THE PENTAGON

Volume XLVIII(48) Fall, 1988 Number 1

CONTENTS

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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics; due, mainly, to its demands for logical and rigorous modes of thought, to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

# SPHERICAL TRIGONOMETRY

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"Find the distance from Chicago, at $41^\circ 50'$N., $87^\circ 37'$W., to Moscow, Russia, at $55^\circ 45'$N., $37^\circ 34'$E., and the bearing of each city from the other. Find the position of the point farthest north on the path between them and its distance from Chicago" (Brink 43).

The problem above is a typical example of one that can be found in certain mathematics books. To make the problem a little more interesting, suppose "Mr. Jones" is a Russian spy. He needs to fly from Chicago to Moscow in his private jet in the shortest possible time. The pilot needs to know the direction in which to fly and the shortest distance to Moscow in order to tell Mr. Jones how long the flight will be. The solution to the problem facing this pilot uses the discipline spherical trigonometry.

Spherical trigonometry involves the solution of triangles on a spherical surface just as plane trigonometry allows for solutions of triangles in a plane. A spherical triangle is formed by the arcs of three great circles that do not all pass through the same point. Thus, it outlines a portion of the surface of a sphere (see Figure 1).

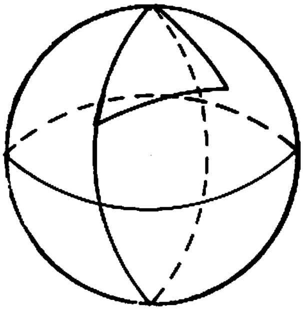

Figure 1

The sides of the triangle are arcs of great circles. Since the length of an arc is measured by its central angle, and since all arcs will be with respect to the same radius, distance on the sphere can best be expressed in degrees or radians, units of angular measure. For example, in Figure 2, if $\angle BOA = 50^\circ$, then $AB = c = 50^\circ$.

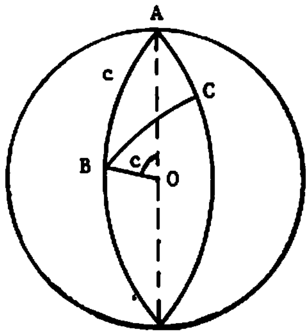


Figure 2

Using the planes containing the great circles of the sides, the included angle of the two sides can be measured. The angle is simply the measure of the dihedral angle formed by these planes (see Figure 3).

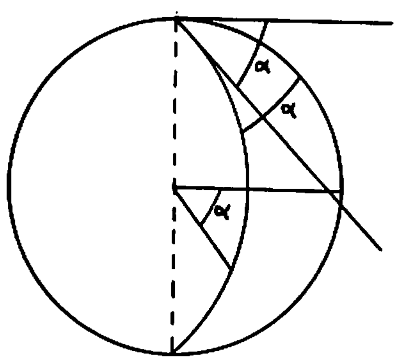


Figure 3

The six parts of a spherical triangle are commonly labeled as shown in Figure 4.

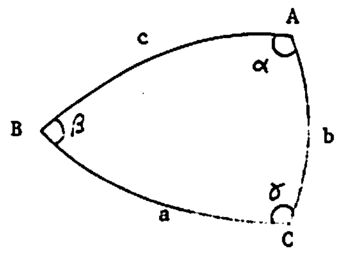


Figure 4

A,B,C are vertices; a,b,c are the sides, and $\alpha, \beta, \gamma$ represent the angle measures.

There are many properties common to both plane and spherical trigonometry. Terms such as right, isosceles, equilateral, and oblique have identical meanings when they are applied to spherical triangles as when they're applied to triangles in a plane. Other properties such as those listed below, also hold for spherical triangles.

1) If two sides of a triangle are unequal, then the angle opposite the greater side is the greater.

2) In an isosceles triangle, the angles opposite equal sides are equal.

Just as in plane trigonometry for right triangles, special solution cases are present for a right spherical triangle. These ten rules are as follows:

1.) $\sin a = \sin c \cdot \sin \alpha$

2.) $\sin b = \sin c \cdot \sin \beta$

3.) $\sin a = \tan b \cdot \cot \beta$

4.) $\sin b = \tan a \cdot \cot \alpha$

5.) $\cos c = \cos a \cdot \cos b$

6.) $\cos c = \cot \alpha \cdot \cot \beta$

7.) $\cos \alpha = \cos a \cdot \sin b$

8.) $\cos \beta = \cos b \cdot \sin \alpha$

9.) $\cos \alpha = \tan b \cdot \cot c$

10.) $\cos \beta = \tan a \cdot \cot c$.

These can be proven using properties of a sphere and its triangles. These ten equations may be more easily remembered with two rules developed by John Napier.

John Napier, Baron of Mechiston (1550-1617), was a Scotsman whose greatest contribution to mathematics was his invention of logarithms (Coolidge 71). Other works of Napier include methods of finding square roots and of multiplication and divison which are now refered to as Napier's rods. Napier also made contributions in algebra, geometrical logistics and trigonometry. His method of solving right spherical triangles involves a device developed previously by Tarporley (Coolidge 80). Napier summarizes the solution of right triangles into two general rules. These rules use a circle graph and a corresponding right triangle as shown in Figure 5.

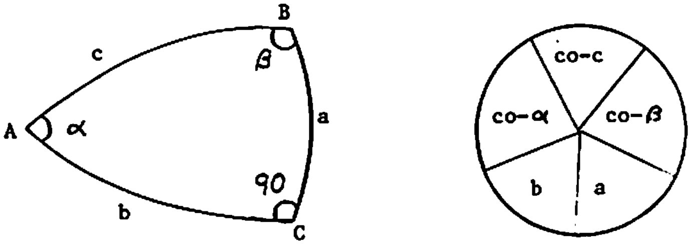


Figure 5

In the figure, $\gamma$ becomes the right angle, and co-$\gamma$ stands for the co-function of $\alpha$. For example, sin(co-$\alpha$) = cos$\alpha$. Napier's rules are as follows.

1.) The sine of any part is equal to the product of the tangents of the adjacent parts. I.e. using the circle, part a is adjacent with part b and part co-$\beta$. Also, part co-$\beta$ is adjacent to part a and part co-c, and so on. So by using this rule, #3 from the ten stated previously can be obtained.

$$\sin a = \cos(\text{co-}\alpha)\ \cos(\text{co-}c)$$

$$\sin a = \text{six}\ \alpha \cdot \sin c.$$

Oblique spherical triangles may be solved using the law of sines and/or laws of cosines.

The Law of Sines: the sines of the sides are proportional to the sines of the respecitvely opposite angles.

I.e. $\dfrac{\sin a}{\sin \alpha} = \dfrac{\sin b}{\sin \beta} = \dfrac{\sin c}{\sin \gamma}.$

The Law of Cosines for Sides: in any spherical triangle, the cosine of any side is equal to the product of the cosines of the other two sides plus the product of those sides times the cosine of their included angle.

I.e. $\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos \alpha$.

The Law of Cosines for Angles: the cosine of any angle is equal to minus the product of the cosines of the other two angles plus the product of the sines of those angles times the cosine of their included side.

I.e. $\cos \alpha = -\cos \beta > \cos \alpha + \sin \beta > \sin \alpha \cdot \cos a$.

Returning now to the original problem, the position of Chicago is $41^\circ 50'$N, $87^\circ 37'$W., and Moscow is $55^\circ 45'$N., $37^\circ 34'$E. Using these two points and the North Pole as a third, a spherical triangle can be drawn (see Figure 6).

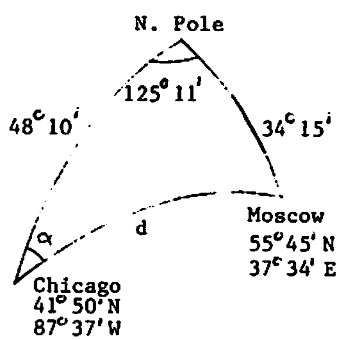


Figure 6

The shortest distance between Chicago and Moscow is one of the sides of this triangle, since the shortest distance between any two points on the surface of a sphere is the arc of a great circle. Thus, the side of the triangle between the North Pole and Chicago is $48^\circ 10'$, which can be obtained by subtracting Chicago's latitude ($41^\circ 50'$N) from the distance between the North Pole and the equator ($90^\circ$). Similarly, the side between the North Pole and Moscow is $90^\circ - 55^\circ 45' = 34^\circ 15'$. The prime meridian, which passes through Greenwich, England, is $0^\circ$ longitude. So, since Chicago is west of the prime meridian, and Moscow is east of it, the east-west angular measure is $87^\circ 37' + 37^\circ 34' = 125^\circ 11'$. Thus, the triangle may be represented as shown.

Now, by using the law of cosines for sides, the distance from Chicago to Moscow (d) can be obtained.

$$\cos d = \cos 48^\circ 10' \cdot \cos 34^\circ 15'$$
$$+ \sin 48^\circ 10' \cdot \sin 34^\circ 15' \cdot \cos 125^\circ 11'$$
$$> \cos d = .5513 + (-.2416) = .3097$$
$$d = 71^\circ 58'.$$

There are 60 nautical miles in each degree on earth, and one nautical mile equals 1.1515 statute miles. Thus, the shortest distance between Chicago and Moscow is 71.9667·(60) = 4318 nautical miles or 4318·(1.1515) = 4972 statute miles. This tells the pilot how far he will have to fly, but it doesn't tell him in which direction.

To find the direction in which to fly, the angle $\alpha$ in the representative triangle must be found. This angle will be the direction east of north that he will fly. $\alpha$ can be found using the law of sines.

$$\frac{\sin 34^\circ 15'}{\sin \alpha} = \frac{\sin 71^\circ 58'}{\sin 125^\circ 11'}$$

$$\Rightarrow \sin \alpha = .4838$$

$$\therefore \alpha = 28^\circ 56'.$$

The pilot now knows he must fly Mr. Jones in a direction of N $28^\circ 56'$E. at take-off. This is a direction that heads toward the southern tip of the Hudson Bay. Normally, one would think Mr. Jones would fly east, but actually he flies more north than east at take-off. He will reach a point farthest north and then travel at a direction southeast. This northernmost point can be found using the right triangle rules discussed earlier. This northern point will be the vertex (C) of a right triangle with other vertices at Chicago and the North Pole (see Figure 7).

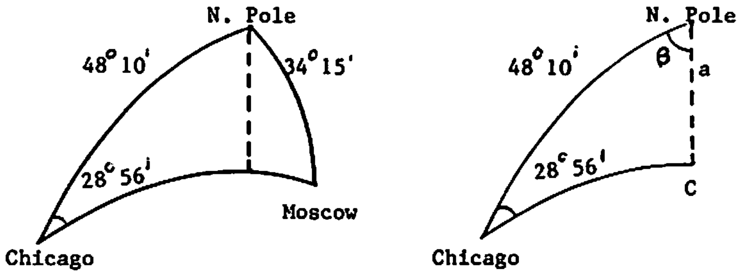


**Figure 7**

**The distance of point C from Chicago (the length of b) can be calculated using rule #9, $\cos\alpha = \tan b \cdot \cot c$.**

$$\cos 28^\circ 56' = \tan b \cdot \cot 48^\circ 10'$$

$$\Rightarrow .8752 = \tan b \cdot (.8952)$$

$$\therefore b = 44^\circ 21'.$$

**Thus, the distance from Chicago is 44.3528·(60) = 2661 nautical miles, or 2661·(1.1515) = 3064 statute miles. However, this doesn't say much about where the corresponding location on earth actually is. To find the line of longitude of this location, $\beta$ must be found. This can be achieved by using rule #8, $\cos\beta = \cos b \cdot \sin\alpha$.**

$$\cos\beta = \cos 44^\circ 21' \cdot \sin 28^\circ 56'$$

$$\Rightarrow \cos\beta = (.7151)\cdot(.4838)$$

$$\therefore \beta = 69^\circ 46'.$$

Since Chicago's longitude is $87^\circ 37'$W., and point C is on the line $69^\circ 46'$ east of this, the longitude of the location of C is $87^\circ 37' - 69^\circ 46' = 17^\circ 51'$W.

To find the line of latitude of the location of point C, the length of a must be found. This can be done by using rule #1, $\sin a = \sin c \cdot \sin \alpha$.

$$\sin a = \sin 48^\circ 10' \cdot \sin 28^\circ 56'$$

$$\Rightarrow \sin a = (.7451)(.4838)$$

$$\therefore a = 21^\circ 8'.$$

Thus, point C is $21^\circ 8'$ from the North Pole so it is $90^\circ - 21^\circ 8' = 68^\circ 52'$ north of the equator. Therefore, the northernmost point on Mr. Jones' journey from Chicago to Moscow is located at $68^\circ 52'$N., $17^\circ 51'$W. This is a point north of Iceland and inside the arctic circle ($66^\circ 30'$N.) by 163 statute miles. So Mr. Jones' flight will include a trip over a portion of Greenland and then Norway and Sweden.

