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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

AMICABLE NUMBERS*

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The theory of numbers is one of the oldest and most widely studied branches of mathematics. Its topics have been pursued by the amateur and the professional alike. Some consider it the epitome of intellectual thought and intrigue, while some consider it frivolous and nonsensical in its inapplicability. Yet no one can write off the challenge it has presented throughout the centuries, as the world's top mathematicians have tackled its various complexities.

Some of the oldest and most interesting problems in the theory of numbers deal with the relationships between numbers and the sums of their divisors. These relationships have led to the theories dealing with perfect numbers, abundant and deficient numbers, multiply perfect numbers, and the topic of this paper, amicable numbers.

We shall denote the sum of all divisors of a number n less than or equal to n (including unity) as S(n).

*A paper presented at the 1983 National Convention of KME and awarded third place by the Awards Committee.

The aliquot divisors of n are those divisors strictly less than n. The sum of all of these divisors shall be denoted s(n). It quickly becomes evident that these are related as

(1) $$S(n)=s(n)+n$$

Two numbers n and m are said to be <u>amicable</u> if

(2) $$s(n)=m \text{ and } s(m)=n$$

Making a quick substitution, we see from (1) and (2) that an alternate form of the definition is that

(3) $$S(n)=S(m)=n+m$$

If we examine the numbers n=220 and m=284, we find that

$$s(220)=1+2+4+5+10+11+20+22+44+55+110=284$$

$$s(284)=1+2+4+71+142=220$$

Also, $S(220)=220+284=S(284)$

Therefore, 220 and 284 fit the definitions given in eqns. (1) and (2), and thus the pair (220,284) is said to be amicable.

From Barnett's text on the theory of numbers [1], we have an easier means of computing S(n). First we write n in its unique prime factorization,

$$n = p1^{a1} \cdot p2^{a2} \cdot \ldots pk^{ak},$$

then

$$(4) \qquad S(n) = \frac{(p1^{a1+1}-1)\cdot(p2^{a2+1}-1)\cdot\ldots\cdot(pk^{ak+1}-1)}{(p1-1)(p2-1)\ldots(pk-1)}$$

This is easily demonstrated by choosing $n=220=2^2\cdot5\cdot11$:

$$S(220) = \frac{(2^3-1)(5^2-1)(11^2-1)}{(2-1)(5-1)(11-1)} = 504 = s(220)+220$$

The history of amicable numbers and the means by which they were discovered is a very interesting subject. The complete history of amicable numbers until the year 1949 was first recorded by L. E. Dickson [2], and much of the following is from this work.

The smallest amicable pair (220,284) has been known since at least the time of the Pythagoreans. Pythagoras related that friends are those who have the power to generate each other, which Aristotle confirmed in his definition of a friend as 'another I'. Thus, numbers bearing this property have since been called amicable or friendly.

The Arab, Thabit ben Korrah was the next to comment on amicable numbers, as he made an important discovery in the 9th century. It is interesting to note that he was considerably ahead of his time, as the great mathematicians Fermat and Descartes made the same discovery eight centuries later.

ben Korrah noted that numbers in the form $2^n pq$ and $2^n r$ are amicable if p,q, and r are primes such that

$$(5) \qquad p=3\cdot 2^n-1,\ q=3\cdot 2^{n-1}-1,\ \text{and}\ r=9\cdot 2^{2n-1}-1$$

While the developments in amicable numbers from the time of the Pythagoreans until the 17th century consisted really only of the aforementioned, people were still very concerned about the importance of these numbers. People at this time believed that the numbers 220 and 284 could help develop relationships between people, making them friendlier with each other. These numbers played a part in various superstitions and myths, always relating the concept that if one carried or consumed some aspect of the number 220, and his friend or lover consumed some aspect of the number 284, then the two would be more compatible. One astronomer even considered these numbers significant in determining whether or not a planet could be considered friendly.

Fermat, working in the 17th century, is credited with discovering the second pair of amicable numbers, and is the first to elaborate on his methods. It is interesting to note the manner of thinking that led to his discovery. He began by writing out the progression

2	4	8	16	. . . .

He then multiplied this row by 3, and wrote it below, yielding

2	1	8	16	. . . .
6	12	24	48	. . . .

Next he subtracted one from this newly computed row, and wrote it above, giving

5	11	23	47	. . . .
2	4	8	16	. . . .
6	12	24	48	. . . .

He then created a final row, taking the product of two successive items in row 3 and then subtracting one, writing this below the third row.

5	11	23	47	. . . .
2	4	8	16	. . . .
6	12	24	48	. . . .
	71	287	1151	. . . .

He then noticed that when a number in the bottom row is a prime, and both the number in the top row directly above it and the number preceding that number are primes, then if these numbers are called r, p and q respectively, then the pair (4r,4pq) is amicable. For example, r=71, p=11 and q=5 fit this form, and thus the pair $(4\cdot 71, 4\cdot 5\cdot 11) = (284, 220)$ is amicable. It was later verified that this rule is equivalent to the rule stated by ben Korrah centuries earlier.

Descartes was the next to rediscover this same rule, and use it to find a new pair of amicable numbers. Thus it was found that there are amicable pairs based on (5) when n=2,4, and 7. It was later verified that these are the only values of n leading to pairs for all n less than 200.

Leonard Euler published a list of 30 pairs in a paper, giving no methods or details of their discovery. A little later, Euler published a list of 62 pairs, and finally informed the world as to his interesting methods. The vast majority of Euler's pairs were in the form (Em,En) where E is a factor common to both numbers and relatively prime to both m and n.

Euler addressed five different cases, the first of which was discussed by Escott [3], and is presented here. This method deals with pairs in the form (Epq,Er) where E is a common factor, and p,q, and r are distinct primes relatively prime to E.

As we stated earlier in (3), when two numbers m and n are amicable

(3) $$S(m)=S(n)=m+n$$

We also know that if m and n are relatively prime, then

(6) $$S(mn)=S(m)\cdot S(n)$$

Using equation (4) for the case of p a prime we have

(7) $$S(p) = \frac{(p^2-1)}{p-1} = \frac{(p+1)(p-1)}{(p-1)} = p+1$$

Using equations (6) and (7):

(8) $$S(Epq)=S(E)\cdot S(p)\cdot S(q)=S(E)\cdot(p+1)\cdot(q+1)$$
$$S(Er) = S(E)\cdot S(r) = S(E)\cdot(r+1)$$

These are equal from eqn. (3). Simplifying we have

(9) $$(p+1)(q+1)=(r+1)$$
$$r=pq+p+q$$

Now let's find another form of $S(Epq)$. Using eqn. (3) we have

$$(10) \qquad S(Epq)=Epq+Er=E(pq+r)$$

And from (8),(9), and (10) we obtain the important result that

$$(11) \qquad S(E)(p+1)(q+1)=E(2pq+p+q)$$

Therefore, the problem has been reduced to finding distinct primes p and q such that both p and q are relatively prime to E, and that equation (11) is satisfied. To better illustrate this method, we look at a specific example.

Let $E=3^2\cdot 7^2\cdot 13$ Then from equation (4), $S(E)=13\cdot 57\cdot 14$ Substituting into equation (11) we have

$$13\cdot 57\cdot 14(p+1)(q+1)=3^2\cdot 7^2\cdot 13(2pq+p+q)$$

Simplifying:

$$38(pq+p+q+1)=21(2pq+p+q)$$

$$38=4pq-17(p+q)$$

Which factors:

$$441=(4p-17)(4q-17)$$

Therefore we equate $4p-17$ and $4q-17$ to unequal factors of 441.

	$4p-17=1$	$4q-17=441$
or	$4p-17=3$	$4q-17=147$
or	$4p-17=9$	$4q-17=49$

These equations yield prime values for p and q only when $4p-17=3$ and $4q-17=147$, which gives $p=5$ and $q=41$. Substituting back into (9), we get $r=251$. Thus we have discovered the pair described by $(E\cdot 5\cdot 41, E\cdot 251)$ where $E=3^2\cdot 7^2\cdot 13$.

We quickly consult our chart of known amicable numbers, and find that we are not the first to discover this pair, but that this pair was listed as the seventh pair found by Euler.

Escott [3] also shows that the previously known rule of Thabit ben Korrah, Fermat and Descartes is actually a special case of this method of Euler's.

Consider $E=2^n$ Since $S(2^n)=2^{n+1}-1$, equation (11) becomes

$$(2^{n+1}-1)(p+1)(q+1)=2^n(2pq+p+q)$$

Simplifying and factoring we obtain

$$(p-(2^n-1))(q-(2^n-1))=2^{2n}$$

Equating the factor on the left hand side of the equation to unequal factors of 2^{2n} we obtain

$$(p-(2^n-1))=2^{n-m} \qquad (q-(2^n-1))=2^{n+m}$$

And therefore

$$p=2^{n-m}(2^m+1)-1 \qquad q=2^n(2^m+1)-1 \qquad r=2^{2n-m}(2^m+1)^2-1$$

If m is equal to one, we see that we have exactly the rule of Fermat, as stated in eqn. (5).

Euler's four other methods are somewhat similar to this one, except that he starts by examining different forms of possible pairs. For example, his second method analyzes pairs in the form (Epq,Ers) where p,q,r, and s are all primes. His other methods include analysis of the forms (Epg,Eqf) where p and q are primes, but g and f are not, (Egpq,Ehr) where p,q, and r are primes, but g and h are not, and lastly (Zap,Zbq) where a and b are given, p and q are unknown primes, and Z is unknown but relatively prime to a,b,p, and q.

Since Euler first published his list, there have been approximately 598 pairs found. Most of them have been accredited to a small and elite group of mathematicians. One interesting exception was the relatively small pair (1184,1210) found by a 16 year old boy named Paganini. This pair escaped the eyes of the great thinkers for years. [2]

With the invention of the computer, a more systematic and complete search has been made possible. Recently a sweep was made of all numbers less than one million. 92 pairs were found, some of them new. [5]

While much research has been invested in the theory of amicable numbers, many questions have been raised which have yet to be answered. [5] Some of these are:

1. Is the number of amicable pairs infinite?

2. Are there any pairs of opposite parity? (None have yet been found. Gioia and Vaidya have done extensive work on this subject, but could not prove or disprove their existence.) [4]

3. Are there any pairs whose common factor is a power of 2 times a power of 3, or only a power of 3? (Many pairs have been found whose common factor was a power of 2 or a power of 3 times something else, but none have been found which satisfy the above.)

The study of amicable pairs has led to a number of various definitions which are related to this field. For example, if we consider $L(n)=S(n)-(n+1)$, the sum of the proper divisors of n, then a reduced amicable pair is defined as one for which $L(m)=n$ and $L(n)=m$.

Dickson [2] refers to several other related definitions, including the imperfectly amicable pair, and the amicable triple. An imperfectly amicable pair is one such that

$$s(n)=k=s(m)$$

For example, (27,35) is an imperfectly amicable pair because

$$s(27)=1+3+9=13=1+5+7=s(35)$$

An amicable triple is a set of three numbers such that the sum of the aliquot divisors of each equals the sum of the remaining numbers. That is, a triple (m,n,q) is amicable if

$$s(m)=n+q \qquad s(n)=m+q \qquad \text{and} \qquad s(q)=m+n$$

Dickson has been credited with finding eight such triples.

