THE PENTAGON

A Mathematics Magazine for Students

Volume 50 Number 2 Spring 1991

CONTENTS

Copyright © 1991 Kappa Mu Epsilon. General permission is granted to *KME* members for noncommercial reproduction in limited quantities of individual articles, in whole or in part, provided complete reference is given as to the source. Printed in the United States of America.

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Manuscripts of interest to undergraduate mathematics majors and first year graduate mathematics students are welcome, particularly those written by students. Submissions should be typewritten (double spaced with wide margins) on white paper, standard notation conventions should be respected and special symbols should be carefully inserted by hand in black ink. All illustrations must be submitted on separate sheets and drawn in black ink. Computer programs, although best represented by pseudocode in the main text, may be included as an appendix. Graphs, tables or other materials taken from copyrighted works MUST be accompanied by an appropriate release from the copyright holder permitting further reproduction. Student authors should include the names and addresses of their faculty advisors. Contributors to *The Problem Corner*, *The Hexagon* or *Kappa Mu Epsilon News* are invited to correspond directly with the appropriate Associate Editor. Electronic mail may be sent to (Bitnet) PENTAGON@LIUVAX.

Domestic subscriptions: $3.00 for two issues (one year) or $5.00 for four issues (two years); foreign subscriptions: $5.00 (US) for two issues (one year). Correspondence regarding subscriptions, changes of address or back copies should be addressed to the Business Manager. Copies lost because of failure to notify the Business Manager of changes of address cannot be replaced.

Microform copies are available from University Microfilms, Inc., 300 North Zeeb Road, Ann Arbor, Michigan 48106-1346 USA.

EDITOR Andrew M. Rockett
Department of Mathematics
C. W. Post / Long Island University, Brookville, New York 11548

ASSOCIATE EDITORS
The Hexagon Iraj Kalantari
Department of Mathematics
Western Illinois University, Macomb, Illinois 61455
The Problem Corner Kenneth M. Wilke
Department of Mathematics
Washburn University of Topeka, Topeka, Kansas 66621
Kappa Mu Epsilon News Mary S. Elick
Department of Mathematics
Missouri Southern State College, Joplin, Missouri 64801

BUSINESS MANAGER Sharon Kunoff
Department of Mathematics
C. W. Post / Long Island University, Brookville, New York 11548

Fun with Planes

John Day, *student*

Missouri Iota

Missouri Southern State College
Joplin, Missouri 64801

Presented at the 1989 National Convention.

The problem I will discuss is due to Bruce Daniels, a professor in the Physics Department of Pittsburg State University. The problem deals with an actual airplane pilot's training exercise. Prof. Daniels, concerned with the validity of a statement made by a flight instructor, set out to prove that the so called "instructor's hypothesis" was wrong. I was given a copy of the problem from Prof. Daniels, worked it out, and here is how it goes.

The pilot exercise our problem deals with consists of an airplane flying around a fixed point P on the ground. During the airplane's turn about P, the pilot must always keep one wing pointed at point P. The "instructor's hypothesis" we wish to prove false states that during this exercise an experienced pilot can maintain a constant altitude and airspeed while flying around point P on the ground (see Figure One).

Our two goals in the problem are: (1) to show that the instructor's hypothesis is realistically impossible and (2) to prove that the resulting path of the airplane, as the pilot attempts this exercise, is a conic section whose eccentricity is a function of the plan's airspeed and the speed of the wind. We will be using cylindrical coordinates. Notice that the airplane's wing is pointed at P. The real, external forces acting on the airplane are labeled $\vec{L}$ and $m\vec{g}$ and we neglect wind at this time.

A vector will be denoted $\vec{x}$ and has both direction and magnitude,

$$\dot{x} = \frac{dx}{dt} \quad \text{and} \quad \ddot{x} = \frac{d^2x}{dt^2}$$

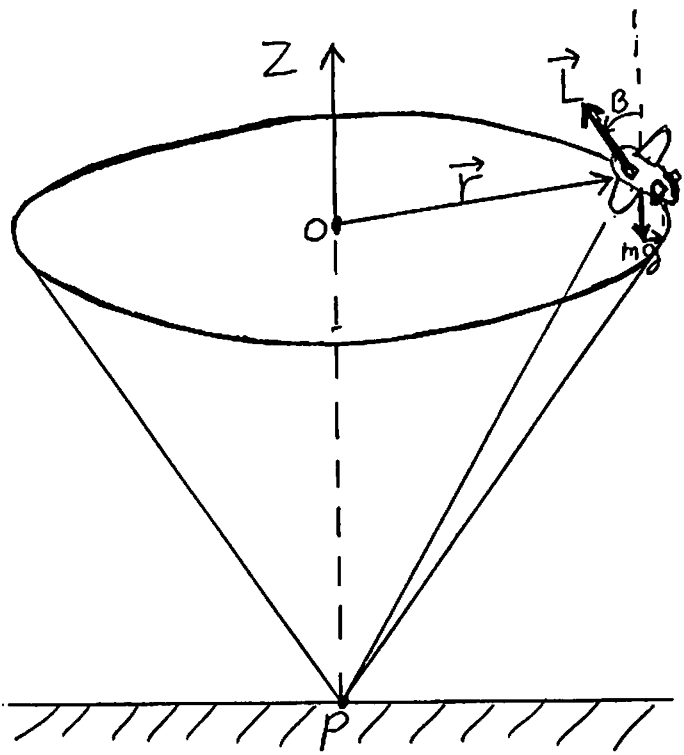


Figure One.

where t denotes time. Further,

h	is the airplane's constant altitude.
$\vec{L}$	is the net lift vector of the airplane. Its direction is always perpendicular to both wings.
$\vec{r}$	is the position vector of the plane from the z-axis in cylindrical coordinates.
$m\vec{g}$	is the net weight vector of the airplane. Its direction is always straight down in the (-z) direction.

We must assume there is exactly enough lift in the z-direction to counter its weigh; i.e., L cos(B) = mg.

Let us now look at an overhead view of the previous diagram, looking down on point P from above (see Figure Two). We will also take wind into our consideration. For simplicity sake, we will define the wind in the $\theta = 0$ direction (parallel to the ground). Also, in this diagram L and mg are not drawn. It should be noted that we make the assumption that the airplane's velocity vector (measured with respect to the air) is always perpendicular to the position vector $\vec{r}$ and in the θ direction. Think of this as the pilot always turning about the point P in a consistent manner.

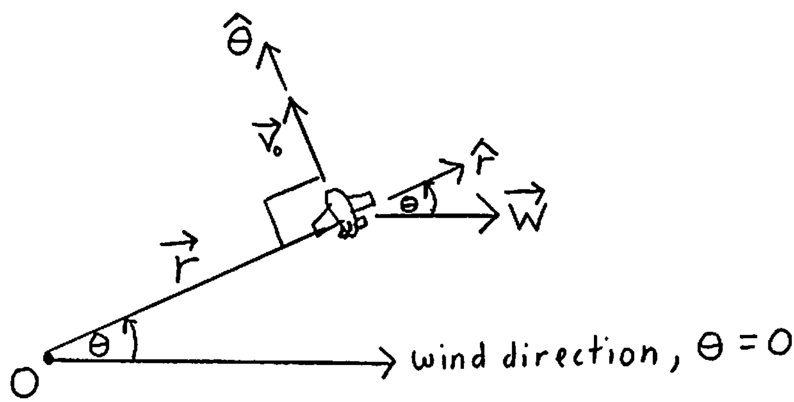


Figure Two.

We use the notations

$\vec{W}$	is the constant wind velocity vector and is measured with respect to the ground.
$\vec{V}_0$	is the airplane's velocity vector and is measured with respect to the wind (air).
$\vec{V}$	is the vector sum of $\vec{W}$ and $\vec{V}_0$ and yields the velocity of the airplane measured with respect to the ground
$\hat{\theta}$	is a unit vector in the θ direction
$\hat{r}$	is a unit vector in the r direction
F_θ	is the magnitude sum of all the forces in the θ direction
F_r	is the magnitude sum of all the forces in the r direction.

From Figure Two we have $\vec{V} = \vec{W} + \vec{V}_0 = (W\cos(\theta))\hat{r} - (W\sin(\theta))\hat{\theta} + (V_0)\hat{\theta}$ and so

(1) $$\vec{V} = \big(W\cos(\theta)\big)\hat{r} + \big(V_0 - W\sin(\theta)\big)\hat{\theta}.$$

Also, an expression for the velocity in a fixed polar coordinate system is

(2) $$\vec{V} = \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

(a proof of equation (2) is given in the Appendix). By equating coefficients in equations (1) and (2), we have:

$$\dot{r} = W\cos(\theta) \quad \text{and} \quad r\dot{\theta} = V_0 - W\sin(\theta).$$

Letting $k = W/V_0$, so that $W = kV_0$, these equations become

(3) $$\dot{r} = kV_0\cos(\theta)$$

and

(4) $$r\dot{\theta} = V_0\big(1 - k\sin(\theta)\big).$$

In the same manner as for the proof of equation (2), we can take the derivative of both sides in (2) and, simplifying, we obtain the acceleration vector $\vec{a}$ in polar coordinates:

(5) $$\vec{a} = \ddot{\vec{r}} = \big(\ddot{r} - r\dot{\theta}^2\big)\hat{r} + \big(r\ddot{\theta} + 2\dot{r}\dot{\theta}\big)\hat{\theta}.$$

We will now focus our attention on the magnitude of the component of acceleration in the θ direction, a_θ, which we have from (5) as

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}.$$

Differentiating both sides of (4) yields

$$\frac{d}{dt}\big(r\dot{\theta}\big) = r\ddot{\theta} + \dot{r}\dot{\theta} = \big(-V_0\, k\cos(\theta)\big)\dot{\theta}$$

which implies

$$r\ddot{\theta} = \big(-V_0\, k\cos(\theta)\big)\dot{\theta} - \dot{r}\dot{\theta} = \big(-V_0\, k\cos(\theta) - \dot{r}\big)\dot{\theta}$$

and, substituting from (3),

$$r\ddot{\theta} = \big(-2V_0\, k\cos(\theta)\big)\dot{\theta}.$$

Also, from (3) we obtain

$$2\dot{r}\dot{\theta} = 2\Big(kV_0 \cos(\theta)\Big)\dot{\theta};$$

hence $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$. Since

$$a_\theta = \frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right)$$

we obtain

$$\frac{d}{dt}\left(r^2\dot{\theta}\right) = 0.$$

By integrating both sides with respect to dt, we get:

$$r^2\dot{\theta} = c$$

where c is a constant. By using (4) to rewrite $r\dot{\theta}$,

$$c = r\Big(V_0(1 - k\sin(\theta)\Big)$$

or

$$r = \frac{c/V_0}{1 - k\sin(\theta)} = \frac{k\left(\frac{c}{kV_0}\right)}{1 - k\sin(\theta)}.$$

Comparing this with the equation of a conic section in polar form,

$$r = \frac{ep}{1 - e\sin(\theta)},$$

we conclude that the path of the airplane is a conic section with focus at the origin, directrix $y = -c/kV_0$ and eccentricity $e = k = W/V_0$. If $V_0 > W$ then $e < 1$ and the path of the airplane is an ellipse; if $V_0 = W$ then $e = 1$ and the path is a parabola; and if $V_0 < W$ then $e > 1$ and the path is a hyperbola. In reality, the airplane's airspeed will be greater than the wind speed ($V_0 > W$) and the path of our airplane is going to be an ellipse (see Figure Three).

Now that we have shown that the path of our airplane is, in general, a conic section, we have completed one of our two goals. We will now concentrate on disproving the "instructor's hypothesis."

We will proceed just as before from equations (3), (4) and (5) only now we will focus our attention on the acceleration in the r direction, a_r, which we have from (5) is $a_r = \ddot{r} - r\dot{\theta}^2$.

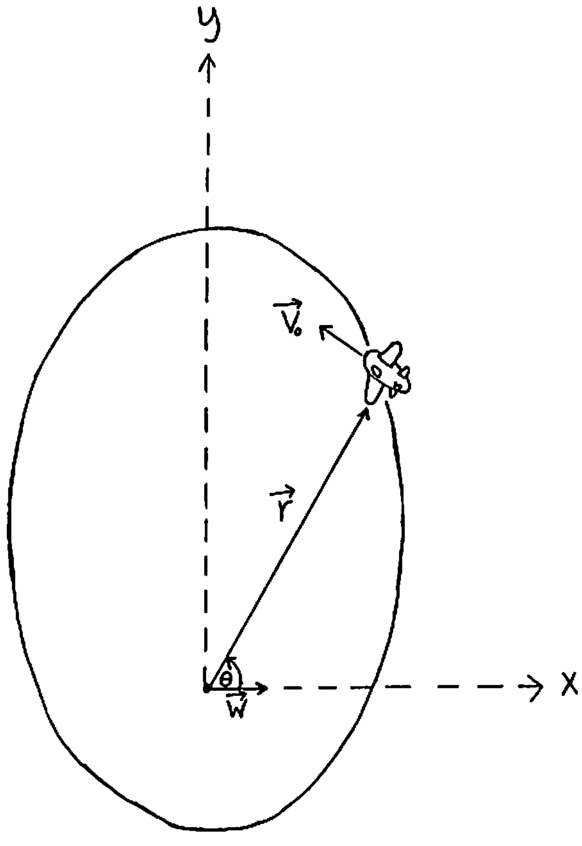


Figure Three.

Differentiating both sides of (3) we obtain

$$\ddot{r} = \frac{d}{dt}\Big(kV_0 \cos(\theta) \Big) = \Big(-kV_0 \sin(\theta)\Big)\dot{\theta}$$

while dividing by r in (4) gives

$$\dot{\theta} = \left(\frac{V_0}{r}\right)\Big(1 - k \sin(\theta)\Big).$$

Combining these two results and simplifying yields

$$\ddot{r} = \Big(-kV_0 \sin(\theta)\Big)\left(\frac{V_0}{r}\right)\Big(1 - k \sin(\theta)\Big)$$

$$= \left(\frac{-kV_0^2}{r}\right)\left(\sin(\theta)\right)\left(1 - k\sin(\theta)\right).$$

Since $r\dot{\theta}^2 = (r\dot{\theta})^2/r$, (4) also gives

$$r\dot{\theta}^2 = \frac{1}{r}\left(V_0\left(1 - k\sin(\theta)\right)\right)^2$$

$$= \left(\frac{V_0^2}{r}\right)\left(1 - k\sin(\theta)\right)^2.$$

Substituting into $a_r = \ddot{r} - r\dot{\theta}^2$, we obtain

$$a_r = \left(\frac{-kV_0^2}{r}\right)\left(\sin(\theta)\right)\left(1 - k\sin(\theta)\right) - \left(\frac{V_0^2}{r}\right)\left(1 - k\sin(\theta)\right)^2$$

$$= -\left(\frac{V_0^2}{r}\right)\left(1 - k\sin(\theta)\right).$$

Now that we have found the acceleration of the airplane in the r direction, let us put it all together and find h, the airplane's altitude, as a function of the known variables. To do this, the side view of Figure One given in Figure Four will be helpful.

From Newton's second law of motion, $F_r = ma_r$ where F_r is the magnitude sum of all the real, external forces in the r direction and so

$$F_r = m\left(\frac{-V_0^2}{r}\right)\left(1 - k\sin(\theta)\right). \tag{7}$$

By our earlier assumption, L cos(B) = mg. We can see now why we must place this restriction on the airplane. Since the lift $\vec{L}$ and the weight $m\vec{g}$ are the only forces on the airplane in the z direction, we must have L cos(B) = mg or else we will have a net acceleration of the airplane in the z direction (by Newton's second law, $F_z = ma_z$). And if the airplane is accelerating in the z direction we cannot have a constant altitude, as proposed in the "instructor's hypothesis." Also, we can see from Figure Four that L sin(B) = F_r. That is, $m\vec{g}$ has no component in the r direction (it is always perpendicular to $\hat{r}$). Only $\vec{L}$ has a component in the r direction; i.e., L sin(B).

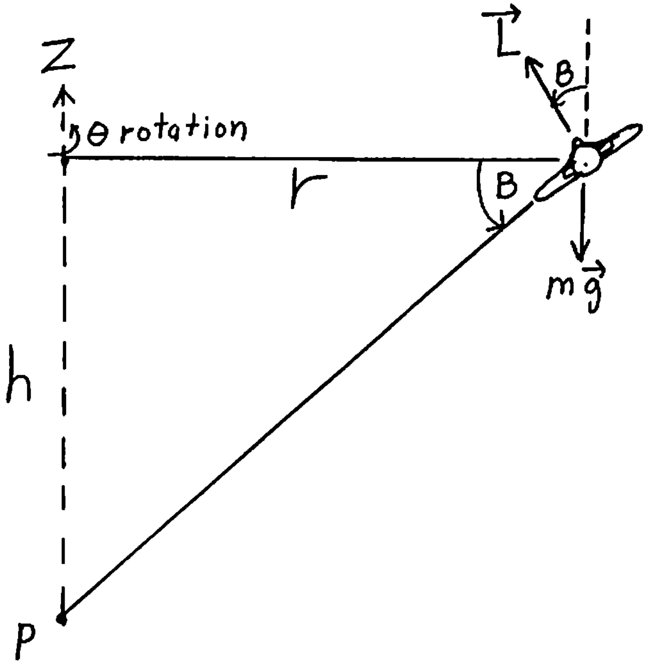


Figure Four.