The trip had a great start and everything looked fine until somewhere inside of the arctic circle where Mr. Jones became so exhausted from the stress of being a double-agent that he collapsed and died. The pilot then decided to change course and spend a week vacationing in the Swiss Alps.

## Works Cited

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# AN APPLICATION OF GRAPH THEORY

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Leonard Euler first used Graph Theory in the mid-eighteenth century to solve the Konigsberg Bridge problem (Haray, 1986, iii). The townspeople of Konigsberg enjoyed taking evening strolls on which they frequently crossed the seven bridges in the town. It bothered some of the townspeople that they could not find a path in which they would cross each bridge only once (Wilson, 265). When this puzzle was presented to Euler, he modelled the situation using a graph and proved that no such path could exist (Wilson, 168). Since this first utilization of graph theory to model real world situations, graphs have become acknowledged as powerful mathematical tools. In the nineteenth century, Cayley used a graph model in his study of chemistry to determine precisely the number of paraffins that existed. In 1947, Kirchoff pioneered electrical engineering utilizing graphs to model electrical systems (Harary, 1986, iii). More recently, graphs have been used to study such diverse areas as group interaction, communication networks, efficient transportation systems, and electrical energy demand (Roberts, 5).

Some Basic Definitions

A directed graph or digraph, D = (V,E), consists of a set of vertices and a set of edges connecting vertices in pairs (Tucker, 202). For example, Figure 1 shows a diagraph modelling the results of a tennis tournament. The four points labelled Laver, Rosewall, Ashe, and Smith make up the vertex set of the digraph, and the directed lines connecting them comprise the edge set (Roberts, 23). The edge set of any graph illustrates a relationship that exists among the elements of the vertex set. For instance, the directed line from Laver to Rosewall indicates that Laver beat Rosewall when they met in the tournament.

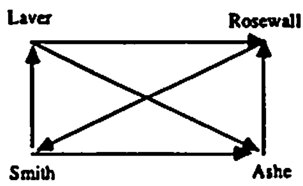


Figure 1 (Roberts,23)

Sometimes, knowing only that some kind of relationship exists is insufficient information. Often, mathematicians find it necessary to differentiate between positive and negative relationships among vertices. In this case, a plus or minus sign is assigned to each edge in the digraph to signify the type of relationship existing between that pair of vertices. This type of digraph is referred to as a <u>signed digraph</u> (Harary, 340). In this paper, a plus sign on an edge from $v_1$ to $v_2$ will mean that an increase in the value of $v_1$ at time zero will lead to a decrease in the value of $v_2$ at time one (Roberts, 186). For example, in the signed digraph which models energy demand (Figure 2), the edge from population to energy use is positive because an increase in population will tend to cause an increase in the use of energy. Similarly, a decrease in population will lead to a decrease in energy use. On the other hand, because an increase in energy use causes the quality of the environment to decrease, a negative sign is placed on the edge leaving energy use and entering quality of environment.

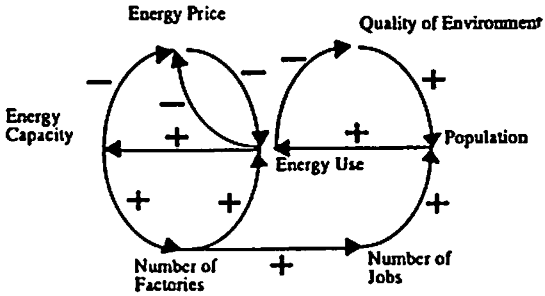


Figure 2( Roberts, 189)

If even more information were required about the relationship existing between two vertices, weights could be added to each edge to signify the relative importance that edge has to the structure of the modelled situation. For instance, in the weighted digraph of Figure 3 modelling the effects sleep loss and noise have on a five choice task test, the weights indicate that sleep loss has a much more significant effect on performance than does noise level (Ringeisen and Shingledecker, 826).

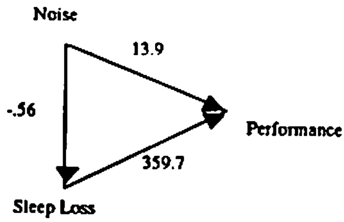


Figure 3 (Ringeisen, 836)

Modelling Real World Situations

Signed and weighted digraphs give a useful and powerful tool for modelling real world problems that consist of many variables interacting and reacting to changes in each other (Roberts, 186). In modelling complex systems, the vertex set consists of the variables significant to the problem being studied. The edges illustrate how the variables interact with one another. For instance, in the energy demand model (Figure 2), the vertices in the digraph represent variables that have an influence on energy use; the edges model the relationships existing between these variables (Roberts, 188).

Since it is usually difficult to study changes in more than one variable within a given system, I will deal only with simple pulse processes: those processes in which an increase of one unit is introduced at a specifically chosen vertex of the system. The propagation of this initial pulse throughout the system is then studied given no additional pulses. For example, in the energy demand model, increasing the value of the population vertex by one unit while all other values remain constant is an example of a simple pulse process.

A Simple Ecosystem

Now, let us consider an ecosystem or food chain consisting of a population of mice, a population of rats, and their predators, a population of eagles. The first step in studying any system involves finding a digraph to model that system. Therefore, let the vertex set consist of three points labelled mice, rats, and eagles. Then, we obtain the edge set for the digraph by analyzing the relationships existing among these species. In the ecosystem, mice and rats compete for the same limited food source. This implies that, if either population were to increase, more animals would compete for the same food and this would cause some animals to die from starvation. Therefore, there will be negative edges both ways between the vertices associated with mice and rats. By the same reasoning, negative loops will also leave and return to these same two vertices. Furthermore, an increase in the mouse or rat population would tend to cause an increase in the eagle population. This relationship would be indicated in the digraph by a positive edge being drawn from the vertex associated with mice to the one for eagles and a positive edge from the vertex associated with rats to the vertex cooresponding to eagles. Increases to the eagle population would result in the consumption of more mice and rats. Therefore, in the digraph negative edges would leave the eagles' vertex and enter the respective vertices corresponding to mice and rats. Assuming that eagles, on the other hand, have an unlimited supply of food, there would be no competition among themselves. So, no loop leaving the eagle's vertex and re-entering it again will exist in the digraph. Using the above analysis, a model for this simple ecosystem appears in Figure 4.

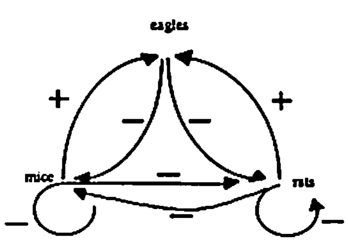


Figure 4 (Roberts, 215)

Now, given this model, suppose we would like to know the effect a simple pulse into the vertex associated with the mice has on the values of the other populations in the ecosystem at some later time period (Roberts, 216). For instance, would an increase in the mouse population cause a population explosion of mice and perhaps even cause the rat population to become extinct? In order to study this type of question, it becomes useful to translate the information contained in the digraph into a more computational form, an adjacency matrix. An adjacency matrix for a digraph has one row and one column for each vertex in the digraph. Therefore, the adjacency matrix will always be an n by n matrix, where n is the number of vertices in the vertex set of the digraph. The $a_{ij}$ entry in the adjacency matrix for an unsigned digraph equals 1 if an edge exists from $v_i$ to $v_j$ and zero if no edge exists (Tucker, 240). If the digraph is signed or weighted, then the $a_{ij}$ entry corresponds to the weight placed on the edge from $v_i$ to $v_j$, if it exists (Harary, 15).

By this definition, the adjacency matrix representing the ecosystem would have three rows and three columns, say row and column 1 labelled "mice", row and column 2 labelled "rats", and row and column three labelled "eagles". Then, the $a_{11}$ entry in the matrix would equal -1 since a negative edge exists from the mice's vertex to the mice's vertex.

Continuing in this manner, the adjacency matrix, A =
$$\begin{array}{c|ccc} & M & R & E \\ \hline M & -1 & -1 & 1 \\ R & -1 & -1 & 1 \\ E & -1 & -1 & 0 \end{array}.$$

The Forecasting Problem

Given this computational form for the digraph, we can now define the Forecasting Problem as follows: predict the value $v_i(t)$ of any vertex $u_i$ at a given time t after introducing an initial pulse into a system (Roberts, 200). In the ecosystem, the Forecasting Problem could translate to determining the effect an initial increase in the mouse population has on the rat population at some later time period, say after three time periods. Although, it may seem obvious what the effect an increase to the mouse population at time zero will be on the other populations in the system at time one, it is not clear what happens to the populations at later time periods because of the multiple pulses that the initial pulse causes as it propogates through the ecosystem. However, the adjacency matrix and the following theorem proved by Roberts provide the tools for solving the Forecasting Problem.

Roberts (211) states that given a digraph, D=(V,E), and its adjacency matrix A, that after an initial pulse at vertex $u_i$, the value $v_j(t) = v_j(\text{start})$ + the i,j entry of the matrix, $M=1+A+A^2+\cdots+A^t$. Therefore, in order to determine the number of species in each population of the ecosystem at t=3, we need to know the number of species in each population at the start of the pulse process, and we need to calculate the matrix $M=1+A+A^2+A^3$.

Assume that before the introduction of the pulse the ecosystem consisted of 10 mice, 10 rats, and 3 eagles. Then at the start of the pulse process the system will contain 11 mice, 10 rats, and 3 eagles. Then at the start of the pulse process the system will contain 11 mice, 10 rats, and 3 eagles because the pulse is introduced to the mice vertex. Calculating the matrix M gives M =

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ 2 & 2 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ -2 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 3 \end{bmatrix}.$$

Therefore, $v(3) = (11,10,3) + (1,0,1) = (12,10,4)$ or at $t=3$ the ecosystem consists of 12 mice, 10 rats and 4 eagles. The vector $(1,0,1)$ is obtained from the row of M corresponding to the vertex which received the initial pulse.

Stability

Although this is a useful technique for small values of t, this process requires the calculation of a large number of matrices, for large values of t. Even with a computer, these calculations become time consuming as the size of the matrix or t becomes large. Therefore, a more useful strategy would involve studying a system in the long run without a specific time frame. Such analysis involves determining if a digraph stabilizes under a simple pulse process.

When discussing stability, two aspects of the model require consideration. First of all, a digraph is considered value stable if the sequence of values, $\{v_i(t),\ t=0,1,2,\ldots\}$ is bounded for every vertex in the digraph, whereas, pulse stability means that the sequence of pulses entering a vertex is bounded (Roberts, 217). Furthermore, it has been shown that testing for stability in a digraph model reduces to asking questions about the eigenvalues associated with the adjacency matrix representation of that digraph (Roberts, 218).

Eigenvalues are defined to be the roots of the equation det$|A-\beta I|$ = 0, where A is the adjacency matrix and $\beta$ is a scalar parameter (Schwenk, 308). To calculate the eigenvalues of a digraph, begin with the adjacency matrix, A, and subtract $\beta$ times the identity matrix. The eigenvalues of this matrix will be the complex roots, a+bi, of the equation obtained from calculating the det$|A-\beta I|$. Furthermore, the eiqenvalues will have magnitudes equal to $\sqrt{a^2 + b^2}$.

Using the adjacency matrix for the ecosystem, $A-\beta I = \begin{bmatrix} -1-\beta & -1 & 1 \\ -1 & -1-\beta & 1 \\ -1 & -1 & -\beta \end{bmatrix}$

Calculating the det $|A-\beta I|$ yields the expression $-2\beta - 2\beta^2 - \beta^3$. Setting this equal to zero and solving for $\beta$, the eigenvalues associated with the digraph of the ecosystem are 0, -1+i, and -1-i, and they have respective magnitudes 0, $\sqrt{2}$, and $\sqrt{2}$.

<u>Theorems Regarding Stability</u>

In <u>Discrete Mathematical Models</u>, Roberts (219) proves two theorems for determining the stability of a digraph which I will present without proof. The first states that "If a weighted digraph D is pulse and value stable under all simple pulse processes, then every eigenvalue of D has magnitude at most unity." Furthermore, Roberts (222) concludes that "a digraph D is value stable under all simple pulse processes if, and only if, D is pulse stable under all simple pulse processes and unity is not an eigenvalue of D." Pulse stability implies that the pulses entering any vertex are bounded over time, whereas value stability implies that the value at any vertex is bounded over time. Therefore, it becomes clear that if the pulses entering every vertex approach zero, then the digraph should be value stable as well as pulse stable. And, as indicated in Roberts' second theorem, a digraph must be pulse stable before it can be considered value stable. Since, the eigenvalues obtained for the ecosystem of mice, rats, and eagles had magnitudes of 0 and $\sqrt{2}$, and $\sqrt{2} > 1$, by Roberts' first theorem the ecosystem is pulse unstable and is therefore value unstable. This implies that the value at some vertex in the digraph will grow without bound over time.

