THE PENTAGON

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NATIONAL OFFICERS

Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics; due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

ON THE IRRATIONALITY OF E

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Aristotle and other early Greek mathematicians proved that $\sqrt{2}$ was not a rational number; Book X of Euclid's Elements is concerned with proofs of the irrationality of other square roots of primes.

In 1737 Euler showed that the number e is irrational. In 1761 Lambert showed that π is irrational. In 1873 Hermite showed that e was not only irrational but transcendental as well. That is, not only is e not the root of a linear equation with integral coefficients, but also e is not the root of any polynomial equation with integral coefficients. In 1882 Lindemann showed that π was transcendental. [P. 602, Boyer]

While the proof of the irrationality of $\sqrt{2}$ is simple enough to be given in many pre-calculus textbooks, the proofs of the irrationality of e and π are considered too advanced for such books.

However, in a book called "Introduction to Analysis" by Bevan Youse, 1972, it is suggested that the irrationality of e can be proved by elementary methods, and the following problem is posed as a project problem:

Let e be given by the series expansion $\sum_{k=0}^{\infty} \frac{1}{k!}$ and define the partial sum $S_n = \sum_{k=0}^{n} \frac{1}{k!}$. Using these prove that e is irrational.

This presentation is my solution to the problem as posed by Dr. Youse.

Let $e = \sum_{k=0}^{\infty} \frac{1}{k!}$

Let $S_n = \sum_{k=0}^{n} \frac{1}{k!}$

Assume that $e = \frac{p}{q}$ where p and q are positive integers whose greatest common divisor is 1. (That is, e is a quotient of two whole numbers reduced to lowest terms.)

1) $e - S_n > 0$.

For $e-S_n = \sum_{k=0}^{\infty} \frac{1}{k!} - \sum_{k=0}^{n} \frac{1}{k!} = \sum_{k=n+1}^{\infty} \frac{1}{k!}$

$$= \frac{1}{(n+1)!} + \frac{1}{(n+2)!} + \frac{1}{(n+3)!} + \ldots > 0.$$

2) The series $\frac{n}{n+1} + \frac{n}{(n+1)(n+2)} + \frac{n}{(n+1)(n+2)(n+3)} + \ldots$

is the term less than the series

$$\frac{n}{n+1} + \frac{n}{(n+1)^2} + \frac{n}{(n+1)^3} + \ldots$$

The latter series is a geometric series with

$a = \frac{n}{n+1}$ and $r = \frac{1}{n+1}$

Thus, it converges, and its sum is

$$\frac{\frac{n}{n+1}}{1 - \frac{1}{n+1}} = \frac{\frac{n}{n+1}}{\frac{q}{n+1}} \ldots = 1$$

3) Therefore, for any positive integer n,

$$\frac{n}{n+1} + \frac{n}{(n+1)(n+2)} + \frac{n}{(n+1)(n+2)(n+3)} + \ldots < 1$$

4) $\frac{1}{n!n} \left(\frac{n}{n+1} + \frac{n}{(n+1)(n+2)} + \frac{n}{(n+1)(n+2)(n+3)} + \ldots \right) < \frac{1}{n!n}$

Hence $\frac{1}{(n+1)!} + \frac{1}{(n+2)!} + \frac{1}{(n+3)!} + \ldots < \frac{1}{n!n}$

That is, $e - S_n < \frac{1}{n!n}$ for all positive integers n.

5) Now recall the assumption that $e = \frac{p}{q}$ where $(p,q)=1$

6) Since $e - S_n < \frac{1}{n!n}$ for all positive integers n, $e - S_q < \frac{1}{q!q}$

7) We have then: $0 < e - S_q < \frac{1}{q!q}$

8) $0 < q!\,(e-S_q) < \frac{1}{q} < 1$

9) But $q!e = \frac{q!p}{q} = (q-1)!p$, the product of two positive integers

10) And $q!S_q = q! \sum_{k=0}^{q} \frac{1}{k!} = q!\,(1+1+\frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{q!})$

As all denominators in the factor on the right divide q!, this product $q!\,(1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{q!})$ is a sum of positive integers.

11) Therefore, both q!e and $q!S_q$ are positive integers. So $0 < q!\,(e-S_q)$ and the right member is a positive integer Z.

12) From 8) above, $0 < Z < 1$. There is an integer Z between 0 and 1.

13) This is a contradiction to a known fact that there is no integer between 0 and 1.

14) Conclusion: the assumption that e is rational is false.

15) Therefore, e is an irrational number.

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KNOT THEORY WITH TOPOLOGY

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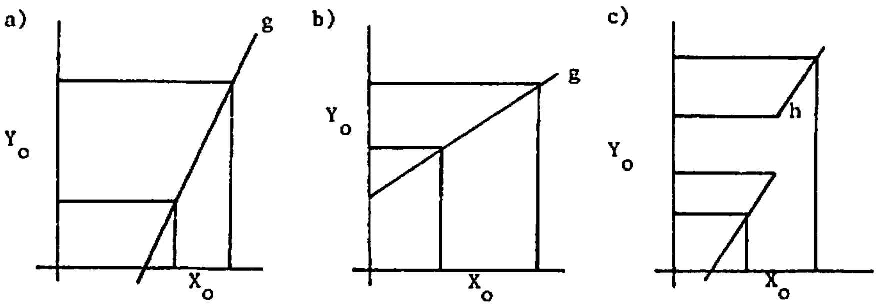
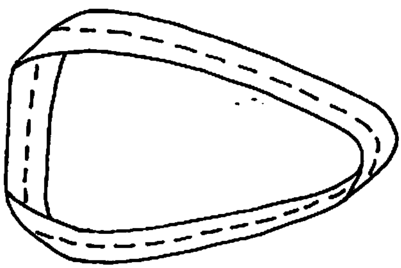
Topology deals with those properties of spaces that remain unchanged no matter how the spaces are deformed, whether they are stretched, pushed, twisted, or bent. So topologists are not interested in the exact shape of a space. Rather, they are interested in the general form of a space. One space is the same as another if all the points in one can be corresponded to points in the other without tearing.



a) g stretches the set X_o into the set Y_o, b) g compresses the set X_o into the set Y_o, and c) h tears the set X_o into the two pieces of the set Y_o.

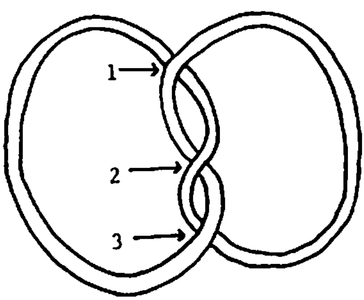
Topological properties are geometrical properties which remain the same in spite of stretching or bending. These properties are called invariants. When a space is badly stretched out of shape, it can be difficult to recognize. Some invariants topologists look at are the number of edges the surface has, the number of sides the object has, and the surface's Betti number. The Betti number is the maximum number of cross cuts that can be made on a surface without dividing it into more than one piece. For example, the Moebius band has only one edge and only one side. If



the band is cut along the dotted line, there will still be only one loop. If it is cut a second time in the same manner, two loops will appear. Therefore, the Betti number of a Moebius strip is one. The Betti number is also called the genus of a surface and will be discussed again later. These three invariants will remain the same as long as the surface is not torn into two or more pieces when it is deformed.

Knot theory is an area of topology and topologists view know theory as an example of a place problem. An example of a place problem would be, "What are the different ways of embedding a circle in a 3-dimensional space?" Topologists look at a space and see how it reacts when different knots are put into it. The fundamental task of knot theory is proving that knots are distinct from one another.

Knots are closed curves embedded in 3-dimensional space. To determine whether two knots are similar, topologists look at the invariants of the knots. One invariant to look at is the minimum number of crossing points of a knot. For



example, the trefoil knot, an overhand knot with the ends joined, is the simplest of all knots with a minimum of three crossings.

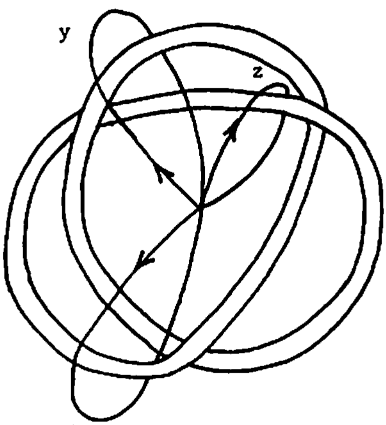
Sometimes it is difficult to determine the minimum number of crossings a knot has. So, topologists usually use another invariant, the knot group. A group is any set with an associative operation defined on its elements in such a way that there is an identity element and an inverse for every element in the set.

In order to prove that the associative property is defined, a definition of a class must be given. Homotopies are paths which can be made similar by deforming one of them by pulling, pushing, or crossing it over itself; however, the end point or beginning point may not be moved out of the complement of the space (the area of the space excluding the knot itself) or broken. Paths that differ by a homotopy are called homotopic. A class, denoted by $[\alpha]$, consists of paths which are homotopic to a particular path α . So, a group has the associative property defined on its elements,

$$([\alpha][\beta])[\lambda] = [\alpha]([\beta][\lambda]).$$

A trivial knot is a closed curve that can be manipulated so that it can be projected on a plane as a curve with no crossing points. In other words, the trivial knot is simply a circle. The trivial knot is denoted by "e". A group must also have an identity element and an inverse for every element in the set. The identity element is $[e]$, which satisfies $[\alpha][e] = [e][\alpha] = [\alpha]$. The inverse is $[\alpha]^{-1}$, i.e. $[\alpha][\alpha]^{-1} = [\alpha]^{-1}[\alpha] = [e]$. ($\alpha^{-1}$ is the path of α traversed in the opposite direction.)

The knot group can always be calculated. It is possible to construct a finite list of objects that will completely describe the group. This list is called a presentation of the group. It includes a list of generators, the number of group elements, relations, and the number of equations.



In the picture above, x,y, and z are homotopy classes. [x], [y], and [z] are generators of the knot group. The way generators are related at crossing points creates an equation called a relation of the knot group. For example, an equation which shows a relationship between the paths and the upper left crossing point in this picture would be $[y]^{-1}[x][y]=[z]$ or $[y]^{-1}[x][y][z]^{-1}=[e]$.

As stated previously, the task of knot theory is to show that knots are different from one another. Topologists have four different ways to classify a knot. First, look to see if it is amphicheiral or nonamphicheiral. If it is possible to manipulate a knot to get its mirror image, the knot is amphicheiral. The trefoil knot is nonamphicheiral. To get its mirror image a new knot must be made. The figure eight knot, the only knot with four crossings, is amphicheiral. Simply turn it over and the mirror image can be seen.

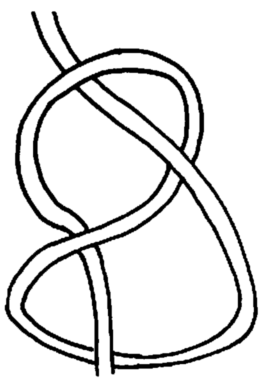
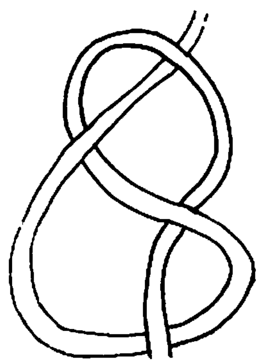


FIGURE EIGHT KNOT



MIRROR IMAGE

A second way to classify a knot is to determine whether it is alternating or nonalternating. To determine this, follow the curve in either direction and if it alternately goes under and over at a crossing point it is alternating.

Determining if a knot is prime or composite is a third way to distinguish knots. A knot is prime if it cannot be manipulated to make two or more separate knots. For example, a square knot is the product of two trefoil knots of opposite handedness. A granny knot is the product of two trefoils of the same handedness.

The last way to classify knots is to distinguish between invertible and noninvertible. If an arrow is painted on a knotted rope to give it direction and it is possible to manipulate that rope so that the structure remains the same but the arrow points in the opposite direction, the knot is invertible. So, topologists use different ways to distinguish knots, namely, whether they are amphicheiral or nonamphicheiral, alternating or nonalternating, prime or composite, or invertible or noninvertible.