A chain of amicable numbers is a set of numbers $(m_1,m_2,...,m_k)$ such that

$$s(m_i)=m_{i+1} \text{ for } i=1,2,...,k-1$$

and

$$s(m_k)=m_1$$

These chains of numbers are often called social numbers. Chains have been found of periods 5 and 28.

The search for amicable numbers continues. Both the aforementioned unanswered questions and these relatively unresearched related topics assure us that the theory of amicable numbers will continue to be a field that excites and challenges number theorists for years to come.

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CONSTRUCTING EFFICIENT CODES*

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Introduction

If data is to be used by a computer it must be converted into a form that the machine can understand. This process of changing information from one representation to another is called *encoding*. Often large amounts of data need to be compactified in order to save money and time. By studying different ways of encoding, information scientists have developed codes that are very efficient in terms of storage and transmission time. This paper explores some desirable properties of codes, ways of determining which codes are more efficient, and shows the basics of the Huffman Algorithm, a method for developing extremely efficient codes.

Some Terminology

If we wanted to store or to transmit data digitally, it would be necessary to encode the information as sequences of binary digits, that is, 0's and 1's. For example, suppose we had an alphabet consisting only of the letters A through H. We could encode the letters as follows:

*A paper presented at the 1983 National Convention of KME and awarded fourth place by the Awards Committee.

A	00
B	100
C	10
D	01
E	011
F	101
G	00
H	11

The letters A through H are known as the *source alphabet*, and the symbols 0 and 1 are called *code alphabet*. The sequence of code symbols used to represent a letter of the source alphabet is called a code word. The *length* of a code word is the number of code symbols it contains. In the above example, the code word for the letter F is 101 and the length of the code word is three. We often use the units *bits* as an abbreviation for binary units when dealing with a binary source.

Properties of Codes

There are some properties that we may want our code to have. First, we want each letter of the source alphabet to have a code word different from any other. That is, we want our code to be distinct, to avoid confusion in decoding. The above code is not distinct since the letters A and G are both represented by the same code word: 00. The following is an example of a distinct code:

Source letter	Code word
A	00
B	01
C	10
D	0

Now suppose we receive the sequence 001010010. If we group the symbols as 0 0 10 10 01 0, we would have the message DDCCBD. If however, we grouped the symbols as 00 10 10 0 10, we would have the message ACCDC. The above code is therefore not uniquely decodable. Some code words cannot be uniquely identified when contained in a string of code words. The following code is uniquely decodable:

Source letter	Code word
A	10
B	100
C	1000
D	1100

Another property that we want our code to have is that it be instantaneously decodable. This means that a code word can be recognized without looking ahead at succeeding symbols. As an example, in the above code the sequence 1001000 would be decoded as BC. However, we do not know that the first code word, 100, ends until the second 1 in the sequence is received. The fourth digit must be received before we can determine whether a B or a C was sent first. Since there is a need to look ahead at the next symbol, the code is not instantaneously decodable. The following code is instantaneously decodable:

Source letter	Code word
A	1
B	01
C	001
D	0001

We can tell a code word ends immediately upon receiving a 1. An instantaneously decodable code is said to have the *prefix property*. That is, no code word is contained as the beginning (prefix) of any other code word.

In a given source alphabet we often know that certain letters occur more frequently than others. In the English alphabet the letter E occurs more frequently than any other letter. If we know the probability of each letter of an alphabet occurring, then we can determine the *average length* of the code words for an encoding of that alphabet. The average length, L, is given by:

$$L = \sum_{i=1}^{n} l_i p_i$$

where:

n = the number of letters in the source alphabet,

l_i = the length of the i^{th} code word, and

p_i = the probability of the i^{th} letter occurring.

For the following code:

Source letter	Probability	Code word
A	.2	1
B	.3	01
C	.1	001
D	.4	0001

the average length of the code words is:

$$L = \sum_{i=1}^{n} l_i p_i = 1(.2) + 2(.3) + 3(.1) + 4(.4)$$

$$= 2.7 \text{ bits}$$

A Measure of Information

As stated previously, we often know that some letters in a source alphabet will occur more often than others. It is desirable to assign shorter length code words to those letters. Such a code word would be efficient in terms of storage and transmission time. Before attempting to create such a code, let's consider probabilities in general.

In everyday life, events such as the sun shining or a dog barking occur frequently, so we say they have a high probability of occurring. Other events, such as the stock market crashing, have a low probability of occurring. We can say that more *information* is gained by the occurrence of those events with lower probabilities. Newspapers use this idea all the time: unusual events are given large headlines, while everyday or uninteresting events usually have smaller ones.

A function which assigns a real number to an event is called the *self-information* of the event, I(E), and is defined as:

$$I(E) = -\log p_E$$

where:

E is the event, and

p_E = the probability of the event.

Note that I has the following properties:

1. As p_E increases, I decreases. This is desirable since less information is gained from the occurrence of an event that is likely to happen.

2. The information gained by the simultaneous occurrence of two independent events is equal to the sum of the information gained by the occurrences of each individual event. This can be easily proven as follows:

Suppose A and B are two independent events, with probabilities p_A and p_B, respectively. The probability of the events occurring simultaneously is:

$$P(A \cap B) = p_A \cdot p_B$$

The information gained when both events occur simultaneously is:

$$\begin{aligned} I(A \cap B) &= -\log(p_A \cdot p_B) \\ &= -\log p_A - \log p_B \\ &= I(A) + I(B) \end{aligned}$$

Thus, when two independent events occur at the same time, the occurrence of one does not add to or detract information from the occurrence of the other.

3. If an event E has a probability of 1, then

$$\begin{aligned} I(E) &= -\log(1) \\ &= 0. \end{aligned}$$

Thus, no information is gained by the occurrence of that event. This is logical, since if $p_E = 1$, then the event E is certain to occur.

Note that no base is placed on the logarithm, since the base can be any arbitrary number greater than 1 for the function I to have the desired properties stated above. Since many computing and coding applications deal with information from a binary source, the base 2 is chosen for the logarithm and the units of I are bits. Any further reference to logarithms in this paper will assume the base is 2.

As an example, let's compute the information gained by the occurrence of one of two equally likely events. In this case, $p_A = p_B = 1/2$, and

$$\begin{aligned} I(A) &= -\log(1/2) \\ &= 1 \text{ bit.} \end{aligned}$$

Entropy

The self-information I gives the amount of information conveyed by the occurrence of a single event. Now suppose we had a system of events: $E_1, E_2, E_3, \ldots, E_n$ with corresponding probabilities: $p_1, p_2, p_3, \ldots, p_n$. We can then find the average information content of the system by taking the sum of the self-information of the individual events, weighted according to their probabilities. This average is called the *entropy* of the system, denoted H, and is formally defined as follows:

If S is a system of events, $S = \{E_1, E_2, E_3, \ldots, E_n\}$ with probabilities p_i, such that $p_i \geq 0$ and $\sum_{i=1}^{n} p_i = 1$, then the entropy of the system is

$$H(S) = -\sum_{i=1}^{n} p_i \log(p_i)$$

where $p_i \log(p_i)$ is defined to be 0 when $p_i = 0$. H has the following properties:

1. It can never be less than zero, but may be equal to zero.

2. If $p_i = 1$ and $p_1 = p_2 = p_3 = \ldots = p_n = 0$, then

$$\begin{aligned} H(S) &= -\sum_{i=1}^{n} p_i \log(p_i) \\ &= -1 \cdot \log(1) + 0 + 0 + \ldots \\ &= 0 \end{aligned}$$

So, in a system where one event is certain to occur, the information content of the system is zero.

Entropy can be thought of as either a measure of uncertainty or a measure of information. If we imagine ourselves after carrying out an experiment, then the entropy is the amount of information gained by the outcome of the experiment. Before the experiment, however, the entropy is the amount of uncertainty concerning the result of the experiment.

As an example, the entropy of a small source is computed here. Suppose the alphabet consisting of the letters A through H has the following probabilities of occurrence.:

Source letter	Probability of occurrence
A	1/8
B	1/8
C	1/8
D	1/16
E	1/4
F	1/8
G	1/16
H	1/8

The entropy of the source:

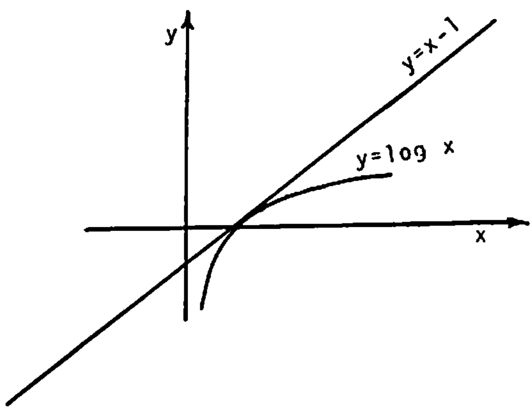
$$H(S) = -\sum_{i=1}^{n} p_i \log(p_i)$$

$$= 2.875 \text{ bits.}$$

A reasonable question one might ask is, "When does a system have maximum entropy?" The following theorem will help us answer that question.

If $p_1, p_2, p_3, \ldots, p_n$ are positive, and $q_1, q_2, q_3, \ldots, q_n$ are non-negative numbers such that $\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} q_i = 1$, then $-\sum_{i=1}^{n} p_i \log(p_i) \leq -\sum_{i=1}^{n} p_i \log(q_i)$, with equality if and only if $p_i = q_i$, for $i = 1, 2, 3, \ldots, n$.

The proof of this theorem uses the fact that $\log x \leq x-1$ for $x > 0$. This inequality is easily seen by looking at the graphs of the functions $\log x$ and $x - 1$. It can also be proved using calculus, but a look at the graphs makes it obvious.



$\log x$ lies below $x-1$ for all positive values of x except when $x = 1$. At this point, the graphs are tangent and they share a common point, (1,0):

We may now prove the theorem by making the substitution $x = q_i/p_i$ in the equality $\log x \leq x - 1$. Then

$$\log(q_i/p_i) \leq (q_i/p_i) - 1$$

Since p_i are positive, and q_i are non-negative, equality will hold when $p_i = q_i$. Multiplying both sides of the above inequality by p_i,

$$p_i \log(q_i/p_i) \leq p_i((q_i/p_i) - 1)$$

Summing both sides,

$$\sum_{i=1}^{n} p_i \log(q_i/p_i) \leq \sum_{i=1}^{n} p_i((q_i/p_i) - 1)$$

Taking the right side of the inequality,

$$\sum_{i=1}^{n} p_i((q_i/p_i) - 1) = \sum_{i=1}^{n} (q_i - p_i)$$

$$= \sum_{i=1}^{n} q_i - \sum_{i=1}^{n} p_i = 0.$$

We now have

$$\sum_{i=1}^{n} p_i \log(q_i/p_i) \leq 0$$

$$\sum_{i=1}^{n} p_i(\log q_i - \log p_i) \leq 0$$

$$\sum_{i=1}^{n} p_i \log q_i - \sum_{i=1}^{n} p_i \log p_i \leq 0$$

$$\sum_{i=1}^{n} p_i \log p_i \leq - \sum_{i=1}^{n} p_i \log q_i$$

where equality holds only when $p_i = q_i$ for $i=1,2,3,\ldots,n$.