## Stabilizing Strategies

When a system is found to be unstable, the next step in studying that system involves finding a way to restructure the digraph model for the system in order to induce stability into the system. Unfortunately, no direct method exists for restructuring an unstable digraph into a stable digraph. In general, however, the following strategies have been used to introduce stability on unstable digraphs:

1) Changing the value of a certain vertex at a given time.
2) Adding a vertex to the existing digraph and new edges from this vertex to existing vertices.
3) Changing the sign on a given edge.
4) Changing the weight on a given edge.
5) Adding a new edge to the digraph.

Furthermore, if by employing trial and error methods, one of the above strategies produces a new, pulse and value stable digraph, then corresponding structural changes could be made to the actual real world situation to ensure the stability of that system over time.

After some work, I found that by employing strategy 5, the unstable ecosystem could be restructured into a stable ecosystem by adding a positive loop from the eagle's vertex to the eagle's vertex. The new adjacency matrix then equals

$$\begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

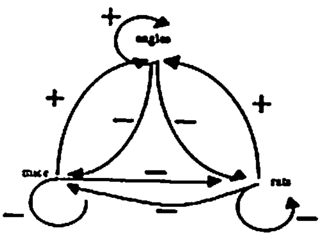


Figure 5 (Roberts, 215)

The eigenvalues associated with this new digraph are 0 and -1, which have respective magnitudes 0 and 1. Since, these eigenvalues have magnitudes at most one, the previous theorem regarding pulse stability gives us no information. Howver, the following theorem, also by Roberts (221), will. "Suppose D is a signed or weighted digraph with all nonzero eigenvalues distinct. If every eigenvalue has magnitude at most unity, then D is pulse stable under all simple pulse processes." Applying this theorem to the new model implies that the ecosystem is pulse stable. Furthermore, since none of its eigenvalues equal 1, by the earlier theorem regarding value stability, this ecosystem is value stable. Thus, this stabilizing strategy has restructured the ecosystem of mice, rats, and eagles to induce stability in the model.

In general, after finding a stable digraph model, the problem then becomes one of finding a way to change the structure of the real world system to fit the model. In the case of the ecosystem, suppose the ecosystem existed within a national park. This would encourage the eagle population to become more productive, warranting the addition of a positive edge to the digraph from the eagles' vertex back to the eagles' vertex.

Applications of Graph Theory

In conclusion, Frank Harary points out in "Graph Theory as Applied Math" in The Journal of Graph Theory (Fall, 1986, iv) that "graph theory serves as the mathematical model for structure in any field." It is conceivable that for any real-world system warranting study, there exists some graph theoretic technique for modelling the situation that would help to make the problem clear and precise. Furthermore, if no technique currently exists, some new method for modelling that situation may be discovered. For, as Roberts states in the introduction to Discrete Mathematical Models (ix), "The interaction between mathematics and any field of application goes two ways

... mathematics can be applied to the field ... or the field can be applied to mathematics." So, as with any area of mathematics, graph theory will continue to expand to fit the needs of old and new technology. And, mathematicians will continue to study the effect that pulse processes have on the stability of structures and will attempt to find specific methods for inducing stability on an unstable structure.

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# A BRIEF INTRODUCTION TO THE PRINCIPLE OF INCLUSION AND EXCLUSION

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## 1. PROBLEM 'A' (PEACE-CORPS VOLUNTEERS)

Suppose that 100 American citizens have volunteered to travel to Europe with the Peace-Corps during the next year. 34 of these volunteers speak German fluently. 19 speak French fluently, and 7 speak both German and French fluently. How many of the volunteers speak neither German nor French fluently?

To answer this problem we must subtract from the 100 volunteers in question all those volunteers who speak German fluently and those who speak French fluently; this is

$$100-34-9.$$

However, we must also remember to consider those 7 volunteers who speak both German and French fluently and have been subtracted twice in the above expression; thus, the correct answer is

$$100-34-19+7,$$

$$= 54.$$

Therefore, there are 54 volunteers who speak neither German nor French fluently.

## II. NOTATION/DEFINITION OF TERMS

In general let us consider a set A with a finite number of elements N, (N is sometimes refered to as the <u>cardinal number</u> of the set A). Let $a_1$, $a_2$, $a_3,\ldots,a_n$ be the list of properties that each object may have. Let $N(a_i)$ be the number of objects of set A having property $a_i$, where $1 \leq i \leq n$. Since each object of set A may have several (or none) of the properties listed, let $N(a_i')$ be the number of objects of set A not having property $a_i$. Notice for each i, $N = N(a_i) + N(a_i')$. Since it is possible for an object to have more than one property, let $N(a_i,a_j)$ be the number of objects in set A that have both properties $a_i$ and $a_j$ where $1 \leq i \neq j \leq n$. Now $N(a_i',a_j')$ will represent the number of objects that have neither property $a_i$ nor $a_j$, and the number of objects having property $a_i$, but not property $a_j$ would be denoted $N(a_i,a_j')$. Using this same idea we can represent each possible combination of the properties

$a_1, a_2, a_3, \ldots a_n$.

## III. PROBLEM 'B' (DIVISIBILITY OF INTEGERS)

How many integers between 1 and 100 are not divisible by any of the prime numbers 2, 3, and 5?

Let $a_1$ be the property that a number is divisible by 2.

Let $a_2$ be the property that a number is divisible by 3.

Let $a_3$ be the property that a number is divisible by 5.

According to the notation that we have defined above, the set A will consist of the integers between 1 and 100, and since there are 98 such integers, $N = 98$. We are looking for $N(a_1', a_2', a_3')$, which is precisely the number of integers between 1 and 100 that are not divisible by any of the prime numbers 2, 3, and 5. To begin we must determine how many integers are divisible by 2 in our list from 2 to 99. This can be determined by noticing that every other integer from 2 to 99 contains a factor of 2. Therefore exactly 98/2 or 49 of the integers are divisible by 2. Similarly, every third integer between 1 and 100 contains a factor of 3. There are 33 such integers. Finally, to find those integers between 1 and 100 that are divisible by 5 we duplicate this process and find 19 such integers. However, as in our previous example, we must also consider those integers that have been counted twice, in other words we must calculate those integers that are divisible by both 2 and 3, or both 2 and 5, or both 3 and 5. To consider these cases we must recall that for an integer to be divisible by both 2 and 3, it must be divisible by 6. Similarly, for an integer to be divisible by both 2 and 5, it must be divisible by 10, and for an integer to be divisible by both 3 and 5, it must be divisible by 15. There are 16, 9 and 6 such integers for these three respective cases. Note there is one group of integers that we have neglected to consider, specifically those integers that are divisible by all of 2,3, and 5. Using the same method as above we can substantiate that there are 3 such integers that are divisible by 30.

We can now use this information to answer our original question. We know there are N integers to start with and we know that we must eliminate all of those integers that are divisible by 2,3, or 5. Thus according to our notation we have;

$$N-N(a_1)-N(a_2)-N(a_3),$$

but we must now take into account all of those integers that have been subtracted twice because they are divisible by two of the primes in question. We must add these integers back into our total;

$$N-N(a_1)-N(a_2)-N(a_3)+N(a_1,a_2)+N(a_1,a_3)+N(a_2,a_3).$$

Finally we must notice that we have added back into our total some integers twice. Particularly those integers that are divisible by all three of the primes. We must take these particular integers and subtract them to give us an accurate count. Therefore,

$$N-N(a_2)-N(a_2)-N(a_3)+N(a_1a_2)+N(a_1,a_3)$$
$$+N(a_2,a_3)-N(a_1,a_2,a_3)$$

is our final expression. Hence,

$$N(a_1',a_2',a_3')$$
$$= N-N(a_1)-N(a_2)-N(a_3)+N(a_1,a_2)+N(a_1,a_3)+$$
$$N(a_2,a_3)-N(a_1,a_2,a_3),$$
$$= 98 - 49 - 33 - 19 + 16 + 9 + 6 - 3,$$
$$= 25.$$

Therefore, there are 25 integers between 1 and 100 that are not divisible by any of the prime numbers 2,3, or 5.

## IV. THE MAIN THEOREM

## (PRINCIPLE OF INCLUSION AND EXCLUSION)

These two problems and the manner in which we have solved them lead us to a generalized theorem for finding the number of objects not having any of $r$ properties, where $r$ is some natural number. The general formula is called the Principle of Inclusion and Exclusion. In the form in which it shall be presented, the principle was discovered by Sylvester about 100 years ago. In an alternate form, it was discovered by DeMoivre some years earlier.

Main Theorem (Principle of Inclusion and Exclusion)

If $N$ is the number of objects in a set $A$, then the number of objects in the set $A$ having none of the properties $a_1, a_2 a_3, \ldots, a_r$ is given by:

$$N(a_1', a_2', a_3', \ldots, a_i', \ldots, a_r')$$

$$= N - \sum_1^r N(a_i) + \sum_{i \neq j}^r N(a_i, a_j) - \sum_{i,j,k \text{ different}}^r N(a_i, a_j, a) + \ldots$$

$$\ldots + (-1)^t \sum^r N(a_i, a_j, a_k, \ldots, a_v) + \ldots + (-1)^r N(a_1, a_2, a_3, \ldots, a_r),$$

$$i, \ldots, v \text{ different}$$

where the first sum is over all $i$ from $(1, \ldots, r)$, and the second sum is over all unordered pairs $(i, j)$, with $i$ and $j$ from $(1, \ldots, r)$, and $i \neq j$. The third sum is over all unordered triples $(i,j,k)$, with $i,j$ and $k$ from $(1,...r)$, and $i,j,k$ distinct. This pattern follows for any general $t^{th}$ term which is of the form $(-1)^t$ times a sum of terms of the form $N(a_i,a_j,a_k,...,a_v)$, where the sum is over all unordered t-tuples $(i,...,v)$, with $i,...,v$ from $(1,...,r)$, and $i,...,v$ distinct. The last term will always be $N(a_1,a_2,a_3,...,a_r)$, since there is only one possible unordered r-tuple $(1,...,r)$ for the final term.

## V. PROOF OF THE MAIN THEOREM

The proof of our theorem follows from a fairly simple concept. Since the left hand side of the theorem counts the number of objects in set A that contain none of the properties $a_1,a_2,a_3,...,a_r$, we shall simply show that the right hand side of the theorem counts the number of objects having none of the properties exactly once, and counts the number of objects having one or more of the properties exactly zero, or no times.

First let us suppose that an object has none of the properties $a_1,a_2,a_3,...,a_r$. Then it is counted only once in determining N and is not counted anywhere else in the theorem, (by definition it cannot be counted by $\Sigma\ N(a_i)$, $\Sigma\ N(a_i,a_j)$, or so on) thus, it is counted only once on the right hand side. (*For the second half of this proof we will reduce the complication of the counting process by using a more sophisticated manner of counting objects which the reader may recognize as combinations.*) Now let us suppose an object has one or more of the properties $a_1,a_2,a_3,\ldots,a_r$. Let p be the number of properties this object has where $1 \leq p \leq r$. As before, the object is counted one time, (denoted by $\begin{bmatrix}p\\o\end{bmatrix}$) in determining N. It is counted exactly p times, (denoted by $\begin{bmatrix}p\\1\end{bmatrix}$) by the summation $N(a_i)$ for each property $a_i$ it has. It is counted $\begin{bmatrix}p\\2\end{bmatrix}$ times by the summation $N(a_i,a_j)$ for every possible pair of properties $a_i$ and $a_j$ that it has. Similarly it is counted $\begin{bmatrix}p\\3\end{bmatrix}$ times by the summation $N(a_i,a_j,a_k)$, and this pattern continues for the number of times an object with p properties is counted, all the way to the final time when it is counted $\begin{bmatrix}p\\p\end{bmatrix}$ times by $N(a_1,a_2,a_3,\ldots,a_p)$. All together the number of times this object is added on the right hand side of the theorem is represented by:

$$\begin{bmatrix}p\\0\end{bmatrix} - \begin{bmatrix}p\\1\end{bmatrix} + \begin{bmatrix}p\\2\end{bmatrix} - \begin{bmatrix}p\\3\end{bmatrix} + \ldots + (-1)^p \begin{bmatrix}p\\p\end{bmatrix},$$

$$= \sum_{k=o}^{p} \begin{bmatrix} p \\ k \end{bmatrix} (-1)^k (1)^{p-k}, \text{(by applying the binomial expansion)}$$

$= (-1 + 1)^p,$

$= 0.$

Thus, an object with one or more of the properties $a_1, a_2, a_3, \ldots, a_r$ is counted zero times total. We have shown what we intended and the theorem is proved. □

I would like to conclude by looking at one final example which is a famous problem, and illustrates the use of the Principle of Inclusion and Exclusion just proven.

