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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

FUN WITH SPLINES*

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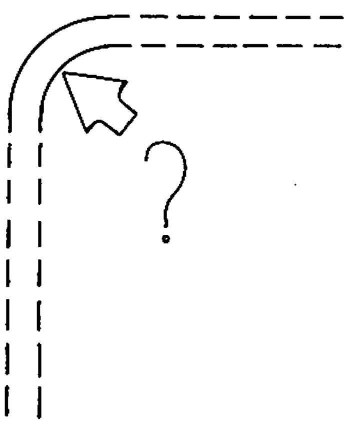
Imagine trying to join two sections of railroad track.



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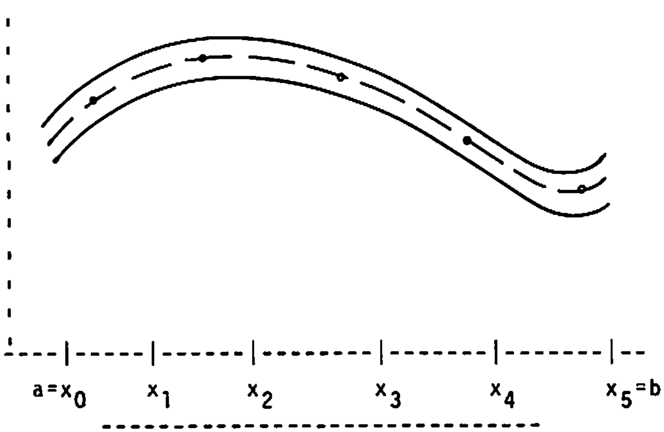
How would you do it? If you were to try and use a semicircle, you would be fired by the head engineer. Anyone who has worked with model railroading would know that that will not work. Because of a lack of continuity in the second derivative, which gives us acceleration, the joining by the use of a second degree curve

*A paper presented at the 1983 National Convention of KME and awarded first place by the Awards Committee.

would cause the train to jerk at the points of joining. The correct answer will become clear as we proceed.

In the following, I will give a brief overview of SPLINE INTERPOLATION in its elementary forms and compare a few splines of varying degrees.

It has long been a question of mathematicians, given certain data, how can a function be determined that will agree with that data, whatever the particular application might be. Often we know certain points through which a function must pass, but we do not know what the function itself is. Or else, we do know what the function is but due to its complexity it is too time consuming or difficult to plot the function, let alone to try and differentiate or integrate it, if that be the particular need. Our problem then, is one of finding a substitute, or interpolating function.



Let us assume that the small circles are the points through which we know a function must pass. If the function we are trying to find is given by the dotted line, we know by the Weierstrass Approximation Theorem that an interpolating function must exist. (Weierstrass's theorem guarantees that we can find an interpolating function within epsilon of the given function.) This epsilon is the 'channel', or band, given by the solid lines in the diagram. Unfortunately, this theorem gives us no clue as to how to find our function. We must look elsewhere.

The use of polynomials to approximate arbitrary functions on a closed interval has long been a favorite method. This is for two very good reasons. First, the form of the polynomial is simple and is thus easy to store in standard computer architecture. We need only keep track of the coefficients. Secondly, integration and differentiation of polynomials can be done easily by computers. However, in some of the standard methods of polynomial approximation, such as Lagrange's method, the order of the approximating polynomial is the same as the number of points of interpolation. We, by design of the method, do go through the interpolatory points, but in between, our function tends to oscillate greatly in small portions of the interval.

A much better alternative to obtain interpolatory functions is to take the collection of subintervals given by the points of interpolation and construct a different approximating polynomial on each subinterval. In effect, we break the function we are trying to approximate into pieces and approximate each of these smaller chunks. When we put these pieces together, we have approximated the entire function. This method, rather aptly, is called piecewise polynomial approximation.

The simplest and most familiar type of piecewise polynomial approximation is called piecewise LINEAR Interpolation. This method consists of merely joining a set of data points by a series of straight lines. For the sake of consistency, I will give a formal definition.

Let us, at this point, set up some terminology; I will use the Pascal notation, ':=', to mean assignment or definition.

DEFINITION -- LINEAR SPLINE:

Given a function f defined on [a,b] and a set of numbers, $a = x_0 < x_1 < \ldots < x_n = b$, a LINEAR SPLINE INTERPOLANT, which we will denote S1, for f is a function that satisfies the following conditions:

a) $S1$ is a linear function denoted $S1_j$, on the subinterval $[x_j, x_{j+1}]$ for each $j=0,1,\dots,n-1$;

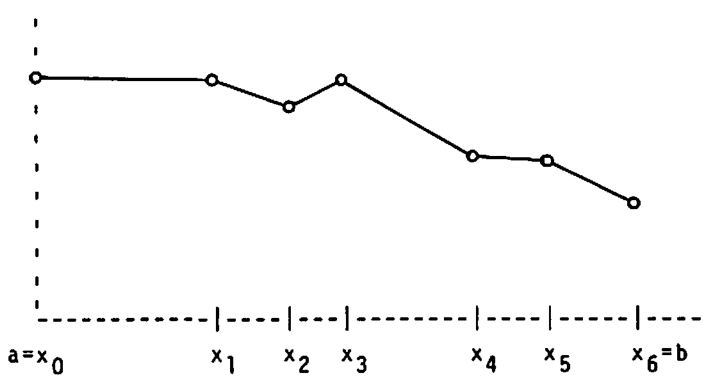
b) $S1(x_j)=f(x_j)$ for each $j=0,1,\dots,n$;

c) $S1_{j+1}(x_{j+1})=S1_j\ (x_{j+1})$ for each $j=0,1,\dots,n-2$;

The general form for the equation of these lines is:

$S1_j(x):=a_j + b_j\ (x-x_j)$ for each $j=0,1,\dots,n-1$;

By definition of the LINEAR spline, the interpolating function must agree with our data points. In addition, the linear spline functions agree with each other at the endpoints of "their intervals." In practical application the process is quite simple, and most mathematics students have run into its application when interpolating logarithmic or trigonometric tables.



***** Linear Interpolation *****

Determining the equations for these lines is quite simple. The coefficient a_j is given by the point originating the line segment, x_j, and the coefficient b_j is the slope of the segment joining x_j and x_{j+1}. (Otherwise given as delta f(x)/delta x.)

The question we must now look at is: what kind of accuracy are we getting with this type of Spline?

Let $h := \max(x_{i+1} - x_i)$ for $i=0,1,\ldots,n-1$;

and

Let $||f(x)|| := \max|f(x)|$ for $a \leq x \leq b$.

The error bound for the Linear Spline, then, is:

$$||f(x) - S1(x)|| \leq (1/8)\, h^2 ||f''(x)||$$

The major drawbacks of this linear approach are, first, although the error bound is good, it could be improved. Second, and more important, there is no assurance of differentiability. In geometric concept this means that the function is not 'smooth'. So, we try for something better.

The type of piecewise polynomial approximation most widely used today is the CUBIC SPLINE INTERPOLANT. Here, in much the same way as the Linear Spline, we construct a Cubic polynomial between each successive pair of nodes. The general cubic involves four constants, so we can ensure that the interpolant has both a continuous first and second derivative on the interval. We now give a proper definition.

DEFINITION -- CUBIC SPLINE:

Given a function f defined on [a,b] and a set of numbers, $a = x_0 > x_1 > \ldots > x_n = b$, a CUBIC SPLINE INTERPOLANT, which we will denote S3, for f is a function that satisfies the following conditions:

a) S3 is a cubic function denoted $S3_j$, on the subinterval $[x_j, x_{j+1}]$ for each $j=0,1,\ldots,n-1$;

b) $S3(x_j)=f(x_j)$ for each $j=0,1,\ldots,n$;

c) $S3_{j+1}(x_{j+1})=S3_j(x_{j+1})$ for $j=0,1,\ldots,n-2$.

d) $S3'_{j+1}(x_{j+1})=S3'_j(x_{j+1})$ for $j=0,1,\ldots,n-2$;

e) $S3''_{j+1}(x_{j+1})=S3''_j(x_{j+1})$ for $j=0,1,\ldots,n-2$;

f) One of the following boundary conditions:

   i) $S3''(x_0)=S3''(x_n)=0$, Natural spline;

   ii) $S3'(x_0)=f'(x_0)$ and $S3'(x_n)=f'(x_n)$, Clamped Spline.

The general form for the equation of these polynomials is:

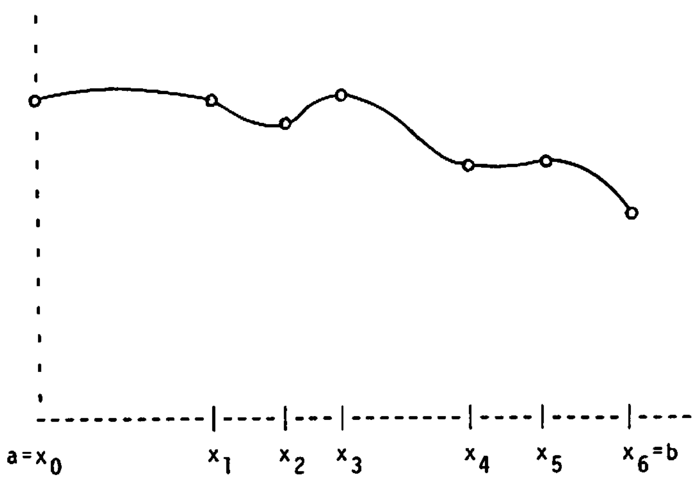
$$S3_j(x) := a_j + b_j(x-x_j) + c_j(x-x_j)^2 + d_j(x-x_j)^3$$

for each $j=0,1,\ldots,n-1$.

Notice that, by our definition, the first and second derivatives of the spline agree with themselves at the nodes. The final conditions impose the natural or clamped cubic spline.

Using these conditions in our definition, along with some matrix manipulation, we can solve the resulting system of equations for the coefficients a_j, b_j, c_j, and d_j. With the use of the computer, we can then easily store the cubics making up the entire spline.

Let us take a look at the same data points as before, now with a cubic spline running through them.



Obviously, we get a much smoother interpolating function this way. How close then, are we to the function?

The error bound for the cubic spline is as follows:

$$||f(x)-S3(x)|| \leq (5/384)\ h^4\ ||\ f^{(iv)}(x)||$$

As long as we keep h small (< 1) we can generally achieve better results than with LINEAR Interpolation. In addition, we have the benefit of continuity in the first and second derivatives of our interpolating function. As a result, we get a very smooth function.

Now, you may ask, "What about splines of even degree (i.e. 2nd or 4th degree splines)?" I'm glad you asked that. The fact of the matter is this. There are significant differences between splines of odd degree and those of even degree. Splines of even degree are more difficult to solve because they have more degrees of freedom. There are more variables than there are equations. Thus, you must impose additional conditions in order to provide for a unique solution. To make this more clear, let us take a look at a Quadratic Spline.

DEFINITION -- QUADRATIC SPLINE

Given a function f defined on $[a,b]$ and a set of numbers, $a = x_0 < x_1 < \ldots < x_n = b$, a QUADRATIC SPLINE INTERPOLANT, which we will denote S2, for f is a function that satisfies the following conditions:

a) S2 is a quadratic function denoted $S2_j$, on the subinterval $[x_j, x_{j+1}]$ for each $j=0,1,\ldots,n-1$;

b) $S2(x_n)=f(x_n)$;

c) $S2(x_0)=f(x_0)$;

d) $S2 \in C'[a,b]$;

e) Additional Condition:

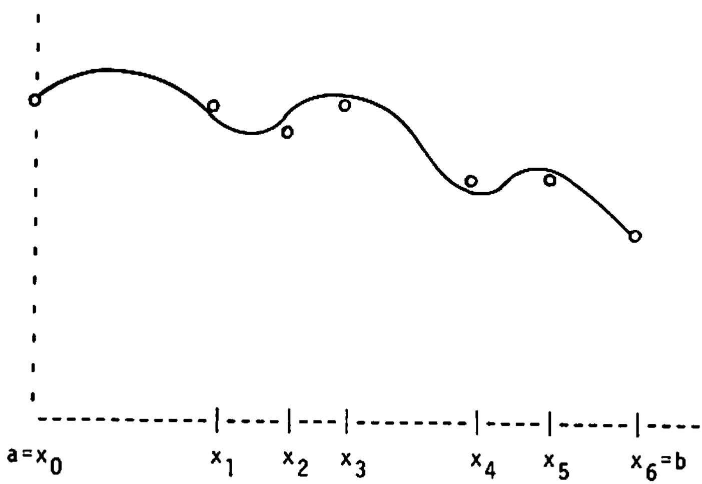
$S2((x_j+x_{j+1})/2)=f((x_j+x_{j+1})/2)$ for $j=0,1,\ldots,n-1$.

The general form for the equation of these polynomials is:

$$S2_j(x) := a_j + b_j(x-x_j) + c_j(x-x_j)^2 \text{ for each } j=0,1,\ldots,n-1;$$

Note that, although the Spline must agree with the function at the initial and final points of the interval, it does not have to agree at the interior nodes. If this were the condition as opposed to condition e), we could not guarantee a unique solution for the coefficients. Instead, I required that the Spline and the function agree at the midpoint between nodes. Again, with some manipulation of matrices, I was able to solve this system of equations and arrive at a Quadratic Spline with a continuous first derivative.