It can now be shown that a system has maximum entropy when the events have equal probabilities. This is formalized in the following theorem:

$$H(S) = - \sum_{i=1}^{n} p_i \log p_i \leq \log n$$

where equality holds if and only if $p_i = 1/n$ for $i = 1,2,3,\ldots,n$.

The theorem is proved by making the substitution $q_i = 1/n$ for $i = 1,2,3,\ldots,n$ in the previous theorem. Then we have

$$H(S) = - \sum_{i=1}^{n} p_i \log p_i \leq - \sum_{i=1}^{n} p_i \log(1/n) = -\log(1/n)$$

or

$$H(S) \leq \log n$$

with equality only when $p_i = q_i = 1/n$ for $i = 1,2,3,\ldots,n$.

In terms of codes, we now know that the average information content of a source symbol is maximum when the source symbols have an equal probability of being transmitted. We also know that this maximum is log n bits.

It can be shown that the entropy of a source gives a bound on the average word length of a uniquely binary source. Stated as a theorem:

> For any source S there exists a uniquely decodable binary code with average word length L, such that $H(S) \leq L \leq 1+H(S)$.

The proof of this theorem is not included in this paper.

We now have a standard for determining the efficiency of a code. We want the average word length of the code to come close to the value of entropy of the source. One code that yields a uniquely decodable code with short average word length is the Huffman Code.

<u>Constructing A Huffman Code</u>

The following are the steps followed in the construction of a Huffman Code for a binary source:

1. Arrange for the source symbols in order of decreasing probability.
2. Add the two smallest probabilities and place the result in the proper place (see example below) in the next column, always keeping the probabilities in descending order in the column.

   Repeat step 2 until only two probability values remain.
3. Put a value of 0 next to one of the remaining probabilities and a value of 1 next to the other. The order of assignment here is arbitrary.
4. Working backwards, break down that probability that was made up of two smaller probabilities.
5. Keeping any bits already assigned to the probabilities, assign a 0 to one of the component values, and a 1 to the other. The order of assignment here must be consistent with that established in step 2. (If a 0 were assigned to the first value in step 2, a 0 must be assigned to the first value now.)

   Repeat steps 4 and 5 until the source alphabet is reached.

As an example, the Huffman Code for the source alphabet on page 16 will be constructed. Recall the source probabilities:

Source symbol	Probability of occurrence
A	1/8
B	1/8
C	1/8
D	1/16
E	1/4
F	1/8
G	1/16
H	1/8

Steps 4, 5, and 6 in the construction of the code are highlighted by arrows for readability.

	b_7	f_1	b_6	f_2	b_5	f_3	b_4	f_4	b_3	f_5	b_2	f_6	b_1	f_7
E	10	1/4	10	1/4	10	1/4	10	1/4	10	1/4	10	1/2	0	1/2
A	010	1/8	010	1/8	11	1/4	11	1/4	11	1/4	10	1/4	1	1/2
B	011	1/8	011	1/8	010	1/8	00	1/4	00	1/4	11	1/4		
C	000	1/8	000	1/8	011	1/8	010	1/8	01	1/4				
F	001	1/8	001	1/8	000	1/8	011	1/8						
H	110	1/8	110	1/8	001	1/8								
D	1110	1/16	111	1/8										
G	1111	1/16												

f columns are constructed from left to right.

b columns are constructed from right to left.

Note that if two probability values are equal, the order of insertion in the column does not matter.

The average word length for the resulting code:

Source symbol	Probability	Code word
A	1/8	010
B	1/8	011
C	1/8	000
D	1/16	1110
E	1/4	10
F	1/8	001
G	1/16	1111
H	1/8	110

is given by:

$$L = \sum_{i=1}^{n} p_i l_i$$

and is found to be 2.875. Recall that this is exactly the value of entropy of the source, so the code is indeed efficient. Note however, that a Huffman Code does not <u>always</u> yield an average word length exactly equal to the entropy of the source, but usually comes very close to it.

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THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August, 1984. The solutions will be published in the Fall 1984 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 367: Proposed by the editor.

Find the two smallest numbers in the decimal system such that each of the numbers is multiplied by 27 by placing the digit 1 on each side of the original number.

Problem 368: Proposed by Charles W. Trigg, San Diego, California.

A square is divided into nine congruent square cells. In each corner cell, place a different odd digit. In each side cell place an even digit, so that

1) the digits along each side of the square and of its reflections about a side form a prime integer, and

2) the sum of the digits in the eight cells is a square integer.

Problem 369: Proposed by Michael W. Ecker, Pennsylvania University, Worthington-Scranton Campus, Scranton, Pennsylvania.

Given a checkerboard which is n x n, find an explicit function of n which counts a) the total number of squares of all sizes in the checkerboard and b) the total number of rectangles of all sizes in the checkerboard.

Problem 370: Proposed by the editor.

Find all positive integers n such that $n^3 + 1$ is exactly divisible by dn - 1 where d is a positive integer.

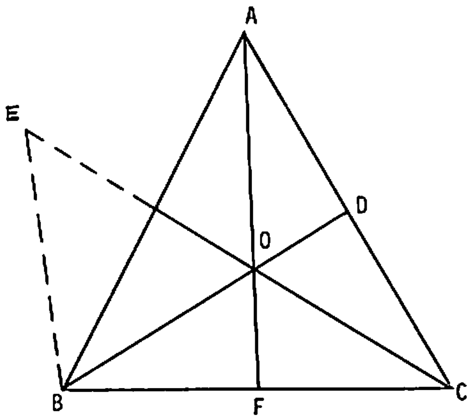
Problem 371: Proposed by the editor.

A long straight rod was dropped causing it to break into two pieces. One of the two pieces is picked up at random. This piece is dropped and it breaks into two pieces. What is the probability that the three pieces of the original rod will form a triangle?

SOLUTIONS

347. Proposed by Roger Izard, Dallas, Texas.

In the figure, line segments BD,AF and EC bisect the angles of triangle ABC. Also $BE^2 \cdot AO^2 = 3 \cdot BC^2 \cdot EO^2$. Prove that EO = OD.



Solution by the proposer.

Through point B in the figure, draw a line which makes an angle with segment AB that is equal to $\angle EOA$. Then extend EC until it meets this line at S. Now triangles SBE and EOA are similar so that BS/AO=BE/EO. Then since $\angle EBO + \angle EOA = 90^\circ$, $BS/BO = \tan(90^\circ - \frac{1}{2}\angle A)$. Therefore $\tan(90^\circ - \frac{1}{2}\angle A) \cdot BO/AO = BE/EO$. The given equation implies that $BE/EO = 3 \cdot BO/AO$. Hence $\angle A = 60^\circ$ and $\angle EOB = 60^\circ$. Thus triangles BEO and ABD are similar so BO/EO = AB/AD. Since AF is an angle bisector, AB/BO = AD/OD and the desired result follows immediately since AB/BO = AD/EO = AD/OD.

348. Proposed by Charles W. Trigg, San Diego, California.

Martin Gardner ("Mathematical Games," Scientific American, April 1964, page 135) showed that 999 is the minimum sum of three three-digit primes composed of the nine non-zero digits. With the nine non-zero digits form other trios of three-digit primes with sums that contain three like digits.

Solution by the proposer.

The maximum sum attainable from three three-digit integers formed from the nine non-zero integers is 963 + 852 + 741 or 2556. The sum of three odd primes is odd. The sum of the nine non-zero digits is 45, so the sum of the three odd primes must be an odd multiple of 9. Consequently, the only possible sums with three like digits are 1161, 1611, and 2223. If the sum is to be 1161, then the unit's digit of the primes will be 1,3,7. The ten's digit will be 2,4,9 with hundred's digits 5 + 6 + 8 > 11; or 2,5,8 with hundred's digits 4 + 6 + 9 > 11; or 4,5,6 with hundred's digits 2 + 8 + 9 > 11.

If the sum is to be 1611, the unit's digits will be 1,3,7. The ten's digits will be 5,6,9 and the hundred's digits will be 2 + 4 + 8 = 14. These digit triads can be matched in $(3!)^2$ ways, among which are the following three sets of primes:

461	491	491
293	257	263
857	863	857
1611	1611	1611

If the sum is to be 2223, the unit's digits will be 1,3,9. The ten's digits will be 6,7,8 with hundred's digits 2 + 4 + 5 < 20; or 2, 4, 5 with hundred's digits 6 + 7 + 8 = 21. These digit triads can be matched in 6^2 ways, from which the following sums of primes emerge:

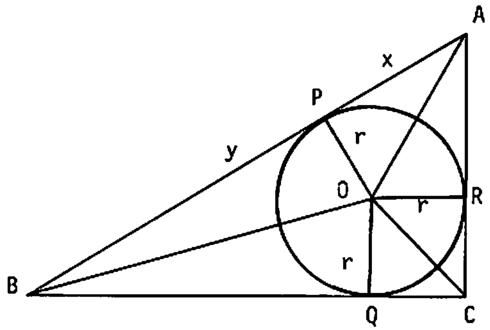
$$751 + 643 + 829 = 2223 = 821 + 743 + 659.$$

Also solved by Nathan Reed, St. Olaf's College, Northfield, Minnesota.

349. Proposed by Fred A. Miller, Elkins, West Virginia.

Prove that the area of a right triangle is equal to the product of the segments determined on the hypotenuse by the inscribed circle.

Composite of solutions by Oscar Castaneda, Edgewood High School, San Antonio, Texas, and Thomas Y. Chu, Macomb Senior High School, Macomb, Illinois.



In the figure, let ΔABC be the right triangle with C denoting the right angle. Let O denote the inscribed circle having center at O and radius r. Let x,y denote the segment on the hypotenuse. Draw the angle bisectors OA,OB,OC and let P,Q, and R denote the points of tangency of the inscribed circle on AB,BC and CA respectively. Hence OP, OQ and OR are perpendicular to AB, BC and CA respectively. Hence $BQ = a-r = y$ and $AR = b-r = x$.

Computing [ABC], the area of triangle ABC, in two different ways one obtains

$$[ABC] = \frac{ab}{2} = r^2 + r(a-r) + r(b-r) = ab - (a - r)(b - r)$$

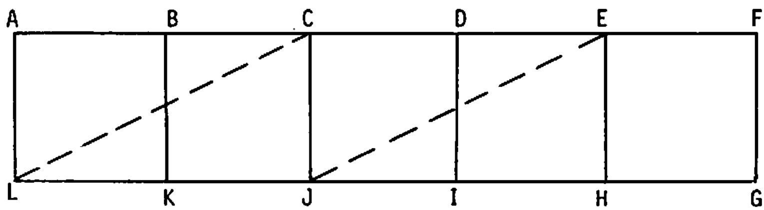
Hence $[ABC] = \frac{ab}{2} = (a - r)(b - r) = xy.$

Also solved by Clayton W. Dodge, University of Maine at Orono, Orono, Maine; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; Charles W. Trigg, San Diego, California, and the proposer.

350. Proposed by Charles W. Trigg, San Diego, California.

A paper rectangle is five times as long as it is wide. With two cuts by scissors, dissect the rectangle into pieces that can be assembled into a square.

Solution by the proposer.



a) Divide the rectangle into squares as shown in figure 1.

