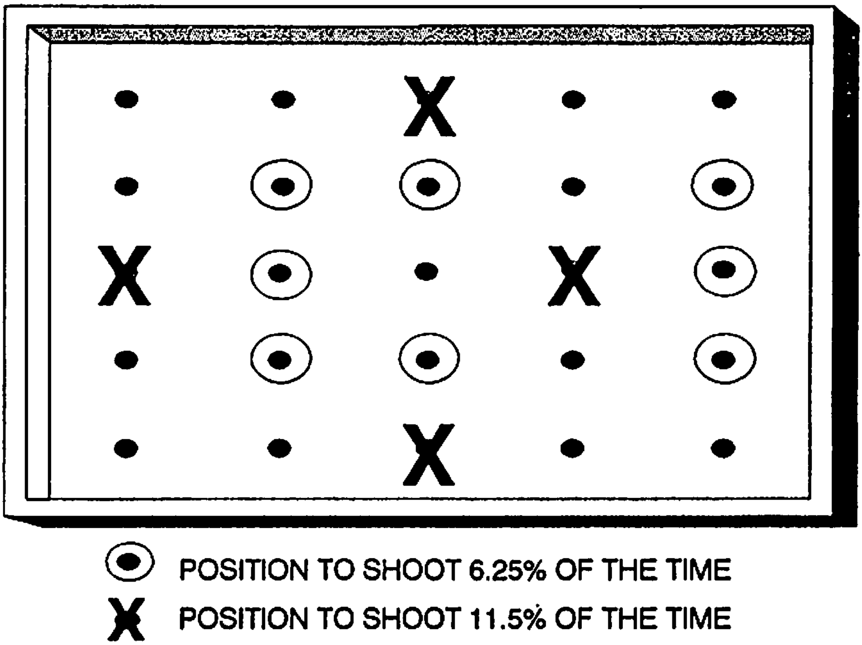

Figure 5.

Acknowledgements. I would like to thank Dr. Eddy Joe Brackin, Dr. Oscar Beck and Mrs. Patricia Roden for their help and support.

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Magical Minimal Mania

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Presented at the 1991 National Convention.

Mother Nature makes things look so easy. It's almost as if she teases us ... she educates us and pushes us into learning and digging deeper. Mother Nature has the answers right away but she leaves it for humans to figure out for themselves. The answers do not come so easily for humans.

One example of a subject that Mother Nature portrays as being so simple but really isn't is know as "Plateau's Problem." Plateau, living from 1810-1883, made many interesting experiments on the subject of his problem. The problem itself is older and goes back to the initial phases of the calculus of variations. In its simplest form, Plateau's Problem is "to find the surface of smallest area bounded by a given closed contour in space" [1, p. 386]. These surfaces have come to be known as minimal surfaces; there are an infinite number of minimal surfaces.

Although minimal surfaces occur in nature, their study can be quite complex. In the words of Ivars Peterson,

> It lies at the intersection of geometry and analysis, where curve and shape meet tangent and area. Analysis includes calculus and differential equations, which can be used to characterize minimal surfaces. These equations define how a surface changes from place to place, providing the coordinates for a kind of topographic map of the surface in space. [3, p. 50]

It is difficult to find a concise definition of minimal surfaces since there are many different was to define them. They can most easily be defined in terms of soap films. For instance, if a circular loop is dipped into a soap solution and then extracted, the soap film that spans the loop has the shape of a flat disk. This is a minimal surface because, of all the surfaces that could conceivably span the wire loop, the flat disk has the least area. Nature forms minimal surfaces because they are physically stable: minimal area means minimal stored energy [2, pp. 128-129]. The tendency is always to seek a state of lowest energy. Stretching a soap film increases its potential energy; work must be done to deform it. A soap film seeks a shape that minimizes its potential energy, and because its surface energy is proportional to its area, it automatically assume the form of a minimal surface [3, p. 48]. Perhaps the simplest definition to work with is: a minimal surface is the smallest area that spans a given curve [4].

The idea of minimal surface also goes into the realm of the infinite. This may seem like a contradiction. After all, how can something that is minimal be infinite at the same time? How can something which minimizes area have an unbounded area? An infinite surface is minimal when any sufficiently small finite region of the surface minimizes area [2, p. 129]. Imagine the plane for a minute. A plane extends forever in all directions, but it is a minimal surface. Take any small finite region of the plane and you can see that it minimizes area. The catenoid is another example of such a surface.

Mathematicians seem to be interested in a very specific type of minimal surface; they are interested in complete, embedded, minimal surfaces of finite topology. A complete surface is one without boundaries. Embedded surfaces do not intersect themselves [3, p. 127]. A surface of finite topology is one which has a finite number of holes and/or punctures.

As you have probably guessed by now, there are many forms that a minimal surface or soap film can take. For this reason, mathematicians have developed schemes for putting surfaces into different categories. One useful classification is based on topological type. Two rubber surfaces are the same, topologically, if one can be deformed into the other by stretching, shrinking, twisting or any other manipulation that doesn't involve ripping, puncturing or filling holes. Categorizing surface includes placing them in a certain topological genus or class. Any topological shape with no holes or handles in it (such as a sphere, a bowl or a coin) is in the same topological class or genus; in this case, genus 0. Any shape with one hole or handle belongs to genus 1. Examples of this are a doughnut, a coffee mug and a sphere with a handle. Shapes with two holes in them are of genus 2. It becomes evident that the genus of the surface is equal to the number of holes or handles in the surface.

The following figures illustrate the idea of topological genus. Figure 1 illustrates objects of genus 0, 1 and 2.

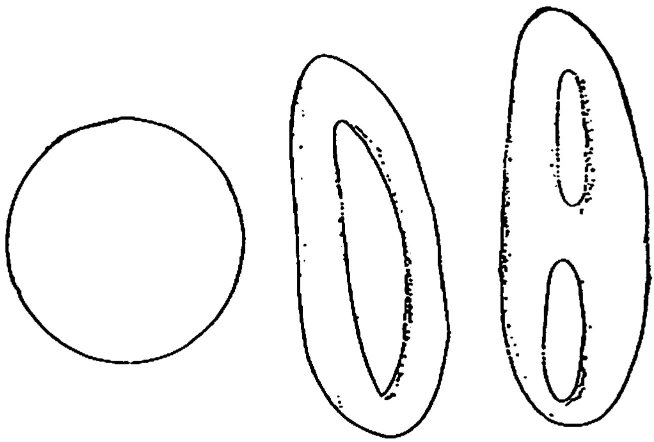


Figure 1.

The next figure (Figure 2) illustrates the coffee mug and the doughnut sharing the same genus, which is genus 1.

Figure 2.

Different sources use the word "hole" differently. It can also mean a puncture point or an end. In this paper, it will be synonymous with "handle."

Topological surfaces may also be punctured in a certain number of places. Each puncture point is called an end. The end can be created by piercing a surface, putting a single tiny hole in it. It is like taking a point out of the surface. This end can be pulled open indefinitely. Figure 3 illustrates how a sphere with one end can be stretched out to form an infinite plane. The second part of the figure illustrates how a sphere with two ends can be transformed into an infinitely extended catenoid.

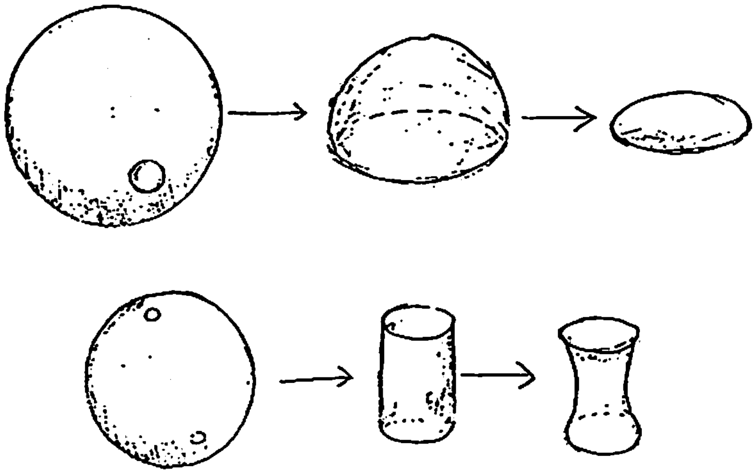


Figure 3.

Topological type and minimal surface type depends on a surface's genus and the number of ends.

Now that we know how minimal surfaces are classified, we can take a look at the history of this fascinating mathematical subject. As stated before, a serious study of minimal surfaces started with Plateau, a Belgian scientist, and we now mention a few highlights of the history of minimal surfaces.

Leonhard Euler, an 18th century Swiss mathematician, discovered an identity dealing with minimal surfaces. He claimed that all non-planar minimal surfaces must be saddle shaped and that the mean curvature at every point must be zero. The solution was shown to exist for many particular cases during the 1800's. The existence of the solution for the general case was proved in the early 1900's by J. Douglas and T. Rado. The mean curvature of a surface at a point P is defined in the following way: Consider the perpendicular to the surface at P and all planes containing it. These planes will intersect the surface in curves which in general have different curvatures at P. Now consider the curves of minimum and maximum curvature respectively. One half the sum of these two curvatures is the mean curvature of the surface at P [1, pp. 386-387]. Figure 4 (adapted from [1]) shows a saddle shaped surface have a mean curvature of zero. The surface has a mean curvature of zero since it curves both downward and upward in the vicinity of every point.

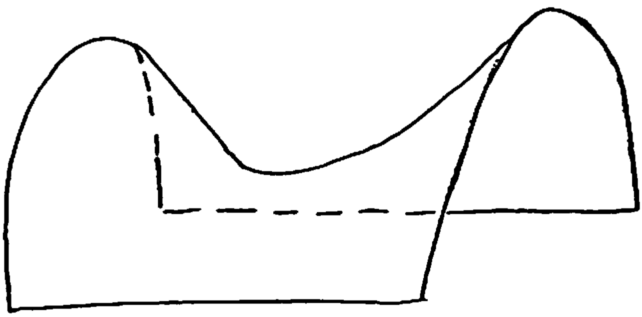
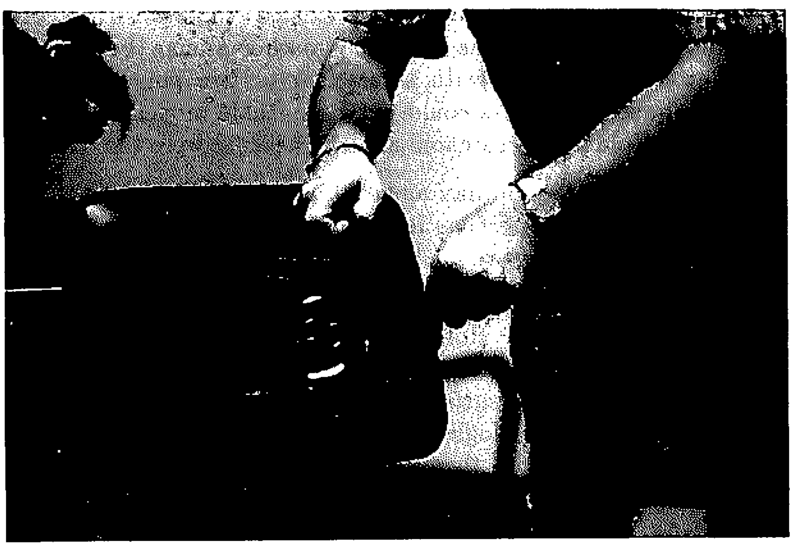


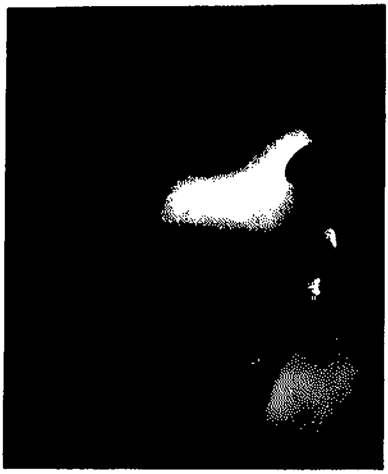
Figure 4.

With regard to soap films, saying that the mean curvature must be zero is equivalent to noting that the pressure on both sides of the soap film is the same.

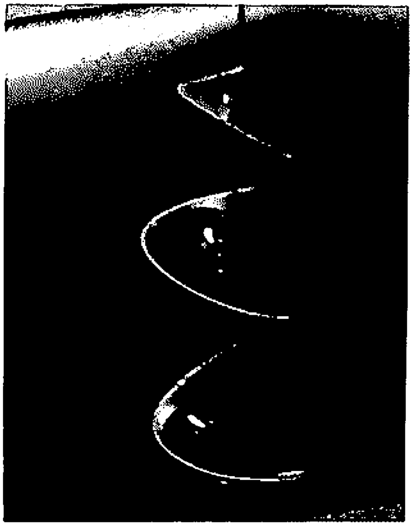
Until recently, only three infinite minimal surfaces had been known that were complete and embedded. These three were the plane, the catenoid and the helicoid. The flat disk soap solution is a piece of the plane. The catenoid, as illustrated earlier, has the same topology as a hollow sphere that has two ends in it. An unbounded helicoid resembles a screw that is extended to infinity. Pictures 1-3 show soap bubble formations of the catenoid, the plane and the helicoid.



