THE PENTAGON

Volume XLVII(47) Fall, 1987 Number 1

CONTENTS

NATIONAL OFFICERS

JAMES L. SMITH........................ President

Muskingum College
New Concord, Ohio 43762

HAROLD L. THOMAS........................ President-Elect

Pittsburg State University
Pittsburg, Kansas 66762

ROBERT L. BAILEY........................ Secretary

Niagara University
Niagara, New York 14109

SISTER JO ANN FELLIN.................. Treasurer

Benedictine College
Atchison, Kansas 66002

M. MICHAEL AWAD........................ Historian

Southwest Missouri State University
Springfield, Missouri 65802

Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics; due, mainly, to its demands for logical and rigorous modes of thought, to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

THE KNIGHT'S TOUR: THE MATHEMATICS BEHIND AN OLD PROBLEM AND ITS NEW IMPLEMENTATION

Connie J. Garces

Ohio Zeta

Muskingum College

Behind the Knight's Tour

The game of chess has been considered as a rather large unsolved mathematical problem.[1] From this challenging sport, numerous other math-related games have been derived and have intrigued mathematicians for over the past two centuries. One such case is the classical problem known as The Knight's Tour. Interestingly enough, various arithmetical approaches to solving for a tour exist. As a result, implementations based on the mathematics of the Knight's tour have been created with the development of the computer. The programs and algorithms that will be presented at the end will show just how the mathematics of this medieval game gives way to the implementation of modern day technology.

The first question one might ask when approaching this problem is: What is the Knight's Tour? Theoretically, it is one dealing with the movement of the knight and the path it makes as it travels from square to square across an 8 x 8 chessboard. This path, or tour, which the knight makes allows him to visit each square, or cell, once and only once until all 64 cells have been traversed. From any one square, the knight can legally move to one of eight cells using three adjacent squares that form an imaginary capital 'L'. This rightside up, backward, or sideway 'L' begins when the knight moves two squares vertically or horizontally from its present position. It then moves one square either to the left or to the right of the second square. Thus, the letter 'L' is formed. Examples of legal moves of the knight are shown in Figure 1.[2]

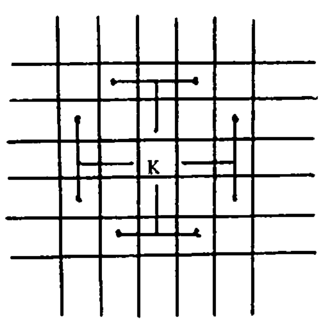

Fig. 1. The knight (K) and its legal moves (----).

To find a solution or a successful tour by brute force tends to create tours that usually end after several unsuccessful attempts. An incomplete tour occurs when the knight encounters a trouble area (i.e. a cell which has only one entry and one exit point).[3] Should the knight land on a square next to an unvisited cell that only has an entry point and no exit, that cell is termed a termination square and it thus ends the tour. It is the avoidance of such trouble spots that led to the development of several algorithms which attempted to simplify the task of finding a tourable path. The general mathematical relation and three early algorithms constructed for the knight's tour will show the various methods of finding a traversable tour.

Algorithms for the Knight's Tour.

During the 18th century, great mathematicians such as De Moivre, Euler, Roget, Vandermonde, Warnsdorff and others attempted to find a practical but also artistic way of finding a tour. The idea of a re-entrant and symmetric tour was conceived. Here, the original square could be reached in one additional move after visiting the last square. For example, if one numbered each square that the knight visited on its tour, a part of the board could appear as in Figure 2[4] below.

20	15	18
1	32	62
46	19	16
43	64	47
48	57	52

Figure 2

From this came mathematical ideas that dealt with finding relationships among the numbered squares of a completed tour. Similarities between odd and even numbers were considered. Obtaining row or column totals consistently led to the idea of a semi-magic square whose diagonal added up to a particular sum. However, no such complete tours have been found to exist through these kinds of relationships.

With these and other ideas being tossed about, one method which arose was proposed by a German mathematician, J.C. Warnsdorff. His algorithm, known as Warnsdorff's Rule or the Double-look-ahead algorithm, proposes that the knight proceed from its present occupying square to one from which "the number of available squares during the next two moves is smallest."[5] Although this method may seem tiresome, no exception to this rule has been found. As a result, this rule tends to be used as a basis for present day algorithms such as the one found in Donald Irwin's computer method [3]. This Double-look-ahead algorithm aids in finding trouble areas that the knight may avoid.

Another method of solution which is based on mathematical analysis was constructed by Euler. Its objective was to create a tour on an 8 x 8 chessboard which would leave very few untraversed squares. Through these 2,3, or 4 cells, Euler was able to create a re-entrant circuit. The author, W.W. Rouse Ball, uses an example that left four spaces unmarked. An original solution with two unvisited cells, a and b, remaining is shown below.

45	30	11	16	43	29	1	14
10	17	44	29	12	15	42	27
31	46	55	52	57	60	13	2
18	9	a	59	54	51	26	41
47	32	53	56	61	58	3	24
8	19	62	b	50	25	40	37
33	48	21	6	35	38	23	4
20	7	34	49	22	5	37	39

Figure 3

Euler's algorithm requires that the above path be made by first noting which cells are one legal move away from the cell visited first (1) and last (62). In the example above, 1 is said to "command" cells 60, 2, and 12 while 62 commands 61,59,49,7,35,9,47,33. As long as there exists two numbers, one from each set, that differ by one, the path can be made re-entrant.[6] Choosing, for example, 60 and 59, one first replaces 62..60 with 60..62. Then, by picking an untraversed cell, say a, Euler works this into the tour by first noting the cells which square a commands. One cell, 57, is chosen arbitrarily and the difference between it and 62 is added to all the squares. Thus, the numbers exceeding 62 are then replaced by 1..5 and cells a and b are made a part of the tour. The path produced is now tourable (see Figures 4 and 5).

50	35	16	21	48	33	6	19
15	22	49	34	17	20	47	32
36	51	60	57	62	67	18	7
23	14	a	64	59	56	31	46
52	37	58	61	66	63	8	29
13	24	65	b	55	30	45	42
38	53	26	11	40	43	28	9
25	12	39	54	27	10	41	44

Fig. 4 Add 5 to each.

50	35	16	21	48	33	6	19
15	22	49	34	17	20	47	32
36	51	60	57	62	5	18	7
23	14	63	2	59	56	31	46
52	37	58	61	4	1	8	29
13	24	3	64	55	30	45	42
38	53	26	11	40	43	28	9
25	12	39	54	27	10	41	44

Fig. 5 Add in a and b

The final mathematical approach to construction of a Knight's Tour is a geometric view. The moves of a knight can be associated with one of six angles ranging from the smallest acute angle, k, to a straight angle (see Figure 6)[7].

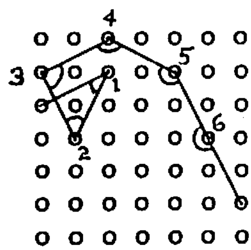

Figure 6.

By using the right angle 3 as a unit equal to 1, each of the other angles can be expressed in terms of k and the right angle. The object of this algorithm is to check the sums of the number of certain angles made for evenness or oddness(parity).[8] Should the number of straight angles have the same parity as the number of right angles, angle 2 or angle 4, jangle 1 or angle 5, then a tour exists. The proof of this algorithm is rather lengthy, as well as complicated, but has been found to be a very challenging method for finding a tourable path.

<u>New Implementations on the Knight's Tour</u>.

As one can see by the algorithms given above, it can be a somewhat tiresome and tedious task to find and compute the number of angles made by the knight as in the angle method. The same can be said for creating and recreating tours that yeild the initial two sets of numbers required by Euler's method, or by trying and retrying the Warnsdorff algorithm of looking ahead for the next traversable square. However, since the computer has entered the lives of both mathematicians and scientists, new algorithms have been developed based on the above, earlier approaches. The following two algorithms are examples of such programs generated using a variation of the Warnsdorff method.

Donald Irwin felt that the idea of "looking ahead" for the next suitable square as Warnsdorff did was very important, yet he removed some restrictions by only looking at squares adjacent to the present position of the knight. This procedure thus omitted problem areas and increased the changes for the computer itself to choose the next cell without having to depend on the user's decisions too heavily.

Irwin's algorithm is somewhat isomorphic in the use of sets and valid binary operations. Beginning with an 8 x 8 set of squares, he labels left to right from 1 to 64 each square and then defines Set S=(1,2,...,64).[9] A binary relation R on set S exists where (a,b) is found in R if and only if a legal move exists from a to b. Figure 7[10] illustrates such a relation with the pair (2,19).

1	2	3	
9	10	11	
17	18	19	

Figure 7

Note that this pair is also symmetrical since (19,2) can also be defined with a relation R. With knowledge of this, Irwin created a 64x64 table showing the legal moves of each square at a given position. A small section of this table is shown in (Figure 8[12]) to illustrate the symmetry and the resulting legal moves available from a to b. These legal moves Irwin calls the EV or exist value of a cell which determines the knight's next move.[11]

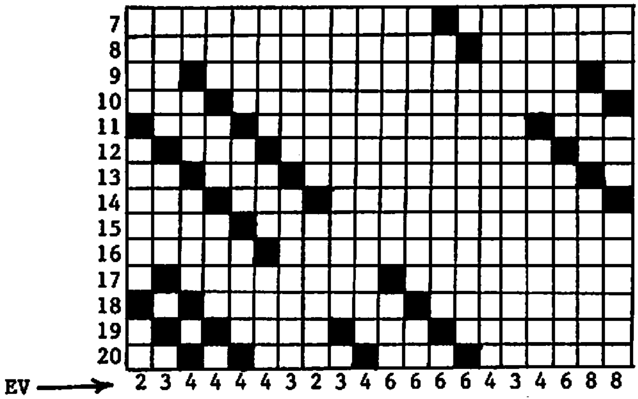

Figure 8

From here, the rest of the algorithm becomes rather simple. Programming a 64x64 matrix into the computer and initializing it with ones (1's) wherever a black square appears and with zeros where blanks occur creates a complete table similar to the one above. A single matrix or array of length 64 is created to hold the corresponding EV's while another array (CB) stores the knight's moves (variable KM).

The computer then is programmed to check for the smallest EV in a given column depending on the knight's current position. It then may choose an EV arbitrarily if there is more than one smallest exit value. The knight then moves to that position and the variable KM is incremented by 1. The process is repeated until the knight's position is equal to 64 or when there are no more available squares to complete a tour.

In the second program, the algorithm also makes use of Warnsdorff's Rule by checking each of the eight directions possible for a traversable square. If such a cell is available, then the function denoted TOUR is true and it repeats its process until the last square is traversed. Should no such square exist, then TOUR is given the value "false" and the path ends. The program also allows for chessboards of size NxN to be tried for a tour. References made to the original program written in the PASCAL language have been simplified and may be found at the end of this paper.

The Pascal Algorithm.

In general, the algorithm begins by accepting N, a positive integer, from the user to determine the board size (step A in program) and an NxN matrix is created and initialized to zero. After accepting an initial row and column value for a square, the function TOUR is called. This recursive procedure checks for boundaries of the matrix to be sure that the starting square actually exists in the two dimensional array (step B).

Since the object is to get the number of squares left to traverse equal to zero, the program checks for this trivial case first (Step C). Should this case not exist immediately, TOUR is given the values of the number of remaining squares not visited and the new coordinates chosen depending on which of the eight conditions is satisfied first (step D). If none of these eight pairs of coordinates allow for another traversable square to be found, then no tour exists and the game is finished. A final printout of a traversable tour is given if one exists for the NxN chessboard used.

Concluding Remarks.

It should not be a surprise to discover what a large role mathematics plays in what may seem to be a trivial chessboard game. The number of implementations and algorithms available are many, and the creation of such methods always is possible. Although one may realize that old medieval problems like the Knight's Tour are purely mathematical, one may forget that the new implementations, such as the computer methods presented, also base their results on the early algorithms of men like Euler and Warnsdorff. The uses of mathematics are endless and the problems which are derived from it continue to intrique both mathematician and scientist.

Ideas such as these are based on the algorithms that stemmed from a medieval game such as chess.

Sample Run of Program

After entering the dimension of the board as a 5 x 5 size, the following corrdinates would be given starting with cell (1,5):

Row	Col
1	5
2	3
3	5
1	4
2	2
4	1
5	3
4	5
2	4
1	2
3	1
5	2
4	4
2	5
3	3
5	4
4	2
2	1
1	3
3	4
5	5
4	3
5	1
3	2
1	1

```
{ The purpose of this PASCAL program is to find a tour through
  a matrix (board) which will allow a knight to visit each
  square once and only once.  The tour, if found, will be
  printed afterwards according to the cell coordinates.}

TYPE
  two_dimension = array[1..25,1..25] of integer;
                  {Largest size chessboard allowed is 25x25}

VAR
  board: two_dimension;          {chessboard to be used}
  row,col:integer;               {coordinates for cells}
  num_of_squares:integer;        {total number of squares}
  max_size: 1..25;               {max. size each side of
                                  board can be}

PROCEDURE Enter_values;
  BEGIN
{

  This procedure gets board size from user and coordinates
  of starting position for tour.}
      .
      .
      .
  readln(max_size);          {initializes Board}
  for row :=1 to max_size do
     for col:= 1 to max_size do
      board[row,col] := 0;
      .
      .
      .


FUNCTION Tour(Var square_left:integer;row,col:integer):boolean;
{This function determines whether a tour exists or not by
 checking each of the eight directions for traversable squares}

 VAR
   tour_found: Boolean;  {true or false value}

 BEGIN
   squares_left:= squares_left - 1;  {arriving at first square}
   IF (row > max_size) OR(row<1) OR (col > max_size) OR
     (col < 1)  THEN
      tour_found:=false      {checking index boundaries}

   ELSE Begin
    If board[row,col] = 1 then
      tour_found:= false       {there is no tour if you are at}
                               {the first square}
     ELSE IF squares_left <= 0 THEN
      tour_found:= true        {trivial case}
     ELSE
      BEGIN
```

```
{         At this point, all directions are checked using
          the following respective command:

          IF (tour(squares_left,row + 1, col + 2))OR
             (tour(squares_left,row - 1, col + 2))OR
             (tour(squares_left,row - 1, col - 2))OR
              etc.
       .
       .
       .
}
       END;
       If tour_found:= true THEN
          writeln(row, col)          {if a tour is found,
                                      the squares coordinates are
                                      printed out}

{MAINLINE}
  Begin
    .
    .      { Calling of above procedures.}
    .
  END.
```

ENDNOTES

[1] Maurice Kraitchik, Mathematical Recreations (New York: Dover Publications, Inc., 1953), p.238.

