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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

COMPUTER GRAPHICS: THREE DIMENSIONAL* REPRESENTATION OF SPHERES

DAVID HARRIS
Student, Central Missouri State University

At the beginning of the summer of 1979 I received a research participation award from the Clark Foundation. The purpose of the award was to gain experience helping scientists at the University of Texas at Dallas with their research. Under the direction of Dr. Christopher Parr I modified a computer program that drew spheres on a high-resolution graphics terminal. The program ran on a PDP 11/45 computer coupled with a Genisco processor that controlled the graphics CRT. The primary use of the program was to draw molecules. These representations of molecules can be used in the study of reaction dynamics and stereochemistry. In this paper I am going to describe the process of drawing the spheres and the mathematics behind their creation.

The process begins with the raw data. The data for the program consists of: The number of spheres; the x, y, z coordinates and the radius of each sphere; the scale factor; the Euler angles theta, phi, and psi which determine the position of the figure relative to the observer; and the parameters that control the roughness of

*A paper presented at the 1981 National Convention of KME and awarded third place by the Awards Committee.

the approximation of the circles. The first step is to scale the data. This just involves multiplying the coordinates and distances by the scale factor.

The next step is to rotate the observer according to the Euler angles. To rotate a point about the z-axis through an angle Θ we use the formulas

$$x' = x\cos\Theta + y\sin\Theta, \quad y' = -x\sin\Theta + y\cos\Theta,$$

and $\quad z' = z,$

which can be represented by the transformation:

$$\langle x\ y\ z\rangle \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \langle x'y'z\rangle.$$

Likewise, rotations about the x-axis and y-axis can be achieved by "permuting the axes in a cyclic fashion." (Newman, p. 335) The other two matrices are shown in Fig. 1.

$$\langle x\ y\ z\rangle \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} = \langle x'y'z'\rangle$$

Rotation about the y-axis

$$\langle x\ y\ z\rangle \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix} = \langle x'y'z\rangle$$

Rotation about the x-axis

Fig. 1

These rotations can be concatenated by matrix multiplication, paying strict attention to the order in which the rotations are made as different orders will give different results. Taking the product of the three matrices in z, y, x order we get

$$R_z R_y R_x = \begin{bmatrix} \cos\theta\cos\phi & -\sin\theta\cos\psi + \cos\theta\sin\phi\sin\psi & \sin\theta\sin\psi + \cos\theta\sin\phi\cos\psi \\ \sin\theta\cos\phi & \cos\theta\cos\psi + \sin\theta\sin\phi\sin\psi & -\cos\theta\sin\psi + \sin\theta\sin\phi\cos\psi \\ -\sin\phi & \cos\phi\sin\psi & \cos\phi\cos\psi \end{bmatrix}$$

and the complete rotation is $\langle x\ y\ z\rangle R_z R_y R_x = \langle x'y'z'\rangle$. At this point the observer is assumed to be looking down the z-axis at the xy plane of the CRT screen with the figure rotated relative to the observer.

Now that the figure is in our view, we compute the visibility of the spheres using a battery of tests. We first check to see if the spheres are behind the observer or off the screen. Then we test to see if the difference in the radii of each pair of spheres is greater than the distance between their centers. If so, one sphere completely engulfs the other. Also, if the difference in radii is greater than the projection to the xy plane of the distance between their centers then one sphere totally eclipses the other. (See Fig. 2). We

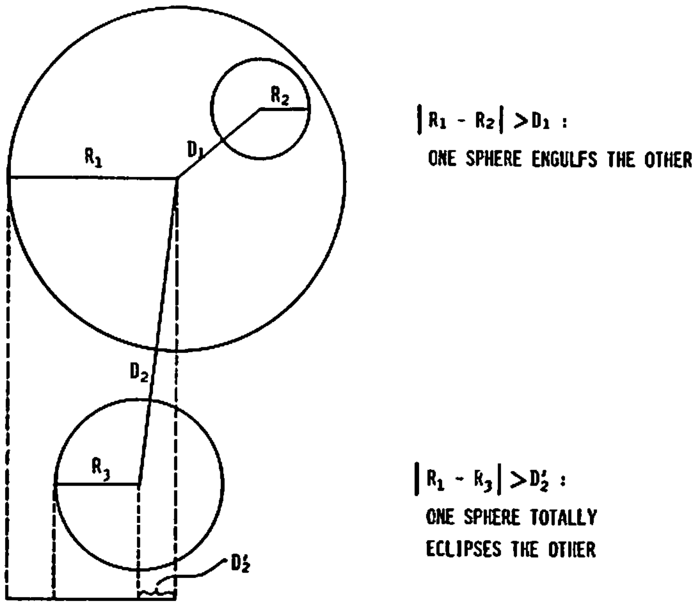

Fig. 2

then take each visible sphere, one at a time, and draw it as a series of concentric circles. Each circle is approximated as a regular polygon. The number of sides of the polygon determines the smoothness of the circle and the number of circles determines the smoothness of the sphere.

To draw the circles we must keep track of the intersections of each sphere with the edges of the screen and the partial eclipses due to the other spheres blocking the view. This time we compare the distance between the projected centers of the circles with the sum of their radii. If the distance is less than the sum of the radii we determine the half-angle of arc deletion by the law of cosines (taking into account the special cases of the angle being $\pi/2$, greater than $\pi/2$, and π.) A subroutine is used to keep the arc deletions in a list of beginnings and endings of arcs. This list is then used to draw the circle. (See Fig. 3)

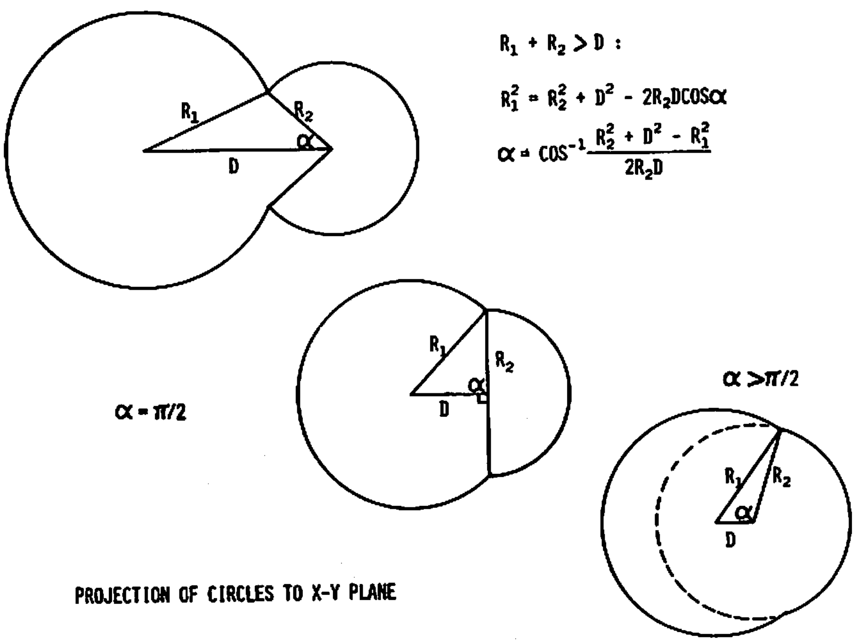

Fig. 3

The last step before drawing the circle is to find out how much light hits each segment so we know how bright it is to be. This is accomplished by finding the dot product of the vector from the center of the sphere to the segment being drawn,with the vector from the segment to the light source. The value of the dot product varies as the cosine of the angle between the vectors. Therefore, when the cosine is one, the segment is at its brightest. As the cosine drops off to zero the brightness diminishes to zero or to a predetermined minimum value.

This process is illustrated with a series of prints reproduced from images on a CRT.* The original slides were taken in a dark room with a camera mounted on a tripod at a distance of 18" from the CRT display screen. The camera was set at a shutter speed of $\frac{1}{4}$ second and the F-stop was set at 4.

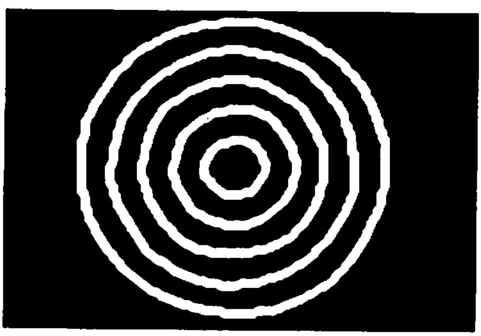

Fig. 4

Figure 4 shows a sphere drawn with very few concentric circles. Each circle is drawn as a polygon with

*Editor's Comment: The black and white figures in this paper were copied from color slides supplied by the author.

a relatively small number of sides.

In Figure 5, another sphere is added close enough to intersect. The intersection of two spheres is a circle; however, projected into two dimensions, the intersection is an ellipse which is seen edge-on here.

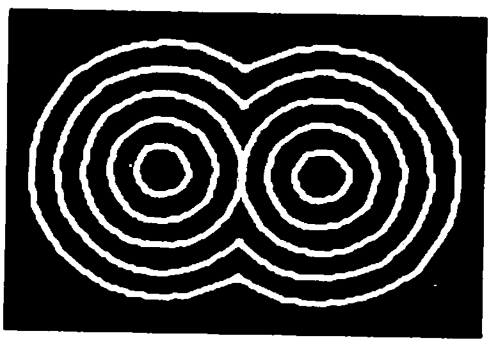

Fig. 5

In Figure 6 we see a partial eclipse caused by rotating the right sphere 30° behind the left one. The ends of the uncompleted circles on the right show the visible part of the ellipse that is the projection of the intersection. The ends don't touch the left sphere due to the rough approximation being used.

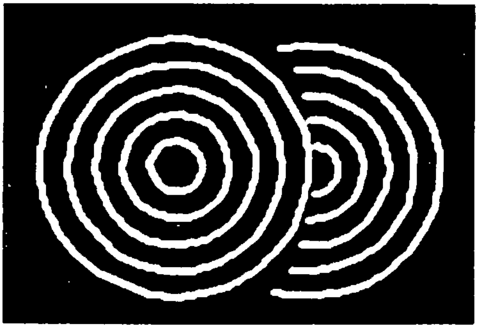

Fig.6

Figure 7 shows a point light source at infinity added in the upper left corner. The sphere is brightest when facing the light source. Intensity decreases as the face of the sphere turns away from the light and finally becomes zero on the far side of the sphere.

Fig.7

In Figure 8, a non-zero minimum value is assigned to all segments in shade so that the entire shape of the figure can be seen.

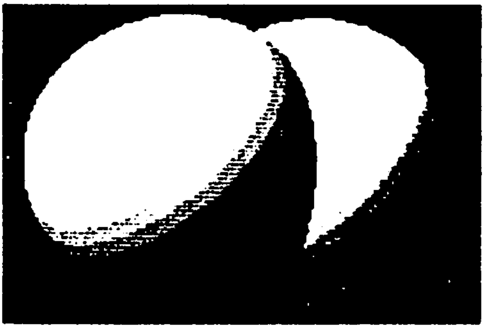

Fig. 8

An application of the program appears as Figure 9: a TCNQ molecule.

Fig. 9

In Figure 10 the observer is rotated to the other side of the molecule and turned 45° off center about one axis and 60° off center about another axis.

Fig.10

In Figure 11, the observer has zoomed in and rotated 45° about each of the three axes.

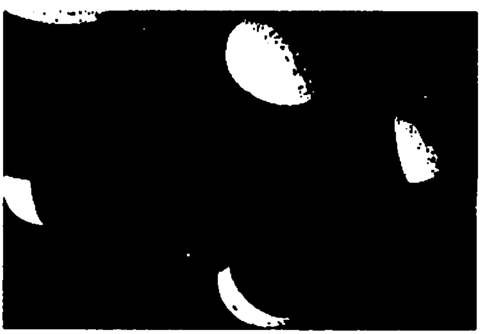

Fig.11

A sense of three-dimensional perspective can be achieved with motion of the figures relative to one another. Dr. Parr has made a film clip of a few chemical reactions whose representations were drawn by this program using a flat-bed plotter. This film was produced, frame by frame, by a computer which controlled the plotter as well as the camera.

Toward the end of my summer at the University of Texas at Dallas, I was working on the problem of spheres casting shadows on other spheres. You may have noticed that the spheres in the pictures cast shadows on themselves but not on the ones around them. The problem involves intersecting a circle with an ellipse which results in a fourth-degree equation. The difficult part is in finding the roots of the equation and then deciding what those roots mean in terms of arc beginnings and endings and whether or not the arc is in shade. This project has not been completed.

With the advent of faster computers, computer graphics is becoming better and easier. There are many ways of achieving the same end and the method I have presented here is one good way of drawing spheres.

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THE FIBONACCI NUMBERS*

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"Mathematics is the alphabet with which God has written the world." (See [12]) When Galileo made this statement, he was emphasizing the importance of Mathematics. Just as the alphabet is the basis for our words, so is Mathematics the basis for the world. Mathematics not only exists in nature, but also in the products of man's civilization. The "alphabet" of Mathematics has many components. One of these is a group of numbers called the Fibonacci sequence.

To begin, the derivation of the Fibonacci sequence occurred in the 13th century. Leonardo of Pisa, called Fibonacci since he was son (figlio) of Bonacci, is considered to be one of the most accomplished mathematicians of the Middle Ages. Fibonacci grew up in the North African city of Bugia, where his father was collector of customs, and acquired his education from the Muslim scholars of the Barbary coast. In 1202, at the age of 27, Fibonacci published a book which introduced the Arabic numerals to the European world. He called his book *Liber Abaci* or *Book of the Abacus* because the use of Arabic numerals was implicit in the "abacus".

*A paper presented at the 1981 National Convention of KME and awarded fourth place by the Awards Committee.

