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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

GENETIC PROBABILITIES

JANET CARPENTER

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Genetics, which is the science of heredity, is a relatively new field. It was developed in 1865 with the work of Gregor Mendel and his garden peas. In order to understand the mathematical models associated with genetics it is first necessary to give a broad overview of the mechanisms of inheritance.

Cells are the building blocks of all biological organisms. Within most cells is a nucleus which in turn contains chromosomes. The chromosomes, which are wavy, hair-like structures, contain genetic determinants known as genes. A gene is the basic unit of heredity and is a complex protein molecule made of DNA. Genes come in pairs, and during reproduction the pairs split with the offspring receiving one member of each pair from each of the parents. This distribution of genes is random. With each given gene is usually associated a particular physical characteristic such as eye color. Alternative forms of a particular gene are known as alleles. An individual who has two similar genes for a particular trait is said to be homozygous for that trait and one who has two different genes for a particular trait is said to be heterozygous.

In a discussion of genetics two other important terms are 'dominant' and 'recessive'. Mendel's work with garden peas dealt with pure traits such as color and texture. Pure traits are those which show up in either one form or another but never a combination of the two. For example when he crossed yellow peas with green peas he found either yellow or green in the next generation but never a paler green or a darker yellow. In fact green and yellow can be described as the two alleles for the gene that determines color. Mendel found that when crossing pure yellow plants with pure green plants the first generation turned out to be 100 percent yellow. It was concluded that the allele for yellow 'masked' the one for green and was thus termed dominant. The allele for green was termed recessive. A simple illustration known as a punnett square clarifies such a phenomenon. Let R represent yellow and r represent green. Crossing a pure yellow plant with a pure green plant can produce combinations as follows:

	R	R
r	Rr	Rr
r	Rr	Rr

Since the R masks r it can be seen that all offspring in the first generation will be yellow. The crossing of the first generation within itself would yield a genetic ratio of $1RR:2Rr:1rr$ which would phenotypically represent three yellow plants to one green plant.

The Hardy-Weinberg Law deals with genetic probabilities in random matings. In working with this law some assumptions must be made.

1. At any particular gene locus there are exactly two alleles, R and r.
2. The three possible genotypes that result from these alleles, namely RR, Rr, and rr, mate randomly.
3. All three of the genotypes are equally biologically fit, meaning that all reproduce at the same rate and have equal probabilities of surviving to reproduce.

Suppose G_0 represents a generation in which the three possible genotypes RR, Rr, and rr have the frequencies u_0 , $2v_0$, and w_0 , respectively, where $u_0 + 2v_0 + w_0 = 1$. If these genotypes mate at random then the genotypes in the next generation, denoted by G_1 , would be in the proportions of $(u_0 + v_0)^2 : 2(u_0 + v_0)(v_0 + w_0) : (v_0 + w_0)^2$. Letting $u_1 = (u_0 + v_0)^2$, $v_1 = (u_0 + v_0)(v_0 + w_0)$, and $w_1 = (v_0 + w_0)^2$, the ratio of the genotypes RR, Rr, and rr would be $u_1 : 2v_1 : w_1$, respectively. This can be easily illustrated

by the following chart.¹

G_0			G_1		
Male	Female	Frequency	Offspring		
			RR	Rr	rr
RR	RR	$u_0 \quad u_0$	u_0^2		
RR	Rr	$u_0 \quad 2v_0$	$u_0 v_0$	$u_0 v_0$	
RR	rr	$u_0 \quad w_0$		$u_0 w_0$	
Rr	RR	$2v_0 \quad u_0$	$u_0 v_0$	$u_0 v_0$	
Rr	Rr	$2v_0 \quad 2v_0$	v_0^2	$2v_0^2$	v_0^2
Rr	rr	$2v_0 \quad w_0$		$v_0 w_0$	$v_0 w_0$
rr	RR	$w_0 \quad u_0$		$u_0 w_0$	
rr	Rr	$w_0 \quad 2v_0$		$v_0 w_0$	$v_0 w_0$
rr	rr	$w_0 \quad w_0$			w_0^2

The process can be repeated again and again and can be generalized by the Hardy-Weinberg Theorem.

Theorem: Let u_0 , $2v_0$, and w_0 be any three non-negative numbers such that no two are zero.

$$\begin{aligned}
 \text{Let } u_{i+1} &= (u_i + v_i)^2 \\
 v_{i+1} &= (u_i + v_i)(v_i + w_i) \\
 w_{i+1} &= (v_i + w_i)^2
 \end{aligned}$$

for $i = 0, 1, 2, \dots$

Then for $i = 1, 2, 3, \dots$

$$u_{i+1} = u_i$$

$$v_{i+1} = v_i$$

$$w_{i+1} = w_i.$$

Proof: Suppose that neither u_0 , v_0 , nor w_0 is zero. It can be shown that the ratios $u_i:2v_i$ and $2v_i:w_i$ are independent of i for $i = 1, 2, 3, \dots$. It is also necessary to use the fact that $u_i+2v_i+w_i = 1$.

It can be observed from the above equations that

$$v_i^2 = u_i w_i \quad \text{for } i = 1, 2, 3, \dots$$

In particular

$$v_1^2 = u_1 w_1$$

Adding $u_1 v_1$ to both sides of the equation yields

$$u_1 v_1 + v_1^2 = u_1 v_1 + u_1 w_1$$

or

$$v_1(u_1 + v_1) = u_1(v_1 + w_1)$$

This implies that

$$\begin{aligned}
 u_1/2v_1 &= (u_1+v_1)/2(v_1+w_1) \\
 &= (u_1+v_1)^2/2(u_1+v_1)(v_1+w_1) \\
 &= u_2/2v_2
 \end{aligned}$$

This says that $u_1:2v_1 = u_2:2v_2$. It can similarly be shown that

$$v_1^2 = u_1w_1$$

implies $2v_1:w_1 = 2v_2:w_2$. But since $u_1+2v_1+w_1 = u_2+2v_2+w_2 = 1$, it follows that $u_1 = u_2$, $v_1 = v_2$, and $w_1 = w_2$. To complete the proof it would be necessary to go through the above process for

$$v_2^2 = u_2w_2.$$

This would imply that $u_2:2v_2 = u_3:2v_3$ and $2v_2:w_2 = 2v_3:w_3$. From this it would follow $u_2 = u_3$, $v_2 = v_3$, and $w_2 = w_3$, and so on through n . The theorem states in general that each generation will be composed of three genotypes RR, Rr, and rr in the ratio $u_i:2v_i:w_i$.²

The Hardy-Weinberg Theorem and the preceding examples are obviously basic but it is necessary to understand them before moving on to more complex and practical applications.

Genetic counseling deals with predicting the occurrence of genetic diseases. A genetic counselor calculates the risk of particular parents transmitting a hereditary genetic disease to their offspring by employing two fundamental principles of probability.

1. The genotype of each child of a particular marriage is independent of the genotypes of any previous children.
2. If two or more events are independent, their probabilities are multiplied to get the probability that they occur in sequence.

A typical case that a genetic counselor might encounter would be the following:

Suppose Mr. and Mrs. Brown had sisters afflicted with cystic fibrosis, a recessive disorder. What is the probability that they will have a child with the disease?

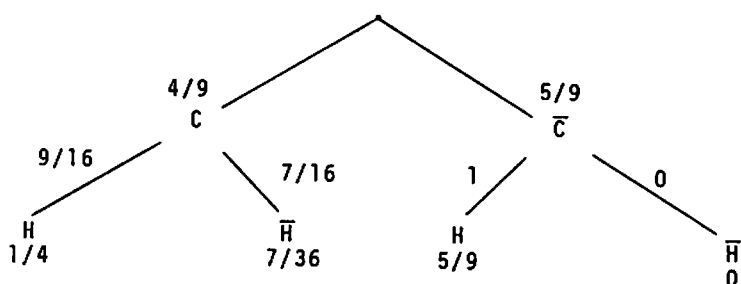
Since Mr. and Mrs. Brown's parents both produced afflicted children, they must both have been carriers. Of the normal children produced by carriers, two out of three are carriers. Thus the probability that Mr. Brown is a carrier is $2/3$ and the probability that Mrs. Brown is a carrier is $2/3$. Along with the fact that the probability two carriers will produce an afflicted child is $1/4$, and the probability that both Mr. and Mrs. Brown are carriers, the probability that their child will be born with the disease is $2/3 \times 2/3 \times 1/4 = 1/9$.³

A slightly more complex example involving conditional probability would be the following:

Suppose Mr. and Mrs. Smith both had sisters who were albinos. (Albinism is a recessive disorder.) If the Smiths have two children, what are the chances that neither will be afflicted with the disease?

In this case there are two probabilities to be considered. For the children to run the risk of being albinos, both parents must be carriers. (If one is not a carrier the children run no risk of getting the disease.)

If Mr. and Mrs. Smith are both carriers the chance that each of their children will be healthy is three out of four. A diagram makes a problem like this clearer.



C represents the event that both Mr. and Mrs. Smith are carriers and \bar{C} represents the complement. Since both of their parents were carriers, Mr. and Mrs. Smith each has a $2/3$ chance of being carriers themselves. The probability of event C is $4/9$. Thus the probability of event \bar{C} is $1 - 4/9 = 5/9$.

H stands for the event Mr. and Mrs. Smith have two healthy

children and \bar{H} is the complement. If \bar{C} is true, the probability of H is 1 and \bar{H} is 0. If C is true, the probability of H is $3/4 \times 3/4 = 9/16$ and \bar{H} is $1 - 9/16 = 7/16$. Thus the probability of event H, that Mr. and Mrs. Smith have two healthy children is $1/4 + 5/9 = 29/36$. Using more mathematically acceptable notation,

$$\begin{aligned} P(H) &= P(C) \cdot P(H|C) + P(\bar{C}) \cdot P(H|\bar{C}) \\ &= 4/9 \cdot 9/16 + 5/9 \cdot 1 \\ &= 29/36^4 \end{aligned}$$

Predicting the incidence of hereditary disease can be accomplished through binomial probability. Here is an example involving Huntington's Disease, a dominant disorder whose symptoms surface around middle-age.