Forming the quotient, we obtain tan(B) = (L sin(B))/(L cos(B)) = F_r/mg so that F_r = mg tan(B). Combining with (7),

$$\text{mg}\tan(\text{B}) = \text{m}\left(\frac{-V_0^2}{r}\right)\left(1 - k\sin(\theta)\right).$$

The negative sign in this equation came about because F is in the -r direction. We can drop the negative sign since we are only interested in the magnitude of the above expression. Therefore, we have

$$\tan(\text{B}) = \left(\frac{V_0^2}{gr}\right)\left(1 - k\sin(\theta)\right)$$

while from Figure Four

$$\tan(B) = \frac{h}{r}.$$

These last two equations result in:

$$h = \left(\frac{V_0^2}{g}\right)\left(1 - k\sin(\theta)\right).$$

From this last equation we see that θ changes independent of B. Therefore, we have proven that the "instructor's hypothesis" is false: it is impossible to keep both V_0 and h constant while pointing the airplane's wing at point P.

Appendix.

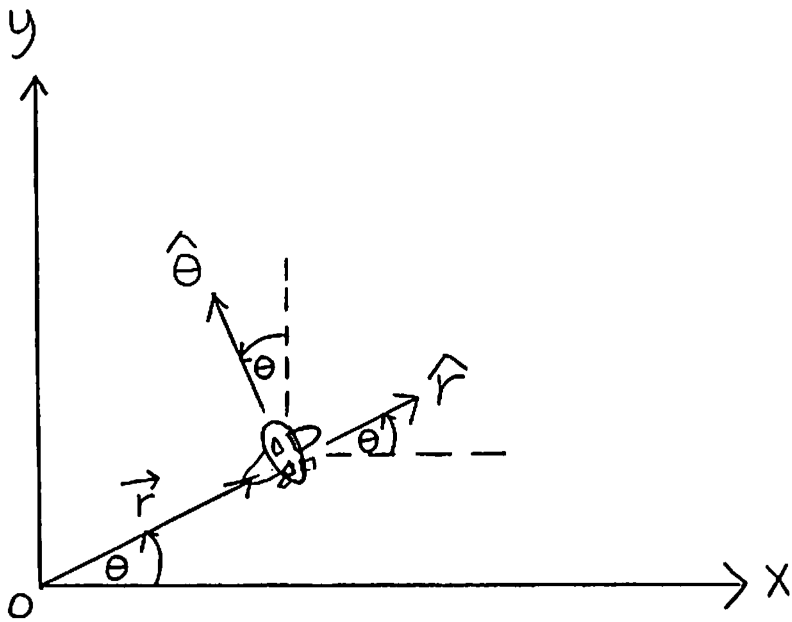


Figure Five.

Let $\hat{i}$ be a unit vector in the x direction and $\hat{j}$ be a unit vector in the y direction (see Figure Five). Then $\vec{r} = r\hat{r}$ and $\vec{V} = \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\hat{r}}$ while from Figure Five

$$\hat{r} = \hat{i}\cos(\theta) + \hat{j}\sin(\theta)$$

and

$$\hat{\theta} = \hat{j}\cos(\theta) - \hat{i}\sin(\theta).$$

Thus

$$\dot{\hat{r}} = -\hat{i}\sin(\theta)\,\dot{\theta} + \hat{j}\cos(\theta)\,\dot{\theta}$$

$$= \dot{\theta}\left(\hat{j}\cos(\theta) - \hat{i}\sin(\theta)\right) = \dot{\theta}\hat{\theta}.$$

Making the substitution of $\dot{\hat{r}}$, we obtain:

$$\vec{V} = \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}n$$

which is our desired result.

Has Your Subscription Expired?

Your *Pentagon* subscription expires with the Volume and Number that appears in the upper right corner of your address label (see back cover). Since this issue is Volume 50 Number 2, if the code 50-2 appears on your label then THIS IS YOUR LAST ISSUE! Please send your renewal check -- just $5 for 4 more issues -- together with your name and address to:

Business Manager
The Pentagon
c/o Department of Mathematics
C. W. Post / Long Island University
Brookville, New York 11548 USA

(Single year subscriptions are $3 per year; foreign subscriptions are $5 (US) per year). Please renew promptly to avoid gaps in your journal collection.

Numerical Integration

Julie Holdorf, *student*

Iowa Alpha

University of Northern Iowa
Cedar Falls, Iowa 50614

Presented at the 1989 National Convention.

From the Fundamental Theorem of Calculus we learn how to evaluate a definite integral by a two step process. First, we treat the integral as an indefinite integral and find the antiderivative of the integrand. Next, we evaluate the antiderivative, also a function, between the limits given by the definite integral by subtracting the antiderivative evaluated at the lower limit from the antiderivative evaluated at the upper limit. However, the first step of the process poses a problem with certain integrals.

Often times it is difficult or even impossible to express the antiderivatives in terms of familiar functions. Consequently, we are unable to compute the exact numerical values of these definite integrals. As a result, we must look to numerical integration methods to find close approximations of these definite integrals. This is achieved by estimating the areas under the corresponding curves between the limits. Two such numerical integration techniques are the "trapezoidal method" and "Simpson's method." With computer applications of these methods, we can speed up the process, avoid tedious computations, and evaluate integrals using large numbers of subdivisions which would be overwhelming by hand. In addition, the estimates can be calculated to any desired degree of accuracy, taking into consideration that a round-off error will limit the capability somewhat.

The simplest (though far from most accurate) method of finding the area under a curve is by approximating that area with a series of trapezoids, known as the "trapezoidal method." Details of the following explanation are taken from [1]. Given a function f that is continuous and nonnegative throughout [a,b], we take a partition that divides the interval into n subintervals of equal length (b-a)/n. We join each pair of points $(x_{k-1}, f(x_{k-1}))$ and $(x_k, f(x_k))$ on the graph of f by a straight line, thereby creating trapezoidal regions (see Figure One). We obtain the sum of the areas of the trapezoids as an approximation of the integral:

$$\frac{b-a}{2n}\Big((f(x_0) + f(x_1)) + (f(x_1) + f(x_2)) + \cdots + (f(x_{n-1}) + f(x_n))\Big)$$

which is simplified into the "trapezoidal rule:"

$$\frac{b-a}{2n}\Big(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\Big).$$

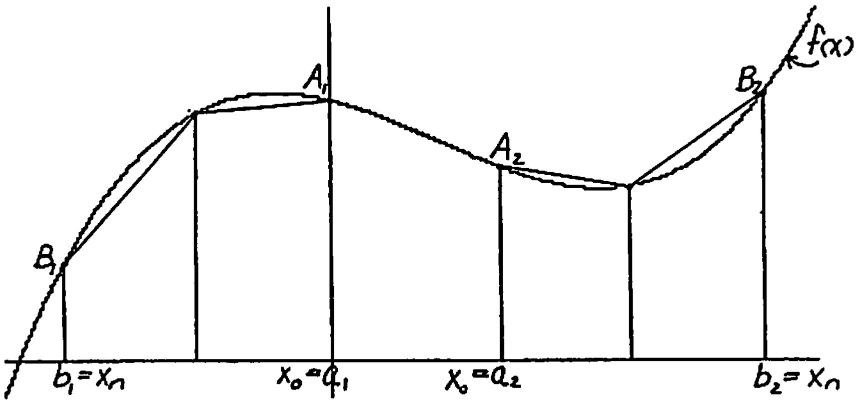


Figure One.

The nth trapezoidal rule error E_n is the error incurred in using this rule to approximate the integral; $E_n = A_n - I$ where I is the exact value of the definite integral and A_n is the approximate area calculated using n subdivisions. The formula for an upperbound of E_n is

$$|E_n| \leq \frac{(b-a)^3 M}{12n^2}$$

where M denotes the maximum absolute value of the second derivative of f over the interval [a,b]. A derivation of this formula, which can be found in [2], involves Taylor series and could be the focus of a different paper. From this formula, we can see that n^2 is inversely proportional to the error term; therefore, as the number of subdivisions n increases, the error term decreases.

Applying these concepts, let us look at two approximate areas resulting from different numbers of subdivisions, for example 10 and 20:

$$A_{10} = I + E_{10} \quad \text{and} \quad A_{20} = I + E_{20}$$

Subtracting these two equations,

$$A_{10} - A_{20} = E_{10} - E_{20},$$

we find that the difference between the areas is directly proportional to the difference between the error terms. As the number of subdivisions increases, the error decreases; moreover, the difference between two error terms resulting from approximations with large values for n will decrease. At the same time, the difference of the areas calculated for these large values of n will decreases as n increases and eventually converge to a close approximation fo the definite integral, if not the exact value. This concept of the difference between successive areas is used in the computer application of this method, as well as in Simpson's method.

Simpson's method, though similar to the trapezoidal method, varies in that we try to approximate the area under the curve by a series of parabolic segments (see Figure Two) as opposed to non-horizontal lines, hoping it will more closely match a given curve. The following description of the method is taken from [2]. Given the function f, we partition it on [a,b] into n subintervals, this time assuming n is an even number. We then pick a point c midway between a and b and construct the following points: A = (a, f(a)), B = (b, f(b)) and C = (c, f(c)). These three points define a unique parabola $y = \alpha x^2 + \beta x + \gamma$ which passes through these points.

Using the trapezoidal method to find the area between x = a and x = c, we would use the formula $(w/2)\big(f(a) + f(c)\big)$ of the general form Pf(a) + Qf(c) where P = Q = w/2. To match the curve with a parabola, we try to derive a formula with a similar form, Pf(a) + Qf(c) + Rf(b). This should give us the area between x = a and x = b if the values of P, Q and R are chosen properly. We use a method called "undetermined coefficients" to find P, Q and R. Simpson's method should give exact answers for any function which is either a constant, a line or a parabola, since the graph of a parabola can match any of these exactly.

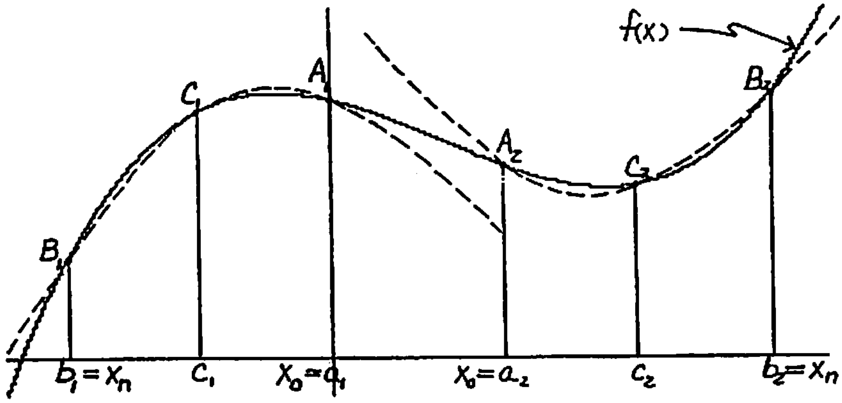


Figure Two.

Specifically, Simpson's method should give exact integral answers for the following three integrals:

$$I_1 = \int_{-w}^{+w} 1\,dx = (\ x\ )\Big[_{w}^{+w} = (+w) - (-w) = 2w$$

$$I_2 = \int_{-w}^{+w} x\,dx = \left(\ \frac{1}{2}x^2\ \right)\Big[_{w}^{+w} = 0$$

$$I_3 = \int_{-w}^{+w} x^2\,dx = \left(\ \frac{1}{3}x^3\ \right)\Big[_{w}^{+w} = \frac{2}{3}w^3$$

We assume a = -w, b = +w and the midpoint c = 0 in each case and try to find the exact area from an equation of the form Pf(a) + Qf(c) + Rf(b), so that for the three functions we have:

$$I_1 = P(1) + Q(1) + R(1) = P + Q + R = 2w$$

$$I_2 = P(-w) + Q(0) + R(+w) = -Pw + Rw = 0$$

$$I_3 = P(-w)^2 + Q(0)^2 + R(+w)^2 = Pw^2 + Rw^2 = \frac{2}{3} w^3.$$

From the second equation Pw = Rw so P = R for any w. Using this equality in the third equation gives us

$$2Pw^2 = \frac{2}{3} w^3$$

so that $P = R = w/3$. From the first equation

$$Q = 2w - P - R = 2w - \frac{2}{3} w = \frac{4}{3} w .$$

Hence, we have derived the equation for Simpson's method to be

$$Pf(a) + Qf(c) + Rf(b) = \frac{w}{3} f(a) + \frac{4w}{3} f(c) + \frac{w}{3} f(b)$$
$$= \frac{w}{3} \left(f(a) + 4f(c) + f(b)\right).$$

It is useful to subdivide the interval to be integrated into an even number n of strips. We can then use Simpson's method to find the area of two adjacent strips at a time lying between x_{i-1}, x_i and x_{i+1} using the equivalents $x_{i-1} = a$, $x_i = c$ and $x_{i+1} = b$. However, evaluating the equation above for every two subdivisions involves many computations for a large number of subdivisions. This equation can be expressed in a more general form to avoid unnecessary calculations:

$$\frac{b-a}{3n} \left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right).$$

As with the trapezoidal method, we expect the difference between the actual value of the integral and the estimated value to decrease as the number of subdivisions increases. The formula for an upperbound of the n^{th} Simpson's method error E_n is

$$|E_n| \le \frac{(b-a)^5 M}{180 n^4}$$

where M denotes the maximum absolute value of the fourth derivative over [a,b]. As with the trapezoidal method, a derivation of this formula involves Taylor series and can also be found in [2]. The same concepts derived from the trapezoidal error term formula can be derived from this formula.

Using the trapezoidal method and Simpson's method, I have written an interactive computer program to approximate definite integrals of polynomial functions (see Appendix One). By evaluating the integrals using both methods, we can compare and contrast the two methods by analyzing the results we obtain. The program operates by recalculating the area under the graph of the function until the desired tolerance is achieved, doubling the number of subdivisions with each successive calculation. To run the program, the user must input the degree of the polynomial, the coefficients of the terms from the highest to the lowest degree term, the left and right endpoints, and the tolerance or error to allow between successive calculated areas.

```
ENTER THE DEGREE OF THE POLYNOMIAL:
1
ENTER THE COEFFICIENT OF THE HIGHEST DEGREE TERM:
4
ENTER THE COEFFICIENT OF THE NEXT TERM:
-1
ENTER THE LEFT AND RIGHT ENDPOINTS SEPARATED BY AT LEAST 1 SPACE:
-2 3
ENTER THE DESIRED TOLERANCE(IF < 0, ENTER 0.___):
0.001

DIVISIONS        AREA(T)     :A(N)-A(N-2):    AREA(S)     :A(N)-A(N-2):

    2            5.000                        5.000
    4            5.000          0.000         5.000          0.000
```

Figure Three.

A sample run of the program to calculate the integral

$$\int_{-2}^{3} 4x - 1\,dx$$

is shown in Figure Three. Using the fundamental theorem of calculus, we obtain the same results:

$$\int_{-2}^{3} 4x - 1\,dx = \left(2x^2 - x\right)\Big|_{-2}^{3} = \left(2(3)^2 - (3)\right) - \left(2(-2)^2 - (-2)\right)$$

$$= (18 - 3) - (8 + 2) = 5 .$$

I used this integral to test the program for execution accuracy. Since the function is a first degree polynomial, one application of the trapezoidal method should be sufficient to obtain the exact value. This is evident if we look at the formula for an upperbound of the error term. M, which is positioned in the numerator of the fraction, is the maximum absolute value of the second derivative over the specified interval. The second derivative of a first degree polynomial is always equal to zero; for example, $f(x) = x$ gives $f'(x) = 1$ and $f''(x) = 0$. Therefore, M will equal zero and thus the error term will be zero, meaning the estimated value of the integral equals the exact value.

DIVISIONS	AREA(T)	|A(N)-A(N-2)|	AREA(S)	|A(N)-A(N-2)|
2	0.3125		0.2500	
4	0.2656	0.0469	0.2500	0.0000
8	0.2539	0.0117	0.2500	0.0000
16	0.2510	0.0029	0.2500	0.0000
32	0.2502	0.0007	0.2500	0.0000
64	0.2501	0.0002	0.2500	0.0000
128	0.2500	0.0000	0.2500	0.0000

Figure Four.

To test Simpson's method, I evaluated the integral of the third degree polynomial x^3 over the interval [0,1]:

$$\int_0^1 x^3\,dx = \left(\frac{1}{4}x^4\right)\Big|_0^1 = \left(\frac{1}{4}(1)^4\right) - \left(\frac{1}{4}(0)^4\right) = \frac{1}{4}$$

and then ran the program using the same function. Theoretically, the exact value should result after one application of Simpson's method for the same reason as the previous example with the trapezoidal method. The only difference is that M is now the absolute maximum of the fourth derivative. From Figure Four, we can see the expected results were obtained and that more than one pass was required using the trapezoidal method as would be expected. Approximations of other integrals are included for comparison in Appendix Two. In deciding which integrals to calculate, I chose examples with varying characteristics to demonstrate the applicability of the program. For example, I chose limits with positive and negative values, constants with real and integer values, polynomials with some degree terms missing, and functions whose graphs lie above the x-axis or waver above and below.

Focusing on the structure of the program, the procedure GETDATA supplies the user with instructions on the data to input and reads in this data. In writing this program, I took advantage of the similarities between the two methods to increase programming efficiency. The number of subdivisions is initialized to two and by doubling the number for each successive calculation remains even throughout execution of the program. This allows us to apply Simpson's method as well as the trapezoidal method to each integral for grounds of comparison. Since n is doubled each time, the width is always half the width of the preceding approximation.

