THE PENTAGON

A Mathematics Magazine for Students

Volume 51 Number 2 | Spring 1992

CONTENTS

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Manuscripts of interest to undergraduate mathematics majors and first year graduate mathematics students are welcome, particularly those written by students. Submissions should be typewritten (double spaced with wide margins) on white paper, standard notation conventions should be respected and special symbols should be carefully inserted by hand in black ink. All illustrations must be submitted on separate sheets and drawn in black ink. Computer programs, although best represented by pseudocode in the main text, may be included as an appendix. Graphs, tables or other materials taken from copyrighted works MUST be accompanied by an appropriate release from the copyright holder permitting further reproduction. Student authors should include the names and addresses of their faculty advisors. Contributors to *The Problem Corner*, *The Hexagon* or *Kappa Mu Epsilon News* are invited to correspond directly with the appropriate Associate Editor. Electronic mail may be sent to (Bitnet) PENTAGON@LIUVAX.

Domestic subscriptions: $3.00 for two issues (one year) or $5.00 for four issues (two years); foreign subscriptions: $5.00 (US) for two issues (one year). Correspondence regarding subscriptions, changes of address or back copies should be addressed to the Business Manager. Copies lost because of failure to notify the Business Manager of changes of address cannot be replaced.

Microform copies are available from University Microfilms, Inc., 300 North Zeeb Road, Ann Arbor, Michigan 48106-1346 USA.

Fractal Geometry: A General Overview

J. Ben Schafer, *student*

Iowa Alpha

University of Northern Iowa
Cedar Falls, Iowa 50613

Presented at the 1991 National Convention and awarded SECOND PLACE by the Awards Committee.

Introduction.

Look at the objects in the room around you and try to describe what you see. You probably see things like rectangular doors and windows, or square tables and chair seats. All of these things are human made objects which can be easily and quite accurately described by Euclidean or "classical" geometry. Our society lives by such geometry. As Michael Barnsley tells us, "Mankind seems to be obsessed with straight lines. In technology straight lines are the basic building block. From our earliest learning moments we are encouraged to manipulate, to rotate and translate, to underline, straighten, measure along straight lines and to use graph paper." [1] From a young age we have been taught to draw straight lines on square paper. We are urged to build elaborate structures with Tinkertoys, Lincoln Logs or simple building blocks. All are things we can represent in the 1, 2, or 3 dimensions described by classical geometry.

However, classical geometry provides us with, at best, an approximation to the structure of the natural world. Polish born mathematician Benoit Mandelbrot poses:

> Why is geometry often described as "cold" and "dry"? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.

> More generally, I claim that many patterns of Nature are so irregular and fragmented, that compared with Euclid, Nature exhibits not simply a higher degree, but an altogether different level of complexity. [2]

It was with this in mind that Mandelbrot, a member of the Physical Sciences Department at the IBM Watson Research Center, set out to define a new form of geometry: "Fractal Geometry."

History.

Although fractal geometry in itself is a fairly new geometry, taking its major developments since 1975, its foundations are deep and draw upon the work of many late 19th and early 20th century mathematicians.

At the turn of this century, two French Mathematicians, Gaston Julia and Pierre Fatou, started working with the iterative properties of rational functions of a complex variable. They classified a complex point z, and a function $T(z)$ according to the outcome of the series z, $T(z)$, $T(T(z))$, $T(T(T(z)))$ and so forth. Recall that if f is a differentiable function and

$$T(z) = z - \frac{f(z)}{f'(z)}$$

then the iterates of T applied to an initial "guess" z_0 converge very often to a zero of f. However, if f has two zeros, this convergence may converge to either of the zeros of $f(z)$, diverge to infinity or alternate between several values. We can thus place a point in the complex plane in one of several subsets based upon the behavior of its iterates. Fatou and Julia became quite interested in the boundaries between these sets. These boundaries are very often complicated, but they were able to prove that the boundary curves produced were self-similar; a view of any subsection, no matter how small, recreates the entire curve. [3] However, their work was quite limited by the absence of computer graphics.

Several decades later, Mandelbrot started exploring certain "irregular sets" found in nature and came across the work of Fatou and Julia. Being surrounded by computers, he began to refine their ideas and draw "pictures" of some of their sets. It was through this work that he developed the set which now bears his name, the Mandelbrot set, and earned him the title of "The Father of Fractal Geometry."

A Definition.

The word "fractal" comes from the Latin words *frangere*, meaning "to break," and *fractus*, meaning "irregular, fragmented." Unfortunately, in addition to fractal shapes being irregular, so is the list of their properties.

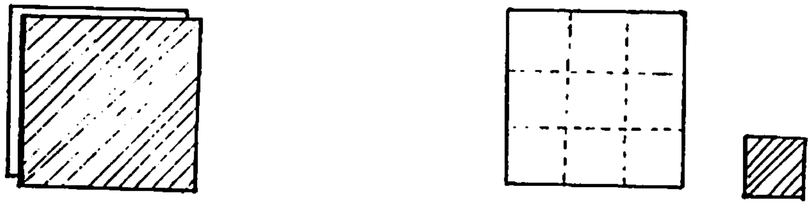
Some fractals consist of curves or surfaces. Others consist of disconnected "dust," that is, a set of points. Still others are so unlike anything we have ever encountered before that there are no good terms in either the sciences or the arts to describe them.

Some of the common attributes of fractals include the fact that they are generally obtained through a series of changes, either systematic or random, and that the simplest of fractals tend to be self-similar or scaling, that is, their degree of irregularity or fragmentation is identical at all scales. But probably the single attribute that identifies a shape as a fractal is the fact that fractals display a dimension — often called the "Hausdorff dimension" — quite different from what we are used to. At this point, we need to digress a bit to examine how we approach the concept of dimension in classical geometry and to develop a generalization on the idea of dimension.

Consider a line segment and a measuring rod of the same length. At this point our line has a length of one unit. If we decrease the magnitude of our measuring rod by three — divide it in thirds — our line segment now has a length of three units. We will note, for reasons that will become clear in a little bit, that $3 = 3^1$.

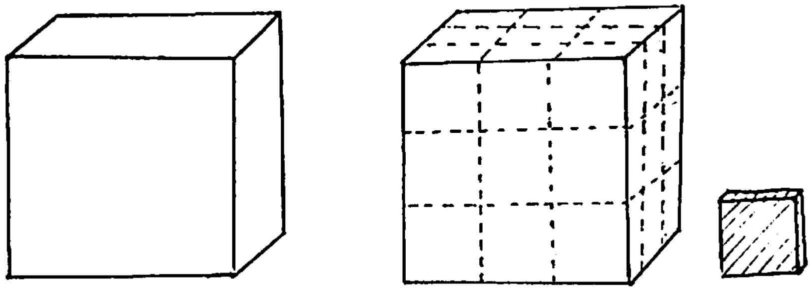
$L = 1$ $M = 3$ $L = 3$

Now consider a square and a "measuring square" of the same area. At this point our square has an area of one square unit. If we decrease the magnitude of the measuring square by three — divide each side of the square by three — our original square now has an area of nine square units. Again, let us note that $9 = 3^2$.



$A = 1$ $M = 3$ $A = 9$

Finally, consider a cube and a "measuring cube" of the same volume. Similar to our first two examples, this cube has an initial volume of one cubic unit. If we decrease the magnitude of our measuring cube by three, our original cube now has a volume of 27 cubic units. Observe that $27 = 3^3$.



$V = 1$ $\qquad M = 3$ $\qquad V = 27$

What we have seen here is that in each case we have an equation consisting of a dimension D, a magnitude M, and a number N of new units, which can be generalized by the equation $N = M^D$. Furthermore, we will notice that in each of these cases, the dimension D has taken on a value which is in accord with the value we expect it to take. Classical geometry teaches us that a line exists in one dimension, a square exists in two dimensions, and a cube exists in three dimensions.

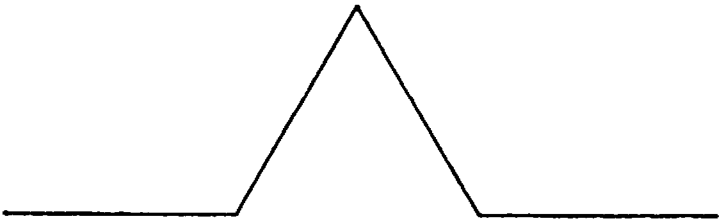
But must all shapes have a dimension whose value is represented by an integer? Are there shapes whose scaling dimension is represented by a rational or even a nonrational number? The answers to these questions are no and yes, respectively, and this is where we start to enter the world of fractals.

A Basic Fractal.

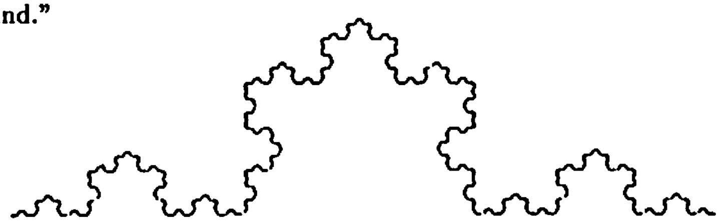
A simple example of a shape whose dimension is not an integer can be created by using the following process [3] (and can be observed by running the program given on pages 455-6 in *An Introduction to Computer Simulation Methods: Applications to Physical Systems, Part 2* by Harvey Gould and Jan Tobochnik, Addison-Wesley, 1988). (1) Start with a line segment:

(2) Divide the segment into three equal parts:

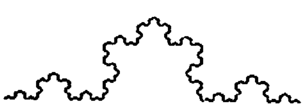
(3) Construct an equilateral triangle over the middle segment and erase the base, or the original portion, of the triangle:



(4) Repeat steps (1)-(3) with each of the new line segments "without bound."



With this shape, the idea of dimension is much different from that developed in classical geometry. The concept of dimension starts to make more sense if we examine the first few iterations in the development of this shape. [4]

	Iteration	M	N
	0	1	1
	1	3	4
	2	9	16
	n	3^n	4^n

Here we have defined M as the factor by which the original line segment has been reduced in making each new line segment, and N as the "length" of the new curve. It may become clear at this point that we can make the generalization that for any iteration number x that M and N can be represented by 3^x and 4^x respectively. Taking $x = 1$, it is clear that we do not have an integer D such that $4 = 3^D$. Solving our equation $N = M^D$ we find that

$$\log(N) = \log(M^D) = D\log(M) \quad \text{so} \quad D = \frac{\log(N)}{\log(M)}.$$

Thus for any iteration number x, the dimension D equals

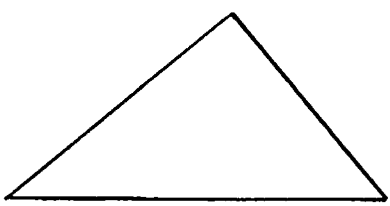
$$D = \frac{\log(N)}{\log(M)} = \frac{\log(4^x)}{\log(3^x)} = \frac{x\log(4)}{x\log(3)} = \frac{\log(4)}{\log(3)} = 1.2619.$$

An interesting thing to note here is that since we have repeated this process an "infinite" number of times, if we were to break off a "branch" of this snowflake, we actually have an exact replica of the original snowflake. This particular fractal is self-similar.

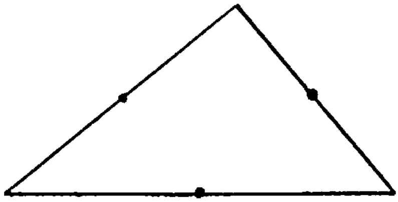
We mentioned earlier that many of the objects which we now call fractals were actually "discovered" long before fractal geometry was "invented." For example, the shape we just talked about is actually quite old. It is a portion of an image Swedish mathematician Helge von Koch produced in 1904 and is now called the "von Koch snowflake." It is an example of a fractal defined by a series of line segments created by systematic changes.

Further Examples.

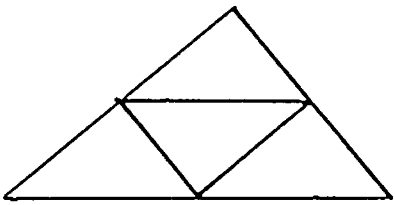
Another fractal, with a similar set of properties and a similar history, is called the "Sierpinski gasket" or the "Sierpinski triangle." This fractal can be obtained as follows [5] (see Appendix 1). (1) Draw a triangle:



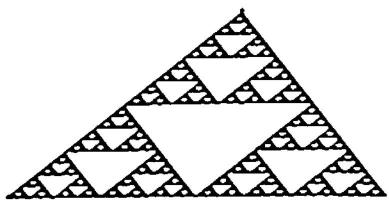
(2) Find the midpoints of the three sides:



(3) Connect the midpoints and "remove" the triangle formed on the interior:



(4) Repeat steps (2)-(3) with each of the new triangles "without bound."



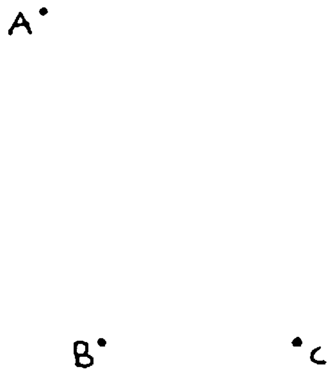
Much in the same way we examined the von Koch snowflake, we can calculate the fractal dimension of the Sierpinski triangle:

$$D = \frac{\log(N)}{\log(M)} = \frac{\log(3^x)}{\log(2^x)} = \frac{x\log(3)}{x\log(2)} = \frac{\log(3)}{\log(2)} = 1.58496.$$

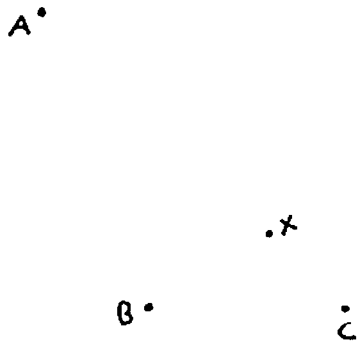
Furthermore, once again we will note that this shape is self-similar.

So far we have examined two very systematic processes for obtaining fractal shapes. However, as was noted earlier, fractals can also be created using a random process. One technique is to plot a point z in a coordinate plane, to randomly apply to it one of several affine transformations of z such that $z' = f(z)$, plotting z', replacing z with z', and then repeating the whole process. At first glance we would expect to see a chaotic mess, yet from these chance transformations often comes order.

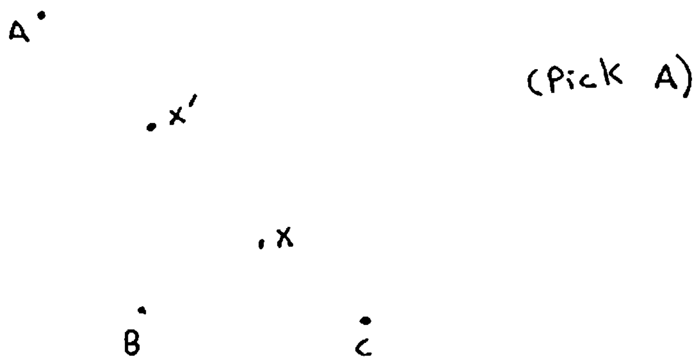
With that in mind, we can follow this second process to produce the Sierpinski gasket [5] (see Appendix 2). (1) Pick three non-collinear points and label them A, B and C:



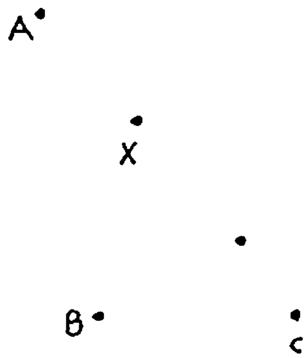
(2) Pick a fourth point X in the same plane as the first three:



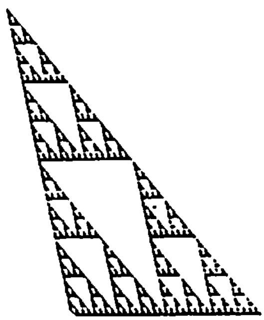
(3) At random, pick one of the initial three points (A, B or C) and plot a point X' which is halfway between X and the point you just picked:



(4) Set $X = X'$:



(5) Repeat steps (3)-(4) "without bound."