Suppose a normal individual marries a carrier of Huntington's Disease and they have three children. To find the possible fates of these children, an equation can be set up using binomial probability. First the probability of the birth of a normal child equals the probability of the birth of an afflicted child. Let $\frac{1}{2}N$ represent the birth of a normal child and $\frac{1}{2}A$ represent the birth of an afflicted child. The eight possible outcomes for three children can be determined as follows:

$$\begin{aligned}
 \left(\frac{1}{2}N + \frac{1}{2}A\right)^3 &= C(3,0)\left(\frac{1}{2}N\right)^3\left(\frac{1}{2}A\right)^0 + C(3,1)\left(\frac{1}{2}N\right)^2\left(\frac{1}{2}A\right)^1 \\
 &\quad + C(3,2)\left(\frac{1}{2}N\right)\left(\frac{1}{2}A\right)^2 + C(3,3)\left(\frac{1}{2}N\right)^0\left(\frac{1}{2}A\right)^3 \\
 &= 1/8N^3 + 3/8N^2A + 3/8NA^2 + 1/8A^3.
 \end{aligned}$$

These results can be translated to say that the probability of three normal children is 1/8; the probability of two normal children and one afflicted child is 3/8; the probability of one normal child and two afflicted children is 3/8; and the probability of three afflicted children is 1/8.⁵

It can be seen that calculating probabilities and combinations plays a major role in genetics, particularly in the field of genetic counseling. Although most of the calculations seem basic, their results can prove to be major factors in family planning.

FOOTNOTES

¹John L. Howland and Charles A. Grobe. A Mathematical Approach to Biology, D. C. Heath and Co., 1972, pp. 60, 61.

²Ibid, pp. 63, 64.

³Nancy S. Rosenberg. Genetic Counseling. Applications of Probability to Medicine, 1980.

⁴Ibid.

⁵Ibid.

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HISTORY OF THE 5TH POSTULATE

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Little is known of Euclid's early life besides the guess of his birthdate to be in 330 B.C. and death in 275 B.C. He seemed to have been well acquainted with Platonic geometry, but not with Aristotle's works, which is the base of the belief that he was educated at Athens. His teaching at Alexandria was highly respected. He "impressed his own individuality to such an extent that, to his successors, and almost to his contemporaries the name Euclid meant (as it does to us) the book or books he wrote and not the man himself. (See [1])

Euclid's followers have praised him for his gentleness, consideration of others, and modesty. Many have quoted his reply to Ptolemy's plea for a "short-cut to geometric knowledge that 'there is no royal road to geometry.'" (See [3]) Tradition carries a story about a boy who had just begun to study geometry. He asked Euclid, "What do I gain by learning all of this stuff?" Euclid's reply was "that knowledge was worth acquiring for its own sake, but made his slave give the boy some pennies, 'since', he said, 'he must make a profit out of what he learns.'" (See [1])

Euclid's knowledge reached far beyond just geometry. He wrote about astronomy, optics, music, porisms, catoptrics, divisions of polygons, and answered questions that arose, for example, in land surveying. (See [8]) His greatest and most influential work was his Elements. It was not devoted to geometry alone, but to number theory and geometric algebra as well. (See [6]) Euclid's goal was to "strengthen our knowledge of points, lines and figures" rather than answer philosophical questions about the significance of geometry. He sometimes took pains "to prove things that his readers thought obvious? because he wanted to present his proofs in a clear and elegant way. He wanted to "organize geometry in a systematic deductive form ... proving beyond all reasonable doubt that certain laws of geometry hold. (See [2])

The impact this work has had on mathematicians and scientists from 300 B.C. until today is overwhelming. The Elements is a compilation of proofs and theorems of earlier writers as well as those original with him. (See [3]) One reason for all of the praise given Euclid is the "simple but logical sequence of theorems and problems" he uses. (See [8]) The thirteen books of the Elements "was at once adopted by the Greeks as the standard text-book on the elements of pure mathematics." (See

[1]) Today American high school plane and solid geometry texts contain much of the material found in the Elements.

The 465 propositions or theorems are derived from five axioms and five postulates. They are:

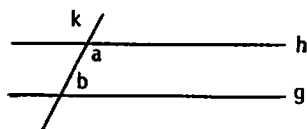
Euclid's Axioms

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the whole are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

Euclid's Postulates

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any center and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines (in the same plane) makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on the side on which the angles are less than the two right angles.

For examples, if $a+b < 180^\circ$,
then h and g will meet on the
right hand side.



"It is seen that the postulates are assumptions of a geometrical nature, whereas the axioms are more general assumptions, applicable to all of mathematics." (See [4]) The geometers who criticized Postulate 5 over the centuries did not question that its content was mathematical fact, but only that it was not brief, simple, and self-evident, as postulates were supposed to be. "This postulate which seems to have been and probably was inserted without much polish, is nevertheless one of the greatest of Euclid's creations, for without such a postulate, Euclidean geometry could not exist. There is nothing more interesting in the history of science than the record of repeated attacks made on this postulate in the hope of reducing it to a proposition deducible from the other four, and which finally resulted in the discoveries of other geometries." (See [7]) Some of these Non-Euclidean geometries, that are just as consistent or free of internal contradictions, have turned out to be enormously useful in, for example, modern physics and cosmology". (See [5])

The Fifth Postulate was not used until Proposition 29; this "tardy utilization, after so much had been proved without it was enough to arouse suspicions with regard to its character...as a consequence, innumerable attempts were made to prove the postulate or eliminate

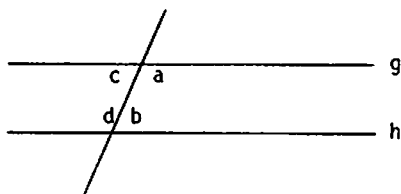
it by altering the definition of parallels." (See [9]) These attempts lasted for 2,000 years. The most important attempts to prove Postulate 5 were made by the following: Ptolemy, Proclus, Saccheri, Lambert, and Legendre. Others who attempted to prove the postulate but obviously failed were Thibaut, Bertrand, Gauss, Bolyai, Lobachewsky, Wallis, and Nasiraddin.

One of the earliest attempts to prove the fifth postulate was made by Ptolemy, a noted astronomer, who lived in Alexandria during the second century A.D. The knowledge of Ptolemy's arguments come to us through the writings of Proclus, "... an Athenian philosopher, mathematician, and historian of the fifth century A.D." (See [4]) Ptolemy's plan was to 1) prove Proposition 29 without using the postulate, and 2) deduce the postulate from the proposition. We remember that the Fifth Postulate was not used in proving Propositions 1-28. Proposition 29 states:

A straight line falling on parallel lines makes the alternate angles equal to one another, the exterior angles equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles.

Ptolemy's reasoning in proving Proposition 29 without the fifth postulate was essentially as follows:

Consider two parallel lines g , h and a transversal.



Suppose that $a+b < 2$ right angles. Since g and h are as parallel on one side of the transversal as on the other, the sum of the interior angles on one side is the same as the sum of the interior angles on the other side. Hence $c+d < 2$ right angles (180^0). We then get $a+b+c+d < 4$ right angles. But this is impossible because Proposition 13 states that $a+b+c+d$ is equal to four right angles. A like contradiction results if we assume that $a+b > 180^0$. Hence $a+b = 180$. But $c+a = 180^0$, hence $c=b$, similarly $a=d$.

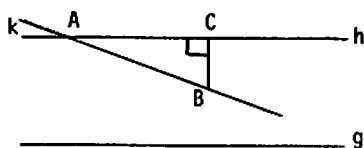
The "as parallel as" idea used in the preceding argument is too vague to serve as mathematical justification for the sum of the interior angles on one side being equal to the sum on the other. Therefore, the statement is an assumption. (See [4]) Proclus pointed out that "Ptolemy really assumed that through a point only one parallel can be drawn to a given line. But this is equivalent to assuming the Fifth Postulate." (See [9])

After criticizing the proof, Proclus offered his own solution of the problem. His plan was to

- 1) prove on the basis E that a line which meets one of two parallels also meets the other. (Basis E includes all of Euclid's definitions, all of his assumptions except Postulate 5 and propositions 1-28.)
- 2) deduce Postulate 5 from this Proposition.

Proclus' argument is as follows:

Let h , g be parallel lines and let another line k meet h in A . From B , a point K situated between g and h , drop a perpendicular to h .



As B recedes indefinitely far from A , its distance BC from h increases and exceeds any value, however great. In particular, BC will exceed the perpendicular distance between g and h . For some position of B , then BC will equal the distance between g and h . When this occurs, k will meet g .

Proclus made a number of assumptions, two of which are:

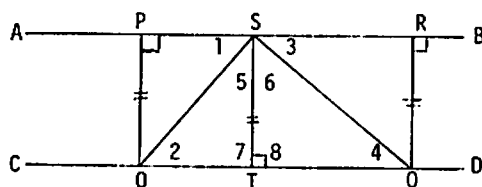
- 1) the distance from one of two intersecting lines to the other increases beyond all bounds as we recede from their common point.

Proclus was safe in making this assumption because it can be proved to be a theorem in basis E.

- 2) that the distance between two parallels never exceeds some finite value. (See [4])

This is where Proclus gets into trouble. Let's prove #2. The Fifth Postulate will follow by first proving that there exists a triangle with the sum of its angles equal to 180° .

Let AB and CD be the two lines everywhere equally distant. From any two points O and Q on CD draw OP and QR perpendicular to AB, and from any point S on AB draw ST perpendicular to CD.



By hypothesis OP, QR, and ST are equal. Since right triangles OPS and OTS are congruent, $\angle 1 = \angle 2$. Similarly, $\angle 3 = \angle 4$. We now have $\angle 1 + \angle 5 + \angle 3 + \angle 6 = 180^\circ$. Substituting $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, gives $\angle 2 + \angle 5 + \angle 6 + \angle 4 = 180^\circ$ so that the sum of the angles of triangle OSQ is

equal to two right angles which is equivalent to the Fifth Postulate. (See [9])

Jumping from the fifth century A.D. to 1733, we find the "first really scientific investigation" of the Fifth Postulate to be published. (See [3]) "While teaching grammar and studying philosophy at Milan, Gerolamo Saccheri had read Euclid's Elements and became interested in his use of the method of reduction ad absurdum. This method consists of assuming, by way of hypothesis, that a proposition to be proved is false; if an absurdity results, the conclusion is reached that the original proposition is true." Eventually, Saccheri tried out his favorite method on the baffling problem of proving the Fifth Postulate. (See [9]) Saccheri seems to have been the first to deny the Postulate in order to observe the consequences.