Picture 1.



Picture 2.



Picture 3.

Only a few years ago it seemed unlikely that a fourth kind of complete, embedded, minimal surface of finite topology existed. In 1981, Rick Schoen at the University of California in San Diego proved that a hollow sphere could serve as a model only for the catenoid. No other surface of the same topological type existed that was free of self-intersection. Also in 1981, the Brazilian mathematician Luquesio Jorge proved that hollow spheres with three, four or five puncture points or ends and no holes or handles could not be suitable models [2, p. 132]. It seemed that the plane, the catenoid and the helicoid were the only ones.

However, David A. Hoffman and William H. Meeks broke the trend. Meeks says, "There were reasons for suspecting that these surfaces exist. If they were to exist, they would have certain symmetries, and from the symmetries one can derive the equations" [4].

In November of 1983, Hoffman learned of a graduate student in Brazil named Celso Costa whose dissertation included some thorny equations for a proposed surface that Costa was able to prove was infinite, minimal and topologically the same as a three ended surface belonging to genus 1. Note that neither Schoen nor Jorge proved that this model could not work.

The major question still unanswered at that point was "Is it embedded or not?" This seemed hopeless to prove. "When you have the equations for a surface," explains Hoffman, "you can't compute some quantity that says 'Yes, it intersects itself' or 'No, it doesn't.' Basically, all you can show is that a particular piece of the surface doesn't intersect another piece" [2, pp. 133-134]. Even this doesn't get you very far when dealing with an infinite surface.

David Hoffman formulated a plan: he would prove what seemed hopeless to prove. He planned to use a computer to find numerical values for the surface coordinates and then draw a picture, from that, of its core. For assistance, he turned to James Hoffman, a graduate student who was working on a new computer graphics program and language. David Hoffman knew that if he saw evidence of the surface intersecting itself, the mathematical quest would be over. The Hoffmans worked together, spending many hours tinkering with the equations and the software to bring the surface to the screen. If there was no evidence of an intersection, then David Hoffman would go ahead and try to prove that the surface was embedded.

The first pictures of the new surface were quite a surprise for the Hoffmans. The pictures indicated that the surface was both embedded and symmetrical. Hoffman said it looked like the surface was made up of eight identical pieces that fit together [3, pp. 57-58]. Figure 5 is a diagram of the new surface.

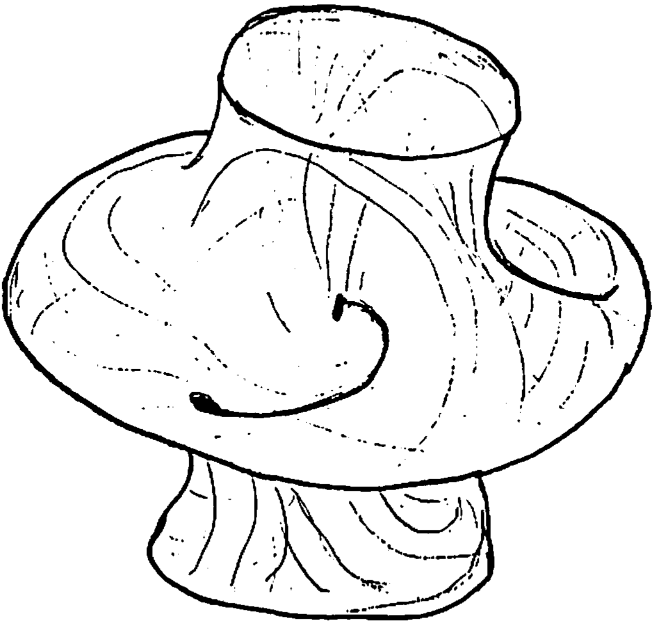


Figure 5.

In [4], Ivars Peterson describes this new minimal surface in this way:

> The new minimal surface has the elegance of a gracefully spinning dancer flinging out her skirt in a horizontal plane. Gentle folds radiate from the skirt's waist. Two holes pierce the lower surface of the skirt and join to form one catenoid that sweeps upward. Another pair of holes, set at right angles to the first pair, lead from the top of the skirt downward into the second catenoid.

The fact that the new surface contains the eight identical pieces and a pair of straight lines made it possible for Hoffman and Meeks to analyze the equations. "This, in turn, led to a mathematical proof that this surface was," David Hoffman says, "in fact, an embedded, complete, minimal surface of a finite type, the first new one to be found in nearly 200 years" [4, p. 169].

The research did not end there. Meeks and Hoffman soon demonstrated the existence of an infinite number of surfaces, each one topologically equivalent to a three ended sphere of any genus number. Once there were only three known complete, embedded, infinite minimal surfaces of finite topology; now there are too many to count.

The history of the study of minimal surfaces will most likely continue. There are still discoveries to be made in this area and there are still many questions left unanswered.

It is important to note the computer's role in Meeks and Hoffman's proof. The computer did not write the proof but it was used as an important tool for the proof. It was most definitely helpful in terms of graphics and also for the structure of the proof.

As in most areas of mathematics, there is a duality in the area of minimal surfaces. There is a pure mathematical aspect to the study as well as an applied aspect. Thus far, the focus has been on the pure aspect. Though minimal surfaces are interesting for their beauty,they also have uses outside the realm of pure mathematics. This only makes sense since "Nature forms minimal surfaces because they are physically stable" [2, p. 129].

Newly discovered forms may be useful for the shape of developing embryos or the structure of certain polymers. A dental surgeon has suggested that such shapes could be used in bone implants for securing false teeth. The implant's design with lots of handles and a least area surface would minimize contact with bone while ensuring a strong bond.

Area minimizing surfaces often occur in physical and biological systems, especially at the boundaries between materials [3, pp. 60-61].

Soap film experiments have been used to design the roofs of several buildings. For instance, architect Frei Otto used soap film experiments to design the roofs of several Olympic buildings in Munich, including the swimming area and the stadium.

These are only a few of the applications of minimal surfaces. There are, no doubt, many more to look forward to. From studying this area of mathematics, it becomes clear that what may take Mother Nature seconds to create takes humans years to figure out.

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Giving Oral Presentations in Mathematics[1]

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These notes will guide you through giving a seminar talk, whether short or long, formal or informal. After you graduate, you will find yourself in many situations where it is important to appear professional and self-confident despite nervousness and uncertainty. You will find the experience of planning a talk to be invaluable later if you make the effort to do it well now. It is normal in public speaking to get stage fright, develop nausea, or go blank (whatever your nightmare is); when you are well-prepared, you can give a successful performance anyway.

Even experienced speakers know that a lot of preparation is required to give a concise and interesting talk. There is a wonderful quote, attributed to Abraham Lincoln: "If you want me to give a ten minute speech, give me two weeks .... If you want a two hour speech, I'm ready now!"

What to include in a talk.

A well-constructed talk has a clear focus. For example, you might present and prove one important theorem, or solve a single interesting problem, or outline a few important results in a single area. Tell the audience enough to hold their interest and to teach them something new, but avoid confusing them with a barrage of detail.

Right at the beginning, state the topic or problem you're going to discuss. State the conclusions or results as early as possible, as well as at

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 A summary of this essay appeared in "Teaching the neglected art of oral presentation," *UME Trends* 3 (December 1991), 2 and it also appeared as an appendix to "An undergraduate seminar emphasizing oral presentation of research mathematics," *Primus* 2 No. 1 (March 1992), 16-32.

the end of the talk. Make sure the audience knows where you are headed before you launch into the details.

Motivate the results you present; let the audience know why the results are interesting or important. For example, give a brief history or an application of a theorem, or show how a problem is related to another well-known problem.

Only give a careful derivation or proof if it will help the audience understand or appreciate a result. It is often better to give only the key ideas rather than a full proof. For example, explaining briefly that an equation can be solved by elementary algebra can be more satisfying than seeing the solution step-by-step. If you are explaining an algorithm, illustrating it with an example can be clearer than writing it out in words.

Keep jargon and symbols to an absolute minimum. Only introduce terms or definitions that you actually use. Remind the audience of the meaning of a new term each time you use it.

Be sure that your talk fits the given time period by timing the talk in advance. Plan the talk so that you can omit or add sections when the talk goes faster or slower than you anticipated. If you are famous or have a spellbinding lecture you may be forgiven for exceeding the time limit; otherwise it is considered rude.

Preparing a talk step-by-step.

1. Write down ideas, ignoring organization and specific wording at first. It is often easier to write short sections rather than trying to write the whole talk at once.
2. Organize the material you have and prepare an outline. If you will be using a chalkboard, write down the things you want to write on the board. If you are making transparencies, work them out roughly on paper; write very large, and don't put any more on each sheet than is necessary.
3. Start over (no kidding)! Decide what is most important, throw away everything else, and rewrite the talk. Pare down what you write on the board or the transparencies to what is absolutely essential. Be merciless: if it does not convey information, leave it out.
4. Prepare what you will say during and between transparencies, or after writing on the chalkboard.

5. Practice and *time* the talk. Revise the talk for length if necessary. Don't talk faster, make the talk shorter! Write up a final draft of your lecture notes and draw final versions of slides on paper.
6. Get constructive criticism from someone experienced (such as your instructor). Generally, you will find people willing and able to help you only after you have made the effort to prepare a good draft. Do not get upset if you are told to start over or make major revisions, no one makes substantial suggestions unless they care about your performance.
7. Revise the talk once more. If using transparencies, transfer your sketches from paper to transparencies. If necessary, rewrite your notes so that you can read them easily; write down reminders to yourself (such as "pause," "check time," or "slow down here"). Number your notes and transparencies so you can reorganize them if you drop them.
8. Dress rehearsal: find a private place to practice the talk in full and time it again. Save some energy for the actual talk, however; too much practice will make a talk sound mechanical.
9. The real thing.
10. Celebrate!

This may seem like a lot of work for a short presentation, but preparation and practice are the secret to appearing knowledgeable and composed. Giving a talk requires rehearsal, just like performing music or acting. The writing and rewriting not only improve the talk, but help you to better understand your topic. If you care about your audience and want them to follow your talk, you will find that the work pays off.

Looking professional.

The primary rule in giving a good talk is to be considerate of your audience. It is surprisingly easy to lose the audience, so keep things simple. The audience would rather understand your talk than be impressed by your extensive knowledge.

Talk only about what you know; if you don't understand it, the audience won't either.

Illuminate results rather than just stating or verifying them. A well-chosen, elementary example is often better than a formal proof.

Avoid messy arithmetic unless it is the point of the talk. You do not have to show all intermediate steps for every derivation. Few listeners will get upset if you say, "And then, by a tedious but straightforward calculation, which I will omit, we get ..."

A talk that begins, "Let me get some definitions out of the way ... ," followed by three slides covered with tiny writing, is deadly for an audience. It is like trying to follow a conversation in a new language after only one glance at a dictionary. Only define a new term if you plan to use it immediately. Give simple examples or pictures to illustrate definitions. Briefly remind the audience of the definition each time you use it.

The audience usually doesn't start concentrating until there is a visual cue, such as a transparency, and may miss that which is only spoken. A good general strategy is to "show" first and then "tell" — that way the audience has two chances to get the point. However, choose visual aids carefully; make sure they are easy to see and as simple as possible. Be creative, but don't exhaust the audience with special effects.

If you want to give an argument step-by-step, a chalkboard is useful, since you can refer back to definitions or earlier steps. Also, writing on the board can help you slow your pace. If you have a complex figure or chart, which takes too long to draw on the board, use transparencies or photographic slides. If you use transparencies alone, organize your talk so that one does not have to remember a previous slide. It is difficult to retrieve a transparency after it has been shown and people have short memories. If you use a physical model, make sure it is large enough.

Write large and legibly, and only write down what is necessary. Repeat and explain everything that you write. Leave transparencies up as long as possible, so people have time to read them.

Let the audience know when you are about to finish the talk: conclude, don't just stop talking. For example, summarize what you have said or discuss the questions left unanswered by the work presented.

Have extra material prepared and decide in advance what you can omit. Anticipate questions and prepare answers in advance. It is hard to think when you are nervous.

Turn up your volume. Talk to the people at the back of the room. Make eye contact, or at least lift your head and face the audience; this makes your voice more audible and animated.

Get rid of potential distractions like keys or bracelets. Expect to sweat and get chalk on your clothes, and dress accordingly. Bring extra chalk, or pens for transparencies. Bring whatever you need to be comfortable, such as a cup of water or tissues.

No matter how terrible you feel, the audience can't tell that your mouth is dry, your palms are sweating, and your knees are shaking — so relax.

Appendix. Standard visual aids.

Using a chalkboard.

If you have never used a chalkboard before, or have a difficult figure to draw, practice in advance. Write large and press hard. If you are left-handed, practice writing until you can write from left to right without accidentally erasing what you've written.

Erase the entire board before you begin. Make sure you have enough chalk before you start. If you want to use colored chalk, bring your own. If the chalk squeaks or vibrates, break it in half.