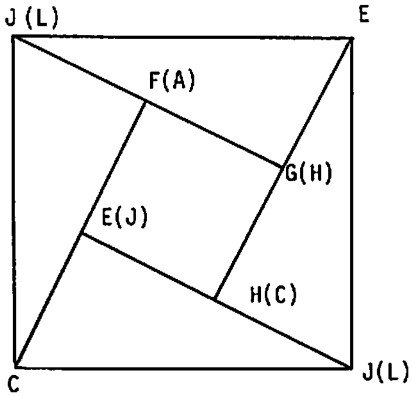
b) Fold the rectangle along DI bringing FG into coincidence with BK and EH into coincidence with CJ.

c) Make the first cut along the CJ - EH line.

d) Unfold to form rectangle CEHJ and place ACJL on top of it with AC on top of CE.

e) Make the second cut along the LC - JE line.

f) Assemble the five pieces into a square as shown in figure 2.



Also solved by Clayton W. Dodge, University of Maine at Orono, Orono, Maine.

351. Proposed by Bruce Sommer, University of Wisconsin-Rock County Campus, Janesville, Wisconsin.

Consider a tangent to the curve $X^n + Y^n = A^n$ where $n \neq 0$ or 1. Suppose that this tangent intersects the x axis at P and the y axis at Q. Show that

$$\frac{1}{P^{n/n-1}} + \frac{1}{Q^{n/n-1}} = \frac{1}{A^{n/n-1}} .$$

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Let (x_0, y_0) be a point where the tangent line intersects the curve $X^n + Y^n = A^n$. Then $dY/dX = -X^{n-1}$ so the slope of the tangent line at that point is $-x_0^{n-1}/y_0^{n-1}$. Also the slope of the tangent line is $-y_0/(P-x_0)$ and also $-Q/P$. It follows that $y_0^{n-1}/x_0^{n-1} = P/Q$ and $P-x_0 = y_0^n/x_0^{n-1}$ so that $P = (x_0^n + y_0^n)/x_0^{n-1} = A^n/x_0^{n-1}$. Thus

$$\frac{1}{P^{n/n-1}} + \frac{1}{Q^{n/n-1}} = \frac{1}{P^{n/n-1}}\left[1 + P^{n/n-1}/Q^{n/n-1}\right]$$

$$\frac{x_0^n}{A^{n^2/(n-1)}}\left(1 + y_0^n/x_0^n\right) = \frac{x_0^n}{A^{n^2/(n-1)}} \cdot \frac{x_0^n + y_0^n}{x_0^n} =$$

$$\frac{A^n}{A^{n^2/(n-1)}} = \frac{1}{A^{n/(n-1)}}\,.$$

Also solved by Fred A. Miller, Elkins, West Virginia and the proposer.

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical *issues* of interest to undergraduate students are discussed in length. Here by *issue* we mean the most general interpretation. Examination of books, puzzles, paradoxes and specific problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to *Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.*

When I tell my students about "$\lim_{n\to\infty} x_n = \ell$" *and give them its definition:* $\forall \varepsilon>0 \exists N_\varepsilon (k>N_\varepsilon \Rightarrow |\ell - x_k|<\varepsilon)$, *I find them uneasy until I use my favorite metaphor to explain it further. My favorite way of explaining it is through the idea of a 'game' played between two people. I have found this metaphor very interesting and I have tried to apply it to similar situations. I first used this method when I was a graduate assistant. Later, I discovered that many mathematicians have used the idea and that several theorems in different subjects are best explained through such 'games'. (I leave it for the reader to discover the game which explains the definition of existence of a limit given above.) This issue's article is about 'games' and about a particularly interesting game.*

IK

AN UNPLAYABLE GAME

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Western Illinois University
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§1. Introduction

Two chimpanzees agree to play a game of Bananas. The game begins with a single bunch of N bananas on a tree. A move consists of a player removing 1, 2 or 3 bananas from the bunch but never is any chimp allowed to completely empty the tree of bananas. (Hence if at some point in the game there are only 3 bananas left, then there are only two legal moves: take 1 banana or take 2 bananas). The players are required to alternate moves. Eventually only one banana will remain on the tree and it will be some unfortunate chimp's turn to move. That chimp is the chump. The other chimp is champ. The chump must give up its bananas to the champ.

In figure 1 we picture all the possible plays of the game which begin with a bunch of five bananas.

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*Professor Weitkamp received his Ph.D. in mathematical logic from Pennsylvania State University and has been with Western Illinois University since 1980. His interests include set theory and logic.

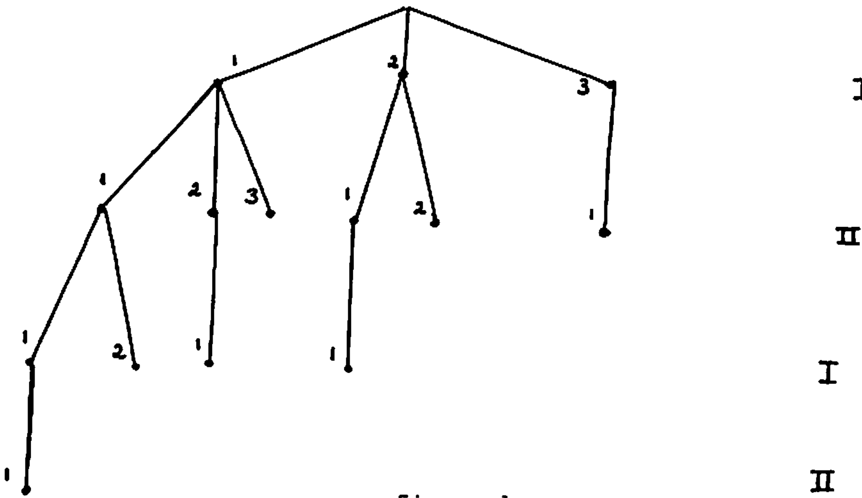

Figure 1

The 'paths' through the 'tree' correspond to possible 'plays' of the game. For example: the path <1,1,2> stands for the play in which the first chimp opens the game by taking 1 banana, the second chimp responds by removing 1 banana and the first chimp wins the game by taking 2 bananas thereby leaving behind just 1 banana on the tree.

Notice that the second chimp to move (if it is clever enough) can always win a game of Bananas which begins with N=5. If player I opens the game by taking 1, 2 or 3 bananas, then player II should respond by taking 3, 2 or 1 bananas respectively. This 'winning strategy' can be represented by the 'subtree' of figure 2.

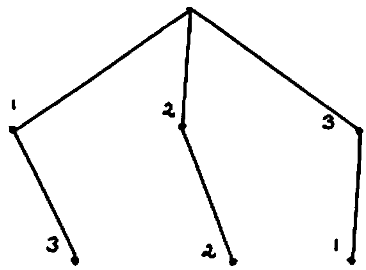

Figure 2

<u>EXERCISE</u>: Prove that the second chimpanzee to move has a winning strategy iff N ≡ 1 (mod 4).

Here's another game. The players are two chipmunks. The two chipmunks have agreed on which is to be first and which second and they have selected to play on the large oak tree in the neighborhood. They climb the trunk together until they reach the first fork. The first chipmunk to move decides which branch to take and together they move along that branch until they either reach another fork or a knot. Now the second chipmunk decides on a branch and together they move out on that branch to the next fork or knot. Thus by alternating roles as navigator they wind their way to the tip of a path through the tree. If from the tip of the chosen path there hangs a single acorn, then the first chipmunk wins. If there hangs more than one acorn, the second chipmunk wins. If there are no acorns at the tip the game ends in stalemate.

Chipmunks call this sort of game a tree-game. Notice that Bananas is equivalent to the tree-game pictured in figure 3.

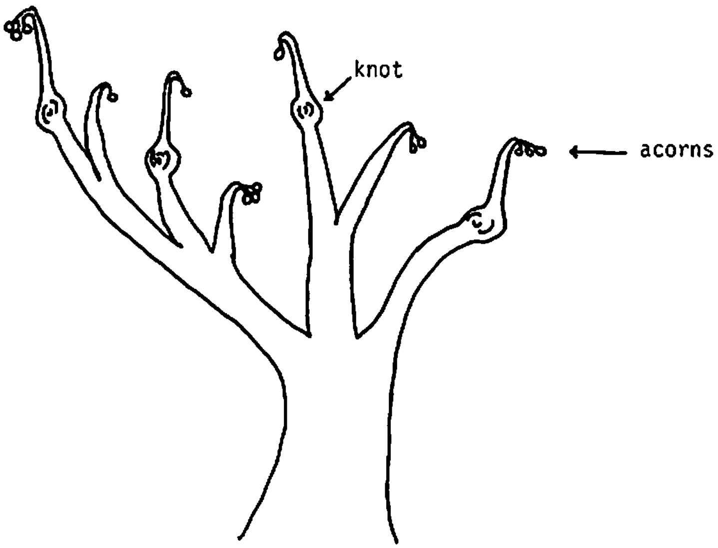

Figure 3

Indeed almost any game of skill between two players who alternate turns is a tree-game.

<u>EXERCISE</u>: Why is checkers a tree-game? Why is chess a tree-game?

§2. Well-founded Games

Let X be a set. A tree over X is a collection T of finite sequences of terms from X which is closed under initial segments. More precisely: T is a collection of finite sequences satisfying

(i) $< > \varepsilon T$

(ii) if $<x_0,x_1,\ldots,x_n> \varepsilon T$, then $x_0 \varepsilon X$, $x_1 \varepsilon X,\ldots,x_n \varepsilon X$

(iii) if $<x_0,\ldots,x_n> \varepsilon T$, then $<x_0,\ldots,x_m> \varepsilon T$ for $0 \leq m \leq n$.

For example, the set {<>,<1>,<2>,<3>,<1,1>,<1,2>, <1,3>,<2,1>,<2,2>,<3,1>,<1,1,1>,<1,1,2>,<1,2,1>,<2,1,1>, <1,1,1,1>} is a tree over {1,2,3}. Notice this tree can be diagrammatically represented by figure 1.

An infinite path through a tree T over X is a function f from the set of whole numbers, ω, into X so that for every $n \varepsilon \omega$, $<f(0),\ldots,f(n)> \varepsilon T$. A tree is said to be well-founded iff it has no infinite paths running through it. An endnode of T is a sequence $<x_0,\ldots,x_n>$ in T with no extensions in T; i.e. for every $x \varepsilon X$, $<x_0,\ldots,x_n,x> \notin T$.

A well-founded game is a well-founded tree T (over some set X) whose endnodes are labeled I, II or Σ. Such a game is played as follows. Two players I and II alternately select elements of X (repetition is allowed) so as to keep the sequence of play in T. Hence player I chooses some x_0 in X so that $<x_0> \varepsilon T$ and player II responds by choosing some x_1 in X so that $<x_0, x_1> \varepsilon T$ (possibly $x_1 = x_0$). They proceed until they generate an endnode $<x_0, \ldots, x_n>$ of T. Since T is well-founded the play will always terminate at an endnode. If the label of $<x_0, \ldots, x_n>$ is I, then I wins. If the label of $<x_0, \ldots, x_n>$ is II, then II wins. Finally if the label is Σ the play ends in stalemate.

Bananas, tree-games, checkers and chess are all well-founded games as defined above.

A <u>strategy</u> for player I is a subtree S of T so that

(i) if $<x_0, \ldots, x_n> \varepsilon S$ and n is even, then for every $x \varepsilon X <x_0, \ldots, x_n x> \varepsilon T \rightarrow <x_0, \ldots, x_n, x> \varepsilon S$.