Using an algorithm written from my solution, I obtained the Quadratic Spline shown below:



****** Quadratic Spline Interpolation ******

The error bound for the Quadratic Spline, assuming three continuous derivatives in the function we are approximating is:

$$||f(x)-S2(x)|| < (11/48)\ h^3\ ||f'''(x)||$$

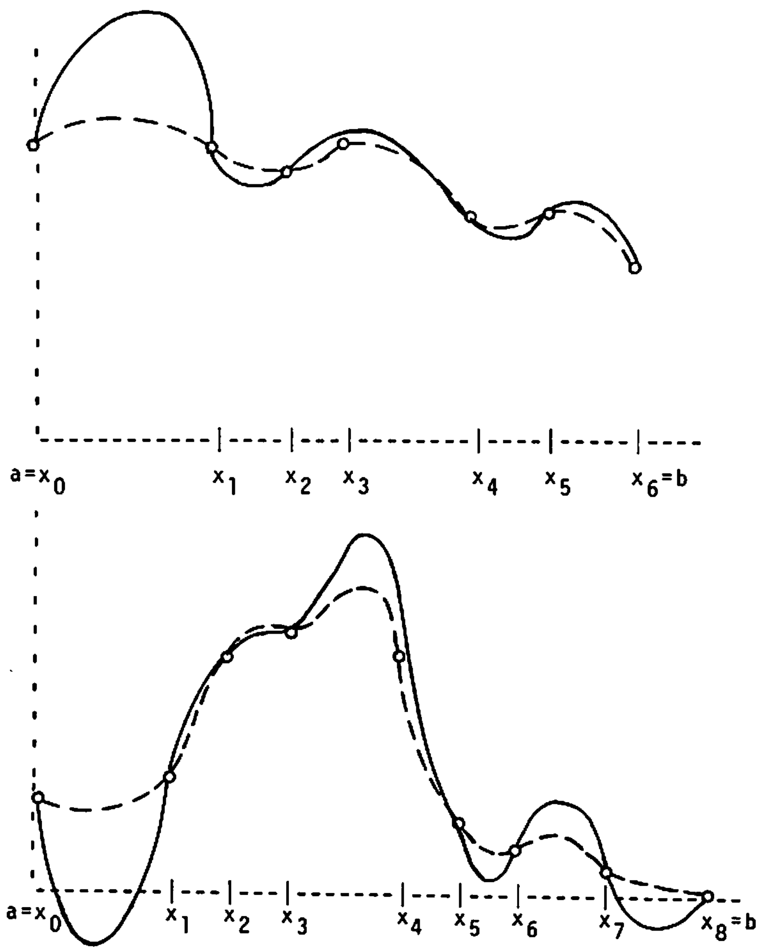
In comparing this error bound with the others that we know, it is about where we would expect it to be--on the order of h^3.

Lastly, we will consider splines of higher degree.

As was indicated earlier, splines of even degree need additional conditions imposed to be able to solve for the coefficients. The Quartic Spline proved to be quite difficult, but the Quintic Spline turned out to be readily solvable. In definition, the Quintic Spline is simply an extension of the Cubic, as one might expect. The end conditions are modified, and the resulting Quintic Spline has four continuous derivatives. The improved smoothness in this spline is not likely to be able to be detected by the human eye.

The error term, as one would also expect, is on the order of h^6. The real question is, are we getting a better interpolation to our function? Using another algorithm which I derived from my solution for the coefficients, the following results were obtained and compared against the Cubic Spline.

In these two examples, the Cubic Spline is given by the dotted line, and the Quintic Spline is given by the solid line. We can see that the Quintic Spline tends to oscillate more than the Cubic. As we get into splines of even higher degree we are getting into the same problem as with the Lagrangian method -- high oscillation. Also, we are gaining very little advantage from the additional number of continuous derivatives and we are taking up roughly two and a half times the computational time. For the gain achieved, it is not normally worth it.



**** A Comparison: Cubic Spline vs. Quintic Spline ****

Where, then does that leave us? It leaves us with the conclusion that the Cubic Spline is a very good method for Interpolating functions. I have pursued these problems with the use of a digitizing screen, and have also applied Cubic Spline Interpolation to reproducing free-hand drawings on a graphics terminal. The applications for Cubic Splines are endless and the field is wide open. Have fun.

BIBLIOGRAPHY

Ahlberg, J.H., E.N. Nilson, and J.L. Walsh (1967), The Theory of Splines and Their Applications, Academic Press, Inc., New York.

Birkhoff, G., and C. de Boor (1964), "Error Bounds for Spline Interpolation," Journal of Mathematics and Mechanics, 13, pp. 827-836.

de Boor, C. (1978), A Practical Guide to Splines, Springer Verlag, Heidelberg.

Greville, T.N.E. (ed.) (1969), Theory and Applications of Spline Functions, Academic Press, Inc., New York.

Spath, H. (1974), Spline Algorithms for Curves and Surfaces, Unitas Mathematica Publications, Winnipeg.

Schultz, M.H. (1973), Spline Analysis, Prentice Hall, Englewood Cliffs, N.J.

WHAT CAN WE SAY ABOUT THE DUFFINIAN NUMBERS?*

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Many mathematical articles are devoted to the subject of Number Theory. Throughout the years, many of the topics of Number Theory, including Diophantine equations, the theory of congruences, and Euler's Phi Function, have appeared in other branches of mathematics as well. A relatively new idea is the "Duffinian" number. The definition of, and initial work on Duffinian numbers, are due to L. Duffy in 1979 (see [2]).

By definition, a *Duffinian number* is a positive, composite integer having the property that the sum of its factors other than itself is not divisible by any of these factors other than 1 (see [2]). Note that the number 1 and the prime numbers are never Duffinian numbers. To become more familiar with the definition, consider as an example the number 100. The proper divisors of 100 are 1, 2, 4, 5, 10, 20, 25, and 50. The sum of these divisors is 117. Dividing 117 by each of these divisors (except 1) yields nonzero remainders. Thus, we conclude that 100 is a Duffinian number.

*A paper presented at the 1983 National Convention of KME and awarded second place by the Awards Committee.

Now that we have defined a Duffinian number, let us turn our attention to determining a general form for such a number, if indeed one does exist. We begin our search by looking for patterns among the first 100 Duffinian numbers. They are as follows:

4	8	9	16	21	25	27	32	35	36
39	49	50	55	57	63	64	65	75	77
81	85	93	98	100	111	115	119	121	125
128	129	133	143	144	155	161	169	171	175
183	185	187	189	201	203	205	209	215	217
219	221	225	235	237	242	243	245	247	253
256	259	265	275	279	289	291	299	301	305
309	319	323	324	325	327	329	333	335	338
341	343	351	355	361	363	365	371	377	381
385	387	391	392	399	400	403	407	413	415

Notice that these include many squares 4, 9, 16, 25, 36, 49, 64, 81, 100, ..., and several cubes 8, 27, 64, 125, ... along with various larger powers. This is accounted for by what Duffy refers to as The First Theorem of Duffinian Numbers.

> <u>The First Theorem of Duffinian Numbers</u>. For any prime p and integer $n > 1$, p^n is Duffinian.

The proof is not complicated; we shall consider it here in an attempt to become more familiar with the definition. Let p be any prime. Then the proper divisors of p^n are $1, p, p^2, \ldots, p^{n-1}$. The sum of these is $S = 1 + p + p^2 + \ldots + p^{n-1} = \frac{p^n-1}{p-1}$. Assume there is at least one factor, other than 1, which divides n. This factor must be a power of p, and so p must divide S. This implies p divides p^n-1. However, since p also divides p^n, then p must divide 1. Thus p is not prime, and we have a contradiction. Therefore, no such factor exists and p^n is Duffinian (see [2], page 113).

Is this the only form which a Duffinian number may take? Looking once again at the first 100 Duffinian numbers, we find the number 21. Clearly, 21 is not a power of a prime. So we must conclude that there are other forms which a Duffinian number may have.

Let us now consider $n = p_1^{a_1} p_2^{a_2} p_3^{a_3} \ldots p_s^{a_s}$ (the unique prime factorization of n) (see [1]).

Let

$\sigma(n) =$

$$(1+p_1+p_1^2+\cdots+p_1^{a_1})(1+p_2+p_2^2+\cdots+p_2^{a_2})\ldots(1+p_s+p_s^2+\cdots+p_s^{a_s})=$$

$$\left(\frac{p_1^{a_1}-1}{p_1-1}\right)\left(\frac{p_2^{a_2}-1}{p_2-1}\right)\cdots\left(\frac{p_s^{a_s}-1}{p_s-1}\right)$$

Now, if S is the sum of all of the positive divisors of n except n itself, then $\sigma(n) = S + n$. To say a factor of n divides S when n is Duffinian, is the same as saying this factor does not divide $\sigma(n)$. This may be formalized as follows:

The Fundamental Theorem of the Duffinian Numbers. Say that p^k is *foreign* to q, for primes p and q and whole number k, if $p^{k+1}-1$ is not divisible by q. If $n = p_1^{a_1}p_2^{a_2}p_3^{a_3}\ldots p_s^{a_s}$ is the prime factorization of n, then n is Duffinian if and only if $p_i^{a_i}$ is foreign to p_j for all integer pairs (i,j), $1 \le i \le s$, $1 \le j \le s$. We shall outline the "only if" part of the proof first. Assume there is an integer pair (r,t) such that $p_r^{a_r}$ is not foreign to p_t; then $\dfrac{p_r^{a_{r+1}}-1}{p_r - 1}$ is divisible by p_t. Then $S+n = \sigma(n)$, as described above as a product of quotients, and n itself, are divisible by p_t. Hence, n is not Duffinian: a contradiction of the Duffinity of n (see [2] page 113). For the "if" part, begin by assuming that n is not Duffinian while the foreign property holds. Then there is a factor of n which divides S. Furthermore, this factor is divisible by some prime p_k, $1 \le k \le s$. Now p_k divides S+n implies p_k divides the product

$$\left(\frac{p_1^{a_1}-1}{p_1-1}\right)\left(\frac{p_2^{a_2}-1}{p_2-1}\right)\cdots\left(\frac{p_s^{a_s}-1}{p_s-1}\right)$$

Since a prime dividing a product of n integers must divide one of the integers, there exists h, $1 \le h \le s$, such that p_k divides $\frac{p_h^{a_{h+1}}-1}{p_h-1}$. Hence $p_h^{a_h}$ is not foreign to p_k--a contradiction. Thus, n is Duffinian.

At this stage, a reasonable question to consider could deal with the distribution and the quantity of Duffinian numbers. First we shall consider how frequently a Duffinian number appears as we scan the positive integers.

With regard to the even Duffinian numbers, and looking once again at the first 100 Duffinian numbers, it appears that as the integers become larger, even Duffinian numbers become less frequent. This may be partially accounted for by the First Theorem which shows that any integral power of 2 will indeed be Duffinian and the fact that the larger powers of 2 are less dense in the higher integers. The infrequency of even Duffinian numbers can be further supported; consider the following:

> <u>Theorem of Even Non-Duffinians</u>. Given an even number n with prime factorization $2^a p_1^{b_1} p_2^{b_2} \cdots p_r^{b_r}$, if b_i is odd for any i from 1 to r, then n is not Duffinian.
>
> The proof of this depends upon the Fundamental Theorem of Duffinians. Let n be an even number with a prime factorization as given. Suppose that for some i, $1 \le i \le r$, b_i is odd. Then, $(1+p_i+p_i^2+\cdots+p_i^{b_i})$, where p_i^k for any integer k is odd, is the sum of b_i+1 odd integers. However, b_i+1 is even and so the sum of b_i+1 odd integers is even. Then $\sigma(n)$ is even also since the above sum is a factor of it. Since n is even, $S=\sigma(n)-n$ is even and hence S, the sum of the proper divisors of n,

is divisible by 2. Therefore, since 2 divides both S and n, n is not Duffinian (see [2], page 114).

From the last theorem, follows a corollary, useful for determining if an even integer may be Duffinian.

<u>Corollary</u> (<u>Even</u> <u>Duffinians</u>). Any even Duffinian must be either a square or twice a square (see [2], page 14).

Even though the proof of this corollary is rather elementary, it is included here since this corollary will be needed later. Let n be an even Duffinian number with a unique prime factorization such as that given in the statement of the Theorem of Even Non-Duffinians. Then, $a > 0$ and b_1 through b_r are all even. If a is even, we have $n=\left(2^{a/2}p_1^{b_1/2}\cdots p_r^{b_r/2}\right)^2$ where all the exponents are integers. Thus, n is a square. If a is odd, then we have $n = 2\left(2^{a-1/2}p_1^{b_1/2}\cdots p_r^{b_r/2}\right)^2$ where the exponents are once again all integers. Thus, n is twice a square.

In his work, Duffy proposed the following question: Does there exist a sequence of four consecutive integers all of which are Duffinian? Duffy believed the tests for Duffinity provided by the theorems and the fact that such a sequence must include two even Duffinian numbers were essential components in this problem of existence although he did not answer the question. Later, the solution to the problem was given in 1981 by Peter Heichelheim (see [3]). He, with the aid of his PET COMMODORE home computer, produced the smallest Duffinian Quintuplet:

202,605,639,573,839,041
202,605,639,573,839,042
202,605,639,573,839,043
202,605,639,573,839,044
202,605,639,573,839,045

Having seen this result, we might ask if there exists a sequence of SIX or more consecutive integers all of which are Duffinian. Once again, the answer to this question was provided by Heichelheim.

Theorem. Five consecutive Duffinian numbers is the maximum possible. Assume there are at least six consecutive integers all of which are Duffinian. Among these would be two squares or two numbers which are twice squares. In other words, there would be two square positive integers which differ by 1,2, or 4. Let n^2 be the smaller of these squares and let $(n+k)^2$ be the larger. Then, $(n+k)^2 - n^2 \geq (n+1)^2 - n^2 = 2n + 1$ must be less than 4. This occurs only when n=1. Thus, $(1+k)^2 - 1 \leq 4$ if and only if k=1 since n and k must both be integers. Hence, the squares are 1 and 4. The sequence can then only be 1, 2, 3, 4, 5, 6 since twice 1 and twice 4 differ by more than 4. However,this, the only possible sequence, contains non-Duffinian numbers. Thus,the theorem is proved (see [3], pages 26-28).

It has been shown that Duffinian integers become less frequent among larger integers. We now turn to the question of how many Duffinian integers there are. The answer is formalized below.