Give the audience as much time as possible to read and absorb what is on the board. Don't erase anything until absolutely necessary. After you write something down, step away from the board so everyone can see. Avoid talking while facing the board. Repeat what you have written after it is complete.

Most people like to fill up the sections of the board from left to right, so you can read them like pages in a book. Try not to write across the vertical lines in the board; many people find this disturbing. Don't bother to correct small errors and don't worry about making the writing look perfect (this can slow your pace enough to be distracting).

Using transparencies.

Transparencies for an over head projector ($8\frac{1}{2}''$ by $11''$), also called slides, foils, or viewgraphs, are common in scientific presentations. You can write on them or transfer figures directly using a copy machine. Transparencies and pens are usually available at office or art supply stores.

For the greatest impact, use relatively few slides, with simple, carefully chosen images. Everything on a slide should be essential — if not, get rid of it. Put only what is *logically* necessary on a single slide, not what is physically possible.

You can use different colors for emphasis or to make a picture clearer. However, changing colors randomly or using too many colors can be distracting.

The audience will be looking at the transparencies from across the room, looking over other people's heads. Write very large; four or five words to a line is about right, using all capital letters. If you prepare your transparencies with a word processor, use a *very* large font. Leave space between lines. Leave at least a one inch margin top and bottom to avoid having parts cut off (most overhead projectors have a square platform). If you have figures or pictures, check that the images are visible from far away.

After you put a slide down, step away from the projector so the audience can see it. Look at the screen to check the picture. To draw attention to part of a slide, point to the screen with a long pointer or pen (with the cap on!). If this is awkward, lay a pen down on the slide pointing in the right direction. Hands tend to shake, so avoid using your finger as a pointer. Holding on to the slide while talking makes it wiggle around and makes it hard to read. If your slides curl up or blow away, weight them down with coins or pens.

Leave slides up as long as possible; many people read slowly. If there are words on a slide, it's a good idea to repeat them all, in case not everyone can see all the words, and so the audience does not worry that they have missed something.

You can overlay transparencies to show a construction or algorithm in stages. If you overlay one or more transparencies, it is a good idea to tape them together into a "book"; it's hard to line up figures when you're nervous! Slides are not perfectly transparent, so if you stack them, make sure you can still see through the stack.

The best all-purpose slides seem to be those from regular copiers; less expensive types tend to curl or stick together; some are covered with a film and cannot be erased. If the talk is to be given more than once, you should use "permanent" pens. These usually can be erased with alcohol (experiment first). I have had good results with both "Stabilo" pens and "Sharpie" laundry markers. Water-soluble pens are easier to erase and are good for writing on a slide during a talk, but run easily.

When writing on slides, keep a piece of paper under your hand to prevent smearing. Avoid handling the slides; protect the faces by putting sheets of paper between them.

Abstracts of Papers Presented at the 1992 Region IV Convention

Edited by Mary Sue Beersman, Region IV Director

The 1992 *KME* Region IV Convention was held 10-11 April 1992 at Emporia State University. The Kansas Beta chapter members were great hosts and "a good time was had by all" ninety-four students and faculty representing fifteen chapters. There were six presentations, five of which have abstracts given below. First prize went to Yvonne Shaw from Missouri Kappa.

Joseph R. CLOUTIER, *The Busy Beaver; Another Perspective.* Kansas Beta (Emporia State University, Emporia).

An explanation of the operation of a Turing Machine (originally described by Alan Turing in 1936) and an examination of the Busy Beaver Turing machine (the result of a game invented by Tibor Rado in 1962) precede an outline of the investigation into the Busy Beaver problem currently underway at Emporia State University. The Busy Beaver is a Turing machine that has two characteristics: (1) it writes more ones to an initially empty tape than any other machine with the same number of states and (2) it halts. The paper explains a method developed at ESU to order the domain of possible Turing machines that, when used on the known state levels (2, 3 and 4-state) led to a current hypothesis that only 25 percent of the domain need be searched when the domain is ordered by this method. The paper also includes a brief description of algorithms used to reject machines that can not produce Busy Beaver machines. These algorithms achieved rejection rates of 99.7% when tested on the 2, 3 and 4-state machines.

Joely S. EASTIN, *Modeling Using Least Squares.* Missouri Iota (Missouri Southern State College, Joplin).

In this paper we consider multiple linear regression and how it applies to modeling. To do this, we write a computer program which utilizes matrix methods for finding the least squares estimates for the parameters. We then use the program to find a good-fitting model for wind chill index as a function of temperature and wind velocity.

Sarah V. GLEASON, *Introduction to Automata.* Kansas Beta (Emporia State University, Emporia).

This paper provides definitions for finite automata and subautomata. Several examples are given to aid in understanding these definitions. In addition, Zamir Bavel's algorithm, which generates all the subautomata of a given finite automaton, is presented and illustrated. Also two significant types of automata are define, retrievable and strongly connected. Furthermore, it is shown that every strongly connected automation is retrievable, but that the converse does not hold. Finally, some applications of automata are discussed.

Michelle D. LAND, *What is Projective Geometry?* Kansas Beta (Emporia State University, Emporia).

This paper is a brief introduction to projective geometry. The development of projective geometry is discussed and some of the consistencies and inconsistencies between Euclidean and projective geometry are considered. Then, following the presentation of a few of the axioms, the Fundamental Theorem of Projective Geometry is presented and proven.

Yvonne SHAW, *Mysterious Modeling.* Missouri Kappa (Drury College, Springfield).

The algebraic structure developed by the Air Force Armament Laboratory of the Air Force Systems Command in conjunction with DARPA (Defense Advanced Research Projects Agency of the Department of Defense in Washington, D.C.) is used to lay a foundation for edge detection within optical images. Images are defined and the concept of an edge detector is discussed. Two different edge detectors are presented, both of which are based upon local gradient operators. The edge detectors are applied to an image and result in an accurate determination of the edge at the given point.

The Hexagon

Edited by Iraj Kalantari

Computers & mathematics, as a combination, is most prevalent in our present and promises to be even more so in the future. The view on the mixture of the two differs quite a bit. As a new direction of *The Hexagon* column (starting with the next issue), we wish to examine the mixture of computers and mathematics as that of the computer offerings its help as a tool to the art of mathematics. Similar to any other tool that has found its use in mathematics, computers open a new and challenging dimension to mathematical ideas and investigations. In particular, the new wealth of computer software (such as Maple, Mathematica, and others), and availability of computers to students of mathematics makes the application of the tool much more inviting.

In the new *Hexagon* we wish to present mathematical investigations with a major role assigned to computing tools. Studies of problems through computer applications, comparison of those applications, methods of inquiry inspired by the computer, and other related ways of doing mathematics will be the focus of *The Hexagon.*

We invite readers (particularly students) to approach, with the computer as a tool, problem solving, investigation of ideas, and inquiries in general; and submit their findings for this column. In particular, we invite work with Maple, Mathematica, or other applications tailored to mathematical needs. Authors should submit articles in electronic and hard-copy forms to the editor of this column: Iraj Kalantari, Department of Mathematics, Western Illinois University, Macomb, Illinois 61455.

About this issue's author ...

Professor Neža Mramor received her Ph.D. in mathematics from the University of Ljubljana in the field of topology in 1989. She was a Visiting Scholar at Indiana University in Bloomington for the year 1991-92. She is a member of the Faculty for Electrotechnical Engineering at the University of Ljubljana, Slovania. Her interests include topology and algebraic topology.

On Spheres, Sandwiches, and Thieves

Neža Mramor, *faculty*

Indiana University
Bloomington, Indiana

The "issue" in this contribution is a well known classical theorem from topology, the Borsuk-Ulam theorem. It is a fundamental mathematical result with consequences that reach far beyond topology, since it has been, and still is, used to solve numerous problems, and prove some deep theorems in algebra, analysis, combinatorics, applied mathematics ... (compare [6], [4]). We will try to illustrate this by describing three problems, which have in common only the fact that the Borsuk-Ulam theorem is the key to their solutions. The first two problems and their solutions are classical, and can be found in many textbooks on either topology or measure theory, while the third problem is a more recent application of the Borsuk-Ulam theorem. But first, a few words about the theorem itself.

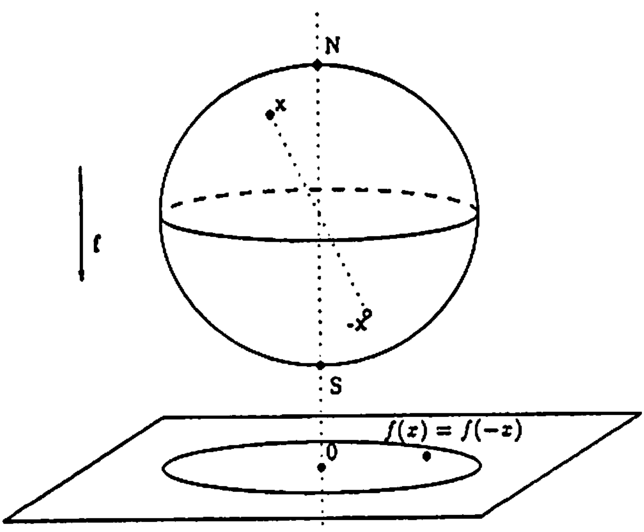

Figure 1.

If we project the sphere $S^2 = \{x \in R^3: |x| = 1\}$ onto the horizontal plane in R^3, then the north and the south pole are both mapped to 0, so there exists a pair of antipodal points on the sphere with the same image. The Borsuk-Ulam theorem states that this is actually true for any map from the sphere to the plane (see Figure 1). More generally:

Borsuk-Ulam Theorem. For any continuous map $f: S^n \to R^n$ from the n-dimensional sphere $S^n = \{x \in R^{n+1}: |x| = 1\}$ to the n-dimensional space R^n there exists a pair of antipodal point $\{x, -x\} \subset S^n$ such that $f(x) = f(-x)$.

In other words, every map from the sphere to the Euclidean space of the same dimension assigns the same value to at least one pair of symmetric points.

The theorem is named after two famous Polish mathematicians, Karol Borsuk who conjectured in 1933 that it is true, and Stanislaw Ulam who gave the first proof a few years later. In 1978, Alexander and Yorke [1] came up with an interesting proof for smooth maps using the *homotopy extension method.* This is a constructive method, which means that it actually provides a way to find a pair of antipodal points with the required property (as opposed to an existential proof, which merely proves the existence of such points without any indication of how to get to them). Since then, this method has become a standard tool in numerical analysis for solving systems of nonlinear equations. Let us describe the basic idea (Figure 2 is an illustration of the case $n = 1$).

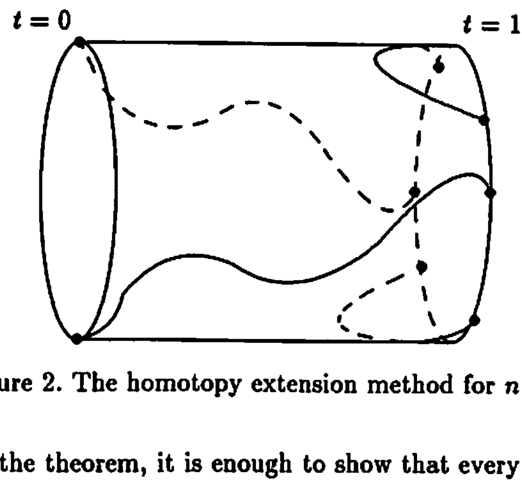

Figure 2. The homotopy extension method for $n = 1$.

To prove the theorem, it is enough to show that every odd map from S^n to R^n has a zero, since this implies that there exists a zero ξ of the odd map $h(x) = f(x) - f(-x)$, so $f(\xi) = f(-\xi)$. Let $\pi: S^n \to R^n$ be the orthogonal projection onto the horizontal "plane" $\{x=(x_0,x_1,\ldots,x_n): x_n=0\}\subset R^{n+1}$ (in Figure 2, where $n=1$, this is just the horizontal line), and let $H:S^n\times[0,1]\to R^n$ be the *linear homotopy* between the maps π and h:

$$H(x,t) = \begin{cases} \pi(x) & \text{if } t=0 \\ h(x) & \text{if } t=1 \\ (1-t)\pi(x)+th(x) & \text{if } 0<t<1 \end{cases}$$

Since

$$h^{-1}(0) = H^{-1}(0) \cap (S^n\times\{1\}),$$

we must show that the set $C=H^{-1}(0)\subset S^n\times[0,1]$ has a nonempty intersection with the $t=1$ level of the cylinder $S^n\times[0,1]$. If h is smooth, also H is a smooth map, and C will very likely be a smooth curve in $S^n\times[0,1]$ (this will be true whenever 0 is a *regular value* of the map H — if this is not true, a slight adjustment of the map H is necessary). C has at least two endpoints — the two points $\{(0,\ldots,0,\pm 1)\}$ in the intersection $C\cap(S^n\times 0)=\pi^{-1}(0)$. (There can of course be more than two endpoints, since there is no reason for C to be a connected curve.) On the other hand, $H(x,t)=H(-x,t)$ for all x and t, and so the curve C is symmetric with respect to x. A bit of simple reasoning tells us that there is no symmetric curve on the cylinder $S^n\times[0,1]$ with only two endpoints, both at $t=0$, and so C must have additional endpoints on the opposite end of the cylinder, at $t=1$. But this implies that $h^{-1}(0)=C\cap(S^n\times\{1\})\neq\emptyset$.

