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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics; due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

AN APPLICATION OF LINEAR INTERPOLATION IN TWO VARIABLES FOR BRAIN WAVE DISPLAYS*

Douglas P. Bogia
Student, Washburn University

I was approached by my supervisor, Dr. Terence Patterson of The Menninger Foundation, Topeka, KS, to do a project that involved linear interpolation in two variables. He wanted to take data from twenty electrodes placed on a subject's head, and turn them into a composite picture of what the brain was doing at a precise time. This requires interpolation of the points over the rest of the head in order to return an approximate knowledge of what is happening over the entire brain. Then this new set of values can be used to draw a picture of what the brain looks like electrically.

First, I will give you some background information on the experiment. Dr. Patterson is studying the difference between normal and schizophrenic people. His hypothesis is that when a schizophrenic person and a normal person are doing the same task, their brain waves will look different. To test this hypothesis he is using a computer that presents a subject with a 3X3

*A paper presented at the 1985 National Convention of KME and awarded third place by the Awards Committee.

matrix of letters on a screen. He starts to sample the brain waves of the subjects 100 milliseconds before the start of the stimulus. This shows him what the brain was doing before it was stimulated, and is represented by the Segment A on Figure 1. Segment B represents the time when the 3x3 matrix is on the screen and lasts for 250 milliseconds. Then in order to control for the length of the visual memory, Dr. Patterson backward-masks the letters. This means that the letters are rearranged so that they are no longer letters and then re-presented. The backward mask for each letter is the same luminosity as the original letter. Segment C represents 250 milliseconds of which the first 10 milliseconds is the backward mask. Segment D is 400 milliseconds and at the beginning of which a pointer appears on the screen. This tells the subject which line of letters to report.

In order to accomplish the precision required by the experimental conditions, Dr. Patterson had special electronic equipment built that allows him to sample twenty electrodes, situated at specific points on the head, every one four-hundredth of a second. The values

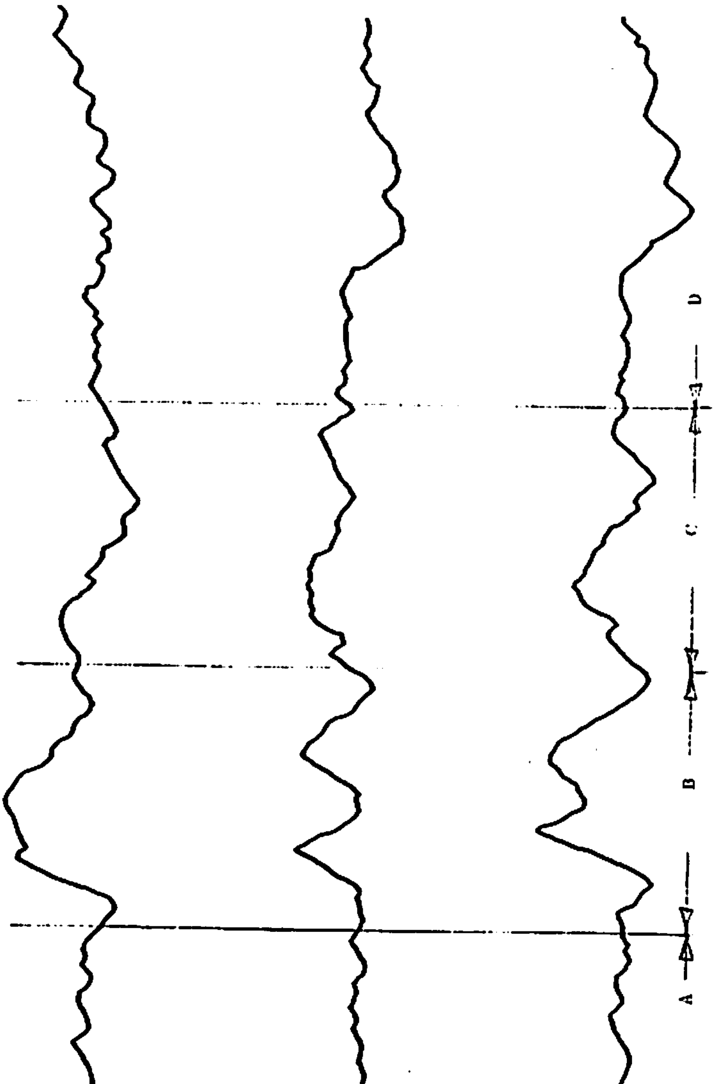


Figure 1

of the twenty electrodes are latched, so all the values represent the same instant on the brain. The computer hardware is run for one second to represent an epoch, returning four hundred data points per channel. The points are then checked for artifacts. An artifact might represent eye movement or muscles in the scalp moving. This type of movement is detected by the hardware. Then if the number of artifacts is less than the maximum number allowed, the data are kept and summed with the other existing data points. The summation is done to get a larger signal-to-noise ratio. It is assumed that, as the trials go along, the noise will not be in synchronization and therefore will be negative as often as it is positive and will cancel out. After fifty good summed trials, the experiment stops and the data are saved. For an example of what the waves from each of the channels might look like from a normal subject, see Figure 1. This type of data is called the **average evoked potential.** The theory behind it is that the electrical activity (potential) generated (evoked) in the brain by any stimulus event will be in phase with the stimulation. The background activity (electroencephalogram or EEG) which is not in phase with the stimulation will "average" to zero, or close to zero, after a certain number of trials. Pilot data in this experiment indicated that about fifty trials were sufficient to allow this to happen.

Then we had the problem of how to represent a picture of a head on the computer graphics system. I decided to use an ellipse to represent a typical head. This would be as if the head were flattened out and the ears were at the end of the long axis. To get a position on each of the electrodes I laid a 64X64 unit grid over the head. Then it is possible to locate each of the channels by an X and Y coordinate. The value derived from the summation would be the Z coordinate. All channels are recorded in a unipolar manner with the electrical reference point being linked earlobes for all leads. Nineteen are active brain channels and the twentieth is an eye movement detecting lead. A ground electrode is placed on the subject's forehead. This technique is conventional for all clinical recording of brain electrical activity. For the location of each of the nineteen probes, see Figure 2. The next problem

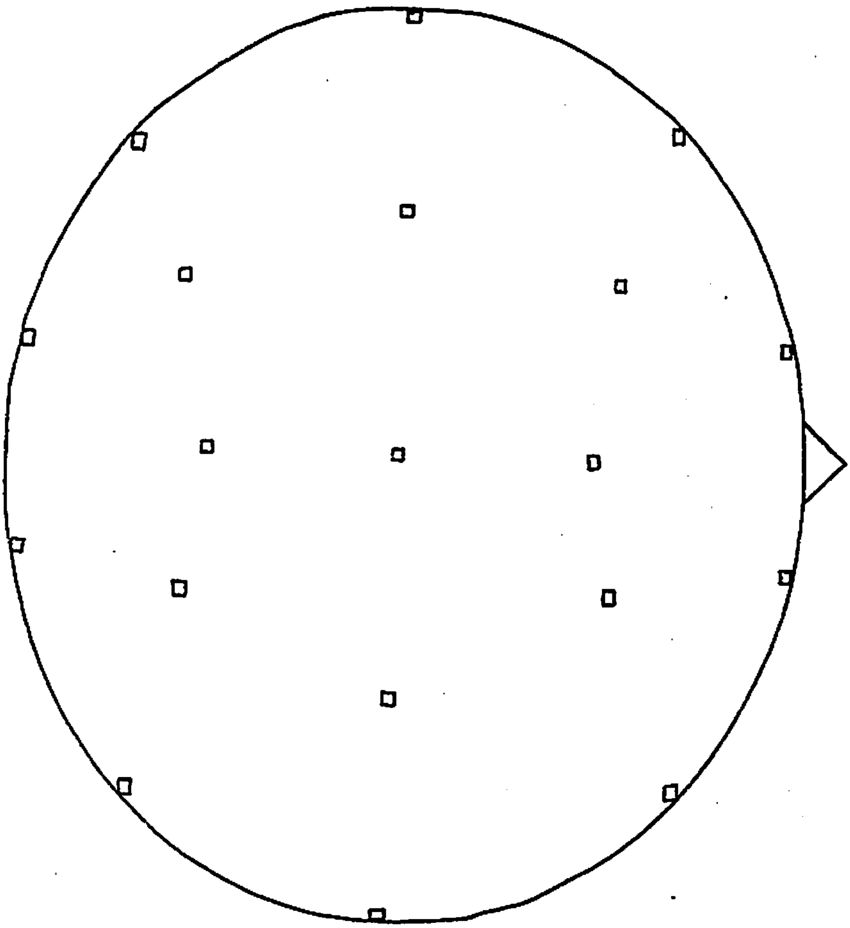


Figure 2

was to compute Z values for the points in the region which are not probe points. This was done by the following method of linear interpolation in two variables which returns a continuous function u(X,Y) over the entire brain region. Continuity is important so there are not large breaks where the triangles join. On the pictures this would show off the individual triangles instead of the brain wave regions. Within a triangle formed by three non-collinear interpolation points, say (X_1,Y_1), (X_2,Y_2), and (X_3,Y_3) we construct a function U(X,Y) of the form

$$U(X,Y) = A + B*X + C*Y$$

If we require that

$$U(X_i,Y_i) = u(X_i,Y_i) = u_i, \quad i = 1,2,3$$

we get

$$(1) \quad \begin{aligned} u_1 &= A + B*X_1 + C*Y_1 \\ u_2 &= A + B*X_2 + C*Y_2 \\ u_3 &= A + B*X_3 + C*Y_3 \end{aligned}$$

Since the three interpolation points are not collinear,

$$D = \begin{vmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ 1 & X_3 & Y_3 \end{vmatrix} = (X_2-X_1)(Y_3-Y_1)-(X_3-X_1)(Y_2-Y_1)$$

is not zero and we may use Cramer's rule to solve (1). Thus,

$$A = \frac{\begin{vmatrix} u_1 & X_1 & Y_1 \\ u_2 & X_2 & Y_2 \\ u_3 & X_3 & Y_3 \end{vmatrix}}{D} = \frac{u_1(X_2Y_3-X_3Y_2)-u_2(X_1Y_3-Y_1X_3) + u_3(X_1Y_2-Y_1X_2)}{D}$$

$$B = \frac{\begin{vmatrix} 1 & u_1 & Y_1 \\ 1 & u_2 & Y_2 \\ 1 & u_3 & Y_3 \end{vmatrix}}{D} = \frac{(u_2 - u_1)(Y_3 - Y_1) - (u_3 - u_1)(Y_2 - Y_1)}{D}$$

$$C = \frac{\begin{vmatrix} 1 & X_1 & u_1 \\ 1 & X_2 & u_2 \\ 1 & X_3 & u_3 \end{vmatrix}}{D} = \frac{(X_2 - X_1)(u_3 - u_1) - (X_3 - X_1)(u_2 - u_1)}{D}$$

Substituting in for A, B, and C in $U(X,Y) = A+BX+CY$, collecting the terms containing u_1, the terms containing u_2, and the terms containing u_3 in separate groups we obtain.

$$U(X,Y) = \frac{u_1((X_2Y_3 - X_3Y_2) + (Y_2 - Y_3)X + (X_3 - X_2)Y)}{D}$$

$$+ \frac{u_2((Y_1X_3 - X_1Y_3) + (Y_3 - Y_1)X - (X_3 - X_1)Y)}{D}$$

$$+ \frac{u_3((X_1Y_2 - Y_1X_2) - (Y_2 - Y_1)X + (X_2 - X_1)Y)}{D}$$

This is not easy to use in computation so with some factoring we can get it into a more workable formula. I will only work with u_1 since u_2 and u_3 will follow in a similar fashion. By multiplying each term out and then adding and subtracting XY we get:

$$u_1(X_2Y_3 - X_2Y - XY_3 + XY - X_3Y_2 + XY_2 + X_3Y - XY)$$

and that is equal to:

$$u_1((X_2 - X)(Y_3 - Y) - (Y_2 - Y)(X_3 - X)$$

Therefore, if we do this to each of the three terms we arrive at:

$$U(X,Y) = A_1u_1 + B_1u_2 + C_1u_3$$

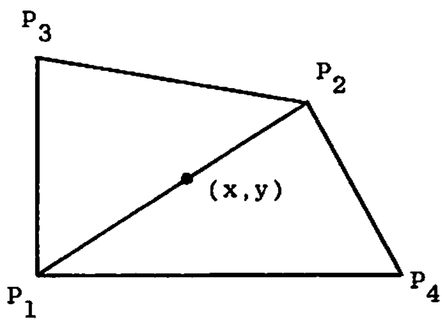
where

$$A_1 = \frac{(X_2 - X)(Y_3 - Y) - (Y_2 - Y)(X_3 - X)}{(X_2 - X_1)(Y_3 - Y_1) - (X_3 - X_1)(Y_2 - Y_1)}$$

$$B_1 = \frac{(X_3 - X)(Y_1 - Y) - (X_1 - X)(Y_3 - Y)}{(X_2 - X_1)(Y_3 - Y_1) - (X_3 - X_1)(Y_2 - Y_1)}$$

$$C_1 = \frac{(X_1 - X)(Y_2 - Y) - (X_2 - X)(Y_1 - Y)}{(X_2 - X_1)(Y_3 - Y_1) - (X_3 - X_1)(Y_2 - Y_1)}$$

If we choose non-overlapping triangles which cover the region of interest, then we obtain in this way a function u(x,y) which is continuous in this region. For, if we have two non-overlapping triangles $P_1P_2P_3$ and $P_1P_2P_4$ with a common side P_1P_2, then the interpolating functions for the two triangles have the same values on P_1P_2. This follows since on any straight line, in particular a side of a triangle of interpolation u(X,Y) is a linear function of the distance along the side. Thus, suppose (X,Y) lies on the side P_1P_2.