[2] Donald P. Irwin, "The Knight's Tour," Creative Computing, May, 1985, p. 64.

[3] Irwin, p. 64.

[4] Irwin, p. 67.

[5] Irwin, p. 67.

[6] W.W. Rouse Ball, Mathematical Recreations and Essays (New York: The Macmillan Co., 1956), p. 177.

[7]Fred Schuh, The Master Book of Mathematical Recreations (New York: The Dover Publications, Inc., 1968), p. 352.

[8] Schuh, p. 352.

[9] Irwin, p. 68.

[10] Irwin, p. 67.

[11] Irwin, p. 68.

[12] Irwin, p. 68.

BIBLIOGRAPHY

1 Ball, W.W. Rouse. Mathematical Recreations and Essays. New York: The Macmillan Co., 1956.

2 Cannon, Robert and Stan Dolan. "The Knight's Tour." The Mathematical Gazette, August, 1986, pp.91-100.

3 Irwin, Donald P. "The Knight's tour." Creative Computing. May, 1985, pp. 64-69.

4 Kraitchik, Maurice. Mathematical Recreations. New York: Dover Publications, Inc., 1953

5 Schuh, Fred. The Master Book of Mathematical Recreations. New York: Dover Publications, Inc. 1968.

6 Spencer, Donald D. Game Playing with Computers. New Jersey: Hayden Book Co., Inc., 1975.

OLD AND NEW RESULTS CONCERNING AMICABLE PAIRS

Timothy J. Melvin

Kentucky Alpha

Eastern Kentucky University

The Theory of Numbers has a unique position in Mathematics. This is due to some of its results which date back to Pythagoras (500 BC). In almost every century since the classical period (800 BC - 1500 BC), there have been new and extremely interesting results in number theory. Many of the great mathematicians (Fermat, Euler, Gauss, Riemann) made contributions to Number Theory.

A school headed by Pythagoras dealt with numbers in a slightly different manner than one might suspect. The Pythagoreans believed that the properties of numbers had mystical qualities that related the numbers to everything in the real world. For example, the number 4 signified justice. They considered it remarkable that the number 6 is equal to the sum of its aliquot divisors (eg. 6 = 1 + 2 + 3). In respect to their mystical way of thinking, numbers such as 6 were called "perfect".

More precisely, a natural number is called perfect if it is equal to the sum of all of its positive divsors, excluding itself. For convenience we will use the function $\sigma(n)$ to denote the sum of the divisors of a natural number n. So for a perfect number n we have $\sigma(n)-n = n$, or $\sigma(n) = 2n$.

Pairs of numbers with the property that one number is equal to the aliquot divisors of another and vice versa are called friendly or amicable. It is believed Pythagorean numerology also dealt with amicable numbers since the first pair (220, 284) was discovered during their time. This is not surprising since when asked what a friend was, Pythagoras replied, "One who is another 1. Such are 220 and 284." Pythagoreans felt this pair of mystical numbers was essential to a good friendship.

This paper reports the methods of discovery of some of the old and new amicable pairs. Also some terminology and ideas for creating new pairs will be mentioned. My inspiration for this paper began with an interest in amicable numbers that started in a Number Theory class and was augmented by ideas from the Number Theorist, Dr. Patrick Costello.

Looking at the first amicable pair, (220,284), we have: $\sigma(220)-220=1+2+4+5+10+20+22+44+55+110=284$ and $\sigma(284)-284=1+4+71+1142=220$.

It follows that each amicable pair is formed by an abundant number and a deficient number. A natural number n is called an abundant number if $\sigma(n)>2n$ and is called a deficient number if $\sigma(n)<2n$. We would also like to give at this time an alternative definition for amicable numbers which is the following:

(m,n) is an amicable pair if $\sigma(m)=m+n=\sigma(n)$ (1)

Checking the pair (220,284) we have:

$\sigma(220) = 220+284 = \sigma(284)$.

During the 9th century an Arab mathmetician, Thabit ibn Kurrah, described how to find amicable pairs. The rule is stated as follows: If

$p = 3*2^{n-1}-1$, $q = 3*2^{n}-1$, and $r = 9*2^{2n-1}-1$ (2)

are all prime numbers, where $n>2$, then $2^{n}pq$ and $2^{n}r$ form an amicable pair. Pair (220,284). Factoring the pair, $(220,284)) = (2^{2}*5*11,2^{2}*71)$,
where $p = 5 = 3*2^{2-1}-1$, $q = 11 = 3*2^{2}-1$, and $r = 71 = 9*2^{4-1}-1$, we note that p,q, and r are prime.

The Arab mathematicians also found many applications for amicable numbers. This included magic, astrology, and sorcery. One Arab, El Madschritt, in the 11th century , believed that eating something labeled with 284 would have an erotic effect on someone who was at the same time eating something labeled 220.

Finally in the 17th century the second pair of amicable numbers was discovered. In 1636 Pierre de Fermat announced the pair (17296,18416) in a letter to Father Mersenne. Fermat had constructed the sequence 5, 11, 23, 47,... by doubling each preceding element and adding unity. If two consecutive terms of the sequence p and q are prime, and if p+q=pq = r is prime, then the pair $(2^n r, 2^n pq)$ is amicable where n is the location of q in the sequence. Notice this is another way of expressing Thabit's rule. For example, when p = 5, and q = 11 we have r = 5+11+55 = 71 which is prime, yielding amicable pair (4*71,4*5*11)=(284,220). Fermat had found the next pair of consecutive primes and thus a new amicable pair. Perhaps Fermat stopped here because of the size of the primes with which he was working.

Two years later Rene Descartes, a French geometer, continued Fermat's table of amicable pairs using consecutive primes 191 and 383 for his choices of p and q. After he determined that r = 73727 was prime he added the pair $(9363584,9437056)=(2^4*191*383,2^4 73727)$, which he described in a letter to Father Mersenne.

Let's take a moment here to relay some useful results about the σ function. Descartes noted that:

$\sigma(p^n)=(p^{n+1}-1)/(p-1)$ where p is prime and

σ is multiplicative, (3)

i.e. $\sigma(mn)=\sigma(m)\sigma(n)$ where $\gcd(m,n)=1$. (4)

These results were combined with the Fundamental Theorem of Arithmetic by the Swiss mathematician, Leonard Euler, to form the results:

For a natural number $n = p_1^{\alpha 1}*p_2^{\alpha 2}*...*p_n^{\alpha n}$, we have

$$\sigma(n) = \left(\frac{p_1^{\alpha 1+1}-1}{p_1-1}\right)\left(\frac{p_2^{\alpha 2+1}-1}{p_2-1}\right)\ldots\left(\frac{p_n^{\alpha n+1}-1}{p_n-1}\right). \quad (5)$$

We illustrate this formula where $n = 12 = 2^2*3$. Here $p_1=2$, $\alpha 1=2, p_2=3$, and $\alpha 2 = 1$, $\alpha(12)=\left(\frac{2^3-1}{2-1}\right)\left(\frac{3^2-1}{3-1}\right)=28$

With numbers containing more factors this formula is very indispensable.

In 1747 Euler published 61 new pairs. He did not reveal his methods until 1750 in his paper "De Numeris Amicabilibus". These 61 new pairs were assumed correct for many years until two pairs were found to be false in 1909 and 1914. Perhaps this was due to the high regard for Euler's work and hence a laxity in the scrutiny of his results.

Euler developed five different forms for amicable pairs based on a factorization method. His first four forms are called regular and employ methods to find pairs (EM,EN) where E is a common factor such that gcd(E,M,N)=1. Euler used different combinations of variables for M and N in his first four forms. An amicable pair of Euler's First Form has M = pq and N = 4 where p,q,r are distinct primes not dividing E. Included with these are the pair (220,284), Fermat's pair, the pair from Descartes, and 13 others discovered by Euler.

Note that if E is a power of 2, then Euler's First Form is the same as Thabit's Form (2).

Euler's other three forms of the regular type and the number of each type he found are given below:

Form	Format	No. Euler found
Second	(Epq,Ers)	26
Third	(Epqr,Est)	17
Fourth	(Epqr,Estu)	1

Euler also discovered two pairs that did not fit any of the first four forms. These were the first two pairs discovered not fitting the (EM,EN) form. Because of the infrequent occurrence of pairs of this form and their odd construction, these pairs were called exotic.

It is interesting to note that all of these early amicable pairs were found before the days of calculators and computers. Even to examine Euler's smallest pair, (2620,2924), which is of his Second Form, is quite a task.

Since Euler's death in 1983, many more amicable pairs have been discovered. Most have been discovered with the help of variations of Euler's methods. As of August 1986 there were 11282 regular pairs known and 620 exotic pairs known. Only a small number have resulted from systematic computer searches, that is, testing all m in an interval to determine whether s(s(m)) = m, where s(m) = σ(m)-m (i.e. the sum of a number's aliquot divisors). Computer searches have verified that the current list of amicable pairs is conclusive up to 10^{10}!

Four pairs with 32-digit, 40-digit , 81-digit, and 152-digit numbers were submitted in a paper [3,p.309] by H.J.J. teRiele in January, 1974. Prior to teRiele's four pairs, the largest known amicable pair of 25 digits, had been discovered by Lee and Madachy, in 1946. teRiele found the 40-digit pair by using a rule of Euler's which was a generalization of Thabit's rule. Most of teRiele's success could be attributed to a fast primality testing program that showed a 28-digit number was prime. His remaining 3 pairs were discovered using known pairs and primality testing of large numbers.

H.J.J.teRiele's four pairs were the largest known amicable pairs until 1983 when he released more very large pairs. His largest pair has 282 digits in each number.

New methods of constructing amicable pairs use clever terminology. One method described by teRiele [2,p.220] concerns "mother-daughter" pairs. teRiele goes on to point out this is a very "prolific" method as 1592 previously known pairs generated 2324 new amicable pairs. One method that yielded about half of the new pairs is the following:

Pick a known pair (au,ap) where gcd(a,u) = gcd(a,p) = 1 and p is prime. If r and s are distinct primes with gcd(a,rs) = 1 satisfying

$$(r-p)(s-p) = (\sigma(a)/a)(\sigma(u))^2 \qquad (6)$$

and a prime q exists where gcd(au,q) = 1 and q = r+s+u, then (auq,ars) is amicable.

teRiele mentioned that if Euler had known this rule, then his pair $(2^4*23*47, 2^4*1151)$ would have produced the pair $(2^4*23*47*9767, 2^4*1583*7103)$. As follows $(2^4*23*47, 2^4*1151) = (2^4*1081, 2^4*1151)$ is our known pair with $\gcd(2^4, 1081) = \gcd(2^4, 1151) = 1$ and 1151 is prime. Now choose primes r=1583, s=7103 and note that $\gcd(2^4, 10544049) = 1$. Checking to see if equation (6) is satisfied, we have

$$(1583-1151)(7103-1151))=(\sigma(2^4)/2^4)(\sigma(23*47))^2$$
$$2571264=31*2^{10}*3^4.$$

Moreover, q = 1583+7103+1081 = 9767 is prime and $\gcd(2^4*1081,9767) = 1$, obtaining the amicable pair $(2^4*23*47*9767, 2^4*1583*7103)$.

It may have been thought that the list of amicable pairs was complete. Today, however, the search continues for new amicable pairs. With the methods mentioned here and elsewhere, number theorists are wondering if perhaps the list of amicable pairs is infinite!

BIBLIOGRAPHY

Costello, Patrick. "Amicable Pairs of Euler's First Form", J. Recreational Mathematics, Vol. 10(3), 1977-78, pp. 183-189.

H.J.J. teRiele, "On Generating New Amicable Pairs from Given Amicable Pairs", Math. Comp., Vol. 42, 1984, pp. 219-223.

H.J.J. teRiele, "Four Large Amicable Pairs", Math. Comp., Vol. 28, 1974, pp. 309-312.

W. Borho, "On Thabit ibn Kurrah's Formula for Amicable Numbers", Math. Comp., Vol. 26, 1972, pp. 303-304.

THE PROBLEM CORNER
EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1988. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1988 issue of THE PENTAGEON, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 175 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROBLEMS 406-409, SOLUTIONS 392, 397-401

Problem 406: Proposed by Bob Prielipp, University of Wisconsin-Oshkosh.

Let a, b, and c be the lengths of the sides of a triangle. Prove that $a^4 + b^4 + c^4 < 2(a^2b^2 + a^2c^2 + b^2c^2)$.

Problem 407: Proposed by the editor.

Prove that the median of a triangle is less than the arithmetic mean of the two adjacent sides of the triangle.

Problem 408: Proposed by the editor.

Dirty Dan had a hot tip on the dog races. He knew that one of four longshots would win the race. If the odds on these four dogs are 3 to 1, 5 to 1, 6 to 1 and 9 to 1 respectively, how much should Dirty Dan bet on each of these four dogs to guarantee making a profit of $143?

Problem 409: Proposed by the editor.

Find one or more solutions in positive integers of the following system of equations:

$$x^2 + 13y^2 = z^2 \quad \text{and} \quad x^2 - 13y^2 = z^2 .$$

SOLUTIONS

Problem 392: Proposed by the editor.

The integer n has exactly 25 divisors. If n can be expressed in the form $a^2 + b^2$ where a and b are relatively prime integers, what is the smallest value of n and what are the corresponding values of a and b ?

Composite of solutions submitted by Bob Prielipp University of Wisconsin-Oshkosh, Oshkosh, Wisconsin and Patrick Pendleton, student - Mount Mary College, Cedar Rapids, Iowa.