In this book, Fibonacci posed the problem: "Someone placed a pair of rabbits in a certain place, enclosed on all sides by a wall, to find out how many pairs of rabbits will be born there in the course of one year, it being assumed that every month a pair of rabbits begin to bear young two months after their own." (See [5]) The solution of this rabbit problem leads to a mysterious series of numbers (Figure 1). At the end of each

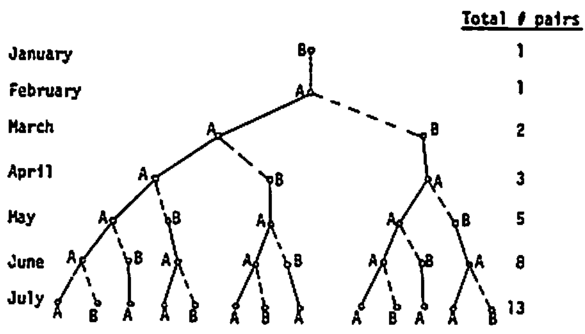

Figure 1

month, the number of pairs of rabbits recorded is 1,1,2, 3,5,8,13,21,34,55,89, and 144. Because of its origin in Fibonacci's rabbit problem, this sequence is called the Fibonacci sequence. This series of numbers follows the recursive equation:

$$F_1 = 1 \text{ and } F_2 = 1, \; F_n = F_{n-1} + F_{n-2} \text{ for } n > 2$$

(Here F_n indicates the nth term of the sequence.)

Every term of this sequence is the sum of the two preceding terms.

Botany illustrates nature utilizing the Fibonacci sequence. The number of petals of a daisy exemplifies the series. For instance, one researcher found that daisies have 21, 34, 55, or 89 petals. (See [13]) The wild rose, cosmos, buttercups, columbine, and trillium are just a few of the flowers that have 2, 3, 5 or 8 petals or petal-like structures. (See [7]) Recall that 2, 3, 5 and 8 are a portion of the Fibonacci sequence.

Furthermore, the botanical phenomenon of phyllotaxis bases the arrangement on leaves on Fibonacci numbers. Take a stalk of a green plant that has leaves on it. Starting at the bottom with a green leaf, move up the stalk, counting the leaves, until reaching the leaf that is directly above the first leaf. (Do not count the first leaf). The result will be a Fibonacci number. Consider the pussy willow, for instance; generally, there are 13 buds arranged between two vertical lines, and to count the buds, you must circle the stalk five times. (See [5]) Two, five and thirteen are all Fibonacci numbers.

Recall again that every term of the Fibonacci sequence is the sum of the two preceding terms. Also, each term of the Fibonacci sequence becomes proportional to the terms surrounding it, namely

$$\lim_{n\to\infty} \frac{F_n}{F_{n+1}} = 0.618034$$

In the ancient times, the Greeks used this ratio of 0.618034 to 1 for the basis of their art and architecture, and called it the "golden mean". The Great Pyramid illustrates the golden proportion in architecture. This is the earliest evidence of human knowledge of the golden mean. The Great Pyramid originally measured 484 ft. 5 in. in height. The height in inches, since it is believed the Egyptians worked in inches, is 5,813 inches. (See [5]) Five, eight and thirteen are numbers in the Fibonacci sequence.

The early Greeks defined the golden mean as "the division of a line segment into two parts, the lengths of which were in a certain proportion to one another. The smaller was in proportion to the larger part as the larger part was to the entire segment. (See [5]) From this description, we can derive the Fibonacci quadratic equation.

From Figure 2, $\frac{a}{b} = \frac{b}{a+b}$ or $\frac{b}{a} = \frac{a+b}{b}$ or $\frac{b}{a} = \frac{a}{b}+1$.

Figure 2

Then letting $x = \frac{b}{a}$ we get $x = \frac{1}{x} + 1$ which reduces to $x^2 - x - 1 = 0$. The roots of this equation are: $\alpha = \frac{1+\sqrt{5}}{2}$ and $\beta = \frac{1-\sqrt{5}}{2}$. It is easily shown that: $\alpha + \beta = 1$, $\alpha - \beta = \sqrt{5}$, and $\alpha\beta = -1$. The positive root, $\alpha = \frac{1+\sqrt{5}}{2}$ (the Golden Ratio), is sometimes given the name ϕ from the first Greek letter in the name of Phidias, a famous Greek sculptor who used the golden proportion often in his work. ϕ is the only number that becomes its own reciprocal by subtracting one i.e. $\phi - 1 = \frac{1}{\phi}$. Furthermore, ϕ can be expressed in several ways as a sum of an infinite series. The Binet form for Fibonacci numbers, named after the French mathematician Binet, is given by:

$$F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} \quad n = 1,2,3,\ldots \quad \text{(See [7]).}$$

The Fibonacci sequence has the habit of popping up where least expected. Pascal's Triangle is an excellent example of this. Recall that the numbers in Pascal's Triangle represent the coefficients of the expansions of the binomial $(x+y)^n$ for $n=0,1,2,3,4\ldots$. The diagonal sums of this triangle follow Fibonacci's sequence as on Figure 3 (See [7]).

F_1 F_2 F_3 F_4 F_5

1 1 2 3 5

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Figure 3

The Fibonacci sequence also appears in geometric figures. One example is the golden rectangle, a rectangle whose length and width are in the golden ratio. The depth of the rectangle is 0.618034 that of its width. Gustav Fechner, a German psychologist, measured thousands of windows, picture frames, playing cards, and books, finding that on the average, their shapes conformed to a golden rectangle. Fechner and his successor, Willheim Max Wundt, also tested hundreds of individuals to determine their preferences for rectangles of various proportions. Seventy-five percent of those tested preferred the golden rectangle. (See [5]) Additionally, the golden rectangle appears frequently in architecture. Examples include the Court of the Lions, the Parthenon and the Cathedral of Notre Dame. (See [5])

Also, from a golden rectangle, "whirling squares" come alive (Figure 4). Draw a square in one end of a golden rectangle. In the smaller golden rectangle that remains, draw still another square. This process continues repeatedly until a succession of "whirling squares" emerges.

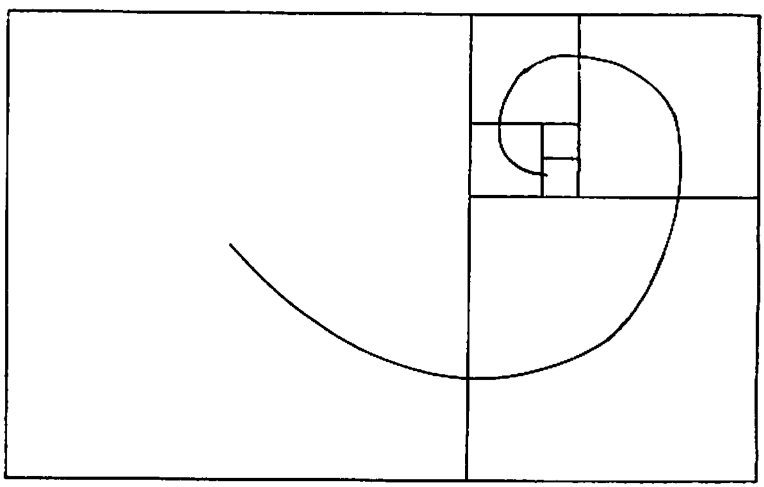

Figure 4

A golden spiral stems from these"whirling squares". Draw lines connecting the centers of the squares. Smooth these lines out and a golden spiral is created. While the spiral coils inward infinitely, it can also be whirled outward.

In addition, a golden triangle possesses sides that follow the golden ratio (Figure 5). Each base angle of this isosceles triangle measures 72^{o}, which doubles the vertex angle of 36^{o}. Drawing golden triangles within

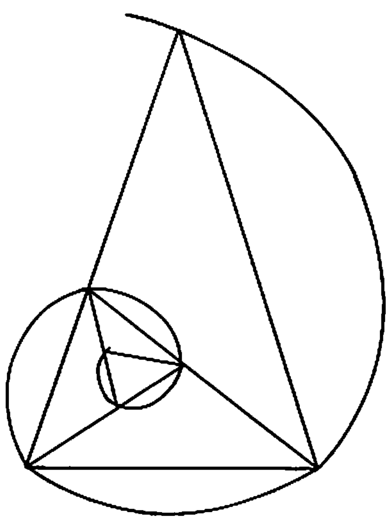

Figure 5

other golden triangles yields "whirling triangles". Bisect the base angle of a golden triangle. This bisector divides the opposite side into the golden ratio. Two smaller golden triangles remain, one of which resembles the original triangle. This process of dividing this triangle by the base-angle bisector method may continue endlessly. These "whirling triangles" form a logarithmic spiral.

The golden spiral, drawn from either the "whirling squares" or the "whirling triangles", acquire the name of logarithmic, or equiangular spiral, from Bernoulli. The reasoning of the naming follows that any line drawn from the center will intersect it at the exact same angle as any other line drawn from the center. Being especially fond of the logarithmic spiral, Bernoulli ordered it engraved on his tombstone. (See [5])

The golden spiral appears frequently in nature. The snail builds his shell according to the spiral. The shell grows in size as the snail grows, but it never loses its shape; it retains the logarithmic spiral. Also, the shell of the chambered nautilus demonstrates the spiral. As the animal grows, a larger compartment is built in the spiral formation. The animal crawls forward, and shuts off the preceding compartment with a layer of mother-of-pearl. The old living quarters are filled with gas and air, making the whole structure buoyant. The outside shell is a spiral, while the inner partitions also conform to the logarithmic spiral. The golden spiral, the only type of spiral that does not alter in shape as it grows, "... seems to be nature's way of building quantity without sacrificing quality." (See [5]) Furthermore, the golden spiral exists in the ram's horn, the elephant's tusk, the lion's claw, the fangs of the saber-toothed tiger, the parrot's beak and the web of the orb weaver spider.

A further example of the golden spiral is found in the human ear. The cochlea, the location of the sound receptors of the ear, forms a golden spiral.

Fibonacci numbers are found throughout geometry--golden rectangles, triangles, spirals, pentagons and pentagrams. These numbers appear in other areas of mathematics as well. For instance, the Fibonacci numbers have some interesting divisibility properties. One example is: No two Fibonacci numbers F_n, F_{n+1} have a prime factor p in common: $(F_n, F_{n+1}) = 1$. A second property of Fibonacci numbers is: F_n is divisible by F_m if and only if n is divisible by m.

The Fibonacci numbers not only have divisibility properties, but also have many identities. One identity involving Fibonacci numbers is:

$$F_1+F_2+F_3+\ldots+F_n = F_{n+2}-1 \qquad n \geq 1$$

From a deviation of the definition of the Fibonacci numbers $F_i = F_{i+2}-F_{i+1}$ we can write the following:

$$\sum_{i=1}^{n} F_i = \sum_{i=1}^{n} F_{i+2} - \sum_{i=1}^{n} F_{i+1} = F_{n+2} - F_2 = F_{n+2} - 1.$$

This identity could also be proven by mathematical induction. (See [10])

Another Fibonacci identity is:

$$F_1^{\,2} + F_2^{\,2} + F_3^{\,2} + \ldots + F_n^{\,2} = F_nF_{n+1}$$

This identity may be pictured geometrically. (See [10] for details.)

A third Fibonacci identity is:

$$F_{n-1}F_{n+1} - F_n^{\,2} = (-1)^n$$

This can be proven by mathematical induction. (See [7]) First we must prove it is true for n=1.

Basis: P(1): $F_0F_2-F_1^{\,2} = 0\cdot 1-1^2 = -1$

Then we assume it true for n=k and attempt to prove it true for n=k+1.

Induction: P(k): $F_{k-1}F_{k+1}-F_k^{\,2} = (-1)^k$ (assumed)

P(k+1): $F_kF_{k+2}-F_{k+1}^{\,2} = (-1)^{k+1}$ (to show)

$$\begin{aligned} F_kF_{k+2} - F_{k+1}^2 &= F_k(F_{k+1} + F_k) - F_{k+1}^2 \\ &= F_k^2 + F_{k+1}(F_k - F_{k+1}) \\ &= F_k^2 - F_{k+1}F_{k-1} \\ &= (-1)(F_{k+1}F_{k-1} - F_k^2) \\ &= (-1)(-1)^k \\ &= (-1)^{k+1} \end{aligned}$$

The identity $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$ is the basis for a geometric paradox by Charles Lutwidge Dodgeson. Cut the square in Figure 6 and rearrange the pieces to form the rectangle in Figure 7. The area of the rectangle is 5 x 13 = 65 while the area of the square is only 8 x 8 = 64. How did we acquire the extra-square unit?

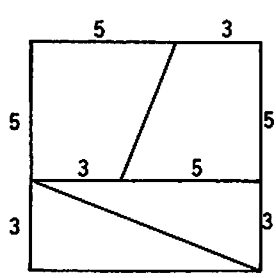

Figure 6

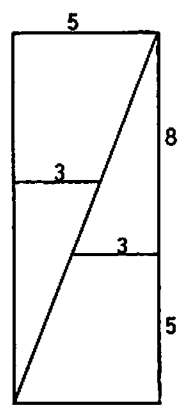

Figure 7

This can easily be shown by drawing these diagrams to a larger scale. The paradox comes about because the sides do not quite meet along the diagonal of the rectangle, but instead for a parallelogram - the magically added unit of area. (See [9])

This can also be shown by the identity $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$. Make the substitution of 2n for n. Therefore $F_{2n-1}F_{2n+1} - F_{2n}^2 = (-1)^{2n} = 1$.

This paradox occurs for any triple of Fibonacci numbers where the middle one has an even subscript. The larger the numbers, the less noticeable is the added parallelogram.

These identities are just a few of the many that exist, and are being discovered each day. Because of this, a group of people who felt that there was still much to learn about the Fibonacci numbers formed the Fibonacci Association in 1963. They began publication of a quarterly journal devoted primarily to the research that is being done on Fibonacci numbers. (See [10])

Indeed, we now realize that the Fibonacci numbers are one of the components of the "alphabet" of Mathematics. We find Fibonacci's numbers in biology and botany, in art and in architecture. This sequence also has many unique mathematical properties and identities.

Fibonacci numbers compose only a small part of the "alphabet" of Mathematics. Yes, Mathematics, is indeed, "the alphabet with which God has written the world".

