THE PENTAGON

Volume XLIV(44) Spring, 1985 Number 2

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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics; due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

"CHRIST AT EMMAUS": A VERMEER MASTERPIECE OR A VAN MEEGEREN FORGERY?

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The Story Behind the "Christ at Emmaus"

The story of the "Christ at Emmaus" began to unfold in May of 1945 when special Allied teams sorted through Herman Goring's art collection. Goring had obtained these paintings by looting art galleries outside of Germany during the German occupation of Europe. Among the paintings in the collection was the "Christ and the Adultress," signed by Jan Vermeer. Further investigation revealed that the painting "Christ and the Adultress" had not been stolen, but actually had been purchased by Goring for approximately $600,000. Subsequent investigation traced the sale to a Dutch citizen, H. A. Van Meegeren.[1]

Van Meegeren, who was born in Holland in 1889,[2] studied to be an architect at the Technological Institute in Delft.[3] In his studies, he acquired a grasp for chemistry, but preferred painting.[4] Although

*A paper presented at the 1985 National Convention of KME and awarded first place by the Awards Committee.

Van Meegeren decided to pursue an art career, his knowledge of chemistry would be put to use in later years.

Van Meegeren's painting received recognition in 1914 when he passed the examination at the Academy of Fine Arts in the Hague by producing a copy of a 17th Century painting. He continued with several exhibits, but his subsequent works did not receive the praise he felt they merited. In 1932, frustrated and hungry, he decided to leave his countrymen and to to France to continue painting. At the outbreak of World War II in 1939, he returned to Holland.[5]

After the war, when it was discovered that Van Meegeran had sold a Vermeer painting to the Nazi Goring, the Dutch were outraged. Vermeer, a 17th Century Dutch painter, was a national hero. Like Rembrandt, his paintings were considered state treasures. Countless streets in Holland bear his name, and even the poorest homes have reproductions of his works.[6] The Dutch were so infuriated that they had Van Meergeren arrested on charges of collaboration.[7]

Upon his arrest, Van Meegeren stated that he had not sold a Vermeer painting, but had actually sold Goring a forgery. He claimed to have forged seven Vermeers between 1937 and 1943. Among the forgeries, he named the famed "Christ at Emmaus." From the sales of these paintings, he accumulated over three-million dollars.[8] To convince people of his story, he began to paint a new forgery, "Jesus in the Temple", while under police surveillance. Collaboration charges were soon dropped, but new charges of forgery were initiated. On November 12, 1947, Van Meegeren was convicted of art forgery and sentenced to prison, where he died on December 30, 1947 at the age of 58.[9] With his death, the answer to the question of the authenticity of the paintings was also buried.

Van Meegeren had used his knowledge of chemistry to aid him in developing his forgeries. To insure that his forgeries would pass all alcohol tests, Van Meegeren was careful to use the same types of oils Vermeer had used. He even purchased inexpensive old paintings to use for canvases so that the age of his canvases would match the age of Vermeer canvases.

Sometimes, he paid as much as $2,000 for a tube of paint to match the pigments available to Vermeer. Van Meegeren gave his works an aged quality by scratching the surfaces of the paintings and baking them in a kitchen stove. He then made up stories about the discoveries of the paintings.[10]

Since most of the alleged forgeries were of inferior quality, people could accept the idea that they were forged. However, the "Christ at Emmaus" had starred in a showing of the Netherland masterpieces during the Queen's Jubilee in 1938 where several critics had called it Vermeer's best![11] Since the "Christ at Emmaus" was so highly praised, the search for conclusive evidence of its origin continued.

Solving the Mystery of The "Christ at Emmaus"

The demand for conclusive proof that the "Christ at Emmaus" was a forgery was met by scientists at Carnegie Mellon University in 1967. They used "radioactivity" to date the painting. The procedure the scientists used is discussed by Martin Braum in his book Differential Equations and Their Applications.

Most of the equations used below are described in Braum's book.

Ernest Rutherford, a British physicist, and his colleagues showed that certain "radioactive" elements are unstable, and that in a certain amount of time the isotopes disintegrate to form new isotopes of different elements. Rutherford showed that this disintegration rate is proportional to the number of isotopes of the substance.

$N(t)$ = number of isotopes of a substance at time t

$\frac{dN}{dt}$ = disintegration rate

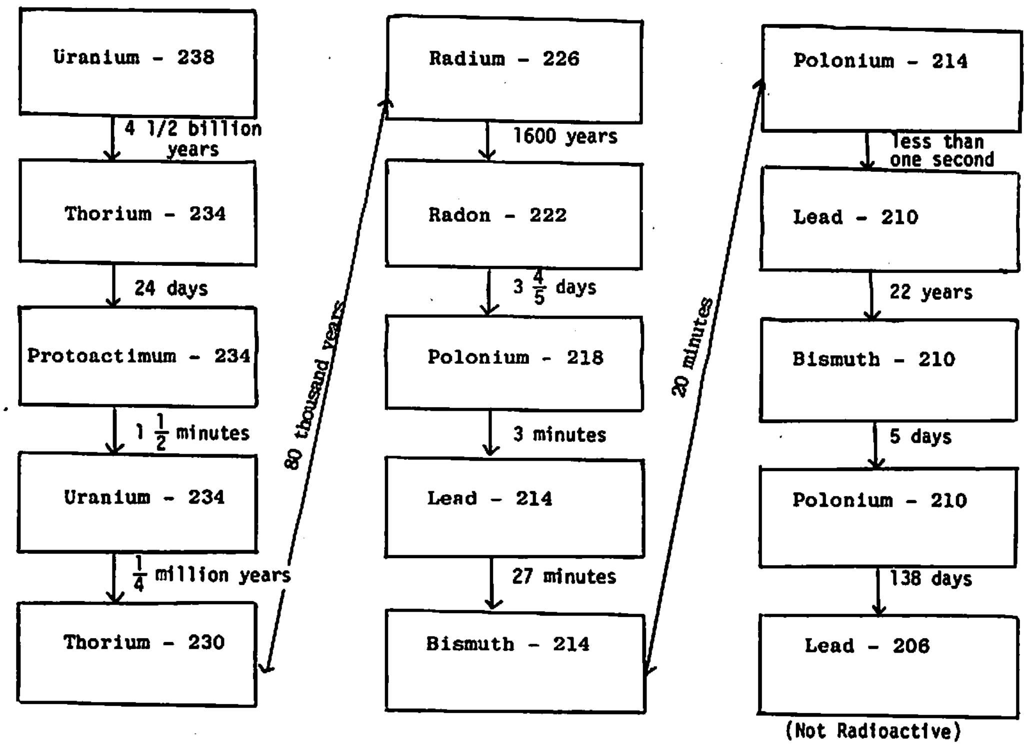
Since the disintegration rate is proportional to the number of isotopes of the substance present, we have

$$\frac{dN}{dt} = -\lambda N$$

where λ = the positive decay constant of the substance.

To apply radioactive dating to paintings, we need some knowledge of the Uranium Series (see Table I). The elements in this series are radioactive: Rutherford discovered as one element decays, it feeds the elements further down in the series. The "half-life" for each element in the series is the time it

The Uranium Series (Times shown on the arrows are the half-lives of each step)



<u>Differential Equations and their Applications</u>

TABLE I

takes for half of a given amount of that element's radioactive isotopes to disintegrate. Since the half-life for Uranium-238 is 4 1/2 billion years, the elements in the series are constantly being fed.

The knowledge of the Uranium series aids in dating a painting, since white lead, a pigment artists have used for 200 years, contains a very small amount of Radium-226 and a slightly larger amount of Lead-210. White lead is extracted from lead ore by smelting. In smelting, the Lead-210 stays with the lead metal, but 90-95% of the Radium-226 is removed. Since the half-life of Lead-210 is only 22 years, there will be rapid decay until the Lead-210 is at radioactive equilibrium with the small amount of Radium-226. Then the disintegration rates of the two elements will be balanced.

With this information, the initial disintegration rate of the Lead-210 for the painting can be calculated. Using the fact that the disintegration rate is proportional to the number of isotopes of the substance, let

$y(t)$ = amount of Lead-210 per gram of white lead at time t

y_o = amount of Lead-210 per gram of white lead at the initial time of manufacturing ($t = t_o$)

$r(t)$ = number of disintegrations of Radium-226 per minute per gram of white lead at time t (disintegration rate)

λ = decay constant of Lead-210

Since Radium-226 decays to Lead-210, we obtain the following initial value problem

$$\frac{dy}{dt} = - \lambda y + r(t), \; y(t_o) = y_o$$

Since the painting is at most 300 years old, we can call the disintegration rate of Radium-226 a constant r. We now have a linear first-order differential equation.

Solving

$$\frac{dy}{dt} + \lambda y = r$$

we obtain

$$ye^{\lambda(t-t_o)} = \frac{r}{\lambda}e^{\lambda(t-t_o)} + C$$

Using the initial condition, $y(t_o) = y_o$ and solving for C, we obtain

$$C = y_o \frac{r}{\lambda}$$

and our solution is

$$ye^{\lambda t}e^{-\lambda t_o} = \frac{r}{\lambda}e^{\lambda t}e^{-\lambda t_o} + y_o - \frac{r}{\lambda}$$

As our goal was to find the initial disintegration rate of Lead-210, we solve for λy_0 and obtain

$$\lambda y_o = \lambda y(t)e^{\lambda(t-t_o)} - r(e^{\lambda(t-t_o)} - 1)$$

We now have an equation for the initial disintegration rate of Lead-210. The present disintegration rate of Lead-210, $\lambda y(t)$, and the disintegration rate of Radium-226, r, are easily measured. The decay constant, λ, can be computed. If we knew the original disintegration rate of Lead-210, λy_0, we could calculate the age of a painting, $t-t_0$. Since the initial disintegration rate of Lead-210 can vary greatly, we cannot accurately calculate an age with this equation.

Fortunately, we have not derived a worthless equation! Recall that when white lead is made, the supply of the Lead-210 is cut off, and what Lead-210 there is decays rapidly until it reaches an equilibrium with the small amount of Radium-226. An actual Vermeer painting is approximately 300 years old. Since Lead-210 has a half-life of 22 years, the Lead-210 would have had the time to reach radioactive equilibrium with the Radium-226 in an old painting. However, if the painting is actually a Van Meegeren forgery, it would only have been about 20 years old at the time of the scientific investigation. The Lead-210 would not have reached radioactive equilibrium with the Radium-226. And, the present disintegration rate of the Lead-210, $\lambda y(t)$, would be large.

Let's take another look at the derived equation.

$$\lambda y_o = \lambda y(t)e^{\lambda(t-t_o)} - r(e^{\lambda(t-t_o)} - 1)$$

If the painting under question is an actual Vermeer, we say that the age $t-t_o$ is about 300 years.

$$\lambda y_o = \lambda y(t)e^{300\lambda} - r(e^{300\lambda} - 1)$$

If our assumption is incorrect, the present disintegration rate of Lead-210, $\lambda y(t)$, would be large. This would make the calculated initial rate of disintegration, λy_o, absurdly large.

Scientists have made measurements on the "Christ at Emmaus", and have determined that the present disintegration rate of Lead-210 for the painting is approximately 8.5 disintegrations per minute per gram of white lead ($\lambda y(t) = 8.5$ dpm/g of Pb), and the disintegration rate of Radium-226 for the painting is $r = 0.8$ dpm/g of Pb. We will use this below to calculate λy_o.

First we find the value for the decay constant of Lead-210. We use the fact that the disintegration rate is proportional to the amount of the substance present. Let

$N(t)$ = number of isotopes of Lead-210 present at time t

N = number of isotopes of Lead-210 present initially ($t = t_o$)

$\frac{dN}{dt}$ = disintegration rate of Lead-210

λ = decay constant for Lead-210

And obtain the initial value problem

$$\frac{dN}{dt} = -\lambda N, \; N(t_o) = N_o$$

Solving the equation, then solving for λ we obtain

$$\lambda = (\ln(\frac{N_o}{N(t)}))/(t - t_o)$$

Since the half-life of Lead-210 is 22 years,

$$\lambda = \frac{\ln 2}{22} \quad .$$

Now we compute λy_o. At the time of smelting, the Lead-210 will be at radioactive equilibrium with the Uranium-238 in the lead ore. Since the half-life of Uranium-238 is 4 1/2 billion years, we can call it a constant for a 300 year period. We then say that the disintegration rate of Uranium-238 will equal the initial disintegration rate of Lead-210.

We let

$N(t)$ = number of isotopes of Uranium-238 in ordinary lead ore at time t

$\frac{dN}{dt}$ = disintegration rate of Uranium-238

λu = decay constant of Uranium-238

λy_o = disintegration rate of Lead-210

and obtain the equation

$$\frac{dN}{dt} = -\lambda_u N = -\lambda y_o$$

We now need to calculate λ_u and N.

The half-life of Uranium-238 is $4.51\text{X}10^9$ years, so

$$\lambda u = \frac{\ln 2}{4.5\text{X}10^9} = 1.54\text{X}10^{-10}$$

We now calculate the number of isotopes of Uranium-238 in ordinary lead ore, N. Since the highest uranium content found in ore is 2-3%, we will use 3% (.03).

$$N = \frac{.03g\,^{238}U}{1g \text{ ore}} \times \frac{1 \text{ mol}\,^{238}U}{238g\,^{238}U} \times \frac{6.022X10^{23}\text{atoms}\,^{238}U}{1 \text{ mol}\,^{238}U}$$

$$= 7.591X10^{19} \text{ atoms } ^{238}U/g \text{ ore}$$

We now compute λy_o.

$$\lambda y_o = \lambda_u N = (1.54X10^{-10})(7.591X10^{19})$$

$$= 1.17X10^{10} \text{ dpy/g of Pb}$$

$$\lambda y_o = (1.17X10^{10} \text{ dpy/g of Pb})(1 \text{ yr}/525600 \text{ min})$$

$$= 2.22X10^{4} \text{ dpm/g of Pb (max)}$$

We assume that any value of λy_o greater than 30,000 dpm/g of Pb is too high.