The Borsuk-Ulam theorem is a theorem about spheres, so it isn't surprising that it has many implications concerning properties of spheres. Probably the best known is the following.

Theorem. In every family of $n+1$ closed subsets $\{M_1,\ldots,M_{n+1}\}$ of the sphere S^n, which cover the sphere, i.e. $M_1\cup\cdots\cup M_{n+1}=S^n$, at least one set contains an antipodal pair of points.

Proof. Assume that the theorem isn't true, and there exists a family $\{M_1,\ldots,M_{n+1}\}$ with the property that $M_i\cap -M_i=\emptyset$ for all $i=1,\ldots,n+1$ (where $-M_i=\{-x:x\in M_i\}$ is the antipodal set to M_i). Since all the sets M_i and $-M_i$ are closed, there exists for every $i=1,\ldots,n$ a continuous map $g_i:S^n\to R$ such that $g_i(M_i)=0$ and $g_i(-M_i)=1$. Let $g=(g_1,\ldots,g_n):S^n\to R^n$. This is a continuous map, so, by the Borsuk-Ulam theorem, $g(z)=g(-z)$ for some point $z\in S^n$. Now, $g(x)\neq g(-x)$ for every $x\in M_i$, $i=1,\ldots,n$, so $z\in M_{n+1}$. But by the same argument. $-z \in M_{n+1}$, and so M_{n+1} contains the antipodal pair $\{z, -z\}$. This contradicts the assumption that $M_{n+1} \cap -M_{n+1} = \emptyset$, and so the statement of the theorem must be true.

This result is actually older than the Borsuk-Ulam theorem, and it is closely related to the concept of *Lusternik-Schnirelmann category*. The interested reader can find more on this topic for example in [5].

Most of you have some experience with sharing sandwiches, and you know that it is practically impossible to cut a ham and cheese sandwich in half in such a way that both parts have the same amount of bread, ham and cheese. Well, difficult as it may seem, it can always be done with one single cut, as the following theorem tells us. This classical application of the Borsuk-Ulam theorem to measure theory was first proved by Banach almost half a century ago.

Ham Sandwich Theorem. Let $M_1, \ldots, M_n$ be a collection of bounded sets in R^n with volumes $m_1, \ldots, m_n$ (this implies of course that all the sets are measurable). Then there exists an $(n-1)$-dimensional plane in R^n which simultaneously divides all the sets.

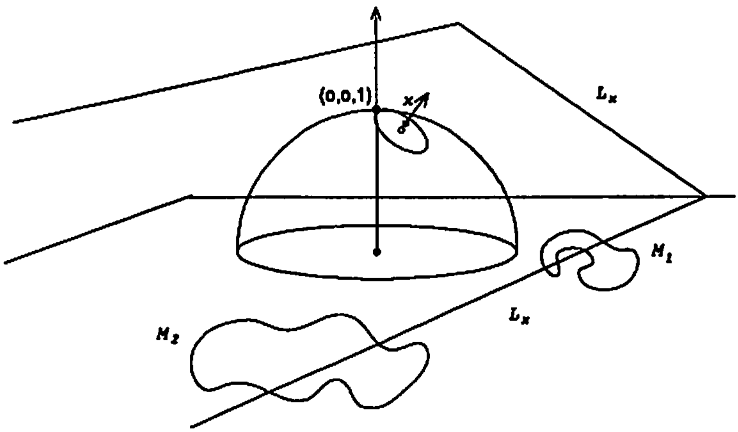

Figure 3. Cutting two sets M_1 and M_2 in the plane.

Proof. Imagine R^n as the "horizontal plane"

$$\{x = (x_1, \ldots, x_{n+1}) \in R^{n+1} : x_{n+1} = 0\}.$$

For every point $x \in S^n$, let L_x be the n-dimensional plane through the point $(0, \ldots, 0, 1)$ which is orthogonal to the unit vector pointing to x. For every $i = 1, \ldots, n$, let f_i be the map which assigns to every $x \in S^n$ the volume of the part of M_i lying on the same side of L_x as the point $x + (0, \ldots, 0, 1)$ and let $f = (f_1, \ldots, f_n): S^n \to R^n$. Since the antipodal points x and $-x$ lie on opposite sides of the plane $L_x = L_{-x}$, the sum $f_i(-x) + f_i(x)$ equals the total volume m_i of M_i. By the Borsuk-Ulam theorem there exists a pair of antipodal points $(z, -z)$ such that $f(z) = f(-z)$ and this implies that the plane L_z cuts all the sets M_i into two equal parts. The intersection $L'_z = L_z \bigcap R^n$ is the $(n-1)$-dimensional plane in R^n that we are looking for. Figure 3 illustrates the case $n = 2$.

If M_1 is the bread, M_2 is the ham and M_3 is the cheese, and your sandwich is conveniently situated in R^3, then all you have to do is cut it along the plane L'_z (after solving the minor problem of finding it).

Well, what about thieves? What can they learn from the Borsuk-Ulam theorem? Imagine two thieves that have managed to steal a lovely necklace made of precious beads of k different colors (and values), with an even number of beads of each color. To be completely fair, they want to split the necklace in such a way that each thief gets the same number of beads of each color, making as few cuts as possible. Obviously one cut will sometimes do, but very rarely. For example, for the four-color necklace in Figure 4, four cuts are necessary. The question is, is there a minimal number of cuts that suffices for any distribution of beads, and if so, what is it? Here is the answer:

Necklace splitting theorem. For any necklace of k different colors, there always exists a choice of k cuts such that the pieces can be collected into two groups with the same number of beads of each color.

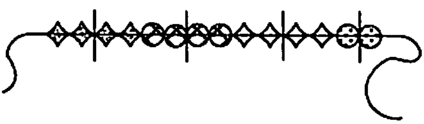

Figure 4. A necklace with 4 kinds of beads which requires 4 cuts.

Proof. Place the necklace on the unit interval I, so that you obtain a coloring of the points of I. For each i, the set $A_i = \{x \in I: x \text{ is of color } i\}$ is a measurable set in I (this simply means that it is a union of intervals, such that the sum of their lengths exists). The trick is to find points $y_0 = 0 \leq y_1 \leq \cdots \leq y_{k-1} \leq y_k = 1$, such that the union of all intervals $[y_{2j}, y_{2j+1}]$, $0 \leq 2j < 1$, contains exactly half of each of the sets A_i. Define a map $f: S^k \to R^k$ in the following way. For every point $x = (x_1, \ldots, x_{k+1}) \in S^k$, let $y_0 = 0$ and $y_j = \sum_{i=1}^{j} x_i^2$, $1 \leq j \leq k+1$.

The points $y_i \in I$ divide I into k segments. Let $m_{i,j}$ be the total length of color i in the j-th segment, and define $f_j(x) = \sum_{i=0}^{k} sign(x_i)\, m_{i,j}(x)$. Then $f = (f_1, \ldots, f_k): S^k \to R^k$ is a continuous function and $f(-x) = -f(x)$, so the Borsuk-Ulam theorem says that there exists a point $\xi = (\xi_0, \ldots, \xi_k)$ such that $f(\xi) = f(-\xi) = 0$. The corresponding points $z_0 = 0 \leq z_1 \leq \cdots \leq z_{k-1} \leq z_k = 1$ determine where the necklace should be cut.

The proof which we have just described is due to Alon and West [2]. Further problems concerning splitting a necklace between l thieves, where $l > 2$, can be found in [3], and an enormous amount of references to other interesting applications of the Borsuk-Ulam theorem in [6].

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The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1993. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1993 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROBLEMS 460-464.

Problem 460. Proposed by the editor. The natural numbers 281 and 1926 have the property that

$$1926^2 + 5 \equiv 0 \pmod{281}$$

and

$$281^2 + 5 \equiv 0 \pmod{1926}.$$

Prove that there are an infinite number of pairs of natural numbers with this property and find an infinite family of solutions.

Problem 461. Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri. A pole of length p is perpendicular to the level ground on which it stands. The pole is surmounted by a sphere with a radius of length r. A person is standing at the "north pole" of the sphere and the person's eyes are at a height m above the north pole. If all distances are measured in feet, find the area of the ground that cannot be seen by the person.

Problem 462. Proposed by J. Sriskandarajah, University of Wisconsin Center-Richland, Richland Center, Wisconsin. Find the sum of the arithmetico-geometric series

$$ag + (a+d)gr + (a+2d)gr^2 + \cdots + (a+(n-1)d)gr^{n-1}$$

and also derive the sum of the corresponding infinite series when $|r| < 1$ where $|\cdot|$ denotes absolute value. Hence find the sum of the infinite series

$$-4 - 2 + 3 - \frac{5}{2} + \frac{7}{4} - \frac{9}{8} + \frac{11}{16} - \cdots.$$

Problem 463. Proposed by Lamarr Widmer, Messiah College, Grantham, Pennsylvania. This problem is proposed in honor of Volume 52 of *The Pentagon* which is being published in 1993. Find positive integers a, b, c and d such that

$$\frac{52}{1993} = \cfrac{1}{a + \cfrac{9}{b + \cfrac{9}{c + \cfrac{3}{d}}}}$$

Problem 464. Proposed by Mary Elick, Missouri Southern State College, Joplin, Missouri. Let C denote the curve given by the equation

$$x^{2/3} + y^{2/3} = 1$$

as shown in Figure 1.

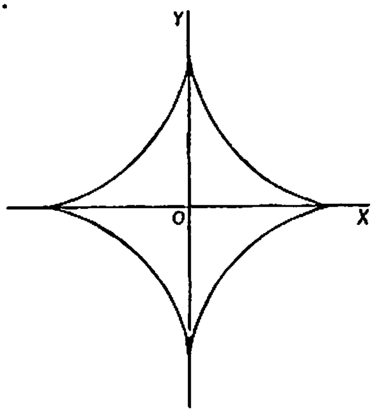

Figure 1.

Suppose that the curve C is made from a flexible material which is attached to the coordinate axes at the points (0,1), (1,0), (0,-1) and (-1,0) and which may be moved without changing the length of C. Let C_p denote the "companion curve" for C which is formed by inflating C with air until it "pops outward" as shown in Figure 2. (a) Find the equation of the companion curve C_p. (b) Find the derivative, if it exists, at the point (0,1) on the curve C_p. (c) Given that the curve C is the circle described by the equation $x^2 + y^2 = 1$, what is the equation of the "popped in" companion curve C_p resulting from "deflating" curve C appropriately?

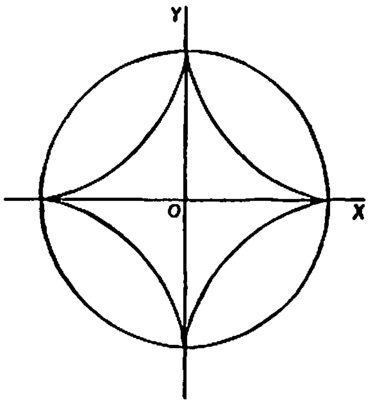

Figure 2.

Please help your editor by submitting problem proposals.

SOLUTIONS 450-454.

Problem 450. Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Determine the smallest odd number $k > 1$ of squares of consecutive positive integers whose sum is a perfect square.

Solution by the proposer.

We shall show that $k = 11$. Let n be a positive integer and N^2 be the sum of $k = 2n+1$ squares of consecutive positive integers. Then

$$\begin{aligned} N^2 &= (a-n)^2+(a-n+1)^2+\cdots+a^2+\cdots+(a+n-1)^2+(a+n)^2 \\ &= (2n+1)a^2+\frac{n(n+1)(2n+1)}{3} \qquad (*) \end{aligned}$$

where a is an integer $\geq n+1$.

When $n=1$, $(*)$ becomes $N^2=3a^2+2$; this equation has no solutions in integers because no square is congruent to 2 (mod 3).

When $n=2$, $(*)$ becomes $N^2=5a^2+10=5(a^2+2)$. This equation has no solutions because it it equivalent to the congruence $a^2 \equiv 3$ (mod 5) which has no solutions.

When $n=3$, $(*)$ becomes $N^2=7a^2+28=4(a^2+4)$. This equation has no solutions because it it equivalent to the congruence $a^2 \equiv 3$ (mod 7) which has no solutions.

When $n=4$, $(*)$ becomes $N^2=9a^2+60$. This equation has no solutions because N must be divisible by 3 which, in turn, requires that we must have $N^2-9a^2=60$ be divisible by 9. This is impossible.

When $n=5$, $(*)$ becomes $N^2=11a^2+110=11(a^2+10)$. In order for this equation to have any integer solutions, we must have $a^2 \equiv 1$ (mod 11); hence $a \equiv 1$ (mod 11) or $a \equiv 10$ (mod 11). Recalling that a must be an integer $\geq n+1=6$ and the restrictions on our choice of a, we can construct the following table:

a	$11(a^2+10)$
10	NOT a perfect square
12	NOT a perfect square
21	NOT a perfect square
23	$5929=77^2$

One can easily check that

$$77^2=18^2+19^2+20^2+21^2+22^2+23^2+24^2+25^2+26^2+27^2+28^2$$

Also solved by Charles D. Ashbacher, Cedar Rapids, Iowa.

