THE PENTAGON

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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

LETTER FROM THE EDITOR

This issue of THE PENTAGON reflects several changes. James Bidwell, after eight years of commendable service as Editor is now pursuing other activities. Jim, you are wished continued success in your new endeavors.

My colleague, Iraj Kalantari, and I met with Douglas Nance and Jim during January to transfer the duties of the Editor. Due to the recommendations of Jim and Doug, beginning with this issue, THE PENTAGON is being produced using a 'camera-ready' method. This technique will help to hold publication costs down while preserving the quality of THE PENTAGON.

We are indebted to our technical typist, Linda McDonald, for her excellent work and valuable suggestions concerning the composition of this first 'camera-ready' issue.

We urge you to submit your work to the different departments of THE PENTAGON for possible publication.

Iraj and I look forward to the challenges and rewards that surely await our efforts on THE PENTAGON.

The Editor

ON MEASURING INACCESSIBLE DISTANCES

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Old mathematics books can be most fascinating at times. Perusal of a "practical treatise" of the early 18th century revealed some intriguing items. The book in question, THE MARROW OF THE MATHEMATICKS, by one W. Pickering, Merchant Adventurer, was published in London in 1710.The title page,reproduced herewith, will supply the flavor of the Age. The book was probably not too well-known, since it is not described by De Morgan in his Arithmetical Books from the Invention of Printing to the Present Time, although the author's name was listed by De Morgan in the Appendix.

The style is quite intimate,and, after the customary foreword addressed "to the ingenious Reader", there follow chapters on arithmetic and geometry. We then come to the chapter on surveying, which discusses the uses of the quadrant, the water-level, the theodolite, and Gunther's Cross-staff. Among the many types of problems given, including triangulation, some of the more quaintly described are: "to take an altitude by a Bowl of Water, or an ordinary Looking-glass"; "to find the distance between any two Forts, and yet come near neither of them"; "to find the Level betwixt any two places, and whether Water may be conveyed from a Spring head, to any appointed place"; etc.

The reader may be amused at the following paragraph, entitled

> To measure an Inaccessible Distance, as the Breadth of a River, with the help of one's Hat only.
>
> "Having your Hat upon your Head, come near to the Bank of the River, and holding your head upright (which may be done by putting a small stick to some one of your buttons to prop up the Chin) pluck down the Brim or Edge of your Hat until you may but see the other side of the Water, then turn about the body in the same posture towards some plain, and mark where the sight by the brim of the Hat glanceth on the Ground: For the Distance from that place to your standing is the Breadth of the River required."

THE

MARROW

OF THE

MATHEMATICKS,

Made Plain and Easie to the understanding of any ordinary Capacity.

CONTAINING

The Doctrines of *Arithmetick*, *Geometry*, *Astronomy*, *Gauging*, the use of the *Sector*, *Surveying*, *Dyalling*, and the Art of *Navigation*, &c. Illustrated with several Cuts for the better Explanation of the whole Matter.

After a New, Compendious, Easy Method, by *W. Pickering*, Merchant-Adventurer.

To which is Added,

Measuring *Surfaces* and *Solids*, such as *Plank*, *Timber*, *Stone*, &c. *Joiners*, *Carpenters*, *Bricklayers*, *Glasiers*, *Painters* and *Paviers* Work: Each Proposition being wrought *Vulgarly*, *Decimally*, *Practically* and *Instrumentally*.

With a Small Tract of *Gauging Wine*, *Ale*, or *Malt*, without Inches, or Division, by which any one may *Gauge* Ten Backs, or Floors of *Malt*, in the same time another shall *Gauge* One, by the way now used: Altogether New, and submitted to the *Censure* of the Honourable *Commissioners* of *Excise*.

By *J. L.* P. M.

LONDON: Printed by *J. W.* for *Eben Tracy* at the *Three Bibles* on *London Bridge*, 1710.