He began with an isoceles quadrilateral with two right angles, say at A and B.



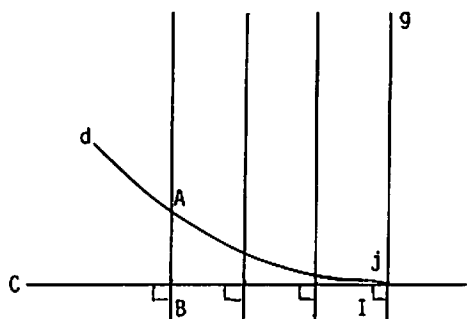
Saccheri proved that C and D are equal, without using the Fifth Postulate and that the line joining the midpoints of AB and CD is perpendicular to both lines.

Calling C and D summit angles, we know they are equal; now we have to decide if they are right, obtuse, or acute. If the Fifth Postulate holds then the angles

are right angles. Saccheri assumed, in turn, that they are obtuse and acute hoping to reach contradictions which could prove that they are right angles and that Postulate 5 holds.


By assuming the infinitude of straight lines, he reached a genuine contradiction with the obtuse angles. However, the acute angles did not bring him the needed contradiction. After a long argument, he states that line AB approaches a limiting line g as A moves without bound to the right,

that g is perpendicular to C , that c and d meet at an infinitely distant point I on g , that j between d and g is right,



and that therefore c and d are both perpendicular to g at I . Saccheri's contradiction is in violating Proposition 12 which says there is a unique perpendicular at each point of the line. Where Saccheri went wrong was in assuming that c and d meet. We know that they get closer and intuitively we know that they meet far off but this cannot be used as a mathematical proof. (See [4])

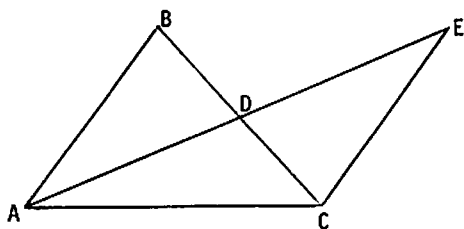
He could have proved that c and d do meet without Postulate 5, then reached a contradiction because A and B are assumed to be parallel. Without realizing it, Saccheri actually was the first to come close in discovering Non-Euclidean geometry.

John Heinrich Lambert (1728-1777) wrote about the discoveries he had made thirty-three years after Saccheri's publication. Lambert followed the same path as Saccheri, considering three hypotheses, calling them the hypothesis of the right angle, the hypothesis of the obtuse angle, and the hypothesis of the acute angle. However, Lambert's figure was that of a quadrilateral with three right angles (or one-half of a Saccheri quadrilateral ) and the fourth angle either right, obtuse, or acute. Like Saccheri, he reached a contradiction with the obtuse angle by assuming that straight lines are infinite. Also like Saccheri, he showed that the angle sum of a triangle is less than two right angles from the hypothesis of the acute angle. Lambert went farther than Saccheri by showing that the sum increases as the area decreases. He also "observed that special segments can be defined, constructed, and named, any two segments of the same name being congruent..." One of Lambert's special segments used as a unit segment

is known as an absolute unit of length. (See [4])

The last important contribution made during the tortorous struggle with Postulate 5 was made by Adrien Marie Legendre (1752-1833). His contribution was not with the originality of his work, but with the style of simplicity and elegance of his proofs. He joined Saccheri and Lambert in using the *reductio ad absurdum* method. Instead of the quadrilateral, his figure was the triangle. He proposed three hypotheses with the angle sum being equal to, greater than and less than two right angles, hoping to reject the last two. He eliminated the geometry based on the second hypotheses ($>$) by assuming the infinitude of straight lines and proving the following theorem:

The sum of the three angles of a triangle cannot be greater than two right angles.



However, he could not dispose of the third hypothesis (<). (See [9])

Independently and at about the same time, the discovery of a logically consistent geometry in which the Fifth Postulate was denied was made by Gauss in Germany, Bolyai in Hungary, Lobachewsky in Russia. Neither Bolyai or Lobachewsky lived to see their work given the recognition it deserved. The reasons were: slow passage of ideas from one part of the world to another, the language barrier, Kantian space philosophy, 2,000 years dominance of Euclid and the relative obscurity of the discoveries of Non-Euclidean geometry. (See [9])

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THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 February, 1983. The solutions will be published in the Fall 1983 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 342: Proposed by Michael W. Ecker, Pennsylvania State University, Worthington-Scranton Campus, Scranton, Pennsylvania.

It is well known that for nonnegative numbers, the geometric mean of the two numbers never exceeds their arithmetic mean (with equality of the two means precisely for equality of the numbers). Characterize all pairs (x,y) of nonnegative integers for which the arithmetic mean exceeds the geometric mean by exactly 1.

Problem 343: Proposed by Charles W. Trigg, San Diego, California.

In the square array of the nine non-zero digits

5 3 8

7 1 4

6 2 9

the sum of the digits in each corner 2-by-2 array is 16. Rearrange the nine digits so that each 2-by-2 corner array has a sum that is nine times the central digit.

Problem 344: Proposed by Charles W. Trigg, San Diego, California.

In what number bases is the repdigit, 33, a triangular number?

Problem 345: Proposed by Willie S.M. Yong, Republic of Singapore.

Show that the sequence of numbers defined by

$$[k + \sqrt{k} + .5], k = 1, 2, 3, \dots$$

i.e. 2, 3, 5, 6, 7, 8, ...

includes all prime numbers. Here $[x]$ denotes the greatest integer function and 1 is not considered to be a prime number.

Problem 346: Proposed by Fred A. Miller, Elkins, West Virginia.

Show that the area of the triangle formed by joining the centers of the three excircles is $abc/2r$ where r is the radius of the inscribed circle.

SOLUTIONS

332: Proposed by Charles W. Trigg, San Diego, California.

In the cryptarithm

$$\text{COOK} + \text{COOK} + \text{COOK} = \text{MEAL}$$

each distinct letter represents a different digit, and

MEAL is a permutation of consecutive digits. Find the unique restoration of this decimal addition.

Solution by Diana Wilson, University of Missouri-Rolla, Rolla, Missouri.

Since MEAL is divisible by three, and since M, E, A and L form a permutation of consecutive digits, $(M, E, A, L) = (6, 7, 8, 9)$, or $(3, 4, 5, 6)$ in some order.

Suppose that MEAL is a permutation of the digits 6, 7, 8 and 9. Then $(K, L) = (2, 6)$ or $(3, 9)$ in order to avoid duplicated digits. Now $(K, L) = (2, 6)$ implies that $A = 9$ and $O = 3$. But we must also have $2263 \leq \text{COOK} \leq 3292$ which requires $C = 3$ also. Suppose $(K, L) = (3, 9)$. Then $A = 6$ and $O = 2$ which requires $C = 2$ also.

Hence MEAL is a permutation of the digits 3, 4, 5 and 6. Reasoning as before $(K, L) = (1, 3)$, $(2, 6)$ or $(8, 4)$ and $1152 \leq \text{COOK} \leq 2181$. $(K, L) = (1, 3)$ implies that $C = 2$, $M = 6$, $O = 8$, $A = 4$ and $E = 6$ which is impossible. Similarly $(K, L) = (2, 6)$ implies that $C = 1$, $L = 6$, $O = 8$, $A = 4$ and $E = 6$ which is impossible. Hence $K = 8$, $L = 4$ and O is 7 or $1.0 = 7$ leads to $A = E = 3$; hence $O = 1$, $A = 5$, $E = 3$, $M = 6$ and $C = 2$. Thus $\text{COOK} = 2118$ and $\text{MEAL} = 6354$ is the unique solution.

Also solved by Fred A. Miller, Elkins, West Virginia; Scott Michael Jeffreys, Hofstra University, Hempstead, New York and the proposer.

333: Proposed by Charles W. Trigg, San Diego, California.

When asked the age of his brother, Ralph replied, "Jim's age, like mine, is one more than eight times the sum of its digits." How old is Jim?

Solution by Paul Hernandez, Southern Methodist University.

Let Jim's age be $10a + b$ for digits a and b and let Ralph's age be $10c + d$ for digits c and d . Now

$$8(a + b) + 1 = 10a + b$$

and a similar relation involving Ralph's age can be obtained by taking $a = c$ and $b = d$.

Hence $2a = 1 + 7b$ or $2a \equiv 1 \pmod{7}$.

Thus $a \equiv 4 \pmod{7}$ and since a is a digit, we must have $a = 4$, $b = 1$ and Jim is 41 years old.

Similarly Ralph is 41.

Also solved by Fred A. Miller, Elkins, West Virginia; Diana Wilson, University of Missouri-Rolla, Rolla, Missouri; Scott Michael Jeffreys, Hofstra University, Hempstead, New York and the proposer.

334: Proposed by Charles W. Trigg, San Diego, California.

In the decimal scale, find a three-term arithmetic progression of three-digit primes in which the first and last terms are permutations of the same set of digits and the three digits of the middle term are distinct.

Solution by the proposer.

There are 75 pairs of three-digit primes in which each member of the pair is a permutation of the same digit set. Eleven of these pairs have a prime arithmetic mean, thus determining the following A.P.'s with common differences:

$d = 72$: 127, 199, 271; 839, 911, 983

$d = 90$: 137, 227, 317; 241, 331, 421; 467, 557, 647;
571, 661, 751; 683, 773, 863; 797, 887, 977

$d = 180$: 593, 773, 953

$d = 234$: 163, 397, 631

$d = 270$: 179, 449, 719

In only one progression are the digits of the middle term distinct, namely: 163, 397, 631.

Also solved by Fred A. Miller, Elkins, West Virginia.

335. *Proposed by the editor. ;*

Given a point P on one side of the triangle ABC, construct a line through P which divides the triangle into two regions of equal area.

Solution by Thu Pham, University of Texas at San Antonio, San Antonio, Texas.