Theorem. There exists an infinitude of Duffinian integers.

Assume there is a finite number of Duffinian integers. Now every number of the form p^2

where p is any prime is a Duffinian number by the First Theorem. Then the set $Q=\{p^2 | p \text{ is a prime}\}$ is a subset of the set of Duffinian numbers and thus must also have a finite number of elements, say k elements. The elements of Q, being a set of integers, has a largest element. Let this be called p_k^2. Associated with this will be the largest prime which was squared to form the elements of Q. Now, we know there exists a prime, p', such that p' is larger than p_k. But, p'^2 is an element of Q with $p'^2 > p_k^2$. This contradicts p_k^2 being the greatest element in Q. Therefore, the theorem holds.

Next, we compare this idea to one which dates back to the Pythagoreans: the *perfect* number. A positive integer n is a perfect number if it is equal to the sum of all its positive divisors, excluding itself. One characteristic of a perfect number is that $\sigma(n)=2n$ (see [1], page 218). We use this fact to arrive at our concluding result.

Theorem. A perfect number is never a Duffinian number and a Duffinian number is never a perfect number.

The proof of the former part begins by assuming that the perfect number n is also Duffinian. Let S be the sum of all the proper divisors of n, as usual. Then the Duffinity of n implies that S and n are relatively prime; that is, they have no common divisor other than 1. And, the perfect nature of n means that $S = \sigma(n) - n = 2n - n = n$. This however is a contradiction of the numbers S and n being relatively prime, and a perfect number is never a Duffinian number. The latter part of the theorem depends on the perfect number n dividing S which contradicts the Duffinity of n.

REFERENCES

[1] David M. Burton, Elementary Number Theory (Boston: Allyn and Bacon, Inc., 1976), pp. 48-49.

[2] L. Richard Duffy, "The Duffinian Numbers," Journal of Recreational Mathematics, 12 (1979-1980).

[3] Peter Heichelheim, "There Exist Five Duffinian Consecutive Integers But Not Six," Journal of Recreational Mathematics, 14 (1981-1982).

"ET CETERA" IN MATHEMATICS

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Before the principle of mathematical induction was formalized, many mathematicians employed it by proving a proposition for <u>one</u> and carrying the proof to <u>two</u>, then asserting *et cetera*. One may find an example of it in a paper of Omar Khayyam [2]:

رسالة فی شرح ما اشکل من مصادرات اقلیدس

We would like to discuss this idea and its history.

Khayyam discovered that a very famous and useful theorem in one of The Thirteen Books of Euclid had been proved only for rational magnitudes. We shall state the theorem first and then talk about Khayyam's proof for it.

<u>Theorem</u>: Let ABC be a triangle (Figure 1). Let H be a point on the line segment AB. Through H we draw a line parallel to BC. This line intersects AC at K. Then $\frac{AH}{HB} = \frac{AK}{KC}$.

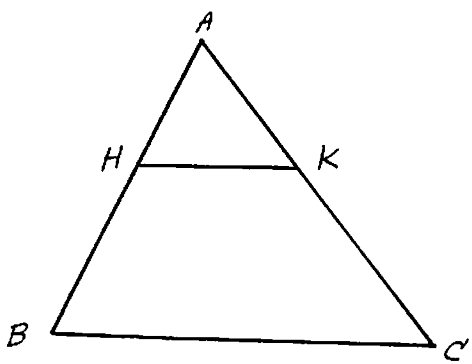


Fig. 1

Almost no one bothers to prove this theorem. In many books the theorem is accepted as an axiom. If one refers to The Thirteen Books of Euclid, one finds a proof for the case when $\frac{AH}{HB}$ is <u>rational</u>.

In order to prove this theorem, Omar gives the following:

Definition: Let $\{a_n\}$ and $\{b_n\}$ be two sequences of magnitudes. Suppose there exists a sequence $\{m_n\}$ of positive integers such that

$$a_{n+2} = a_n - m_n a_{n+1} \text{ if and only if } b_{n+2} = b_n - m_n b_{n+1}$$

for $n = 1,2,\ldots$. Then we define $\frac{a_2}{a_1} = \frac{b_2}{b_1}$.

Franklin [1] through continued fractions, proved that Omar's definition was equivalent to Eudoxus' axiom.

In fact, we shall not supply details for the proof of the theorem in order to make it more interesting for the reader. But we would like to explain what Khayyam has done in his proof. His idea is to produce a finite sequence of line segments parallel to BC the last of which is HK. Thus he starts as:

Suppose, without loss of generality, AH < HB. We choose

$$AH = HH_1 = \dots = H_{m_1-1}H_{m_1}$$

until we have $0 \leq H_{m_1}B < AH$ (Figure 2). Then clearly lines through $H_1,\dots,H_m$ parallel to BC intersects AC respectively at $K_1,\dots,K_{m_1}$. Obviously

$$AK = KK_1 = \dots = K_{m_1-1}K_{m_1}.$$

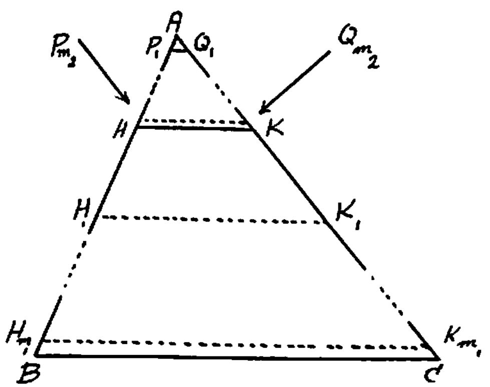


Fig 2

Let AH = a_2 and HB = a_1. Similarly let AK = b_2

and $KC = b_1$. We shall let $H_{m_1}B = a_3$ and $K_{m_1}C = b_3$. Then we observe that $a_3 = a_1 - m_1 a_2$ if and only if $b_3 = b_1 - m_1 b_2$. Two cases may occur.

(i) If $a_3 = 0$, then $b_3 = 0$. In this case

$$\frac{a_2}{a_1} = \frac{b_2}{b_1} \text{ or } \frac{AH}{HB} = \frac{AK}{KC} .$$

(ii) If $a_3 \neq 0$, then we choose $P_1, \ldots, P_{m_2}$ on AH such that

$$AP_1 = P_1P_2 = \ldots = P_{m_2-1}P_{m_2} = a_3,\ 0 \leq P_{m_2}H < a_3.$$

Lines through $P_1, \ldots, P_{m_2}$ parallel to BC intersect AK respectively at $Q_1, \ldots, Q_{m_2}$ (Figure 2). Clearly

$$AQ_1 = Q_1Q_2 = \ldots = Q_{m_2-1}Q_{m_2}$$

Let $P_{m_2}H = a_4$ and $Q_{m_2}K = b_4$. Then

$$a_4 = a_2 - m_2a_3 \text{ if and only if } b_4 = b_2 - m_2b_3.$$

This is where Omar says: "Et cetera".

The reader may finish the proof by using the principle of mathematical induction (induct on the value of the denominator of $\frac{AH}{HB}$ which is a rational number). Khayyam had in mind that finally, during the process described above, a line parallel to BC would have to be HK.

Indeed, a converse is also suggested. We shall leave it to the reader.

Next we would like to study another example of et cetera. This is an interesting theorem which was studied in hope of an application of it to the famous problem of *cube-duplication*. Later it became an exercise for mathematical induction.

Theorem: Let ABC be a right triangle, where A is a right angle (Figure 3). Let AH be the altitude corresponding to BC. We draw HB_1 and HC_1, respectively, perpendicular to AB and AC. Then

$$\frac{BB_1}{CC_1} = \left(\frac{AB}{AC}\right)^3 .$$

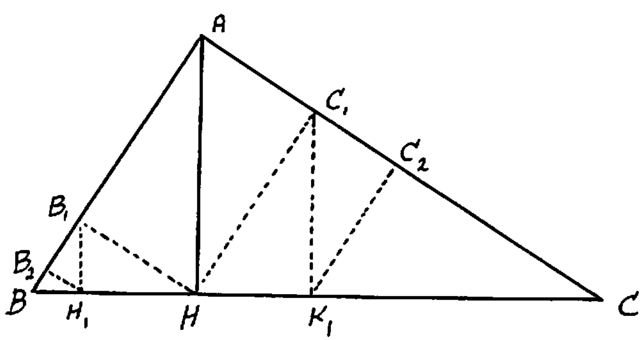


Fig. 3

The proof is simple and left to the reader. It isn't known to whom the theorem is due. The exponent three has attracted attention, but it could not solve the cube-duplication problem.

Now let H_1 and K_1 be the feet of perpendiculars from B_1 and C_1 to BC respectively. Then we draw H_1B_2 and K_1C_2 respectively perpendicular to AB and AC. It can be proved that

$$\frac{BB_2}{CC_2} = \left(\frac{AB}{AC}\right)^5 .$$

We shall again leave the proof to the reader. But we may appeal to *et cetera* as follows. If we repeat these constructions n times, then we obtain B_n and C_n such that

$$\frac{BB_n}{CC_n} = \left(\frac{AB}{AC}\right)^{2n+1}$$

Indeed, one can prove the proposition by use of the principle of mathematical induction.

Now we study another *et cetera* problem in connection with the *squaring of a circle*. It is well-known that one can construct a square whose area is equal to the area of a given rectangle. This is an application of the fact that in a right triangle the altitude corresponding to the hypotenuse is the geometric mean of the two parts it separates in the hypotenuse. Thus the squaring of a flat area means, first changing it to a rectangle, then to a square. Of course this problem is solved for any polygon of n sides. Let $A_1A_2 \ldots A_n$ be a polygon of n sides (Figure 4). Then, for example, we

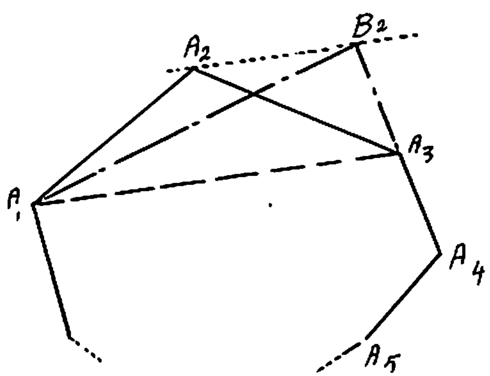


Fig. 4

consider the line segment A_1A_3. We draw the line parallel to A_1A_3 through A_2. This line intersects A_3A_4 at B_2. We observe that the triangles $A_1A_2A_3$ and $A_1B_2A_3$ have the same area. This way we change the polygon to one of n-1 sides. Again in most old geometry books one finds that the proof is considered complete by asserting *et cetera* to make the case for the general form.

We shall leave the proof of above using the principle of mathematical induction to the reader. We also hope the reader will search for other interesting *et cetera* problems.

REFERENCES

[1] S. P. Franklin, Eudoxus, Omar, and continued fractions, Scripta Mathematica, 25 (1960) pp. 353-55.

[2] Omar Khayyam, Discussion of difficulties in Euclid, Scripta Mathematica, 24 (1959) pp. 275-303.

[3] Gholam-Hossein Mossohob, Khayyam's Algebra and History of Mathematics (in Persian), Chopkhanek Markazi, Teheran, Iran (1938).

THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August, 1984. The solutions will be published in the Fall 1984 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

CORRECTION

Problem 338. The solution to this problem appears in the Fall 1982 issue [Vol. XLII (42), No. 1, p. 38]. Part (c) of the printed solution contains some extraneous language and should be corrected to read as follows:

(c) On a line, locate B and C so that BC = b. With CB extended as a side and using B as a vertex, construct an angle = α. Bisect this angle with bisector BF. Lay off BA = a. Draw AC. On the side remote from B erect perpendiculars AG and CH to AC. With A and C as vertices and AG and CH, respectively, as sides, draw AM and CN between the parallels AG and CH so that GAM = $\frac{\alpha}{2}$ = HCN. (see the figure) Denote the intersection of AM and CN as D. Then ABCD is the desired quadrilateral with maximum area.

The proof is obvious.

PROPOSED PROBLEMS

Problem 352: Proposed by Charles W. Trigg, San Diego, California.

Gargantuan Gastronomy

A	R O M E	restaurant
	O N C E	made history by
serving	M C M L	
	E E L S	at a single meal.

In the word square above, each word represents a square decimal integer. Each different letter uniquely represents a digit. Convert the word square into a square of squares.

Problem 353: Proposed by Charles W. Trigg, San Diego, California.

Find a product of three consecutive integers which has the form abcabc.

Problem 354: Proposed by Fred A. Miller, Elkins, West Virginia.

In a circle with O as its center, draw the fixed diameter AOB and the chord BC. Extend the chord BC to a point D such that BC = CD. Find the locus of the point of intersection of OD and AC.

Problem 355: Proposed by the editor.

Find all three digit numbers N such that N is the arithmetic mean of all numbers formed by permuting the digits of N; exclude the trivial case where all three digits are the same.

Problem 356: Proposed by R. S. Luthar, University of Wisconsin Center, Janesville, Wisconsin.

If a,b and c are the sides of a triangle, prove that

$$\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c} + \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} > 6.$$

SOLUTIONS

342. Proposed by Michael W. Ecker, Pennsylvania State University, Worthington-Scranton Campus, Scranton, Pennsylvania.

It is well known that for nonnegative numbers, the geometric mean of the two numbers never exceeds their arithmetic mean (with equality of the two means precisely for equality of the numbers). Characterize all pairs (x,y) of nonnegative integers for which the arithmetic mean exceeds the geometric mean by exactly 1.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Let x and y be nonnegative integers such that $\frac{x + y}{2} - xy = 1$. Then $xy = \frac{x + y - 2}{2}$ so that $4xy = x^2 + y^2 + 4 + 2xy - 4x - 4y$. Hence $(x-y-2)^2 = 8y$ which implies that $y = 2z^2$ for some integer z. Thus $x - 2z^2 = 4z$ so that $x = 2(z + 1)^2$.