Then,

$$\frac{Y - Y_1}{Y_2 - Y_1} = \frac{X - X_1}{X_2 - X_1} = s$$

Since $U(X_1,Y_1) = u_1$ we have

$$u_1 = A + BX_1 + CY_1$$

Hence

$$U(X,Y) = U(X_1 + s(X_2 - Y_1),\ Y_1 + s(Y_2 - Y_1))$$
$$= A + B(X_1 + s(X_2 - X_1)) + C(Y_1 + s(Y_2 - Y_1))$$

and

$$U(X,Y) - u_1 = s(B(X_2 - X_1) + C(Y_2 - Y_1))$$

so that U is a linear function of s. Since the distance from P_1 to (X,Y) is

$$d = \sqrt{(X-X_1)^2 + (Y-Y_1)^2} = s\sqrt{(X_2-X_1)^2+(Y_2-Y_1)^2}$$

it follows that $U(X,Y) - u_1$ is a linear function of d.

It is easy to section off the brain into non-overlapping triangular regions using the known probe points (see Figure 3). This allows each of the points between the known points to be interpolated and given a value. Once there is a value for each of the units in the 64X64 unit grid, the picture can be drawn in whatever method the programmer desires. In this case,

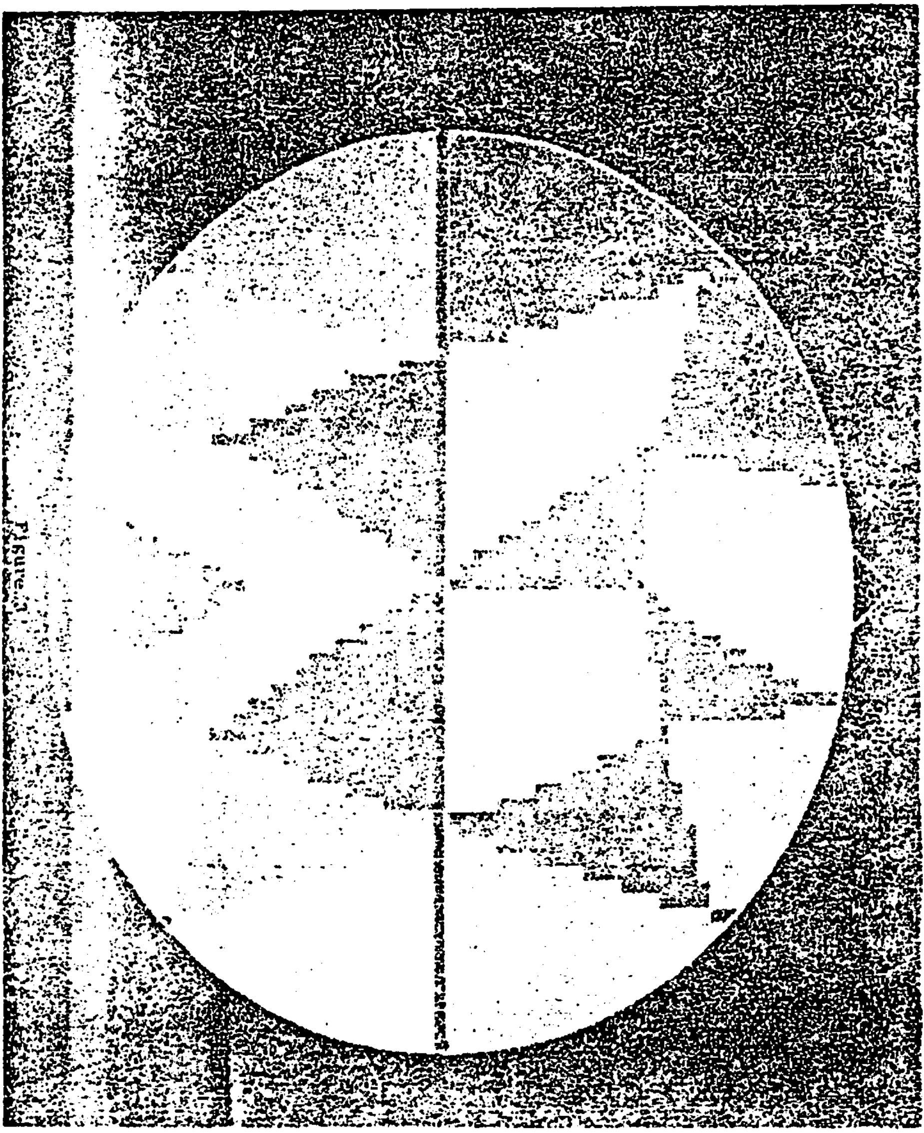
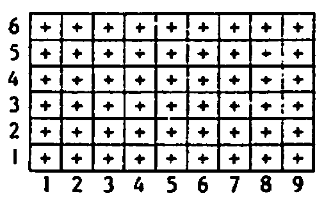


Figure 3

Dr. Patterson requested that it be done using colors to represent the different electrical values (see Figure 4 for a sample; although the figures were produced in color, here they are reproduced in black and white where different shades represent different colors.) In order to do this, the midpoint of each of the boxes is the X,Y coordinate of the box and the point that is sent to the linear interpolation function. Each of the probe points also lies in the center of a specific box. Then when the picture is drawn, the entire box region is shaded with the color that the midpoint represents.



With these pictures Dr. Patterson can look at any one of the four hundred time-frames and see

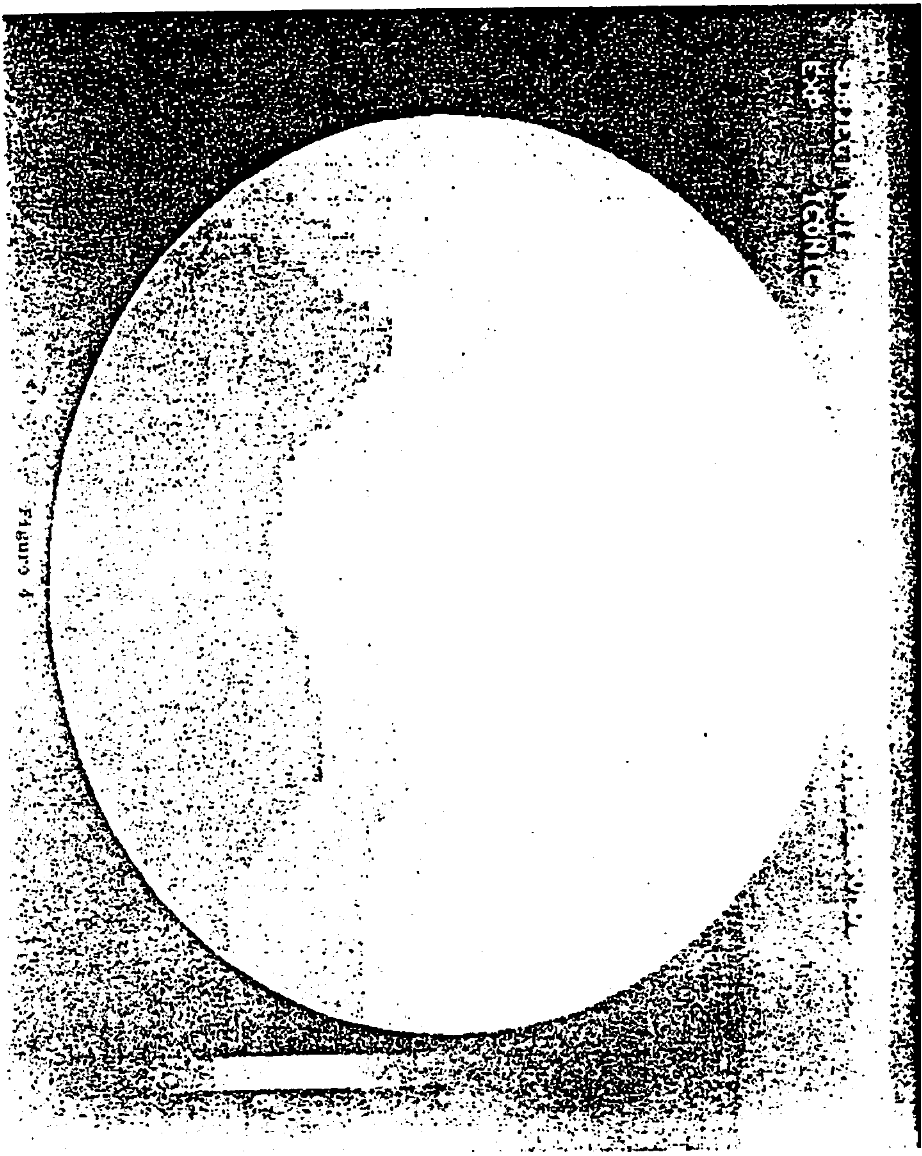


Figure 4

approximately what is happening in the brain. This is easier than trying to interpret what is happening with twenty different probes by simply looking at line drawings.

This method of illustrating what the brain is doing is extremely helpful for the researcher and linear interpolation is what makes it possible. This helps the researcher "see" the difference between brain waves and, hopefully, speed up the process of deciding which theories are worth pursuing further and which theories should be discarded.

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THE NUDE NUMBERS*

Roberto A. Ribas
Student, Central Missouri State University

Among the infinitude of natural numbers, one group that has caught my eye is the nude numbers.[1] A nude number is one that, by its decimal representation, exposes some of its factors.

<u>**Definition**</u> Let N be a natural number with decimal representation

$$N = d_n d_{n-1} \ldots d_2 d_1.$$

Then N is <u>nude</u> if and only if d_i divides N for each $i=1,2,3,\ldots n$.

Some examples of such numbers are:

12 is nude since 1 divides 12 and 2 divides 12.

312 is nude since 3 divides 312, and 1 divides 312, and 2 divides 312.

Note that there exists an infinitude of nude numbers. For example, each of 1, 11, 111,... is nude.

*A paper presented at the 1985 National Convention of KME and awarded fourth place by the Awards Committee.

In fact, it is clear that any number consisting of repeated digits other than 0 is nude. For example 22, 333 and 7777 are each nude. Thus the following theorem:

<u>Theorem</u> Any natural number consisting entirely of repeated non-zero digits is nude.

<u>Proof</u> Let $(d)_n = d_n d_{n-1} \dots d_1$ where

$$d_n = d_{n-1} = \dots = d_1.$$

Then $(d)_n/d = (1)_n$ and thus $(d)_n$ is nude.

A more interesting question is, "What is the largest number of digits a nude number can have such that all the digits are distinct?" Since zero cannot be a digit in a nude number, because division by zero is prohibited, a nude number with distinct digits can have only 1, 2, 3, 4, 5, 6, 7, 8, 9, in its decimal representation. Note that if 5 and any of 2, 4, 6, 8 are digits of a number, the number is divisible by 10 and has a zero for the final digit. To be nude and have a digit of 5 means only 5, 1, 3, 7, 9 may be used. Surely a number with more than five digits can be found. Drop 5 from the list. This leaves the 8 digits

1, 2, 3, 4, 6, 7, 8, 9. The sum of the remaining eight is 40. Since 40 is not divisible by 3 or 9, any permutation of these eight digits will likewise not be divisible by 3 or 9. Two options are presented.

1. Remove the 3, 6, and 9, leaving five digits. (No improvement!)
2. Remove another digit, so that the sum of the remaining digits is a multiple of 9, leaving seven digits.

Since the object is to maximize the number of digits in the nude number, follow option 2. If the 4 is taken, the sum of the remaining seven digits is 36 = 4X9. Thus a permutation of these seven digits will be divisible by 3 and 9. Generating the 7! (5040) permutations, one finds that 105 are nude.[2] The largest number of distinct digits that a nude number can have is seven.

<u>Example</u> 1293768 = N

N/2 = 646884	N/7 = 184824
N/9 = 143752	N/6 = 215628
N/3 = 431256	N/8 = 161721

In searching for other properties of nude numbers notice that 11 and 12, consecutive natural numbers, are both nude. This raises the question, "Can three or more consecutive nude numbers exist?" A start on the answer is found in the following theorem.

__Theorem__ The only digit that can differ in consecutive nude numbers is the last (d_1) digit.

__Proof__ Let $d_n d_{n-1} \ldots d_2 d_1 = X$

$$d_n d_{n-1} \ldots d_2 (d_1+1) = X+1$$

For the addition (d_1+1) to affect d_2, d_1 must equal 9. Adding 1 causes the d_1 digit to change to 0, but 0 is not allowed as a digit in a nude number.

Since $d_1 \neq 0$ and $d_1 \neq 9$, an upper bound on the number of possible consecutive nude numbers is 8.

Further analysis yields more surprising details. Consider two consecutive nudes:

$$d_n d_{n-1} \ldots d_1 = X$$

$$d_n d_{n-1} \ldots (d_1+1) = X+1$$

By the definition of nudes $d_t | X$ and $d_t | X+1$ for $1<t\leq n$. But this implies $d_t = 1$. Therefore, each digit of X, with the exception of the last (d_1) digit, must be equal to 1. Examining the impossible last digits gives the possible consecutive nude numbers.

<u>Decimal form</u> $X = 111...1d_1$ n digits.

The number X can be expressed as the following geometric series:

<u>Open form</u> $X = 10^{n-1}+...+10+d_1$ n digits

<u>Closed form</u> $X = \frac{(10^{n-1}-1)10}{9} + d_1$ n digits

Using the closed form, the consecutive nude numbers can now be fully characterized.