If we erase the first eight points that we plotted [6], we discover that the image obtained is actually another production of the Sierpinski gasket.

$$X_{n+1} = (A_i * X_n) + (B_i * Y_n) + E_i$$

$$Y_{n+1} = (C_i * X_n) + (D_i * Y_n) + F_i$$

i	A	B	C	D	E	F
1	0.00	0.00	0.00	0.16	0.00	0.00
2	0.85	0.04	−0.04	0.85	0.00	1.60
3	0.20	−0.26	0.23	0.22	0.00	1.60
4	−0.15	0.28	0.26	0.24	0.00	0.44

Table 1.

All of the shapes we have seen so far are all very fascinating, but as of yet, we have neglected to create a fractal which mimics nature. Since this was the original intent of Mandelbrot's work, we should examine such a fractal [1] (see Appendix 3). The complete data for this fractal is contained in Table 1. (1) Plot a point t in the coordinate plane:

• t (x, y)

(2) At random, pick one of several affine transformations ($i = 1, 2, 3$ or 4), let $t' = f_i(t)$ and plot t'.

• t (x, y)

• t' $(a_i x + b_i y + e_i,\ c_i x + d_i y + f_i)$

(3) Set $t = t'$.

•

• t (x, y)

(4) Repeat steps (2)-(3) "without bound."

Once again, from a random set of transformations, we have developed an image with order; in this case, the image of a fern. It is interesting to note that this image is also a self similar image: If we were to break off one of the leaves, we would discover that it is an exact replica of the plant from which it was broken.

Beyond Pretty Pictures.

After observing a few fractal images, it would be easy for someone to say "Well, they are really interesting, but do they really have any practical purposes?" The answer is a resounding "yes."

In the early 1980's, Loren Carpenter, a simulation programmer for Boeing Aircraft, became aware of Mandelbrot's work. Carpenter felt that by using fractals in his programming he could create more realistic images for the simulators used by Boeing pilots. He became so good at this that he was soon an often sought programmer by Hollywood. Probably his most well known work was for the movie *Star Trek II: The Wrath of Khan.* Several computer generated sequences of the film involve fractal landscapes, the best known of which is the Genesis planet transformation sequence.

Again, one could argue that this does not prove the importance of the study of fractal geometry. But there are more uses. Michael Barnsley has used fractals to study ways of compressing digitized images [7]. For example, to recreate the fern which we saw earlier, without the use of fractals, I would have to tell you that pixel $(1,1)$ on my computer is off, pixel $(1,2)$ is on, and so forth for hundreds of thousands of pixels, before you could produce the image. This process would take an enormous amount of data space. However, with a knowledge of fractals and the four transformations that I used to create the fern, you would be able to reproduce it.

Furthermore, many different people can use fractals in their work. Below is just a partial list of some of the uses found for fractal theory.

astronomers	location of matter in the universe
biologists	effects of acid rain
chemists	energy transfer
economists	rise and fall of prices
geologists	seismic faults
material scientists	cracks in metals
medical doctors	heart behavior
meteorologists	weather patterns
military	automatic target detection
physicists	fluid turbulence

Yet, despite the apparent benefits of fractal geometry, many mathematicians are still reluctant to accept its significance. Just as many mathematics students question whether the works of Riemann, Lobachevski and Bolyai are anything more than intriguing mind games, many question whether Mandelbrot's work is anything more than intriguing pictures.

Whether fractal geometry will ever gain scientific credibility has yet to be determined, but D. E. Thomsen wraps it up well when he wrote:

> Yet there have always been large tracts of science where these simple analytic methods hardly applied. The natural phenomena were just too complex. It is in these realms that fractals are finding application after application. Either the development of fractals is just a fascinating mathematical game, or it is one of the most significant steps that scientific analysis has taken. [8]

References.

[1] Barnsley, Michael. *Fractals Everywhere.* Academic Press, Inc., 1988.

[2] Mandelbrot, Benoit. *The Fractal Geometry of Nature.* New York: W.H. Freeman and Company, 1983.

[3] Bridger, Mark. "Looking at the Mandelbrot Set," *The College Mathematics Journal* 19 (1988), 353-363.

[4] Barcellos, Anthony. "The Fractal Geometry of Mandelbrot," *The College Mathematics Journal* 15 (1984), 98-114.

[5] Peitgen, H.O., and Richter, P.H. *The Beauty of Fractals.* Berlin: Springer-Verlag, 1986.

[6] Peitgen, H.O., and Saupe, Dietmar. *The Science of Fractal Images.* New York: Springer-Verlag, 1988.

[7] Marxen, Donald. Lecture at Malcom Price Lab School, Cedar Falls, Iowa, on 23 November 1989.

[8] Thomsen, Dietrick E. "Fractals: Magical Fun or Revolutionary Science?" *Science News*, 21 March 1987, 184.

The programs given in the following appendices are written in "TrueBASIC."

Appendix 1.

```
PROGRAM Serp_Gask
call initial(lx,ly,rx,ry,tx,ty,n)
do
     let k=0
     call draw(lx,ly,rx,ry,tx,ty,n)
     do
          get key k
     loop until k<>0
     let n=n+1
     clear
loop
end

sub inital(lx,ly,rx,ry,tx,ty,n)
     let lx=5
     let ly=5
     let rx=95
     let ry=10
     let tx=50
     let ty=90
     set window 0,100,0,100
end sub

sub draw(lx,ly,rx,ry,tx,ty,n)
     if n>0 then
          let lmx=(lx+tx)/2
          let lmy=(ly+ty)/2
          let rmx=(rx+tx)/2
          let rmy=(ry+ty)/2
          let bmx=(lx+rx)/2
          let bmy=(ly+ry)/2
          call draw(lx,ly,bmx,bmy,lmx,lmy,n-1)
          call draw(bmx,bmy,rx,ry,rmx,rmy,n-1)
          call draw(lmx,lmy,rmx,rmy,tx,ty,n-1)
     else
          plot lines:lx,ly;rx,ry
          plot lines:rx,ry;tx,ty
          plot lines:tx,ty;lx,ly
     end if
end sub
```

Appendix 2.

```
PROGRAM Serp_Gask_Alt
dim x(1 to 3), y(1 to 3)
mat read x,y
data 5,95,50,5,10,90
let x0=rnd*100
let y0=rnd*100
set window 0,100,0,100
do
     let point=int(rnd*3)+1
     let x0=(x0+x(point))/2
     let y0=(y0+y(point))/2
     plot points:x0,y0
loop
end
```

Appendix 3.

```
PROGRAM Fern
dim trans(4,6)
mat read trans
     data 0,0,0,0.16,0,0
     data 0.85,0.04,-0.04,0.85,0,1.6
     data 0.2,-0.26,0.23,0.22,0,1.6
     data -0.15,0.28,0.26,0.24,0,0.44
set window 0,100,0,100
let x0=0
let y0=0
do
     let pt=int(rnd*4)+1
     let xn=trans(pt,1)*x0+trans(pt,2)*y0+trans(pt,5)
     let yn=trans(pt,3)*x0+trans(pt,4)*y0+trans(pt,6)
     let x0=xn
     let y0=yn
     plot points:15*x0+50,10*y0
loop
end
```

Divisibility in Bases

Ruth L. Owen, *student*

Messiah College
Grantham, Pennsylvania 17027

In his "Mathematical Games" column in the December 1978 issue of *Scientific American*, Martin Gardner gave the following problem: *Find a 9 digit number using each of the digits 1 through 9 once and only once such that the first n digits, when taken by themselves, form a number divisible by n.* This problem was first posed by Lea Gorodisky of Argentina and asked for a Social Security number with these properties. The number 147328695 would fulfill the requirement in the first three digits because 1 is divisible by 1, 14 is divisible by 2, and 147 is divisible by 3, but it would fail in the fourth digit because 1473 is not divisible by 4. To check each of the many possible permutations of the digits 1 through 9 would require quite a bit of work. Obviously, it would be helpful if we can eliminate some of the possibilities so we have fewer numbers to check. This can be done by using some "divisibility rules." One example is that to be divisible by 5, a number in base 10 must end with either a 5 or a 0, and since we want to use only the digits 1 through 9, we now know the fifth digit must be a five. There are other rules which would narrow our search still further. Divisibility rules obviously play an important role in solving this problem efficiently.

This "nine digit problem" can also be generalized to bases other than 10, and in doing so one can find divisibility rules which apply in any base. I have discovered three theorems of divisibility rules in trying to find solutions to this problem in bases up to 10.

The first theorem is: *In a numeration system with an odd base, a number is even if and only if it has an even number of odd digits.* As an example, let us look at base 5. A four digit number in base 5 which would be written in the form $ABCD$ has the value $125A + 25B + 5C + D$. Since 5 is odd, each of the place values (which are powers of 5) is also odd. We know that an odd number times an even number yields an even number and an odd number times an odd number yields an odd number. We also know that an odd number plus an odd number yields an even number, an odd number plus an even number yields an odd number, and an even number plus an even number yields an even number. Because of these facts, a number of the form $ABCD$ in base 5 which has exactly two or four even digits would be even, but if it has one or three even digits it would be odd because it would have an odd number of odd digits. Thus for any odd base, an even number must have an even number of odd digits.

In our "generalized nine digit problem" we want numbers ending with the second, fourth, sixth, etc. digits to be divisible by 2, 4, 6, etc., respectively, which means that those numbers will be even. So the first two digits in an odd base must either both be even or both be odd, and then the next two digits must also be both even or both odd, etc. Thus a solution for an odd base, if one exists, would have each pair of digits being of the same parity.

The second theorem to aid us in solving this problem is: *If d divides b, then a number in base b notation is divisible by d if and only if its last digit is a multiple of d.* A familiar example of this is in base 10. We learn very early in our mathematical education that a number in base 10 is divisible by 2 if the last digit is a 0, 2, 4, 6 or 8 (i.e., divisible by 2), as well as the fact that it is divisible by 5 if the last digit is a 0 or 5. The reason for this is that the place value for the last digit is 1, while the rest of the place values are 10 or higher powers of 10 and thus have 2 and 5 as factors, which means that divisibility by 2 or 5 depends only on the last digit. For our "generalized nine digit problem" this means (among other helpful things) that a number in an even base must have its second, fourth, sixth, etc. digits being even (i.e., divisible by 2). Using all the even digits in the even positions leaves the odd digits to be used in the odd positions, so solutions in an even base will begin with an odd digit and the parity of the digits will alternate.

The third theorem is: *A number written in base b notation is divisible by $b-1$ if and only if the sum of its digits is divisible by $b-1$.* This will be shown using a five digit number $ABCDE$ with base b. The value of this number is

$$Ab^4 + Bb^3 + Cb^2 + Db + E.$$

We can then manipulate this expression algebraically:

$$= A(b^4 - 1 + 1) + B(b^3 - 1 + 1) + C(b^2 - 1 + 1) + D(b - 1 + 1) + E$$

$$= A(b^4-1)+A+B(b^3-1)+B+C(b^2-1)+C+D(b-1)+ D+E$$

$$= A(b^4-1)+B(b^3-1)+C(b^2-1)+D(b-1)+(A+B+C+D+E).$$

Because n^k-1 can be factored as $(n-1)(n^{k-1}+n^{k-2}+\ldots+1)$, we can see that each of the terms in parentheses has a factor of $b-1$. Therefore, for the entire expression to have a factor of $b-1$, the sum $A+B+C+D+E$ must have a factor of $b-1$.

This third theorem has a powerful effect on this problem: it shows that for any odd base there are no solutions to the "generalized nine digit problem." To show this, we will look at a well-known theorem which can be easily proven by induction:

$$1+2+3+\ldots+k=\frac{k(k+1)}{2}.$$

In our problem, we want the sum of all the digits for a given number in base b to be divisible by $b-1$ because there will be $b-1$ digits. So let $k=b-1$. Then the sum of the $b-1$ digits is $(b-1)b/2$. If the base b is even there is no problem because the 2 divides into b and the sum is still divisible by $b-1$. However, if b is odd then b is not divisible by 2 and $b-1$ is. Once one factor of $b-1$ (that is, the 2) has been divided out by the 2 in the denominator, the resulting sum of the digits is no longer divisible by $b-1$. Thus there is no number in an odd base satisfying the requirements of the "generalized nine digit problem."

Using these divisibility rules to eliminate many permutations of the digits and checking the remaining ones, I found these as the only solutions for the problem in bases from 2 to 10:

Base b	Solution(s)
any odd base	no such number exists
2	1
4	123 and 321
6	14325 and 54321
8	3254167, 5234761 and 5674321
10	381654729

In generalizing this problem, three theorems were discovered for divisibility in bases. These theorems were discovered by trying to simplify our specific problem, but they can be used for any situation as general divisibility rules for bases other than 10.

Acknowledgement. I would like to thank my faculty advisor, Dr. Lamarr C. Widmer, for his encouragement during my investigations.

Has Your Subscription Expired?

Your *Pentagon* subscription expires with the Volume and Number that appears in the upper right corner of your address label (see back cover). Since this issue is Volume 51 Number 2, if the code 51-2 appears on your label then THIS IS YOUR LAST ISSUE! Please send your renewal check -- just $5 for 4 more issues -- together with your name and address to:

Business Manager
The Pentagon
c/o Department of Mathematics
C. W. Post / Long Island University
Brookville, New York 11548 USA

(Single year subscriptions are $3 per year; foreign subscriptions are $5 (US) per year). Please renew promptly to avoid gaps in your journal collection.

Pascal's Triangle Revisited

Michelle Creola, *student*

Pennsylvania Lambda

Bloomsburg University of Pennsylvania
Bloomsburg, Pennsylvania 17815

Presented at the 1990 Region I Convention and
awarded THIRD PLACE by the Awards Committee.

Pascal's triangle seems to turn up in places where we would least expect it. For example, examine Figure 1 (below) and try to find the number of ways to read the word ABRACADABRA starting at the top A, ending with the bottom A and doing no back tracking (one possibility is illustrated).

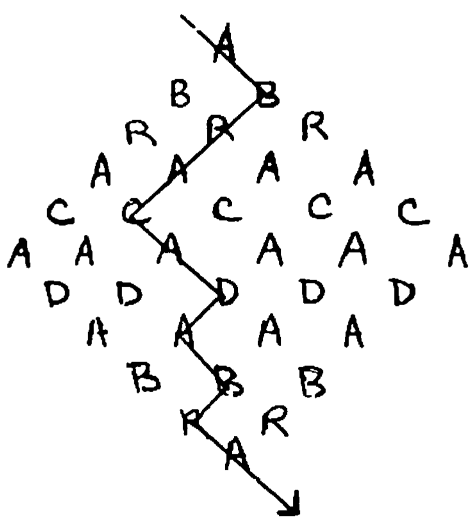


Figure 1

One approach to the problem would be to exhaust all possibilities by counting the number of ways to read ABRACADABRA. However, without breaking the problem down into smaller tasks, it would probably become quite confusing.