Another invariant a topologist might use is the genus of the knot. The genus is determined by the number of nonintersecting closed or completely circular cuts that can be made on the surface without cutting it into two or more pieces. In other words, the genus of a surface is given according to the number of holes, or handles, it has. Surfaces with the same genus belong to the same group. Topologists are often described as not being able to tell the difference between a doughnut and a coffee cup. To a topologist, these objects are the same with respect to their surfaces. The surface of a doughnut can easily be transformed into the surface of a coffee cup without tearing it. Both surfaces have a genus of one.

As a brief summary, topology is the study of space shapes and the determination of which spaces are similar. One area of topology is knot theory. The main function of knot theory is in determining which knots are distinct from one another. Topologists accomplish this by the use of invariants, properties that remain the same in a space when it is deformed. Four invariants topologists look for in a knot are the states of being amphicheiral, alternating, prime, or invertible. A weak invariant of knots is the minimum number of crossings a knot has. So, the knot group is examined.

Topology is a growing branch of mathematics and there has been some recent research which has related topology to other fields. A lot of new discoveries lie ahead in the area of topology.

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FINDING POSITIVE INTEGRAL SOLUTIONS TO LINEAR DIOPHANTINE EQUATIONS

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INTRODUCTION

For thousands of years, mathematicians have intrigued themselves with the solving of a wide variety of puzzles. Many of these problems require more than merely finding the solutions of some equation. Some problems are worded in such a way that the solutions must satisfy certain conditions. For example, suppose that Mr. Smith is starting a small farm, and he buys a certain number of cows at \$70 each and pigs at \$50 each. If he wants to spend exactly \$530, how many cows and how many pigs can he buy? [3, p. 62]

Before attempting to write an equation to represent this problem, it is obvious that the solutions will be restricted to the set of positive integers. (That is assuming that Mr. Smith will buy only whole animals.). A problem such as this, in which the solutions are restricted to the set of positive integers, is called a Diophantine problem, and the resulting equation is a Diophantine equation. Sometimes the solutions of these equations may be expanded to include the set of rational numbers as well as integers.

Diophantine equations get their name from Diophantus, a Greek mathematician who lived during the third century A.D. His major work, entitled 'Arithmetica', contains numerous problems dealing with the solution of indeterminate equations in rational numbers and integers. Although Diophantus did not discuss linear equations, the term 'diophantine equation' today includes linear indeterminate equations as well as a variety of complex quadratic forms. There is no general method for solving all diophantine equations. However, the complete solution of any diophantine equation consists of two parts:

(1) Determining under what conditions the solution in integers is possible.

(2) Finding a method for determining all such integer solutions.

The linear diophantine equation in two unknowns has the general form,

$$ax + by = c,$$

where x, y are unknowns and a, b, c are given integers, such that $a \neq 0$ and $b \neq 0$. Also, let d be the greatest common divisor of a and b. Before attempting to solve this equation, there are several useful principles worth noting:

(1) If d is the greatest common divisor of a and b, then we can find integers x_1, y_1 such that $ax_1 + by_1 = d$. [8, p. 55]

(2) The equation $ax + by = c$ has at least one solution if and only if the greatest common divisor of a and b divides c. [8, p. 55]

(3) There exists at least one positive integral solution for the equation $ax + by = c$, if d divides c and $dc > ab$. [5, p. 106]

The basic method for solving a linear diophantine equation is to find one set of solutions x_0, y_0, from which a general solution may be formed. Having found x_0, y_0, satisfying $ax_0 + by_0 = c$, and given the general equation $ax + by = c$, then $ax_0 + by_0 = ax + by$, so:

$$ax + by - (ax_0 + by_0) = 0,$$

$$a(x - x_0) + b(y - y_0) = 0,$$

$$(a/d)(x - x_0) + (b/d)(y - y_0) = 0,$$

$$(a/d)(x - x_0) = -(b/d)(y - y_0).$$ [8, p. 56]

Since all solutions x, y and x_0, y_0 are required to be integers, then $y - y_0$ must be an integer. Thus $(a/d)(x - x_0)$ must be divisible by (b/d). In addition, (b/d) and (a/d) are relatively prime, so $x - x_0$ must be divisible by (b/d); thus $x - x_0 = (b/d)t$ or $x = x_0 + (b/d)t$, for some integer t. By a similar argument, $-(y - y_0)$ is divisible by (a/d) so that $y-y_0 = -(a/d)t$ or $y = y_0 - (a/d)t$. Thus, any integer solution x, y for $ax + by = c$ can be represented as:

$$x = x_0 + (b/d)t, \qquad y = y_0 - (a/d)t,$$

for t some arbitrary integer. These expressions give all possible positive and negative integral solutions for x and y.

However, if we want only positive solutions, as the Farmer Smith problem requires, then we must go one step further and restrict the values of t in such a way that x and y will be positive. Given the general solution $x = x_0 + (b/d)t$, we want $x > 0$, so:

$$0 < x_0 + (b/d)t \text{ or } t > -(d/b)x_0 \,.$$

Likewise, from the general solution $y = y_0-(a/d)t$ we want $y > 0$ so:

$$0 < y_0 - (a/d)t \text{ or } t < (d/a)y_0.$$

Therefore, in order to get only positive solutions, t is restricted to the range $-(d/b)x_0 < t < (d/a)y_0$. To find values for x_0 and y_0, we must first solve the equation $ax_1 + by_1 = d$.

USING THE EUCLIDEAN ALGORITHM

From principle (1), it is possible to find x_1, y_1 such that $ax_1 + by_1 = d$. The Euclidean algorithm provides a convenient method for finding x_1, y_1. For integers $a > 0$ and $b > 0$, the division algorithm is applied repeatedly to obtain a series of equations:

$$\begin{aligned} a &= bq_1 + r_1, \\ b &= r_1q_2 + r_2, \\ r_1 &= r_2q_3 + r_3, \\ &\cdots \\ r_{n-2} &= r_{n-1}q_n + r_n, \\ r_{n-1} &= r_nq_{n+1}. \quad [5, \text{p. } 7] \end{aligned}$$

From this series of equations we may determine the greatest common divisor of a, b to be r_n, and by working backward through this algorithm, we may obtain values for x_1, y_1, such that $d = r_n = ax_1 + by_1$.

Once numerical values have been found for x_1 and y_1, it is simple to find x_0, y_0 such that $ax_0 + by_0 = c$. The equation $ax_1 + by_1 = d$ may be multiplied by c to give $a(cx_1) + b(cy_1) = cd$. Then, dividing by d gives $a((c/d)x_1) + b((c/d)y_1) = c$, which is equivalent to $ax_0 + by_0 = c$ with $x_0 = (c/d)x_1$ and $y_0 = (c/d)y_1$.

Now we are prepared to solve Farmer Smith's problem. Letting w be the number of cows and p the number of pigs that Mr. Smith can purchase, then the equation to be solved is:

$$70\,w + 50\,p = 530, \text{ or equivalently, } 7w + 5p = 53.$$

Since the greatest common divisor of 7 and 5 is 1, which also divides 53, then we know, from principle (2) that this problem has at least one solution. In addition, since $53 \cdot 1 > 7 \cdot 5$ there is, according to principle (3), at least one positive integral solution.

Using the Euclidean algorithm with $a = 7$, and $b=5$ gives:

$$7 = 5 \cdot 1 + 2$$

$$5 = 2 \cdot 2 + 1$$

$$2 = 1 \cdot 2$$

and then working backward we get:

$$\begin{aligned} 1 &= 5 - 2\cdot 2 \\ &= 5 - 2(7 - 5\cdot 1) \\ &= 3\cdot 5 - 2\cdot 7. \end{aligned}$$

Thus $x_1 = -2$ and $y_1 = 3$. Subsequently, $x_0 = (53)(-2) = -106$ and $y_0 = (53)(3) = 159$, so that $w = -106 + 5t$ and $p = 159 - 7t$. Wanting only positive integer values for w and p, we restrict the value of t to the range $21 < t < 23$. Therefore, the only possible value for t is 22, which gives the acceptable solution $w = 4$, $p = 5$, so that Mr. Smith can purchase 4 cows and 5 pigs to start his farm.

SOLVING LINEAR EQUATIONS IN MORE THAN TWO UNKNOWNS

THE LINEAR EQUATION IN THREE UNKNOWNS

Suppose that Mr. Smith is now expanding his farm. He can buy a certain number of chickens at \$15 each, pigs at \$50 each, and cows at \$70 each. He wants to spend \$755. Assuming that he wants no more than 20 chickens, how many of each animal can Mr. Smith buy?

This problem leads to a linear indeterminate equation in three unknowns, which must be solved over the set of positive integers. (That is, assuming that Mr. Smith continues to buy only whole animals!). The basic goal in solving a linear equation in three unknowns is to reduce it to a linear equation in two unknowns, which is solvable by the previous process.

Consider the general equation in three unknowns:

$$ax + by + cz = e,$$

where x, y, z are unknowns, and a,b,c,e are given integers. Furthermore, a,b,c are all different from zero, since if any one of these three coefficients is zero the equation becomes one in two unknowns, and if any of these coefficients equal zero at the same time, then the solution is trivial. The principles governing the solution of such equations are similar to those for linear equations in two unknowns. Most importantly:

> The equation $ax + by + cz = e$ has at least one solution if and only if the greatest common divisor d of a, b, c also divides e. [5, p. 107]

To solve the equation $ax + by + cz = e$, we first define two of the variables, say y and z, each in terms of two parameters s and u, such that $y = As + Bu$ and $z = Cs + Du$, where A, B, C, D are integers to be determined later. Substituting these expressions for y and z into original equation gives the new equation:

$ax + b(As - Bu) + c(Cs + Du) = e$, or combining like terms:

$$ax + s(Ab + Cc) + u(Bb + Dc) = e.$$

Letting the coefficient of u be zero results in an equation in two unknowns, with $Bb + Dc = 0$. Note that $(-c)b + (b)c = 0$, and dividing this by g, the greatest common divisor of b and c, gives $(-c/g)b + (b/g)c = 0$. Thus, we choose values for B and D such that $B = (-c/g)$ and $D = (b/g)$.

Now, if we choose values for A and C such that $AD - BC = 1$, we will have $A(b/g) - (-c/g)C = 1$ or $Ab + Cc = g$, which gives the coefficient of variable s. Therefore, the original equation is expressible as $ax + gs = e$. This is a linear equation in two unknowns, which has general solutions $x = x_0 + (g/d)t$ and $s = s_0 - (a/d)t$ for x_0, s_0 a solution for $ax + gs = e$. Then, substituting into the definitions for y and z gives the results:

$$\begin{aligned} y &= As + Bu \\ &= A(s_0 - (a/d)t) + Bu \\ &= As_0 - (a/d)At + Bu \\ &= y_0 + Mt + Bu, \text{ where } y_0 = As_0 \\ &\qquad \text{and } M = -(a/d)A, \end{aligned}$$

$$\begin{aligned} \text{and}\quad z &= Cs + Du \\ &= C(s_0 - (a/d)t) + Du \\ &= Cs_0 - (a/d)Ct + Du \\ &= z_0 + Nt + Du, \text{ where } z_0 = Cs_0 \\ &\qquad\qquad \text{and } N = -(a/d)C. \end{aligned}$$

Thus, the general solution for a linear equation in three unknowns is the set of three equations in two unknowns:

$$\begin{aligned} x &= x_0 + Gt \text{ (where } G = (g/d)) \\ y &= y_0 + Mt + Bu \\ z &= z_0 + Nt + Du \end{aligned}$$

where t, u are arbitrary integers, G, M, B, N, D are all known integers, and x_0, y_0, z_0 are known integers which make up one solution for the equation $ax+by+cz=e$.

Now, we can solve Farmer Smith's problem. Let k be the number of chickens, p the number of pigs, and w the number of cows which Mr. Smith can buy. Then, the equation to be solved is $15k + 50p + 70w = 755$, with $a = 15$, $b = 50$, $c = 70$, $d = (a,b,c) = 5$, and $g = (b,c) = 10$.

Note first of all, that this equation has at least one solution, since $d = 5$ divides 755. To find the general solution, first let $p = As+Bu$ and $w = Cs+Du$.