Because n is a positive integer having exactly 25 positive divisors, either $n = p^{24}$ or $n = p^4q^4$ where p and q are distinct primes. It is well known [1] that a positive integer n is the sum of the squares of two relatively prime positive integers if and only if n is not divisible either by 4 or by an integer of the form 4k + 3. Now $n^4 = 5^4 * 13^4 = 17850625$. A short computer program yields the following twelve representations of n in the form $a^2 + b^2$ where a and b are positive integers with $a < b$: $n =$ (1) $468^2 + 4199^2 =$ (2) $580^2 + 4185^2 =$ (3) $615^2 + 4180^2 =$ (4) $1040^2 + 4095^2 =$ (5) $1183^2 + 4056^2 =$ (6) $1625^2 + 3900^2 =$ (7) $2016^2 + 3713^2 =$ (8) $2047^2 + 3696^2 =$ (9) $2145^2 + 3640^2 =$ (10) $2535^2 + 3380^2 =$ (11) $2652^2 + 3289^2 =$ (12) $2975^2 + 3000^2$.

Clearly 5 divides each of a and b in (2), (3), (4), (6), (9), (10) and (12). 13 divides each of a and b in (1), (5) and (11). One can easily verify that (2016, 3713) = (2047, 3696) = 1; i.e. that each pair is relatively prime and provide the required values of a and b.

[1]. Sierpinski, W. Elementary Theory of Numbers, Hafner Publishing Company, New York, 1964, p.362.

Problem 397: Proposed by the editor.

Which of the following expressions is larger $\sqrt{10} + \sqrt{29}$ or $\sqrt{73}$? Verify your answer without using any table, calculator or computer.

Solution by Zhong Su Chen, student member New York Chapter, Pace University, New York, New York.

Raising both sides to the fourth power, we have

$$(\sqrt{10} + \sqrt{29})^4 = [(\sqrt{10} + \sqrt{29})^2]^2 = (39 + 2 * \sqrt{290})^2$$
$$= 2681 + 156 * \sqrt{290} \geq 2681 + 156 * \sqrt{289}$$
$$= 5333 > 5329 = (\sqrt{73})^4.$$

Hence $\sqrt{10} + \sqrt{29} > \sqrt{73}$.

Also solved by Fred A. Miller, Elkins, West Virginia and Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. One incorrect solution was received.

Problem 398: Proposed by the editor.

Fred noticed two different triangular scraps of wood on the floor beside his saw. He didn't think anything about them until his son noticed that each of them have sides which are an integral number of inches. Then he noticed that for each piece the area is 3/4 of the perimeter. What are the dimensions of each triangle and are there any others which have the same ratio between the area and the perimeter?

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Let (a,b,c) be a triangle of the required type and let $s = (a + b + c)/2$. Then $4\sqrt{s(s - a)(s - b)(s - c)} = 3(2s)$. It follows that $(2s - 2a)(2s - 2b)(2s - 2c) = 9(2s)$. (1)

Let $x = 2s - 2a$, $y = 2s - 2b$ and $z = 2s - 2c$. Then $x + y + z = 2s$. Substituting this into (1) yields

$$xyz = 9(x + y + z). \qquad (2)$$

Without loss of generality, we may assume $x \geq y \geq z$. Then

$y(\frac{1}{9}yz - 1) \leq x(\frac{1}{9}yz - 1) = y + z \leq 2y$ so $\frac{1}{9}yz - 1 \leq 2$.

Thus $z^2 \leq yz \leq 27$, making $z = 1, 2, 3, 4$ or 5.

For $z = 1$, $xy = 9(x + y + 1)$ or $(x - 9)(y - 9) = 90$. Setting $x - 9$ and $y - 9$ equal to divisors of 90, one can find corresponding values of x, y, z, s and c. No pair of divisors yields a solution in this case.

For $z = 2$, $2xy = 9(x + y + 2)$ or $(2x - 9)(2y - 9) = 117$. Setting $2x - 9$ and $2y - 9$ equal to divisors of 117, a solution occurs for $2x - 9 = 39$ and $2y - 9 = 3$. Here $(x,y,z) = (24,6,2)$ whence $s = 16$, $a = 4$, $b = 13$ and $c = 15$.

For $z = 3$, $3xy = 9(x + y + 3)$ or $(x - 3)(y - 3) = 18$. Proceeding as before, no new solutions are found.

For $z = 4$, $4xy = 9(x + y + 4)$ or $(4x - 9)(4y - 9) = 225$. Proceeding as before, the only solution found corresponds to $4x - 9 = 15$ and $4y - 9 = 15$. Here $(x,y,z) = (6,6,4)$ whence $s = 8$, $a = 5$, $b = 5$ and $c = 6$.

For $z = 5$, $5xy = 9(x + y + 5)$ or $(5x - 9)(5y - 9) = 306$. Proceeding as before, no new solutions are found. Hence the only solutions are given by $(a,b,c) = (5,5,6)$ and $(4,13,15)$.

Problem 399: Proposed by Bill Olk, University of Wisconsin-Madison, Madison, Wisconsin.

Find all right triangles whose sides are integers and whose inscribed circles have prime radii.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Let (x,y,z) denote a right triangle whose sides are integers. Then x, y and z are positive and $x^2 + y^2 = z^2$. Let x be even. Then it is well known that $x = k(2st)$, $y = k(t^2 - s^2)$, and $z = k(t^2 + s^2)$ where k, s and t are positive integers with s t, $(s,t) = 1$ and s and t have opposite parity. Let r denote the radius of the inscribed circle of (x,y,z). Then

$$r = \frac{\text{area}}{\text{semi-perimeter}} = \frac{xyz}{x + y + z} = ks(t - s).$$

If $r = p$ where p is a prime number, then:

(1) $k = 1$, $s = 1$ and $t = p + 1$;

(2) $k = 1$, $s = p$ and $t = p + 1$ or

(3) $s = 1$, $t = 2$ and $k = p$.

Correspondingly, these cases yield right triangles

(1) of the form $(2p + 2, p^2 + 2p, p^2 + 2p + 2)$.

(2) of the form $(2p^2 + 2p, 2p + 1, 2p^2 + 2p + 1)$.

(3) of the form $(4p, 3p, 5p)$.

Also solved by the proposer.

Problem 400: Proposed by the editor.

Fred was calculating the area of the ellipse $144x^2 + 256y^2$ 36864 when his friend Al commented that he could produce a closed curve which had exactly the same perimeter as Fred's ellipse and enclosed an area of exactly 16 more square units. Show how this can be done without performing any calculations.

Since no solution has been received, this problem will remain open until the next issue at which time a solution will be supplied.

Problem 401: Proposed by Bill Olk, University of Wisconsin-Madison, Madison, Wisconsin.

Show that if $b^2 < 3ac$ for real numbers a, b and c, then the equation $x^3 + ax^2 + bx + c = 0$ has one real root and two complex roots.

Solution by John Oman, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Let q, r and s be the roots of the given equation. Then $a = -(q + r + s)$, $b = qr + qs + rs$ and $c = -qrs$. Thus

$$\begin{aligned} b^2 - 3ac &= (qr + qs + rs)^2 - 3(q + r + s)(qrs) \\ &= q^2r^2 + q^2s^2 + r^2s^2 - q^2rs - r^2qs - s^2qr \\ &= [\, q^2(r - s)^2 + r^2(q - s)^2 + s^2(q - r)^2 \,]/2. \end{aligned}$$

Hence $b^2 < 3ac$ is equivalent to $q^2(r - s)^2 + r^2(q - s)^2 + s^2(q - r)^2 < 0$. Obviously this can't occur when q, r and s are all real numbers. Consequently the given equation can't have three real roots. Then since the given equation must have at least one real root, it must have one real root and two non-real roots.

Also solved by the proposer.

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical issues of interest to undergraduate students are discussed in length. Here by issue we mean the most general interpretation. Examinations of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the readers are encouraged to write responses to the discussion and submit them to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.

When one is in pursuit of an explicit formula for the nth term of a sequence (which is given recursively) most introductory textbooks on the subject describe a procedure which is not satisfactory. For example, for the Fibonacci sequence we have $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = a_{n+1}+a_n$. Then we are to assume that the explicit solution is of the form $a_n = cr^n$ for appropriate r & c to be found. Substitutions in the recursive formula lead to the characteristic equation: $r^2-r-1=0$. We solve this to get

$$r = \frac{1\pm\sqrt{1+4}}{2}\ ;\ r_1 = \frac{1+\sqrt{5}}{2}\ \&\ r_2 = \frac{1-\sqrt{5}}{2}.$$

Then we are to claim: therefore $a_n = c_1r_1^n + c_2r_2^n$,

where c_1 & c_2 are to be calculated using initial conditions $a_1 = a_2 = 1$. This is unsatisfactory because we are told to assume $a_n = cr^n$ and then to accept $a_n = c_1r_1^n + c_2r_2^n$. Most introductory sources tell us that an exact analysis is out of the scope for them. The following article is for the student who prefers a more complete answer.

I.K.

ON GENERAL SOLUTIONS OF LINEAR HOMOGENEOUS RECURRENCE RELATIONS

Abdollah Darai*

1. Introduction

There is a standard technique of finding the general solution of a linear homogeneous recurrence relation with constant coefficients. Such relations may be defined as in Section 2. This technique is found in most texts which deal with topics in combinatorics [1].

In this section we shall demonstrate the method for a second order recurrence relation. It is essentially based on assuming a particular form for the solution. This is then substituted in the recurrence relation and a new equation, usually called the characteristic equation, is obtained. If the solution of this equation consists of several linearly independent terms, then the general solution is taken to be a linear combination of

---

*Professor Darai received his Ph.D. in mathematics from the University of Manitoba at Winnipeg and is currently a member of the mathematics department of Western Illinois University. His interests include applied classical analysis and theory of elasticity.

these linearly independent solutions. However one does not often find, in the above mentioned texts, a justification for the transition from one particular form of the solution to a linear combination of a set of linearly independent solutions.

In this paper we shall demonstrate why the solution obtained by the above technique is indeed the general solution of the recurrence relation.

First we shall use this method to solve a second order problem.

Let the recurrence relation be given by

$$x_n = 5x_{n-1} + 6x_{n-2} \qquad \text{for } n \geq 2.$$

We want to find a sequence x_n which satisfies this equation.

Suppose the solution is of the form

$$x_n = r^n \qquad \text{where } r \neq 0.$$

Substituting in the original equation and simplifying we obtain the characteristic equation

$$r^2 - 5r - 6 = 0$$

where solutions are $r_1 = 6$ and $r_2 = -1$.

Since 6^n and $(-1)^n$ are linearly independent, i.e. there is not a constant c such that $6^n = c(-1)^n$, then the general solution of the recurrence relation is given by

$$x_n = A6^n + B(-1)^n$$

where A and B are arbitrary constants.

2. The General Solution of a Linear Recurrence Relation

A linear homogeneous recurrence relation of order k is an expression of the form

$$x_n = \sum_{i=1}^{k} a_i x_{n-i} \qquad (1)$$

where $n \geq k$, $a_i \in \mathbb{R}$ and $a_k \neq 0$.

In general the coefficients a_i may depend on n as well as on i in which case we shall assume that $a_n \neq 0$ for $n=0,1,2,\ldots$.

A solution of equation (1) is a sequence of real numbers $\{x_n\}$ which satisfies (1) for all non-negative integers n.

If $x_0, x_1, \ldots, x_{k-1}$ are known, then clearly a unique solution to the initial value problem exists.

In what follows we shall concentrate on second order linear homogeneous recurrence relations, but the results can easily be extended to general recurrence relations of order k.

Consider the equation

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} \qquad (2)$$

where $a_2 \neq 0$ for $n = 0,1,2,\ldots$. If a_1 and a_2 are not constants (that is to say not independent of n) then there is no general method leading to a general solution. However if one solution is known, a second linearly independent solution may be obtained by a method similar to the "variation of parameters" used in finding a second solution of a homogeneous second order linear differential equation. On the other hand if a_1 and a_2 are constants, then the general solution of equation (2) depends on the nature of the roots r_1 and r_2 of the characteristic equation

$$r^2 = a_1 r + a_2.$$

If $r_1 \neq r_2$ then the general soltuion of (2) is given by

$$x_n = c_1 r_1^n + c_2 r_2^n$$

where c_1 and c_2 are arbitrary constants. In the case of complex roots, the usual representation of powers of complex numbers is employed. A more detailed discussion can be found in [2].

In the case of repeated roots, $r_1 = r_2$, the general solution of (2) takes the form

$$x_n = c_1 r^n + c_2 n r^n.$$

It is clear that in each case the pair of solutions r_1^n, r_2^n when $r_1 \neq r_2$ and r^n, nr^n when $r_1 = r_2$, are linearly independent.

Here we shall show the following two results for the general linear second order homogeneous recurrence relation:

(i) If y_n and z_n are two solutions of (2), then the linear combination $Ay_n + Bz_n$ with arbitrary constants A and B, also satisfies (2).

(ii) If y_n and z_n are two linearly independent solutions of (2) and t_n is also a solution of (2), then t_n can be expressed as a linear combination of y_n and z_n.

<u>Proof of (i)</u>:

From (2) $\quad Ay_n = Aa_1y_{n-1} + Aa_2y_{n-2}$

and $\quad Bz_n = Ba_1z_{n-1} + Ba_2z_{n-2}$

Adding the above equations we get

$$(Ay_n+Bz_n) = a_1(Ay_{n-1}+Bz_{n-1}) + a_2(Ay_{n-2}+Bz_{n-2}).$$

Hence Ay_n+Bz_n satisfies equation (2).

<u>Proof of (ii)</u>:

Suppose y_n and z_n are two linearly independent solutions of (2) and t_n also satisfies (2). First we shall show that

$$y_0z_1 \neq y_1z_0.$$

Suppose $y_0z_1 = y_1z_0$; then y_0 and z_1 cannot both be zero, for if $y_0 = z_1 = 0$, then either $y_1 = 0$ or $z_0 = 0$. If $y_1 = 0$ and $y_0 = 0$, then from (2) $y_n = 0$ for all n, and if $z_0 = 0$ and $z_1 = 0$, then $z_n = 0$ for all n. In either case we have a contradiction of the assumption that y_n and z_n are linearly independent.