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MATHEMATICAL INDUCTION AND GUESSING

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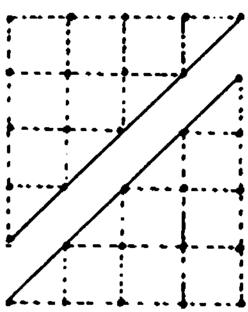
Certain finite series appear in connection with obtaining the area under a curve. We wonder where the sum comes from, for example, why $\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$? In this note we study some Pythagorean series and give some generalizations.

1. Triangular numbers: The sum $\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$ is sometimes called a triangular number since to it corresponds a triangle (Fig. 1). If we put two of these triangles next to each other as in Figure 1, we get a rectangle,



$$2 \sum_{j=1}^{5} j$$

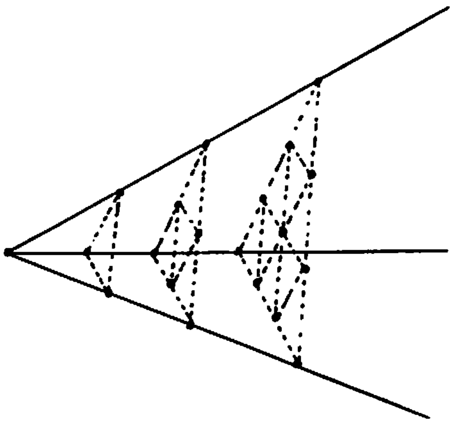
Figure 1

and n(n+1) gives the area of this rectangle. Thus one gets

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2} \tag{1}$$

We would like to carry this idea to a three dimensional configuration and then to an n-dimensional one.

2. <u>Tetrahedral numbers</u>: If we put triangular numbers next to one another, we get a configuration that we may call a tetrahedron. For example, for $n = 5$ we have Fig. 2 . This idea suggests that three of these



$$\sum_{j=1}^{4} \frac{j(j+1)}{2}$$

Figure 2

tetrahedrons will give a prism. Thus one would try

$$(2) \qquad \sum_{j=1}^{n} \frac{j(j+1)}{2} = \frac{n(n+1)(n+2)}{(2)(3)} .$$

Using mathematical induction one can easily prove the above equality. We omit the proof.

3. <u>Hyper-Pyramid Numbers</u>: Now we generalize (2). Let $p \geq 3$ be a fixed natural number. Then

(3) $$\sum_{j=1}^{n} \frac{j(j+1)\ldots(j+p-1)}{p!} = \frac{n(n+1)\ldots(n+p-1)(n+p)}{(p+1)!}$$

Proof: It is clear that the formula is true for $n = 1$.

Suppose (3) is true for $n = k$. Then for $n = k+1$

$$\sum_{j=1}^{k+1} \frac{j(j+1)\ldots(j+p-1)}{p!}$$

$$= \sum_{j=1}^{k} \frac{j(j+1)\ldots(j+p-1)}{p!} + \frac{(k+1)(k+2)\ldots(k+p)}{p!}$$

$$= \frac{k(k+1)\ldots(k+p-1)(k+p)}{(p+1)!} + \frac{(k+1)\ldots(k+p)}{p!}$$

$$= \frac{(k+1)\ldots(k+p)}{p!} \left[\frac{k}{p+1} + 1\right]$$

$$= \frac{(k+1)\ldots(k+p)(k+1+p)}{(p+1)!} .$$

Therefore the equality (3) is true.

4. Some Applications: Suppose we would like to compute $\sum_{j=1}^{n} j^2$. We observe that (2) is equivalent to

$$\sum_{j=1}^{n} (j^2+j) = \frac{n(n+1)(n+2)}{3}$$

Thus

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(n+2)}{3} - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1)}{6} .$$

Now to compute $\sum_{n=1}^{n} j^3$ we consider (3) for the case $p=3$, i.e.,

$$\sum_{j=1}^{n} \frac{j(j+1)(j+2)}{3!} = \frac{n(n+1)(n+2)(n+3)}{4!}$$

From this equality we get

$$\sum_{n=1}^{n} (j^3+3j^2+2j) = \frac{n(n+1)(n+2)(n+3)}{4}$$

Consequently

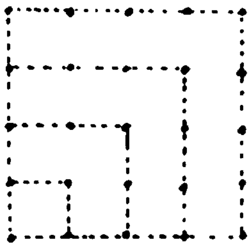
$$\sum_{j=1}^{n} j^3 = \frac{n(n+1)(n+2)(n+3)}{4} - 3\sum_{j=1}^{n} j^2 - 2\sum_{j=1}^{n} j = [\frac{n(n+1)}{2}]^2 .$$

One can continue and obtain other formulas.

5. Square Numbers: It is well known that

(4) $$\sum_{j=1}^{n} (2j-1) = n^2.$$

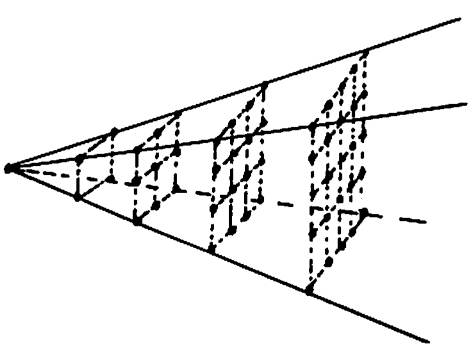
This is called a square number (Fig. 3). As was done



$$\sum_{j=1}^{5}(2j-1)$$

Figure 3

in previous sections we may ask other questions such as how does one obtain a formula for $\sum_{j=1}^{n} j^2$? We may call this sum a pyramid number (Fig. 4). We



$$\sum_{j=1}^{5} j^2$$

Figure 4

have already done this, but we would like to get other generalizations. Thus we start with

$$j^2 = j(j+1) - j$$

(which incidentally may be found from (4)).

We sum over both sides of this equality and we get

$$\sum_{j=1}^{n} j^2 = 2 \sum_{j=1}^{n} \frac{j(j+1)}{2} - \sum_{j=1}^{n} j$$

$$= 2\frac{n(n+1)(n+2)}{3!} - \frac{n(n+1)}{2} = \ldots$$

$$= \frac{n(n+1)(2n+1)}{3!}$$

using (3) and simplifying.

Now we look for the next sum, i.e., what is a formula for

$$\sum_{j=1}^{n} \frac{j(j+1)(2j+1)}{3!} \ ?$$

Let us have a guess (based on our trend so far) such as

$$\sum_{j=1}^{n} \frac{j(j+1)(2j+1)}{3!} = \frac{n(n+1)(n+2)(2n+a)}{4!} .$$

If this is to be true for n = 1 we must have a = 2. Thus we try

(5) $$\sum_{j=1}^{n} \frac{j(j+1)(2j+1)}{3!} = \frac{n(n+1)(n+2)(2n+2)}{4!} .$$

One can prove this equality by mathematical induction. We omit the proof.

6. <u>Another Hyper-Pyramid Number</u>: A generalization of (5) will be as follows.

Let $p \geq 2$ be a fixed positive integer. Then

$$\sum_{j=1}^{n} \frac{[j(j+1)(j+2)\ldots[j+(p-2)]][2j+(p-2)]}{p!} \tag{6}$$

$$= \frac{[n(n+1)\ldots[n+(p-1)]][2n+(p-1)]}{(p+1)!} \; .$$

This can easily be proved by mathematical induction.

One may test cases of $p = 2,3,\ldots$. For example for $p = 2$ we get

$$\sum_{j=1}^{n} \frac{j(2j)}{2!} = \sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{3!} \; .$$

There is a set of interesting formulas to be obtained from (6). There is also a set of other generalizations for finding a formula for

$$\sum_{j=1}^{n} \left[\frac{j(j+1)}{2}\right]^2 .$$

We invite the reader to try to answer some of the questions.

THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August, 1982. The solutions will be published in the Fall 1982 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 337: Proposed by Dmitry P. Mavlo, Moscow, USSR.

(a) Find all solutions of the equation

$$\sqrt[3]{a_1a_2a_3a_4} = a_1+a_2+a_3+a_4$$

where a_1, a_2, a_3, a_4 are natural numbers or zero, $a_1 > 0$, and $a_1a_2a_3a_4$ is a four digit number in the decimal system.

(b) Generalize the problem in part(a)to n variables.

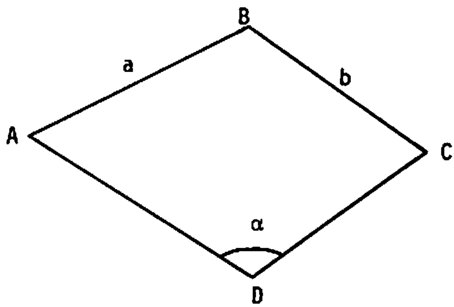
Problem 338: Proposed by Dmitry P. Mavlo, Moscow, USSR.

Given the angle $\angle ADC = \alpha$ and the two opposide sides $AB = a$ and $BC = b$ in quadrangle ABCD, as shown in the figure,

(a) find the quadrangle having the maximum area;

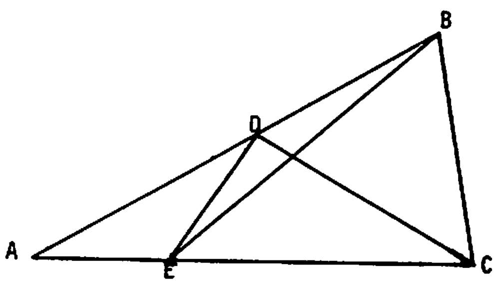
(b) express this maximum area in terms of a,b, and α; and

(c) give the Euclidean construction of this quadrangle of maximum area.



Problem 339: Proposed by Fred A. Miller, Elkins, West Virginia.

In triangle ABC as shown in the figure, AB = AC, $\angle B = \angle C = 80^\circ$, $\angle BCD = 50^\circ$, and $\angle BEC = 40^\circ$. Find $\angle DEB$.



Problem 340: Proposed by Charles W. Trigg, San Diego, California.

Find all two-digit integers in all bases that are three times the number formed by reversing the digits.

Problem 341: Proposed by Charles W. Trigg, San Diego, California.

Can each x in the diagram be replaced by an odd digit in such a fashion that the three digits on each side of the pentagon form a prime when read in either direction using ten distinct primes?

```
            X
        X       X
    X               X
     X             X
        X    X    X
```

SOLUTIONS

327. Proposed by the editor.

Young Leslie Morely, while participating in a stock-car race, noticed a peculiar fact as he sped around the oval track. He noticed that 5/12 of the number of racers in front of him plus 3/5 of the number of racers behind him add up to the total number of participants in the race. If Leslie Morely placed second, how many racers did he beat?

Solution by Douglas K. Sorenson, Western Illinois University, Macomb, Illinois.

First we must define "in front of" and "behind". Since he was on an oval track, "in front of" refers to the region from the front bumper of Leslie's racer to the back bumper of the racer. Similarly, the same region applies in defining "behind". If there are x cars in this region, we obtain the equation

$$5x/12 + 3x/5 = x+1.$$

Hence $x = 60$ and since there were 61 cars in the race, Leslie Morely beat 59 of them.

Also solved by: Kent Gross and Andy Schmidt (jointly), Fort Hays State University, Hays, Kansas; Scott Michael Jeffries, Hofstra University, Hempstead, New York; Mark Schultz, University of Wisconsin-Marathon County, Wausau, Wisconsin; Robert A. Stump, Hopewell, Virginia; Charles W. Trigg, San Diego, California; and Michael Wallace, California State Polytechnic University, Pomona, California. Two incorrect solutions were received.

Editor's Comment

This simple problem contains a pitfall which must be avoided to reach a solution, hence the reason for its selection. Michael Wallace characterized the pitfall perfectly when he remembered an old song played sometimes on the "Captain Kangaroo Show" entitled "Horace, the

Horse on the Merry-go-round".

The song tells of Horace,who believes he is competing with other horses on the merry-go-round. Some of the lines were:

"Horace tried and tried, but he just could never win.
Horace tried and tried, but all the other horses were ahead of him.
One day though, ...
...
Horace looked around and then said, "Gosh! Oh gee! I'm the very first horse on the merry-go-round; the others are following me!"

If one remembers "Horace", Leslie's problem becomes simple!

328. Proposed by Robert A. Stump, Hopewell, Virginia.

Evaluate $\prod_{i=k+1}^{\infty} \frac{i^2 - k^2}{i^2}$ where k is any positive integer.

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Let $P = \prod_{i=k+1}^{N} \frac{i^2 - k^2}{i^2}$

Examining the cases k = 1,2,3, suggests that

$$P = \frac{(k!)^2(N+1)(N+2)\ldots(N+k)}{(2k!)(N-k+1)(N-k+2)\ldots N} \quad (1).$$

By a straightforward induction, the validity of the expression for P can be established.