<u>Theorem</u> Let X be a natural number of the form

$$X = \frac{(10^{n-1}-1)10}{9} + d_1 \quad \text{n digits}$$

where d_1 is a non-zero digit. Then X is nude if and only if at least one of the following conditions hold.

1) $d_1 = 1$

2) $d_1 = 2$

3) $d_1 = 3$ and $3|n-1$

4) $d_1 = 4$ and $n = 1$

5) $d_1 = 5$

6) $d_1 = 6$ and $3|n-1$

7) $d_1 = 7$ and $6|n-1$

8) $d_1 = 8$ and $n = 1$

9) $d_1 = 9$ and $9|n-1$

__Proof__ Given a non-zero value for d_1, for what possible values of n does the following consequence hold?

$$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod{d_1}$$

This test determines whether X is nude or not. It follows immediately that

$$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod{1}$$

$$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod{2}$$

and $$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod{5}$$

for all n since 11...10 is always divisible by 1, 2 or

5. For example, 11...11 is nude, 11...12 is nude and 11...15 is nude. Thus, for $d_1 = 1,2,5$ and $n \geq 1$, X is a nude number.

On the other hand, tests for divisibility by 4 imply that

$$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod 4$$

and

$$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod 8$$

only if n = 1. For example, 10, 110 nor 1110 are not divisible by 4 and hence not divisible by 8. Therefore 14, 114 and 1114 are not divisible by either 4 or 8 and the congruence fails.

Similarly, tests for divisibility by 3 and 9 imply

$$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod 3$$

and

$$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod 6$$

if and only if 3 divides n-1, and

$$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod 9$$

if and only if 9 divides n-1. The condition here amounts to forcing the 1's to appear in groups of three (or nine). For example 1110 has n=4 digits so n-1 = 3 if the digits are 1's. Then 1113 is divisible by 3.

Now we need to investigate $d_1 = 7$. That is, what values of n make the following congruence true?

$$\frac{(10^{n-1}-1)10}{9} \equiv 0 \pmod 7$$

Since 10 and 7 are relatively prime, this congruence is true if and only if

$$10^{n-1} \equiv 1 \pmod 7$$

The order of 10 modulo 7 is 6, therefore 6 must divide n-1.

The above theorem implies that the maximum number of consecutive nude numbers is three if n > 1. They occur at

$$\underbrace{11\ldots1}_{n-1}1 \quad \underbrace{11\ldots1}_{n-1}2 \quad \underbrace{11\ldots1}_{n-1}3 \quad \text{iff 3 divides } (n-1)$$

and

$$\underbrace{11\ldots1}_{n-1}5 \quad \underbrace{11\ldots1}_{n-1}6 \quad \underbrace{11\ldots1}_{n-1}7 \quad \text{iff 6 divides } (n-1)$$

For all values of d_1 except 7, the conditions of the theorem can be observed without the use of modular arithmetic. Clearly 111...1, 111...2, and 111...5 are nude numbers. Also, 111...3, and 111...6 are nude numbers if the sums of their digits can be divided by 3, thus 3 divides (n-1). Note that 111...9 is nude only if 9 divides the sum of its digits, 9 divides (n-1). It can also be demonstrated that 111...4 and 111...8 are not nude numbers, except for the trivial case of n=1 digit. Consider

$$111...14 = 10^n + 10^{n-1} + \dots \quad 10 + 4.$$

In order to be nude, the number must be evenly divisible by 4. This implies

$$10^n + 10^{n-1} + \dots + 10 + 4 = 4Xt \qquad t \in N$$

$$\frac{10^n + 10^{n-1} + \dots + 10 + 4}{2X2} = t$$

$$\frac{2^{n-1}X\ 5^n + 2^{n-2}X\ 5^{n-1} + \dots + 5 + 2}{2} = t$$

(dividing by 2)

$$\frac{5(10^{n-1} + 10^{n-2} + \dots + 10 + 1)}{2} = t$$

However, the numerator is an odd number, and cannot be evenly divided by 2. A similar proof exists for 111...18.

Earlier, the infinitude of the nude numbers was demonstrated. Now, consider the "density" of the nude numbers. Using a simple program[3], the nude numbers from 1 to 999999 were generated. The number of nudes by category and their densities are compiled in Table 1.

Densities

Digits	Number of Nudes	10^n	Density
1	9	10	.9
2	13+9	100	.22
3	57+13+9	10^3	.079
4	261+57+13+9	10^4	.034
5	1,307+261+57+13+9	10^5	.01647
6	7,395+1307+261+57+13+9	10^6	.009042

Table 1

Density was calculated as follows:

$$\text{Density} = \frac{\text{number of nudes with n digits or less}}{10^n}$$

Notice in Table 1 that the density of the nude numbers decreases with each increase in digits. The next theorem means that the density of the nudes of n digits or less, as n increases without bound, approaches 0.

Theorem The density of the nudes is zero.

Proof Remember that 0 can never be a digit in a nude number. Thus an upper bound for the density is:

Digits	Number of numbers with non-zero digits	Upper bound density
1	9	$9/10$
2	$9^2 + 9$	$(9^2+9)/10^2$
3	$9^3 + 9^2 + 9$	$(9^3+9^2+9)/10^3$
...	...	...
n	$9^n + \ldots + 9$	$(9^n+9^{n-1}+\ldots+9)/10^n$

The following limit should suggest the conclusion of the theorem if it is zero.

$$\lim_{n\to\infty} \frac{9^n+9^{n-1}+\ldots+9^2+9}{10^n}$$

The numerator is a geometric series with sum

$$\frac{9(1-9^n)}{-8}$$

Thus, the limit becomes

$$\lim_{n\to\infty} \frac{9(1-9^n)}{10^n(-8)} = \lim_{n\to\infty} \frac{9^{n+1}}{10^n X 8} - \lim_{n\to\infty} \frac{9}{10^n X 8}$$

$$= \lim_{n\to\infty} \frac{9^{n+1}}{10^n X 8} - 0$$

$$= \frac{9}{8} \lim_{n\to\infty} \left(\frac{9}{10}\right) = 0.$$

With a limit of 0 and the fact that densities cannot be negative, the density of the nude numbers is 0.

These are the answers to a few interesting questions that can be raised about nude numbers. I would now like to propose some further problems.

1) Can the nude numbers be characterized? If not, can they be proven?

2) What is the expected value of the nudes? Can

$$\lim_{k\to\infty} \frac{1}{k} \sum_{n=1}^{k} a_n$$

be solved?

3) Define degree of nudity as follows:

Total of digital, digital pair, digital triple, etc. factors.

<u>Example</u> 1248

1248/1=1248; 1248/2=624; 1248/4=312

1248/8=156; 1248/12=104; 1248/24=52

1248/48=26.

Thus, the degree of nudity of 1248 is 7.

<u>Question</u> What are the nudest 2,3,4,5, digit numbers?

<u>Conjecture</u> For n= a non-prime number of digits, the repeated digits ($d_n=d_{n-1}=d_{n-2}\ldots=d$) will have the greatest degree of nudity.

Notes

[1]The name "nude number" was introduced by Yoshinao Katagiri, to whom I am greatly indebted for the idea. He brought it up in a letter to the editor of the Journal of Recreational Mathematics, Vol. 15(4), 1982-83. I would like to note, however, that no properties or conjectures were proposed, and in fact, no definition of a nude number was published.

[2]Generated by a simple permutation program. Each result was then checked to insure that all seven digits could be divided evenly.

[3]Waterloo Basic Program to generate 4 digit nude

numbers. Four nested loops from 1 to 9, each representing one digit of the number. The interior of the loops check the number generated by using the MOD function on each of the four digits.

THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1986. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1986 issue of The Pentagon, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

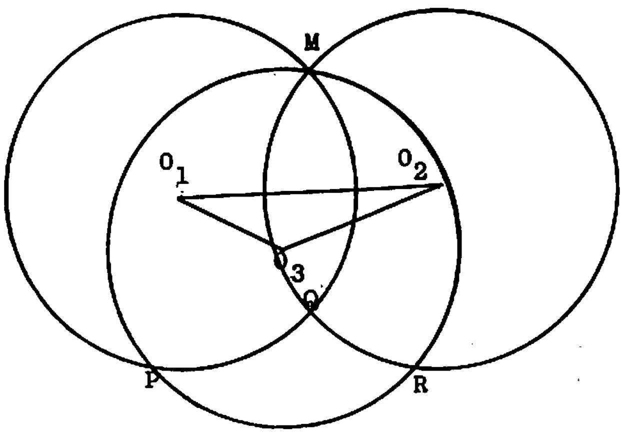
PROPOSED PROBLEMS

Problem 387: Proposed by Charles W. Trigg, San Diego, California.

Two n-digit primes are said to be complementary if their sum is 10^n. Show that with the single exception of 3, 7 both primes must be of the form 6k-1.

Problem 388: Proposed by the Dmitry P. Mavlo, Moscow, U.S.S.R.

Three circles of equal radii r with respective centers O_1, O_2, and O_3 have the common point M as shown in the figure below. Denote by P the point of intersection of circles O_1 and O_3, by Q the intersection of the circles O_1 and O_2 and by R the point of intersection of the circles O_2 and O_3. Denote by $S_{O_1O_2O_3}$ the area of the triangle $O_1O_2O_3$ and by S_{PQR} the area of the "curved triangle": PQR. Prove that $S_{PQR} = 2\, S_{O_1O_2O_3}$.



Problem 389: Proposed jointly by Ambati Jaya Krishna, Johns Hopkins University, and Mrs. Gomathi S. Rao, Orangeburg, New Jersey.

$$\text{Let } M = 1 + \frac{1}{3} + \frac{2!}{3\cdot 5} + \frac{3!}{3\cdot 5\cdot 7} + \cdots .$$

Does the following sum converge?

$$\sum_{n=1}^{\tan M} (-1)^{n+1} \sum_{s=0}^{n} \sum_{i=0}^{s} (-1)^{s-1}\ {}_nC_s\ {}_sP_i .$$

Problem 390: Proposed by Fred A. Miller, Elkins, West Virginia.

Bisect the area under one arch of the curve $y = \sin x$ by drawing a line from the origin to the curve.

Problem 391: Proposed by the editor.

Let r be a real number such that r^{1835} and r^{1986} are both integers. Prove that r is an integer also.

SOLUTIONS

377: Proposed by Charles W. Trigg, San Diego, California.

The only square integer that is the concatenation of two two-digit squares is $1681=41^2$. Find a six digit square integer, N^2, that is the concatenation of three two-digit squares.

Solution by A. Jaya Krishna, Johns Hopkins University, Baltimore, Maryland.

If N satisfies the given requirements, then there exist single digit positive integers, A, B, C, a, b, and c, such that

$$N^2 = 10^4A^2 + 10^2B^2 + C^2 = (10^2a+10b+c)^2.$$

The quantity $d = 10b + c \leq 99$. Thus $A = a$, for otherwise we have

$N = (10^2a + d)^2 \leq 10^4a^2 + 2(100)99a + 99^2$

$\leq 10^4(a + 1)^2 \leq N$.

Thus $8181 \geq 10^2B^2 + C^2 = N-10^4a^2 =$

$10^3(2ab)+ 10^2(b^2 + 2ac) + c^2$.

In particular, $2ab \leq 8$. Thus either $b = 0$ or $b = 1$ and $a = 4$. Since no digit of N is zero, $a \geq 4$. In the latter case, $N = (410a + c)^2$ with $c \geq 1$. Since no value of c produces a solution, we must have $b = 0$.

Therefore, $N = (10^4a^2+c)^2 = 10^4a^2 + 10^2(2ac)+c^2$. Thus, $a \geq 4$, $c \geq 4$ and 2ac is an even two-digit square. Hence either $a = 4$ and $c = 8$ with $N = 646416$ or $a = 8$ and $c = 4$ with $N = 166464$.

Also solved by: Oscar Castenada, St. Mary's University, San Antonio, Texas; Fred A. Miller, Elkins, West Virginia; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; and the proposer.

Editor's Comment: Several solvers noted the following "near misses" provided by the squares of: 102, 108, 201, 204, 209, 306, 402, 603, 801, and 902. These are allowed if zero is acceptable as a ten's digit in a two-digit integer.

378: Proposed by Charles W. Trigg, San Diego, California.

Show that in every system of notation with an even positive base there is a repdigit triangular number.

Solution by Oscar Castenada, St. Mary's University, San Antonio, Texas.

Revised by the editor. The general formula for a triangular number is given by

$$\frac{n(n+1)}{2} .$$

By taking $n = 2t$ in thus formula, one obtains

$$\frac{2t(2t+1)}{2} = t(2t) + t = t\,t_{(\text{base } 2t)} .$$

Thus for $n = 2t$ where t is a positive integer, $t\,t$ is a repdigit triangular number in base $2t$.

Also solved by: Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin and the proposer.

379: Proposed by the editor.

Let $< 5 >$ denote the fractional part of $\sqrt{5}$. Then $\sqrt{5} = \sqrt{5} - \sqrt{4}$. Also $(< 5 >)^2 = \sqrt{81} - \sqrt{80}$ and $(< 5 >)^3 = \sqrt{1445} - \sqrt{1444}$.

Prove that for each positive integer n, there is an integer T_n such that (< 5 > $)^n = \sqrt{T_n} - \sqrt{T_n - 1}$ and show how T_n can be found.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

We shall use the following known results:

(1) All solutions of $x^2 - 5y^2 = -1$ are given by (x_n, y_n) where $x_n - y_n = (2 - \sqrt{5})^n$ = with n = 1,3,5, ... and all solutions of $x^2 - 5y^2 = 1$ are given by the same (x_n, y_n) with n = 2,4,6, ...[1]

(2) $\sqrt{5}$ = (2; 4) where (2; 4) denotes the continued fraction expansion of $\sqrt{5}$.