The mystery of the "Christ at Emmaus" can now be solved! We again examine the equation

$$\lambda y_o = \lambda y(t)e^{\lambda(t-t_o)} - r(e^{\lambda(t-t_o)} - 1)$$

We have

$(t-t_o) = 300$ (assume painting is a Vermeer)

$\lambda y(t) = 8.5$ dpm/g of Pb (scientifically determined)

$r = 0.8$ dpm/g of Pb (scientifically determined)

$\lambda = \frac{\ln 2}{22}$ (calculated earlier)

Substituting these values into

$$\lambda y_o = \lambda y(t)e^{\lambda(t-t_o)} - r(e^{\lambda(t-t_o)} - 1)$$

we obtain

$$\lambda y_o = 8.5e^{(\ln 2/22)(300)} - 0.8(e^{(\ln 2/22)(300)} - 1)$$

$$\lambda y_o = 8.5e^{(150/11)\ln 2} - 0.8(e^{(150/11)\ln 2} - 1)$$

$$\lambda y_o = (8.5)2^{150/11} - (0.8)(2^{150/11} - 1)$$

$$\lambda y_o = 98{,}050$$

Since $\lambda y_o = 98{,}050 > 30{,}000$, the initial disintegration rate calculated is unacceptable. Therefore, the "Christ at Emmaus" is a forgery! The radioactive elements of the paint pigments gave the origin of the painting away.

Footnotes

[1]Wallace Irving, "The Man Who Swindled Goring," The Reader's Digest, March 1947, p. 6.

[2]Kilbracken, Van Meegeren: Master Forger (New York: Charles Scribner's Sons, 1967), p. 81.

[3]Kilbracken, p. 84.

[4]Piere Biasnconi and John Jacob, The Complete Paintings of Vermeer (New York: Harry N. Abrams, In. Publishers, 1967), p. 100.

[5]Biasnconi and Jacob, p. 100.

[6]Wallace Irving, p. 6.

[7]Martin Braum, Differential Equations and Their Applications (New York: Springer-Verlag, 1978), p. 11.

[8]"Masterpieces Only," Time, July 30, 1945, p. 65.

[9]Wallace Irving, p. 12.

[10]Wallace Irving, p. 9.

[11]Wallace Irving, p. 10.

[12]Information for this part of the paper was taken from Martin Braum, pp. 12-17.

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MODEST NUMBERS, A MATHEMATICAL EXCURSION*

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The field of number theory primarily deals with the natural numbers and their properties. Many sequences of numbers which have unique properties have been studied. Some of these are well-known, such as the prime, triangular and amicable numbers; and some are more obscure, such as the Duffinian, Niven and Kaprekar numbers. In the problem section of a recent edition of The Journal of Recreational Mathematics, a new type of number was introduced called the Modest number. The problem was stated as follows.

> If a given number can be sectioned into two parts in such a way that the division of the numerical value of the second part into the given number yields a remainder equal to the first part, then that number may be said to be modest. It immediately follows that the numerical value of the second part must exceed that of the first part if a given sectioning is to be "allowable." If a number is modest with respect to every allowable sectioning of it (assuming there is more than one), then that number may be said to be extremely modest.
>
> Some of the thirteen 2-digit modest numbers are 13,19,23,46,79. (For example, 46/6 yields a remainder of 4.) Some of the seventy-one 3-digit numbers are 103,218,327,515,666,711,818,981. (For example, 327/27 yields a remainder of 3.)

*A paper presented at the 1985 National Convention of KME and awarded second place by the Awards Committee.

1333 is an extremely modest number since 1333/333 yields a remainder of 1 and 1333/33 leaves a remainder of 13. 1333/3 is not an allowable division since the sectioning 133 and 3 is not allowable (3 does not exceed 133).

*a. Characterize all modest numbers.
*b. Characterize all extremely modest numbers. More specifically, do any exist that do <u>not</u> end in some repetition of the digits 3,6, or 9? (The first 4-digit extremely modest numbers are 1333,2333,2666,4666,1999,2999,...,8999).) 1.

The stars next to the problem letters indicate that a solution was found by neither the proposer of the problem nor the editors of the magazine at that time. The following discussion will attempt to solve these problems as well as prove some of the properties of the modest numbers.

Before I begin to analyze the modest numbers, I need to define some terminology which I will use throughout this discussion. One symbol which will be used is $\lceil\ \rceil$. $\lceil n \rceil$ is defined as the least integer greater than or equal to n. (In other words, this is essentially the symbol for rounding up.) For example, $\lceil \pi \rceil = 4$ and $\lceil 2 \rceil = 2$.

Another notation which will be used is d_n where d is a single digit and n is a positive integer. It represents the number formed by repeating the digit d, n times. This is called a repdigit of order n. For example, $6_3 = 666$. Also, the symbol * will stand for concatenation (of two numbers.) For example, $1 * 4 = 14$ and $3 * 5_3 = 3555$.

Since the problem is to characterize all of the modest numbers, a good place to start might be to generate and observe some of them. The first 102 are listed in Figure 1.

13	19	23	26	29	39
46	49	59	69	79	89
103	109	111	133	199	203
206	209	211	218	222	233
266	299	309	311	327	333
399	406	409	411	412	418
422	433	436	444	466	499
509	511	515	533	545	555
599	609	611	618	622	627
633	654	666	699	709	711
721	733	763	777	799	809
811	812	818	822	824	833
836	844	866	872	888	899
911	927	933	981	999	1003
1009	1011	1015	1018	1022	1027
1030	1033	1037	1045	1055	1066
1090	1099	1111	1133	1199	1218

Figure 1.

At a first glance, there doesn't appear to be much of a pattern to the modest numbers. However, there does seem to be a large occurrence of repeated digits. Also, every 3-digit repdigit (111,222,...,999) and 1111 are on the list. Upon further generation, it can be seen that all of the four, five and six digit repdigits are modest, and furthermore, the five and six digit ones are extremely modest. These facts were the impetus for the first theorem on modest and extremely modest numbers.

<u>The Theorem of Modest Repdigits:</u>

All repdigits of length greater than two are modest; and of length greater than four, extremely modest.

Proof: Clearly, any repdigit with one or two digits cannot be a modest number because they have no allowable sectionings.

Let d_n be an arbitrary repdigit with $n \geq 3$. Consider any sectioning which satisfies the condition that there are more digits on the right than the left (to insure that it is "allowable".) This divides d_n

into two sections d_s and d_{n-s} such that $d_n = d_s * d_{n-s}$. When the division is done, the fraction:

$$\frac{d_n}{d_{n-s}} = \frac{(d_{n-2} \times 10^s + d_s}{d_{n-s}} = 10^s + \frac{d_s}{d_{n-s}}$$

and therefore the remainder is d_s which is the left section. Thus, for any allowable sectioning of d_n, the number is modest. Furthermore, by this finding and the definition of extremely modest numbers, d_n is extremely modest whenever d_n has more than one allowable sectioning. This occurs whenever $n > 4$.

As well as characterizing some of the modest numbers, this theorem answers part of problem (b) which asks whether any extremely modest number ends in any digit besides 3, 6 or 9. Now it has been shown that there are extremely modest numbers which end in each of the digits one through nine. This theorem also leads to two corollaries.

<u>Corollary 1:</u>

There are an infinite number of extremely modest numbers.

Proof: Assume that N is the largest extremely modest number, and let the number of digits in N be d. Now, consider 1_{d+5}. Since 1_{d+5} has more digits than N, it follows that $1_{d+5} \geq N$. However, by the Theorem of Modest Repdigits, 1_{d+5} is extremely modest which is a contradiction since N was defined as the largest extremely modest number. Therefore, the assumption is false, and there is no largest extremely modest number.

<u>Corollary 2:</u>

There are an infinite number of modest numbers.

Proof: This follows directly from Corollary one and the fact that the extremely modest numbers are a subset of the modest numbers. Again referring to Figure 1, observe that numbers which end in 99 (199, 299, ..., 1199) also appear. Looking at larger modest numbers, it can be seen that this pattern holds until 9999. However, for five and six digit numbers, the number must end in three nines to be modest. This finding is generalized in the following theorem.

<u>The Theorem of Nines:</u>

If a positive integer N has d digits, and at least the last $\lceil d/2 \rceil$ (the last half) are nines, then the number is modest.

Proof: Let the sectioning occur just to the left of the $\lceil d/2 \rceil^{th}$ nine from the right, and let the number represented by the left section of N be called L. By this sectioning, it is clear that, $N = L * 9_{\lceil d/2 \rceil}$ and $L \leq 9_{\lceil d/2 \rceil}$. Consider the last statement in its two separate cases.

Case 1: $L = 9_{\lceil d/2 \rceil}$.

This would imply that $N = 9_{\lceil d/2 \rceil} * 9_{\lceil d/2 \rceil} = 9_d$ which was proved to be a modest number in the Theorem of Modest Repdigits.

Case 2: $L < 9_{\lceil d/2 \rceil}$

It was stated that $N = L * 9_{\lceil d/2 \rceil}$ so:

$$\frac{N}{9_{\lceil d/2 \rceil}} = \frac{L * 9_{\lceil d/2 \rceil}}{9_{\lceil d/2 \rceil}} = \frac{(L \times 10^{\lceil d/2 \rceil}) + 9_{\lceil d/2 \rceil}}{9_{\lceil d/2 \rceil}} = \frac{L \times 10^{\lceil d/2 \rceil}}{9_{\lceil d/2 \rceil}} + 1 =$$

$$\frac{L \times (9_{\lceil d/2 \rceil} + 1}{9_{\lceil d/2 \rceil}} + 1 = L + \frac{L}{9_{\lceil d/2 \rceil}} + 1 = L + 1 + L/9_{\lceil d/2 \rceil} \cdot$$

Therefore, L is the remainder of $N/9_{\lceil d/2 \rceil}$ and thus, N is modest.

An interesting corollary of this result pertains to the lengths of arithmetic progressions of modest numbers.

<u>Corollary 3:</u>

Given any positive integer k, there exists an arithmetic progression of modest numbers of length k.

Proof: Let q be the positive integer such that $10^{q-1} \leq k < 10^q$. Let $a_0 = 9_q$ and $a_i = a_0 + (i \times 10^q)$. By the Theorem of Nines, a_i will be a modest number as long as $i \leq 9_q$. This means that $\{a_i\}$ is an arithmetic progression of length $9_q + 1 = 10^q$. Since $k < 10^q$, there is clearly an arithmetic progression of length k and the proof is complete.

Although some of the modest numbers have now been classified, there are still many unclassified. Looking at the remaining modest numbers, it is interesting to note that many of the numbers which are modest are actually multiples of other modest numbers. As a matter of fact, if the two types of modest numbers which I have discussed are discounted, of the 82 remaining modest numbers in Figure 1, 47 of them are multiples of other modest numbers in the table. This realization leads to another important theorem.

<u>The Theorem of Multiple Modest Numbers:</u>

If N is a modest number with respect to some sectioning, then kN is a modest number of any positive integer k such that a carry does not cross over the sectioning line.

Example: 1333 is modest with respect to the line sectioning down the middle as shown in the statement of the problem. Then, applying this theorem, 2666 and 3999 are modest since when 1333 was multiplied by 2 and 3 respectively, and no carry crossed the sectioning line. However, 5332 is not modest and a carry did cross over.

Proof: Let N be a modest number. Denote the left side of N with respect to some sectioning as L and the right side R. Since N is modest, it must be true that:

$$\frac{N-L}{R}$$

is an integer. Now consider kN such that when N was multiplied by k, a carry did not cross the sectioning line. This would imply that the new right section would just be kR and that the left section would be kL. In order for kN to be modest, it must be true that:

$$\frac{kN-kL}{kR}$$

is an integer, which is true since it was shown that (N-L)R was an integer. Therefore, kN is modest.

Now, most of the modest numbers have been classified. All that is left is to find a general form for the "Non-multiple" modest numbers. For reference, the first 50 three and four digit non-multiple modest numbers are printed in Figure 2.

103	109	203	209	211
233	311	409	411	433
509	511	533	611	709
711	733	809	811	833
911	1003	1009	1011	1027
1033	1037	1333	1433	1633
1733	1933	2003	2009	2027
2033	2037	2111	2333	2533
2633	2833	2933	3037	3133
3233	4009	4027	4037	4111

Figure 2

Note to Figure 2: The arrow accompanying each of the numbers marks the sectioning line which makes each number modest.

It can be observed from the chart that all of the three digit non-multiple modest numbers end in either 3, 9, 11 or 33. Note that all of these are factors of 99. Also, the four digit numbers end in either 3, 9, 11, 27, 33 or 37. Note that all of these are either factors 99 or 999. By further analyzing the four digit numbers, it can be seen that if the sectioning is done after the first digit, then the number ends in either 3, 9, 27 or 37; which are the factors of 999. If the sectioning is done after the second digit, then the numbers end in either 11 or 33; each of which divides 99. This leads to the final major characterization theorem.

<u>The Theorem of Non-Multiple Modest Numbers:</u>

If a positive integer N, which is sectioned just to the left of the m^{th} digit from the right, is not a multiple modest number, then N is modest iff $R|9_m$ and $L < R$. (L and R are defined as before.)

Proof: Let N be a modest number which is sectioned as stated above and is not a multiple modest number. Since N was divided at the m^{th} digit, it can be written: $N = (L \times 10^m) + R$. Also, since N is modest, it can be written as some multiple p of R plus the remainder L: $N = pR + L$. Thus, equating the two:

$$N = N$$

$$\Longleftrightarrow \quad (L \times 10^m) + R = pR + 1$$

$$\Longleftrightarrow \quad (L \times 10^m) - L = pR - R$$

$$\Longleftrightarrow \quad L \times (10^m - 1) = (p-1)R$$

$$\Longleftrightarrow \quad L(9_m) = (p-1)R$$

$$\Longleftrightarrow \quad \frac{L(9_m)}{R} = (p-1) \text{ which is an integer since p is.}$$

Therefore, $R|L(9_m)$. Since N is a non-multiple modest number, R and L have no factors in common. Thus, $R|9_m$. (The fact that $L < R$ is obvious from the fact that N is modest.)

The proof of the converse is practically a reversal of the steps above and is not given here.

Now that all of the modest numbers have been taken into account, they can be placed into one of the four categories below:

1. All repdigits
2. All numbers provided for in the Theorem of Nines
3. All multiples of modest numbers
4. All numbers which can be sectioned such that $L < R$ and $R | 9_m$.

Although this does encompass all of the modest numbers, these four groups are not mutually exclusive. For instance, any repdigit d_n where $2 \le d \le 9$ is a multiple of 1_n. Obviously, 1_n is not a multiple modest number, but since for any sectioning (say, just to the left of the m^{th} digit from the right as in the Theorem of Non-Multiple Modest Numbers) it is clear that $1_m | 9_m$, any repdigit of order 1 is taken care of by group 4. Therefore group 1 is included in groups 2 or 3.

Also, for group two, some of the numbers taken into account in the Theorem of Nines are multiple modest numbers (like 3999.) Further, consider the same sectioning used in the Theorem of Nines. If a number N falls in this group, and it is not a multiple modest number, then it still must be modest by the Theorem of Non-Multiple Modest Numbers because the sectioning was defined so that L < R and clearly $9_{\lceil d/2 \rceil} | 9_{\lceil d/2 \rceil}$. Thus, group two need not be listed in the final characterization.

