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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics; due, mainly, to its demands for logical and rigorous modes of thought, to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

THREE DIMENSIONAL HYPERBOLIC GEOMETRY: AN EXPOSITION

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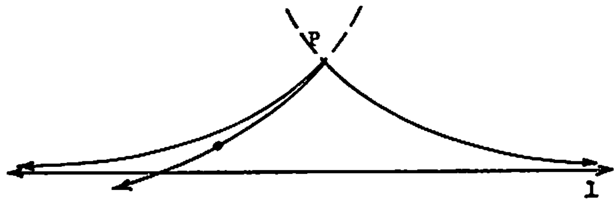
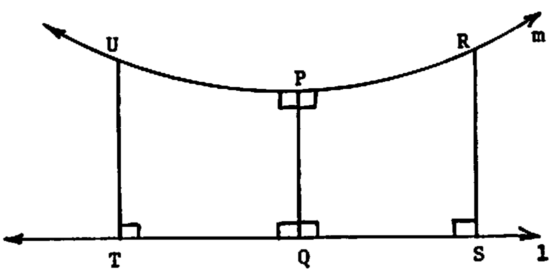
Hyperbolic geometry is a very interesting field of geometry. It is full of little twists which usually are not seen in Euclidean geometry. Gauss once tried to determine whether our universe was hyperbolic or Euclidean. By measuring the angles of a triangle made up of three mountain peaks, he attempted to answer this question. If all of the angles added to less than 180 degrees then this would be evidence that the universe might be hyperbolic. With the accuracy available to him he measured the angle sum as 180. Does this mean that the universe is Euclidean? Not quite. The present scales we are able to measure may not be large enough to detect the difference between the two universes. Either way, this expository paper deals with some basic plane geometry which is then developed into three dimensional geometry.

Before moving into three dimensions, it would be helpful to cover some definitions and properties of plane hyperbolic geometry. The main difference between Euclidean geometry and hyperbolic geometry is the parallel postulate. It is well known in Euclidean geometry that for a given line, l, and a point, P, not on line l, there exists a unique line through the point P which does not intersect line l. In hyperbolic geometry there are many lines through point P, which do not intersect line l. Two of the lines which are parallel to line l have a special relationship to line l. This relationship is called limiting parallel. These lines which are limiting parallel to line l have the property that they approach line l in one direction but never actually intersect line l. A limiting parallel line has the property that any ray emanating from point P which points in the direction of parallelism and contains a point which lies between the two parallel lines will intersect line l.



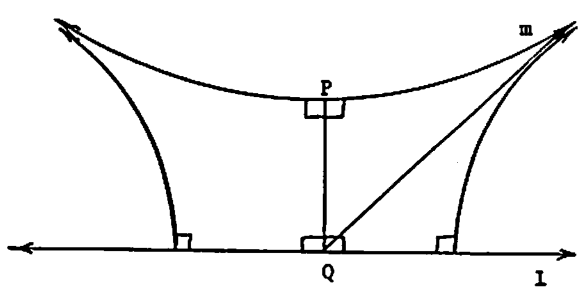
In essence the limiting parallel line represents the limit to which one may push one line toward another line before they intersect. Let S and T be any two points on line l, and then let Q be a point such that ray $\overrightarrow{PQ}$ is limiting parallel to ray $\overrightarrow{ST}$. The resulting figure is called an asymptotic triangle with base $\overline{PS}$. The asymptotic triangle is essential when attempting proofs in three dimensions.

There exist two quadrilaterals in hyperbolic geometry which have interesting properties and are useful in proving theorems in both plane and space geometry. The first is called a Saccheri quadrilateral. It can be constructed by taking any two lines l and m which are both non-intersecting and non-limiting parallel in either direction. From now on I will refer to these as parallel lines, and limiting parallel lines will be specified as such. The next step is to find the unique common perpendicular of these parallel lines l and m. Let the intersection of the perpendicular and line m be called P, and likewise let the intersection on line l be called Q. Now choose two distinct points U and R on line m such that segment $\overline{UP}$ is congruent to $\overline{PR}$. From points U and R drop perpendiculars to line l. The resulting figure RSTU is a Saccheri quadrilateral.

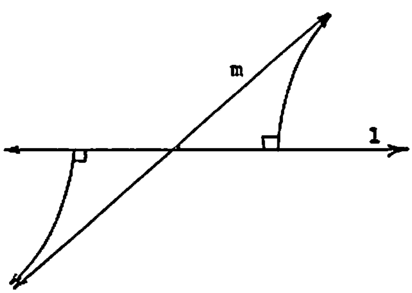


Although it will not be proved here, the angles associated with vertices U and R are both acute and are congruent. The quadrilateral RSQP is known as a Lambert quadrilateral. Note that it has three right angles and one acute angle; this is the greatest number of right angles which any quadrilateral may have in hyperbolic geometry.

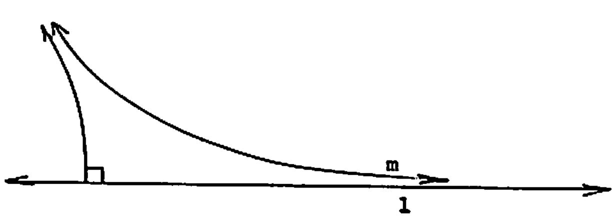
Projecting lines onto other lines proves to be of interest, and the understanding of the resulting projections is crucial when considering projections in three dimensions. In the following cases two lines will be considered; name them l and m. Points on line m will be projected onto line l, and all projecting lines will be perpendicular to line l. Given that l and m are parellel, the projection of line m onto line l is an open segment on m. The proof is relatively easy, and the strategy involves the use of asymptotic triangles. Refer to the figure below, and note the common perpendicular segment $\overline{PQ}$, where P lies on line m. From point Q draw a ray which is limiting parallel to m. Then with some previous experience with asymptotic triangles one can construct a right asymptotic triangle with the ray just drawn and line l.



The proof is then finished by showing that the points of m correspond to the points on the open segment on l, and conversely that the points on the open segment correspond to the points of line m. This is obviously not a proof but it does give some idea as to the attack that may be used. The case where l and m intersect gives the same result as before, an open segment on line l.

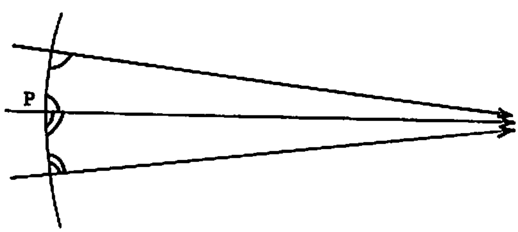


The case where the lines are limiting parallel to each other is a different story. Here line m projects to an open ray on line l which points in the direction of parallelism.



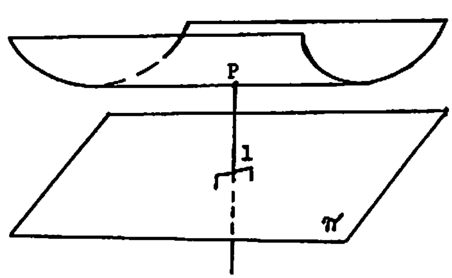
It is interesting indeed how the projections in hyperbolic geometry are so radically different from the projections we are used to observing in Euclidean geometry.

There is another figure which needs to be discussed before going on to three space. A curve called a horocycle will appear in three space projections. Given a set of lines all limiting parallel to each other in one direction, choose a point P on one of the lines. Then the set of points on the other lines which correspond to the point P form a horocycle. Another way to think of a horocycle is to take the chosen point P and construct isosceles asymptotic triangles with the set of limiting parallel lines. The set of all the resulting vertices lie on a horocycle.

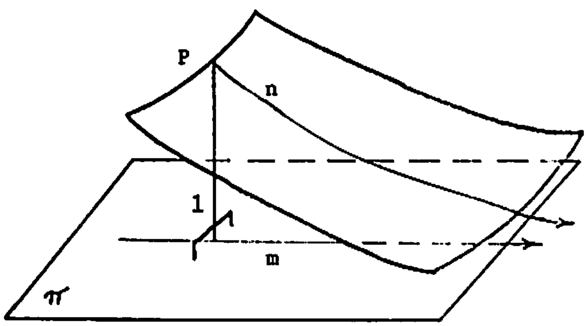


There is one more way, and perhaps the easiest way, to think of a horocycle. Start with a line l and a point P not on l. Now construct a circle about point P which is tangent to line l. To make the horocycle, move the center of the circle, point P, out an infinite distance from the line l along a line parallel to l. The resulting circle in the limit is a horocycle. Here it is interesting to note that in Euclidean geometry, this procedure would merely turn the circle into line l.

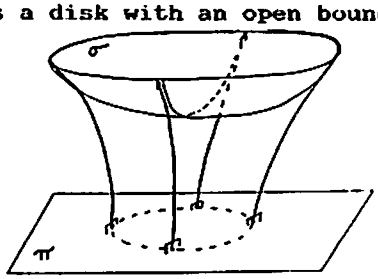
Now we are ready to move on to three dimensional geometry. Consider a plane π and a point P not on the plane. There exists a line l, containing the point P, which is perpendicular to the plane π. Now it is easy to find a plane which contains point P, does not intersect the plane π, and has line l as a perpendicular. This plane is said to be parallel to the plane π.



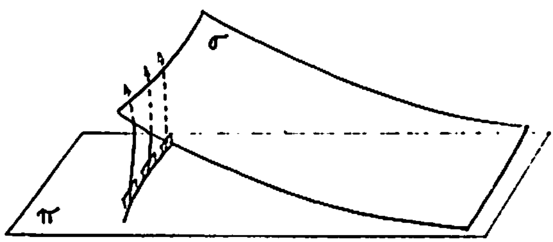
There also exist limiting parallel planes. These planes are easily visualized by constructing a line m on plane π through the perpendicular l. Now through point P construct a line n which is limiting parallel to line m. Given line n, it is easy to find the plane which contains n and does not intersect the plane π. The plane just found is limiting parallel to plane π.



Just as projections in plane geometry proved to be interesting, projections in three space are also interesting, probably more so. This time a plane σ will be projected onto a plane π. First consider the case when plane σ is parallel to plane π. Here σ is projected as a disk with an open boundary.

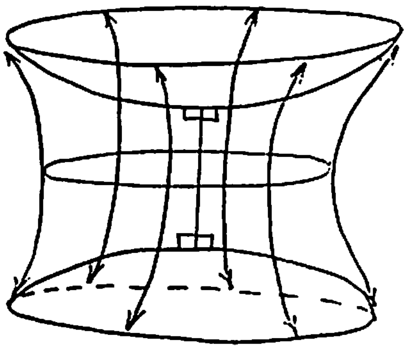


Now assume that plane σ is limiting parallel to plane π. The projection gives a half plane with the boundary as an open horocycle.