In order to approach the problem systematically, first find the number of shortest paths to the letters which are one space away from the top A. For example, there is only one possible path to each of the two B's which are one space away from the top A. Next, find the number of paths to letters which are two spaces away from the top A; namely, the next row of three R's. There is only one possible path to each of the two R's on the boundaries but there are two possible paths to the center R from the top A. If we continue this process for letters which are three spaces away, four spaces away, and so on, we should notice a pattern. Substituting the number of paths to each of these letters from the top A for the letters themselves, we have the numbers shown in Figure 2.

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

Figure 2

Remarkably, this figure appears to be part of Pascal's Triangle, in which each entry is the sum of the two numbers diagonally above it. If we use this recursion to fill in the number of paths to each of the remaining letters in the ABRACADABRA figure, we can easily find the number of paths from the top A to the bottom A.

In Figure 3, notice that when the number of paths to each of the letters in the ABRACADABRA figure is filled in using the recursion, there are 252 paths from the top A to the bottom A. Here we have the answer to the original question: there are 252 ways to read ABRACADABRA in the diamond figure (see [1]).

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

6 15 20 15 6

21 35 35 21

56 70 56

126 126

252

Figure 3

Another problem which relates to Pascal's triangle is that of finding the number of paths from the origin (0,0) to any point (x,y) with integer coordinates in the Cartesian plane. Examining Figure 4, we can reason that the number of paths to a point (x,y) must be the sum of the number of paths to $(x\text{-}1,y)$ and $(x,y\text{-}1)$, since all paths to (x,y) must pass through either $(x\text{-}1,y)$ or $(x,y\text{-}1)$.

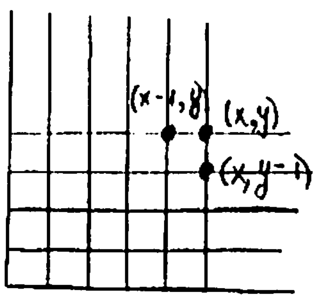


Figure 4

This is an important detail, since if we turned the coordinate system so that the origin was the apex of a triangle, we would observe that each number would be the sum of the two numbers diagonally above it. This is the same recursion we found in Pascal's triangle and we can use it to fill in the number of paths to each point (x,y), as shown in Figure 5.

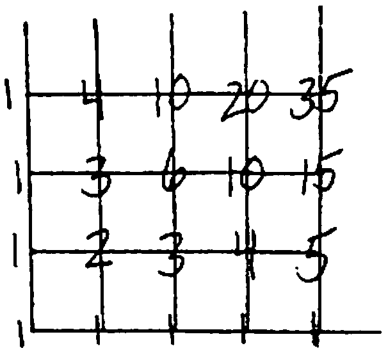


Figure 5

Since we could find the answer to this problem by simply finding the row and position that the point would occupy in Pascal's triangle, it would be convenient to find a formula which converts a point (x,y) to the corresponding position in the triangle. Recall that each number in Pascal's triangle can be computed by finding the combination $\binom{n}{r}$, where n is the row in the triangle and r is the position within that row. Figure 6 illustrates the problem using the point (2,3) as the specific example and Figure 7 shows the coordinate system turned at an angle to resemble Pascal's triangle.

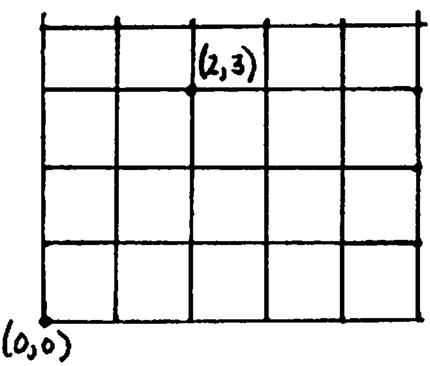


Figure 6

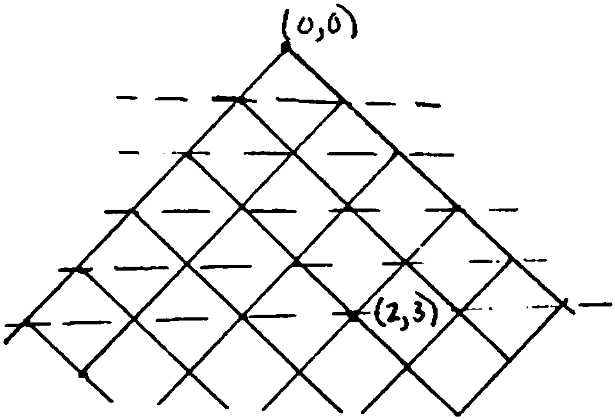


Figure 7

The row in Pascal's triangle in which a point (x,y) is positioned can be found by adding the x and y values, since this gives the distance from the origin or the apex of the triangle. The y value gives the position within any row of the triangle. Note in Figure 8 that for the point (2,3), the position in Pascal's triangle would be the third position in the fifth row. The value in this position is $\binom{2+3}{3} = 10$. Therefore, if we wish to find the number of paths to any point (x,y) in the Cartesian coordinate system, we need only to find $\binom{x+y}{y}$, where $x+y$ is the row and y is the position within the row in Pascal's triangle.

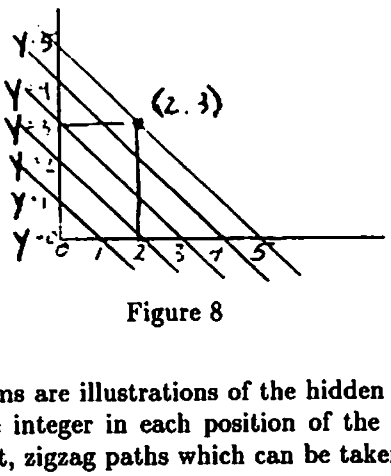


Figure 8

These two problems are illustrations of the hidden geometry found in Pascal's triangle. The integer in each position of the triangle represents the number of shortest, zigzag paths which can be taken from the apex to that point.

Pascal's triangle came about as a result of Blaise Pascal's work in probability. He used the binomial coefficients in the triangle to solve problems involving card and dice games (see [2]). Although we commonly refer to the triangle of binomial coefficients as "Pascal's triangle," Chinese mathematicians used similar diagrams as a source of the method of extracting higher degree roots as early as the fourteenth century [3]. It is not known for certain whether or not Pascal was aware of these discoveries in the Far East; however, he can be credited with introducing the triangle to the West.

Blaise Pascal listed nineteen properties or "numerical correspondences" dealing with the triangle (see [4]) and we will examine a few of these properties. First, observe in Figure 9 the pattern found when summing the integers in the rows of the triangle. For any row n in the triangle, the sum of its entries is 2^n.

row 0	1	$= 2^0$
row 1	$1+1$	$= 2^1$
row 2	$1+2+1$	$= 2^2$
row 3	$1+3+3+1$	$= 2^3$
row 4	$1+4+6+4+1$	$= 2^4$

Figure 9

This property can be proven using mathematical induction, or it can be verified using a combinatorial argument. Recall that for any set of n elements there are 2^n subsets of that set. We can also reason that there are $\binom{n}{0}$ subsets of size zero, $\binom{n}{1}$ subsets of size one, $\binom{n}{2}$ subsets of size two, ..., and $\binom{n}{n}$ subsets of size n. If we add these together, we must come up with the total number of subsets, which we already know is 2^n. However, the sum $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots+\binom{n}{n}$ is nothing more than the sum of row n in Pascal's triangle. Therefore, we know that the sum of row n in Pascal's triangle must be 2^n. Knowing this, we can also observe that the sum of any row is one greater than the sum of all previous rows. The proof of this property follows easily by mathematical induction.

$$
\begin{array}{cc}
1 & \\
1-1 & =0 \\
1-2+1 & =0 \\
1-3+3-1 & =0 \\
1-4+6-4+1 & =0
\end{array}
$$

Figure 10

If we sum the rows in the triangle once again, only this time alternating plus and minus signs, we can observe another interesting property, illustrated in Figure 10. For each row in Pascal's triangle, the sum is zero. In order to verify that this property is always true, we can expand the binomial $(1+(-1))^n$

$$= \binom{n}{0} 1^n + \binom{n}{1} 1^{n-1}(-1) + \binom{n}{2} 1^{n-2}(-1)^2 + \cdots + \binom{n}{n}(-1)^n$$
$$= \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots \pm \binom{n}{n}$$

This is the sum we are looking for in each row of the triangle and it is equal to $(1+(-1))^n$, which is nothing other than zero. Therefore, we know that the sum of each row of Pascal's triangle with alternating plus and minus signs is zero.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \\ & 4 & & 6 & & 4 & \\ & & 10 & & 10 & & \\ & & & 20 & & & \end{array}$$

Figure 11

Next, examine the diamond shaped portion of Pascal's triangle shown in Figure 11 and find the sum of the integers shown in the figure (see [1]). We may observe another pattern in Figure 11 which makes it easier to find the sum of the integers. The sum of each diagonal, which we will call an avenue, is found in the last position of the next avenue. For example, the sum of the first avenue is four and this number is found in the last position of the second avenue. This property holds due to the recursion on which the triangle is based. Each number in the last position can be written in terms of numbers in the previous avenue using this recursion formula. For example, the sum of the third avenue is 20 and this number can be found in the last position of the fourth avenue. But $20 = 10 + 10$ and $10 = 4 + 6$ so that $20 = 10 + 6 + 4$. We also know from the recursion that $4 = 3 + 1$ and $1 = 1 + 0$, so we have by substitution that $20 = 10 + 6 + 3 + 1$, which is all the numbers in the third avenue.

$$
\begin{array}{ccccccccc}
 & & & & \frac{1}{1} & & & & \\
 & & & \frac{1}{2} & & \frac{1}{2} & & & \\
 & & \frac{1}{3} & & \frac{1}{6} & & \frac{1}{3} & & \\
 & \frac{1}{4} & & \frac{1}{12} & & \frac{1}{12} & & \frac{1}{4} & \\
\frac{1}{5} & & \frac{1}{20} & & \frac{1}{30} & & \frac{1}{20} & & \frac{1}{5}
\end{array}
$$

Figure 12

We have discussed a few interesting patterns and properties of Pascal's triangle, but let us now turn our attention to a lesser know triangle of numbers. Figure 12 displays what is known as the "Harmonic Triangle of Leibnitz." Its boundaries are the reciprocals of successive integers and there is a recursion in this triangle which is somewhat opposite to that which we observed in Pascal's triangle. In Pascal's triangle each entry is the sum of the two entries diagonally above it, but in Leibnitz's triangle each entry is the sum of the two diagonally below it. Figure 13 shows a comparison between the recursions of Pascal's and Leibnitz's triangles.

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
\qquad
\begin{array}{ccccccc}
 & & & \frac{1}{1} & & & \\
 & & \frac{1}{2} & & \frac{1}{2} & & \\
 & \frac{1}{3} & & \frac{1}{6} & & \frac{1}{3} & \\
\frac{1}{4} & & \frac{1}{12} & & \frac{1}{12} & & \frac{1}{4}
\end{array}
$$

Figure 13

Although the two triangles appear to be very different from one another, there exists an interesting relationship between corresponding entries. Using the same notation previously used for row and position, each entry in Leibnitz's triangle is

$$\frac{1}{\binom{n}{r}(n+1)} = \frac{1}{\frac{n!}{r!(n-r)!}(n+1)} = \frac{r!(n-r)!}{(n+1)!}$$

and so the entry in Leibnitz's triangle in row n position r can be computed using the formula $r!(n-r)!/(n+1)!$

We can prove that this formula is valid by showing that it fits the recursion previously defined for Leibnitz's triangle. If we use the notation $L(n,r)$ to represent the value in row n position r of Leibnitz's triangle, the recursion formula may be written

$$L(n,r) = L(n+1,r+1) + L(n+1,r).$$

Now, if we substitute our formula into the right side of the equation, we have

$$\begin{aligned} L(n,r) &= \frac{(n+1)!\big(n+1-(r+1)\big)!}{(n+2)!} + \frac{r!(n+1-r)!}{(n+2)!} \\ &= \frac{(r+1)!(n-r)! + r!(n-r+1)!}{(n+2)!} \\ &= \frac{r!(n-r)!\big((r+1)+(n-r+1)\big)}{(n+1)!(n+2)} \\ &= \frac{r!(n-r)!(n+2)}{(n+1)!(n+2)} = \frac{r!(n-r)!}{(n+1)!}, \end{aligned}$$

which is the conjectured formula for the entry $L(n,r)$ in Leibnitz's triangle.

row 0	$\frac{1}{1}$	$=\frac{1}{1}$
row 1	$\frac{1}{2}-\frac{1}{2}$	$=0$
row 2	$\frac{1}{3}-\frac{1}{6}+\frac{1}{3}$	$=\frac{1}{2}$
row 3	$\frac{1}{4}-\frac{1}{12}+\frac{1}{12}-\frac{1}{4}$	$=0$
row 4	$\frac{1}{5}-\frac{1}{20}+\frac{1}{30}-\frac{1}{20}+\frac{1}{5}$	$=\frac{1}{3}$

Figure 14

Earlier, we observed several properties of Pascal's triangle dealing with summing rows. When we alternately add and subtract entries in Leibnitz's triangle, we can observe the pattern shown in Figure 14. Sums of the odd rows shown are zero and sums of the even rows are reciprocals of consecutive integers. I do not have a complete proof of this conjectured property at present, but it appears to be an interesting comparison to a similar property of Pascal's triangle.

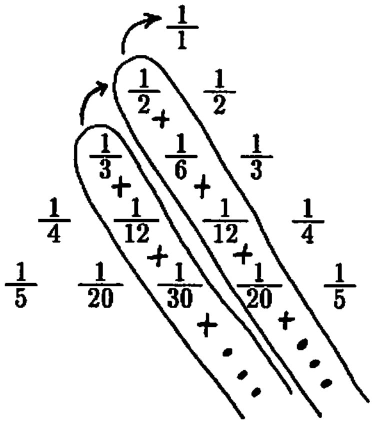


Figure 15

We also noted that in Pascal's triangle the sum of an avenue of finite length is found in the last position of the next avenue. In Leibnitz's triangle the infinite sum of an avenue is found in the first position of the previous avenue (see Figure 15). For example,

$$1 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots$$

since the partial sums S_n are $S_n = 1 - \frac{1}{n+1}$ and $\lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right) = 1.$ Also,

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{12} + \frac{1}{30} + \cdots$$

and

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{20} + \frac{1}{60} + \cdots.$$

The partial sums of the infinite sum equal to one half lead to the formula

$$S_n = \frac{1}{2} - \frac{1}{\binom{n}{r}(n+1)}$$

and since

$$\frac{1}{\binom{n}{r}(n+1)} \leq \frac{1}{n+1} \quad \text{and} \quad \lim_{n \to \infty} \frac{1}{n+1} = 0,$$

we have that

$$\lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(\frac{1}{2} - \frac{1}{\binom{n}{r}(n+1)} \right) = \frac{1}{2}.$$

We can use the same method to verify that the infinite sum of the fourth avenue is equal to one third. I have not finished a complete proof of this property at this time, but we can see a parallel to a similar property in Pascal's triangle.