Since the non-multiple modest numbers were defined as not being multiple modest numbers, these two groups are mutually exclusive, and thus form the following theorem.

<u>The Fundamental Theorem of Modest Numbers</u>

A positive integer N may be said to be modest iff one of the following conditions holds:

1) N can be sectioned into two parts L and R (the left and right parts) such that L < R and $R | 9_m$ where m is the number of digits in N minus the number of digits in L.

or

2) N is a multiple of a modest number such that a carry did not cross the sectioning line.

Earlier, it was proved that there exist extremely modest numbers which end in each of the digits one through nine. Now, the final case will be dealt with. Does any extremely modest number end in zero? These final theorems will answer this question.

The Theorem of Terminating Zero in Modest Numbers:

A positive integer N which ends in zero may be modest with respect to a given sectioning only if the left section is a multiple of ten.

Proof: Let N be a positive integer which ends in zero. Let L and R be defined as before. Obviously, since N ends in zero, so does R, and thus they are both multiples of ten. This implies that there exist k_1 and k_2 such that $N = 10K_1$ and $R = 10k_2$. In order for N to be modest, L must be the remainder of N/R. Using the division algorithm, this implies:

$$N = pR + L \text{ for some integer } p$$

$$10k_1 = p10k_2 + 1$$

$$10(k_1 - pk_2) = L$$

and thus, L is a multiple of ten.

The Theorem of Terminating Zero in Extremely Modest Numbers:

No extremely modest number may end in zero.

Proof: Let N be an extremely modest number that ends in zero. Since N is extremely modest, it must be modest for every possible sectioning. Thus, consider the sectioning just to the right of the first digit. By the Theorem of Terminating Zero the left digit must be a multiple of ten. However, it is impossible for a single digit to be a multiple of ten (unless the number is zero, but then it would not have been the first digit) and thus N is not modest with respect to that sectioning which is a contradiction. Therefore, no extremely modest number may end in zero.

Although the questions which were asked about modest numbers have been answered, as with most research in mathematics, these solutions also lead to new questions. A few of these are:

1) How many consecutive modest numbers are there? (By examining the numbers in Figure 1, there are only two pairs of consecutive modest numbers: 411,412 and 811,812.)
2) What are the other forms besides the repdigits for the extremely modest numbers? (For example, 1333,2666,3999 and 8999 are all extremely modest

but they are not all repdigits.)

3) Is there a function A(N) which for any positive integer N will give the number (or a close approximation) of modest numbers less than or equal to N.

Bibliography

1. Joseph S. Madachy, Problems and Conjectures, Problem 1291, 16 (1983-1984).

2. Joseph S. Madachy, Solutions to Problems and Conjectures, Solution 1291, 17(1984-1985).

THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 February 1986. The solutions will be published in the Spring 1986 issue of **The Pentagon**, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 382: Proposed by Fred A. Miller, Elkins, West Virginia.

Given a quadrilateral ABCD, draw a line which divides the given quadrilateral into two quadrilaterals of equal area.

Problem 383: Proposed by Charles W. Trigg, San Diego, California.

More than half of the integers that are permutations of a certain set of four consecutive integers have squares with digit sums of 37. Identify those integers.

Problem 384: Proposed by Steve Ligh, University of Southern Louisiana, Lafayette, Louisiana.

Find all integers n such that $2n = (a + 1)(b + 1)(c + 1)$ where n is the product of a, b and c.

Problem 385: Proposed by the editor.

A preschool nursery class has three girls. The boys have not yet been counted. An hour later a new child is brought into the nursery. Then a child is selected at random to be photographed. If the child who was selected to be photographed is a girl, what is the probability the last addition to the nursery class was a boy?

Problem 386: Proposed by the editor.

On the first of the month, an eccentric millionaire started a new bank account with a deposit. On the first of the next month he made a second deposit to the account. On the first of each of the following months he made a deposit to the account which was the sum of the amounts deposited in the two preceding months. If the seventeenth deposit was $500,000.00 and if all deposits were integral amounts of dollars, what were the amounts of the two initial deposits?

SOLUTIONS

372: Proposed by the editor.

Consider the sequences [nA] and [nB] where [x] denotes the greatest integer function and n is a natural number. Let $A = 3 + \sqrt{3}$ and $B = 3 - \sqrt{3}$. Then the sequence [nA] = 4,9,14,18,... and the sequence [nB] = 1,2,3,5,6,... . Show that each natural number appears in either [nA] or [nB] but not both.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

We shall establish the following more general result: If A and B are positive irrational numbers such that $1/A + 1/B = 1$, then the two sequences [nA] and [nB] for n = 1, 2, 3,... contains all positive integers without repetition. Since A and B are positive with $1/A + 1/B = 1$, it follows that both $A > 1$ and $B > 1$.

(1) First we show that there is no repetition of integers in the sequences [nA] and [nB],

$n = 1, 2, 3, \ldots$.

(a) In the sequence $[nA]$ there is no repetition since we have $[nA] < nA < (n+1)A - 1 < [(n+1)A]$. The first of these inequalities follows because nA is irrational; the second follows from $1 < A$. Similarly there is no repetition in the sequence $[nB]$.

(b) For all positive integers n and m, $[nA] \neq [mB]$. If we suppose that the integer $x = [nA] = [mB]$ for some positive integers n and m, then

$$nA - 1 < x < nA \quad \text{or} \quad n - 1/A < x/A < n$$

$$\text{and} \quad mB - 1 < x < mB \quad \text{or} \quad m - 1/B < x/B < m.$$

Adding the second set of inequalities in each line and using the fact that

$1/A + 1/B = 1$, yields $n + m - 1 < x < n + m$

so that the integer x lies between two consecutive integers. This contradiction proves that no integer appears in both sequences $[nA]$ and $[mB]$.

(2) Next we shall show that no positive integer x is omitted in both sequences $[nA]$ and $[mB]$. Suppose that x is such an integer. Then there must be

integers n and m such that

$$[nA] < x < [(n + 1)A] \text{ and } [mB] < x < [(m + 1)B].$$

Thus $nA < [nA] + 1 \le x \le [(n + 1)A] - 1 < (n + 1)A-1$

or $$n < x/A < n + 1 - 1/A$$

and $mB < [mB] + 1 \le x \le [(m + 1)B] - 1 < (m + 1)B-1$

or $$m < x/B < m + 1 - 1/B.$$

Adding the second set of inequalities of each line and using the fact that

$1/A + 1/B = 1$, yields $n + m < x < n + m + 1$

which is an obvious contradiction.

(1) and (2) together establish the desired result.

373: Proposed by the editor.

Let $f(x)$ be a polynomial having integer coefficients. For the distinct integers a,b,c, and d, we have $f(a) = b$, $f(b) = c$, $f(c) = d$, and $f(d)=a$. Find all such polynomials or show that none exist.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

We shall show that no such $f(x)$ exists.

Suppose there is a polynomial $f(x) = \sum_{i=0}^{i=n} a_i x^i$ which satisfies the requirements of the problem.

Then $b - c = f(a) - f(b) = \sum_{i=0}^{i=n} a_i(a^i - b^i) =$ $(a - b)Q(a,b)$ where $Q(a,b)$ is an integer. Similarly, $c - d = f(b) - f(c) = (b - c)Q(b,c)$; $d - a = f(c) - f(d) = (c - d)Q(c,d)$; and $a - b = f(d) - f(a) = (d - a)Q(d,a)$ where $Q(b,c)$, $Q(c,d)$ and $Q(d,a)$ are all integers. Thus $(b-c)(c-d)(d-a)(a-b) = [(a-b)Q(a,b)][(b-c)Q(b,c)][(c-d)Q(c,d)][(d-a)Q(d,a)]$. Hence $Q(a,b) \cdot Q(b,c) \cdot Q(c,d) \cdot Q(d,a) = 1$. It follows that the absolute value of each of $Q(a,b)$, $Q(b,c)$, $Q(c,d)$, and $Q(d,a)$ is 1. Hence $b-c = a-b$ and $c-d = b-c$ and $d-a = c-d$ and $a-b = d-a$ so $a-b = b-c = c-d = d-a$. Because $a-b = b-c$, $a-b = b-c$ or $a-b = -(b-c)$. This latter is impossible since $a \neq c$. Similarly, $c-d = d-a$ implies that $c-d = d-a$. Thus $(a-b) + (d-a) = (b-c) + (c-d)$, making $b = d$. This contradicts the hypothesis that $b \neq d$. Hence no polynomial $f(x)$ exists which satisfies the conditions of the problem. [If the hypothesis that $f(x)$ have integer coefficients is deleted, then the polynomial

$$f(x) = \frac{(x-b)(x-c)(x-d)}{(a-b)(a-c)(a-d)}\, b + \frac{(x-a)(x-c)(x-d)}{(b-a)(b-c)(b-d)}\, c$$
$$+ \frac{(x-a)(x-b)(x-d)}{(c-a)(c-b)(c-d)}\, d + \frac{(x-a)(x-b)(x-c)}{(d-a)(d-b)(d-c)}\, a$$

satisfies the remaining requirements of the problem. This polynomial was obtained by using the Lagrange interpolation formula.]

374: Proposed by the editor.

Solve the following system of interrelated alphametics: ABCD/AE = FD; AG + AH = CB; ABBA/CI = CC; ABCD - AG = ABBA; AE + AH = CI; and FD - CB = CC.

Solution by Charles W. Trigg, San Diego, California.

We reorder the six cryptarithms as [1] (CC)(CI)=ABBA; [2] AE + AH = CI;[3] CB + CC = FD; [4] ABCD - ABBA= AG; [5] AG + AH = CB; and [6] ABCD = (FD)(AE). There are four solutions of [1] with distinct digits, namely: (33)(37) = 1221, (44)(49) = 2112, (66)(69) = 4554, and (77)(78) = 4554. From [2], we must have C $\geq$ 2; this eliminates the last two solutions of [1].

From [3] and the first two solutions of one, we have:

A = 1, B = 2, C = 3. I = 7, F = 6, and D = 5; or

A = 2, B = 1, C = 4, I = 9, F = 8, and D = 5.

Substituting this last set of values in [4] gives A = G = 3, a duplication. The other set of values gives G = 4, and H = 8 from [5] and E = 9 from [2]. Thus the solutions of the cryptarithms in order are: [1] (33)(37) = 1221;[2] 19+18 = 37;[3] 32 + 33=65; [4] 1235 - 1221 = 14; [5] 14 + 18 = 32; and of the superfluous [6] 1235 = (65)(19).

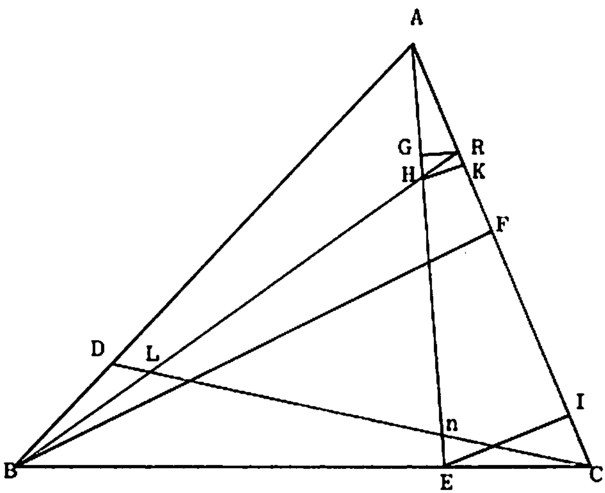
Also solved by Fred A. Miller, Elkins, West Virginia.

Editor's Comment:

Trigg correctly points out that since the letter combinations used in this problem do not make related words or meaningful phrases, they are more properly classified as cryptarithms.

375: Proposed by the editor.

Consider a triangle each of whose sides have been divided into four equal parts. Proceeding in a clockwise direction, from each vertex draw a line to the first division to the right of the next vertex on the opposite side as shown in the figure. How does the area of the inner triangle compare with the area of the original triangle? What is the corresponding result if 4 is replaced by n?



(Figure 1)

Solution by Oscar R. Castenada, St. Mary's University, San Antonio, Texas.

Revised by the editor. In the triangle ABC, let points D, E and R be located on lines AB, BC and CA respectively so that BD = z, AB = nz, CE = y,

$BC = ny$, and $AR = x$, $AC = nx$ for n a positive integer. From E draw a line perpendicular to AC cutting AC at I. From H draw a line perpendicular to AC cutting AC at K. From B draw a line perpendicular to AC cutting AC at F. From R draw a line parallel to CE cutting AE at G. Let [ABC] denote the area of triangle ABC.

Triangles AEC and ABC have the same altitude and their respective bases both lie on AC, so

$[AEC]/[ABC] = EC/BC = 1/n.$ (1)

Similarly for triangles ABR and BCD,

$$[ABR]/[ABC] = [BCD]/[ABC] = 1/n.$$

Next we show that $[AHR] = [BDL] = [CME]$.

Since $GR \parallel EC$, triangles ARG and AEC are similar; thus

$$GR/CE = AG/AE = AR/AC = 1/n \qquad (2)$$

Also we have $AG/GE = AR/RC = x/(n-1)x = 1/(n-1)$ (3)

Also triangles RGH and BEH are similar; thus

$$RH/HB = GH/HE = GR/BE = GR/CE \cdot CE/BE$$

$$= (y/n)/(n-1)y = 1/n(n-1) \qquad (4)$$

Hence $GH = HE/n(n-1)$. (5)

But $GE = GH + HE = HE(1 + 1/n(n - 1))$

$$= HE(n^2 - n + 1)/(n(n - 1)) \qquad (6)$$

By (3), (5) and (6) $AH = AG + GH = HE(n)/(n-1)^2$ (7)

Since HK and EI are both perpendicular to AC, these lines are parallel and triangles AHK and AEI are similar. Thus using (7) we have

$$AK/AI = HK/EI = AH/AE = AH/(AH+HE) = n/(n^2-n+1) \qquad (8)$$

Since BF is perpendicular to AC, $BF \parallel EI$ and triangles BFC and EIC are similar. Hence

$$BF/EI = FC/IC = BC/EC = ny/y = n. \qquad (9)$$

Comparing the areas of triangles of triangles ARH and ACE, we find

$[ARH]/[ACE] = (AR \cdot HK)/(AC \cdot EI) = AR/AC \cdot (HK/EI)$

which by (2) and (8) is equal to $1/(n^2 - n+1)$. (10)

Hence $[ARH/ABC] = [ARH]/[ACE] \cdot [ACE]/[ABC]$

$$= 1/n(n^2 - n + 1). \qquad (11)$$

Similarly $[BDL]/[ABC] = [CEM]/[ABC] = [ARH]/[ABC]$

$$= 1/n(n^2 - n + 1). \qquad (12)$$

Finally

$$[HLM]=[ABC]-[ABR]-[BCD]-[AEC]+[ARH]+[BDL]+[CEM]$$

$$=[ABC]\cdot(1-3/n+3/(n^2-n+1))=[ABC]\cdot(n-2)^2/(n^2-n+1).$$

In the given problem, n=4 so that [HLM]/[ABC]=4/13 .