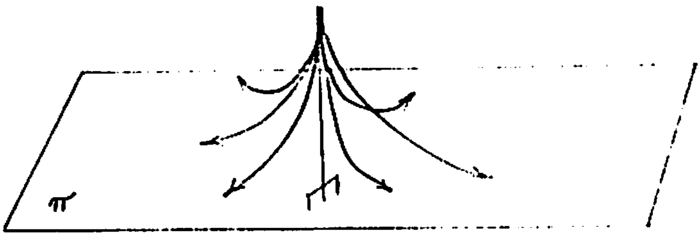
This may seem strange at first, but with a little thought it seems reasonable. I have yet to deal with two planes which intersect, but at this time my intuition tells me that in such a case, plane σ would project to a strip open on both sides and with the boundary being either a hypercycle (equidistant curve) or a line.

There are a few figures in hyperbolic three space which are worth mentioning. The first is a cylinder, which is easily described. Take any pair of parallel planes and find the common perpendicular. These two planes will be the ends of the cylinder, and the common perpendicular will be the axis of symmetry. The cylinder is then completed by finding all of the lines which are parallel to the axis and are limiting parallel to both planes.

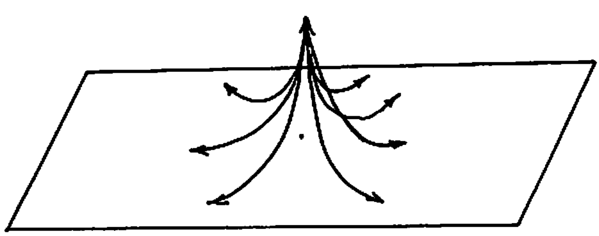


One interesting exercise is to project the side of the cylinder onto one of the ends of the cylinder. This creates a large donut of sorts. That is, the entire plane is covered except for a disk which is centered on the axis of symmetry.

There are two more figures which are not difficult to construct. The first figure is a cone. Visualize the plane with a point P not on the plane. The cone is now defined by all of the rays emanating from P which are limiting parallel to the plane π. This is the only figure so far which when projected onto the plane actually covers the entire plane.



The next figure, in some ways similar to the cone, is the limiting cone. In this case imagine the vertex, point P, lying an infinite distance from the plane. Stated more precisely, the cone consists of lines all limiting parallel to each other in one direction and limiting parallel to the plane in the other direction. The projection of a limiting cone is similar to the projection of the cone, but now one point is left open. The open point is of course the intersection of the axis of symmetry.



This is the end of the brief introduction to unexpected surprises. Just think of what is in store for space travelers if our universe is indeed hyperbolic. There is much to be explored in three dimensions, and I am looking forward to continuing my study. (Note: Proofs for most of these results are available.)

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Professor John Longnecker, my advisor and instructor in my studies in hyperbolic three space.

COMPUTATIONAL COMPLEXITY

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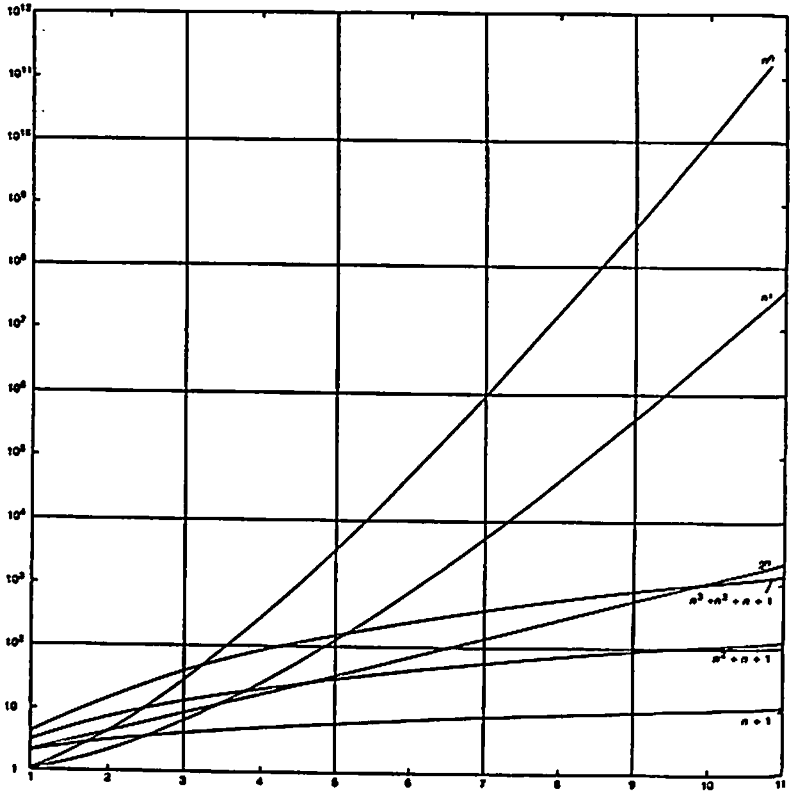
One has a general intuitive sense of the notion of complexity; procedures, ideas, theories, proofs, or functions can all be called complex. But what do we really mean by complexity? "In mathematics and in the sciences we tend to consider overly complex theories as being unclean and temporary, not really representative of nature...the closer we get to ultimate truth, the less contrived will our explanations appear." [3, p.140] Thus, complexity could be viewed as not fully understanding all the principles behind the problem and its nature. However, it is important to note that a problem can be very easy to understand and offer methods of solution, but to solve it "efficiently" is very complex. Thus, complexity can also be viewed as intrinsic difficulties of mathematically posed problems. Complexity here deals with the nature of the problem as we perceive it in this era with what theories and concepts we have available.

Thus, in speaking of the complexity of a problem, we will use a scale of measurement in terms of time or computer space. Surprisingly enough, "perfect" methods of solution to problems are not always enough. We need solutions that can be implemented in a general method called an algorithm; "it is a precisely stated procedure or set of instructions that can be applied in the same way to all instances of a problem." [10, p.96] Algorithms need to operate in a reasonable amount of time; that is, we need solutions that conserve time - a scarce resource.

Before continuing, we need to decide what will qualify as a "good" algorithm. The rate of growth in execution time of an algorithm can be described as a function of polynomial time complexity or exponential time complexity. To offer some explanation of these terms we say polynomials are of the form n^2, n^3, n^4 or linear combinations of such functions. What distinguishes them from exponential functions is that in polynomials a variable never appears as an exponent. Therefore, if the number of operations to solve the problem is an exponential function then the algorithm has exponential time complexity; if the number of operations is a polynomial function it is said to have polynomial time complexity. We will define the algorithm as "good" if it has polynomial time complexity. If, however, the rate is a function of exponential time, the algorithm is considered inefficient and of little practical value.

Also, we will define all polynomial time algorithms efficient and we will not determine which among the class is most efficient. The problems we will concentrate on are such that finding any solution in polynomial time would be an incredible breakthrough. By looking at the graph, (figure 1), we can see that it is imperative to find polynomial time algorithms if we want solutions that we can use in this life time. Exponential functions grow at such an exorbitant rate that even the most powerful computer imaginable could not calculate rapidly enough to arrive at a solution for problems with large n in a "reasonable" amount of time.

Graphs of Polynomial and Exponential Functions



It can also be seen in figure 1 that n! seems to increase very rapidly. Later we will use the fact that for $n \geq 4$, $n! \geq 2^n$. (The proof can be carried out by induction.)

Now that we have defined complexity and what will qualify as a "good" algorithm, we can roughly classify all algorithms in terms of three classes before looking at models to more clearly understand complexity. The first class consists of those problems that have solutions and polynomial time algorithms such that the problems can be solved mechanically. For this group of problems we can say there exist upper and lower bounds on their solutions; this is the property that allows us to maximize and minimize. We know where the bounds on the solutions exist which allows us to determine the "value", good or bad, of our algorithm in terms of efficiency. The second general class of problems are those that can be proven to have no polynomial or even inefficient exponential time solutions.

Thirdly, the class on which we will concentrate, are those problems that have solutions, but the algorithms are of exponential time complexity and to arrive at these solutions takes an inordinate amount of time for problems with large n. "Some kinds of computational problems require for their solution a computer as large as the universe running for at least as long as the age of the universe. They are nonetheless solvable in principle." [10, p.96]

Now, we will look in more detail at our first general class of problems - those which have solutions and efficient algorithms. An example in the first class should help us to better grasp the nature of computational complexity. As we know, there are numerous ways to solve a system of linear equations; we are going to look at two and compare their efficiency. One should note the luxury of many times having multiple algorithms for solution in this first class.

The first method we will examine is the procedure learned in algebra called elimination. Consider the following system of n equations with n unknowns:

<u>System of Linear Equations</u>

$$
\begin{array}{ccccccccc}
a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & c_1 \\
a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & = & c_2 \\
\vdots & & \vdots & & & & \vdots & & \vdots \\
a_{n1}x_2 & + & a_{n2}x_2 & + & \cdots & + & a_{nn}x_n & = & c_n
\end{array}
$$

We are all familiar with this method, and to obtain solutions efficiently one might multiply the first equation by $1/a_{11}$ and then add multiples of the first equation to the others to "zero out" the rest of column 1. This procedure would be repeated for $x_2, x_3, \ldots, x_{n-1}$, leaving x_n with a solution. Then the values can be back-substituted into the equations. A count of the number of multiplications gives us approximately $1/3*n^3$ multiplications.

The second way this problem could be solved would be to use Cramer's Rule, where x_1 can be given by the quotient of two determinants as in the following:

Cramer's Rule

$$x_1 = \frac{\begin{vmatrix} c_1 & a_{12} & \cdots & a_{1n} \\ c_2 & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ c_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}$$

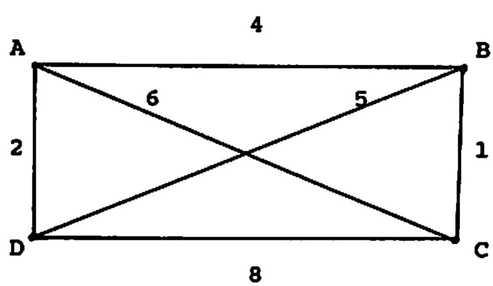
If one were to implement an algorithm to solve these determinants in a very straightforward manner - expansion from the definition, evaluation of a determinant such as the numerator or denominator would involve adding n! terms. The complete solution would involve $N^2*N!$ multiplications and divisions.

For comparison's sake let us suppose that we want to solve a system of 100 linear equations. The elimination method would take 1/3*1,000,000 multiplications while the determinant method would take 10^{160} multiplications. Thus, the first method would be manageable by the modern computer; however, the naive approach to the solution of Cramers' Rule would take more that 3 X 10^{20} years on the fastest computer. This is 10^{10} times greater than the estimated life of the universe! [2, p.381] Here we have examined a simple model of linear equations with which we are all familiar. The vastness of exponential time complexity and the need for efficiency seem to be of new importance. Note the determinants arrived at from the algorithm, Cramer's Rule, can be solved in polynomial time by use of Gaussian elimination, but if one is not prudent in the choice of algorithms, such as our naive choice to solve the determinants of Cramer's Rule by definition, we can induce complexity where it need not exist.