The function DECPLACES is used to find the number of decimal places in the tolerance. This is then used to round the approximations and to adjust the spacing in the tables. The function FUNC evaluates the function at a specific x value by building up the polynomial in a loop. To reduce the number of times the computer has to evaluate a function at certain values of x, temporary variables are used. For example, the value of the function at the left endpoint plus the value at the right endpoint is stored in ATEMP because it is used in the original calculations of the integrals (when $n = 2$) for both methods, and in each successive calculation with different values of n. MID is only used for each initial calculation of the function (when $n = 2$), and stores the value of the function at the midpoint, which is them multiplied by two for the trapezoidal method and multiplied by four for Simpson's.

A loop is used to recalculate the area until the approximation is within the desired tolerance. Each approximation is calculated in the following way. From 1 to n-1, TEMP stores the value of the function at the current value of x. This is multiplied by two and added to the variable ANEWT, which already contains the sum of the function evaluated at the previous subdivisions and includes ATEMP (the sum of the values at the endpoints). If the current number of subdivision is odd, then two times TEMP is added to ANEWS, which is the variable that at this point has only stored two times the function value at each odd-numbered subdivision. Having processed each subdivision, the final approximation value for Simpson's method is found by adding ANEWT (the trapezoidal value before dividing by two and multiplying by the width) plus ANEWS to get the sum of the function values at each x value appropriately taken times two (at the even-numbered subdivisions) or four (at the odd-numbered subdivisions) or left alone (at the endpoints). This value is then multiplied by the width and divided by three. The final approximate value using the trapezoidal method is found by multiplying ANEWT by the width and dividing by two. The error is then calculated using both methods by subtracting the area of the previous approximation from the area just calculated, unique for each method. The present values calculated, ANEWT, ANEWS and the error terms, are then added to the table.

Having completed the program to approximate definite integrals for given polynomial functions, I then revised the program by replacing the function FUNC with one whose antiderivative is impossible to find,

$$f(x) = \sin(x) + e^{x^2} .$$

This is a distinct advantage of the computer application. The results are shown in Figure Five.

DIVISIONS	AREA(T)	|A(N)-A(N-2)|	AREA(S)	|A(N)-A(N-2)|
2	2.02166		1.93559	
4	1.94798	0.07368	1.92342	0.01217
8	1.92881	0.01917	1.92242	0.00100
16	1.92397	0.00484	1.92235	0.00007
32	1.92275	0.00121	1.92235	0.00000
64	1.92245	0.00030	1.92235	0.00000
128	1.92237	0.00008	1.92235	0.00000
256	1.92236	0.00002	1.92235	0.00000
512	1.92235	0.00000	1.92235	0.00000

Figure Five.

From the results obtained by approximating the three integrals given as examples (and the others in Appendix Two), it is evident that Simpson's method is a more accurate approximation than the trapezoidal method; therefore, less subdivision are required to find the area within the desired tolerance. For this reason, the program was written to recalculate the area until the tolerance is achieved using the trapezoidal method. The methods discussed are only two of the possible approximation methods which can be used to estimate the area under the graph of a function. Other estimation techniques to research are "Romberg integration" and "Gauss quadrature."

Appendix One.

```
{****************************************************************************
 *                                                                          *
 *  PROGRAMMER:  JULIE HOLDORF                                              *
 *  DATE WRITTEN:  11/16/87                                                 *
 *                                                                          *
 *  PURPOSE:   GIVEN A POLYNOMIAL FUNCTION, THIS PROGRAM ESTIMATES THE      *
 *             AREA UNDER THE GRAPH OF THE FUNCTION USING THE TRAP-         *
 *             EZOIDAL RULE AND SIMPSON'S RULE.  THIS IS DONE BY            *
 *             DOUBLING THE NUMBER OF SUBDIVISIONS UNTIL THE DIFFER-        *
 *             ENCE BETWEEN TWO CONSECUTIVE ESTIMATIONS USING THE           *
 *             TRAPEZOIDAL RULE IS WITHIN A GIVEN TOLERANCE.                *
 *                                                                          *
 ****************************************************************************}

PROGRAM INTEGRAT(INPUT,OUTPUT);

TYPE
   REALARRAY = ARRAY [1..20] OF REAL;

VAR
   DECPL,           {NUMBER OF DECIMAL PLACES TOLERANCE CONTAINS}
   N,               {NUMBER OF SUBDIVISIONS}
   DEG,             {DEGREE OF POLYNOMIAL INPUT}
   I: INTEGER;      {COUNTER/LOOP CONTROL VARIABLE}
   AOLDT,           {PREVIOUS ESTIMATION USING TRAPEZOIDAL RULE}
   AOLDS,           {PREVIOUS ESTIMATION USING SIMPSON'S RULE}
   ANEWT,           {CURRENT ESTIMATION USING TRAPEZOIDAL RULE}
   ANEWS,           {CURRENT ESTIMATION USING SIMPSON'S RULE}
   W,               {WIDTH OF EACH SUBDIVISION}
   DIFFS,           {DIFFERENCE BETWEEN CURRENT & PREVIOUS ESTIMATE USING SIMP}
   DIFFT,           {DIFFERENCE BETWEEN CURRENT & PREVIOUS ESTIMATE USING TRAP}
   TOL,             {TOLERANCE INPUT}
   L,               {LEFT ENDPOINT INPUT}
   R,               {RIGHT ENDPOINT INPUT}
   MID,             {STORES THE FUNCTION EVALUATED AT THE MIDPOINT OF [A,B]}
   ATEMP,           {STORES SUM OF THE FUNCTION EVALUATED AT BOTH ENDPOINTS}
   TEMP: REAL;      {STORES THE FUNCTION EVALUATED AT A SPECIFIC X VALUE}
   C: REALARRAY;    {STORES THE COEFFICIENTS OF THE POLYNOMIAL TERMS}


 {****************************************************************************
  *                                                                          *
  *  THIS PROCEDURE DISPLAYS THE PROMPTS AND READS IN THE NECESSARY          *
  *  DATA:  DEGREE OF THE POLYNOMIAL, COEFFICIENTS OF POLY. TERMS,           *
  *  LEFT AND RIGHT ENDPOINTS, AND DESIRED TOLERANCE.                        *
  *                                                                          *
  ****************************************************************************}

 PROCEDURE GETDATA(VAR DEG: INTEGER; VAR C: REALARRAY; VAR L,R,TOL: REAL);

 VAR
    I: INTEGER;

 BEGIN
    WRITELN('ENTER THE DEGREE OF THE POLYNOMIAL:');
    READLN(DEG);
    WRITELN('ENTER THE COEFFICIENT OF THE HIGHEST DEGREE TERM:');
    READLN(C[1]);
    FOR I := 2 TO (DEG + 1) DO
       BEGIN
          WRITELN('ENTER THE COEFFICIENT OF THE NEXT TERM:');
          READLN(C[I])
       END;
    WRITELN('ENTER THE LEFT AND RIGHT ENDPOINTS SEPARATED BY A SPACE:');
    READLN(L,R);
    WRITELN('ENTER THE DESIRED TOLERANCE(IF < 0, ENTER 0.___):');
    READLN(TOL)
 END;     {GETDATA}
```

```
{*************************************************************************
 *                                                                       *
 *  THIS FUNCTION RETURNS THE NUMBER OF DECIMAL PLACES IN THE GIVEN     *
 *  TOLERANCE TO USE FOR ROUNDING THE APPROXIMATIONS AND SPACING IN     *
 *  IN THE TABLES.                                                       *
 *                                                                       *
 *************************************************************************}

FUNCTION DECPLACES(TOL: REAL): INTEGER;

VAR
   DEC: INTEGER;

BEGIN
   DEC := 0;
   WHILE (TOL < 1) DO
      BEGIN
         TOL := TOL * 10;
         DEC := DEC + 1
      END;
   DECPLACES := DEC
END;     {DECPLACES}


 {*************************************************************************
  *                                                                       *
  *  THIS FUNCTION RETURNS THE VALUE OF THE GIVEN POLYNOMIAL FUNCTION    *
  *  EVALUATED AT A SPECIFIC X VALUE.                                     *
  *                                                                       *
  *************************************************************************}

FUNCTION FUNC(C: REALARRAY; DEG: INTEGER; X: REAL): REAL;

VAR
   F: REAL;
   I: INTEGER;

BEGIN
   F := C[1];
   FOR I := 1 TO DEG DO
      F := F * X + C[I+1];
   FUNC := F
END;     {FUN}


BEGIN    {INTEGRAT}
   GETDATA(DEG,C,L,R,TOL);
   N := 2;
   W := (R - L) / 2;
   DECPL := DECPLACES(TOL);
   WRITELN('DIVISIONS':9,'AREA(T)':((26+DECPL)DIV 2),'|A(N)-A(N-2)|':(14+DECPL)
          'AREA(S)' :((17+DECPL)DIV 2),'|A(N)-A(N-2)|' :(14+DECPL));
   WRITELN;
   ATEMP := FUNC(C,DEG,L) + FUNC(C,DEG,R);
   MID := FUNC(C,DEG,L+W);
   AOLDS := W * (ATEMP + 4 * MID) / 3;
   AOLDT := W * (ATEMP + 2 * MID) / 2;
   WRITELN(N :6, AOLDT :(13+DECPL):DECPL, AOLDS :(20+2*DECPL):DECPL);
   REPEAT
      N := N * 2;
      W := W / 2;
      ANEWT := ATEMP;
      ANEWS := 0;
      FOR I := 1 TO (N-1) DO
         BEGIN
            TEMP := FUNC(C,DEG,L+I*W);
            ANEWT := ANEWT + 2 * TEMP;
            IF (I MOD 2 = 1) THEN
               ANEWS := ANEWS + 2 * TEMP
         END;
      ANEWS := (ANEWT + ANEWS) * W / 3;
      ANEWT := ANEWT * W / 2;
      DIFFT := ABS(ANEWT - AOLDT);
      DIFFS := ABS(ANEWS - AOLDS);
      WRITELN(N :6, ANEWT :(13+DECPL):DECPL,DIFFT :(10+DECPL):DECPL,
                    ANEWS :(10+DECPL):DECPL,DIFFS :(10+DECPL):DECPL);
      AOLDT := ANEWT;
      AOLDS := ANEWS
   UNTIL (DIFFT < TOL)
END.     {INTEGRAT}
```

Appendix Two

Example One. $f(x) = 0.9x^2 - 3.6x - 2.3$ with $[a,b] = [1,4]$.

DIVISIONS	AREA(T)	$\lvert A(N)-A(N-2)\rvert$	AREA(S)	$\lvert A(N)-A(N-2)\rvert$
2	-13.98750		-15.00000	
4	-14.74688	0.75938	-15.00000	0.00000
8	-14.93672	0.18984	-15.00000	0.00000
16	-14.98418	0.04746	-15.00000	0.00000
32	-14.99604	0.01187	-15.00000	0.00000
64	-14.99901	0.00297	-15.00000	0.00000
128	-14.99975	0.00074	-15.00000	0.00000
256	-14.99994	0.00019	-15.00000	0.00000
512	-14.99998	0.00005	-15.00000	0.00000
1024	-15.00000	0.00001	-15.00000	0.00000
2048	-15.00000	0.00000	-15.00000	0.00000

Example Two. $f(x) = x^4 - 1.9x^3 - 2x^2 - 1$ with $[a,b] = [-2,3.5]$.

DIVISIONS	AREA(T)	$\lvert A(N)-A(N-2)\rvert$	AREA(S)	$\lvert A(N)-A(N-2)\rvert$
2	82.6096		50.2878	
4	28.8789	53.7307	10.9687	39.3191
8	13.6031	15.2758	8.5112	2.4574
16	9.6690	3.9341	8.3576	0.1536
32	8.6783	0.9907	8.3480	0.0096
64	8.4301	0.2481	8.3474	0.0006
128	8.3681	0.0621	8.3474	0.0000
256	8.3526	0.0155	8.3474	0.0000
512	8.3487	0.0039	8.3474	0.0000
1024	8.3477	0.0010	8.3474	0.0000
2048	8.3475	0.0002	8.3474	0.0000
4096	8.3474	0.0001	8.3474	0.0000

Example Three. $f(x) = x^6 + 3x^5 - 4x^4 - 2x^2 + x - 1$ with $[a,b] = [-1,1]$.

DIVISIONS	AREA(T)	$\lvert A(N)-A(N-2)\rvert$	AREA(S)	$\lvert A(N)-A(N-2)\rvert$
2	-7.000000		-5.333333	
4	-5.234375	1.765625	-4.645833	0.687500
8	-4.793701	0.440674	-4.646810	0.000977
16	-4.684093	0.109608	-4.647558	0.000748
32	-4.656735	0.027359	-4.647615	0.000057
64	-4.649898	0.006837	-4.647619	0.000004
128	-4.648189	0.001709	-4.647619	0.000000
256	-4.647761	0.000427	-4.647619	0.000000
512	-4.647655	0.000107	-4.647619	0.000000
1024	-4.647628	0.000027	-4.647619	0.000000
2048	-4.647621	0.000007	-4.647619	0.000000
4096	-4.647620	0.000002	-4.647619	0.000000
8192	-4.647619	0.000000	-4.647619	0.000000

References.

[1] Ellis, Robert, and Denny Gulick. *Calculus with Analytic Geometry (3rd edition).* Orlando, Florida: Harcourt Brace Jovanovich, 1986.

[2] Stark, Peter A. *Introduction to Numerical Methods.* New York: The Macmillan Company, 1970.

Application of Number Theory: Cryptosystems

Mala Renganathan, *student*

Kansas Alpha

Pittsburg State University
Pittsburg, Kansas 66762

Presented at the 1989 National Convention.

Introduction.

Cryptography is the study of schemes to transform a plaintext p into encoded or cipher text c. The encoding process involves constructing a function F from P (the set of admissible plaintexts p) to C (the set of encoded texts c). The following notation is helpful.

$$F : P \longrightarrow C \quad \text{with} \quad p \mapsto c = F(p).$$

The encoding map F usually involves a key k, known only to the person who sends the plaintext p and the person who receives the ciphertext c. The basic problem of cryptography is to devise schemes (P, F, C) and keys K so that the decoding transformation

$$F^{-1} : C \longrightarrow P \quad \text{with} \quad c \mapsto p = F^{-1}(c,k)$$

is easily computed when the key k is known and is difficult to decipher when the key k is not known.

Recently, cryptography has evolved into a precise mathematical subject. The main new idea is to base the security of a cryptosystem upon well known but intractable problems of number theory. Primality testing and factoring large positive integers are distinct problems. Primality testing asks the question: "Is the positive integer n a prime?" Certainly if n can be factored then n is not a prime. Much of the current cryptography exploits the fact that testing whether or not a positive integer n is a prime is a more tractable problem than factoring n. This is particularly true of the RSA encryption scheme discussed in this paper.

In Part 1, I will give some historical examples of cryptosystems and introduce the concept of integers modulo n. In Part 2, I will discuss Euler's theorem from elementary number theory and an algorithm to compute the inverse of a unit in the ring of integers modulo n. In Part 3, I will treat the RSA encryption scheme in detail. Finally in Part 4, I will indicate directions for further study and exploration in this field.

Part 1. Caesar codes, affine codes and the ring of integers modulo n (see [1], [2]).

Julius Caesar used a substitution cipher shifting each letter in the plaintext p by 3 with X, Y, Z mapping around to A, B, C, respectively. The letters of the English alphabet A, ..., Z can be identified with the integers 0, ..., 25 modulo 26. Caesar's encoding scheme can then be described as a translation $p \mapsto c = F(p) = p + k \bmod 26$ with $k = 3$ as the key. As an illustration of Caesar's method of encoding a plaintext, consider:

Plaintext:	BUILD CASTLES IN THE AIR
Remove punctuations:	BUILDCASTLESINTHEAIR
Identify with $\mathbb{Z}_{26}$:	01 20 08 11 03 02 00 18 19 11 04 18 08 13 19 07 04 00 08 17
Translate by $k = 3$:	04 23 11 14 06 05 03 21 22 14 07 21 11 16 22 10 07 03 11 20
Return to English alphabet (ciphertext):	EXLOGFDVWOHVLQWKHDLU

Translation codes can be broken by counting the frequency of the characters in the ciphertext and comparing these frequencies with the frequency of English letters in the ordinary English texts. The Appendix is a listing of my computer program that I used to encode the "Gettysburg Address" using Caesar's method. From the observed frequencies (see Figure One), we would guess that an E in the plaintext corresponds to an H in the ciphertext. This gives $F(E) = H$ since $c = F(p) = p + k \bmod 26$. This suggests $7 = 4 + k$ and consequently $k = 3$.