Also solved by Fred A. Miller, Elkins West, Virginia and Charles W. Trigg, San Diego, California.

376: Proposed by the editor.

Egyptian fractions were based upon the representation of a given fraction as the sum of a finite number of fractions whose numerators are equal to 1. Erdos has conjectured that the fraction 4/n can be expressed as the sum of three or fewer such fractions having distinct denominators for all natural numbers $n > 2$. Prove that the conjecture is correct for at least 95% of the natural numbers.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

We begin by considering the case when n is even. If $n = 2k$ for some positive integer k, $\frac{4}{n} = \frac{1}{k} + \frac{1}{k}$.

But $$\frac{1}{k} = \frac{1}{k+1} + \frac{1}{k(k+1)} \quad . \tag{1}$$

Thus, $$\frac{4}{n} = \frac{1}{k} + \frac{1}{k+1} + \frac{1}{k(k+1)} \tag{2}$$

with each of the unit fractions on the right side of (2) being distinct. This shows that one half of the rational numbers of the form have the required expansions. Hence for $n = 24k$, $24k + 2$, where k is a positive integer and for $24j + d$ where $d = 4, 6, 8, 10, 12, 14, 16, 18, 20$, or 22 and j is a non-negative integer, equation (2) provides the required expansion.

What happens when n is odd? Obviously $\frac{4}{n} = \frac{1}{n} + \frac{3}{n}$ and $\frac{3}{n}$ will be a unit fraction whenever $n = 3k$ for some positive integer k. Thus by using (1) we obtain

$$\frac{4}{n} = \frac{1}{3k} + \frac{1}{k + 1} + \frac{1}{k(k + 1)} \tag{3}$$

The denominators $3k$, $k+1$, and $k(k+1)$ are not distinct only if $k = 1$ or 2.

For $k = 2$, $n = 6$ and we have by (2)

$$\frac{4}{6} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12}$$

For $k = 1$, $n = 3$ and we have

$$\frac{4}{3} = \frac{1}{1} + \frac{1}{4} + \frac{1}{12} .$$

Thus the rational number $\frac{4}{n}$, $n \geq 3$, has the required expansion for n having the forms $24j + 3$, $24j + 9$, $24j + 15$, and $24j + 21$. We can use the decomposition $\frac{4}{n} = \frac{1}{n} + \frac{3}{n}$ to take care of some additional cases. By applying (1) to $\frac{3}{n}$ we find that

$$\frac{4}{n} = \frac{1}{n} + \frac{3}{3+1} + \frac{3}{n(n+1)} \tag{4}$$

If 3 divides n + 1, the latter two fractions in (4) will be distinct. So, if n = 12k + 5 for some non-negative integer k, then

$$\frac{4}{n} = \frac{1}{12k+5} + \frac{1}{4k+2} + \frac{1}{(4k+2)(12k+5)} \tag{5}$$

and if n = 12k + 11 for some non-negative integer k, then

$$\frac{4}{n} = \frac{1}{12k+11} + \frac{1}{4k+4} + \frac{1}{(4k+4)(12k+11)} \tag{6}$$

Thus when n has one of the forms 24j+5, 24j+17, 24j+11, and 24j+23 the rational number $\frac{4}{n}$, $n \geq 3$, has an expansion of the type sought.

By (1) $$\frac{4}{n} = \frac{4}{n+1} + \frac{4}{n(n+1)} .$$

The last two fractions will be distinct if 4 divides n+1. In particular, if n = 24j + 7, then by using (1) appropriately we have

$$\frac{4}{n} = \frac{1}{(24j+7)(6j+2)} + \frac{1}{6j+3} + \frac{1}{(6j+2)(6j+3)} \tag{7}$$

and if n = 24j + 19, then similarly we have

$$\frac{4}{n} = \frac{1}{(24j+19)(6j+5)} + \frac{1}{6j+6} + \frac{1}{(6j+5)(6j+6)} \qquad (8)$$

Up to this point we have demonstrated that $\frac{22}{24} = \frac{11}{12}$ of the rational numbers $\frac{4}{n}$, $n \geq 3$, have the required expansion. Only values of n of the forms 24k + 1 and 24k + 13 have not been considered. Observe that in either case 4 divides n + 3 while in the second instance 8 divides n + 3.

But $$\frac{4}{n} - \frac{4}{n(n+3)} = \frac{12}{n(n+3)} = \frac{8}{n(n+3)} + \frac{4}{n(n+3)} \quad .$$

Thus if n = 24j + 13 for some nonnegative integer j, then

$$\frac{4}{n} = \frac{1}{(6j+4)} + \frac{1}{(24j+13)(3j+2)} + \frac{1}{(6j+4)(24j+13)} \qquad (9)$$

We have established $\frac{23}{24} \geq 95\%$ of the rational numbers $\frac{4}{n}$, $n \geq 3$, have expressions of the type desired. To prove the conjecture of Erdos it remains to find an expansion of the desired type when n has the form 24k + 1. To date no one has been able to do it for every positive integer k.

Editor's comment:

A solution for this problem appears in "Crux

Mathematicorum" Vol. 5, No. 1, (January 1979) at page 26. Yamamoto [1] has verified that the conjecture holds for all integers n such that $3 \leq n \leq 10^7$. For other material and a similar conjecture see [2].

1. L. J. Mordell, Diophantine Equations, Academic Press, London, 1969, pp. 287-290.

2. B. M. Stewart, Theory of Numbers, Macmillan, New York, 1964, pp. 198-207.

THE HEXAGON

This department of THE PENTAGON is intended to be a forum in which mathematical issues of interest to undergraduate students are discussed in length. Here by **issue** we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to Iraj **Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.**

In this issue's Hexagon, I wish to introduce to you an unsolved problem.

Problem 1) Suppose g is a function from the positive integers to positive integers defined by

$$g(n) = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ 3n+1 & \text{if } n \text{ is odd.} \end{cases}$$

This function was introduced by Lothar Collatz who also conjectured that:

for every n, there exists a k such that $g^k(n)=1$.

(Here $g^2(n) = g(g(n))$, $g^3(n) = g(g(g(n)))$ etc.)

This conjecture is neither proved nor refuted as of yet. Although, it has been verified, with the help of computers, that it holds for $n \leq 10^9$.

The purpose of this article is to invite the reader to write and verify (or refute) the truth of the conjecture for larger segments of the positive integers. The findings are to be sent to Hexagon. Best findings and most clever programs will be published in this column. Similar problems are listed below and the reader is invited to approach them with the same questions. Once a program is written for one of them, it would be easy to modify it for the others.

Problem 2) Let $h(n) = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ 5n+1 & \text{if } n \text{ is odd.} \end{cases}$

Does, for every n, there exist a k such that

$$h^k(n) = 1 \;?$$

Problem 3) Let

$$k(n) = \begin{cases} n/3 & \text{if } n \text{ is a multiple of 3,} \\ 2n+1 & \text{if } n \text{ is a multiple of 3 plus 1,} \\ 2n-1 & \text{if } n \text{ is a multiple of 3 plus 2.} \end{cases}$$

Does, for every n, there exist a m such that

$$k^m(n) = 1.$$

Problem 4) Let n be a d-digit positive integer in base 10. Let

$f_1(n)$ = the digits of n in non-increasing order (left to right) read as a d-digit number,

$f_2(n)$ = the digits of n in non-decreasing order (left to right) read as a d-digit number,

(for example $f_1(3724) = 7432$

$f_2(3724) = 2347$).

Finally $$F(n) = f_1(n) - f_2(n).$$

Fix d. Does there exist a 'small' list of d-digit numbers, S, such that for every n, there exists a k with $F^k(n)$ in S? The reader should start with d=1 and proceed to d=2,3,4 etc.

Problem 5) Investigate Problem 4 in an arbitrary base b.

Problem 6) Generate your own problem similar to the problems above.

Send all solutions, proofs and findings to the editor of this column and examine the next issue for a report on these problems.

IK

THE CURSOR
Edited by Jim Calhoun

This issue of THE CURSOR is special because for the first time a student paper is presented. The paper was originally presented by Damon Antos at the 25th Biennial Convention of Kappa Mu Epsilon held April 11-13, 1985 at Southern Methodist University. Damon is an undergraduate at California Polytechnic State University in San Louis Obispo, California and his paper deals with the construction of "intelligent" programs which emulate the playing of such games as chess, checkers and backgammon.

We encourage readers, particularly students, to submit papers which deal with the relationship between mathematics and computer science.

GAME PLAYING AND ARTIFICIAL INTELLIGENCE
by
Damon Antos

Although Artificial Intelligence (AI) has existed as a field of research for the past thirty years, only recently has it gained general public attention. One focus of AI research deals with the playing of games. This focus began in 1963 with Samual's work with the game of checkers and was continued by others to include the games of chess, Othello, Reversi, Go and Backgammon. Typically such work involves the construction and implementation of an algorithm which is intended to emulate the near perfect strategy of a master of the game.

One may ask, "Why should game playing be of interest to AI?"

it would seem that it would be no problem to write a program to play a game with perfect strategy and thus always win against a human opponent. However, a game playing program has yet to be written which can match the best human play. What are the constraints which account for this fact? A game of strategy consists of a sequence of moves, each of which is an occasion for a choice between certain alternatives, made by players of the game. Such games depend on well defined rules and strategies which specify at each move which player is to make the move and what his alternatives are. A computer program written to play such a game would hope to construct a complete game tree where each node corresponds to a state in the play and the nodes which are the successors of that node form the set of alternatives from which the indicated player must choose. Figure 1 shows a sketch of a complete game tree for a checkers game in which it is assumed that the "red" player moves first. Note that the levels of the tree alternate play between the "red" player and the "black" player and that each state S_i corresponds to a state which could be reached upon the i^{th} move of the game. The actual number of alternatives is represented symbolically since an accurate calculation depends on precise knowledge of the state of the checker board. With respect to a given player, each path which identifies the complete play of a game can be classified as a "win", "loss" or "tie" path. Ideally, by traversing the tree in reverse using a minimax search procedure it is possible to find a winning line of play.

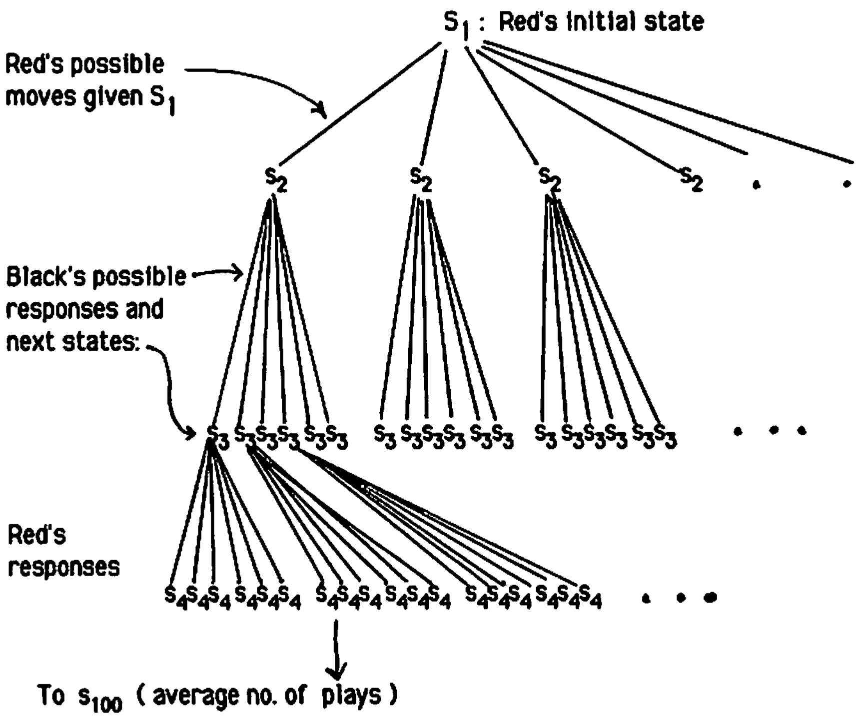


Figure 1: A checkers game tree drawn from red's point of view, red's move based on the average number of legal move available at each play.

A program using this result would play a theoretically perfect game because for each of its moves it would be able to rule out those choices which could ultimately allow the opponent to win. However, because of the enormous number of nodes involved in the construction of the game tree, this technique works only for the simplest of games. In fact for games such as checkers, chess and Go it is so large that even the ablest computers cannot handle it. For example, a game tree for checkers would include approximately 10^{78} nodes and require 10^{60} centuries of computing time for construction and evaluation even if the computation were done on one of today's supercomputers. Since the number of nodes is enormous , the computer must use huerlstics or "rules of thumb" in order to reduce the search to a manageable subtree of the total game tree.

The use of huerlstics in a game tree approach has achieved some success with games which do not involve chance. However, for games such as backgammon which do involve chance, the game tree approach has only limited use. In backgammon there are an average of four hundred different moves available at each play (the roll of the dice produces 21 different results, each of which can be played an average of 20 different ways). On a given roll of the dice with a given state of the board the player must determine the set of legally available moves. This evaluation process is static in the sense that it evaluates only the current state of the game and ignores all previous, as well as future moves. The remainder of this paper focuses on the construction of a "Static Evaluation Function" which is used to construct the set of legal moves and determine which move is best. The game tree of Figure 2 shows the application of the Static Evaluation Function to produce the next states. Note that the evaluation function not only identifies the next states but returns a value for each. It is the state with the "best" return value which is selected as the next state.

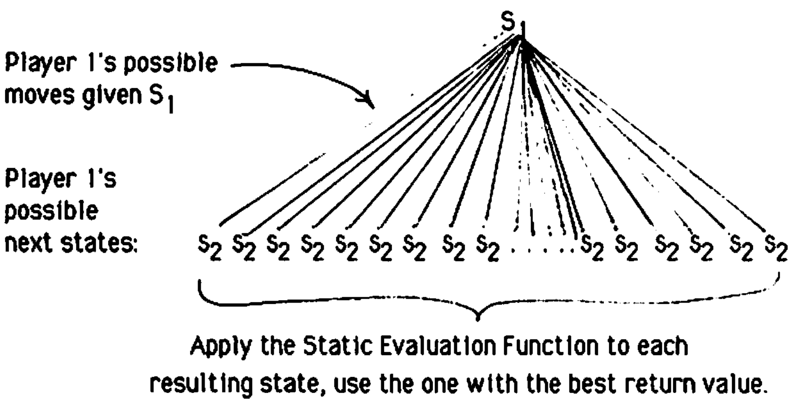


FIGURE 2: A backgammon game tree to be used with a Static Evaluation Function drawn from player 1's point of view based on the average number of legal moves available at each play.