Finally $$\prod_{i=k+1}^{\infty} \frac{i^2 - k^2}{i^2} = \lim_{N\to\infty} P = \frac{(k!)^2}{(2k!)}$$

because $$\lim_{N\to\infty} \frac{N+1}{N-k+1} = \ldots = \lim_{N\to\infty} \frac{N+k}{N} = 1.$$

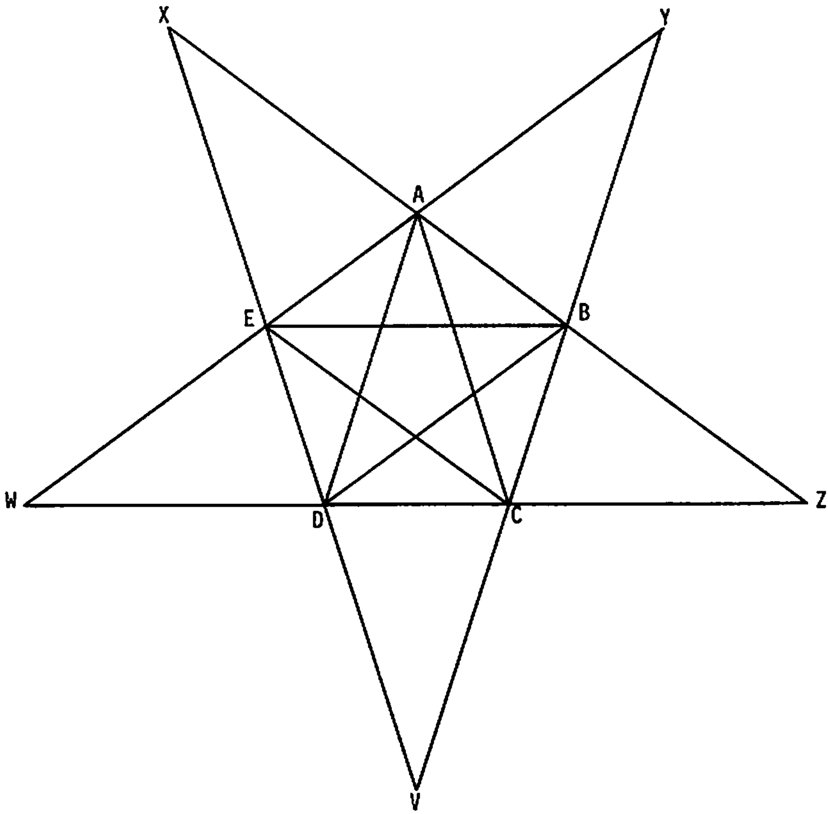
Also solved by Richard Gibbs, Fort Lewis College, Durango, Colorado; Mark Schultz, University of Wisconsin-Marathon County, Wausau, Wisconsin, and the proposer.

329. Proposed by Charles W. Trigg, San Diego, California.

Find the length of the diagonals of both a regular pentagon and a regular hexagon in terms of the sides of the respective polygons without the aid of explicit functions of the angles. (The Pythagorean theorem is an implicit function of 90°).

Solution by Robert A. Stump, Hopewell, Virginia.

For the regular pentagon ABCDE, as shown in the figure, let s denote the length of the sides and d denote the length of the diagonals.

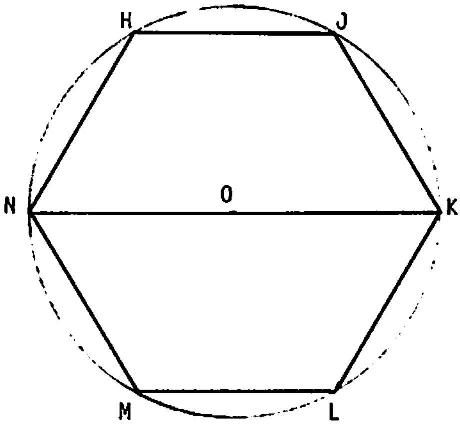


Triangles ABY and CYE are similar, hence

$$\frac{AB}{EC} = \frac{AY}{EY} \text{ or } \frac{s}{d} = \frac{d}{s+d} .$$

Therefore $d^2 - sd - s^2 = 0$ or $\frac{d}{s} = \frac{1 + \sqrt{5}}{2}$.

Note that the negative root has been discarded.



Given the regular hexagon HJKLMN inscribed in circle O, it is readily apparent from the construction of the hexagon that the length of each side of the hexagon is $r = OK$, the radius of the circle. The length of each major diagonal is $2r$. It remains to determine the length of the minor diagonal HK. Since the area of the rhombus $HOKJ = mr/2$, where $m = HK$ and $r = OJ$, also equals twice the area of the equilateral triangle HOJ, we obtain $r^2 \sqrt{3} = mr$ whence $m = r \sqrt{3}$ is the length of the minor diagonal.

Also solved by: Mark Schultz, University of Wisconsin-Marathon County, Wausau, Wisconsin; John A. Winterink, Albuquerque Technical-Vocational Institute, Albuquerque New Mexico, and the proposer.

330: Proposed by Robert A. Stump, Hopewell, Virginia.

Prove that every integer (positive, negative,or zero) can be expressed as the sum of at most five integer cubes.

Solution by Richard A. Gibbs, Fort Lewis College, Durango, Colorado.

First note that

$$6i = (i+1)^3+2(-i)^3+(i-1)^3 \qquad (1)$$

for any integer i.

Next note that $i^3-i = (i+1)(i)(i-1)$ is the product of three consecutive integers and hence i^3-i is divisible by $6 = 3!$ for any integer i. Therefore any integer n can be expressed in the form $6k+i$ where $0 \leq i < 6$ and we have the following representation of n as the sum of five integral cubes

$$n = i^3+6(k-R) = i^3+(k+1-R)^3+2(R-k)^3+(k-1-R)^3$$

where $\qquad 6R = i^3-i.$

Also solved by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin and the proposer. A partial solution was submitted by H.O. Eberhart, Columbia, Maryland.

331: Proposed by Willie S. M. Yong, Republic of Singapore.

The numbers A, B and C each contain three digits and each of the non-zero digits appears exactly once in A, B or C. Each digit of A is less than the corresponding digit of B. If A + B = C and C is a power of a prime, find the values of A, B and C.

Solution by Charles W. Trigg, San Diego, California.

The sum of the nine non-zero digits is 45. Consequently, $A + B + C \equiv 0 \pmod 9$. Since $A + B = C$, then $C \equiv 0 \pmod 9$ and C is a power of 3. Clearly, the hundreds' digit of $C \geq 1 + 2$, so $C = 729$. Thus the sum of the units' digits of A and B is 9; the sum of their tens' digits is 12; and the sum of their hundreds' digits is $6 = 1 + 5$. Immediately, $9 = 3 + 6$, and $12 = 4 + 8$. Finally $A = 143$, $B = 586$, and $C = 729$.

Also solved by Scott Michael Jeffries, Hofstra University, Hempstead, New York; Fred A. Miller, Elkins, West Virginia; John Oman and Bob Prielipp (jointly), University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; Randall J. Schadt, Kearney State College, Kearney, Nebraska; Robert A. Stump, Hopewell, Virginia; Diana Wilson, University of Missouri-Rolla, Rolla, Missouri; and the proposer.

Late solutions were received from Jeff Teeters, University of Wisconsin-River Falls, River Falls, Wisconsin for problems 322, 323 and 324 and from H.O. Eberhart, Columbia, Maryland for problems 324 and 326.

THE MATHEMATICAL SCRAPBOOK

EDITED BY RICHARD L. BARLOW

Readers are encouraged to submit Scrapbook material directly to the Scrapbook editor. Material submitted will be used whenever possible and acknowledgement will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Dr. Richard L. Barlow, Department of Mathematics, Statistics and Computer Science, Kearney State College, Kearney, Nebraska 68847.

The study of prime numbers is an interesting area of number theory upon which much research has been done. Even in the most elementary of mathematics classes, the student usually becomes acquainted with the Sieve of Eratosthenes, developed two thousand years ago as a device for determining prime numbers from a subset of the set of positive integers. In Martin Gardner's Sixth Book of Mathematical Games from Scientific American, other approaches are examined with some unusual results. One such procedure follows.

In 1963, Stanislaw M. Ulam of the Los Alamos Scientific Laboratory developed a spiral grid of horizontal and vertical lines (see Figure 1). After positioning the positive integers on a spiral grid as shown, he circled all the prime numbers in the spiral grid and discovered that the primes appeared to have a tendency to form straight lines as indicated in Figure 1. As usual, the integer 1 is not to be considered a prime number.

Around the center of the spiral grid the primes should be expected to form straight lines due to the great density of the smaller primes. But to Ulam's surprise, the primes also formed lines away from the center of the spiral grid as shown in Figure 1.

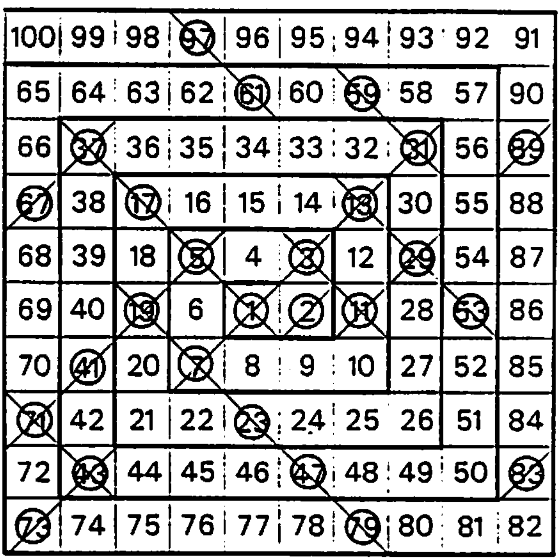


Figure 1

Ulam and others further examined this problem using the first 10,000 positive integers and the first 65,000 positive integers. These results are shown in Figures 2 and 3. In each case, a MANIAC computer was programmed to display the primes on a similar spiral grid. One will note that even the positive integers around the extremities of the grid also line up into straight lines.

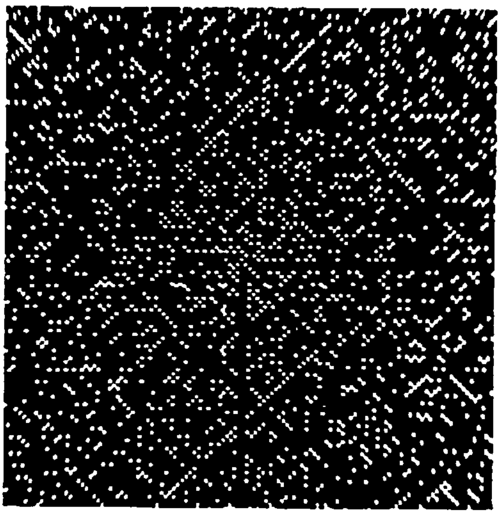


Figure 2

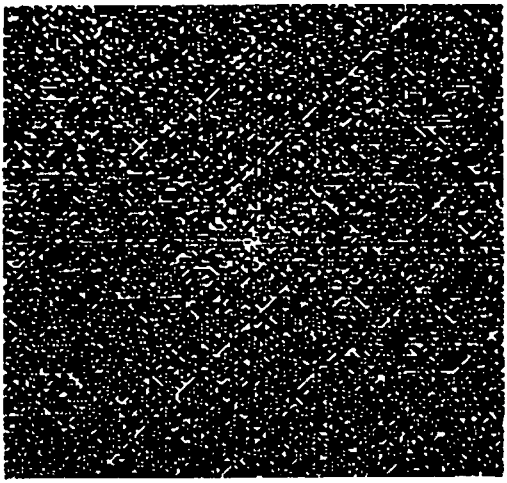


Figure 3

The straight lines that are diagonal lines are apparent, but there are also a number of horizontal and vertical lines. Straight lines in all directions bear numbers that are the values of quadratic expressions of the form $4x^2 + ax + b$ where a and b are constants. For example, the diagonal sequence of primes 5, 19, 41, 71 in Figure 1 is given by the values of the quadratic expression $4x^2 + 10x + 5$ where $x = 0, 1, 2, 3$, respectively, and the diagonal sequence 31, 59 is given by $4x^2 + 24x + 31$ for $x = 0, 1$, respectively.

A natural question to ask is, whether similar results occur when the subset of positive integers begins with a positive integer greater than 1. Consider the grid formed by starting with 20, a composite number, as in Figure 4. The primes are circled.

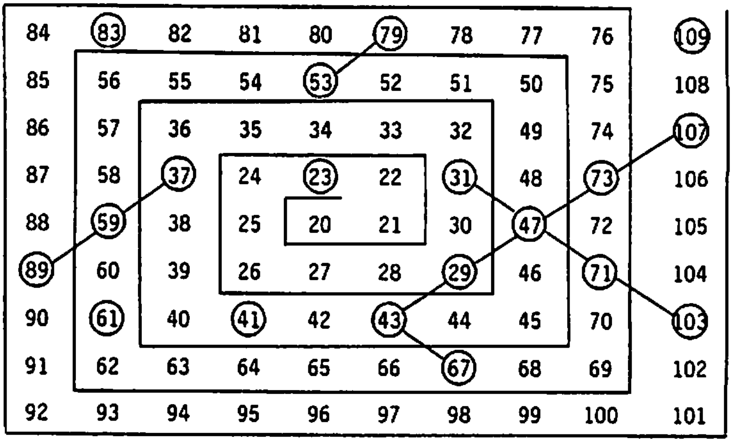


Figure 4

Examining the prime sequence 31, 47, 71, 103 one notes that $4x^2 + 12x + 31$ yields these primes for

$x = 0, 1, 2, 3$, respectively.

Euler's most famous prime generator, $x^2 + x + 41$ can be derived on a spiral grid which starts with 41, as in Figure 5.

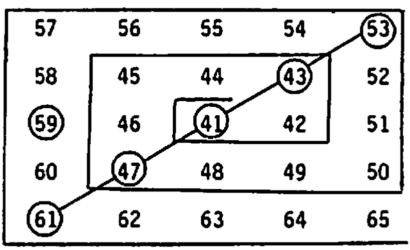


Figure 5

By extending this grid,we can obtain an unbroken sequence of 40 primes along the diagonal of a 40 x 40 square. Historically, it has been known that of the first 2398 integers generated by $x^2 + x + 41$, exactly half are prime. Ulam, Stein and Wells found the proportion of primes to all integers produced by this formula which are less than ten million to be approximately .475.

Ulam's spiral grids present an unusual approach to the determination of primes and a search for patterns. Starting a spiral grid with the integer 17, can you find the sequence of primes generated by Euler's formula, $x^2 + x + 17$ for $x = 0, 1, 2, \ldots, 15$?

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical *issues* of interest to undergraduate students are discussed in length. Here by *issue* we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to *Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.*

The issue that we wish to bring up through the following article, is that of Artificial Intelligence. The field of Artificial Intelligence addresses to questions such as "Is there a notion of natural intelligence?" "If so, is it fixed and defined by nature alone?" "How much of natural intelligence can we simulate by Artificial Intelligence?" "Is Artificial Intelligence only dependent upon the external cues supplied to the simulator?"