## VI. PROBLEM 'C' (THE HAT-CHECK PROBLEM)

Suppose that n people attend a party and check their hats at the door. During the course of the evening the doorman has too much to drink and forgets which hat belongs to which person. At the end of the evening the doorman returns the hats to the guests at random. What is the probability that no person receives their own hat back?

Let A be the set of all possible combinations of ways that the hats can be returned, and N is the size of A. Then N = n!, since there are n people to choose from in returning the first hat, n-1 people to choose from in returning the second hat, n-2 people to choose from in returning the third hat, and so on. To calculate each of $\Sigma N(a_i)$, $\Sigma N(a_i,a_j)$, $\Sigma N(a_i,a_j,a_k)$,... we use the combination of n items chosen t at a time, multiplied by $(n-t)!$, where t is the number of $a_y$'s for that particular term. This gives us a permutation for each t term which represents the number of possible combinations of ways that the hats can be returned so that t particular people will receive their respective hats back. To determine the number of possible combinations of ways that the hats can be returned so that no person receives their own hat back we know by the Principle of Inclusion and Exclusion that: $N(a_1', a_2', a_3', \ldots, a_t', \ldots, a_n')$

$$= n! - \begin{bmatrix} n \\ 1 \end{bmatrix}(n-1)! + \begin{bmatrix} n \\ 2 \end{bmatrix}(n-2)! - \begin{bmatrix} n \\ 3 \end{bmatrix}(n-3)! + \ldots$$

$$\ldots + (-1)^t \begin{bmatrix} n \\ t \end{bmatrix}(n-t)! + \ldots + (-1)^n \begin{bmatrix} n \\ n \end{bmatrix}(n-n)!,$$

$$= n! - \frac{n!}{1!} + \frac{n!}{2!} - \frac{n!}{3!} + \ldots + (-1)^t \frac{n!}{t!} + \ldots + (-1)^n \frac{n!}{n!},$$

$$= n![1-1+\frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^t \frac{1}{t!} + \ldots + (-1)^n \frac{1}{n!}],$$

$$= n![\frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^t \frac{1}{t!} + \ldots + (-1)^n \frac{1}{n!}].$$

Therefore, the probability that no person receives their own hat back is $N(a_1',a_2',a_3',\ldots,a_t',\ldots a_n')$ divided by the number of possible ways that the hats can be returned, which is n!. This gives a probability of:

$$\frac{n!\left[\frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^t \frac{1}{t!} + \ldots + (-1)^n \frac{1}{n!}\right]}{n!}$$

$$= \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^t \frac{1}{t!} + \ldots + (-1)^n \frac{1}{n!} .$$

Notice this answer for the probability that no person receives their own hat back is dependent on the value of n, but after considering some of the smaller cases one may observe that the above answer converges rapidly to $\frac{1}{e}$ as n approaches infinity. In fact even for n as small as 7, the probability only differs from $\frac{1}{e}$ in the fifth decimal place.

I would like to recognize and acknowledge the contributions of three persons to the construction of this paper.

To begin, I highly recommend the book Applied Combinatorics, by Fred S. Roberts. From it I based my notation for and construction of the Principle of Inclusion and Exclusion. Also, I frequently referred to its context to verify information that was used in the writing of this paper.

Secondly, Professor K. Smith, whose class in applied combinatorics brought about my interest in this topic, and led me to the "hat-check problem" on derangements.

Finally, Professor A. Hammel, KME Michigan Beta chapter advisor/corresponding secretary, who encouraged me to write this paper and submit it to the KME selection committee for judging, regarding the possibility of its presentation at the 26th Biennial Convention.

I am very appreciative to these three mathematicians for their influence and support of my work during the writing and submitting of this paper.

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# MOTIONS AND MATRICES

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## Section I. Introduction

The fundamental concept of Euclidean geometry is congruence. In plane geometry the basic theorems deal with congruence of triangles. These theorems are established by superimposing one triangle on another. This means that one triangle is moved until it coincides with the other. In this paper the motions that produce congruent triangles will be discussed within the framework of modern and linear algebra.

In the next section the motions will be defined and some of their properties discussed. The object is to develop the group concept by way of transformations and matrices.

In the third section the concept of an abstract group is defined. The associated ideas of subgroup and commutative group will also be introduced.

In the final section it will be shown that the group concept can be used to give a very general definition of geometry. This approach was due to

Felix Klein and is known as the Erlanger Program. It was once thought that all of geometry could be included in this way. It is now known that this is not the case. Nevertheless, it will be seen to be an enlightening way to approach the subject.

Section 2. The Motions

The motions in the plane that preserve congruence of plane figures are rotation of the points in the plane about a fixed point called the center of rotation, reflection of the points in the plane about a fixed line, translation of the points in the plane a certain fixed distance in a fixed direction, and motions formed by a finite sequence of the previous motions in succession.

The first two types of motions are one-to-one linear transformations of the plane onto itself. In algebraic form, this method means that they can be expressed as:

$$\begin{aligned} x' &= a_{11}x + a_{12}y \\ x' &= a_{21}x + a_{22}y \end{aligned} \qquad \text{where} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0.$$

The linear trnsformation then maps the point $(x,y)$ onto the point $(x',y')$. Such a linear transformation may be written in the form

$X' = AX$ where $X' = \begin{pmatrix} x' \\ y' \end{pmatrix}$, $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$,

and $X = \begin{pmatrix} x \\ y \end{pmatrix}$. The condition $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ ,means that $A^{-1}$ exists. If the transformations are done in succession, then $X' = AX$, $X'' = BX'$ and the resulting transformation is $X'' = (BA)X$ which amounts to a change of variable in the original equations by substitution.

Rotation of the plane about the origin yields the equations

$$x' = x\cos w - y\sin w$$

. See Figure 1.

$$y' = x\sin w + y\cos w$$

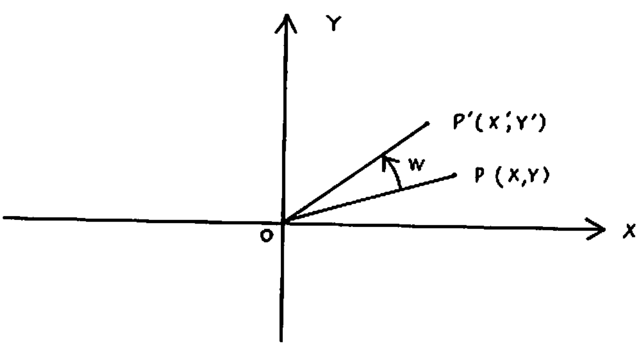


Figure 1.

It is a convenient shorthand to write $R(w) = \begin{pmatrix} \cos w & -\sin w \\ \sin w & \cos w \end{pmatrix}$ . R(w) is called the matrix of the transformation. R(W) has the following properties:

1) $R(w)R(t) = \begin{pmatrix} \cos w & -\sin w \\ \sin w & \cos w \end{pmatrix} \begin{pmatrix} \cos w & -\sin t \\ \sin t & \cos t \end{pmatrix}$

$$= \begin{pmatrix} \cos w \cos t - \sin w \sin t & -(\sin w \cos t + \cos w \sin t) \\ \sin w \cos t + \cos w \sin t & \cos w \cos t - \sin w \sin t \end{pmatrix}$$

$= \begin{pmatrix} \cos(w+t) & -\sin(w+t) \\ \sin(w+t) & \cos(w+t) \end{pmatrix}$. That is, these matrices are closed under multiplication. Geometrically interpreted, this means that a rotation about 0 followed by a second rotation about 0 is another rotation about 0.

2) The set of all n x n matrices obeys the associative law of multiplication. Here, n = 2. The rotation matrices are closed under multiplication and so it follows that they also obey the associative law of multiplication.

3) $R(o) = I_2$, the multiplicative identify for 2 x 2 matrices.

4) R(-w) is the multiplicative inverse of

$R(w)$ since by (1) $R(w)R(-w) = R(-w)R(w) = R(o) = I_2$. That is, each rotation matrix has a multiplicative inverse which is a rotation matrix.

These four properties are the theme of this section. If the family of motions is chosen in the correct way, it will be discovered to have these four properties.

The matrix of the transformation that reflects points about a fixed line through 0 is $L(w) = \begin{pmatrix} \cos 2w & \sin 2w \\ \sin 2w & -\cos 2w \end{pmatrix}$. See Figure 2.

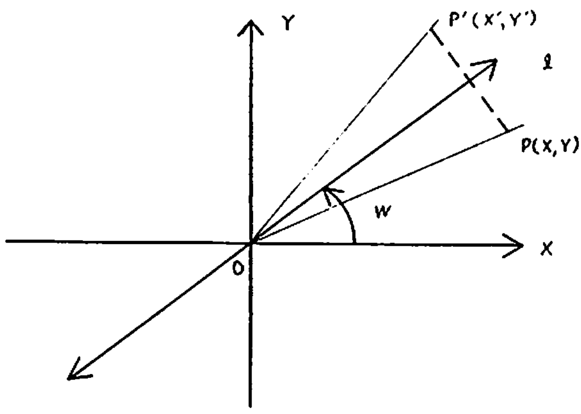


Figure 2.

If one reflection is followed by another, it follows that $L(\frac{w}{2})\ L\ (\frac{t}{2}) = R\ (w-t)$. That is, a reflection followed by a reflection is a rotation. This family of motions will not yield the above four properties.

However, all is not lost! It is an easy computation to verify that if A is the matrix of a rotation about 0 or a reflection about a line through 0, then $A^tA = I_2$ where $A^t$ is the transpose of A. The four properties mentioned above hold for this family of motions since:

1) If A and B are any two such matrices, then $(AB)^t(AB) = (B^tA^t)\ (AB) = B^t\ (A^tA)B =$ $B^t\ (I_2)B = B^tB = I_2$.
2) As above, the associative law holds.
3) $I_2^tI_2 = I_2$.
4) If $A^tA = I_2$, then $A^{-1}(A^{-1})^t = I_2$ since $(A^{-1})t = (A^t)^{-1}$ so that $A^{-1}$ is a member of the family if A is.

   Moreover, it is not difficult to show that if $A^tA = I_2$, then A is either a matrix of a rotation or a reflection of the kind mentioned above.

For translations of the plane the equations are

$x' = x + h$ See Figure 3.
$y' = y + k$.

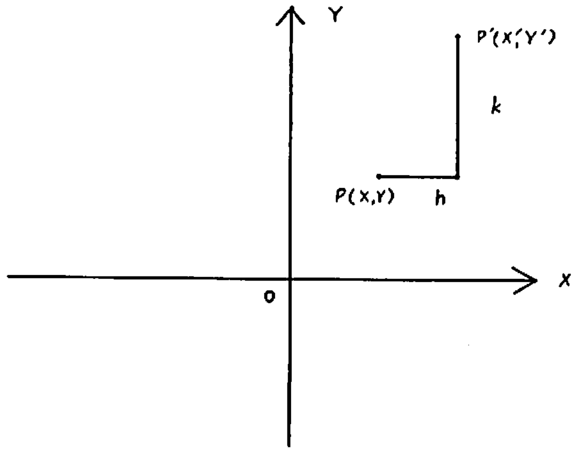


Figure 3.

Translations are not linear transformations, of course, but they can be cast in the form of a matrix equation: $X' = I_2X + K$ where $K = \binom{h}{k}$. This equation can also be written as follows by taking transposes of all the matrices:

$$(X')^t = X^tI_2 + K^t.$$

A convenient matrix for the second equation, which is equivalent to the first is $T(K) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ h & k & 1 \end{pmatrix}$ .

Again, the multiplication of matrices corresponds to a change of variable by substitution. This is the algebraic equivalent of one translation followed by another. It is now easy to verify that the four properties hold:

1) $T(K) \cdot T(L) = T(K+L)$.

2) As above, the associative law holds for this family of motions.

3) $T(0) = I_3$.

4) $[T(K)]^{-1} = T(-K)$ so that $[T(K)]^{-1}$ is a translation matrix.

It is now an easy matter to combine all of the above motions into one equation. This equation is $X' = AX + K$ where $AA^t = I_2$. The equivalent transposed equation is $(X')^t = X^tA^t + K^t$.

As for translations, a convenient matrix is $E(A,K) = \begin{pmatrix} A^t & 0 \\ K^t & 1 \end{pmatrix}$. The usual properties hold:

1) $E(A,K)\, E(B,L) = E(BA, BK + L)$ so that the family is closed under multiplication.

2) Hence, as usual, the association law holds for this family of motions.

3) $E(I_2, 0) = I_3$.

4) The inverse of $E(A,K)$ is $[E(A,K)]^{-1} =$
$E(A^{-1}, -A^{-1}K) = \begin{pmatrix} (A^t)^{-1} & 0 \\ -K^t(A^t)^{-1} & 1 \end{pmatrix}$.
Hence, $[E(A,K)]^{-1}$ belongs to the family.