(3) All positive integer solutions of $x^2 - 5y^2 = \pm 1$ are to be found among $x = h_n$, $y = k_n$, where h_n/k_n are the convergents of the continued fraction expansion of $\sqrt{5}$. More specifically, by [2]. $x = h_{n-1}$, $y = k_{n-1}$ give all positive integer solutions of $x^2 - 5y^2 = -1$ when n is odd and all positive integer solutions of $x^2 - 5y^2 = 1$ when n is even.

The initial convergents of the continued fraction expansion of 5 are:

$h_0/k_0 = 2/1$; $h_1/k_1 = 9/4$; $h_2/k_2 = 38/17$; $h_3/k_3 = 161/72$ and $h_4/k_4 = 682/305$. It follows that

$$< 5 > = \sqrt{5} - 2 = \sqrt{5} - \sqrt{4}$$

$$(< 5 >)^2 = 9 - 4\sqrt{5} = \sqrt{81} - \sqrt{80}$$

$$(< 5 >)^3 = 17\sqrt{5} - 38 = \sqrt{1445} - \sqrt{1444}$$

$$(< 5 >)^4 = 161 - 72\sqrt{5} = \sqrt{25921} - \sqrt{25920}$$

$$(< 5 >)^5 = 305\sqrt{5} - 682 = \sqrt{465125} - \sqrt{465124}$$

and so on.

[1] Niven and Zuckerman, An Introduction to the Theory of Numbers, Fourth Edition, Wiley and Sons, New York, 1980, p. 214 ex. 1.

[2] Ibid., Theorem 7.25, pp. 211-212.

Also solved by: Oscar Castenada, St. Mary's University, San Antonio, Texas.

380: Proposed by the editor.

A golden rectangle is one whose side a and b satisfy the proportion a:b = b:(a - b). Suppose that this golden rectangle has been drawn on a coordinate axis with the longer side lying on the x axis. We mark off on the left side of the rectangle the largest possible square; a smaller golden rectangle remains. We then rotate the figure 90 degrees counterclockwise so that the longer side of the smaller golden rectangle lies along the x axis. Again we mark off the largest possible square on the left side of this new rectangle. By repeating this process, continually smaller golden rectangles are constructed until only one point in the original rectangle remains. What is the location of this unique point?

No solution was received for this problem. If no solution is received prior to preparation of the next column, a solution will be published at that time.

381: Proposed by the editor:

Little Euclid has a toy box in the shape of a rectangular parallelopiped. The length of each side and the length of the diagonal of each lateral face is an integral number of inches. If the sum of the three side lengths is 401 inches, what are the dimensions of the box?

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

In [1] Sierpinski reports that if we take $a = x(4y^2 - z^2)$, $b = y(4x^2 - z^2)$, and $c = 4xyz$, where x, y, and z are positive integers such that $x^2 + y^2 = z^2$, then

$a^2 + b^2 = z^6$, $a^2 + c^2 = x^2(4y^2 + z^2)^2$, and $b^2+c^2 =$
$= y^2(4x^2 + z^2)^2$.

Thus from a given solution of the equation $x^2 + y^2 = z^2$ in positive integers, we obtain positive integers a, b, and c such that the sum of the squares of any two of them is the square of a positive integer. The numbers a, b, and c are then the sides of a rectangular parallelopiped such that the diagonals of its faces are positive integers. In particular, putting $x = 3$, $y = 4$, and $z = 5$ yields $a = 117$, $b = 44$, and $c = 240$, $a^2 + b^2 = 125^2$, $a^2 + c^2 = 267^2$, $b^2 + c^2 = 125^2$, and $a + b + c = 401$. This particular solution was found by P. Halcke in 1719.

[1] W. Sierpinski, Elementary Theory of Numbers, Haefner Publishing Co., New York, 1964.

Also solved by Oscar Castenada, St. Mary's University, San Antonio, Texas and Charles W. Trigg, San Diego, California.

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical issues of interest to undergraduate students are discussed in length. Here by issue we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.

The following article is a continuation of the article printed in THE HEXAGON of the Fall '83 issue of THE PENTAGON. Both articles are on 'mathematical games', a convenient forum for description of some very interesting results. The central idea of the games described in the following article was invented in 1928 by the Polish mathematician S. Mazur. Later, S. Banach worked on the idea and now the games are known as Banach-Mazur games.

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INFINITE GAMES IMMORTALS PLAY

Galen Weitkamp*
Department of Mathematics
Western Illinois University
Macomb, Illinois 61455

§4. Olympic Games

Two greek gods, Apollo, the god of beauty and form, and Dionysus, the god of drink and joy, together decide to play the following game.

Apollo begins. On each of his turns Apollo selects a real number from the interval [0,1]... repetition allowed. We let a_i denote his i^{th} selection.

Dionysus on his turn selects an integer from the set $\{0,1,2,\ldots,9\}$. We denote the i^{th} response by d_i.

The gods decide to use a game clock. The first move of the game must be completed within one hour. The second within one half hour. The third within one quarter hour, etc. The entire game is over within $2(=\sum_{i=0}^{\infty} 1/2^i)$ hours.

*Professor Weitkamp received his Ph.D. in mathematical logic from Pennsylvania State University and has been with Western Illinois University since 1980. His interests include set theory and logic.

At the game's end Apollo will have generated the set $A = \{a_1, a_2, \ldots\}$ and Dionysus will have created the real number $d = \sum_{i=1}^{\infty} d_i/10^i$ in $[0,1]$. We say Apollo wins iff $d \in A$. Thus in the course of play Apollo tries to anticipate the number d and enumerate it into A. Dionysus tries to avoid being listed in A.

The game above we call ENUMERATION.

The most interesting feature of ENUMERATION is that it is determined. In fact Dionysus can always win! In order to win Dionysus only needs to use the following algorithm:

On his i^{th} move Dionysus first examines the canonical decimal expansion of

$$a_i = \sum_{i=1}^{\infty} a_{ij}/10^j \quad (a_{ij} \in \{0,1,\ldots,9\}).$$

Then Dionysus chooses $d_i \in \{0,1,2,\ldots,8\}$ so that $d_i \neq a_{ii}$.

By following this prescription the real d cannot be in the set $\{a_1, a_2, \ldots\}$. Indeed $d \neq a_i$ because the i^{th} decimal place of d is different from that of a_i.

The 'diagonal strategy' described above was first invented by Georg Cantor to prove the following theorem.

Theorem: The set [0,1] is uncountable.

Proof: If [0,1] were countable, then Apollo could list each and every element in [0,1] and thereby win every play of ENUMERATION!

We now define in general what we mean by a game. A game is a tree T (not necessarily well-founded) whose endnodes and whose infinite paths are labeled I, II or Σ.

A winning strategy of I for T from the position $\langle x_0,\ldots,x_n\rangle$ is a subtree S of T so that

(i) $\langle x_0,\ldots,x_n\rangle \in S$;

(ii) if $\langle x_0,\ldots,x_n,\ldots,x_m\rangle \in S$ and n is even, then for every $x \in X$, $\langle x_0,\ldots,x_n,\ldots,x_m,x\rangle \in T$ implies $\langle x_0,\ldots,x_n,\ldots,x_m,x\rangle \in S$;

(iii) if $\langle x_0,\ldots,x_n,\ldots,x_m\rangle \in S$ and n is even and if $\langle x_0,\ldots,x_n,\ldots,x_m\rangle$ is not an endnode of T, then it's not an endnode of S;

(iv) No endnode of S nor any infinite path through S is labeled II.

Similarly one may define a winning strategy II for T from $\langle x_0,\ldots,x_n \rangle$.

<u>EXERCISE</u>: Define the diagonal strategy of Dionysus for ENUMERATION from the position $\langle\ \rangle$ as a subtree S of T.

We say a game is <u>determined</u> if some player has a winning strategy.

§5. <u>Gale-Stewart Games</u>

Let $\mathcal{N}$ denote the collection of all functions from ω into ω, and let $A \subseteq \mathcal{N}$. Let G_A be the tree $\{\langle x_1,\ldots,x_n \rangle : n, x_1,\ldots,x_n$ are whole numbers$\}$. Note G_A has lots of infinite paths and no endnodes. Imagine that the infinite paths through G_A in A are labeled and those not in A are labeled II.

Intuitively Apollo and Dionysus alternately and ever more quickly select whose numbers together creating an infinite sequence $x_0, x_1, x_2, \ldots$. This sequence can be codified by an element f of $\mathcal{N}$, namely $f(n) = x_n$. Apollo wins iff $f \in A$. Thus Apollo tries to direct the sequence into A and Dionysus tries to steer the sequence clear of A.

Games of the form G_A are called Gale-Stewart games. The theory of these games is very rich and (as we shall see) very perplexing.

Given a sequence $\langle x_0,\dots,x_n\rangle$ of whole numbers define $N(x_0,\dots,x_n) = \{f \in N : \forall i \le n \quad f(i) = x_i\}$. It is not difficult to see that the set of all $N(x_0,\dots,x_n)$ (as $(x_0,\dots,x_n)$ ranges through the class of all whole number sequences) forms a basis for a topology on N. Taken with this topology N is called the Baire space.

EXERCISE: Let ω be given the discrete topology and the countable cartesian product $\omega \times \omega \times \dots$ be given the corresponding product topology. Prove this product is homeomorphic to the Baire space.

If T is a tree over ω let [T] denote the set of all infinite paths through T. Note that $[T] \subseteq N$.

Theorem: A subset A of N is closed iff there is a tree T over ω so that A = [T].

Proof: Let T be a tree over ω and let f be a 'point' of N not in [T]. Since f is not a path through T there is an $n \in \omega$ so that $\langle f(0),\dots,f(n)\rangle \notin T$. This implies that if $U = \mathcal{N}(f(0),\ldots,f(n))$ then $U \cap [T] = \emptyset$. Hence every point f in $\mathcal{N}-[T]$ is contained in a neighborhood U which is a subset of $\mathcal{N}-[T]$. This means $\mathcal{N}-[T]$ is open and therefore [T] is closed.

<u>EXERCISE</u>: Complete the proof. Let $T = \{ \langle f(0),\ldots,f(n)\rangle : n \in \omega \text{ and } f \in A\}$. Show $A = [T]$ and prove T is a tree.

<u>Theorem</u>: If $A \subseteq \mathcal{N}$ is closed, then G_A is determined.

<u>Proof</u>: Suppose that player II doesn't have a winning strategy for G_A. Then we shall prove player I does.

Let T be a tree over ω so that $A = [T]$. Define $S = \{\langle x_0,\ldots,x_n\rangle$: II does not have a winning strategy for G_A from position $\langle x_0,\ldots,x_n\rangle\}$. Then S is a tree.

If $\langle x_0,\ldots,x_n\rangle \notin T$, then <u>no</u> infinite sequence f extending $\langle x_0,\ldots,x_n\rangle$ is in A(i.e. if $f(i) = x_i$ for all $i \leq n$, then $f \notin [T] = A$). Hence player II has a winning strategy for G_A from this position. In fact at this point we know II will win even if he plays randomly. So $\langle x_0,\ldots,x_n\rangle \notin S$. Hence, if $\langle x_0,\ldots,x_n\rangle \in S$, then $\langle x_0,\ldots,x_n\rangle \in T$ and so $S \subseteq T$.

Consequently $[S] \subseteq [T] = A$. Thus if player I can manage to keep every initial segment of play inside S, then he shall win the game.

<u>EXERCISE</u>: Use the proof in §1 as a hint to complete the proof above that S is a winning strategy for player I.

A very difficult theorem in the theory of games asserts the determinacy of Borel games. The <u>class of Borel sets</u> is the smallest family of sets which contains the open subsets of N and is closed under countable unions and complementation. A <u>Borel game</u> is any game of the form G_A where A belongs to the class of Borel sets.

<u>Theorem of Borel Determinacy</u>: If A belongs to the class of Borel sets, then G_A is determined.

Before the proof of Borel determinacy was discovered Harvey Friedman proved that Borel determinacy, if provable at all, could not be proven from Z alone. He ventured the prediction that it could be proven from ZFC. Not long after this announcement D. Martin proved Borel determinacy by making essential use of the replacement axiom. Borel determinacy is one of the very few theorems of ordinary mathematics (mathematics which is not directly concerned with higher set theory) which cannot be carried out in Z alone.

§6. <u>Epistimology and the Theory of Games</u>

In topology a point p is an <u>isolated point</u> of the set A if $p \in A$ and there is a neighborhood U of p which contains no other points of A. A set is <u>perfect</u> if it is non-empty, closed and contains no isolated points.

<u>Theorem</u>: Suppose S is a winning strategy of some player for some Gale-Stewart game. Then [S] is perfect.

<u>Proof</u>: [S] is clearly non-empty and closed. Without loss of generality we may assume S is a strategy for player I. Let f be an arbitrary element of [S], and let U be any neighborhood of f. Then U must contain as a subset some basic open neighborhood of f. Hence there is an $n \in \omega$ so that $f \in N(\langle f(0),\ldots,f(n)\rangle) \subseteq U$. Indeed if $m \geq n$ then $f \in N(\langle f(0),\ldots,f(m)\rangle) \subseteq U$. So

take m to be even. Then by definition of winning strategy for I, $\langle f(0),\ldots,f(m),0\rangle$ and $\langle f(0),\ldots,f(m),1\rangle$ are both in S, and both extend to infinite paths through S. One of these paths g must be different from f. Hence $g \in [S] \cap U$ and $g \neq f$. Hence f is not an isolated point of [S].

<u>Theorem</u>: If G_A is determined, then A either contains or is disjoint from a perfect set (or both).

