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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics; due, mainly, to its demands for logical and rigorous modes of thought, to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

HOMOTHETIC PROOF OF THE NINE-POINT CIRCLE*

LORI BASKINS

Student, Southwest Missouri State University

Probably the most significant of the discoveries leading to renewed interest in classical geometry early in the nineteenth century was the discovery of the nine-point circle, sometimes credited to the German mathematician Feuerbach in 1822. To this day, most mathematicians rely upon the classical approach, as used by James Smart in Modern Geometries, to prove the existence of the nine-point circle. However, I find the homothetic transformation technique, outlined by David Kay, to be more appealing. In this paper, I would first like to define and give examples of a homothetic transformation. I will then define the nine-point circle, and lastly I will prove homothetically its existence.

*A paper presented at the 1987 National Convention of KME and awarded first place by the Awards Committee.

Consider a point O and a nonzero real number C. For each point X in the plane, define TX = X' as that point on line OX such that, in magnitude and sign, OX' = COX. The mapping thus defined is called a homothetic transformation, with O as center and C as dilation factor. A homothetic transformation is said to be direct if the dilation factor is positive and opposite if the dilation factor is negative. Since a homothetic transformation is uniquely determined by its center O and dilation factor C, it will be denoted by T = H(O,C). I will give two examples.

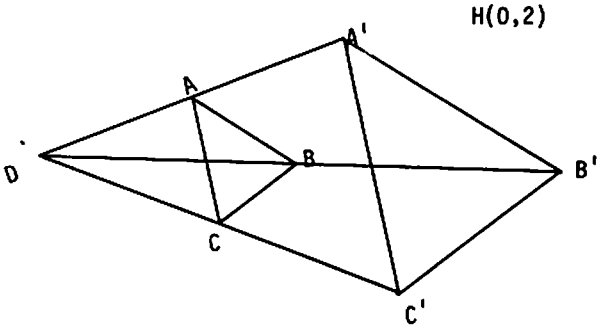

Figure 1

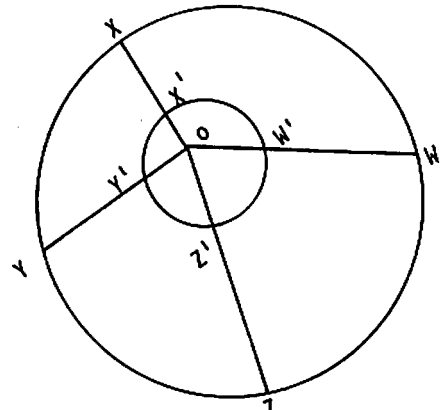

Figure 11

These examples illustrate some characteristics of homothetic transformations. It is apparent from Figure 1 that a homothetic transformation maps a triangle into another triangle which is similar to the first with factor of proportionality 'C|. In the figure, triangle ABC is mapped into triangle A'B'C'. In fact, a homothetic transformation will always maintain the shape of the figure. It maps a triangle into a triangle, a circle into a circle, a rectangle into a rectangle, and so on. Also, a homothetic transformation shifts the figure and either enlarges it or shrinks it, depending upon the value of C. (C' = 1 is an exception. The

figure is then unchanged.) In Figure 2, the circle containing points X, Y, Z, and W is mapped into the smaller circle containing points X', Y', Z', and W', and the center is shifted.

Before applying the homothetic transformation to the nine-point circle, I need to define four more terms.

1. The orthocenter of a triangle is the point of concurrency of the altitudes of a triangle.
2. The line dropped from a vertex of a triangle perpendicular to the opposite side meets the opposite side in a point called the foot.
3. The circumcircle of a triangle is the unique circle containing the three vertices of the triangle.
4. The circumcenter, the center of the circumcircle, is the point of concurrency of the perpendicular bisectors of the sides of the triangle.

Now consider triangle ABC in Figure 3 with orthocenter H, and let the following three groups of points be determined:

a. L, M, N - the midpoints of the sides,

b. D, E, F - the feet of the altitudes on the sides, and

c. X, Y, Z - the midpoints of the three segments joining the orthocenter and each vertex. In the figure, X, Y, and Z are the midpoints of HA, HB, and HC, respectively.

It can be established that these nine points all lie on a circle.

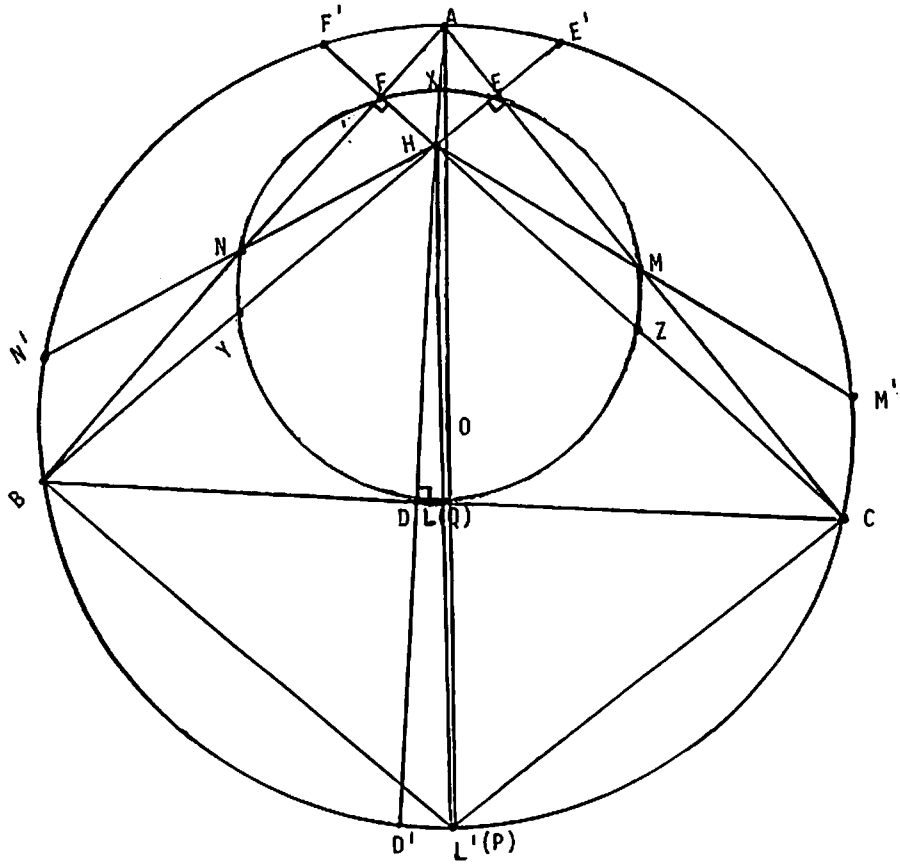

Figure III

My objective is to prove the existence of the nine-point circle using a homothetic transformation. To do this, I will denote points D', E', F', L', M'. and N' as the points of intersection of the circumcircle (with center O) with the rays $\overrightarrow{HD}$, $\overrightarrow{HE}$, $\overrightarrow{HF}$, $\overrightarrow{HL}$, $\overrightarrow{HM}$, and $\overrightarrow{HN}$.

Consider now the homothetic transformation $T = H(H, \frac{1}{2})$ which maps the circumcircle into some circle W, with O being mapped into the center of W. Proving the existence of the nine-point circle is now equivalent to proving that X, Y, Z, D, E, F, L, M. and N are on circle W.

By definition, $HX = \frac{1}{2}\ HA$, $HY = \frac{1}{2}\ HB$, and $HZ = \frac{1}{2}\ HC$, so that T(A) = X, T(B) = Y, and T(C) = Z. Therefore, X, Y, and Z are points of W. Hence, it remains to be proved that D,E, and F are the midpoints of $\overline{HD'}$, $\overline{HE'}$, and $\overline{HF'}$, respectively, and that L, M, and N are the respective midpoints of $\overline{HL'}$, $\overline{HM'}$, and $\overline{HN'}$. Since the proofs are analogous, I will prove this for only D and L.

Let P be the point of intersection of $\overleftrightarrow{AO}$ with the circumcircle and take Q as the point of intersection of $\overleftrightarrow{HP}$ and $\overleftrightarrow{BC}$.

Now, a point is on a semicircle if and only if it forms a 90° angle with two endpoints of the diameter. So, $\angle ACP = \angle ABP = 90^{\circ}$ since B and C lie on the circumcircle and $\overline{AP}$ is the diameter. In addition, $\angle BEC = 90^{\circ}$ and $\overleftrightarrow{BH}//\overleftrightarrow{PC}$. Likewise, since $\angle BFC = 90^{\circ}$, $\overleftrightarrow{HC}//\overleftrightarrow{BP}$. Therefore, it follows that BHCP is a parallelogram. So Q is the midpoint of $\overline{HP}$ and $\overline{BC}$, and so Q = L, and P = L'. This proves that L is the midpoint of $\overline{HL'}$.

Observe also that $\angle AD'L' = \angle AD'P = \frac{\pi}{2}$ and hence $\overleftrightarrow{D'L'}//\overleftrightarrow{DL}$. Then, since L is the midpoint of $\overline{HL'}$, D is the midpoint of $\overline{HD'}$. It follows that T(D') = D, T(L') = L, T(M') = M, T(E') = E, T(F') = F, and T(N') = N, which shows that the original nine points lie on the homothetic image of the circumcircle with center O. Therefore, they lie on a circle, called the nine-point circle of the triangle.

I have thus proved: The nine-point circle of a triangle is homothetic to the circumcircle of the triangle under the homothetic transformation which has the orthocenter of that triangle as center and $\frac{1}{2}$ as dilation factor.

References

Kay, David. College Geometry. New York: Holt, Rinehart, and Winston, 1969.

Smart, James. Modern Geometries. Monterey, California: Brooks/Cole Publishing, 1978.

INWARD SPIRALS IN TURTLE GEOMETRY*

William W. Rademaker

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PROBLEM BACKGROUND

As a Mathematics major and a Data Processing minor, I am currently preparing to engage in secondary mathematics and computer science education. A major portion of that preparation involves a teaching internship. A component of my internship is teaching a beginning computer course to junior high school students. A significant part of that course deals with turtle geometry in the programming language LOGO.

In order to understand the concept of turtle geometry, think of the turtle as a computer controlled animal that lives on the display screen and responds to LOGO commands that make it do the following:

1. Move forward and backward.
2. Rotate left and right.

* A paper presented at the 1987 National Convention of KME and awarded second place by the Awards Committee.

As the turtle moves, it leaves a trace of its path, and in this way, can be used to make drawings on the display screen. For example, the turtle will draw a square of side 100 units if it repeats the following commands four times:

```
FORWARD 100
RIGHT 90.
```

A procedure to produce this square is:

```
TO SQUARE
    REPEAT 4 [FORWARD 100 RIGHT 90]
END.
```

In order to help better prepare to teach turtle geometry, I recently undertook an independent study that focused on turtle geometry. The subject of this paper is an interesting finding I've made in my study of a particular LOGO procedure which Abelson and diSessa[1] call INSPI. INSPI is an abbreviation for inward spiral. The procedure is:

```
TO INSPI :SIDE :ANGLE :INC
     FD :SIDE
     RT :ANGLE
     INSPI :SIDE (:ANGLE + :INC) :INC
END
```

The diagram in Figure 1 illustrates what INSPI does. The diagram is constructed with side input = 10, angle input = 30, and increment = 10. For purposes of illustration, only the first six sides generated by INSPI are shown. From its initial position, the turtle goes forward 10 units and turns right 30 degrees. On its second step, it procedes forward 10 units and turns 30 + 10 = 40 degrees. From that position, it travels 10 units and turns 40 + 10 = 50 degrees. As this process continues, the figure begins the characteristic inward spiral from which the name INSPI is derived.

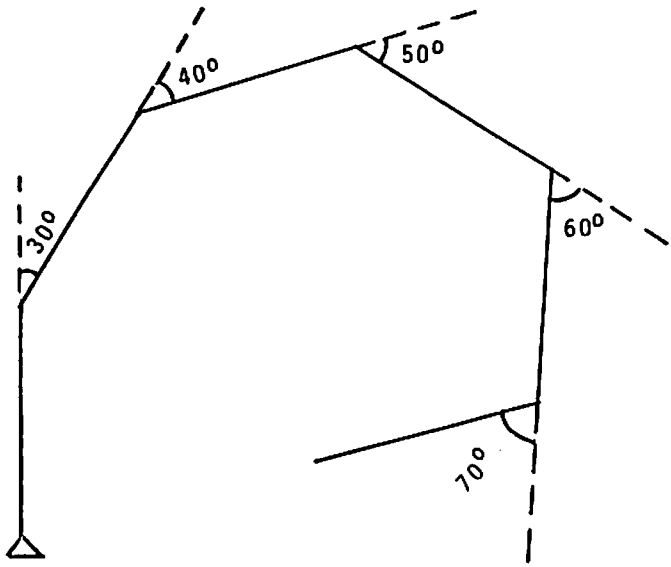
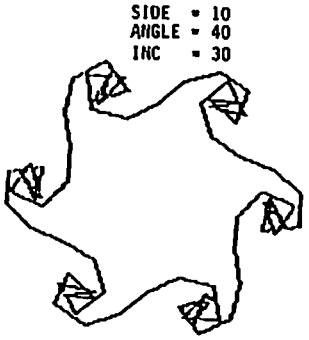
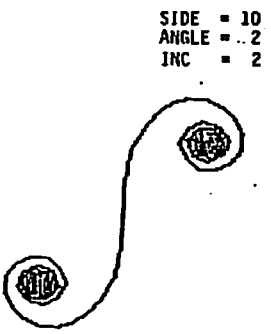
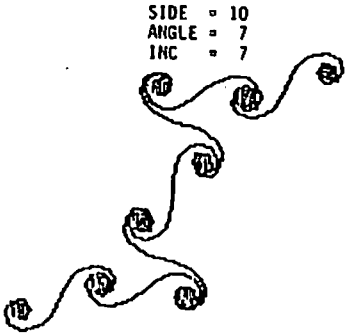
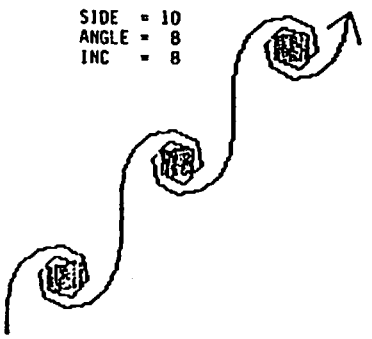


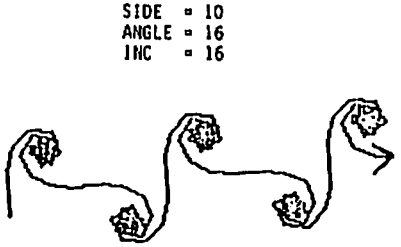
Figure 1

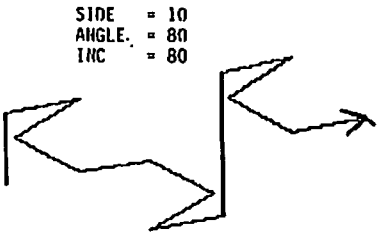












(Examples of INSPI with a variety of inputs.)

FIGURE 2

Careful analysis of these figures reveals that some of the figures are closed and some are not. In other words, some figures return to their initial position and heading while others do not.

PROBLEM DEFINITION

To determine which angle inputs to the INSPI procedure produce closed figures, and which do not.

An examination of the following figure will help in determining which figures are closed, and which ones are not. The figure was produced with the INSPI procedure with the following inputs: SIDE = 10, ANGLE = 60, INC = 60.