If $x = 2(z + 1)^2$ and $y = 2z^2$ for some integer $z > 0$, then x and y are nonnegative integers and $\frac{x + y}{2} - xy = (2z^2 + 2z + 1) - (2z^2 + 2z) = 1$.

Thus all pairs of nonnegative integers (x,y) for which the arithmetic mean exceeds the geometric mean by exactly 1 is $[2(z + 1)^2, 2z^2]$ where z is an integer.

Also solved by: Joseph Bonin, Assumption College, Worcester, Massachusetts; Clayton W. Dodge, University of Maine at Orono, Orono, Maine; Fred A. Miller, Elkins, West Virginia; Charles W. Trigg, San Diego, California; and the proposer.

343. *Proposed by Charles W. Trigg, San Diego, California.*

In the square array of the nine non-zero digits

$$\begin{matrix} 5 & 3 & 8 \\ 7 & 1 & 4 \\ 6 & 2 & 9 \end{matrix}$$

the sum of the digits in each corner 2-by-2 array is 16. Rearrange the nine digits so that each 2-by-2 corner array has a sum that is <u>nine</u> times the central digit.

Solution by Clayton W. Dodge, University of Maine at Orono, Orono, Maine.

Let the elements of the square be

$$\begin{matrix} a & b & c \\ d & e & f \\ g & h & i. \end{matrix}$$

Then we have

$$a+b+d+e = b+c+e+f = d+e+g+h = e+f+h+i = 9e \qquad (1)$$

Hence

$$a+b+d+e+f+h+i = 17e \qquad (2)$$

In (2) the sum on the left lies between $1+2+...+7=28$ and $3+4+...+9 = 42$. Hence $17e = 34$ and $e = 2$.

If $a = 9$, then $b+d = 7$. If either b or d is 6 or if b and d are 3 and 4, then the unused digits cannot satisfy the necessary $c+f+g+h = 25$. Hence 9 cannot occupy a corner square.

Thus take $b = 9$. Then $a+d = c+f = 7$. If $a = 1$, then $g+h = 10$ which is impossible. Thus $a = 6$ and $d = 1$. Using 3 and 4 for c and f respectively, we obtain the two solutions

6	9	3		6	9	4
1	2	4	and	1	2	3
8	7	5		7	8	5.

All other solutions are rotations or reflections of these two arrays.

Also solved by: Joseph Bonin, Assumption College, Worcester, Massachusetts; Fred A. Miller, Elkins, West Virginia; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin and the proposer.

344. Proposed by Charles W. Trigg, San Diego, California.

In what number bases is the repdigit, 33, a triangular number?

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Let b be a number base in which 33 is a triangular number. Then b is an integer ≥ 2 and $3b+3 = n(n+1)/2$. Since 3 divides the left side of the preceding equality, then either 3 divides n or 3 divides $n+1$.

(1) If $n = 3k$ for some positive integer k, then $b+1 = k(3k+1)/2$ so that $b = (k+1)(3k-2)/2$ for $k = 2,3,\ldots$.

Some examples (in decimal notation) are:

k	2	3	4	5
b	6	14	25	39
triangular number	21	45	78	120.

(2) If $n+1 = 3j+1$ for some positive integer j, then $b+1 = j(3j-1)/2$ so that $b = (j-1)(3j+2)/2$ for $j = 2,3,\ldots$.

Some examples (in decimal notation) are:

j	2	3	4	5
b	4	11	21	34
triangular number	15	36	66	105.

Also solved by: Clayton W. Dodge, University of Maine at Orono, Orono, Maine; Fred A. Miller, Elkins, West Virginia and the proposer.

345. Proposed by Willie S.M. Yong, Republic of Singapore.

Show that the sequence of numbers defined by

$$[k + \sqrt{k} + .5],\ k = 1,2,3,\ldots$$

i.e. $\quad 2,3,5,6,7,8,\ldots$

includes all prime numbers. Here $[x]$ denotes the greatest integer function and 1 is not considered to be a prime number.

Solution by Richard A. Gibbs, Fort Lewis College, Durango, Colorado.

We will prove the stronger result that the sequence omits only the perfect squares from the sequence of positive integers.

Let $$f(x) = x + \sqrt{x} + \frac{1}{2}.$$

Now for m = 2,3,... we have the inequalities

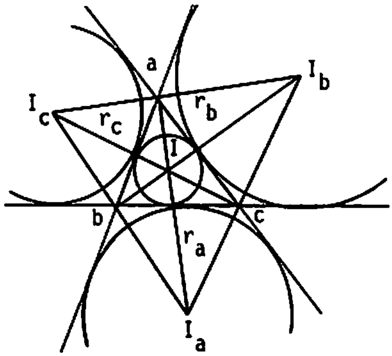
$$(m - \frac{3}{2})^2 \leq m^2 - m < (m - \frac{1}{2})^2$$

which when squared, yield $m^2 - 1 \leq f(m^2 - m) < m^2$. Thus $[f(m^2 - m)] = m^2 - 1$ and $[f(m^2 - m + 1)] = m^2 + 1$ is established similarly. Thus as x assumes the 2m values from $m^2 - m + 1$ to $(m + 1)^2 - (m + 1)$, $[f(x)]$ assumes the 2m distinct values from $m^2 + 1$ to $(m + 1)^2 - 1$. Hence the only values omitted by $[f(x)]$ are the perfect squares.

Also solved by: Joseph Bonin, Assumption College, Worcester, Massachusetts; Michael W. Ecker, Pennsylvania State University, Worthington-Scranton Campus, Scranton, Pennsylvania and the proposer.

346. Proposed by Fred A. Miller, Elkins, West Virginia.

Show that the area of the triangle formed by joining the centers of the three excircles is abc/2r where r is the radius of the inscribed circle.



Solution by Charles W. Trigg, San Diego, California.

Let the area of the triangle, with sides a,b, and c, be A, its semiperimeter be $s = (a+b+c)/2$, and the radii of its excircles be r_a, r_b, and r_c. Then $A = s(s-a)(s-b)(s-c)$. Also by [1] $A = rs = r_a(s-a) = r_b(s-b) = r_c(s-c)$.

The bisectors of the exterior angles of a triangle pass through the vertices of the triangle, and intersect at the excenters. The larger triangle thus formed consists of the original triangle and the three triangles with bases a,b,c and altitudes r_a, r_b and r_c, respectively. Thus the area of the larger triangle T gives

$$T = rs+(a_{ra}+br_b+cr_c)/2$$

$$= rs +\left(\frac{ars}{s-a}+\frac{brs}{s-b}+\frac{crs}{s-c}\right)/2 = \frac{rs}{2}\left(\frac{a}{s-a}+\frac{b}{s-b}+\frac{c}{s-c}+2\right)$$

$$= \frac{rs}{2}\left(\frac{a}{s-a}+1+\frac{b}{s-b}+1+\frac{c}{s-c}\right) = \frac{rs}{2}\left(\frac{s}{s-a}+\frac{s}{s-b}+\frac{c}{s-c}\right)$$

$$= \frac{rs}{2}\left(\frac{s(s-c)(2s-a-b)+c(s-a)(s-b)}{(s-a)(s-b)(s-c)}\right) - 2$$

Now since $2s-a-b = c$, $s(s-c) = (s-a)(s-b) = ab$, we have

$$T = \frac{rs}{2} \cdot \frac{abc}{r^2 s} = \frac{abc}{2r}\ .$$

[1] N. Altschiller-Court, College Geometry, Johnson Publishing Co., 1925, p. 71.

Also solved by: Clayton W. Dodge, University of Maine at Orono, Orono, Maine; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin and the proposer (two solutions).

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical *issues* of interest to undergraduate students are discussed in length. Here by *issue* we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to *Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.*

We write programs to do chores and computations for which we do not have the time or the power. Computers can be quick, neat and accurate only if we would write a correct program for what we wish to be done.

To check if a program does the job it is supposed to do is a time consuming chore. Would it not be great if someone wrote a program to do that? That is, would it not be useful if there was a program which could examine any given program to see if it did what it is supposed to do?

The following article by Professor Welch is related to this question.

IK

"ALAN IN COMPUTERLAND"

Lawrence Welch*

Alan considered himself a lucky guy. It was May 31, college was out for the summer, and tomorrow was

*Professor Welch received his Ph.D. in mathematics from University of Illinois at Urbana and is currently a member of the mathematics department of Western Illinois University. His interests include music composition and mathematical logic.

his twentieth birthday. Furthermore, he had just landed a summer job as a computer programmer with a small company near home. This meshed nicely with his education; he had declared himself a mathematics major this last semester,and knew that he wanted to spend his life doing math or computer science. But the best thing of all was the birthday present that he expected to receive. For his parents had decided it was time he had a computer of his own, and clearly nothing micro or mini would do for <u>their</u> son. A bright guy like Alan deserved the very best.

Sure enough, the next day a computer appeared in the family room -- taking up most of the floor space of the room, in fact. Where his parents had stored it the night before was a mystery to Alan. He read the brand-name label on it. "Universal II Maxicomputer," it said. Alan had never heard of a Universal II Maxicomputer before. He picked up one of the many instruction manuals that came with it, and thumbed through it to find a description of the machine's capabilities. The specifications were truly amazing. It boasted a Nonstandard Memory System which utilized Monadic Magnetobubbles (trademark pending) to give the user "literally galaxies of program and data memory," as the advertisement said. Furthermore, it used "the newly developed Two-Ring processor--the fastest on the market." ("I thought processors processed bits, not rings," thought Alan. "I wonder what those two rings are." But for that matter,he didn't know what a Monadic Magnetobubble was, either.)

Despite its extraordinary power, the Universal II was basically an easy machine to use, and Alan mastered it in a week. To test his knowledge, he decided to write a really big, complicated program. This would be good preparation for his job, he thought. As the job began on June 21, and as this was June 7, he had two weeks to spend on his project. He already knew what he wanted that project to be. He wanted to build an error detecting routine which could later be incorporated into a debugging program. But his routine wasn't to be just any error detector -- no, indeed. It was meant to determine in advance whether a given program would halt or not.

Now, to really understand what Alan wanted to do, you have to know how a Universal II works. We need not spend time right now discussing the peripherals, or the adjuncts to memory such as the Galactic Compiling Buffer, or the mechanism of the Two-Ring processor. Only three parts of memory concern us: program memory, data memory, and the output buffer. Because these parts of memory use Monadic Magnetobubbles, they contain literally infinitely more memory than Alan--or you or I, for that matter--would ever need for any one program. When a program is fed into the computer it is translated by the compiler into binary machine code, and is placed into the program memory as a string of zeros and ones, which we can think of as representing a positive integer in base two. Data on which the program is to operate is stored in data memory, also in binary code; again, we can think of the string of zeros and ones that represents the data as standing for a single positive integer in base two, if we wish. For convenience, let us write P(n; x) to mean program #n with data input #x (that is, the program whose binary string stands for the number n,with data input whose binary string stands for the number x). Once a program and its data are put into the computer -- say program and data P(n; x)--and the machine is given the RUN command, it computes (very fast) and places its output in the output buffer,which, like all other areas of the memory, is plenty large to accommodate any output for any program anyone would ever want to run. <u>Only after all computation is done</u> does the computer print the output. Thus if a program contains a bug that causes it to run forever, no output will ever be printed. (There is a peripheral device for printing the output while the computer is still running, but we will discuss that later.)

Alan's idea was simply this: If $P(n; x)$ runs for what seems an unduly long time, it <u>could</u> be because the computation is more complex and involved than expected. or it <u>could</u> be because the program contains a bug and will never stop running. If the program never stops running, it will never print an output, as we have seen. Suppose f is a function such that $f(n, x) = 1$ if $P(n; x)$ halts, and $f(n, x) = 0$ if $P(n; x)$ runs forever. If we can write a program for f, then we have a powerful error detector.

It was this that Alan wanted to do--to write a program for f. And it was here that his troubles began. For all his mathematical abilities, Alan could not figure out how to do it. So he restricted his work to what he thought would be an easier part of the project. He decided to write a program for the function $g(n) = f(n, n)$ which determines whether or not $P(n; n)$ (program #n with data input #n) halts. Finally, after a few days of struggling, he had a program which worked for <u>some</u> inputs: If n was input to it the program (which we shall call program G, to give it a name) would try to calculate $g(n)$ by seeing if $P(n; n)$ halted. If it did, then program G declared that g(n) equaled 1. But if P(n; n) did not halt ... . Well, there was the snag. For if P(n; n) did not halt, neither did G; instead, it waited forever for the halting of P(n, n) which never came.

Hence Alan set about a related project. He decided to write a new program to do precisely the opposite of what G did--that is, to print the output 1 if P(n;n) failed to halt and to run forever if P(n; n) halted. For he said to himself, "This program could surely be written easily if only I had a program for the function g, because I would need only to require output 1 if g(n) = 0, and to send the computer into an infinite loop if g(n) = 1. So writing this program may give me a feeling for how to carry out the project I originally had in mind." Let us call Alan's new project F (for Failure-to-halt Project).

By June 12, Alan had yet to succeed in writing a workable program for project F. Furthermore, he began to get the distinct feeling that something was fishy. Suppose he succeeded sometime in writing his program. Its binary code would represent a number, say m. Now using project F to check the halting of P(n; n) would be equivalent to putting n into this program -- that is, it would be equivalent to running P(m; n). But what if the data input were m itself? Did P(m; m) halt or not? Suppose P(m; m) did halt. Then g(m) = 1, so the program for F, with input m, would go into an infinite loop and run forever. Now of course, the program for F with input m was just P(m; m) itself. So if P(m; m) halted, it would run forever. This type of contradiction was not what Alan was hoping for. Very well, then, P(m; m) would not halt. But then g(m) = 0, so the program for F with input m (which is P(m; m), remember) would halt and output 1 -- another contradiction. It looked like a program for F could not exist. But this meant that a program for computing the function g could not be written, either, since a program for g would make it possible (easy, even) to write one for F.