Now we will mention a problem in the second class, problems that have been proven impossible to solve efficiently. This type lends us helpful information even though we can't solve the problems. When we identify a problem in this class we know not to try to solve it; approximations are the best that one can hope for, so by this knowledge and classification we can save ourselves time and great frustration. The Halting Problem is a classical example of this type. The Halting Problem states that the input is a computer program; an algorithm is needed to determine if the input program will eventually halt. The problem lies in the fact that this is an unbounded search. If the program terminates when run, you have the solution, but when does it become logical to stop the search and say it isn't going to halt? "Turing constructed a proof that no algorithm will ever exist that can handle all instances of the Halting Problem." [8, p.103]

Next, we want to gain insight into the third general group. This group includes the interesting and rather large class of problems with solutions, but no polynomial time algorithms to solve these problems. Many of these decision problems can be related to graph theory and also optimization problems. To once again grasp an idea of computational complexity -

which this time we have no way to avoid - we will look at a classical optimization problem, The Traveling Salesman or Chinese Postman Problem, which has frusterated many a mathematicaian for years. The problem is simply posed: the mailman is assigned houses to which he must deliver mail, let us say 4; the problem is to find the shortest route, if one exists, that the mailman can take that will pass all the houses exactly once and enable him to deliver the mail and return home in the shortest amount of time.



Using graph theory we will assign each house to a corresponding node of a graph and the streets to edges connecting the nodes. Each edge will be labeled with a weight that corresponds to the distance between the two houses that it connects. Thus, we want to find the tour such that the sum of the weights of this tour, that being the total distance, is minimal.

I now tell you that I have a solution to this problem in the third class that can be generalized for any such optimization problem. Thus, I have a solution to this seemingly simple and straightforward problem; so what could be the complexity involved? As I leave this question to be answered by our development, I will offer the solution to the four house case.

To find the possible routes that the postman could take we can use an enumeration technique. We will list the permutations of all the vertices with A as the starting and ending point. This is the set of all the possible paths. So among them exists the shortest path. To find the shortest path, we add all the weights of the edges corresponding to the tour and the minimum sum will correspond to the shortest path.

Possible Tours:	Length of Tours:
A B C D	4+1+8+2 = 15
A B C D	4+5+8+6 = 23
A B C D	6+1+5+2 = 14
A B C D	6+8+5+4 = 23
A B C D	2+5+1+6 = 14
A B C D	2+8+1+4 = 15

Thus, ACBD and ADBC are the shortest routes for the postman to take in this particular problem. This was very simple and not complex in our ordinary way of thinking, but let us expand to a larger case. We will still have a relatively small number, for example, 30 houses to visit, and we implement this algorithm. To arrive at the possible tours would involve generating (30-1)! or approximately 8.8 X 10^{30} permutations. A simple algorithm to generate the possible tours and locate the minimum should be discussed. Each step contains four parts and could be as follows:

Generate permutations	Compute Sums	Test	Store

When the first permutation is generated, let us assume that the sum is computed and stored. Then the next permutation and sum are generated and the new sum is checked against the first; if it is smaller it will be stored as the new minimum. Let us also assume that the complexity of each of these steps is a constant C, but this procedure must be repeated (n-1)! times to generate all the possible tours of (n-1) nodes. As previously seen, C*(n-1)! is an exponential function.

Thus, as with the algorithm for solving Cramer's Rule, this algorithm would induce combinatorial results and thus, require more computational time than the universe is likely to contain. However, this type of problem with 30 nodes would not be uncommon and although this algorithm is an extreme case it demonstrates the type of computational complexity that we are discussing.

Before we continue in our discussion of complexity and its importance in our world, we need to break down our classes to better understand the nature of these problems. In 1971 Cook defined a new system of classes in his famous paper, "On the Complexity of Theorem-Producing Procedures." This system is now widely accepted; it includes the classes P, NP, and NP-complete. Informally, P consists of all problems that can be solved in polynomial time. The problems in the class NP are solvable in principle and have algorithms, but for now only ones of exponential time are known. However, proposed solutions can be checked in polynomial time. The problem in this area is that there is no proof that "good" algorithms exist or not. Problems such as the Chinese Postman fall into the class designated by the letters NP. NP-Complete problems form a subset of NP, with the special property that all NP Problems are reducible to each problem in the set.

Also, it is important, in terms of understanding this class, to realize that NP signifies "nondeterministic polynomial". Thus, nondeterministic refers to the fact that we are dealing with a theoretical nondeterministic turing machine, that being one which makes guessing computations instead of logical decision steps with which we are accustomed. In using this turing machine, it is assumed in theory, that if the correct answer exists the machine will luckily pick the minimal tour in polynomial time. This is a conceptual idea not proven to exist, but used in theory to explain the type of machine that would solve this type of problem. In other words this machine will guess and that venture will be correct. "A mathematical procedure defined in terms of lucky guesses may seem bizarre, but it is a quite legitimate approach to defining the class of problems in NP." [10, p.104] This is because a search such as in the Chinese Postman problem is so inefficient that a sequence of guesses, hopefully one of which is lucky, can't be any worse than an exhaustive search.

Also, it is important to know that optimization problems are reducible to decision or "yes, no" problems; this allows one to use Cook's classification system on them. For problems in the class NP an efficient means of answering the "yes, no" question need not exist. However, it is interesting that if the answer is yes then polynomial time algorithms exist to check if the yes solution is correct.

Now let us consider why the Chinese Postman problem is in NP. For the present, only exponential time algorithms are known, making it extremely difficult to find the shortest tour; however, we can reduce this to a "yes, no" problem by setting the stipulation that the number of miles that the postman can travel must be less than or equal to T. Then if a tour is proposed, an algorithm can easily check if the tour is less than or equal to T. Therefore, the Chinese Postman problem is in the class NP.

Furthermore, it is quite easy to show that all problems in P are also in NP. By definition, if a problem is in the class P an algorithm exists to solve it in polynomial time. Thus, if a yes answer is given it can certainly be discerned if the solution is correct.

The importance of this classification is that one of the big existing questions in mathematics is: does $NP \overset{?}{=} P$. If P were to equal NP the consequences would be astounding: "it would mean that every problem for which solutions are easy to check would also be easy to solve." [8, p.104] Also, the huge menagerie of optimization problems would have polynomial time solutions!! However, the proof that NP=P or NP≠P has yet to be produced. The guessing involved in nondeterministic turing machines leads many experts to think that P=NP.

The area of research is open and challenging and may join that select group of mathematical enigmas that remains unsolved for decades. There are many applicable problems in the area of optimization, and much research is being done, but the frustration continues as researchers work in this area of unknown bounds. They can not prove solvability in polynomial or exponential time exculusively. Thus, they strive for approximate solutions close to an assumed optimal solution - that they are uncertain of on even the most simplistic models. "When all is said and done, the design of practical combinatorial optimization algorithms remain as much an art as it is a science." [10, p.109] One might agree that this is due to the intrinsically complex nature of these problems.

In our brief glance of complexity we sought to define its properties and gain an understanding of its intrinsic nature through models such as the Chinese Postman problem. "The aim of the study of computational complexity is to develop techniques for discovering better algorithms and to explain why some computational tasks are difficult, no matter what algorithm is applied to them." [4] We also divided decision problems into classes to better understand the different nature of problems and their solvability. Complexity faces us as mathematicians, but by having an understanding of its existence we can - if not contribute to its resolution - not be baffled or confused by it. In our recognition we will realize what confronts us and thus look for approximations. Although efficient approximations exist, from the standpoint of mathematics, the important question is whether $NP \neq P$. "There is new suspicion that they are not identical, but the proof of their distinctiveness may be beyond present mathematical capabilities...the solution may have to await the development of new methods of mathematics." [10, p.109]

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NP-COMPLETENESS AND THE TRAVELING SALESMAN PROBLEM

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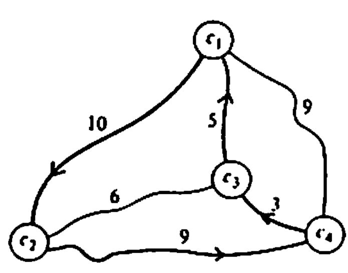
Professor Stephen Cook once made the observation that "a good part of computer science research consists of designing and analyzing enormous numbers of efficient algorithms" (402). It seems somehow ironic that Professor Cook also happens to be one of the foremost researchers into problems for which no efficient algorithm is known. This class of problems was formally dubbed "NP-Complete, by Professor Cook in 1971 (404). Within one year, said Cook, Professor Richard Karp "proved 21 problems were NP-Complete, thus forcefully demonstrating the importance of the subject" (404). Since so many of the problems that fall into this class occur frequently in business and industry, Cook has observed, much research in the past decade has been devoted to finding ways to deal with these apparently unsolvable problems (405). According to J.F. Traub, merely designing a "faster" computer will not change anything, because the difficulty in solving a problem, or its complexity, lies within the problem itself, independent of the algorithm or model used (Pagels 13). Traub goes on to raise the following questions: Does any algorithm exist to solve these problems, or are they so difficult as to require unreasonable amounts of computer resources (Pagels 13)? Rather than deal with the entire NP-Complete class, let us restrict ourselves to one problem, known commonly as the Traveling Salesman Problem, and explore past, present, and future efforts to find an efficient solution.

Mathematicians have been struggling with the idea of NP-Completeness since early in this century. The first real breakthrough was made in 1937 when Alan Turing developed his "Turing machine," which Cook explains to be a formal model of functions that are computable in a reasonable amount of time by a computer algorithm (401). This model, observed Cook, has become the tool for Turing and many others to prove that some problems have no efficient algorithmic solution (401). Cook continues by describing the work of J. Hartmanis and R.E. Stearns who, in 1965, defined computational complexity as a function of the time required to process a given problem on a Turing machine (401). About the same time, said Cook, Alan Cobham first described a class of problems whose complexity was a polynomial function of the length of the input data (402). Cook points out that although this observation had been made more than 10 years before by Von Neumann, it was Cobham who showed that "the class was well-defined and independent of which computer model was chosen" (402). Edmonds was the first to equate Cobham's idea of "polynomial time computability" with solvability, Cook continues, and Karp later formally termed this class of problems P (402). It seems natural to say that those problems not in P, those not having a polynomially-time-bounded solution, are not solvable, or are much more difficult to solve. M.O. Rabin addressed the very heart of computational complexity by asking, "What does it mean to say [a function] f is more difficult to compute than [a function] g?" (Cook 401) This led to Cook's classification of certain problems as NP-Complete.