Now suppose $y_0 \neq 0$, then

$$z_1 = hy_1 \quad \text{where } h = \frac{z_0}{y_0}.$$

From (2) $z_2 = a_1z_1 + a_2z_0$

$$= a_1hy_1 + a_2hy_0$$

$$= hy_2.$$

It may be shown by induction that

$$z_n = hy_n \quad \text{for every } n,$$

again contradicting the assumption that y_n and z_n are linearly independent.

Similarly if $z_1 \neq 0$ then it can be shown that y_n and z_n are linearly dependent.

Hence $y_0z_1 \neq y_1z_0$.

This guarantees that the linear system

$$y_0b_1+z_0b_2 = t_0$$

$$y_1b_1+z_1b_2 = t_1$$

has a unique solution b_1 and b_2.

Next we define the sequence

$$t_n^* = b_1y_n+b_2z_n \quad \text{for } n\geq 0.$$

Then
$$t_n^* = a_1t_{n-1}^* + a_2t_{n-2}^*,$$

$$t_0^* = t_0 \text{ and } t_1^* = t_1.$$

Since we already know that this initial value problem has a unique solution, then $t_n^* = t_n$. Consequently we can write the solution t_n in the form

$$t_n = b_1 y_n + b_2 z_n$$

3. An Alternative Method - Hardy's Solution.

Here we shall briefly discuss a method for solving a linear homogeneous recurrence relation with constant coefficients. This method which employs the use of infinite power series, in a formal sense, is given by G.H. Hardy [3].

Consider equation (1) where for $i = 1, \ldots, k$, a_i is independent of n.

Let $\sum_{n=0}^{\infty} x_n z^n$ be the power series expansion of a rational function $f(z)$.

If we mutliply this series by

$$1-(a_1 z + a_2 z^2 + \ldots + a_k z^k)$$

and apply relation (1) to the result, we obtain a polynomial of degree $k-1$,

$$A_0 + A_1 z + A_2 z^2 + \ldots + A_{k-1} z^{k-1}$$

where $$A_0 = x_0$$

and $$A_i = x_i - \sum_{j=1}^{i} a_j x_{i-j}$$

for $$1 \le i \le k-1.$$

Then

$$f(z) = \frac{A_0 + A_1 z + A_2 z^2 + \ldots + A_{k-1} z^{k-1}}{1 - (a_1 z + a_2 z^2 + \ldots + a_k z^k)} . \tag{3}$$

Now assuming that the denominator of the right hand side of (3) can be expressed as a product of linear factors, then the series $\sum_{n=0}^{\infty} x_n z^n$ will be represented in partial fractions.

Finally, adding the binomial expansion of each term on the right hand side and comparing the coefficients of the resulting series with those of the expansion of f(z), we obtain the solution of (1).

Using the example of Section 1, suppose we want to solve

$$x_n = 5x_{n-1} + 6x_{n-2},$$

then

$$\sum_{n=0}^{\infty} x_n z^n = \frac{x_0+(x_1-5x_0)z}{1-5z-6z^2}$$
$$= \frac{B_1}{(1-6z)} + \frac{B_2}{(1+z)}$$

Now

$$(1-6z)^{-1} = 1 + 6z + (6z)^2 + (6z)^3 + \ldots$$

and

$$(1+z)^{-1} = 1-z + z^2 - z^3 + \ldots$$

so $$\sum_{n=0}^{\infty} x_n z^n = \sum_{n=0}^{\infty} [B_1 6^n + B_2(-1)^n]z^n.$$

Comparing coefficients of z^n, we obtain the following solution

$$x_n = B_1 6^n + B_2(-1)^n.$$

REFERENCES

1. E.M. Reingold, Jurg Nievergelt, Narsingh Deo, Combinatorial Algorithms Theory and Practice, Prentice-Hall, 1977.

2. W.R. Derrick, S.I. Grossman, Elementary Differential Equations with Applications, Addison-Wesley, 1981.

3. G.H. Hardy, A Course of Pure Mathematics, Cambridge University Press, 1967.

THE CURSOR
Edited by Jim Calhoun

This issue of THE CURSOR presents a paper which uses Operations Research to show the close relationship between Mathematics and Computer Science. Dr.'s Cicero and Meehan from Illinois Benedictine College are its authors. Both authors have a background in operations research while Dr. Cicero has additional interests in computer vision and networking and Dr. Meehan in statistics and probability. Illinois Benedictine is located in the Naperville, Illinois area which is one of the most rapidly growing areas of that state because of its high concentration of "high-tech" companies.

Readers, both students and faculty, are encouraged to submit papers which deal with the relationship between mathematics and computer science for consideration for inclusion in the Cursor section of The Pentagon.

A MATHEMATICAL AND SIMULATION MODEL OF A COMPUTER COMMUNICATION NETWORK

John A. Cicero and James M. Meehan
Department of Mathematical and Computer Sciences
Illinois Benedictine College
5700 College Road
Lisle, Illinois 60532

ABSTRACT

The need for mathematical and simulation models is discussed. A description of a computer communication network is presented. The physical behavior of the network is then obtained by constructing and evaluating both mathematical and simulation models. The results of the two models are compared to verify their accuracy. The reader is shown that it is nearly impossible to construct a mathematical model for more complex systems. However, simulation models can be easily obtained for more complex systems. Finally, the reader is encouraged to apply the simulation model to a more complex system.

INTRODUCTION

The area of operations research clearly illustrates the close relationship between mathematics and computer science. A common area of investigation is to predict the behavior of a physical system by both mathematical and statistical models. One gains a fair degree of confidence that the behavior of the physical system is clearly understood when the results of the mathematical simulation models closely agree. It is suggested here that as the physical system becomes more complex it becomes much more difficult to construct the mathematical model. However, it is usually quite simple to construct a simulation model for almost any physical system.

There is a strong emphasis on problem solving throughout operations research. Problems which have "real world" implications are chosen. One such problem is the calculation of expected blocking time for the interconnection network described below. First, a description of the problem is presented. Then, mathematical and simulation models are developed. Next, the results obtained from each of these models are compared to verify their correctness. Finally, the reader is advised to apply these models to more complicated systems in order to see the need for simulation.

DESCRIPTION OF THE INTERCONNECTION NETWORK

Consider the communication network of Figure 1 (the figures and program follow the body of the text of this paper). Any computer on the left side of the network must have the capability of communicating with any computer on the right side of the network. C. Clos [CLOS 53] introduced the interconnection network shown in figure 2. In that network, every intersection represents a crosspoint that can be activated by some central control unit. For example, if computer A needed to communicate with computer Z, the central control unit would activate the crosspoints at the intersection of row one, column four. The problem with that type of network is that an N x N network (where N is equal to the number of computers on the left or on the right side of the network) requires N^2 crosspoints. The cost of implementing such a network increases proportionally to the number of crosspoints. Consequently, Clos introduced the three stage interconnection network shown in figure 3. That network is made up of three stages of switching elements (the small boxes in the figure). Each switching element is a small interconnnection network. The advantage of the multiple stage network over the single stage network is that the number of crosspoints is reduced as the network size is increased. For example, a single stage 16 x 16 network requires 256 crosspoints, while a three stage 16 x 16 network with four 4 x 4 switching elements per stage requires only 192 crosspoints. As the network size increases the difference becomes more significant.

A disadvantage of the multiple stage network is that the phenomenon known as blocking can occur when an unused computer on the left side of the network cannot be connected to an unused computer on the right side of the network. For example, consider the network in Figure 4. Assume that computer A is connected to computer W through the path (i.e., the bold line) across the top of the network and computer D is connected to computer Z through the path across the bottom of the network. Now, if computer B needs to communicate with computer Y or if computer C needs to communicate with computer X, there is no available path, hence, blocking occurs. At this point, some network rearrangement scheme must be applied (see [BENE 61] and [PAUL 61]).

Assuming an initially empty network, the average amount of time it takes for blocking to occur is called the expected blocking time. We develop here a mathematical and simulation model for predicting this time in a 4 X 4 interconnection network. This small network was chosen because it makes both the analytical analysis and the simulation time reasonable.

OPERATION OF THE INTERCONNECTION NETWORK

Assume that the three stage interconnection network shown in Figure 3 is initially empty. Then, a path will be added to the network with probability p, and no action will occur with probability 1-p. If a path is to be added to the network, a randomly chosen computer on the left side of the network is connected to a randomly chosen computer on the right side of the network. Notice that for an initially empty network there are two paths between any two computers at the opposite sides of the network. Therefore, an interconnection algorithm is used to randomly select one of the two possible paths. (See lines 108 through 142 in the program in the appendix.) Once a path is established, all of the interconnections required to complete that path are saved so that at some future time the path can be removed.

Next, assume that the network is full and that all of the interconnections have been saved. Then a path will be removed from the network with probability q, and no action will occur with probability 1-q. If a path is to be removed it is chosen randomly. (See lines 61 through 71 in the program.) All of the interconnectionsthat were used in the path that was removed are returned to a pool of available resources.

Finally, assume that the network has an occupancy (i.e., number of paths) such that 0 < occupancy < N. Then, a path will be added to the network with probability (1-occupancy/N)*p, a path will be removed from the network with probability (occupancy/N)*q and no action will occur with probability

1 - (1- occupancy/N)*p - (occupancy/N)*q.

MATHEMATICAL MODEL FOR A 4 x 4 INTERCONNECTION NETWORK

For the 4 x 4 interconnection network the expected blocking time can be determined by using the theory of Markov chains. The material for this discussion can be found in any one of a number of undergraduate texts on probability such as the one by Breiman [BREI

69]. The Markov chain for this network consists of a finite number of states. During each unit of time the chain is observed to be in exactly one of these states. The transition probability P(i,j) is the conditional probability that the chain will be in the state j during the next unit of time given that it is currently in state i. (In a Markov chain the occurrence of future states depends only on the present state, not on the past.) The behavior of the chain is determined by its set of transition probabilities.

At first glance it may be tempting to reduce the state space for this chain to a set of six states: one representing the condition of blocking and each of the other five representing the number of paths (0 through 4) in the network. The problem is not that simple, however, since the various connection configurations for a given number of paths behave differently.

For this reason, it is necessary to expand the state space of the Markov chains so that it includes 11 distinct states. These will be referred to as states 0, 1, 2, 2'. 2''. 2''', 3, 3',4, 4', and S (where state S represents the condition of blocking). Examples of configurations of each of the states (except the trivial states 0 and S) are given in Figure 5. Each configuration is labeled according to the state to which it belongs, and the arrows indicate which configurations lead to one another. Two configurations are said to lead to one another if one can be obtained from the other by connecting or disconnecting a single path. Two states are said to lead to one another if there are two configurations, one in each state, which lead to one another. It is possible for the chain to remain in a given state (actually a particular configuration) for more than one unit of time if there are no connections or disconnections.

Notationally, the integer value in the name of a state represents the number of paths in each of its configurations, while the number of primes for states with the same number of paths is an indicator of the proximity to blocking. For example, state 2' and state 2'' each consist of configurations with two paths but the configurations in state 2'' have a higher probability of blocking within a short period of time than the configurations in state 2'. However, state 3' configurations, which tend to block sooner than those of state 3, also tend to block sooner than those of state 2''. The configurations of state 2''' are exactly those which lead directly to blocking. That is, it is only when the chain is in state 2''' that blocking can occur in the next unit of time. In all, there are four configurations in each of the states 2', 2'', and 2'''; two configurations in each of states 4 and 4' ; and eight configurations in each of states 1, 2, 3 and 3'. All of these facts become more apparent when the entire set of configurations is studied. It is a good exercise for the reader to complete Figure 5 by finding all remaining configurations, determining which configurations lead to one another, and then classifying the configurations according to states.

From the information in Figure 5, transition probabilities can be determined. Notice, for instance, that two of the disconnections from the configurations at the top of the state 3 column lead to state 2 configurations, while the other disconnection leads to a state 2' configuration. On the other hand, disconnections from the state 3' configuration at the bottom of the third column lead to configurations in states 2', 2'', and 2'''. Now consider the transition probability P(3,2). In order to move from state 3 to state 2 a disconnection must occur. As described in the previous section a disconnection will occur with probability 3/4*q. Given that the chain is in state 3 and a disconnection occurs, the probability of moving to state 2 is 2/3, since two of the possible equally likely disconnections lead to state 2. Assuming that the occurrence of a disconnection and the determination of the path are independent events then P(3,2) = 3/4*q*2/3 = q/2. Similarly, it can be argued that P(3',2') = P(3',2'') = P(3',2''') = 3/4*q*(1/3) = q/4. Going in the other direction, it can be shown that P(2',3) = P(2'.3') = p/4.

A state diagram indicating the set of strict transition probabilities for this Markov chain appears in Figure 6. A transition is said to be strict if a change in configuration actually occurs. Figure 6 does not include probabilities of transition from a state to itself (i,e,, those of the form P(i,i)). Each P(i,i) can be determined from the information in Figure 6 by subtracting from one the sum over j of the strict probabilities, P(i,j). In order to compute P(2',2'), for instance, notice that it is possible to move from state 2'' to states 1, 3, or 3'. Therefore P(2',2') = 1- P(2',1) - P(2',3) - P(2',3') = 1 - p/2 - q/2.

Having obtained the set of transition probabilities, the value of T, the expected blocking time given that the chain is currently in state i, can be found. The T_i's are related by equations of the form

$$T_i = 1 + \sum_j P(i, j)*T_j$$

where the summation is taken over all states in the state space. Roughly speaking, each such equation indicates that the time until blocking from state i is one unit of time longer than it would take to block from state j if the chain moved directly from state i to state j.

Since T_s=o there are 10 unknown values of T_i and 10 such equations for our simple model. For example, the equation for $T_{2'}$ is

$$T_{2'} = 1 + q/2*T_1 + (1-p/2-q/2) * T_{2'} + p/4*T_3 + p/4* T_{3'}$$

Note that P (2',j) = 0 for all states j other than states 1, 2', 3 and 3'. Since the set of equations is linear, T_0 can be found using Cramer's Rule. The resulting values of T_0 for various values of p and q will be compared with simulation results.