*PRINT_FREQUENCIES			NO. OF CHARACTERS=		1148
		EXP. FREQ	ACTUAL FREQ		
0:	A	8.0000	0	0.0436	
1:	B	1.5000	10	0.9146	
2:	C	3.0000	0	0.0436	
3:	D	4.0000	101	8.8415	
4:	E	13.0000	14	1.2631	
5:	F	2.0000	31	2.7439	
6:	G	1.5000	58	5.0958	
7:	H	6.0000	166	14.5035	
8:	I	6.5000	27	2.3955	
9:	J	0.5000	28	2.4826	
10:	K	0.5000	80	7.0122	
11:	L	3.5000	68	5.9669	
12:	M	3.0000	0	0.0436	
13:	N	7.0000	3	0.3049	
14:	O	8.0000	42	3.7021	
15:	P	2.0000	13	1.1760	
16:	Q	0.2000	77	6.7509	
17:	R	6.5000	93	8.1446	
18:	S	6.0000	15	1.3502	
19:	T	9.0000	1	0.1307	
20:	U	3.0000	79	6.9251	
21:	V	1.0000	44	3.8763	
22:	W	1.5000	125	10.9321	
23:	X	0.5000	21	1.8728	
24:	Y	2.0000	24	2.1341	
25:	Z	0.2000	28	2.4826	

Figure One.

Affine encoding schemes involve a slight modification of translation codes and the ability to compute inverses modulo 26. Let $c = F(p) = ap + b$ where a has an inverse modulo 26. By again using frequency counts, we can "guess" that $F(p_1) = c_1$ and $F(p_2) = c_2$. This yields the linear equations

$$ap_1 + b = c_1 \quad \text{and} \quad ap_2 + b = c_2.$$

Subtracting the first from the second, we obtain $a(p_2 - p_1) = (c_2 - c_1)$ so that $a = (p_2 - p_1)^{-1}(c_2 - c_1)$ and then $p = F^{-1}(c) = a^{-1}(c - b)$. I also encoded the "Gettysburg Address" using $c = F(p) = 7p + 10$ and the corresponding frequency table is given in Figure Two.

*PRINT_FREQUENCIES NO. OF CHARACTERS= 1148

		EXP. FREQ	ACTUAL FREQ	
0:	A	8.0000	28	2.4826
1:	B	1.5000	24	2.1341
2:	C	3.0000	3	0.3049
3:	D	4.0000	0	0.0436
4:	E	13.0000	93	8.1446
5:	F	2.0000	58	5.0958
6:	G	1.5000	44	3.8763
7:	H	6.0000	80	7.0122
8:	I	6.5000	28	2.4826
9:	J	0.5000	42	3.7021
10:	K	0.5000	101	8.8415
11:	L	3.5000	15	1.3502
12:	M	3.0000	166	14.5035
13:	N	7.0000	125	10.9321
14:	O	8.0000	68	5.9669
15:	P	2.0000	0	0.0436
16:	Q	0.2000	13	1.1760
17:	R	6.5000	14	1.2631
18:	S	6.0000	1	0.1307
19:	T	9.0000	27	2.3955
20:	U	3.0000	21	1.8728
21:	V	1.0000	0	0.0436
22:	W	1.5000	10	0.9146
23:	X	0.5000	77	6.7509
24:	Y	2.0000	31	2.7439
25:	Z	0.2000	79	6.9251

Figure Two.

From our frequency counts, we conjecture that F(E) = M and F(T) = N and then

$$M = F(E) = aE + b \quad \text{and} \quad N = F(T) = aT + b$$

or

$$4a + b = 12 \quad \text{and} \quad 19a + b = 13.$$

Subtracting, $15a = 1$. The inverse of 15 modulo 26 is 7. Therefore, $a = 15^{-1} = 7$. Finally, $b = 12 - 4a = 12 - 28 = -16 = 10$ modulo 26. Therefore, $c = F(p) = 7p + 10 \bmod 26$ and $p = 7^{-1}(c - 10) \bmod 26$ or

$$p = 15(c + 16) \bmod 26.$$

This example show the necessity of studying the integers modulo n.

We will denote the ring of integers modulo n by $\mathbb{Z}_n = \{0, 1, \ldots, n\text{-}1\}$. Addition and multiplication are performed modulo n. $\mathbb{Z}_n$ is a commutative ring with unity. It can be shown that the following three statements are equivalent: (1) n is a prime, (2) the ring $\mathbb{Z}_n$ is an integral domain, and (3) the ring $\mathbb{Z}_n$ is a field. A unit x of $\mathbb{Z}_n$ is an element of $\mathbb{Z}_n$ which has a multiplicative inverse. An element x of $\mathbb{Z}_n$ is a unit if and only if it is non-zero and relatively prime to n. We let U_n denote the set of all units of $\mathbb{Z}_n$. U_n is a multiplicative group with $\phi(n)$ elements, where $\phi(n)$ stands for Euler's totient function. Many number theory texts derive the expression

$$\phi(n) = n\left(1 - \frac{1}{p_1}\right)\cdots\left(1 - \frac{1}{p_k}\right)$$

where $p_1, \ldots, p_k$ are the distinct prime divisors of n.

Figure Three shows the multiplication table for U_{26}, the units of $\mathbb{Z}_{26}$. In Part 2, I will discuss an algorithm for computing the inverse of any given unit x of $\mathbb{Z}_n$.

×	1	3	5	7	9	11	15	17	19	21	23	25
1	1	3	5	7	9	11	15	17	19	21	23	25
3	3	9	15	21	1	7	19	25	5	11	17	23
5	5	15	25	9	19	3	23	7	17	1	11	21
7	7	21	9	23	11	25	1	15	3	17	5	19
9	9	1	19	11	3	21	5	23	15	7	25	17
11	11	7	3	25	21	17	9	5	1	23	19	15
15	15	19	23	1	5	9	17	21	25	3	7	11
17	17	25	7	15	23	5	21	3	11	19	1	9
19	19	5	17	3	15	1	25	11	23	9	21	7
21	21	11	1	17	7	23	3	19	9	25	15	5
23	23	17	11	5	25	19	7	1	21	15	9	3
25	25	23	21	19	17	15	11	9	7	5	3	1

Figure Three.

Part 2. Some results from number theory.

In this part, I will discuss Euler's theorem and an algorithm that computes the inverse of any unit of $\mathbb{Z}_n$. I shall refer to several basic facts from elementary group theory (see [6] and [7]).

Euler's theorem is a special case of Lagrange's theorem for finite commutative groups. Let G be a finite commutative multiplicative group with $|G|$ elements. $|G|$ is called the order of the group G. A subgroup H of G is a subset of G that is closed under multiplication. Lagrange's theorem for groups asserts that for every subgroup H of a finite group G, the order of H divides the order of G; that is, $|G|$ is a multiple of $|H|$. For any element x of G, the set of positive powers of x form a subgroup $<x> = \{1, x, x^2, x^3, \ldots\}$ referred to as the subgroup of G generated by x. The order $|<x>|$ of the subgroup $<x>$ is called the order of x and will also be denoted by $|x|$. We note that $|x|$ is the smallest power k of x such that $x^k = 1$. It follows from Lagrange's theorem that $|x|$ divides $|G|$. We can now state Euler's theorem and see its relation to elementary group theory.

EULER'S THEOREM. Let x be a unit of $\mathbb{Z}_n$. Then $x^{\phi(n)} = 1$.

That Euler's theorem is a special case of Lagrange's theorem now becomes clear. A unit x of $\mathbb{Z}_n$ is by definition an element of $\mathbb{U}_n$. Hence its order $|x|$ divides $\phi(n) = |\mathbb{U}_n|$; that is, $\phi(n)$ is a multiple of $|x|$. Let

$$\phi(n) = k \cdot |x|$$

then

$$1 = 1^k = \left(x^{|x|}\right)^k = x^{(k \cdot |x|)} = x^{\phi(n)}.$$

We close our discussion with an example. Consider the unit $x = 5$ of $\mathbb{Z}_{26}$.

$$<5> = \{1, 5, 5^2 = 25, 5^3 = 21\}$$

since $5^4 = 1$ again. Thus $|5| = |<5>| = 4$ and 4 divides $\phi(26) = 12$.

We now consider the question of how to compute the inverse of a unit x of $\mathbb{Z}_n$ (see [4]). In the example above, $5 \cdot 21 = 1$ in $\mathbb{Z}_{26}$ so that the inverse of $x = 5$ modulo 26 is 21. In fact, this example indicates one method for computing the inverse of a unit x modulo n. Simply take powers of x modulo n until 1 occurs. The last power of x before 1 occurs is the inverse of x. We now discuss a more efficient algorithm.

Let a be a unit of $\mathbb{Z}_n$. Then a and n are relatively prime and so the linear Diophantine equation $nx + ay = 1$ has a solution. Thus the inverse of a in $\mathbb{Z}_n$ is y modulo n. A modification of Euclid's algorithm enables us to compute the integral solutions x and y.

Let $r_0 = n$ and $r_1 = a$ so that $r_0 > r_1$. Then Euclid's algorithm is:

$$
\begin{array}{ll}
r_0 = q_2 r_1 + r_2 & \text{where } 0 < r_2 < r_1 \\
r_1 = q_3 r_2 + r_3 & \text{where } 0 < r_3 < r_2 \\
\ddots & \\
r_{p-2} = q_p r_{p-1} + r_p & \text{where } r_p = 1
\end{array}
$$

and the additional computations are:

$$
\begin{array}{ll}
r_0 = nx_0 + ay_0 \ (= n), & x_0 = 1,\ y_0 = 0 \\
r_1 = nx_1 + ay_1 \ (= a), & x_1 = 0,\ y_1 = 1 \\
r_2 = nx_2 + ay_2, & x_2 = x_0 - q_2 x_1,\ y_2 = y_0 - q_2 y_1 \\
\ddots & \\
r_p = nx_p + ay_p &
\end{array}
$$

Thus the inverse of a in $\mathbb{Z}_n$ is y_p modulo n.

For example, let $a = 15$ and $n = 26$. Then the computation of the inverse of 15 modulo 26 proceeds as follows:

	r_i	r_{i+1}	q_{i+2}	r_{i+2}	x	y	
					1	0	
					0	1	
	26	15	1	11	1	-1	
	15	11	1	4	-1	2	
	11	4	2	3	3	-5	
	4	3	1	1	-4	7	
(	3	1	3	0	15	-26	)

and so the inverse of 15 modulo 26 is 7.

Part 3. The RSA public encryption scheme (see [4], [5], [6]).
§3.1. The use of keys and the problem of security.

To illustrate the use of keys, I will briefly discuss Vigenère codes. Vigenère codes apply a sequence of translations to encode a plaintext. The sequence of translations to be used is determined by the key. We will identify the letters A to Z with the integers 0 to 25 modulo 26. The key PSUMATH corresponds to the integers 15 18 20 12 00 19 07 and determines the sequence of translations $P(x) = x + 15$, $S(x) = x + 18$, $U(x) = x + 20$, ..., $H(x) = x + 7$. If the plaintext has more than 7 characters, the sequence $P(x)$, ..., $H(x)$ is used cyclically. All this sound complicated and can be greatly simplified by the use of a Vigenère table (see Figure Four).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B:	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C:	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D:	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E:	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F:	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G:	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H:	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I:	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J:	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K:	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L:	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M:	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N:	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O:	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P:	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q:	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R:	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S:	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T:	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U:	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V:	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W:	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X:	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y:	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z:	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Figure Four.

As an example of Vigenère encoding, we consider:

Plaintext:	BRIEF HAPPY TIME
Key:	PSUMATH
Cipher:	QJCQF AHEHS FIFL

§3.2. The RSA encryption scheme of Rivest, Shamir and Adleman.

I have discussed Vigenère codes to illustrate the use of keys. If both the sender and the receiver of a message know the key, then Vigenère encoding and decoding make sense. Many cryptosystems use keys. The difficulty with the use of keys is security.

The innovation of RSA public encryption schemes is that the decoding transformation $F^{-1} : C \to P$ is unknown to someone who knows the encoding transformation $F : P \to C$.

Let p and q be two primes. Let $n = pq$ and then $\phi(n) = (p - 1)(q - 1)$. Let e be a unit of $\mathbb{Z}_{\phi(n)}$ with inverse d.

To encode a plaintext, we remove all spaces and punctuation and identify the letters A to Z with 0 to 25 modulo 26. Let p be a segment of the plaintext. Encode p as follows:

$$c = p^e \bmod n.$$

To decode the ciphertext c, we raise c to the d^{th} power modulo n; that is,

$$p = c^d \bmod n.$$

I will discuss three aspects of RSA encryption systems: (a) the inherent simplicity, (b) the scheme works, and (c) security.

§3.2a. The inherent simplicity of RSA cryptosystems.

We illustrate the steps stated above with an example.

Let $p = 3$ and $q = 11$ so $n = 33$ and $\phi(33) = (3 - 1)(11 - 1) = 20$. Since $e = 7$ is a unit in $\mathbb{Z}_{33}$, we can compute the inverse d of 7 in $\mathbb{Z}_{\phi(33)}$ by Euclid's algorithm to find that $d = 3$.

Plaintext:	NO CLASS
Remove punctuation:	NOCLASS
Numerical equivalent:	13 14 02 11 00 18 18

To encode, we compute modulo 33: $13^7 = 7$, $14^7 = 20$, $2^7 = 29$, $11^7 = 11$, $0^7 = 0$, $18^7 = 6$ and $18^7 = 6$ to obtain

Encoded text:	07 20 29 11 00 06 06

To decode, we compute modulo 33: $7^3 = 13$, $20^3 = 14$, $29^3 = 2$, $11^3 = 11$, $0^3 = 0$, $6^3 = 18$ and $6^3 = 18$ to obtain

Decoded text:	13 14 0 11 00 18 18

Thus, we get our original plaintext back!

We note that there are algorithms which compute $b^x \bmod y$ efficiently. An example of such an algorithm can be found in [4].

§3.2b. The scheme works.

To prove that the RSA scheme works, we apply Euler's theorem. Since ed = 1 in $\mathbb{Z}_{\phi(n)}$,

$$ed = 1 + k \cdot \phi(n).$$

Therefore,

$$\begin{aligned}(x^e)^d &= x^{1+k\phi(n)} \bmod n \\ &= (x^1).(x^{\phi(n)})^k \bmod n \\ &= (x) \cdot (1) \bmod n \qquad \text{by Euler's theorem} \\ &= x \bmod n\end{aligned}$$

as desired.

§3.2c. Security: Who knows what? How safe are RSA schemes?

First, I'll discuss who knows what. The receiver knows p, q, d and C, the sender knows P and everybody knows n and e. Since everyone knows n and e, anyone can send an encoded message. The receiver knows d and can decode the ciphertext. The primes p and q are kept secret to prevent an eavesdropper from computing $\phi(n) = (p - 1)(q - 1)$ and the inverse of e in $\mathbb{Z}_{\phi(n)}$.

Second, we ask: How secure are RSA schemes? The security of an RSA system rests upon several "hard" problems from number theory. "Hard" problems are mathematically solvable but involve prohibitively large amounts of computations; that is, current algorithms for solving hard problems are very costly in terms of computer time as the "size" of the problem increases. The "size" of the problem for RSA cryptosystems is determined by the number of binary digits of n. As the number of binary digits in n grows, breaking an RSA code becomes so costly that the code is for all practical purposes secure.

What are some of the approaches to breaking RSA codes? We discuss three classical problems of number theory and their relation to RSA codes: (1) factoring n, (2) computing $\phi(n)$, and (3) finding large primes p and q.

That a positive integer n can be uniquely factored into a product of primes was known to Euclid. What is amazing is the fastest known algorithm for factoring n requires approximately

$$e^{\sqrt{\ln(n)\,\ln(\ln(n))}}$$

steps (see [5], page 126). Mackiw [5] gives the following table of estimates:

Number of binary digits in n	Number of steps in algorithm	Running time of algorithm
50	1.4×10^{10}	3.9 hours
100	2.3×10^{15}	74 years
200	1.2×10^{23}	3.8×10^{9} years

Thus, while we can reasonably expect to declare in a short time whether a 100 digit number is prime, it does not seem reasonable to expect to be able to factor it using current techniques. The outlook for factoring a 200 digit number is much worse!

How hard is it to compute $\phi(n)$? If $\phi(n)$ were known, then p and q could be calculated as follows: (1) since $\phi(n) = (p - 1)(q - 1) = n - (p + q) + 1$, we have that $p + q = \phi(n) - n + 1$; (2) since $(p - q)^2 = (p + q)^2 - 4n$, we can now find $p - q$; and finally (3) we have $p = \big((p + q) + (p - q)\big)/2$ and $q = \big((p + q) - (p - q)\big)/2$. Thus any attempt to compute $\phi(n)$ would yield p and q and consequently factor $n = pq$. So computing $\phi(n)$ would be equivalent to factoring, which is a "hard" problem.

Are RSA schemes easy to construct? Can we find large primes? Are they plentiful? What progress is being make in computational number theory? A complete answer to these questions would involve another paper and would necessitate an in depth knowledge of current number theory. But a brief picture of the state of the art is possible. Primality testing is much easier than factoring. Even a brief introduction to pseudoprimes, strong-pseudo primes, Miller's test, and the probabilistic primality test of Rabin, Solovay and Strassen shows that there are efficient algorithms for deciding whether or not a given integer is prime. Primality testing may be a separate topic but it is fruitful and intriguing. The ability to identify large primes makes RSA cryptosystems practical. The difficulty of factoring makes RSA schemes secure.

Part 4. Further explorations.

In addition to primality testing, I would suggest and am personally interested in two directions: current algorithms for factoring integers and implementing RSA schemes and the algorithms of computational number theory on personal computers. This would involve developing multiple precision arithmetic packages and further study -- both of which I am undertaking. I hope you have enjoyed our excursion into RSA cryptosystems and the application of the bastion of pure mathematics known as number theory. But read my lips: "Number theory can be applied (Horrors)!"