TURN	ANGLE
1	60
2	120
3	180
4	240
5	300
6	360
7	60
8	120
9	180
10	240
11	300
12	360

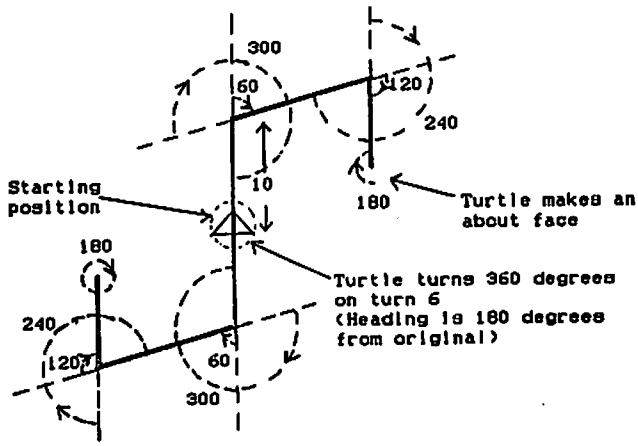


FIGURE 3

The figure is closed because the turtle returns at some point to its initial position and heading. An essential clue to the determination of closure or non-closure of the figure can be seen when the turtle makes its first turn which is a multiple of 360 degrees. At that point, the turtle's heading is 180 degrees from its original heading. This characteristic was seen in the above figure and any other closed figure that I generated. Any non-closed figure that I generated did not possess this characteristic.

LOOKING AT A SPECIAL CASE

In the original problem, any combination of positive integers for the angle input and increment could be used. This makes it very difficult to determine which angle inputs produce closed, and which produce non-closed figures because the outcome depends not only on the angle input, but on the increment as well. Therefore, I've decided to look at a special case of the problem at hand. The restriction for this special case is that the angle input (A) must equal the increment (I). This means the turn is always nA for some integer n.

A PROGRAM TO AID CALCULATIONS

I stated earlier that closed figures seem to have the unique characteristic in which the heading is 180 degrees from the original heading when the turn reaches a multiple of 360 degrees for the first time. The calculation of the heading when the turn reaches a multiple of 360 degrees can be very time consuming. Therefore, I created a computer program to aid the number crunching process. The program, which is written in the LOGO language, follows:

```
TO INSPICALC :ANGLE :INC
  MAKE "TOTALTURN 0
  MAKE "TURN :ANGLE
  MAKE "TOTALTURN :TOTALTURN + :TURN
  IF NOT((REMAINDER :TURN 360) = 0 )
        THEN INSPILOOP :TURN :INC
        ELSE (PRINT :TURN :TOTALTURN)
END
TO INSPILOOP :TURN :INC
  MAKE "TURN :TURN + :INC
  MAKE "TOTALTURN :TOTALTURN + :TURN
  IF NOT ((REMAINDER :TURN 360) = 0)
         THEN INSPILOOP :TURN :INC
         ELSE (PRINT :TURN :TOTALTURN)
END.
```

The above program follows the steps of an INSPI procedure, calculating the heading and turn that the turtle is about to make. It terminates when the turtle turns through an angle of 360n, where n is a positive integer. At this point, it prints out the last turn made by the turtle and its heading at that point. As stated earlier, if the heading is 180 degrees from the original heading at that point, then the figure is a closed figure. The table below shows results using a variety of angle inputs. For ease of viewing, the angle inputs are grouped according to closed figures and non-closed figures.

CLOSED FIGURES

ANGLE	INCREMENT	TURN	HEADING
1	1	360	64,980 = 180 * 361
2	2	360	32,580 = 180 * 181
3	3	360	21,780 = 180 * 121
7	7	2,520	454,860 = 180 * 2,527
11	11	3,960	714,780 = 180 * 3,971
12	12	360	5,580 = 180 * 31
14	14	2,520	228,060 = 180 * 1,267
17	17	6,120	1,104,660 = 180 * 6,137
19	19	6,840	1,234,620 = 180 * 6,859
21	21	2,520	152,460 = 180 * 847
22	22	3,960	358,380 = 180 * 1,991
23	23	8,280	1,494,540 = 180 * 8,303
25	25	1,800	65,700 = 180 * 365
27	27	1,080	22,140 = 180 * 123

NON-CLOSED FIGURES

8	8	360	8,280 = 360 * 23
16	16	760	16,560 = 360 * 46
24	24	360	2,880 = 360 * 8
32	32	1,440	33,120 = 360 * 92
40	40	360	1,800 = 360 * 5
48	48	720	5,760 = 360 * 16

These experiments lead to the following two conjectures:

1. All closed figures have a heading of 180 degrees from the original heading when the turn reaches a multiple of 360 degrees.
2. The only angle inputs that produce non-closed figures are multiples of 8.

I've stated the second conjecture as my main theorem and the first as a lemma to the theorem. The lemma and theorem and proofs for each follow.

LEMMA: For the INSPI procedure with angle input = increment, the generated figure is closed if and only if, at some point the turn = 180k, where k is an odd integer.

Let A = the initial angle input to INSPI.

Any odd multiple of 180 degrees results in a heading change of 180 degrees. Since INSPI produces turns of nA, where n is a positive integer, then the turn just before the 180 degree turn is 180 - A, and the turn just after the 180 degree turn is 180 + A.

Let us call the original heading prior to the 180 - A turn 0 degrees.

Then, the heading just after the turn 180 + A is (180k - A) + (180k) + (180k + A) = 540K = 180 degrees (mod 360).

Since the forward distance traveled is constant, the turtle is at the same point, P2, just after the 180 - A turn and just after the 180 + A turn, except that the heading is now 180 degrees from the original heading.

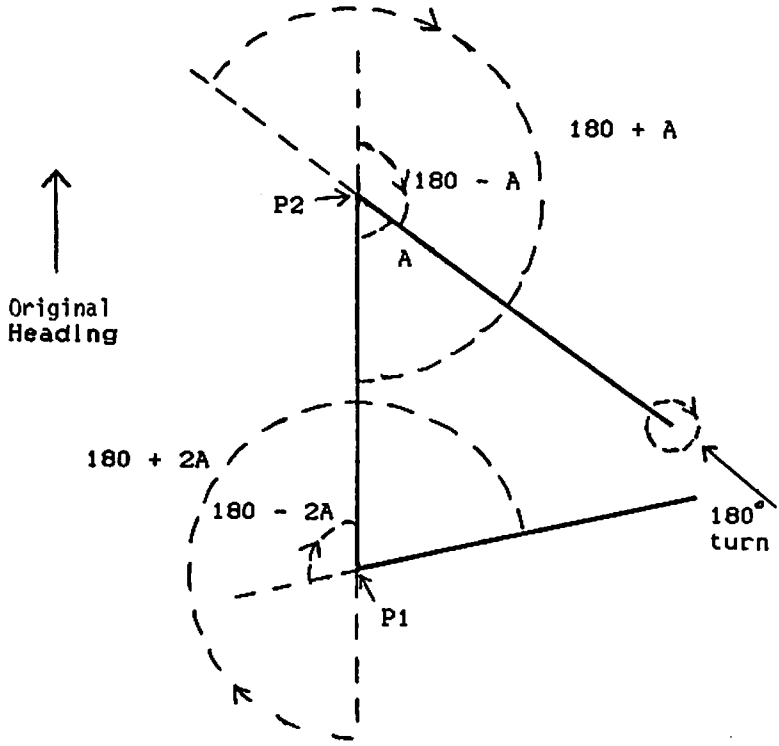


FIGURE 4

Since the angle = increment, the turn just before the 180 - A turn is 180 - 2A. Then, the heading just after

the 180 + 2A turn is (180k - 2A) + (180k - A) + (180k) + (180k + A) + (180k + 2A) = 900k = 180 degrees(mod 360). Likewise, at each point where a turn occurs, the new heading for departing the point will differ by 180 degrees (mod 360) from the previous heading upon arriving at the point. Thus, the turtle will begin to trace the previous portion of the figure in reverse. Hence, the figure continues to retrace itself.

To prove the second part of the two part lemma, I will prove the contrapositive. If the turn is never = 180k, where k is an odd integer, then the figure is never closed.

If the turn is never an odd multiple of 180, then it must be, at some point, an even multiple of 180. This means that it is a multiple of 360. If that is the case, the turn will never make an about face which allows it to retrace itself. Therefore, it will never close.

THEOREM: For the INSPI procedure with the angle input = the increment, all angle inputs will produce closed figures except inputs of 8m, where m is a positive integer.

PROOF: From the previous lemma, a turn of nA = 180k produces a closed figure, if k is an odd integer, and n is an integer. In the proof of of the theorem, it can be assumed that n and k are the smallest n and k for which nA = 180k. Then, k and n have no common factor except 1.

A. Angle inputs of 8m produce non-closed figures.

(8m)n = 180k

(2m)n = 45k

This implies that k is even.

By the previous lemma, k must be odd to produce closed figures.

Therefore, multiples of 8 produce non-closed figures.

B. Inputs other than 8m produce closed figures.

1. If A is odd then n must be even.

For nA = 180k:

Since n and k have no factor in common, k must be odd.

Therefore, odd angle inputs produce closed figures.

2. If A is an even number not equal to 8m then A cannot have a factor of 8, so it must have a factor of 2 or 4.

   a. If A has a factor of 2. Then n(A/2) = 90k. The integer A/2 is not even, therefore n is even and so k is odd.

   b. If A has a factor of 4, then n(A/4) = 45k. The integer A/4 is not even, therefore both n and k must be odd.

Therefore, inputs other than 8m produce closed figures.

REFERENCES

1. Harold Abelson and diSessa, Turtle Geometry, London: MIT Press, 1982.

FROM ALICE TO ALGEBRA*

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1. Introduction

Everyday, people are frightened by situations, ideas, and objects. Often times this fear stems from nothing more than a misunderstanding or no understanding at all. Through the centuries, men have used humor to reject ideas they fear or which contradict their "perception of the world", often recording jokes, puns, and sarcasms for posterity. The pages of literature are filled with satire belying the underlying fear or misunderstanding of the author. Obviously, new ideas and questions beget mathematical progress often rejected by many. Thus, mathematical studies open themselves to this ridicule.

During the late eighteenth and early nineteenth centuries, mathematicians hotly debated concepts that

*A paper presented at the 1987 National Convention of KME and awarded third place by the Awards Committee.

today are widely accepted. Many of the controversies found preservation in literary works of the period. A surprising example of such, the well-known Lewis Carroll tales, Alice's Adventures In Wonderland and Through the Looking-Glass, argue against the new symbolical algebra of the age. Familiarity with the setting and the author will allow us to perceive Carroll's hidden messages.

2. The opposition to negative numbers

With regard to the time setting, we first examine the field of mathematics. Continuing to expand its horizons, the mathematics of the mid to late eighteenth century spawned developments in geometry and in algebra. Prior to 1820, algebra focused only on arithmetical operations. Thus, symbols such as (a,b,c) stood for concrete and abstract positive numbers. Now into this useful science entered the sign '-' in front of symbols to denote "negativity". Evolution of such a sign is understandable when inspecting the application of algebra in the natural sciences where the need to denote direction or possibly temperature exists. As a new element in algebra, the negative required a definition. Many mathematicians employed two common definitions for "negative": (1) "quantities less than nothing" and (2) "quantities obtained by taking a greater quantity from a

lesser". These definitions spawned a great deal of antagonism.

Among the opposition, Francis Maseres and William Frend fought valiantly for total rejection of the negative number. Maseres, a fourth Wrangler at Clara College, Cambridge, and Frend, a second Wrangler at Christ's College, charged that negatives lacked an adequate definition. In modern times, one may still appreciate the charge of insufficience. For instance, give to person #1 a quantity of milk equal to nothing (i.e., no milk). Now, give person #2 less milk than person #1 possessess. The invalidity of the second definition follows from the definition usually given for subtraction. Subtraction was most commonly defined as the taking of a lesser quantity from a greater. The proponents of the negative number defended "quantities less than nothing" through examples such as debts and lines drawn in certain directions. Frend's response was a challenge to such imprecise thinking: "When a person cannot explain the principles of a science without reference to a metaphor, the probability is, that he has never thought accurately upon the subject" (Frend 1796 1, x in Pycior , "Origins" 28). Maseres stated his opinion even more adamantly, saying that "the science of

algebra has been disgraced and rendered obscure and difficult, and disgusting to men of a just taste for accurate reasoning" (Measeres 1800, iv in Pycior, "Origins" 28). Ultimately, Frend and Maseres demanded rejection of negatives and the governing of subtraction to allow only the taking of a lesser from a greater.

As always, a flip side to the negative dispute existed. Although most scholars agreed with Frend and Maseres that the definitions lacked in efficiency, few could accept giving up the negative number, which had already proven useful. There came as a result the idea voiced by William Greenfield, a professor of rhetoric at the University of Edinburgh and a part-time mathematician, to "exert...industry and ingenuity, rather to confirm than to destroy, rather to demonstrate, how far we might rely on the method of negative quantities, than to overturn at once so great a part of the labours of the modern algebraists" (Greenfield 1788, 136 in Pycior, "Origins" 30). Backing this statement, Robert Woodhouse, a Lucasian Professor of Mathematics at Cambridge, argues, "If operations with any characters or signs lead to just conclusions, such operations must be true by virtue of some principle or other" (Woodhouse 1801, 90 in Pycior, "Origins" 30).

With this ideology in mind, many set out to justify the use of the negative.

British mathematicians went to work on the problem, obtaining more than one solution. One solution, however, became the most popular and most accepted one--symbolical algebra. In 1830, George Peacock's Treatise on Algebra became the answer to the negative question. In this two volume book, Peacock treats arithmetical algebra and symbolical algebra separately, explaining the differences in this manner: "In arithmetical algebra, the definition of the operations determine the rules; in symbolical algebra, the rules determine the meaning of the operations, or more properly speaking, they furnish the means of interpreting them" (Peacock 1833, 200 in Pycior, "Origins" 36).

3. Symbolical algebra

As symbolical algebra unfolds on the opening pages of Treatise on Algebra Vol II, the distinction between the symbols of arithmetical and symbolical algebra become apparent. Those employed in arithmetical algebra are specific in value, representing "numbers whether abstract or concrete, whole or fractional" (Peacock 1940, 1). In the symbolic approach, these same general

symbols may stand for "quantities of all kinds, and of all relations of magnitude" (Peacock 1940, 1). It follows from the inclusion of all quantities that there must exist signs '+' and '-' to precede the symbols. For if these signs did not exist how would one differentiate between a line segment in one direction and the one of the same magnitude in the opposite direction. Now, these symbols "Preceded by the signs + and - are called 'positive' and 'negative' symbols, or 'positive' and 'negative' quantities: such symbols are also said to be affected with the signs + and -" (Peacock 190, 2).

To create an algebra of these unrestricted symbols there must exist operations to perform with regard to their combinations. Since symbolical algebra was to be identical to arithmetical algebra in all common matters, this meant the operations of addition, subtraction, multiplication, division, and so on had to be included. However, to make the extension to these symbols not included in the arithmetical approach, restrictions had to be lifted. This change brought about the operations in a new form--"the definitions of those operations must regard the laws of their combination only"(Peacock 1830, ix in Pycior, "Origins" 35). These laws were set up in

such a way that their results would be identical to those achieved through arithmetical algebra, if application were possible in the arithmetical approach. Thus, all of symbolical algebra was created on the assumption that whatever applied in arithmetical algebra also applied in this study. In Peacock's words:

> Whatever algebraical forms are equivalent when the symbols are general in form but specific in value, will be equivalent likewise when the symbols are general in value as well as in form. (Peacock 1940, 59)

This principle is known as "the principle of the permanence of equivalent forms" and is the basis for "the science which treats the combinations of arbitrary signs and symbols by means of defined though arbitrary laws" (Peacock 1830, 71 in Pycior, "Origins" 35).

By following arithmetical algebra through the principle of the equivalent forms to symbolical algebra, one discovers the difference between arithmetical and symbolical is quickly observed in the subtraction of monomials, since subtraction is restricted to subtrahends smaller than minuends in the first case and not in the second. This means that the simple equation

7a - 4a = 3a is possible in both approaches, but 4a - 7a = -3a is possible or valid only in the symbolical approach.

The distinction when subtracting polynomials is less pronounced. The initial steps are identical (references are to Peacock's treatise):

Steps for subtracting polynomials --

1. change the signs of the subrahend, + to - and - to + (Article 23, Article 549)
2. collect like terms (Articles 28, 547)
3. combine like terms
   i) if the like terms have the same sign, add the coefficients and subjoin the common symbol (Articles 28, 29)
   ii) if the like terms have different signs, add those preceded by the same sign, subtract the resulting coefficients and prefix to the remainder the sign of the greater (Article 31)

At this point the difference begins:

4. Arrange the terms in any convenient order, remembering that when working in arithmetical algebra the first term must be preceded by + or no sign at all. This is because in the

arithmetical approach the sign - represents subtraction and thus must be preceded by a greater number. If such an arrangement is not possible then the operation is said to be "impossible" in arithmetical algebra. On the other hand, any arrangement whatsoever will suffice in symbolical algebra.