The first step toward the creation of a model for the simulation of this game is to identify the rules (although you need not be familiar with backgammon to follow the major ideas presented here). Once the rules have been established in the model, the stucture of the backgammon program can be created. As noted before, each play has an average of twenty legal moves. The task of the program is to find these legal moves from among the 24^2 (24^4 if doubles are thrown) possible moves. Most of the illegal moves can be easily identified and ruled out by the following set of backgammon rules:

(a) A player may not move a man around the playing board if he has one or more men on the bar

(b) A player may only move from positions on which he has men

(c) A player may not land a man on a position which is occupied by two or more of the opponent's men

(d) A player may not "bear off" a man unless all of his stones which are left in play are on his inner table

The application of these rules to determine the set of twenty or so legal moves is easy, but the process of selecting the best move from among the set of the legal moves is not. The difficulty arises because an effective solution requires the emulation of human thinking.

To determine the best move, one must select from among all available legal moves the one which will advance play to a state from which the player has the best chance of producing a win. That is, we seek the move whose "mathematical expectation" is highest. To do this, one formulates several sub-strategies which are then used to improve the Static Evaluation Function. The following list identifies such sub-strategies:

(1) Minimize the number of unprotected men (blots)

(2) Maximize protected men (blocks)

(3) Seek the most effective arrangement of blocks

(4) Land on the opponent's blots

(5) When blots must be left, leave them where they are least likely to be hit by the opponent

Although these sub-strategies are available to the human player as well, the computer has the advantage of being able to quickly and accurately calculate the probability that the opponent will receive a particular dice roll. With such information it is possible to rule out moves which are potentially detrimental to one's position. The human player can only approximate such values.

Each sub-strategy including (1) through (5) above is used to construct a mathematical function F_i which represents the extent to which a particular move follows that sub-strategy. Of course a particular move may do very well with respect to one sub-strategy and poorly to another. These functions are used to construct a Static Evaluation Function of the form:

$$\text{Evaluation Polynomial } P = F_1 + F_2 + \ldots + F_n$$

```
DO UNTIL end of game
   Roll Dice
   IF the player has a man on the bar and
         it cannot be put back into play THEN
      skip his turn
   OTHERWISE
      DETERMINE all possible ways of moving
         based on the current roll, remembering only those moves
         which are legal according to the predefined model
      CALL the Static Evaluation Function for each of the remembered
         moves; denote the move with the highest return value as the
         optimal move
      PERFORM the optimal move found above
END UNTIL
```

FIGURE 3a : THE ALGORITHM FOR THE MAIN PROGRAM

The value returned by P is a collective measure of the extent to which the move satisfies all the sub-strategies. Although the polynomial P is a good first step, it suffers because it doesn't take into consideration the fact that the significance of a sub-strategy may vary with the progress of the game. A study of the strategy of the best human backgammon players quickly points out that plays which are excellent in the middle of the game will most likely be poor if used toward the end of the game. To achieve appropriate weighting of the sub-strategies, coefficients A_i are applied to the functions F_i altered during the course of play. The objective is to modify the Static Evaluation Function so that it adjusts to suit the conditions.

STATIC EVALUATION FUNCTION

(* During course of play assign coefficients to function in such a way to attain an appropriate strategy for a player's board position *)

<u>IF</u> at the start of game:

Player on offensive: Assign a higher value to hitting opponent's unprotected men than other sub-strategies.

Player on defensive: Same as offensive.

<u>OTHERWISE</u>

<u>IF</u> sometime in middle of game:

Player on offensive: Concentrate on moving men to the inner table

Player on defensive: Take more risks to get back on the offensive; hit opponent's unprotected men whenever possible disregarding own unprotected men.

<u>OTHERWISE</u> (* at end of game *)

Player on offensive: Maximize protected men on inner table, hit opponent's unprotected men at any time.

Player on defensive: Player on defensive: Hit opponent's unprotected men disregarding own protected men.

<u>END FUNCTION</u>

FIGURE 3b : THE STATIC EVALUATION FUNCTION

The coefficients are to be mathematical representations of the following conditions:

(1) Game phases : beginning, middle, end or somewhere between

(2) The ease of winning with the current board position (ready to bear off, position on the inner table)

(3) The degree to which the winning side is winning

Each coefficient A_i is based on one or more of conditions (1) through (3) and is used to produce the revised evaluation polynomial:

$$P = A_1F_1 + A_2F_2 + \ldots + A_nF_n$$

The ideas discussed in this paper are incorporated in the algorithms shown in Figures 3a and 3b. Together they represent a computer program which plays backgammon. In contrast to chess playing programs whose success depends on computer search speed, the backgammon program depends on the execution efficiency of the Static Evaluation Function whose computations can be lengthy.

The implementation presented in this paper is not the definitive algorithm for backgammon. There are a number of interesting possibilities for further research. One is to modify the present program so it adjusts the coefficients of the Evaluation Function based on past experience. A second one is the construction of a new algorithm based on theories of learning which would allow the program to learn from experience. Both of these proposals express the need to explore further the relationship between the methods used in programs to emulate human thinking and those believed to be used by humans. Improvements in the emulation of human thinking are certain to add to our understanding of how the human mind works.

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KAPPA MU EPSILON NEWS
Edited by Harold L. Thomas, Historian

News of chapter activities and other noteworthy KME events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department, Pittsburg State University, Pittsburg, Kansas 66762.

CHAPTER NEWS

Alabama Gamma, University of Montevallo, Montevallo
Chapter President - Debbie Evans
7 actives, 2 initiates

The Chapter held a fall student-faculty cookout. Other 1984-85 officers: Vicki Simmons, vice president; Lauri Stevens, secretary; Cathy Johnson, treasurer; Joseph Cardone, corresponding secretary and faculty sponsor.

Alabama Zeta, Birmingham-Southern College, Birmingham
Chapter President - Judy Tanquary
39 actives

At the spring initiation of new members, Dr. David Johnson spoke on "Graph Theory." Other activities have included a talk by Dr. Robert Kaufman from the University of Alabama in Birmingham on "Graduate Programs in Mathematics," informal pizza lunches with professors and students, participation in the College Scholars Bowl, spring picnic and Christmas party with other science honoraries. Other 1984-85 officers: Keyna Warren, vice president; Karin Johnson, secretary; Richard Sturgeon, treasurer; Lola Kiser, corresponding secretary and Sarah E. Mullins, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President - Harland Duncan
54 actives, 29 initiates

Weekly meetings were held with speakers from business and industry. A Christmas social and pledge ceremony were held. A joint meeting (regional meeting) was held on November 16 with California Delta at Vandenberg Air Force Base. Other 1984-85 officers: Julie Justice & Damon Antos, vice president; Laura Melody, secretary; Charles Hughes, treasurer; George R. Mach, corresponding secretary; Adelaide T. Harmon-Elliott, faculty sponsor.

California Delta, California State Polytechnic University-Pomona, Pomona
Chapter President - Randall Swift
28 actives, 8 initiates

The chapter held a regional meeting with California Gamma Chapter at Vandenberg Air Force Base. Supplementary tutorial services were provided for the Mathematics Department. Pre-Putnam competition seminars were held and a team was sponsored by KME. Other 1984-85 officers: Michele Carter, vice president; Lisa Finkbiner, secretary; Holly Young, treasurer; Richard Robertson, corresponding secretary; Judy McKinney, faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins
Chapter President - Terri M. Woods
19 actives

Four regular meetings were held with programs being presented by mathematics professors and by an industrial mathematician. Other 1984-85 officers: Curt Bennett, vice president; Kim Pokorney, secretary & treasurer; Arne Magnus, corresponding secretary and faculty sponsor.

Connecticut Beta, Eastern Connecticut State University, Willimantic
Chapter President - Michael Rosseau
15 actives

In the Spring of 1984, Connecticut Beta installed twenty new members during a dinner/dance which was attended by many math department alumni. During the fall semester, the chapter sponsored two campus-wide contests in the form of mathematical puzzles which generated or reawakened much interest in mathematics and in KME. The first puzzle was a robotics and vector analysis problem and the second a logic problem. A cash prize was awarded to the author of the best solution in each case. The winners were then honored with an invitation to present their solutions at a meeting of the chapter. Other 1984-85 officers: Debra Adams, vice president; Keven Ratcliffe, secretary & treasurer; Stephen Kenton, corresponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Maureen Ramey
25 actives

The Georgia Alpha Chapter met on October 22, 1984, and planned a Christmas party for November 28, 1984. Members brought presents to the party which were later donated to a local Training Center for mentally retarded people. Twenty-seven persons attended the party. Several members ordered jewelry from the National Office. Also, several members ordered KME jerseys from the Campus Bookstore. Other 1984-85 officers: Sandra Hyde, vice president; Leanne Perry, secretary; Beth Hyde, treasurer; Joe Sharp, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston
Chapter President - Dave Bryden
45 actives

The Chapter met once a month with the EIU Math Club. Spring semester activities were planned at these meetings. Other 1984-85 officers: Greg Oberlog, vice president; Pat Winkler and Jeff Nettles, secretary; Ken Mills, treasurer; Lloyd L. Koontz, corresponding secretary and faculty sponsor; June Shanholtzer, faculty sponsor.

Illinois Zeta, Rosary College, River Forest
Chapter President - Sheila M. Schultze
8 actives

Illinois Zeta members sponsored a combined plant and bake sale to help finance the trip to the national convention in April, 1985. A questionnaire was sent to mathematics majors of the past fifteen years and data is being compiled on their careers. The information will be used for discussion by present mathematics and computer science majors. Other 1984-85 officers: Mary Martino, secretary; Lisa Lynn Behnke, treasurer; Sister Nona Mary Allard, corresponding secretary and faculty sponsor.

Illinois Eta, Western Illinois University, Macomb
Chapter President - Tamara Lakins
12 actives, 2 initiates

Regular meetings were held throughout the semester. Faculty speakers at the meetings included Dr. Iraj Kalantari, Dr. Galen Weitkamp, and Dr. Ross Wilkinson. Dr. Michael Moses gave a short talk on a course in Graph Theory which he will teach in the spring. The Math Club contributed a great deal in the execution of our two fund raisers: The Annual Faculty Chili Lunch and a bake sale. Two members were initiated in December when department chairman, Dr. Larry Morley was the speaker. A pizza party followed the initiation. Other 1984-85 officers: Doug Faries, vice president; Lissa Pope, secretary and treasurer; Alan Bishop, corresponding secretary; Iraj Kalantari, faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
Chapter President - Dan Cripe
20 actives

Other 1984-85 officers: Ryan McBride, vice president; Todd Saunders, secretary; Mike Ober, treasurer; Ralph McBride, corresponding secretary; James Rowe, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President - Kande Hooten
34 actives, 7 initiates

The annual Homecoming Breakfast for Iowa Alpha members, faculty, and alumni was held September 22, 1984 at the home of Dr. and Mrs. Hamilton. This event was well attended with about six alumni joining the local crowd. At the September meeting, Iowa Alpha member, Kevin Junck, presented his paper on the Schroedinger Wave Equation. At the October meeting, Lisa Naxera spoke on Negative Number Bases and Kelly Donlin provided a talk on Angle Trisection at the December initiation banquet held at Nino's. Other 1984-85 officers: Lisa Naxera, vice president; Kelly

Donlin, secretary; Scott Kibby, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President - Scott Rothfus
12 actives

A "get-acquainted" picnic was held in September with Freshman and transfer students as guests. At the regular fall meetings, papers were given by Scott Rothfus and Ruth Gornet. Other 1984-85 officers: Tracy Parks, vice president; Sheryl Shapiro, secretary; Ruth Gornet, treasurer; Joseph Hoffert, corresponding secretary; Lawrence Naylor, faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President - Gary Friedrichsen
24 actives

The Chapter held four regular monthly meetings during the Fall semester. In September, reports were given on Summer Internships. A. M. Fink from Iowa State University gave a talk on Grundy Functions in October. The November program was given by Paul Fjelstad of St. Olaf College about Perplex Numbers. The semester closed with a December Christmas party at Dr. Fenneman's home. Other 1984-85 officers: LeAnn Hobbs, vice president; Steven Harr, secretary; Deanna Bauman, treasurer; Glenn C. Fenneman, corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President - Sue Pyles
40 actives, 12 initiates

The Chapter held monthly meetings in October, November, and December. In addition, a fall picnic was hosted for all mathematics and physics students. Fall initiation for new members was held at the October meeting. Twelve new members were initiated at that time. The October program was given by Kendall Draeger. He related some of his summer experiences with Weyerhaeuser. Sue Pyles and M. Norbani presented the November meeting on the "Romberg Integration Procedure." In December, a special Christmas meeting was held at the home of Dr. Helen Kriegsman, Mathematics Department Chairman. Mary Slobaszewski shared some of her experiences she had during her student teaching. Other 1984-85 officers: David Pennington, vice president; Earlena Brownewell, secretary; Tami Dodds, treasurer; Harold L. Thomas, corresponding secretary; Helen Kriegsman and Gary McGrath, faculty sponsors.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Mary Jo Muckey
16 actives, 17 initiates

The Chapter began the year with the annual Fall picnic at the home of faculty member Larry Schultz on September 21. In October, two fund raising events were held. The first was a book sale. Many students purchased math texts that had been donated by the faculty. The second activity was a scavenger hunt. Participants had to solve mathematical problems in order to determine how many of each item they were to find. On November 13, Dave Dover, a 1974 graduate of the college, came to campus as a guest speaker for the chapter. His topic, "Math and Air Traffic Control" was very familiar to him because of his many years of experience as an air traffic controller. The semester ended with the traditional Christmas Wassail party on December 7. Other 1984-85 officers: Rita Lundstrom, vice president; Patricia Patterson, secretary and treasurer; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President - Doug Bogia
20 actives

Two papers were presented at fall meetings which had been prepared for the national meeting in the spring. Other 1984-85 officers: Ward Canfield, vice president; Gina Estes, secretary and treasurer; Robert Thompson, corresponding secretary; Allan Riveland, faculty sponsor.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President - Bev Musselwhite
25 actives

Fall semester activities included a picnic on September 10, a Halloween party on October 26, and monthly meetings with student presentations. Other 1984-85 officers: Todd Deines, vice president; Michelle Ferland, secretary and treasurer; Charles Votaw, corresponding secretary; Jeffrey Barnett, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President - Phillip Hamilton
35 actives