(ii) if $<x_0, \ldots, x_n> \varepsilon S$, n is odd and $<x_0, \ldots, x_n>$ is not an endnode of T, then there is at least one $x \varepsilon X$ so that $<x_0, \ldots, x_n, x> \varepsilon S$.

We say S is a <u>non-losing strategy</u> for I in the game iff none of the endnodes of S is labeled II.

Thus if player I has a non-losing strategy S for a game T he may use it as follows and never lose a game.

First player I opens the game by selecting an x_0 in X satisfying $<x_0> \varepsilon S$. Player II responds with some x_1 in X so that $<x_0,x_1> \varepsilon T$. By (i) of the definition of a strategy this implies $<x_0,x_1> \varepsilon X$ and so by (ii) player I chooses an x_2 so that $<x_0,x_1,x_2> \varepsilon S$... and so on.

What is the definition of 'a non-losing strategy for II in the game T'?

<u>EXERCISE</u>: A tree is <u>finite</u> if it has only finitely many elements. Prove that every finite tree is well-founded.

<u>EXERCISE</u>: Let $T = \{<x_0,\ldots,x_n>: x_0 = \ldots = x_n > n, n \varepsilon \omega\}$
$= \{<1>,<2>,<2,2>,<3>,<3,3>,<3,3,3>, \ldots \}$

Prove T is an infinite but well-founded tree!

<u>Theorem</u>: If T is a well-founded game, then either player I or player II (or both) has a non-losing strategy for T.

Before giving the proof we need one further definition. We say I has a winning strategy for T from the position $<x_0,\ldots,x_n>$ if there is a strategy S for I with $<x_0,\ldots,x_n> \varepsilon S$ and every endnode of S which extends $<x_0,\ldots,x_n>$ is labeled I.

<u>Proof of the theorem</u>: Suppose player I doesn't have a

winning strategy for T from position $<>$. It will be sufficient to prove that under these hypotheses, player II must have a non-losing strategy for T.

Define

$$S = \{<x_0,\ldots,x_n> \in T: \text{ I doesn't have a winning strategy for } T \text{ from position } <x_0,\ldots,x_n>\}.$$

Notice that S is a tree over X and $S \subseteq T$. Let $<x_0,\ldots,x_n> \in T$.

(i) If n is odd, then $<x_0,\ldots,x_n>$ represents a partial play of the game in which it's player I's turn to move. Since player I doesn't have a winning strategy from position $<x_0,\ldots,x_n>$ there is nothing player I can do at this stage to gain such a strategy. Hence for every $x \in X$, if $<x_0,\ldots,x_n,x> \in T$, then $<x_0,\ldots,x_n x> \in S$.

(ii) If n is even, then $<x_0,\ldots,x_n>$ is a partial play of the game in which it is II's move. Since player I doesn't have a winning strategy from this position either $<x_0,\ldots,x_n>$ is an endnode of T or it is possible for II to select an $x \in X$ so that player I still does not have a winning strategy from $<x_0,\ldots,x_n,x>$. Hence $<x_0,\ldots,x_n,x> \in S$.

We've just shown S is a strategy of player II for T. Now suppose $<x_0,\ldots,x_n>$ is an endnode of S. Then by (i) and (ii) above it is also an endnode of T. Since I doesn't have a forced win from $\langle x_0, \ldots, x_n \rangle$, this position certainly isn't labeled I. Hence no endnode of S is labeled I. This means S is a non-losing strategy of II for T.

An immediate corollary of this theorem is that at least one of the players (white or black) has a non-losing strategy for the game of chess. Similarly for checkers, Othello, go and other games of strategy which are equivalent to well-founded games as defined in this section.

<u>EXERCISE</u>: Define what it means for player I or player II to have a winning strategy for a game T.

A game G is <u>determined</u> if either player I or II has a winning strategy for G. Finally we say a game is <u>two-valued</u> if stalemates are impossible.

<u>COROLLARY</u>: Every well-founded two-valued game is determined.

<u>EXERCISE</u>: Prove the corollary above.

§3. <u>A Paradox of Games</u>

What distinguishes well-founded games from other tree-games is the fact that no play of a well-founded game can go on forever. The rules of a well-founded game are such that every play of the game eventually terminates.

William Zwicker invented the following game which I prefer to call SELECTION. SELECTION is played by two individuals who move alternately as before. The first player for the opening move is required to select a well-founded game G. The two players then play a game of G in which our second player is the first to move. Whichever player wins G, wins SELECTION.

Is SELECTION a well-founded game? Well, imagine any play of SELECTION. At the first move a well-founded game G is chosen. The players then sit down to a game of G. Since G is well-founded the players eventually complete their play of the game G. At this point their play of SELECTION also terminates. Thus we see that

> every play of the game of SELECTION eventually terminates.

Now imagine the following rather interesting play of SELECTION. Our two inexhaustible players meet to play a game of SELECTION. Player I is required to open by choosing a well-founded game. Feeling a bit perverse he selects SELECTION. Now player II becomes the first to move in this new game of SELECTION. Thus player II is required to select a well-founded game. Feeling a bit piqued II selects SELECTION too! And so it goes; each player in its turn selecting SELECTION ad infinitum. Here we've described a perfectly legal play of the game of SELECTION which never ends, and yet we've also seen that every play of the game of SELECTION must end! What's going on here?

First let's examine some less dangerous versions of SELECTION. Define $S_0, S_1, \ldots$ by induction as follows.

$$WF = \{T: T \text{ is a well-founded tree over } \omega \text{ whose endnodes are labeled I, II or } \Sigma\}.$$

$$S_0 = \{<T, x_1, \ldots, x_n>: T \in WF \text{ and } <x_1, \ldots, x_n> \in T\}.$$

$$S_1 = \{<T, x_1, \ldots, x_n>: T \in WFU\{S_0\} \text{ and } <x_0, \ldots, x_n> \in T\}.$$

$$\vdots$$

$$S_{m+1} = \{<T, x_1, \ldots, x_n>: T \in WFU\{S_0, \ldots, S_m\} \text{ and } <x_0, \ldots, x_n> \in T\}.$$

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.
.

$S_\omega = \{<T, x_1, \ldots, x_n>: T \varepsilon WFU\{S_m : m \varepsilon \omega\}$ and $<x_1, \ldots, x_n> \varepsilon T\}$

.
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$S_{\omega+1} = \{<T, x_1, \ldots, x_n>: T \varepsilon WFU\{S_m : m \varepsilon \omega\} U \{S_\omega\}$ and $<x_1, \ldots, x_n> \varepsilon T\}$.

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Notice that S_0 is a game which is very similar to SELECTION. In the opening move of a play of S_0 player I must choose a well-founded tree T over ω whose end-nodes are labeled I, II or Σ. Player II responds by selecting an integer x_1 so that $<x_1> \varepsilon$ T. From this position on the play remains in T. Notice too that since player I must open by selecting a game T in WF it cannot select T = S_0 because S_0 is a well-founded tree over WFUω, not over ω. For this reason the S_0 is not paradoxical.

Suppose players I and II decide to play S_ω. Player I must first select x_0 from WFU$\{S_m : m < \omega\}$. Suppose the selection is $x_0 = S_{98}$. Now II must choose x_1 from

WFU $\{S_0, \ldots, X_{97}\}$. Clearly within 100 moves I and II will be down to selecting nothing but whole numbers. So again the paradox is averted.

Similarly each S_α is a well-founded version of SELECTION which is free from paradox. The moral is that in order to unambiguously describe a game one must present an accurate definition of the game as a well-founded labeled tree. This we have done for each of the games S_α, but we have given no such description of SELECTION.

At this point it is tempting to mathematically define SELECTION as the tree S (with the obvious labeling)

$$S = \{\langle T, x_1, \ldots, x_n\rangle : \text{ T is any well-founded game and } \langle x_1, \ldots, x_n\rangle \in T\}. \qquad (*)$$

As we have argued earlier SELECTION is a well-founded game and so S is an allowable opening selection for player I in the game of S. Hence $\langle S\rangle \in S$. By the usual definition of 'ordered n-tuples' (see the next exercise) $S \in \langle S\rangle \in S$. Therefore S is an element of an element of itself!

A glance at the definition of the less paradoxical games S_α reveals that they are sets which may in turn contain other similar sets as elements. Thus any given set represents a hierarchy of sets. It is important to realize that in order to specify any level of that hierarchy it is first necessary to specify the lower levels of the hierarchy. This is exactly how the games of 'α-selection' are defined — one level at a time through the transfinite ordinals. In short, the elements of a proposed set must be shown to exist before the set can be shown to exist. Since begging a question is illegal it is illegal for any set to be an element of itself. Similarly no set is an element of an element of itself, etc. Hence S is ill-defined.

You may well wonder exactly what is and isn't legal in the world of sets? The laws of set theory occur (with two exceptions) in two varieties: the existence axioms and the set formation axioms. They are generally couched in a formal first order language. However, at the risk of some imprecision we shall sketch them in English.

0) (Extensionality): If x and y have the same elements, then x = y.

1) (Empty set): The empty set exists.

2) (Infinity): There exists an infinite set.
3) (Foundation): If a is a set, then there is a $b \in a$ so that for every $x \in a$, $x \notin b$.
4) (Pairing): If x and y are sets, so is $\{x,y\}$.
5) (Union): If x is a set, then $\{y: \exists z \in x \cdot y \in z\}$ is a set.
6) (Separation): If x is a set and P is a definable property of sets, then $\{y \in x$: y has property P$\}$ is also a set.
7) (Power): If x is a set then so is $\{y: y \subseteq x\}$.
8) (Replacement): If x is a set, and F a definable functional with domain x, then $\{F(y): y \in x\}$ is a set.
9) (Choice): If x is a set of non-empty sets then there is a function f with domain x so that for every $y \in x$, $f(y) \in y$.

This axiom scheme is known as Zermelo-Fraenkel set theory with the axiom of choice (ZFC). If we omit the axiom of replacement (#8) the result is the system Z of Zermelo set theory.

The system Z is incredibly powerful. It was first proposed by E. Zermelo as a foundation for mathematics. As such it is quite successful. Most all of classical mathematics makes use only of the axioms in Z. ZFC was later invented (A. Fraenkel added replacement) in order to give a more satisfactory account of 'higher set theory' (i.e. that portion of set theory which deals with infinite cardinal and ordinal arithmetic).

<u>EXERCISE</u>: Define

$$\begin{aligned}
\langle\ \rangle &= \emptyset \\
\langle x_1\rangle &= \{x_1\} \\
\langle x_1,x_2\rangle &= \{\{x_1\},\{x_1,x_2\}\} \\
\langle x_1,\ldots, x_{n+1}\rangle &= \langle\langle x_1,\ldots,x_n\rangle, x_{n+1}\rangle.
\end{aligned}$$

Notice the definition of $\langle x_1,\ldots,x_n\rangle$ is by recursion on n. Thus for example

$$\begin{aligned}
\langle 1,3,5\rangle &= \langle\langle 1,3\rangle, 5\rangle \\
&= \{\{\langle 1,3\rangle\}, \{\langle 1,3\rangle, 5\}\} \\
&= \{\{\{\{1\},\{1,3\}\}\}, \{\{\{1\},\{1,3\}\}, 5\}\}
\end{aligned}$$

Show that for every positive integer n,

$\langle x_1,\ldots,x_n\rangle = \langle y_1,\ldots,y_n\rangle$ iff $x_i=y_i$ for every $i=1,\ldots,n$.