Alan was not to be dissuaded by the impossible, however. He decided to try a new tack. One of his peripherals, which we mentioned above, in passing, was the Serial Printer, which printed whatever was placed in the output buffer at the time it arrived there. You may remember that the computer itself printed the contents of the output buffer only after it had finished computing. The Serial Printer was useful for checking outputs as the computer ran. It was an independent machine, and was not under the Universal's control.

Alan decided the thing to do was to write a program which never halted, but which listed into the output buffer the numbers n for which P(n; n) failed to halt. Let us call his new project idea L, for Lister.

A few days later, on June 15, Alan sadly realized that project L, too, was hopeless. Why? Well, suppose a program for L could be written. Keep in mind, while we are discussing this, that Alan had already determined that no program could be written to compute the function g. Now take this hypothetical program for L, and combine it with the program G (which outputs 1 if g(n) = 1, and does not halt if g(n) = 0) to get a new program which works as follows: Given any input n, run G and L by time sharing, doing first a little of G, then a little of L, then a little more of G, etc. Continue this until one of two things happens -- <u>either</u> G gives output 1, signaling that P(n; n) halts, <u>or</u> n turns up among the output list of L, signaling that P(n; n) does not halt. In the first case, print 1; in the second case, print 0. Then stop running.

The program just described seems simple enough, provided that a program for L can be written. But there is a problem, because what we have just described is a program to compute the function g, which, as we have seen, does not exist. So the listing program L must be nonexistent, too.

Alan was somewhat dismayed by these failures, but suddenly, in the wee hours of the morning of June 16, he had a brainstorm. (If you are a mathematician or scientist, you, too, have probably had brainstorms in the wee hours of the morning. So you know how unreliable the great discoveries made at such times seem when you enthusiastically describe them to your friends in the light of day.) Alan's idea was this: The "halting problem," as he had begun to call his original project, seemed rather stubborn, but maybe the computer could be outfoxed.

"So," thought Alan, "forget about trying to write the nonexistent listing program L. Instead, make a different lister -- one which lists those numbers n for which P(n; n) does halt. (This can be done directly, or by using program G.) Employ the Serial Printer to print the list, of course, and just inspect the list to see which numbers are not on it. Those will be the numbers of the nonhalting computations."

There were several snags to be overcome in making the new program, of course, but fortunately, after so many dreadful failures, Alan was able to write it and get it to run properly by June 20. The main difficulty was that time-sharing had to be built in, and not just ordinary time-sharing,but time-sharing among infinitely many computations. For the most efficient way Alan could think of to construct his new listing program was to try running P(n; n) for every number n, and list those for which a halting occurred. But for infinitely many inputs n, P(n; n) would run forever, glopping up the works. (Using program G would have led to the same difficulty, because if P(n; n) did not halt, neither did G with input n.)

Alan overcame the difficulty in this way: A computer, when executing a program does it one step at a time, reading the first instruction, changing its memory's contents accordingly, then reading the next instruction, etc. Furthermore, the computer's operations are controlled by a clock, and the computer does at most one thing in one tick of the clock. Let us use the word <u>step</u> technically, to mean what is done in one tick of the clock (which may actually be nothing at all, if the computer is finished computing, or if it is just "resting" temporarily). Alan wrote his new program to compute <u>zero</u> steps of P(0; 0); then to start all over again and compute <u>one</u> step of P(0; 0) and one step of

P(1; 1); then to start all over again and compute <u>two</u> steps of P(0; 0), two of P(1; 1), and two of P(2; 2); then to start all over again and compute <u>three</u> steps each of P(0; 0), P(1, 1), P(2; 2), and P(3; 3); then to start all over again, etc. If program P(n; n) ever halted, then it would clearly do so after computing had continued on the program for some definite number of steps, say m. Suppose q were a number bigger than m or n. The process just described would sooner or later run P(0; 0), P(1, 1), ... , P(q; q) for q steps; when that occurred, P(n; n) would be among the computations tried at that stage, and clearly, since q was bigger than m, P(n; n) would halt at that time. So Alan simply had his new program print the output n when it became clear that P(n; n) halted. He then had the computer remember what numbers had already been printed, so that if n were printed once, the machine could avoid having to print it again every time P(n; n) was run from scratch for a number of steps large enough to cause it to halt.

Alan was very proud of his new program, for it had been terribly hard to write. He couldn't wait to run it to see what numbers were <u>not</u> listed, so that he would know which computations P(n; n) failed to halt.

But as soon as he ran it, he noticed still one more problem. The numbers were listed in jumbled order, rather than from smallest to largest. The printout started: 2, 1436, 15, 283, 576, ... Alan noticed that 100 did not show up as long as he ran the program. But of course, the program would run and give output forever, if allowed to. Was 100 absent because P(100; 100) did not halt, or was it just because he had not waited long enough to see it appear in the list? Clearly the list would have to be straightened out to give its output in increasing order, so that Alan would know, if 100 were skipped over, that it would never be enumerated.

For most of the day Alan puzzled over this difficulty. Then he came abruptly to the realization that this, too, would be impossible. For let us call the set of numbers listed by this program K, so that K is the set of all numbers n such that P(n; n) halts. If the Universal II could list K in increasing order (with help from the Serial Printer, of course) then it could easily remember what numbers it had already printed at each stage, just as the present program for listing K in jumbled order was having it do. Therefore it would be a simple task to ask the machine not actually to print these numbers, but just to remember them, and to print instead the numbers in between the ones it was remembering. The output would thus be a listing of the complement of K. But notice that this new program would then be the program L which Alan knew could not exist, as he had realized on June 15.

Near midnight Alan decided to give up his project for good. The next day, June 21, his summer job would begin, and he probably would not want to run a computer for eight hours a day and then come home to do the same thing in the evening. So he unplugged his computer and went to bed. But for a long time he lay awake mulling over what he had learned: The set K of numbers n such that P(n; n) halts can be listed by a computer program, without any number in the list being repeated. But its complement <u>cannot</u> be listed by a computer. Nor can a program be written to determine of each number n whether or not n is in K--that is, whether or not P(n; n) halts --for the characteristic function g of K cannot be programmed. And although K can be listed by a computer without repetitions, no computer program can cause it to be listed in increasing order. <u>No</u> computer program can <u>ever</u> do <u>any</u> of these things, on <u>any</u> computer. Not even on such an ideal wonder as the Universal II Maxi-computer.

In fact, Alan's "halting problem" <u>cannot be solved</u>, and this story has been the proof of its unsolvability, and of the unsolvability of some closely related problems.

* * * * * *

In real life, the Alan who proved the unsolvability of the halting problem was the mathematical logician, Alan M. Turing, and the infinitely large computer that he imagined using is called a Turing Machine. A program which computes P(n; x) when n and x are input into it is called a Universal Turing Machine Program. Often the halting problem is known under its German name, Entscheidungsproblem.

REFERENCES

1. Bell, John and Machover, Moshe, A Course in Mathematical Logic, North Holland Publishing Company, Amsterdam, 1977. (Parts of chapter 6 are relevant.)
2. Enderton, Herbert B., A Mathematical Introduction to Logic, Academic Press, New York, 1972, chapter 3.8.
3. Rogers, Hartley, Jr., Theory of Recursive Functions and Effective Computability, McGraw-Hill Book Company, New York, 1967, chapter 1.

4. Turing, Alan M., "On Computable Numbers, with an Application to the Entscheidungsproblem," in The Undecidable: Basic Papers on Undecidable Propositions, Unsolvable Problems and Computable Functions, ed. Martin Davis, Raven Press, Hewlett, N.Y., 1965, pp. 115-154.

KAPPA MU EPSILON NEWS

EDITED BY HAROLD L. THOMAS, HISTORIAN

News of chapter activities and other noteworthy KME events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department, Pittsburg State University, Pittsburg, Kansas 66762.

CHAPTER NEWS

Alabama Zeta, Birmingham-Southern College, Birmingham
Chapter President - Alison Pool
35 actives

The first presentation of the Kappa Mu Epsilon Senior Award during Honors Day at Birmingham-Southern College will be the culmination of much of the work done at the Alabama Zeta Chapter during the Fall Semester. Faculty members who are also members of KME will vote on the recipient of the award. Student members of KME have a part in the decision process through submitting nominations, recommendations and resumes. Among the more relaxing activities of the fall semester was the Christmas party sponsored jointly by all the Science Clubs on campus. The "scientific" decorations kept everyone amused and the holiday atmosphere helped bring the successful semester to a festive close. Other 1982-83 officers: Thomas Herring, vice president; Debi Sievers/ Donna Brown, secretary; Robert Sipes, treasurer; Lola F. Kiser, corresponding secretary; Sarah Mullins, faculty sponsor.

Arkansas Alpha, Arkansas State University, State College
Chapter President - Nhan Truong
19 actives, 5 initiates

At the beginning of the semester, Arkansas Alpha held a faculty-student drop-in. Through the support of the members and the faculty, the drop-in turned out to be a huge success. During the fall semester, programs and topics presented were as follows: Dr. Kachoon Yang on "A Fair Coin and a Fair Tosser", Dr.Lawrence A. Mink on "His Summer Visit to Vanderbilt"; Dr. Richard L. Tangeman on "The Game of Life". A guest speaker from the Air Force also talked about careers in the Air Force. To close out the semester, the chapter sponsored a Christmas party for the Computer Science, Mathematics and Physics Departments, Kappa Mu Epsilon members, and their families. Other 1982-83 officers: W. Dora Rios, vice president; Trena L.Richardson,secretary/treasurer, Kirby Roe, reporter; J. L. Linnsteadter, corresponding secretary; R. P. Smith, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President - Dan Weeks
40 actives, 10 initiates

Weekly meetings were held with speakers from business and industry. A Christmas social and pledge ceremony were held at the end of the fall quarter. At the beginning of the winter quarter the seventh annual Math Sciences Career Day was sponsored by the chapter. It featured eight California Gamma KME graduates now in business and industry and was attended by well over 100 students. Other 1982-83 officers: Mark Lucovsky & Dennis Ikenoyama, vice president; Nancy Lott,secretary; Fremont Bainbridge, treasurer; George R. Mach, corresponding secretary; Adelaide Harmon-Elliott, faculty sponsor.

Connecticut Beta, Eastern Connecticut State College, Willimantic
Chapter President - Raymond Hill, Jr.
37 actives

Other 1982-83 officers: Karen Johnson, vice President; Kathy Evans, secretary/treasurer; Ann Curran, corresponding secretary; Steve Keaton, faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Bob Ingle
17 actives

On October 19,1982, the Georgia Alpha Chapter of KME met to plan the 1982-83 activities. It was decided that meetings would be quarterly. The possibility of joint meetings with other allied honor societies was also discussed. As usual, initiation of new members will occur in the spring quarter of 1983. Other 1982-83 officers: Darla House, vice president; Cindy Holladay, secretary; Steve Townsend, treasurer; Joe Sharp, corresponding secretary/faculty sponsor.

Illinois Zeta, Rosary College, River Forest
Chapter President - Brad Erickson
15 actives

Plans for the national convention occupied members of the Illinois Zeta chapter during some of the fall meetings. Activities culminated with a Christmas party. Other 1982-83 officers: Joan Novak, vice president; Jean Rexroat, secretary; Laura Voss, treasurer; Sister Nona Mary Allard, corresponding secretary/faculty sponsor.

Illinois Eta, Western Illinois University, Macomb
Chapter President - Ann Bogue
5 actives

Fall semester programs included a talk by Dr. Iraj Kalantari, "Learning from Rubik's Cube" and two films shown entitled "Donald in Mathmagicland" and "Space Filling Curves". The chapter held a bake sale in October and donated $30 to Project SACK for math education materials for the children's library. Other 1982-83 officers: Dianne Wille, vice president; Judy Smithhisler, secretary/treasurer; A. Bishop, corresponding secretary; M. Ingrassia, faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
Chapter President - Jim Brumbaugh
15 actives

Other 1982-83 officers: Deb Hansen, vice president; Tammy Ulery, secretary; Kent Workman, treasurer; Ralph B. McBride, corresponding secretary; Tom S. Hudson, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President - Margaret Chizek
28 actives, 5 initiates

Two students presented papers at local KME meetings--Lori Maruth on "The Probability of Backgammon" and Don McCluskey on "The Computer as Envelope Generator in Musical Synthesizers." Professor Emeritus and Mrs. E.W. Hamilton hosted the KME Homecoming Breakfast on October 9, 1982. Professor John Longnecker presented a talk on the mathematics of a gift exchange at the December initiation banquet. Professor and Mrs. Dotseth invited KME and faculty to their home for the annual Christmas party. Many members ordered KME polo shirts at semester's end. Other 1982-83 officers: Chuck Daws, vice president; Kirk Montgomery, secretary; J'ne Day, treasurer; John S.Cross, corresponding secretary/faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President - Grant Izmerlian
8 actives

Other 1982-83 officers: Connie Johnson, vice president; Tom Potempa,secretary; Sheryl Shapiro, treasurer; Wayne Woodworth, corresponding secretary; Alex Kleiner, faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President - Darren Smith
40 actives, 6 initiates

The Chapter held monthly meetings in October, November and December. In addition, a fall picnic was hosted for all mathematics and physics students. Fall initiation for new members was held at the October meeting. Six new members were received at that time. The October program was given by Darren Smith on "Two Predator-One Prey Population Models." Patrick Lopez spoke at the November meeting about "A Property of a^x." In December, a special Christmas meeting was held at the home of Dr. Helen Kriegsman, Mathematics Department Chairman. Rebeca Graham gave a talk about "Plastics." Other 1982-83 officers: Patrick Lopez, vice president; Debbie Birney, secretary; Jeanine Carver, treasurer; Harold L. Thomas, corresponding secretary; Helen Kriegsman and Gary McGrath, faculty sponsors.