Arto Salomaa points out that even though two problems are both theoretically solvable, in practice one might be so difficult to compute that it acts like an intractable, or unsolvable one (139). By studying the complexity of the problem, we may classify it in one of two classes. P, short for polynomially-time-bounded, is the collection of problems for which an algorithm exists to solve each in a polynomially-time-bounded amount of time (polynomial time, for simplicity) on a Turing machine. According to Salomaa, these problems are generally considered tractable, or solvable (165). Those problems not in P belong to the class NP, short for nondeterministically-polynomially-time-bounded; these problems are computable in polynomial time only on a nondeterministic Turing machine, that is, one that can make an arbitrary choice between two courses of action and follow up on both simultaneously. According to Karp, these guesses, which are input strings of 1's and 0's, are commonly called "languages" (Miller 91). Algorithms for NP problems generally consist of a guessing stage, according to Michael Garey and David Johnson, and a subsequent checking stage (28). Salomaa illustrates the difference between these two classes in that problems in P are all tractable, but only the checking stage of NP problems is tractable (165). The class NP needs to be broken down further, though; if a language L is in NP, say Harry Lewis and

Christos Papadimitriou, it may also be NP-Complete if there exists, for every language L' in NP, a polynomial transformation from L' to L (341). A polynomial transformation is simply a function mapping every instance of L' to one of L, which is computable in polynomial time. The theory of NP-Completeness, asserts Garey and Johnson, is defined to include only decision problems that could include many diverse disciplines, which have only the answers "yes" or "no" as possible solutions (18). The guessing stage, they continue, consists of specifying an arbitrary problem instance and the checking stage of a yes-or-no question in terms of the possible solutions (Garey 18). To illustrate the implications of NP-Completeness, let us look at one classic problem.

Suppose a traveling salesman needs to visit 10 cities on his next tour, passing through each once and only once (see plate 1).



In order to save money, the company would like to minimize the mileage he is to cover. It is fairly simple to design an algorithm that would systematically construct and calculate the distances of all possible routes. However, for any n cities, the number of routes to check is (n - 1)!, or in this example, 9! = 362,880, assuming a direct route between each pair of cities exists. This could be accomplished, but what if the number of cities jumped to 30? The number of possibilities is now 29!, which is even greater than 10^{29}. According to Lewis and Papadimitrious, "even if we could examine a billion tours per second -- a pace far beyond the capabilities of existing or projected computers -- the required time for completing this calculation would be more than a billion human lifetimes" (312). Clearly, the Traveling Salesman Problem (TSP, for simplicity) is intractable, in that the number of possible guesses at each stage in the tour grows exponentially (2^n, for example), as there are at least two choices for the next city to visit. Garey and Johnson observe that a nondeterministic algorithm can be constructed for TSP using a guessing stage to supply an arbitrary sequence of the given cities and a checking stage to verify in polynomial time whether the distance calculated from that sequence is above or below the fixed limit (29). Hence, without rigorous proof, TSP is one member of NP.

The fact that TSP is in NP would imply that there is a nondeterministic algorithm that solves it in polynomial time; however, as Lewis and Papadimitriou point out, "the fact that a problem is solvable in theory does not immediately imply that it can be realistically solved" (312). A problem may appear to be solvable when, in fact, comments Salomaa, it either operates in exponential time or uses enormous quantitites of computer space (176). In order to determine if TSP can be solved realistically, we need to examine its computational complexity. We have already observed that TSP has an exponential complexity. Salomaa suggests asking oneself questions such as: Is there one algorithm that is better than another in the sense that one uses less computer time, or does TSP have a "best," or most efficient, algorithm (140)? Karp found that most researchers consider a problem like TSP to be well-solved when a polynomial time algorithm is found to solve it (Miller 85).

This would seem to indicate, however, that no problem in NP, including TSP, will ever be well-solved. NP problems can be solved in polynomial time only when using a nondeterministic model which, Garey and Johnson are quick to point out, is unfortunately more of a "definitional device...then a realistic method for solving decision problems" (29). It appears that for any problem in NP to be well-solved, it must also be a member of P. But this contradicts our working definition of NP; it has yet to be proven, however, that P and NP are in fact separate and distinct classes. According to Lewis and Papadimitriou, "the importance of the class P resides in a ... somewhat controversial opinion that P coincides with the class of problems that can be realistically solved by computers" (329). Salomaa makes the observation that P is contained in NP; however, he continues, there are a number of frequently-occuring problems in NP which cannot be proven or disproven to be in P (166). This would lead one to ask what Salomaa calls "the most celebrated open problem in the theory of computation": Is P properly contained in NP, or rather, does P = NP (166)?

Clearly, says Salomaa, a proof of P = NP would make all NP problems tractable (166). No one has yet been able to prove or disprove this question; this does lead us, however, back to our definition of the subset NP-Complete within NP. If we cannot show a problem is easy to solve, say Alfred Aho, John Hopcroft, and Jeffrey Ullman, we can show that it is as "hard" as any other in NP, that is, the problems are of equivalent complexity and thus NP-Complete (373). According to Salomaa, NP-Complete languages "represent the hardest problems in NP" (167). He goes on to say that all we need is one NP-Complete language that is in P in order to be able to solve the whole class, since they are all equivalent in complexity (Salomaa 167). Therefore, conclude Aho, Hopcroft, and Ullman, "either all NP-Complete languages are in P or none are, "echoing our earlier dilemma of, if P = NP (374). Since much effort to find better algorithms, or improve existing ones, has proven fruitless, most researchers, reports Salomaa, accept the view that P does not equal NP (168).

Working under this assumption, one could make the claim that TSP is thus intractable; no exact solution exists, other than calculating all possible paths and choosing the shortest one. John Litke observes that for any more than 10 cities, the number of possibilities grows too quickly to reasonably check them all (1227). What is important to note is the rate of growth of the time function, since NP and NP-Complete problems have only exponential time functions. Lewis and Papadimitriou note that "any exponential time function grows strictly faster than any polynomial function" (323). Consequently, they add, we need to set an upper limit on the time function, thus reflecting not the general behavior of the algorithm but the worst possible case allowed (Lewis 315). It now becomes evident that, for a small number of cities, an exponential time algorithm for TSP can actually be relatively efficient (see plate 2).

Time Complexity Function	Size n 10	20	30	40	50	60
n	.00001 second	.00002 second	.00003 second	.00004 second	.00005 second	.00006 second
n^2	.0001 second	.0004 second	.0009 second	.0016 second	.0025 second	.0036 second
n^3	.001 second	.008 second	.027 second	.064 second	.125 second	.216 second
n^5	.1 second	3.2 seconds	24.3 seconds	1.7 minutes	5.2 minutes	13.0 minutes
2^n	.001 second	1.0 second	17.9 minutes	12.7 days	35.7 years	366 centuries
3^n	.059 second	58 minutes	6.5 years	3855 centuries	2×10^8 centuries	1.3×10^{13} centuries

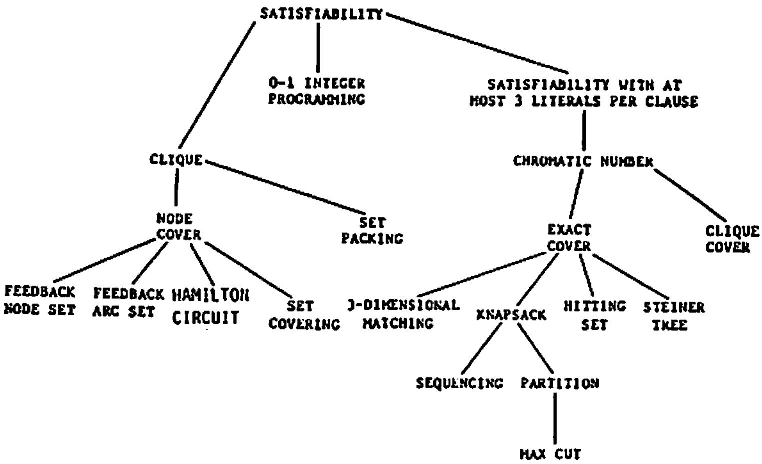
"Nevertheless," Aho, Hopcroft, and Ullman point out, "the growth rate of an exponential function is so explosive" that the problem is considered intractable if there is no other exact solution (364).

If we could prove TSP is NP-Complete, and therefore in all likelihood is intractable, we could shift our attention from solving the problem in polynomial time to verifying an arbitray guess is a solution. "It is this notion of polynomial time 'verifiability' that the class NP is intended to isolate," according to Garey and Johnson (28). They go on to point out, however, that:

> polynomial time verifiability does not imply polynomial time solvability. In saying that one can verify a "yes" answer for a TSP instance in polynomial time, we are not counting the time one might have to spend in searching among the exponentially many possible tours for one of the desired form. (Garey 28)

Knowing a problem is NP-Complete, therefore, does not provide any added clues for finding an exact solution, but rather motivation to search out an alternative solution.

Usually it is not immediately apparent if a problem is NP-Complete; assuming P does not equal NP, Garey and Johnson point out, the problem may be in NP and not tractable or NP-Complete (78). Some have speculated, but it is not yet known, whether NP = NP-Complete, so we must provide for the case they are not equal. In order to prove TSP NP-Complete, Garey and Johnson outline four steps that need to be satisfied: show TSP is a member of NP, select a known NP-Complete problem, construct a mapping from the known problem to TSP, and prove that mapping can be done in polynomial time (45). This method follows directly from Cook's Theorem, named for its author Stephen Cook, and the definition of NP-Completeness. Cook was the first to prove a problem NP-Complete, one which he named Satisfiability (SAT, for simplicity); his theorem states that any language in NP is polynomially transformable to SAT (Miller 92). (see plate 3).



Therefore, if TSP is polynomially transformable to SAT, or some other known NP-Complete problem which is transformable to SAT, then TSP must also be NP-Complete. This can be tricky, say Garey and

Johnson, since there may be no known NP-Complete problem that obviously resembles the problem at hand (46). "Even though in theory <u>any</u> known NP-Complete problem can serve just as well as any other for proving a new problem NP-Complete, in practice certain problems do seem much better suited for this task," they continue (Garey 46). To prove TSP is NP-Complete, we shall use the NP-Complete problem Hamilton Circuit (HC, for simplicity).

Following Garey and Johnson's four-step method, we first make the observation that TSP is a member of NP, as shown when we first defined the problem earlier. Second, they suggest the use of HC as the source of the transformation (Garey 35). The HC problem consists of a graph with a set of vertices V and a set of edges E connecting the vertices. A HC is a simple circuit which includes every vertex in V; a simple circuit is merely some arbitrary sequence $\{v_0, v_1, \ldots, v_n\}$ of vertices where $v_0 = v_n$, every v_i is unique, and each edge v_{ij} is a distinct element of E. Next, we will need to specify a function f that maps each instance of HC to a corresponding instance of TSP. Finally, we will then need to show that this function satisfies the two properties required of a polynomial transformation, as defined by Garey and Johnson: f is computable by a polynomial time algorithm, and every instance of HC is a solution if and only if the function on that instance is a soluton of TSP (35).