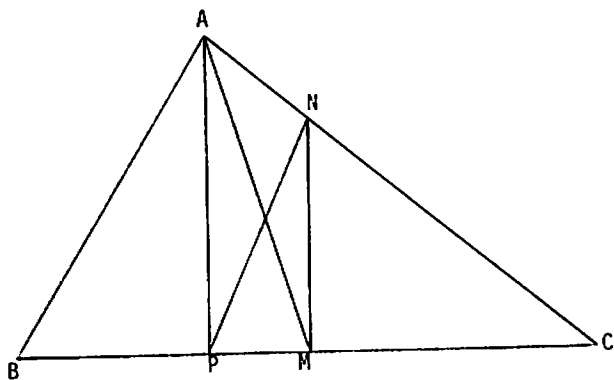


Figure. Problem 335

Suppose the point we choose is on side BC and that M is the midpoint of BC . Draw AP and AM . Let $S(ABC)$ denote the area of triangle ABC . Then

$$S(ABM) = S(AMC) = \frac{1}{2} S(ABC).$$

From the point M draw $MN \parallel AP$ and let this line intersect AC at N . Then the line PN is the desired line.

To prove the validity of the construction, note that $MN \parallel AP$ and that triangles APN and APM have equal altitudes to the common base AP . Hence $S(APM) = S(APN)$. Finally $S(ABP) + S(APN) = S(ABM) = \frac{1}{2} S(ABC)$. If $P = M$, then PN is the median AM ; if $P = B$ (or C), then PN is the median from B (or C) to AC (or AB) respectively.

Also solved by Fred A. Miller, Elkins, West Virginia; Charles W. Trigg, San Diego, California; L. J. Upton, Mississauga, Ontario and John A. Winterink, Albuquerque Technical-Vocational Institute, Albuquerque, New Mexico.

336: Proposed by Fred A. Miller, Elkins, West Virginia.

Let A, B, C and D be any four concyclic points such that C and D are separated by A and B . If P_1, P_2 and P_3 denote the lengths of the perpendiculars from D to the line AB, BC and CA respectively, show that

$$\frac{AB}{P_1} = \frac{BC}{P_2} + \frac{CA}{P_3}$$

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

From the figure below, we have the following relations:

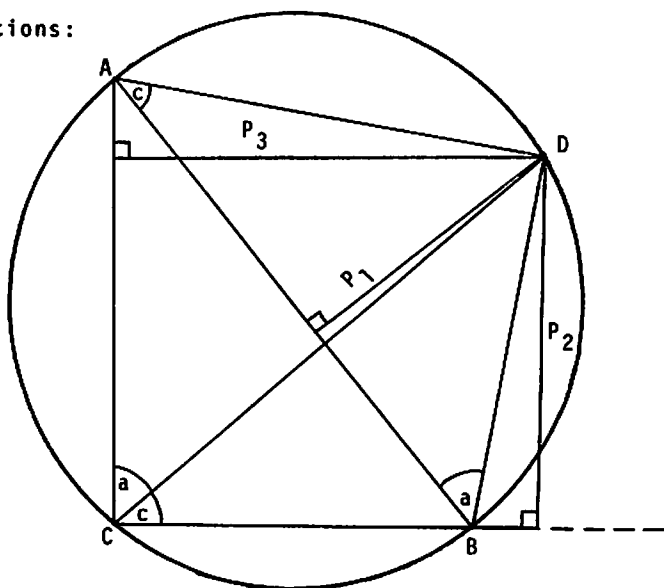


Figure. Problem 336

$$P_1/BD = \sin a = P_3/CD \text{ and } P_1/AD = \sin c = P_2/CD.$$

$$\text{Hence } AD = CD \cdot (P_1/P_2) \quad \text{and} \quad BD = CD \cdot (P_1/P_3).$$

Ptolemy's Theorem states that in a cyclic quadrilateral, the product of the diagonals is equal to the sum of the products of the opposite sides.

$$\text{Thus} \quad AB \cdot CD = AD \cdot BC + CA \cdot BD. \quad (1)$$

Substituting the expressions for AD and BD into equation (1) and simplifying one obtains the desired result.

Also solved by Thu Pham, University of Texas at San Antonio, San Antonio Texas, Tho Pham, Tom C. Clark High School, San Antonio, Texas and the proposer (two solutions).

THE MATHEMATICAL SCRAPBOOK

Edited by Richard L. Barlow

Readers are encouraged to submit Scrapbook material directly to the Scrapbook editor. Material submitted will be used whenever possible and acknowledgement will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Dr. Richard L. Barlow, Department of Mathematics, Statistics and Computer Science, Kearney State College, Kearney, Nebraska 68847.

Through the years many interesting mathematical problems have been handed down from generation to generation. One such series of problems which arouses mathematical curiosity is the so-called cannibal-missionary problem. The solutions of this problem result in interesting applications of graph theory, number theory, geometry, and probability.

We first consider the following problem.

Three missionaries and three cannibals are traveling together and wish to cross a river. The only mode of transportation at their disposal is a boat which will transport at most two people. In crossing the river, at least one person must be in the boat. However, at no time (not even for an instant) can the missionaries be outnumbered by the cannibals on either bank. How can the missionaries and the cannibals cross the river?

The solution to the above problem can be considered using several approaches. A graphical solution to the problem is as follows.

Let us consider the problem with respect to the

situation on the first bank. Let c be the number of cannibals and m be the number of missionaries on the first bank. Hence, the ordered pair (c,m) denotes the number of cannibals and missionaries on the first bank at any instant. Also, since c and m must be integers with $0 \leq c \leq 3$ and $0 \leq m \leq 3$, we start with 16 ordered pairs as indicated in the lattice of points below in figure 1.

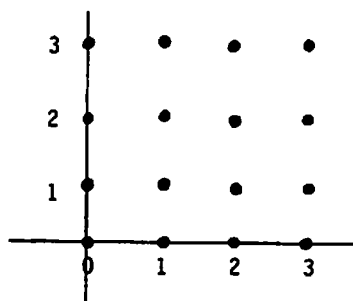


Figure 1

We first eliminate from consideration the points in Figure 1 which must be ignored due to the rules of the problem. The cannibals outnumber the missionaries on the first bank with points $(2,1)$, $(3,1)$, and $(3,2)$ and on the second bank with points $(0,1)$, $(0,2)$, $(1,2)$. Hence, the permissible points are as indicated in Figure 2.

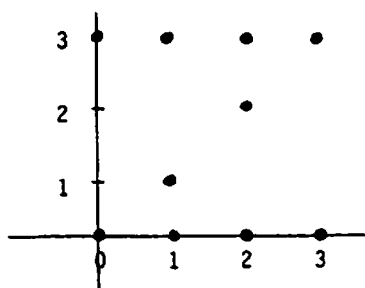


Figure 2

We start on the first bank with the point (3,3) and eventually reach (0,0) through a sequence of boat rides following our set of rules: These rules are as follows:

- (1) If we are at state (c_i, m_i) and the boat is at the first bank, then we can reach permissible state (c_{i+1}, m_{i+1}) provided $c_i \geq c_{i+1}, m_i \geq m_{i+1}$ and $(c_i + m_i) - (c_{i+1} + m_{i+1}) \leq 2$.
- (2) If we are at state (c_i, m_i) and the boat is at the second bank, then we can reach permissible state (c_{i+1}, m_{i+1}) provided $c_{i+1} \geq c_i, m_{i+1} \geq m_i$ and $(c_{i+1} + m_{i+1}) - (c_i + m_i) \leq 2$.
- (3) The overall sequence of states should be minimized.

The solution to the problem can be accurately

represented using a directed graph which connects the ten permissible points in Figure 2. One such representation is as in Figure 3, where the directed paths are indicated with arrows. Note that we must begin with $(3,3)$ and proceed to $(0,0)$ using the three rules above.

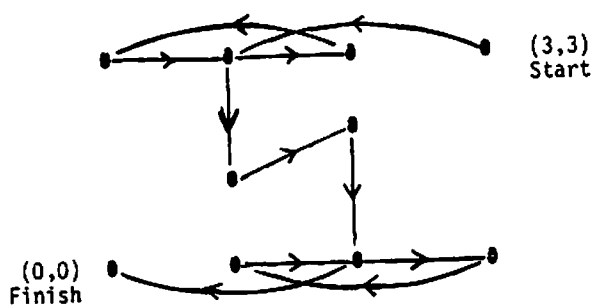


Figure 3

Figure 3 may be summarized with the following table where the odd numbered boat crossings are river crossings from the first bank to the second bank and the even numbered boat crossings are from the second bank to the first bank.

Boat Crossing Number	Boat Termination Point	As Boat Crosses...		
		On First Bank	In Boat	On Second Bank
0	(3,3)	ccmmmm	--	--
1	(1,3)	cmmmm	cc	--
2	(2,3)	cmmmm	c	c
3	(0,3)	mmmm	cc	c
4	(1,3)	mmmm	c	cc
5	(1,1)	cm	mm	cc
6	(2,2)	cm	mc	mc
7	(2,0)	cc	mm	mc
8	(3,0)	cc	c	mmmm
9	(1,0)	c	cc	mmmm
10	(2,0)	c	c	mmmc
1	(0,0)	--	cc	mmmc

One will note it takes 11 boat crossings to complete the required action. Graph theory provides a similar solution using directed signed graphs. It can be shown that there is no solution to this problem which requires less

than 11 boat crossings. Can you show this? Can you find any other solutions involving exactly 11 crossings?

With the same rules but with only two missionaries and two cannibals the following solution is possible.

Boat Crossing Number	Boat Termination Point (c,m)	As Boat Crosses...		
		On First Bank	In Boat	On Second Bank
0	(2,2)	ccmm	--	--
1	(0,2)	mm	cc	--
2	(1,2)	mm	c	c
3	(1,0)	c	mm	c
4	(2,0)	c	c	mm
5	(0,0)	--	cc	mm

Are there any other solutions with five or less boat crossings?

For further consideration, attempt the problem with four cannibals and four missionaries. Also, the following is a slight change from the three missionaries and three cannibals problem.

Three married couples must get into town in a small sport car which holds only two people. How can they accomplish this so that no wife is ever left with either of the other husbands unless her own husband is also present?

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical issues of interest to undergraduate students are discussed in length. Here by *issue* we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.