We believe that the story of computers' ability to play games is a good forum for discussion of this topic. The game of chess has been a particularly challenging one to teach to computers.

The following article is a reprint from THE MATHEMATICAL INTELLIGENCER Volume 3, Number 2, 1981 and is reproduced here with kind permission of its editor Prof. Roberto Minio and its author Prof. Filman.

Readers are invited to respond to this article.

IK

COMPUTERS AND CHESS

Robert E. Filman*

There has been a recent upsurge in popular interest

*Professor Filman received his Ph.D. in Computer Science from Stanford and is currently a member of the Computer Science Department at Indiana University. His fields of interest include Artificial Intelligence.

in Computer Chess. This has been primarily a function of two events: the emergence of chess programs that can compete with good human players, and the marketing of small chess playing machines. These toys are competent enough to challenge the average chess player and inexpensive enough to make themselves quite numerous.

Along with scientific advance comes the popular literature of that advance. The books that provide the best perspective on the progress of computer chess are a series by David Levy. The most recent of these volumes, More Chess and Computers (by David Levy and Monroe Newborn [1]) recently crossed our desk, and prompted this review of the state of computer chess. (The first volume in this series was Chess and Computers [2]. This was a good book, notable for its fine history of chess playing machines and computer chess. The second and third were 1975 - U.S. Computer Chess Championship [3] and 1976 - U.S. Computer Chess Championship [4]. These volumes were essentially transcripts of the games from those competitions.)

Many people have made a career out of programming computers to play chess. Levy is special: he has made a career out of playing chess against computers. David Levy first gained the attention of the chess programmers (and the Artificial Intelligence community)with a famous bet. As Levy relates the story [2]:

> In August of 1968 - I attended a conference on Artificial Intelligence at Edinburgh University's Department of Machine Intelligence and Perception. At a cocktail party one evening during the conference, I happened to be playing a friendly game of chess with John McCarthy,a professor of Artificial Intelligence at Stanford University and one of the world's leading authorities in the field. I won the game, and he

> remarked that although he was not strong enough for me, he thought that within ten years there would be a computer program that could beat me. You can imagine my reaction. I was the reigning Scottish Chess Champion at the time, and here was this inexpert player telling me that in only a few years I would succumb to a computer program! I said something roughly equivalent to (but more polite than) 'put your money where your mouth is,' and I offered to bet Professor McCarthy £500 (then worth $1,250) that he was wrong.

Between McCarthy and others (Computer Science Professors Donald Michie, Seymour Papert and Ed Kozdrowicki) the bet soon swelled to £1,250.

For many years,it appeared that the Computer Scientists had no hope of winning their wager. There were no programs that could compete with a skilled chessplayer, certainly not with an International Master like David Levy. However, in 1976 events took a dramatic turn, and a potential challenger emerged: Chess 4.5. Chess 4.5 had been written by a pair of systems programmers from Northwestern University, David Slate and Larry Atkin. That year, it won the class B section of the Paul Masson tournament. It followed that title in early 1977 with the crown from the Minnesota Open Championship.

The program was not invincible. Its Open victory was succeeded a week later by a crushing defeat in the Closed Championship. Much of its performance, Levy asserts, was due not so much to its chess ability, as to the emotional failings of its human opponents,who seemed "psyched out" at playing a computer. Hans Berliner has pointed out another major advantage the machine had in the Paul Masson victory [5]: "The tournament had been played out-of-doors in a vineyard with wine being served between the rounds; a condition which could impair a human's performance considerably more than a machine's."

Nevertheless, the quality of Chess 4.5's play was a dramatic improvement over its predecessor chess programs. While it seemed that the program was not of Levy's caliber a series of matches was arranged to decide the bet.

The final challenge match was played by Chess 4.7, a successor of Chess 4.5, in August of 1978. The competition took place in Toronto, at the Canadian National Exhibition. The match was for six games, with one point awarded for a win, and one half for a draw. Levy needed three points to win his bet, that no program would beat him within the ten years.

In 1950, Claude Shannon identified three possible ways of constructing a chess playing program [6]. A Type A program would search all possible moves to some predefined depth, and evaluate the resulting positions. A Type B program would prune from the search tree those moves that did not appear worth pursuing. Type C programs would be goal oriented, though Shannon did not specify how such a system was to be realized.

Almost all the current "good" chess programs, and Chess 4.7 in particular, are Type A programs. They combine two techniques in deciding what move to play. The first is essentially a "minimax" search of the "tree" of all possible move sequences to some depth. The second is a "static evaluation" of the board at the leaf of the search tree when the search reaches that depth. That is, for any board position the computer considers all of the possible moves it might make. For each of these moves, it "puts itself in the position of its opponent", and repeats the process recursively. The program does not, of course, continue down to the bottom of the "tree", where the result of the game is decided. (For chess, this "tree" of all possible games is considerably larger than the number of electrons in the universe.) Instead, at some depth the machine concludes that it has followed a particular path far enough, and performs a "static evaluation" on the resulting board. This depth may be a predefined value that remains constant throughout the entire running of the program,or the program may be more clever, varying this depth dynamically during the course of the program execution. The decision as to when to cut off search and statically evaluate can be based upon factors such as the developing game situation, the time remaining to the machine for the rest of its moves, and the time consumed during the search. The depth may even vary within a single move, if the circumstances warrant. For example, the best chess programs will not stop searching in the middle of a capturing or checking sequence.

Static evaluation implies looking at the board, without searching, and deciding which side is ahead. Current static evaluation theory places primary weight on material difference (who has more or better pieces on the board). It also gives credit to such factors as piece mobility, pawn organization and square control. The programmer of the system has considerable discretion in deciding what factors to include in the static evaluation function. The major constraint is that the static evaluation function will be computed often, so it needs to be quick. The result of static evaluation is a number, the machine's estimate of how favorable the particular position is. The larger this number, the greater the machine's belief in the likelihood of its victory.

At the next to the bottom level of the tree, the numbers returned by static evaluation of each possible move are compared. If this move was considered for the machine, the maximum of the various static evaluations returned is selected as the appropriate move. If the move was for the opponent, then the minimum is used. These values are passed successively upwards, maximizing and minimizing at alternative levels, until the topmost, "root" of the tree, where the move whose value is highest is selected. This process is aptly called "minimax".

There is an important variation on full minimax, called the alpha-beta heuristic, where those branches of the tree that could not possibly be useful are not searched. This pruning is justified by the following observation: if your opponent has a move that will crush you, it does not matter what could happen with his other moves; you do not need to consider any of them. If the number of different moves at any level of the tree is about B (the branching factor), and one wishes to search to a depth D, then minimax will visit on the order of B^D terminal nodes. On the other hand, properly applied alpha-beta will consider only about $B^{\sqrt{D}}$ static evaluations.

The Northwestern chess programs employ this methodology in their play. Chess 4.7, which played Levy in the challenge match, sometimes examined over a million and a half terminal positions when considering a move; it typically looked to depth 8 (eight half moves) in the middle game, and to depth 12 in the end game (where the branching factor is smaller).

Almost all books on computer chess describe the alpha-beta heuristic in some detail; Levy's first volume

[2] and Nilsson's book on Artificial Intelligence [7] are good sources for further explanation. An excellent article by Slate and Atkin [8] contains a complete enough description of Chess 4.5 as to allow for its programming. The variety of factors used in the static evaluation function in Chess 4.5 makes for particularly interesting reading.

The tree search paradigm employed by Chess 4.7 results in a highly tactical type of play. It is difficult to trap the machine through a complicated combination, because the machine is looking at all possible continuations. However, the tree search and static evaluation methodology provides no basis for strategy. The machine plays. It knows to avoid losing material, but it lacks a wider vision or goals. What little strategic knowledge such a system possesses is weakly embodied in its static evaluation functions: a little credit for keeping the king in relative safety, or for controlling some particular squares. The idea of mounting a "king side attack" or "gaining control of the center" is not part of Chess 4.7's language. It makes no such plans. Rather, it makes no plans at all.

Levy is the expert at playing chess programs, and was well aware of these limitations when facing his challenge match. He understood that the way to beat the machine is to play defensively, avoiding sharp tactical situations, and to simplify until the positional advantage of planning and strategy becomes overwhelming. As he describes his technique [1], "I was following my dictum of doing nothing but doing it well, and waiting for the program to dig its own grave."

Nevertheless, the computer managed a draw in the first game. This in itself was an accomplishment; no machine had drawn with an international master under tournament conditions before. Levy avoided such difficulties in the next two games, winning handily. He now had 2 1/2 of the three points he needed to "avoid losing". With three games left to play, Levy decided to experiment. Casting aside caution, his plan was to try "sharp, tactical chess ... endeavoring to outanalyze Chess 4.7."

Levy failed. To everyone's surprise, and the programmer's delight, the machine now had achieved not only a draw against an International Master, but also a victory!

Levy soon made amends. He returned to his earlier defensive style and defeated the machine in the fifth game, winning the match 3 1/2 to 1 1/2. With the match came the bet: Levy was several hundred pounds sterling richer. Levy relates that with his check, John McCarthy sent a note that expressed the sentiment that, had Levy lost to a brute force program McCarthy would not have felt that the science of Artificial Intelligence was responsible for the defeat. McCarthy's view is perhaps too pessimistic;one could also argue that the Artificial Intelligence research of twenty years ago has been transformed (in the natural progression) into the engineering of today.

Levy has since offered to renew the bet for up to $10,000 to 1984, but has indicated that he expects to be beaten after that. He has also offered a prize of $1,000 to the first program that defeats him in a tournament-condition match; a contribution from OMNI magazine has swelled that sum to $5,000.

Recently, the Fredkin foundation has offered a prize of $100,000 to the first program that defeats the world chess champion. To encourage work on the problem, a series of smaller prizes can be won against human competition. The first of these matches occurred last August, at the First National Artificial Intelligence Conference. A randomly chosen "expert", Paul Benjamin, with United States Chess Federation rating of about 2050 was selected to play Chess 4.9 in a two game match for $1,500. The match was split, with Benjamin winning the first game through solid, defensive, strategic play, and losing the second when he tried an attacking, tactical game. Readers interested in winning the various prizes for chess playing programs are referred to [9] for the details of the contests.

It is also possible to program Type B chess systems that do not look at every possible move in every position. Rather, at any level in the tree, such programs select only a few "best" moves for further continuation. However, there is both an execution time cost in deciding which moves bear expanding, and a severe penalty in a highly tactical game like chess for missing some move in a sequence. Programs designed on this principle have not been as successful chess players as the full-width search variety.

Advanced full-width search chess systems, such as Chess 4.7 (and its latest descendant, Chess 4.9) can examine millions of positions in deciding upon which piece to play. Psychological studies have shown that even the greatest human grandmasters seem to look at only about 100 positions per move. The grandmaster sees which positions need expanding, and which do not. He has "reasons" for his choice of moves; when a particular terminal node is reached, the "cause" of its success or failure is communicated to the rest of the search. (An article by Neil Charness [10] is an excellent summary of the research on human chess skill.) A Shannon Type C program would behave in a similar fashion. Few chess programs have any comparable ability, though there have been attempts to incorporate this kind of knowledge representation into such systems. The most successful example of instilling reasoning and planning in a chess program has been David Wilken's program PARADISE (PAttern Recognition Applied to DIrecting SEarch) [11]. By the use of pattern directed production rules and planning sequences, PARADISE was able to solve 97% of the difficult, tactical middle game problems from Win at Chess [12]. In doing so, it would visit tens of terminal positions, rather than hundreds of thousands.

While programs such as PARADISE can sometimes make expert plays in difficult positions, their expertise fails them in the course of prolonged competition, where most positions are not difficult problems. Nevertheless, the idea of strategic play, where strategies and causality are communicated during the reasoning process, remains the prime research topic in computer chess.

The Northwestern chess program has been recently deposed as king of computer chess. The program Belle, written by Ken Thompson and Joe Condon of Bell Laboratories, recently claimed the world computer chess championship. Belle, like the Northwestern programs, is a Type A system. However, it possessed the advantage of special-purpose computer hardware to perform lightning swift search. The world computer chess championships allow three minutes per move; Belle was able to examine almost 30 million positions in that time. This proved to be a decisive advantage.

And what are the prospects for a computer grandmaster? Levy, in summary of his experience, expresses his contempt for chess playing machines with the conclusion [1]:

> Until Artificial Intelligence makes giant strides in the realm of concept formation it will be impossible for chess programs to exhibit the understanding of a Fischer.

He continues

> With its present level of intelligence and a very fast typewriter, the monkey can type innumerable crude sonnets, but without increasing its I.Q. it will never write Hamlet.
>
> With their present level of sophistication, and running on very fast computers, the best chess programs can play innumerable crude games, but without increasing their 'understanding' of chess they will never play with the subtlety of a World Champion.

Levy himself displays a lack of understanding with such criticism. It may be true that computers will never possess the concept formation ability to play like Fischer. Then again, neither will Levy. Levy seems to be expressing the sentiment, "If a computer can do it, then it isn't intelligence." Once something is well enough understood to program it, it ceases to impress us. This sentiment is expressed in Tesler's Theorem: "A.I. is whatever hasn't been done yet" [13]. "Crude" brute force search techniques endow the machine with greater chess skill than all but a fraction of a percent of all human players. Levy seems unwilling to admit that this is an accomplishment; that while the machine was unable to beat him, it came a lot closer than he would have expected. Of course, he probably wouldn't be impressed with a monkey who could only do say, Neil Simon, rather than Shakespeare.

Another relevant sentiment was expressed by Hans Berliner, himself a former World Postal Chess Champion, and a researcher on computer chess [5]:

> From this I must conclude that human chess players largely delude themselves in believing that chess is a 'conceptual' game. Apparently a large part of chess can be solved by exhaustive searching... .