After examining several properties of Pascal's triangle and comparing them to some properties of Leibnitz's triangle, it is clear that the triangle is much more than a useful tool for finding the binomial coefficients. By studying Pascal's triangle we can learn and discover relationships between powers of binomials, aspects of probability, and we may even find solutions to problems which at first seem to be unrelated to the triangle. Roger Hazelton [4] points out the significant role that Pascal's work played in the evolution of Leibnitz's integral calculus and the theory of probability and that even now Pascal's ideas are being implemented "... in such diverse fields as genetics or life insurance."

References.

[1] Polya, George. *Mathematical Discovery.* New York: John Wiley & Sons, Inc., 1981.

[2] Bishop, Morris. *Pascal: The Life of Genius.* New York: Greenwood Press, 1968.

[3] Yan, Li, and Shiran, Du. *Chinese Mathematics: A Concise History.* Oxford: Clarendon Press, 1987.

[4] Hazelton, Roger. *Blaise Pascal: the genius of his thought.* Philadelphia: The Westminster Press, 1974.

Bibliography.

Berman, Gerald, and Fryer, K. D. *Introduction to Combinatorics.* New York: Academic Press, 1972.

Bishop, Morris. *Pascal: The Life of Genius.* New York: Greenwood Press, 1968.

Bogart, Kenneth. *Introductory Combinatorics.* New York: Harcourt Brace Joanovich, 1990.

Hazelton, Roger. *Blaise Pascal: the genius of his thought.* Philadelphia: The Westminster Press, 1974.

Polya, George. *Mathematical Discovery.* New York: John Wiley & Sons, Inc., 1981.

Tucker, Alan. *Applied Combinatorics.* (2nd ed.) New York: John Wiley & Sons, Inc., 1984.

Yan, Li, and Shiran, Du. *Chinese Mathematics: A Concise History.* Oxford: Clarendon Press, 1987.

New Tools for Mathematics Students

Lisa Whitis, *student*

Kentucky Alpha

Eastern Kentucky University
Richmond, Kentucky 40475

Presented at the 1991 National Convention.

Most people will agree that carpenters, mechanics, and plumbers all need tools. Of course, these professionals might be able to do at least some of their tasks without tools, but their work would tend to be much more difficult. In similar fashion, mathematicians can perform their jobs more easily and more quickly if they have the right "tools." While a hammer might not be particularly helpful in finding a derivative or calculating the length of an arc, a symbolic manipulation package could prove to be useful.

Derive and *Mathematica* are two of the top symbolic manipulation packages on the market today, and both can be very productive tools for the mathematician. Just as a hammer is a better tool for driving nails than is a wrench, *Derive* and *Mathematica* each have advantages over the other in different situations. In general, *Derive* is a better tool in the classroom setting, while *Mathematica* is a better research tool. Regardless of the user's purposes, both programs can often save the user time and energy. Leibniz once said, "It is unworthy of excellent men to lose hours like slaves in the labor of calculation" ([2], 6). Certainly, *Derive* and *Mathematica* can readily perform many calculations to save users hours of labor.

Derive is a menu-driven program from Soft Warehouse. It is a low-cost package (around $200) that requires only 512K of computer memory. It runs on a PC-XT or any more powerful IBM-compatible machine and does not use a coprocessor. *Derive* performs many of the functions that *Mathematica* does, and it generally does the simpler ones in less time. The program is very user-friendly — this facet of *Derive* is primarily due to its menu. Commands may be chosen by highlighting with the space bar and entering a command or by typing the corresponding capital letter of the command; the Author command will allow the user to type in most commands directly. *Derive* has excellent graphing capabilities — it performs two-dimensional and three-dimensional plots very nicely. A movable cross, whose coordinates are indicated at the bottom of the screen, is displayed in the first quadrant of the two-dimensional plot window. The cross may be moved, and the center of the window may then be positioned over the cross with the Center command. Zooming magnifies surface details, and plotting accuracy and the plot scale may be adjusted. A window may be split so that expressions can be viewed beside graphs. It is possible on computers attached to color monitors to make use of colors; for instance, one color may be used on the upper side of a three-dimensional graph with another color on the lower side. It is also possible to graph several functions, each a different color, in the same two-dimensional window; this is not possible on *Mathematica*. Examples of interesting *Derive* plots include a couple of three-dimensional graphs that resemble a volcano and a sombrero (see Figure 1). In an article in *PC Magazine*, Barry Simon praises *Derive* for its graphics, low cost, "basic hardware requirements," fast numerics, and friendly user interface ([6], 326). Certainly, *Derive* merits a lot of praise for its usefulness as a tool.

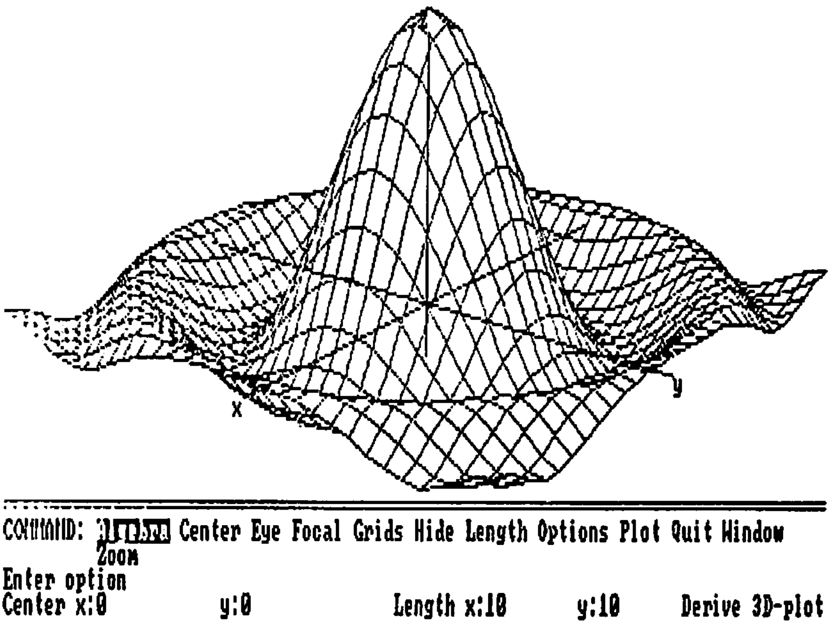


Figure 1.

Not unlike other tools, *Derive* is not suitable for every task. Simon emphasizes *Derive*'s lack of a programming language as a flaw ([6], 326). The recently released *Derive*, Version 2.02, does have a programming language, although not a very sophisticated one. Simon points to another flaw that still holds true: "*Derive* offers no direct printer support; you are limited to what is provided by DOS graphics or third-party programs" ([6], 326). This lack of printer support may or may not be a problem, depending on the user's needs and access to other resources. *Derive* is also somewhat limited in equation solving. While it can handle a system of linear equations or one nonlinear function of a single variable, it is not equipped to handle nonlinear equations in several variables (unless the user converts them to several problems involving one variable at a time). *Derive* does not numerically evaluate very many infinite sums (it does do geometric series), and it is not very useful in performing symbolic calculations dealing with more sophisticated functions. Essentially, *Derive* is an excellent tool for users who do not require a package equipped for complicated programming or high-end calculations with more exotic functions.

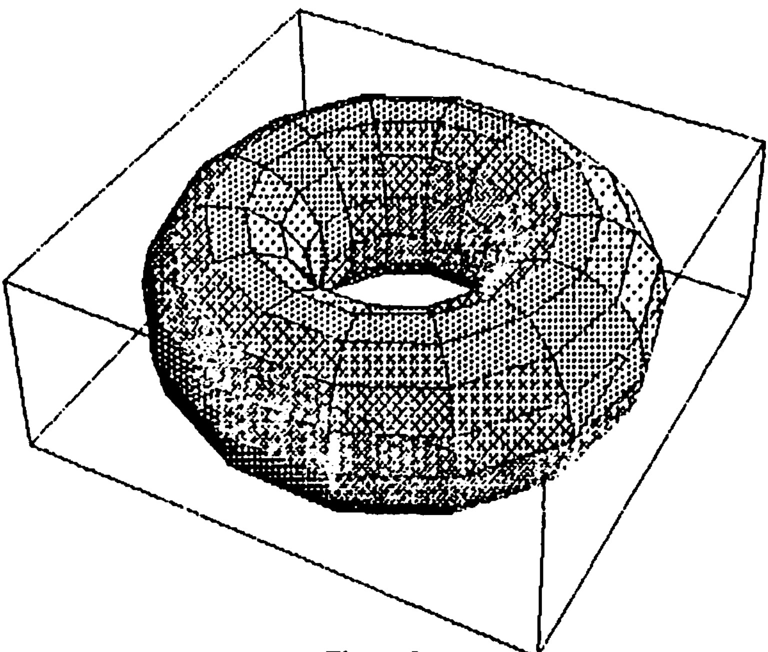


Figure 2.

Mathematica, Version 1.2, is a very powerful program from Wolfram Research. The IBM version of *Mathematica* sells for around $700, and the

Mac version sells for about $400. It requires 2MB of computer memory and uses a command-line interface. *Mathematica*, while essentially command-line driven, has a command completion feature; typing the beginning of a command and hitting F2 produces a list of all commands that start that way. A question mark followed by a command produces information about the command. *Mathematica* also has superb graphing capabilities (see Figure 2). Graphs may be rotated to change the viewpoint, and the animation of graphs is also possible. *Mathematica* is comprised of a kernel that does computations and a front end that deals with user interaction. Many front ends have "notebooks" that contain text, graphics, and definitions. Notebooks can allow the user to readily manipulate work done previously. *Mathematica* has exceptional programming abilities; its programming language can be used to perform tasks like list processing and generation. *Mathematica* is a tool for those who need a lot of power.

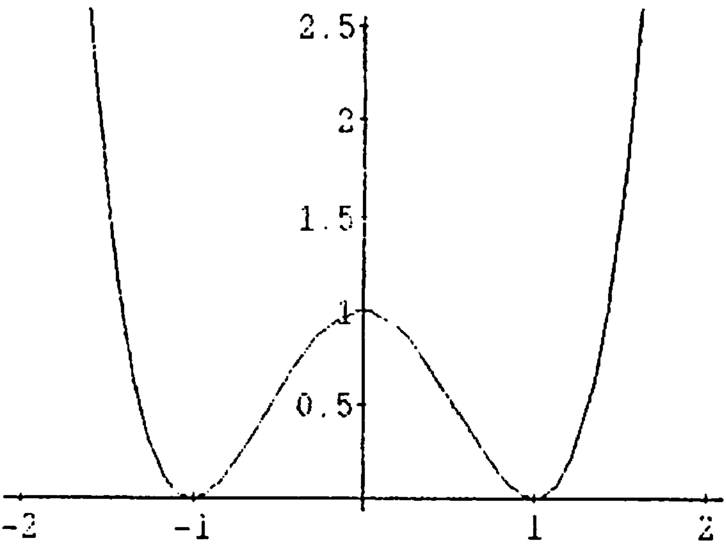
Of course, *Mathematica*, like *Derive*, is not a tool that is ideal for every purpose. In November of 1990, at the Ohio State Conference on Technology in Calculus, Wade Ellis said that "a software package for use in a mathematics class must have a short learning curve." Most assuredly, *Mathematica*, in spite of its command-completion feature, has a fairly steep learning curve. Its user manual is a very thick text to which the user must refer often. According to Bob Pervine of Murray State University, the average student requires around 6 hours to learn how to use the Mac and basic *Mathematica*. Unlike *Derive*, *Mathematica* is sometimes more time- and thought-consuming because of its input requirements. The first letter of a command must be capitalized, and square brackets, instead of the standard parentheses, are used to enclose arguments of functions. If cost is a consideration (it usually is something to consider when purchasing a tool), *Mathematica* may be expensive enough that the user might want to consider a less powerful program that costs less. Basically, *Mathematica* is a suitable tool for users who need power and have some time and money to spare.

Both *Derive* and *Mathematica* can serve a lot of purposes, so let us examine a little more closely what these two tools can do with regard to a specific aspect of mathematics: calculus. After all, calculus does involve many of the features like algebra, trigonometry, and graphics. Before Barry Simon wrote his article that included evaluations of the two programs, he sent a set of problems to their makers and requested procedures for optimal results. I have also tried working certain problems with the programs, and some of Simon's and my results will be presented.

First of all, let us look at a few notable examples of tasks that *Derive* and *Mathematica* can perform for the user. *Derive* can be used to Author the expression 1/(sqrt(4x^2 - 8x - 5)). Choosing the Calculus, Integrate, and Simplify commands produces the result of the indefinite integral of the expression. Since this is a problem for which a trigonometric substitution would be employed for a by-hand solution, obviously letting the computer do the work is much easier. Notice that "sqrt" can be in lower-case letters and that an asterisk is not necessary for multiplication. *Mathematica* does not require the asterisk either; it interprets spaces as multiplication, too. (An asterisk can be used for either program.) Using *Mathematica* and typing Integrate[x^n, x] results in producing (x^(1 + n))/(1 + n). Examples such as this may help calculus students to reproduce and learn integration or differentiation formulas. Again using *Mathematica*, entering D[f[x]^n, x] results in the output of the derivative nf[x]^(1 + n)f'[x]. While *Derive* will not perform such an operation that involves a function f(x), it can be used to evaluate several basic examples of integration or differentiation from which the user might be able to deduce the formulas being employed (in a classroom setting, this might even be preferable). On *Derive*, several files containing examples of calculus problems may be loaded. The CALCULUS.MTH file includes examples of calculations of derivatives, integrals, limits, and sums. *Derive* also includes files including examples of two-dimensional, three-dimensional, polar, and parametric plots that the user interested in calculus may find helpful or interesting. The *Derive* user may also Author an expression like x^4 - 2x^2 + 1 and use a split window to view the graph beside the expression. The user can employ the Calculus, Differentiate, Simplify, and Solve commands to find the critical points and then Plot to examine the points on the graph; the Manage and Substitute commands will give the value of y at a critical point. With *Mathematica*, the user may type x^4 - 2x^2 + 1, then D[%, x] (% stands for the last result, %% means the next-to-the-last result, and %n means the n-th output), and finally Solve[% == 0, x] to find critical points. Keying in Plot[x^4 - 2x^2 + 1, {x, -2, 2}] graphs the function below the other results (see Figure 3). Overall, both packages will perform an extensive set of calculus and graphical operations.