<u>EXERCISE</u>: Prove the theorem above and its corollaries below.

<u>COROLLARY</u>: Every Borel subset of $\mathcal{N}$ either contains or is disjoint from a perfect set.

A classical theorem of topology (which uses the axiom of choice) asserts

<u>Theorem</u>: There is a set $A \subseteq \mathcal{N}$ which neither contains nor is disjoint from a perfect set.

For a proof of this fact see Oxtoby [1980] pg. 23.

Hence

COROLLARY: Not every Gale-Stewart game is determined.

We are now faced with the obvious question: 'For which subsets A of N is G_A determined?' The question is easier asked than answered.

A subset A of N is analytic if it is the image of a Borel subset B of N under a continuous function $F: N \to N$: i.e. $A = \{F(f): f \in B\}$. It can be shown that every Borel set is analytic but not every analytic set is Borel. Analytic determinacy (abbreviated AND) is the assertion that if A is analytic, then G_A is determined. Is analytic determinacy true or false? No one knows. Worse, it may be the case that we will never know! At present the situation is this.

It is known that

1) AND is not provable from ZFC (i.e. if ZFC is consistent, then so is ZFC + $\neg$AND where $\neg$AND is the negation of AND).

2) If one assumes ZFC is consistent, one cannot prove that ZFC + AND is consistent.

It is not known

3) whether ZFC implies the negation of analytic determinacy (i.e. for some analytic A, G_A is not determined).

Most mathematicians suspect that ZFC does not imply the negation of AND. Hence it is their opinion that ZFC+AND is consistent and so AND is independent of ZFC. The reason for this belief is that AND is a consequence of ZFC+(some other higher axioms). Since early in the century some mathematicians have been investigating axioms that might be added to ZFC. These axioms assert the existence of various kinds of large infinite cardinals and are consequently called 'higher axioms of set theory.' One such variety of cardinals are known as <u>measurable</u> cardinals. Few mathematicians accept these higher axioms because it can be shown that it is impossible to prove that it is even consistent to assume they exist! Yet no one has ever derived an inconsistency using them. It turns out that ZFC+(there is a measurable cardinal) proves AND.

If in fact AND is consistent with ZFC, then we have a situation which is similar to the question of Euclidean vs non-Euclidean geometry. One could accept

ZFC+¬AND as the axioms of set theory, or one could use ZFC+AND! However the situation with '¬AND vs AND' isn't exactly analogous to Euclidean vs non-Euclidean geometry. If one assumes Euclidean geometry is consistent, then one can prove non-Euclidean geometry is consistent. However, if one assumes ZFC+¬AND is consistent one cannot prove ZFC+AND is consistent! The dilemma should now be clear. If ZFC+AND is inconsistent, then there will always be the hope of proving it by finding the inconsistency. But if ZFC+AND is consistent we'll never be able to prove it. The interesting question is: 'if ZFC+AND is consistent (as most mathematicians suspect), then how will we ever know?'

References

Halmos, P. R. Naive Set Theory, D. Van Nostrand Company Inc., Princeton, New Jersey-Toronto-London-New York, 1960.

Oxtoby, J. C. Measure and Category, 2nd Ed., Springer-Verlag, New York-Heidelberg-Berlin, 1980.

THE CURSOR

Edited by Jim Calhoun

The topics presented here can be broadly classified as belonging to computer science but their emphasis is more narrowly defined. Like most of the applied sciences, computer science depends heavily upon a large body of mathematical theory. It is the aim of this department to present discussions which help to define the interface between the disciplines of mathematics and computer science. Specifically, it seeks to relate ideas from the theory of mathematics to an understanding of concepts in computer science. Readers are encouraged to submit articles on any topic which seems directed toward this goal. Address all correspondence to Jim Calhoun, Computer Science Department, Western Illinois University, Macomb, IL 61455.

SHANNON'S THEOREM FOR ENTROPY AND ITS USE IN THE HUFFMAN CODING PROCEDURE

Susan L. Andrews
Student, Bloomsburg University

The statistics of language are based on a characteristic of all physical systems known as "entropy." All fluent speakers of a language utilize the concept of language entropy daily without realizing it, because the entropy of a language is determined by the rules of grammar and the spelling system of that language. Although English uses the same alphabet as many other languages, our spelling system differs considerably. This is why words written in a foreign language, or in code using the characters of the English alphabet, look strange to us.

The concept of entropy in language was introduced by Claude E. Shannon, a forerunner in the field of mathematical communication theory. In Physics, "entropy" describes the amount of UNCERTAINTY or DISORDER in a system. Shannon borrowed this term to describe the LACK OF STRUCTURE in a language. (His work was based on English.)

Entropy, H(X), is mathematically defined to be a summation:

$$H(X) = -\sum_{i=1}^{q} P(x_i) * \log_2[P(x_i)],$$

where X = the set of all source characters,

q = the number of source characters,

and $P(x_i)$ = the probability of the i^{th} source character.

Language entropy goes to zero only when $P(x_i) = 1$, since $\log_2(1) = 0$. This situation occurs when only ONE message can ever be transmitted, and there is NO uncertainty in the system. But then NO INFORMATION is conveyed by the message, because it was completely specified at the beginning.

The fewer the number of restrictions placed on letter permutations acceptable in word construction, the greater is the language entropy. A language in which MAXIMUM entropy is achieved is called an "URN" language. In an urn language, <u>no</u> restrictions are placed on acceptable letter permutations; the text is purely random, and all letters are dictated by chance. Every character in the source alphabet is equally likely to occur at any position in a word; therefore, each source character has an <u>equal probability</u>. Under such conditions, the average amount of information conveyed by each individual character is maximized, and the number of characters required to transmit a message is minimized.

For an urn language of q source characters,

$$P(x_i) = 1/q, \ i = 1,2,3,\ldots,q.$$

$$H_{max}(X) = -\sum_{i=1}^{q} P(x_i)*\log_2[P(x_i)] = -q*(1/q)*\log_2[1/2]$$

$$= -\log_2[1/q] = -[\ln(1/q)/\ln 2].$$

<u>Example</u>:

For an urn language with a 10-character source alphabet, $q = 10$. The probability of every source character is: $P(x_i) = 1/10 = 0.1$. Therefore,

$H_{max}(X_{q=10}) = -[\ln(0.1)/\ln 2] = -(-3.32) =$ <u>3.32 bits/letter</u>.

Unfortunately, no spoken language is constructed in a completely random manner. The rules of grammar and spelling restrict allowable letter combinations in word construction. Successive letters are not selected independently, but rather, the probability of each letter depends on all the letters which precede it. This letter leads to a REDUCTION in language entropy.

To actually CALCULATE the entropy of a language would require that frequency counts be performed first on all 1-letter words, then on all 2-letter words, then on 3-letter words, etc. These frequency counts quickly get out of hand.

<u>Example</u>:

<u>Frequency count on 1-letter English words</u>:

A = 1

I = 1

Frequency of all other letters = 0.

Frequency count on 2-letter words:

Based on the sample including: AT, TO, BE, OF, HE, AS, IN, IT, WE, DO, & ON,

A = 2	F = 1	O = 4
B = 1	H = 1	S = 1
D = 1	I = 2	T = 3
E = 3	N = 2	W = 1

Frequency of all other letters = 0.

Frequency count on 3-letter words:

Just a SMALL sample includes: ACE, ACT, ADO, AFT, AGE, AGO, AID, AIM, AIR, ALE, ALL, ALP, AND, ANT, APE, APT, ARC, ARE, ARK, ARM, ART, ASK, ATE, ...

It is obvious that such an endeavor would be endless!

But Shannon observed that "anyone speaking a language possesses, implicitly, an enormous knowledge of the statistics of that language."[1] He discovered a clever, indirect route by which to ESTIMATE the entropy of the English language by performing 2 simple experiments using average English speakers as his subjects.

<u>EXPERIMENT 1</u>: Shannon counted the number of guesses a subject required to select the NEXT letter in an unknown text.

<u>Example</u>: The dog ran home.

The principles behind this experiment are the same as those we use each time we play the game, "Hangman." Assuming the unknown text is written in English, we normally guess the three most frequently occurring vowels (E, A, and I) first, followed by the consonants having the highest frequencies (T, R, N, and S). Each correct guess narrows the possibilities for the remaining undiscovered text, because we are familiar enough with the English spelling system to realize that not all permutations of letters form acceptable English words.

If all permutations were allowed, there would be 1.68×10^{25} 4-letter words alone!

$$_nP_r = {}_{26}P_4 = 26!/4! = 26!/24 = 1.68 \times 10^{25}.$$

Because certain 4-letter permutations are prohibited by the English spelling system, we must resort to words LONGER than 4 letters to express our ideas.

Example:

CLPD

This is a perfectly acceptable permutation of 4 letters of the English alphabet. Yet, we do not recognize "CLPD" as a word; instead, we substitute a much longer string of letters, "ENCYCLOPEDIA."

English speakers are so indoctrinated with this spelling system that when they are asked to create a list of NONSENSE words, the words they most often create still follow the conventions of standard English spelling. They do not begin words with the diagraphs, "NG" or "DS", nor do they place several consonants or several vowels together in sequence.

EXPERIMENT 2: Shannon gave his subjects the opportunity to reconstruct a phrase from which the vowels and spaces had been omitted.

This raises an important point:

THE "SPACE" IS A CHARACTER IN THE ENGLISH SPELLING SYSTEM.

Separating words by inserting a "space" between them is NOT a characteristic of all languages.

The text Shannon used:

"FCTSSTRNGRTHNFCTN"

can be reconstructed after only a few moments of consideration.

Try: "PCTRSWRTHTHSNDWRDS."

The concept of language entropy may be put into practical use to construct a binary code which conveys a maximum amount of information using a minimum number of binary digits ("bits"). Such a code is called an OPTIMAL code.

Because letters occur with different frequencies in every language, each letter may be assigned a probability. Since one interpretation of PROBABILITY is the number of successes in a given number of trials, letters which occur most often in text are assigned the highest probabilities, and those which occur seldomly are assigned lower probabilities.

A systematic procedure discovered by D. A. Huffman takes advantage of this concept by assigning short code words to letters having high probabilities and longer ones to the letters having lower probabilities, and thus, it yields an optimal binary code.

<u>Steps in the Huffman Coding Procedure</u>:

1. All characters in the source alphabet are placed in order of DECREASING probabilities with their corresponding probabilities listed to the right.
2. Beginning with the 2 lowest probabilities in the table, pairs of probabilities are ADDED together, and the probability listing is RE-ORDERED to place the resulting sum in the correct position relative to the rest of the table.
3. Since the BINARY alphabet ("0" and "1") is used in Huffman coding, this process is repeated, moving always to the right, until only <u>2</u> probabilities remain. They correspond to the 2 main branches of the "Huffman coding

tree," and they MUST SUM TO 1 (within acceptable error).

4. Conventionally, "0" is assigned to the upper branch, and "1" is assigned to the lower, but this assignment is arbitrary as long as one is consistent.

5. Working backwards, the end of the previous code word receives a "0" on the upper branch and a "1" on the lower branch each time a DECISION POINT is reached.

The following Huffman code was constructed based on the most widely-used frequency table for English.

X	P(X)	C(X)	•	X	P(X)	C(X)
SPACE	0.1589	000	•	N	0.0574	1001
A	0.0642	0100	•	O	0.0632	0110
B	0.0127	011111	•	P	0.0152	011110
C	0.0218	11111	•	Q	0.0008	0111001101
D	0.0317	01011	•	R	0.0484	1101
E	0.1031	101	•	S	0.0514	1100
F	0.0208	001100	•	T	0.0796	0010
G	0.0152	011101	•	U	0.0228	11110
H	0.0467	1110	•	V	0.0083	0111000
I	0.0575	1000	•	W	0.0175	001110
J	0.0008	0111001110	•	X	0.0013	0111001100
K	0.0049	01110010	•	Y	0.0164	001111
L	0.0321	01010	•	Z	0.0005	0111001111
M	0.0198	001101	•	Avg. Word Length = 4.12 bits/letter 2		

The 26 letters of the English alphabet have been classified in terms of frequency as follows:

E, T, A, O, N, I, R, S, H = High-frequency letters
D, L, U, C, M = Medium-frequency letters
P, F, Y, W, G, B, V = Low-frequency letters
J, K, Q, X, Z = Rare letters.

It is obvious that the letter "Z", which occurs more seldomly than the letter "E" is assigned a much longer code word.

To see the true value of the Huffman coding procedure, it is necessary to understand the concept of REDUNDANCY. Redundancy in information theory refers to the transmission of more characters or words in a message than are actually NECESSARY to convey the information.

All spoken languages exhibit redundancy; the PERCENT REDUNDANCY of a language is determined by comparing the entropy of that language with the MAXIMUM entropy obtained by an "urn" language having an equal number of source characters.

% REDUNDANCY =

[1 - (entropy of lang.under/entropy of urn lang. with)]*100%.
consideration same # of characters

<u>For English</u>:

$$H_{max}(X_q = 27) = -[\ln(27)/\ln(2)] = 4.75 \text{ bits/letter}$$

Shannon's estimate for the entropy of English = 1.0 bit/letter

∴ % REDUNDANCY of English = [1 - (1/4.75)] * 100%
= [1 - 0.21] * 100%
= 0.79 * 100%
= 79%

This means that over 3/4 of all English text is unnecessary!

Redundancy exists in a code if any code word acts as the PREFIX of any other. For example, "111" and "1110" would be redundant code words, because "111" is a prefix of "1110."

The manner by which a Huffman code is constructed prevents this from occurring. Each time a decision point is reached, where the probabilities of 2 source characters must be divided, one branch receives a "0", and the other receives a "1." If redundancy were to exist, both branches at a decision point would have to be assigned the SAME binary digit.