The following article is presented by Prof. Michael W. Ecker. After reading it, we tried to see what issue does it stir up which we could point out for the reader of the Hexagon to think about and perhaps respond to. We found a few.

One of the references of the article is a most delightful book by Ross Honsberger: Mathematical Gems (same can be said for his second book). It is an absolute must for every mathematic major to browse through the thirteen topics from elementary mathematics examined there and read the appealing ones carefully. There are, undoubtedly, many chapters appealing to everyone.

Another point of interest in the following article is examples of the fact that some apparently unrelated problems are approachable by the same mathematical method. There must be more of such examples.

Finally, we (the mathematics instructors) are always in an eternal challenge with that invincible monster: 'the word problem' (both of the problems discussed in the article to follow are word problems). Many students (both in algebra and in calculus) are heard stating their 'can do all' and then 'but have trouble with word problems'. This instructor is of the belief (and always tells his students) that much of mathematics is word problems. We must never shy away from a 'word problem' but in fact approach any subject with an eye toward how it developed to answer a question described in words, i.e. a word problem.

The readers (especially students) are invited to share their thoughts on these matters (particularly the word problems).

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SOME RECREATIONAL APPLICATIONS
OF ELEMENTARY NUMBER THEORY

Michael W. Ecker*

Consider any army battalion of unspecified numerical size, for which it is known that the soldiers may be arranged into equal-sized rows and equal-sized columns in exactly 64 ways. Counting j rows by k columns as distinct from k rows by j columns, what is the smallest possible size of this battalion? What about a largest possible size? (While the existence of a minimum is intuitively plausible, that of a maximum is not so plain.)

The question involving minimality was posed to me not long ago by a mathematically-inclined non-mathematician friend, Edward Leader of New York City. Ed had seen it in [1], with answer but no full solution provided there. Incidentally, this reference credited the question to Geoffrey Chaucer's Canterbury Tales. After some explorations, Ed and I compared methods of solution and found that we had each handled the question quite differently. This article considers both solutions as well as attempts at reconciliation and generalization.

*Professor Ecker received his Ph.D. in mathematics from City University of New York and is currently a member of the mathematics department of Pennsylvania State University, Worthington Scranton Campus. His interests include problem solving, recreational mathematics and analysis.

First, however, consider this apparently unrelated question: An eccentric jailer offers the following partial amnesty to his 1000 imprisoned-for-life convicts, who are each alone in the 1000 closed cells. On day #1 every cell is opened. On day #2, every second cell is reversed. (i.e. cells 2, 4, 6, ..., 1000 are closed.) On day #3, every third cell is reversed (open cells among 3, 6, 9, ... are now closed and closed ones are now opened). In general, on day #k, for $k=1,2,3, \dots, 1000$, every k^{th} cell is reversed. No prisoner may leave his cell on any of the 1000 days, but here's the deal: Any prisoner in an open cell after 1000 days may go free. How many prisoners go free? Which ones?

It should be clear that both of the two questions relate to divisibility. The latter question is a gem which I first saw in a mathematics newsletter; later on, a colleague reminded me that the problem appears in [2], part of an excellent series by Honsberger. I include it here because of its connection to the first problem and because some readers may be unfamiliar with Honsberger.

The reader is invited to try his hand at one or both of these questions before reading the solutions that follow.

Preliminaries: Since divisors of natural numbers are involved, we consider the number-theoretic function $\tau(n)$ ("tau of n "), which is defined to be the number of divisors of n (counting 1 and n as divisors). To avoid trivialities, we assume $n > 1$. By the fundamental theorem of arithmetic, every positive integer n is expressible uniquely, except for order, as a product of primes raised to powers:

$$n = p_1^{a_1} \cdot p_2^{a_2} \dots p_k^{a_k}. \quad (1)$$

(The uniqueness will be used implicitly hereafter.)

Since a divisor necessarily has the form $p_1^{b_1} \cdot p_2^{b_2} \dots p_k^{b_k}$, where each b_j has one of the values $0, 1, 2, \dots, a_j$, there are as many divisors of n as there are ways to choose k -tuples (b_1, b_2, \dots, b_k) . There are $a_1 + 1$ choices for b_1 ; $a_2 + 1$ choices for b_2 ; and so on. By the fundamental counting principle, then, there are $(a_1+1)(a_2+1) \dots (a_k+1)$ choices of k -tuples (b_1, b_2, \dots, b_k) . Hence,

$$\tau(n) = (a_1+1)(a_2+1) \dots (a_k+1). \quad (2)$$

As an immediate consequence of this, we have an interesting and initially surprising result: The only natural numbers n for which $\tau(n)$ is odd are the perfect squares. Reason: $\tau(n)$ is odd if and only if every factor

$a_j + 1$ is odd; this in turn is true if and only if every a_j is even, which is equivalent to saying that n is a perfect square, since we can write

$$p_1^{2c_1} \cdot p_2^{2c_2} \cdot \dots \cdot p_k^{2c_k} = (p_1^{c_1} \cdot p_2^{c_2} \cdot \dots \cdot p_k^{c_k})^2.$$

Solution to jailer problem: A prisoner's cell is open if and only if it has been reversed an odd number of times. Recall that all the cells were initially closed, that every cell was reversed on day #1, every second one on day #2, etc. The only ones finally open are ones reversed precisely once, or thrice, etc. So, cell n is open after 1000 days if and only if $\tau(n)$ is odd. By the previous paragraph, cell n is open if and only if n is a perfect square. As $31^2 < 1000 < 32^2$, there are 31 prisoners who get to go free.

Solution to army problem: The key here is the recognition that there are precisely as many ways to form an army of size n as there are divisors of n . To verify this, note that we have a correspondence, where $n = a \times b$, as follows:

'a = number of rows of battalion', corresponds to
'a = divisor of n'.

This works even if n is a perfect square, q^2 , since we would not count q columns by q rows twice and likewise would not count q twice as a divisors of n . It follows

that what we seek is the smallest positive integer n satisfying $\tau(n) = 64$. Using (2), we see that n , which has factored form (1), must satisfy $(a_1+1)(a_2+1)\dots=2^6$. For minimality, we should choose our primes p_j in (1) to be as small as possible. Thus, we take $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, ..., the usual sequence of primes, for any choice of corresponding exponents a_j satisfying $(a_1+1)(a_2+1)\dots=2^6$. Note that the number of primes is unspecified, since we do not yet know n . However, it is further clear that the sequence a_j is nonincreasing, for, otherwise, we would be able to obtain a smaller solution n by simply interchanging exponents so that the larger power was on the smaller prime.

We have several possibilities at this point. We might have each $a_j+1 = 2$, for $j = 1, 2, 3, 4, 5, 6$, so that each $a_j = 1$. We could also have, say, $a_1+1 = 4$ and all other $a_j + 1 = 2$, for $j = 2, 3, 4, 5$. (Note the one fewer a_j .) Or perhaps we should even take $a_1 + 1 = 8$, with $a_2+1 = 8$. Or $a_1 + 1 = 8$ and all other $a_j + 1 = 2$, $j = 2, 3, 4$; etc.

Eventually, I surmised that the smallest n was obtained upon using $a_1 + 1 = 4 = a_2 + 1$, and $a_3 + 1 = 2 = a_4 + 1$. This gives $n = 2^3 \cdot 3^3 \cdot 5^1 \cdot 7^1 = 7560$. But I regard it dissatisfying to resort to so much trial and error.

Is there another way?

This brings us to Ed Leader's solution. Proceeding from scratch somewhat intuitively, Ed observed that $2^6 = \binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}$, where the symbols $\binom{6}{j}$ denote the usual binomial coefficients which count the number of ways of choosing j objects from a set of 6. He argued that because this is the number of divisors altogether, it is possible to form a number having 6 divisors among the natural numbers beginning with 2, provided the natural number is not a product of previous natural numbers in the list. That is, beginning with 2, one could write the product of the first 6 natural numbers, in order, such that no natural number is a product of previous ones. Thus, we get $n = (2)(3)(4)(5)(7)(9)$, which also gives 7560. (Note that 6 and 8 were included since each is a product of earlier factors.)

This works, Ed argued, because divisors correspond to choices of factors from $(2)(3)(4)(5)(7)(9)$ taken several at a time, with the divisor of 1 corresponding to the understood factor of 1 not shown here, corresponding to choosing 0 of 6 factors. The $\binom{6}{1} = 6$ choices refer to 2, 3, 4, 5, 7, and 9. The $\binom{6}{2}$ choices refer to $(2)(3)$, $(2)(4)$, $(2)(5)$, etc.

After a scrutiny, during which I persuaded myself

of the correctness of this method, I realized how the method compares to the solution I indicated earlier. I then considered Ed's approach, which applies to any power of 2, noting the way that the product $n = (2)(3) \dots$ was formed. If there are 3 factors or more, then 4 will be next as a factor of n . If there are only 2 factors, however, 2 occurs only to the power of 1. If there are at least 3 factors, 2 occurs to at least the power of 3. When one gets an example in which $\tau(n) = 2^p$ with p at least 9, then one reaches $n = 2 \cdot 3 \cdot 4 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 16 \dots$. So, now 2 occurs to at least the power 7. The alert reader will note that these are precisely the powers I found as candidates for the a_j 's earlier: 1, 3, 7, etc. Similar remarks apply to powers of 3 and other primes: each must be to these powers by virtue of every $a_j + 1$ being a power of 2.

The question of generalizing Leader's method when $\tau(n)$ is not a power of 2 is open, as far as I know at this time. Experimenting with a few numbers does not lead to an immediate generalization.

As to the remaining question of a largest size army, I suggest that the reader settle the question negatively by showing that there are infinitely many numbers m with the property that $\tau(m) = 64$. A similar question in a

slightly more general form appears in [3].

I close with the hope that some reader will be able to extend the limited insights provided in this article.

REFERENCES

- [1] Shirley Cunningham, The Pocket Entertainer, Picket Books, 1942, p. 8.
- [2] Ross Honsberger, Mathematical Gems, Mathematical Association of America, 1973, p. 68. (For a second nice proof concerning $\tau(n)$'s parity, see p. 64 as well.)
- [3] Niven and Zuckerman, An Introduction to the Theory of Numbers, second edition, John Wiley & Sons, 1968, p. 95, exercise #13.