Then, $B = -(c/g) = -7$ and $D = (b/g) = 5$. $Ab + Cc = g$ so $50A + 70C = 10$, an equation which may be solved, either by inspection or through the use of Euclidean's algorithm, to give $A = 3$ and $C = -2$. The original equation to be solved is now $15k + 10s = 755$. A solution k_0, s_0 for this equation may be found by inspection or by the process for solving linear equations in two unknowns. Choosing $k_0 = 25$ and $s_0 = 38$ gives the general solution $k = 25+2t$ and $s = 38-3t$. Then, substituting these solutions into the expressions for p and w gives the following solution:

$$p = 114 - 9t - 7u$$
$$w = -76 + 6t + 5u$$
$$k = 25 + 2t$$

Since Mr. Smith wants no more than 20 chickens, $k \le 20$, so $25 + 2t \le 20$ or $t \le (-5/2)$ and $k > 0$ so $t > (-25/2)$, that is t is restricted to integers in the range $(-25/2) < t \le (-5/2)$. Possible values for t may be substituted, one at a time, into the expression for k. Each resulting value of k may then be substituted into the original equation $15k + 50p + 70w = 755$, to give an equation in two unknowns. Apparently, there are a number of different combinations in which Mr.

Smith can buy his animals for $755. For example, two possible solutions are to buy 19 chickens, 8 pigs, and 1 cow, or to buy 5 chickens, 8 pigs, and 4 cows.

THE LINEAR EQUATION IN MORE THAN THREE UNKNOWNS

A linear equation in more than three unknowns has the general form:

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = c.$$

As with the other linear diophantine equations, if d is the greatest common divisor of $(a_1,a_2,\dots,a_n)$, then the equation has at least one solution if and only if d divides c. [5, p. 107] To solve such an equation, the method used with linear equations in three unknowns is repeated until it produces an equation in two unknowns. For the equation $a_1x_1+a_2x_2+\dots+a_{n-1}x_{n-1}+a_nx_n=c$, in n unknowns, x_n and x_{n-1} are defined in terms of parameters t and u to produce an equation in n-1 unknowns: $a_1x_1+a_2x_2 + \dots + (a_n,a_{n-1})t = c$, in which (a_n,a_{n-1}) is the greatest common divisor of a_n and a_{n-1}. This equation, likewise, can be reduced to an equation in n-2 unknowns. This process is continued until an equation in two unknowns is obtained.

For example, in the previous problem, if Mr. Smith had also wanted to purchase sheep at $45 each, then for s the number of sheep he can buy, the equation is $15k + 50p + 70w + 45s = 755$. Defining $w = At + Bu$ and $s = Ct + Du$ and choosing values $A = 2$, $B = -9$, $C = -3$, and $D = 14$ leads to the following equation in three unknowns: $15k + 50p + 5t = 755$. Then defining $t = Fv + Gx$ and $p = Mv + Nx$, and choosing values $F = -9$, $G = 10$, $M = 1$, and $N = -1$ produces the following equation in two unknowns: $15k + 5v = 755$. Now it is possible to find a solution k_0, v_0 for this equation, using the method for an equation in two unknowns. Letting $k_0 = 40$ and $v_0 = 31$ leads to a general solution: $k = 40 + y$ and $v = 31 - 3y$. From these expressions we can get solutions for the four unknowns:

$$k = 40 + y$$

$$p = 31 - 3y - x$$

$$w = -558 + 54y + 20x - 9u$$

$$s = 837 - 81y - 30x + 14u.$$

Thus, putting values for y into the expressions for k and then putting values for x into the expression for p, the resulting values can be substituted into the original equation to get a linear equation in two unknowns. One possible solution for this equation is

$k = 5$, $p = 4$, $w = 3$, and $s = 6$, meaning that Mr. Smith can buy 5 chickens, 4 pigs, 3 cows, and 6 sheep for $755.

A NOTE CONCERNING THE GENERAL SOLUTION

There is an interesting note to make about the general solution of any linear diophantine equation. As we have seen, the solution of a linear equation in two unknowns can be expressed as two equations with a single parameter t. Also, the solution of a linear equation in three unknowns can be expressed as three equations in two parameters, t and u. In general, the solution of a linear equation in n unknowns can be expressed as n equations in $n-1$ parameters. [5,p.108]

CONCLUDING REMARKS

Although the examples discussed here have been fairly simple, they serve to show a few of the methods by which linear diophantine equations can be solved. Certainly, these are not the only methods. For example, linear congruences often prove useful in solving linear diophantine equations. Nonetheless, it is obvious, that diophantine equations play a major part in number theory, and they continuously show up a wide variety of mathematical puzzles and games.

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THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 15 January 1987. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Spring, 1987 issue of **The Pentagon**, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 392: Proposed by the editor.

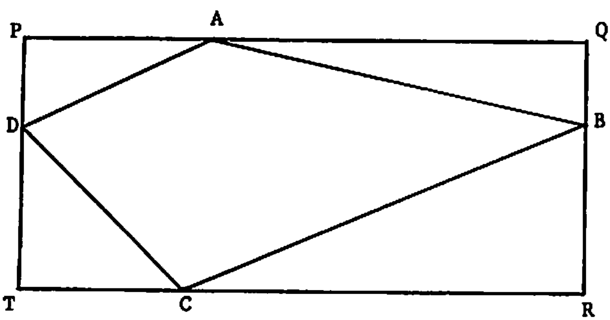
The integer n has exactly 25 divisors. If n can be expressed in the form $a^2 + b^2$ where a and b are relatively prime integers, what is the smallest value of n and what are the corresponding values of a and b ?

Problem 393: Proposed by Rich Hanlon, Ambler, Pennsylvania.

We define a two-tined fork as a pair of positive integers i,i separated by a distance of i units. We define a packing of size 2n as an interlocking arrangement of two-tined forks which has no gaps; e.g. 3 1 2 1 3 2 and 4 1 3 1 2 4 3 2 are packings of sizes 6 and 8 respectively. Note that in each case, there are exactly i digits between the two digits i. For what integers n can packings of size 2n exist?

Problem 394: Proposed by Dmitry P. Mavlo, Moscow, USSR.

Prove that the perimeter P_r of the rectangle PQRT which is circumscribed over an arbitrary convex quadrangle ABCD, as shown in the figure below, satisfies the inequality $P_r \leq P\sqrt{2}$ where P is the perimeter of the quadrangle ABCD and equality holds if and only if the quadrangle ABCD is a rectangle and PQRT is a square.



Problem 395: Proposed by Fred A. Miller, Elkins, West Virginia.

Given triangle ABC, draw a line which cuts two sides of the triangle so that the resuling quadrilateral has a common base with the triangle and also has its other three sides equal.

Problem 396: Proposed by the editor.

Which of the following expressions is larger?

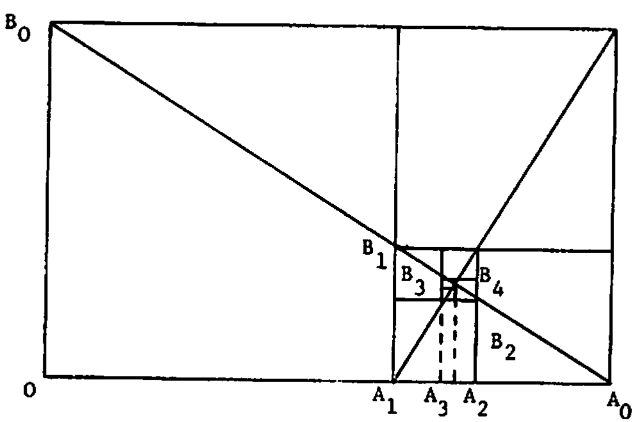
$$5 + \cfrac{5}{6 + \cfrac{5}{6 + \cdots}} \quad \text{or} \quad 5 + \cfrac{5 + \cfrac{5 + \cdots}{6}}{6}$$

Problem 380: Proposed by the editor.

A golden rectangle is one whose side a and b satisfy the proportion a : b = b : (a - b). Suppose that this golden rectangle has been drawn on a coordinate axis with the longer side lying on the x axis. We mark off on the left side of the rectangle the largest possible square; a smaller golden rectangle remains. We then rotate the figure 90 degrees counterclockwise so that the longer side of the smaller golden rectangle lies along the x axis. Again we mark off the largest possible square on the left side of this new rectangle. By repeating this process, continually smaller golden rectangles are constructed until only one point in the original rectangle remains. What is the location of this unique point ?

Solution by the editor.

In the figure, the golden rectangle lies in the first quadrant with one corner at the origin. The counterclockwise rotation of the figure to locate the successive squares is equivalent to starting with the largest square on the left side of the figure and taking squares in a clockwise direction. Let O denote the origin and let $OA_n = x_n$ and $A_nB_n = y_n$ for $n = 1, 2, 3, \ldots$. Note that $x_1, x_3, \ldots$ form an increasing sequence while $x_2, x_4, \ldots$ form a decreasing sequence.



Furthermore both sequences approach the desired x coordinate. Now from the stated conditions on a and b we have $b = aq$ where $2q = \sqrt{5} - 1$. Thus the side of the first square is aq and the side of the side of the nth square is aq^n. Noting that the squares spiral inward in a clockwise direction, we find that $OA_1 = aq$, $A_1A^3 = aq^5$, $A_3A_5 = aq^9$, etc., form an infinite geometric progression whose sum is

$$\frac{aq}{1 - q^4} = x \text{ , the x coordinate of the desired point.}$$

Proceeding similarly, the segments

$y_1 = A_1B_1$, $y_3 = A_3B_3$, $y_5 = A_5B_5$, . . . form a decreasing sequence, and the segments $y_2 = A_2B_2$, $y_4 = A_4B_4$, $y_6 = A_6B_6$, . . . form an increasing sequence. Again both sequences approach the same limit, this limit being the y coordinate of the desired point. We have

$$y = aq^4 + aq8 + aq^{12} + \ldots = \frac{aq^4}{1 - q^4} .$$

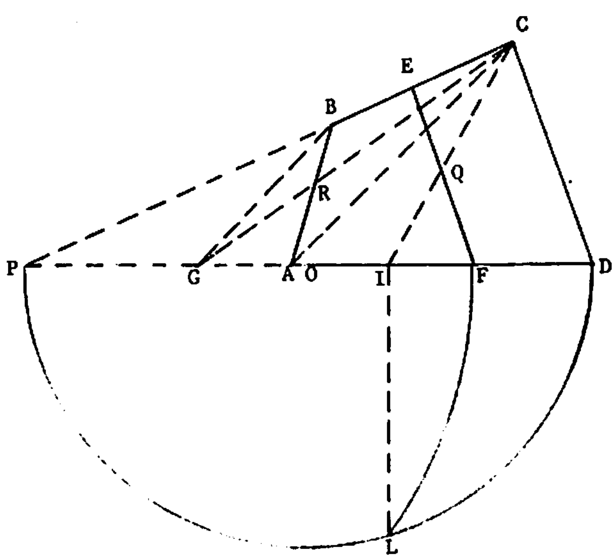
Hence the only point which will remain in the rectangle has the coordinates (x,y) as described above. One can verify that this point is the intersection of the two diagonals shown in the figure. This problem and its solution appear in Steinhaus' <u>One Hundred Problems in Elementary Mathematics</u>, Dover Publications Inc., New York, 1979, pp. 16, 79.

Problem 382: Proposed by Fred A. Miller, Elkins, West Virginia.

Given a quadrilateral ABCD, draw a line which divides the given quadrilateral into two quadrilaterals of equal area.

Solution by the proposer.

Given quadrilateral ABCD, draw AC and draw BG || AC. Draw GC and denote its intersection with AB by R. Let I be the midpoint of GD. Extend CB and DA until they meet at point P. Let I be the midpoint of GD. Let O be the center of a circle having PD as its diameter. Through I construct a line perpendicular to PD and denote its intersection with Circle O by L. Draw PL and LD. Using P as a center of a circle locate a point F on line PD such that PF = PL. Construct FE || CD. Line EF is the desired line. In the following, let [PCD] denote the area of triangle PCD.



Proof: Triangles PLI and PLD are similar so that

$$PL^2 = PF^2 = PI * PD. \quad (1)$$

Since triangles PEF and PCD are similar, we have

$$[PEF]/[PCD] = (PF)^2 /(PD)^2 = (PD * PI)/(PD * PD) = PI/PD. \quad (2)$$

Since triangles PCI and PCD have the same altitude, we have

$$[PCI]/[PCD] = PI/PD \text{ ; hence by (2), } [PEF] = [PCI] . \quad (3)$$

Furthermore, subtracting the area of the common quadrilateral PEQI from each of the triangles PEF and PCI, we find

$$[CEQ] = [IFQ]. \quad (4)$$

Hence the area of quadrilateral CEFD = [CID] = [CGD]/2 . (5)

It remains to show that the area of quadrilateral ABCD = [GCD]. Since GB||AC, triangles GBA and GBC have a common base and the same altitude. Hence [GBA] = [GBC] and thus [GRA] = [BRC]. Finally we have the area of quadrilateral ABCD = [CAD] + [CRA] + [BRC] = [CAD] + [CRA] + [GRA] = [GCD] . Hence the desired result follows from equation (5).