Three examples similar to Peacock's follow:

Example 1: from $7a + 4b$ subtract $6a + 5b$

$7a + 4b - (6a + 5b)$

$7a + 4b - 6a - 5b$ Step 1

$7a - 6a + 4b - 5b$ Step 2

$a - b$ Step 3

This is a valid result in both algebras if $a > b$, whereas the answer arranged as $-b + a$ is acceptable only in the symbolical approach.

Example 2: from $7a + 4b$ subtract $8a + 6b$

$7a + 4b - (8a + 6b)$

$7a + 4b - 8a - 6b$ Step 1

$7a - 8a + 4b - 6b$ Step 2

$-a - 2b$ Step 3

This is strictly a result of symbolical algebra.

Example 3: from 2a - 4b subtract 3b - 6a

2a - 4b - (3b - 6a)

2a - 4b - 3b + 6a Step 1

2a + 6a - 4b - 3b Step 2

8a - 7b Step 3

Just like example 1, the validity of this result in arithmetical algebra depends on the order and the value of a and b.

Obviously, many instances occurred prior to symbolical algebra where subtraction was "impossible."

Another example of the differences between the two algebras is the handling of exponents. Since symbols are unrestricted in symbolical algebra, the term 5^a can have many interpretations. In the arithmetical approach exponents were restricted to positive whole numbers and were used only as a shorthand notation. Thus 4^3 was considered a convenient way to write 4x4x4. Of course, this usage has no bearing if the exponent is a fraction. To find a definition for such an expression, the new approach relies once again on the principle of equivalent forms, and on the arithmetical fact that $a^m \times a^n = a^{m+n}$. Then, the product of $a^{1/3} \times a^{1/3} \times a^{1/3} = a^{1/3 + 1/3 + 1/3} = a$, so $a^{1/3}$ has

the same meaning as the cube root of a. This is the manner in which the meaning of the expression $a^{1/n}$ is derived. Negative exponents are defined by a similar process.

A third major difference between arithmetical and symbolical algebra is brought about by the unrestrictive symbols in expressions involving the radical sign, with the symbolical approach able to deal with such expressions as "the square root of negative one."

The few examples we have discussed in no way cover the differences between the two algebras. They do, however, set the stage for an understanding of what symbolical algebra is, and how as a new idea in mathematics, it was open to misunderstanding and ultimately to man's use of humor as a defense.

4. Alice and symbolical algebra

Obviously, since symbolical algebra responded to the negative questions, it included the sign '-', and so the negative opponents rejected the approach. The previously discussed arguments against the definitions of negative rear their heads in Lewis Carroll's Alice's Adventures in Wonderland and Through the Looking-Glass. As Alice travels through "Wonderland" and "Looking-Glass House", she encounters at least three instances which

directly refer to the discussion of negatives.

The first instance, the "Mad Tea Party", incorporates the first defintion of negative.

> "Take some more tea," the March Hare said to Alice very earnestly.
>
> "I've had nothing yet," Alice replied in an offended tone: "so I can't take more."
>
> "You mean you can't take less," said the Hatter: "it's very easy to take more than nothing."
>
> (Carroll 71)

In the next instance, Alice is conversing with the Mock Turtle and the Gryphon. The Mock Turtle expounds on his once-upon-a-time life as a Real Turtle and how he attended school everyday.

> "And how many hours a day did you do lessons?" said Alice, in a hurry to change the subject.
>
> "Ten hours the first day," said the Mock Turtle: "nine the next, and so on."
>
> "What a curious plan!" exclaimed Alice.
>
> "That's the reason they're called lessons," the Gryphon remarked: "because they lessen from day to day."
>
> .... "Then the eleventh day must have been a holiday?"

"Of course it was," said the Mock Turtle.

"And how did you manage on the twelfth?" Alice went on eagerly. (Carroll 91-92)

At this point, the Gryphon decidedly changes the subject, leaving no chance to explore the possibility of a quantity less than nothing. One more reference to the negative problem addresses the "taking of a greater from a lesser" definition. In Through the Looking-Glass, the infamous Red Queen and White Queen do illogical (to the reader) and 'backwards' things since they seemingly live in a mirror. As Alice works toward the attainment of a royal position, she must pass a test administered by the two queens.

"Can you do addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?"

"I don't know," said Alice "I lost count."

"She can't do Addition," the Red Queen interrupted. "Can you do Subtraction? Take nine from eight."

"Nine from eight I can't, you know," Alice replied very readily: "but--"

"She can't do subtraction," said the White Queen. "Can you do division? Divide a loaf by a knife--what's the answer to that?"

"I suppose--" Alice was beginning, but the Red Queen answered for her.

"Bread- and -Butter, of course. Try another Subtraction sum. Take a bone from a dog: what remains?" (Ans: the dog's temper) (Carroll 220)

With such outlandish application of addition, subtraction, and division it seems obvious that the taking of nine from eight is meant to be impossible with respect to the "real world view" (outside the mirror).

5. More objections to symbolical algebra

a. arbitrariness

Of course, others voiced objections, aside from the negative protest, to symbolical algebra. In Peacock's Treatise on Algebra, article 544, the symbols "represent quantities of all kinds, and of all relations of magnitude." Many mathematicians interpreted this restriction (or lack of it) as arbitrariness. Some of these same scholars viewed arbitrariness as a loss of meaning since with such a symbol, interpretation follows rather then precedes the symbol and the operations performed on it. "Arbitrary" quantities thus became a new target for the opposition. In his reveiw of the Treatise on Algebra, Augustus De Morgan claimed, "At first sight it appeared to us something like symbols

bewitched, and running about the world in seach of a meaning." The author of Alice's Adventures in Wonderland, Charles Lutwidge Dodgson, pen name Lewis Carroll, also criticized the approach. "There is one preliminary step, that is absolutely indispensable before the humban intellect can accept any Axiom whatever: and that is, it must attch some meaning to it. We cannot, relationally, either assent to, or deny, any Proposition the words of which convey to us no idea."

Arbitrariness abounds throughout Alice's Adventures in Wonderland and Through the Looking-Glass. Once "Down the Rabbit Hole", Alice proceeds to eat and drink substances which alter her size. As a result, Alice begins to wonder who she truly is. She, like the symbols of the "new method" possesses no particular 'definition'. To discern "who she is", Alice tests what she knows.

> "I'll try if I know all the things I used to know. Let me see: four times five is twelve, and four times six is thirteen, and four times seven is -- oh dear! I shall never get to twenty at that rate! However, the Multiplication-Table doesn't signify. Let's try Geography."

(Carroll 26)

Exemplifying the arbitrariness of multiplication, the operation is carried out in a varying base. The base of ten may be the usual choice, but ultimately it is only an arbitrary though convenient one. Also in this passage, Alice discovers that certainty is not an innate quality of mathematics; therefore, mathematics cannot rescue her from her crazy underground world. This attacks symbolical algebra directly, exemplifying that the allowance of arbitrariness into mathematics yields a meaningless and powerless entity.

Aside from his review of Peacock's work, De Morgan also included a reference to symbolic algebra in the introduction of Chapter II of his Trigonometry and Double Algebra of 1849. He wrote, "With one exception, no word nor sign of arithmetic or algebra has one atom of meaning throughout this chapter, the object of which is symbols, and their laws of combination, giving a symbolic algebra" (De Morgan 101 in Pycior, "Humor" 149). Alice parrots this quote during the trial of the Knave. "I don't believe," Alice declared, "there's an atom of meaning in it" (Carroll 112).

b. universality

The symbols of Peacock's symbolical algebra were not only arbitrary but also general. This "general"

trait is translated by many to universality; it is on this point that a new set of objections stand. Osborne Reynolds, a graduate of Queens' College, Cambridge, attached this aspect of the symbol. He claimed that for symbols to be universal it must be assumed "that there are properties common to every species of abstract and concrete quantity. This seems to me an unphilosophical assumption, for no attempt is made to prove it by any evidence of the fact either previous or subsequent to the assumption itself" (Anon. 1837, 8 in Pycior, "Criticisms" 404).

Alice seems to agree with the idea that one cannot assume the existence of resemblances or qualities in common between two words. This conviction follows from her disagreement with the Dormouse at the "Mad Tea Party". The Dormouse tells a story in which the main characters draw treacle from a treacle-well. Alice questions the validity of such a process only to be rebuffed by the Mad Hatter. "You can draw water out of a water-well," said the Hatter; "so I should think you could draw treacle out of a treacle-well -- eh, stupid?" (Carroll 72). In symbolism, this could be expressed as "you can draw b out of a b-well". Structure therefore does not guarantee meaning. On this point, many

defenders of symbolical algebra may have replied that "nonsense results only when symbols are accepted as absolutely universal in interpretation" (Pycior, "Humor" 167). This, however, left the opposition free to ask yet another question. Who determines appropriate interpretations of symbols?

Humpty Dumpty displays the absurdity of the existence of a being with the power to determine interpretations. In conversing with Humpty Dumpty, Alice finds herself confused by the use of a word in a sentence which does not 'fit'.

> "But 'glory' doesn't mean a 'nice knock-down argument'," Alice objected.
>
> "When I use a word," Humpty Dumpty said, in a rather scornful tone, "it means just what I choose it to mean--neither more nor less."
>
> "The question is," said Alice, "whether you <u>can</u> make words mean so many different things." (Carroll 186)

This passage also refers to the question of using the same term in symbolical algebra to mean different things. (For example, some rejected the use of "multiply" with respect to the operation on such diverse quantities as lines and numbers.

These factors, inclusion of negatives, and arbitrary and universal symbols make for the most vehement onslaught against the symbolic approach to algebra. As already recorded, each charge presents itself in Lewis Carroll's fairytales.

6. Dodgson the mathematician

Now, many may say, "anyone can make a writing look like it says some thing it was never intended to say." This writer believes that Charles Dodgson did intend for Alice's Adventures in Wonderland and Through the Looking-Glass to have hidden mathematical implications. Though not overwhelming, the evidence of this is undeniable. First and foremost, by profession Dodgson was a mathematical lecturer at Christ Church, Oxford. In this capacity, he was familiar with the arguments and also educated enough to harbor an opinion of his own. Next, that he would record his opinion for future generations is an indisputable assumption. Charles Dodgson published many mathematically related works including Euclid and His Modern Rivals, Curiosa Mathematica, Notes on the First Two Books of Euclid, and A Guide to the Mathematical Student. Would he use a 'non-reporting' style such as the humor in Alice's

Adventures in Wonderland when discussing a serious subject like mathematics? Certainly, Euclid and His Modern Rivals is a play in defense of Euclidean Geometry--the setting is hell. Dodgson's sense of humor in relation to mathematics is also apparent in the advice he gave to a building committee.

> "It may be sufficient for the present to enumerate the following requisites, others may be added as the funds permitted:
>
> "A. A very large room for calculating greatest common measure. To this a small one might be attached for least common multiple; this, however, might be dispensed with.
>
> "B. A piece of open ground for keeping roots and practising their extraction; it would be advisable to keep square roots by themselves, as their corners are apt to damage others.
>
> "C. a room for reducing fractions to their lowest terms. This should be provided with a cellar for keeping the lowest terms when found.
>
> "D. A large room, which might be darkened and fitted with a magic lantern for the purpose of exhibiting circulating decimals in the act of circulation.

> "E. a narrow strip of ground, railed off and carefully leveled, for testing practically whether parallel lines meet or not; for this purpose it should reach, to use the expressive language of Euclid, 'ever so far'." (White 201)

The last question, "was Dodgson an opponent of the symbolic approach", is easily answered. As already stated, with the symbolic approach came a lack of meaning and uncertainty. On this matter, Dodgson was greatly dissappointed. In Curiosa Mathematica Part I, he wrote:

> It may well be doubted whether, in all the range of science, there is any field so fascinating to the explorer--so rich in hidden treasures--so fruitful in delightful surprises--as that of Pure Mathematics. The charm lies chiefly, I think, in the absolute certainty of its results..."

In his Guide to the Mathematical Student, which advises the order in which mathematical subjects should be learned, Dodgson listed "higher algebra" twenty-third on a list of twenty-six. No reference was made to symbolical algebra (Pycior, "Humor" 162). As quoted earlier, Dodgson also opposed the lack of meaning

inherent to the symbols of this approach. In a letter to Mary Dodgson, Charles Dodgson discusses his inability to reconcile the use of the term "multiplication" on such differing objects as lines and numbers. "What likeness is there in the two operations to justify our calling them by the same name? I am not nearly satisfied yet on the subject" (Pycior, "Humor" 162). Another work by Dodgson, "The New Method of Evaluation as Applied to π" was a satirical pamphlet which offered pseudo-mathematical solutions to the controversy of raising the salary of the Regius Professor of Greek. In it Dodgson included a solution based on "the priciple known as 'the permanence of equivalent formularies', an obvious spoof of the "principle of the permanence of equivalent forms" (Dodgson 1127 in Pycior, "Humor" 161).

All of these references point to the conclusion that Charles "Lewis Carroll" Dodgson channeled his criticisms of the symbolic approach to algebra through the mad journeys of Alice--his lack of understanding through humor. His resolution of the dispute also follows--get rid of the arbitrary and universal symbols, bring back meaning and certainty.

> "Who cares for you?" said Alice (she had grown to full size by this time). "You're

nothing but a pack of cards!" (Carroll 114)

With the rejection of the "symbol" people, Alice returns from Wonderland to a land where words have designated meanings.

WORKS CITED

Anonymous [O. Reynolds]. 1837. Strictures on Certain Parts of "Peacock's Algebra" by a Graduate. Cambridge.

Carroll, Lewis. Alice's Adventures in Wonderland and Through the Looking-Glass New York: New American Library, 1960.

De Morgan, Augustus. Trigonometry and Double Algebra. London: Taylor, Walton, Maberly, 1849.

Dodgson, Charles. "The New Method of Evaluation as Applied to π" in The Complete Works of Lewis Carroll. New York: Random House, 1936.

Frend, William. The Principles of Algebra. London,1796.

Greenfield, W. "On the Use of Negative Quantities in the Solution of Problems by Algebraic Equations". Transactions of the Royal Society of Edinburgh (1788): 131-145

Maseres, F. (ed.) "Tracts on the Resolution of Affected Algebraick Equations". London, 1800.

Peacock, G. "Report on the Recent Progress and Present State of Certain Branches of Analysis". Report of the Third Meeting of the British Association for

the Advancement of Science (1833): 185-352

---. A Treatise on Algebra. London: 1830

---. A Treatise on Algebra Vol. II. New York: Scripta Mathematica, 1940

Pycior, Helena M. "Early Criticism of Symbolical Algebra" Historia Mathematica 9 (1982): 392-412

---. "George Peacock and the British Origins of Symbolical Algebra" Historia Mathematica 8(1981): 23-45

---. "At the Intersection of Mathematics and Humor: Lewis Carroll's 'Alices' and Symbolical Algebra" Victorian Studies (1984): 149-170

Woodhouse, R. "On the Necessary Truth of Certain Conclusions Obtained By Means of Imaginary Quantities" Philosophical Transactions of the Royal Society of London (1801): 89-119

FRACTALS: THE MANDELBROT SET*

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Fractals are giving people a new method of looking at mathematics. They are placing the importance of images along with numbers, theories, and formulas. Although fractals have been experimented with since the turn of the century, they had not been seriously studied and applied until the last few years. They are now being used in practical applications ranging from the sciences to television advertisements.

Fractals form a bridge between mathematics and computer science. They are defined by mathematical formulas. These formulas, while usually simple, must be used iteratively to give fractals their characteristic depth. This paper will examine the characteristics of fractals. The algorithm for generating a fractal known as the Mandelbrot set will then be provided.

*A paper presented at the 1987 National Convention of KME and awarded fourth place by the Awards Committee.

WHAT ARE FRACTALS?