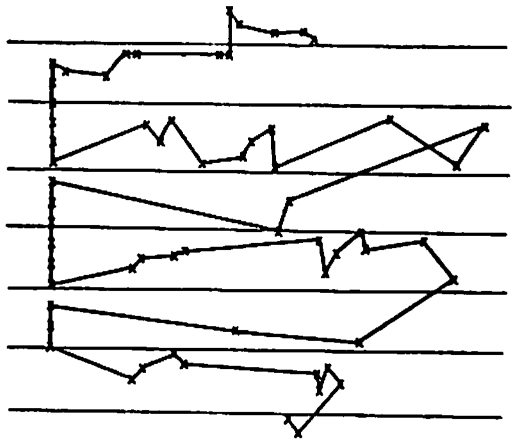
Garey and Johnson define the desired function quite simply: suppose a graph satisfying HC consists of a set V of vertices and a set E of edges, with m defined as the total length traversing the circuit (35). They continue, "the corresponding instance of TSP has a set of cities C that is identical to V" (Garey 35). Given any pair of cities in C, Garey and Johnson define the distance between them to be 1 if the pair makes up an edge in E, and 2 otherwise (35). They set the limit on the total tour distance to be m (Garey 35).

Without rigorous proof, f is clearly a polynomial transformation. The first property is satisfied, Garey and Johnson observe, since f can surely be calculated within polynomial time; one merely needs to compare each of the $1 + 2 + \ldots + m = m(m + 1)/2$ distances between arbitrary pairs of cities with the graph to determine inclusion in the set of edges E(35). "To verify that the second requirement is met," continue Garey and Johnson, "we must show that G [the graph] contains a HC if and only if there is a tour of all the cities in f(G) that has total length no more than the upper bound" (35). First, they propose $\{v_0, v_1, \ldots, v_m\}$ is a HC for G, with $v_0 = v_m$; because it picks up every point, it must also be a tour in f(G) having total length of m, the upper limit, as each pair is an edge in E and therefore has a distance of 1 (Garey 35). Conversely, they propose $\{v_0, v_1, \ldots, v_m\}$ to be a tour in f(G), also having total length no more than the limit m; because exactly m distances are summed to get the total of m, and those distances have been defined to be either 1 or 2, this implies that each pair must have a distance of exactly 1 (Garey 36). Thus, conclude Garey and Johnson, by definition each pair has an edge in E, and $\{v_0, v_1, \ldots, v_m\}$ must be a HC (36). Therefore, all conditions having been met, TSP is NP-Complete.

Knowing TSP belongs to this class, we can direct our efforts away from searching for an exact efficient algorithm to solve it, which in all likelihood does not exist. According to Garey and Johnson, the next best approach in dealing with these problems is to find an "approximation algorithm" that has been proven to always produce a solution, although not always the best one, within a given range of the optimal solution (Traub 41). If the algorithm consistently produces its own best possible solution, they call it an "optimization algorithm" (Garey 123). For TSP, in practice a minimization problem, Garey and Johnson observe that an approximation algorithm is required only to "find some permutation of the given set of cities," but an optimization algorithm must always find one that has a tour length within the allowable limit (123). Depending upon the circumstances, either algorithm could produce the desired results.

B. Golden and others compared a number of these algorithms that are in wide use and found three basic forms. A tour construction procedure creates a tour that is close to optimal; tour improvement procedures start with a given tour and try to find a better one (Golden 695). They also found many composite procedures, which combine the previous two procedures, building an initial tour and then trying to improve upon it (Golden 695). All three forms seem to be comparable to each other in efficiency, and useful without a loss of computation time. A standard tour construction procedure can produce a tour, according to Golden, to within 5 - 7% of the optimal solution (708). The complexity of most tour improvement procedures, they found, was comparable to the best of the tour construction procedures (Golden 708). The composite procedure has the potential to be the best of the three alternatives, however. Golden observed consistent results within 2 - 3% of optimality, and repeating this procedure can yield a solution within 1 - 2% of the best possible solution (708). They point out, however, that although "a very accurate solution for [large instances of TSP] can be obtained in a reasonable amount of computing time by repeated use of a three-step composite procedure..., none of the procedures discussed can be guaranteed to find the minimal length TSP tour" (Golden 709).

Another alternative solution that has met with considerable success is a variation on the band sort. Litke describes the band sort as a method that "divides the field of points into equally spaced bands, and the algorithm proceeds from left to right in the first band, right to left in the second band, etc., until all points are visited by the path" (1227). (see plate 4)



An example of an elementary band sort path. Notice the proximity of the points at the band divisions that could have been grouped together, thereby reducing the overall path length. From Litke (1229).

According to Litke, the Photocircuits Division of Kollmorgan Corporation had little success with this method on TSP; due mainly to differences in the band width, total path length was sometimes reduced by 10% but occasionally increased as much as 300%, so a consistent reduction of even 0.5% was not possible (1227). A new variation was tried, attempting to simulate the way the human eye would estimate the shortest path. The process is broken down into two steps, adds Litke, joining small "clumps" together and then connecting those clumps together (1229). Litke reports that this new resulting algorithm has been in use at Photocircuits since October of 1982; after being applied to over 1,000 jobs, an average reduction of 44% has been observed (1229). The computing cost seems to be easily adjustable for this procedure and, adds Litke, "it outperforms previous algorithms we tried by more than 10:1 on a statistical basis" (1236).

Even though there is no exact solution for TSP, some very useful and efficient approximate solutions can be found. It remains to be seen if an exact, polynomial time solution can ever be found for TSP and other NP-Complete problems; due to the importance and frequency of these problems, the search will certainly continue. J.F. Traub once observed, "If information is limited and inexact, you cannot solve a problem exactly" (Pagels 15). In the case of TSP, we have the information, but it is not feasible to use it. Until a reasonable solution is found, as Traub put it, "we choose to live with uncertainty to decrease complexity" if we are to find any kind of solution at all (Pagels 15).

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THE PROBLEM CORNER
EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 January 1990. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring 1990 issue of THE PENTAGEON, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROBLEMS 420-424, SOLUTIONS 408, 410 -413

Problem 420: Proposed by the editor.

Berwick once proposed a classic arithmetic restoration problem in which only seven 7's appear. For our version find the smallest cube of a positive integer which ends in 7777777 ;i.e. seven 7's.

Problem 421: Proposed by the editor.

While working on her homework, a student noticed the following peculiar relationship between two sets of consecutive squares:

$$\frac{600^2 + 601^2 + 602^2 + 603^2 + 604^2}{144^2 + 145^2 + 146^2 + 147^2 + 148^2} = 17.$$

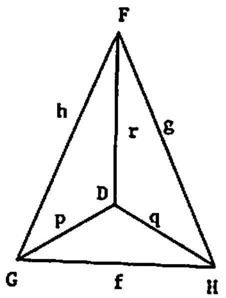
She wants to know if this relationship is unique or does 17 have other such representations?

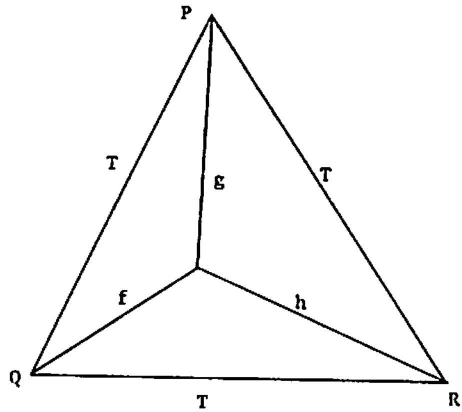
Problem 422: Proposed by the editor.

Consider the two triangles FGH and PQR shown below with

$\angle FDH = \angle FDG = \angle GDH = 120^\circ$.

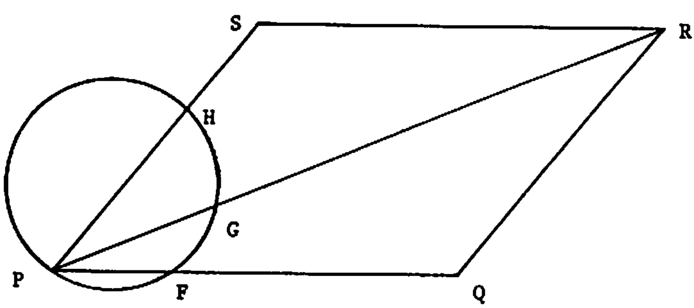
Let the line segments be denoted as marked. Prove that $T = p + q + r$. (Third USA Mathematical Olympiad 1974)





Problem 423: Proposed by the editor.

Let a circle cut two adjacent sides and a diagonal of parallelogram PQRS at points F, G, and H as shown in the figure below. Prove that $\overline{PF}\cdot\overline{PQ} + \overline{PH}\cdot\overline{PS} = \overline{PG}\cdot\overline{PR}$.



Problem 424: Proposed by the editor.

Completely factor the number $2^{34} + 1$ using only a pencil and paper. (No computers please!) This number has four distinct prime factors.

PLEASE HELP YOUR EDITOR BY SUBMITTING PROBLEM PROPOSALS.

Problem 408: Proposed by the editor.

Dirty Dan had a hot tip on the dog races. He knew that one of four longshots would win the race. If the odds on these four dogs are 3 to 1, 5 to 1, 6 to 1 and 9 to 1 respectively, how much should Dirty Dan bet on each of these four dogs to guarantee making a profit of \$143?

Solution by the editor. Let x , y, z and w denote the amounts bet at the respective odds 3 to 1, 5 to 1, 6 to 1 and 9 to 1 . Then we have the following four equations in four unknowns:

$$4x = 6y = 7z = 10w = x + y + z + w + 143$$

where $4x$, $6y$, $7z$ and $10z$ denote the respective amounts won for each bet and $x + y + z + w + 143$ denotes the total amount bet plus the expected profit. The solution of this system is $x = \$105$, $y = \$70$, $z = \$60$ and $w = \$42$. Thus for a total "investment" of \$277, Dirty Dan wins \$420 making a "clean" profit of $143.

This works only because

$$\frac{1}{4} + \frac{1}{6} + \frac{1}{7} + \frac{1}{10} = \frac{277}{420} < 1.$$

Problem 410: Proposed by the editor.

Let A, G, and H be the arithmetic mean, the geometric mean and the harmonic mean respectively of the divisors of an even perfect number. Prove or disprove that $G^2 = A \cdot H$.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

We shall show that $G^2 = A \cdot H$.

First we need the following result.

Lemma: If n is an even perfect number, then the sum of the reciprocals of the positive integer divisors of n is 2.