Also solved by A. Jaya Krishna, student, Johns Hopkins University, Baltimore, Maryland and the proposer (second solution).

Problem 383: Proposed by Charles W. Trigg, San Diego, California.

More than half of the integers that are permutations of a certain set of four consecutive integers have squares with digit sums of 37. Identify those integers.

Solution by the proposer.

If the digit sum, S, of N^2 is $37 \equiv 1 \pmod 9$, then $N \equiv 1$ or $8 \pmod 9$. Hence, the digit set is 1, 2, 3, 4 or 5, 6, 7, 8.

Of the twenty-four integers belonging to the 1, 2, 3, 4 set, three, 1432, 32431, and 3421, have an associated S of 19; nine, 1234, 1243, 1342, 1423, 2341, 3412, 4132, 4231, and 4321, have an associated S of 28; three, 3124, 4123, and 4213, have an associated S of 46; while only nine, 1324, 2134, 2143, 2314, 2413, 2431, 3142, 3214, and 4312, have an associated S of 37.

Of the twenty-four integers belonging top the 5, 6, 7, 8 set, two, 6578 and 6785, have an associated S of 28; eight, 5786, 6587, 7586, 7685, 8567, 8576, 8657, and 8756, have an associated S of 46 whereas the remaining fourteen have an associated S of 37 as may be confirmed below:

N	N^2	N	N^2		
5678	32239684	6875	47265625	(*)	(#)
5687	32341969	7568	57274624		(#)
5768	33269824	7685	59059225	(*)	
5867	34421689	7856	61716736		
5876	34527376	7865	61858225	(*)	
6758	45670564	8675	75255625	(*)	(#)
6857	47018449	8765	76825225	(*)	

We note that 5876^2 consists of consecutive digits with two repeated. The five squares marked with an asterisk (*) end with three-digit squares. Those squares marked (#) end with a four-digit square which in each case is the square of the last two digits of N.

Also solved by Fred A. Miller, Elkins, West Virginia.

Problem 384: Proposed by Steve Ligh, University of Southern Louisiana, Lafayette, Louisiana.

Find all integers n such that $2n = (a + 1)(b + 1)(c + 1)$ where n is the product of a, b and c.

Since the proposer submitted the answers without a solution, the folowing solution is supplied by the editor.

The problem seeks solutions in positive integers of the equation,

$$2n = 2abc = (a + 1)(b + 1)(c + 1) \qquad (1)$$

or

$$2 = \frac{(a + 1)}{a} * \frac{(b + 1)}{b} * \frac{(c + 1)}{c} \quad . \qquad (2)$$

Without loss of generality, we take $a \leq b \leq c$. (3)

From (2), at least one of the factors, say $\frac{a + 1}{a}$ must be greater than $\sqrt[3]{2}$ whence a = 2, 3 or 4.

Case I. a = 2

Equation (2) becomes $4bc = 3(b + 1)(c + 1)$.

Hence

$$c = \frac{3b + 3}{b - 3} = 3 + \frac{12}{b - 3} \,. \qquad (4)$$

Testing positive values of b which allow b - 3 to divide 12 yields the solutions (a,b,c) = (2,4,15), (2,5,9) and (2,6,7).

Case II. a = 3

Equation (2) becomes $3bc = 2(b + 1)(c + 1)$.

Hence

$$c = \frac{2b + 2}{b - 2} = 2 + \frac{6}{b - 2} \,. \qquad (5)$$

Testing positive values of b which allow b - 2 to divide 6 yields the solutions (a,b,c) = (3,3,8) and (3,4,5).

Case III. $a = 4$

Equation (2) becomes $8bc = 5(b + 1)(c + 1)$.

Hence $c = \dfrac{5b + 5}{3b - 5} \geq 4$ or $25 \geq 7b$, a contradiction.

Hence there are no solutions in this case.

The solutions are:

$$2*120 = 2*2*4*15 = 3*5*16$$

$$2*90 = 2*2*5*9 = 3*6*10$$

$$2*84 = 2*2*6*7 = 3*7*8$$

$$2*72 = 2*3*3*8 = 4*4*9$$

$$2*60 = 2*3*4*5 = 4*5*6 .$$

A partial solution was received from Fred A. Miller, Elkins, West Virginia.

Problem 385: Proposed by the editor.

A preschool nursery class has three girls. The boys have not yet been counted. An hour later a new child is brought into the nursery. Then a child is selected at random to be photographed. If the child who was selected to be photographed is a girl, what is the probability the last addition to the nursery class was a boy?

No solution was received for this problem. If no solution is received by the time the next column is prepared, a solution will be provided at that time.

Problem 386: Proposed by the editor.

On the first of the month, an eccentric millionaire started a new bank account with a deposit. On the first of the next month he made a second deposit to the account. On the first of each of the following months he made a deposit to the account which was the sum of the amounts deposited in the two preceding months. If the seventeenth deposit was $500,000.00 and if all deposits were integral amounts of dollars, what were the amounts of the two initial deposits?

Composite of solutions by Fred A. Miller, Elkins, West Virginia; A. Jaya Krishna, student, Johns Hopkins University, Baltimore, Maryland and Dmitry P. Mavlo, Moscow, USSR.

Let A and B denote the first and second deposits respectively. Then the seventeenth deposit is $610\,A + 987\,B$ which yields the linear diophantine equation

$$610\,A + 987\,B \;=\; 500000\;. \qquad (1)$$

By the Euclidean algorithm or otherwise, we find a solution of the given equation to be

$$A = 8900000 - 987t \quad \text{and} \quad B = -5500000 + 610t \qquad (2)$$

for any integer t. Since both A and B must be positive integers, $t = 9017$ yielding $A = 221$ and $B = 370$.

Also solved by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Editor's comment: Since the deposits after the two initial ones obey the Fibonacci relation $F_{n+2} = F_{n+1} + F_n$, the ratio of successive deposits approaches @ = $(1 + \sqrt{5})/2$ as n increases. Hence by dividing d_{17} by @ and taking d_{16} as the nearest integer to this quotient, we find $d_{16} = 309017$. It is now an easy matter to backtrack to the initial deposits of \$370 = d_2 and \$221 = d_1 respectively. We have denoted the nth deposit by d_n. After proposing this problem, the editor found a similar problem in Martin Gardner's Wheels, Life And Other Mathematical Amusements, Freeman and Co., New York, 1983, at page 22.

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical **issues** of interest to undergraduate students are discussed in length. Here by **issue** we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to Iraj **Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.**

VECTOR FIELDS AND THE EULER CHARACTERISTIC

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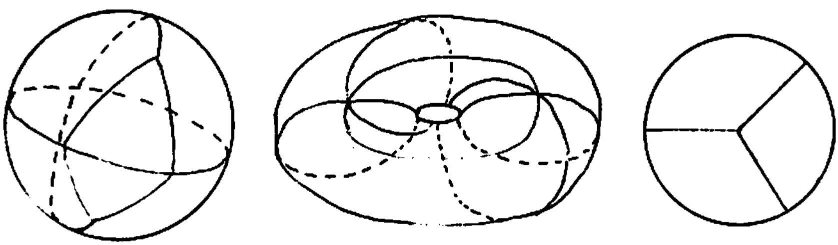
In this article we are going to study vector fields on surfaces and the Euler characteristic. As an application we shall indicate a proof of the Brouwer fixed point theorem.

*Professor Kranjc received his Ph.D. in geometric topology from the University of California at Los Angeles and has been at Western Illinois University since 1985. His interests include 4-dimensional manifolds and knot theory.

1. Surfaces

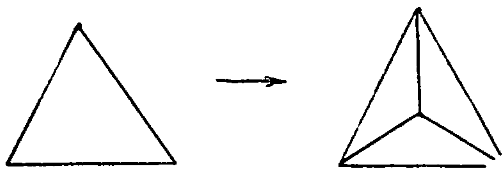
One way to study surfaces is to triangulate them and to look for numerical invariants associated with the triangulations. Some examples of triangulations are given below.



sphere torus disc

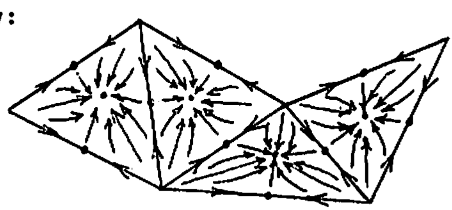
Figure 1

Suppose a surface F is triangulated. Let b_0 be the number of vertices, b_1 the number of edges, and b_2 the number of triangles. Of course these numbers can be arbitrarily big if only the triangulation is fine enough. One way to refine a triangulation is by repeating the following operation:

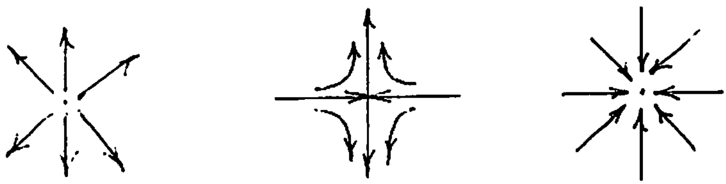


Even though b_0, b_1, and b_2 can be arbitrarily big, it does not seem unreasonable to expect that the numbers b_0, b_1 and b_2 are related in some way. For example, during the operation described in figure 2 the number $b_0-b_1+b_2$ did not change (b_0 increased by one, b_1 by three, and b_2 by two). It turns out that $b_0-b_1+b_2$ does not depend on a particular triangulation but only on the surface F. $b_0-b_1+b_2$ is called the Euler characteristic of F and is denoted by $\chi(F)$. From figure 1 we can see that $\chi(\text{sphere})=4-8+6=2$, $\chi(\text{torus})=2-6+4=0$, and $\chi(\text{disc})=4-6+3=1$.

Suppose now that F is without boundary. Then each triangulation gives rise to a vector field as illustrated below:

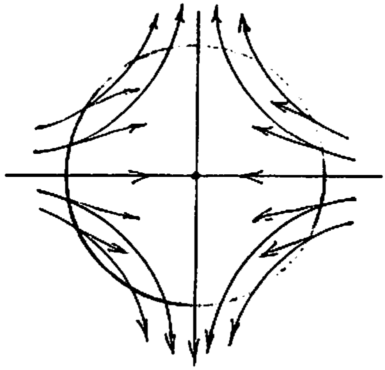


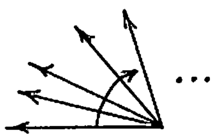
This vector field has singularities (i.e.: zeros) of the following types:



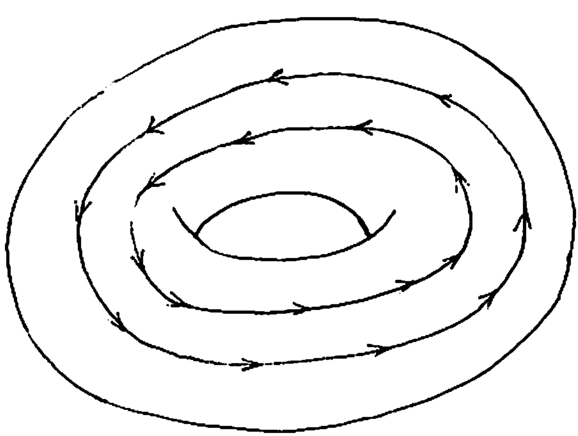
source saddle sink

If we draw a small circle C around an isolated singularity s, such that the vector field has no zeros on C and such that s is the only singularity inside C, then we can count the number of turns made by the vectors of the vector field as we travel along C in the counterclockwise direction. This number is called the index of the singularity s and is denoted by ind(s). If s is either a source or a sink, then ind(s)=1, while it is equal to -1 if it is a saddle:



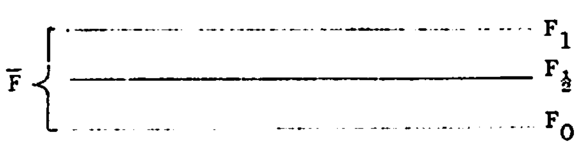


By construction of a vector field from a triangulation there is a source at each vertex, a saddle in the middle of each edge and a sink in the center of each triangle. Therefore $\chi(F)$ is equal to $\sum \mathrm{ind}(s)$ where the sum is over all singularities of the vector field (b_0+b_2=the number of sinks and sources, b_1=the number of saddles). It is natural to ask if this formula is true for every vector field which has only isolated singularities. A way to try to prove it would be to construct a triangulation from a vector field. This, however, is not always possible. On a torus, for example, there exists a vector field without singularities:

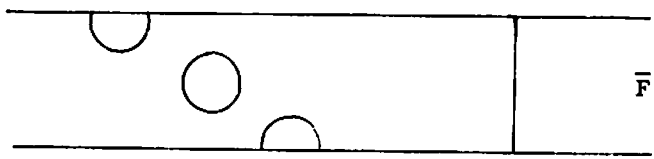


Since this field has no singularities, the formula $\chi(F) = \sum \mathrm{ind}(s)$ is still true in this case. Thus one could try proving it by deforming a given vector field into a vector field corresponding to some triangulation without changing the sum of indices. This is indeed possible: first one can deform any vector field into any other vector field by first shrinking it to zero and then growing it in the direction of the other vector field. By slightly perturbing the deformation one can make it "nice" as described below:

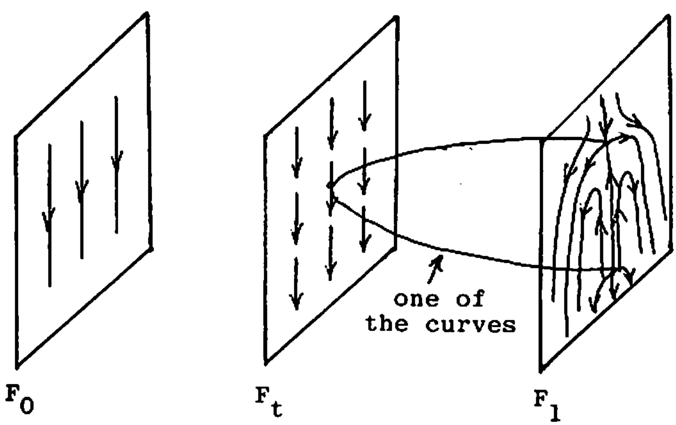
Suppose the deformation takes place from time 0 to time 1. We can picture it on a thickened surface $\bar{F}$ of thickness 1: at time t the situation is drawn on the surface F_t which is at distance t from one side:



Then the deformation can be made such that at each time the vector field has only isolated singularities which trace through time a union of curves in $\bar{F}$ with boundaries lying in $F_0 \cup F_1$. Thus we have the following four possibilities for these curves:



For example, a deformation could look like this:



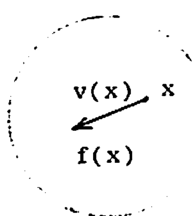
By further examining the indices one can see that the indices of two singularities joined by an arc have sum 0 if both lie on one side of $\overline{F}$, while if one lies in F_0 and the other in F_1 they are equal. This shows that $\sum$ ind(s) does not change through deformation.

2. Brouwer fixed point theorem

Suppose a table is completely covered by a tablecloth. If the tablecloth is removed and thrown back on the table so that it does not extend beyond the edge, then the following is true: there is always at least one point on the tablecloth which was covering the same spot in the beginning and in the end! This fact is a special case of the Brouwer fixed point theorem:

Every continuous mapping f of a disc to itself has (at least one) fixed point, i.e.: there is (at least one) point x on the disc such that $f(x)=x$.

We shall prove the theorem by contradiction. Suppose a continuous mapping of a disc D to itself has no fixed point. Then a non-zero vector field v can be constructed on D as follows:

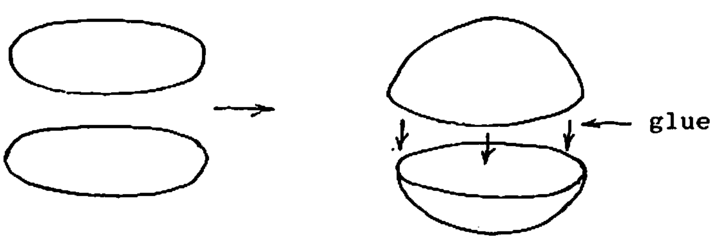


$$v(x) = f(x) - x$$

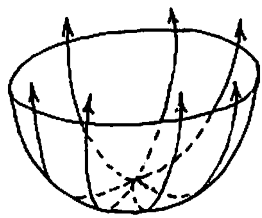
We can assume that v points inwards along the boundary. (Since f is continuous, so is $|v|$, therefore it has a minimum value m at some point x_0.

Since $v(x_0) \neq 0$, m has to be positive. Let $\bar{f}(x) = (1-m/2)f(x)$. Then $|\bar{f}(x)-x| = |f(x)-x-(m/2)f(x)| = |v(x)-(m/2)f(x)| \geq |v(x)|-(m/2)|f(x)| \geq |v(x)|-m/2 \geq |v(x_0)|-m/2 = m/2 > 0$. Thus f has no fixed point and clearly $|f(x)| < 1$ for all x. Therefore the vector from x to $\bar{f}(x)$ points inwards if x is on the boundary. Let the new v be defined by $v(x) = \bar{f}(x)-x$.) By adjusting v along the boundary we can make it point toward the center at each point of the boundary.

Intuitively it is quite obvious that such a vector field cannot exist. We are going to prove this by using the Euler characteristic formula. If we glue two copies of a disc together along their boundaries we get a sphere:



Construct a vector field w on the sphere by defining it to be v on the upper hemisphere while on the lower hemisphere let it look as illustrated:



We can certainly make w continuous by matching both vector fields along the equator. w has only one singularity of index 1. This contradicts the fact that (sphere)=2.

3. Final remarks

The ideas described above can be generalized to all dimensions, therefore the following Brouwer fixed point theorem is true:

Every continuous mapping of the n-ball to itself has a fixed point (an n-ball is the set $\{(x_1,\ldots x_n) | \sum_{i=1}^{n} x_i^2 \leq 1\}$).

For the readers who want more information we recommend the following books:

J. Milnor: Topology from the Differentiable Viewpoint. University of Virginia Press, Charlottesville, 1966.

M. W. Hirsch: Differential Topology, Springer-Verlag, 1976.

J. Milnor Morse Theory, Annals Study 5, Princeton University Press, Princeton, 1963.

Brocker & Janich; Einfuhrung in die Differentialtopologie, Springer-Verlag, 1973.

THE CURSOR
Edited by Jim Calhoun

Like most applied sciences, computer science depends heavily upon a large body of mathematical theory, and it is our goal that the ideas presented in this department explore relationships between these two disciplines. Readers are encouraged to submit articles directed toward this goal.

THE CURSOR was chosen as the name of this department because of the role that mathematics plays in pointing the way toward the understanding of important concepts of computer science.

REDUCING RECURSIVE CALLS

by
L. Carl Leinbach and Alex L. Wijesinha
Gettysburg College, Gettysburg, PA.

INTRODUCTION

There is a large class of problems which are elegantly solved by means of self-recursive algorithms (algorithms which call themselves). In many cases such recursion is the simplest and most natural means of expressing the solution. Examples of problems of this kind include the generation of Pascal's triangle, permutations of order n, and the Towers of Hanoi puzzle. The execution behavior of these algorithms, however, is often inefficient because of the number of calls (procedure or function) which must be made. In particular, it is often necessary to repeatedly make the same recursive call.

The familiar Fibonacci sequence, {1,1,2,3,5,8,...}, can be generated by the algorithm shown in figure 1. Note that the solution it represents depends on the realization that a problem is either primitive, as is the case with FIBO(0) and FIBO(1), and is immediately solvable, or depends directly on solving the two subproblems FIBO(n-1) and FIBO(n-2). The inefficiency of the algorithm is a result of a large number of redundant calls. As an illustration consider the partial tree of calls for the computation of the 6th Fibonacci number (see figure 2). The calculation of FIBO(6) requires five separate calls to FIBO(2). As a more dramatic example, note that the calculation of FIBO(20) requires 34 calls to FIBO(12).

```
ALGORITHM FIBO( n )
   IF ( n = 0 ) OR ( n = 1) THEN
      FIBO := 1  (* return value of 1 *)
   ELSE  (* make two recursive calls *)
      FIBO := FIBO( n-1) + FIBO (n-2)
END { algorithm for FIBO( n )
```

FIGURE 1 - FIBONACCI ALGORITHM

More formally, we assume that, given a recursive problem, its set of parameters can be divided into a vector **n** of non-negative integers and a vector **q** of other parameters. We also assume for this discussion that the value of **n** determines whether the problem is primitive (i.e., immediately solvable without further recursive calls).

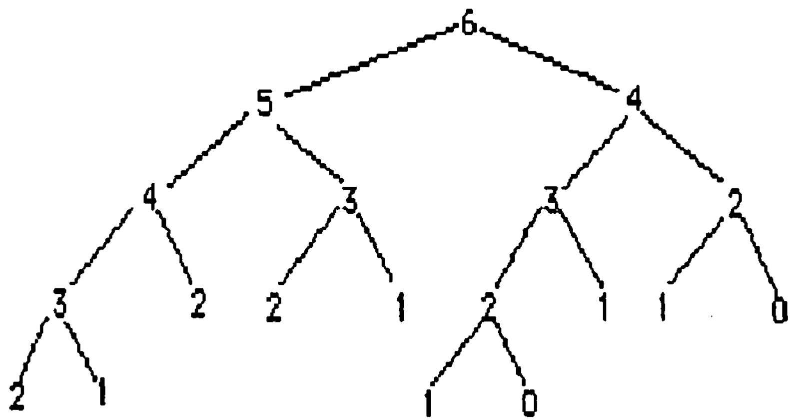


FIGURE 2 - Tree of Calls for FIBO(6)

We also assume the existence of an operator, T, and transition functions T_i $i=1,\ldots,t$ such that for each pair (**n**, **q**), the problem P(**n**, **q**) is mapped into an ordered sequence of subproblems $P(n_1, q_1), \ldots, P(n_t, q_t)$, where $(n_i, q_i) = T_i(\mathbf{n}, \mathbf{q})$ for $i=1,\ldots,t$. The solution of these subproblems, in order, implies the solution of the original problem P(**n**, **q**). If the tree of calls produced by this decomposition has terminal nodes (leaves) which are primitive subproblems, then a recursive algorithm is easily formulated (the general algorithm is given in figure 3 below). It should be obvious that the general algorithm P(**n**, **q**), like FIBO(**n**), can involve a great number of calls and that redundancy can be a problem.

```
Algorithm P( n, q )
    IF P( n, q ) is primitive THEN
      SOLVE P( n, q )
    ELSE (* solve subproblems *)
      P( n1, q1), ..., P( nt, qt)
  END { algorithm P( n, q )}
```

FIGURE 3 - GENERAL RECURSIVE ALGORITHM

Note: In the above algorithm intervening statements which do not involve references to P are ignored. It is also assumed that n_i and q_i are determined using transition function T_i.

```
Algorithm P( n, q )
  TOP <--- 0
  PUSH(STACK, n, q, Local_Variables )
  WHILE (TOP > 0) ) DO
    IF primitive( n, q ) THEN
      SOLVE P( n, q )
      POP(STACK, n, q, Local_Variables)
    ELSE
      k <--- stack[TOP] . counter
      IF k ≤ t THEN
        n <--- nk
        q <--- qk
        PUSH(STACK, n, q, Local_Variables)
      ELSE
        POP(STACK, n, q, Local_Variables)
  END {ALGORITHM P( n, q )}
```

FIGURE 4a - NONRECURSIVE ALGORITHM

```
PROCECEDURE PUSH(STACK,N,Q,Local_Variables)
    TOP <--- TOP + 1
    STACK[TOP].counter <--- 1
    SAVE(STACK,N,Q,Local_Variables)
END {PROCEDURE PUSH}
PROCEDURE POP(STACK,N,Q,Local_Variables)
    TOP <--- TOP - 1
    IF TOP >0 THEN
        STACK[TOP].COUNTER <--- STACK[TOP].COUNTER + 1
        RESTORE(N,Q,Local_Variables,STACK)
END{PROCEDURE POP}
```

FIGURE 4b - STACK OPERATIONS

RECURSION ELIMINATION

To improve efficiency, one approach is to eliminate all recursion. In this approach, a recursive algorithm is translated into an equivalent nonrecursive one (see figures 4a,b). This is a direct translation in which recursive calls and returns are replaced with the stack operations of push and pop. The push stores the local environment (local variables) and current parameter values and the pop recovers them. Although this direct translation eliminates all recursive calls, it does not eliminate redundancy of calculation. In addition, it may suffer by being less understandable than the recursive algorithm. Note that to avoid the overhead in parameter passing resulting from calls to PUSH and POP, one could replace the POP and PUSH statements in the algorithm by the explicit lines of code (shown in figure 4 b) for these operations.