SIMULATION MODEL FOR A 4 X4 INTERCONNECTION NETWORK

We have included our simulation program at the end of the paper because computer simulation is such a major component of our Operations Research course. The program was written in Turbo Pascal and was run on a PC XT. The program has two major components: a stopping rule used to estimate the expected blocking time within a specified precision, and the actual interconnection experiment itself.

The main program (lines 150 through 186) will estimate the expected blocking time of the network for various values of p and q. This is accomplished by continuously running the interconnection experiment until the estimated blocking time is obtained within a specified precision. The stopping rule used to obtain this precision is described in detail on page 292 of [LAW 82].

The actual interconnection experiment is the procedure 'blocking time' (lines 11 through 148). Each time the experiment is run it will return a single blocking time. This time is used by the main program (as described above) to estimate the expected blocking time. This procedure is described in detail in the following paragraphs.

In lines 25 through 55, the initialization of the experiment is performed. The lines connected to the left and right sides of the network and interconnection lines within the network are set to unused. The path matrix which records all the interconnecting lines for a given path is set to empty, and the time to block and the system occupancy are set to zero.

In lines 56 through 147, the experiment is run until a block occurs. Specifically, lines 60 through 79 represent a disconnection. This is accomplished by randomly selecting a path from the path matrix then restoring all the lines associated with that path to unused.

In lines 81 through 144, a connection is attempted. First, unused lines on the left and right sides of the network are randomly chosen. Then, an interconnection path is attempted between these two lines. If the path is established, the experiment continues, other wise, an alternate path is attempted. If the alternate path is successful the experiment continues, otherwise the experiment terminates in a blocked condition.

In line 145, the time click is updated whether there was a connection, a disconnection, or no action at all.

COMPARISON OF SIMULATION AND MATHEMATICAL RESULTS

An important objective of Operations Research studies is to present results in an acceptable manner. Therefore, graphical presentation is encouraged.

Figure 7 gives a comparison of the results obtained from the simulation and mathematical models for different values of p and q. It can be seen that the results from the two models are almost identical. This should provide the reader with a high degree of confidence in the simulation model. This is important because as the network size increases the reader still needs to obtain expected blocking time results even though the mathematical model is probably not available.

CONCLUSION AND FURTHER INVESTIGATION

The reader is encouraged to apply both models to a 9 x 9 interconnection network. That network will have three 3 x 3 switching elements per stage. The reader will soon discover that increasing the size of the analytical model is very difficult, but increasing the size of the simulation model is very simple.

The reader is also encouraged to try to increase the expected blocking time for an interconnection network by replacing the random connection algorithm with a strategic connection algorithm. This new algorithm can be obtained by observing the state diagram in Figure 6. An algorithm that attempts to keep the chain out of state 2'" will extend the expected blocking time.

FIGURES 1-7

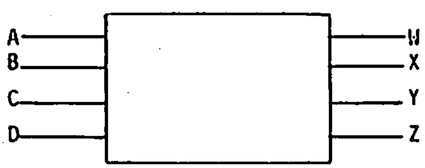

Figure 1. An Interconnection Network.

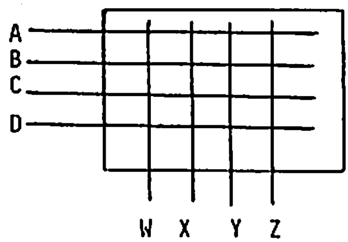

Figure 2. A Single Stage Interconnection Network.

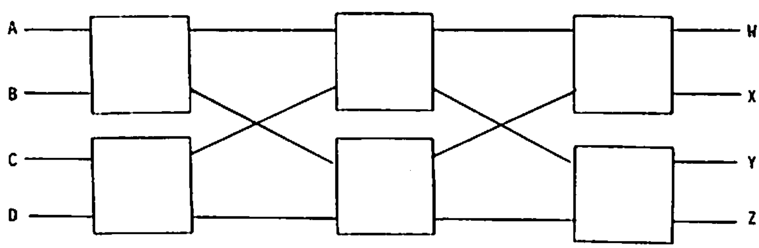

Figure 3. A Three-Stage Interconnection Network.

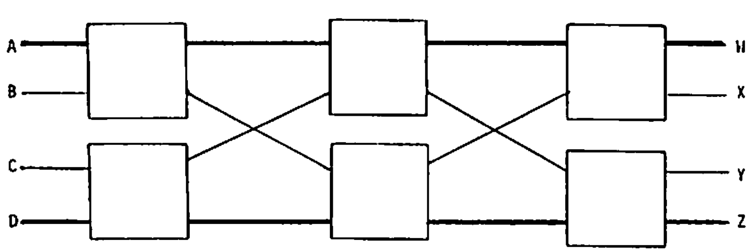

Figure 4. A Three-Stage Interconnection Network with the Potential for Blocking.

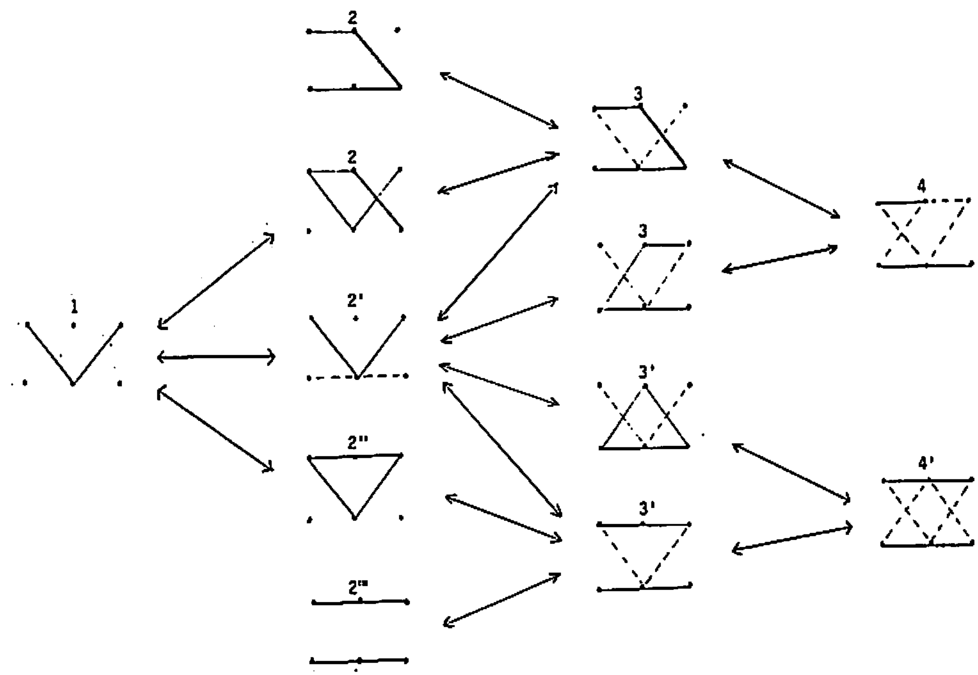

Figure 5. Examples of Configurations in a 4x4 Three Stage Network.

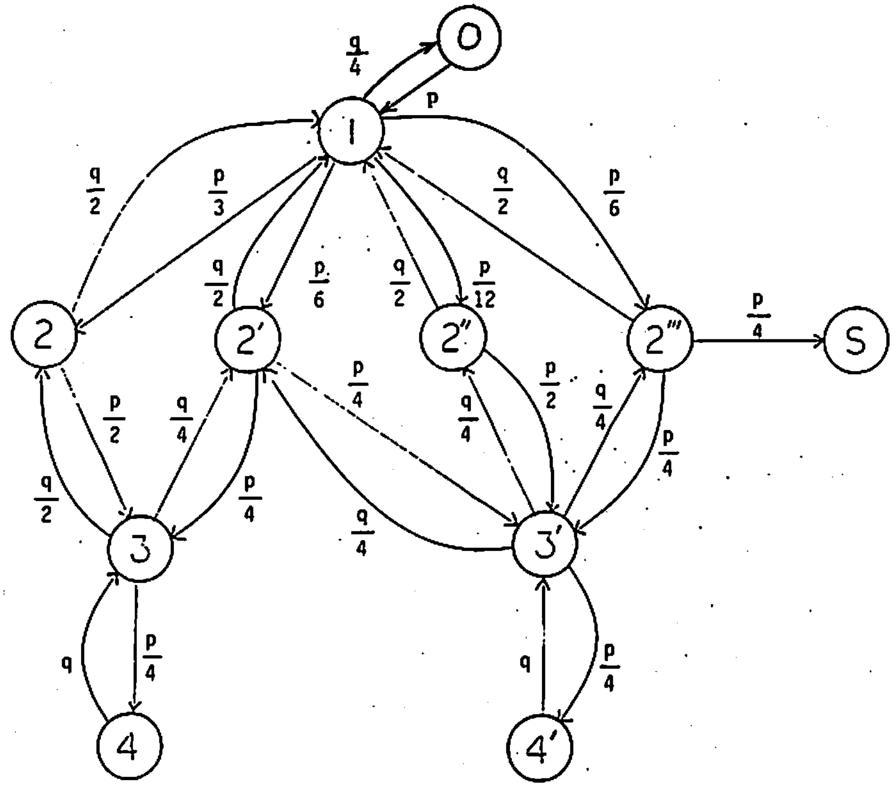

Figure 6. State Diagram with Transition Probabilities in a 4x4 Network.
Note: The probability of moving from a state to itself is not indicated.

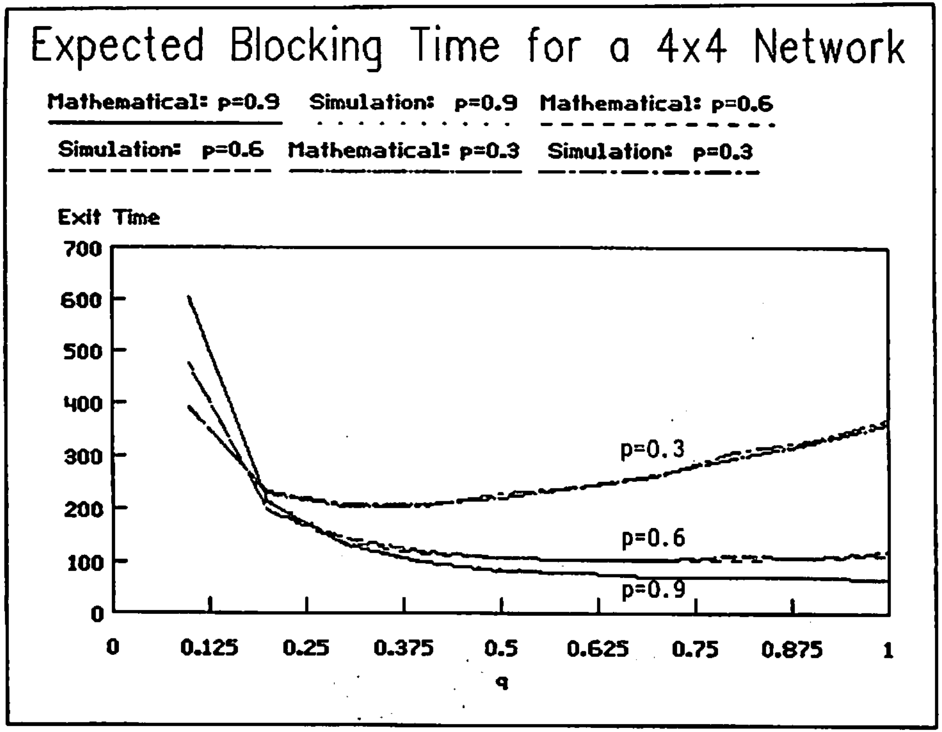

Figure 7. Expected Blocking Time for a 4x4 Network.