The non-prefix rule solves a problem which is inherent in all variable-length codes like the Huffman: the inability to recognize the termination character of a code word. Because there is no fixed count which indicates that the last bit in a code word has been transmitted (as in the case of a "block" code, where all code words are of equal length), some other means must be devised. But because no code word begins the same way as any other, a message transmitted in a Huffman code can be decoded in ONLY ONE WAY.

Another characteristic of an optimal code is that the average code word length should be minimal. The AVERAGE CODE WORD LENGTH, $\overline{n}$, is the WEIGHTED sum of the individual code word lengths:

$$\overline{n} = \sum_{i=1}^{q} l_i * P(x_i),$$

where q = the number of characters in the source alphabet,

l_i= the number of bits in the code word assigned to the i^{th} source character,

and $P(x_i)$ = the probability of the i^{th} source character.

Example:

X	P(X)	C(X)
A	0.45	1
B	0.25	01
C	0.15	000
D	0.15	001

q=4

$$\overline{n} = \sum_{i=1}^{4} l_i*P(x_i) = 1*0.45+2*0.25+3*01.5+3*0.15$$

$$= 0.45+0.50+0.45+0.45$$

$$= \underline{1.85} \text{ \underline{bits/letter}}.$$

Notice that the UNITS for AVERAGE WORD LENGTH are the same as those for ENTROPY.

This point leads us to Shannon's central concept. Shannon's Theorem for Entropy makes 2 statements:

1. The AVERAGE WORD LENGTH for a code is always greater than or equal to the entropy of that code. These quantities may be EQUAL only if the probability of every source character is a power of 2.
2. Given any set of source characters and the probability associated with each character, it is possible to construct a Huffman code such that the difference between the average code word length and the entropy is arbitrarily small.

<u>Demonstration</u>:

I.

X	P(X)	R1	C(X)
A	0.50	0.50	1
B	0.25	0.50	00
C	0.25		01

$q = 3$

$$H(X) = - \sum_{i=1}^{q} P(x_i) * \log_2[P(x_i)]$$

$$= - [0.50*\log_2(0.50) + 0.25*\log_2(0.25) + 0.25*\log_2(0.25)]$$

$$= - [-0.50 + (0.50) + (-0.50)] = -[-1.50]$$

$$= \underline{1.50 \text{ bits/letter}}.$$

$$\bar{n} = \sum_{i=1}^{q} l_i * P(x_i)$$

$$= 1*0.50 + 2*0.25 + 2*0.25 = 0.50 + 0.50 + 0.50$$

$$= \underline{1.50 \text{ bits/letter}}.$$

II.

X	P(X)	R1	C(X)
A	0.50	0.50	1
B	0.30	0.50	00
C	0.20		01

$q = 3$

$$H(X) = -[0.50*\log_2(0.50) + 0.30*\log_2(0.30) + 0.20*\log_2(0.20)]$$

$$= -[-0.50 + (-0.52) + (-0.46)] = [-1.48]$$

$$= \underline{1.48}\ \underline{\text{bits/letter}}.$$

$$n = 1*0.50 + 2*0.30 + 2*0.20 = 0.50 + 0.60 + 0.40$$

$$= \underline{1.50}\ \underline{\text{bits/letter}}.$$

The difference here is only 0.02 bits/letter!

FOOTNOTES

[1] Kahn, David, *The Codebreakers: The Story of Secret Writing*, p. 761.

[2] Ingels, Franklin M., *Information and Coding Theory*, p. 129.

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KAPPA MU EPSILON NEWS

Edited by M. Michael Awad, Historian

News of chapter activities and other noteworthy KME events should be sent to Dr. M. Michael Awad, Historian, Kappa Mu Epsilon, Mathematics Department, Southwest Missouri State University, Springfield, Missouri 65804-0094.

CHAPTER NEWS

Arkansas Alpha, Arkansas State University, State University
Chapter President - Jack L. Jackson II
11 actives, 11 initiates

Our biggest activity for the spring semester was operating a concession stand at the regional mathematics contest. We obtained two Coca-Cola wagons and sold sodas and doughnuts. In spite of the terrible rainy weather we were able to turn a small profit. This activity was beneficial because it gave the organization a coordinated activity to do and provided refreshments for area high school math students. Our other main activity was a fish fry at Dr. Bishop's house. This provided a needed social function and an opportunity to spend some time together as a group. While at Dr. Bishop's home, we initiated 11 new members, doubling our active membership, and we elected officers for the next year. Other 1985-86 officers: Karyn J. Infield, vice president; Rena McFatter, secretary and treasurer; J. L. Linnstaedter, corresponding secretary; R. P. Smith and Tom Bishop, faculty sponsors.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President - Karen McKenzie
40 actives, 39 initiates

The chapter assisted the Mathematics Department with the annual Poly Royal Mathematics Contest which attracted over 500 high school students to the campus. Weekly meetings featured alumni and industry speakers. Five students and one faculty member attended the biennial convention. Two students presented papers. Karen McKenzie was the fourth recipient of the Arthur Andersen & Co. Professional Performance Award. Les Grove was the first recipient of the new Founders Award. Roseann Luan was the recipient of the KME Scholarship. Other 1985-86 officers: Tony Beardsley and Yvonne Pederson, vice president; Jane McGuire, secretary; John Burt, treasurer; George R. Mach, corresponding secretary; Adelaide T. Harmon-Elliott, faculty sponsor.

Colorado Gamma, Fort Lewis College, Durango
Chapter President - Glen Hodges
26 actives, 26 initiates

Our chapter was inducted on March 29. Dr. Merle Mitchell from the University of New Mexico was scheduled to perform the ceremony, but a storm prevented her travel. The Fort Lewis College Vice President for Academic Affairs, Dr. William Langworthy, filled in at the last minute. A banquet followed the ceremony. One faculty and two student members attended the biennial convention at SMU. Other 1985-86 officers: Susan Jocham, vice president; Sheri Postier, secretary; Johnny Snyder, treasurer; Richard Gibbs, corresponding secretary and faculty sponsor.

Connecticut Beta, Eastern Connecticut State University, Williamantic
Chapter President - Debra Adams
25 actives, 18 initiates

The chapter sponsored three mathematics contests for the college. There were fortnightly colloquial meetings and travel to area colloquia. A spring picnic was attended by 100. Other 1985-86 officers: Stephen Kenton, corresponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Sandra Hyde
20 actives, 8 initiates

The Georgia Alpha Chapter held its annual initiation meeting on May 7, 1985. We had a dinner together and then initiated eight new members, bringing the total number of names on our roll to 100. We then elected chapter officers for 1985-86. Afterwards, it was announced that three KME members would receive academic scholarships for 1985-86: Sandra Hyde, Robin Eason, and Martin Kemp. The meeting was then adjourned. Other 1985-86 officers: Joan Lyons, vice president; Robin Eason, secretary; Doug Ginn, treasurer; Joe Sharp, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston
Chapter President - Jeff Nettles
45 actives, 17 initiates

The spring semester included activities such as the KME initiation and mixer on April 11, the honors banquet on April 14, and the spring picnic on April 28 which took place in Morton Park. Other 1985-86 officers: Mike North, vice president; Linda Pfeiffer, secretary; Karie Andreina, treasurer; Lloyd L. Koontz, Jr., corresponding secretary and faculty sponsor.

Illinois Zeta, Rosary College, River Forest
Chapter President - Jill Rothermel
16 actives, 10 initiates

The Illinois Zeta Chapter initiated ten new members on February 26, 1985. A questionnaire on career and educational advancement, sent to 120 graduates in mathematics over the past twenty years, elicited a response of 67, many of whom are willing to counsel present math and computer science students. Other officers: Mary Ann Liebner, vice president; Denise Bastas, secretary; Gina Suareo, treasurer; Sister Nona Mary Allard, corresponding secretary and faculty sponsor.

Illinois Eta, Western Illinois University, Macomb
Chapter President - Heidi Kush
10 actives, 7 initiates

The main event of the spring, 1985, semester was the biannual Faculty Chili Lunch, which was a great success. Faculty speakers during the semester included Dr. Larry Welch and Dr. Larry Morley. A volleyball tournament with Pre-Engineering, Chemistry, and Physics was held, and Mathematics took second place. Student speaker for the semester was President Tamara Lakins. Five students and one faculty member attended the National Convention in Dallas, Texas, and Tami Lakins was selected to present her paper at the Convention. Faculty sponsor for the semester was Dr. Michael Moses. Other 1985-86 officers: Elizabeth Wood, vice president; David Herman, secretary and treasurer; Alan Bishop, corresponding secretary.

Indiana Alpha, Manchester College, North Manchester
Chapter President - Ryan McBride
20 actives, 5 initiates

A spring picnic was held, with installation of new members as one of the activities. Other 1985-86 officers: Mark Cawood, vice president; Norman Rohrer, secretary; Lisa Jerva, treasurer; Ralph McBride, corresponding secretary and faculty sponsor.

Indiana Gamma, Anderson College, Anderson
Chapter President - Mark Colgan
7 initiates

Other 1985-86 officers: Vicki Mills, vice president; Carol Smatlak, secretary; Kevin Pitts, treasurer; Stanley L. Stephens, corresponding secretary.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President - Scott Kibby
43 actives, 6 initiates

Five students from Iowa Alpha attended the KME National Convention in Dallas: Denise Cutler, Kande Hooten, Scott Kibby, Lisa Naxera, and Barbara Soucek, along with faculty members J. Bruha, J. Cross and G. Dotseth. Students presenting papers at local KME meetings were: Annette Bahlmann on "Geometry - A Basic Insight," David Kester on "A Maximization Problem," Joe Dittmer on "Multiprocessors and Software," and Nancy Nordbrock on "Fibonacci Numbers." Scott Kibby addressed the KME initiation banquet in May on "Probabilities of Duplication." Other 1985-86 officers: Austin Jones, vice president; Tracy Konrad, secretary; Tim Roegner, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President - Tracy Parks
16 actives, 3 initiates

Other 1985-86 officers: Ruth Gornet, vice president; Bill Schreiber, secretary; Craig Kalman, treasurer; Alex Kleiner, Jr., corresponding secretary; Lawrence Naylor, faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President - Sue Pyles
50 actives, 13 initiates

The spring semester began with a banquet and initiation for the February meeting. Thirteen new members were initiated at that time. After the initiation, Bryan Dawson gave the program on "Pythagorean Triples and their Respective Perimeters." Earlena Brownewell spoke at the March meeting. Her paper, "The Branch and Bound Method," was also accepted for presentation at the National Convention in Dallas. The April meeting program was given by Dr. Helen Kriegsman of the Mathematics Department faculty. She talked about the history and development of the slide rule. The annual Robert M. Mendenhall awards for scholastic achievement were also presented to Earlena Brownewell and Mary Slobaszewski. They received KME pins in recognition of this honor. The chapter also assisted the Mathematics Department faculty in administering and grading tests given at the annual Math Relays, April 23, 1985. Several members worked for the Alumni Association's Spring Phon-o-thon. Their efforts were well rewarded when it was announced that Kansas Alpha won a second prize of $75.00 for the most money raised by student organizations. Ten students and three faculty members attended the National Convention in Dallas. The final meeting of the semester was a social event held at Professor McGrath's home. Homemade ice cream and cake were served to those attending. Officers for the 1985-86 school year were elected. Other 1985-86 officers: Laura Rea, vice president; Sharon Million, secretary; Cathy Brenner, treasurer; Harold L. Thomas, corresponding secretary; Helen Kriegsman and Gary McGrath, faculty sponsors.

Kansas Beta, Emporia State University, Emporia
Chapter President - Bob Schif
25 actives, 8 initiates

The Kansas Beta Chapter had eight new members join during their spring initiation banquet. Six club members attended the KME National Convention in Texas. Steve Sodergren's paper was selected for presentation at the National Convention. In addition, KME sweatshirts were designed and sold to club members. The semester was concluded with a spring picnic. Other 1985-86 officers: Dawn Slavens, vice president; Barb Applegarth, secretary; Shelly Redeker, treasurer; George Downing, corresponding secretary; Thomas Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Mary Jo Muckey
27 actives, 11 initiates

Initiation to Kansas Gamma followed a dinner in Riccardi Center on 11 February. Eleven students were added to the roll of national Kappa Mu Epsilon membership. First-year teacher, Tamara Lasseter, gave a short talk on Platonic Solids and the Euler characteristic after the induction ceremony. Many Kansas Gamma members and associates joined together to make the 15th Mathematics Tournament for around 150 area high school students a success on 23 March. Attending the national convention in Dallas, Texas, on 12-13 April were students Sheila O'Brien, Rita Lundstrom, Mary Jo Muckey, Lisa Hatcher and faculty member Sister Jo Ann Fellin. Kansas Gamma celebrated its 45th anniversary on 21 April. Honored with a bud corsage were the four charter members present for the festivities - Sister Helen Sullivan, Sister Jeanette Obrist, Mary Margaret Downs Intfen and Muriel Thomas Jared. The program followed Mass and Brunch and included a slide presentation narrated by Sister Jo Ann Fellin which traced the history of the chapter from its founding to the present. Receiving Sister Helen Sullivan Scholarships at the Honors Banquet were senior Rita Lundstrom and juniors Lisa Huerter and Rhonda Leonard. Awards were given by the chapter to these three scholarship recipients as well as members Sheila

O'Brien, Mary Jo Muckey, and Carla Cihal. Other 1985-86 officers: Rita Lundstrom, vice president; Patricia Patterson, secretary and treasurer; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

Kansas Delta, Washburn University, Topeka
25 actives, 11 initiates

Four students and two faculty members attended the national KME convention at SMU in Dallas. Doug Bogia won third place for his presentation of a paper "An Application of Linear Interpolation in Two Variables for Brain Wave Displays." Chapter officers for 1985-86 will be elected during the fall, 1985, semester. Other 1985-86 officers: Robert H. Thompson, corresponding secretary; A. Allan Riveland, faculty sponsor.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President - Michelle Ferland
23 actives, 8 initiates

During the spring semester a presentation was given by the Steel Institute, and both the annual spring banquet and the annual spring picnic were held. Other 1985-86 officers: Janet Schuetz, vice president; Mary Doxon, secretary and treasurer; Charles Votaw, corresponding secretary; Jeffrey Barnett, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President - Lorie Barker
31 actives, 21 initiates

The chapter sold over 2000 boxes of M&M's to pay for the trip to the national convention in Dallas. Thirteen students and two faculty members attended the national convention. Before the convention, we had Mr.