ELIMINATE REDUNDANT CALLS

Another approach attempts to retain recursion but avoid redundancy of calls. This is done by dynamically varying the set of primitive subproblems. In particular, once a subproblem is solved for the first time, in the course of a calculation, it is added to the list of primitives and its solution stored in a table. Each subsequent call to solve the same subproblem results in the subproblem being identified as primitive and its solution extracted by table look-up.

To further illustrate the problem and identify the power of this approach, consider again the Fibonacci example. If F_k is the k^{th} Fibonacci number and $k \leq N$, then FIBO(N) computes F_k a total of F_j times where $j = N - k + 1$. Thus the total number of self-calls is $\sum_{k \leq N} F_k$, a number which can quickly get beyond our control. Using the table look-up approach in the calculation of FIBO(20), only the first call to FIBO(12) involves further recursive calls. Table look-up is utilized on the remaining 33 calculations. The overall result is 20 self-calls, a considerable savings over the 28,656 calls which would be required without table look-up. We encourage the reader to show that, in general, the modification reduces the number of subroutine calls for the calculation FIBO(N) to N. The general algorithm shown in figure 3 can be expanded to incorporate this approach. The resulting algorithm is shown in figure 5.

```
Algorithm P( n, q )
    IF P( n, q ) is primitive THEN
      SOLVE P( n, q )
    ELSE (* solve subproblems *)
      P( n1, q1), ..., P( nt, qt)
    Add P( n, q ) to the set of primitives
  END { algorithm P( n, q )}
```

FIGURE 5 - RECURSIVE, TABLE LOOK-UP ALGORITHM

The dimensions of the table, of course, depend on the number of coordinates of the vector **n**. The recursive, table look-up algorithm for the Fibonacci problem needs only a one-dimensional array to represent the table.

As a second concrete example, consider the problem of calculating binomial coefficients (e.g., the cofficients of $(X + Y)^n$). The following recursive formulae can be deduced from the fact that the coefficients for the n^{th} power can be derived from those of the $(n-1)^{st}$ power:

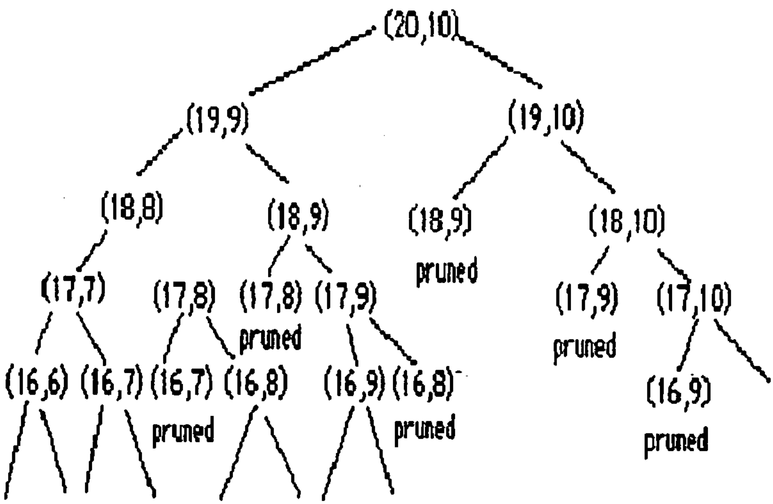
$$C_{n,k} = C_{n-1,k-1} + C_{n-1,k}$$

$$C_{n,0} = C_{n,n} = 1$$

Also by the symmetry property of the binomial coefficients we have:

$$C_{n,(n-k)} = C_{n,k} \quad , k = 0,\dots,n$$

Note that a straightforward recursive algorithm for this problem does result in redundant calls. If, however, we store the values of the coefficients as they are generated, a significant improvement in execution time can be realized. This is the case even for some relatively small values of n and k. For example, calculating $C_{12,6}$ without table look-up requires 1847 calls, but the use of table look-up in conjunction with symmetry reduces this to less than 40 calls. More dramatic results can be obtained for larger values of n and k. For example in computing $C_{20,10}$ only 101 calls are needed as compared to a total of 369,511 without table look-up. Figure 6 represents the pruned tree of calls for $C_{20,10}$. It can be shown, in general, that the number of subroutine calls in calculating $C_{n,k}$ is reduced to $1+2(k-1)(n-k)$. Symmetry considerations will further reduce this number. Note that the look-up table required must be two dimensional because **n** is an ordered pair.



PRUNED TREE OF BINOMIAL COEFFICIENTS

FIGURE 6

In summary, we have shown that a significant improvement in execution efficiency of selected recursive algorithms can be gained by elimination of redundant calls and utilization of problem specific information such as the symmetry of binary coefficients. For further information on classification of recursive problems and additional techniques for improved efficiency see the more complete work of Leinbach and Wijesinha [5].

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5. L. Carl Leinbach & Alex L. Wijesinha, "On Classifying Recursive Algorithms", ACM SIGCSE Bulletin, Vol. 18, Number 1, (1986), pp. 186-190.

KAPPA MU EPSILON NEWS

Edited by M. Michael Awad, Historian

News of chapter activities and other noteworthy KME events should be sent to Dr. M. Michael Awad, Historian, Kappa Mu Epsilon, Mathematics Department. Southwest Missouri State University, Springfield, Missouri 65804.

CHAPTER NEWS

Alabama Zeta, Birmingham-Southern College, Birmingham
Chapter President - Brian G. Cole
32 actives, 14 initiates

Fourteen new initiates were inducted into KME at the 1985 initiation ceremony held in the Olin Building, our new Mathematics/ Computer Science facility. At the initiation, Mrs. Ouida B. Kinzey gave her "Grand National Award" winning slide presentation, "The Nature of Mathematics." During the semester we had another honored speaker, Dr. Henry Hoffman, chairman of admissions at UAB medical school. He spoke on how to get accepted to medical school and the value of a positive attitude. Our activities off-campus included a Christmas party for the elderly at Oak Knoll Nursing Home near the campus. During our January interim term of 1986 our president and vice president will be traveling and studying in the Soviet Union. We all look forward to a busy spring term. Other 1985-86 officers: Leroy Beyer, vice president; Meredith Folland, secretary; Kathy Ray, treasurer; Lola F. Kiser, corresponding secretary; Sarah E. Mullins, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President - Karen McKenzie
50 actives, 28 initiates

Weekly meetings were held with speakers from business and industry. We visited the space shuttle site at Vandenberg Air Force Base. A Christmas social and pledge ceremony was held with a speaker from Hewlett-Packard. We had a picinc with the Math Club. Other 1985-86 officers: Tony Beardsley and Yvonne

Pederson, vice president; Jane McGuire, secretary; Jon Burt, treasurer; George R. Mach, corresponding secretary; Adelaide T. Harmon-Elliott, faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins
Chapter President - Barbara D'Ambrosia
5 actives

We held three meetings of the Mathematics-KME Club. Other 1985-86 officers: Robyn Thoelke, vice president; Willis Gorthy, Secretary and treasurer; Arne Magnus, corresponding secretary.

Colorado Gamma, Fort Lewis College, Durango
Chapter President - Glen Hodges
17 actives

All of the fall meetings were devoted to writing and adopting our chapter bylaws. An initiation ceremony is planned for early in 1986. Other 1985-86 officers: Susan Jochum, vice president; Sheri Postier, secretary; Johnny Snyder, treasurer; Richard A. Gibbs, corresponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Sandra Hyde
25 actives

Our Second Annual Christmas Social was held at a local Steak House on December 5, 1985, with Sandra Hyde presiding. Members brought Christmas gifts which were collected and taken to the Carroll County Training Center for Mentally Retarded Adults. A fine time was had by all. Other 1985-86 officers: Joan Lyons, vice president; Robin Eason, secretary; Doug Ginn, treasurer; Thomas J. Sharp, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston
Chapter President - Jeff Nettles
35 actives

The first Math Club/KME meeting was held on Sept. 17th and the Math Club fall picnic was held Oct. 3rd at the Morton Park Pavillion. Dennis Bury from Illinois Power spoke at our Oct. 15th meeting; Russ Colberg was the speaker at our Nov. 19th meeting. Our Christmas party took place on Dec. 6th. Other 1985-86 officers: Mike North, vice president; Linda Pfeiffer, secretary; Karie Andreina, treasurer; Floyd Koontz, corresponding secretary; Jane Bradfield, faculty sponsor.

Illinois Zeta, Rosary College, River Forest
Chapter President - Jill Rothermell
12 actives

Members of Illinois Zeta met jointly with the Science Club to hear a lecture by Sister Mary O'Donnell on the use of the Fast Fourier Transform in her research on thunder and lightning. Other 1985-86 officers: Mary Ann Liebner, vice president; Denise Bastas, secretary; Gina Suareo, treasurer; Sister Nona Mary Allard, corresponding secretary; Mordechai Goodman, faculty sponsor.

Illinois Eta, Western Illinois University, Macomb
Chapter President - Heidi Kush
12 actives, 5 initiates

The chapter met about every three weeks with the WIU Math Club. Faculty speakers at the meetings included Dr. Michael Moses and Dr. Marko Kranjc. The Math Club was successful in raising money through having a raffle. Five members were initiated in December; a reception followed. Other 1985-86 officers: Debbie Tanner, vice president; Elizabeth Wood, secretary and treasurer; Alan Bishop, corresponding secretary; Iraj Kalantari, faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
Chapter President - Ryan McBride
20 actives

Picnic and organizational meeting. Meeting with presentation on Halley's Comet by Dr. Dwight Beery. Joint Christmas meeting (party) with A.C.S. and S.P.S. Other 1985-86 officers: Mark

Cawood, vice president; Norman Rohrer, secretary; Lisa Jerva, treasurer; Ralph B. McBride, corresponding secretary and faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President - Scott Kibby
41 actives, 4 initiates

The annual KME homecoming breakfast was held in October hosted by Professor and Mrs. E. W. Hamilton. There was a good turnout of alumni, faculty, and student members. Students presenting papers at Iowa Alpha meetings include: Jeremy Philips reporting on summer work experience, Tim Roegner on The Duplication of the Cube. F. Austin Jones presented a paper on The Potential in a Rectangle at the initiation and banquet in December. Other 1985-86 officers: F. Austin Jones, vice president; Tracy Konrad, secretary; Tim Roegner, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President - Tracy Parks
7 actives

Other 1985-86 officers: Ruth Gornet, vice president; Linda Delaney, secretary; Sean Downing, treasurer; Alex Kleiner, corresponding secretary; Larry Naylor, faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President - Ron Waltmann
23 actives, 8 initiates

A general meeting took place at the beginning of the year at which interns discussed their experiences from the past summer. Other activities which took place during the course of the semester were a Quiz Bowl (students vs. faculty), a Major Merger Dinner with speaker, and a Christmas party. Other 1985-86 officers: Deanna Baumann, vice president; Scott Hatteberg, secretary; Eric Stahlberg, treasurer; Greg Diercks, corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President - Sue Pyles
50 actives, 7 initiates

The Chapter held monthly meetings in October, November, and December. In addition, a fall picnic was hosted for all mathematics and physics students. Fall initiation for new members was held at the October meeting. Seven new members were initiated at that time. The October program was given by Bryan Dawson. He spoke on "Least Squares." Tammy Horn presented the November meeting on the "Revised Simplex Algorithm." In December, a special Christmas meeting was held at the home of Dr. Helen Kriegsman, Mathematics Department Chairman. Melinda Powers gave the program entitled, "The Chemistry of Integrated Circuits." Other 1985-86 officers: Laura Rea, vice president; Sharon Million, secretary; Cathy Brenner, treasurer; Harold L. Thomas, corresponding secretary; Helen Kriegsman and Gary McGrath, faculty sponsors.