Problem 451. Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Find the value of the following limit.

$$\lim_{n\to\infty} e^{-n}\left(1+\frac{1}{n}\right)^{n^2}$$

Solution by Sean Forbes, student, Drake University, Des Moines, Iowa.

Let

$$y = e^{-n}\left(1+\frac{1}{n}\right)^{n^2}$$

Then

$$\ln y = -n + n^2 \ln\left(1+\frac{1}{n}\right)$$

Thus we can rewrite ln y as

$$y = \frac{-\frac{1}{n} + \ln\left(1+\frac{1}{n}\right)}{\frac{1}{n^2}}$$

which is a "0/0" form to which L'Hospital's rule applies. Applying L'Hospital's rule and simplifying, we get

$$\lim_{n\to\infty} \ln y = \lim_{n\to\infty} \frac{n^{-2} + (-n^{-1})(1+(1/n)\big)^{-1}}{-2n^{-3}}$$

$$= \lim_{n\to\infty} \frac{-n(1/n)}{2(1+(1/n))} = -\frac{1}{2}$$

Therefore

$$\lim_{n\to\infty} y = e^{-1/2} = \frac{1}{\sqrt{e}}$$

Also solved by The Alma College Problem Solving Group, Alma College, Alma, Michigan and the proposer. Two incorrect solutions were received.

Problem 452. Proposed by Russel Euler, Northwest Missouri State University, Maryville, Missouri. Evaluate

$$\int \frac{\cos(x)}{\sin(x)+\cos(x)}\,dx$$

using as few steps as possible.

Solution by The Alma College Problem Solving Group, Alma College, Alma, Michigan.

Denote the given integral by I. Then I can be written as

$$\mathrm{I} = \frac{1}{2}\int\left(\frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)} + 1\right)dx$$

$$= \frac{1}{2}\left(\ln\left|\sin(x) + \cos(x)\right| + x\right) + C.$$

Solution by Sean Forbes, student, Drake University, Des Moines, Iowa.

Denote the given integral by I. Then I can be written as

$$\mathrm{I} = \int \frac{\cos^2(x) + \sin(x)\cdot\cos(x)}{(\cos(x)+\sin(x))^2}\,dx$$

$$= \frac{1}{2}\int \frac{1+\cos(2x)+\sin(2x)}{1+\sin(2x)}\,dx = \frac{1}{2}\int\left(1+\frac{\cos(2x)}{1+\sin(2x)}\right)dx$$

$$= \frac{x}{2} + \frac{1}{4}\ln\left|1+\sin(2x)\right| + C.$$

Solution by Joseph Possert, student, Oral Roberts University, Tulsa, Oklahoma.

Denote the given integral by I. Then I can be written as

$$\mathrm{I} = \int \frac{\cos^2(x) - \sin(x)\cdot\cos(x)}{\cos^2(x) - \sin^2(x)}\,dx$$

$$= \int \frac{\cos^2(x)}{\cos(2x)}\,dx - \frac{1}{2}\int \frac{\sin(2x)}{\cos(2x)}\,dx$$

$$= \frac{1}{2}\int \sec(2x)\,dx + \frac{1}{2}\int dx - \frac{1}{2}\int \tan(2x)\,dx,$$

since $2\cos^2(x) = 1+\cos(2x)$, and then

$$\mathrm{I} = \frac{1}{4}\ln\left|\sec(2x)+\tan(2x)\right| + \frac{x}{2} + \frac{1}{4}\ln\left|\cos(2x)\right| + \mathrm{C}.$$

Also solved by Charles D. Ashbacher, Cedar Rapids, Iowa; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; Scott H. Brown, Stuart Middle School, Stuart, Florida; Mohammad K. Azarian, University of Evansville, Evansville, Illinois; and the proposer.

Editor's Comment. The variety of approaches used to solve this problem is amazing!

Problem 453. Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Prove that if a function f satisfies the equation $f(x+y) = f(x)\cdot f(y)$ for all real numbers x and y and if $f(x) = 1 + x \cdot g(x)$ for all real numbers x where $\lim g(x) = 1$ as $x \to 0$, then $f'(x)$ exists for each real number x and $f'(x) = f(x)$.

Solution by The Alma Problem Solving Group, Alma College, Alma, Michigan.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x)\cdot f(\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x)\big(f(\Delta x) - 1\big)}{\Delta x}$$

$$= f(x) \lim_{\Delta x \to 0} \frac{\big(1 + \Delta x \cdot g(\Delta x)\big) - 1}{\Delta x} = f(x) \lim_{\Delta x \to 0} \frac{\Delta x \cdot g(\Delta x)}{\Delta x}$$

$$= f(x) \lim_{\Delta x \to 0} g(\Delta x) = f(x).$$

Also solved by Charles D. Ashbacher, Cedar Rapids, Iowa; Russell Euler, Northwest Missouri State University, Maryville, Missouri; John R. Hughes Jr., Frostburg State University, Frostburg, Maryland; Matthew Amoroso (student), St. Bonaventure University, Saint Bonaventure, New York; and the proposer.

Problem 454. Proposed by Charles Ashbacher, Hiawatha, Iowa. On page 29 of the book *Unsolved Problems in Number Theory* by Richard K. Guy there is the following problem: "Graham asks if $s(n) = [n/2]$ implies that n is 2 or a power of 3." (a) Prove the converse of this statement. (b) Prove that if $n = 2^j 3^k$ with $j, k > 1$ then n cannot satisfy the conditions of Guy's problem. Here $s(n)$ is the sum of the aliquot divisors of n (excluding n) and $[x]$ is the greatest integer function.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Part (a). If $n = 2$ then $s(n) = s(2) = 1 = [2/2] = [n/2]$. If n is a power of 3, then $n = 1$ or $n = 3^k$ for some positive integer k. If $n = 1$ then $s(n) = s(1) = 0 = [1/2] = [n/2]$. If $n = 3^k$ for some positive integer k, then $s(n) = (3^k - 1)/2 = [3^k/2] = [n/2]$.

Part (b). Let $n = 2^j 3^k$ where j and k are integers such that $j > 1$ and $k > 1$. Suppose that

$$s(n) = [n/2] = 2^{j-1}3^k.$$

Then since $s(n) = \sigma(n) - n$, where $\sigma(n)$ denotes the sum of the positive divisors of n, and because $\sigma(n)$ is a multiplicative function,

$$\begin{aligned} 2^{j-1}3^k &= [n/2] = s(n) = \sigma(2^j 3^k) - 2^j 3^k \\ &= \sigma(2^j)\sigma(3^k) - 2^j 3^k = (2^{j+1} - 1)\left(\frac{3^{k+1} - 1}{2}\right) - 2^j 3^k \\ &= 2^{j+1}3^k - 2^j - \frac{3^{k+1} - 1}{2} \qquad (*) \end{aligned}$$

It follows that 2^{j-1} divides $(3^{k+1} - 1)/2$. Let $(3^{k+1} - 1)/2 = 2^{j-1}q$, where q is a positive integer. Then by dividing the terms of $(*)$ by 2^{j-1}, we obtain

$$3^k = 4 \cdot 3^k - 2 - q.$$

Hence $3^{k+1} - 1 = q + 1$ and $(3^{k+1} - 1)/2 = (q + 1)/2$, making $2^j q = q - 1$. Hence $(2^j - 1)q = 1$ so that $q = 1$ and $2^j - 1 = 1$ which implies that $j = 1$. This contradicts the fact that $j > 1$ and completes the proof of part (b).

Also solved by The Alma College Problem Solving Group, Alma College, Alma, Michigan; Russell Euler, Northwest Missouri State University, Maryville, Missouri; and the proposer.

Kappa Mu Epsilon News

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy *KME* events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, Missouri 64801.

INSTALLATION OF NEW CHAPTERS

South Dakota Alpha

Northern State University, Aberdeen, South Dakota

The installation of the South Dakota Alpha Chapter of Kappa Mu Epsilon was held on May 3, 1992, in the Missouri River Room on the campus of Northern State University. Dr. Harold L. Thomas, National President of Kappa Mu Epsilon, conducted the installation ceremony. Dr. Lynn Hodgson, Chair of the Department of Mathematics and Natural Sciences served as Conductor during the ceremony. Seven students and three faculty constituted the founding group of the new chapter at Northern State University. Those initiated were:

> *Students:* Joseph Brooks, Vicky Hanson, Audrey Hartman, Brenda Rook, Marci Rozell, Cheryl Tewksbury, and Ann Vidoloff.
>
> *Faculty:* Dr. Abid Elkhader, Dr. Lynn Hodgson, and Dr. Raj Markanda.

Following the initiation ceremony, Dr. Thomas gave a brief history of honor societies in colleges and universities and, in particular, the founding of Kappa Mu Epsilon. Several Northern State University administrators attended the 12:30 p.m. installation which was preceded with brunch at 11:30 a.m. Dr. Thomas O. Flickema, Vice President for Academic Affairs, extended congratulations to the group on behalf of the university.

Officers installed during the ceremony were Ann Vidoloff, president; Joseph Brooks, vice-president; Marci Rozell, secretary; and Brenda Rook, treasurer. Faculty members Abid Elkhader and Raj Markanda accepted the responsibilities of the corresponding secretary and faculty sponsor, respectively.

New York Nu

Hartwick College, Oneonta, New York

The installation of the New York Nu Chapter of Kappa Mu Epsilon was held on May 14, 1992, in the Eaton Lounge of Bresee Hall on the campus of Hartwick College. Dr. Robert L. Bailey, National Secretary of Kappa Mu Epsilon, conducted the installation ceremony. Dr. Gary E. Stevens of the Department of Mathematics and originally a student member of Ohio Alpha served as Conductor during the ceremony. Fourteen students and four faculty constituted the founding group of the new chapter at Hartwick College. Those initiated were:

> *Students:* Jennifer Bennett, Claudia-Angelica Chiarenza, Jennifer Cote, Eric DeJager, Justin Fermann, Peter Flinders, Timothy French, Alexey Gerasimov, Noreen Hussey, Joanne Lazarou, Jacalyn O'Connor, Christina Osvoldik, John Pape, and David Piccirilli.
>
> *Faculty:* Dr. Ronald Brzenk, Ms. Maureen Gallagher, Dr. Gerald Hunsberger and Dr. Charles Scheim.

Also in attendance at the 4:00 p.m. installation ceremony were Bryant Cureton, Provost of Hartwick College, and Timothy Keating, Associate Dean of Hartwick College. Officers installed during the ceremony were Eric Dejager, president; Jacalyn O'Connor, vice-president; Timothy French, secretary; and John Pape, treasurer. Faculty member Gary Stevens accepted the responsibilities of both corresponding secretary and faculty sponsor.

A banquet was held at 5:15 p.m. in Dewar Hall Campus Center following which Dr. Bailey gave a brief history of honor societies in colleges and universities and, in particular, the founding of Kappa Mu Epsilon.

CHAPTER NEWS

Alabama Gamma University of Montevallo, Montevallo

Chapter President - Beverly Smith

Other 1992-93 officers: Lucretia Weeks, vice president; Terry Harper, secretary; Marsha Oden, treasurer; Charles F. Coats, corresponding secretary; Gene Garza, faculty sponsor.

California Gamma California Polytechnic State University, San Luis Obispo

Chapter President - Eric Bauer

25 actives, 2 associates

California Gamma held its Annual Banquet on June 6, 1992 at Marie Callender's Restaurant in scenic Pismo Beach. Thirteen pledges were initiated and the annual *KME*-related awards were presented. In the absence of Jeff Stought from Arthur Andersen Company, Dr. Steve Weinstein, newly-elected Chair of the Mathematics Department, presented the Arthur Andersen Award for Academic Excellence to Eric Gordon. Dr. W. Boyd Judd, Professor Emeritus, presented the scholarship which bears his name to Jennifer Huskey. Professor Raymond D. Terry presented the *KME* Founders' Award to Jennifer Courter. The guest speaker was Nancy Monson of Lawrence Livermore Laboratory. Nancy, a former *KME* officer and 1986 graduate of Cal Poly, gave a lively after-dinner address. The banquet culminated a year of hard work marked by weekly meetings from 6:00 p.m. to 7:30 p.m. each Thursday. Invited speakers during the winter and spring quarters included Mick Wets from Fair Isaac Company, Bill Cross from San Luis Obispo High School and other on-campus speakers. The Annual Booksale, held in March, was quite successful. The Annual Tee Shirt Sale was replaced by an on-going sale of sweatshirts bearing an intricate design in which each department in the School of Science and Mathematics was represented by some unique symbol. *KME* was ready, willing and committed to help the Mathematics Department with its Annual Math Contest. However, due to a low response by California high schools this year, the contest was cancelled, thus bringing to a close a 39 year tradition. One reason for the decreased participation is the depressed economy of the nation and the California budget shortfall in excess of 12 billion dollars. In Spring 1992 the active members of the chapter voted to donate \$500 to the Mathematics Department Discretionary Fund to be used for the purchase of hardware/software for the Charles J. Hanks Memorial Computer Lab, which is overseen by Professors Michael Colvin and Donald Hartig. The transfer of funds was effected through internal bookkeeping. However, as part of the banquet program, Susan Rehn, the 1991-92 President of the California Gamma Chapter, presented a symbolic oversized check to Professor Mike Colvin. Other officers for 1992-93: Jennifer Courter, vice president; Henry Mesa, treasurer; Raymond D. Terry, corresponding secretary/faculty sponsor.