Imagine a typical coastline. How long is it? To find out, we might first draw it and then use calipers to measure it. This will give us an approximation of the length of the coastline. This approximation, however, will underestimate the coastline's true length. The reason is that calipers estimate the details of the coastline with straight lines, ignoring small peninsulas and bays. This effect is especially noticeable under magnification as shown in Figure 1. In effect, we're taking a series of short cuts.

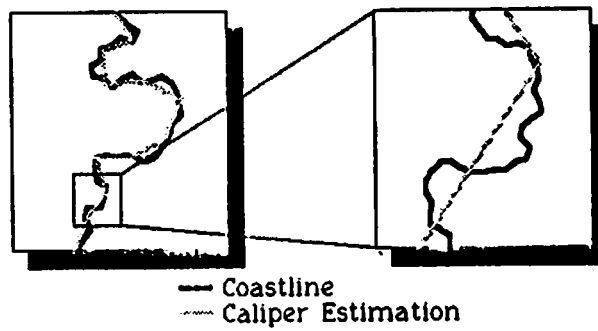


Figure 1

To account for the bays and peninsulas, expand the coastline to its full size and step out the length by foot. This method would seem to give a much more accurate and certainly much longer estimation of the length of the coastline. However, we're still estimating details of the coastline with straight lines - our feet - ignoring the effect that small rocks have upon the result.

Measure around each rock. Measure around each grain of sand. Measure around each atom. Each level of detail traversed yields yet a longer coastline. Continue this process indefinitely and you will find that the coastline has an infinite length. Extend this process to natural landscapes or worlds and you will find infinite areas and infinite volumes respectively. This is a basic property of fractals: a fractal has an infinite size although it may be contained within a finite space.

A more subjective quality of fractals is self-duplication. A fractal looks, at least roughly, the same no matter the scale upon which you look at it. A coastline without references (such as trees or people)

looks the same whether you look at a kilometer of it or one hundred kilometers of it. Mathematician Helge Von Koch designed one of the simplest fractals, now called the "Koch Curve,," at the turn of the century. The basis for the curve is four equally long line segments arranged as shown in Figure 2a. Each segment is then made an image of this basis, resulting in Figure 2b. Each segment of this curve is then made an image of the basis, resulting in Figure 2c. The Koch curve is a result of this process continued indefinitely. With each segment being an image of the basis, the Koch curve looks exactly the same no matter how close you look.

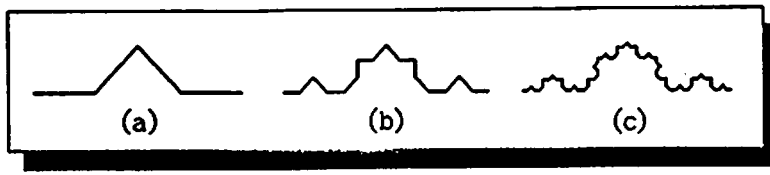


Figure 2

Fractals also display a curious quality in the dimensions. Imagine that you have the task of determining the number of sections that a straight line will have if you were to assure that each section was an arbitrary number of times smaller than the line, three

times smaller for example. The answer is three sections, of course. Now do the same for a square. With each section of the square three times smaller than the square itself, the square will have nine sections. The relationship between the number of sections the object will have, call this N, and the number to times smaller each section is from the object, call this r, is contained in Equation 1.

$$N = r^d \quad \text{(Equation 1)}$$

d is the number of dimensions the object has, one for a straight line, two for a square, three for a cube, and so on. A summary is given in Figure 3.

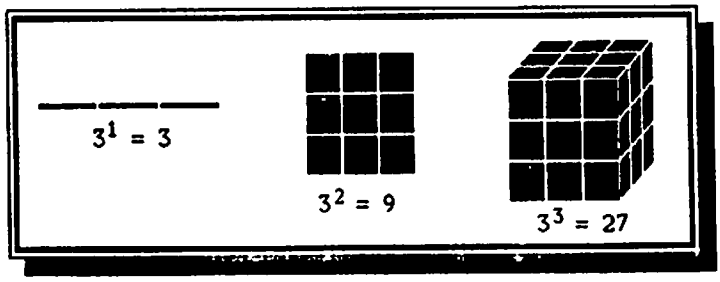


Figure 3

How many dimensions does the Koch curve have? Making each piece three times smaller results in four segments as shown in Figure 4. Assuming that the Koch curve has one dimension, the number of segments is 3^1 or three. Assuming that it has two dimensions, the number of segments is 3^2 or nine. Then how many dimensions does it have? To find out, solve equation 1 for d. The result is in Equation 2. Now plug in N = 4 and r = 3. d turns out to be 1.261859507. The Koch curve has 1.261859507 dimensions.

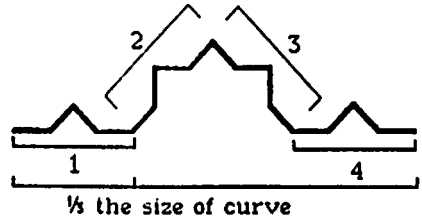


Figure 4

$$d = \frac{\log N}{\log r} \quad \text{(Equation 2)}$$

All fractals display this curious quality of fractional dimensions. The dimensions aren't integral, but rather flow between the integers. In fact, fractional dimensions are used to compare fractals. All fractals have infinite size, so comparing sizes is meaningless. However, we can compare dimensions. The higher the dimension a fractal is, the rougher and more broken it looks. Coastlines have dimensions between 1.15 to 1.25. Natural landscapes vary between 2.3 to 2.4 dimensions.

THE FRACTAL MARKETING MANAGER

There is no one discoverer of fractals, but the acknowledged father of fractal geometry is Benoit B. Mandelbrot. Born in Warsaw in 1924, Mandelbrot displayed a penchant for mathematical images, solving mathematical problems geometrically rather than by the numeric methods that were encouraged.

He began noticing that the apparent randomness of nature was not entirely random, but rather contained a structure. Comparisons of the graphs of readings of pollution levels, electrical noise, or river levels revealed great similarities. One could look at these graphs at any scale and they looked the same.

It was during a study of the annual levels of the Nile river that Mandelbrot became aware of the power of fractal geometry. No one had been able to predict the level of the Nile from year to year. Mandelbrot examined the graphs of Nile river level readings and constructed fractal models which looked similar to the graphs. Upon showing his models to hydrologists, he found them unable to discern them from graphs of actual Nile readings.

Mandelbrot then began to use fractal models to construct coastlines. By varying the parameter of fractional dimension to the coastline model, he was able to generate islands ranging from featureless blobs to realistic coastlines to exceptionally rough, alien-looking landforms.

Mandelbrot has had an impressively diverse occupational history. "Very often when I listen to the list of my previous jobs, I wonder if I exist," he said. "The intersection of such sets is surely empty." From studying the Nile river to analyzing the source of

electrical noise to studying stock market prices, he has seen fractals develop from the mathematical abstractions of Helge Von Koch to a new branch of geometry.

THE MANDELBROT SET

Now let us take a look at a fractal called the Mandelbrot set. The Mandelbrot set is a set of numbers on the complex number plane whose shape is a fractal.* It is easily generated on any computer capable of graphics, although that can be time consuming.

A point on the complex number plane is either in the Mandelbrot set, or it isn't. To test a point to see if it's in the set, we must put the point into an iterative algorithm.

*The complex number plane is constructed by plotting the real parts of complex numbers along the horizontal axis and the complex part along the vertical axis.

Call the point c. Now we need a running complex variable: call it z. Set z to 0 + 0i. The iterative loop is then entered. The whole process is outlined below:

1. z ← z^2 + c
2. Count ← Count + 1
3. Size ← size of z
4. Go back to step 1 until we're done

What does "we're done" means? There are two conditions which indicate that we are done. Which of the two conditions that is taken determines whether the point is in the Mandelbrot set or not. One of the conditions is that |z| becomes larger than two. Complex number theory assures that z will eventually diverge to infinity as a necessary and sufficient condition of |z| being greater than two. If this condition is satisfied, the point is not in the Mandelbrot set. The other condition is that we have iterated many times and |z| never became larger than two. The Count variable keeps track of the number of times that we have iterated. The number of times to iterate is fairly arbitrary, but it usually is between 100 and 1000. The higher the number, the more accurate the calculation.

If we do this process for an array of points in the plane, the Mandelbrot shape emerges. The whole Mandelbrot set is shown in Figure 5. The imaginary part is plotted on the vertical axis and the real part is plotted on the horizontal axis. The black areas are the points within the Mandelbrot set and the white areas are points outside of the Mandelbrot set.

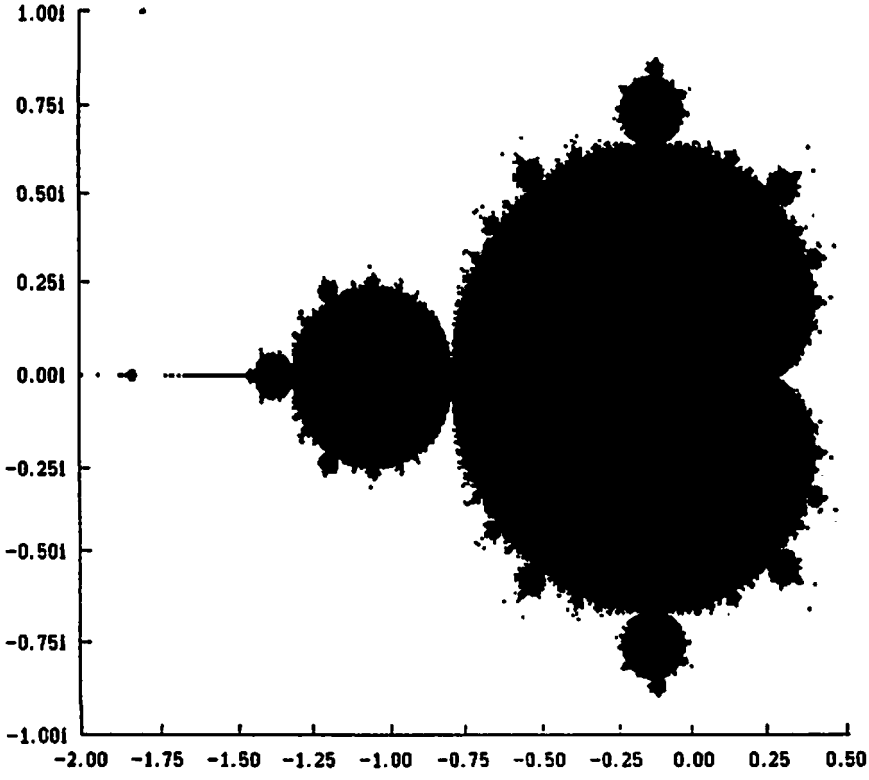


Figure 5

The edge is infinitely detailed. If we examine the edge at high magnification, we find the characteristic maintenance of detail.

FRACTAL APPLICATIONS

Fractals are increasingly playing an important part in the sciences. Physics was the first major player in fractal applicatins. A complicated physical problem such as the study of fluid turbulence, was far beyond the ability of physicists until the last few years. Now fractals are a standard method of studying turbulence. Fractal models for capillary action have been built. Some economics is being studied with fractals as a replacement for trend analysis. Although the process of using fractals in science varies from problem to problem, the basic method is to find a fractal with a similar dimension to the problem to be analyzed. The fractal is then subjected to quantitative analysis and the results are applied to the real problem.

Even music is finding itself subjected to scrutiny-by-fractal. Fractal models have been found that mimic rhythm and melodies of human-written music. Music written by computers using fractals will sound convincingly "human."

By far the most stunning use of fractals has been in the area of computer graphics. Since natural landscapes are fractals, it makes sense to use fractals to draw artificial landscapes. One common method of doing this starts with a triangle. This triangle is divided up into several triangles with random displacements. Each resulting triangle is divided up similarly, and so on. Continue this process indefinitely and a remarkably real-looking landscape will form. Fractals used for computer-generated graphics are beginning to be shown to the general public in movies, such as Star Trek II and The Last Starfighter, and in television and print advertisements.

SUMMARY

Fractals are objects displaying three fundamental qualities: infinite size, self-duplication at any scale, and fractional dimension. Fractals are found throughout nature. The next time you go into your kitchen, look at a broccoli stalk or a ginger root. The fractal effect is obvious; a small stalk of broccoli looks like the original stalk and a bit of the ginger root still looks like a whole ginger root. In fact, very convincing broccoli stalks and ginger roots have been generated by fractal computations.

Fractals will undoubtedly become more pivotal in the future. Some sciences which have displayed a need for fractal analysis have held back thus far due to conservatism and scientific skepticism. Once fractals are accepted into these sciences, roads to other uses of fractals will be paved. The future will reveal to us the uses and products of fractals which we can't even begin to think of now.

REFERENCES

Dewdney, A.K., "Computer Recreations," Scientific American, August, 1985

Mandelbrot, Benoit, The Fractal Geometry of Nature. New York: W. H. Freeman and Company, 1983

Solomon, Les, "Computer Hardware," Computers & Electronics, December 1984

Steen, Lynn Arthur, "Fractals: A World of Nonintegral Dimensions," Science News, August 20, 1977

Thomsen, Dietrick E., "A Place in the Sun for Fractals," Science News, January 9, 1982

"Interview: Benoit B. Mandelbrot," Omni. February, 1984

THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 15 January 1988. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring, 1988 issue of The Pentagon, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 402: Proposed by the editor.

Evaluate the product

$$\prod_{k=1}^{n} \cos \frac{k\pi}{2n+1}$$

Problem 403: Proposed by the editor.

Young Eulcid pondered a triangle in which one side is 12 feet longer than another. The angle formed by these two sides is 55 . If two circles are drawn with these respective sides as diameters, one of the points of intersection of the circles is the common vertex where these two sides meet. What is the locus of the other point of intersection?

Problem 404: Proposed by the editor.

A certain number N is the product of three primes. If the sum of the cubes of these primes is 2645187 and if the sum of the divisors of N (including N and 1) is 104328, identify the three primes.

Problem 405: Proposed by the John A. Winterink, Albuquerque, New Mexico.

In triangle ABC, AB = 25, points D and E lie on AC and BC respectively, and DE = 26. Let AD : DC = 1 : 2 and BE : EC = 1 : 6. Determine the smallest integer values for AD and BE such that AB + BC + CA $\angle$ 190.

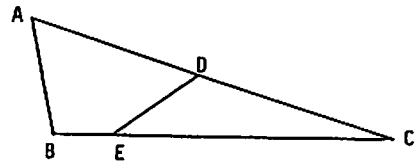


Figure Problem 405

Problem 392: Proposed by the editor.

The integer n has exactly 25 divisors. If n can be expressed in the form $a^2 + b^2$ where a and b are relatively prime integers, what is the smallest value of n and what are the corresponding values of a and b ?

Since only one incorrect solution has been received, this problem will remain open until the next issue at which time a solution will be supplied.

Problem 393: Proposed by Rich Hanlon, Ambler, Pennsylvania.

We define a two-tined fork as a pair of positive integers i,i separated by a distance of i units. We define a packing of size 2n as an interlocking arrangement of two-tined forks which has no gaps; e.g. 3 1 2 1 3 2 and 4 1 3 1 2 4 3 2 are packings of sizes 6 and 8 respectively. Note that in each case, there are exactly i digits between the two digits i. For what integers n can packings of size 2n exist?

Solution by the proposer.

Let the positions of the tines be numbered 1, 2, ... , 2n. Suppose that the first appearance (from the left) of the number 1 occurs at position p_1 which is the first tine of the fork F_1. Then the other tine of the fork F_1 occurs at position $p_1 + 2$. Generally the tines of the fork F_i occur at the positions p_i and $p_i + i + 1$. Let $P = p_1 + p_2 + \ldots + p_n$. Summing the numbers denoting the positions of the tines yields

$2P + (2 + 3 + \ldots + (n + 1)) = 2P + n(n + 3)/2 = n(2n + 1)$.

Hence $P = n(3n - 1)/4$ where P is an integer. This requires that $n \equiv 0$ or 3 (mod 4). Thus packings of size 2n occur only when $n \equiv 0$ or 3 (mod 4).

Editor's Comment: For a similar problem, see Problem 22 in Ross

Honsberger's Mathematical Morsels, Dolciani Mathematical Expositions Volume 3, Mathematical Association of America (1978), pp. 45-47.