KAPPA MU EPSILON NEWS

EDITED BY HAROLD L. THOMAS, HISTORIAN

News of chapter activities and other noteworthy KME events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department, Pittsburg, Kansas 66762.

CHAPTER NEWS

Alabama Gamma, University of Montevallo, Montevallo
Chapter President - Linda Rinehart
12 actives

The chapter held several meetings during the fall semester including an evening meal together. Plans were also begun for the spring semester's annual high school mathematics tournament. Other 1981-82 officers: D'Andrea Cardone, vice president; Charlene Garrett, secretary; Ruth Smith, treasurer; Joseph Cardone, corresponding secretary; Angela Hernandez, faculty sponsor.

Alabama Zeta, Birmingham - Southern College, Birmingham
Chapter President - Lawrence R. Shoemaker
25 actives

The fall semester, 1981, was devoted primarily toward establishing organizational principles by which our chapter will operate. At our first meeting a change of officers from those selected at the end of last year was performed. Also, plans were discussed regarding the year's activities. The first and most important business was to create and adopt a system of bylaws. A committee was established which created a set of bylaws, which after several revisions, was adopted at a subsequent chapter meeting. KME sponsored a guest speaker from the University of Alabama in Birmingham, Dr. Kirk, who discussed mathematical applications in the field of biomedical statistics. KME also sponsored speakers in conjunction with the other local campus science clubs: American Chemical Society, Beta Beta Beta, and Alpha Epsilon Delta. Dr. Beaton, also from U.A.B., spoke on "Naturally Occurring Hallucinogens", and Dr. Porter, from

Brookwood Medical Center in Birmingham, lectured on Immunology. Nomination and election of officers to replace departing ones for the coming spring term occupied the last official business for the fall term. The fall term ended with a Christmas party for the science and math students and teachers, organized and set up by the campus math and science clubs. KME was actively involved in organizing this party. Other 1981-82 officers: Charles E. Runnels, vice president; Renee Brown, secretary; Cheng Fon Lin, treasurer; Lola F. Kiser, corresponding secretary; Sarah E. Mullins, faculty sponsor.

Arkansas Alpha, Arkansas State University, State University

Chapter President - John Fiala
19 actives, 17 initiates

The chapter held biweekly meetings during the fall semester at which members of the faculty spoke on topics varying from "The History of Mathematics" to "The Uses of Microprocessors in Math Education." In addition, a social for the Math and Physics Department faculties and the university administration was held. A Christmas party was held at Dr. R. P. Smith's home in December. Other 1981-82 officers: Kathy Steffy, vice president; Trena Richardson, secretary-treasurer; Jerry Linnstaedter, corresponding secretary; R. P. Smith, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President - Lori Canter
40 actives, 24 initiates

A fall faculty-student picnic began the semester. Chapter meetings featured speakers from business and industry. Workshops were held to produce materials for the county-wide Junior High Math Field Day to be held in the spring. A Christmas social and pledge ceremony were held at the end of the quarter. Other 1981-82 officers: Maryjean Rende and Jim Carley, vice presidents; Nancy Miller, secretary; Tom Crump, treasurer; George R. Mach, corresponding secretary; Adelaide Harmon-Elliott and Dina Ng, faculty sponsors.

Colorado Alpha, Colorado State University, Fort Collins
Chapter President - Michael Thomas
20 actives, 2 initiates

Fall semester activities included freshman orientation ice-breaker in September and Dr. Du Chateau's magic show in October. The chapter assisted at the Math Department's annual Math Day in November. Colorado high school students compete individually and on teams to solve high school math problems for scholarships and prizes. Other 1981-82 officers: Mary Jo Black, vice president; Paul Magnus, secretary; Anne Murray, treasurer; Arne Magnus, corresponding secretary and faculty sponsor.

Connecticut Beta, Eastern Connecticut State College, Willimantic
Chapter President - Michael Lamb
26 actives

Other 1981-82 officers: Carole Grenier, vice president; April Schulze, secretary; Ann M. Curran, corresponding secretary; Steve Kenton, faculty sponsor.

Florida Beta, Florida Southern College, Lakeland
Chapter President - Beth Brisbin
18 actives, 4 initiates

Other 1981-82 officers: Marie Oliver, vice president; Peter Langendorff, secretary and treasurer; Henry Hartje, corresponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Tammy Woodworth
18 actives

An October meeting of the chapter consisted of two short films, "IBM Peepshow" about mathematical principles throughout history and "Some Call it Software" about computers. Refreshments were served during the films. Other 1981-82 officers: Annelle Colevins, vice president; Darla House, secretary; Wendy Muse, treasurer; Thomas J.

Sharp, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston
Chapter President - Marcus Maier
25 actives

The fall semester calendar included several joint meetings with the math club. Specific activities included a picnic in October, a Halloween party, candy sales, career night, and a Christmas party. Other 1981-82 officers: Pat Edwards, vice president; Jane Fischer, secretary; Shirley Hall, treasurer; Lloyd Koontz, corresponding secretary; Delmar Crabill, faculty sponsor.

Illinois Epsilon, North Park College, Chicago
Chapter President - Peter Nelson
15 actives, 9 initiates

Fall activities included a dinner meeting at which time new members were initiated. The chapter discussed potential future activities of the chapter as well as the meaning of the different symbols in the Kappa Mu Epsilon emblem. Other 1981-82 officers: Roger Wickstom, vice president; Kathleen Reed, secretary; Marilyn Meyer, treasurer; Alice Iverson, corresponding secretary and faculty sponsor.

Illinois Eta, Western Illinois University, Macomb
Chapter President - Hans Hamilton
7 actives

Fall semester activities included a lunch for mathematics department members to raise money, a presentation by Dr. Gerald White of the WIU mathematics department on the TRS-80 microcomputer, a volleyball game with the physics and chemistry clubs (which KME won!), a speaker from Bankers Life and Casualty Insurance Company speaking on careers as an actuary, and an end of the semester pizza party. Other 1981-82 officers: Karen Seehafer, vice president; Arthur Zenner, secretary and treasurer; Alan Bishop, corresponding secretary; Iraj Kalantari, faculty sponsor.

Illinois Zeta, Rosary College, River Forest

Chapter President - Michael Renella

8 actives, 3 initiates

Members met to discuss initiation of new members, ways of raising money to attend the Region II convention and plans for a mathematics clinic to be of service to students facing examinations. Other 1981-82 officers: Mary Potaczek, vice president; Mary T. Brady, secretary; Brad Erickson, treasurer; Sister Nona Mary Allard, corresponding secretary and faculty sponsor.

Indiana Alpha, Manchester College, North Manchester

Chapter President - Ramona Seese

23 actives

The chapter meets regularly with the Math Club. Other 1981-82 officers: Craig Stine, vice president; Karen Rund, secretary; Larry Holston, treasurer; Ralph McBride, corresponding secretary and faculty sponsor.

Indiana Delta, University of Evansville, Evansville

Chapter President - Wadieh Hawa

30 actives

Fall semester programs included "Perfect Numbers" by Wadieh Hawa, "Mathematical Thinking about Politics" by Dr. Duane Broline, and "Three Philosophical Theories of the Existence of Mathematical Entities" by Dr. Bruce Paternoster. Other 1981-82 officers: Brett Barnett, vice president; Brent Moore, secretary and treasurer; Melba Patberg, corresponding secretary; Duane Broline, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President - Kay Sacquitne

35 actives, 2 initiates

The following students presented papers at chapter meetings this fall: Greg Rasmussen on "Micro-Computers",

Pam Ryerson on "A Different Shade of Geometry" and Douglas Lane on "General Systems Theory". Professor Greg Dotseth hosted the annual Christmas party in his home. There was a large turnout for the party this year. Iowa Alpha organized sales of jackets with the name of the University and the Department stenciled on them. Members of the chapter also had "KME" sewn on their jackets. Other 1981-82 officers: Darla Dettmann, vice president; Margaret Chizek, secretary; Charles Daws, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President - Karen Waltmann
25 actives

A program was given by Tony Hogge who outlined a general procedure for solving Rubik's cube. In preparation for the chalk talk at Math Field Day, Allan Guetzlaff presented the rudiments of Boolean algebra. At the final meeting for the semester, Kathy Schultz challenged the group with several interesting "brain teasers". Other 1981-82 officers: Allan Guetzlaff, vice president; Jean Movall, secretary; Edmond Bonjour, treasurer; Lynn Olson, corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President - Brenda Brinkmeyer
40 actives, 14 initiates

The Chapter held monthly meetings in October, November and December. In addition, a fall picnic was hosted for all mathematics and physics students. Fall initiation for new members was held at the October meeting. Fourteen new members were received at that time. The October program was given by Janet Schwenke on "Math Models." Professor William Self from the Mathematics Department spoke at the November meeting about "Euler." In December, a special Christmas meeting was held at the home of Dr. Helen Kriegsman, Mathematics Department Chairman. Patrick Lopez presented a history of the abacus and demonstrated "Fun with Numbers." Other 1981-82 officers: Linda McCracken, vice president; Hazel Kent, secretary; Paige

Chilton, treasurer; Harold L. Thomas, corresponding secretary; J. Bryan Sperry, faculty sponsor.