THE PROGRAM

```
program expected_blocking_time;

{This program will obtain the expected blocking time of a 4x4     }
{ interconnection network within a specified precision.           }

      var done:boolean;
           average, average_squared,variance : real;
           n,delta,p,q,gamma,time : real;
           pp,qq : integer;

procedure blocking_time(var time: real: p,q : real);
{This procedure will return the amount of time that it takes to   }
{reach a blocked state given that the network is initially empty  }

      var k,m,j,: integer;
           a,b : array(0..3) of integer;
           c   : array(0..3,0..3) of integer;
           d   : array(0..3,0..1) of boolean;
           left_side,right_side,counter : integer;
           stage_1,stage_3_2,stage_2_to_1,stage_3 : integer;
           done,blocking_occurs : boolean;
           x : real;
           occ : integer;

begin
{The 'a' vector indicates the unused lines entering the left      }
{side of the network.                                             }
{The 'b' vector indicates the unused lines leaving the right}
{side of the network.                                             }
{The 'c' matrix records all the interconnections for one          }
{complete path.                                                   }
{The 'd' matraix indicates the unused lines between the first     }
{and second stages and between the second and third stages.       }

      for j:=0 to 3 do begin
         a(j):=true;  {Initially,m set all lines entering the left    }
                      {side of the network to unused.                 }
         b(j):=true;  {Set all lines leaving the right side of the    }
                      {network to unused.                             }
         for k:=0 to 3 do c(j,k):=0;  {Initially, there are no        }
      end;                            {recorded connections.          }
      for j:=0 to 3 do begin {Set all of the lines between the        }
         d(j,0):=true;       {first and second stages and between }
         d(j,1):=true;       {the second and third stages to unused}
      end;
      {time will count how long it takes before blocking occurs.  }
      time:=0.0;

      {blocking_occurs is true when blocking occurs               }
      blocking_occurs:=false;
```

```
{Initially, the occupancy of the network is zero or empty.   }
occ := 0;

repeat  {Run the experiment until blocking occurs.            }
     x:=random;  {x is a random number such that 0 <=  x < 1}
  {There is either a connection, a disconnection, or no action}

     if 1x>(1-(occ/4)*q)) then begin{There is a disconnection}
        j:=trunc(random occ);{Randomly, pick an existing path}
                                        {from the path matrix.}
       k:=c(j,1);        {Set the interconnection line between }
       d(k,0):=true;     { the first and second stage to unused. }
       k:=c(J,2);        {Set the interconnection line between }
       d(k,1):=true;     {  the second and third stage to unused.}
       k:=c(j,0);        {Set the line entering the left side of  }
       a(k):=true;        {  the network to unused.              }
       k:=c(j,3);         {Set the line leaving the right side of }
       b(k):=true;       {the network to unused.                 }
       occ:=occ-1;       {Reduce the network occupancy by one.}

        if (occ>0) and (j<occ) then {If there are any remaining}
          for m:=j to occ-1 do begin {paths, readjust the path}
             for k:=0 to 3 do begin   {matrix so that the first }
               c(m,k):=c(m+1,k);      {'occ' entries in the path }
             end;                 {matrix contain all valid      }
           end;                         {paths.                  }
end {This is the end of a disconnection.

else if x<((1-occ/4)*p) then begin    {There is a connection.}

   done:=false;
   while not done do begin                {Find an unused line  }
       left_side := trunc(random 4);      {coming into the left }
       if a(left_side) then done:=true; {  side of the network.}
   end; {while not done}

  done:=false;
   while not done do begin                 {Find an unused line }
        right _side := trunc(random 4);  { leaving the right     }
        if b(right_side) then done:=true;{side of the network.}
   end; {while not done}

c[occ,0] := left_side;    {Record the line entering the left    }
c[occ,3] := right_side;   { side of the netework and the line   }
                          { leaving the right side of the       }
                          { network in the path matrix.         }
a[left_side] := false;    { Set the line entering the left      }
b[right_side] := false    { side of the network and the line    }
                          {leaving the right side of the        }
done := false;            {network used                         }
{Calculate the switching element used in stage one: 0 or 1  }
```

```
        stage_1  := trunc(( left_side)/2);
        {Calculate the switching element used in stage three: 0 or 1}
        stage_3 := trunc((right_side/2))
        j := 0;
        {once the stage three switching element is determined, a    }
        { counter is used to randomly select a stage two switching}
        { element.                                                    }
        counter := trunc( random*2);

        {Next, attempt to establish a path from the third stage to  }
        { the first stage.  There are two possible paths to take;   }
        { if the first path is available, use it; otherwise, try    }
        { the second path.                                          }
   while (j<2) and  (not done)  do  begin
        j := j + 1;
        {Locate a line from stage three to stage two.              }
        stage_3to_2 := stage_3 * 2*+ counter;

        {Given the line from stage three to two ther is only       }
        { one posible line from stage two to one.                   }
        Stage_2_to_1 := (stage_3_to_2 mod  2) *2 +_ stage_1;
        { If there is a complet path from stage three to one, set  }
        { the line from stage three to two and the line from       }
        { stage two to one to be used  record the connections in the}
        { path matrix; increase the network occupancy by one; and  }
        { set the done flag to true.                                }
        if (d[stage_3_to_2, 1]  ) and (d([stage_2_to_1,0]) then begin
             d[stage_3_to_2, 1) := false;
             c[occ, 2] := stage_3_2;
             d[stage_2_to_1, 0] := false;
             c[occ, 1] := stage_2_to_1;
             done := true;
             occ := occ + 1;
        end {if}
        {Else, try the other path between stage three and one      }
        else counter := (counter + 1) mod 2;
        end;  {while (j<2 and not done}
        { Blocking occurs if no path exists after the second try    }
        blocking_occurs := not  done;
     end; { This is the end of a connection attempt.               }
        time := time + 1.0; {  Update the clock whether there was a }
                  {disconnection, a connection, or no event at all.  }
  until  blocking_occurs;
 end;  {procedure blocking_time}


 begin { expected_blocking_time}
   {This main routine will calculate the expected blocking time    }
   { within a specified precision. The stopping rule that is ued in }
   { The experiments are run for various values of p and q.         }
        for  pp:=10 downto 1 do begin
             p := oo/a0;  { p is the probability that an empty system}
```

```
                        { will add a path.                        }
            for qq:= 10 downto 1 do begin
                q := qq/10; { q is the probability that a full system}
                            { will remove a path                    }
                blocking_time(time, p, q); { For a given value of p  }
                        { and q find the blocking time of one trial. }
                average := time;
                average_squared := sqr(time);
                done := false;
                n := 2;

                while not done do begin
                    blocking_time(time,p,q);
                    average := (average*(n-1) + time)/n;
                    average_squared:= average_squared+sqr(time);
                    variance:=(average_squared-
                                        N*sqr(average))/(n-1 );
                    delta := 2*sqrt(variance/(n));
                    gamma := delta/average;
                    if ( gamma < 0.06 ) then begin
                        write (lst, 'p=',p:7:2,' q= ',q:7:2,
                        ' n=',n:7:1,' expected blocking_time
                                        =',average:7:2);
                        writeln(lst,'[',(average-delta):7:2,'-',
                                        (average+delta):7:2,']');
                        done := true;
                    end;
                    else n:= n + 1;
                end {while not done};
            end; {end for qq}
        end; { end for pp
    end. {expected_blocking_time}
```

BIBLIOGRAPHY

[BENE 61] Benes, V.E., "On Rearrangeable Three-Stage Connecting Networks", The Bell System Technical Journal, Vol. XL, September 1961, pp 1481-1491.

[BREI 69] Breiman, L., Probability and Stochastic Processes: With a View Toward Applications, Houghton Mifflin Publishing Company, 1969, Boston,MA.

[CLOS 53] Clos,C., "A Study of Non-Blocking Switching Networks", The Bell System Technical Journal. Vol. XXXLL, March 1953, pp406-424.

[LAW 82] Law, A.M. and Kelton, W.D., Simulation, Modeling and Analysis, McGraw-Hill Book Company, 1982, New York,NY.

[PAUL 61] Paull, M.C., "Reswitching of Connection Networks", The Bell System Technical Journal, Vol. XL, May 1961, pp. 833-855.

KAPPA MU EPSILON NEWS

Edited by M. Michael Awad

News of chapter activities and other noteworthy KME events should be sent to Dr. M. Michael Awad, Historian, Kappa Mu Epsilon, Mathematics Department, Southwest Missouri State University, Springfield, MO 65804.

CHAPTER NEWS

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President - Erik Harder
62 actives, 37 initiates

The chapter assisted the Mathematics Department with the annual Cal Poly Math Contest which attracted over 500 high school students. We spent most of our efforts preparing to host the 26th Biennial Convention. Rick Kalinyak was the recipient of the Arthur Andersen & Co. Professional Performance Award. June Aoki and Steven Sorensen were joint recipients of the Founders Award. Other 1987-88 officers: Rex McAfee, Stefan Steiner, Janet Garrett, Ed Soliman, David Martin, vice presidents; Susan Daijo, secretary; Kelly Abbott, treasurer; Raymond D. Terry, corresponding secretary; Adelaide T. Harmon-Elliott, faculty sponsor.

Colorado Beta, Colorado School of Mines, Golden
Chapter President - Jeff Jackson
20 actives, 8 initiates

Special activities during the spring semester 1986-87 included a very successful "book sale" of old textbooks that had accumulated in the department over a period of years and a scrumptiously successful picnic in honor of all the seniors scheduled to graduate in May or during the summer of 1987. Other 1987-88 officers: Todd Versaw, vice president; Sherri

Anzlovar, secretary; Amber Lusk, treasurer; Ardel J. Boes, corresponding secretary; Aldo V. Vecchia, faculty sponsor.

Colorado Gamma, Fort Lewis College, Durango
Chapter President - Johnny Snyder
13 actives, 5 initiates

Two meetings were held in the spring. At one of the meetings, Dr. Gibbs spoke on "The Divergence of the Harmonic Series" and at the other, Amy Getz spoke on "Probability and the Reimann Zeta Function." The high school tutoring program continued with some success. Mike Vair, Denise Kobel, Art Smith and faculty sponsor, Dr. Gibbs, attended the biennial meeting at Cal Poly and had a great time! Five new members were inducted on April 13. On April 24-25 two members preesented short talks at the Rocky Mountain section meeting of the Mathematical Association of America in Pueblo, Colorado. Amy Getz spoke on "Probability and the Reimann Zeta Function" and Art Smith spoke on "Fun with Logarithms." Other 1987-88 officers: Brian Sherfey, vice president; Amy Getz, secretary; Kevin Marushack, treasurer; Richard A. Gibbs, corresponding secretary and faculty sponsor.

Connecticut Beta, Eastern Connecticut State University, Willimantic
15 actives, 12 initiates

Officers are to be chosen during the fall, 1987, semester. We had three guest lecturers, traveled to outside colloquia, had a picnic, and sponsored a math tutoring lab. Other 1987-88 officers: Stephen Kenton, corrsponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Tammy Gresham
25 actives, 13 initiates

On May 20, we had our annual initiation meeting and inducted 13 new members. After the ceremony, there was a reception in honor of the new initiates. At the reception it was announced that two KME members will receive mathematics scholarships for 1987-88: Dwain Hooten and Keisha Cantrell. Earlier on May 1, the KME members assisted with Math Day activities when area high school math students visited the campus. Other 1987-88 officers: David Abeita, vice president; Keisha Cantrell, secretary; Kristi Milam, treasurer; Joe Sharp, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston
Chapter President - Tricia Setzke
46 actives, 11 initiates

There were a number of KME meetings held throughout the semester. The KME initiation was held on April 9 and the honors banquet took place on April 12. The spring picnic was held on April 25. Other 1987-88 officers: David Wasser, vice preesident; Dorothy Graham, secretary; Debra English, treasurer; Floyd Koontz, corresponding secretary.

Illinois Delta, College of St. Francis, Joliet
Chapter President - Jeanette Rogers
22 actives, 13 initiates

During the spring semester of 1987 Illinois Delta sponsored an information seminar in conjunction with CNA Insurance Corporation of America. Representatives of CNA provided pertinent information relative to job opportunities in the actuarial sciences, computer science and business areas of their many companies. Interviews were organized and several of our May graduates are now employees of CNA. On March 31 thirteen new members were inducted into our chapter.

Our guest speaker for the evening was Dr. Bruce Berndt, Professor of Mathematics, University of Illinois - Urbana, and his presentation was "Ramanujan's Notebooks." Dr. Berndt has devoted all of his research efforts for the last ten years editing and proving the theorems of Ramanujan. The corresponding secretary and one member of our chapter attended the 26th Biennial Convention at Cal Poly State University, April 2-4. Other 1987-88 officers: Emily Schmit, vice president; Pamela Damore, secretary; Peter Kohl, treasurer; JoAnn Lopykinski, historian; Sister Virginia McGee, corresponding secretary and faculty sponsor.

Illinois Zeta, Rosary College, River Forest
Chapter President - Patty Cara
26 actives, 11 initiates

Meetings were held monthly. One of our members delivered a paper to a local math society (ACCA). We conducted a daily tutoring lab for any student in need of tutoring in math and raised sufficient funds to take the whole club to a restaurant for lunch on the last day of class. Other 1987-88 officers: Natalie Perri, vice president; Mariola Janek, secretary; Kathleen Schmidt, treasurer; Mordechai Goodman, corresponding secretary and faculty sponsor.

Illinois Eta, Western Illinois University, Macomb
Chapter President - David Miller
18 actives, 12 initiates

During the semester, we held biweekly meetings with speakers, Spring Bowling Party, Faculty Chili Lunch, Spring Banquet, Spring Initiation, Contest and displays for Mathematics Awareness Week. Other 1987-88 officers: Janet Kester, vice president; Darren Schultz, secretary; Alan Bishop, corrsponding secretary.

Illinois Theta, Illinois Benedictine College, Lisle
Chapter President - Jacqueline Haeflinger
15 actives

The chapter met every three weeks during the semester. The major activity was hosting the eighth annual high school mathematics contest. One-hundred students from twenty high schools participated on February 21, 1987. The next initiation will be held in October, 1987. Other 1987-88 officers: Paul Toussaint, vice president; Tracey, secretary; Jill, treasurer; James Meehan, corresponding secretary and faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
Chapter President - Dawn Crum
16 actives, 9 initiates

Other 1987-88 officers: Verne Leininger, vice president; Julie Eichenauer, secretary; Dan Byler, treasurer; Ralph McBride, corresponding secretary; Deborah Hustin, faculty sponsor.

Indiana Beta, Butler University, Indianapolis
Chapter President - Amy Odell
2 actives, 15 initiates

Other 1987-88 officers: Jan Messersmith; vice president; Keith Fuller, secretary; Gregory Francis, treasurer; Jeremiah Farrell, corresponding secretary and faculty sponsor.

Indiana Gamma, Anderson College, Anderson
Chapter President - Kevin Pitts
10 actives, 4 initiates

Other 1987-88 officers: Rachel Deal, vice president; Susan Duncan, secretary; Lisa Rozevink, treasurer; Stanley Stephens, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President, Diane Strachan
31 actives, 6 initiates

Students David Bishop, Tracy Konrad, Jeremy Phillips, Brad Story, and Diane Strachan attended the KME National Convention at Cal Poly accompanied by faculty members J. Bruha, J. Cross, and J. Longnecker. Both Bishop and Phillips presented papers at the convention. Students preesenting papers at local KME meetings included Joe Inmann on "Two in Succession - A Probability Problem" and Jeremy Phillips on "The Superreals." Phillips also provided the address for the KME initiation banquet held on April 30 at the Cedar Falls Woman's Club. Other 1987-88 officers: Robert Hauser, vice president; Greg Mehrl, secretary; Suzanne Buckwalter, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President - Susan Poppen
44 actives, 22 initiates

The Iowa Delta Chapter of KME and the Department of Mathematics and Computer Science conducted the tenth annual Wartburg Math Field Day on March 14, 1987. Eighty-eight students participated representing twelve schools. Three schools were recognized for participating in each of the ten math field days at Wartburg. On March 28, 1987, twenty-two new members were initiated into the Iowa Delta Chapter at the annual initiation banquet. Two members were supported by the chapter to be delegates to the 26th Biennial Convention held at California Polytechnic State University in San Luis Obispo, California on April 2-4, 1987. The year's activities ended with the annual picnic on May 11, 1987. Other 1987-88 officers: Candy Saunders, vice president; Terry Letsche, secretary; Brian Isaacs, treasurer; August Waltmann, corresponding secretary; Lynn Olson, faculty sponsor.