It should be observed that this last family includes all of the other families with these four properties. It includes the translations if $A = I_2$, the second family is $K = 0$, and the rotations if $K = 0$ and $A = R(w)$ for some $w$.

There is one more question to address: What happens if the rotations are about some point other than the origin and the line of reflection does not pass through the origin? The amazing simple answer is that the $K$ in $X' = AX + K$ can be interpreted to mean translation of coordinates (not the plane). So a translation is made to the desired point as origin and the motions resumed.

Section 3. <u>Groups</u>

The above families of motions in the plane suggest that some general concept is illustrated by them. The concept is that of an abstract group. An abstract group is a non-empty set with an operation, $\cdot$, that has the following properties:

1) Closure: If $a \in S$ and $b \in S$, then $a \cdot b \in S$.
2) Associativity: For all $a, b, c \in S$, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
3) Identity: There is a unique $i \in S$ such that $a \cdot i = i \cdot a = a$.

4) Inverse: For each $a \in S$, there is a unique $a^{-1} \in S$ such that $a \cdot a^{-1} \cdot a = i$.

If, in addition, $a \cdot b = b \cdot a$, the group is said to be commulative. In the above S is an appropriate set of matrices and $\cdot$ denotes matrix multiplication.

These matrix groups have names. The group of rotations is written as SO(a) and is called the special orthogonal group of 2x2 matrices. The set of all 2x2 matrices such that $AA^t = I_2$ is written as O(2) and is called the orthogonal group of 2x2 matrices. Notice that O(2) includes SO(2) as a proper subset. The translations form what is called the translation group. The last group is called the Euclidean group and includes the others as proper subsets (making due allowance for the change from 2x2 to 3x3 matrices). Any non-empty subset of a group that is a group with respect to the same operation is called a subgroup of the given group. Thus, O(2), SO(2), and the translation group are subgroups of the Euclidean group. The Euclidean group and O(2) are not cummulative but the translation group and SO(2) are.

## Section 4. The Erlanger Program

Plane Euclidean geometry may now be characterized by the group concept. The three essential elements in this description are the plane, a group of one-to-one mappings of the plane onto itself which are called transformations (the group itself is called a transformation group) and a defining property of the plane that is left unchanged by the elements of the transformation group. This defining property is, of course, congruence of plane figures. Another way of expressing the same thing is to say that the distance between any two points is unaffected by elements of the transformation group. The transformation group is the Euclidean group which until now has been the central topic of discussion. The fact that the distance between any two points is left unchanged by the elements of the Euclidean group if expressed as follows: the distance between any two points in the plane is an invariant of this group. As stated at the outset, this idea can be greatly generalized. This generalization is known as Felix Klein's Erlanger Program.

Before proceeding to a description of the Erlanger Program, it may interest the reader to know something of its historical development. The nineteenth century was a time of immense creativity in geometry. It has been called the Heroic Age of Geometry.

Until the nineteenth century geometry meant either synthetic or analytic Euclidean geometry. But now a great revolution was to occur. There is now not one geometry but many. Among them are projective geometry, inversive geometry, n dimensional geometries, and non-Euclidean geometries. These are not mutually exclusive categories.

Projective geometry was the joint effort on the part of mathematicians in France and Germany. Brianchon discovered the dual of Pascal's theorem, Poncelet introduced ideal and imaginary points into geometry and claimed to have discovered the principle of duality, while Chasles emphasized the cross ratio of four collinear points or four concurrent lines. In Germany, von Staudt constructed a projective geometry lacking reference to number or magnitude. Julius Plücher, the most prolific of the geometers of the analytic school, introduced abbreviated notation and rediscovered homogeneous coordinates. He also formulated the analytic version of the principle of duality.

Inverse geometry was the work of Jacob Steiner who has been called the greatest geometer of modern times. He did not publish his results in this area, and they were rediscovered several times by others, including Lord Kelvin. Steiner's work on inverse geometry was generalized by Cremona into the theory of Cremona transformations.

Geometries of n dimensions were investigated by Cayley, Grassman, and Riemann. Arthur Cayley was primarily an algebraist but he invented a geometry of n dimensions using determinants. Hermann Grassman introduced an n dimensional vecter analysis which he called a "calculus of magnitudes". G.F.B. Riemann, in his 1854 lecture "On the Hypotheses which lie at the Foundation of Geometry" saw geometry as the study of n dimensional manifolds. His geometry was really the study of curved metric spaces and eventually made possible the theory of relativity.

Riemann's geometry is non-Euclidean. It was preceded in the 1920's by the hyperbolic geometry of Bolyai and Lobochevsky. It was non-Euclidean in a different sense in that it was not based simply on a denial of the parallel postulate but was an n dimensional geometry.

The nineteenth century also witnessed the beginnings of group theory. This theory has its sources in number theory, the theory of equations, and geometry. At the time that Klein was Plucker's assistant, the subject had become one of considerable importance.

In his famous inaugural address at Erlangen in 1872 when he became a professor, he proposed to characterize all geometries in terms of groups. He proposed to do for every geometry what has been done here for plane Euclidean geometry. The three necessary ingredients to accomplish this task are a non-empty set, S; a set of one-to-one mappings of S onto itself, called tranformations; and the invariants of the group of transformations.

For example, in affine geometry length and area are not invariants but the property that a conic is of a given type is. In projective geometry the property of a curve being a conic (not of a specified type) and the cross ratio are invariants. If point set topology is viewed as geometry, then its invariants are those properties preserved by bicontinuous one-to-one maps of the topological space onto itself (homeomorphisms). For example, compactness and connectness are topological invariants.

Klein was seeking to unify all of the different geometries of his time. He did not succeed. The geometry for which the group theoretic approach would not work is the Riemannian Geometry of Einstein's General Theory of Relativity. The grandeur of his conception is our heritage.

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# THE PROBLEM CORNER
## EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1989. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1989 issue of THE PENTAGEON, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 175 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROBLEMS 414-419 SOLUTIONS 402, 403, 406-409

Problem 414: Proposed by the editor.

Let S denote sum of the infinite series

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{8} + \frac{1}{10} + \ldots$$ in which each

denominator contains no prime factor except 2 or 5. What is the value of S?

Problem 415: Proposed by the editor.

Find two or more sets of twenty-three consecutive natural numbers such that the sum of their squares is itself the square of a natural number.

Problem 416: Proposed by the editor.

Find r and s such that $rx^{15} - sx^{14} + 1$ is divisible by $x^2 - x - 1$.

Problem 417: Proposed by the editor.

Find two or more positive integers k such that $3^k$ terminates in k or show that none exist.

Problem 418: Proposed by the editor.

Consider the sets $\{1\}$, $\{4,9,16\}$, $\{25,36,49,64,81\}$, $\{100,121,144,169,196,225,256\}$ . . . in which each set contains two more consective squares than the preceding set. Find a formula for the sum of the members of the nth set.

Problem 402: Proposed by the editor.

Evaluate the product

$$\prod_{k=1}^{n} \cos \frac{k\pi}{2n+1} .$$

Solution by the proposer.

Consider the 2n + 1 th roots of unity excluding 1. then

$$\frac{x^{2n} - 1}{x - 1} = \prod_{k=1}^{2n} \left(x - \cos \frac{2k\pi}{2n+1} + i \sin \frac{2k\pi}{2n+1}\right). \qquad (1)$$

Taking x = -1 in (1) and applying the standard double angle formulas $\cos 2a = 2 \cos^2 a - 1$ and $\sin 2a = 2 \sin a \cdot \cos a$, we obtain,

$$1 = \prod_{k=1}^{2n} \left(-2\cos\frac{k\pi}{2n+1}\right) \prod_{k=1}^{2n} \left(\cos\frac{k\pi}{2n+1} + i\sin\frac{k\pi}{2n+1}\right). \quad (2)$$

But

$$\prod_{k=1}^{2n} \left(\cos\frac{k\pi}{2n+1} + i\sin\frac{k\pi}{2n+1}\right) = \cos n\pi + i\sin n\pi = (-1)^n. \quad (3)$$

Then since $\cos\dfrac{(2n+1-k)\pi}{2n+1} = -\cos\dfrac{k\pi}{2n+1}$, (4)

we can apply (3) and (4) to simplify (2) to obtain

$$1 = (-1)^n \cdot (2)^{2n} \cdot (-1)^n \prod_{k=1}^{n} \left(\cos\frac{k\pi}{2n+1}\right)^2 .$$

Finally since $\cos\dfrac{k\pi}{2n+1} > 0$ for $k = 1, 2, \ldots, n$, we have

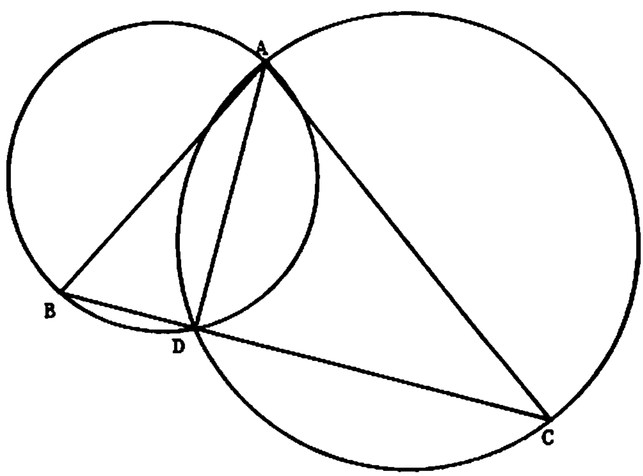
$$\prod_{k=1}^{n} \cos\frac{k\pi}{2n+1} = 2^{-n}.$$

<u>Editor's Comment:</u> Bob Prielipp found a solution to this problem on pp. 337-338 of <u>The USSR Olympiad Problem Book</u> by D.O. Shlarsky, N.N. Chentzov, and I.M. Yaglom. This book was published by W.H. Freeman and Company, San Francisco and London, in 1962.

Problem 403: Proposed by the editor.

Young Eulcid pondered a triangle in which one side is 12 feet longer than another. The angle formed by these two sides is $55^{\circ}$. If two circles are drawn with these respective sides as diameters, one of the points of intersection of the circles is the common vertex where these two sides meet. What is the locus of the other point of intersection?

Solution: W.L.O.G. let the circles be drawn with sides AB and AC as diameters as shown in the figure. Then the required locus of the other point of intersection of the circles lies on the third side BC; in fact, the other point of intersection of the circles coincides with the foot of the altitude from A to the side BC.



Proof: Since both circles pass through vertex A of the triangle, let D denote the other point where the circles intersect. Then since angle ADB is inscribed in a semicircle, it is a right angle. The same is true of angle ADC. Thus BDC is the sum of two right angles and must be a straight line. Hence D lies on the line BC and D must be the foot of the altitude from A to side BC. The result holds regardless of the size of angle A.

Problem 406: Proposed by Bob Prielipp, University of Wisconsin-Oshkosh.

Let a, b, and c be the lengths of the sides of a triangle. Prove that $a^4 + b^4 + c^4 < 2(a^2b^2 + a^2c^2 + b^2c^2)$.

Solution by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.

Since a, b and c are the sides of a triangle, we have

$$b + c > a, \quad c + a > b \quad \text{and} \quad a + b > c.$$

Thus,

$$b + c - a > 0, \quad c + a - b > 0 \quad \text{and} \quad a + b - c > 0.$$

Also since $a + b + c > 0$, we get,

$$(a + b + c)(b + c - a)(c + a - b)(a + b - c) > 0. \quad (1)$$

Multiplying the factors on the left side of (1) and simplifying the result yields the desired inequality.

Also solved by the proposer (two solutions).

Problem 407: Proposed by the editor.

Prove that the median of a triangle is less than the arithmetic mean of the two adjacent sides of the triangle.

Composite of solutions submitted by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin and Fred A. Miller, Elkins, West Virginia.

It is known [1] that if $m_a$ is the median to side a, then

$$m_a^{\,2} = \frac{1}{2}(b^2 + c^2) - \frac{1}{4}a^2 \qquad (1)$$

To complete the solution is suffices to establish that

$$\frac{1}{2}(b^2 + c^2) - \frac{1}{4}a^2 < \left(\frac{b + c}{2}\right)^2. \qquad (2)$$

Each of the following inequalities is equivalent to (2).

$$2b^2 + 2c^2 - a^2 < b^2 + 2bc + c^2$$

$$(b - c)^2 < a^2$$

$$|b - c| < a$$

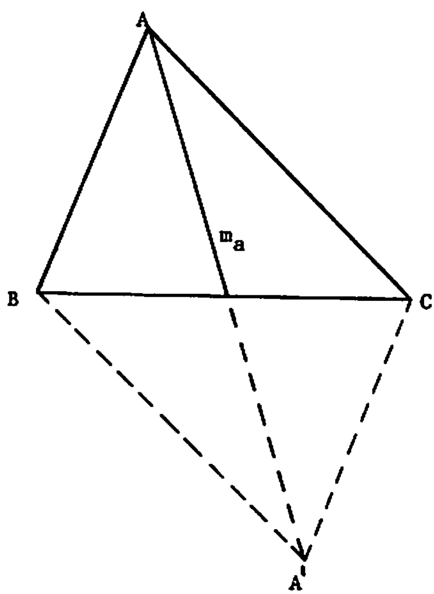
$$-a < b - c < a$$

$$c < a + b \quad \text{and} \quad b < a + c. \qquad (3)$$

Then because we have a triangle, the statement (3) holds and the desired result follows immediately.