With all the excitement about chess playing machines, the reader may be tempted to go out and buy one for him or herself. One chapter of More Computers and Chess is devoted to the small micro-computer chess toys that are available for consumer purchase. These gadgets now cost less than $100; their prices will fall, just as the prices of pocket calculators fell. This chapter, written by the co-author, Monroe Newborn, is notable for its layman's explanation of micro-processors and micro-computers. The comparative analysis of the various available brands is also enlightening. However, computer technology is a field of rapid change; I fear that the specific purchase recommendations listed in that chapter may already be out of date. More current reviews of the marketplace may be periodically found in the magazines that have emerged for computer hobbyists, such as Creative Computing, Byte, and Personal Computing.

Strangely enough, computers have made contributions to the theory of chess. Levy [1] relates an opening innovation that Chess 4.6 was able to discover. Two of my favorite "computer" contributions have been adjustments to the rules of chess. The first was discovered early, when people first programmed machines to play. In chess, one is allowed to claim a draw if the same position is repeated three times. The programmers, with their ability to distinguish pieces at a non-semantic level (this is the queen's rook, and that the king's rook) asked the question: if the two rooks (knights) have changed places in reaching the repetition, is the position still drawn? The rules committee decided: a rook is a rook is a rook.

The second discovery also concerns drawn games, and was found by programmers researching endgame problems. It was the rule that if no pawn advances, and no piece is captured within fifty moves, then one is allowed to demand a draw. The number fifty was selected because it was believed that any "won" position could be claimed by correct moves within that span. However, work on endgame programs has revealed that proper play can prolong some king and rook versus king and knight endgames so long as to force the player with the rook to use up to fifty four moves before it can obtain victory.

The work on game playing has also contributed to the mathematics of algorithms. A very recent discovery by Dana Nau is that there is a certain class of games for which deeper minimax (alpha-beta) search produces poorer performance [14]. For many, this is a quite counterintuitive result; the performance of chess programs has monotonically improved as the search depth increases. It is possible that Nau's work will lead to an improvement or replacement for alpha-beta; it will be interesting to see what form that substitute takes.

Work continues on improving the performance of chess programs. With the continuing decline in computer costs and the trend towards multiple processor machines, we will see machines that search deeper and farther. Gains are also being made in the field of chess knowledge representation and concept formation. We await the (not too distant)time when a computer will be the world chess champion.

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This is a new section of THE PENTAGON which will be devoted to topics related to Computer Science. If you have a topic of interest you are invited to submit your composition for publication in this section. Preference will be given to articles authored or co-authored by students. Although many titles have been suggested for this section, none so far seems quite appropriate, so we urge you to give us your ideas. The chapter with the best idea will be duly recognized in the next issue of THE PENTAGON. Address all correspondence to *Kent Harris, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.*

THE EIGHT QUEENS PROBLEM

Hans Hamilton, student Western Illinois University

and The Editor

The Queen is the most powerful piece in the game of chess, in that it can capture any other piece which is on its row, column or diagonal. The Eight Queens Problem is to place eight Queens on a chessboard (8 squares by 8 squares) so that no Queen is in a position to capture another Queen.

Can this be done? If so, in how many different ways? The reader is urged to experiment with 4 Queens on a 4x4 board and 5 Queens on a 5x5 board before continuing through our explanation.

Before explaining the mechanics of the solution, a little history of the problem is in order.

According to Campbell [1], the Eight Queens Problem first appeared in the Sept. 1848 issue of the chess newspaper, "Schachzeitung". The problem was posed by the famous German chessplayer Max Bezzel. The same problem was posed again in 1850 by Franz Nauck in an issue of "Illustrirte Zeitung" and it was this article which involved Gauss in the problem. The problem was never completely solved by Gauss although Gauss was responsible for: 1) the reformulation of the problem by relating it to the representation of the complex numbers, and also 2) an initial estimate of 72 solutions to the problem. A more complete account of Gauss's involvement can be found in Campbell's paper [1].

Today, this problem whose complete solution eluded Gauss,(perhaps because of the tedium involved in solving the problem by hand?) makes an interesting problem for students interested in programming.

To begin our analysis of the program solution it should be noted that several different types of data structures could be used to represent the position of the Queens on the board.

1) The position of the Eight Queens could be represented in a two-dimensional array of 64 elements.

2) The positions could be represented in a one-dimensional array of 8 elements.

3) The positions could be represented using 3 one-dimensional arrays— one of 8 elements (as in 2), the other two of 15 elements, being used to indicate the

presence or absence of Queens on the diagonal.

Wirth [4] uses a variation of 3) which employs 4 one-dimensional arrays, three of which are Boolean (i.e. True or False).

Our algorithm depends on 2), using a simple one-dimensional array A[1],..., A[8] in which the Queen in column I is in row A[I]. As an example consider the Four Queens Problem:

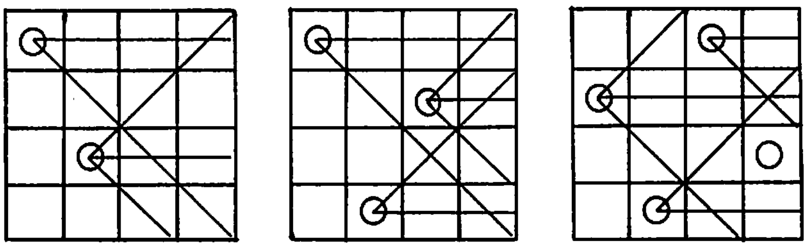

Figure 1

As can be seen, the method is by "trial and error", proceeding from the first column to the right, always moving the Queen down in the rightmost column in which it is possible to do so. If the bottom of a column is reached and no solution is possible, the search backtracks (i.e. moves into the column to the left), then moves down in that column looking for a 'feasible' position. This method illustrates backtracking, a general technique in algorithm design. (See Horowitz [2] for more on this topic.)

An alternative to using the concept of backtracking would be "brute force". In other words we would determine the total number of possible solutions and check each one. If the reader wonders why we didn't use "brute force", he/she should consider that there are $8^8 = 16,777,216$ elements in the solution space. (Actually this can easily be reduced to $\frac{1}{2} \times 8! = 20160$ elements.)

The backtracking technique used in the Four Queens example owes its efficiency to the elements that are never examined. The tree in figure 2 shows the subtrees that need never be checked due to row or diagonal constraints. The symmetry of the problem allows us to work with the left half subtree.

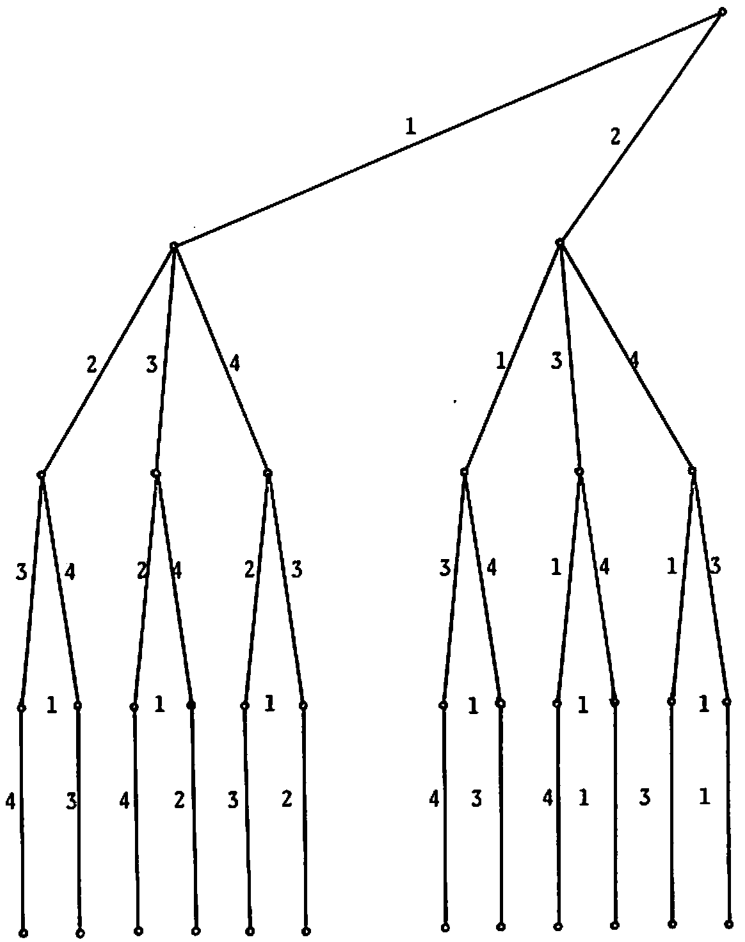

Figure 2

In the tree shown, the leftmost subtree contains all possible solutions with A[1] = 1, i.e. the Queen in column I is in row 1. The leftmost subtree of this subtree contains all possible solutions with A[2] = 2, i.e.

the Queen in column II is in row 2. Each digit represents a value of A[I]. The 1 and 2 at level one represent values of A[1], digits at level two represent values of A[2], etc. The sequence 2, 4, 1, 3 represents the solution previously mentioned.

Before we proceed to our program solution, let us look again at the Four Queens Problem. Suppose we have assigned values to A[1], A[2], and A[3] in accordance with the row and diagonal constraints. For A[4] to satisfy a row constraint, it must be true that A[4]≠A[I] for I = 1,2,3. For A[4] to satisfy the diagonal constraints, it must be true that $A_4 \neq A_1 \pm (4-1)$, $A_4 \neq A_2 \pm (4-2)$, and $A_4 \neq A_3 \pm (4-3)$. That is, $A_4 \neq A_I \pm (4-I)$ for I=1,2,3. This can also be written $A_4 - A_I \neq \pm (4-I)$, or $|A_4 - A_I| \neq 4-I$ for I=1,2,3.

In general, these conditions are:

1) $A_K \neq A_I$ for I=1,...K-1,

2) $|A_K - A_I| \neq K-I$ for I=1...K-1

For more on this analysis, see Tucker [3].

Now let us turn to our algorithm (Figure 3) and the output of the Pascal program (Figure 4). There are 92 solutions, only 12 of which are non-symmetric.

```
N←0; K←1; A[1]←1
Repeat
   K←K+1; A[K]←1
   Repeat
      While (A[K] < 8 AND COMP(K)=FALSE) DO
         A[K]←A[K]+1
         IF A[K]=9 THEN
            IF K>1 THEN BACKTRACK
            ELSE END
         ELSE  IF K<8 THEN MOVE TO TOP OF NEXT COL
               ELSE (*A solution has been found.*)
                  Test for Repetition
                  Output the Non-Repetitious Solution
                  Backtrack
   Until The Column Has Been Exhausted
Until All Solutions Have Been Found

Function COMP(K)
   COMP←TRUE
   FOR I=1 TO K-1 DO
      IF (A[I]=A[K]) OR (ABS(A[K]-A[I])=K-I) THEN
         COMP=FALSE

Procedure BACKTRACK
   K←K-1; A[K]←A[K]+1

Procedure Repetition
   (*Preliminary Computation Used to Compute B[4,K] *)
   FOR TARGET=1 TO 8 DO
      I←1
      WHILE A[I]<>TARGET DO
         I←I+1
      S[TARGET]←I
   (*Compute the Eight Transformations of A[1..8]   *)
   FOR K=1 TO 8 DO
      B[0,K]←A[K]
      B[1,K]←A[9-K]
      B[2,K]←9-A[K]
      B[3,A[K]]←K
      B[4,K]←9-S[9-K]
      B[5,K]←9-B[4,K]
      B[6,K]←9-B[1,K]
      B[7,K]←9-S[K]
   Compare Each Transformation With Those
Solutions Already Found
```

Figure 3

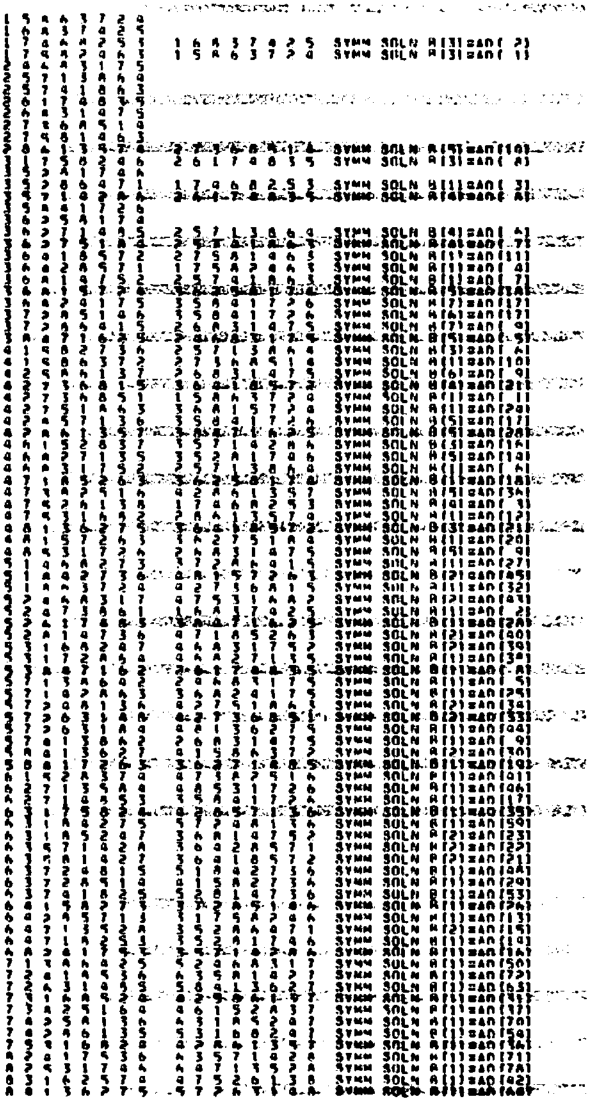

Figure 4

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KAPPA MU EPSILON NEWS

EDITED BY HAROLD L. THOMAS, HISTORIAN

News of chapter activities and other noteworthy KME events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department, Pittsburg State University, Pittsburg, Kansas 66762.