Colorado Gamma Fort Lewis College, Durango

Chapter President - Rachel Zeller
20 actives

Colorado Gamma inducted 14 new members during the spring semester. Three students, Duane Brown, Steve Luxa, and Rachel Zeller, traveled with Dr. Gibbs (in his 1967 Cadillac limousine) to the first regional convention at Southwest Oklahoma State University in March. The end of the year nomination meeting featured pizza and videos of the regional convention. Other 1992-93 officers: Mary Wright, vice president; Stacy Maples, secretary; Jim Zieff, treasurer; Richard Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

Colorado Delta Mesa State College, Grand Junction

Chapter President - Christina Files
38 actives, 28 associates

The chapter viewed a film entitled "Non-wellfounded sets and Their Applications," and heard a lecture on statistics given by Dr. Tim Novotny. Other 1992-93 officers: Eric Linville, vice president; Lori M. Wynkoop, secretary; Kathy Inman, treasurer; W. Harold Davenport, corresponding secretary; Clifford C. Britton, faculty sponsor.

Georgia Alpha West Georgia College, Carrollton

Chapter President - Debbie Ingle
25 actives, 6 associates

The Georgia Alpha chapter initiated ten new members on June 3, 1992. Following the initiation ceremony, officers for 1992-93 were elected and a reception was held in honor of the new initiates. At the reception the winners of the three Departmental Academic Scholarships for the coming year were announced. All are members of *KME*: Stacy Hobbs (Crider Scholarship), Debbie Ingle (Cooley Scholarship), and Denise Askin (Whatley Scholarship). Other 1992-93 officers: Denise Askin, vice president; Kevin Gill, secretary; Joy McCallie, treasurer; Joe Sharp, corresponding secretary/faculty sponsor.

Illinois Beta — Eastern Illinois University, Charleston

Chapter President - Laura Tougaw
42 actives

Four meetings were held during the semester. The program for one of these meetings featured Eastern graduates who spoke concerning their experiences with graduate school and/or teaching. Additional activities included the ICTM Conference in March and the *KME* initiation of new members and the *KME* Honors Banquet in April. Other 1992-93 officers: Rodney Johnson, vice president; Wendy Coplea, secretary; Andrew Rice, treasurer; Lloyd Koontz, faculty sponsor.

Illinois Delta — College of St. Francis, Joliet

Chapter President - Mark Mitchell
20 actives, 5 associates

Four members of Illinois Delta attended the regional meeting in Bowling Green, Ohio, in April. Receipts from the sale of taffy apples in the fall helped with travel and lodging expenses. Induction of eight new members was held on April 14. Other 1992-93 officers: Molly Sullivan, vice president; Jennifer Colwell, secretary; Carrie Briscoe, treasurer; Sister Virginia McGee, R.S.M., corresponding secretary/faculty sponsor.

Illinois Eta — Western Illinois University, Macomb

Chapter President - Deana Bobzien
12 actives, 6 associates

MAA visiting lecturer, Dr. T. Christine Stevens of St. Louis University, was the featured speaker for the initiation of nine new members in April. Dr. Stevens' topic was "Mathematics and Science, Gender and Government." In addition, the organization held monthly meetings involving faculty presentations, viewed mathematics related videos, and enjoyed two faculty-student picnics. Other 1992-93 officers: Jeanne Campbell, vice president/treasurer; James Easley, secretary; Larry Morley, corresponding secretary/faculty sponsor.

Iowa Alpha — University of Northern Iowa, Cedar Falls

Chapter President - Julie Beck
35 actives

The Departments of Mathematics and Computer Science moved back into a completely remodeled Wright Hall in January, 1992. Both students and faculty are enjoying the new modern facilities. Students presenting papers at local *KME* meetings included Beth Ehresman on "Long Run Cost Functions," Tascha Yoder on "Experiences in China," Karen Brown on "Error Correcting Codes," and Ted Juhl on "Applications of Lagrange Multipliers in Economics." Jason Auxier addressed the *KME* initiation banquet held at The Broom Factory on April 28 on the topic "Non-linear Dynamics of a Springy Pendulum." Professor Carl Wehmer retired this spring. Iowa Alpha hopes to maintain close contact with both him and his wife, Wanda; they have long been active supporters of the organization. Other 1992-93 officers: Ted Juhl, vice president; Jason Auxier, secretary; Karen Brown, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

Iowa Gamma Morningside College, Sioux City

Chapter President - Doug Rants
12 actives

Other 1992-93 officers: Vimal Kumar, vice president; Taylor Guo, secretary; Mike Murray, treasurer; Steven Nimmo, corresponding secretary/faculty sponsor.

Iowa Delta Wartburg College, Waverly

Chapter President - Nancy Wirth
59 actives, 5 associates

The new term for the chapter began with a pizza party in January. The February program presented by Dr. William Slough and Professor Josef Breutzmann consisted of demonstrations of the mathematics text editors LATEX and MAC. During March, the chapter co-sponsored the fifteenth annual Wartburg Math Field Day and initiated 18 new members. The annual May Term picnic was held at a local park. Other 1992-93 officers: Jeffrey Isaacson, vice president; Melissa Dodd, secretary; Nicole Lang, treasurer; August Waltmann, corresponding secretary; William Waltmann, faculty sponsor.

Kansas Alpha Pittsburg State University, Pittsburg

Chapter President - Ed Morris
40 actives, 17 associates

The spring semester activities started with a pizza party and initiation in February for seventeen new members. Following the initiation ceremony, Rebecca Newcomb presented a program on game activities. The program for the March meeting was given by Tom Bartkowiak, a computer engineer employed in the electronics division of the Eagle Picher Company. Tom spoke about mathematical applications encountered in his work. Seven students and two faculty from KS Alpha attended the Region 4 convention at Emporia State University. An enjoyable time was had by all. The chapter assisted the Mathematics Department faculty in administering and grading tests given at the annual Math Relays, April 21, 1992. Several members also worked on the Alumni Association's Annual Phon-a-thon. The final meeting of the semester was an ice cream and cake social event held at the University Lake. Reports were given by those that attended the Emporia convention in April. Officers for the 1992-93 school year were elected. The annual Robert M. Mendenhall award for scholastic achievement was presented to Jon Sherrod. Other 1992-93 officers: Barry Smith, vice president; Thein Maung, secretary; Jerri Lott, treasurer; Harold Thomas, corresponding secretary; Bobby Winters, faculty sponsor.

Kansas Beta **Emporia State University, Emporia**

Chapter President - David Herrs
18 actives, 6 associate

The focus of the spring semester was the Region 4 *KME* Conference which Kansas Beta hosted in April. By all accounts it was a successful enjoyable convention. Other 1992-93 officers: Sheila Nutter, vice president/secretary; Crystal Meyer, treasurer; Connie S. Schrock, corresponding secretary; Larry S. Scott, faculty sponsor.

Kansas Gamma **Benedictine College, Atchison**

Chapter President - David Klenke
13 actives, 14 associates

The initiation of ten new members was held on February 10. Election of officers followed the initiation. A week later the group gathered to hear faculty member Linda Herndon, OSB, talk about "Implementing a Programming Language." In early March Radu Oprea produced another campus edition of *The Exponent* in which he outlined activities remaining in the spring semester. Associate members were received into the group during Italian Night held in the Roost March 19. In early March members assisted with the Math Tournament for high school students. Students David Klenke, Shelly Kerwin, and Mike Seebeck, along with moderator, Jo Ann Fellin, OSB, attended the Regional meeting in Emporia April 10-11. In late April senior Pamela Clearwater shared information on "Symmetry" which she researched during the spring semester for a seminar course. Eight Kansas Gamma members were awarded a Sister Helen Sullivan Scholarship Award at the Honors

Banquet. A May picnic culminated the spring activities. Other 1992-93 officers: Pamela Clearwater, vice president; Tiffany Opsahl, secretary/treasurer; Jo Ann Fellin, OSB, corresponding secretary/faculty sponsor.

Kansas Delta — Washburn University, Topeka

23 actives

Thirteen students and one faculty member were initiated at the annual initiation dinner on March 29. Several other meetings were held during the semester to conduct chapter business. Student officers will be elected in the fall. Allan Riveland is corresponding secretary; Ron Wasserstein, faculty sponsor.

Kansas Epsilon — Fort Hays State University, Hays

31 actives

In addition to monthly business meetings, a spring banquet was held in late April. Officers will be elected in the fall. Charles Votaw is corresponding secretary.

Kentucky Alpha — Eastern Kentucky University, Richmond

Chapter President - Eddie Robinson

The annual initiation ceremony included an interesting talk by Dr. Kirk Jones which incorporated the use of Mathematica and which was entitled "Jump, I Dare You: An Analysis of Bungee Cord Jumping." The traditional party afterwards was in the student center. The other major event of the semester was the regional convention which the chapter hosted. The day ended with a trip to Keeneland Race Track in Lexington where the Bluegrass Stakes race was being held. The musician, Hammer, just happened to be in the box seat directly above the Kentucky Alpha group. While his horse, Dance Floor, did not win that day, he did let people take pictures of him. Other 1992-93 officers: James Mattingly, vice president; Susan Popp, secretary; Crystal Pendygraft, treasurer; Patrick Costello, corresponding secretary; Kirk Jones, faculty sponsor.

Maryland Alpha — College of Notre Dame of Maryland, Baltimore

Chapter President - Susan Miller

12 actives, 10 associates

On May 4 four new initiates, Danielle Kulick, Susan Miller, Vickie Petrone, and Pamela Rick, were inducted into Maryland Alpha. Following dinner and the induction ceremony, Sandy Burgess, the outgoing president, gave a slide presentation entitled "Computer

Graphics." Other 1992-93 officers: Vickie Petrone, vice president; Sharon Pesto, secretary; Pamela Rick, secretary; Sister Marie Dowling, corresponding secretary; Joseph Di Rienzi, faculty sponsor.

Maryland Beta — Western Maryland College, Westminster

Chapter President - Todd Wisotzkey

12 actives

Chapter activities included the induction of five new members in April. Other 1992-93 officers: Daniel Pace, vice president; Sin Yee Wu, secretary; William Yankosky, treasurer; James E. Lightner, corresponding secretary; Linda Eshleman, faculty sponsor.

Maryland Delta — Frostburg State University, Frostburg

Chapter President - Steven Smith

43 actives, 1 associate

Seventeen new members were inducted into the Maryland Delta chapter in early March. At the induction, student Paul Browning gave a presentation on computers and music. Other spring semester programs were given by faculty Karen Parks ("Green Globs") and Marcella Bessman ("Women of Mathematics.") Other 1992-93 officers: Christine Bittinger, vice president; Thomas Currier, secretary; Diana Beisel, treasurer; Edward White, corresponding secretary; John Jones, faculty sponsor.

Michigan Beta — Central Michigan University, Mt. Pleasant

Chapter President - Matt Ayotte

38 actives

An initiation banquet held in March featured eleven new initiates. Guest speakers were Jim and Marilyn Bidwell who spoke about Jim's sabbatical in Australia during 1990-91. Student presentations during the semester were made by Betsy Bacon, Pete Shavinski, Tom DeClark, Sheree Matthews, and Matt Ayotte. One meeting, held at The Student Activities Center, included wally ball playing. To commemorate the 50th anniversary of The Michigan Beta Chapter a committee headed by Pete Shavinski prepared a plaque to be hung in the *KME* showcase. The plaque includes names of present members along with an inscription thanking all previous members and faculty. Ten student members and their advisor enjoyed a good time at the Region 2 Convention at Bowling Green University. Matt Ayotte and Tom DeClark were among the student speakers. The semester ended with a picnic, shared with the mathematics faculty and members of Gamma Iota Sigma, the local actuary club. Other 1992-93 officers: Dave Koester, vice president; Jenny Blake, secretary; Tami Hanson, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

Mississippi Gamma University of Southern Mississippi, Hattiesburg

Chapter President - Tracy Tiel

25 actives

Other 1992-93 officers: Luis Dopico, vice president; Debera Stogner, secretary; Alice W. Essary, treasurer/corresponding secretary; Barry Piazza and Lida McDowell, faculty sponsors.