The Chapter is planning to send a delegation to the national convention, so several Fall activities were aimed at raising money to help finance the trip. The students sold over 400 long-stemmed roses during the fall semester. They have also sold M&M's. There were three speakers that presented talks to the chapter. Dr. Donald Greenwell spoke on "Games and Graphs." Dr. Patricia Costello gave a talk on "Statistics and Polls" (just before election). Dr. James Patterson discussed "Graduate Mathematics" with the students. Social activities included a picnic for both KME members and faculty of the department, a flag football game against the faculty, and a volleyball game with the faculty. Other 1984-85 officers: Vince Leopold, vice president; Barb McGrath, secretary; Dana Baxter, treasurer; Patrick Costello, corresponding secretary; Donald Greenwell, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore
Chapter President - Michele Ritter
7 actives, 8 initiates

In November, the members of the Math Society sponsored a film on the art of M. C. Escher entitled, "Ventures in Perspective." Most of the time has been spent in making arrangements for the annual Mathematics Olympiad to be held for high school students. This contest will be held on February 16 in the Knott Science Center. Other 1984-85 officers: Donna Woods, vice president; Nikki Simmers, secretary; Sister Marie A. Dowling, corresponding secretary; Sister Del Marie Rysavy, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President - Cliff Martin
17 actives

One new member was initiated during the Fall semester. A fondue party was held for all mathematics and science majors in December. Other 1984-85 officers: Steve Coffman, vice president; Julie Winkler, secretary; Wende Reeser, treasurer; James Lightner, corresponding secretary; Linda Eshleman, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President - Kurt Lemmert
27 actives

The Fall 1984-85 semester of MD Delta began with an organizational meeting in August. In September, a get-acquainted picnic was held at New Germany State Park. Other activities for the semester included a talk by Fred Cunningham (student) concerning a very beneficial internship experience at COMSAT; a presentation by Dr. Richard Weimer, Head of the Frostburg State College Mathematics Department, entitled, "Neat Aspects of Correlation;" and Dr. James Fey of the University of Maryland spent an entire day on the FSC campus discussing the "Educational Use of Computers" with faculty and students. Other 1984-85 officers: Lisa McIntosh, vice president; Alana Fatkin, secretary; Teresa Neville, treasurer; Donald Shriner, corresponding secretary; John Jones, faculty sponsor.

<u>Michigan Beta</u>, Central Michigan University, Mt. Pleasant
Chapter President - Larry Ludwig
60 actives

Michigan Beta began the Fall semester with a picnic for student members and faculty. Professor James Angelos spoke at the November meeting on "Approximation Theory." Problem help sessions for freshman-sophomore mathematics classes were again held for 3 nights a week. The semester closed with a Christmas party at the home of Professor Hammel. Other 1984-85 officers: Mike Vieau, vice president; Deidre Lentz, secretary, Lisa Burnell, treasurer; Arnold Hammel, corresponding secretary and faculty sponsor.

<u>Mississippi Alpha</u>, Mississippi University for Women, Columbus
Chapter President - Susan Furlow
10 actives, 5 initiates

Other 1984-85 officers: Becky Flowers, vice president; Elizabeth Douglas, secretary and treasurer; Jean Parra, corresponding secretary; Carol Ottinger, faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President - John McDonald
51 actives, 20 initiates

KME initiated 20 new members at the Fall cookout which was held at the home of the sponsor, Virginia Entrekin. The Taco-Fest was attended by math department faculty members as well as old and new members. "Mucho" tacos were consumed by all along with the many side dishes contributed by the group. KME continues to sponsor the undergraduate mathematics colloquiums organized by Dr. Temple Fay. These challenge sessions in mathematics are given by members and faculty. Dr. Steve Doblin spoke to a large number of the club on "Careers in Mathematics -- What One Can Do with a Degree in Math!" A new project for KME this year is the newsletter, "MatheMADics," which is sent periodically to members with informative club material and fun problems in mathematics. Other 1984-85 officers: Twila Marie Hendry, vice president; Deanna Caveny, secretary and treasurer; Alice Essary, corresponding secretary; Virginia Entrekin, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President - Sharyn Birkenbach
37 actives, 12 initiates

Other 1984-85 officers: Joy Farr, vice president; Mary Smith, secretary; Mike Kimzey, treasurer; M. Michael Awad, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President - Lynn Hitchcock
32 actives, 4 initiates

The Chapter began the Fall semester with a pot luck supper in conjunction with the fall initiation. Besides inducting regular initiates, two associate initiates were also inducted. Associate membership is on a trial basis for the 1984-85 school year. An associate member has the same qualifications as a regular member for invitation into Kappa Mu Epsilon with the following exception: The student is not required to have completed at least three semesters of school. The associate member is allowed to attend all meetings and activities, but is not allowed to vote. The associates have representatives that attend all executive meetings. These representatives are trained in Kappa Mu Epsilon ideals and objectives. The new associate membership has increased meeting attendance and activity attendance for the year. The Chapter's other activities included a float trip in September, and a field trip to Marion Laboratories in Kansas City, MO. KME also attended annual Halloween and Christmas parties. Some guest speakers for the monthly meetings included Dave Greenley from TWA, a speaker from AT&T, and Richard Gibson. Other 1984-85 officers: Roberto Ribas, vice president; Cheryl Harris, secretary; Brenda Enke, treasurer; Martha Coats, historian; Homer Hampton, corresponding secretary; Larry Dilley, Gerald Schrag and Rhonda McKee, faculty sponsors.

Missouri Gamma, William Jewell College, Liberty
Chapter President - Eric Conrad
11 actives

Monthly meetings were held during the Fall of 1984. Spring initiation and banquet will be held in April. Other 1984-85 officers: Brian Wells, vice president; Laurie Honeyfield, secretary and treasurer; Joseph T.

Mathis, corresponding secretary and faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President - Cheryl Mathewson
11 actives

Other 1984-85 officers: Stacy Garrett, vice president; Robin Hamil, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

Missouri Zeta, University of Missouri, Rolla
Chapter President - Kevin Davis
67 actives, 21 initiates

Monthly meetings were held in September, October, November and December. Program topics included "Crayons, Points and Lines" by Johnny Roberts, "The Two Color Theorem" by Dr. Purcel, and "Gomory's Theorem" by Dr. Hicks. The annual faculty-student picnic was held on September 7. Regular semi-weekly help sessions were held for students in all math classes through Differential Equations. The annual banquet was held on November 24, at Chub & Jo's Restaurant. The guest speaker was Dr. Wright and his topic was "Statistical Law." Other 1984-85 officers: Paul Whitten, vice president; Roberta Bateman, secretary; Tim Allen, treasurer; Tom Powell, corresponding secretary; Jim Joiner, faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville
Chapter President - Bob Clark
24 actives, 13 initiates

Regular bi-monthly meetings with speakers were held during the Fall semester. The Chapter hosted an open house for seventy high school seniors. Other activities included a volleyball challenge round with the mathematics department faculty and a Christmas party. As a money raising project, the members designed and sold note pads on campus. Other 1984-85 officers: Nancy Schmitt, vice president; Yvonne Hall, secretary; Rebecca Hutton, treasurer; Sam Lesseig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin
Chapter President - Susan Petty
16 actives

The Chapter began the fall semester with the annual float trip on Shoal Creek. Other activities during the semester included working concessions at a football game as a money making project, regular monthly business meetings, viewing a film on space-filling curves, free tutoring of pre-calculus students, attendance at the installation of Missouri Kappa at Drury College, a Christmas party at Mrs. Elick's house, and initial preparation for the celebration of the tenth anniversary of the chapter in the spring of 1985. Other 1984-85 officers: Carol Lazure, vice president; Steve Brock, secretary and treasurer; Cheryl Ingram, historian; Mary Elick, corresponding secretary; Jo Shields, faculty sponsor.

Missouri Kappa, Drury College, Springfield
Chapter President - Pauline Hart
12 actives

On November 30, 1984, the Missouri Kappa Chapter of Kappa Mu Epsilon was installed at Drury College in Springfield, Missouri. Eleven participants, including Drury students and faculty, were initiated as charter members. The installation was conducted by the Southwest Missouri State University (Missouri Alpha) with Dr. L. Thomas Shiflett (faculty sponsor) and Dr. Michael Awad (corresponding secretary) presiding. Also present at the ceremony were representatives from Missouri Southern State College (Missouri Iota) and Evangel College (Missouri Theta). The newly installed chapter evolved from the Drury College Math Club that was established during the spring of 1983. The club has been quite active since its conception: provided a free tutoring service for all math classes, sponsored a campus-wide math contest, had guest speakers from different departments explaining the application of mathematics to their disciplines, and has had a social function each semester. The charter members are: Julie Blumhost, Sami Long, Clint Brown, Scott Rollins, Jean Netzer, Sonya Hauck, Barbara Robinson, Pauline Hart, Mike Shackelford, Dr. Steve Rutan (Math Department), and Mr. Ted Nickle (sponsor). Other 1984-85 officers: Barbara Robinson, vice president; Julie Blumhost, secretary; Sami Long, treasurer; Charles S. Allen, corresponding secretary; Ted Nickle, faculty sponsor.

<u>Nebraska Alpha</u>, Wayne State College, Wayne
Chapter President - Annette Schmit
32 actives

To make money throughout the fall semester, club members have monitored the Math-Science Building in the evenings. Sandra Sunderman and Doug Anderson each were awarded a $25.00 book scholarship which is given to a <u>KME</u> member each semester by the club. The club participated in the college homecoming activities by painting and erecting a billboard. The club also participated in the building student-faculty picnic and softball game. The <u>KME</u>-Computer Club team won the softball game. With a grant from the Wayne State College Student Senate, <u>KME</u> and the Computer Club purchased an X-Y Plotter which is available to all students. Other 1984-85 officers: Lori Schulenberg, vice president; Sandy Sunderman, secretary-treasurer; Jerry Wiesler, historian; Fred Webber, corresponding secretary; James Paige and Hilbert Johs, faculty sponsors.

Nebraska Beta, Kearney State College, Kearney
Chapter President - Christine Moses
30 actives, 14 initiates

The Chapter sold CRC Math Handbooks to math statistics, computer science and physics students as a fall money-making project. It was very successful. In January, members helped the Chemical Engineer Society with the Mathcounts contest for the region. Other 1984-85 officers: Gloria Liljestrand, vice president; Anne Wilson, secretary; Laura Issac, treasurer; Charles J. Pickens, corresponding secretary and faculty sponsor.

Nebraska Gamma, Chadron State College, Chadron
Chapter President - Annette Stumpf
7 actives, 3 initiates

Other 1984-85 officers: David Mundt, vice president; Terri Scofield, secretary; David Mundt, treasurer; James A. Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

New Jersey Beta, Montclair State College, Upper Montclair
Chapter President - Phyllis Blasi
6 actives, 20 initiates

Other 1984-85 officers: Lisa Herbst, vice president; Kristin Peterson, secretary; Beth Olohan, treasurer; John G. Stevens, corresponding secretary and faculty sponsor; Thomas Devlin, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President - Gina D'Antonio
50 actives

Other 1984-85 officers: Eric Brown, vice president; Jennifer Tyler, secretary; Richard Metzler, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Alpha, Hofstra University, Hempstead
Chapter President - Paul Orgera
8 actives, 7 initiates

The Chapter sponsored and participated in the mathematics contest with schools in the area. A mathematics contest for Hofstra freshmen was also held. Regular meetings were held with programs given by faculty and invited speakers. Other 1984-85 officers: Ethel Kleinhans, vice president; Kathrin Phillips, secretary; Nariman Ayyad, treasurer; Stanley Kertzner, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara
Chapter President - Chris Reilly
12 actives

During the Fall semester, the members met to elect officers for the school year and to consider activities that would generate revenue for the upcoming national convention in Dallas. Other 1984-85 officers: Ray Mueller, vice president; Amy Davis, secretary; Mary Ertl, treasurer; Robert L. Bailey, corresponding secretary and faculty sponsor.

New York Lambda, C. W. Post Center - Long Island University, Greenvale
Chapter President - Caroline Diffley
24 actives, 4 initiates

The Chapter was active in helping students in the math lab. A wine and cheese party was held at the beginning of the semester to welcome new math majors. An end of term party and installation was held on December 11. Other 1984-85 officers: Demetrios Zavallis, vice president; Susan Blaurock, secretary; Patricia Cairns, treasurer; Sharon Kunoff, corresponding secretary and faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green
Chapter President - Dixie Poppe
20 actives

The Chapter held a panel discussion on the job opportunities in Mathematics. Faculty members from different areas came in and talked about what possibilities exist and what course backgrounds would be beneficial. Other 1984-85 officers: Susan Kaeck, vice president; Cheryl Noe, secretary; Cynthia Jean Guest, treasurer; Frederick Leetch, corresponding secretary; Wallace Terwilliger, faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President - John Baker
22 actives, 9 initiates

The Chapter held monthly meetings during the Fall semester. The September 19 meeting featured student Steve Harris speaking on "The Works of Joseph LaGrange." October 17 saw the initiation of nine new members, who, after the ceremony, each gave a mathematical/historical talk of 3-5 minutes length. On

November 28, two talks were given by Dr. Chull Park and Dr. John Skillings, respectively, who came up from Miami University, Oxford, OH for the occasion. Both talks were well received and each generated discussion. The final meeting December 9 was a Christmas party at Smith's. Other 1984-85 officers: Ryan Harvey, vice president; Debra Clausing, secretary; Kimberly Lutz, treasurer; James L. Smith, corresponding secretary; Russell Smucker, faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President - Doug Walters
25 actives, 10 initiates

Fall activities began with a back-to-school picnic in September. A guest speaker gave a program for the Chapter. Fund-raising projects were held to raise money to send some members to the national convention. Other 1984-85 officers: Mike Ragan, vice president; Latrrica Anderson, secretary and treasurer; Wayne Hayes, corresponding secretary; Raymond McKellips, faculty sponsor.

Pennsylvania Beta, LaSalle University, Philadelphia
Chapter President - Karen Bruno
20 actives, 9 initiates

The Fall semester induction ceremony took place October 18, 1984, with nine members initiated. Dr. Errol Pomerance of the mathematical sciences department, spoke on the fundamentals of topology. Other 1984-85 officers: Mary McGee, vice president; Lisa Tresnan, secretary; Leon Weiner, treasurer; Hugh N. Albright, corresponding secretary; Carl McCarty, faculty sponsor.

Pennsylvania Gamma, Waynesburg College, Waynesburg
Chapter President - Laura Arrington
23 actives

The Chapter held an ice cream party at Professor Jackson's house to acquaint math students with KME. Other 1984-85 officers: Sherry Scott, vice president; Mary Beth Huffman, secretary and treasurer; Rosalie Jackson, corresponding secretary; David Tucker, faculty sponsor.