Naturally, there are some tasks that tools cannot perform and some that they attempt and perform with errors. Thus, in the cases of *Derive* and *Mathematica*, there are some instances in which one or both tools fail in the area of calculus. Notably, both programs produce the answer of -2 when faced with the integration of (1/x^2)dx from -1 to 1, despite the fact that the integral does not exist across x = 0. Both also give the answer of ln x ("LN(x)" on *Derive* and "Log[x]" on *Mathematica*) rather than ln|x| for the indefinite integral of (1/x)dx. Both programs give (2/3)(x - 1)^(3/2) - (2/3)(1 - x)^(3/2) as the answer to the indefinite integral of (sqrt(1 - x) + sqrt(x - 1))dx, even though the integrand is only defined at x = 1. In calculus problems involving the square root of x, *Derive* simplifies sqrt(x^2) to |x| while *Mathematica* produces the result of x. According to Barry Simon, one of the problems submitted to the makers of the programs was the following: "Consider a function F(x,y) with y a function of x, y(x). Compute the total derivative DF/Dx and the partial derivative with respect to the first variable." *Derive* does not do partial derivatives for general functions but could handle the total derivative. The problem was solved on *Mathematica* by entering Dt[f[x, y], x] and D[f[x, y], x] for the total and partial derivatives, respectively. Simon also asked for the derivative of (x^10)*cos(x^5*ln(x)) and for the integral of the result (and its simplification). Both programs could take the derivative, but neither could produce the answer to the integration ([5], 863). It is worthwhile to note that these packages are not infallible — the user must check results for accuracy using common sense and knowledge about mathematics.

```
In[9]:=
  Plot[x^4 - 2x^2 + 1, {x, -2, 2}]
```




```
Out[9]=
  -Graphics-
```

Figure 3.

Finally, let us compare *Derive* and *Mathematica* with regard to their performance in handling the problem of "The Great Marble Race." Let a marble race begin at the origin and end at the point (π,-2). Using elementary physics, the time required for a marble to roll down a path given by (x(t),y(t)), 0<=t<=1 from the origin to (pi,-2) can be shown to be equal to 1/sqrt(g) times the integral from 0 to 1 of

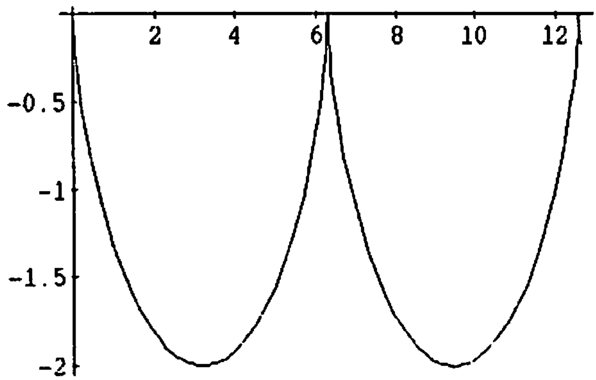
sqrt(((x'(t))^2 + (y'(t))^2)/(2y(t)))dt

(g is a constant of gravitational attraction.) Let us examine three different paths for the marble to follow: the straight line (pi*t,-2t), 0<=t<=1; the parabolic arch (pi*t, 2(t-1)^2 - 2), 0<=t<=1; and a rather interesting curve, the brachistochrone (pi*t - sin(pi*t), cos(pi*t) - 1), 0<=t<=1. On *Derive*, key in Author, and 1/sqrt(g) times the integral given (including t, 0, 1), making appropriate substitutions for each curve. π may be entered by typing pi or Alt-P. Simplify and approX(imate) give the correct results of 3.72419/sqrt(g), 3.27079/sqrt(g), and 3.14159/sqrt(g) (*Derive* indicates dubious accuracy for the second result). For *Mathematica*, 1/Sqrt[g]NIntegrate[appropriate expression, {t, 0, 1}] gives correct numerical approximations for the brachistochrone and the line, but *Mathematica* is unable to find the time for the parabolic arch. Thus, the programs (at least *Derive*) can show that the marble rolls down the brachistochrone curve in less time. (It is worthwhile to note that the integrals for the line and the brachistochrone can be evaluated fairly easily by hand; however, the integral for the parabolic arch cannot be evaluated in closed form. (The integral is not found in the table of integrals in [4].) Both programs also easily perform calculations that show that the marble arrives at the bottom in the same time, no matter where it starts on the brachistochrone curve, short of the finish line. (The brachistochrone problem was taken from [3].) With *Derive* a plot of the brachistochrone curve can be produced with Author, [Alt-P t - sin(Alt-P t), cos(Alt-P t) - 1], and Plot. The curve can be seen better if the cross is used to move the center and the scale is changed. The curve can be graphed with *Mathematica* by keying in ParametricPlot[{Pi t - Sin[Pi t], Cos[Pi t] - 1}, {t, 0, 4Pi}] (see Figure 4).

Now I will point out a few things regarding the brachistochrone problem that cannot be shown within the text of this paper. First of all, entering the commands and expressions is simply less time-consuming on *Derive*. The output of *Derive* is labeled in a slightly simpler manner and tends to look more like standard mathematical notation. For instance, *Derive* prints the Greek letter for pi, while *Mathematica* prints "Pi." *Derive* also seems to be much more flexible concerning input; its userfriendliness is a plus. *Mathematica* demands adherence to its guidelines concerning capital letters and bracket usage. Overall, I favor *Derive* in solving such a problem as this.

In[19]:=

```
ParametricPlot[{Pi t - Sin[Pi t], Cos[Pi t] - 1},
{t, 0, 4/3Pi}]
```



Out[19]=

-Graphics-

Figure 4.

In conclusion, both *Mathematica* and *Derive* are marvelous tools for the mathematician, particularly one concerned with calculus. Obviously, *Derive* is easier to use and less expensive, while *Mathematica* has more power and better printing capabilities. Although *Mathematica* would be the better choice for someone pursuing serious research, *Derive* is certainly ideal for educational purposes. Welcome to the world of mathematical tools — maybe *Derive* or *Mathematica* will be just what you need in your toolbox!

Acknowledgement. I wish to thank Dr. Patrick Costello, my faculty advisor, for all his help and encouragement throughout this project.

Bibliography.

[1] Custer, Linda and Minna Levine. "Number Crunching." *MacUser* November 1990, 154-62.

[2] Edwards, C. H. *Calculus and the Personal Computer.* New Jersey: Prentice-Hall, 1986.

[3] Evans, Benny and Jerry Johnson. *Uses of Technology in the Mathematics Curriculum.* Oklahoma State University, 1990.

[4] Hurley, James. *Calculus.* Belmont: Wadsworth, 1987.

[5] Simon, Barry. "Four Computer Mathematical Environments." *Notices of the American Mathematical Society.* September 1990, 861-868.

[6] ---. "The New World of Higher Math." *PC Magazine.* May 1990, 323-336.

[7] Soft Warehouse. *Derive User Manual.* Honolulu: Soft Warehouse, 1990.

[8] Wolfram, Stephen. *Mathematica: A System for Doing Mathematics by Computer.* Redwood City: Addison-Wesley, 1988.

Acknowledgement

The editor thanks the following individuals for their service during the past two years as referees of papers submitted to *The Pentagon* for possible publication.

V. Sagar Bakhshi

Virginia State University
Petersburg, Virginia

Robert P. Boner

Western Maryland College
Westminster, Maryland

Gerald M. Church

Southwestern Oklahoma State University
Weatherford, Oklahoma

Patrick Costello

Eastern Kentucky University
Richmond, Kentucky

Martin Erickson

Northeast Missouri State University
Kirksville, Missouri

Donald F. Shult

Ohio Northern University
Ada, Ohio

Joe Yanik

Emporia State University
Emporia, Kansas

Mathematical Mayhem

Reviewed by

Sean Forbes, *student*
and
H. K. Krishnapriyan, *faculty*

Iowa Beta

Drake University
Des Moines, Iowa 50311

Mathematical Mayhem is a journal for high school and university students enthusiastically produced by high school and university students in Toronto with the support of the University of Toronto. It is published from the "desktop" and, according to the September/October 1991 issue which we have before us, has a circulation of 92.

In its 33 pages, the journal has a report on the International Mathematics Olympiad, articles by students and a problem column, which has problems ranging from elementary high school level to ones that should challenge junior and senior mathematics majors in college. We found all the material interesting and were awed by the amount of work that has gone into it. The following description of the contents should give a good idea of the sort of material that finds its way into *Mathematical Mayhem*.

After a brief introduction, there is a page devoted to "noxious notation," a description of notation used in the journal, some standard ($[a, b]$ for the closed interval from a to b) and some which seemed non-standard (WOLOG for "without loss of generality"). This is followed by a "Shreds and Slices" column containing a contribution called "π in your face" by Ravi Vakil. As the punning title suggests, the article is a collection of some curious facts about π. After this is a report on the International Mathematics Olympiad (in which the founding editors and at least one of the current editors have competed). The report includes the problems of the 1991 IMO and solutions are solicited with a promise of publication.

We then come to articles on "A Generalization of a Problem by Ravi Vakil" (by Jeff Biesecker, Dave Smith and Lamarr Widmer), "The Meaning of 'Never'" (by Ravi Vakil), "European Express: Polynomials mod p" (by Plamen Koev, the European editor) and "Even Calculus Can Be Fun!" (by Ravi Vakil). These are followed by the problem section in which the problems are classified as "high school," "advanced" and "challenge board." To suggest the level of difficulty of these classes, we give below three problems, one from each class.

High School

Evaluate $\log(\tan 1°) + \log(\tan 2°) + \cdots + \log(\tan 89°)$.

Advanced

Show that

$$1 < \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{3n+1} < 2$$

for each integer $n > 1$.

Challenge Board

Let $f(x)$ be a polynomial of degree n with real coefficients and such that $f(x) \geq 0$ for every real number x. Show that

$$f(x) + f'(x) + f''(x) + \cdots + f^{(n)}(x) \geq 0$$

for all real x.

As can be seen from the above description, the focus of the journal is on pure mathematics and problem solving. The style of the journal is clear, informal and chatty.

Mathematical Mayhem is supported financially by the University of Toronto, subscription revenues and individual donations. We hope that the University of Toronto will continue to provide support for this project, both financially and by locating enthusiastic successors to the current group of editors.

We strongly recommend the journal to anyone interested in problem solving and especially to undergraduates interested in pure mathematics. At \$15 for five issues per year (\$25 for institutions), it is well worth the money. For those who want to subscribe, the following address should provide more information.

Mathematical Mayhem
c/o J. P Grossman
128 Ava Road
Toronto, Ontario, CANADA
M6C 1W1

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 January 1993. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 1993 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROBLEM 453 *Corrected* and PROBLEMS 455-459.

Problem 453 (Corrected). Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Prove that if a function f satisfies the equation $f(x+y) = f(x) \cdot f(y)$ for all real numbers x and y and if $f(x) = 1 + x \cdot g(x)$ for all real numbers x where lim $g(x) = 1$ as $x \to 0$, then $f'(x)$ exists for each real number x and $f'(x) = f(x)$.

Problem 455. Proposed by Russell Euler, Northwest Missouri State University, Marysville, Missouri. Rays r_1 and r_2 are concurrent at O. Let $\{a_i\}$ and $\{b_i\}$ be increasing sequences of points on r_1 and r_2 respectively, such that $d(\mathrm{O}, a_i) = d(\mathrm{O}, b_i)$ for $i = 1, 2, 3, \ldots$. If M_i is the midpoint of the line segment a_ib_i, prove that the points $\{M_i\}$ are collinear.

Problem 456. Proposed by the Editor. Hy Potenuse, president of the Society of Pythagoreans, announced that starting this year all members would celebrate certain special days as "Pythagorean Days." By definition, a Pythagorean Day occurs when the numerical value of the month and day are the legs of a right triangle whose sides are all integers. How many Pythagorean Days are there in a year and when do they occur?

Problem 457. Proposed by Albert White, Saint Bonaventure University, Saint Bonaventure, New York. In a standard bridge deck, assign the values 11, 12 and 13 to the jack, queen and king, respectively. Aces may assume the value 1 or 14. If four cards are selected, what is the probability that the cards are of the same suit and the numbers of the cards are consecutive with the first card having the smallest value? What is the probability if the cards do not have to be of the same suit?

Problem 458. Proposed by Michael White, Portville, New York and Albert White, Saint Bonaventure University, Saint Bonaventure, New York. Find

$$\lim_{n\to\infty} \sum_{k=0}^{n} \frac{k}{k!}\left(1-\frac{(-1)^{n-k}}{(n-k)!}\right).$$

Problem 459. Proposed by the Editor. A Heronian triangle has integral sides and integral area. Find an infinite family of such triangles which have two consecutive integers and a third odd integer for sides and such that the sides are not in arithmetic progression. Are there any Heronian triangles whose sides include two primes and two consecutive integers? For the purpose of this problem, right triangles also are excluded from consideration. An example of a right triangle which satisfies the problem is (11, 60, 61).

Please help your editor by submitting problem proposals.

SOLUTIONS 445-449.

Late solutions for problems OBG1-OBG5 were received from J. Sriskandarajah, University of Wisconsin Center-Richland, Richland, Wisconsin.

Problem 445. Proposed by Dave Smith, Messiah College, Grantham, Pennsylvania. Dirk, a junior math major, visited the campus post office to pick up his key for the new year. When he found his mailbox, he noticed that every year his mailbox had been in the same row in the large rectangle that was formed by all of the mailboxes. Hours later, the only thing he remembered about his current mailbox number was that it was somewhere in the 920's. He recalled that during his freshman and sophomore years, the numbers of his mailbox were #837 and #897, respectively. He also remembered that his roommate's mailbox number was #65 and that it was located seven boxes above the bottom row. If the post office numbers the mailboxes consecutively from top to bottom starting in the upper left corner, what is the number of Dirk's mailbox?

Solution by Paul J. Velek, Liberty University, Lynchburg, Virginia.

Since the roommate's mailbox (#65) is seven above the bottom row, mailbox #72 is the last one in that column and the number of boxes in each column must be a factor of 72 and be $\geq$ 8. Furthermore, since box #897 and box #837 are in the same row, the number of boxes in each column must also be a factor of $897 - 837 = 60$. Since the only common factor of 72 and 60 which is also $\geq$ 8 is 12, there are 12 boxes in each column. Since $837 \equiv 897 \equiv 921 \equiv 9 \pmod{12}$, the number of Dirk's current mailbox must be 921.

Also solved by Kendall Bailey, Drake University, Des Moines, Iowa; Charles Ashbacher, Hiawatha, Iowa; and the proposer.

Problem 446. Proposed by Lamarr Widmer, Messiah College, Grantham, Pennsylvania. The composite integer $1991 = 11 \cdot 181$ is palindromic as are all of its prime factors. What is the next integer after 1991 which has the same property if (a) single digit primes are allowed? (b) single digit primes are not allowed?

Solution by Charles Ashbacher, Hiawatha, Iowa.

Part (a). The next palindromic integer is 2002. This value must be discarded because it has 13 as a factor and 13 is not palindromic. The next possibility is $2112 = 2^6 \cdot 3 \cdot 11 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 11$ where each factor is a palindrome.

Part (b). The only two digit palindromic prime is 11. The next few palindromic primes are 101, 131, 151, 181, 191, 313, 353, 373 and 383. Now $11 \cdot 181 = 1991$, $11 \cdot 191 = 2101$ and $11 \cdot 313 = 3443$, which satisfies the conditions of the problem. Since the smallest possible product of two three digit numbers is $101^2 = 10201$ and $10201 > 3443$, 3443 is the desired solution because $11 \cdot p$ where $p = 101$, 131 or 151 produce products < 1991.

Also solved by Chris Clark, Drake University, Des Moines, Iowa (Part(a) only) and by the proposer.