CHAPTER NEWS

Alabama Zeta, Birmingham-Southern College, Birmingham
Chapter President-Elizabeth Curry
36 Actives

The chapter sponsored a seminar on careers in actuarial science. A spring picnic was held jointly with honorary organizations in biology and chemistry. Other 1981-82 officers: Linda Eckert, vice president; Carolyn Millican, secretary; Cheng Lin, treasurer; Lola Kiser, corresponding secretary; William Boardman, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President-Lori Canter
46 Actives, 28 Initiates

Annual Math Sciences Career Day for the campus was sponsored by the chapter. Math Field Day was held for all junior high school students in the county. The chapter assisted the Mathematics Department faculty with Poly Royal (annual open house) which included a mathematics contest that attracted over 700 high school students. Dr. Mach and four students drove round trip to the Twenty-Third Biennial Convention in Springfield, Mo. A booksale was held to raise money for the convention travel expenses. Chapter meetings included alumni and industry speakers. The annual initiation banquet had about 115 members and guests in attendance. Other 1981-82 officers: Maryjean Rende and Jim Carley, vice presidents; Nancy Miller, secretary; Tom Crump, treasurer; George R. Mach, corresponding secretary; Adelaide Harmon-Elliott and Dina Ng, faculty sponsors.

California Delta, California State Polytechnic University, Pomona
Chapter President-Jay Edson Ebling
14 Actives, 4 Initiates

Spring activities included a mathematics display for "Poly Vue," Cal Poly's annual open house and a picnic held at San Dimis Park. The annual spring banquet was held June 14, 1981, at the Arbor Restaurant. Mrs. Jean Pedersen spoke on geometric solids. The chapter plans to be active during the summer. A beach party will be held. Other 1981-82 officers: Mariangela Muggia, vice president and treasurer;Richard Robertson, corresponding secretary; Cameron Bogue, faculty sponsor.

Colorado Alpha, Colorado State University, Ft.Collins
Chapter President-Mike Thomas
10 Actives

Other 1981-82 officers: Mary Jo Black, vice president; Paul Magnus, secretary; Ann Murray, treasurer;Arne Magnus, corresponding secretary and faculty sponsor.

Connecticut Beta, Eastern Connecticut State College, Willimantic
Chapter President-Michael Lamb
0 Actives, 26 Initiates

KME's newest chapter was installed May 2, 1981. The initiation was part of the Math Alumni Reunion Banquet which was attended by 140 people. The chapter initiated 26 new members. Other 1981-82 officers: Carole Grenier, vice president; April Schulze, secretary and treasurer; Ann Curran, corresponding secretary; Stephen Kenton, faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President-Tammy S. Woodworth
16 Actives, 9 Initiates

The annual initiation ceremony was held April 28,1981. Nine pledges were initiated and chapter officers were elected for 1981-82. A reception was held in honor of the new members. A May program was given by Dr. Leland

Gustafson, economics professor at West Georgia College, entitled "Mathematics of Retirement Planning." Other 1981-82 officers: Annelle Colevins, vice president; Darla House, secretary; Wendy Muse, treasurer; Thomas J. Sharp, corresponding secretary and faculty sponsor.

Illinois Zeta, Rosary College, River Forest
Chapter President-Michael Renella
13 Actives, 2 Initiates

Members continued the practice of presenting problems at chapter meetings for the amusement and challenging of fellow members. Two new members were initiated on March 24. The chapter extended financial support to three members who attended the national convention in Springfield, Mo. Other 1981-82 officers: Mary Brady, secretary; Brad Erickson, treasurer; Sister Nona Mary Allard, corresponding secretary and faculty sponsor.

Illinois Eta, Western Illinois University, Macomb
Chapter President-Hans Hamilton
21 Actives, 13 Initiates

Other 1981-82 officers: Karen Seehafer, vice president; Tom Maher, secretary and treasurer. Alan Bishop, corresponding secretary; Iraj Kalantari, faculty sponsor.

Illinois Theta, Illinois Benedictine College, Lisle
Chapter President-Judith Meismer
22 Actives, 10 Initiates

Spring activities included a mathematics contest for West Suburban Chicago High School students on February 21, 1981. Mr. Irwin Katzman of Katzman and Associates, Actuarial Consultants, spoke on Career Opportunities in Actuarial Science at the April 17 meeting. Initiation ceremony was held May 1st. The chapter continues to sponsor free tutorial service for college mathematics courses. Other 1981-82 officers: Susan Milnamow, vice president; Jeanne Stablein, secretary; Robert Gullett, treasurer; James Meehan, corresponding secretary and faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
Chapter President-Ramona Seese
27 Actives, 7 Initiates

Initiation ceremonies for seven new members was held at a dinner at the home of Dr. McBride, faculty sponsor. Thirty-five members and guests were in attendance. Other 1981-82 officers: Craig Stine, vice president; Karen Rund, secretary; Larry Holston, treasurer; Ralph B. McBride, corresponding secretary and faculty sponsor.

Indiana Gamma, Anderson College, Anderson
Chapter President, Terri Reynolds
16 Actives, 9 Initiates

Other 1981-82 officers: Brian Nogar, vice president; Lauri Van Norman, secretary; Douglas Skipper, treasurer, Stanley Stephens, corresponding secretary and faculty sponsor.

Indiana Delta, University of Evansville, Evansville
Chapter President-Wadieh Hawa
27 Actives, 13 Initiates

Other 1981-82 officers:Brett Barnett, vice president; Brent Moor, secretary; Melba Patberg, corresponding secretary; Duane Broline, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President-Kay Sacquitne
37 Actives, 3 Initiates

The following students presented papers at chapter meetings: Rebecca Borseth on "Catastrophe Theory," Charles Daws on "Projectile Motion with Drag Forces," Margaret Chizek on "Computer Assisted Instruction," and Ruth Appleby on "The Byte in Crime (Computer Capers)." In February a group from Iowa Alpha traveled to Waverly, Iowa for a joint meeting with Iowa Delta at Wartburg College. Hopefully such an exchange will become an annual event. The mathematics faculty and graduating KME members held a "seminar" at Tony's Lounge on the last day of classes. Professor Augusta Schurrer was presented the College of Natural Sciences Dean's award for Outstanding Achievement as a faculty member. She was nominated by the chapter for this award. Other officers for 1981-82: Darla Dettmann, vice president; Margaret Chizek, secretary; Charles Daws, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President-Barbara Schuck
10 Actives, 3 Initiates

Dr. Dean Sandquist, a former KME member and currently a researcher at the University of Iowa Medical School, spoke at one of the spring meetings on "Applications of Mathematics to Medicine." Other 1981-82 officers: Susan Burmont, vice president and treasurer; Dan Whitnah, secretary; Lawrence Naylor, corresponding secretary; Alexander Kleiner, faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President-Karen Waltmann
15 Actives, 19 Initiates

Two films on integrated circuits and microprocessors were viewed for the January meeting. In February a joint pizza and games party was held with the Iowa Alpha chapter from the University of Northern Iowa. Wartburg Math Field Day was sponsored by the chapter in March. One-hundred thirty-five high school students participated along with faculty advisors. A banquet was held March 28, 1981, at which time nineteen new members were initiated. Dr. John Chellevold, Professor Emeritus, presented a talk on the International Congress of Mathematicians. Rain once again forced the May picnic to be held in Dr. Olson's house. Other 1981-82 officers: Al Guetzlaff, vice president; Jean Movall, secretary; Edmond Bonjour, treasurer; Lynn J. Olson, corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President-Brenda Brinkmeyer
40 Actives, 3 Initiates

Kansas Alpha began the spring semester with a banquet and initiation for the February meeting. Three new members were initiated at that time. Linda McCracken presented the program about"Little Known Facts about Mathematics." The March program was given by Pamela Duncan on "How to Cut a Cake." Paige Chilton gave the April program on "The Rubik's Cube." The chapter also assisted the mathematics department faculty in administering and grading tests given at the annual Math Relays, April 28, 1981. Six students and faculty attended the National Convention held in Springfield, Mo., the first weekend in April. The final meeting of the spring semester was held at Professor Sperry's home. It was highlighted by election of officers for the 1981-82 school year. In addition, the annual Robert M. Mendenhall award for scholastic achievement was presented to Terri Hoseney. She received a KME pin in recognition of this honor. Other 1981-82 officers: Linda McCracken, vice president; Hazel Kent, secretary; Paige Chilton, treasurer; Harold L. Thomas, corresponding secretary; J. Bryan Sperry, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President-Steve Pahls
13 Actives, 13 Initiates

Kansas Gamma started off their second semester activities with a computer dance on January 17 to raise funds to support other chapter activities. The chapter sponsored a speaker in February from a local bank. Peter Gabroysky, who holds a doctorate in computer science, spoke on "What is the Meaning of Mathematics?" Kay Kreul, Lisa Kolb, Dan McChesney, and Becky Hollis were initiated on March 12. Following initiation final plans were made for the 11th Mathematics Tournament held on March 14 for approximately 200 area high school students. Officers Amy Duffy and Rick Desko co-directed this event. Eight students attended the national convention with corresponding secretary Sister Jo Ann Fellin on April 2-4. Dianne Hickert was named the recipient of the Sister Helen Sullivan Scholarship at the honors banquet. The chapter closed its activities with a steak picnic on April 23 in honor of its senior members. Other 1981-82 officers: Terri Beye, vice president; Kay Kreul, secretary; Tom Gallagher, treasurer; Leslie Peabody, historian; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President-Cindy Dietrich
20 Actives, 5 Initiates

Other 1981-82 officers: Kathy King, vice president; John Brad Lichtenhon, secretary; Kevin Heideman, treasurer; Robert Thompson, corresponding secretary; Billy Milner, faculty sponsor.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President-Don Jesch
27 Actives, 9 Initiates

Monthly meetings were held with students presenting programs. Other 1981-82 officers: Maxine Arnoldy, vice president; Sally Irvin, secretary and treasurer; Charles Votaw, corresponding secretary; Jeffrey Barnett, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond
Chapter President-Kevin Preston
26 Actives, 18 Initiates

Members held free weekly tutorial sessions for all university students. Meetings were held bi-weekly with guest speakers presenting varied mathematical topics. A campus-wide backgammon tournament was conducted to raise money. A spring party with initiation was held at Dr. Yeager's house. Two students and two faculty attended the national convention in Springfield, Mo. Other 1981-82 officers: Judy Dusing, vice president; Andrea Norris, secretary; Beth Stewart, treasurer; Dorian Yeager, corresponding secretary; Don Greenwell, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore
Chapter President-Cecilia Wintz
10 Actives, 3 Initiates

Three new members were initiated at the annual induction meeting. Marie D. Eldridge ('47) spoke about her statistical work at the Department of Education in Washington, D.C. Officers for 1981-82 will be elected in September, 1981. Sister Marie Augustine Dowling, corresponding secretary; Sister Delia Dowling, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President-Sally Carlson
22 Actives, 3 Initiates

In April the chapter hosted a career night for all mathematics majors and featured talks by four KME alumni members from the fields of actuarial science, operations research, teaching, and health statistics. The chapter also held several money-raising activities to help pay the expenses of a delegate to the national convention in April. The March meeting was annual banquet and initiation of new members, and the May meeting was a chapter picnic at the home of a faculty member. Other 1981-82 officers: Lisa DelPrete, vice president; Pamela Huffington, secretary; Gail Waterman, treasurer; Sally Townsend and Sarah Hellstrom, historians; James E. Lightner, corresponding secretary; Robert Boner, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President-Douglas Cannon
27 Actives, 13 Initiates

Thirteen new members were initiated February 1, 1981. During the semester, talks were given by Dr. George Plitnik, Physics Department and an organist, on the mathematics of music, and by Dr. Arthur Cohen, Rutgers University, a statistician. The chapter sponsored a trip to the Smithsonian Institution. Three members received departmental honors at Honors Convocation. A pizza party marked the last day of classes. Other 1981-82 officers: John Wagner, vice president; Kathy Hardy, secretary; Timothy Lambert, treasurer; Agnes Yount, corresponding secretary; John Jones, faculty sponsor.

Michigan Alpha, Albion College, Albion

The chapter reports that Brian Winkel, previous corresponding secretary and faculty sponsor, has resigned from Albion to take a position at Rose-Hullman Institute. Raymond Greenwell will assume responsibility of corresponding secretary and faculty sponsor. Other officers for 1981-82 were not identified.

Michigan Beta, Central Michigan University, Mt. Pleasant
Chapter President-Judy MacGrayne
50 Actives, 18 Initiates

Chapter members provide tutorial help for students in undergraduate mathematics courses three evenings per week. After much planning throughout the semester, thirteen students and three faculty attended the national convention at Springfield, Mo. Speaker for the Spring Initiation service was Dr. Douglas Nance, Business Manager of the Pentagon. He spoke about KME and what KME membership means. In April, Dr. Daniel Thornton, Central Michigan Economics Department,gave a talk on "Regression in Economics." Several members visited Dow Chemical Company in Midland, Michigan, in March. Dow personnel in analysis, mathematical modeling, and statistics spoke with the group. Other 1981-82 officers: Dan Franck, vice president, Sandra Dolde, secretary; Jolyn Cornell, treasurer; Arnold Hammel, corresponding secretary and faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President-Jayne Ward
58 Actives, 9 Initiates

Spring semester activities were primarily devoted to preparing for and hosting the 23rd biennial convention of KME. The report of this convention was published in the Spring 1981 Pentagon. Other 1981-82 officers: Donna Garoutte,vice president; Debra Oehlschlaeger, secretary; Michael Eidson, treasurer; M. Michael Awad,corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President-Ellen Frieze
35 Actives, 20 Initiates

The chapter held two initiations, a total of six regular meetings, a Christmas party, and an honors banquet. Seven members attended the national convention in

Springfield, Mo. One of these, David Harris, was awarded third prize for his paper entitled"Computer Graphics: A Three-dimensional Representation of Spheres." Other 1981-82 officers: Rita Rotert, vice president; Lisa Oshima,secretary; Lisa Weidinger, treasurer; Homer Hampton, corresponding secretary; Alvin Tinsley, faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
3 Actives, 8 Initiates

Officers for 1981-82 will be elected in the fall. William D.McIntosh is corresponding secretary and faculty sponsor.