Pennsylvania Zeta, Indiana University of PA, Indiana
Chapter President - Toni Frick
19 actives, 5 initiates

The Chapter held meetings in October, November, and December. New members were initiated in October. Blaine Crooks, member of the Mathematics Department Faculty, gave a talk on statistics. In November, Dr. John Matolyak, member of Physics Department faculty presented a talk about applications of mathematics in physics. Officers were elected in December. Allan Williams, student member of KME, spoke on "Prisoner's Dilemma." Other 1984-85 officers: Joseph Ramsey, vice president; Amy Page, secretary; Carolyn Horrell, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Theta, Susquehanna University, Selinsgrove
Chapter President - Steve McGinty
15 actives, 7 initiates

The Chapter provided tutoring services Sunday through Thursday evenings during the semester. Seven new members were initiated on January 28, 1985. Other 1984-85 officers: Geraldine Gorman, vice president; Doris Cook, secretary and treasurer; Carol Harrison, corresponding secretary and faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President - Linda Rafferty
10 actives

Field trips to the Franklin Institute on November 10 and November 17 were a great success. Tutoring and problem solving are always an important activity of the Chapter. Plans were made for the initiation ceremonies to be held on March 11, 1985. The initiation is the highlight of KME and much preparation is made for it. Other 1984-85 officers: Linda Czajka, vice president; Christine Michaels, secretary and treasurer; Sister M. Grace, corresponding secretary.

Pennsylvania Lambda, Bloomsburg University, Bloomsburg
Chapter President - Wayne Hilker
35 actives, 15 initiates

Fall activities included a hayride, a faculty-student volleyball night, and a combinatorics lecture by Joan Hutchinson of Smith College. The members are actively trying to raise money for the Dallas convention in April, 1985. Other 1984-85 officers: Craig Funt, vice president; Lisa Cummings, secretary; Lori Snyder, treasurer; James C. Pomfret, corresponding secretary; Joseph Mueller, faculty sponsor.

Pennsylvania Mu, St. Francis College, Loretto
Chapter President - Ivan Jareb
8 actives

Other 1984-85 officers: Lori Paulekovsky, vice president; Susan Andrews, secretary; Gerard Weaver, treasurer; Adrian Baylock, corresponding secretary and faculty sponsor.

South Carolina Gamma, Winthrop College, Rock Hill
Chapter President - Kenneth Peay
17 actives, 3 initiates

Kappa Mu Epsilon and athletics go together at Winthrop. Three KME members have been named to the NAIA - Academic All American Teams: Phil Blankstein, baseball; Pam Garrett, basketball; and Larry Tavino, soccer. Larry Tavino was also a Rhodes Scholarship nominee. As a group, the Chapter participated in Winthrop's Phonathon in October and assisted with the annual Winthrop-Wylie Mathematics Tournament in December. Featured speakers for the Fall semester were Dr. Komkov, who spoke on definitions of convergence and continuity; Dr. Goolsby, who spoke on the language of mathematics and demonstrated interesting games and tricks; and Karen Huff, who told her famous card story for the December program. Initiation was also held during the December meeting. KME had a great Fall semester and looks forward to an even more spectacular Spring semester. Other 1984-85 officers: Phil Blankstein, vice president; Angie Breland, secretary; Norma H. Robinson, treasurer; Don Aplin, corresponding secretary; Kay Creamer, faculty sponsor.

Tennessee Alpha, Tennessee Technical University, Cookeville
Chapter President - Elisa Gould
35 actives

Other 1984-85 officers: Derek Lane, vice president; Anita Hasty, secretary; Edmund Prater, treasurer; Edmond D. Dixon, corresponding secretary; Richard Savage, faculty sponsor.

Tennessee Beta, East Tennessee State University, Johnson City
Chapter President - Jeff Hightower
10 actives

The Chapter made plans to attend the Mathematical Association of America meeting at Wake Forest U. An initiation service will be held in the Spring. A social was held at a local Mexican restaurant. Other 1984-85 officers: Tammy Gillenwater, vice president; Suzanne Walters, secretary; Lyndell Kerley, corresponding secretary and faculty sponsor.

Tennessee Gamma, Union University, Jackson
Chapter President - Emily Garrett
17 actives

Two Chapter meetings were held during Fall semester. At the second meeting, Dr. Matthew Gould, from Vanderbilt University spoke on the topic of Lattice Theory. New members will be inducted in April. Other 1984-85 officers: Brenda Ross, vice president; Charlotte Stockton, secretary; DeAnne Jarvis, treasurer; Don Richards, corresponding secretary; Dwayne Jennings, faculty sponsor.

Tennessee Delta, Carson-Newman College, Jefferson City
Chapter President - Jeff Knisley
23 actives

The Chapter kicked off the Fall semester with a cookout and volleyball match with the Society of Physics students. The two organizations joined forces again later in the semester for a hike in the Smoky Mountains. On Tuesday, November 13, Dr. Billy Bryant of Vanderbilt University spoke to KME members and mathematics majors. Covered in his lectures were the topics of "Babylonian Mathematics" and "Using a Computer to Guess at Algebra Theorems." KME concluded the fall semester with a progressive dinner provided by the mathematics faculty. Other 1984-85 officers: Jeff Kinsler, vice president; Art Blevins, secretary; Susan Williams, treasurer; Albert Myers, corresponding secretary; Carey Herring, faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President - Linda Haire
20 actives

A get-acquainted party was held at the home of Dr. and Mrs. Edwin J. Hewett. The purpose and activities of the society were explained to prospective members and recently inducted members received their shingles. Other 1984-85 officers: Ben Barris, vice president; Donna George, secretary; Mike Cagle, treasurer; Mary Wagner, corresponding secretary; Charles Robinson and Ed Hewett, faculty sponsors.

Virginia Beta, Radford University, Radford
Chapter President - Sharon Goad
20 actives

Other 1984-85 officers: Tammi Altice, vice president; Susan Weeks, secretary; Kathleen Cargo, treasurer; Coreen Mett, corresponding secretary; J. D. Hansard, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President - Linda Schmidt
6 actives, 6 initiates

Wisconsin Alpha sponsored the annual Mathematics Contest for junior and senior high school girls on November 17. Approximately 90 high school girls were in attendance. Time was spent discussing ways to raise funds. A doughnut sale was held. Other 1984-85 officers: Betty Zaborske, vice president and treasurer; Linda Schmidt, secretary; Sister Adrienne Eickman, corresponding secretary; Sister Petronia Van Straten, faculty sponsor.

Wisconsin Beta, University of Wisconsin-River Falls
Chapter President - Daniel Thiel
21 actives

This past fall, new officers were elected for 1984-85. The Chapter held monthly meetings. One of the activities enjoyed by all was a Christmas party on December 12. Also, on January 1, 1985, at the annual KME meeting, Bruce Williamson, mathematics professor at UW-River Falls, spoke on "The Golden Number." Other 1984-85 officers: Archie Ecker, vice president; Janice Pete, secretary; Janet Socha-Huppert, treasurer; Lyle Oleson, corresponding secretary; Don Leake, faculty sponsor.

Wisconsin Gamma, University of Wisconsin-Eau Claire, Eau Claire
Chapter President - Susan Kelly
37 actives, 13 initiates

The semester began with an initiation for new members held at Kristen's on September 22, 1984. At the ceremonies, Dr. Carroll Rush gave a talk entitled, "The Pluses and Minuses of Symbol-Mindedness." Student speakers for the fall semester have been Chris Heywood, Wendy Horeck, Kathy Webb and Scott Schneck. Activities for next semester include helping with a high school math meet and working with the math faculty on the annual Math Bowl. More student talks are also planned. Other 1984-85 officers: David Hasse, vice president; Sue Krueger, secretary; Erin Kelly, treasurer; Tom Wineinger, corresponding secretary.

REPORT ON THE TWENTY-FIFTH BIENNIAL CONVENTION

The Twenty-fifth Biennial Convention of Kappa Mu Epsilon was held April 1-13, 1985 on the campus of Southern Methodist University, Dallas, Texas, with Texas Beta the host chapter.

On Thursday evening, April 11, following registration in the lobby of the Holiday Inn, North Park Plaza, there was a mixer for delegates in the Atrium of the Holiday Inn. The National Council and the Regional Directors met in the Stephen F. Austin Board Room of the Holiday Inn.

On Friday morning, April 12, registration continued in the Umphrey Lee Student Center. The first general session was held in the Assembly Room of the Umphrey Lee Student Center, commencing at 9:15 a.m. with Ida Z. Arms of Pennsylvania Zeta, National President, presiding. Dr. R. Hal Williams, Dean of Dedman College, Southern Methodist University, gave an address of welcome and Dr. James L. Smith of Ohio Zeta, National President-Elect, responded for the society.

Roll call of the chapters was made by George R. Mach of California Gamma, National Secretary. Thirty three chapters and about 222 members were in attendance. Delegate certification forms were checked, travel vouchers were filed, and delegate voting cards were issued.

L. Thomas Shiflett of Missouri Alpha reported for the nominating committee. The committee nominated: James C. Pomfret of Pennsylvania Lambda and Harold L. Thomas of Kansas Alpha for the office of National President-Elect and M. Michael Awad of Missouri Alpha and Patrick Costello of Kentucky Alpha for the office of National Historian. The candidates were introduced and vita sheets were distributed. Nominations were requested from the floor. There being none, nominations were closed.

Dr. James L. Smith of Ohio Zeta, National President-Elect, presided during the presentation of the following student papers:

1. "Game Playing and Artificial Intelligence," Damon Antos, California Gamma, California Polytechnic State University.

2. "Always the One," Joni Brockschmidt, Missouri Eta, Northeast Missouri State University.

3. "The Discovery and Development of Non-Euclidean Geometries," Sheila O'Brien, Kansas Gamma, Benedictine College.

4. "The Power Method for Symmetric Matrices," Tamara J. Lakins, Illinois Eta, Western Illinois University.

At noon, a group picture was taken on the steps of McFarlin Auditorium. Convention committees and the National Council met during lunch.

The convention reconvened at 1:30 p.m. in the Assembly Room of the Umphrey Lee Student Center. James L. Smith of Ohio Zeta, National President-Elect, presided during the presentation of the following student papers:

5. "An Application of the Newton-Raphson Algorithm," Roberta Bateman, Missouri Zeta, University of Missouri-Rolla.

6. "The Nude Numbers," Roberto A. Ribas, Missouri Beta, Central Missouri State University.

7. "An Application of Linear Interpolation in Two Variables for Brain Wave Displays," Douglas P. Bogia, Kansas Delta, Washburn University.

At 2:45 p.m. a student section met in the Assembly Room with Steve Owens, President of Texas Beta, presiding and a faculty section met in a conference room with Ida Z. Arms of Pennsylvania Zeta, National President, presiding.

The convention reconvened at 3:45 p.m. in the Assembly Room of the Umphrey Lee Student Center. James L. Smith of Ohio Zeta, National President-Elect, presided during the presentation of the following student papers:

8. "The Branch and Bound Method," Earlena Brownewell, Kansas Alpha, Pittsburg State University.

9. "Christ at Emmaus': A Vermeer Masterpiece or a Van Meegeren Forgery?," Susan E. Kelly, Wisconsin Gamma, University of Wisconsin Gamma, University of Wisconsin Eau Claire.

10. "Shannon's Theorem for Entropy and its Use in the Huffman Coding Procedure," Susan L. Andrews, Pennsylvania Lambda, Bloomsburg University.

A Bar-B-Que Dinner was held in the rotunda and on the steps and lawn of Dallas Hall.

At 7:00 p.m. in the Assembly Room of the Umphrey Lee Student Center, Steve Owens, President of Texas Beta, introduced the guest speaker, Dr. David Y. Yun, Chairman of the Department of Computer Science, School of Engineering and Applied Science, Southern Methodist University, who gave the address, "Toward the Automation of Calculus."

The convention reconvened at 8:30 a.m. on Saturday, April 13, in the Assembly Room of the Umphrey Lee Student Center. James L. Smith of Ohio Zeta, National President-Elect, presided during the presentation of the following student papers:

11. "Modest Numbers, A Mathematical Excursion," Richard Gibson, Missouri Beta, Central Missouri State University.

12. "Motivation and Maturity-Mathematical Sciences," Diane Formea, California Gamma, California Polytechnic State University.

13. "The Chinese Remainder Theorem," Kimberly A. Lutz, Ohio Zeta, Muskingum College.

14. "On the Irrationality of e," Steven Daniel Sodergren, Kansas Beta, Emporia State University.

15. "Finding Positive Integral Solutions to Linear Diophantine Equations," Lisa K. Elderbrock, Ohio Zeta, Muskingum College.

The second general session (business meeting) was held at 10:30 a.m. in the Assembly Room of the Umphrey Lee Student Union with Ida Z. Arms of Pennsylvania Zeta, National President, presiding. The following national officers presented reports (copies attached):

Business Manager, THE PENTAGON	- Douglas Nance Michigan Beta
Editor, THE PENTAGON	- Kent Harris Illinois Eta
National Historian	- Harold L. Thomas Kansas Alpha
National Secretary	- George R. Mach California Gamma
National President-Elect	- James L. Smith Ohio Zeta
National President	- Ida Z. Arms Pennsylvania Zeta

Election of officers was conducted by L. Thomas Shiflett of Missouri Alpha, a member of the nominating committee.

Wayne Hayes of Oklahoma Gamma reported for the auditing committee that the National Treasurer's records were found to be accurate.

Homer Hampton of Missouri Beta reported for the resolutions committee. The following resolutions were adopted:

"Whereas, Kappa Mu Epsilon held its 25th Biennial Convention on the campus of Southern Methodist University and whereas, this convention has been an exciting and profitable experience for all, be it resolved that the 25th Biennial Convention of Kappa Mu Epsilon express a deep feeling of gratitude:

1. To Hal Williams who, on behalf of the president and the provost of Southern Methodist University, welcomed the convention.

2. To all those at Southern Methodist University who rendered many services to the officers and chapters.

3. To Monte Monzingo, convention coordinator, Steve Owens, president of Texas Beta, George Reddien, advisor, and the members of the host chapter for their Texas-size hospitality.

4. To Professor David Yun, Chairman of the Department of Computer Science at Southern Methodist University, for his inspiring talk, "Toward the Automation of Calculus."

Whereas, there is much effort and sacrifice on the part of many who help carry out the numerous and varied activities of Kappa Mu Epsilon, be it resolved that the 25th Biennial Convention of Kappa Mu Epsilon express its appreciation:

1. To Ida Z. Arms, Pennsylvania Zeta, who has served diligently as president of Kappa Mu Epsilon, maintaining high ideals and providing direction and leadership for a period of eight years.