Kansas Beta, Emporia State University, Emporia
Chapter President - Julie Romine
30 actives, 6 initiates

The Chapter held monthly meetings throughout the semester at which math topics were discussed. A formal initiation of the new members included a banquet and a speaker whose topic was on the job of a systems analyst. Other 1982-83 officers: John Vogt, vice president; Phyllis Tidd, secretary; Patricia Herrick, treasurer; John Gerriets, corresponding secretary; Tom Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Kay Kreul
13 actives, 12 initiates

Two fund raisers were sponsored by the chapter. The first was a Book Sale -- books provided by the math faculty -- and the second was the annual computer dance. At one of the fall meetings, several students shared information about their summer employment in math/computer related jobs. Mid-semester the group heard a talk on "The Mathematical Properties of the Snowflake Curve" presented by Dr. Gary McDonald of Northwest Missouri State University, Maryville, MO. Social activities included a fall picnic, a volleyball game with biology students, and the Christmas wassail party. Other 1982-83 officers: John Agnew, vice president; Ann Devoy, secretary; Jane Feltmann, treasurer; Sister Jo Ann Fellin, corresponding secretary/faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President - Cindy Dietrich
21 actives

Other 1982-83 officers: Kevin Heideman, vice president; Kathy King,secretary; Brad Lichtenhan, treasurer; Robert Thompson, corresponding secretary; Billy Milner, faculty sponsor.

Kansas Epsilon, Fort Hays Kansas State University, Fort Hays
Chapter President - Betty Burk
23 actives

The students of KME, in conjunction with some other math students, sponsored a Christmas dinner to mark their appreciation to the faculty. Santa Claus presented gag gifts to the faculty with appropriate purported letters from the faculty providing the rationale for the individual gifts. Other 1982-83 officers: Arron VonSchriltz, secretary; Charles Voltaw, corresponding secretary; Jeffrey Barnett, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President - Beth Stewart
26 actives, 5 initiates

During fall semester, the chapter took steps to prepare for the upcoming national KME convention to be held at Eastern. Committees were formed and their responsibilities decided upon. Some committees started carrying out some of these (for example, a banquet speaker was chosen in November). Besides planning for the convention, departmental faculty members were invited to give talks at regular meetings. Dr. Amy King gave a talk on complex variables and Dr. Paul Bland gave a talk on fallacies in mathematics. Just before semester break, a Christmas party was held for the entire faculty of the department. Other 1982-83 officers: Monica Feltner, vice president; Carole Stagnolia, secretary; Karen Applegate,treasurer; Patrick Costello, corresponding secretary; Don Greenwell,faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President - Judy Van Duzer
15 actives, 4 initiates

Other 1982-83 officers: Cynthia Bowden, vice president; Amy Polashuk,secretary; Millard Mazer, treasurer; James Lightner, corresponding secretary; Jack Clark, faculty sponsor.

Michigan Beta, Central Michigan University, Mount Pleasant
Chapter President - Sandra Dolde
20 actives, 21 initiates

The fall started with a Kappa Mu Epsilon picnic for members and faculty. Fall initiation was held in November with Arnold Hammel speaking on the Coupon Collector Problem from Probability. A homecoming Alumni coffee hour was held the morning of the homecoming football game. Kappa Mu Epsilon members also conducted help sessions for undergraduate mathematics classes. Other 1982-83 officers: Diane Francisco, vice president; Laurie Park, secretary; Dan Franck, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President - Johnny Graves
40 actives, 18 initiates

The annual fall cookout and initiation were held on October 1, 1982. Eighteen new members were initiated. Other 1982-83 officers: Charles Orr, vice president; Marie Turnage, secretary; Liz O'Neal, public relations; Alice W. Essary,corresponding secretary;Mylan Betounes, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President - Kathy Merlo
31 actives, 15 initiates

Monthly meetings were held at which time a special speaker presented an interesting program on current mathematics. Other 1982-83 officers: Belinda Butcher, vice president; Craig McCowen, secretary; Shari Birkenbach, treasurer; M. Michael Awad, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President - Jennifer Koch
23 actives, 5 initiates

Activities for the past year were 8 regular meetings (including 2 initiations), a Christmas party, a canoe float trip, an honors banquet, and the Annual Klingenberg Lecture. Other 1982-83 officers: Vince Edmondson, vice president; Ruth Lichte, secretary; Randy Bush, treasurer; Homer F. Hampton, corresponding secretary; Larry Dilley, faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President - Michael Hanson
6 actives

Other 1982-83 officers: Judy Frazer, vice president; Kirk Meyer, secretary/treasurer; William D. McIntosh, corresponding secretary/faculty sponsor.

Missouri Zeta, University of Missouri-Rolla, Rolla
Chapter President - Eva Taylor
25 actives, 20 initiates

The chapter began the fall semester with a carryover of enthusiasm from the preceding semesters. The annual fall picnic was held at the Lions Club park in September. Although inclement weather held down the crowd, a good time was had by those that did attend. General meetings were held in September, October, and November. Programs for these meetings were given by faculty members of the Mathematics and Statistics Department. In September, Professor Tim Wright spoke on Statistical Convex Cones. Professor Tom Powell discussed Differential Geometry and Relativity at the October meeting. Professor Charles Hatfield presented a talk on some interesting combinatorial problems in November. The chapter agreed to increase local dues to $12.50 per year effective with the Fall 1982 semester. Twenty new members were inducted at the initiation banquet held November 4 at Zeno's Restaurant. Professor Thomas Sager of the Computer Science Department gave a talk-slide presentation entitled, Some Observations on the Modernization of China. Professor Sager taught computer science in the People's Republic of China. The chapter also discussed the biennial convention to be held in Richmond, Kentucky in April 1983. Several members plan to submit papers for the meeting. New officers were elected for the Spring Semester in November. These include: Denise Rost, vice president; Tom Shannon, secretary; Steven Phillips, historian; Johnny Henderson, corresponding secretary; James Joiner, faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville
Chapter President - Neil Meyer
8 actives, 18 initiates

Fall semester activities began with a picnic for new initiates and closed with a Christmas party for mathematics students. In addition, senior students began their senior presentations. Members also participated in tutoring service and provided demonstrations of micro-computing systems. Work was also started on a spring math contest for high school students. Other 1982-83 officers: Sandy Nelson, vice president; Cindy Strait, secretary; Peggy Shippen, treasurer; Sam Lesseig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

<u>Missouri Iota</u>, Missouri Southern State College, Joplin
Chapter President - Larry Hicks
12 actives

The chapter enjoyed a fall float trip down the Elk River. Tutorial assistance was provided for pre-calculus students. As a service to the college and as a money making project, members worked concessions at one of the home football games. One of the regular meetings included a talk given by Ken Buzzard on probabilities associated with the game of monopoly. To wrap up the semester, a Christmas party was held for <u>KME</u> and Math Club members. Other 1982-83 officers: Sherri Plagmann, vice president; Karen Foster, secretary/treasurer; Mary Elick, corresponding secretary; Joe Shields, faculty sponsor.

<u>Nebraska Alpha</u>, Wayne State College, Wayne
Chapter President - Mike Ronspies
18 actives, 11 initiates

Money making projects throughout the fall semester included monitoring the Math-Science Building in the evenings and selling discs for the Apple II Computer to college students. The club also had made and many members purchased Kappa Mu Epsilon T-shirts. Kelli Goodner was the winner of the club's $25.00 scholarship which can be used to purchase text books or pay fees. The award is given each semester to one of the members. The recipient is selected by club members using a secret ballot. Wayne State College Student Senate awarded the Kappa Mu Epsilon chapter a $200.00 grant which the club is using to purchase computer software that will be utilized by students. The major piece of software to be purchased will be a word processor which can be used for term paper production. Other 1982-83 officers: Kelli Goodner, vice president;Brenda Mandel, treasurer; Ken Burns, historian; Fred Webber, corresponding secretary; James Paige and Hilbert Johs, faculty sponsors.

Nebraska Beta, Kearney State College, Kearney
Chapter President - Martha Haeberle
15 actives, 19 initiates

Nebraska Beta chapter held initiation in October. Nineteen new members were initiated. Membership Chairman, Bill Meyer supervised the new initiates in updating the Kappa Mu Epsilon bulletin boards. Jim Welborn and Bill Meyer worked on revising the chapter's constitution. In November, members, along with the faculty, met with high school seniors to explain KSC's math program. December found members taking time out from studying in order to attend the Kappa Mu Epsilon wine and cheese party. Other 1982-83 officers: Sharon Hostler, vice president; John Klimek, secretary; Melvin Joy, treasurer; Charles Pickens, corresponding secretary; Nelson Fong, faculty sponsor.

Nebraska Gamma, Chadron State College, Chadron
Chapter President - Evonnda Sharp
4 actives, 8 initiates

Other 1982-83 officers: David Mundt,vice president; Diana Thomas, secretary; Janine Spracklen, treasurer; James Kaus, corresponding secretary; Monty Fickel, faculty sponsor.

New Jersey Beta, Montclair State College, Montclair
Chapter President - Elizabeth Pachella
6 actives, 14 initiates

During the fall semester the chapter held several meetings. Bagel sales were organized to raise money. Dr. C. Bredlau gave a lecture entitled "Show and Tell with the Microcomputer." Members were given the opportunity to experiment with the Apple II microcomputers in the computer science department throughout the semester. The initiation dinner was held at the Burns Country Inn in Clifton, NJ on Friday evening, November 12, 1982. Other 1982-83 officers: Susan Stecewicz, vice president; Anjali Sangani, secretary; Teresa Matarazzo, treasurer; Marilyn Saiewitz, acting treasurer; Dr. Thomas Devlin and Dr. Carl Bredlau, faculty sponsors.

<u>New Mexico Alpha</u>, University of New Mexico, Albuquerque
Chapter President - Dave Hilland
60 actives, 23 initiates

Fall semester activities included an initiation banquet, quiz for initiates, and a speaker. In an attempt to improve attendance, a pizza party was held in the mathematics departmental lounge one afternoon. Members enjoyed the change although attendance didn't improve significantly. Other 1982-83 officers: Martin Murphy, vice president; Stephanie Hendrix, secretary; Rebecca Gore, treasurer; Merle Mitchell, corresponding secretary/faculty sponsor.

<u>New York Eta</u>, Niagara University, Niagara
Chapter President - Bridgette Baldwin
16 actives

The chapter has been meeting concurrently with the Math Club on campus to plan various social activities and fund-raising events. The highlight of the semester was a visit by Dr. Erik Hemmingsen of Syracuse University who spoke on the topic of iteration and also discussed graduate school and career opportunities. Other 1982-83 officers: Tom Copeland, vice president; Jim Krzyzanowski, secretary; Cathy Difonzo, treasurer; Robert Bailey, corresponding secretary; James Huard, faculty sponsor.

New York Theta, Saint Francis College, Brooklyn
Chapter President - Lorraine DeCicco
5 actives, 2 initiates

At meetings, the chapter has spent most of the time in going over questions on the GRE Advanced Test in Mathematics. Other 1982-83 officers: Elaine Powers, vice president; Elizabeth Blum, secretary; Kathleen Mador, treasurer; Rosalind Guaraldo, corresponding secretary/faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green
Chapter President - Deb Steffens
50 actives, 29 initiates

Fall activities included a panel discussion on "The Crisis in Math Education." The chapter selected Harry R. Mathias, former KME National vice president, as recipient of the 1983 KME award for excellence in teaching mathematics. Other 1982-83 officers: Joan Bowman, vice president; Larry Zaborski, secretary; Dan Kleber, treasurer;Dr. Frederick Leetch,corresponding secretary; Wallace Terwilliger, faculty sponsor.

Ohio Gamma, Baldwin-Wallace College, Berea
Chapter President - Larry Mills
15 actives

Dr. Harvey Salkin of Case Western Reserve University gave a talk to the chapter on "Analytic Approaches to Investing and/or Gambling in the Stock Market." Dr. Salkin gave the talk as a SIAM visiting lecturer. Other 1982-83 officers: Martin Porter, vice president; Pamela Botson, secretary; Rich Hughes, treasurer; R. E.Schlea, corresponding secretary/faculty sponsor.

Ohio Epsilon, Marietta College, Marietta
Chapter President - Joanna Urbano
22 actives, 9 initiates

The chapter held an initiation which was followed by a party. Other 1982-83 officers: Richard Clark, vice president; Steven Mizer, treasurer; John R. Michel, corresponding secretary/faculty sponsor; Neil Bernstein, faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President - Gail Yoder
40 actives, 6 initiates

Grant Gainey presented a talk at the September meeting on "Georg Cantor -- A Man of Infinite Worth." Six new members were initiated in October. The chapter sponsored two speakers from Miami University in November as well as a placement panel which included two faculty and two alums. The annual Christmas party was held at Smith's in December. Other 1982-83 officers: Tom Bressoud, vice president; Shayne Fawcett, secretary; Paula Gomory, treasurer; James L. Smith, corresponding secretary/faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President - Robert Estes
25 actives, 5 initiates

Fall semester activities included a cookout for members and initiates, a bake sale for Math Day visitors, initiation of new members, and a field trip to Haliburton Oil Company's computer center. Other 1982-83 officers: Mark Craig, vice president; Debbie Biehler, secretary/treasurer; Wayne Hayes, corresponding secretary; Kelvin Casebeer, faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington
Chapter President - Carl Schartner
34 actives

Fall activities began with a picnic for all mathematics and computer science majors. Annette Trivilino, a mathematics major who spent the spring of 1982 in France, gave a talk to the chapter. The members also enjoyed a dinner at Dr. Faires' house. Other 1982-83 officers: Rob Streeter, vice president; Kirsten Pealstrom, secretary; Nicholas Kounavelis, treasurer; Miller Peck, corresponding secretary; Barbara Faires, faculty sponsor.