(Hint: Use induction.)

<u>EXERCISE</u>: Use Axioms 3 and 4 to prove that for every set x, $\langle x \rangle \notin x$.

We've already mentioned that the definition (*) of S implies $\langle S \rangle \in S$. Since this contradicts the exercise above we conclude that there is no set which satisfies (*). Hence SELECTION is not a well-defined game!

KAPPA MU EPSILON NEWS

Edited by Harold L. Thomas, Historian

News of chapter activities and other noteworthy KME events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department, Pittsburg State University, Pittsburg, Kansas 66762.

CHAPTER NEWS

Alabama Zeta, Birmingham-Southern College, Birmingham
Chapter President - Thomas Donald Herring
28 actives, 16 initiates

Sixteen new members were inducted into KME at the 1983 initiation ceremony. At the initiation, Professor Natwarlal Bosmia spoke on "Cardinal Numbers." The chapter set up the Kappa Mu Epsilon Award, which is given annually on Honors Day to a senior member selected on the basis of scholarly attainment in mathematics and service to Kappa Mu Epsilon. This year, the award, a Kappa Mu Epsilon Key, was presented to Alison Pool, outgoing president. Other activities included a lecture given by the chair of mathematics at the University of Alabama in Birmingham, Dr. Peter O'Neil. His unique discussion centered around the topic of "Random Walks through Crystals." The end of the Spring term was celebrated by a party sponsored by KME and other science honor societies. Other 1983-84 officers: Judy Tanquary, vice president; Carol Anderson, secretary; Richard Sturgeon, treasurer; Lola F. Kiser, corresponding secretary; Sarah E. Mullins, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President - Ken Hoyt
40 actives, 25 initiates

The chapter assisted the Mathematics Department with the annual Poly Royal Math Contest which attracted over 500 high school students. Chapter meetings featured alumni and industry speakers. The 25th anniversary of the chapter was celebrated at the annual Spring banquet. The founder of California Gamma, Dr. George R. Mach, is still the corresponding secretary. Tina Masatini was the second recipient of the annual Arthur Andersen & Company Professional Performance Award. Other 1983-84 officers: Jennifer Martin & Tina Masatini, vice president; Diane Formea, secretary; Greg Kucala, treasurer; George R. Mach, corresponding secretary; Adelaide T. Harmon-Elliott, faculty sponsor.

Connecticut Beta, Eastern Connecticut State University, Willimantic
Chapter President - Hans Weidig
37 actives, 9 initiates

The chapter held only one meeting this year since the faculty sponsor, Steve Kenton, has been on leave. Other 1983-84 officers: Virender Gupta, vice president; Michael Rousseau, secretary/treasurer; Ann Curran, corresponding secretary; Steve Kenton, faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Yolanda Hasty
17 actives, 7 initiates

The Georgia Alpha Chapter of KME held its annual initiation meeting on Wednesday, May 25, 1983. After the initiation ceremony, an election was held to determine the 1983-84 chapter officers. A reception in honor of the seven 1983 pledges followed. At the reception, it was announced that a $500 scholarship was being awarded to Bob Ingle for the 1983-84 academic year. Also, a $300 scholarship was awarded to Maureen Ramey for 1983-84. Special congratulations to these two KME members! Other 1983-84 officers: Kim Huckeba, vice president; Maureen Ramey, secretary; Bob Ingle, treasurer; Joe Sharp, corresponding secretary/faculty sponsor.

<u>Illinois Beta</u>, Eastern Illinois University, Charleston
Chapter President - Nancy Martin

Spring semester activities included a mixer and initiation, honors banquet, field trips to Illinois Power and Staley, picnic, and a reception for graduates. Other 1983-84 officers: Dawn Hoskins & Kathy Jordan, vice president; Chor Mathies, secretary; Allen Rogers. treasurer.

<u>Illinois Zeta</u>, Rosary College, River Forest
Chapter President - Deborah Wagner
12 actives, 4 initiates

Members of the Illinois Zeta chapter held a plant sale to help finance attendance at the national convention. Four members, students Brad Erickson and Joan Novak, alumnus Michael Renella and faculty sponsor, Sister Nona Mary Allard, made the trip to Eastern Kentucky University. Jacqueline Nocek reported on her semester in London and on her study of the life and times of Isaac Newton at the initiation of new members held on February 25. Other 1983-84 officers: Sheila Ruh, secretary; Monica Behnke, treasurer; Sister Nona Mary Allard, corresponding secretary/faculty sponsor.

<u>Illinois Eta</u>, Western Illinois University, Macomb
Chapter President - Judith Smithhisler
7 actives, 10 initiates

Other 1983-84 officers: Tamara Lakins, vice president; Alan Bishop, corresponding secretary.

Illinois Theta, Illinois Benedictine College, Lisle
Chapter President - Steve Becker
16 actives, 3 initiates

In addition to meeting once every three weeks, the chapter held a math contest for high school students in February and an initiation ceremony in May. Other 1983-84 officers: Mike Cooney, vice president; Annette Markun, secretary; Michelle Szum, treasurer; James M. Meehan, corresponding secretary/faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
Chapter President - James Lehman
14 actives, 13 initiates

Spring activities included a picnic held May 1, 1983. New initiates were also received at a banquet. Dr. Francis Jones from Huntington College spoke on "Recreational Aspects of Game Theory." Other 1983-84 officers: Daniel Cripe, vice president; Tammy Ulery, secretary; Kent Workman, treasurer; Ralph McBride, corresponding secretary; Stan Beery, faculty sponsor.

Indiana Delta, University of Evansville, Evansville
Chapter President - Emily Reisinger
35 actives, 20 initiates

Dr. Hernan Rivera Rodas presented a talk entitled "Finite Geometries" at the Spring initiation banquet. Other 1983-84 officers: Blake Middleton, vice president; Suzzie Halwes, secretary; Melba Patberg, corresponding secretary; Duane Broline, faculty sponsor.

Indiana Gamma, Anderson College, Anderson
Chapter President - Douglas Skipper
20 actives, 7 initiates

Other 1983-84 officers: Janet Lopp, vice president; Vicki Mills, secretary/treasurer; Stanley L. Stephens, corresponding secretary.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President - Kirk Montgomery
34 actives, 7 initiates

The major event for the spring semester was the KME national convention in Richmond, KY. Margaret Chizek presented her paper on "The Heptagon" and Charles Daws presented his paper on "The Motion of Light Weight Projectiles" at the convention. Others attending were students Susan Anderson, J'ne Day, Darla Dettmann and Scott Pierce and four UNI faculty members. The following students presented papers at local meetings of Iowa Alpha: Kevin Fifo on "Mathematical Induction" and Kande Hooten on "Women in Mathematics." Other 1983-84 officers: Andrea Bean, vice president; Lori Maruth, secretary; Kande Hooten, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President - Coni Johnson
10 actives, 2 initiates

Other 1983-84 officers: Sheryl Shapiro, vice president; Mary Bernard, secretary; Craig Kalman, treasurer; Joseph Huffert, corresponding secretary; L. Naylor, faculty sponsor.

Iowa Gamma, Morningside College, Sioux City
Chapter President - Marlene Gieselman
8 actives, 6 initiates

Other officers have not been elected for 1983-84. Douglas A. Swan is faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President - Ron Stahlberg
30 actives, 9 initiates

Other 1983-84 officers: Teresa Tehven, vice president; Sarah Dieck, secretary; Gary Friedrichsen, treasurer; Mark Reinhardt, corresponding secretary/faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President - Brad Averill
40 actives, 9 initiates

The chapter began the Spring Semester with a banquet and initiation for the February meeting. Nine new members were initiated at that time. The March program was given by Brad Averill on "Mathematical Challenges." Jeanine Carver gave the April program on "Gaussian Quadrature." The Chapter also assisted the Mathematics Department faculty in administering and grading tests given at the annual Math Relays, April 26,1983. Several members also worked for the Alumni Association's Spring phonothon. Their efforts were well rewarded when it was announced that Kansas Alpha won the top prize of $100 for the most money raised by student organizations. In addition, Sharon Hunt and Cheryl Burns received $50.00 each for being individual students that raised the most money. Seven students and faculty attended the National Convention held in Richmond,Kentucky, April 21-23, 1983. The final meeting of the Spring Semester was a social event held at Professor McGrath's home. It was highlighted by election of officers for the 1983-84 school year. In addition, the annual Robert M. Mendenhall awards for scholastic achievement were presented to Darren Smith and Russ Jewett. They received KME pins in recognition of this honor. Jeanine Carver also received a monetary award in honor of Professor R. G. Smith, former Mathematics Department Chairman, who is now retired. Other 1983-84 officers: Kendall Draeger, vice president; Rebeca Graham, secretary; Lisa Burgan, treasurer; Harold L. Thomas, corresponding secretary; Helen Kriegsman & Gary McGrath, faculty sponsors.

Kansas Beta, Emporia State University, Emporia
Chapter President - Mary Kuestersteffen
35 actives, 9 initiates

The chapter held monthly meetings at which mathematical topics were discussed. New members were initiated at a banquet which included a guest speaker. Some members attended the National Convention in April at Richmond, Kentucky. The final meeting of the semester was a picnic at which time new officers were elected for the coming year. Other 1983-84 officers: Galen Zirnstein, vice president; Elisabeth Henkle, secretary; Sherri Spade, treasurer; John Gerriets,corresponding secretary; Tom Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Jenny Farrell
14 actives, 10 initiates

Second semester activities began with the initiation of three sophomores into Kansas Gamma -- Pat Gallagher, Deanna Haddad, and George Hickert. Eight students became associate members. President Kay Kreul presented information on the Agnesi curve from a paper she had prepared for seminar. On 12 March, chapter members became very active as they conducted the 13th Math Tournament for about 150 area high school students. Dr. Adele Marie Rothan, csj was the guest speaker for a March meeting. Dr. Rothan from Avila College in Kansas City, MO spoke on one sample non-parametric tests based on Kolmogorov-Smirnov Statistics which she investigated in her dissertation completed in 1982 at the University of Montana. April activities included the election of officers, a steak picnic in honor of the seniors, and attendance at the National Convention in Richmond by students Kay

Kreul, Therese Bendel, and Ann Devoy and faculty moderator, Sister Jo Ann Fellin. Co-recipients of the Sister Helen Sullivan Scholarship award for the 1983-84 academic year are Jenny Farrell and Ann Devoy. Other 1983-84 officers: Ann Devoy, vice president; Karen Henneberry, secretary/treasurer; Sister Jo Ann Fellin, corresponding secretary/faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President - Cindy Dietrich
15 actives

Officers for 1983-84 will be elected in the fall of 1983. Robert Thompson is corresponding secretary and Billy Milner is faculty sponsor.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President - Jeff Sadler
23 actives, 15 initiates

Regular chapter meetings were held with student speakers at three of them and a Philosophy Department faculty member at the other. Other Spring activities included initiation and banquet on March 10 and a picnic for the Mathematics Department on April 25. Other 1983-84 officers: Betty Burk, vice president; Bev Musselwhite, secretary/treasurer; Charles Votaw, corresponding secretary; Jeffrey Barnett, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President - Karen Applegate
25 actives, 36 initiates