Appendix.

```
PROGRAM CAESAR1; {MALA RENGANATHAN - KME NATIONAL CONVENTION - SPRING 1989}
USES PASPRINTER,SANE;

TYPE IT=INTEGER;LIT=LONGINT;ET=EXTENDED;AT=ET;BT=BOOLEAN;TT=TEXT;
     STR1=STRING[1];STR20=STRING[20];STR80=STRING[80];

     SA=ARRAY[0..25] OF STR1;

VAR
    OUTF:TT;FNAME:STR80;OUT_OPT:IT;PROG_NAME:STR80;

PROCEDURE GET_ALPHABET(VAR IO:TT;VAR ALPHABET:SA);
VAR S:STR80;I,L:IT;DEBUG:BT;
BEGIN
  DEBUG:=TRUE;
  IF DEBUG THEN WRITELN(IO,'*GET_ALPHABET');
  S:='ABCDEFGHIJKLMNOPQRSTUVWXYZ';L:=LENGTH(S);
  FOR I:=1 TO L DO ALPHABET[I-1]:=COPY(S,I,1);
END;

PROCEDURE FIND_CHAR(VAR IO:TT;ALPHABET:SA;C:STR1;VAR K:IT);
VAR I:IT;
    QUIT,FOUND,DEBUG:BT;
BEGIN
  DEBUG:=FALSE;
  IF DEBUG THEN WRITELN(IO,'*FIND_CHARACTER  C=',C);
  I:=0;QUIT:=FALSE;FOUND:=FALSE;
  REPEAT
    IF C=ALPHABET[I] THEN FOUND:=TRUE;
    IF FOUND OR (I=25) THEN QUIT:=TRUE ELSE I:=I+1;
  UNTIL QUIT;
  IF FOUND THEN K:=I ELSE K:=-1;
END;

PROCEDURE GET_PLAINTEXT(VAR IO:TT;ALPHABET:SA;
                        VAR G:TT;VAR GNAME:STR80;VAR FLAG:IT);
VAR F:TT;
    S,T,FNAME:STR80;C:STR1;
    I,J,K,L:IT;
    CONT,FOUND,DEBUG:BT;
BEGIN
  DEBUG:=FALSE;
  WRITELN(IO,'*GET_PLAINTEXT');

 {SAMPLE DATA FILE:
  PLAINTEXT1
  NOW IS THE TIME FOR ALL
  GOOD MEN TO COME DO THE
  AID OF THEIR COUNTRY}

  CONT:=TRUE;
  WHILE CONT DO
    BEGIN
      WRITE(' FILE NAME ? ');READLN(FNAME);
      IF FNAME='' THEN BEGIN FLAG:=1;CONT:=FALSE; END
      ELSE
        BEGIN
          FLAG:=-1;
          WRITELN(IO,' FILE NAME = ',FNAME);
          WRITE(' CORRECT FILE NAME ? (Y/N) ');READLN(S);
          IF S='' THEN BEGIN CONT:=FALSE;FLAG:=1;END
          ELSE
            BEGIN
              S:=COPY(S,1,1);
              IF (S='N') OR (S='n') THEN BEGIN CONT:=FALSE;FLAG:=1; END
              ELSE
                IF ((S='Y') OR (S='y')) THEN BEGIN CONT:=FALSE;FLAG:=0;END;
            END;
```

```
        END;
      END;

  IF FLAG=0 THEN
   BEGIN
     GNAME:=CONCAT(FNAME,'.T');
     REWRITE(G,GNAME);
     WRITELN(G,GNAME);

     RESET(F,FNAME);
     READLN(F,S);IF DEBUG THEN WRITELN(IO,' ',S);
     I:=0;
     WHILE NOT EOF(F) DO
       BEGIN
         I:=I+1;
         READLN(F,S);IF DEBUG THEN WRITELN(IO,' ',S);
         L:=LENGTH(S);T:='';
         IF L>0 THEN
           FOR J:=1 TO L DO
             BEGIN
               C:=COPY(S,J,1);
               IF C<>' ' THEN
                 BEGIN
                   FIND_CHAR(IO,ALPHABET,C,K);
                   IF K=-1 THEN FLAG:=1;
                   T:=CONCAT(T,C);
                 END;
             END;
         {WRITELN(IO,' ',T);}
         WRITELN(G,T);

       END;
     WRITELN(IO);
     CLOSE(F);CLOSE(G);

     {
     RESET(G,GNAME);
     WHILE NOT EOF(G) DO
       BEGIN
         READLN(G,S);WRITELN(IO,' ',S);
       END;
     WRITELN(IO);
     CLOSE(G);
     }

     {
     WRITELN(IO,' DATA FILE NAME:  ',FNAME,'  FLAG=',FLAG);
     WRITE(' STOP FOR VIEWING !  CONTINUE ? ');READLN(S);WRITELN;
     }
   END;
END;

PROCEDURE ENCODE(VAR IO:TT;ALPHABET:SA;VAR G:TT;GNAME:STR80);
VAR E:TT;S,T,ENAME:STR80;C:STR1;
    I,K,L:IT;
BEGIN
  WRITELN(IO,'*ENCODE');WRITELN(IO);

  ENAME:=CONCAT(COPY(GNAME,1,LENGTH(GNAME)-2),'.E');
  REWRITE(E,ENAME);
  WRITELN(E,ENAME);
  RESET(G,GNAME);
  READLN(G,S);WRITELN(IO,' ',S);
  WHILE NOT EOF(G) DO
    BEGIN
      READLN(G,S);WRITELN(IO,' ',S);L:=LENGTH(S);T:='';
      FOR I:=1 TO L DO
        BEGIN
```

```
          C:=COPY(S,I,1);FIND_CHAR(IO,ALPHABET,C,K);
          K:=K+3;        {CAESAR'S CODE}
          {K:=7*K+10;} {AN AFFINE CODE}
          K:=K MOD 26;
          C:=ALPHABET[K];T:=CONCAT(T,C);
        END;
      WRITELN(E,T);
    END;
  WRITELN(IO);
  CLOSE(D);CLOSE(E);

  RESET(E,ENAME);
  WHILE NOT EOF(E) DO
    BEGIN
      READLN(E,S);WRITELN(IO,' ',S);
    END;
  WRITELN(IO);
  CLOSE(E);

  WRITELN(IO,' DATA FILE NAME:  ',GNAME);
  {WRITE(' STOP FOR VIEWING ! CONTINUE ? ');READLN(S);}
END;

PROCEDURE DRIVER(VAR IO:TT;PROG_NAME:STR80;OUT_OPT:IT);
VAR G:TT;GNAME:STR80;
    ALPHABET:SA;FLAG:IT;
    DEBUG:BT;
BEGIN
 IF OUT_OPT<>0 THEN WRITELN('>WELCOME TO  ',PROG_NAME);
                    WRITELN(IO,'>WELCOME TO  ',PROG_NAME);

 GET_ALPHABET(IO,ALPHABET);
 GET_PLAINTEXT(IO,ALPHABET,G,GNAME,FLAG);

 IF FLAG=0 THEN
   BEGIN
     ENCODE(IO,ALPHABET,G,GNAME);
   END;
END;

BEGIN
 PROG_NAME:='CAESAR1';
 OUT_OPT:=0;

 CASE OUT_OPT OF
  0:DRIVER(OUTPUT,PROG_NAME,OUT_OPT);
  1:DRIVER(PRINTER,PROG_NAME,OUT_OPT);
  2:BEGIN
     FNAME:=PROG_NAME+'.OUT';REWRITE(OUTF,FNAME);
     DRIVER(OUTF,PROG_NAME,OUT_OPT);
     WRITELN(OUTF);
     CLOSE(OUTF);
    END;
 END{CASE};

 WRITELN;WRITE('>EXIT ',PROG_NAME,'  (PLEASE PRESS RETURN)  ');READLN;
END.
```

References.

[1] Denning, Dorothy. *Cryptography and Data Security*. Addison-Wesley, 1982.

[2] Kranakis, Evangelos. *Primality and Cryptography*. John Wiley, 1986.

[3] Rivest, R. L., A. Shamir and L. Adleman, "A method for obtaining digital signatures and public-key cryptosystems." *Communications of the ACM*. 21 No. 2 (February 1978), 120-126.

[4] Wilf, Herbert S. *Algorithm and Complexity*. Prentice-Hall, 1986.

[5] Mackiw, George. *Applications of Abstract Algebra*. John Wiley, 1985.

[6] Rosen, Kenneth. *Elementary Number Theory and Its Applications* (2nd edition). Addison-Wesley, 1988.

[7] Niven and Zuckerman. *An Introduction to the Theory of Numbers* (4th edition). John Wiley, 1979.

[8] Riesel, Hans. *Prime Number and Computer Methods for Factoring*. Birkhauser, 1985.

Recreational and Educational Computing

Reviewed by
Brent Marcum, *student*

Kentucky Alpha

Eastern Kentucky University
Richmond, Kentucky 40475

Today's student of the mathematical and computer sciences often becomes so devoted to study that the current computer technology often goes unnoticed. A person can easily become overwhelmed by publications containing software or other computer information. Even if a person is fortunate enough to find a source of quality information, the problem then becomes finding the time to decide what to read. A possible solution to this dilemma is the *Recreational and Educational Computing* (or *REC*) newsletter. It provides an overview of the latest popular software releases as well as interesting articles and problems in mathematics and computer science. I have read and will comment on two recent issues of *REC*.

This newsletter has several attractive features that make it enjoyable and informative to read. One of the most outstanding qualities is the exuberant enthusiasm of the editor, Dr. Michael W. Ecker. Throughout the newsletter, he inputs ideas, comments and suggestions to enhance the overall effectiveness of the publication. His enthusiasm is first evident in the letters department. Here, Dr. Ecker responds to almost every reader letter in such a way as to promote new ideas and challenges for the subscribers. Similarly, he often adds comments following articles submitted to *REC* that enhance the ideas of the article.

Another characteristic of *REC* that is worth mentioning is the continuous encouragement of subscriber participation. *REC* is pervaded with challenges presented to the readers. These often take the form of programming problems or puzzle challenges. For both, subscribers are requested to present solutions and ideas to *REC*. Fortunately, subscriber contributions do not go unnoticed and are published with praise and credit given to the author. The net result of this subscriber/editor interaction is higher quality solutions of problems and puzzles. It also encourages further subscriber participation.

REC is also very understandable at the student level. While most of the articles could be read and understood by someone who has not had an extensive mathematics/computer science background, a large number are of special interest to mathematics and computer science majors. For example, in the two issues I read there were articles on mathematical black holes, magic squares, fractals and how to solve puzzles. This makes it ideal for students in that it presents a variety of topics in a form that can be interpreted without too much effort.

A similar benefit to students is evident in the software reviews contained in *REC*. There were several programs that were reviewed, including Mandelbrot 3, Express Publisher, Derive and New Basic. In addition, a few game programs were reviewed. These reviews were generally short and less in depth than one might expect but they were informative. Often the software package being considered is compared to existing, well-known programs. This gives the reader a chance to draw on his/her own experience and compare a new package to one already known. In all cases, complete information was provided concerning the price of the software and where an order could be placed.

Students should also find it helpful to see programming used as a problem solving tool. *REC* subscribers frequently submit solutions to mathematical problems that involve simple programs that closely approximate actual solutions. This presents a whole new spectrum of programming challenges to the individual who is bored with the standard classroom programming assignments. This could help computer science majors improve their programming skills along with providing pastime entertainment.

Experimenting with programs presented in *REC* should be little problem to anyone with a computer that has BASIC. A person with very little programming knowledge should have no trouble keying in the example programs and experimenting with the code since most of the examples are written in BASIC. However, in some cases a more thorough knowledge of BASIC is required to completely understand what is taking place. In particular, programs that involve graphics soon become complicated to a person with just a working knowledge of BASIC.

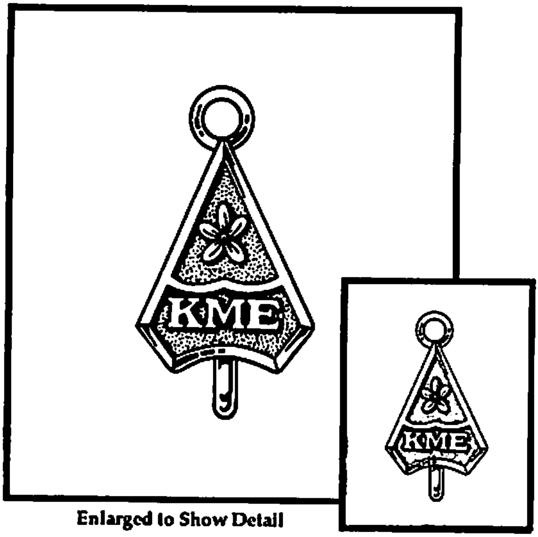
One weak trait of *REC* is that sometimes a programming solution is given when an exact mathematical solution would be more appropriate.

This could cause a dependence on programming skills for problem solving and weaken a reader's mathematical skills. One example of such a case was presented in a reader's letter to the editor. The reader criticized the use of a program to solve a problem since a mathematical solution was easier. The original problem was to use initial conditions to find a solution for the millionth case of the scenario that was presented. This was solved by another reader with a one line program that quickly provided a solution. However, the first reader solved the problem using binomial coefficients and then explained that if the original program was altered to find the trillionth case, the run time would be phenomenal whereas the manual calculations could be done in a few minutes.

Overall, *REC* is an entertaining publication that suits the needs and interests of anyone who enjoys working in mathematics or computer science. The material contained in each issue is both enjoyable to read and very informative. When this is combined with the enthusiasm of the editor and the comments and inputs provided by the subscribers, *REC* becomes a very interesting and enjoyable publication.

Editor's comment. To subscribe for one year (8 issues) of *Recreational and Educational Computing,* send $27 (US introductory rate only) to: *REC,* Att: Dr. M. Ecker, 909 Violet Terrace, Clarks Summit, Pennsylvania 18411. Three sample issues are available for $10, creditable towards subscription.

Kappa Mu Epsilon Jewelry



Enlarged to Show Detail

Actual Size

The Kappa Mu Epsilon Key (Shown above) is available as a charm, pin or tie - tac.

	GOLD PLATE	STERLING SILVER	10K Gold
#805 KEY CHARM	$ 4.75	$ 7.50	$42.00
# 806 KEY PIN	$ 5.25	$ 8.00	$49.50
# 807 KEY TIE - TAC	$ 5.25	$ 8.00	$49.50

Manufactured by

J.O. POLLACK Co.

Prices subject to change without notice

(Prices include sales tax)

Orders for Kappa Mu Epsilon Jewelry MUST be placed through your chapter's corresponding secretary.

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 January 1992. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 1992 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROBLEMS 445-449.

Problem 445. Proposed by Dave Smith, Messiah College, Grantham, Pennsylvania. Dirk, a junior math major, visited the campus post office to pick up his key for the new year. When he found his mailbox, he noticed that every year his mailbox had been in the same row in the large rectangle that was formed by all of the mailboxes. Hours later, the only thing he remembered about his current mailbox number was that it was somewhere in the 920's. He recalled that during his freshman and sophomore years, the numbers of his mailbox were #837 and #897, respectively. He also remembered that his roommate's mailbox number was #65 and that it was located seven boxes above the bottom row. If the post office numbers the mailboxes consecutively from top to bottom starting in the upper left corner, what is the number of Dirk's mailbox?

Problem 446. Proposed by Lamarr Widmer, Messiah College, Grantham, Pennsylvania. The composite integer $1991 = 11 \cdot 181$ is palindromic as are all of its prime factors. What is the next integer after 1991 which has the same property if (a) single digit primes are allowed? (b) single digit primes are not allowed?

Problem 447. Proposed Don Tosh, Evangel College, Springfield Missouri. The usual Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, ... and any term may be found by adding together the two preceding terms. Formally we have $f_1 = 1$, $f_2 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for integers $n > 2$. It is well known that the ratio of consecutive terms in the Fibonacci sequence converges to $r = (\sqrt{5} + 1)/2$, the golden ratio; i.e. $\lim f_n/f_{n-1} = r$ as $n \to \infty$. Next, we define a generalized Fibonacci sequence $\{x_n\}$ by choosing any two real numbers a and b (neither of which is zero) and then setting $x_1 = a$, $x_2 = b$ and $x_n = x_{n-1} + x_{n-2}$ for integers $n > 2$. Prove that the ratio of consecutive terms in this generalized Fibonacci sequence still converges to r; i.e. $\lim x_n/x_{n-1} = r$ as $n \to \infty$.

Problem 448. Proposed by Fred A. Miller, Elkins, West Virginia. Let A, B, C, and D be four concyclic points in the plane such that C and D are separated by A and B. If p_1, p_2 and p_3 are the lengths of the perpendiculars from D to lines AB, BC and CA respectively, show that

$$\frac{AB}{p_1} = \frac{BC}{p_2} + \frac{CA}{p_3}.$$

Problem 449. Proposed by Albert White, Saint Bonaventure University, Saint Bonaventure, New York. Assume that a square is inscribed in a circle whose radius is r. Then a circle is inscribed in the square. A square is inscribed in this circle and this pattern continues ad infinitum. Find the sum of the circumferences of all the circles and the sum of the perimeters of all the squares.

Please help your editor by submitting problem proposals.

SOLUTIONS 426 and 430-434.