Joe Sharp speak to us on "Skydiving, Savings Accounts, and a Simple Differential Equation." After the convention, we held our annual initiation with Dr. Aughtum Howard as speaker, and a party afterwards. Other 1985-86 officers: John Carroll, vice president; Barb McGrath, secretary; Dana Baxter, treasurer; Patrick Costello, corresponding secretary; Bill Janeway, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore
Chapter President - Donna Woods
7 actives, 6 initiates

February: The members of the chapter contributed to the smooth running of the Fourth Mathematics Olympiad held for high school students. At the luncheon Michele Ritter '85, President of our chapter, spoke on "Let's Party with Mathematics," April: Open meeting at which Mr. J. Edward Johnston, Jr., President of Forecast Research Systems, Inc., spoke on the topic "Stock Market: A Mathematical Model." May: Annual dinner. Initiation of six new members into our chapter. Mrs. Margaret Cermak, Assistant Professor of Management, gave an address on "The Computer World: Where Women Excel." Other 1985-86 officers: Angela Baccala, vice president; Etherine Lee, treasurer; Sister Marie A. Dowling, corresponding secretary; Sister Del Marie Rysavy, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President - Catherine LaPointe
20 actives, 21 initiates

Two events highlighted the Maryland Delta chapter's spring, 1985, activities list. In February, 21 new members were inducted into the chapter making it one of the largest groups ever to join; and in April, KME co-sponsored the Fifteenth Annual FSC Mathematics

Symposium entitled: "Discrete Mathematics: The Gifted and Talented Student." More than 125 mathematics personnel from the tri-state area (MD-PA-WVA) attended the event. Other 1985-86 officers: Teresa Neville, vice president; Donna Pope, secretary; John Galleher, treasurer; Don Shriner, corresponding secretary; John Jones, faculty sponsor.

Massachusetts Alpha, Assumption College, Worcester
Chapter President - Nancy Testa
5 actives, 6 initiates

Six new members were initiated on April 25, 1985. Following a dinner in honor of the new members, James Heffernan, a student member, spoke on "Applications of Fourier Transforms to Image Reconstruction." Other 1985-86 officers: Deborah Gosselin, vice president; Justine McEvoy, secretary; Charles Brusard, corresponding secretary.

Michigan Beta, Central Michigan University, Mt. Pleasant
Chapter President - Mike Carlson
45 actives, 39 initiates

Throughout the semester, Michigan Beta members held help sessions for freshman-sophomore level mathematics courses. During past years we have had initiation each fall and winter semester. This year we decided to have just one initiation. This was held early in the winter semester with a Friday night banquet. Our members really liked it and want to make it an annual happening. Our guest speaker was Professor Peter Hilton of the State University of New York at Binghampton. He gave a fascinating talk on Cryptanalysis in World War II. Professor Hilton was in the Michigan area that particular week speaking at several colleges. The speaker at our April meeting was Professor Wayne Osborn of the CMU Astronomy Department.

He spoke on Mathematics in Astronomy. We concluded the school year with a KME picnic for members and faculty. Other 1985-86 officers: Mary Sayler, vice president; Sue Hirtzel, secretary; Lisa Scheuerman, treasurer; Arnold Hammel corresponding secretary and faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President - Sharon Crook
32 actives, 21 initiates

The Mississippi Gamma chapter has initiated new members and named officers. New members are: Monty Bedi, Linda Brown, Roxanne G. Burrus, Margaret Carpenter, Janet C. Carr, Sharon Crook, Doris Culberson, Terri Leigh Green, Michelle Jeanfreau, Linda R. Jones, Glenn Dell Kalper, Debbie A. Marlow, Harold Vincent Miller, Pam Montgomery, John W. Myatt, IV, Mark P. Ortiz, Vivian Robinson, Phyllis Renee Rhodes, Izumi Yoshida Sullivan, Michelle Wells. The initiation and election took place at the home of Virginia Entrekin. A taco dinner was held following the business meeting. Dr. Steve Doblin, Chairman of the Mathematics Department, presented the KME Freshman Mathematics Award to Christopher Todd Mullins of Niceville, Alabama. Other 1985-86 officers: Johnny Myatt IV, vice president; Linda Brown, secretary; Alice Essary, corresponding secretary; Virginia Entrekin, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President - Marylynne Abbott
52 actives, 11 initiates

Three regular meetings of Missouri Alpha were held during the 1985 spring semester. Presentations were made by Dr. David Lehmann on Continued Fractions and by

Mr. James Downing on Linear Difference Equations. A spring banquet was held at which Shari Birkenbach was presented the KME Merit Award, Dr. L. T. Shiflett was presented with a silver platter in recognition of his service to Missouri Alpha, and six students were recognized as Outstanding Freshman Mathematics Students. Missouri Alpha in cooperation with Missouri Zeta, Missouri Iota, Missouri Kappa, and Missouri Theta, chartered a bus to attend the National KME Convention in Dallas. From these chapters, 42 students and faculty attended the convention at which Dr. M. Michael Awad of Missouri Alpha was elected National Historian. Other 1985-86 officers: Mark Payton, vice president; Susan Holt, secretary; Jean Ann Gay, treasurer; John Kubicek, corresponding secretary.

<u>Missouri Beta</u>, Central Missouri State University, Warrensburg
Chapter President - David Naas
57 actives, 24 initiates

Missouri Beta held four regular meetings during the spring semester. Speakers included a representative from AT & T of Kansas City and Alice Hisk. During the election ceremonies, five new executive associates were elected into office. After the ceremonies we had a pot luck supper. Some activities during the semester included: an ice skating party and a float trip. The ninth annual Klingenburg Lecture was held in April with Rick Drummond as guest speaker. We also held a book and bake sale during the annual CMSU Math Relays. An extra activity this year included attending the KME National Convention held in Dallas, Texas. Other 1985-86 officers: Rhonda Schneiders, vice president; Cheryl Harris, secretary; Kim Ward, treasurer.

<u>Missouri Gamma</u>, William Jewell College, Liberty
Chapter President - Blane Baker
11 actives, 14 initiates

The Chapter held monthly meetings throughout the school year, with presentations by student members. The spring initiation and banquet for new members was held on April 9, 1985, with Dr. Wallace A. Hilton as speaker. One of the largest single groups of initiates was added in the spring (14). Other 1985-86 officers: Laurie Honeyfield, vice president; Remy Blanchaerdt, secretary; Joseph Mathis, treasurer, corresponding secretary, and faculty sponsor.

<u>Missouri</u> <u>Epsilon</u>, Central Methodist College, Fayette
Chapter President - Hector Bencomo
11 actives, 1 initiate

Other 1985-86 officers: Keith Young, vice president; Greg Faust, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

<u>Missouri</u> <u>Zeta</u>, University of Missouri, Rolla
Chapter President - Kevin Davis
44 actives, 16 initiates

Monthly meetings were held in January, February, March and April. The pizza party to acquaint new members with officers was held on February 17. Regular biweekly help sessions were held for students through Differential Equations. For our March meeting, Dr. Haddock gave a speech on "Adventures of the Mind" and for our April meeting we viewed the television video of NOVA Math: A Mystery Tour. Our annual banquet was held April 23 with Dr. Wilderson speaking on proving theorems by computers. Our Chapter started a Math Challenge which was printed bimonthly in the school newspaper and awarded $5 to the winner. Seven members and faculty attended the National Convention at SMU in Dallas with one member presenting a paper. Other 1985-86 officers: Tim Allen, vice president; Pravin Ruktasire, secretary; Rana Jones, treasurer; Tom

Powell, corresponding secretary; Jim Joiner, faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville
Chapter President - Sara Morley
35 actives, 11 initiates

The chapter conducted a mathematical contest for high school students and hosted a Mathematics Expo for 125 selected high school students. Nine chapter members attended the National Convention in Dallas. Senior Joni Brockschmidt presented a paper at the Nationals. The chapter also hosted a spring picnic for the members and the mathematics faculty. Other 1985-86 officers: Tammy Erickson, vice president; Nancy Wolff, secretary; Lynda Sullivan, treasurer; Sam Lesseig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin
19 actives, 7 initiates

This semester marked the tenth anniversary of the Missouri Iota Chapter. All past presidents of the organization were invited to the annual spring initiation banquet and to participate the next day in a career seminar for current math students. The students initiated at the banquet were Jeff Jenness, Angela Noyes, Melinda Robinson, Brenda Stokes, Michael Stokes, Todd Thelen, and Jeff Williams. The program was presented by the organization's charter president now Dr. Cindy Carter Haddock of St. Louis University, whose topic was "The Metamorphosis of a Kappa Mu Epsilon President." Other presidents who returned to campus for the tenth anniversary events were Terri O'Dell, Robyn Housman Caruthers, Tricia DeWitt McKay, Rhonda McKee, Rickey Richardson, Larry Hicks, Sherri Plagman Hicks, and Charles Metz. Former charter sponsor, Professor Charles Allen of Drury College, was also present for the career seminar. Attendance at the National Convention was another highlight of the semester. Eight students and two faculty members attended this convention. Steve Brock served as a student member of the Awards Committee. The spring social was a swimming pool party held at the home of Professor Carolyn Wolfe. Other 1985-86 officers: Mary Elick, corresponding secretary; Joe Shields, faculty sponsor.

Missouri Kappa, Drury College, Springfield
Chapter President - Sami Long
12 actives, 1 initiate

The meetings this year have been centered around the use of mathematics in the various departments across campus. The spring program included talks from the Music Department and the Psychology Department. The chapter attended its first National Convention held in Dallas and finished the semester with a faculty-student picnic. Other 1985-86 officers: Scott Rollins, vice president; Barbara Robinson, secretary; Lynn Ruehle, treasurer; Charles Allen, corresponding secretary; Ted Nickle, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President - Dan Stalp
43 actives, 14 initiates

To make money throughout the spring semester, club members have monitored the Math-Science Building in the evenings and raffled off a VCR at a home basketball game in January. The club administered the annual test to identify the outstanding freshman majoring in mathematics. The award went to Siow Ling Tan whose home is in Malaysia. The award includes the recipient's name being engraved on a permanent plaque, payment of KME National dues, one year honorary membership in the local KME chapter, and announcement of the honor at the annual spring banquet. At the annual spring banquet, sponsor Dr. James Paige of the Mathematics Department was named Outstanding Professor in the Mathematics-Science Division for the 1984-85 academic year. The selection of the Outstanding Professor is by secret ballot where students majoring or minoring in the sciences and mathematics are eligible to vote. Speaker at the annual banquet was Dr. Lee Simmons who is the curator of the Henry Doorly Zoo in Omaha, Nebraska. The local KME chapter sponsored two College Bowl teams in April. One team was eliminated in the semifinals and the other placed second in the finals. Team participants were Doug Anderson, Marlyn Roth, Jim Urbanec, Deborah Ernesti, Gwen Hartman, Kelli Krutz, Colleen Spieker, Dan Stalp, Joan West, and Thad Book. Members Annetee Schmit, Doug Anderson, Dan Stalp, Tim Ott, Kurt Meisinger, Colleen Spieker, Betty Kuan, Dana Hungerford, Kelli Krutz, Greg Holm, Marlyn Roth and Jim Urbanec, and sponsor Dr. James Paige attended the 25th National KME Convention at Southwest Methodist University in Dallas, Texas in April. Club members assisted the Wayne State College mathematics faculty with the Eleventh Annual WSC Mathematics Contest on May 6, 1985, kept the KME bulletin board current, and sponsored some social functions for club members. Other 1985-86 officers: Sandra Sunderman, vice president; Tammy Strand, secretary-treasurer; Doug Anderson, historian; Fred Webber, corresponding secretary; James Paige and Hilbert Johs, faculty sponsors.

Nebraska Beta, Kearney State College, Kearney
Chapter President - Lori Smith
26 actives, 11 initiates

Other 1985-86 officers: Laura Isaac, vice president; Ann Wilson, secretary; Jeanette Hinkle, treasurer; Charles Pickens, faculty sponsor.

Nebraska Gamma, Chadron State College, Chadron
Chapter President - Terri J. Scofield
8 actives, 10 initiates

The Nebraska Gamma Chapter of KME has had an interesting spring semester. We initiated ten new members into our chapter. An informal initiation was held during which we had some fun with the initiates. Following this, there was a formal initiation dinner where the new members were sworn in. Our chapter sponsored the Fourth Annual Purple Shaft Award. This award, which is voted on by the students, is given to the professor who has made the students work extra hard for their grades. The whole Math/Science division has fun with this event. The chapter was also responsible for presenting the Outstanding Freshman Math Student Award at the Annual Ivy Day Ceremony. This year, the award - a CRC book of math tables - was presented to Kurt Palik. Other 1985-86 officers: Noelle D. Strang, vice president; Deborah Gaswick, secretary; Jim Paloucek, treasurer; James A. Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President - Jennifer Tyler
60 actives

Other 1985-86 officers: Cecilia DeBlasi, vice president; Suzanne Fehrenbach, secretary; Richard Metzler, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara
12 actives, 6 initiates

Other than our annual banquet and initiation ceremony held in April, most of the chapter's energies were focused on fund raising activities for the national convention in Dallas. We were able to raise enough money to allow four students to attend. Election of officers for 1985-86 will take place in early fall. Robert L. Bailey is corresponding secretary.