Missouri Beta Central Missouri State University, Warrensburg

Chapter President - Jay Rowland

20 actives, 6 associates

Missouri Beta enjoyed several interesting meetings and activities throughout the spring semester. In January Matt Doliver from Kansas City Life spoke to the group concerning his job as an actuary. Fourteen new members were initiated in February. Following initiation a graphing calculator demonstration was presented. The semi annual *KME* sale was also held in February. Kim Ward presented The Klingenburg Lecture and was also given the Distinguished Alumni Award in March. Four students and one sponsor attended the regional convention at Emporia State University on April 10-11. Michael Prock won third place in the paper competition with his paper, "Applications of Linear Programming in Economics." New officers were elected and awards presented at the end-of-year pizza party. Other 1992-93 officers: Russell Savage, vice president; Jennifer Ritzo, secretary; Tracy Rouchka, treasurer; Rhonda McKee, corresponding secretary; Larry Dilley, faculty sponsor.

Missouri Epsilon Central Methodist College, Fayette

Chapter President - Mary Ann Neal

5 actives, 2 associates

Other 1992-93 officers: Roselyn Magosha, vice president; Ed LaValle, secretary; Holly Toler, treasurer; William D. McIntosh, corresponding secretary; Linda O. Lembke, faculty sponsor.

Missouri Eta Northeast Missouri State University, Kirksville

Chapter President - Jason Lott

33 actives, 4 associates

In addition to monthly meetings, the chapter co-sponsored a student/faculty picnic with the actuary club and MAA student chapter. A day of math contests for high school students was also held and several members attended the regional *KME* convention. Other 1992-93 officers: Scott Neimeyer, vice president; Deanne Reber, secretary; Angela Hahn, treasurer; Mary Sue Beersman, corresponding secretary; Shelte Palaski, faculty sponsor.

Missouri Iota Missouri Southern State College, Joplin

Chapter President - Melissa Sherrel
18 actives, 10 associates

Highlights of the spring semester included a guest speaker, Dr. Guido Weiss of Washington University, who spoke on Fourier Analysis, initiation of nine new members in late March, and attendance at the Region 4 convention at Emporia State University in April. At the convention Joely Eastin presented her student paper entitled "Modeling with Linear Regression." Officers for 1992 will be elected in the fall. Mary Elick is corresponding secretary; Linda Hand Noel, faculty sponsor.

Missouri Lambda Missouri Western State College, St. Joseph

Chapter President - Shawn Crawford
25 actives, 12 associates

Other 1992-93 officers: Tammy Resler, vice president; Tracy Schemmer, secretary; Denise Fuller, treasurer; John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

Nebraska Alpha Wayne State College, Wayne

Chapter President - Amy Anderson
30 actives

Throughout the semester, club members monitored the Math-Science building in the evenings to earn money for the club. The club administered the competitive examination to identify the outstanding freshman in mathematics. The competition ended in a tie with two awards going to Travis Herrick of Brady, NE, and Mitchell Mann of Broken Bow, NE. The award includes the recipient's name being engraved on a permanent plaque, payment of *KME* national dues, and one year honorary membership in the local *KME* chapter. Members Amy Anderson and Jill Brehm were awarded the $25.00 book scholarships which are given to *KME* members each year by the club. Members Amy Anderson, Susan Sorenson, Jaime Tiller, Doug Roberts and Bryce Sands, and faculty members Jim Paige and Fred Webber attended the regional

Kappa Mu Epsilon convention at Emporia State University in Emporia, Kansas, April 10-11, 1992. Other 1992-93 officers: Susan Sorenson, vice president; Jaime Tiller, secretary/treasurer; Wendy Stanley, historian; Freb Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

Nebraska Delta — Nebraska Wesleyan University, Lincoln

Chapter President - Matthew Meyer
16 actives

Students currently involved in Internships presented the program for one of the regular meetings. In conjunction with the Annual Spring Picnic fourteen new members were inducted. Other 1992-93 officers: Joseph C. Roth, vice president; Kenneth L. Guiberson, secretary; John R. Heckman, treasurer; Muriel Skoug, corresponding secretary/faculty sponsor.

New York Alpha — Hofstra University, Hempstead

13 actives

New York Alpha enjoyed a volleyball night and a talk by M. Weiss entitled "The Mandelbrot Set and Self Similiarity." Officers for 1992-93 have not yet been elected. Aileen Michaels is corresponding secretary and faculty sponsor.

New York Eta — Niagara University, Niagara University

12 actives, 8 associates

Four new members were initiated during the spring semester. The annual end-of-year faculty/student picnic was held at a faculty member's cottage near Lake Ontario. Plans are underway for hosting the 29th Biennial Convention in April of 1993. Officers for 1992-93 have not yet been elected. Robert Bailey is corresponding secretary/faculty sponsor.

New York Kappa — Pace University, New York

Chapter President - Angeliki Kazas
20 actives, 10 associates

The chapter sponsored a weekly mathematics seminar consisting of invited lecturers and discussion sessions. Featured speaker at the induction dinner and ceremony on May 20 was Dr. Joseph E. Houle of Pace University. His topic was "On Some Aspects of Seduction and Obsession in Mathematics." Other 1992-93 officers: Mei Ho, vice president; Geraldine Taiani, corresponding secretary; Blanche Abramov and John W. Kennedy, faculty sponsors.

North Carolina Gamma Elon College, Elon College

Chapter President - Varun Vijay Rao

30 actives, 5 associates

Induction ceremony for thirteen new members took place on April 22 in the Fine Arts Isabella Canen Room. Guest speaker for the evening was Dr. Stephen Davis from Davidson College who is presently the North Carolina State Director of the MAA. Other 1992-93 officers: Miguel Johnston, vice president; Kristie Collins, secretary; Charles Touron, treasurer; Rosalind Reichard, corresponding secretary; Graham Gersdorff, faculty sponsor.

Ohio Alpha Bowling Green State University

Chapter President - Kevin P. Davis

36 actives, 17 associates

The main focus of the spring was preparing for and hosting the Region 2 Convention on April 10-11. Forty-six students and faculty from seven chapters in six states participated. Other 1992-93 officers: Diana Nietz, vice president for initiation; Nancy Price, vice president for programming; Holly McDaniel, secretary/treasurer; Waldemar Weber, corresponding secretary; Neal Carothers, faculty sponsor.

Ohio Zeta Muskingum College, New Concord

Chapter President - Janet Gongola

35 actives

The first meeting in January featured talks by new members Craig McKendry, Brian Hess, Sabrina Fuller, Jim Buddenberg and Pat Geschwent. Initiation of new members was held in February. In April Kim Forgrave gave a preview of her paper she had prepared for the region 2 convention. Also in April new officers were elected. Eight delegates, including the faculty advisor, attended the regional convention at Bowling Green State University April 10-11. Other 1992-93 officers: Steve Miller, vice president; Sabrina Fuller, treasurer; Jim Buddenberg, treasurer; James Smith, corresponding secretary; Russ Smucker, faculty sponsor.

Oklahoma Alpha Northeastern State University, Tahlequah

Chapter President - Stephanie Brumfield

The chapter sponsored a talk by Dr. Richard Resco, University of

Oklahoma mathematics professor. The title of his talk was "Cryptography, Complexity and Abstract Algebra." The spring 1992 initiation ceremonies for eight students were held in the banquet room of a local restaurant in Tahlequah. During Math Awareness Week the students put up posters, banners, and biographies of mathematicians in the Math-Science building. They also showed several math videos, such as "Not Knot." They offered a $50.00 prize to the NSU student who could provide a well-written, correct solution to three math problems, which were written by Dr. Darryl Linde. The annual ice cream social, held prior to finals week, was a big hit. Students and faculty played "Jeopardy," using math questions written by Mrs. Linda Collins. It was wild! The students in the NSU Kappa Mu Epsilon and MAA chapters presented Dr. Joan E. Bell with the annual "Mathematics Teacher of the Year" award. Other 1992-93 officers: S. Kalen Dodson, vice president; Okcha Cockrum, secretary; Donna Baughman, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

Oklahoma Gamma Southwestern Oklahoma State University

Chapter President - Melissa Kirkland

25 actives

Oklahoma Gamma hosted the Region 5 spring convention; four chapters were represented. Other 1992-93 officers: Dixie Harris, vice president; Jeremy Osmus, secretary; Jodi Lubinus, treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Pennsylvania Alpha Westminster College, New Wilmington

Chapter President - Monica Mundo

23 actives

Other 1992-93 officers: Amy Shannon, vice president; Jennifer Peelor, secretary; Kelly Hughes, treasurer; J. Miller Peck, corresponding secretary; Warren Hickman, faculty sponsor.

Pennsylvania Beta La Salle University, Philadelphia

Chapter President - Michael A. Scafidi, Jr.

12 actives

This spring, using local talent, Pennsylvania Beta Chapter sponsored two lectures: Dr. Richard DiDio spoke on "Chaos" and Dr. Errol Pomerance spoke on "Geodesics and General Relativity." Other 1992-93 officers: Angela Rowbottom, vice president; Joseph Evangelist, secretary; Richard Wojnar, treasurer; Hugh Albright, corresponding secretary; Carl McCarty, faculty sponsor.

Pennsylvania Delta Marywood College, Scranton
Chapter President - Kelly Curtin
13 actives

The chapter sponsored the Annual Mathematics Contest for high school students in two parts: The written exams on April 11 and the orals on April 26. Induction of new members was held in May. Other 1992-93 officers: Alice Ward, vice president; Paul Harewood, secretary; Kathleen Hanlon, treasurer; Sister Robert Ann vonAhnen, IHM, corresponding secretary/faculty sponsor.

Pennsylvania Eta Grove City College, Grove City
Chapter President - Kristi Kowalski
39 actives

The initiation of new members and election of new officers was held March 31. Based on the results of a competitive exam, *KME* selected an outstanding Freshman mathematics student, the winner being announced at The Parent's Day Award Ceremony on May 2. Activities ended with a Spring Picnic held May 3 at the Grove City Park. Other 1992-93 officers: Vajeera Dorabawila, vice president; Tracy Plieninger, secretary; Steve Swartzlander, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Iota Shippensburg University, Shippensburg
Chapter President - Marnie Paul

Nine new members were initiated into the chapter on April 26 in the Cumberland Union Building. Following the initiation, the spring banquet was held at a local restaurant. Other 1992-93 officers: Jeff Rady, vice president; Lisa Nesbit, secretary; Fred Nordai, treasurer; Susan Waltimeyer, historian; Michael Segfried, corresponding secretary/faculty sponsor.

Pennsylvania Kappa Holy Family College, Philadelphia
Chapter President - Kevin Carsley
10 actives, 6 associates

Pennsylvania Kappa continued to provide tutoring as a service to math students. Plans were made for the initiation to be held March 15, 1993. Again, speakers will be former *KME* members who will share experiences in industry, insurance and teaching with those being initiated. Other 1992-93 officers: David McCabe, vice president/secretary /treasurer; Sister Mary Grace Kuzawa, corresponding secretary/faculty sponsor.

Pennsylvania Lambda — Bloomsburg University, Bloomsburg

Chapter President - Kathleen Szymczak
15 actives

Five new members were inducted in March. Glasses etched with the letters *KME* were distributed as gifts to all current members as well as to the new inductees. To raise funds for the organization a candy sale was held, each member selling one case of thirty candy bars consisting of Plain and Peanut M & M's, Kit Kats, and Reese's Peanut Butter Cup. On March 13, Dr. James Pomfret, Dr. John Riley, and student president Karen M. Russell attended the *KME* regional convention at St. Francis College in Loretto, Pennsylvania, where Karen took first place in the student talks with her paper "Comparison of Two Populations." Also in March chapter members assisted the Mathematics Department in distributing information on majors, minors, and career concentrations in mathematics. In April Nancy Wyshinski, a 1978 graduate and former chapter president, visited the campus and gave a presentation on continued fractions. To conclude the year, a picnic was held at the farm home of senior Todd Reichart. Other 1992-93 officers: Thaddea Puzio, vice president; Kathryn Yarrington, secretary; Todd Rider, treasurer; James Pomfret, corresponding secretary; John Riley, faculty sponsor.

Pennsylvania Mu — Saint Francis College, Loretto

Chapter President - Amy Miko
34 actives, 5 associates

Twelve students were inducted on February 24, bringing total membership in Pennsylvania Mu to 115. The induction ceremony followed mass and dinner. The chapter also hosted a successful Region 1 Convention on March 13-14. A marathon game of Chicago Softball (no mitts!) was enjoyed along with the annual picnic April 28. Other 1992-93 officers: Kim Cool, vice president; Paula Knaze, secretary; Sergio Bascon, treasurer; Peter Skoner, corresponding secretary; Adrian Baylock, faculty sponsor.

Pennsylvania Nu — Ursinus College, Collegeville

Chapter President - Deborah Collinge
20 actives, 2 associates

During the spring semester Pennsylvania Nu co-sponsored three speakers. On February 4 Professor Samuel Merrill of Wilkes University spoke on "Game Theoretic Models of Final-Offer Arbitration." Professor Douglas M. Stokes of The Shipley School presented a March 23 lecture entitled "Does ESP Exist? A Critical Examination of the Mathematical Evidence for the Existence of Psi Phenomena." The final presentation was that of Professor Thomas Bartlow of Villanova University whose topic was "Solution of Polynomial Equations." Other 1992-93 officers: Beth Carkner, vice president; Jennifer Mauer, secretary; Reid Gilbert, treasurer; Jeff Neslen, corresponding secretary; Richard Bremiller, faculty sponsor.