Problem 426. Proposed by Dmitry P. Mavlo, Moscow, U.S.S.R. Prove that an arbitrary plane closed curve of length L as shown in the figure below can be completely placed into a pentagon having perimeter P where P is not greater than $(\sqrt{5} - 1) \cdot L$. Consider all cases in which the equality $P = (\sqrt{5} - 1) \cdot L$ holds.

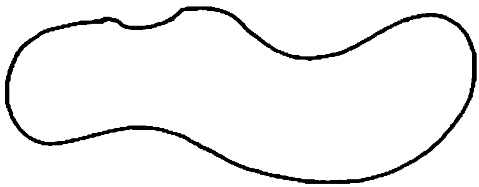


Figure 1.

Solution by the proposer.

First we circumscribe the given arbitrary plane curve with an equiangular pentagon as shown in Figure 2. This procedure is always possible. Let the points T_i (for i = 1, 2, 3, 4 and 5) denote the vertices of the circumscribing pentagon; let the points A_i (for i = 1, 2, 3, 4 and 5) denote the points where the curve contacts the pentagon; let R denote the perimeter of the formed pentagon $A_1A_2A_3A_4A_5$; and let P denote the perimeter of the circumscribing equiangular pentagon (notice that each angle of this pentagon measures 108°) (see Figures 2 and 3).

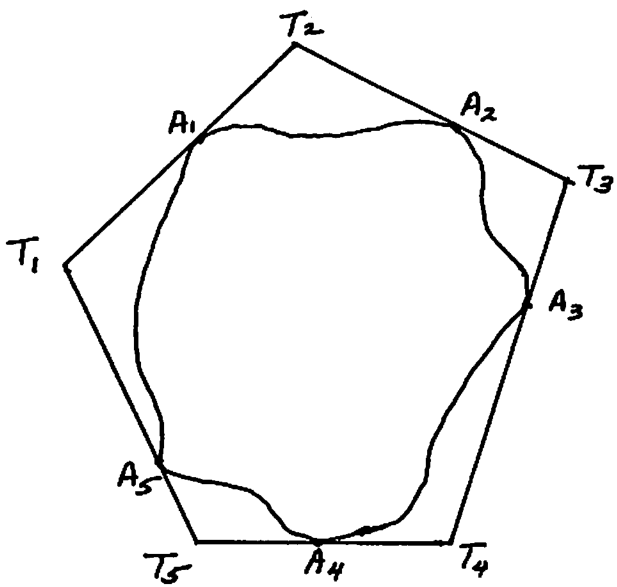


Figure 2.

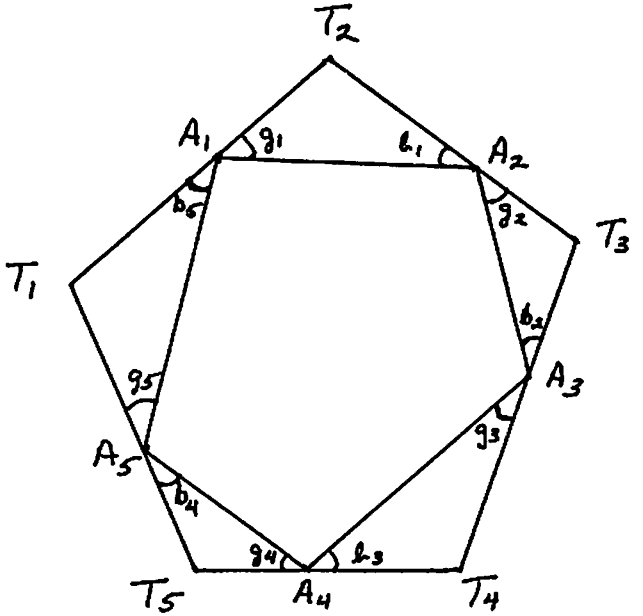


Figure 3.

In the notation of Figure 3 we have

$$P = [T_1A_1 + T_1A_5] + [T_2A_1 + T_2A_2] + [T_3A_2 + T_3A_3]$$
$$+ [T_4A_3 + T_4A_4] + [T_5A_4 + T_5A_5]$$

$$= \frac{|A_5A_1|}{\sin(108^\circ)}\Big(\sin(b_5) + \sin(g_5)\Big) + \frac{|A_1A_2|}{\sin(108^\circ)}\Big(\sin(b_1) + \sin(g_1)\Big)$$
$$+ \frac{|A_2A_3|}{\sin(108^\circ)}\Big(\sin(b_2) + \sin(g_2)\Big) + \frac{|A_3A_4|}{\sin(108^\circ)}\Big(\sin(b_3) + \sin(g_3)\Big)$$
$$+ \frac{|A_4A_5|}{\sin(108^\circ)}\Big(\sin(b_4) + \sin(g_4)\Big)$$

$$= \frac{|A_5A_1|}{\sin(54^\circ)}\cos(b_5 - g_5) + \frac{|A_1A_2|}{\sin(54^\circ)}\cos(b_1 - g_1) + \frac{|A_2A_3|}{\sin(54^\circ)}\cos(b_2 - g_2)$$

$$+ \frac{|A_3A_4|}{\sin(54°)} \cos(b_3 - g_3) + \frac{|A_4A_5|}{\sin(54°)} \cos(b_4 - g_4)$$

$$\leq \frac{1}{\sin(54°)} \left(|A_5A_1| + |A_1A_2| + |A_2A_3| + |A_3A_4| + |A_4A_5| \right)$$

$$= \frac{R}{\sin(54°)}$$

so that

(1) $$P \leq \frac{4R}{1 + \sqrt{5}}.$$

To complete the proof, we should recall that the length of the curve is greater than or equal to the length of the segment connecting the same points. It follows immediately that $L \geq R$ and in view of (1), we have $L \geq R \geq \left((1 + \sqrt{5})/4\right) \cdot P$ so that

(2) $$P \leq (\sqrt{5} - 1) \cdot L.$$

The proposer states that the analysis of the necessary and sufficient conditions under which equality holds in (2) is very simple and is therefore omitted.

Editor's Comment. It appears that equality holds only when the original curve is itself an equiangular pentagon; however, I am unable to prove this assertion. Perhaps some reader can supply the missing details or the correct conditions for equality.

Problem 430. Proposed by the editor. John and his brother Bill have ages which when added together produce a perfect cube. Furthermore when John was half as old as Bill is now, Bill's age equaled the square of John's age when Bill was born. Find their current ages.

Solution by Jamie Konrad, Rockford College, Rockford, Illinois.

Let x be Bill's current age, y be John's current age, and z be John's age when Bill was born. Note that $z = y - x > 0$. The first sentence yields the equation

(1) $$x + y = n^3$$

for some integer $n > 0$. Facts from the second sentence, "... when John was half as old as Bill is now ($x/2$), Bill's age equaled the square of John's age (z^2) when Bill was born," yield the equation $x/2 - z^2 = z$ or

(2) $$x = 2z(z+1).$$

Substituting (2) into (1) and using $z = y - x$, yields the equation

$$2z(z+1) + \left(2z(z+1) + z\right) = n^3$$

which reduces to

(3) $$z(4z+5) = n^3.$$

Thus z divides n so that $n = rz$ for some integer $r > 0$. Substituting rz for n in (3) and rearranging yields

(4) $$z(r^3 z - 4) = 5$$

which implies that $z = 1$ or $z = 5$. The choice $z = 1$ requires $r^3 = 9$ which is impossible. The choice $z = 5$ requires that $r = 1$ which yields the solution $n = 5$, $x = 60$ and $y = 65$.

Also solved by Charles Ashbacher, Hiawatha, Iowa; Kendall Bailey, Drake University, Des Moines, Iowa; Jill Carnahan, Eastern Kentucky University, Richmond, Kentucky; Joel Derstine, Messiah College, Grantham, Pennsylvania; Melinda Dolen, Eastern Kentucky University, Richmond, Kentucky; Onecia Gibson, Eastern Kentucky University, Richmond, Kentucky; New York Lambda Problem Solvers, Long Island University, C. W. Post Campus, Brookville, New York; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; and Chao Yang, Central Missouri State College, Warrensburg, Missouri.

Problem 431. Proposed by the editor. In a high school math contest, the answer sheet stated that the equation

$$6^{3x^2 - 11x - 4} = 11^{x^2 - 3x - 4}$$

had only the solution $x = 4$. Prove or disprove the truth of this statement when x is a real number.

Solution by Richard Giza, Illinois Benedict College, Lisle, Illinois.

We shall disprove the statement that the given equation has only one solution. By taking natural logarithms of both sides of the given equation, we obtain

$$(1)\qquad \left(3x^2 - 11x - 4\right)(\ln 6) = \left(x^2 - 3x - 4\right)(\ln 11).$$

Let $a = (\ln 6)/(\ln 11)$. Substituting a into equation (1) and rearranging, we have the following quadratic equation in x (since a is not zero),

$$(2)\qquad (3a - 1)x^2 + (3 - 11a)x + 4(1 - a) = 0.$$

From the quadratic formula, we find

$$(3)\qquad x = \frac{(11a - 3) \pm |13a - 5|}{2(3a - 1)}$$

$13a - 5 > 0$ is guaranteed by our choice of a because it can be verified easily that $a = (\ln 6)/(\ln 11) > 5/13$ which is equivalent to $13a - 5 > 0$. The plus sign in (3) yields the answer $x = 4$ while the minus sign yields the solution

$$x = \frac{1 - a}{3a - 1} = \frac{\ln 11 - \ln 6}{3\ln 6 - \ln 11} = \frac{\ln (11/6)}{\ln(216/11)}.$$

Also solved by Charles Ashbacher, Hiawatha, Iowa; Dave Aschbrenner, Drake University, Des Moines, Iowa; Richard A. Gibbs, Fort Lewis College, Durango, Colorado; New York Lambda Problem Solvers, Long Island University, C. W. Post Campus, Brookville, New York; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; Kenneth L. Price, Western Illinois University, Rushville, Illinois; Dave Smith, Messiah College, Grantham, Pennsylvania; and Chao Yang, Central Missouri State College, Warrensburg, Missouri.

Problem 432. Proposed by the editor. Let G(x) be a function over the real numbers such that

$$(x^2 + 3)\cdot G(x) - x^2\cdot G(2 - x) = 4x^3 - x^2 + 6.$$

Determine G(x).

Solution by Chao Yang, Central Missouri State University, Warrensburg, Missouri.

Let G(x) be the desired function such that

$$(1) \qquad (x^2 + 3)\cdot G(x) - x^2\cdot G(2 - x) = 4x^3 - x^2 + 6.$$

Substituting 2 - x for x in the given equation yields the equation

$$(2) \qquad -(2-x)^2\cdot G(x) + \left((2-x)^2 + 3\right)\cdot G(2-x) = 4(2-x)^3 - (2-x)^2 - 6.$$

Thus equations (1) and (2) form a system of linear equations having G(x) and G(2 - x) as unknowns which can be solved for G(x). Using Cramer's Rule, we have

$$D = \begin{vmatrix} x^2 + 3 & -x^2 \\ -(2-x)^2 & (2-x)^2 + 3 \end{vmatrix} = 3\left(2x^2 - 4x + 7\right).$$

Similarly,

$$D_{G(x)} = \begin{vmatrix} 4x^3 - x^2 - 6 & -x^2 \\ 4(2-x)^3 - (2-x)^2 - 6 & (2-x)^2 + 3 \end{vmatrix}$$

$$= 3(x^2 - 2)\left(2x^2 - 4x + 7\right).$$

Therefore

$$G(x) = \frac{D_{G(x)}}{D} = \frac{3(x^2 - 2)\left(2x^2 - 4x + 7\right)}{3\left(2x^2 - 4x + 7\right)} = x^2 - 2.$$

Also solved by Charles Ashbacher, Hiawatha, Iowa; Sean Forbes, Drake University, Des Moines, Iowa; Richard A. Gibbs, Fort Lewis College, Durango, Colorado; New York Lambda Problem Solvers, Long Island University C. W. Post Campus, Brookville, New York; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; Kenneth L. Price, Western Illinois University, Rushville, Illinois; and Dave Smith, Messiah College, Grantham, Pennsylvania.

Problem 433. Proposed by the editor. Consider the family of hyperbolas

$$\frac{x^2}{a_i^2} - \frac{y^2}{b_i^2} = 1$$

where a_i and b_i satisfy the relation

$$\frac{1}{a_i^2} - \frac{1}{b_i^2} = 5$$

for i = 1, 2, ..., n. Find all points which the hyperbolas have in common or prove that none exist.

Solution by Sean Forbes, Drake University, Des Moines, Iowa.

The family of curves

$$\frac{x^2}{a_i^2} - \frac{y^2}{b_i^2} = 1,$$

where i = 1, 2, ..., n, are hyperbolas. Since these curves also satisfy the condition

$$\frac{1}{a_i^2} - \frac{1}{b_i^2} = 5,$$

we must have

$$\frac{x^2}{a_i^2} - \frac{y^2}{b_i^2} = \frac{1}{5a_i^2} - \frac{1}{5b_i^2}.$$

For this to be true, we must have $x^2 = y^2 = 1/5$. Thus there are four common points of intersection for the family of hyperbolas; i.e. $(1/\sqrt{5}, 1/\sqrt{5})$, $(-1/\sqrt{5}, 1/\sqrt{5})$, $(1/\sqrt{5}, -1/\sqrt{5})$ and $(-1/\sqrt{5}, -1/\sqrt{5})$.

Also solved by Charles Ashbacher, Hiawatha, Iowa; New York Lambda Problem Solvers, Long Island University, C. W. Post Campus, Brookville, New York; and Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Problem 434. Proposed by the editor. Let r be a positive rational number. Prove that $(8r + 21)/(3r + 8)$ is a better approximation to $\sqrt{7}$ than r is.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

It suffices to establish that

$$\left|\frac{8r + 21}{3r + 8} - \sqrt{7}\right| < |r - \sqrt{7}|,$$

where r is a positive rational number. We next construct the following collection of equivalent inequalities.

$$\left|\frac{8r + 21}{3r + 8} - \sqrt{7}\right| < |r - \sqrt{7}|$$

$$\left|\frac{8r + 21 - 3r\sqrt{7} - 8\sqrt{7}}{3r + 8}\right| < |r - \sqrt{7}|$$

$$\left|\frac{(r - \sqrt{7})(8 - 3\sqrt{7})}{3r + 8}\right| < |r - \sqrt{7}|$$

$$|r - \sqrt{7}|\left(\frac{8 - 3\sqrt{7}}{3r + 8}\right) < |r - \sqrt{7}|$$

since $8 - 3\sqrt{7} > 0$ and $3r + 8 > 0$. Also, $(8 - 3\sqrt{7})/(3r + 8) < 1$ because r is a positive rational number, $r - \sqrt{7} \neq 0$. Finally, $8 - 3\sqrt{7} < 3r + 8$ since $-\sqrt{7} < 0 < r$ and the solution is complete.

Also solved by Charles Ashbacher, Hiawatha, Iowa; Richard A. Gibbs, Fort Lewis College, Durango, Colorado; New York Lambda Problem Solvers, Long Island University, C. W. Post Campus, Brookville, New York; Kenneth L. Price, Western Illinois University, Rushville, Illinois; and Doug Staz, Messiah College, Grantham, Pennsylvania.

The following public service announcement was prepared by the Peace Corps especially for *The Pentagon*'s student readers.

Mathematics Majors and the Peace Corps

Peace Corps - Public Response
1990 K Street, N. W.
Washington, D. C. 20526
((202) 606-3000, extension 755)

The Peace Corps is one of many options available to mathematics undergraduates at graduation time. Volunteers with training in mathematics and the physical sciences are in high demand in many of the seventy-three countries where Volunteers serve. The Peace Corps offers opportunities to take a break from the rigors of academia without joining the conventional working world. U. S. citizens over the age of eighteen (though few under twenty-one qualify) are eligible. With the experience gained through Peace Corps service one is better prepared to take advantage of options that present themselves later in life.

Volunteers with undergraduate degrees in mathematics are eligible for many kinds of overseas assignments. The primary placement is direct classroom teaching of mathematics to students of varying ages and abilities, offering opportunities to teach in settings where teachers are respected and honored. Or one may train teachers so that they can be more effective.

Mastery of basic mathematics, algebra, geometry and trigonometry is required to qualify for entrance into universities. A body of professionals educated in mathematics (such as engineers, architects and urban planners) is essential for a developing country wanting to improve its own situation. "The necessity of a strong mathematics background in such disciplines as engineering, which is of importance to development ... helped to justify the work I was involved in" remembers former Volunteer Janet Lea Barnett. A mathematics course load could be combined with teaching other classes in a skill that the Volunteer possesses such as handicrafts or gardening.

Teaching jobs are not the only possible placements for a person with a degree in mathematics. With a minor in botany and nursery experience, a Volunteer could be placed in a forestry assignment. An assortment of responsibilities fall under this category including managing a national park or wildlife preserve, designing and executing forest management plans, establishing and managing a nursery, establishing woodlots, or directing reforestation. With skills and an interest in the environment, one could develop an environmental education curriculum.

Fisheries assignments are another option for those with a minor in chemistry or biology. Volunteers in this field provide advice to farmers on fertilizing, feeding and stocking ponds, plan production of fish and rice crops in the same area, select suitable pond sights, survey irrigation and drainage structures, and supervise construction of dikes, trenches and ponds. In addition, a Volunteer could train the host country nationals in fisheries techniques. The connections between mathematics and the sciences are strong enough to make these options viable to many people with a bachelor's degree in mathematics.