<u>Kansas Alpha</u>, Pittsburg State University, Pittsburg
Chapter President - Darbi Frieden
50 actives, 12 initiates

The spring semester began with a dinner and initiation for the February meeting. Twelve new members were initiated at that time. Following the initiation ceremony, Dr. Will Self, PSU Mathematics Department, presented the program entitled "Artificial Intelligence." Todd Tarter spoke at the March meeting. His talk was titled "PROLOG, An Artificial Intelligence Language." The April meeting program was given by Tom Skahan on "Banded Matrices." The chapter assisted the Mathematics Department faculty in administering and grading tests given at the annual math relays, April 3, 1987. Several members also worked for the Alumni Association's annual Phon-o-thon. They received 5th prize for amount of money raised by student organizations. The final meeting of the semester was a social event held at Professor McGrath's home. Homemade ice cream and cake were served to those attending. Carla VanCleave told of her experiences in attending the <u>KME</u> National Convention in San Luis Obispo, California. Officers for the 1987-88 school year were elected. The annual Robert M. Mendenhall awards for scholastic achievement were presented to Tim Flood, Jennifer Munson, Carla VanCleave, and Ronald Warstler. They received <u>KME</u> pins in recognition of this honor. Other 1987-88 officers: Marcia Allmond, vice president; Kymberly Booth, secretary; Tim Flood, treasurer; Harold Thomas, corresponding secretary; Helen Kriegsman and Gary McGrath, faculty sponsors.

<u>Kansas Beta</u>, Emporia State University, Emporia
Chapter President - Deanne Eberhart
31 actives

Other 1987-88 officers: Teri Hallman, vice president; Doris Prothe, secretary and treasurer; George Downing, corresponding secretary; Larry Scott, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Lisa Brox
23 actives, 7 initiates

An all-night volleyball tournament was sponsored by KS Gamma on February 6 to raise funds for attending the National Convention at Cal Poly. Initiated into KS Gamma on February 19 were Scott Beaven, Frank Feuerborn, Rosemary Mattas, Susanne Piper, Denise Quigley, Elizabeth Zahrt, and Matthew Strathman. Dinner preceded the initiation ceremony which was followed by a preesentation of "As the Water Swirls" by Pat Hirsch. Many students helped with the chapter's 17th Mathematics Tournament for high school students on March 21st. Juniors Lisa Brox and Pat Hirsch attended the April National Convention at Cal Poly with Sr. JoAnn Fellin. Lisa presented a paper on "Computational Complexity" and Pat presented a paper entitled "As the Water Swirls." Sr. JoAnn Fellin was elected National Treasurer. Holding Sister Helen Sullivan Scholarship awards during the 1987-88 academic year will be senior Lisa Brox and Junior Susanne Piper. Senior members were presented key chains and wishes of good luck at the April 24th picnic which closed the chapter activities for the year. Other 1987-88 officers: Susanne Piper, vice president; Matthew Strathman, secretary and treasurer; Betsy Zahrt, historian; Sister JoAnn Fellin, corresponding secretary and faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President - Kelly Eisenbarth
15 actives, 3 initiates

Kelly Eisenbarth and Ron Wasserstein attended the San Luis Obispo convention. Kelly's paper, "From Alice to Algebra," won third place. Other 1987-88 officers: Kara Richey, vice president; Jill Rasmussen, secretary and treasurer; Robert Thompson, corresponding secretary; Ron Wasserstein, faculty sponsor.

Kansas Epsilon, Fort Hays State University, Hays
36 actives, 6 initiates

Our spring picnic was held on May 2. Officers for 1987-88 have not yet been selected. Charles Votaw is the corresponding secretary; Mary Kay Schippers is the faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President - Brenda Coble
20 actives, 25 initiates

The chapter spent a good part of the spring semester raising money for the trip to the national KME convention. Candy bars and donuts were the main sources of income. Seven students and Dr. C made the trip to the convention and had a great time. Before the convention, there were two guest speakers at regularly-scheduled chapter meetings. Tim Melvin gave his talk on amicable numbers that he presented at the convention. Dr. Aughtum Howard presented an interesting talk on "What Can Be Done with a Mathematics Degree After Graduation." New members were initiated after the convention at a ceremony where Dr. Amy King gave an inspirational talk on Grace Murray Hopper. Other 1987-88 officers: Donnie Steinberg, vice president; Carrie Lash, secretary; Dave Boldery, treasurer; Patrick Costello, corresponding secretary; Bill Janeway, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore
Chapter President - Mollie Toole
6 actives, 10 initiates

On May 11, the annual dinner and induction of new members took place. As part of the program there was a presentation given by Mary Ann Hoppe, Michele Garcia and Suzanne Zeiler on their teamwork for the Mathematics Modeling contest sponsored by COMAP. They spoke on the Optimization of Parking Spaces. Other

1987-88 officers: Donna Parker, vice president; Dale Woods, secretary; Suzanne Zeiler, treasurer; Sister Marie Augustine Dowling, corresponding secretary; Joseph DiRienzi, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President - Michele A. Lawyer
12 actives, 2 initiates

We held a career night in the early part of the semester, inviting back several alumni members to talk about their work. We also held our initiation of new members and President Michele Lawyer presented a paper which she then presented again at the National KME Meeting in California in early April. We held our annual picnic at our department chairman's house in May. We had a booth at May Day, selling soft pretzels as a fund raiser. Other 1987-88 officers: Christopher Conklin, vice president; Elaine Joyce, secretary; Andrew Raith, treasurer; James E. Lightner, corrsponding secretary; Linda R. Eshleman, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President - Bradley Richards
30 actives, 15 initiates

On February 15, 1987, fifteen new members were inducted into the Maryland Delta Chapter of KME: Leslie Baker, Patrick Crouse, Suzanna Dotson, Michael Duncan, Laura Dudley, Cheryl Ison, Wendi Johnson, Mary Jones, Lisa Lewis, Lisa Mackay, John Myers, Holly Robinson, Susan Starr, Edward White, and Gregory Wolodkin. During the spring semester, members Kevin Lowery and Chris Judge presented talks entitled, respectively, "Biorhythms," and "The Coffee Cup or the Doughnut." The chapter also co-sponsored the Frostburg State College Mathematics Symposium: "A New Angle on Geometry." Other 1987-88 officers: Lisa Lewis, vice president; Laura Dudley, secretary; Judy Anderson, treasurer; Edward T. White, corresponding secretary; John P. Jones, faculty sponsor.

<u>Massachusetts</u> <u>Alpha</u>, Assumption College, Worcester
Chapter President - Christopher Abreu
4 actives, 5 initiates

Five new members were initiated on April 1, 1987. Following a dinner in honor of the new members, Dr. Joyce Brown, of the Assumption faculty; spoke on "Cents and Irrationality: The Mathematics of Musical Temperament." Other 1987-88 officers: Lisa Maher, vice president; Charles Brusard, corresponding secretary.

<u>Michigan</u> <u>Alpha</u>, Albion College, Albion
Chapter President - Douglas LeMaster
17 actives, 17 initiates

We have had an active program for the past several years. Only this spring, however, did we renew our formal contacts with <u>KME</u> by initiating 17 members. New officers will be elected in the fall. Other 1987-88 officers: Susan Krampe, vice president; Kenneth Alberts, secretary; Holly Stroud, treasurer; Robert Messer, corresponding secretary and faculty sponsor.

<u>Mississippi</u> <u>Alpha</u>, Mississippi University for Women, Columbus
Chapter President - Katherine Adams
15 actives

<u>KME</u> assisted in hosting the meeting of the Louisiana-Mississippi Section of the Mathematical Association of America on our campus. Other 1987-88 officers: Michele Whitley, vice president; Sherri Duncan, secretary and treasurer; Jean Parra, corresponding secretary; Carol Ottinger, faculty sponsor.

<u>Mississippi Gamma</u>, University of Southern Mississippi, Hattiesburg
Chapter President - Mark Huff
30 actives, 10 initiates

Our chapter held its initiation and patio supper on April 9th at the home of Dr. and Mrs. Mullins. There was a short initiation ceremony and officers for the 87-88 school year were nominated. Other 1987-88 officers: Amy Duvall, vice president; Joyce Deer, secretary; Alice Essary, treasurer and corresponding secretary; Virginia Entrekin, faculty sponsor.

<u>Missouri Alpha</u>, Southwest Missouri State University, Springfield
Chapter President - Kevin Keltner
44 actives, 11 initiates

The Missouri Alpha Chapter held three regular meetings in the spring which included presentations by two students. We sent a delegation of six students and two faculty members to the national convention in San Luis Obispo. Lori Baskins received the first place award for her paper, "Homothetic Proof of the Nine-Point Circle," at the convention. A spring banquet was held at which Lori Baskins and Darlena Jones were honored for their contributions to <u>KME</u>. Other 1987-88 officers: Anita Shockley, vice president; Melissa Hays, secretary; Sharon Kruse, treasurer; John Kubicek, corresponding secretary; Simon Bernau, faculty sponsor.

<u>Missouri Beta</u>, Central Missouri State University, Warrensburg
Chapter President - David Beard
45 actives - 12 initiates

This year we were represented at the national convention by four students and three faculty members. Gayla Benson presented a paper. Dr. Louis Leithold from Pepperdine College spoke at our annual spring honors banquet. Michael Fallon, a former CMSU student who now works for AT&T, was our guest speaker for the annual Klingenberg Lecture. Other 1987-88 officers: Richard Wohletz, vice president; Gayla Benson, secretary; Kelly Elwell, treasurer; Homer Hampton, corresponding secretary; Larry Dilley, faculty sponsor.

Missouri Gamma, William Jewell College, Liberty
Chapter President - Greg Dance
23 actives, 15 initiates

Chapter meetings were held monthly throughout the year. The annual initiation ceremony and banquet were held in April, with guest speaker Dr. Glenn Burnett of Evangel College. Other 1987-88 officers: Laura Kephart, vice president; Susan Brannen, secretary; Joseph T. Mathis, treasurer, corresponding secretary, and faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President - Lynn Stacy
3 actives, 10 initiates

Other 1987-88 officers: Suzie Conley, vice president; Mark Briesacher, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President - Mike Seih

Throughout the spring semester, club members monitored the Mathematics and Science Building in the evenings to earn money for the club. The club purchased a permanent campus park bench with a plaque in memory of Dr. Frank Prather, Professor of Mathematics and Chairman of the Mathematics-Science Division, who passed away on April 7, 1987. The club administered the annual test to identify the outstanding freshman majoring in mathematics. The award went to Julie Moffett whose home is Omaha, Nebraska. The award includes the recipient's name being engraved on a permanent plaque, payment of KME national dues, and one year honorary membership in the local KME chapter. The speaker at the annual banquet was Dr. Jeffery Martin, a local physician. Dr. Fred Webber, Corresponding Secretary of Nebraska Alpha and Professor of Mathematics, was selected as the Outstanding Professor in the Mathematics-Science Division for 1986-87. Students majoring or minoring in the sciences and mathematics are eligible to vote by secret ballot. The $25 book scholarship sponsored by the club was won by Kelly Krutz. Club members assisted the Wayne State College mathematics faculty with the Thirteenth Annual WSC Mathematics Contest on May 11, 1987, kept the KME bulletin board current, and sponsored some social functions for club members and guests. Meetings this semester were highlighted with the "Meet the Professor Series." At each meeting one professor gave a brief synopsis of his background and up-bringing and then fielded questions from members. The club participated in the annual WSC College Bowl. KME team members were Jim Fisher, Darin Moon, Rusty Sadler, Stacey Blakemore, and Renee Wolfe. The highlight of the spring semester was when sponsor Jim Paige, along with members Dana Hungerford, Stacey Blakemore, Ken Mestl, Darin Moon, Rusty Sadler, Kelly Krutz, Colleen Spieker, Iris Mestl, Michelle Dubas, and Cindy Rutten, flew to the KME National Convention in San Luis Obispo, California, April 2-5, 1987. Hostesses Katja Arvin and Susan Ridenour made their convention days very pleasant and memorable. Other 1987-88 officers: Jim Fisher, vice president; Heather Ballard, secretary and treasurer; Michelle Dubas, historian; Fred Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

Nebraska Gamma, Chadron State College, Chadron
Chapter President - Hortensia Soto
16 actives, 3 initiates

We had formal initiation on February 24 and initiated three new members. They were Amy Goodrich, Richard Reed, and Bruce Ford. Our money making project went over very well. Every spring we award the purple shaft award and this year we made $160. Other 1987-88 officers: Tracy Gifford, vice president; Kathy Hall, secretary; Bruce Ford, treasurer; James A. Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

<u>Nebraska Delta</u>, Nebraska Wesleyan University, Lincoln
Chapter President - Nicole Austin
18 actives, 8 initiates

This was the first full year as an active chapter for Nebraska Delta. We had two fund-raising projects: a car wash in October, and a Valentine's Day computer match-up in February. In April we sponsored a High School Math and Computer Contest. Four schools and about 30 students participated in five events. We felt it was a success and plan to continue and expand this event. Initiation of new members was held on March 24, 1987. Seven student members and one faculty member were initiated. This was followed by election of officers for the 1987-88 year, and a pizza party. Other 1987-88 officers: Trevor Jares; vice president; Diane Humphrey, secretary; Nancy Nichols, treasurer; Muriel Skoug, corresponding secretary; Daniel Kaiser, faculty sponsor.

<u>New Mexico Alpha</u>, University of New Mexico, Albuquerque
Chapter President - Mohammed Akbarzadeh
100 actives, 17 initiates

Other 1987-88 officers: Sheryl Henry, vice president; Chris Kaye, secretary; Rachel Vickrey, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

<u>New York Alpha</u>, Hofstra University, Hempstead
Chapter President - Susan Genzardi
4 actives, 8 initiates

Other 1987-88 officers: Sandra McGrath, vice president; Jonathan Krasner, secretary; Pamela Caplette, treasurer; Stanley Kertzner, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara
Chapter President - Elmer Bauer
12 actives, 4 initiates

The Initiation Banquet was held on Friday, March 27. Special speaker: Prof. Erik Hemmingsen (April 6) spoke on "The Strange World of Iteration of Functions." Prof. Hemmingsen is from Syracuse University in New York. Our chapter president, Elmer Bauer, received the Mathematics Department's award for excellence in mathematics. Elections for 1987-88 officers will be held during the fall, 1987, semester.