<u>Ohio Alpha</u>, Bowling Green State University, Bowling Green
Chapter President - Lynne Howard
23 initiates

Other 1985-86 officers: Rosie Yao, vice president; Cathy Raimer, secretary; Peter Lorenzetti, treasurer; J. Frederick Leetch, corresponding secretary; James H. Albert, faculty sponsor.

<u>Ohio Gamma</u>, Baldwin-Wallace College, Berea
Chapter President - David Bradshaw
21 actives, 13 initiates

Other 1985-86 officers: Jim Kerr, vice president; Lynne Martin, secretary; Bill Kastak, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.

<u>Ohio Zeta</u>, Muskingum College, New Concord
Chapter President - Lisa Elderbrock
32 actives, 9 initiates

On January 30, Dr. Robert Teese of our physics faculty gave a talk on the applications of mathematics in physics. On February 13, the initiation of nine new members took place, following which, each new member gave a short mathematical talk. New members are: Tim Coss, Pam Crooks, Monette Dulkoski, Geoff Eubank, Sherri Finnell, Gretchen Hedin, Chris Datona, Sue Kinjorski, and Fay Rezabek. The election of officers took place on March 13; Lisa Elderbrock presented her paper, "Finding Positive Integral Solutions to Linear Diophantine Equations," and Kim Lutz presented her paper on "The Chinese Remainder Theorem." On April 11-13, twelve students plus Dr. Smith attended the Biennial Convention in Dallas. One Sunday evening in March pizza was sold to help make expense money for the convention. On May 1, a convention report and party were given at Dr. Smith's home. Other 1985-86 officers: Kim Lutz, vice president; Monette Dulkoski, secretary; Barry Gowdy, treasurer; James L. Smith, corresponding secretary; Russ Smucker, faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President - Nandana Silva
20 actives, 10 initiates

Other 1985-86 officers: Niel Christensen, vice president; Lisa Peters, secretary and treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington
Chapter President - David Gore
36 actives, 9 initiates

Spring activities included the initiation banquet, a chess tournament, assistance with the Math/CS Lecture Series, and tutorial sessions for the Math/CS Department. Other 1985-86 officers: Barbara Forbes, vice president; Jody Vaccaro, secretary; Tracey Boyce, treasurer; Miller Peck, corresponding secretary, James B. Hall, faculty sponsor.

Pennsylvania Beta, LaSalle University, Philadelphia
Chapter President - Robert Morgan
20 actives, 17 initiates

The spring meeting to elect officers and induct new members was held on April 25. Other 1985-86 officers: Leon Wiener, vice president; Lisa Tresnan, secretary; Edward Dzialo, treasurer; Hugh N. Albright, corresponding secretary; Carl McCarty, faculty sponsor.

Pennsylvania Gamma, Waynesburg College, Waynesburg
Chapter President - Mark Keller

Other 1985-86 officers: Vincent Perdos, vice president; Mary Beth Huffman, secretary-treasurer; Rosalie B. Jackson, corresponding secretary; David S. Tucker, faculty sponsor.

Pennsylvania Delta, Marywood College, Scranton
3 actives, 13 initiates

Initiation of new members took place on May 9. A math contest for high school students in the area was held on March 30 (written portion) and April 21 (oral portion). New officers will be elected at the beginning of the fall, 1985, semester. Sr. Robert Ann Von Ahnen is corresponding secretary and faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana
Chapter President - Toni Frick
28 actives, 10 initiates

February meeting: Initiation of ten new members. Dr. George Matous, a member of the Physics Department of IUP, spoke on "The Philosophy of Mathematics." March meeting: Dr. George Mitchell, a member of the Mathematics Department at IUP spoke on "Differentiation." May meeting: A banquet was held at which time Dr. Melvin Woodard, a member of the

Mathematics Department at IUP, spoke about the "History of the Mathematics Department at IUP." A delicious meal prepared under the supervision of Mr. Raymond Gibson, with help from student members, was enjoyed by everyone. Other 1985-86 officers: Joseph Ramsey, vice president; Amy Page, secretary; Carolyn Horrell, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President - Mike Leonzo
33 actives, 11 initiates

The spring picnic was held at the Grove City Community Park on May 5, at which time Mark Snavely was announced as the winner of the Philip N. Carpenter Award. The award is given in recognition of the senior math major displaying the most promise in the field of mathematics and is named after a former chairman of the Mathematics Department. Other 1985-86 officers: Tawni Emanuel, vice president; Jeanne Mauersberg, secretary; Ami Waldschmidt, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Theta, Susquehanna University, Selinsgrove
Chapter President - Doris Cook
22 actives, 4 initiates

Mrs. Carol Harrison attended the national KME meetings in Dallas. Dr. Karl Klose spoke at our Spring Banquet and Initiation. All members tutored for the Mathematics Department. The club is paid for this tutoring service and the money is used towards the banquet and the end of the term pizza party. Other 1985-86 officers: Jeff Lockard, vice president; Robert Walker, secretary-treasurer; Carol Harrison, corresponding secretary and faculty sponsor.

Pennsylvania Iota, Shippensburg University, Shippensburg
Chapter President - R. Scott Sigel
27 actives, 19 initiates

Six members attended the national meeting in Dallas and returned with enthusiastic encouragement for other members to consider going to the next national meeting. Other 1985-86 officers: Kelly J. Lamey, vice president; Linda L. Cooper, secretary; Howard T. Bell, treasurer; Carl E. Kerr, corresponding secretary; Rick E. Ruth, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President - Linda Rafferty
8 actives, 6 initiates

The KME members participated in many activities; however, the most important and most successful was the installation of Debra Arena, Nadine Hillgen, Barbara Beckelman, Christina Mescier, Maryann McKeogh and Kathleen Kumos into membership of KME. The speakers were KME members: Frances Logue '64, Margaret Jankowski '75, Joanna Dellavalle '75 and Linda Czajka '83. Thediscussion that followed was very enlightening and the speakers explained how they used math in their careers. A pizza party followed. KME members continue to tutor students in need of help - free of charge. Other 1985-86 officers: Christina Mescier, vice president; Maryann McKeogh, secretary-treasurer; Sister M. Grace, corresponding secretary.

Pennsylvania Lambda, Bloomsburg University, Bloomsburg
Chapter President - Natalie Homick
35 actives, 10 initiates

The chapter made and sold hoagies to help send eight members and two faculty to Dallas. The chapter co-sponsored two speakers with the Women in Mathematics group and had an end of the year picnic. Other 1985-86 officers: Todd Adams, vice president; Lori Wagner, secretary; Ann Pinamonte, treasurer; Jim Pomfret, corresponding secretary; Joe Mueller, faculty sponsor.

Tennessee Beta, Eastern Tennessee State University, Johnson City
Chapter President - Jeff Hightower
17 actives, 11 initiates

The annual initiation banquet was held with eleven people being initiated. The KME officers conducted the initiation. Dr. Al Tirman gave a lecture on paradoxes. Other 1985-86 officers: Tammy Gillenwater, vice president; Suzanne Walters, secretary; Lyndell Kerley, corresponding secretary and faculty sponsor.

Tennessee Gamma, Union University, Jackson
Chapter President - Suzanne Morgan
17 actives, 11 initiates

Other 1985-86 officers: Stacey Sheppard, vice president; Suzanne F. Pack, secretary; Mary Anne Stephenson, treasurer; Don Richard, corresponding secretary; Dwayne Jennings, faculty sponsor.

Tennessee Delta, Carson-Newman College, Jefferson City
Chapter President - Jeffrey Drinnen
22 actives, 10 initiates

Dr. Steve Serbin was a visiting lecturer from the University of Tennessee. We viewed two films on the computer revolution. Jeff Knisley attended the KME convention at SMU. We had the annual initiation banquet at the Ramada Inn; we held the annual spring cookout at Panther Creek State Park. Other 1985-86 officers: Joanne Raye, vice president; Michael Souleyrette, secretary; Patricia Snowden, treasurer; Albert Myers, corresponding secretary; Carey Herring, faculty sponsor.

<u>Texas Alpha</u>, Texas Tech University, Lubbock
Chapter President - Warren Koepp
35 actives, 24 initiates

Other 1985-86 officers: Louis Gritzo, vice president; Van Gravitt, secretary; Donald Dotson, treasurer; Robert Moreland, corresponding secretary and faculty sponsor.

<u>Texas Eta</u>, Hardin-Simmons University, Abilene
Chapter President - Laura Watson
20 actives, 7 initiates

The Texas Eta Chapter of Kappa Mu Epsilon, the national mathematics honor society at Hardin-Simmons University, held its eleventh annual induction banquet March 1, 1985. There were seven members inducted: Cindy Brown and Nancy Wilson from Abilene, Texas; Richard Bergum from Monticello, Wisconsin; Mabel Kaloupek from Belle Plain, Iowa; James Lee from Seoul, Korea; Danny Richardson from El Paso, Texas; and Stephanie Thomas from Eldorado, Texas. With the induction of these members, membership in the local chapter stands at 94. Dr. Edwin J. Hewett, professor of mathematics at Hardin-Simmons University, addressed the chapter on the subject "Anecdotes from Mathematics." Special guests included Dr. Ronald Smith, Vice President for Academic Affairs at H-SU, and his wife, Patricia. Leading the induction ceremonies were Linda Haire, chapter president; Benjamin Barris, vice president; Donna George, secretary; and Mike Cagle, treasurer. Other 1985-86 officers: Sam Shin, vice president; Stephanie Thomas, secretary; Mike Cagle, treasurer; Mary Wagner, corresponding secretary;

Charles Robinson and Ed Hewett, faculty sponsors.

Virginia Alpha, Virginia State University, Petersburg
Chapter President - Pamela Miller
16 actives, 4 initiates

Other 1985-86 officers: Vincent Robenson, vice president; Benjamin Obhotaye, secretary; Molunder Tewari, treasurer; LaVerne Goodridge, corresponding secretary; Emma Smith, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President - Sandra Theiler
6 actives, 2 initiates

Four KME members and pledges and one faculty member attended the Biennial Convention at Southern Methodist University in Dallas, Texas. Those present included: Linda Schmidt, chapter president, Nora Hughes and Sandra Theiler, 1985 initiates, and Ann Brandt, freshman pledge, and Sister Adrienne Eickman, corresponding secretary. Nora Hughes and Sandra Theiler were initiated into Wisconsin Alpha Chapter on April 23, 1985. Chapter members and pledges were among those hosting Expanding Your Horizons in Mathematics and Science, a conference for young women of high school age and junior high age, held on March 30, 1985. Linda Schmidt, KME president, has received a teaching assistantship to Marquette University for the coming academic year. Other 1985-86 officers: Betsy Zaborske, vice president; Sandra Theiler, secretary; Betsy Zaborske, treasurer; Sister Adrienne Eickman, corresponding secretary and faculty sponsor.

Wisconsin Beta, University of Wisconsin-River Falls, River Falls
15 actives, 32 initiates

In March, our chapter had a film festival; we had three math-oriented films. We had a very successful bake sale on February 20. Our initiation meeting with guest speaker and reception was held on May 14. Overall, this year was very successful because we maintained our main goal which was to rejuvenate the Wisconsin Beta Chapter of Kappa Mu Epsilon. Chapter officers for 1985-86 will be elected in the fall, 1985, semester. Lyle Oleson is the corresponding secretary; Don Leake is the faculty sponsor.

REPORT OF THE PRESIDENT-ELECT*
TWENTY-FIFTH BIENNIAL CONVENTION
KAPPA MU EPSILON
April 11-13, 1985
Dallas, Texas

One of the responsibilities of the President-Elect is to serve as coordinator of regional activities of the Society through the Regional Directors. During Spring, 1984 to Fall, 1984 there were three conventions held in:

Region I at Pennsylvania Iota, Shippensburg University, with 52 participants (4 chapters); James Pomfret, Reg. Dir.

Region II at Michigan Beta, Central Michigan University, with 50 participants (2 chapters); J. Frederick Leetch, Reg. Dir.

Region VI at Calif. Gamma, Calif. Polytechnic State U., 47 participants (2 chapters); Adelaide T. Harmon-Elliott, Reg. Dir.

Programs at each convention site included student papers, guest talks, and good social times. We extend our sincere thanks to the host chapters, Regional Directors, and all who participated in this regional activity.

It is another of the President-Elect's responsibilities to make arrangements for the presentation of student papers at the National Convention. I am pleased to report that thirty students, representing fourteen Chapters and eight states, submitted papers for this Convention. Fifteen undergraduate students will present papers at the Convention. On behalf of our entire Society I am pleased to extend special thanks to the members of the Paper Selection Committee who read and ranked the papers: Dr. Patrick Costello (Kentucky Alpha), Dr. Sharon Kunoff (New York Lambda), and Dr. Gary McGrath (Kansas Alpha). In addition, I am especially pleased to express our thanks to the thirty students who prepared and submitted papers. These papers are the most important component in helping to make my Kappa Mu Epsilon Convention a success.

Respectfully submitted,
James L. Smith

*Due to an error by the editor, this report was omitted from the Spring, 1985 issue of the PENTAGON.

The Editor

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