Peace Corps service has many benefits, both tangible and intangible. During a Volunteer's service all expenses are paid and complete medical and dental care is provided. Other financial incentives include partial cancellation of eligible "Perkins" loans and deferment of payment on most other government-sponsored college loans. Academic credit could be earned for participation in select certified pre-service and in-service training programs. Upon return to the U. S., a former Peace Corps Volunteer receives a readjustment allowance of $5,400 for a typical twenty-seven month tour. The returned Volunteer is eligible for special graduate school scholarships and assistantships. A job search is facilitated by job hunting assistance and easier access to Federal jobs through a preferential hiring status called "non-competitive eligibility."

The real rewards for service are the knowledge that others have been helped and an increased awareness of oneself and the world. Many returned Volunteers tell of increased self-reliance and confidence in their own leadership, creativity and human relations. Employers view these qualities with added consideration and respect. As expressed by Mike McCormick, a former Volunteer in Ghana, "two years as a Volunteer was incredibly enriching and I'm more confident than ever about marketing my skills. Previous experience tells me that employers want to know that we handled school well and can accomplish complex technical tasks.

Beyond that, they look for maturity, adaptability, judgment, self-motivation, communications skills, and a perspective that reaches beyond one's technical field – attributes that Peace Corps service only enhances."

The Peace Corps prepares you to capitalize on any desirable options that come about after service. Former Volunteers are available to answer your questions and talk about their experiences. Call or write: Peace Corps, Public Response, 9th Floor, 1990 K Street, N. W., Washington, D. C. 20526; (202) 606-3000 extension 755, or check listings in your area for one of the fifteen regional offices.

Kappa Mu Epsilon News

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy *KME* events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, Missouri 64801.

INSTALLATION OF NEW CHAPTERS

Pennsylvania Xi Chapter

Cedar Crest College, Allentown, Pennsylvania

The Pennsylvania Xi Chapter of Kappa Mu Epsilon was established at Cedar Crest College in Allentown, Pennsylvania at 6:30 p.m. on October 30, 1990. The installing officer was James C. Pomfret, Region I director. Charter members of Pennsylvania Xi inducted in this ceremony were:

> *Students:* Jennifer Christmer, Sandra Fry, Dee Dee Geijer, Karen Haase, Tammy Hawkins, Susan Kleckner, Carol Kobayashi, Stacey Nelson, and Laura Witucki.
>
> *Faculty:* Charles Chapman.

Officers installed during the ceremony were: Susan Kleckner, president; Sandra Fry, vice-president; Karen Haase, secretary; Carol Kobayashi, treasurer. Faculty members Regina Brunner and Charles Chapman accepted the responsibilities of corresponding secretary and faculty sponsor respectively.

The induction ceremony was followed by a banquet for new members and guests. A total of 18 were in attendance. An after dinner talk on

"Tessellations of the Plane" was given by Professor Doris Schattscheider of Moravian College.

Missouri Lambda Chapter

Missouri Western State College, St. Joseph, Missouri

The installation of the Missouri Lambda Chapter of Kappa Mu Epsilon was held on February 10, 1991, in Room 110 of the Student Center on the campus of Missouri Western State College. Dr. Jo Ann Fellin, OSB, National Treasurer of Kappa Mu Epsilon, conducted the installation ceremony. Associate professors John Atkinson and Jerry Wilkerson, Kappa Mu Epsilon members initiated by the KS Beta and MO Beta Chapters, respectively, and currently on the faculty at Missouri Western, also participated in the ceremony. Dr. Atkinson served as Conductor. Thirteen students and seven faculty in addition to Atkinson and Wilkerson constituted the founding group of the new Chapter at Missouri Western. Those initiated were:

> *Students:* Anita K. Chancey, Audrey G. Davis, Robin Fowler, Douglas A. Gibson, Wanda S. Gibson, Julie Hansbrough, Kevin R. Heyde, Susan K. Nichols, Gena J. Puckett, Roy E. Rhinehart, Eric Toot, Tammy Steinkamp and David Vlieger.
>
> *Faculty:* Jennifer S. Austin, Christopher P. Godfrey, Bill L. Huston, David John, Kenneth W. Lee, Don Mahaffy and Leo H. Schmitz.

Dr. Fellin, OSB, began the afternoon ceremony with a short history of Kappa Mu Epsilon. Following the installation of officers, William J. Nunez, III, the Dean of Liberal Arts and Sciences at Missouri Western State College, congratulated the group and spoke on the importance of honor societies in higher education. Several relatives and friends of the initiates were present at the 3 p.m. installation which was followed by a reception.

Officers installed during the ceremony were: Susan K. Nichols, president; Julie Hansbrough, vice-president; Gena J. Puckett, recording secretary; and Douglas A. Gibson, treasurer. Faculty members John

Atkinson and Jerry Wilkerson accepted the responsibilities of the corresponding secretary and faculty sponsor, respectively.

Texas Kappa Chapter

University of Mary Hardin-Baylor, Belton, Texas

The installation of the Texas Kappa Chapter of Kappa Mu Epsilon was held on February 21, 1991, in Hardy Hall on the campus of the University of Mary Hardin-Baylor. Dr. Harold L. Thomas, National President of Kappa Mu Epsilon, conducted the installation ceremony. Sherry O'Neal, president of Delta Psi Theta, the petitioning club, served as Conductor during the ceremony. Twenty-four students and three faculty constituted the founding group of the new Chapter at the University of Mary Hardin-Baylor. Those initiated were:

Students: Florence Akinyi, Abeer Al-Naji, Garry Bartek, Curtis Breaux, Claudia Drayton, Charles Fewless, Helene Gaede, Kerry Geiger, Don Henslee, Roger Hoelscher, Melinda Hollan, Neil Ling, Susannah Marshall, Regina Noles, Sherry O'Neal, Taeko Osterman, Jacqueline Pilkey, Bernice Reeves, Melissa Santana, Karen Scott, Shane Scott, Darren Seifer, Edward Tunstall and Stephanie Williams.

Faculty: Prof. Peter Chen, Dr. William Harding and Prof. Maxwell Hart.

Following the installation ceremony, Dr. Thomas gave a brief history of honor societies in colleges and universities and, in particular, the founding of Kappa Mu Epsilon. Several University of Mary Hardin-Baylor administrators attended the 4:30 p.m. installation as well as many relatives and friends of the initiates. A large group enjoyed dinner together with Dr. Thomas at Frank's Lakeview Inn after the formal installation.

Officers installed during the ceremony were: Karen Scott, president; Don Henslee, vice-president; Jacqueline Pilkey, recording secretary; and Abeer Al-Naji, treasurer. Faculty members Peter Chen and Maxwell Hart accepted the responsibilities of the corresponding secretary and faculty sponsor, respectively.

CHAPTER NEWS

Alabama Beta University of North Alabama, Florence

Chapter President - Stacy Barringer
40 actives, 14 initiates

Other 1990-91 officers: Kristin Vandiver, vice president; Kellye Thompson, secretary/treasurer; Eddy Joe Brackin, corresponding secretary; Patricia Roden, faculty sponsor.

Alabama Zeta Birmingham-Southern College, Birmingham

Chapter President - Mark Kent
40 actives, 19 initiates

The fall initiation program was given by Mrs. Ouida Kinzey, Birmingham-Southern retired mathematics faculty member. Her slide presentation, "How Do You See Your World?" challenged members to look for mathematics in the world around them. Other 1990-91 officers: Pamela Brantley, vice president; Laura Francie, secretary/treasurer; Lola F. Kiser, corresponding secretary; Shirley Branan, faculty sponsor.

California Gamma California Polytechnic State University, San Luis Obispo

Chapter President - Andrew Skrylov
40 Actives, 12 initiates

California Gamma held almost-weekly meetings which featured speakers from business, industry and academia. Denise Meyers from Compaq Computers made a presentation to the club in October. Also in October, Kelly Abbott, former California Gamma Treasurer, now working with D. H. Wagner & Associates, made himself available for several hours for informal conversation with club members. In November, Professor Jim Delaney of Cal Poly gave a semi-formal colloquium to the club in which he showed how the solution of a problem posed by a former student touched on numerous areas of mathematics and eventually reduced to a topic in chaotic dynamical systems. Ten pledges were introduced at the Ice Cream Social on October 4. Formal induction occurred on November 2 in a ceremony held at Embassy Suites Hotel in San Luis Obispo. The induction ceremony was unique in that it was the climax of a belated Halloween masquerade party. On November 25, interested members of the club gathered to sing Christmas carols to patients of several hospitals in the San Luis Obispo city limits. On

November 5, Professor Terry attended the semi-annual Chevron dinner at Pesenti Wineries in Paso Robles. The dinner, attended by numerous club advisors, climaxed with the presentation of speeches and gifts to Sandra Leister (of Chevron, San Ramon, CA) who has led the Chevron interview team for many years. Chevron's participation in the Cal Poly Cooperative Program is deeply appreciated. Numerous past members of California Gamma have participated in a Chevron co-op. Other 1990-91 officers include: Andrew Schaffner, vice president; Scott Langfeldt, secretary; Derek Bernhardt, treasurer; Julie Smeltzer and Cindy Walter, Pledgemasters; Leo Flores, representative to the SOSAM Council; and Raymond D. Terry, corresponding secretary/faculty sponsor.

Colorado Gamma — **Fort Lewis College, Durango**

Chapter President - David Beazley
30 actives, 7 initiates

Colorado Gamma held two meetings during the fall semester, at one of which the movie, "Stand and Deliver," was shown. The chapter raised $119 during the College Alumni Phone-a-Thon. The Dean of the School of Arts and Science and the Vice President for Academic Affairs were present to congratulate seven new members who were initiated on November 14, 1990. Other 1990-91 officers: Jeff Johnson, vice president; Todd Sehnert, secretary; Duane Brown, treasurer; Richard A. Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

Illinois Zeta — **Rosary College, River Forest**

Chapter President - Glenn Jablonski
14 actives

Other 1990-91 officers: Joseph Pignataro and Nicholas Amendola, vice presidents; Patricia Rubio, secretary; Anna Lazenby, treasurer; Sister Mary T. O'Malley, corresponding secretary/faculty sponsor.

Iowa Alpha — **University of Northern Iowa, Cedar Falls**

Chapter President - Bill Pothoff
38 actives, 5 initiates

The annual *KME* Homecoming Coffee was held October 6, 1990, at the home of Professor Emeritus and Mrs. E. W. Hamilton in Cedar Falls with 32 members, alumni, and guests in attendance. Students presenting papers at local *KME* meetings included Lori Scott on "Applications of

Calculus to Thermodynamics" and Bill Kruse on "Mayan Mathematics." Shari Blum addressed the December initiation banquet on "Evolutes and Involutes of Curves." Rachel Britson was awarded a student membership in the Mathematics Association of America. Other 1990-91 officers: Mark Bohan and Mike Hirsch, vice presidents; Rachel Britson, secretary; Ben Schafer, treasurer; John Cross, corresponding secretary/faculty sponsor.

Iowa Delta Wartburg College, Waverly

Chapter President - Daniel Nettleton
37 actives

Ron Stahlberg, a Wartburg alumnus on the University of Iowa computer research staff, presented a talk entitled "Computer Graphics" at the September 10 meeting. The video "A Mathematical Mystery Tour" was the program for the October 8 meeting. On November 12, Ms. Kathy Brackemyer, a new faculty member, presented information about a Non-Euclidean Geometry unit she developed and used in her geometry course at Vinton High School. The Iowa Delta Chapter held its traditional Christmas dinner party on December 3, complete with a mathematical Christmas song and games. A new feature at each of the meetings this year has been a mathematical thought problem or puzzle. Other 1990-91 officers: Todd Letsche, vice president; Stephanie Hurley, secretary; Jerrod Staack, treasurer; August W. Waltman, corresponding secretary/faculty sponsor.

Kansas Alpha Pittsburg State University, Pittsburg

Chapter President - Jason Williams
40 actives, 4 initiates

The chapter held monthly meetings in October, November, and December. Fall initiation was held at the October meeting. Four new members were initiated at that time. The meeting was preceded with a pizza party. The October program focused on preliminary plans for the chapter to attend the national convention in the Spring. Dr. Elwyn Davis, Mathematics Department Chairman, gave the November program. His presentation was on "The Mathematics of Christopher Columbus." The final meeting of the semester in December featured a guest lecturer, Dr. Donald Teets from the South Dakota School of Mines and Technology Mathematics Department. He spoke on "A Generalization of Runge's Example" or "Has Polynomial Interpolation Gone to the Dogs?" Other 1990-91 officers: Mark Stewart, vice president; Brenda Beat, secretary; Lori Bruns, treasurer; Harold L. Thomas, corresponding secretary; Gary McGrath, faculty sponsor.

Kansas Gamma — Benedictine College, Atchison

Chapter President - Matthew McIntosh
8 actives, 16 initiates

The second annual "Make and Eat" pizza party was held on October 3 at the home of three senior students -- Cheryl Koelsch, Karen Dreiling, and Julie Stenger. A campus edition of the chapter newsletter, *The Exponent,* announced the event in its September edition. Several freshmen were attracted to the chili party held at the College Roost on November 11. Senior Julie Stenger spoke that evening about her experiences as an actuary intern in Oklahoma during the summer of 1990. Contest and door prizes were handled by Cheryl Koelsch and Nancy Sheble. The second campus edition of *The Exponent* came out in November. It included amusing problems as well as an invite to the Christmas Wassail which took place at the home of faculty member James Ewbank on December 9. The group was entertained by its local magician and chapter president, Matt McIntosh. Senior Karen Dreiling was initiated on November 14. Other 1990-91 officers: Julie Stenger, vice president; Nancy Sheble, associate vice president; Ty Anderson, secretary; Ken VanSpeybroeck, treasurer; Jo Ann Fellin, OSB, corresponding secretary/faculty sponsor.

Kansas Delta — Washburn University, Topeka

Chapter President - Mary J. Wilson
20 actives

Fall activities included viewing the movie "Stand and Deliver" at the October meeting. Other 1990-91 officers: Jody Whitaker, vice-president; Jonette Oestreich, secretary/treasurer; A. Allen Riveland, corresponding secretary; Ronald Wasserstein, faculty sponsor.

Kansas Epsilon — Fort Hays State University, Hays

Chapter President - Sharon Richards
18 actives, 12 initiates

Fall chapter activities including a picnic, Halloween party, and Christmas party. Other 1990-91 officers: Carl Keith, vice president; Jana Maryman, secretary/treasurer; Charles Votaw, corresponding secretary; Mary Kay Schippers, faculty sponsor.

Maryland Alpha College of Notre Dame of Maryland, Baltimore

Chapter President - Celine Burque

15 actives

The members of the chapter together with members of the Mathematics Society and departmental faculty worked to bring the annual Mathematics Olympiad for junior and senior high girls to a successful completion. The contest, held in late October, consists of four rounds of progressively difficult problems, prepared by faculty and students. The participating teams were eager and did extremely well. Other 1990-91 officers: Cheryl Gates, vice president; Ann Marie Webster,secretary; Marta Blotny, treasurer; Sister Marie Augustine Dowling, corresponding secretary; Joseph DiRienzi, faculty sponsor.

Maryland Beta Western Maryland College, Westminster

Chapter President - Tammy Mahan

13 actives, 2 initiates

The chapter held several planning meetings as well as the annual fall induction of two new members. Other 1990-91 officers: Andrea Pinkham, vice president; Laura Balikir, secretary; Deanna Dailey, treasurer; James Lightner, corresponding secretary; Linda Eshleman, faculty sponsor.

Maryland Delta Frostburg State University, Frostburg

Chapter President - Wayne Squillari

23 actives

Maryland Delta met once in the fall, enjoying pizza and the film "Mathematics of the Honeycomb." Other 1990-91 officers: Carla Saville, vice president; Brenda Moore, secretary; Andrew Kaylor, treasurer; Edward White, corresponding secretary; John Jones, faculty sponsor.

Massachusetts Alpha Assumption College, Worcester

Chapter President - Michael Drude

5 actives

Seven new members were initiated on May 8, 1990. Following a dinner in honor of the new members, Professor Vincent Cioffari of the Assumption faculty spoke on "Calendars and the Determination of Easter." Other 1990-91 officers: Margaret Rice, vice president; Valerie Tolosko, secretary/treasurer; Charles Brusard, corresponding secretary/faculty sponsor.

Michigan Beta Central Michigan University, Mt. Pleasant

Chapter President - Deidre McClelland

25 actives

Michigan Beta chapter conducted mathematics help sessions for freshman/sophomore mathematics classes. The October meeting featured a pizza party and help for CMU students who wanted assistance in scheduling mathematics courses for the winter semester. Laurie Raven gave a talk at the November meeting. Some members are also members of the Actuarial Club at CMU. This club was recently installed as the Nu Chapter of Gamma Iota Sigma, the collegiate insurance fraternity. Other 1990-91 officers: Tom De Clark, vice president; Laurie Raven, secretary; Mary Langeveld, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

Mississippi Alpha Mississippi University for Women, Columbus

Chapter President - Sean Hays

12 actives, 7 initiates

Mississippi Alpha provided daily free tutoring for math students in need. Members also organized a mathematics trivia game and began preparation for spring initiation and for sending representatives to the national convention in April. Other 1990-91 officers: Stacy Peacock, vice president; Beth Tilghman, secretary/treasurer; Margaret Memory, corresponding secretary/faculty sponsor.