Proof of Lemma: Since n is an even perfect number, $\sum d = 2n$. All sums are taken over the divisors d of n.

Then $$\sum \frac{1}{d} = \sum \frac{1}{n/d} = \frac{1}{n} \sum d = \frac{(2n)}{n} = 2.$$

Proof of main result:

Let P be an even perfect number. Then $P = 2^{m-1}(2^m-1)$ where 2^m-1 is a prime number. Thus P has the following 2m positive divisors: $1, 2, \ldots, 2^{m-1}$, 2^m-1, $2(2^m-1), \ldots$, and $2^{m-1}(2^m-1)$.

Then it follows that $A = \dfrac{2P}{2m} = \dfrac{2^{m-1}(2^m-1)}{m}$.

By our lemma, $H = \dfrac{2m}{2} = m$.

Finally $G^2 = [(1\cdot 2\cdot \cdots \cdot 2^{m-1})^2(2^m-1)^m]^{1/m} =$
$(2^{1+2+\ \ldots\ +(m-1)})^{2/m}(2^m-1) = (2^{(m-1)m/2})^{2/m}(2^m-1) = 2^{m-1}(2^m-1)$.

Thus $G^2 = A\cdot H$.

Problem 411: Proposed by Dmitry P. Mavlo, Moscow, USSR.

Let a, b, and c be positive real numbers and let k be a positive integer. Prove the following inequality and determine all cases when equality occurs:

$(a + b + c)^k - (a^k + b^k + c^k) \geq [3^k - 3]\,(abc)^{k/3}$.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Since a, b, and c are positive real numbers and k is a positive integer, we shall show that

$$(a + b + c)^k \geq a^k + b^k + c^k + (3^k - 3)(abc)^{k/3} . \quad (1)$$

This solution is based upon the following case of the Arithmetic Mean - Geometric Mean Inequality: If x, y and z are positive real numbers, then $x + y + z \geq 3(xyz)^{1/3}$ with equality if and only if $x = y = z$.

When k = 1, (1) is the equality a + b + c = a + b + c. Assume that $(a + b + c)^k \geq a^k + b^k + c^k + (3^k - 3)(abc)^{k/3}$ (2)

is true for the positive integer k. Then $(a + b + c)^{k+1} \geq$

$(a^k + b^k + c^k)(a + b + c) + (3^k - 3)(abc)^{k/3}(a + b + c) =$

$(a^{k+1} + b^{k+1} + c^{k+1}) + (b^k c + c^k a + a^k b) + (bc^k + ca^k + ab^k)$

$+ (3^k - 3)(abc)^{k/3}(a + b + c) \geq$

$(a^{k+1} + b^{k+1} + c^{k+1}) + 3(abc)^{(k+1)/3} + 3(abc)^{(k+1)/3} +$

$(3^k - 3)^{k/3} \cdot 3(abc)^{1/3}$ which by the Arithmetic Mean - Geometric Mean Inequality $= (a^{k+1} + b^{k+1} + c^{k+1})$

$+ 6(abc)^{(k+1)/3} + (3^{k+1} - 9)(abc)^{(k+1)/3} =$

$a^{k+1} + b^{k+1} + c^{k+1} + (3^{k+1} - 3)(abc)^{(k+1)/3}$. Now (1) follows by mathematical induction.

Also equality holds in (1) if and only if $k = 1$ or $a = b = c$ which is guaranteed by the Arithmetic Mean - Geometric Mean Inequality.

Also solved by the proposer.

Problem 412: Proposed by the editor.

Fred and Pete were duck hunting one day when a lone mallard flew by within range. Pete is three times more likely to hit his target than Fred is. Assuming that the duck has an even chance to survive, what are Pete and Fred's respective probabilities of hitting the duck?

Solution by Jim Mercurio, Student, University of Michigan.

Let F denote the probability that Fred hits his target. Then the probability that Pete hits his target is P = 3F. The probability that the duck survives is (1 - F)(1 - 3F). Hence we have the equation

$$(1 - F)(1 - 3F) = .5 \quad \text{or} \quad 6F^2 - 8F + 1 = 0.$$

By the Quadratic Formula, we have $F = \dfrac{4 - \sqrt{10}}{6}$.

Discarding the root which exceeds 1, we have

$$F = \frac{4 - \sqrt{10}}{6} \approx .13962 \quad \text{and} \quad P = \frac{4 - \sqrt{10}}{2} \approx .41886$$

Problem 413: Proposed by the editor.

While studying the function $F(n) = n! + n^2 - 1$, where n is a positive integer, with his computer, a student noticed that F(n) is prime when n = 2. Unfortunately the precision of his computer limits the number of cases in which he can accurately produce a value for F(n). He would like to know other values of n for which F(n) is prime. Find other values of n for which F(n) is prime or show that none exist.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

We shall show that F(n) is prime only when n = 2. For n = 1,

$F(n) = 1$ which is not prime. For $n = 2$, $F(n) = 5$. Finally, for $n \geq 3$, $F(n) = n! + n^2 - 1 = (n - 2)!(n - 1)(n) + (n - 1)(n + 1)$ $= (n - 1)[(n - 2)!(n) + (n + 1)]$ which establishes that $F(n)$ is composite for $n \geq 3$.

KAPPA MU EPSILON NEWS

Edited by M. Michael Awad

News of chapter activities and other noteworthy KME events should be sent to Dr. M. Michael Awad, Historian, Kappa Mu Epsilon, Mathematics Department, Southwest Missouri State University, Springfield, MO 65804.

CHAPTER NEWS

Alabama Gamma, University of Montevallo, Montevallo
Chapter President – Aimee K. Thornton
11 actives, 4 initiates

Other 1988–89 officers: Kevin R. Harris, vice president; Deanna M. Miller, secretary and treasurer; Gene Garza, corresponding secretary and faculty sponsor.

Alabama Zeta, Birmingham–Southern College, Birmingham
Chapter President – Jennifer Millican
40 actives, 17 initiates

Other 1988–89 officers: Ashita Tolwani, vice president; Anamaria Vickery, secretary; Charles Montague, treasurer; Lola F. Kiser, corresponding secretary; Shirley M. Branan, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President – Donald Priest
44 actives, 8 initiates

The Chapter assisted the Mathematics Department with the annual Phon–a–thon to raise funds for the School of Science and Mathematics. Weekly meetings featured alumni and industry speakers, including representatives from IBM, Chevron, Fair Isaac and the Department of the Navy. Speakers at the Pledge Induction Ceremony in November were Joanie Carew and Jill Terry who illuminated the members concerning their experiences teaching in the California public school system and attending Cal Poly as graduate students. On November 30, 1988, KME sponsored a Christmas Party at the Village Retirement Center in San Luis Obispo. The preparation for the festivities was extensive and included the purchase of an eight foot Christmas tree. The pledges and social chairpersons prepared the refreshments. Entertainment consisted of singing Christmas carols. A good time was had by all. Other 1988–89 officers include: Susan Daijo and Athan Spiros, 1st vice presidents; Terry Bly and Jerry Burch, 2nd vice presidents; Dede Guevara and Warren Fernandes, pledgemasters; Stefan Steiner, treasurer; Sarah Parks, publicist; Rachel Jeffries and Kathy Perino, social coordinators; Lisa Fjeldal, alumni representative; Joni Otoshi, School Council representative; Chris Lucke, Poly Royal representative; Lonnie Smith, Curriculum Committee representative; Raymond D. Terry; faculty sponsor and corresponding secretary. The office of recording secretary remains vacant to date.

California Delta, California State Polytechnic University, Pomona
Chapter President – Kee Kragness
25 actives

Other 1988–89 officers: Pat Dunn, vice president; Julia Chu, secretary; Michelle Stratton, treasurer; Richard Robertson, corresponding secretary; Jim McKinney and Scott Sportsman, faculty sponsors.

Colorado Gamma, Fort Lewis College, Durango
Chapter President – Amy Getz
22 actives, 4 initiates

We held three business meetings and one induction ceremony in the fall. Members participated in the Alumni Phon–a–Thon to raise money for the Chapter. We published three issues of a department newsletter. Other 1988–89 officers: Earl Edwards, vice president; Carol Kjar, secretary; Kevin Marushack, treasurer; Richard A. Gibbs, corresponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President – Lynn Harris
25 actives

At our fall planning meeting on October 26, 1988, a social was scheduled at a local restaurant for November 9. We also sponsored a Charity Food Drive where canned goods were collected and given to the Community Food Bank to feed the poor. At the social on November 9, there were fifteen people present. A good time was had by all. Other 1988–89 officers: Tracy Tepp, vice president; Anne Salchow, secretary; Tammy Gresham, treasurer; Joe Sharp, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston
35 actives

There were a number of KME/Math Club meetings held throughout the semester. Our fall picnic took place on September 27th at Morton Park. We had two speakers: Dr. Keith Wolcott and Dr. Allen Davis. Our Christmas Party took place on Friday, December 2nd. Other 1988–89 officers: Wayne Watkins, vice president; Rita Stinde, secretary; Melissa Tracy, treasurer; Lloyd Koontz, corresponding secretary and faculty sponsor; Allen Davis, faculty sponsor.

Illinois Delta, College of St. Francis, Joliet
Chapter President – Jo Ann Lopykinski
24 actives

We had a Saturday A.M. tour of Areonne National Laboratory with twenty in attendance. We also organized a fund–raiser during Homecoming week. Proceeds of over $200 will help finance attendance at Spring Conference in Topeka, Kansas. Other 1988–89 officers: Pamela Damore, vice president; Rita Drab, secretary; Debra Becker, treasurer; Sister Virginia McGee, corresponding secretary and faculty sponsor.

Illinois Epsilon, North Park College, Chicago
Chapter President – David R. Johnson
26 actives

During the first half of the 1987–88 year, our Chapter visited Argonne National Laboratories in suburban Chicago and viewed the "Star of Wonder" Christmas planetarium star show in downtown Chicago. Other 1988–89 officers: Kimberlee J. Roth, vice president; Carol M. Uhl, secretary; Dena L. Pachucki, treasurer; Alice Iverson, corresponding secretary and faculty sponsor.

Illinois Zeta, Rosary College, River Forest
Chapter President – Mark Crosbie
20 actives

Other 1988–89 officers: Chad Husting and Natalie Perri, vice presidents; Mariloa Janek, secretary; Kathy Schmidt, treasurer; Sister Mary T. O'Malley, corresponding secretary and faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
Chapter President – Julie Eichenauer
30 actives

Our Chapter sponsored a fall picnic for all students interested in mathematics or computer science on September 18. On October 17, Dr. Dan Pritikin, Miami University, spoke on "Subtly erroneous proofs and other pseudo mathematical confusions." Dr. Richard Ringeiser, Clemson University, visited the college and discussed "Printed Circuits, Graphs and Brick Factories," on November 3. Other 1988–89 officers: Jenny Newton, vice president; Cindy Bull, secretary; Lauri Robison, treasurer; Ralph B. McBride, corresponding secretary; Deborah L. Hustin, faculty sponsor.