Problem 447. Proposed Don Tosh, Evangel College, Springfield Missouri. The usual Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, ... and any term may be found by adding together the two preceding terms. Formally we have $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for integers $n > 2$. It is well known that the ratio of consecutive terms in the Fibonacci sequence converges to $r = (\sqrt{5} + 1)/2$, the golden ratio; i.e. $\lim f_n/f_{n-1} = r$ as $n \to \infty$. Next we define a generalized Fibonacci sequence $\{x_n\}$ by choosing any two real numbers a and b (neither of which is zero) and then setting $x_1 = a$, $x_2 = b$ and $x_n = x_{n-1} + x_{n-2}$ for integers $n > 2$. Prove that the ratio of consecutive terms in this generalized Fibonacci sequence still converges to r; i.e. $\lim x_n/x_{n-1} = r$ as $n \to \infty$.

Solution by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.

The general solution for any second order linear homogeneous difference equation with constant coefficients

$$(1) \qquad y_n + ay_{n-1} + by_{n-2} = 0$$

depends upon the roots of the characteristic polynomial

$$(2) \qquad r^2 + ar + b = 0$$

associated with the difference equation (1). If (2) has two distinct real roots r_1 and r_2, then the general solution for (1) is given by

$$y_n = c_1 r_1^n + c_2 r_2^n,$$

where c_1 and c_2 are arbitrary constants. But the characteristic polynomial for both the Fibonacci relation $f_n = f_{n-1} + f_{n-2}$ and the more general relation $x_n = x_{n-1} + x_{n-2}$ is

$$(3) \qquad r^2 - r - 1 = 0.$$

The two distinct roots of (3) are $r_1 = (1 + \sqrt{5})/2$ and $r_2 = (1 - \sqrt{5})/2$. Now, for arbitrary constants c_1, c_2, d_1 and d_2,

$$f_n = c_1\left((1 + \sqrt{5})/2\right)^n + c_2\left((1 - \sqrt{5})/2\right)^n$$

and

$$x_n = d_1\left((1 + \sqrt{5})/2\right)^n + d_2\left((1 - \sqrt{5})/2\right)^n.$$

Since

$$\lim_{n\to\infty} \left((1 - \sqrt{5})/2\right)^n = 0,$$

we clearly have

$$\lim_{n\to\infty} \frac{f_n}{f_{n-1}} = \lim_{n\to\infty} \frac{x_n}{x_{n-1}} = (1+\sqrt{5})/2.$$

Also solved by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; Russell Euler, Northwest Missouri State University, Marysville, Missouri; Charles Ashbacher, Hiawatha, Iowa; Michael Shepard, Southwest Missouri State University, Springfield, Missouri; and the proposer.

Problem 448. Proposed by Fred A. Miller, Elkins, West Virginia. Let A, B, C, and D be four concyclic points in the plane such that C and D are separated by A and B. If p_1, p_2 and p_3 are the lengths of the perpendiculars from D to lines AB, BC and CA respectively, show that

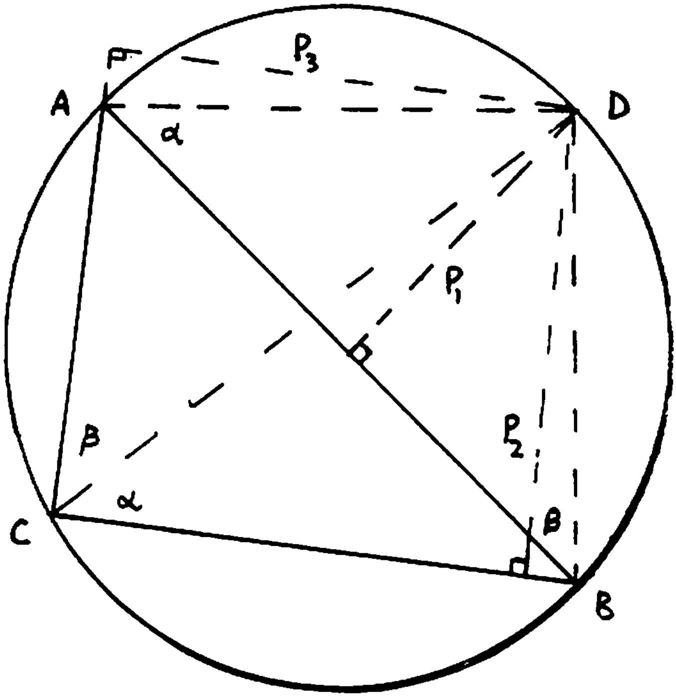
$$\frac{AB}{p_1} = \frac{BC}{p_2} + \frac{CA}{p_3}.$$

Solution by Scott H. Brown, Stuart Middle School, Stuart, Florida.

Let α be the common measure of angles BCD and BAD and let β be the common measure of angles DCA and DBA as shown in the figure. Then $p_1 = BD \sin(\beta)$, $p_2 = CD \sin(\alpha)$ and $p_3 = CD \sin(\beta)$. Now, according to Ptolemy's Theorem,

$$AB \cdot CD = AD \cdot BC + BD \cdot CA.$$

Hence



$$\left(\frac{AD}{CD}\right)\cdot BC+\left(\frac{BD}{CD}\right)\cdot CA = \left(\frac{p_1}{p_2}\right)\cdot BC+\left(\frac{p_1}{p_3}\right)\cdot CA = AB$$

and thus

$$\frac{AB}{p_1} = \frac{BC}{p_2} + \frac{CA}{p_3}.$$

Also solved by the proposer.

Editor's Comment. The featured solver also found this problem as Problem 3849 in the *School Science and Mathematics Journal.*

Problem 449. Proposed by Albert White, Saint Bonaventure University, Saint Bonaventure, New York. Assume that a square is inscribed in a circle whose radius is r. Then a circle is inscribed in the square. A square is inscribed in this circle and this pattern continues ad infinitum. Find the sum of the circumferences of all the circles and the sum of the perimeters of all the squares.

Solution by Sean Forbes, Drake University, Des Moines, Iowa.

Since the radius of the outside circle is r and the length of one side of the outermost square is $r\sqrt{2}$, each succeeding radius will be $\sqrt{2}/2$ times the radius of the preceding circle. The same relation holds for the lengths of the sides of the squares.

Thus, using the formula for the sum of an infinite geometric series (since $\sqrt{2}/2 < 1$), we have the sum of the circumferences is

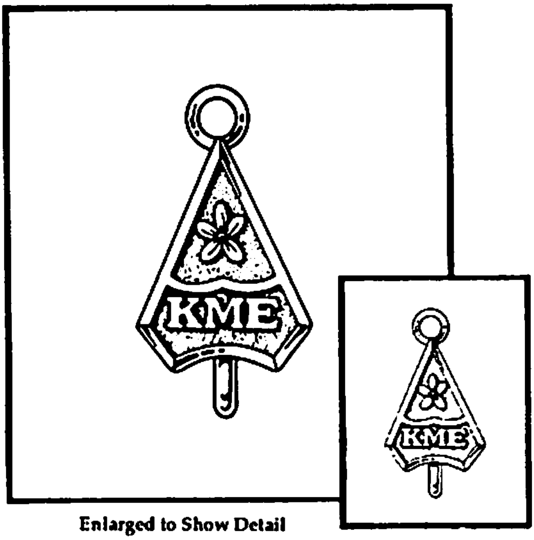
$$\frac{2\pi r}{1-(\sqrt{2}/2)} = (4 + 2\sqrt{2})\pi r.$$

Similarly, the sum of the perimeters is

$$\frac{4r\sqrt{2}}{1-(\sqrt{2}/2)} = (8 + 8\sqrt{2})r.$$

Also solved by Russell Euler, Northwest Missouri State University, Marysville, Missouri; Charles Ashbacher, Hiawatha, Iowa; Kendall Bailey and Diane Cummins (jointly), Drake University, Des Moines, Iowa; Scott H. Brown, Stuart Middle School, Stuart, Florida; and the proposer.

Kappa Mu Epsilon Jewelry



Enlarged to Show Detail

Actual Size

The Kappa Mu Epsilon Key (Shown above) is available as a charm, pin or tie - tac.

	GOLD PLATE	STERLING SILVER	10K Gold
#805 KEY CHARM	$ 5.00	$ 8.00	$45.00
# 806 KEY PIN	$ 5.50	$ 8.50	$52.00
# 807 KEY TIE - TAC	$ 5.50	$ 8.50	$52.00

Manufactured by

J.O. POLLACK Co.

Prices subject to change without notice

(Prices include sales tax)

Orders for Kappa Mu Epsilon Jewelry MUST be placed through your chapter's corresponding secretary.

Announcement of the Twenty-Ninth Biennial Convention

The Twenty-Ninth Biennial Convention of Kappa Mu Epsilon will be hosted by the New York Eta Chapter and will be held 22-24 April 1993 at the Niagara University in Niagara University, New York. Each attending chapter will receive the usual travel expense reimbursement from the national funds as described in Article VI, Section 2, of the Kappa Mu Epsilon constitution.

A significant feature of this convention will be the presentation of papers by student members of Kappa Mu Epsilon. The mathematical topic selected by each student speaker should be of interest to the author and of such scope that it can be given adequate treatment in a timed oral presentation. Student speakers will be chosen by the Selection Committee on the basis of written papers submitted prior to the convention. At the convention, the Awards Committee (composed of four students and four faculty members representing as many chapters as possible) will judge the speakers on both content and presentation and will select the prize winners.

Who may submit a paper?

Any undergraduate or graduate student member of Kappa Mu Epsilon may submit a paper for use on the convention program. A paper may be co-authored. If selected for presentation at the convention, the paper must be presented by one (or more) of the authors. Graduate students will not compete for prizes with undergraduates.

Presentation topics.

Papers submitted for presentation at the convention should discuss material understandable by undergraduate mathematics majors, preferably those who have completed differential and integral calculus. The Selection Committee naturally will favor papers within this limitation and which can be presented with reasonable completeness within the time allotted.

Presentation time limits.

The presentation of the paper must take at least 15 minutes and no more than 25 minutes.

How to prepare a paper.

Five copies of your paper, together with a description of any charts, models or other visual aids you plan to use during the presentation, must be submitted. The paper should be typewritten in the standard form of a term paper. It should be written as it will be presented, including length. A long paper (such as an honors thesis) must not be submitted with the idea that it will be shortened later when you present it! Appropriate references and bibliography are expected.

The first page of your paper must be a "cover sheet" giving the following information: (1) title, (2) author (your name must not appear elsewhere in the paper), (3) your student status ("undergraduate" or "graduate"), (4) both your permanent and school addresses, (5) the name of your *KME* Chapter and school, (6) a signed statement giving your approval that your paper be considered for publication in *The Pentagon*, and (7) the signed statement of your Chapter's Corresponding Secretary that you are indeed a member of Kappa Mu Epsilon.

How to submit a paper.

You must send the five copies of your paper to:

Dr. Arnold D. Hammel
KME National President-Elect
c/o Department of Mathematics
Central Michigan University
Mt. Pleasant, Michigan 48859

no later than 5 February 1993.

Selection of papers for presentation.

The Selection Committee will review all papers submitted to the National President-Elect and will choose approximately fifteen papers for presentation at the convention; all other papers will be listed by title and author in the convention program and will be available as "alternates." The National President-Elect will notify all authors of the status of their papers after the Selection Committee has completed its deliberations.

Criteria used by the Selection and Awards Committees.

The paper will be judged on (1) topic originality, (2) appropriateness to the meeting and audience, (3) organization, (4) depth and significance of the content, and (5) understanding of the material. The presentation will be evaluated on (1) style of presentation, (2) maintenance of interest, (3) use of audio-visual materials (if applicable), (4) enthusiasm for the topic, (5) overall effect, and (6) adherence to the time limits.

Prizes.

All authors of papers presented at the convention will be given two-year extensions of their *Pentagon* subscriptions. Authors of the four best papers presented by undergraduate students, as determined by the Awards Committee, will be awarded cash prizes of $100, $80, $60 and $40, respectively. If enough papers are presented by graduate students then one or more prizes will be awarded in this category.

Publication.

All papers submitted to the convention are considered as submitted for publication in *The Pentagon* (see page 2 for further information). Prize winning papers will be published after any necessary revisions have been completed and all other papers will be considered for publication. All authors are expected to schedule brief meetings with the Editor during the convention to review their manuscripts.

Kappa Mu Epsilon News

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy *KME* events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, Missouri 64801.

CHAPTER NEWS

Alabama Beta — University of North Alabama, Florence

Chapter President - Stacy Barringer
33 actives

Other 1991-92 officers: Kim Weems, vice president; Vicky Locker, secretary; Eddy Joe Brackin, corresponding secretary; Patricia Roden, faculty sponsor.

Alabama Gamma — University of Montevallo, Montevallo

Chapter President - Cynthia Pruitt
7 actives

Other 1991-92 officers: Marsha Oden and Beverly Smith, co-vice presidents; Kim Wilson, secretary; Brenda Valentine, treasurer; Charles Coats, corresponding secretary; Gene Garza, faculty sponsor.

Alabama Zeta — Birmingham-Southern College, Birmingham

Chapter President - Heath Gatlin
30 actives

The program for the fall initiation of 16 new members was presented by speaker Brian Cain, a high school mathematics teacher and alumnus of Alabama Zeta. Other 1991-92 officers: Erica Taylor, vice president; Julia Spazzarini, secretary/treasurer; Lola F. Kiser, corresponding secretary; Shirley Branan, faculty sponsor.

California Gamma California Polytechnic State University, San Luis Obispo

Chapter President - Susan Rehn

30 actives, 7 associates

The 1990-91 Annual Banquet was held at Embassy Suites Hotel in June 1991. Sixteen pledges were initiated, and new officers were honored. George Mach presented the Founders' Award to Leo Flores. W. Boyd Judd presented the University Scholarship which bears his name to Arnold Chan (formerly Arnold Tan). On behalf of Jeff Stought of Arthur Anderson & Company, Raymond Terry presented the Arthur Anderson Award for Excellence to Susan Rehn. The first two weeks of the Fall Quarter were devoted to the Fall Pledge Drive. In conjunction with general meetings in October, chapter members heard a presentation by Greg Becerra from IBM and a talk by Kristy McMahon from Hewlett Packard. In late October a Symposium from Lawrence Livermore Laboratory, led by Nancy Monson, was sponsored jointly by *KME*, UPE, and SWE. A Pledge Recognition Event was held November 1 at The Vista Grande Restaurant Conference Room. Speakers at general meetings in November included Dr. Williamson who spoke on "Coding Theory," and a team of four from Chevron who discussed their work activities with that company. Members of the Chevron team included Tom Neumayer, Merrill Lynch, Doug Harter, and Donna Seeger. A number of *KME* members also attended an Information Night sponsored by Anderson Consulting at Embassy Suites in November. California Gamma wishes to thank all of the afore mentioned speakers and companies for their contributions to the chapter. Additional thanks go to Mick Wets from Fair, Isaac for her continuing interest in *KME* and to Peter L. McCorkell, Vice President/Corporate Affairs, for Fair, Isaac's contribution of $500.00 to the club's work. Other 1991-92 officers: Philip Banks, vice president; Leo Flores, secretary/publicity chair; Shellie Hoyt, treasurer; Jennifer Courter and Jennifer Huskey, pledgemasters; Andrew Schaffner, SOSAM representative; Raymond D. Terry, corresponding secretary/faculty sponsor.