Most of this Spring semester was spent preparing for the national convention which was hosted by Kentucky Alpha. However, two faculty talks were fit into a very busy schedule. Dr. Patti Costello gave a talk on the benefits of Statistics and Dr. Alvin McGlasson gave a talk on compound interest. Dr. McGlasson, who has been a member of KME since 1971, retired this year and was presented a plaque thanking him for his many years of support of Kentucky Alpha. Other 1983-84 officers: Monica Feltner, vice president;Greg Allender, secretary; Philip White, treasurer; Patrick Costello, corresponding secretary; Donald Greenwell, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland,Baltimore
Chapter President - Debbie Chaney
6 actives, 2 initiates

The chapter sponsored the Second Annual Mathematics Olympiad for high school students in March. Two new members, Debbie Chaney and Patti McCarron, were initiated at the annual dinner in May. Patricia Tillman, Data Processing Manager was guest speaker. Her topic was "The Computerization of the College." Other 1983-84 officers: Michele Ritter, vice president; Jane Bisasky, secretary; Kim Kvech, treasurer; Sister Marie A. Dowling, corresponding secretary; Sister Delia M. Dowling, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President - Stephen Coffman
15 actives, 4 initiates

The annual Spring banquet and initiation of new members was held in March. A career night was also held featuring Diane Briggs Martin,'65, and David Baugh, '70, who spoke on their work with the mathematics and value of a liberal arts education in the preparation for future work. The chapter sponsored a movie to provide funds for a new mathematics award,the David Cross award, in memory of Davey Cross, '81, who died of Cystic Fibrosis last August. Davey was a KME member who graduated with honors. May activities included the annual picnic held at a professor's home and sponsoring a pretzel booth in the May Day carnival. Other 1983-84 officers: Wende Reeser, vice president; Michael Armacost, treasurer; James E. Lightner, corresponding secretary; Jack Clark, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President - Vincent Costello
35 actives, 17 initiates

Programs were presented by faculty members, Dr. Donald Shriner and Mr. Lance Revennaugh. Members assisted the Mathematics Department in sponsoring the annual symposium for teachers: this year's topic was Microcomputers in Education. Members provided individual tutoring to math students. A team of KME members competed with other campus honor societies in a True Greek Night Bowl, receiving first place honors. Five members received departmental honors at the College's Honors Convocation: Margaret Neville, Mathematics; John Wagner, Physics; Mark Nelson, Chemistry; Vincent Costello, General Science - Earth Science; and Barbara Dyker, General Science-Physical Science. John Wagner received a John Allison Outstanding Senior Award. A ski trip and a picnic were also held. Other 1983-84 officers: Joseph Olah, vice president; Susan Hurt, secretary; Lynn Harpold, treasurer; Donald Shriner, corresponding secretary; John P. Jones, faculty sponsor.

Massachusetts Alpha, Assumption College, Worcester
Chapter President - Joseph Bonin
9 actives, 5 initiates

Five new members were initiated on March 22, 1983. Following a dinner in honor of the new members, Dr. Robert Fry, a member of the Assumption faculty, spoke on "The Four Color Theorem." Other 1983-84 officers: Joseph Kirby, vice president; Robert Andrews, secretary; Charles Brusard, corresponding secretary/faculty sponsor.

Michigan Beta, Central Michigan University, Mt. Pleasant
Chapter President - John Porterfield
60 actives, 39 initiates

Dr. Richard Fleming, Chairman of CMU Mathematics Department, was guest speaker for the Spring initiation. He spoke on "A Transcendental Meditation." Two officers of the Michigan Society of Actuaries, Eugene Chang and Fred Sievert, spoke to the chapter on the actuarial profession. Two student members also gave talks during the semester. Tracy Anderson spoke on a problem in Operations Research and Sandy Dolde on the Central Limit Theorem. Tutorial help sessions were held three evenings a week for students in freshman-sophomore mathematics courses. A Spring picnic was held to finish the semester. Other 1983-84 officers: Bill Ruelle, vice president; Kathy Baker, secretary; Dan Roche, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

Mississipi Alpha, Mississippi University for Women, Columbus
Chapter President - Melesia Coleman
17 actives

The chapter took an active role in recruiting students for next year by writing letters to all prospective math or pre-engineering students. Other 1983-84 officers: Susan Furlow, vice president; Rhonda Wercinski, secretary/treasurer; Jean Ann Parra, corresponding secretary; Carol B. Ottinger, faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President - Jack Davenport
40 actives, 10 initiates

Other 1983-84 officers: Lisa Hurt, vice president; Tracey Tinnon, public relations; Alice Essary, corresponding secretary; Mylan Betounes, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President - Shelly L. Robbins
68 actives, 11 initiates

Other 1983-84 officers: Sharyn Brikenbach, vice president; Ellen Capehart, secretary; Keith A. Huffman, treasurer; M. Michael Awad, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President - Alice Hink
30 actives, 15 initiates

The Chapter held four regular meetings with one initiation. The William Klingenberg Lecture was held in April and an honors banquet during the month of May. Other 1983-84 officers: Barry Haden, vice president; Janette Mize, secretary; Brenda Enke, treasurer; Homer F. Hampton, corresponding secretary; Larry Dilley, faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President - Michael Hanson
4 actives, 6 initiates

Officers for 1983-84 will be elected in the fall. William D. McIntosh is corresponding secretary and faculty sponsor.

Missouri Zeta, University of Missouri - Rolla, Rolla
Chapter President - Trent Eggleston
25 actives, 21 initiates

At the first general meeting on February 9, discussion centered on the type of prize the chapter would award to the winning math exhibit at the annual Spring Science Fair. A CRC book of tables was selected as the prize, provided some exhibit proved worthy. Other discussion at this meeting was concerned with the KME annual Spring pizza party, the upcoming Biennial Convention, a change in chapter bylaws, and the election of a treasurer. With regard to the latter topic, Tom Lonski was elected by acclamation to the office of treasurer. Trent Eggleston then passed out shingles to last semester's initiates after which Prof. Marc Raphael of the Mathematics and Statistics Department gave a very interesting talk on "Spectral Theory." At the next general meeting of the chapter, details of the Spring initiation banquet were finalized. Italo's Restaurant was selected as the setting for the banquet and the date was set at April 27. The meeting concluded after Prof. John Carstens of the Cloud Physics Department presented some understandable and intriguing concepts concerning cloud physics. In March, three Missouri Zeta members were informed by Prof. James Smith that their papers had been selected for presentation at the 24th Biennial Convention. The 3 authors and the titles are respectively; Trent Eggleston - "Taylor's Formula With Blumenthal Remainder," Thomas Shannon - "The Divisibility of Seven and Other Numbers," and Steven Siems - "Some on $\frac{1}{\sin x}$ and $\frac{1}{x}$ on $(0, \frac{\pi}{2}]$." The third general meeting in April was highlighted by the presentation of papers by those who had been selected for such at the Biennial Convention. Steven Siems was unable to attend the convention, and Denise Rost was selected from the list of alternates to replace him. Denise's paper was entitled "Pi-Some Derivations and Coincidences." On the evening of April 14, 1983, a special meeting was held. On that evening, KME jointly sponsored, with the local chapter of ACM, an address given by the distinguished numerical analyst Prof. George M. Andrews from the University of St. Andrews, St. Andrews, Scotland. The title of his talk was "The Historical Development of Logarithms." Due to the positive response, KME is going to encourage such joint endeavors in the future. Five students and two faculty attended the 24th Biennial Convention. They were Denise Rost, Trent Eggleston, Thomas Shannon, Paul Whitten, James Farley, Jim Joiner, and Johnny Henderson. On April 27, 1983, twenty-one initiates were inducted into KME at the initiation banquet. Prof. Sam Geonetta of the Speech Department delivered the address which dealt with the timely topic. "Freedom of Communication." The last general meeting of the semester was held on May 4, 1983 with the election of officers for the fall semester being the main item of business. After the election, those present at the meeting viewed the award winning films, "I Maximize" and "Infinite Acres." Other 1983-84 officers: James Farley, vice president; Kim Borgmeyer, secretary; Sharri Riggs, treasurer; Thomas Powell, corresponding secretary; James Joiner, faculty sponsor; Paul Whitten, historian.

Missouri Eta, Northeast Missouri State University, Kirksville
Chapter President - Sandy Nelson
28 actives, 10 initiates

The Chapter hosted the annual academic contests in mathematics in March with 756 participants. Help was also given in organizing Special Olympics on campus and in providing tutorial help for the freshman level mathematics courses. A KME team participated in the Campus Bowl. A delegation attended the national convention at Richmond, Kentucky. The Chapter is trying to organize a Homecoming breakfast for all alumni. Other 1983-84 officers: Ed Jurotich, vice president; Rebecca Hutton, secretary; Robert Clark, treasurer; Sam Lesseig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin
Chapter President - Charles Metz
18 actives, 11 initiates

Regular meetings were held during the semester. Eleven new members were initiated. Seven students and two faculty attended the national convention in Richmond, Kentucky. The Chapter also provided tutorial help for pre-calculus students. The annual Spring picnic was held at Professor Joe Shields' home. Other 1983-84 officers: Susan Petty, vice president; Steve Brock, secretary/treasurer; Mary Elick, corresponding secretary; Joe Shields, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President - Annette Schmit
17 actives, 17 initiates

Money making projects throughout the Spring semester included monitoring the Math-Science Building in the evenings and selling discs for the Apple II Computer to college students. The club also administered the annual test to identify the outstanding freshman majoring in mathematics. The award went to Miles McGinnis whose home town is LaVista, Nebraska. The award includes the recipient's name being engraved on a permanent plaque, payment of KME National dues, one year honorary membership in the local KME chapter, and announcement of the honor at the annual Spring banquet. Members Mike Ronspies, Annette Schmit, Sue Kohno, Kelli Goodner and sponsor Dr. Hilbert Johs attended the 24th KME National Biennial Convention held at Eastern Kentucky University in Richmond, Kentucky on April 21-23, 1983. At the convention, Kelli Goodner presented her paper entitled "Perfect Numbers." During the semester, other club activities included entering two teams in the annual Wayne State College - College Bowl, maintaining the KME bulletin board, assisting the Wayne State College mathematics faculty with the Ninth Annual Wayne State College Mathematics Contest on May 12, 1983, and sponsoring socials including an ice cream party and a pizza party. Other 1983-84 officers: Kim Prchal, vice president; Deb Lofton, secretary/treasurer; Mike Rood, historian; Fred Webber, corresponding secretary; James Paige & Hilbert Johs, faculty sponsors.

Nebraska Gamma, Chadron State College, Chadron
Chapter President - Evonnda Sharp
10 actives, 4 initiates

Spring activities included providing tours of the Math and Science Building during the Annual High School Scholastic Contest, March 25. Initiation ceremonies were held for 4 new members on April 20. Other 1983-84 officers: David Mundt, vice president; Diana Thomas, secretary; Janine Spracklen, treasurer; James Kaus, corresponding secretary; Monte Fickel, faculty sponsor.