[1] Davis, Modern College Geometry, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts (1949) p.60 .

Editor's Comment: Consider the triangle ABC in which $m_a$ denotes the median from the vertex A. Another solution can be obtained by reflecting the triangle through the opposite side BC. The resulting figure is a parallelogram having one diagonal equal to twice the median $m_a$. Then the desired result follows immediately from the triangle inequality.




Problem 408: Proposed by the editor.

Dirty Dan had a hot tip on the dog races. He knew that one of four longshots would win the race. If the odds on these four dogs are 3 to 1, 5 to 1, 6 to 1 and 9 to 1 respectively, how much should Dirty Dan bet on each of these four dogs to guarantee making a profit of \$143?

Since no solution has been received, this problem will remain open. If no solution is received, a solution will be supplied by the editor.

Problem 409: Proposed by the editor.

Find one or more solutions in positive integers of the following system of equations:

$$x^2 + 13y^2 = z^2 \quad \text{and} \quad x^2 - 13y^2 .$$

Solution by William D. Nolte Jr., Mount Mercy College, Cedar Rapids, Iowa.

Clearly z can not be zero because that would require either $x^2$ or $y^2$ in the first equation to be negative. Similarly $x^2$ can not be zero because $z^2$ would be negative in the second equation. Finally, subtracting the second equation from the first yields $13y^2 = 0$ whence $y = 0$ contrary to the conditions of the problem. Thus the given system of equations has no solution. If $y = 0$ is allowed, then we have the infinite family of solutions $(x,y,z) = (1,0,1)$, $(2,0,2)$, $(3,0,3)$, etc.

Also solved by Bob Prielipp, University of Wisconsin- Oshkosh, Oshkosh, Wisconsin.

Editor's Comment: This problem results from an amusing "typo":

The problem was originally intended to require the solution of the system of equations $x^2 + 13y^2 = z^2$ and $x^2 - 13y^2 = t^2$. The smallest solution in positive integers of the system is $x = 10629$, $y = 19380$, $z = 127729$, and $t = 80929$. There are an infinite number of solutions of this system.

# KAPPA MU EPSILON NEWS

Edited by M. Michael Awad

News of chapter activities and other noteworthy KME events should be sent to Dr. M. Michael Awad, Historian, Kappa Mu Epsilon, Mathematics Department, Southwest Missouri State University, Springfield, MO 65804.

## REPORT ON THE 1988 REGION III CONVENTION

Eastern Kentucky University hosted the Regional Convention jointly with the Fourth Annual Eastern Kentucky University Symposium in the Mathematical Sciences on March 24–25, 1988.

## REPORT ON THE 1988 REGION IV CONVENTION

Two Regional Conventions were held in Region IV. Kansas Alpha hosted a Regional Convention on March 25, 1988. Seven Chapters were represented (Missouri Alpha, Missouri Beta, Missouri Theta, Kansas Alpha, Kansas Beta, Kansas Delta and Kansas Gamma). There were about nine faculty members and 35 student members in attendance. Five papers were presented. Each paper received a certificate, three papers received awards, and one paper received honorable mention.

Iowa Alpha hosted the Second Regional Convention on April 23, 1988. Six papers were presented. Each paper received a certificate; three papers received awards.

## CHAPTER NEWS

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President – Donald Priest
20 actives, 20 initiates

The Chapter assisted the Mathematics Department with the annual Cal Poly Mathematics Contest which attracted over 500 students to the campus. Weekly meetings featured speakers from business and industry. fund raising events included the annual KME Book Sale (of books donated by faculty of the Mathematics, Statistics and Computer Science Departments) in January and the sale of Mathematics Department/Math Contest tee–shirts during Poly Royal in April. The annual Banquet was held on May 13, 1988. The invited dinner speaker was Kent Hoyt of Hewlett–Packard, A Cal Poly alumnus and former President of the California Gamma Chapter. Donald Priest was the recipient of the Arthur Andersen & Co. Professional Performance Award. Erik Harder was the recipient of the Founders' Award. Joseph Seebach, a fall quarter pledge, was the recipient of the W. Boyd Judd Scholarship. The Banquet was attended by Dr. and Mrs. Judd and by Mrs. Grace Warten who presented the Founders' Award. Some officers include: Jerry Burch, 2nd vice president; Marie Guevara, pledgemaster; Sarah Parks, publicity representative; Rachel Jeffries, social director; Lisa Fjeldal, alumni representative; Joni Otoshi, school council representative; Chris Lucke, Poly Royal representative; Lonnie Smith, curriculum committee representative. Other 1988–89 officers: Susan Daijo, vice president; Stefan Steiner, treasurer; Raymond D. Terry, corresponding secretary and faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins
Chapter President – Thomas Painter
8 actives, 16 initiates

Other 1988–89 officers: Ruth Lindquist, secretary; Arne Magnus, corresponding secretary and faculty sponsor.

Colorado Gamma, Fort Lewis College, Durango
Chapter President – Amy Getz
21 actives, 10 initiates

The high school tutoring program was continued throughout the spring. A second edition of the KME–Math/Cs Club Newsletter was published. Other 1988–89 officers: Earl Edwards, vice president; Carol Kjar, secretary; Kevin Marushack, treasurer; Richard Gibbs, corresponding secretary and faculty sponsor.

Connecticut Beta, Eastern Connecticut State University, Willimantic
Chapter President – Robert Rossow

Connecticut Beta held four meetings during the semester, offered a math tutoring service, and attended an end of the year banquet. Other 1988–89 officers: Joseph Gregorio, vice president; Lynne Croteau, secretary and treasurer; Sally Keating, corresponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President – Lynn Harris
30 actives, 12 initiates

On April 29, 1988, the KME actives assisted with the annual Math Day activities as area high schools sent teams of their best students to visit the WGC campus. On May 31, 1988, we initiated twelve new members and had a reception in their honor. At the reception it was announced that two KME members will receive major academic scholarships for next year: Tracy Tepp received the David Cooley Scholarship and Lynn Harris received the Marion Crider Scholarship. Other 1988–89 officers: Tracy Tepp, vice president; Anne Salchow, secretary; Tammy Gresham, treasurer; Joe Sharp, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston
Chapter President – Tricia Setzke
39 actives, 14 initiates

There were a number of KME/Math Club meetings held throughout the semester. Our initiation of new members was held on April 14 and the KME Honors Banquet took place on Sunday, April 17. The semester's activities ended with the spring picnic in Morton Park. Other 1988–89 officers: Wayne Watkins, vice president; Rita Stinde, secretary; Melissa Tracy, treasurer; Lloyd Koontz, corresponding secretary and faculty sponsor.

Illinois Delta, College of St. Francis, Joliet
Chapter President – Jo Ann Lopykinski
30 actives, 17 initiates

We hosted the Region II Conference here on March 18–19th, 1988. Several members attended the NCTM Convention held in Chicago April 5–9th. The speaker at our Initiation was Gail Digate, Executive Director for the Corridor Partnership for Excellence in Education. Other 1988–89 officers: Pamela Damore, vice president; Rita Drab and Debra Becker, secretary/treasurer; Sister Virginia McGee, corresponding secretary and faculty sponsor.

Illinois Zeta, Rosary College, River Forest
Chapter President – Natalie Perri
21 actives

We participated in a Walk–a–Round to raise funds for the school union; a bake sale was also held for this purpose. The Great America amusement park was the site of our Club outing. A free tutoring service was provided for students requiring help. Other 1988–89 officers: Mark Crosbie and Chad Husting, vice presidents; Mariola Janek, secretary; Kathy Schmidt, treasurer; Mordechai S. Goodman, corresponding secretary and faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
Chapter President – Julie Eichenauer
24 actives, 11 initiates

The Spring Banquet and Initiation were held April 14, 1988. An address was given by Dr. David Neuhouser, Taylor University, entitled "Beauty and Perfection in Mathematics. Other 1988–89 officers: Jenny Newton, vice president; Cindy Bull, secretary; Lauri Robison, treasurer; Ralph B. McBride, corresponding secretary; Debbie Huston, faculty sponsor.

Indiana Beta, Butler University, Indianapolis
Chapter President – Blayne Carroll
20 actives, 7 initiates

Other 1988–89 officers: Gregory Francis, vice president; Rita Fuller, secretary; Prem Sharma, treasurer; Jeremiah Farrell, corresponding secretary and faculty sponsor.

Indiana Delta, University of Evansville, Evansville
Chapter President – Jennifer Seckinger
32 actives, 20 initiates

We met monthly (four times) and we had a speaker at each meeting. Our last meeting coincided with our Initiation and Banquet. Other 1988–89 officers: Laura L. Locke, vice president; Mary Singleton, secretary; Melba Patsberg, corresponding secretary; Mohammad K. Azarian, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President – Suzanne Buckwalter
34 actives, 6 initiates

The major event for Iowa Alpha Chapter during the spring semester was hosting the KME Region IV Convention on April 22–23, 1988. The Convention program featured six student papers including two from our chapter: "Two Heads in Succession" by Joe Inman (Third Place Award) and "Size–biased Sampling and the Negative Binomial Distribution" by Kerris Renken. In March the members and faculty traveled to the Chicken House Restaurant in Albion, Iowa for dinner – the ribs were "awesome." Students presenting papers at local KME meetings include Bill Kruse on "It Might be a Hit," Julie Holdorf on "Numerical Integration," Joe Inman with an early version of "Two Heads," and Susan Strong who addressed the initiation banquet on April 28 on "Is the Pythagorean Theorem Valid in Hyperbolic Geometry?" Iowa Alpha President Robert Hauser was chosen to present the student address at the UNI Commencement on May 14, 1988. In summary, a busy and very successful semester! Other 1988–89 officers: Kerris Renken, vice president; Julie Holdorf, secretary; William Kruse, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President – Curtis Eide
42 actives, 16 initiates

The Iowa Delta Chapter of KME met in January, February, March and May during the spring semester. A sledding party with pizza was held in January. A Math Trivia Contest with students teamed against faculty was enjoyed by all in February. Math Field Day committee reports were the primary business throughout the year. Seventy–seven high school students competed in that event on March 12. Dr. Charles Jepsen, a 1962 Wartburg graduate, was the featured speaker at the Initiation Banquet held on March 26th at which 16 new members joined KME. In May the Chapter picnic with volleyball was held in the park. Other 1988–89 officers: Patricia Glawe, vice president; Terry Letsche, secretary; Kaaren Hemmingson, treasurer; August Waltmann, corresponding secretary; Josef Breutzmann, faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President – Jon Beal
50 actives, 9 initiates

The spring semester began with a dinner and initiation for the February meeting. Nine new members were initiated at the time. Following the initiation ceremony, Jon Beal presented the program entitled "The Ring of Two x Two Matrices Over the Integers." Three students, Mala

Renganathan, Seyi Inoue, and Huey-Fang Chang, gave the program entitled, "International Mathematics Education and Examinations," for the March meeting. Kansas Alpha hosted a Region IV convention on March 25-26. Eight chapters in the Region participated. Papers were presented by students from five different chapters. The April Meeting Program was given by Tim Flood, Chapter treasurer, entitled "A Tinkertoy Farm Implement." The Chapter assisted the Mathematics Department faculty in administering and grading tests given at the annual Math Relays, April 26, 1988. Several members also worked for the Alumni Association's annual Phon-o-thon. They received first prize for amount of money raised by student organizations. The final meeting of the semester was a social event held at Professor Thomas' home. Homemade ice cream and cake were served to those in attendance. Officers for the 1988-89 school year were elected. The annual Robert M. Mendenhall awards for scholastic achievement were presented to Kym Wright, Jeff Benelli, Jon Beal, and Todd Tarter. They received KME pins in recognition of this honor. Other 1988-89 officers: Mala Renganathan, vice president; Lora Woodward, secretary; David Beach, treasurer; Harold L. Thomas, corresponding secretary; Helen Kriegsman and Gary L. McGrath, faculty sponsors.

Kansas Delta, Washburn University, Topeka
7 initiates

Planning sessions for the 1989 national meeting to be held on the Washburn campus were held regularly. Ron Wasserstein and students attended the spring regional meeting. Election of officers for 1988-89 will be held during the fall, 1988, semester. Other 1988-89 officers: Robert Thompson, corresponding secretary; Ron Wasserstein, faculty sponsor.