Colorado Gamma Fort Lewis College, Durango

Chapter President - Rachel Zeller

21 actives

The program for the first meeting of the semester was given by Duane Brown, who spoke on "The Post Office Box Problem." Members attending the second meeting assembled polyhedra models and enjoyed a pizza party. The chapter participated in the college's annual phon-a-thon and earned over $190.00. Other 1991-92 officers: Duane Brown, vice president; Gretchen Rothschopf, secretary; Steve Luxa, treasurer; Richard A. Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

Colorado Delta Mesa State College, Grand Junction

Chapter President - Karen L. Hughes

Fall semester programs included a film on the history and applications of pi and a film on symmetry. Other 1991-92 officers: Daniel L. Carroll, vice president; Duncan T. Thompson, secretary; Karl J. Castleton, treasurer; Harold Davenport, corresponding secretary; Cliff Britton, faculty sponsor.

Georgia Alpha West Georgia College, Carrollton

Chapter President - Peaches Leonard
25 actives

During the fall semester the Georgia Alpha Chapter of *KME* sponsored its Fourth Annual Food Drive for the needy. Food was collected and delivered to the Community Food Bank. On November 23, seventeen students and eight faculty members enjoyed a Fall Social at a local restaurant. Other 1991-92 officers: Kelly Graham, vice president; Michelle Williams, secretary; Paul McKinzey, treasurer; Joe Sharp, corresponding secretary/faculty sponsor.

Illinois Beta Eastern Illinois University, Charleston

Chapter President - Diane Cramer
42 actives

Programs scheduled for fall semester included a talk by Dr. Rahman entitled "A Trip to Russia & Scandinavia," Dr. Kinderman's "Let's Make a Deal, or What's Behind the Green Door?", a presentation by Dr. Rosenholtz entitled "Cats, Coloring Triangles, & Immovable Objects," and a talk by Mrs. Roma Dey. Math Club/*KME* also held a book sale to raise revenues for the club, discussed t-shirt designs, and enjoyed a fall picnic sponsored by the faculty, as well as a Christmas party in early December. Other 1991-92 officers: Laura Tougaw, vice president; Laura Yegge, secretary; Joe Klingler, treasurer; Lloyd Koontz, corresponding secretary/faculty sponsor; Pat Coulton/Rosemary Schmalz, faculty sponsors.

Illinois Eta Western Illinois University, Macomb

Chapter President - Tracey Merrill

14 actives, 20 associates

Chapter activities began in late August with a booth at the University Fair. The organization viewed the video "Math ... Who Needs It?" and heard Dr. Kalantari talk on mathematical paradoxes. A picnic and volleyball game in Everly Park was enjoyed by students, faculty, staff, and friends. The December 5 initiation of three new members was followed by a pizza party. Other 1991-92 officers: Kerry E. Pettengill, vice president/treasurer; Beth Litke, secretary; Larry J. Morley, corresponding secretary/faculty sponsor.

Iowa Alpha University of Northern Iowa, Cedar Falls

Chapter President - J. Ben Schafer

38 actives

The annual *KME* Homecoming Coffee held on October 5 was well attended by alums, students, and faculty. Students presenting papers at local *KME* meetings included Steve Walk on "Combinatorial Logic" and Charlotte Roth on "Euclid's Proof." Chris Mefford addressed the *KME* initiation banquet held at the Broom Factory on December 9 on "Public Key Crypto-systems." Tascha Yoder was awarded a student membership in the MAA. The Department of Mathematics and Computer Science are looking forward to moving back into a totally refurbished (and finally air conditioned) Wright Hall in January 1992. Other 1991-92 officers: Steve Walk, vice president; Julie Beck, secretary; Mary Bond, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

Iowa Delta Wartburg College, Waverly

Chapter President - Stephanie Hurley

53 actives

The September, October, November and December meeting programs were, respectively, the Car-Goat Problem, a panel about graduate schools, a talk by Cliff Lee (a Wartburg graduate employed as a statistician at CAT Company), and the traditional Christmas party. The chapter also formed a mathematics reading and discussion group, sold Roy's Egg-Cheeses at Homecoming, and organized committees to prepare for the fifteenth annual Math Field Day, which it cosponsors with the Mathematics and Computer Science Department. Five members were given one-year memberships in the Association for Women in

Mathematics based on their expressed interests in women's issues in mathematics. Other 1991-92 officers: Richard Brooks, vice president; Julie Rhoades, secretary; Nancy Wirth, treasurer; Augie Waltmann, corresponding secretary; Lynn Olson, faculty sponsor.

Kansas Alpha — Pittsburg State University, Pittsburg

Chapter President - Pam Vandervoort
50 actives

Fall semester activities focused on the initiation of ten new members in October. The initiation was preceded by a pizza party and followed by a magic show presented by Matt McIntosh, graduate assistant in the mathematics department. Kansas Alpha chapter sponsored a guest speaker, Joe Swartz of the Martin Marietta Astronautics Group, for the November meeting. He spoke on "The Finite Element Method and Maxwell's Equations." He also discussed opportunities in mathematics. Other 1991-92 officers: Kris Mengarelli, vice president; Rebecca Newcomb, secretary; Regina Hulvey, treasurer; Harold L. Thomas, corresponding secretary; Bobby Winters, faculty sponsor.

Kansas Beta — Emporia State University, Emporia

Chapter President - Susan Hurt
28 actives

Other 1991-92 officers: Sarah Gleason, vice president; Michelle Land, secretary; Dave Herrs, treasurer; Connie Schrock, corresponding secretary; Larry Scott, faculty sponsor.

Kansas Gamma — Benedictine College, Atchison

Chapter President for Associates - Radu Oprea
4 actives, 18 associates

Kansas Gamma began its semester activities in September with a "Make and Eat" Pizza Party at the home of Sister Jo Ann Fellin. Other social activities were the November Chili Party in the Roost (chili provided by Pam Clearwater, Jill Weigand, and Holly Dorlac) and the traditional Christmas Wassail at the home of Jim Ewbank. Doyle Dreiling, a 1978 Benedictine grad and member of KS Gamma, returned to campus on November 14 to share with the group. Having gained experience working for a large firm in the KC area and having now completed a masters degree in computer science, he currently owns his own computing firm with four other principals. Dreiling had some valuable information for the undergrads both in terms of employment and for profiting from their undergraduate study. Other 1991-92 officers: Jill Weigand, stugo representative; Jo Ann Fellin, OSB, corresponding secretary/faculty sponsor.

Kansas Delta Washburn University, Topeka

Chapter President - Jennifer Hudson
29 actives

Kansas Delta held one business meeting in November. Other 1991-92 officers: Jessica Dyck, vice president; Frank Duran, secretary; Michelle Reed, treasurer; Allan Riveland, corresponding secretary; Ron Wasserstein, faculty sponsor.

Kansas Epsilon Fort Hays State University, Hays

Chapter President - Chuck Neuschafer
30 actives

Fall semester activities included monthly meetings and two socials, a Halloween Party and a Christmas Party. Other 1991-92 officers: Fabian Leiker, vice president; Donna Weninger, secretary/treasurer; Charles Votaw, corresponding secretary; Mary Kay Schippers, faculty sponsor.

Kentucky Alpha Eastern Kentucky University, Richmond

Chapter President - Kevin Huibregtse
22 actives, 7 associates

The fall semester began with a faculty/*KME* picnic held on the Metcalf farm. Those attending enjoyed good food, games, and wading in the creek. In late September, Dr. Janeway, Dr. Jones and Dr. Costello gave a panel discussion on graduate school. In October three students took the Virginia Tech Math Exam. The December program featured Dr. Amy King, who included numerous anecdotes about her favorite mathematicians in her presentation entitled "A Short History of Math." A Christmas party and white elephant exchange closed out a successful semester. Other 1991-92 officers: James Hannis, vice president; Monica Klein, secretary; Gary Cline, treasurer; Patrick Costello, corresponding secretary/faculty sponsor.

Maryland Alpha College of Notre Dame of Maryland, Baltimore

Chapter President - Marta Blotny

10 actives, 8 associates

Other 1991-92 officers: Sandy Burgess, vice president; Lisa Myers, secretary; Judy Urban, treasurer; Sister Marie Dowling, corresponding secretary; Joseph DiRienzi, faculty sponsor.

Maryland Beta Western Maryland College, Westminster

Chapter President - Brenton Squires

12 actives

Four new members were inducted raising the active membership to 12. Other 1991-92 officers: Jennifer Boggs, vice president; Jesse Taylor, secretary; William Yankoskv, treasurer; James Lightner, corresponding secretary; Linda Eshleman, faculty sponsor.

Maryland Delta Frostburg State University, Frostburg

Chapter President - Beth Stallings

26 actives

Maryland Delta Chapter held monthly meetings during the fall semester. In September, Prof. Kathy Elder informed the group about "The Life of an Actuary;" in October students and faculty enjoyed "Pizza, Puzzles and Pi," and in November Prof. Edward White made a presentation entitled "Some Mathematical Fallacies, or Ten Proofs That I am Arnold Schwarzeneggar." Other 1991-92 officers: Steven Smith, vice president; Christine Bittinger, secretary; Diana Beisel, treasurer; Edward White, corresponding secretary; John Jones, faculty sponsor.

Massachusetts Alpha Assumption College, Worcester

Chapter President - Lynn Monaco

8 actives, 3 associates

Six new members were initiated on May 1, 1991. Following a dinner in honor of the new members, Professor Thomas Slavkovsky, of the Assumption Faculty, spoke on "Chaos and Complexity." Other 1991-92 officers: Sheila Gomula, vice president/secretary; Charles Brusard, corresponding secretary/faculty sponsor.

Michigan Beta Central Michigan University, Mount Pleasant

Chapter President - Laurie Raven

30 actives

The National Convention in Alabama last spring generated much excitement and enthusiasm which carried over into the Fall '91 semester. Chapter members successfully turned a service project into a fund raising event in assisting faculty member Linda Wagner to prepare and sell MacIntosh disks to students in her classes in Microcomputer for the Secondary (and Elementary) Mathematics Classroom. CMU faculty member Ken Smith was the guest speaker at the initiation of new members in October. He spoke about the role of mathematics at the National Security Agency where he spent a sabbatical in 1990-91. He presently is teaching Number Theory and Abstract Algebra, using a problem solving approach — students pose problems, look at open problems, and work in teams on proposed problems. Members and Dr. Hammel had fun answering the phone for the CMU Public Television fund raiser. The chapter hosted a problem session for the Calculus I classes and devoted one meeting to helping mathematics students plan their schedules for the semester. Students Laurie Raven, Dave Parks, Mary Wheeler, and Tom DeClark, all gave talks at chapter meetings. The semester ended with a Christmas get-together at the home of advisor Arnie Hammel and family. Other 1991-92 officers: Tom DeClark, vice president; Betsy Bacon, secretary; Pete Shavinski, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

Mississippi Alpha Mississippi University for Women, Columbus

Chapter President - Mary Beth Falcon

16 actives

Other 1991-92 officers: Rebecca Cagle, vice president; Teresa Loper, secretary; Mary Jane Chambers, treasurer; Jean Parra, corresponding secretary; Saochen Yang, faculty sponsor.

Mississippi Gamma University of Southern Mississippi, Hattiesburg

Chapter President - Brian Hraborsky

30 actives

Other 1991-92 officers: Joy Adams, vice president; Brenda Seal, secretary; Alice W. Essary, treasurer/corresponding secretary; Karan Thrash, Barry Piazza, faculty sponsors.

Missouri Alpha Southwest Missouri State University, Springfield

Chapter President - Christina Daniel

21 actives, 12 associates

Missouri Alpha had a very active Fall 1991, semester. Monthly meetings inaugurated a new era of cooperation with the student chapter of the Mathematical Association of America. Since there is considerable overlap of membership in *KME* and MAA, it was decided that joint meetings were not only possible, but desirable. As a result, *KME* invited the MAA students to each of its meetings and vice-versa. In addition, the two organizations joined forces for the two social events held during the semester: the annual fall picnic, and the end-of-semester pizza party. The benefits of cooperation include larger audiences for invited speakers, a sense of cooperation with others, and the opportunity to share the benefits of each organization. Other 1991-92 officers: Susan Gibiser, vice president; Mark Peters, secretary; Karen Martin, treasurer; Ed Huffman, corresponding secretary; John Kubicek, faculty sponsor.

Missouri Beta Central Missouri State University, Warrensburg

Chapter President - Michael Prock

20 actives, 10 associates

Missouri Beta held monthly meetings during the fall semester with invited presentations on topics ranging from modulus surfaces to study abroad. Other activities included a picnic and volleyball game in September, a book sale and Halloween party in October, a field trip to the Linda Hall Library in Kansas City in November and a Christmas party in December. Sixteen new members were initiated in October. *KME* members donated approximately 12 hours per week to help tutor in The Math Clinic. Other 1991-92 officers: Mica Johnson, vice president; Kirk Monsees, secretary; Sarah Moss, treasurer; Rhonda McKee, corresponding secretary; Larry Dilley, Homer Hampton and Debbie Moran, faculty sponsors.

Missouri Gamma William Jewell College, Liberty

Chapter President - James A. Mathis

16 actives

Other 1991-92 officers: Tim Anderson, vice president; Steve Swenson, secretary; Joseph T. Mathis, corresponding secretary/faculty sponsor.

Missouri Epsilon Central Methodist College, Fayette

Chapter President - Richard Courter

7 actives

Other 1991-92 officers: William D. McIntosh, corresponding secretary /faculty sponsor; Linda O. Lambke, faculty sponsor.

Missouri Zeta University of Missouri-Rolla, Rolla

Chapter President - Steve Klump

Rick Whittaker, research mathematician from McDonnell Douglas Research Lab, Saint Louis, gave a talk at the fall *KME* initiation ceremony on the role of a research mathematician in industry. Other 1991-92 officers: Chris Case, vice president; Melissa Ince, secretary; Victor Poland, treasurer; Jennifer Marino, historian; Clint Scott, social chairman; Roger Hering, corresponding secretary; Jim Joiner, faculty sponsor.

Missouri Eta Northeast Missouri State University, Kirksville

Chapter President - Scott Niemeyer

27 actives, 11 associates

In addition to regular monthly meetings, Missouri Eta sponsored a student/faculty softball game and picnic, and celebrated the holiday season with a Christmas party. Other 1991-92 officers: Becky Evans, vice president; Jason Lott, secretary; Angela Hahn, treasurer; Mary Sue Beersman, corresponding secretary; Shelle Palaski, faculty sponsor.

Missouri Iota Missouri Southern State College, Joplin

Chapter President - Melissa Sherrel

17 actives, 12 associates

Members and sponsors again worked concession stands at football games throughout the fall semester to raise revenue for chapter activities. Monthly meetings were held. The chapter co-sponsored with The Mathematics Department a guest speaker, Joe Swartz of Martin Marietta Astronautics Group, who provided interesting information concerning his work as an applied research mathematician. Socials included a fall cookout/volleyball game and an end of semester Christmas pizza party. Other 1991-92 officers: Terri Findley, vice president; John Borchardt, secretary/treasurer; Tricia Leake, historian; Mary Elick, corresponding secretary; Linda Noel, faculty sponsor.