Kansas Beta, Emporia State University, Emporia
Chapter President - Robert R. Schif
25 actives, 8 initiates

The chapter assisted with Math Day activities. There were presentations on various topics given at our annual banquet and at meetings. The chapter helped promote a "Math Paper" contest. A new bulletin board was designed and made which gives general information on KME at Emporia State and shows what KME is about. Other 1985-86 officers: Dawn Slavens, vice president; Barb Applegarth, secretary; Shelly Redeker, treasurer; George Downing, corresponding secretary; Tom Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Mary Jo Muckey
21 actives, 17 initiates

Kansas Gamma began the 1985 fall semester with its annual fall picnic on September 18. Several KME and faculty members attended and enjoyed the cookout and volleyball games. On October 22, KME member Lisa Huerter presented a discussion of ideas on elementary concepts of mathematics. It was very interesting to math and education students. The mathematician Gauss, portrayed by Junior Jim Wiggs, spoke about his life and some of his more famous discoveries at an early November meeting. Later in the month, Sr. Clarisse Lolich, an employee of NASA, came to talk about "Math and Space." We found out about all the projects going on at NASA this year. To end the first semester, we held our traditional Wassail Party on December 3 at the home of faculty member Richard Farrell. Other 1985-86 officers: Rita Lundstom, vice president; Patricia Patterson, secretary and treasurer; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President - Doug Bogia
15 actives

Other 1985-86 officers: David Ratner, vice president; Gina Estes, secretary and treasurer; Robert Thompson, corresponding secretary; Allan Riveland and Ron Wasserstein, faculty sponsors.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President - Michelle Ferland
20 actives, 5 initiates

A Christmas party was held for the Math Club, KME and the department. There were two student presentations. Other 1985-86 officers: Janet Schuetz, vice president; Mary Doxon, secretary and treasurer; Charles Votaw, corresponding secretary; Jeffrey Barnett, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President - Lorie Barker
22 actives

The semester's activities started with a picnic for KME members and faculty of the department. Long-stemmed roses were again sold this year to increase chapter funds. Three speakers presented talks to the chapter. Dr. Charles Franke (the new department chairman) gave a talk on "Computer Round-off Errors." Dr. James Patterson gave an interesting talk on "Lattices Points of an n-dimensional Orthant." The third speaker was Mr. Lafferty from the Navy Recruiting Office who spoke on "Careers in the Navy." Some of the other activities included a touch football game against the faculty and a Christmas party that included the appearance of a "modern" Santa. Other 1985-86 officers: John Carroll, vice president; Jackie Back, secretary; Dana Baxter, treasurer; Patrick Costello, corresponding secretary; Bill Janeway, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President - Nancy Sekira
14 actives, 1 initiate

The chapter sold soft pretzels at the Westminster Fall-Fest Days to raise money for the Spicer Award and Duren Scholarship. The induction featured a charter member of the chapter, Dr. Charles Miller, a teacher at Camden County Community College (N.J.) who spoke on the topic: Will We Ever Need It (Mathematics)? One of our members, Ronald Gavlin, gave a talk at another meeting; his topic was systems management and he described his summer work at Hewlett Packard. The chapter also sponsored a Fondu Party for all mathematics students during November. Other 1985-86 officers: Julie Winkler, vice president; Nettie Barrick, secretary; Brian Russo, treasurer; James E. Lightner, corresponding secretary; Linda Eshleman, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President - Catherine LaPointe
27 actives

The fall 1985 semester was a busy and exciting one for MD Delta. A get acquainted picnic was held at Rocky Gap State Park in September. During the semester talks were presented by Drs. Barnet, Biggs, and White of the mathematics faculty on the topics of Computer Graphics, Number Theory, and Logic, respectively. Robert Katz, a student KME member, spoke about his summer work experiences at the Naval Research Laboratory. In addition, MD Delta will host the Spring 1986 KME Sectional and planning in this regard commenced. Other 1985-86 officers: Teresa Nevelle, vice president; Donna Pope, secretary; John Galleher, treasurer; Don Shriner, corresponding secretary; John Jones, faculty sponsor.

Michigan Alpha, Albion College, Albion
Chapter President - Doug LeMaster
11 actives

Mathematics Colloquium Series: August 28, Robert Messer, "Antoine's Necklace and the Amazing Cantor Set;" September 11, John Wenzel, "Computability, Turing Machines, and the Halting Problem;" September 25, Tom Phillips, "The Secret Life of the Actuary;" October 9, Putnam Exam Practice Session; October 23, Ron Fryxell, "Taxicab Geometry;" November 13, Mark Meershaert, "Forgetful Models are the Best;" November 20, Putnam Exam Practice Session; December 4, Organizational Meeting for Spring Semester; December 7, Putnam Exam. Other 1985-86 officers: Susan Krampe, vice president; Kevin Randall, secretary; Ken Alberts, treasurer; Bob Messer, corresponding secretary; Norm Loomer, faculty sponsor.

Mississippi Alpha, Mississippi University for Women, Columbus
Chapter President - Judy E. Dye
3 actives, 12 initiates

We hosted a seminar on Knowledge-Based Expert Systems for the Division of Science and Mathematics. Other 1985-86 officers: Deborah Bland, vice president; Rissa Lawrence, secretary and treasurer; Jean Ann Parra, corresponding secretary; Carol B. Ottinger, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President - Marylynne Abbott
35 actives, 15 initiates

During the fall semester the Missouri Alpha Chapter held three monthly meetings and organized a picnic for all KME members and faculty. Other 1985-86 officers: Mark Payton, vice president; Sue Holt, secretary; Jean Ann Gay, treasurer; John Kubicek, corresponding secretary; David Lehmann, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President - David Naas
82 actives, 11 initiates

The Missouri Beta Chapter held three monthly meetings during the fall 1985 semester, and hosted a guest speaker at each. During September, we sponsored a float in the homecoming parade and set up an information booth at an open house in the Mathematics Department. We initiated 11 new members in October and also presented an autographed copy of our calculus book to the recipient of the "Top Freshman Award." Two other organizations joined us for a Halloween party on October 30. Our semi-annual book sale in November was a big success. Other 1985-86 officers: Rhonda Schnieders, vice president; Cheryl Harris and Nancy Schondelmaier, secretaries; Kim Ward, treasurer; Rhonda McKee, corresponding secretary; Gerald Schrag and Larry Dilley, faculty sponsors.

Missouri Gamma, William Jewell College, Liberty
Chapter President - Blane Baker
19 actives

Our chapter holds a regularly scheduled meeting each month. This fall the students have been working on a fund raising proposal to raise money for our activities fund. A spring initiation ceremony and banquet is planned. Other 1985-86 officers: Laurie Honeyfield, vice president; Remy Blanchaert, secretary and treasurer; Joseph T. Mathis, corresponding secretary and faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President - Hector Bencomo
5 actives

Other 1985-86 officers: Keith Young, vice president; William D. McIntosh, corresponding secretary and faculty sponsor.

Missouri Zeta, University of Missouri, Rolla
Chapter President - Tim Allen
35 actives, 17 initiates

Meetings were held every month with a speaker or in one case a video on the "Wonders of Mathematics." Help sessions were held twice a week for students in all courses through differential equations. The KME Barbecue, in early September, served as a fine mixer between students and faculty. The Initiation Banquet took place on November 24. Professor Roy Utz spoke about "Mathematics in Literature." Other 1985-86 officers: Pravin Ruktasiri, vice president; Rana Jones, secretary; Angie Wallace, treasurer; Thomas Powell, corresponding secretary; James Joiner, faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin
Chapter President - Carol Lazure
13 actives

The Missouri Iota Chapter won first place for obtaining the most money pledged to March of Dimes in conjunction with a Halloween Dance-a-thon. Members participating were Angela Noyes and Melinda Robinson. Other fall activities included the annual float trip, regular meetings, a talk by Professor David Foreman on "The Game of Nim," and working concessions at a football game to raise revenue. Other 1985-86 officers: Melinda Robinson, vice president; Cheryl Ingram, secretary and treasurer; Mary Elick, corresponding secretary; Joe Shields, faculty sponsor.

Missouri Kappa, Drury College, Springfield
Chapter President - Sami Long
6 actives

The Missouri Kappa Chapter conducted a free tutoring service for all math students during the fall semester. The Chapter also ran a campus wide math contest. The lower division was won by Diane Prior; the upper division by Lynne Ruehle. A pizza party was held for the contestants and prize money awarded to the winners. Student and faculty math talks were given during the semester. Scott Rollins gave a talk on a Mathematical Model of Osmosis and Dr. Allen talked on Model Theory. Other 1985-86 officers: Scott Rollins, vice president; Steve Rutan, secretary; Lynne Ruehle, treasurer; Charles Allen, corresponding secretary; Ted Nickle, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President - Dan Stalp
23 actives

Activities for the fall semester began with a building picnic for faculty members, students, and potential KME members. The picnic was held in conjunction with the Biology Club, the Computer Club, and Lambda Delta Lambda. In early October the same four clubs sponsored a hayrack ride and bonfire. Later the club participated in the college homecoming activities by painting and erecting a billboard. Club members also participated in a pre-Parents Day "all campus cleanup" sponsored by the Wayne State College Student Senate. In December the fall banquet was held. The featured speaker was Mr. John Wrenholt, president of the Big Red Apple Club of Norfolk, Nebraska. Former Nebraska Alpha KME president, Bob Nissen, and another representative of Continental Oil Company, Oklahoma City, OK, spent two days on campus. They spoke to club members about their experiences since leaving college. They also interviewed seniors for potential employment with Continental Oil Company. With a grant from the Wayne State College Student Senate, KME and the Computer Club purchased a color printer which is available for student use. Throughout the semester, club members have monitored the Math-Science Building in the evenings to earn money for the club. Other 1985-86 officers: Sandra Sunderman, vice president; Tammy Strand, secretary-treasurer; Doug Anderson, historian; Fred Webber, corresponding secretary; James Paige and Hilbert Johs, faculty sponsors.

Nebraska Gamma, Chadron State College, Chadron
Chapter President - Terri Scofield
7 actives, 5 initiates

For Chadron State College Homecoming in October, the Chapter decorated the Math and Science Building on the campus. A five-foot rabbit was hung from the pendulum wire, stars were hung from the ceiling, and the banners read "Formulas Work Magic."

All of this led to the awarding of $25.00 from the Student Senate for a first place prize. In the new year the Chapter plans to form a math handbook, including guidelines and tests, and possibly tutor future teachers who have a difficult time in the field of mathematics; for example, a health education major. Other 1985-86 officers: Noelle Strang, vice president; Debra Gaswick, secretary; James Paloucek; James A. Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President - Jennifer Tyler

Other 1985-86 officers: Cecilia DeBlasi, vice president; Susan Fehrenbach, secretary; Richard Metzler, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Alpha, Hofstra University, Hempstead
Chapter President - Ethel Kleinhans
10 actives

Sponsored a student talk on "A Topological Proof of the Fundamental Theorem of Algebra." Other 1985-86 officers: John Cartmell, vice president; Shawn Dawson, secretary; Carol Bishop, treasurer; Stanley Kertzner, corresponding secretary and faculty sponsor.

New York Lambda, C. W. Post Center - Long Island University, Greenvale
Chapter President - Caroline Diffley
35 actives, 16 initiates

Our main meeting was a holiday-party installation. We also held a career night in conjunction with the Math Club. Other 1985-86 officers: Cynthia Ferro, vice president; Sung-Yong Um, secretary; Ioannis Pappas, treasurer; Sharon Kunoff, corresponding secretary and faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green
Chapter President - Lynne Howard

Activities during the fall semester included (1) Halloween-Pizza Party for faculty, graduate students and undergraduates; (2) Dr. Waldemar Weber spoke at a special meeting concerning career opportunities for those with a degree in mathematics. Activities during the spring semester will include (1) Tee-shirts with math-related logos will be sold; (2) annual spring banquet to initiate new members. "Teacher of the Year" Award will be presented. Entertainment by the Logarythms, a barber-shop quartet made up of math professors. The spring banquet will also feature a topic of interest presented by Dr. Fred Rickey; (3) combined meeting of KME and campus computer club with Dr. Clifford Long presenting a topic of common interest. (4) Spring volleyball game is planned for faculty and students. Other 1985-86 officers: Peter Lorenzetti, vice president; Cathy Raimer, secretary; Susan Cartwright, treasurer; James Albert, faculty sponsor.