Kansas Epsilon, Fort Hays State University, Hays
25 actives, 3 initiates

Election of officers for 1988-89 will be held during the fall, 1988, semester. Our chapter held monthly meetings. The Spring Banquet took place on April 20, 1988.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President - Brenda Coble
23 actives, 21 initiates

This was a semester filled with interesting mathematical talks. As vice president, Beckman Eldridge managed to arrange for several speakers to come to our regular meetings. Dr. Scott Metcalf gave a multimedia presentation on "The Mandelbrot Set and Other Fractals." Dr. Patricia Costello gave a talk on "Using Statistics in Presidential Polls" just before the major primaries. Mr. Jackson Lackey talked on a number of interesting problems from the book Games for the Superintelligent. Mr. Cliff Swauger wrapped up the semester with "Problems from Calculus" (and an outline of a calculus book he is writing). The major event of the semester, however, was the regional convention that we hosted. It was a two-day convention with talks scheduled for a Thursday afternoon and Friday morning and a square dance on Thursday evening. We had eight student talks and the top three were presented with monetary awards. The featured speaker was Phil Brashear of SofTech Corporation who spoke on difficult and impossible problems for the computer. This year's initiation ceremony included a talk by our chairman, Dr. Charles Franke, on the mathematics involved in "Weighted Voting Games" followed by the traditional party at the Costello house. Other 1988-89 officers: Beckham Eldridge, vice president; Carrie Lash, secretary; Sean Nicol, treasurer; Pat Costello, corresponding secretary; Bill Janeway, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore
Chapter President – Mary Agnes McCarron
11 actives, 2 initiates

The main event of the spring semester was the celebration of the 25th anniversary of the installation of our chapter. This was held on May 10, 1988. At this time we had a buffet dinner followed by induction of two new members. The evening concluded with a very enjoyable talk given by Dr. James Lightner, MD Beta member and former President of KME. Dr. Lightner entertained the 38 attending members (alumnae and present members) with his investigation of the "Mathematicians Who Are Human But Are Not Too Serious." Other 1988-89 officers: Teena DeWalk, vice president; Elizabeth Lee, secretary; Lisa Waidner, treasurer; Sister Marie A. Dowling, corresponding secretary; Joseph DiRienzi, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President – Mary Beth VanPelt
13 actives, 4 initiates

During the spring the chapter sponsored a Career Night for all mathematics majors and invited back four mathematics alumni to speak about their careers: an elementary teacher who because of her mathematics major has become a mathematics specialist for several New Jersey schools; a Ph.D. in statistics who has taught in a university and is now a statistician for an environmental agency; an actuary who as a very recent graduate is still going through the rigors of the various actuarial exams; and a recent graduate who told of his long, frustrating but rewarding efforts to get a job as a mathematician with the government (DOD). The session was well attended by current majors. The chapter also ran a booth selling soft pretzels as a fund raiser for our annual donation to the Duren Scholarship Fund. This was part of the annual college May Day Carnival. The chapter sponsored a year–end picnic for all mathematics majors in May. Other 1988–89 officers: Deborah A. Camara, vice president; Beth A. Trust, secretary; Lisa K. Brown, treasurer; James E. Lightner, corresponding secretary; Linda R. Eshleman, faculty sponsor.

Maryland Delta, Frostburg State University, Frostburg
Chapter President – Laura Dudley
33 actives, 15 initiates

On February 27, 1988, fifteen new members were inducted into Maryland Delta Chapter of KME: Marcelle Bessman (faculty), William Byers, Michelle Glotfelty, Patricia Hout, Raymond Hughes, Brenda Iseminger, Joanne Kanellakos, Lana Lease, Melissa Ravenscroft, Angela Roque, Marnie Ross, Mark Shore (faculty), Cynthia Stein, Christa White and Pamela Yantz. During the semester the chapter sponsored talks by faculty members Dr. Horton Tracy ("LOGO: the Rodney Dangerfield of Computer Languages") and Dr. George Plitnik ("Is Time Travel Possible?"), and by student David Webb ("Introduction to Fractals"). Maryland Delta also co–sponsored the Eighteenth Annual Frostburg State University Mathematics Symposium: "Innovation: The Cutting Edge of Mathematics Education." Officers for spring, 1988, were Lisa Lewis, president; Leslie Baker–Dunmire, vice president; Laura Dudley, secretary; and Judy Anderson, treasurer. Other 1988–89 officers: Mary Jones, vice president; Michelle Glotfelty, secretary; Christa White, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

Michigan Beta, Central Michigan University
Chapter President – Theresa Budzinski
40 actives, 27 initiates

Some of the activities for the spring semester, 1988, were pizza after a meeting, bowling after a meeting, and a spring picnic. KME had a co–ed basketball team and a women's volleyball team during the winter semester.

Michigan Beta assisted the Department of Mathematics in their activities during Mathematics Awareness Week in April. KME sponsored a book sale (books donated by mathematics faculty) for CMU students in February. The Math Help Sessions for freshman/sophomore math classes were held again this semester. Students from the Secondary Education Math Methods class also helped. Two of our KME members, Vicki Gerus and Scott Wood, were named co–valedictorians of CMU's Winter 1988 graduating class. Eight students and their advisor attended the Region II regional convention at Illinois Delta. Tim Allen, Mark Longman, Erich Hauenstein and Brian Varney gave talks there. Other 1988–89 officers: Nancy Haskell, vice president; Agnes Hausbeck, secretary; George Lasecki, treasurer; Arnold Hammel, corresponding secretary and faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President – Stuart Hartfield
26 actives, 11 initiates

The spring initiation and cookout was held in conjunction with the 11th Conference on Undergraduate Mathematics on April 15–16. Nine undergraduate students from across the nation presented talks of about 20 minutes each. In addition, three faculty members gave hour–long talks. Dr. Temple Fay invited the conference participants and KME members to his home for a Picknick: an outdoor crawfish boil, barbequed chicken, and all the trimmings. Initiation of new members and election of officers for 1988–89 took place preceding the meal. Other 1988–89 officers: Patsy Saucier, vice president; Beth Page, secretary and treasurer; Alice Essary, corresponding secretary; Virginia Entrekin, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President – Sherri Renegar
40 actives, 10 initiates

The Missouri Alpha chapter had a student present a paper at the regional meeting in Pittsburg, Kansas. In addition two faculty members attended the meeting. The chapter held three regular meetings and a banquet during the semester. Other 1988–89 officers: Gayla Evans, vice president; Ellen Hurst, secretary; Lynette Top, treasurer; John Kubicek, corresponding secretary; Mike Awad, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President – Sharon Johnson

15 actives, 3 initiates

There were five regular meetings, two initiations (fall and spring), and an honors banquet during the past year. Other 1988–89 officers: Ray Flach, vice president; Angela Duncan, secretary; Sandy Dietz and David Beard, treasurers; Homer F. Hampton, corresponding secretary; Larry Dilley and Gerald Schrag, faculty sponsors.

Missouri Gamma, William Jewell College, Liberty
Chapter President – Susan Brannen
24 actives, 12 initiates

Students participated in monthly meetings at which time presentations by the students were made. The annual spring initiation and banquet were held in April with 12 new initiates. Our speaker was Mr. Greg Dance, Chapter President, whose talk on a computer controlled car anti–theft alarm, designed by himself, incorporated elements of logic, computer science and physics. Other 1988–89 officers: Alysia Hicks, vice president; Sarah Littlewood, secretary and treasurer; Joseph T. Mathis, corresponding secretary and faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President – Laura Knight
9 actives, 10 initiates

Other 1988–89 officers: Lesa Stoecklin, vice president; John Callaway, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor; Linda O. Lembke, faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville
Chapter President – Jim Daues
20 actives, 5 initiates

Hosted the Fourth Annual Math Expo for local high schools. Monitored and gave exams at annual academic festival (980 high school math students). Attended regional convention at UNI. Other 1988–89 officers: Wesley Clifton, vice president; Shelle Palaski, secretary; David Smead, treasurer; Sam Lesseig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin
21 actives, 9 initiates

Members of Missouri Iota heard Dr. Charles Allen speak on "What is Mathematics?" They competed and won in a college-wide competition of "Win, Lose, or Draw." They prepared an entry for the time capsule that was sealed on May 5th as part of the 50th anniversary celebration of Missouri Southern State College. In this entry they attempted to depict the life of a typical 1988 college math major. The capsule is to be opened in 2038. Two students prepared papers for the regional convention at Cedar Falls, Iowa. John Day won first place with his paper "Fun with Planes," and Robert Stokes won second place with his paper "The Cutting Edge." A spring social at Dr. Joe Shields' house concluded the semester activities. Officers for next year have not yet been elected.

Missouri Kappa, Drury College, Springfield
Chapter President – Melissa Arnold
9 actives 5 initiates

The chapter started the spring semester with a fund raiser by asking Drury faculty to donate textbooks for resale. The project raised $125. In March, Dr. Allen gave a talk on Egyptian mathematics followed by the initiation ceremony. Five new members were initiated: Lara Denouden, Connie Keith, Pat Kenney, Scott Steubing, and Jay Swartz. In preparation for the upcoming regional conventions the SMSU, Evangel, and Drury chapters held two joint meetings – one at SMSU and the other at Drury. The highlight of the semester was the regional convention at Cedar Falls, Iowa. Two papers were presented by Missouri Kappa: Investigations of the Cross Product on $R^7$ by Donna Luetkenhaus and Infinite Infinities by Scott Steubling, presented by Jeff Staley. The chapter celebrated the end of a successful semester with the annual spring picnic hosted by Dr. Rutan. Other 1988–89 officers: Donna Luetkenhaus, vice president; Chris Hutchison, secretary; Scott Steubing, treasurer; Charles S. Allen, corresponding secretary; Ted Nickle, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President – Jim Fisher
40 actives, 14 initiates

Throughout the spring semester club members monitored the Mathematics–Science Building in the evenings to earn money for the club.

The club also participated in the annual Wayne State College Foundation Phone–a–thon and in the annual WSC College Bowl. The club administered the competitive examination to identify the outstanding freshman in mathematics. The award went to Brenda Spieker whose home is Petersburg, Nebraska. The award includes the recipient's name being engraved on a permanent plaque, payment of KME national dues, and one year honorary membership in the local KME chapter. An additional honor came to the WSC Mathematics–Science Division when Dr. James Paige, Professor of Mathematics and sponsor of the local chapter, was named Outstanding Professor at Wayne State College for the 1987–88 academic year. Students from throughout the campus are eligible to vote by secret ballot for their favorite instructor. Speaker at the annual spring banquet was Val Collins, now employed by Applied Communication, Inc. in Omaha and former member of the Nebraska Alpha Chapter. Tom Hochstein was awarded the $25 book scholarship which is given to a KME member each semester by the club. Club members assisted the WSC mathematics faculty with the 14th Annual WSC Mathematics Contest on May 9, 1988, kept the KME bulletin board current, and sponsored some social functions for club members and guests. Members Sherry Linnerson, Darin Moon, and Jim Fisher, along with faculty member Margaret Lundstrom, attended the regional KME convention at the University of Northern Iowa in Cedar Falls, Iowa, April 22 and 23, 1988. Other 1988–89 officers: Darin Moon, vice president; Sherry Linnerson, secretary–treasurer; Renae Harre, historian; Fred Webber, corresponding secretary; James Paige and Hilbert Johs, faculty sponsors.

Nebraska Delta, Nebraska Wesleyan University, Lincoln
Chapter President – Diane Humphrey
14 actives, 9 initiates

Meetings were used to plan and prepare for two major activities. In February we sponsored a Computer Match–up before Valentine's Day. In April we held our Second Annual Mathematics and Computer Contest. Students from area high schools competed in five different events. A spring picnic was held at the home of Professor Wampler. Other 1988–89 officers: Lisa Stoehr, vice president; Cheryl Olsen, secretary; Michael Mead, treasurer; Muriel Skoug, corresponding secretary; Daniel Kaiser, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President – Mahomet Akbarzadeh
11 initiates

Other 1988–89 officers: Mark Andrews, vice president; Ben Martinez, secretary; Joe McCanna, treasurer; Richard Metzler, corresponding secretary and faculty sponsor.

New York Alpha, Hofstra University, Hempstead
Chapter President – Pamela Caplette
10 actives, 4 initiates

Other 1988–89 officers: Michelle Lisi, vice president; Carol Ann Sutherlin, secretary; Kimberly Selby, treasurer; Stanley Kertzner, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara
Chapter President – Christine Carbone
12 initiates

The highlight of the semester was our annual initiation banquet during which 12 new members were inducted. Our speaker was Walter Mazurowski, an alumnus who is employed by Comptek, a firm specializing in computer applications. Other 1988–89 officers: Laura Plyter, vice president; Amy Potter, secretary; Theresa Toenniessen, treasurer; Robert Bailey, corresponding secretary; Kenneth Bernard, faculty sponsor.