Problem 394: Proposed by Dmitry P. Mavlo, Moscow, USSR.

Prove that the perimeter P_r of the rectangle PQRT which is circumscribed over an arbitrary convex quadrangle ABCD, as shown in the figure below, satisfies the inequality $P_r \leq P\sqrt{2}$ where P is the perimeter of the quadrangle ABCD and equality holds if and only if the quadrangle ABCD is a rectangle and PQRT is a square.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

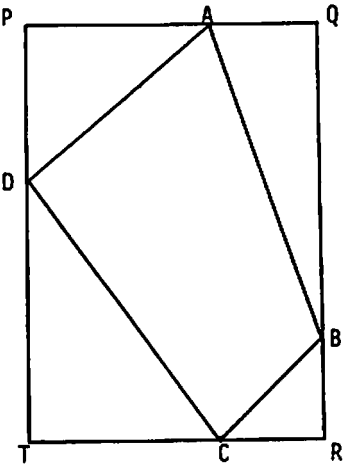


Figure 1 Problem 394

First we establish the following result.

Lemma: If triangle ABC has a right angle at C, then $a + b \leq c\sqrt{2}$.

Proof of lemma: Since $c^2 = a^2 + b^2 = (a + b)^2/2 + (a - b)^2/2$, $2c^2 \geq (a + b)^2$. Thus $a + b \leq c\sqrt{2}$. Equality holds if and only if $(a - b)^2 = 0$; i.e. if and only if $a = b$.

Proof of main result: It follows from the lemma that

$$P_r = (PA + PD) + (AQ + QB) + (RB + RC) + (CT + TD) \leq (DA + AB + BC + CD)\sqrt{2} = P\sqrt{2}.$$

Also from the lemma, note that if $P_r = P\sqrt{2}$, then PA = PD, AQ = QB, RB = RC, and CT = DT. Then since PQRT is a rectangle, PT = [(PD + DT) + (QB + BR)]/2 = [(PA + CT) + (AQ + RC)]/2 = PQ. Thus PQRT is a square. Also when PQRT is a square, triangles PDA and TDC are isosceles right triangles, so that angles PDA and TDC are 45^o angles and angle CDA is a right angle. Similarly the other interior angles of quadrangle ABCD are right angles; hence ABCD is a parallelogram and a rectangle.

In order to complete the problem as stated, we need to show that if quadrangle ABCD is a rectangle and PQRT is a square, then $P_r = P\sqrt{2}$. Figure 2 shows that this need not be the case when $x \neq y$.

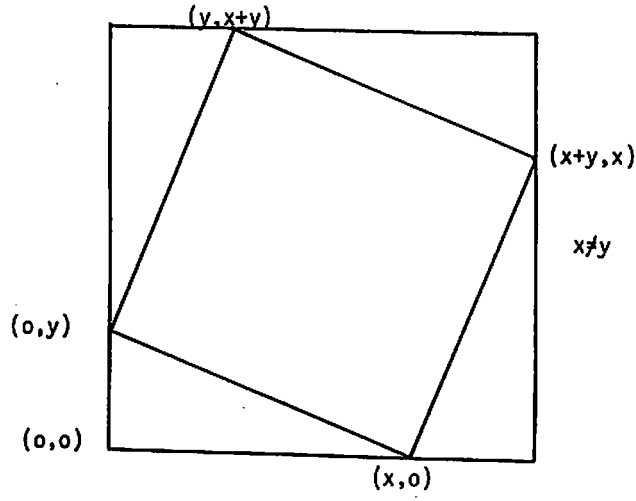


Figure 2 Problem 394

Solution by the proposer: Given quadrangle ABCD and the circumscribing rectangle PQRT as shown in Figure 3, let A_2 be the reflection of A in the line QR, A_1 be the reflection of A in the line PT, and B_1 and A_3 be the respective reflections of B and A in the line TR. Let PQ = a and QR = b so that A_1A_2 = 2a and A_2A_3 = 2b. Note that the length A_1A_3 depends only upon a and b, not on the location of A, B, C, and D. Applying the Pythagorean Theorem to the right triangle $A_1A_2A_3$, we get

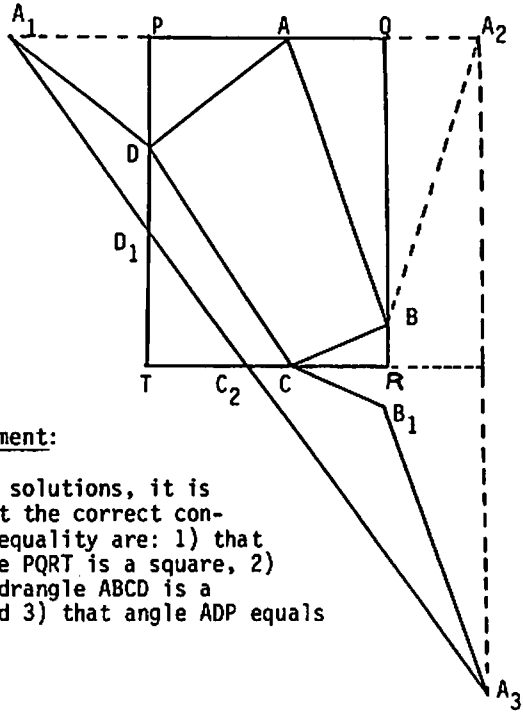
$$P \geq A_1A_3 = 2\sqrt{a^2 + b^2} \qquad (1)$$

since P = the length of the broken line $A_1DCB_1A_3$.

Now since $(a - b)^2 \geq 0$ for nonnegative quantities a and b, this is equivalent to $2\sqrt{a^2 + b^2} \geq (a + b)\sqrt{2}$.

But this is equivalent to $P_r \leq P\sqrt{2}$. (2)

Equality in (2) holds if and only if the following conditions hold: 1) the broken line $A_1DCB_1A_3$ coincides with the segment A_1A_3 and 2) a = b. Hence the circumscribed rectangle is a square and thus triangle $A_1A_2A_3$ is an isosceles right triangle and angle $A_2A_1A_3 = 45^\circ$. The coincidence of the broken line $ADCB_1A_3$ with A_1A_3 is equivalent to the coincidence of D with D_1 and C with C_2. Hence angles PDA and TDC are both equal to 45° so that angle ADC equals 90°. Hence the quadrangle ABCD is a rectangle.



Editor's Comment:

From our two solutions, it is apparent that the correct conditions for equality are: 1) that the rectangle PQRT is a square, 2) that the quadrangle ABCD is a rectangle and 3) that angle ADP equals 45°.

Figure 3 Problem 394

Problem 395: Proposed by Fred A. Miller, Elkins, West Virginia.

Given triangle ABC, draw a line which cuts two sides of the triangle so that the resuling quadrilateral has a common base with the triangle and also has its other three sides equal.

Solution by the proposer.

As in the figure below, given triangle ABC assume that ABDE is the required quadrilateral. Then ABDE is homothetic to KBCH with B as the homothetic center. Locate a point F on AC such that AF = BC. Through F draw a line parallel to AB and on this line locate a point H such that CH = BC. Through H draw a line parallel to AC and let this line cut the extension of line AB at K. Draw BH and let this line intersect AC at E. Through E a line parallel to HC and let this line intersect BC at D.

Now quadrilateral ABDE is homothetic to quadrilateral KBCH. Since AFHK is a parallelogram, we have constructed BC = AF = KH = CH. Thus BD = DE = EA.

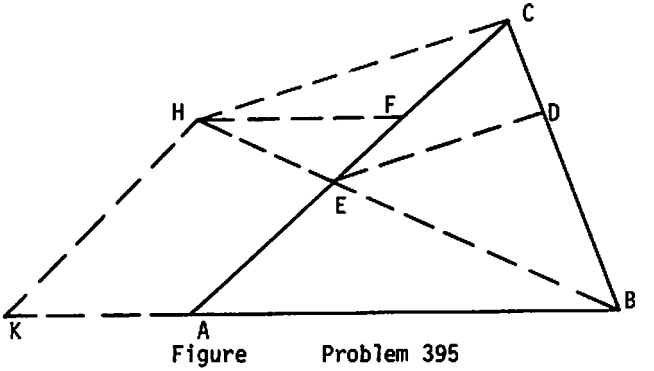


Figure Problem 395

Problem 396: Proposed by the editor.

Which of the following expressions is larger?

$$5 + \cfrac{5}{6 + \cfrac{5}{6 + \ldots}} \quad \text{or} \quad 5 + \cfrac{5 + \ldots}{5 + \cfrac{5}{6}}{6}$$

Composite of solutions submitted by Rodney R. Saunders, Mount Mercy College, Cedar Rapids Iowa, and Thomas J. LeCompte, Illinois Theta Chapter, Illinois Benedictine College, Naperville, Illinois.

Let $r = 5 + \cfrac{5}{6 + \cfrac{5}{6 + \ldots}} = 5 + \cfrac{5}{6 + k}$ where $k = \cfrac{5}{6 + \ldots}$.

Since $k > 0$, then we have $r < 5 + \cfrac{5}{6}$. Proceeding similarly,

let $s = 5 + \cfrac{5 + \cfrac{5 + \ldots}{6}}{6} = 5 + \cfrac{5 + l}{6}$ where $l = \cfrac{5 + \ldots}{6}$.

Since $l > 0$, we have $s > 5 + \cfrac{5}{6}$. Hence $r < 5 + \cfrac{5}{6} < s$.

Editor's Comment: Since $r - 5 = k$, we have the equation $k = \frac{5}{6 + k}$ or $k^2 + 6k - 5 = 0$. The positive root of this is $-3 + \sqrt{14}$ so that $r = 2 + \sqrt{14}$. Similarly since $s - 5 = 1$, we have the equation $s = 5 + \frac{s}{6}$ or $s = 6$. Hence $r \angle s$.

KAPPA MU EPSILON NEWS

Edited by M. Michael Awad

News of chapter activities and other noteworthy KME events should be sent to Dr. M. Michael Awad, Historian, Kappa Mu Epsilon, Mathematics Department. Southwest Missouri State University, Springfield, MO 65804.

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence
Chapter President - Angela Jackson
36 actives, 14 initiates

Other 1986-87 officers: Karen Weems, vice president; Susan Nale, secretary and treasurer; Eddy Joe Brackin, corresponding secretary; Patricia Roden, faculty sponsor.

Alabama Gamma, University of Montevallo, Montevallo
Chapter President - Karen Ronning
7 actives, 8 initiates

Other 1986-87 officers: Susan Pinegar, vice president; Barry LaGrone, secretary; Mary Manuel, treasurer; Joe Cardone, corresponding secretary; Gene Garza, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President - Jeanmarie Short
50 actives, 19 initiates

Weekly meetings have featured alumni and industry speakers. A reception was held for E. J. Newton, a 1963 member of our chapter, who was honored as a Cal Poly Distinguished Alumnus of the year. Other 1986-87

officers: Anna Heimgartner, Jacquie O'Mera, and Susan Ridenour, vice presidents; Stephanie Logan, secretary; Jon Fingold, treasurer; George R. Mach, corresponding secretary; Adelaide T. Harmon-Elliott, faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins
Chapter President - Robyn Tholke
15 actives

Other 1986-87 officers: Valerie van Felt, vice president; Thomas Painter, secretary and treasurer; Arne Magnus, corresponding secretary and faculty sponsor.

Colorado Gamma, Fort Lewis College, Durango
Chapter President - Johnny Snyder
42 actives, 8 initiates

On October 22, Chapter members participated in the college alumni phone-a-thon to raise funds for the Chapter. The Chapter has begun weekly tutoring sessions at Durango Senior High School. On November 19, eight new members were indicted into the Chapter. Other 1986-87 officers: Amy Getz, vice president; Jennifer Mabry, secretary; Tom Wyman, treasurer; R. A. Gibbs, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston
Chapter President - Vanessa Miller
34 actives

Other 1986-87 officers: Tricia Setzke, vice president; Dorothy Graham, secretary; David Wasser, treasurer; Lloyd Koontz, corresponding secretary; Lloyd Koontz and Duane Broline, faculty sponsors.

Illinois Zeta, Rosary College, River Forest
Chapter President - Jim Blondin
15 actives

Members are serving as tutors in mathematics. At each Chapter meeting a member presents a problem on a paper. In preparation are plans for a tour of an historically interesting spot in Chicago and a money raising hot dog sale. Other 1986-87 officers: Gina Suareo, vice president; Rosa Somma, secretary; Linda Russo, treasurer; Sister Nona Mary Allard, corresponding secretary; Mordechai S. Goodman, faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
Chapter President - Mark Cawood
15 actives

Other 1986-87 officers: Norman Rhorer, vice president; Lisa Jerva, secretary; Dawn Crum, treasurer; Ralph B. McBride, corresponding secretary; Stanley Beery, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President - Tracy Konrad
28 actives, 5 initiates

The following students presented papers at Iowa Alpha Chapter meetings: Tony Hays on his Apple II program Creative Function Plotter, Jeremy Phillips on Non-Standard Analysis, and Steven Shaff on the Geometry of Axial Reflexions. Diane Strahan will provide the program for the December 2 initiation banquet with her presentation on Spherical Trigonometry. The annual KME Homecoming Breakfast was held on September 27 at the home of Professor Emeritous and Mrs. Hamilton. No alumni showed, but the "locals" had a good time. Other 1986-87 officers: Jeremy Phillips, vice president; Tony Hays, secretary; C. David Bishop, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President - Carla VanCleave
50 actives, 9 initiates

The Chapter held monthly meetings in October, November,and December. In addition, a fall picnic was hosted for all mathematics and physics students. Fall initiation for new members was held at the November meeting. Nine new members were initiated at that time. The October program was given by Tom Skahan. His topic was "Solutions for the Instant Insanity Puzzle." The November meeting was preceded with a pizza party. Lester Adams presented the program on "Patterns in Problem Solving." In December, a special Christmas meeting was held at the home of Dr. Helen Kiregsman, Mathematics Department Chairman. Carla VanCleave gave the program entitled "Minitab Applications." Other 1986-87 officers: Tom Skahan, vice president; Tammy Horn, secretary; Jennifer Munson, treasurer; Harold L. Thomas, corresponding secretary; Helen Kriegsman and Gary McGrath, faculty sponsors.