South Carolina Delta — Erskine College, Due West

Chapter President - Jodi Dixon Long
13 actives, 2 associates

South Carolina Delta held three meetings during the spring and initiated six new members. Other 1992-93 officers: Dawn Alison Smith, vice president; Lindi Latham, secretary/treasurer; Ann F. Bowe, corresponding secretary/faculty sponsor.

Tennessee Alpha — Tennessee Technological University, Cookeville

Chapter President - Molly Slaughter
30 actives

Other 1992-93 officers: Leannie Link, vice president; Jennifer Kite, secretary; Lori. Robbins, treasurer; Frances Crawford, corresponding secretary; Ed Dixon, faculty sponsor.

Tennessee Beta — East Tennessee State University, Johnson City

18 actives

Officers Debra Pearcy, Lisa Jones, and Lynn Hall officiated at the annual spring initiation banquet. Two *KME* members, Lisa Blankenship and Robin Blankenship, were selected as the outstanding senior mathematics students at the University. Officers for 1992-93 have not yet been elected. Lyndell Kirley is faculty sponsor.

Texas Alpha — Texas Tech University, Lubbock

Chapter President - Troy R. Smith
10 actives, 46 associates

Other 1992-93 officers: Chris Norden, vice president; Nina Nelson, secretary; Brian Ashcraft, treasurer; Robert Moreland, corresponding secretary/faculty sponsor.

Texas Eta Hardin-Simmons University, Abilene

Chapter President - Louis Revor

18 actives

The Texas Eta Chapter held its 18th annual induction banquet March 12, 1992. With the induction of four new members, membership in the local chapter stands at 136. The program for the event was provided by Louis Revor who spoke on his studies and experiences at the Argonne National Laboratory. Louis, a junior with a dual major in Math and computer science, had an opportunity to apply his problem solving skills in a biological area. Leading the induction ceremonies were Charles Reed, president; Louis Revor, vice president; and Jill Sims, secretary. Other 1992-93 officers: Jill Sims, vice president; Christine Hieronymus, secretary; Amy Garrison, treasurer; Edwin Hewett and Charles Robinson, faculty sponsors; Mary Wagner-Krankel, corresponding secretary.

Texas Kappa University of Mary Hardin-Baylor, Belton

Chapter President - Becky Hunt

10 actives, 15 associates

Peter Chen attended the *KME* Region 5 convention at Southwestern Oklahoma State University in Weatherford on March 20-21. Other 1992-93 officers: Tim Collins, vice president; Shirley Feild, secretary; Scott Callaway, treasurer; Peter H. Chen, corresponding secretary; Maxwell Hart, faculty sponsor.

Virginia Alpha Virginia State University, Petersburg

Chapter President - Joycelyn Josey

20 actives, 5 associates

Highlights of spring meetings included presentations of research papers by student members and a special seminar by Dr. Walter Elias, Jr., Distinguished Professor of Mathematics, entitled: "A Walk Past Infinity." The Annual Initiation Ceremony, with nine initiates, featured remarks by Dr. James C. Nelson, a mathematician and Dean of The School of Natural Sciences at Virginia State University. A senior member of Virginia Alpha Chapter, Linwood Jarratt, is one of four students nationwide selected to participate in the Summer Program in Parallel Computing at California Institute of Technology sponsored by National Science Foundation. The chapter, in conjunction with The Walter Johnson Mathematics Club and the mathematics faculty at Virginia State, sponsored a retirement tribute and banquet for three Distinguished Professors on May 21. The honorees, Dr. Loretta M. Braxton, Dr. Walter

Elias, Jr., and Dr. Vivian G. Howard, retired after 30, 34, and 22 years, respectively, of service in the Mathematics Department of Virginia State University. President Joycelyn Josey presented each retiree a pewter cup engraved with the recipient's name and the name Kappa Mu Epsilon. Other 1992-93 officers: Kevin Taylor, vice president; Yolanda Pierce, secretary; Earl Ingram, treasurer; Emma B. Smith, corresponding secretary; V. Bakshi, faculty sponsor.

Wisconsin Alpha Mount Mary College, Milwaukee

Chapter President - Jill Rogahn
5 actives, 4 associates

Four students and two faculty members attended the Region 2 Convention of Kappa Mu Epsilon at Bowling Green State University April 10-11. Jill Rogahn presented a student paper at the convention; her paper dealt with problem solving in the lives of Rene Descartes and Isaac Newton. Other 1992-93 officers: Jill Rogahn, secretary/treasurer; Sister Adrienne Eickman, corresponding secretary/faculty sponsor.

Wisconsin Gamma University of Wisconsin-Eau Claire, Eau Claire

Chapter President - Laura Whitehead
31 actives, 17 associates

Other 1992-93 officers: Jeff Ion, vice president; Jacqueline Hoffman, secretary; Stacy Goodyer, treasurer; Tom Wineinger, corresponding secretary/faculty sponsor.

1992 Regional Conventions

Edited by Arnold D. Hammel, President-Elect

There were five *KME* Regional Conventions during the Spring of 1992. The following reports were prepared from materials submitted by the regional directors.

Report of the 1992 Region I Convention

James Pomfret, Regional Director

The Region I Convention was held March 13-14 at St. Francis College in Loretto, Pennsylvania. Two chapters (Pennsylvania Lambda, Bloomsburg University; and Pennsylvania Mu, St. Francis College) were represented with a total registration of twenty-seven students and faculty. The hosts at Pennsylvania Mu did a terrific job and were led by Jim Kelly, President; John Miko, Vice President; Amy Miko, Secretary; Kim Cool, Treasurer; and Dr. Peter Skoner, Corresponding Secretary.

The meeting began at 5:00 pm, March 13, in the John F. Kennedy Student Center with welcoming remarks by Jim Kelly and Dr. Richard Crawford, Assistant to the President of St. Francis College. After Dr. Crawford's interesting recount of the history of St. Francis College, participants enjoyed an excellent dinner and an after dinner talk by Dr. David Bressoud of Pennsylvania State University. Dr. Bressoud's talk entitled "Factoring large integers: What can we do and why do we care?" was a fascinating introduction to public key cryptography and included the idea that some number theorists earn money by selling large prime numbers.

In the evening, John Miko hosted a game of "Mathematics Jeopardy" which was won by a combined team of faculty and students from both Bloomsburg University and St. Francis College.

On Saturday morning, Dr. Peter Skoner of St. Francis College hosted the student paper competition. The papers were:

Fourier Analysis
Jim Kelly (St. Francis College)

Comparison of Two Populations
Karen Russell (Bloomsburg University)

An Introduction to the Van Hiele Model of Geometric Thought
Edward Lenz (St. Francis College)

The Mathematics behind the Physics of Baseball
John Miko (St. Francis College)

Judging the contest were students Brian Hebert and Paula Knaze of St. Francis College and Professors John Riley and Adrian Baglock from Bloomsburg University and St. Francis College, respectively. The top two papers were judged to be those given by Karen Russell and John Miko, each of whom received an award of $25.00.

Report of the 1992 Region II Convention

Sister Adrienne Eickman, Regional Director

The Region II Convention was held April 10-11 at Bowling Green State University in Bowling Green, Ohio. The sponsoring chapter was Ohio Alpha under the able direction of Waldemar Weber, Corresponding Secretary. There were seven chapters represented (Illinois Delta, College of St. Francis; Michigan Beta, Central Michigan University; New York Eta, Niagara University; Ohio Alpha, Bowling Green State University; Ohio Zeta, Muskingum College; Pennsylvania Zeta, Indiana University of Pennsylvania; and Wisconsin Alpha, Mount Mary College). There were forty-six students and faculty in attendance.

The banquet started on Friday evening with initiation of new members of the Ohio Alpha chapter. After an excellent meal we heard Dr. Richard Putney from the Toledo Museum of Art who spoke on "Notions of harmony and proportion in medieval art and architecture." This was followed by the barbershop music of the Logarythms from Bowling Green State University's mathematics department. Later in the evening there was ice skating.

Saturday morning started with an address on "How can we see black holes? Simulating their effects using the Ohio supercomputer" by Comer Duncan of the Department of Physics and Astronomy at Bowling Green. The following student papers were presented:

A Brief Look at Ancient Mathematics
Tom DeClark (Central Michigan University)

The Hunt for the Red Parabola
Kevin Davis (Bowling Green State University)

P.S. I Love You (A Look at Problem Solving and Two Mathematicians)
Jill Rogahn (Mount Mary College)

A Solution to Problem E 3450 of the American Mathematical Monthly 98:6 (June-July 1991)
Matthew Ayotte (Central Michigan University)

Exploration of Functional Iteration
Kim Forgave (Muskingum College)

Fractal Dimensions
Mark King (Bowling Green State University)

Students and faculty also had the opportunities to meet to discuss issues of interest. Ohio Alpha videotaped much of the conference. After editing, the plan is to distribute the tape to other chapters in Region II.

Report of the 1992 Region III Convention

Patrick Costello, Regional Director

The Region III Convention was held April 11 at Eastern Kentucky University in Richmond, Kentucky. Corresponding Secretary Patrick Costello and the members of Kentucky Alpha Chapter had plans in place to host the meeting. Because of travel distance, tight budgets, etc., no other chapters could attend the convention. Kentucky Alpha is to be congradulated for going ahead with their plans and having a "local" regional convention.

The conference started with talks by three students:

An M and M's Experiment
Lisa Whitis (Eastern Kentucky University)

Fractals and Mathematica
James Hannis (Eastern Kentucky University)

Zombies, Chinese Rooms and Talking Gyms — Speculations about Artificial Intelligence
Kevin Huibregtse (Eastern Kentucky University)

The members then viewed the VCR tape of Howard Eves giving a talk entitled "Mathematically Motivated Designs." After lunch at Arbys, the group car pooled to Keeneland Race Track in Lexington to observe the Bluegrass Stakes. All had a good time.

Report of the 1992 Region IV Convention

Mary Sue Beersman, Regional Director

The Region IV Convention was held April 10-11 at Emporia State University in Emporia, Kansas. There were fifteen chapters represented (Kansas Alpha, Beta, Gamma, Delta and Epsilon from Pittsburg State University, Emporia State University, Benedictine College, Washburn University and Fort Hays State University; Missouri Alpha, Beta, Eta, Theta, Iota, Kappa and Lambda from Southwest Missouri State University, Central Missouri State University, Northeast Missouri State University, Evangel College, Missouri Southern State College, Drury College and Missouri Western State College; Nebraska Alpha, Beta and Gamma from Wayne State College, University of Nebraska at Kearney and Chadron State College). There were ninety-four in attendance — seventy students and twenty-four faculty. The members of Kansas Beta and Corresponding Secretary Connie Schrock put in much effort to make this well-attended conference a success.

There was a mixer with refreshments on Friday evening. Time was spent looking at pictures of previous Region IV Conventions.

The papers presented at the Saturday morning session were:

Economic Applications of Linear Programming
Michael Prock (Central Missouri State University)

The Busy Beaver: Another Perspective
Joseph R. Cloutier (Emporia State University)

Modeling with Linear Regression
Joely Eastin (Missouri Southern State College)

What is Projective Geometry?
Michelle D. Land (Emporia State University)

Mysterious Modeling
Yvonne Shaw (Drury College)

Introduction to Automata
Sarah V. Gleason (Emporia State University)

The convention concluded with a noon luncheon and guest speaker, David Janzen of Lawrence, Kansas. He spoke on "A mathematics exchange in Budapest, Hungary."

Report of the 1992 Region V Convention

Dick Gibbs, Regional Director

The Region V Convention was held March 20-21 at Southwestern Oklahoma State University in Weatherford, Oklahoma. Three chapters (Colorado Gamma, Fort Lewis College; New Mexico Alpha, University of New Mexico; and Oklahoma Gamma, Southwestern Oklahoma State University) were represented with a total registration of twenty-two students and faculty. Those attending were very impressed with the work of the Oklahoma Gamma Chapter and Corresponding Secretary Wayne Hayes.

There was a reception Friday night. As an ice-breaker, a brain teaser was handed out and when it was time to show a movie, everyone was so wrapped up in the brain teaser that no one watched the movie. On Saturday morning, the following student papers were presented:

How to Design a Model Rocket
Duane Brown (Fort Lewis College)

Behind Closed Doors
Joe Charles (Southwestern Oklahoma State University)

Pi is Irrational
Steve Luxa (Fort Lewis College)

Professor Radwan Al-Jarrah of Southwestern Oklahoma State University spoke on "Mathematics education in the Arab world." This was followed by a luncheon and awards.

Kappa Mu Epsilon National Officers

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation.

Chapter	Location	Installation Date
OK Alpha	Northeastern Oklahoma State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State College	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State College, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949

IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	Kearney State College, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri - Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin - River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Northeast Missouri State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel College, Springfield	12 Jan 1971

PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop College, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	West Georgia College, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin - Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Illinois Benedictine College, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C. W. Post Center of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry College, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992