Indiana Delta, University of Evansville, Evansville
Chapter President – Jenifer Seckinger

Mr. William Houser, Dr. Mohammad K. Azarian, Dr. Papadopoulos, and Dr. Martin Jones were the speakers for the fall meetings. Other 1988–89 officers: Mary Singleton, acting vice president and secretary; Melba Patberg, corresponding secretary; Mohammad K. Azarian, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President – Suzanne Buckwalter
38 actives, 11 initiates

The annual KME Homecoming Coffee was hosted by Professor Emeritus and Mrs. E. W. Hamilton at their home in Cedar Falls with 33 alumni, members, and guests in attendance. Students presenting papers at local Iowa Alpha KME meetings include Mark Bohan on "Archimedes' Work on the Tangent to a Spiral," Jody Barrick on "Magic Squares," and Susan Paustian on "Eulerian and Hamiltonian Circuits." Mary Meier gave the address at the December initiation banquet on the topic "The Mathematics of Surveying." Susan Strong was awarded a student membership in the Mathematics Association of America. Other 1988–89 officers: Kerris Renken, vice president; Julie Holdorf, secretary; William Kruse, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President – L. Kidder
5 actives, 2 initiates

Some fall activities: Three regular meetings with student presentations, fall picnic, and December study break to watch mathematical films. Other 1988–89 officers: J. Seal, vice president; P. Wiedemeier, secretary; P. Waschbush, treasurer; A. Kleiner, corresponding secretary; L. Naylor, faculty sponsor.

Iowa Gamma, Morningside College, Sioux City
Chapter President – Dan Kruger
8 actives

Other 1988–89 officers: Lanette Curry, vice president; Kim Ashby, secretary; Matt Carney, treasurer; Doug Swan, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President – Curtis Eide
32 actives

The first 1988–89 meeting of the Iowa Delta Chapter featured members sharing about the internship that they had during the late spring and summer months. Dr. Sherry Nicol from the Mathematics Department of the University of Wisconsin at Platteville spoke about "Women in Mathematics" during the October meeting. The November program, presented by Ronald Nelson, Director of International Marketing for Decision Data, Inc., was a slide show on the history of computing and the evolution of the corporation he works for. As is traditional, the December meeting was a Christmas party, this year featuring a variation of Pictionary related to holidays. The Chapter again prepared and sold egg–cheese sandwiches at the Renaissance Faire event of the Wartburg College Homecoming. This booth is becoming a tradition and earned the Chapter approximately one hundred dollars. Other

1988–89 officers: Patricia Glawe, vice president; Terry Letsche, secretary; Kaaren Hemmingson, treasurer; August Waltmann, corresponding secretary; Josef Breutzmann, faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President – Jon Beal
40 actives, 4 initiates

The Chapter held monthly meetings in October, November and December. Fall initiation was held at the October meeting. Four new members were initiated at that time. This meeting was preceded by a pizza party. The October program was presented by Professor John Iley of the PSU Department of Industrial Arts and Technology. He demonstrated Thunderskan and MacVision software. Mala Renganathan gave the November meeting program. She discussed the actuarial profession and illustrated some of the problems from previous actuarial exams. In December, a special Christmas meeting was held at the home of Dr. Helen Kriegsman, Mathematics Department Chairperson. Professor Joe Siler, PSU Mathematics Department faculty member gave the program entitled "The Golden Numeration System." Other 1988–89 officers: Mala Renganathan, vice president; Lora Woodward, secretary; David Beach, treasurer; Harold L. Thomas, corresponding secretary; Helen Kriegsman and Gary L. McGrath, faculty sponsors.

Kansas Beta, Emporia State University, Emporia
Chapter President – Cathrine Barnes
18 actives, 2 initiates

During the fall semester, our Chapter held regular monthly meetings. At the September meeting, we met and welcomed new faculty members in the Mathematics Department. The December program was presented by Larry Hannah, the Director of our Career Development and Placement Office. He spoke on the careers available to math majors and offered valuable advice. Dr. Marion Emerson honored us by presenting a talk on problem solving at our November Initiation Banquet. Other 1988–89 officers: Laura Kincaid, vice president; Susan Streeter, secretary; Ed West, treasurer; George Downing, corresponding secretary; Larry Scott, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President – Susanne Piper
11 actives

Kansas Gamma started the fall semester with a meeting to elect officers. The election was preceded by a video of the spring picnic. Elaine Tatham, who owns her own consulting firm in Kansas City, was the November meeting guest speaker. She spoke on the many uses of mathematics in her work. In December the traditional Wassail party was enjoyed by a large turnout of people at the home of Jim Ewbank. Other 1988–89 officers: Elizabeth Zahrt, vice president, secretary and treasurer; Richard Farrell, corresponding secretary; Sister Jo Ann Fellin, faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President – Larry LaMee
18 actives, 4 initiates

Final plans are being made for the biennial convention to be held at Washburn, April 6–8, 1989. Other 1988–89 officers: Bryan Elrichs, vice president; Denise Winfrey, secretary and treasurer; Robert H. Thompson, corresponding secretary; Allan Riveland and Ron Wasserstein, faculty sponsors.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President – Julie Schmitt
21 actives, 7 initiates

Some events held during the fall semester: Fall picnic, Halloween Party, helped with the Math Relays sponsored by the Department. Other 1988–89 officers: Brian Kinsey, vice president; Marty Orth, secretary and treasurer; Charles Votaw, corresponding secretary; Mary Kay Schippers, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President – Wally Siddiqui
17 actives

Fall semester activities started off with a faculty/KME picnic in September at Dr. Metcalf's house. In October there was a KME alumni reunion breakfast at the Costello house. On a cold weekend at the end of October, Dr. Costello and a brave bunch of students camped out at Natural Bridge park. The semester ended with a Christmas party just before finals. There were two talks presented during the semester. A representative of Electronic Data Systems gave a talk on "Technology Trends." Dr. Costello gave a talk on "Some of My Favorite Math Puzzles." Other 1988–89 officers: Stacy Fluegge, vice president; Russ King, secretary; Bobby Hart, treasurer; Patrick Costello, corresponding secretary; Bill Janeway, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President – Mary Beth Van Pelt
13 initiates, 3 actives

Other 1988–89 officers: Deborah Camara, vice president; Beth Trust, secretary; Lisa Brown, treasurer; James E. Lightner, corresponding secretary; Linda R. Eshleman, faculty sponsor.

Maryland Delta, Frostburg State University, Frostburg
Chapter President – Laura Dudley
20 actives

Maryland Delta Chapter held its fall programs jointly with the newly-formed Mathematics Club at Frostburg State. The semester activities began with a picnic in early October and ended with a Christmas get-together in December. At meetings between these events, student members Mary Jones and Cynthia Stein described their mathematics-related summer employment, Dr. Lance Revennaugh gave a talk on "Sensitive Sampling," and the Secondary Mathematics Teaching Methods class demonstrated their final projects. The Chapter also sponsored a mock presidential election in early November (George Bush won). We look forward to inducting new members in the spring. Other 1988–89 officers: Mary Jones, vice president; Michelle Glotfelty, secretary; Christa White, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

Michigan Beta, Central Michigan University, Mt. Pleasant
Chapter President – Theresa Budzynski
40 actives, 13 initiates

Some activities of Michigan Beta were: KME conducted math help sessions for Freshman–Sophomore mathematics classes. Professor Donna Ericksen of CMU was our Fall Initiation speaker. Her topic was "Students' Conceptions of Variables and Their Uses for Generalization of Mathematical Patterns." At some of our meetings we enjoyed refreshments of pizza and/or cookies. KME had a co–ed volleyball team in the fall and will have a basketball team in the winter. We finished the fall semester with a Christmas party at the home of our advisor, Arnold Hammel, and family. During the first week of the winter semester, a faculty member and a student from Oakland University spoke to KME on their Master's Degree program in Statistics and Quality Control and its link with Ford Motor Company. Other 1988–89 officers: Nancy Haskell, vice president; Agnes Hausbeck, secretary; George Lasecki, treasurer; Arnold Hammel, corresponding secretary and faculty sponsor.

Mississippi Alpha, Mississippi University for Women, Columbus
Chapter President – Michelle D. Whitley
7 actives, 11 initiates

Other 1988–89 officers: Connie Hudson, vice president; Karen Scott, secretary and treasurer; Jean Parra, corresponding secretary; Carol Ottinger, faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President – Stuart Hartfield
24 actives, 8 initiates

Other 1988–89 officers: Patsy Saucier, vice president; Beth Page, secretary; Alice Essary, treasurer and corresponding secretary; Virginia Entrekin, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President – Sherri Renegar
34 actives, 8 initiates

The Missouri Alpha Chapter held three regular monthly meetings during the fall semester, each of which had a faculty guest speaker. In addition, the Chapter hosted a picnic for all mathematics faculty, staff and students. Other 1988–89 officers: Gayla Evans, vice president; Ellen Caldwell, secretary; Lynette Top, treasurer; John Kubicek, corresponding secretary; M. Michael Awad, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President – Sharon Johnson
15 actives, 4 initiates

The Missouri Beta Chapter started the fall semester by challenging the instructors to games of Win, Lose or Draw and Outburst. Then business began for the members, who again volunteered time to the KME Math Clinic (a tutoring clinic for algebra students). Our first regular meeting featured Dr. Craven, a CMSU instructor, who talked about her summer trip to an international teachers conference in Europe. For our October meeting, Dr. Davenport, the head of the Department of Mathematics and Computer Science at CMSU, spoke about the graduate school opportunities. The end of the month we had a Halloween Party. November was busy with the book sale (our major fund raiser). At the regular meeting, four new members were initiated. For the program, two CMSU graduates came and spoke to KME about their jobs as computer programmer and actuary. As the semester came to an end KME members enjoyed a Christmas party, before taking finals. Other 1988–89 officers: Ray Flach, vice president; Angela Duncan, secretary; Sandy Dietz and David Beard, treasurers (fall and spring, respectively); Homer Hampton, corresponding secretary; Larry Dilley and Gerald Schrag, faculty sponsors.