Kansas Beta, Emporia State University, Emporia
Chapter President - Lisa Wagner
19 actives, 6 initiates

Other 1986-87 officers: Carole Hildreth, vice president; Donald Buster, secretary; Allison Harbaugh, treasurer; George L. Downing, corresponding secretary; Larry Scott, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Lisa Brox
21 actives, 8 initiates

Other 1986-87 officers: Patrick Hirsch, vice president; Monica Halstead, secretary and treasurer; Richard Farrell, corresponding secretary; Tamara Lasseter, faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President - Bryan Selby
20 actives

A dinner meeting was held in September. KME members will help with the Washburn University Math Day in November. Other 1986-87 officers: Joe Schisa, vice president; Torrey Lumpe, secretary; Robert Thompson, corresponding secretary; Ron Wasserstein, faculty sponsor.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President - Leann Brozek
36 actives, 4 initiates

During the fall semester we held a department picnic and a Halloween party. Other 1986-87 officers: Don Hager, vice president; Lisa Walker, secretary and treasurer; Charles Votaw, corresponding secretary; Mary Kay Schippers, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President - Teri Terrell
25 actives

The semester started off with a picnic at Dr. Janeway's house. Another social activity was the weekly softball games with the faculty. Since the Chapter is hoping to send a delegation to the national convention, several fund-raising activities were organized. Roses were sold at the University's Fall Festival and donuts were sold every Tuesday morning. There were three faculty members and one student who presented talks to the Chapter. Dr. Benny Hall gave a talk on "I Don't Know Puzzles." Dr. Paul Bland gave a talk on "Puzzles in Math" and Dr. Pat Costello talked about "Weird Numbers." Kenneth Lester spoke on "Database Systems." Other 1986-87 officers: Tim Daniel, vice president; Pam Adams, secretary; Karen Cardwell, treasurer; Patrick Costello, corresponding secretary; Bill Janeway, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President - Donna Pope
22 actives

During the fall semester, Maryland Delta Chapter has enjoyed a series of talks by students and faculty. At the September and October meetings, students Kevin Lowery, John Bernas and Rita Fowler recounted their summer work experience in math-related positions, and in November, Dr. Richard Weimer, Chairman of the Mathematics Department, presented a talk detailing mathematical applications of LOTUS software. We look forward to welcoming new members at our induction in February. Other 1986-87 officers: Brad Richards, vice president; John Bernas, secretary; Kevin Lowery, treasurer; Edward White, corresponding secretary; John Jones, faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President - Mark Huff
45 actives, 15 initiates

The USM Chapter of KME held its fall initiation and patio super on October 30th at the home of Dr. and Mrs. Newt Fawcett. Other 1986-87 officers: Amy Duvall, vice president; Joyce Deer, secretary and treasurer; Alice Essary, corresponding secretary; Virginia Entrekin, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President - Darlena Jones
50 actives, 8 initiates

The Chapter has had two meetings with one more planned for the fall semester. The annual fall picnic was held in conjunction with the Society of Physics students. We are also busy calling alumni to raise funds for KME and the Mathematics Department. Other 1986-87 officers: Doug Starkey, vice president; Lori

Baskins, secretary; Kevin Keltner, treasurer; John Kubicek, corresponding secretary; Simon Bernau, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President - Douglas Haas
36 actives, 7 initiates

The new members, initiated on October 2, 1986, are Charles Cole, Kirby Cass, Stan Detrick, Pamela Hughes, Rhea Moore, Susan Seipp, and Mary Viebrock. Speakers at the meetings spoke on the subjects of internships and job opportunities in math and computer science. We had a book sale and a car wash to raise money to send interested students to the national convention. The Math Clinic, supported by the members, was a big success. A field trip to Harmon Industries concluded the events for this semester. Other 1986-87 officers: Joseph Ratterman, vice president; Gayla Benson, secretary; David Stevenson, treasurer; Homer Hampton, corresponding secretary; Larry Dilley, Gerald Schrag, and Debbie Detrick, faculty sponsors.

Missouri Gamma, William Jewell College, Liberty
Chapter President - Don Marolf
22 actives

The Chapter, in addition to its regularly scheduled monthly meeting, is presenting a movie each month, for the entertainment of not only members, but also to anyone else who wishes to attend. The films are chosen on topics suited to the audience (usually a type of science fiction, but with an emphasis on young people doing scientific things). Attendance has been good, and refreshments are also served. Other 1986-87 officers: Jim Ed Wooldridge, vice president; Greg Dance, secretary; Joseph T. Mathis, treasurer, corresponding secretary, and faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President - Roberta Burger
5 actives

Other 1986-87 officers: Christina Drummond, vice president; Malcolm Hower, secretary; Debbie Sellmeyer, treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin
Chapter President - Melinda Robinson
12 actives

The Missouri Iota Chapter began the fall semester with the annual canoe trip down Shoal Creek. The Chapter then started working concessions at Missouri Southern's home football games as a money-making project for the year's activities. Dr. Shields gave a talk about a mathematical model for highway design. The year concluded with a Christmas party at the Park Place Club House. Other 1986-87 officers: Melissa Landers, vice president; Angela Noyes, secretary and treasurer; Susan Paulson, historian; Mary Elick, corresponding secretary; Joe Shields, faculty sponsor.

Missouri Kappa, Drury College, Springfield
Chapter President - Sami Long
8 actives, 3 initiates

The Missouri Kappa Chapter began its semester with a wiener roast-bonfire party. The Chapter continued to conduct a free tutoring service for all math students and also ran a campus-wide math contest. The lower division was won by Scott Steubing and the upper division by Matt Bear. A pizza party was held for the contestants and prize money awarded to the winners. Dr. Allen presented a math talk on "Peeking into the Fourth Dimension" and a student talk is planned for December. On October 30th, the Chapter conducted an initiation in which Andrea Ehrsam, Christine Hutchison, and Donna Luetkenhaus joined the society. A Christmas

party is also planned for December. Other 1986-87 officers: Joe Sherrill, vice president; Christine Hutchison, secretary; Donna Luekenhaus, treasurer; Charles Allen, corresponding secretary; Ted Nickle, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President - Dana Hungerford
24 actives

Throughout the semester club members have monitored the Math-Science Building in the evenings to earn money for the club. In October club members participated in a pre-Parents Day "all campus clean-up" which was sponsored by the Wayne State College Student Senate. The club participated in the college homecoming activities by painting and erecting a billboard. The club also sold pop, hot chocolate and popcorn at the Homecoming Carnival. With an allocation from the Wayne State College Student Senate, KME and the Computer Club purchased two printer interfaces, each of which allows four computers to be hooked up to one printer. Other 1986-87 officers: Colleen Spieker, vice president; Karen Devine, secretary and treasurer; Kenneth Mestl, historian; Fred Webber, corresponding secretary; James Paige and Hilbert Johs, faculty sponsors.

Nebraska Gamma, Chadron State College, Chadron
Chapter President - Beth Kilday
15 actives, 5 initiates

This semester, members of KME decorated the Math/Science Building for homecoming. We again won first place for our effort. Our homecoming theme was "Building on the Past ... Looking to the Future." We decided to string up a rocket, between second and third floors, shooting it toward the planets. On second floor we filled a glass display cabinet with old math books and slide rules. On third floor we placed pictures of modern rockets and other advanced technological items in a glass display case. Donna Wehling also made up posters of "future" math formulas

to put in the case. On November 4, KME informally initiated five new members: Froozan Afiat, Tracy Gibbons, Kathleen Hall, Rick Logan, and Hortensia Soto. Our formal initiation is planned for November 18 at the home of Mr. James Kaus. Dr. Monty Fickel submitted a money-making plan to KME at our last meeting. KME intends to sell LOGO disks with documentation to students in the class "Math for Elementary Teachers." This package will be sold at a reduced cost to benefit the students as well as make a small profit for KME. Other 1986-87 officers: Rod Tyma, vice president; Donna Wehling, secretary; Deb Gaswick, treasurer; James Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President - Cecilia De Blasi
100 actives

Other 1986-87 officers: Susan Fehrenbach, vice president; Sheryl Henry, secretary; Richard Metzler, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara
Chapter President - Elmer Bauer
12 actives

The Chapter is presently reorganizing at this time, but tentative plans have been made for a Christmas party with the faculty. An outside speaker will be invited to a spring meeting which has not been scheduled as yet. Other 1986-87 officers: Christina Bonnette, vice president; Linda Andrews, secretary and treasurer; Robert L. Bailey, corresponding secretary; Kenneth Bernard, faculty sponsor.

New York Kappa, Pace University, New York
Chapter President - Maureen Wong
33 actives, 16 initiates

Other 1986-87 officers: Tracey Morrison, vice president; Zhong-Su Chew, secretary and treasurer; Louis V. Quintas, corresponding secretary; Martin Kotler and John W. Kennedy, faculty sponsors.

New York Lambda, C. W. Post Center - Long Island University, Greenvale
Chapter President - Paula Punis
40 actives

Our big push is to have students present talks at the biennial convention. Several cake sales are planned to raise funds. A career day program is being planned. Other 1986-87 officers: Cynthia Ferro, vice president; Louis Sassano, secretary; Vida Moniriarani, treasurer; Annmary Esposito, historian; Sharon Kunoff, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President - Connie Garces
35 actives, 7 initiates

A business and convention discussion was held on September 17. Shabbir Jasdanwala presented a talk on Leontief modelling. Our initiation was held on October 15 and talks were given by the new members: Greg Files, Timothy Hiles, Ranjit Malik, Vijit Malik, Jason Ting, and Dana Woodland. Invited talks by two Miami University mathematicians were given on November 19: Don Weber - "Probability, Statistics, and the Law," and Rich Laatsch - "Polyominoes for Fun and Profit." The annual Christmas party was held in December. Other 1986-87 officers: Bonnie Kieffer, vice president; Karen Linn, secretary; Fred List, treasurer; James L. Smith, corresponding secretary; Russ Smucker, faculty sponsor.

Oklahoma Alpha, Northeastern Oklahoma State University, Tahlequah
Chapter President - Anne Fischer
37 actives, 22 initiates

The fall initiation ceremonies were held in the banquet room of the Western Sizzlin' Steak House in Tahlequah. In addition to being welcomed to the Honor Society by the present members, officers and faculty, the 22 initiates were honored by the presence of the past sponsor, Mike Reagan, who recently retired from Northeastern State University. Other 1986-87 officers: Chris Denney, vice president; Patricia McGinn, secretary and treasurer; Joan E. Bell, corresponding secretary and faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington
Chapter President - Tracey Boyce
16 actives, 3 initiates

The Pennsylvania Alpha Chapter of KME has continued its tutoring sessions twice a week. We also have three new associate members, have had a party for them, and are currently planning a career seminar in December. Other 1986-87 officers: Karen Haney, vice president; Rhonda Smith, secretary; David Jarrett, treasurer; J. Miller Peck, corresponding secretary; Barbara Faires, faculty sponsor.

Pennsylvania Gamma, Waynesburg College, Waynesburg
Chapter President - Paula Puskarich
15 actives

A KME meeting was held in September to plan for the upcoming events of the fall semester, 1986-87. A get together was held on October 15 at Dr. Tucker's house to introduce the new freshmen students to KME and make them aware of what KME could offer them. Our KME Chapter along with the American Chemical Society Student Affiliate at the College are each co-sponsoring Mr. David Wildman of the Pittsburgh Energy Technology

Center to come and speak about coal combustion research to the students at Waynesburg College. Mr. Wildman is a 1974 graduate of Waynesburg College and received his Master in Applied Math at Purdue University. The speech is scheduled for October 28. Some special news for our Chapter is that Dr. Walker, one of our KME faculty members, was able to attend Clarkson University in New York this past summer where he gained some retraining in computer science. Other 1986-87 officers: Steve Grudi, vice president; Kim Simon, secretary and treasurer; Rosalie Jackson, corresponding secretary; David Tucker, faculty sponsor.

Pennsylvania Delta, Marywood College, Scranton
Chapter President - Elizabeth Kerrigan
13 actives

Some members attended the NCTM Regional Convention in Hartford, Connecticut and the MAA Regional meeting at Drexel University in Philadelphia. Other 1986-87 officers: Susan Pacanowski, vice president; Peggy Rekus, secretary; Diane Babilow, treasurer; Sr. Robert Ann von Ahnen, corresponding secretary and faculty sponsor.

Pennsylvania Epsilon, Kutztown University, Kutztown
Chapter President - Pamela J. Dotterer
11 actives

One program was entitled "A Mathematician's View of a Snowflake," dealing with a theoretical boundary without finite length. Other 1986-87 officers: Kevin J. Olsen, vice president; Glen R. Naregang, secretary; Chad S. Benner, treasurer; William E. Jones, Jr., corresponding secretary; Edward W. Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana
Chapter President - Anne Polito
30 actives, 4 initiates

October meeting: Initiation of new members. Talk by Professor Doyle McBride on Symbolic Logic. November meeting: Presentation by Professor Thomas Giambrone demonstrating the use of Apple IIE computers in learning mathematics. Members were encouraged to write papers for possible presentation at the National Convention in April, 1987. Other 1986-87 officers: Pamela Weisgarber, vice president; Bonnie Jacko, secretary; Denise Rogers, treasurer; George Mitchell, corresponding secretary; Ida Z. Arms, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President - Pam Davis
43 actives, 21 initiates

Twenty-one new members were initiated at a dinner meeting on October 27. At the same meeting a new member of the Mathematics Department, Ms. Susan Frank, gave a brief talk on Graph Theory; it was very well received. Later, on November 16, the group gathered for an afternoon of volleyball. As the semester draws to a close, we look forward to the annual Christmas party at the home of Mr. Schlossnagel (Math Department Chairman). Other 1986-87 officers: Sally Hamburg, vice president; Sue Hockenberry, secretary; Paige Ackelson, treasurer; Marvin Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Iota, Shippensburg University, Shippensburg
Chapter President - Julie A. Chronister
37 actives, 9 initiates

The Pennsylvania Iota Chapter had an eventful fall, 1986, semester. After a reorganizational meeting in early September, the Chapter sponsored a program on the

actuarial field with alumnus Jeff Myers as the guest speaker. The official kickoff occurred the next day at the annual fall Homecoming Picnic. A few members also made a banner for the football game to show everyone our KME spirit. The Pentagons, our intramural volleyball team, proved that mathematicians are more than just spectators when it comes to athletics. They disſed (disintegrated) their oponents, placing second in the co-ed competition. November activities of the Chapter included a Math/Computer Science Career Day and a panel discussion on internship opportunities and procedures. Throughout the semester KME members were busy volunteering their services as tutors, making plans for the trip to California and initiating the fall pledge class. The membership did approve a constitutional change requiring a higher overall GPA for membership, effective spring, 1987. This semester will close with the initiation ceremony and Christmas Party. Other 1986-87 officers: Timithy A. Bream, vice president; Kathy Good, secretary; Howard T. Bell, treasurer; Donald E. Spickler, Jr., historian; Carl E. Kerr, corresponding secretary; Rick E. Ruth, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President - Nadine Hillgen
6 actives, 10 initiates

Monthly meetings (3rd Wednesday of every month) are held and sets of problems, which are distributed on the first of the month, are discussed and solved. The annual field trip was to The Franklin Institute which had dedicated their main exhibit to Women in Science. Besides the exhibit, there also was a one-act play, "Sara, the Scientist," presented. Plans were made for the initiation of new members on March 16, 1987. More details for the program will be discussed in December, 1986. Other 1986-87 officers: Constance Hefner, vice president and secretary; Sherri Teti, treasurer; Sister Mary Grace, corresponding secretary; Linda Czajka, faculty sponsor.

South Carolina Beta, South Carolina State College, Orangeburg
Chapter President - Nadine Ford
8 actives

Other 1986-87 officers: Patrice Marshall, vice president; Rodney Wright, secretary; Janice Brunson, treasurer; Frank M. Staley, Jr., corresponding secretary; Manuel Keepler, faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President - Stephanie Thomas
15 actives

A get-acquainted party was held in Room 141 of the Sid Richardson Science Center. The purpose and activities of the society were explained to prospective members and recently inducted members received their shingles. The speaker for the evening was Dr. James Bradford, Professor of Mathematics at Abilene Christian University in Abilene. Dr. Bradford spoke on current job opportunities in the field of mathematics. Other 1986-87 officers: John Dailey, vice president; Mike Cagle, secretary and treasurer; Mary Wagner, corresponding secretary; Charles Robinson and Ed Hewett, faculty sponsors.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President - Ann Brandt
5 actives, 6 initiates

On November 22, 1986, the Chapter sponsored its annual Mathematics Contest for high school junior and senior young women. Seventy-five students participated in this event. Awards included a trophy for the winning school team and a $1000 renewable scholarship to Mount Mary College for the winning individual student. Wisconsin Alpha sponsored a fund raiser on November 20th to add to funds supporting Chapter members' attendance at the upcoming convention. Other 1986-87 officers: Michelle Wielebski, vice president

and treasurer; Ann Brandt, secretary; Sister Adrienne Eickman, corresponding secretary and faculty sponsor.

Wisconsin Beta, University of Wisconsin - River Falls, River Falls
Chapter President - Thomas R. Scott
40 actives

We have had officers' meetings, full meetings, a Halloween Party, and we have put on a presentation for local high school seniors at the annual University of Wisconsin-River Falls Math/Science Day. Other 1986-87 officers: Thomas Weber, vice president; Christine Farwick, secretary; Sarah Flood, treasurer; Lyle Oleson, corresponding secretary; Don Leake, faculty sponsor.

Wisconsin Gamma, University of Wisconsin - Eau Claire, Eau Claire
Chapter President - Susan Hatlen
42 actives, 15 initiates

Fall semester in our KME Chapter has brought with it a major change of monthly to bimonthly meetings. This new policy has been very productive for our Chapter and has helped to create member interest. Our club has been fortunate enough to participate in a few fund raisers such as popcorn sales, cake sales, and lawn raking. These activities have not only raised a considerable amount of money but have been great fun for those of us involved. Finally, we are all looking forward to the coming of Thanksgiving, which brings with it our annual KME dinner involving both faculty and students. Other 1986-87 officers: Lisa Swerman, vice president; Sarah Sass, secretary; Jim Fischer, treasurer; Thomas Wineinger, corresponding secretary; Robert Langer, faculty sponsor.