Missouri Kappa Drury College, Springfield

Chapter President - Robert Hayden

9 actives

The first activity of the semester for the chapter was a bonfire weiner roast held at Dr. Allen's house. The winners of the Annual Campus Math Contest were Lynette Pember (Calculus I and below) and Cindy Schwab (Calculus II and above). Prize money was awarded to the winners at a Pizza party held for all contestants. At a luncheon for the chapter, Kathy Vincent, Yvonne Shaw, and Christine Oelrichs gave reports on their undergraduate research projects (hopefully to be presented at the Region 4 *KME* Convention). The end of the semester was celebrated with a Christmas Party. The math club ran a tutoring service for both the day school and the evening college as a money making project. Other 1991-92 officers: Yvonne Shaw, vice president; Kathy Vincent, secretary; Matt Henderson, treasurer; Charles Allen, corresponding secretary; Ted Nickle, faculty sponsor.

Missouri Lambda Missouri Western State College, St. Joseph

Chapter President - Robin Fowler

22 actives, 5 associates

Initiation ceremonies were held in October for five new members. Dr. David John provided the program. Chapter members participated in Homecoming and Parent's Day activities. Two bake sales were held for fundraising. The semester ended with a holiday covered dish dinner for students and faculty. Other 1991-92 officers: Audrey Davis, vice president; Susan Nichols, secretary; Roy Rhinehart, treasurer; John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

Nebraska Alpha Wayne State College, Wayne

Chapter President - Amy Anderson

18 actives

Throughout the semester club members monitored the Math-Science Building to earn money for the club. They also participated in homecoming activities by manning a booth at the Homecoming Carnival. With a grant from the Wayne State College Student Senate, *KME* and Computer Club purchased instructional VCR tapes for mathematics and computer science topics. Social activities included a fall picnic with the Math-Science faculty and other clubs in the building and a pizza-movie party at Dr. Paige's home. Other 1991-92 officers: Jaime Tiller, vice president; Jill Brehm, secretary-treasurer; Susan Sorensen, historian; Fred Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

Nebraska Beta Kearney State College, Kearney

Chapter President - Liane Schroeder

Nebraska Beta members were involved in the ESU #10 Math Fun Day in Kearney. Also a Christmas supper was held by *KME* members and faculty for graduating seniors. Other 1991-92 officers: Chris Mueher, vice president; Dawn James, secretary; Anita Lutz, treasurer; Charles Pickens, corresponding secretary; Lutfi Lutfiyya, faculty sponsor.

Nebraska Delta Nebraska Wesleyan University, Lincoln

Chapter President - Matthew R. Meyer
16 actives

The club met jointly with the Mathematics/Computer Science organization. Speakers for the fall semester included an actuary and a computer programmer. The program for one meeting was given by students relaying information about their experiences while in summer internships. Other 1991-92 officers: Joseph C. Roth, vice president; Kenneth L. Guiberson, secretary; John R. Heckman, treasurer; Muriel Skoug, corresponding secretary/faculty sponsor.

New Mexico Alpha University of New Mexico, Albuquerque

Chapter President - William C. Grover
25 actives, 5 associates

A fall semester activity for the New Mexico Alpha Chapter was helping with the Albuquerque exhibit of paintings by M. Escher. Other 1991-92 officers: David Morrow, vice president; David Black, secretary; Marjorie Bond, treasurer; Richard C. Metzler, corresponding secretary /faculty sponsor.

New York Alpha Hofstra University, Hempstead

Chapter President - Karin Grossu
9 actives, 20 associates

One activity enjoyed by the chapter during the fall semester was a movie night and dinner. Other 1991-92 officers: Shannon Wagner, vice president; Dawn Eriksen, secretary; Scott Drucker, treasurer; Aileen Michaels, corresponding secretary/faculty sponsor.

New York Kappa Pace University, New York

Chapter President - Angeliki Kazas
25 actives, 15 associates

New York Kappa hosts an annual induction dinner and co-sponsors The Pace University Mathematics Seminar. Other 1991-92 officers: Mei L. Ho, vice president; Jeff Rubens, corresponding secretary; John W. Kennedy and Martin Kotler, faculty sponsors.

New York Lambda C.W. Post/Long Island University, Brookville

Chapter President - Nicholas Ramer
20 actives, 2 associates

Chapter activities included election of officers and preparation for spring initiation. Other 1991-92 officers: Jonathan Stevens, vice president; Myleen Rojano, secretary; Brigid Rice, treasurer; Sharon Kunoff, corresponding secretary; Andrew Rockett, faculty sponsor.

Ohio Alpha Bowling Green State University, Bowling Green

Chapter President - Malcolm Shrimplin
50 actives

The annual student/faculty fall picnic and volleyball game was held in September. The October meeting featured pumpkin carving and problem solving. In November members enjoyed a Planetarium Show with Dr. James Smith discussing celestial cycles and coordinate systems, and concluding with the showing of "It's About Time." The chapter, along with Bowling Green Teachers of Mathematics, enjoyed their third annual Christmas Party in December. Plans were also made to host a Region 2 Convention in April, 1992. Other 1991-92 officers: Kim Kukla, vice president/programming; Deb Lutz, vice president/initiations; Angela Baumgard, secretary/treasurer; Waldemar Weber, corresponding secretary; Neal Carothers, faculty sponsor.

Oklahoma Alpha Northeastern State University, Tahlequah

Chapter President - Michael Seals

36 actives, 15 associates

A fall initiation was held for twelve new members. The guest speaker was Dr. Pavel Bosin, a visiting professor from Russia, who spoke on "The Educational System in the Soviet Union." The annual book sale fundraiser produced close to $100. The chapter is enjoying joint activities with NSU's student chapter of MAA. The group heard Dr. James R. Choike of Oklahoma State University speak on the topic, "Archimedes and the Volume of a Sphere." The December meeting was a Christmas party with pizza provided by *KME* and the MAA. According to the students, the students annihilated the faculty in the games. Other 1991-92 officers: Luke Foster, vice president; Leslie Tramell, secretary; Susan Dismore, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

Oklahoma Gamma Southwest Oklahoma State University, Weatherford

Chapter President - Melissa Kirkland

22 actives

The focus of the fall semester was on preparation for the Region 5 Convention which Oklahoma Gamma will host in March 1992. Other 1991-92 officers: Dixie Harris, vice president; Jeremy Osmus, secretary; Jodi Lubinus, treasurer; Wayne Hayes, corresponding secretary; Robert Morrill, faculty sponsor.

Oklahoma Delta Oral Roberts University, Tulsa

Chapter President - Jason Graves

17 actives

In September the chapter sponsored a barbecue and swim party for all math and engineering students. A group of members met weekly throughout the fall to prepare for the Putnam Exam. The organization, in conjunction with The Computer Club, celebrated the holidays with a Christmas party. Other 1991-92 officers: Richard Kirby, vice president; Elizabeth Jacobsen, secretary; Mark Gollahon, treasurer; Debra Oltman, corresponding secretary; Roy Rakestraw, faculty sponsor.

Pennsylvania Alpha Westminster College, New Wilmington

Chapter President - Denise Ullom

24 actives, 2 associates

In addition to meetings held during the semester, Pennsylvania Alpha provided tutoring at Westminster College Learning Center from October 2 to December 4. The combined local chapters of *KME* and MAA enjoyed a late October ice cream social. Other 1991-92 officers: Jacque McCrory, vice president; Brian Staudt, secretary; Kimberly Lary, treasurer; J. Miller Peck, corresponding secretary; Warren Hickman, faculty sponsor.

Pennsylvania Beta La Salle University, Philadelphia

Chapter President - Jason DiVirgilio

15 actives

Other 1991-92 officers: Kathleen Robinson, vice president; Charles Nyce, secretary; Geralyn Bowers, treasurer; Hugh N. Albright, corresponding secretary; Carl McCarty, faculty sponsor.

Pennsylvania Gamma Waynesburg College, Waynesburg

Chapter President - Bill Guappone

10 actives, 3 associates

Two new members were inducted into the chapter on December 5. Other 1991-92 officers: Scott Sherwood, vice president; Nhan Huynh, secretary; Mohsin Nagvi, treasurer; A. B. Billings, corresponding secretary/faculty sponsor.

Pennsylvania Delta Marywood College, Scranton

Chapter President - Teresa Larkin

5 actives

Other 1991-92 officers: Janel Caporali, vice president; Rhoda Dellecane, secretary/treasurer; Sister Robert Ann von Ahnen, IHM, corresponding secretary/faculty sponsor.

Pennsylvania Epsilon Kutztown University, Kutztown

Chapter President - Doris Sagl
12 actives, 2 associates

Other 1991-92 officers: Ursula Jorhend, vice president; Andrea Schaeffer, secretary; Chris Hartman, treasurer; Cherry C. Mauk, corresponding secretary; Randy Schaeffer, faculty sponsor.

Pennsylvania Eta Grove City College, Grove City

Chapter President - Kim Ayers
34 actives

Fall activities of Pennsylvania Eta Chapter of *Kappa Mu Epsilon* included a talk in September by John Thomas, a fellow in The Actuarial Society of America and charter member of Pennsylvania Eta, the fall initiation of new members in November, and the annual Christmas party held December 9 at the home of the department chairman, Mr. Jack Schlossnagel. Other 1991-92 officers: Julie Cambell, vice president; Jeanette Jordan, secretary; Krista LaComb, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Iota Shippensburg University of Pennsylvania, Shippensburg

Chapter President - Marianne Paul
25 actives

Pennsylvania Iota chapter co-sponsored the annual fall picnic for math and computer science students and faculty. The picnic was held at the picnic grounds on campus and was a great success. The fall initiation of 12 new members took place at the home of faculty advisor, Dr. Rick Ruth. The event included refreshments and was enjoyed by all present. Other 1991-92 officers: Jeff Rady, vice president; Debra Callender, secretary; Fred Nordai, treasurer; Michael Segfried, corresponding secretary; Rick Ruth, faculty sponsor.

Pennsylvania Kappa Holy Family College, Philadelphia

Chapter President - Melissa Kershes
10 actives, 5 associates

Tutoring by *KME* members continued as a service to the college community. Problem sessions also were held. Departmental activity has centered on preparation for Middle States Evaluation and the Pennsylvania Department of Education Evaluation to be held in March

1992. Other 1991-92 officers: David McCabe, vice president; Kevin Carsley, secretary/treasurer; Sister M. Grace Kuzawa, corresponding secretary/faculty sponsor.

Pennsylvania Mu Saint Francis College, Loretto

Chapter President - Jim Kelly
19 actives, 13 associates

Six members attended Kutztown State University's "Career Seminar on Mathematics" on September 28. Plans were began to host a Region 2 *KME* Conference March 13-14, 1992. Other 1991-92 officers: John Miko, vice president; Amy Miko, secretary; Paula Knaze, treasurer; Peter Skoner, corresponding secretary; Adrian Baylock, faculty sponsor.

Pennsylvania Nu Ursinus College, Collegeville

Chapter President - Charles Kullmann
16 actives

The chapter sponsored a lecture by Professor George Rosenstein of Franklin and Marshall College entitled "Wallis' Formula for Pi." Other 1991-92 officers: Jill Ramsland, vice president; Beth Carkner, secretary; Kevin Acken, treasurer; Jeff Neslen, corresponding secretary; Richard Bremiller, faculty sponsor.

Tennessee Delta Carson-Newman College, Jefferson City

Chapter President - Melissa Bryant Smith
20 actives

Chapter activities included a picnic with SPS at Cherokee Lake in September, a lecture on "Fractals and Chaos" by Shannon Lee in October, and a movie at Dr. Herring's house in December. Other 1991-92 officers: Laurie G. Plunk, vice president; Eugenia Beth Lee, secretary; Kendra Dawn Canup, treasurer; Verner T. Hansen, corresponding secretary; Carey R. Herring, faculty sponsor.

Texas Eta Hardin-Simmons University, Abilene

Chapter President - Charles Reed
17 actives

Texas Eta chapter of *Kappa Mu Epsilon* co-sponsored with the Mathematics Department an appreciation party for all students currently enrolled for mathematics courses at HSU. Other 1991-92 officers: Louis Revor, vice president; Jill Sims, secretary/treasurer; Mary Wagner-Krankel, corresponding secretary; Charles Robinson and Ed Hewett, faculty sponsors.

Texas Kappa University of Mary Hardin-Baylor, Belton

Chapter President - Karen Scott
10 actives, 8 associates

Members of Texas Kappa attended The Texas Geometry Conference at the University of Texas in Austin on October 19. The Chapter sponsored member Abeer Al-Naji as a contestant in The Miss MHB Pageant at the University on November 15-16. Four new members were initiated at the December meeting. Other 1991-92 officers: Donald R. Henslee, vice president; Stephanie L. Williams, secretary; Abeer Al-Naji, treasurer; Peter H. Chen, corresponding secretary; Maxwell M. Hart, faculty sponsor.

Virginia Gamma Liberty University, Lynchburg

Chapter President - Kathleen Bowers
20 actives, 7 associates

Activities included regular meetings, a car wash fundraiser, and an end of semester Christmas caroling. James Ward, actuary and graduate of Liberty University, talked to the chapter about opportunities in actuarial science. Other presentations were made by Dr. Saami Shaibani of the physics department, and Carole Anne Lindquist, senior honor student, who spoke on the topic, "Newtonian Methodology." Other 1991-92 officers: Michael Sarver, vice president; Nicole S. Boodram, secretary; Brian Renshaw, treasurer; Glyn Wooldridge, corresponding secretary; Sandra Rumore, faculty sponsor.

Wisconsin Alpha Mount Mary College, Milwaukee

Chapter President - Jill Rogahn
4 actives, 10 associates

Meetings of Wisconsin Alpha are held monthly. In November the chapter co-sponsored with the Mathematics Department a Mathematics

Contest for junior and senior high school women. The winner of the contest receives a partial scholarship to Mount Mary College. Other 1991-92 officers: Sister Adrienne Eickman, corresponding secretary /faculty sponsor.

Wisconsin Gamma University of Wisconsin-Eau Claire, Eau Claire

Chapter President - Jackie Hoffman

29 actives, 20 associates

Wisconsin Gamma held monthly meetings highlighted by student presentations. The organization sold popcorn in the student union as a successful fundraiser. A midsemester bowling and pizza party was enjoyed by chapter members and a Christmas party marked the end of the semester. Other 1991-92 officers: Jeff Ion, vice president; Penny Lee, secretary; Tammy Christel, treasurer; Tom Wineinger, corresponding secretary/faculty sponsor.

Kappa Mu Epsilon National Officers

Harold L. Thomas *President*

Department of Mathematics
Pittsburg State University, Pittsburg, Kansas 66762

Arnold D. Hammel *President-Elect*

Department of Mathematics
Central Michigan University, Mt. Pleasant, Michigan 48859

Robert L. Bailey *Secretary*

Department of Mathematics
Niagara University, Niagara University, New York 14109

Jo Ann Fellin *Treasurer*

Mathematics and Computer Science Department
Benedictine College, Atchison, Kansas 66002

Mary S. Elick *Historian*

Department of Mathematics
Missouri Southern State College, Joplin, Missouri 64801

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation.

Chapter	Location	Installation Date
OK Alpha	Northeastern Oklahoma State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State College	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State College, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949

IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	Kearney State College, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri - Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin - River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Northeast Missouri State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel College, Springfield	12 Jan 1971

PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop College, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	West Georgia College, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin - Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Illinois Benedictine College, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C. W. Post Center of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry College, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991