Kansas Beta, Emporia State University, Emporia
Chapter President - Robert Baumer
27 actives, 9 initiates

The chapter held monthly meetings during the fall semester. In addition, a banquet was held for new initiates. At the fall banquet, a guest speaker from the Physical Science Department gave a talk on mathematics and its relationship to energy. Plans were also made for Dr. Marion Emerson, ESU mathematics professor, to give a talk in the spring on "The Rubik's Cube: It's Solution using Group Theory." Other 1981-82 officers: Jeff Roberts, vice president; Mary Beuerlein, secretary; Julie Romine, treasurer; Susan K. Vopat, historian; John Gerriets, corresponding secretary; Thomas Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Steve Pahls
17 actives, 19 initiates

Kansas Gamma began the new fall term by sponsoring a picnic for members and interested mathematics students. Liz O'Brien and Jim Graham were initiated September 30, 1981. In November the chapter heard career related talks by Barb Holder, Systems Analyst for Benedictine College, and by Elaine Tatham, Vice-President of a Management Consultant Firm in Kansas City (on leave from her position as Director of Institutional Research for Johnson County Community College). Fall activities culminated with the traditional Wassail Party held this year at the home of Richard Farrell on the evening of December 9. Other 1981-82 officers: Terri Beye, vice president; Kay Kreul, secretary; Tom Gallagher, treasurer; Leslie Peabody, historian; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President - Cindy Dietrich
20 actives

Two fall semester meetings were held. Ken Wilke gave a presentation on the relationship between the law and mathematics at one of the meetings. At the other, a faculty member discussed the format he was using in his systems analysis class. Other 1981-82 officers: Kathy King, vice president; Kevin Heideman, secretary; John Lichtenhan, treasurer; Robert Thompson, corresponding secretary; Billy Milner, faculty sponsor.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President - Don Jesch
19 actives

Fall activities included a picnic in September and Halloween party in October. The chapter also assisted the mathematics department with the Math Relays. Student presentations on logic and visualization and the film "Challenging Conjectures" were given at meetings. Other 1981-82 officers: Maxine Arnoldy, vice president; Sally Irvin, secretary and treasurer; Charles Votaw, corresponding secretary; Jeffrey Barnett, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President - Kevin Preston
26 actives

Chapter meetings were held on a bi-weekly basis. Professor Alvin McGlasson gave a talk on the Golden Ratio, and Professor Paul Blond talked on Mathematical Fallacies. A T-shirt design contest was won by Dr. Don Greenwell. Members conducted weekly tutoring sessions for undergraduate students. Social activities included a picnic at Boonesborough State Park, a Halloween party at the home of Dr. Dorian Yeager, and a Christmas party for Mathematical Sciences faculty and students. Other 1981-82 officers: Judy Dusing, vice president; Andrea Norris, secretary; Beth Stewart, treasurer; Dorian P. Yeager, corresponding secretary; Don L. Greenwell, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore
Chapter President - Barbara Slezak
6 actives, 2 pledges

Other 1981-82 officers: Rita McCardell, vice president and treasurer; Charlotte Corini, secretary; Sister Marie A. Dowling, corresponding secretary; Sister Delia Dowling, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President - Sally Carlson
22 actives, 3 initiates

Fall programs included: Charles Wheatley '80 (graduate student at George Washington University) who spoke on "Operations Research" and a career panel which consisted of four former KME members from classes of '70, '74, '75, and '76. The panelists returned to speak on their mathematically-related jobs: teacher, computer systems analyst (DOD), computer analyst (private research firm), accountant and financial consultant/manager. The chapter initiated three new members including new faculty member Richard Dillman. Speaker for the initiation was Dr. Jack Clark who spoke on "Complex Eigenvalues." Other activities included sponsoring a weekly math puzzle column in the college newspaper which awards prizes to correct solvers, sponsoring a welcoming picnic for all mathematics majors at beginning of school, selling "academic" T-shirts to raise funds, and sponsoring several movies on mathematical subjects on one evening. Other 1981-82 officers: Lisa Del Prete, vice president; Pamela Huffington, secretary; Gail Meadows, treasurer; James Lightner, corresponding secretary; Robert Boner, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President - Douglas Cannon
19 actives

The chapter assisted the Department of Mathematics with its annual symposium for mathematics educators on November 6, 1981. On the evening preceding the symposium, KME sponsored Sheila Tobias in a talk "Overcoming Math Anxiety", attended by many students on campus. The chapter continued to provide a tutoring session each week for mathematics students. Three KME members of the class of 1981 were among the eight seniors chosen to

receive the John Allison Outstanding Senior Award. They were Marcia Hahn, James Martens and Luther Yost. Other 1981-82 officers: John Wagner, vice president; Kathy Hardy, secretary; Timothy Lambert, treasurer; Agnes Yount, corresponding secretary; John P. Jones, faculty sponsor.

Michigan Beta, Central Michigan University, Mt. Pleasant
Chapter President - Judy Macgrayne
45 actives, 16 initiates

The fall started with a KME picnic for members and faculty. Fall initiation was held in November with Professor James Bidwell speaking on the history of the quadratic formula. A homecoming Alumni coffee hour was held the morning of the homecoming football game. KME members conducted help sessions for undergraduate mathematics classes. Other 1981-82 officers: Don Franck, vice president; Sandy Dolde, secretary; Jolyn Cornell, treasurer; Arnold Hammel, corresponding secretary and faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President - Jayne Ward
47 actives, 14 initiates

Other 1981-82 officers: Donna Garoutte, vice president; Debra Oehlschlaeger, secretary; Michael Eidson, treasurer; M. Michael Awad, corresponding secretary; L. Thomas Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President - Ellen Frieze
21 actives, 8 initiates

Fall semester activities included three regular meetings, an initiation ceremony, and a Christmas party. Other 1981-82 officers: Rita Rotert, vice president; Lisa Oshima, secretary; Lisa Weidinger, treasurer; Homer Hampton, corresponding secretary; Larry S. Dilley, faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President - Kathy Bremer
8 actives

Other 1981-82 officers: Sarah Abnstedt, vice president; Kirk Meyer, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

Missouri Zeta, University of Missouri-Rolla, Rolla
Chapter President - Bill Pulse
25 actives, 14 initiates

The chapter held its first meeting for the fall semester in September. General meetings are held on the second Tuesday of each month while officers meet on the first and third Tuesdays. The annual fall outing was a picnic at Schuman Park. It was well attended by KME members and departmental faculty and families. Professor Johnny Henderson assumed corresponding secretary responsibilities replacing Tim Wright who is on a leave of absence with the National Science Foundation in Washington, D.C. A committee worked on construction of a movable KME sign to be used to publicize the chapter. Peter Ho, of the Computer Science Department, gave a program on computer robotics and their uses in industry. He demonstrated with a working miniature robot. Professor Troy Hicks, of the Mathematics/Statistics Department, talked about some old number theory problems and history related to them. Fourteen new initiates were received at the November meeting held at Chub and Jo's Restaurant. Professor Arlan DeKock, of the Computer Science Department, gave the program on artificial intelligence. Other 1981-82 officers: Michelle Maes, vice president (fall); Eva Taylor, vice president (spring); Donna Miller, secretary (fall); Peggy Baker, secretary (spring); Bob Nespodzany, treasurer (fall); Anne Burton, treasurer (spring); Lynn Brammeier, historian (fall); Karen Anderson, historian (spring); Johnny Henderson, corresponding secretary; Jim Joiner, faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville
Chapter President - Ruth Dare
15 actives, 6 initiates

Fall semester activities included open house for all mathematics students sponsored by KME, Halloween party, Christmas party, participation in United Fund Drive on campus, and a Christmas project for needy children. Other 1981-82 officers: Anita Fashing Kiska, vice president; Mary Nelson, secretary; Neil Myer, treasurer; Sam Lesseig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin
Chapter President - Rick Richardson
12 actives

The chapter held regular business meetings. Other activities were a fall social which included a 6-mile float trip down the Elk River, preparation of a homecoming banner, working concessions at ball games to raise money, serving as tutors for pre-calculus students and as proctors for monthly high school mathematics contests. A film on statistics was seen at one of the meetings. Other 1981-82 officers: Larry Hicks, vice president; Rhonda McKee, secretary and treasurer; Mary Elick, corresponding secretary; Joseph Shields, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President - Rita Lübbe
19 actives

Brenda Mandel of Dodge, Nebraska was named Outstanding Freshman in Mathematics for the 1980-81 school year. The selection is based on results of a competitive examination administered to freshmen enrolled in mathematics courses who are recommended by the mathematics faculty. The award includes the recipient's name being engraved on a permanent plaque, one year honorary membership in the local KME Chapter and announcement of this honor at the annual banquet. To earn money, club members have been selling floppy discs to students who use the Apple II computers. The club is also paid for proctoring the Math-Science building in the evenings. This responsibility is shared with two other clubs. Club members participated as a club in the slave auction in order to

raise money for the Juvenile Diabetes Foundation. Members Mike Ronspies and Jody Meisinger competed for homecoming king and queen respectively in the fall homecoming activities. Other 1981-82 officers: Cynthia Gregory, vice president; Marian Rhods, secretary and treasurer; Cheryl Wamberg, historian; Fred Webber, corresponding secretary; James Paige and Hilbert Johns, faculty sponsors.

Nebraska Beta, Kearney State College, Kearney

Chapter President - Jeff Lodi

21 actives, 12 initiates

Initiation was held October 8, 1981 at which time twelve new members were initiated. Other 1981-82 officers: Steve Kominek, vice president; Martha Haeberle, secretary; Bob Gentzler, treasurer; Charles Pickens, corresponding secretary; Marilyn Jussel, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President - Carmen Montoya

60 actives, 16 initiates

The chapter experimented with an afternoon meeting instead of an evening meeting. The attendance was surprisingly good. A Christmas party was held after exams were over. Other 1981-82 officers: Debra Lang, vice president; Greg Parets, secretary; Becky Gore, treasurer, Merle Mitchell, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara

Chapter President - Cindy McDonald

17 actives

Other 1981-82 officers: Bridgette Baldwin, vice president; Brian Talladay, secretary, Tom Copeland, treasurer; Robert Bailey, corresponding secretary; James Huard, faculty sponsor.

New York Theta, St. Francis College, Brooklyn
Chapter President - Anne Marie Plantemoli
3 actives

The chapter meets approximately every week in an informal seminar organized around the book, "Gödel, Escher, Bach: An Eternal Golden Braid," by Douglas Hofstadter. Other 1981-82 officers: Lorraine DeCicco, vice president; Elaine Powers, secretary; Rosalind Guaraldo, corresponding secretary and faculty sponsor.

New York Kappa, Pace University, New York
Chapter President - Maria Tripodi
25 actives

Thirteen new members were initiated on April 6, 1981. The evening was highlighted by a talk given by Michael Schiano. His topic was "Some Applications of Graph Theory". Other 1981-82 officers: Patricia Ann Lewis, vice president; Michael Kazlow, corresponding secretary; Martin Kotler, faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green
Chapter President - Mark Worline
30 actives

The chapter held a joint meeting November 10, 1981 with the BGSU chapter of the Association for Computing Machinery. The speaker was Dr. Phillip Tuchinsky of the Ford Motor Company. His topic was his work with computers following his education as a mathematician. Other 1981-82 officers: Gwen Hagemeyer, vice president; Larry Zaborski, secretary; J. Frederick Leetch, corresponding secretary and faculty sponsor; L. David Sabbagh, faculty sponsor.