REPORT ON THE TWENTY-SIXTH BIENNIAL CONVENTION

The Twenty-sixth Biennial Convention of Kappa Mu Epsilon was held April 2-4, 1987 on the campus of California Polytechnic State University, San Luis Obispo, California, with California Gamma the host chapter.

On Thursday afternoon and evening, April 2, California Gamma member hosts and hostesses met the arriving delegates. Following registration in the conference room of the Sands Motel there was a mixer for delegates at the Sands. The National Council and the Regional Directors met in Room 219, Julian A. McPhee University Union.

On Friday morning, April 3, registration continued in the lobby of the Julian A. McPhee University Union. The first general session was held in Chumash Auditorium of the Julian A. McPhee University Union, commencing at 8:32 a.m. with James L. Smith of Ohio Zeta, National President, presiding. Dr. Warren J. Baker, President of California Polytechnic State University, gave an address of welcome and Harold L. Thomas of Kansas Alpha, National President-Elect responded for the Society. Kevin Swanson, President of Associated Students Incorporated, California Polytechnic State University, presented a greeting to the delegates.

A roll call of the chapters was made by George R. Mach of California Gamma, National Secretary. 36 chapters and about 210 members were in attendance. Travel vouchers were filed and delegate voting cards were issued. The following new chapters installed during the 1985-87 biennium were recognized: Colorado Gamma at Fort Lewis College, installed on March 29, 1985, and Nebraska Delta at Nebraska Wesleyan University, installed on April 18, 1986.

James L. Smith of Ohio Zeta, National President, announced that chapter petitions had been endorsed by the National Council from the following:

St. Thomas Acquinas College, Sparkill, NY
Ursinus College, Collegeville, PA
Liberty University, Lynchburg, VA
McMurry College, Abilene, TX.

Data Sheets prepared from each petition were distributed to the voting delegates. It was announced that the delegates would vote on these petitions at the Second Business Meeting in accordance with Article VI, Section 1 of the constitution.

Ida Z. Arms of Pennsylvania Zeta, chairperson of the Nominating Committee, reported for the committee. The committee nominated: Robert L. Bailey of New York Eta and James E. Lightner of Maryland Beta for the office of National Secretary and Patrick J. Costello of Kentucky Alpha and Sister JoAnn Fellin, OSB of Kansas Gamma for the office of National Treasurer. The nominees were introduced and vita sheets were distributed to the voting delegates. Nominations were requested from the floor. There being none, nominations were closed.

Harold L. Thomas of Kansas Alpha, National President-Elect, presided during the presentation of the following student papers:

1) "Non-standard Analysis Using Infinitesimals," Jeremy Phillips, Iowa Alpha, University of Northern Iowa

2) "Some Investigations into the Primes of Generalized Integers," Rana Jones, Missouri Zeta, University of Missouri, Rolla

3) "Three Dimensional Hyperbolic Geometry: An Exposition," C. David Bishop, Iowa Alpha, University of Northern Iowa

4) "The Use of Computers to Solve Games of Logic," Joe Allen, Ohio Zeta, Muskingum College

5) "Can You Get to There from Here Without Crossing Wires?," Gayla Benson, Missouri Beta, Central Missouri State University

6) "Concept of Unique Factorization," Elizabeth Shettler, Pennsylvania Zeta, Indiana University of Pennsylvania

The convention reconvened at 1:35 p.m. in Chumash Auditorium of the Julian A. McPhee University Union. Harold L. Thomas of Kansas Alpha, National President-Elect, presided during the presentation of the following student papers:

7) "Coding Theory," Michael Lee Barlow, Ohio Zeta, Muskingum College

8) "Homothetic Proof of the Nine-Point Circle," Lori Baskins, Missouri Alpha, Southwest Missouri State University

9) "As the Water Swirls," Patrick J. Hirsch, Kansas Gamma, Benedictine College

At 2:45 PM a group picture was taken on the steps below the War Memorial Plaza. At 4:00 PM buses took the delegates to Morro Bay for a tour of the Embarcaderos and then to Cayucos for a beach party, barbeque, and dance at the Veterans Memorial Building at the pier. At 9:05 AM on Saturday a student section met in Chumash Auditorium of the Julian A. McPhee University Union with Jeanmarie Short, President of California Gamma, presiding. A faculty section met in Room 220 of the University Union with James L. Smith of Ohio Zeta, National President, presiding.

The convention reconvened at 10:10 AM in Chumash Auditorium, with Harold L. Thomas of Kansas Alpha, National President-Elect, presiding during the presentation of the following papers:

10) "An Application of Graph Theory," Michele Ann Lawyer, Maryland Beta, Western Maryland College

11) "Inward Spirals in Turtle Geometry," William W. Rademaker, Wisconsin Gamma, University of Wisconsin-Eau Claire

12) "Fractals: The Mandelbrot Set," Forrest Y. Tanaka, California Gamma, California Polytechnic State University, San Luis Obispo

13) "From Alice to Algebra," Kelly Eisenbarth, Kansas Delta, Washburn University of Topeka

14) "NP-Completeness and the Traveling Salesman Problem," Melanie K. Breaker, Missouri Eta, Northeast Missouri State University

At noon in the Chumash Auditorium of the Julian A. McPhee University Union the California Gamma Chapter presented its Eleventh Annual Mathematical Sciences Career Conference for the convention delegates. The conference, provided by California Gamma alumni and their employers, featured a sandwich and salad luncheon, a presentation "Practically Organized," by Amanda Scott Lawhern, IBM Southwest Marketing Division, and opportunities for the delegates to talk with representatives of the 17 sponsoring companies.

The convention reconvened at 2:30 p.m. in Chumash Auditorium of the Julian A. McPhee University Union. Harold L. Thomas of Kansas Alpha, National President-Elect, presided during the presentation of the following student papers:

15) "The Knight's Tour: The Mathematics Behind an Old Problem and Its New Implementations," Connie J. Garces, Ohio Zeta, Muskingum College

16) "Computational Complexity," Lisa D. Brox, Kansas Gamma, Benedictine College

17) "A Brief Introduction to the Principle of Inclusion and Exclusion," Erich Laurence Hauenstein, Michigan Beta, Central Michigan University (Graduate Paper)

18) "Old and New Results Concerning Amicable Pairs," Timothy J. Melvin, Kentucky Alpha, Eastern Kentucky University (Graduate Paper)

The second business meeting was held at 4:00 p.m. in Chumash Auditorium of the Julian A. McPhee University Union with James L. Smith of Ohio Zeta, National President, presiding. The delegates voted to establish new chapters at:

St. Thomas Acquinas College, Sparkill, NY
Ursinus College, Collegeville, PA
Liberty University, Lynchburg, VA
McMurry College, Abilene, TX

The delegates adopted the following resolution:

"Recognizing that the voting body of the National Convention of Kappa Mu Epsilon has the ultimate responsibility for maintaining the health of the organization, and

Acknowledging that many important decisions, which are the responsibility of the national convention, cannot be made without having adequate information available to all delegates on the status of the organization in financial and other matters,

the Twenty-sixth Biennial Convention of Kappa Mu Epsilon requests that a copy of the reports of each of the national officers be distributed to all voting delegates before the opening of business at all subsequent conventions."

The following national officers presented reports (copies attached):

Business Manager, THE PENTAGON	- Gerald White, Illinois Eta
Editor, THE PENTAGON	- Kent Harris, Illinois Eta
National Historian	- M. Michael Awad, Missouri Alpha
National Treasurer	- Sister Nona Mary Allard, Illinois Zeta
National Secretary	- George R. Mach, California Gamma
National President-Elect	- Harold L. Thomas, Kansas Alpha
National President	- James L. Smith, Ohio Zeta

The election of a National Secretary and a National Treasurer was conducted by Ida Z. Arms of Pennsylvania Zeta, chairperson of the Nominating Committee.

James L. Smith of Ohio Zeta, National President, announced that the National Council had donated $100 on behalf of Kappa Mu Epsilon to the Sister Mary Petronia Van Straten Scholarship Fund at Mount Mary College and he presented the check to Sister Adrienne Eickman of Wisconsin Alpha.

Patrick Costello of Kentucky Alpha, chairperson of the Auditing Committee reported that the National Treasurer's records were found to be accurate.

Janey McGuire of California Gamma reported for the student section meeting. Ronald Wasserstein of Kansas Delta reported for the faulty section meeting.

Invitations to host the Twenty-seventh Biennial Convention in 1989 were extended by: Missouri Alpha, Southwest Missouri State University; Kansas Delta, Washburn University; and Alabama Beta, University of North Alabama.

Convention evaluation forms were distributed to the delegates and collected by the host chapters. Copies of the reports of all the national officers were made available to the delegates.

At 7:30 p.m. the convention banquet was held in Chumash Auditorium of the Julian A. McPhee University Union with Jeanmarie Short, President of California Gamma, as mistress of ceremonies. A musical interlude was provided by the Cal Poly Brass Quintet. An address, "A Simple Recipe," was given by Donald Gibson, Hewlett-Packard Co.

Following the address, the third business meeting was held at 10:12 p.m. with James L. Smith of Ohio Zeta, National President, presiding.

Carol Harrison of Pennsylvania Theta, chairperson of the Awards Committee, reported for the committee. Certificates of participation were presented to all 18 student speakers. The following student paper awards were announced and presented:

First Place	($60)	- Lori Baskins, Missouri Alpha
Second Place	($40)	- William W. Rademaker, Wisconsin Gamma
Third Place	($30)	- Kelly Eisenbarth, Kansas Delta
Fourth Place	($20)	- Forrest Y. Tanaka, California Gamma.

Ida Z. Arms of Pennsylvania Zeta, chairperson of the Nominating Committee, announced that the following officers were elected for the next four years, 1987-1991:

National Secretary - Robert L. Bailey, New York Eta
National Treasurer - Sister JoAnn Fellin, Kansas Gamma.

James L. Smith of Ohio Zeta, National President, installed the newly elected officers.

Sister JoAnn Fellin of Kansas Gamma, chairperson of the Resolutions Committee, reported for the committee. The following resolutions were adopted:

"Whereas Kappa Mu Epsilon held its 26th Biennial Convention on the campus of California Polytechnic State University in San Luis Obispo, California after being so graciously invited to the west at many previous conventions and whereas this convention has been highly valued by all in attendance, be it resolved that we the participants of this 26th Biennial Convention of Kappa Mu Epsilon express our gratitude:

1. to Warren Baker, President of California State Polytechnic State University for his welcome and to the university for the use of its facilities,

2. to California Gamma officers, convention committees, members and alumni of the host chapter who rendered invaluable services to the national officers and visiting chapters,

3. to California Gamma for providing a distinctive flavor for this 26th convention - gracious hospitality, detailed planning, and natural beauty,

4. to Amanda Scott Lawhern of IBM Southwest Marketing Division for her practical presentation on careers and to Donald Gibson of Hewlett-Packard Co. who cooked up a good deal of interest with his address "A Simple Recipe."

Whereas two of our national officers are terminating their official duties at this convention after giving so freely of their time and gifts in the continuation of the activities of Kappa Mu Epsilon, be it resolved that we the participants of this 26th Biennial Convention of Kappa Mu Epsilon express our appreciation:

1. to George R. Mach, California Gamma, for his meticulous service as secretary over the past ten years,

2. to Sister Nona Mary Allard, Illinois Zeta, for her diligent keeping of the accounts during the past four years.

Be it further resolved that we recognize the ongoing services of our membership. Our thanks:

1. to James L. Smith, president; Harold L. Thomas, president-elect; M. Michael Awad, historian; Kent Harris, PENTAGON editor; and Gerald L. White, PENTAGON business manager,

2. to the faculty and students who served with distinction on the nominating committee, selection committee, awards committee, and audit committee,

3. to the twenty-four students who submitted papers to the selection committee and the eighteen students who presented papers to the convention."

On behalf of the society, James E. Lightner of Maryland Beta presented a plaque to George R. Mach of California Gamma which contained the following inscription:

PRESENTED TO

GEORGE R.MACH

IN APPRECIATION FOR
21 YEARS OF SERVICE ON
THE NATIONAL COUNCIL

NATIONAL VICE-PRESIDENT	1966-1969
NATIONAL PRESIDENT	1969-1973
NATIONAL PAST PRESIDENT	1973-1977
NATIONAL SECRETARY	1977-1987

26TH BIENNIAL CONVENTION

KAPPA MU EPSILON

APRIL 4, 1987

Dr. Lightner then read the following two citations:

CITATION FOR GEORGE R. MACH
KAPPA MU EPSILON BANQUET
APRIL 4, 1987

At this gala banquet which concludes the Twenty-Sixth Biennial Convention (and the first ever held in California), the National Council wishes to recognize the outstanding service of one of Kappa Mu Epsilon's dedicated members and officers, without whose efforts our Society would surely not be meeting here in California tonight. But this convention is merely a fitting capstone of twenty-one years of devoted service to the Society at the national level, twenty-nine years at the local chapter level as Corresponding Secretary, and thirty-nine years as a member. Three others during our fifty-six year history have served as national officers for slightly longer periods of time, but none has served in such varied capacities and with such wide-ranging responsibilities.

He became a national officer in 1966 when he became National Vice President. After three years in the post, he was elected in 1969 to the office of National President, a role he performed with distinction for four years (the maximum allowed by our Constitution). Then for the next four years, he played the role of "elder statesman", remaining on the Council as Past-President. After eleven years as a Council member, he probably was quite ready to relax a bit and not worry about Kappa Mu Epsilon quite so much, but he was prevailed upon to assume the demanding office of Secretary which he willingly did in 1977, being re-elected in 1979 and 1983. He has now completed ten years of service in this office, serving the two full terms our new Constitution will allow anyone to inflict upon themselves, and so at this convention he steps off the National Council for the first time since 1966. This means that either as President or as Secretary, he has signed about 17000 membership certificates(fortunately not all individually, or he would have severe writer's cramp!).

Our honoree will long be remembered for his sincere dedication to the Society and its ideals, for his meticulous attention to detail and accuracy and organization in whatever office he held, and for his never-failing concern for his students, especially those he often personally transported to national conventions around the country since he helped to establish the chapter here at California Polytechnic State University in 1958. He has worked tirelessly here in his own chapter to make it one of our largest and most active, one which so graciously and efficiently hosted this convention.

This native Iowan, who as an undergraduate was inducted into Kappa Mu Epsilon in the Iowa Alpha Chapter at Iowa State Teachers College, Cedar Falls, as member #181 on February 11, 1948, has spent most of his professional career in California as a dedicated professor of mathematics and as a loyal and articulate spokesman for Kappa Mu Epsilon. He has served his profession well and our Society superbly, and it is for this that we express our sincere appreciation tonight as we honor him on his retirement from the Council and as Corresponding Secretary of the California Gamma Chapter.

In recognition of his exemplary service on our behalf for these past twenty-one years, the National Council (in a move WITHOUT his knowledge or consent!) has established a special award in his honor:

THE GEORGE R. MACH DISTINGUISHED SERVICE AWARD

for exemplary service to Kappa Mu Epsilon, which will be awarded each biennium to a person who has made major contributions to the Society.

We wish him the very best for the future and thank him for bequeathing to his successors a strong, well-organized, vital Society which will continue to honor students of mathematics for decades to come. God speed, George.

Citations for Recipients of
The George R. Mach Distinguished Service Award

The inagural recipients of the George R. Mach Distinguished Service Award are two former national officers who devoted many years of exemplary service to our Society.

LAURA Z. GREEN

Miss Green began her service to Kappa Mu Epsilon nationally when she was elected Historian in 1951. Four years later she was elected Secretary and served in this demanding office for eighteen years until her retirement in 1973. A long-time faculty member in mathematics at Washburn University in Topeka, Kansas, Miss Green is remembered for her quiet and efficient manner of dealing with the many demands of her office. Her memory and record of prior Council policy and activity greatly assisted those in other offices, and her presence gave stability to the Society in a period of growth and change. Miss Green is now retired from professional service and still resides in Topeka. We take pleasure in recognizing her twenty-two years of service to Kappa Mu Epsilon by presenting to her the George R. Mach Distinguished Service Award.