Pennsylvania Gamma, Waynesburg College, Waynesburg
Chapter President - Mark Corwin
18 actives

Other 1982-83 officers: John Stewart, vice president; Karen Connolly, secretary/treasurer; Rosalie Jackson, corresponding secretary; David Tucker, faculty sponsor.

Pennsylvania Epsilon Kutztown State College, Kutztown
Chapter President - Melodie Schumacher
20 actives, 4 initiates

Other 1982-83 officers: Steve Caufield, vice president;Cynthia Rapp, secretary; Jeffery Herbein, treasurer; Irving Hollingshead,corresponding secretary/faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana
Chapter President - Ann Leiberton
24 actives, 2 initiates

Dr. Melvin Woodard, member of Mathematics Department faculty presented a talk on number theory at the October meeting. The November meeting was held in conjunction with the annual Mathematics Department Career Night. Two IUP Alumni and an applied mathematician from Pratt-Whitney Aircraft Corporation presented talks concerning careers in mathematics. In December, officers were elected. Mr. David Wilson, Mathematics Department faculty member presented a talk on "Knots." Other 1982-83 officers: Diane Kirchner, vice president; Claudia Christner, secretary; Sue Ann Kaufold, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President - Mylene Klipa
18 actives, 8 initiates

The annual Christmas party was held on December 7 at the home of the Mathematics Department Chairman, Jack Schlossnagel. Other 1982-83 officers: Leslie Demarest, vice president; Ron Ellenberger, secretary; Susan Kay, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Theta, Susquehanna University, Selinsgrove
Chapter President - Dave Cianfarini
6 actives, 9 initiates

Other 1982-83 officers: Lisa Kapustay, vice president; Donna Kratzer, secretary; Sue Coates, social chairman; Carol Harrison, corresponding secretary/faculty sponsor.

Pennsylvania Iota, Shippensburg State College, Shippensburg
Chapter President - Mark E. Kriaski
35 actives, 12 initiates

The semester began with a picnic which was open to all math and computer science students and faculty. During the pledging of the twelve fall initiates, a special joint meeting was held with Western Maryland's KME chapter. Each area presented a number of interesting talks and refreshments were served. The term finished with a fall initiation ceremony held at the Math Department Chairman's house. Other 1982-83 officers: Joel M. Whitesel, vice president; Annette DeRenzi, secretary; Howard T. Bell, treasurer; Carl E. Kerr, corresponding secretary; J. Winston Crawley, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President - Linda Czajka
5 actives, 3 initiates

During the monthly meetings, Sister M. Grace lectured on "Probability." Problems which appeared in math journals were discussed and solved. Also, volunteer tutoring was done by the members. Plans for initiation of new members were formulated at the last meeting of the fall semester. Initiation will take place on March 16, 1983. Other 1982-83 officers: Christine Michaels, vice president/secretary; Teresa McKeon, treasurer; Sister M. Grace Kuzawa, corresponding secretary/faculty sponsor.

Pennsylvania Mu, Saint Francis College, Loretto
Chapter President - Nancy Dudziec
7 actives

Other 1982-83 officers: Penney D. Horner, vice president/treasurer; Peter Laird, secretary; Father John Kudrick, corresponding secretary; Adrian Baylock, faculty sponsor.

South Carolina Gamma, Winthrop College, Rock Hill
Chapter President - Donna Davis
14 actives, 2 initiates

Fall semester programs included a talk on numbers expressed in base one-half, and a Christmas program based on the article "The Twelve Days of Christmas and Pascal's Triangle" in the December issue of The Mathematics Teacher. Members submitted questions to be used in the Winthrop College Mathematics Tournament for high school students and assisted with the tournament on December 4, 1982. The chapter president, Donna Davis, also submitted the T-shirt design which was used for the tournament -- a flow chart on how to solve a mathematics problem. Other 1982-83 officers: Mary Jones, vice president; Anita Anderson, secretary; Thad Jennings, treasurer; Don Aplin,corresponding secretary; Kay Creamer, faculty sponsor.

Tennessee Alpha, Tennessee Technological University, Cookeville
Chapter President - Sharon S. Lovett
125 actives

During fall quarter, 1982, the Tennessee Alpha Chapter held an organizational meeting which included a guest speaker and lunch. The featured speaker was Mr. Frank Bush, the lead Systems Programmer for Tennessee Tech's Computer Center. Mr. Bush spoke concerning the history and future of TTU's computer systems. Preparation is underway for spring initiation, and for the awarding of a $150.00 scholarship to a new spring KME initiate. Other 1982-83 officers: Sherri G. Menees, vice president; Carolyn F. Talbert, secretary; Shelley Bynum,treasurer; Edmund Dixon, corresponding secretary; S. B. Khleif, faculty sponsor.

Tennessee Delta, Carson-Newman College, Jefferson City
Chapter President - Vicki Jarett
20 actives

Other 1982-83 officers: Ruby Jane Cooke, vice president; B. Alden Starnes, secretary; Leslie Mackichan, treasurer; Albert Myers, corresponding secretary; Carey Herring, faculty sponsor.

Texas Beta, Southern Methodist University, Dallas
Chapter President - Brad Neugebauer
40 actives, 7 initiates

Other 1982-83 officers: Dave Logan & Kathy Heizer, vice president; Alisa Armitage, secretary; Victoria Bychok, treasurer; Susie Shull, corresponding secretary/faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President - Tammy Bridgewater
15 actives

A get-acquainted party was held at the home of Mr. and Mrs. Ben Bentley. Prospective members were informed of the activities of Kappa Mu Epsilon. Other 1982-83 officers: Bobbie Chesser, vice president; Dana Teer, treasurer; Mary Wagner, corresponding secretary; Charles Robinson and Ed Hewett, faculty sponsors.

Virginia Beta, Radford University, Radford
Chapter President - Laura Robertson
30 actives, 12 initiates

Fall semester speakers at regular meetings were Louise McDonald from McDermott, Inc., and Dr. Donna Brogan from Emory University. Other 1982-83 officers: Rebecca Mehaffey, vice president; Blanche Fralin, secretary; Linda Eanes, treasurer; Coreen Mett, corresponding secretary; J. D. Hansard, faculty sponsor.

Wisconsin Alpha Mount Mary College, Milwaukee
Chapter President - Catherine Schueller Drasch
6 actives

Wisconsin Alpha chapter sponsored the annual mathematics contest for junior and senior high school girls on November 20, 1982. Time was spent discussing means of raising funds. A doughnut sale was held for this purpose. One of the pledges gave a talk on prominent women mathematicians. Veronica Walicki included Hypathia, Sonya Corvin-Krukovsky Kovalevsky and Emmy Noether in her presentation of Women in Mathematics. Other 1982-83 officers: Bonnie Best, vice president; Catherine Schueller Drasch, secretary; Bonnie Best, treasurer; Sister Adrienne Eickman, corresponding secretary; Sister M. Petronia Van Straten, faculty sponsor.

REPORT ON THE TWENTY-THIRD BIENNIAL CONVENTION

The Twenty-fourth Biennial Convention of Kappa Mu Epsilon was held April 21-23, 1983 on the campus of Eastern Kentucky University, Richmond, Kentucky, with Kentucky Alpha the host chapter.

On Thursday evening, April 21, following registration in the lobby of the Perkins Building, delegate activities included a game room in the Perkins Building, a panel discussion on "Career Opportunities in Mathematics" in the Perkins Building, and a square dance in the Stratton Gym. The National Council and the Regional Directors met in Room 224 of the Perkins Building.

On Friday morning,April 22, registration continued in the lobby of the Perkins Building. The first general session was held in Room A of the Perkins Building, commencing at 8:52 a.m.with Ida Z. Arms of Pennsylvania Zeta, National President, presiding. Dr. J. D. Rowlett, Vice-President for Academic Affairs of Eastern Kentucky University, gave an address of welcome and James L. Smith of Ohio Zeta, National President-Elect, responded for the Society.

Roll call of the chapters was made by George R. Mach of California Gamma, National Secretary. 27 chapters and about 167 members were in attendance. Delegate certification forms were checked, travel vouchers were filed, and delegate voting cards were issued.

James L. Smith of Ohio Zeta, National President-Elect,presided during the presentation of the following student papers:

1. "The Construction of a Heptagon," Margaret Chizek, Iowa Alpha, University of Northern Iowa.

2. "Fun With Splines," Thomas Bressoud, Ohio Zeta, Muskingum College.

3. "Taylor's Formula With Blumenthal Remainder," Trent Eggleston, Missouri Zeta, University of Missouri, Rolla.

4. "Amicable Numbers," Mark Woodard, Pennsylvania Zeta, Indiana University of Pennsylvania.

At noon, a group picture was taken in front of the Perkins Building. Convention committees and the National Council met during lunch.

The convention reconvened at 1:30 p.m. in Room A of the Perkins Building. James L. Smith of Ohio Zeta, National President-Elect, presided during the presentation of the following student papers:

5. "The Perfect Math Topic," Kelli Goodner, Nebraska Alpha, Wayne State College.

6. "What Can We Say About Duffinian Numbers?" Rose Mary Zbiek, Pennsylvania Zeta, Indiana University of Pennsylvania.

7. "Degree Distributions and Percolations for Random Subgraphs of the Square Lattice," Michael Klein, New York Kappa, Pace University.

At 2:45 p.m. a student section met in Room A of the Perkins Building with Beth Stewart, President of Kentucky Alpha, presiding and a faculty section met in Room 212 of the Perkins Building with Ida Z. Arms of Pennsylvania Zeta, National President, presiding.

The convention reconvened at 3:45 p.m. in Room A of the Perkins Building. James L. Smith of Ohio Zeta, National President-Elect, presided during the presentation of the following student papers:

8. "The Divisibility of Seven and Other Numbers," Thomas Shannon, Missouri Zeta, University of Missouri, Rolla.

9. "Calculation of the Motion of Light Projectiles Using Euler's Methods," Charles Daws, Iowa Alpha, University of Northern Iowa.

10. "Constructing Efficient Codes," Nancy Zebraitis, Pennsylvania Lambda, Bloomsburg State College.

The convention banquet was held on Friday evening, April 22, in the Banquet Room of the Perkins Building with Beth Stewart, President of Kentucky Alpha, as master of ceremonies. Musical entertainment was provided by "The Logarithms," a barbershop quartet from Bowling Green University. Guest speaker, Professor Arnold Ross, Ohio State University, gave the address, "Discovery and Development of Mathematical Talent: Nature or Nurture?"

The convention reconvened at 8:30 a.m. on Saturday, April 23, in Room A of the Perkins Building. James L. Smith of Ohio Zeta, National President-Elect, presided during the presentation of the following student papers:

11. "Trilaterals and Triangles," Belinda Butcher, Missouri Alpha, Southwest Missouri State University.

12. "Computer Image Processing," Jeffrey Brent, Pennsylvania Lambda, Bloomsburg State College.

13. "The Mathematics of Musical Scales," Cynthia Bowden, Maryland Beta, Western Maryland College.

14. "Pi - Some Derivations and Coincidences," Denise Ann Rost, Missouri Zeta, University of Missouri-Rolla.

15. "Hegel and Math," Ann Bogue, Illinois Eta, Western Illinois University.

The second general session (business meeting) was held in Room A of the Perkins Building. The following national officers presented reports (copies attached):

Business Manager, THE PENTAGON	- Douglas Nance Michigan Beta
Editor, THE PENTAGON	- Kent Harris Illinois Eta (presented by Douglas Nance)
National Historian	- Harold L. Thomas Kansas Alpha

National Treasurer	- Wilbur J. Waggoner Michigan Beta
National Secretary	- George R. Mach California Gamma
National President-Elect	- James L. Smith Ohio Zeta
National President	- Ida Z. Arms Pennsylvania Zeta

M. Michael Awad of Missouri Alpha reported for the auditing committee that the treasurer's financial report as of March 24, 1983 was accurate. The committee recommended the following:

1. On voided checks, tear off bottom corner and leave check in book,so nobody can use the voided checks.

2. Instruct the National Treasurer to request the bank to submit in writing to the Chairperson of the Auditing Committee the balance of the KME account as of the closing date of the Biennial Report.

3. Double check that carbon paper of the checks makes legible copy for records.

John Cross of Iowa Alpha reported for the resolutions committee. The following resolutions were adopted:

"Whereas, Kappa Mu Epsilon has been holding its 24th Biennial Convention on the beautiful campus of Eastern Kentucky University and whereas this convention has been a pleasurable and profitable experience for all of us, be it resolved that we express our appreciation:

1. To the host chapter, Kentucky Alpha, especially its faculty advisor Don Greenwell and corresponding secretary Pat Costello, its president Beth Stewart, its officers and members, and to the administration of Eastern Kentucky University for their gracious hospitality and efficient organization which has been so important to the success of this convention.

2. To each of the national officers: President Ida Z. Arms, President-Elect James L. Smith, Secretary George Mach and Historian Harold Thomas for their efficiency in the performance of their respective duties throughout the past biennium and especially to Treasurer Wilbur Waggoner, who will retire from office this year following 6 years as national treasurer and 20 years as business manager of THE PENTAGON.

3. To Kent Harris, the editor of THE PENTAGON and Douglas Nance, the business manager of THE PENTAGON.

4. To Professor Arnold E. Ross who provided the interesting program at the convention banquet and to "The Logarithms" for the musical entertainment.

5. To the fifteen students who prepared and presented papers to the convention and the four students whose papers were chosen as alternates.

6. To the members of the convention committees, both local and national, who gave so unselfishly of their time.

Robert L. Bailey of New York Eta reported for the nominating committee. The committee nominated: George R. Mach of California Gamma for National Secretary and Sister Nona Mary Allard of Illinois Zeta and L. Thomas Shiflett of Missouri Alpha for National Treasurer. Nominations were requested from the floor. There being none, nominations were closed and ballots were distributed to the voting delegates and collected.