2. To Harold L. Thomas, Kansas Alpha, who has maintained account of the activities of the various chapters of Kappa Mu Epsilon as the historian for the past six years.

3. To James L. Smith, Ohio Zeta, for his professional and spirited supervision of the selection and presentation of student papers during two biennial conventions, and further that he succeed to the office of president with our collective support.

4. To George R. Mach, secretary, Nona Mary Allard, treasurer, Douglas Nance, PENTAGON business manager, and Kent Harris, PENTAGON editor, for their services to Kappa Mu Epsilon.

5. To the faculty and students who served with distinction on the nominating committee, selection committee, awards committee, and audit committee.

6. To the thirty students who submitted papers to the selection committee and the fifteen students who presented papers to the convention."

Steve Owens of Texas Beta reported for the student section meeting. Carol Harrison of Pennsylvania Theta reported for the faculty section meeting.

Invitations to host the Twenty-sixth Biennial Convention in 1987 were extended by: Kansas Delta, Washburn University; Nebraska Beta, Kearney State College; Missouri Alpha, Southwest Missouri State University; Wisconsin Alpha, Mount Mary College; Kansas Beta, Emporia State University; and California Gamma, California Polytechnic State University.

Mary Elick of Missouri Iota reported for the awards committee and presented the following student paper awards:

First Place	($60) -	Susan E. Kelly Wisconsin Gamma
Second Place	($40) -	Richard Gibson Missouri Beta
Third Place	($30) -	Douglas P. Bogia Kansas Delta
Fourth Place	($20) -	Roberto A. Ribas Missouri Beta

James L. Smith of Ohio Zeta, National President-Elect, distributed certificates to all students who had presented papers at the convention.

The election results were announced by L. Thomas Shiflett of Missouri Alpha. The following officers were elected for the next four years 1985-1989:

National President-Elect	- Harold L. Thomas Kansas Alpha
National Historian	- M. Michael Awad Missouri Alpha

Ida Z. Arms of Pennsylvania Zeta, National President, installed the newly elected officers and also installed James L. Smith of Ohio Zeta, National President-Elect, as National President for 1985-1989.

The convention voted that an engraved plaque be given to Ida Z. Arms in recognition of her 8 years of service to the Society.

Travel allowances were paid to the delegates by Nona Mary Allard of Illinois Zeta, National Treasurer. Reports of the national officers and delegate participation certificates were distributed and convention evaluation forms were collected by the host chapter. The convention adjourned at 11:55 a.m.

George R. Mach

REPORT OF THE NATIONAL PRESIDENT
April 13, 1985

Since my election as National President at the 23rd Biennial Convention, our Society has added four new chapters, bringing the total number to 100. Connecticut Beta at Eastern Connecticut State College, Willimantic, was installed by Professor Loretta K. Smith on May 2, 1981. New York Lambda at C. W. Post Center-Long Island University, Greenvale, was installed by Dr. John Weidner on May 2, 1983. Missouri Kappa at Drury College, Springfield, was installed by Dr. Thomas Shiflett on November 30, 1984 and our newest chapter, Colorado Gamma, at Fort Lewis College, Durango, was installed on March 29, 1985 by Dr. William Langworthy, substituting for Merle Mitchell who was unable to be there because of a snowstorm.

The National Officers continue to support the regional organization. Several regional conventions have been held since the last biennial convention as reported by President-Elect James L. Smith. Directors from Regions I, III, and V, James C. Pomfret, Joseph Sharp, and Wayne F. Hayes, respectively, complete their terms of office at this convention. I would like to express my thanks to each of them for having served in this capacity. Appointment of Regional Directors for these regions will be made shortly after this convention. Regional Directors for Regions II, IV and

VI, J. Frederick Leetch, Homer F. Hampton, and Adelaide Harmon-Elliott, respectively, continue to serve for the next two years.

I want to express my personal thanks to all those members who agreed to serve and have served on the various convention committees. Almost without exception any Kappa Mu Epsilon member who was asked to serve did so. This cooperation certainly makes the job of serving as President a lot easier. I want to recognize each of the Committee members at this time. Also, I would like to express my sincere appreciation to all the students who submitted papers for the convention. The presentation of papers is the highlight of our conventions and this year a record number was submitted. A special thanks to all the members of Texas Beta and especially Dr. Monzingo and Dr. Reddien for all the work involved in hosting a National Convention.

Thanks also to the National Officers, the Editor and Business Manager of The Pentagon. A very special note of appreciation to Douglas Nance who has served as Business Manager for the past eight years, to Harold Thomas who has served as Historian for the past six years, and James L. Smith, who today completes the first four years of a commitment he made to Kappa Mu Epsilon when he was elected to the office of President-Elect. Today he will be installed as President.

I can truly say that the experiences that I have had while serving as a National Officer for the past eight years have provided me with a great amount of satisfaction and pleasure. Kappa Mu Epsilon has very worthwhile objectives and each member contributes to the achievement of those objectives. I believe the future holds opportunity for further success.

Ida Z. Arms

REPORT OF THE NATIONAL SECRETARY
April 13, 1985

During the last biennium two new chapters of Kappa Mu Epsilon were installed. They are: New York Lambda at C. W. Post Center of Long Island University, installed on May 2, 1983, and Missouri Kappa at Drury College, installed on November 30, 1984. After the close of the biennium another new chapter was installed. It is Colorado Gamma at Fort Lewis College, installed on March 29, 1985. The Society now has 100 active chapters in 30 states.

During the last biennium 2,542 members were initiated. The 99 chapters active during the biennium have a combined membership of 40,547 and the 26 inactive chapters have a combined membership of 6,153, making the total membership of Kappa Mu Epsilon 46,700 at the end of the biennium on March 3, 1985.

As National Secretary, I maintain permanent files on all active and inactive chapters, including reports of all initiations. I order membership certificates for all new members and I stock all supplies, including forms, invitations, and jewelry. I assist corresponding secretaries in any ways that I can and I take the minutes of National Council meetings and Biennial Conventions.

George R. Mach

FINANCIAL REPORT OF THE NATIONAL TREASURER

Biennium: March 24, 1983 through March 20, 1985

Receipts

1.	Cash on hand March 24, 1983			$25,378.36
2.	Receipts from Chapters			
	Initiates (2542)	38,452.00 (includes 322.00		
	Jewelry	626.80 in overpayments)		
	Supplies	563.33		
			39,642.13	
3.	Miscellaneous Receipts			
	Interest	3,883.06		
	Chapter Installations	99.34		
	Chapter Petition	60.00		
			4,042.40	
4.	Total Receipts			43,684.53
5.	Total Receipts plus cash on hand			69,684.89

Expenditures

6.	National Officers Expense		3,709.02	
7.	Jewelry (Pollack)		588.99	
8.	Printing (Herff-Jones)		7,476.98	
9.	Pentagon (4 issues)		11,503.45	
10.	Biennial Convention - 1983 and Regional Conventions - 1984		7,204.38	
11.	Association of College Honor Societies		176.25	

12.	Miscellaneous			
	Refunds	322.00		
	Money Management Fees	92.34		
	National Council Meeting	355.12		
	Postage	359.28		
	Treasurer's Bond	80.00		
	Supplies	10.38		
			1,219.12	
13.	Total Expenditures			31,878.19
14.	Cash on hand - March 20, 1985			37,184.70

Proof of cash:	In checking	4,003.70
	In money market account	33,181.00
		38,184.70

Sister Nona Mary Allard

REPORT OF THE NATIONAL HISTORIAN 1983-1985

The files of the National Historian are being maintained and continually updated with the reports received from the chapters about their events and activities; with information received from Regional Directors about regional conventions and items of interest related to the regions; and with material received from the National Officers which has historial significance.

News items have been solicited from the corresponding secretaries semi-annually, in January and in May. The responses are then edited for publication in the chapter news section of The Pentagon.

During the past biennium, 79 of the active chapters responded at least once to the chapter news request. Special mention goes to the following 32 chapters for their cooperation in responding to all four inquiries: AL Zeta, CA Gamma, CT Beta, GA Alpha, IL Zeta, IL Eta, IA Alpha, IA Beta, KS Alpha, KS Gamma, KS Delta, KS Epsilon, MD Beta, MD Delta, MI Beta, MS Alpha, MO Alpha, MO Beta, MO Epsilon, MO Eta, NE Alpha, NE Beta, NE Gamma, NM Alpha, NY Eta, NY Lambda, OH Gamma, PA Zeta, SC Gamma, TN Delta, TX Eta, and WI Alpha. I would urge chapters to reply to the requests for chapter news even if it is to just identify chapter officers. This would provide chapters with a permanent record of their local officers in the event they do not retain that information within their own chapter.

The only regional convention report for the past biennium was received from Region II and published in the Fall, 1984, issue of The Pentagon.

I want to extend thanks to all with whom I have corresponded relative to this office -- the National Officers, the Regional Directors, the Editor of The Pentagon, Corresponding Secretaries, and individual KME members. It has been a pleasure to serve as your historian for this past biennium.

Harold L. Thomas

REPORT OF THE EDITOR OF THE PENTAGON

During the past year a new section, THE CURSOR, edited by James Calhoun of The Mathematical Systems Department, Sangamon State University, has appeared in THE PENTAGON. The subject of The CURSOR is the interface between the disciplines of mathematics and computer science, specifically an attempt to use mathematics to better understand the concepts of computer science. The Spring, 1984 issue of The PENTAGON marks the first appearance of The CURSOR and contains a description of its intended purpose.

Since the last national convention, eight student papers and seven faculty papers have been published in The PENTAGON. These papers were submitted through the national or regional meetings, directly to the Editor, to Iraj Kalantari, associate editor at Western Illinois University, or to James Calhoun, associate editor at Sangamon State University. Articles of interest to mathematics students or teachers are always welcome for possible publication.

My thanks go to the associate editors of The PENTAGON, Richard Barlow, Harold Thomas, Kenneth Wilke, Iraj Kalantari, and James Calhoun, whose efforts contribute so much to The PENTAGON. A special thank you to Douglas Nance who has served as Business Manager of The PENTAGON for the past eight years. It has been my pleasure to know and work with Doug during the past five years, and I wish him the best in his new ventures.

Kent Harris

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON
April, 1985

It is a pleasure to make my fifth and final Business Manager's report during this 25th Biennial Convention. As many of you know, the Business Manager's primary responsibility is to see that THE PENTAGON gets mailed to members of Kappa Mu Epsilon who have current subscriptions. Mailing dates for THE PENTAGON are approximately June and December.

During this past biennium, we mailed an average of 2600 Pentagons per issue. The mailing list includes subscribers in forty-two states and twenty-six foreign countries. States receiving the most copies of THE PENTAGON are, in descending order, Pennsylvania, Missouri, Ohio, Illinois and Michigan.

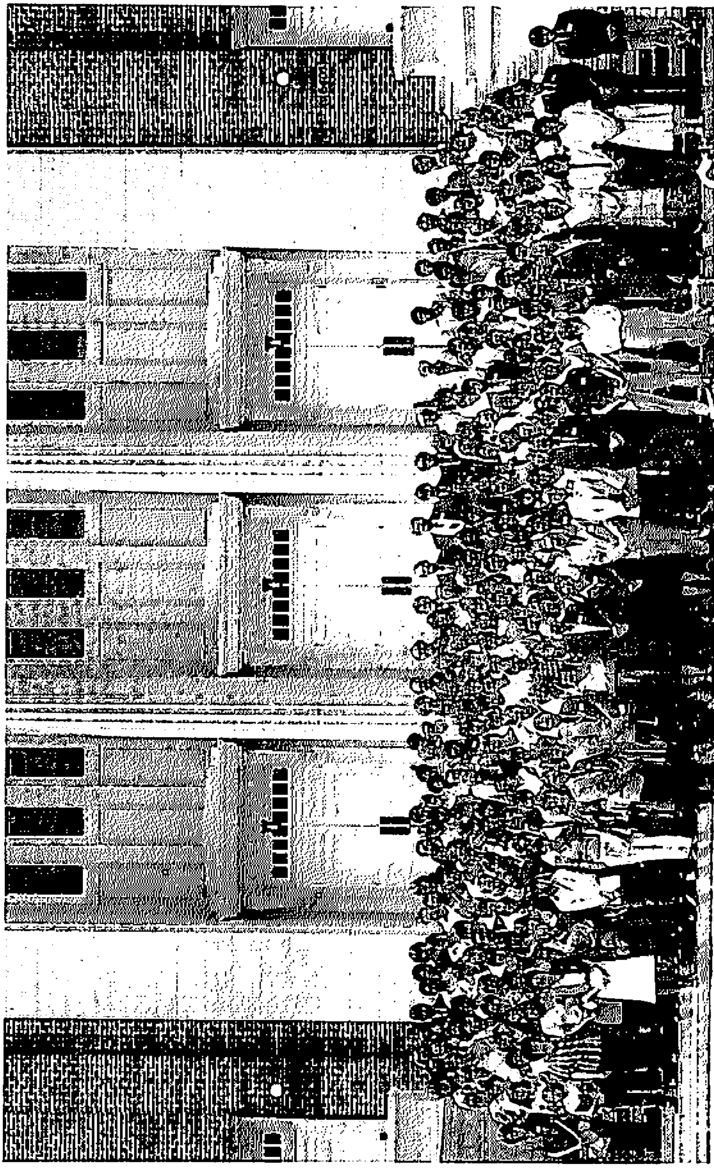
During each semi-annual mailing, approximately forty Pentagons are returned to the office of the Business Manager by the postal service as undeliverable due to incorrect address. Please inform your chapter members that to receive their journal they must keep a <u>current</u> address on file. If a subscriber has any problem with receiving THE PENTAGON, please contact the office of the Business Manager.

Complimentary copies of THE PENTAGON are sent to the library of each college or university with an <u>active</u> chapter of Kappa Mu Epsilon. Also, complimentary copies are sent to authors of articles in THE PENTAGON. Speakers at this convention will automatically have their subscriptions extended for two years.

During this past biennium, I have received cooperation and support from Editor Kent Harris, National Secretary George Mach, student assistant Michelle Giuliano and the mathematics department at Central Michigan University. This is gratefully acknowledged.

Finally, I would like to thank Kappa Mu Epsilon for the opportunity to serve this organization for the past eight years. It has been a pleasure to work with students, faculty and National Board members of this honorary organization. The dedication and commitment to excellence shown by all affiliated with KME has been rewarding and heart warming to witness. I am grateful to have been able to work with this group.

Douglas W. Nance



Kappa Mu Epsilon, Twenty-Fifth Biennial Convention
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