New Jersey Beta, Montclair State College, Upper Montclair
Chapter President - Donna Jean Cedio
19 actives, 15 initiates

Spring activities included general meetings, Spring Picnic, and Fall '82 Induction Service. The principle activity for the members was programming experience on the Apple Micro-computer. Other 1983-84 officers: Elizabeth Olohan, vice president; Raquel Hernandez, secretary; Lisa Rutkowski, treasurer; John G. Stevens, corresponding secretary; Thomas Devlin, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President - Martin Murphy
30 actives

The initiation banquet was held in April at a restaurant in the nearby Sandia Mountains. The initiates were put through their usual quiz and the guest speaker spoke about the university. Other 1983-84 officers: Stephanie Hendrix, vice president; Terry Hardin, secretary; Richard Metzler, treasurer; Merle Mitchell, corresponding secretary/faculty sponsor.

New York Eta, Niagara University, Niagara
Chapter President - Bridgette Baldwin
19 actives, 4 initiates

Students were involved in various fund-raising activities. A former faculty member, Dr. Sean O'Reilly, spoke to the students at the initiation banquet. His topic centered around job opportunities for graduates. A delegation represented the chapter at the National Convention in April. Officers for 1983-84 will be elected at the beginning of the Fall semester.

New York Lambda, C. W. Post Center - Long Island University, Greenvale

KME's newest chapter was installed May 2, 1983. Eight students and ten faculty were initiated at that time. Dr. John Weidner, corresponding secretary of New York Alpha, was installing officer. Guest speaker for the installation was Dr. James Peters of the Mathematics Department faculty at the C.W. Post Center.

Ohio Gamma, Baldwin-Wallace College, Berea
Chapter President - Pamela Botson
17 actives, 13 initiates

Other 1983-84 officers: Alice Kruzel, vice president; Kelly Flood, secretary; Mark Maceyko, treasurer; Robert Schlea, corresponding secretary/faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President - Jeff Seitter
62 actives, 16 initiates

Tom Bressoud presented his paper, "Fun With Splines," at the January meeting. He also gave it at the national convention in April and received first place prize for it. Six members and National President-elect, Jim Smith, attended the convention. Sixteen new members were initiated in February. In March, officers were elected and Dr. Smith spoke on "Where in the World is Mathematics." A Spring picnic was held in May. Myron Duklowski was awarded one of ten NCAA Academic Fellowships. He will attend the University of Montana at Missoula. Other 1983-84 officers: Paula Gomory, vice president; Sandy Cutler, secretary; Andy Herron, treasurer; Russ Smucker, corresponding secretary/faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President - Cynthia Phillips
20 actives

Spring activities included monthly meetings, a trip to Haliburton's Oil in Duncan, Oklahoma, and a Spring picnic. Dr. Frank Chimenti gave a talk to the Chapter on "Image Processing." Other 1983-84 officers: Robert Estes, vice president; Scott Goeringer, secretary/treasurer; Wayne Hayes, corresponding secretary; Kelvin Casebeer, faculty sponsor.

Pennsylvania Beta, LaSalle College, Philadelphia
Chapter President - Joanne Kelly
22 actives, 6 initiates

Initiation of new members took place on April 26, 1983. Other 1983-84 officers: Karen Bruno, vice president; Mary McGee, secretary; John Englehart, treasurer; Hugh Albright, corresponding secretary; Stephen Andrilli, faculty sponsor.

Pennsylvania Epsilon, Kutztown State University, Kutztown
Chapter President - Jeff Herbein
20 actives, 10 initiates

Other 1983-84 officers: Mel Schumaker, vice president; Kelley Green, secretary; Brian Balthaser, treasurer; I. Hollingshead, corresponding secretary; E. Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana
Chapter President - Ann Leiberton
31 actives, 8 initiates

Monthly meetings were held in February, March and April. In February, Dr. Robert Stonebraker, member of the Economics Department Faculty, spoke about the uses of mathematics in economic theories. Mr. Thomas Giambrone, member of the Mathematics Department Faculty, demonstrated the Apple II computer with several specific examples of its use in the mathematics classroom at the March meeting. In April, the annual banquet was held at the University Lodge. The meal was prepared by student members under the supervision of Mr. Raymond Gibson. Mr. Art Spencer,a mathematician with Westinghouse Corp., was the speaker. Students who had attended the National Convention also reported on convention activities. Other 1983-84 officers: Diane Kirchner, vice president; Claudia Christner, secretary; Sue Ann Kaufold, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President - Mary Ann Friedrich
30 actives, 10 initiates

The annual Spring picnic was held in the Grove City Recreation Center on May 4. Those who attended included all of the faculty, but only a hand full of the more dedicated students. Other 1983-84 officers: Sue Lowin, vice president; Diane J. Brosius, secretary; Marci L. Barkich, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Theta, Susquehanna University, Selinsgrove
Chapter President - Lisa Kapustay
14 actives, 12 initiates

Other 1983-84 officers: Herbert Conover III, treasurer; Carol N. Harrison, corresponding secretary/faculty sponsor.

Pennsylvania Iota, Shippensburg State University, Shippensburg
Chapter President - Joel Whitesel
38 actives, 23 initiates

Other 1983-84 officers: Jim Shoop, vice president; Gina Jackson, secretary; Howard T. Bell, treasurer; Carl E. Kerr, corresponding secretary; Richard B. Ruth, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President - Linda Rafferty
7 actives, 2 initiates

Initiation ceremonies were held on March 16, 1983. Linda Rafferty and Christine Michaels were initiated. The speakers were former KME members, Susan Capozio,'78, Suzanne Carroll, '64, Linda Chinn, '81, and Judith Washburn, '78. A reception followed the induction ceremony. KME members also attended a Seminar on "Making it Count" on February 16, 1983 sponsored by the Boeing Computer Services Company. At each meeting held, "Probability" was the topic of discussion. Other 1983-84 officers: Teresa McKeon, vice president; Christine Michaels, secretary/treasurer; Sister M. Grace, corresponding secretary.

South Carolina Gamma, Winthrop College, Rock Hill
Chapter President - Anita Anderson
10 actives, 5 initiates

Other 1983-84 officers: Phil Blankstein, vice president; Ken Peay, secretary; Chuck Baldwin, treasurer; Don Aplin, corresponding secretary; Kay Creamer, faculty sponsor.

Tennessee Alpha, Tennessee Technological University, Cookeville
Chapter President - Sherri Menees
75 actives, 37 initiates

Other 1983-84 officers: Lynne Matthews, vice president; Elisa Gould, secretary; Mark Sessions, treasurer; Edmond D. Dixon, corresponding secretary; S.B. Khleif, faculty sponsor.

Tennessee Delta, Carson-Newman College, Jefferson City
Chapter President - Dennis Beal
18 actives, 8 initiates

Other 1983-84 officers: Susan Saylor, vice president; Jeff Knisley, secretary; Susan Williams, treasurer; Albert L. Myers, corresponding secretary; Carey R. Herring, faculty sponsor.

Texas Beta, Southern Methodist University, Dallas
Chapter President - Victoria Bychok
47 actives, 15 initiates

The annual spring initiation banquet was held March 4, 1983. Fifteen new members were inducted, including fourteen students and one faculty member. Dr. George W. Reddien, Jr., Chairman of the Department of Mathematics, spoke at the banquet. Also attending the banquet were several alums of the Texas Beta chapter. Texas Beta's main project was sponsoring the 5th annual S.M.U. Math Competition. On April 9, 1983, over 150 Texas high school students competed in hour long tests in the areas of Algebra I through Calculus. One special event held during the competition was the Number Sense test, which measures speed and accuracy of mathematical calculations in a twenty minute time period. Trophies were given to the students with the three highest scores on each test and to the three schools with the best overall performances. The movie "Flatland" was shown at the awards ceremony. Other 1983-84 officers: David Weltman, vice president; Nilesh Naik, secretary; Linda Fernicola, treasurer; Carolyn S. Shull, corresponding secretary/faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President - Randall Bradshaw
25 actives, 10 initiates

The Chapter held its ninth annual induction banquet March 31, 1983. Ten new members were inducted: Tim Avis, Marie Baldwin, Carol Salzwedal, and Sybile Schwarz all from Abilene, Texas; Stephanie Brooks from Corpus Christi, Texas; Donna George from Post, Texas; Linda Haire from Lubbock, Texas; LaQuita Lorance from Cisco, Texas; Ron Ryan from Green River, Wyoming; and Seok Sagong, a member of the faculty of the Department of Mathematics. With the induction of these new members, membership in the local chapter stands at 78. Dr. David Hughes, Chairman of the Department of Mathematics at Abilene Christian University, addressed the chapter on the subject, "Rabbits and Foxes." Special guests included Dr. Ronald Smith, Vice President for Academic Affairs at H-SU and Mrs. B.C. Bentley, a former mathematics faculty member at H-SU. Induction ceremonies were conducted by the officers. Other 1983-84 officers: Dana Teer, vice president; Linda Haire, secretary; Robbie Chesser, treasurer; Mary Wagner, corresponding secretary; Charles Robinson & Ed Hewett, faculty sponsors.

Virginia Beta, Radford University, Radford
Chapter President - Dave Derowitsch
20 actives, 12 initiates

Spring activities included a banquet in April and a picnic in May. Other 1983-84 officers: Sharon Napper, vice president; Sharon Goad, secretary; Cynthia Carr, treasurer; Coreen Mett, corresponding secretary; J.D. Hansard, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President - Bonnie Best
6 actives, 5 initiates

Four pledges (Gail Paul, Barbara Schilter, Linda Schmidt, and Veronica Walicki) were initiated into Wisconsin Alpha on March 15. Prior to initiation, each pledge had given a talk to the members and pledges of the chapter. During Spring semester, Barbara Schilter discussed computer assisted instruction, Linda Schmidt looked at patterns in card tricks, and Gail Paul spoke about the work of Pascal. Initiates Veronica Walicki, Linda Schmidt, and Barbara Schilter and pledge Jenie Puerzer, accompanied by corresponding secretary Sister Adrienne Eickman, attended the KME Convention at Eastern Kentucky University. Other 1983-84 officers: Veronica Walicki, vice president; Linda Schmidt, secretary; Barbara Schilter, treasurer; Sister Adrienne Eickman, corresponding secretary; Sister Mary Petronia Van Straten, faculty sponsor.

INSTALLATION OF NEW CHAPTERS

Loretta K. Smith

NEW YORK LAMBDA CHAPTER

Long Island University, C. W. Post Center, Greenvale, New York

New York Lambda Chapter was installed on May 2, 1983, by Corresponding Secretary, Dr. John Weidner of New York Alpha Chapter, Hofstra University. Ten faculty and eight students are charter members of this chapter:

Faculty

Dr. Geoffrey Berresford	Dr. Robert McKane
Dr. Elliott Bird	Dr. James Peters
Dr. Anne Burns	Dr. Andrew Rockett
Dr. Neo Cleopa	Dr. Sheldon Rothman
Dr. Sharon Kunoff	Dr. Steven Wagner

Students

Ron Biagini	Debbie Galosich
Steven Buonincontri	Alex Karpov
Joanne Carlough	Kathy Purcell
Colleen Gallivan	Raymond Westwater

Dr. Elliott Bird gave the opening remarks at the installation. The Chapter was then installed by Dr. Weidner. Dr. Sharon Kunoff introduced the initiates after which Dr. James Peters spoke on "How I spent my Spring Vacation (What Is Happening Mathematically at the I.A.S.)." Refreshments were served at the close of the program.

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