Ohio Epsilon, Marietta College, Marietta
Chapter President - Sharon Castillo
25 actives, 11 initiates

The KME initiation ceremony took place on November 15, 1985. A party followed. Other 1985-86 officers: Patrick Cassity, vice president; Jacqueline Hurner, treasurer; John R. Michel, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President - Lisa Elderbrock
29 actives, 8 initiates

Our fall meeting consisted of the following: In September, Lisa Elderbrock presented "The Golden Rule: A Precious Jewel." October: Initiation of eight new members and mathematical talks by each. Sponsored faculty talk by Russ Smucker on "Periodic Properties of Animal Populations." November: Sponsored two talks by Drs. Dave Groggel and Dan Pritikin of the Miami University Mathematics Department. December: Christmas party, tree decorating, and spring regional convention planning time at the Smucker home. Other 1985-86 officers: Kim Lutz, vice president; Monette Dulkoski, secretary; Barry Gowdy, treasurer; James L. Smith, corresponding secretary; Russell Smucker, faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President - Nandana Silua
25 actives, 12 initiates

We have met several times and discussed having a regional convention. We have a pizza party planned before Christmas. We hope to begin a "Problem of the Month" competition next month. We also plan to identify the outstanding freshman math student. Other 1985-86 officers: Niel Christensen, vice president; Lisa Peters, secretary and treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Pennsylvania Beta, La Salle University, Philadelphia
Chapter President - Robert Morgan
30 actives, 6 initiates

The Chapter met on Thursday, November 21, 1985, to induct six initiates. Mr. Raymond Kirsch of the department faculty spoke on the "Topology of Multiprocessors." Other 1985-86 officers: Leon Wiener, vice president; Lisa Tresnan, secretary; Edward Dzialo, treasurer; Hugh N. Albright, corresponding secretary; Carl McCarty, faculty sponsor.

Pennsylvania Gamma, Waynesburg College, Waynesburg
Chapter President - Mark Keller
11 actives

The members of our Chapter sponsored a homemade pizza party for the freshman mathematics and computer science majors. This party was held at our chapter advisor's home. It allowed for perspective KME members to find out about our fraternity. We ate pizza and played Trivial Pursuit. It allowed for us to get to know new students and it gave the Chapter a chance to get together before the semester got under way. Other 1985-86 officers: Vince Perdos, vice president; Mary Beth Huffman, secretary and treasurer; Rosalie Jackson, corresponding secretary; David Tucker, faculty sponsor.

Pennsylvania Delta, Marywood College, Scranton
Chapter President - Peggy Rekus
11 actives

The regional MAA meeting at Temple University which took place on November 23, 1985, was attended by several of our members. Other 1985-86 officers: Mary Kelly, vice president; Laurie Wood, secretary; Susan Pacanowski, treasurer; Sister Robert Ann von Ahnen, corresponding secretary and faculty sponsor.

Pennsylvania Epsilon, Kutztown University, Kutztown
Chapter President - Susan A. Peters
17 actives

Titles of four talks given by students at our fall, 1985, meetings were: Fallacious Proofs in Geometry, Testing Divisibility, Geometry and the Computer, and Blackjack by Counting Cards. Other 1985-86 officers: Stephanie L. Fung, vice president; Kelly Robaton, secretary; Pi-ling Tsou, treasurer; William E. Jones, Jr., corresponding secretary; Edward W. Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of PA, Indiana
Chapter President - Anne Polito
25 actives, 9 initiates

In October we held our initiation of new members. Ida Z. Arms, adviser, reported on her attendance at an MAA Short Course "The Total Role of the Mathematician" which was held at the University of Maine-Orono during June, 1985. In November Dr. Daniel Boone, a member of the Philosophy Department at IUP, gave a talk on some aspects of mathematics and philosophy. In December Joseph Ramsey, student member, who is both a philosophy and mathematics major, presented a talk on "Godel's Theorem." Other 1985-86 officers: Lucy Sgrignoli, vice president; Bonnie Jacko, secretary; Daniel Besecker, treasurer; Ida Z. Arms, corresponding secretary; George R. Mitchell, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President - Mike Leonzo
38 actives, 16 initiates

The fall semester always has one eagerly anticipated occasion: The Christmas "dessert" at the home of the Mathematics Department Chairman, Mr. Schlossnogel. This semester was no exception and everyone enjoyed the gathering. Other 1985-86 officers: Tawni Emmanuel, vice president; Jeanne Maliersberg, secretary; Ami Waldschmidt, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Theta, Susquehanna University, Selinsgrove
Chapter President - Doris Cook
18 actives, 5 initiates

Some activities which took place during the fall semester included: a trip to Goddard Space Center and Baltimore Inner Harbor; fall banquet with initiation of new members and a talk by Dr. Trudy Cunningham, Assistant Dean of Engineering at Bucknell; tutoring for the mathematics department. Other 1985-86 officers: Jeff Lockard, vice president; Robert Walker, secretary and treasurer; Carol Harrison, corresponding secretary; Karl Klose, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President - Linda Rafferty
5 actives, 10 initiates

During the meetings Sister Grace lectured on "Correlation: What Makes a Perfect Pair?" and "Tesselations: Patterns in Geometry." Members received NCTM Student Math Notes on the above and were encouraged to complete the questions and hand them in for grading. The field trip was discussed. The Franklin Institute, the Planetarium and the Art Museum were decided upon. Tutoring (free of charge) is one of the services the KME members continue to offer to students requesting help. Have a successful year 1986! Other 1985-86 officers: Christine Mescier, vice president and secretary; Nadine Hillgen, treasurer; Sister M. Grace, corresponding secretary.

Tennessee Beta, East Tennessee State University, Johnson City
Chapter President - Tammy Gillenwater
12 actives

In addition to our regularly scheduled meetings, we had a social at a local restaurant. Also, the group had its picture taken for the college yearbook and discussed plans for the spring semester. Other 1985-86 officers: Suzanne Walters, vice president; Gina Comer, secretary; Jeff Byington, treasurer; Lyndell Kerley, corresponding secretary and faculty sponsor.

Tennessee Gamma, Union University, Jackson
Chapter President - Suzanne Morgan
20 actives

For our November meeting, we had Professor Jonson from Vanderbilt University discuss paradoxes. We learned a formal definition for a paradox: a paradox is a result that conflicts with our intuition or a strongly held belief. Professor Jonson presented us with mathematical and logical paradoxes. One paradox that we all particularly enjoyed was the Paradox of the Liar. All Cretans are liars. Written by Epidamedes the Cretan. Other 1985-86 officers: Stacey Sheppard, vice president; Suzanne Pack, secretary; Mary Anne Stevenson, treasurer; Pattie Crane, historian; Mr. Richard, corresponding secretary; Mr. Jennings, faculty sponsor.

Tennessee Delta, Carson-Newman College, Jefferson City
Chapter President - Jeff Drinnen
19 actives

Some activities for the fall semester included a picnic to Panther Creek Park, a bike ride in the Smoky Mountains, and a presentation by a visiting lecturer from Vanderbilt University. Other 1985-86 officers: Joanne Raye, vice president; Michael Souleyrette, secretary; Trish Snowden, treasurer; Albert Myers, corresponding secretary; Carey Herring, faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President - Laura Watson
15 actives

A get-acquainted party was held at the home of Dr. and Mrs. Charles Robinson. The purpose and activities of the society were explained to prospective members and recently inducted members received their shingles. Other 1985-86 officers: Sam Shin, vice president; Stephanie Thomas, secretary; Mike Cagle, treasurer; Mary Wagner, corresponding secretary; Charles Robinson and Ed Hewett, faculty sponsors.

Virginia Alpha, Virginia State University, Petersburg
Chapter President - Pamela Miller
16 actives

Other 1985-86 officers: Vincent Robenson, vice president; Benjamin Ebhojaye, secretary; Mohinder Tewari, treasurer; LaVerne Goodridge, corresponding secretary; Emma Smith, faculty sponsor.

Virginia Beta, Radford University, Radford
Chapter President - Karla Cooper
16 actives

In November, 1985, we offered a tutoring service for precalculus students. Other 1985-86 officers: Joyce Reush, vice president; Ann Coleman, secretary; Mike Rietz, treasurer; Coreen Mett, corresponding secretary; J. D. Hansard, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President - Sandra Theiler
5 actives, 3 initiates

Wisconsin Alpha sponsored the annual Mathematics Contest for high school junior and senior girls. Approximately ninety young women attended. The top scoring participant in the contest receives a partial scholarship to Mount Mary College. Time was spent discussing means of raising funds. A doughnut sale was held. Other 1985-86 officers: Betsy Zaborske, vice president; Sandra Theiler, secretary; Betsy Zaborske, treasurer; Sister Adrienne Eickman, corresponding secretary and faculty sponsor.

Wisconsin Beta, University of Wisconsin-River Falls, River Falls
Chapter President - Thomas Weber
20 actives

In September, the chapter elected new officers for the 1985-86 school year. This fall, we were able to acquire a lounge where our chapter holds it's monthly meetings and where the mathematics students can get together to study. One of the activities enjoyed by all was a Christmas party in December. Other 1985-86 officers: Jody Speer, vice president; Janice Pete, secretary; Sarah Flood, treasurer; Lyle Oleson, corresponding secretary; Don Leake, faculty sponsor.

ANNOUNCEMENT OF TWENTY-SIXTH BIENNIAL CONVENTION

The 26th Biennial convention of Kappa Mu Epsilon will be held on April 2-4, 1987 at California Polytechnic State University, San Luis Obispo, California. Each chapter that sends a delegation will be allowed some travel expenses from National Kappa Mu Epsilon funds. Travel funds are disbursed in accordance with Article VI, Section 2 of the KME constitution.

A significant feature of this convention will be the presentation of papers by student members of KME. The mathematics topic which the student selects should be in his/her area of interest, and of such scope that he/she can give it adequate treatment within the time allotted.

Who May Submit Papers? Any student member of KME, undergraduate or graduate, may submit a paper for use on the convention program. A paper may be co-authored; if selected for presentation at the convention it must be presented by one or more of the authors. Graduate students will not compete with undergraduates.

Subject: The material should be within the scope of the understanding of undergraduates, preferably those who have completed differential and integral calculus. The Selection Committee will naturally favor papers within this limitation, and which can be presented with reasonable completeness within the time limit.

Time Limit: The minimum length of a paper is 15 minutes; the maximum length is 25 minutes.

Form of Paper: Four copies of the paper to be presented, together with a description of the charts, models, or other visual aids that are to be used in the presentation, should be presented in typewritten form, following the normal techniques of term paper presentation. It should be presented in the form in which it will be presented, including length. (A long paper should not be submitted with the idea it will be shortened for presentation.) Appropriate footnoting and bibliographical references are expected. A cover sheet should be prepared which will include the title of the paper, the student's name (which should not appear elsewhere in the paper), a designation of his/her classification in school (graduate or undergraduate), the student's permanent address, and a statement that the author is a member of Kappa Mu Epsilon, duly attested to by the Corresponding Secretary of the Student's Chapter.

Date Due: January 16, 1987

Address to Send Papers:

Dr. Harold L. Thomas
Mathematics Department
Pittsburg State University
Pittsburg, KS 66762

Selection: The Selection Committee will choose about fifteen papers for presentation at the convention. All other papers will be listed by title and student's name on the convention program, and will be available as alternates. Following the Selection Committee's decision, all students submitting papers will be notified by the National President-Elect of the status of their papers.

Criteria for Selection and Convention Judging:

A. The Paper

1. Originality in the choice of topic
2. Appropriateness of the topic to the meeting and audience
3. Organization of the material
4. Depth and significance of the content
5. Understanding of the material

B. The Presentation

1. Style of presentation

2. Maintenance of interest

3. Use of audio-visual materials (if applicable)

4. Enthusiasm for the topic

5. Overall effect

6. Adherence of the time limit

Prizes: The author of each paper presented at the convention will be given a two-year extension of his/her subscription to The Pentagon. Authors of the four best papers presented by undergraduates, based on the judgment of the Awards Committee, composed of faculty and students, will be awarded cash prizes of \$60, \$40, \$30, and \$20 respectively. If enough papers are presented by graduate students, then one or more prizes will be awarded to this group.

Prize winning papers will be published in The Pentagon, after any necessary editing. All other submitted papers will be considered for publication at the discretion of the Editor.

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