New York Iota, Wagner College, Staten Island
Chapter President – Carla Pantophlet
12 actives, 5 initiates

Other 1988–89 officers: Kelly Doty, vice president; David Reimertz, secretary; Linda Raths, treasurer; Zohreh Shahvar, corresponding secretary and faculty sponsor.

New York Lambda, C.W. Post Campus of Long Island University, Brookville
Chapter President – Kevin O'Reilly
37 actives, 7 initiates

Six students and one faculty member were initiated at our annual banquet on March 23, 1988. Other 1988–89 officers: Jodie Salasny, vice president; Debra A. Caputo, secretary; Cynthia Ferro, treasurer; Frankie

Mohammed, historian; Andrew M. Rockett, corresponding secretary and faculty sponsor.

New York Mu, St. Thomas Aquinas College, Sparkill
Chapter President – Geri Hausner
5 actives, 5 initiates

Our induction ceremony for our new pledges was held on April 22. Our guest speaker was Sr. Kathleen Ann Gorres who is a member of the alumni association of STAC. A reception was held afterward for all the new members, their families, faculty and administration. Due to the excellent presentation that was made in the fall semester, the computer graphics program was repeated for the benefit of those faculty who were unable to attend the first presentation. Other 1988–89 officers: Dianna Bricker, vice president; Anna Yuen, secretary; Harsha Pujara, treasurer; Mary Ellen Ferraro, corresponding secretary and faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green
Chapter President – Ty Damon
33 initiates

By sponsoring hay rides, planetarium visits, and alumni visits, the chapter tries to involve as many people and their guests as possible. Other 1988–89 officers: Laura Herrington, vice president; Sara Mason, secretary; Cheryl Richmond, treasurer; Waldemar Weber, corresponding secretary; Thomas Hern, faculty sponsor.

Ohio Gamma, Baldwin–Wallace College, Berea
Chapter President – Mike Jakupca
25 actives, 10 initiates

Other 1988–89 officers: Kim Hinkle, vice president; Eric Angyal, secretary; Cheryl Soltis–Muth, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President – Gina Aluerson
40 actives, 6 initiates

Regular monthly meetings were held by the chapter. Our initiation banquet took place in February and we held a pretzel sale in April. Other 1988–89 officers: Sophie Asghar, vice president; Julie Clark, secretary; Karen Allender, treasurer; Carolyn Crandell, corresponding secretary; Russell Smucker, faculty sponsor.

Oklahoma Alpha, Northeastern State University, Tahlequah
Chapter President – Michelle Harper
53 actives, 19 initiates

Our KME chapter helped sponsor a visiting lecturer from the Mathematical Association of America. On February 19, Dr. Christine Stevens from the National Science Foundation gave several presentations to math students. We sponsored a "Best Legs" contest among the male faculty members in the Science and Math Division as a fund raiser. The spring, 1988, initiation ceremonies were held in the banquet room of the Western Sizzlin' Steak House in Tahlequah. Dr. Herbert V. Monks, who retired from the math department at NSU in December, 1987, was honored as the KME mathematics teacher of the year. He was a member of KME for 40 years. Also honored as the KME student of the year was Patricia McGinn. The annual ice cream social was held April 27. Other 1988–89 officers: Suzanne Blackwell, vice president; Shelli Phillips, secretary and treasurer; Joan E. Bell, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President – Kellie Logan
20 actives, 9 initiates

We had one speaker this spring. We initiated nine new members and had a spring pizza party. Other 1988–89 officers: Amy Bagwell, vice president; Rhonda Hollrah, secretary; Ajith Dharmawardhana, treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington
Chapter President – Mary Joyce
13 initiates

Other 1988–89 officers: Matthew S. Mrozek, vice president; Jennifer Hannon, secretary; David Chapnell, treasurer; J. Miller Peck, corresponding secretary; Warren Hickman, faculty sponsor.

<u>Pennsylvania Gamma</u>, Waynesburg College, Waynesburg
Chapter President – Todd Gibson
8 actives

Two meetings were held during the semester. A get–together was held at the home of Dr. David Tucker. Mrs. Rosalee Jackson retired. Dr. David Tucker accepted a position at Midwestern State University of Texas. Mr. Jack R. Westwood became corresponding secretary. Other 1988–89 officers: David Help, vice president; Brenda Barnhart, secretary and treasurer; Jack R. Westwood, corresponding secretary.

<u>Pennsylvania Kappa</u>, Holy Family College, Philadelphia
Chapter President – Scott Kromis
10 actives, 6 initiates

The membes continued to tutor (free of charge) students that sought help. Meetings consisted of problem solving; points were given to a member who was able to solve a designated problem quickly. Awards were given on the last meeting, April 21, 1988, to the three members (Scott Kromis, Sherry Teti and Eric Mehler) who scored highest during the semester. Plans were made for the installation of new members – March 13, 1989. Speakers again will be former members of <u>KME</u>. Other 1988–89 officers: Eric Mehler, vice president; Constance Hefner, secretary and treasurer; Sister Mary Grace Kuzawa, corresponding secretary.

<u>Pennsylvania Lambda</u>, Bloomsburg University, Bloomsburg
Chapter President – Ann Vnuk

Other 1988–89 officers: Joshua Payne, vice president; Teresa Creasy, secretary; Karen Billingham, treasurer; James Pomfret, corresponding secretary; John Riteg, faculty sponsor.

<u>Pennsylvania Mu</u>, Saint Francis College, Loretto
Chapter President – Karen Kumpon
24 actives, 5 initiates

Pennsylvania Mu met April 18 for induction ceremonies. The evening began with a mass celebrated by Fr. Hebert (Math Professor), included a formal meal, and was concluded by induction of 5 new members. Other

1988–89 officers: Chris Bifano, vice president; Jeff Mendenhall, secretary; Kelly Subasic, treassurer; Adrian Baylock, corresponding secretary; Peter Skoner, faculty sponsor.

Pennsylvania Nu, Ursinus College, Collegeville
Chapter President – B. Timothy Evans
20 actives, 13 initiates

On March 17, 1988, our initiation ceremony and reception took place for new members. On March 25, 1988, we had a guest speaker, Dr. Michael Echer from Penn State–Wilkes Barre. His lecture was entitled "Recreational Mathematics and Problem Solving." Dr. Ecker was selected from a list of available speakers provided by the EPADEL section of the Mathematical Association of America. This event was jointly sponsored with Sigma Xi. A video presentation of the NOVA program entitled "The Man Who Loved Numbers" was held on May 2, 1988. Other 1988–89 officers: Trevor Feldman, vice president; James Doyle, secretary; Tracey Hitchner, treasurer; Jeff Neslen, corresponding secretary; John Shuck, faculty sponsor.

Tennessee Alpha, Tennessee Technological University, Cookeville
Chapter President – Michael Allen
20 actives, 41 initiates

$100 scholarships were given to two new initiates. Other 1988–89 officers: Chris Roden, vice president; Sarwat Kashmire, secretary; Curt Griggs, treasurer; Frances Crawford, corresponding secretary; John Mason, faculty sponsor.

Texas Alpha, Texas Tech University, Lubbock
Chapter President – Karen Engel
30 actives, 21 initiates

Other 1988–89 officers: Gregory Henderson, vice president; Paula Kajs, secretary; Scott Ellett, treasurer; Robert Moreland, corresponding secretary and faculty sponsor.

Texas Eta, Hardin–Simmons University, Abilene
Chapter President – Randal Schwindt
12 actives, 10 initiates

Other 1988–89 officers: Stephen Cody, vice president; Ricky Davis, secretary; Russell Perkins, treasurer; Mary Wagner, corresponding secretary; Edwin J. Hewett and Charles D. Robinson, faculty sponsors.

Virginia Alpha, Virginia State University
Chapter President – Charleen Mitchell
20 actives, 3 initiates

The Annual Louise Stokes Hunter Scholarship Award in honor of the founder of the Virginia Alpha chapter was presented to Wayne Saunders, former KME president, during Honor's Night activities at Virginia State University in April, 1988. The induction ceremony for new members of KME was held on May 3, 1988. Two of our members, Wanda Gay and Cynthia Holston, were Langley–AeroSpace Summer Scholars during the summer, 1987. Cynthia Holston, a 1987 graduate of Virginia State University, is now a computer analyst with UNISYS, a NASA contractor in Hampton, Virginia. Wanda Gay is studying for a dual degree in Mathematics at Virginia State University and Engineering at Old Dominion University in Norfold, Virginia. Other 1988–89 officers: Jacqueline N. Payton, vice president; Gerald L. Burton, secretary; Mohinder D. Tewari, treasurer; Dorothy R. Stevenson, corresponding secretary; Emma B. Smith, faculty sponsor.

Virginia Gamma, Liberty University, Lynchburg
Chapter President – John Wilson
16 actives, 15 initiates

We did not have a busy spring semester but had a good turnout at our initiation ceremony on April 26, 1988. Fifteen new members were inducted while friends and family looked on. Other 1988–89 officers: Wayne Whitaker, vice president; Melissa Damon, secretary; Lisa Barwick, treasurer; Glyn Wooldridge, corresponding secretary; Robert Chasnov, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President – Maureen Pastors
6 actives, 4 initiates

Five KME members and pledges (Ann Brandt, Maureen Pastors, Lea Salkowski, Julie Elver, Mary Porter) and the faculty sponsor attended the regional KME convention at the College of St. Francis in Joliet, Illinois on

March 18–19, 1988. On May 1st pledge Julie Elver was initiated into KME. Her initiation was preceded by a presentation on tesselations to the members of the chapter. Other 1988–89 officers: Julie Elver, vice president and treasurer; Maureen Pastors, secretary; Sister Adrienne Eickman, corresponding secretary and faculty sponsor.

Wisconsin Gamma, University of Wisconsin – Eau Claire, Eau Claire
Chapter President – Karen Tallafuss
35 actives, 7 initiates

The meetings for the spring semester 1987–88 were highlighted by four student presentations. In addition the club had two successful fund raisers selling popcorn and a used book sale. The semester was concluded with a picnic. Other 1988–89 officers: Renee Koslowski, vice president; Brian Vlcek, secretary; Jennifer Linn, treasurer; Tom Wineinger, corresponding secretary.

## ANNOUNCEMENT OF TWENTY-SEVENTH BIENNIAL CONVENTION

The 27th Biennial convention of Kappa Mu Epsilon will be held on April 6-8, 1989 at Washburn University, Topeka, Kansas. Each chapter that sends a delegation will be allowed some travel expenses from National Kappa Mu Epsilon funds. Travel funds are disbursed in accordance with Article VI, Section 2 of the KME constitution.

A significant feature of this convention will be the presentation of papers by student members of KME. The mathematics topic which the student selects should be in his/her area of interest, and of such scope that he/she can give it adequate treatment within the time allotted.

Who May Submit Papers? Any student member of KME, undergraduate or graduate, may submit a paper for use on the convention program. A paper may be co-authored; if selected for presentation at the convention it must be presented by one or more of the authors. Graduate students will not compete with undergraduates.

Subject: The material should be within the scope of the understanding of undergraduates, preferably those who have completed differential and integral calculus. The Selection Committee will naturally favor papers within this limitation, and which can be presented with reasonable completeness within the time limit.

Time Limit: The minimum length of a paper is 15 minutes; the maximum length is 25 minutes.

Form of Paper: Four copies of the paper to be presented, together with a dscription of the charts, models, or other visual aids that are to be used in the presentation, should be presented in typewritten form, following the normal techniques of term paper presentation. It should be presented in the form in which it will be presented, including length. (A long paper should not be submitted with the idea it will be shortened for presentation.) Appropriate footnoting and bibliographical references are expected. A cover sheet should be prepared which will include the title of the paper, the student's name (which should <u>not</u> appear elsewhere in the paper), a designation of his/her classification in school (graduate or undergraduate), the student's permanent address, and a statement that the author is a member of Kappa Mu Epsilon, duly attested to by the Corresponding Secretary of the Student's Chapter.

<u>Date Due</u>: January 18, 1989

<u>Address to Send Papers</u>:

Dr. Harold L. Thomas
Mathematics Department
Pittsburg State University
Pittsburg, KS 66762

<u>Selection</u>: The Selection Committee will choose about fifteen papers for presentation at the convention. All other papers will be listed by title and student's name on the convention program, and will be available as alternates. Following the Selection Committee's decision, all students submitting papers will be notified by the National President-Elect of the status of their papers.

<u>Criteria for Selection and Convention Judging</u>:

A. The Paper

1. Orginality in the choice of topic
2. Appropriateness of the topic to the meeting and audience
3. Organization of the material
4. Depth and significance of the content
5. Understanding of the material

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The Problem Corner (see p. 59).

Please consider submitting an article or a problem to the appropriate editor.