Helen Kriegsman of Kansas Alpha reported for the faculty section meeting. Carole Stagnolia of Kentucky Alpha reported for the student section meeting.

Invitations to host the Twenty-fifth Biennial Convention in 1985 were extended by: California Gamma, California Polytechnic State University;Missouri Alpha, Southwest Missouri State University; Wisconsin Alpha, Mount Mary College; and Texas Beta, Southern Methodist University.

James L. Smith of Ohio Zeta, National President-Elect, distributed certificates to all students who had presented papers at the convention. Sister Nona Mary Allard of Illinois Zeta reported for the awards committee and announced the following student paper awards:

First Place ($60)-Thomas Bressoud, Ohio Zeta
Second Place ($40)-Rose Mary Zbiek, Pennsylvania Zeta
Third Place ($30)-Mark Woodard, Pennsylvania Zeta
Fourth Place ($20)-Nancy Zebraitis, Pennsylvania Lambda

The election results were announced by Robert L. Bailey of New York Eta. The following officers were elected for the next four years, 1983-87, and they were installed by Ida Z. Arms of Pennsylvania Zeta, National President:

National Secretary-George R. Mach, California Gamma
National Treasurer-Sister Nona Mary Allard, Illinois Zeta

Travel allowances were paid to the delegates by Wilbur J.Waggoner of Michigan Beta, National Treasurer. Reports of the national officers were distributed and convention evaluation forms were collected by the host chapter. The convention adjourned at 11:58 a.m.

George R. Mach

REPORT OF THE NATIONAL PRESIDENT

During the last biennium the Society and its membership have been quite active. One new Chapter, Connecticut Beta at Eastern Connecticut State College, was installed. One Chapter, Illinois Delta at The College of St. Francis, has been reactivated. Four Chapters have been declared inactive: North Carolina Alpha at Wake Forest University, North Carolina Beta at Western North Carolina University, Illinois Alpha at Illinois State University, Normal, and Texas Epsilon at North Texas State University. The National Council and Chapters have approved the petition of C.W.Post Center-Long Island University for a new chapter. New York Lambda will be installed on May 2, 1983.

Several successful Regional Conventions have been held during the biennium and the National Council strongly supports Regional activity. I would like to express my appreciation and that of the National Council members to each Regional Director and particularly to Sister Nona Mary Allard, Dr. John Cross, and Professor Adelaide Harmon-Elliott,Directors of Regions II, IV and VI respectively, who complete four year terms as of this date. Directors for these Regions will be appointed for four year terms at this convention.

At the last National Convention a resolution was adopted that the National Council consider redefining national requirements for membership in Kappa Mu Epsilon in particular with regard to accepting computer science and/or statistics courses as part of the mathematics requirements. The National Council addressed this resolution at a meeting in October, 1981 and voted to adopt the following statement of policy and transmit it to the chapters:

> "The spirit of the Kappa Mu Epsilon Constitution recognizes a basic proficiency in mathematics in Article II, Section 2c. The National Council suggests that each Chapter, in its decision as to which courses it will accept in satisfying the required minimum in Article II, Section 2c, be guided by both the maturity and proficiency implied in Article II, Section 2c. Within this structure it is considered reasonable that courses in computer science and/or statistics which are strongly based in mathematics would be acceptable courses to satisfy the minimum three course requirement specified. The chapters are also reminded that in Article IV, Section 4, it is stated that chapters are not prevented from adding additional requirements."

I want to express my thanks to all those members who agreed to serve on the various convention committees. Much of their work goes on behind the scenes before or during the convention so that the convention activities proceed in an orderly manner. I especially want to thank the faculty and student members of Kentucky Alpha who have worked so diligently for a long time to make the convention a success. I want to thank all the members of the National Council and particularly Dr. Wilbur Waggoner who has served Kappa Mu Epsilon in an official capacity for over twenty years. He has performed his duties with great care and we wish him good health and happiness in the future.

The success of this Society depends upon each member. I am confident that the future of Kappa Mu Epsilon can be as good, if not better, than it has been in the past. I challenge each of you to make this happen.

Ida Z. Arms

REPORT OF THE PRESIDENT-ELECT

One of the responsibilities of the President-Elect is to serve as coordinator of regional activities of the Society through the Regional Directors. During the 1981-82 academic year there were three conventions held in:

Region I	at Pennsylvania Lambda, Bloomsburg State College, with 75 participants (5 chapters); James Pomfret, Reg. Dir.
Region II	at Ohio Alpha, Bowling Green State University, with 89 participants (5 chapters); Sr. Nona Mary Allard, Reg. Dir.
Region IV	at Nebraska Beta, Kearney State College, 83 participants (12 chapters); John S. Cross, Reg. Dir.

Programs at each convention site included student papers, guest talks, and good social times. We extend our sincere thanks to the host chapters, Regional Directors, and all who participated in this important regional activity.

It is another of the President-Elect's responsibilities to make arrangements for the presentation of student papers at the National Convention. I am pleased to report that nineteen students, representing ten

Chapters and eight States, submitted papers for this Convention. Fourteen undergraduate students and one graduate student will present papers at the convention. On behalf of our entire Society I am pleased to extend special thanks to the members of the Paper Selection Committee who read and ranked the papers: Dr. Oscar Beck (Alabama Beta), Dr. Barbara Faires (Pennsylvania Alpha), and Dr. Wayne F. Hayes (Oklahoma Gamma). In addition, I am especially pleased to express our thanks to the nineteen students who prepared and submitted papers. These papers are the most important component in helping to make any Kappa Mu Epsilon Convention a success.

James L. Smith

REPORT OF THE NATIONAL SECRETARY

During the last biennium, one new chapter of Kappa Mu Epsilon was installed. Connecticut Beta was installed on May 2, 1981 at Eastern Connecticut State College. (A petition from the C.W. Post Center of Long Island University has been approved and a new chapter will be installed there next month.) Illinois Delta at College of St. Francis was reactivated on October 30, 1981. On April 2, 1981 the National Council declared New York Epsilon at Ladycliff College to be inactive due to the closing of the college. On December 8, 1981 Illinois Alpha at Illinois State University was declared inactive at its own request. On June 15, 1982 the National Council declared the following chapters to be inactive: North Carolina Alpha at Wake Forest University, North Carolina Beta at Western Carolina University, and Texas Epsilon at Texas State University. The Society now has 97 active chapters in 30 states.

During the last biennium, 2,372 members were initiated. The 97 active chapters have a combined membership of 38,005 and the 26 inactive chapters have a combined membership of 6,153, making the total membership of Kappa Mu Epsilon 44,158 at the end of the biennium on March 19, 1983.

As National Secretary, I maintain permanent files on all active and inactive chapters, including reports of all initiations. I order membership certificates for new members and I stock all supplies, including forms, invitations, and jewelry. I take minutes of National Council meetings and biennial conventions.

George R. Mach

FINANCIAL REPORT OF THE NATIONAL TREASURER
Biennium March 2, 1981 to March 24, 1983

<u>Receipts</u>

1.	Cash on Hand March 2, 1981			13,440.78
2.	Receipts from Chapters			
	Initiates (2377)	35,715.00		
	Jewelry	917.64		
	Supplies	353.40		
			36,986.04	
3.	Miscellaneous Recepts			
	Interest	3,190.14		
	Chapter Installations	219.30		
	Royalties	27.20		
	Travel Refund	300.00		
	PENTAGON	500.00		
	Other (Uncashed check $60)	77.50		
			4,314.14	
4.	Total Receipts			41,300.18
5.	Receipts plus cash on hand			54,740.96

<u>Expenditures</u>

6.	National Officers Expenses		5,455.96
7.	Jewelry (Balfour and Pollack)		692.80
8.	Printing Blake, Herff Jones, Brochures		6,511.55
9.	Pentagon (4 issues)		10,441.00
10.	Biennial Convention-1981 and Regional Conventions 1982		4,746.41
11.	ACHS		829.30

12. Miscellaneous			
Refunds	26.99		
Money Management fees	202.82		
National Council Mtg.	300.67		
Postage	155.10		
		685.58	
13. Total Expenditures			29,362.60
14. Cash on Hand-March 24, 1983			25,378.36

Wilbur J. Waggoner

REPORT OF THE NATIONAL HISTORIAN

The files of the National Historian are being maintained and continually updated with the reports received from the chapters about their events and activities; with information received from Regional Directors about regional conventions and items of interest related to the regions; and with material received from the National Officers which has historical significance.

News items have been solicited from the corresponding secretaries semi-annually, in January and in May. The responses are then edited for publication in the chapter news section of The Pentagon.

During the past biennium, 81 of the active chapters responded at least once to the chapter news request. Special mention goes to the following 27 chapters for their cooperation in responding to all four inquiries: AL Zeta, CA Gamma, CT Beta, GA Alpha, IL Zeta, IN Alpha, IA Alpha, KS Alpha, KS Gamma, KS Delta,KS Epsilon, KY Alpha, MD Beta, MI Beta, MO Alpha, MO Beta, MO Epsilon, MO Zeta, NE Alpha, NY Eta, OH Gamma, OH Zeta, PA Alpha, PA Epsilon, PA Zeta, PA Kappa and WI Alpha. I would urge chapters to reply to the requests for chapter news even if it is to just identify officers. This would provide chapters with a permanent record of their local officers in the event they do not retain that information within their own chapter.

Regional convention reports from Regions II and IV were received and published in the Fall, 1982, issue of The Pentagon.

I want to extend thanks to all with whom I have corresponded relative to this office -- the National officers, the Regional secretaries, the editor of The Pentagon, corresponding secretaries, and individual KME members. It has been a pleasure to serve as your historian for this past biennium.

Harold L. Thomas

REPORT OF THE EDITOR OF THE PENTAGON

During the past two years, one joint student-faculty paper and nine student papers have been published in THE PENTAGON. You are all urged to submit your work for possible publication in THE PENTAGON. Please submit regional convention papers as well as national convention papers to be considered for publication in THE PENTAGON.

Many interesting and unusual papers were received from a variety of individuals during the last two years. These include papers on "A Proof of Fermat's Last Theorem" (2), "Trisecting An Angle" (2), "Factoring Powers of Ten", "A Unification Theory of All of the Forces in the Universe", and others which include some very original assumptions.

My thanks go to my associates, Richard Barlow, Kenneth Wilke, Harold Thomas, Iraj Kalantari, and Douglas Nance for their fine efforts which go into THE PENTAGON. Thanks to all of you for supporting and contributing to THE PENTAGON.

Kent Harris

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON

It is a pleasure to make my fourth Business Manager's report during this 24th Biennial Convention. As many of you know, the Business Manager's primary responsibility is to see that THE PENTAGON gets mailed to members of Kappa Mu Epsilon who have current subscriptions. Mailing dates for THE PENTAGON are approximately June and December.

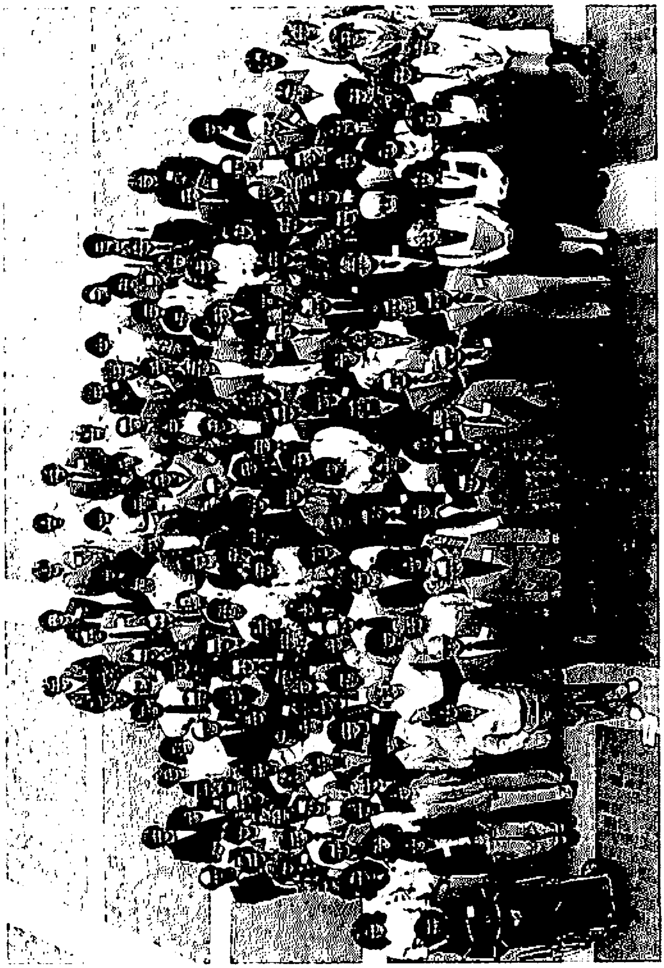
During this past biennium, we mailed an average of 2400 Pentagons per issue. The mailing list includes subscribers in forty-four states and seventeen foreign countries. States receiving the most copies of THE PENTAGON are, in descending order, Pennsylvania, Missouri, Ohio and Illinois.

During each semi-annual mailing, approximately forty Pentagons are returned to the office of the Business Manager by the postal service as undeliverable due to incorrect address. Please inform your chapter members that to receive their journal they must keep a current address on file. If a subscriber has any problem with receiving THE PENTAGON, please contact the office of the Business Manager.

Complimentary copies of THE PENTAGON are sent to the library of each college or university with an active chapter of Kappa Mu Epsilon. Also, complimentary copies are sent to authors of articles in THE PENTAGON. Speakers at this convention will automatically have their subscriptions extended for two years.

During this past biennium, I have received cooperation and support from former business manager Wilbur Waggoner, editor Kent Harris, national secretary George Mach and the mathematics department at Central Michigan University. This cooperation is gratefully acknowledged.

Douglas W. Nance



Kappa Mu Epsilon, Twenty-Third Biennial Convention
April 21-23, 1983, Richmond, Kentucky

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