Ohio Gamma, Baldwin - Wallace College, Berea
Chapter President - Janet Gosche
31 actives

The fall quarter program was given by Mr. Nick Lauer of Glidden Durkee and the American Statistical Association. His topic was "Some Applications of Statistics in the Real World." Other 1981-82 officers: Michael Mazzone, vice president; Nancy Lewis, secretary; Larry Mills, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord

Chapter President - Lael Wall

32 actives, 16 initiates

An initiation banquet was held October 19, 1981 for sixteen new members. The initiates gave talks on mathematicians whose identity they assumed for the occasion. The chapter sponsored two visiting speakers from Miami University: Gary Gilbert who spoke on "Graphs as Models for Psychiatric Strategies" and Lyman Peck whose topic was "The Mathematics of the Stone Circles of Great Britain." A December Christmas party was held at Dr. Smith's home. Other 1981-82 officers: Cathy Roby, vice president; Candace Truscott, secretary; Gail Yoder, treasurer; James L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford

Chapter President - Dean Meharg

30 actives, 12 initiates

Fall activities included sponsoring a bake sale for "Math Day" when area high school students visit the mathematics department. Initiation was held on October 13, 1981. The chapter took a field trip to Magnetic Peripherals, Inc. in Oklahoma City on December 2nd. This company manufactures peripheral products for computers. A picnic was held the afternoon of "Math Day" for all initiates and actives. Other 1981-82 officers: Kathe Searcey, vice president; Vicki Pruitt, secretary and treasurer; Wayne Hayes, corresponding secretary; Kelvin Casebeer, faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington
Chapter President - Candy Yarnell
39 actives, 3 initiates

A fall picnic was held for all mathematics and computer science majors on September 23, 1981. The chapter assisted with the annual Westminster College High School Mathematics Competition in October and participated in Careers Night in November. Other activities include sponsoring a talk on the actuarial profession, a chess tournament, and a film titled "Mind Machine." Other 1981-82 officers: Carl Schartner, vice president; Kathy Christman, secretary; Joel Balleza, treasurer; J. Miller Peck, corresponding secretary; Barbara T. Faires, faculty sponsor.

Pennsylvania Beta, La Salle College, Philadelphia
Chapter President - Donna Malloy
25 actives, 11 initiates

At a special meeting for initiation, Dr. Samuel Wiley spoke on "External Sorting and the Fibonacci Sequence". At other regular meetings, guest speakers from industry spoke about the world of work. These included Mr. Thomas Swen, Arthur Anderson Co., Mr. C. Ernie Busby, IBM, and Mr. Rich Meyer, National Security Agency. Other 1981-82 officers: GERALYN MOTZ, vice president; Diane Balzereit, secretary; Susan Krembs, treasurer; Hugh Albright, corresponding secretary; Samuel Wiley, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown
Chapter President - Lori Klee
20 actives, 9 initiates

The chapter held five meetings with student talks given as programs. Other 1981-82 officers: Donna Mrazik, vice president; Kate Ramsey, secretary; Lisa Schell, treasurer; I. Hollingshead, corresponding secretary; William Jones, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania,
Indiana

Chapter President - Rose Mary Zbiek
19 actives, 3 initiates

Dr. Gerald Buriok, Chairman of the Computer Science Department, demonstrated use of an APPLE II computer and programmable Hewlett-Packard calculator at the October meeting. The November meeting was held in conjunction with the annual Mathematics Department Career Night. Four IUP Mathematics Majors Alumni told of their job experiences. Also, Mr. John Tunney, Technical Training Specialist, Pittsburgh Plate Glass Co., gave information about opportunities in mathematics-related jobs. In December, Mr. Joseph Peters, Mathematics Department faculty member, explained the types of projects that students can do in independent study. Kelly Barber and Donna Reed, KME members, explained what they had done in independent study during the fall semester. Other 1981-82 officers: Mark Woodard, vice president; Edna Keba, secretary; Wendy Stilwell, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Gamma, Waynesburg College, Waynesburg

Chapter President - Connie Machek
18 actives, 1 initiate

Other 1981-82 officers: Eric Wright, vice president; Linda Jackson, secretary and treasurer; Rosalie B. Jackson, corresponding secretary; David S. Tucker, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia

Chapter President - Linda Czajka
7 actives, 5 initiates

During the fall semester the chapter attended the mathematics exhibits at The Franklin Institute in Philadelphia and the lecture: "Everybody Has Problems", given by Professor Paul R. Halmos at Kutztown State College on October 14, 1981. They also viewed the PACS 81 Philadelphia Area Computer Show held November 12-14 at the

Sheraton Hotel. During regular chapter meetings problems are presented, discussed, and solved.

Virginia Alpha, Virginia State University, Petersburg
Chapter President - Jonathan B. Ransom
33 actives

Virginia Alpha Chapter has established the Louise Stoke Hunter Award to honor its Founder, Dr. Louise Stokes Hunter, Professor Emeritus of Virginia State University. The cash award is made annually to a member of Kappa Mu Epsilon who has an average of not less than 3.0 in mathematics classes, including Calculus, and who has exhibited the character, ability and diligence to pursue a career in some area of mathematics. Other 1981-82 officers: Bernice Peery, vice president; Betty Saunders, secretary; Martina Lewis, treasurer; LaVerne Goodridge, corresponding secretary; Emma B. Smith, faculty sponsor.

West Virginia Alpha, Bethany College, Bethany
Chapter President - Steve Petrovich
27 actives, 13 initiates

Other 1981-82 officers: Ken Romanski, vice president; Donna Gates, secretary; James Allison, corresponding secretary.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President - Marie Schwerm
7 actives

A donut sale was held for a fund raising project. The chapter sponsored the annual Mathematics Contest for junior and senior high school girls on November 21, 1981. Members viewed and discussed two films: "Sandra, Zella, Dee, and Clair: Four Women in Science" and "Math and Science Connection: Educating Young Women for Today." These films looked at mathematics and science careers for young women and stressed the importance of mathematics in the high school. Other 1981-82 officers: Catherine Schueller, vice president; Prudence Kelly, secretary; Heather Shelton, treasurer; Sister Adrienne Eickman,

corresponding secretary; Sister Mary Petronia Van Straten, faculty sponsor.

Wisconsin Gamma, University of Wisconsin-Eau Claire, Eau Claire

Chapter President - Linda Kelly

15 actives, 12 initiates

Other 1981-82 officers: Mike Kelly, vice president; Karl Wellnitz, secretary; Glen Wetzel, treasurer; Tom Wineinger, corresponding secretary; Wilbur Hoppe and Bob Langer, faculty sponsors.

ANNOUNCEMENT OF TWENTY-FOURTH BIENNIAL CONVENTION

Eastern Kentucky University
Richmond, Kentucky

April 21-23, 1983

The 24th Biennial Convention of Kappa Mu Epsilon will be held on April 21-23, 1983 at Eastern Kentucky University; Richmond, Kentucky. Each chapter that sends a delegation will be allowed some travel expenses from National Kappa Mu Epsilon funds. Travel funds are disbursed in accordance with Article VI, Section 2 of the KME constitution.

A significant feature of this convention will be the presentation of papers by student members of KME. The mathematics topic which the student selects should be in his area of interest, and of such scope that he/she can give it adequate treatment within the time allotted.

Who May Submit Papers? Any student member of KME, undergraduate or graduate, may submit a paper for use on the convention program. A paper may be co-authored; if selected for presentation at the convention it must be presented by one or more of the authors. Graduate students will not compete with undergraduates.

Subject: The material should be within the scope of the understanding of undergraduates, preferably those who have completed differential and integral calculus. The Selection Committee will naturally favor papers within this limitation, and which can be presented with reasonable completeness within the time limit.

Time Limit: The minimum length of a paper is 15 minutes; the maximum length is 25 minutes.

Form of Paper: Four copies of the paper to be presented, together with a description of the charts, models, or other visual aids that are to be used in the presentation, should be presented in typewritten form, following the normal techniques of term paper presentation. It should be presented in the form in which it will be presented, including length. (A long paper should not be submitted with the idea that it will be shortened for presentation.) Appropriate footnoting and bibliographical references are expected. A cover sheet should be prepared which will include the title of the paper, the student's name (which should not appear elsewhere in the paper), a designation of his classification in school (graduate or undergraduate), the student's permanent address, and a statement that the author is a member of Kappa Mu Epsilon, duly attested to by the Corresponding Secretary of the student's Chapter.

Date Due: January 21, 1983

Address to Send Papers: Professor James L. Smith
Mathematics & Computing
Science Department
Muskingum College
New Concord, OH 43762

Selection: The Selection Committee will choose about fifteen papers for presentation at the convention. All other papers will be listed by title and student's name on the convention program, and will be available as alternates. Following the Selection Committee's decision, all students submitting papers will be notified by the National President-Elect of the status of their papers.

Criteria for selection and convention judging:

A. The Paper

1. Originality in the choice of topic
2. Appropriateness of the topic to the meeting and audience
3. Organization of the material
4. Depth and significance of the content
5. Understanding of the material

B. The Presentation

1. Style of presentation
2. Maintenance of interest
3. Use of audio-visual materials
(if applicable)
4. Enthusiasm for the topic
5. Overall effect
6. Adherence to the time limit

Prizes: The author of each paper presented at the convention will be given a two-year extension of his/her subscription to The Pentagon. Authors of the four best papers presented by undergraduates, based on the judgment of the Awards Committee, composed of faculty and students, will be awarded cash prizes of \$60, \$40, \$30, and \$20 respectively. If enough papers are presented by graduate students, then one or more prizes will be awarded to this group. Prize winning papers will be published in The Pentagon, after any necessary editing. All other submitted papers will be considered for publication at the discretion of the Editor.

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KME members are reminded that pins, keys, and tie-tacs are available and may be ordered through corresponding secretaries.

THE AMERICAN

At the time of the first issue of the American, the country was in a state of confusion and the people were in a state of ignorance. The first issue of the American was published in 1847 and was the first of a series of publications that were to follow.

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