Missouri Zeta, University of Missouri-Rolla, Rolla
Chapter President-William L. Pulse
25 Actives, 16 Initiates

Other officers for 1981-82: Michelle Maes, vice president; Donna Miller, secretary; Robert Nespodzary, treasurer; Lynn Brammeies, historian; Tim Wright,corresponding secretary; James Joiner, faculty sponsor.

Missouri Eta, Northeast Missouri State, Kirksville
Chapter President-Ruthie Dare
21 Actives, 7 Initiates

Six students, two faculty, and one alumnus attended the national convention in Springfield, Mo. Other spring activities included the 11th annual spring picnic for mathematics faculty and students, participation in the Campus Bowl competition with two teams entered, hosted the high school mathematics contest for 780 students,and presentation of the annual award to the outstanding freshman mathematics student. Other officers for 1981-82: Anita Fashing, vice president; Mary Nelson, secretary; Neil Meyer, treasurer; Sam Lesseig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

Missouri Iota, Missouri Southern State College, Joplin
Chapter President-Rick Richardson
12 Actives, 6 Initiates

In addition to monthly meetings where student members give talks for the entire math club, the chapter also held an initiation dinner and a spring picnic. Eight members attended the national convention in Springfield, Mo. Other officers for 1981-82: Larry Hicks, vice president; Rhonda McKee, secretary; Joel Callicott, treasurer; Mary Elick, corresponding secretary; Joseph Shields, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President-Rita Liibbe
19 Actives, 9 Initiates

The second semester began with a money making project when a hand-held calculator was raffled off by the club at one of the home basketball games. Throughout the semester the club also sold disc's for the Apple II computers to college students to earn money. Another money making activity throughout the semester has been monitoring the Math-Science building in the evenings. This responsibility is shared with two other organizations on a three week rotation basis. The club entered two teams in the Annual Wayne State College Bowl. One of the two teams finished in the semi-finals. The club also administered the annual test to identify the outstanding freshman in mathematics. The award went to Brenda Mandel of Dodge, Nebraska. The award includes the recipient's name being engraved on a permanent plaque, one year honorary membership in the local KME chapter, and announcement of the honor at the annual banquet. The Nebraska Alpha Chapter was represented at the National KME Convention in Springfield, Missouri, by members Cheryl Wamberg, Rita Liibbe and Howard Maricle and by mathematics faculty member Margaret Lundstrom. Four chapter members, Rita Liibbe, Cheryl Wamberg, Marian Rhods, and Janice Gahan assisted the mathematics department faculty in administering the Seventh Annual Wayne State College Mathematics Contest. Approximately 425 high school students from 60 high schools participated in the contest. Other officers for 1981-82: Cindy Gregory, vice president; Marian Rhods, secretary and treasurer; Cheryl Wamberg, historian; Fred Webber, corresponding secretary; James Paige and Hilbert Johs, faculty sponsors.

New Jersey Beta, Montclair State College, Upper Montclair
Chapter President-Dan Nigro
22 Actives, 11 Initiates

Spring activities included a trip to Atlantic City, an initiation dinner, a Christmas party, and a high school math contest. Other officers for 1981-82: Lisa Parrillo, vice president; Diane Forgione, secretary; Dawn Miller, treasurer; Phillip Zipsze, corresponding secretary; William Koellner, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President-Carmen Montoya
60 Actives, 17 Initiates

Other officers for 1981-82: Debra Long, vice president; Greg Parets, secretary; Becky Coté, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Alpha, Hofstra University, Hempstead
8 Actives, 6 Initiates

During the month of February, an animated math film festival was sponsored by the chapter. Students from local high schools as well as university students were invited. A math contest for freshmen was held in March. Prizes were awarded to the top three students. Officers for 1981-82 will be elected at the first meeting in the fall. Professor John Weidner is corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara
13 Actives, 9 Initiates

Spring activities revolved around preparing for the national convention in Springfield, Mo. Chapter members helped raise funds for the trip. Two senior members, John Michalek and Karen Young, coauthored a paper on "The Characterization of a Finite Power Set as a Ring." The paper was presented at the convention by Karen. Officers for 1981-82 will be elected in the fall. Robert Bailey is the corresponding secretary.

New York Iota, Wagner College, Staten Island
Chapter President-Gloria Presti
10 Actives, 3 Initiates

The chapter held the annual spring induction and annual dinner on May 1st. Professor Sydney Welton is retiring after the spring semester. Iota Chapter extends best wishes to him and welcomes Phil Ratner as a new faculty member in the mathematics department. Other officers for 1981-82: Karen McKendry, vice president; Victor Lindberg, secretary; Anthony Castellano, treasurer; William Horn, corresponding secretary and faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green
Chapter President-Mark Worline
50 Actives

Spring activities included a puzzle night, the annual initiation banquet, and the spring picnic. Other officers for 1981-82: Gwen Hagemeyer, vice president; Lawrence Zaborski, secretary; W. C. Weber, corresponding secretary; W. A. Kirby, faculty sponsor.

Ohio Gamma, Baldwin-Wallace College, Berea
Chapter President-Janet Gosche
29 Actives, 20 Initiates

Other officers for 1981-82: Mike Mazzone, vice president; Nancy Lewis, secretary; Larry Mills, treasurer; Robert Schlea, Corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President-Lael Wall
38 Actives, 9 Initiates

Dr. Ralph Hollingsworth, NCR Advanced R & D Section, gave a talk on applications of computer technology at the January 28 meeting. In February, an initiation dinner was held followed by short talks by the new inductees. The March meeting included election of officers, convention preview, and a student talk by Toshio Maruo. The chapter was also responsible for the all-campus cookie hour. Six students with advisor, Dr. Smith, attended the 23rd Biennial Convention in Springfield, Mo. Dr. Smith was elected to the national office of President-Elect. The semester closed with a Spring picnic with homemade ice cream at Dr. Knight's home. Other officers for 1981-82: Cathy Roby, vice president; Candace Truscott, secretary; Gail Yoder, treasurer; James L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President-Doris Pyles
10 Actives

Spring activities included sponsoring a bake sale for "Math Day," a day when area high school students visit the mathematics department. One of the regular meetings featured a talk by Dr. David Taylor about computer graphics. Officers for 1981-82 will be elected in the fall. Wayne Hayes is the corresponding secretary and David Taylor and Rosalle Taylor are faculty sponsors.

Pennsylvania Alpha, Westminster College, New Wilmington
Chapter President-Candace Jo Yarnell
42 Actives, 22 Initiates

During the spring semester, the chapter conducted a tutorial service for students enrolled in Calculus I,II, and III. Twenty-two new members were also initiated at the annual initiation banquet. Some members also attended the regional MAA meeting. Other officers for 1981-82: Carl Schartner, vice president; Kathy Christman, secretary; Joel Ballezza, treasurer; J. Miller Peck, corresponding secretary; Barbara Faires, faculty sponsor.

Pennsylvania Beta, La Salle College, Philadelphia
Chapter President-Donna Malloy
30 Actives, 3 Initiates

In early April a guest speaker spoke on "Computer Scientists in Industry." Later in the month, the initiation meeting was held and officers elected for the coming year. Other officers for 1981-82: Geralyn Motz, vice president; Diane Balzereit, secretary; Susan Krembs, treasurer; Hugh Albright, corresponding secretary; Carl McCarty, faculty sponsor.

<u>Pennsylvania Epsilon</u>, Kutztown State College, Kutztown
Chapter President-Loria Klee
25 Actives, 8 Initiates

Spring activities included an initiation banquet on March 24th, student speakers on April 6th, and a picnic April 26th. Other officers for 1981-82: Donna Mrazik, vice president; Kate Ramsay, secretary; Lisa Imendorf, treasurer; I. Hollingshead, corresponding secretary; William Jones, faculty sponsor.

<u>Pennsylvania Zeta</u>, Indiana University of Pennsylvania, Indiana
Chapter President-Kelly Barber
23 Actives, 10 Initiates

All activities and programs were governed by the fact that two members, Mary Markert and James Benner were to present papers at the National Convention. At the February meeting, Jim presented his paper, and Mary presented her paper at the March meeting. A hoagie sale was held to help raise money to defray expenses for the students. The annual banquet was held at the University Lodge with an excellent meal being prepared by student members with the assistance of Mr. Raymond Gibson, member of the mathematics faculty. Students reported on convention activities. Other officers for 1981-82: Donna Reed, vice president; Tracy Snyder, secretary;Rose Marie Zbiek,treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

<u>Pennsylvania Eta</u>, Grove City College, Grove City
Chapter President-Karen Konarski
25 Actives, 6 Initiates

The annual spring picnic was held at the Grove City Country Club on May 4, 1981. Fine weather, good food prepared by faculty wives, volleyball games were enjoyed by all. KME also sponsored a mathematics competition during the spring semester. A copy of the CRC Math Handbook was awarded to the outstanding freshman mathematics student. Other officers for 1981-82: Bill Kerr, vice president; Mary Jo Donivan, secretary; Ann Music, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President-Linda Czajka
6 Actives, 5 Initiates

Five new members were initiated on April 15, 1981. Speakers at the ceremony were Jo Ann Dellavalle, Marguerite Leicht, and Margaret Jankowski, former graduates and KME members at Holy Family College. A pizza party followed the initiation giving new members and friends the opportunity to visit further with the guest speakers about how they use their mathematics in industry. The chapter also held a field trip to Reading, PA's museum to view "Mathematics in Art." Other officers for 1981-82: Linda Chinn, vice president; Theresa McKeon, secretary and treasurer; Sister Mary Grace, corresponding secretary and faculty sponsor.

Pennsylvania Mu, St. Francis College, Loretto
Chapter President-John Harris
13 Actives, 9 Initiates

Other officers for 1981-82: Grace Farrell, vice president; Nancy Dudziec, secretary; Rebecca Stanisha, treasurer; Rev. John Kudrick, TOR, corresponding secretary; Adrian Baylock, faculty sponsor.

South Carolina Beta,South Carolina State College,Orangeburg
Chapter President-Wendell Fortune

Other officers for 1981-82: Loretta Conyers, vice president; E. Marie Washington,secretary; James Littles, treasurer;Frank M. Staley, Jr., corresponding secretary, C. Allen Jones, faculty sponsor.

South Carolina Gamma, Winthrop College, Rock Hill
Chapter President-June High
21 Actives-5 Initiates

Several math faculty members presented programs at chapter meetings. Topics discussed included the Solution to the Instant Insanity puzzle, finding the oddly weighted golf ball in a dozen balls in no more than three weighings, powers of primes of $n!$, and alternate definitions for defining properties of groups. Other officers for 1981-82: Donna Jo Davis, vice president; John Imholtz, secretary; Mary Robinson, treasurer; Donald Alpin, corresponding secretary; Kay Creamer, faculty sponsor.

Tennessee Beta, East Tennessee State University, Johnson City
Chapter President-Eric Bowman
25 Actives, 10 Initiates

The chapter held its annual initiation service on April 24, 1981. Following dinner, an address related to activities of NASA was given by Dr. Bill Curran, ETSU Computer Science Department faculty, who had worked for NASA for five years. The initiation service was then conducted by the officers. Ten new members were initiated. Other officers for 1981-82: Betty Carpenter, vice president; Debbie Crawford, secretary; Lyndell Kerley, corresponding secretary and faculty sponsor.

Texas Alpha, Texas Technical University, Lubbock
Chapter President-John Prindle
25 Actives, 13 Initiates

Spring activities included various programs given by faculty members, a picnic and softball game, annual awards and initiation banquet. Other officers for 1981-82: Val Stokes, vice president; Jennifer Smith, secretary; Lance Cary, treasurer; Robert Moreland, corresponding secretary and faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President-David Proctor
57 Actives, 7 Initiates

The annual spring induction banquet was held March 7, 1981. Seven new members were initiated. The program included a talk, "All You Need are Four Colors and a Computer," given by Dr. Leon W. Harkleroad of McMurry

College. Other officers for 1981-82: Debbie Smith, vice president; Nancy Chege, secretary and treasurer; Anne B. Bentley, corresponding secretary; Charles Robinson and Edwin Hewett, faculty sponsors.

West Virginia Alpha, Bethany College, Bethany
Chapter President-Steve Petrovich
16 Actives, 13 Initiates

Other officers for 1981-82: Ken Romanski, vice president; Donna Gates, secretary and treasurer; James Allison, corresponding secretary and faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President-Marie Schwerm
9 Actives, 2 Initiates

Fund raising activities were considered to raise money to send members to the 23rd Biennial Meeting. A doughnut sale was held. Two new members were initiated March 12, 1981. They each presented a talk. Catherine Schueller spoke on "Gauss and Congruences." Prudence Kelly spoke on "Dienes Blocks" and illustrated their use. After the initiation, the group went out for dinner. They were joined by some KME alumnae. Prospective secondary teachers taught lessons to the KME members. Sister Mary Kay Brooks taught a lesson in geometry and a discovery lesson. Marie Schwerm taught a lesson in trigonometry and a discovery lesson. Members also played "Petals Around the Rose." Other officers for 1981-82: Catherine Schueller, vice president; Prudence Kelly, secretary; Heather Shelton, treasurer; Sister Mary Petronia Van Straten, corresponding secretary and faculty sponsor.

Wisconsin Gamma, University of Wisconsin, Eau Claire
Chapter President-Linda Kelley
25 Actives, 13 Initiates

Spring activities included a tour of Cray Research (computers), picnic for all KME members, and initiation ceremony followed by a banquet. Dr. Walter Tape, faculty member, spoke on "Folds, Pleates, and Halos." Two delegates attended the National Convention in Springfield, Mo. Other officers for 1981-82: Mike Kelley, vice president; Karl Wellnitz, secretary; Glen Wetzel, treasurer;Alvin Rolland, corresponding secretary; Thomas Wineinger, faculty sponsor.

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KME members are reminded that pins, keys, and tie-tacs are available and may be ordered through corresponding secretaries.