Mississippi Gamma University of Southern Mississippi, Hattiesburg

Chapter President - Theresa Kelly

35 actives, 8 initiates

Other 1990-91 officers: Lisa Carroll, vice president; Jane Blackledge, secretary; Alice W. Essary, treasurer/corresponding secretary; Karen Thrash and Barry Piazza, faculty sponsors.

Missouri Alpha Southwest Missouri State University, Springfield

Chapter President - David McWilliams

26 actives, 11 initiates

The Missouri Alpha Chapter of Kappa Mu Epsilon began the 1990 fall semester with an annual picnic for all mathematics students, faculty and staff at Phelps Grove Park. Two monthly meetings were held during the semester, each highlighted by a faculty presentation. Dr. Susan

Palmer spoke on "The Existence of Miracles" and Dr. Les Reid delivered a presentation on "Polygonal and Polyhedral Dissections." In the fall initiation ceremony, membership was extended to 9 new members and 2 faculty members. This increased the total Missouri Alpha Chapter membership to 1,356 members. A successful semester was brought to a close with 28 members, faculty and guests attending an end-of-semester pizza party at Mazzio's restaurant. Other 1990-91 officers: Mark Gerke, vice president; Marc Meyer, secretary; Rhonda Crites, treasurer; Vera B. Stanojevic, corresponding secretary; M. M. Award, faculty sponsor.

Missouri Gamma William Jewell College, Liberty

Chapter President - Kevin Tanner
18 actives

Regularly scheduled meetings were held during the fall semester. The chapter is making plans for its annual spring initiation and banquet to be held in April. Other 1990-91 officers: James Mathis, vice president; Catherine Pagacz, secretary; Joseph T. Mathis, treasurer/corresponding secretary/faculty sponsor.

Missouri Epsilon Central Methodist College, Fayette

11 actives

Officers for 1990-91: John Slovensky, vice president; Jeff Wilcox, secretary/treasurer; William D. McIntosh, corresponding secretary/faculty sponsor; Linda O. Lambke, faculty sponsor.

Missouri Eta Northeast Missouri State University, Kirksville

Chapter President - Julie Ridlen
25 actives, 10 initiates

Fall semester activities included softball and volleyball games with the faculty and competition in the NMSU College Bowl. Plans were also made for the annual high school competition, Math Expo, which the chapter will host on February 16. Other 1990-91 officers: Ann Novitske, vice president; Lisa Aukee, secretary; Rhonda Gibler, treasurer; Mary Sue Beersman, corresponding secretary; Mark Faucette, faculty sponsor.

Missouri Iota Missouri Southern State College, Joplin

Chapter President - Wayne Cripps

12 actives

Missouri Iota once again worked the concession stands at football games as a money making project. All those who worked were rewarded with a pizza party at the end of the football season. Regular monthly meetings were held. Members heard Dr. John Knapp of the Physical Science Department speak on earthquakes at the December 4 meeting and students Melissa Sherrel and Liesl Bode presented Putnam Exam problems at the November meeting. The organization supported Melissa Sherrel as the Math Club homecoming queen candidate. Other activities included a fall float trip and a Christmas party. Dr. Cindy Carter Haddock, who served as charter president of Missouri Iota in 1975-76, was recognized as the MSSC 1990 Outstanding Alumna. Dr. Haddock is currently an associate professor at the University of Alabama with the Department of Health Services Administration. Other 1990-91 officers: Melissa Sherrel, vice president; Terri Findley, secretary/treasurer; Mary Elick, corresponding secretary; Linda Hand, faculty sponsor.

Missouri Kappa Drury College, Springfield

Chapter President - Sharon Rowe

5 actives

Semester activities began with a bonfire wiener roast at Dr. Allen's house. Prize money was awarded to the winners of the Annual Campus Math Contest, Shannon Koonce (Calculus I and below) and Matt Henderson (Calculus II and above), at a pizza party held for all contestants. In conjunction with a chapter luncheon, Robert Hayden and Mark Wampler gave reports on their undergraduate research projects. Math Club provided tutoring for the evening college as a money making project. A Christmas party marked the end of the semester. Other 1990-91 officers: Jim Rutan, vice president; Mark McDonald, secretary; Monty Towe, treasurer; Charles Allen, corresponding secretary; Ted Nickle, faculty sponsor.

Nebraska Alpha Wayne State College, Wayne

Chapter President - Brenda Spieker

18 actives

Throughout the semester club members monitored the Math-Science building in the evenings to earn money for the club. The club participated in the college homecoming activities by manning a booth at the Homecoming Carnival. With a grant from the Wayne State College Student Senate, *KME* and Computer Club purchased an overhead projector computer panel which will be used by computer science classes. Social activities included a fall picnic with the Math-Science faculty and other clubs in the building and a pizza-movie party at Dr. Paige's home. Other 1990-91 officers: Rory Rut, vice president; Julie Gottschalk, secretary/treasurer, Monte Gilliland, historian; Fred Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

Nebraska Beta Kearney State College, Kearney

Chapter President - Ann L. Gibson
17 actives

In addition to regular meetings, Nebraska Beta enjoyed two special events: A presentation about Electronics Data Systems regarding employment opportunities and a Christmas party at which the movie "Stand and Deliver" was viewed. Other 1990-91 officers: Jim Nissen, vice president; Dawn James, secretary; Teresa Volcheck, treasurer; Charles Pickens, corresponding secretary; Lutfi Lutfiya, faculty sponsor.

Nebraska Gamma Chadron State College, Chadron

Chapter President - Lanelle Henderson
18 actives, 3 initiates

Nebraska Gamma continued fund raising efforts. Members were encouraged to work on papers for the national *KME* conference. Fall initiates were Desiree Ingraham, Danette Jackson, and Courtney Schaffert. A Christmas party was held at the end of the semester. Other 1990-91 officers: Marla Soester, vice president; Laura Dooley, secretary; Maya Leicht, treasurer; James A. Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

Nebraska Delta Nebraska Wesleyan University, Lincoln

Chapter President - Mary Rose Philpot
21 actives

Other 1990-91 officers: Shelley Bolduan, vice president; Michele Spale, secretary; Halcyon Foster, treasurer; Muriel Skoug, corresponding secretary/faculty sponsor.

New York Alpha Hofstra University, Hempstead

Chapter President - Karin Grossu
7 actives

Other 1990-91 officers: Christopher Rosenblatt, vice president; Diana Beaudette, secretary; Deanna De'Liberto, treasurer; Aileen Michaels, corresponding secretary.

New York Eta Niagara University, Niagara University

Chapter President - James Wysocki
11 actives

Other 1990-91 officers: Joseph Scherer, vice president/secretary; James Wysocki, treasurer; Robert L. Bailey, corresponding secretary; Kenneth J. Bernard, faculty sponsor.

New York Lambda C. W. Post Campus/Long Island University, Brookville

Chapter President - Coleen O'Boyle

The chapter is sponsoring a problem solving group which is actively working on problems from "The Problem Corner." Additionally, plans are underway for the spring initiation and banquet. Other 1990-91 officers: Jacqueline Mansuetta, vice president; Keven O'Reilly, secretary; Cynthia Ferro, treasurer; Sharon Kunoff, corresponding secretary; Andrew Rockett, faculty sponsor.

North Carolina Gamma Elon College, Elon College

Chapter President - Kristen McMillan
25 actives

Installed in May 1990, North Carolina Gamma held its first official meeting in September. Discussion centered on fund raisers, service projects, workshops and speaker ideas. It was decided the organization would sponsor a team in the 1991 Mathematical Modeling Contest and would provide refreshments for a fractal speaker sponsored by Elon's chapters of ACM and MAA. Information concerning the *KME* 60th Anniversary Convention, the Math Modeling Contest, Problem Corner entries, and math winter term courses were published in a chapter newsletter. Plans were also made for initiation of new members during Math Awareness Week in April. Other 1990-91 officers: Mathew Wright, vice president; Julia Morris, secretary; Jennifer Lee, treasurer; Rosalind Reichard, corresponding secretary; Jeffrey Clark, faculty sponsor.

Ohio Alpha Bowling Green State University, Bowling Green

Chapter President - Jenny Laveglia

53 actives

Chapter activities began with the annual fall picnic and volleyball game. The organization heard Dr. Neal Carothers' colloquium presentation, "Pi a la Mode," in December. They also bought tee shirts, painted the university rock, and had a group picture taken for BGSU's yearbook, *The Key*. Other 1990-91 officers: Tracie Wedell, vice president; Malcolm Shrimplin, secretary; Travis Doom, treasurer; Waldemar Weber, corresponding secretary; Thomas Hern, faculty sponsor.

Ohio Zeta Muskingum College, New Concord

Chapter President - Kristi Pritchett

28 actives, 8 initiates

Fall semester activities got underway with a presentation by students Jennifer Suschil and Eric Poorman entitled "Mathematics of Betting on Horses." Presentations by former initiaties were featured at the October initiation of eight new members. Dr. Douglas Ward and Dr. Charles Holmes from Miami University, Oxford, Ohio, were special speakers in November. A Christmas party ar Dr. Smith's home closed out a successful semester. Other 1990-91 officers: Jon Ransom, vice president; Kim Forgrave, secretary; Tom Myers, treasurer; James L. Smith, corresponding secretary; Javad F. Habibi, faculty sponsor.

Oklahoma Alpha Northeastern State University, Tahlequah

Chapter President - Monique Harrison

34 actives, 12 initiates

This fall the Oklahoma Alpha chapter sponsored a presentation by Dr. David Lawrence of Rogers State College, Claremore, Oklahoma. Dr. Lawrence showed the video "Math! A Four Letter Word" to an audience of over 150 students and faculty. A lively discussion of the fear of math followed. Math professors again donated used textbooks to the *KME* booksale. The Fall '90 initiation ceremonies for twelve students were held in the banquet room of the Western Sizzlin' restaurant in Tahlequah. The December meeting was a Christmas pizza party. Entertainment included "Scattergories," with mathematical topics. Other 1990-91 officers: Lisa Singer, vice president; Rebecca Smith, secretary; Lori Austin, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

Oklahoma Gamma Southwestern Oklahoma State University, Weatherford

Chapter President - Jeanna Day

25 actives

Other 1990-91 officers: Melicia Kirkland, vice president; Karen Cochran, secretary; Kristen Casebeer, treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Oklahoma Delta Oral Roberts University, Tulsa

Chapter President - Dave Largent

15 actives

Oklahoma Delta, one of *KME*'s newest chapters, was busy this fall establishing the format of the organization and beginning the traditions to sustain the chapter and its growth in the years to come. An actuary was invited to speak to the group and plans were made to invite other professionals to present programs during the spring semester. Dr. Rakestraw organized a weekly problem solving session; top performers then competed on the Putnam Exam in December. Other 1990-91 officers: Bill Orth, vice president; Margaret Schultz, secretary; Melissa Fulbright, treasurer; Debra Oltman, corresponding secretary; Roy Rakestraw, faculty sponsor.

Pennsylvania Alpha Westminster College, New Wilmington

Chapter President - Kimberly A. Hoener

20 actives

The organization continued to tutor math and related topics at the student learning center and plans were made for a spring career night. Pennsylvania Alpha is sponsoring a new math club on campus this term, a student chapter of the Mathematical Association of America. Other 1990-91 officers: Christy Heid, vice president; Lori Metsger, secretary; Jeannette Huczko, treasurer; J. Miller Peck, corresponding secretary; Warren Hickman, faculty sponsor.

Pennsylvania Gamma Waynesburg College, Waynesburg

Chapter President - Angela Stewart

10 actives, 6 initiates

Other 1990-91 officers: Ronald Shaffer, vice president; Jennifer Thyreen, secretary; Nhan Huynh, treasurer; Monica McGervey, corresponding secretary/faculty sponsor; A. B. Billings, faculty sponsor.

Pennsylvania Iota Shippensburg University, Shippensburg

Chapter President - John Swingle

5 initiates

A fall picnic for math and computer science majors was held at Shippensburg Memorial Park. The December initiation was held at the home of Department Chairman, Dr. Howard T. Bell. Other 1990-91 officers: Thomas Goebler, vice president; Candy Staub, secretary; Fred Nordai, treasurer; Michael D. Seyfried, corresponding secretary; Rick Ruth, faculty sponsor.

Pennsylvania Kappa Holy Family College, Philadelphia

Chapter President - Monica Magilton

10 actives, 10 initiates

Free tutoring continues to be provided by members of Pennsylvania Kappa. A topic for discussion this semester has been the impact of mathematics upon civilization during the early centuries, 3000 B.C. up to 1300 A.D. Plans for the spring initiation were begun. Other 1990-91 officers: David McCabe, vice president; Paul Hiller, secretary/treasurer; Sister M. Grace Kuzawa, corresponding secretary/faculty sponsor.

Pennsylvania Mu Saint Francis College, Loretto

Chapter President - Kris Miller

14 actives

The chapter sponsored a Career Exploration Day in October for local high school students. A presentation by NASA astronaut Steven Oswald highlighted the event. Other 1990-91 officers: Antonine Gatto, vice president; John Miko, secretary; Brian Hebert, treasurer; Peter Skoner, corresponding secretary/faculty sponsor.

Texas Eta Chapter Hardin-Simmons University, Abilene

Chapter President - Charles Reed

10 actives

At a Get-Acquainted Party for prospective members the purpose and activities of Kappa Mu Epsilon were explained. Professional opportunities in the mathematical sciences were discussed as well as degree programs and awards available in the mathematics area at HSU. Faculty members also demonstrated mathematical applications using the Casio Graphics Overhead Projector Calculator OH-7000G. Members inducted in the spring of '90 received their shingles. Other 1990-91 officers: Tondi Jeter, vice president; Kristen Knebel, secretary/treasurer; Mary Wagner-Krankel, corresponding secretary; Charles Robinson and Ed Hewett, faculty sponsors.

Texas Iota McMurry University, Abilene

Chapter President - Rusty Teeter
18 actives

Fall semester activities included a Get Acquainted Mixer featuring a math careers discussion and a pizza party. Other 1990-91 officers: Charles Converse, vice president; Randy McCarble, secretary; Jacqueline Bryan, treasurer; Dianne Dulin, corresponding secretary; Bill Dulin, faculty sponsor.

Tennessee Delta Carson-Newman College, Jefferson City

Chapter President - Kim Caldwell Atchley
17 actives

Tennessee Delta sponsored a beginning-of-the-year picnic at Kesley Moore's lakeside home and a visit to Gatlinburg for a night of ice skating. Other 1990-91 officers: Lisa Bryant Smith, vice president; Shannon Lee, secretary; Kesley Tucker Moore, treasurer; Verner Hansen, corresponding secretary; Carey Herring, faculty sponsor.

Virginia Beta Radford University, Radford

Chapter President - Patches Johnson
15 actives, 12 initiates

Other 1990-91 officers: Cheryl Dixon, vice president; Melissa Reedy, secretary/treasurer; Steve Corwin, corresponding secretary; J. D. Hansard, faculty sponsor.

Wisconsin Gamma University of Wisconsin-Eau Claire, Eau Claire

Chapter President - James Kelley
45 actives, 26 initiates

Twenty-six new members were inducted at a formal initiation followed by a banquet and speaker. The club held monthly meetings highlighted by four student speakers. A bake sale and popcorn sale were held to raise money. Near the Thanksgiving break several members got together for a Thanksgiving dinner with all the trimmings. In addition the club began preparing for the forthcoming national convention. Other 1990-91 officers: Julia Folsom, vice president; Kim Anderson, secretary; Theodore Herzog, treasurer; Tom Wineinger, corresponding secretary.

Copies of articles from this publication are now available from the UMI Article Clearinghouse.

For more information about the Clearinghouse, please fill out and mail back the coupon below.

The UMI Article Clearinghouse offers articles from more than 11,000 copyright-cleared periodicals in a wide range of subjects. You can place your orders electronically, as well as by phone, mail, and telefacsimile. For more information, please complete and mail this coupon to UMI Article Clearinghouse, 300 North Zeeb Road, Box 11, Ann Arbor, MI 48106 USA. Or call toll-free for an immediate response: 800-521-0600. From Alaska and Michigan call collect 313-761-4700. From Canada, call toll-free 800-343-5299.

U·M·I

A Bell & Howell Company
300 North Zeeb Road
Ann Arbor, MI 48106 USA

YES! I'd like to know more about UMI Article Clearinghouse.

Name________________

Title________________

Company/Institution________________

Address________________

City/State/Zip________________

Telephone (______)________________

Kappa Mu Epsilon National Officers

Harold L. Thomas *President*
Department of Mathematics
Pittsburg State University, Pittsburg, Kansas 66762

Arnold D. Hammel *President-Elect*
Department of Mathematics
Central Michigan University, Mt. Pleasant, Michigan 48859

Robert L. Bailey *Secretary*
Department of Mathematics
Niagara University, Niagara University, New York 14109

Jo Ann Fellin *Treasurer*
Mathematics and Computer Science Department
Benedictine College, Atchison, Kansas 66002

Mary S. Elick *Historian*
Department of Mathematics
Missouri Southern State College, Joplin, Missouri 64801

Kappa Mu Epsilon, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation.

Chapter	Location	Installation Date
OK Alpha	Northeastern Oklahoma State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State College	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State College, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949

IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	Kearney State College, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri - Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin - River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Northeast Missouri State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel College, Springfield	12 Jan 1971

PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop College, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	West Georgia College, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin - Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Illinois Benedictine College, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C. W. Post Center of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry College, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991