New York Kappa, Pace University, New York
Chapter President - Thomas Sinkulet
30 actives, 15 initiates

Our annual induction was held in March, 1987. We held biweekly mathematics seminars on the New York campus. Other 1987-88 officers: Zhong-Su Chen, vice president; Ann Marie Maccini, secretary; Panagiotis Nikolakos, treasurer; Louis V. Quintas, corresponding secretary; John W. Kennedy and Martin Kotler, faculty advisors.

Ohio Gamma, Baldwin-Wallace College, Berea
Chapter President - John MacDougall
20 actives, 13 initiates

Other 1987-88 officers: Lisa Renker, vice president; John Waters, secretary; Tracy Glodziak, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.

<u>Ohio Zeta</u>, Muskingum College, New Concord
Chapter President - Connie Garces
38 actives, 9 initiates

On January 28 talks were given by Dr. Javad Habibi, Greg Files and Dana Woodland. Initiation of nine new members took place on February 18: Karen Allender, Regina Alverson, Jeffrey Arnold, Julie Harper, Rajat Kumar, Nancy McMillan, Diwaker Rana, Pamela Roehling, and Timothy Rogers. On March 25 pre-national convention talks were given by Connie Garces, Mike Barlow, and Joe Allen. Eleven students and two faculty members attended the national convention in San Luis Obispo; three students presented papers. A convention report was given on April 21 and on May 10 we held our picnic. Other 1987-88 officers: Kevin Dunn, vice president; Gina Alverson, secretary; Tim Coyne, treasurer; Carolyn Crandell, corresponding secretary; Russ Smucker, faculty sponsor.

<u>Oklahoma Alpha</u>, Northeastern State University, Tahlequah
Chapter President - Patricia McGinn
52 actives, 23 initiates

The spring, 1987, initiation ceremonies were held in the banquet room of the Western Sizzlin' Steak House in Tahlequah. Dr. Hansen was the speaker. Several members helped publicize Mathematics Awareness Week, April 12 - April 18. An ice cream social was held May 5 to welcome new members and to plan activities for the coming year. Other 1987-88 officers: Scott Forester, vice president; Catherine Carlin, secretary and treasurer; Joan E. Bell, corresponding secretary and faculty sponsor.

<u>Oklahoma Gamma</u>, Southwestern Oklahoma State University, Weatherford
Chapter President - Bobby Boyd
15 actives

Oklahoma Gamma sponsored a three-on-three basketball tournament to raise money. Other 1987-88 officers: Allison Roberts, vice president; Sherri Norton, secretary and treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

<u>Pennsylvania Alpha</u>, Westminster College, New Wilmington
Chapter President - Karen Haney

The Initiation Banquet was held on March 3 and on March 26 we sponsored a Pizza Study Break. On April 28 Karen Haney gave a presentation, with video, on the national convention at Cal Poly. Other 1987-88 officers: T. R. Warters, vice president; Theresa Stamos, secretary; Mary Joyce, treasurer; Miller Peck, corresponding secretary; Barbara Faires, faculty sponsor.

<u>Pennsylvania Kappa</u>, Holy Family College, Philadelphia
Chapter President - Scott Kromis
5 actives, 6 initiates

Our installation was held on March 16, 1987. Six mathematics majors who met the qualifications were initiated into <u>KME</u>. Former graduates spoke for about ten minutes each and encouraged the members to continue to strive for excellence in math. "It pays off," they said. A pizza party followed. Meetings were held every third Wednesday of the month; math problems were discussed and solved. Other 1987-88 officers: Eric Mehler, vice president; Constance Hefner, secretary; John Androwski, treasurer; Sister M. Grace, corresponding secretary; Linda Czajka, faculty sponsor.

Pennsylvania Gamma, Waynesburg College, Waynesburg
Chapter President - Steve Grudi
11 actives, 5 initiates

Other 1987-88 officers: Kim Simon, vice president; Brenda Barnhart, secretary and treasurer; Rosalie B. Jackson, corresponding secretary; David S. Tucker, faculty sponsor.

Pennsylvania Delta, Marywood College, Scranton
Chapter President - Karen Borusovic
13 actives, 4 initiates

Our initiation ceremony and dinner were held on May 12, 1987. Along with the math club, we sponsored the seventh annual high school math contest for area high schools. Forty-four schools were invited to participate. Other 1987-88 officers: Laurie Bartol, vice president; Mary Roginski, secretary; Thomas Powell, treasurer; Sister Robert Ann von Ahnen, corresponding secretary and faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana
Chapter President - Karen Ashby
32 actives, 5 initiates

During the spring semester of 1987 our chapter had three regular meetings and sent five students and two faculty members to the National Convention in San Luis Obispo. At our regular meeting on February 17 we inducted five new members and Dr. George E. Mitchell presented a talk on research in mathematics. At our meeting on March 24 we planned our annual spring banquet and Dr. Tom Giambrone presented a talk on the use of computers in the teaching of mathematics. Our last meeting of the spring semester was the banquet on April 8. The speaker at this meeting was Dr. James Reber who gave a talk on his sabbatical leave at the University of Arizona. Also at this banquet meeting the student members honored Professor Ida Z. Arms for her long and dedicated service to KME and the PA Zeta Chapter. Ms. Arms was presented with a dozen roses, a piece of luggage and several travel logs. Other 1987-88 officers: Todd Weaver, vice president; Susan Jack, secretary; Kathleen Ford, treasurer; George E. Mitchell, corresponding secretary; Ida Z. Arms, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President - Anne Kister
32 actives, 7 initiates

Pennsylvania Eta held its spring picnic on Saturday, May 9, 1987, at the Grove City Community Park. Students and faculty enjoyed the charcoal-grilled hamburgers and the volleyball games both before and after the meal. Other 1987-88 officers: Terri Mauersberg, vice president; Cara Masquelier, secretary; Janet White, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Theta, Susquehanna University, Selinsgrove
Chapter President - William Purnell
10 actives, 5 initiates

Mrs. Harrison attended the national meeting in California. Seven students went to Moravian College to hear the talk, "A Mathematicians View of Esher," and to view the Esher exhibit. Other 1987-88 officers: Kerri Linker, vice president; Stephen Schneeweis, secretary; James Turner, treasurer; Carol N. Harrison, corresponding secretary.

Pennsylvania Iota, Shippensburg University, Shippensburg
Chapter President - Juliann M. Peterson
50 actives, 7 initiates

Other 1987-88 officers: Sandra Gorka, vice president; Meg Masterson, secretary; Howard Bell,

<u>Texas Eta</u>, Hardin-Simmons University, Abilene
Chapter President - Susan Petersen
18 actives, 8 initiates

The Texas Eta Chapter of <u>KME</u> held its thirteenth annual induction banquet April 10, 1987. There were eight members inducted: Peter Barten and Denise Kuciemba from Abilene, Texas; William Baker from Mt. Pleasant, Texas; Ricky Davis from Lawn, Texas; Carol Kemph from Great Falls, Montana; Susan Petersen from League City, Texas; Stephen Watson from DeSoto, Texas; and Kellie Webb from Keller, Texas. With the induction of these members, membership in the local chapter stands at 106. Ms. Mary Wagner, Assistant Professor of Mathematics at Hardin-Simmons University, addressed the chapter on the subject, "Whole-Brain Learning Implications for Mathematics Teaching and Using Computers in the Classroom." Leading the induction ceremonies were Stephanie Thomas, chapter president; John Dailey, vice president; and Mike Cagle, secretary and treasurer. Other 1987-88 officers: John Dailey, vice president; Kellie Webb, secretary; Ricky Davis, treasurer; Mary Wagner, corresponding secretary; Charles Robinson and Ed Hewett, faculty sponsors.

<u>Virginia Alpha</u>, Virginia State University, Petersburg
Chapter President - Wayne Saunders
18 initiates

The induction ceremony was followed by a banquet with an invited mathematician as speaker. On Honors Night two plaques were presented to graduating members with high achievement in mathematics. A scholarship award in the name of Dr. Louise Stokes Hunter was given to a junior member of <u>KME</u>. Papers were presented by student members. Our chapter participated in the NTA symposium. A picnic was sponsored jointly by <u>KME</u> and the Math Club. Other 1987-88 officers: Michael Parker, vice president; Alison Codrington, secretary; Mohinder Tewari, treasurer; Dorothy Stevenson, corresponding secretary; Emma Smith, faculty sponsor.

<u>Wisconsin Alpha</u>, Mount Mary College, Milwaukee
Chapter President - Ann Brandt
4 actives, 5 initiates

Two <u>KME</u> members, Ann Brandt and Michelle Wielebski, and one faculty member, Sister Adrienne Eickman, attended the Biennial Convention in San Luis Obispo, California, in April. Meeting time was spent discussing means of raising funds. A doughnut sale was held in April. Other 1987-88 officers: Michelle Wielebski, vice president; Ann Brandt, secretary; Michelle Wielebski, treasurer; Sister Adrienne Eickman, corresponding secretary and faculty sponsor.

<u>Wisconsin Beta</u>, University of Wisconsin-River Falls, River Falls
Chapter President - Thomas R. Scott
15 actives, 9 initiates

We had a few informal get-togethers this spring along with our annual <u>KME</u> Film Festival. Our initiation of new members was a success and featured a guest speaker on the subject of Painting and Mathematics. Thomas Weber, Sarah Flood, Chris Farwick, and Mary Gessner represented us with talks at the Pi Mu Epsilon Conference at St. Johns in Collegeville, Minnesota. Other members attending the conference were Nancy Socha and Thomas Scott. Faculty members attending the conference included Don Leake, James Senft, and Ed Rang. Other 1987-88 officers: Thomas Weber, vice president; Christine Farwick, secretary; Sarah Flood, treasurer; Lyle Oleson, corresponding secretary; Don Leake, faculty sponsor.

<u>Wisconsin Gamma</u>, University of Wisconsin-Eau Claire, Eau Claire
Chapter President - Karsten Hangen
40 actives

The mainstay of the spring semester was the bimonthly meetings at which there were one to two student speakers. There were also several fund raisers including bake sales and the selling of popcorn. Bill Radermaker and Jim Fischer attended the national convention in California. Bill received second place for his talk, "Inspirals in Turtle Geometry." During honors week in May three of the local KME members presented talks to a general audience. The semester ended with a picnic. Other 1987-88 officers: Shelly Lundgren, vice president; Kathy Hammar, secretary; John Maierhofer, treasurer; Tom Wineinger, corresponding secretary.

IF YOUR SUBSCRIPTION HAS EXPIRED

We hope you have found THE PENTAGON both interesting and helpful. Your suggestions are always welcome and may be written on this form. They will be forwarded to the Editor.

If you wish to renew your subscription for two years, please send $5* to THE PENTAGON, Department of Mathematics, Western Illinois University, Macomb, Illinois 61455.

Please give your name below in the manner in which you wish your mail addressed (Miss, Mrs., Prof., etc). If your name has changed, please indicate also the name to which THE PENTAGON was sent.

Name____

Address____

City____

*Foreign subscriptions require an additional $1 per year for postage.

Papers and problems are needed for :

The Hexagon (see p. 36),
The Cursor (see p. 46), and
The Problem Corner (see p. 29).

Please consider submitting an article or a problem to the appropriate editor.

treasurer; Donna Plank, historian; Leonard Jones, corresponding secretary; Rick Ruth, faculty sponsor.

<u>Pennsylvania Lambda</u>, Bloomsburg University, Bloomsburg
Chapter President - Michelle Frye
40 actives, 12 initiates

The president, vice president, and corresponding secretary had a fruitful trip to San Luis Obispo for the National Convention. Other 1987-88 officers: Michael Jarus, vice president; Linda Davarus, secretary; Karen Billingham, treasurer; James C. Pomfret, corresponding secretary; Joseph Mueller, faculty sponsor.

<u>South Carolina Gamma</u>, Winthrop College, Rock Hill
Chapter President - Martha Atkins
16 actives, 3 initiates

Dr. Ray Wiley, retired chairman of the Department of Mathematics at Furman University, gave a talk on finite differences at the April meeting. Other 1987-88 officers: Denise Nibarger, vice president; Rebecca Turpin, secretary; Cindy Nicholson, treasurer; Donald Aplin, corresponding secretary; Edward Guettler, faculty sponsor.

<u>Tennessee Alpha</u>, Tennessee Technological University, Cookeville
Chapter President - Craig Morrow
60 actives, 46 initiates

Two $100 scholarships were given to new initiates. The recipients were Jonathan B. Pace and Blaine G. Brint. Other 1987-88 officers: Amy Scott, vice president; Patrick Godin, secretary; Joe Coen, secretary; Frances Crawford, corresponding secretary; Evelyn Brown, faculty sponsor.

Tennessee Gamma, Union University, Jackson
Chapter President - Melodi Myers
10 actives, 11 initiates

Other 1987-88 officers: Beth Dennis, vice president; Jennifer Powers, secretary; Joseph Hunter, treasurer; Don R. Richard, corresponding secretary; Dwayne Jennings, faculty sponsor.

Tennessee Delta, Carson-Newman College, Jefferson City
Chapter President - James Day
17 actives, 4 initiates

Bonnie Barnard and Trish Snowden were our delegates to the Biennial Convention at Cal Poly. Our initiation banquet was held at Justin's in Morristown and our picnic took place at Panthen Creek State Park. Other 1987-88 officers: Elizabeth Nations, vice president; Rhonda Neihardt, secretary; Trevor Roberts, treasurer; Albert Myers, corresponding secretary; Carey Herring, faculty sponsor.

Texas Alpha, Texas Tech University, Lubbock
Chapter President - Cathy Cain
10 actives, 18 initiates

KME members took a very active part in helping with the activities of University Research Day in April. Eighteen pledges were initiated during the Departmental Awards Banquet on April 15, 1987. Other 1987-88 officers: Karen Engel, vice president; Gregory Henderson, secretary; Roger Frazier, treasurer; Robert A. Moreland, corresponding secretary and faculty sponsor.