Missouri Gamma, William Jewell College, Liberty
Chapter President – Susan Brannen
19 actives

Regular monthly meetings of the Chapter were held in the fall of 1988. An area high school teacher gave a talk at one of the meetings, and demonstrated how he uses a file card set of applications problems to arouse interest among his students to the applications of mathematics. Other

1988–89 officers: Alysia Hicks, vice president; Sarah Littlewood, secretary: Joseph T. Mathis, treasurer, corresponding secretary and faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President – Laura Knight
14 actives

Other 1988–89 officers: Lesa Stoecklin, vice president; John Callaway, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor; Linda O. Lembke, faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville
Chapter President – Jim Daues
14 actives, 12 initiates

Other 1988–89 officers: Wes Clifton, vice president; Shelle Palaski, secretary; Dave Smead, treasurer; Mary Sue Beersman, corresponding secretary; Mark Faucette, faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin
Chapter President – Robert Stokes
12 actives

During the semester, programs presented at regular meetings included a talk by Dr. Patrick Cassens on coding, a talk by Susan Paulson on mathematical illusion, and a talk by Bryan Campbell on areas under curves. Activities included raffling off the college president's parking place as a money making project to support United Way. Both a Halloween Party and a Christmas Party were held. Other 1988–89 officers: Susan Paulson, vice president; Julie Stuewalt, secretary and treasurer; Mary Elich, corresponding secretary; Joe Shields, faculty sponsor.

Missouri Kappa, Drury College, Springfield
Chapter President – Missy Arnold
6 actives

The first activity of the semester for the Chapter was a bonfire weiner roast held at Dr. Allen's house. The winners of the Annual Campus Math

Contest were Mark Duncan (Calculus I and below) and Connie Keith (Calculus II and above). Prize money was awarded at a pizza party held for all the contestants. At one of the Chapter's monthly meetings, Dr. Allen gave a talk on the Muslim contribution to Mathematics. The end of the semester was celebrated with a Christmas Party. The Chapter continued to run a free Math Tutoring Service for all math students at Drury. Other 1988–89 officers: Donna Luetkenhaus, vice president; Connie Keith, secretary; Scott Steubing, treasurer; Charles S. Allen, corresponding secretary; Ted J. Nickle, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President – Jim Fisher
30 actives

Throughout the semester club members have monitored the Math–Science Building in the evenings to earn money for the club. The club participated in the college homecoming activities by painting and erecting a billboard. Club members also manned a "Fish for Suckers" booth at the Homecoming Carnival. Tom Hochstein was awarded the $25.00 book scholarship which is given to a KME member each semester by the club. With a grant from the Wayne State College Student Senate, KME and the Computer Club purchased a MacIntosh computer. At Christmas time, KME, LDL, Computer Club, and Biology Club treated the Math–Science faculty to a dinner out at the Wagon Wheel in Laurel. Inexpensive gifts were exchanged. Other 1988–89 officers: Darin Moon, vice president; Sherry Linnerson, secretary and treasurer; Renee Harre, historian; Fred Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

Nebraska Gamma, Chadron State College, Chadron
Chapter President – Kim Sedlacek
15 actives, 3 initiates

The Chapter helped to host the fall convention for the Nebraska Association of Teacher's of Mathematics (NATM). The convention was a huge success. We had 28 presentations during the day and 108 teachers attended. There were also many CSC faculty members and students who attended the sessions. We initiated three members last semester. Other 1988–89 officers: Michelle Dodd, vice president; Pat Reilly, secretary; Betty Rudnick, treasurer; James A. Kaus, corresponding secretary; and Monty G. Fickel, faculty sponsor.

New York Alpha, Hofstra University, Hempstead
Chapter President – Pamela Caplette
8 actives, 4 initiates

Other 1988–89 officers: Michelle Lisi, vice president; Carol Ann Sutherlin, secretary; Howard Robinson, treasurer; Stanley Kertzner, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara University
Chapter President – Christine Carbone
15 actives

We have been undergoing a reorganization this school year and were able to have only one fall meeting. Our problem at this time is finding a meeting time which will fit the various students' schedules. We are currently planning our annual banquet/initiation and hope to have an alumnus as speaker. Other 1988–89 officers: Laura Plyter, vice president; Any Potter, secretary; Theresa Toenniessen, treasurer; Robert Bailey, corresponding secretary; Kenneth Bernard, faculty sponsor.

New York Kappa, Pace University, New York
Chapter President – Antonia Marzella

Other 1988–89 officers: Marya R. Doery, vice president; Louis V. Quintas, corresponding secretary; John W. Kennedy and Martin Ketle, faculty sponsors.

Ohio Gamma, Baldwin–Wallace College, Berea
Chapter President – Michael Jakupca
16 actives

Other 1988–89 officers: Kimberly Hinkle, vice president; Eric Angyal, secretary; Cheryl Soltis–Muth, treasurer; Robet Schlea, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President – Regina Dolick
24 actives, 5 initiates

Some fall activities: September – organizational meeting; October – initiation with an outside speaker; November – two outside speakers. Other 1988–89 officers: Sophia Asghar, vice president; Julie Clark, secretary; Karen Allender, treasurer; Carolyn Crandell, corresponding secretary; Russell Smucker, faculty sponsor.

Oklahoma Alpha, Northeastern State University, Tahlequah
Chapter President – Michelle Harper
51 actives, 11 initiates

This fall the Oklahoma Alpha Chapter sponsored a talk by Dr. Stanley Eliason, Head of the Mathematics Department at the University of Oklahoma. He used mathematical induction to discuss "how to cut a cake fairly." Suzanne Blackwell, Chapter vice president, was a finalist for the 1988 Northeastern State University homecoming queen. She was sponsored by our Chapter. Mathematics professors donated textbooks to KME's very successful book sale. Our motto was "50 cents per inch" (thickness of the book). The fall, 1988, initiation ceremonies were held in the banquet room of the Sirloin Stockade Restaurant in Tahlequah. At our December Christmas party, we awarded $5.00 to the person telling the best math joke. Other 1988–89 officers: Suzanne Blackwell, vice president; Shelli Phillips, secretary and treasurer; Joan E. Bell, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President – Kellie Logan
20 actives

The Oklahoma Gamma Chapter of KME participated in the homecoming parade this fall. We had a fund–raising project to help raise money to send students to the national convention. Other 1988–89 officers: Amy Bogwell, vice president; Rhonda Hollrah, secretary; Ajith Dharmawardhana, treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Pennsylvania Delta, Marywood College
2 actives

Induction of new members and election of officers will take place in spring, 1989, semester. Sister Robert Ann von Ahnen is corresponding secretary.

Pennsylvania Iota, Shippensburg University, Shippensburg
Chapter President – Elizabeth Weller
14 actives, 9 initiates

Other 1988–89 officers: John Thompson, vice president; Tina Fiory, secretary; Fred Nordai, treasurer; Lenny Jones, corresponding secretary; Rick Ruth, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President – Scott Kromis
7 actives, 10 initiates

KME members hold their meetings every third Thursday of each month. Problems are solved and discussed. Plans are being made for the initiation into KME on March 13, 1989. Members continue to tutor (free of charge) in mathematics. The demand from the Freshmen at the College to be tutored is very great. The members have to decide how to handle this. Other 1988–89 officers: Eric Mebler, vice president; Constance Hefner, secretary and treasurer; Sister M. Grace, corresponding secretary; Linda Czajka, faculty sponsor.

Pennsylvania Lambda, Bloomsburg University, Bloomsburg
Chapter President – Ann Vnuk
19 actives, 7 initiates

Dr. Edward Kerlin presented his Graphics Editor at one of our meetings. Other 1988–89 officers: Joshua Payne, vice president; Theresa Creasy, secretary; Karen Billingham, treasurer; James Pomfret, corresponding secretary; John Riley, faculty sponsor.

Tennessee Alpha, Tennessee Technological University, Cookeville
Chapter President – Michael Allen
10 actives

Co–op opportunities were discussed at a fall meeting along with plans for a January ski trip. (No snow, no trip.) Other 1988–89 officers: Chris Roden, vice president; Sarwat Kasmiri, secretary; Curt Griggs, treasurer; Frances E. Crawford, corresponding secretary; Ed Dixon, faculty sponsor.

Tennessee Delta, Carson–Newman College, Jefferson City
Chapter President – Trevor Roberts
17 actives

Other 1988–89 officers: Sabrina Hall, vice president; Shannon Langley, secretary; Jackie Kearney, treasurer; Verner Hansen, corresponding secretary; Carey Herring, faculty sponsor; Denver R. Childress, interim sponsor.

Texas Alpha, Texas Tech University, Lubbock
Chapter President – Karen Engel
25 actives

Guest speakers spoke on several topics in applied mathematics at meetings. Other 1988–89 officers: Gregory Henderson, vice president; Paula Kajs, secretary; Scott Ellett, treasurer; Robert Moreland, corresponding secretary and faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President – Maureen Pastors
3 actives, 7 initiates

Wisconsin Alpha KME members and pledges and faculty sponsored and coordinated a mathematics competition for high school junior and senior young women on November 19, 1988. Top individual prize is a partial scholarship to Mount Mary College. Other 1988–89 officers: Julie Elver, vice president; Maureen Pastors, secretary; Julie Elver, treasurer; Sister Adrienne Eickman, corresponding secretary and faculty sponsor.

Wisconsin Gamma, University of Wisconsin – Eau Claire, Eau Claire
Chapter President – Renee Kozlowski
49 actives, 28 initiates

The fall semester of the 1988–89 year was started off with our annual fall initiation. As has been the case for the last several years, this initiation is held off–campus at one of the nicer restaurants in town. Well attended by relatives and friends of the initiates, the ceremony was concluded with a formal dinner and a presentation by a member of the Math Department.. Monthly meetings at which one or more students gave mathematical talks to the club were the mainstay of the rest of the semester. In addition, we had a successful fundraiser selling popcorn and at Thanksgiving time the club got together for a Thanksgiving dinner and celebration. Other 1988–89 officers: Renee Wagner, vice president; Brian Vlcek, secretary; Jennifer Linn, treasurer; Tom Wineinger, corresponding secretary.

IN MEMORIAM

Carl V. Fronabarger

Dr. Carl V. Fronabarger died August, 1988.

Dr. Fronabarger taught mathematics at Southwest Missouri State University for many years after joining the faculty in 1941. He served as head of the Department of Mathematics from 1965 until 1967, when he became Director of the Division of Science and Technology. The title was later changed to Dean. He held that position until his retirement in June, 1974.

Among the professional duties he enjoyed and did well were editing The Pentagon, the magazine of the Kappa Mu Epsilon Society, from 1953 to 1959, and serving as National President of that Society from 1959 to 1963. Kappa Mu Epsilon is the national honor society in undergraduate mathematics.

Louise Stokes Hunter

Dr. Louise Stokes Hunter died December, 1988

Dr. Hunter took the necessary steps to establish the Virginia Alpha Chapter of KME in 1955 at Virginia State University (the first chapter in the State of Virginia). She was awarded the rank, Professor Emeritus, by the Board of Visitors of Virginia State University on May 5, 1974, upon her retirement as Professor of Mathematics.

On May 31, 1978, the Virginia Alpha Chapter of KME established an annual award titled "The Louise Stokes Hunter Award". The first award was presented by Dr. Hunter, to a Mathematics major on Honors Day in 1979.

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