WILBUR J. WAGGONER

When he was appointed Business Manager of THE PENTAGON in 1957, Dr. Waggoner never suspected that he would hold that office for twenty years. But this he did, most capably and efficiently, working closely with four different Editors to make our journal one of which we could be proud. His careful attention to the details of the ever-increasing mailing list and the ever-changing postal regulations was exemplary. Then, in 1977, at a time of crisis in our Society, he became Treasurer and worked most diligently to put our treasury and our financial procedures in sound order. His wise and thoughtful counsel was always sought in National Council deliberations, and his representation of our Society in the Association of College Honor Societies brought us recognition and stature. Dr. Waggoner is now semi-retired from professional service as Professor of Mathematics at Central Michigan University and still resides in Mount Pleasant, Michigan. We take pleasure in recognizing his twenty-six years of service to Kappa Mu Epsilon by presenting to him the George R. Mach Distinguished Service Award.

Travel allowances were paid to the delegates by Sister Nona Mary Allard of Illinois Zeta, National Treasurer. The convention adjourned at 10:44 p.m.

Respectfully submitted,

George R. Mach
National Secretary, 1977-1987

REPORT OF THE NATIONAL PRESIDENT
APRIL 4, 1987

Since my election as National President at the 25th Biennial Convention, our Society has added one new chapter and placed two chapters on the inactive list, making the total number of 99 active chapters. Added to the active chapter list is Nebraska DELTA (Nebraska Wesleyan Univ.), which was inducted into Kappa Mu Epsilon on April 18, 1986 with Sr. Jo Ann Fellin of Kansas GAMMA (Benedictive College) serving as the installing officer. The two chapters placed on the inactive list are Connecticut ALPHA (South Conn. State Col.) and Texas THETA (Southwest Texas State Univ.) At this meeting we are acting on petitions from four institutions, which could bring the total of active chapters to 103. There is one more petition in process, which could be handled by a mail vote as early as next fall. Twelve other institutions are currently considering the possibility of establishing a chapter of Kappa Myu Epsilon on their campuses.

The National Offices continue to support the regional organization. Please refer to the report by President-Elect Harold L. Thomas for the report of regional conventions. With much gratitude we recognize the steady work of our three Regional Directors whose terms expire with this convention: namely, Region II, J. Frederick Leetch, Ohio ALPHA (Bowling Green State University); Region IV, Homer Hampton, Missouri BETA (Central Missouri State Univ.); and Region VI, Adelaide Harmon-Elliott, California GAMMA (Calif. Polytechnic State Univ.). These people have served our Society well and deserve the thanks of each of us. The appointment of the next class of Regional Directors for Regions I,III, and V, James C. Pomfret, Thomas J. Sharp, and Wayne F. Hayes, respectively, will continue to serve for the next two years.

Almost without exception, everyone who was asked to serve on a convention committee willingly agreed to do so. This kind of response, both at convention time and throughout my tenure so far, certainly makes the job of being your President much easier. Special thanks is extended by all of us to each of the corresponding secretaries and faculty members who encouraged and assisted your students in the preparation of the excellent papers we have on the convention program this year. Even more so, we all express our gratitude to each of the students who did the work, endured the

stress, and prevailed in the submitting and presenting of your papers to this convention. Without the student papers, the major focal point of the convention does not exist. Thank you to each !!. We are further indebted to the diligent and super people who did all of the committee work necessary to bring this convention to fruition, and to staff the committees at the convention. To all of you at California GAMMA (organized by Adelaide Harmon-Elliott and George Mach), and to each convention committee member, please accept our most sincere thanks for jobs well done.

During the past biennium, I have been increasingly active on your behalf in the affairs of the Association of College Honor Societes (ACHS). At the most recent meeting, Feb. 1987, I chaired one of the discussion sessions as part of the program. The following few notes have been stimualted by ACHS attendance, and provide some ideas upon which we may wish to move during the next biennium: Regional Directors as members of the National Council; fellowship/scholarhip programs; endowed fund; correspond with members annually establish national office; send President-Elect with President to last ACHS meeting of current President's tenure; continue active participation in ACHS; use list of chapter officers w/addresses and communicate with students directly; monthly newsletter on campus produced by students; for every 15 members, offer a free membership (as scholarship) if someone is financially strapped, students choose outstanding faculty member, honor in December, and that person becomes more active, and subsequently invite to be table host at initiation banquet; apple-polisher social using apple cider and apple pie and to which invite favorite professor; we are honor society, so whatever we do, do it well; start math club for all interested in mathematics and let KME be part of it; send out letter to parents and give parents the opportunity to pay the 1st year dues; give grandparents opportunity to buy KME pin; for KME as whole, institute a best chapter award; need dedicated faculty advisor; perhaps establish chapter at large for people who not have active chapter on their campus; increase interest by conducting meaningful initiation ceremonies; with extra KME funds give A MATTER OF HONOR to each undergraduate member; Sunday supper for old and new officers when trade off responsibilities; and be watchful of our GNP - Growing New People. These notes are provided with the intent of stimulating discussion on the local and national level, but do not represent

any policies already in place. There are three kinds of people: those who make things happen, those who watch things happen, and those who wonder what happened. Let's put ourselves in the first category and help Kappa Myu Epsilon move forward towards being an even better and strong National Honor Society.

Before closing this report, we all recognize the fantastic job which is being done by those who work with, manage, write for, and produce our publication, THE PENTAGON. We are most appreciative of the editorial leadership of Kent Harris, Illinois ETA and the sound business management given to us by Gerald White, Illinois ETA (Western Illinois Univ.). We thank these two, and all the people staffing the low profile jobs who truly give us a journal which is respected nationwide by our professional mathematics community.

Finally, we are having a transition in National officers at this meeting as we elect a new National Secretary and a new National Treasurer. We have special indebtedness to the very capable and conscientious persons who permitted themselves to be nominated for positions of national leadership. Be assured that whoever is elected, Kappa Mu Epsilon will continue in strength because of the elections. Sr. Nona Mary Allard, as our out-going National Treasurer, has done a most conscientious job (I know! - we have corresponded almost weekly over the past two years). Sr. Nona, thanks tons!! We wish you all the best in your new responsibilities which will take you to Strasbourg, France.

For George R. Mach, our out-going National Secretary, along with our deepest thanks, perhaps the best way we can continue to honor you is to strive ever so persistently to take the baton you have firmly given us and carry it forward with continued efforts towards excellence, quality, and dignity. Thank you George for the leadership, dedication, and the part of your life which you (and your wife through you) have shared and given to Kappa Mu Epsilon!!

Respectfully submitted,

James L. Smith

REPORT OF THE PRESIDENT-ELECT

One of the responsibilities of the President-Elect is to serve as coordinator of regional activities of the Society through the regional directors. During the spring of 1986, there were four regional conventions held in:

Region I at Maryland Delta, Frostburg State College, April 25-26, James Pomfret, Regional Director.

Region II at Ohio Zeta, Muskingum College, March 14-15, J. Frederick Leetch, Regional Director.

Region IV at Missouri Kappa, Drury College and Missouri Theta, Evangel College, March 14, Homer Hampton, Regional Director.

Region IV at Nebraska Beta, Kearney State College, April 11-12, Homer Hampton, Regional Director.

This is the first time that two regional conventions have been held in the same region. Because of travel distances involved, it proved to be quite successful. Programs at the regional conventions included student papers, guest talks, and good social times. We extend our sincere thanks to the host chapters, regional directors, and all who participated in this regional activity. We also appreciate the efforts of the other Regional Directors, Tom Sharp, Region III, Wayne Hayes, Region V, and Adelaide Harmon-Elliott, Region VI, in attempting to have regional conventions in their regions.

It is another of the President-Elect's responsibilities to make arrangements for the presentation of student papers at the National Convention. I am pleased to report that twenty-four students, representing sixteen chapters and eleven states, submitted papers for this convention. Nineteen papers were written by undergraduates and five by graduate students. Sixteen undergraduate papers and two graduate papers are being presented at this convention. On behalf of our entire Society, I want to extend special thanks to the members of the Paper Selection Committee who read and ranked the papers: Professor Alan Bishop (Illinois Eta), Professor Richard Gibbs (Colorado Gamma), and Professor Arnold Hammel (Michigan Beta). In addition, I want to express our sincere thanks to the twenty-four students who prepared and submitted papers. It is these papers and the work of the Selection Committee which are the most important components in having a successful convention.

Respectfully submitted,
Harold L. Thomas
President-Elect

REPORT OF THE NATIONAL SECRETARY
APRIL 4, 1987

During the last biennium two new chapters of Kappa Mu Epsilon were installed. They are: Colorado Gamma at Fort Lewis College, installed on March 29, 1985, and Nebraska Delta at Nebraska Wesleyan University on April 18, 1986. On September 18, 1986 the National Council declared the following chapters to be inactive: Connecticut Alpha at Southern Connecticut State College and Texas Theta at Southwest Texas State University. The Society now has 99 active chapters in 30 states.

During the last biennium 2,679 members were initiated. The 99 active chapters have a combined membership of 43,065 and the 28 inactive chapters have a combined membership of 6,314, making the total membership of Kappa Mu Epsilon 49,379 at the end of the biennium.

As National Secretary, I maintain permanent files on all active and inactive chapters, including reports of all initiations. I order membership certificates for all new members and I stock all supplies, including forms, invitations, and jewelry. I assist corresponding secretaries in any ways that I can and I take minutes of National Council meetings and Biennial Conventions.

George R. Mach, Secretary

FINANCIAL REPORT OF THE NATIONAL TREASURER

Biennium: March 21, 1985 through March 10, 1987

Receipts

1. Cash on hand March 21, 1985				$37,184.70
2. Receipts from Chapters				
Initiates (2679)	40,185.00			
Jewelry	1,055.25			
Supplies	303.75			
			41,544.00	
3. Miscellaneous Receipts				
Interest	4,068.95			
Chapter Installations	85.96			
Chapter Petition	240.00			
Refunds & Overpayments	319.66			
			4,714.57	
4. Total Receipts				46,258.57
5. Total Receipts plus cash on hand				83,443.27

Expenditures

6. National Officers Expense			240.46	
7. Jewelry (Pollack)			754.47	
8. Printing (Herff-Jones)			7,780.30	
9. Pentagon (4 issues)			11,176.11	
10. Biennial Convention - 1985 and Regional Conventions - 1986			9,420.17	
11. Association of College Honor Societies			974.67	
12. Travel			4,607.34	
13. Miscellaneous				
Refunds	70.80			
Chapter Installations	77.24			
National Council Meeting	576.27			
Postage	259.00			
Plaques	127.94			
14. Total Expenditures				36,064.77
15. Cash on hand - March 10, 1987				47,378.50
Proof of cash: In checking		20,411.44		
In money market account		26,967.06		
		47,378.50		

REPORT OF THE NATIONAL HISTORIAN
1985-1987

The files of the National Historian are being maintained and continually updated with the reports received from chapters about their events and activities; with information received from Regional Directors about regional conventions and items of interest related to the region; and with material received from the National Officers, which has historical significance.

News items have been solicited from corresponding secretaries semi-annually, in January and May. The responses are then edited for publication in the Chapter News section of The Pentagon.

During the past biennium, 77 of the active chapters responded at least once to the chapter news request. Special mention goes to the following 25 chapters for their cooperation in responding to all four inquiries: CA Gamma, CO Gamma, IL Zeta, IN Alpha, IA Alpha, KS Alpha, KS Gamma, KS Delta, KS Epsilon, KY Alpha, MD Delta, MO Alpha, MO Beta, MO Gamma, MO Epsilon, NE Alpha, NE Gamma, OH Zeta, PA Beta, PA Delta, PA Zeta, PA Kappa, TX Eta, WI Alpha, WI Beta. I urge chapters to reply to the requests for chapter news even if it is only to identify chapter officers. This will provide chapters with a permanent record of their local officers in the event they do not retain that information within their own chapter.

I want to extend thanks to all with whom I have corresponded relative to this office - the National Officers, the Regional Directors, the Editor of The Pentagon, Corresponding Secretaries, and individual KME members. It has been a pleasure to serve as your historian for this past biennium.

M. Michael Awad

REPORT OF THE EDITOR OF THE PENTAGON

Since the last national convention, eleven student papers and five faculty papers have been published in the PENTAGON. The student papers were submitted through the national and regional meetings or were submitted to one of the editors. The faculty papers were submitted to the editor, to James Calhoun who edits the Cursor section, or to Iraj Kalantari who edits the Hexagon section.

We have a shortage of papers available to publish in these latter two sections and urge both students and faculty to submit papers of interest, to the appropriate editor. As a reminder, the Hexagon covers a wide spectrum, including books, puzzles, paradoxes, history, and many special topics. The Cursor, beginning its third year, is intended to explore the relationships between mathematics and computer science. We earnestly solicit your work for these two sections of the PENTAGON.

My thanks go to the associate editors of the PENTAGON; Kenneth Wilke, Michael Awad, Iraj Kalantari, and James Calhoun, as well as to the Business Manager, Gerald White. Their efforts are very much appreciated.

Kent Harris

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON

APRIL, 1987

After several years of dedicated service, Douglas W. Nance from the Mathematics Department at Central Michigan University retired from the position of Business Manager of THE PENTAGON. As he was outlining for me the tasks that lay ahead, he indicated that there was one responsibility that made it all worthwhile -- being a part of the Biennial Conventions. It is with pleasure that I make my first report as Business Manager at this 26th Biennial Convention.

My primary responsibilities are to arrange for printing and to see that THE PENTAGON is mailed out to members of Kappa Mu Epsilion for the two initial years of membership and to those who wish to continue subscriptions. The goal is to have issues mailed in December and May of each academic year. During this past biennium the mailings averaged 3030 copies. The mailing list includes subscribers in forty-three states and twenty-six foreign countries. States receiving the largest number of copies are, in descending order, Pennsylvania, Missouri, Illinois and Ohio.

The cost of printing in 1984-85 was just over $1 per copy. Some savings were available by using the lowest bid from printing companies in Illinois near our office. However, making use of the printing services on the campus of Western Illinois University produced a savings of over $800 more for the year. When the printing service adjusted procedures based on last year's experience, the cost for Fall of 1986 was less than $.50 per copy. However, turn around time on campus has proven to be a problem as this type of work is given lowest priority. In particular, the Editor established a submission deadline in early October and delivered the final form at the end of October, but printing was not completed until early February. The late mailing has increased the number of copies that do not reach members. After each mailing, about eighty copies are returned by the US Postal Service as undeliverable due to incorrect address or lack of forwarding address.

Please inform your chapter members that it is important that they provide us at THE PENTAGON with a current address. If it is convenient, a new member could indicate a home address rather than a school address since a school address is more likely to be changed. Any subscriber that does not receive their copy should contact the Business Manager.

Complimentary copies of THE PENTAGON are sent to the library of each college or university with an active chapter of Kappa Mu Epsilon. Anyone contributing an article for an issue will receive a free copy. Speakers at this 26th Biennial Convention will have their subscriptions extended for two years.

I would like to thank Kent Harris, Editor of THE PENTAGON, who is responsible for getting me assigned to this position, James L. Smith, President; George R. Mach, Secretary; and Sister Nona Mary Allard, Treasurer, whose cooperation have made things move smoothly. The Mathematics Department at Western Illinois University has also been supportive and I would like to thank Debra Vorderer, student assistant, who does such a great job of keeping the everyday details so well organized.

Gerald White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BIENNIAL
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APRIL 1987

IF YOUR SUBSCRIPTION HAS EXPIRED

We hope you have found THE PENTAGON both interesting and helpful. Your suggestions are always welcome and may be written on this form. They will be forwarded to the Editor.

If you wish to renew your subscription for two years, please send $5* to THE PENTAGON, Department of Mathematics, Western Illinois University, Macomb, Illinois 61455.

Please give your name below in the manner in which you wish your mail addressed (Miss, Mrs., Prof., etc.). If your name has changed, please indicate also the name to which THE PENTAGON was sent.

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