THE SENSITIVITY OF POLYNOMIAL EQUATIONS TO PERTURBATIONS IN DATA

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Introduction

In solving numerical problems on a high-speed computer, one common concern is the algorithm to be used for the solution of the problem. Of particular importance is the cost of using a specific algorithm to solve a given problem. This is determined by two interrelated factors: the computational complexity involved in a single iteration, and the number of iterations required to solve the problem with a suitable degree of accuracy, i.e., the rate of convergence.

Another matter which is of equal importance is the accuracy of the solution attained, which may or may not be dependent upon the algorithm. Cases in which the accuracy is algorithm-independent are strongly associated with the sensitivity of the solution of equations to perturbations in the input data. This sensitivity can best be described as the accuracy in the input data necessary to produce a desired accuracy in the output. The purpose of this paper is to study the sensitivity of polynomials to perturbations in the coefficients and to propose methods for coping with the problems encountered.

Hurwitz' Theorem and Wilkinson's Example

Consider the polynomial

$$p(x) = a_0x^n + a_1x^{n-1} + \dots a_{n-1}x + a_n, \qquad (1)$$

with $a_0 \neq 0$ and $n \geq 1$. By the Fundamental Theorem of Algebra, the polynomial p can be expressed in the alternate form

$$p(x) = a_0(x-z_1)(x-z_2)\dots(x-z_{n-1})(x-z_n), \qquad (2)$$

where $z_1, z_2, \dots, z_n$ are n (not necessarily distinct) complex numbers. The complex numbers $z_1, z_2, \dots, z_n$ are called the roots of the equation $p(x) = 0$.

In practice, polynomials are usually not found in factored form, but in form (1). It is a common problem in numerical analysis to find the roots of the equation $p(x) = 0$. In such problems, the coefficients are commonly obtained from experimental data, and as such are subject to experimental error. The question which arises naturally is the following: how do inaccuracies in the input data (that is, the coefficients) affect roots of the equation?

Fortunately, this question is answered by Hurwitz' Theorem. A consequence of this theorem is that the roots of an nth degree polynomial are continuous functions of the n+1 coefficients. That is, small changes in the coefficients of a polynomial of given degree will bring about small changes in the roots. But, in the words of G. E. Forsythe [1], "... one's intuition may not be very good at guessing how small these smallnesses are."

Let us examine an example due to J.H. Wilkinson. Let

$$p(x) = (x-1)(x-2)\dots(x-19)(x-20).$$

When converted to normal polynomial form (1), p is a twentieth degree polynomial whose first two terms are x^{20} and $-210x^{19}$.

Suppose now that this polynomial were first encountered in form (1), and the problem was to find its roots. Furthermore, suppose the coefficients were obtained from experimental data and were at best extremely good approximations to the actual coefficients. As a simplification of this situation, let us consider the case in which all the coefficients are exact except for the coefficient of x^{19}, which, instead of -210, is $-210-2^{-23}$. (The number 2^{-23}, which is approximately 1.2×10^{-7}, is a consequence of the computing system on which Wilkinson was working at the time. This was the smallest change that could be made in the coefficient of x^{19} without resorting to double precision computations.) This represents an experimental error of less than 6×10^{-8}%, which is several orders of magnitude better than could be expected under ordinary circumstances.

In view of the results stated previously, one would expect the differences between the roots of this equation and the corresponding roots of $p(x) = 0$ to be "small". However, when Wilkinson calculated these roots, he found them to be those in Table 1, correct to the number of digits shown.

Table I

1.000	10.095 ± .644i
2.000	11.794 ± 1.652i
3.000	13.992 ± 2.519i
4.000	16.731 ± 2.813i
5.000	19.502 ± 1.940i
6.000	
7.000	
8.007	
8.917	
20.847	

The change in the coefficient of x^{19} has caused ten of the roots to move off the real axis, and has caused some to move by as much as three units!

In view of Hurwitz' Theorem, it would seem that such catastrophic behavior could not be found in a class of functions as well-behaved as polynomials. However, the difficulty is numerical in nature, and Hurwitz' Theorem, while containing no logical flaws, is of no practical value in this particular problem.

The difficulty can be identified by examining the definition of continuity in a metric space. Using the Euclidean metric and assuming that only the coefficient of x^{19} varies, Hurwitz' Theorem implies that, for every $\varepsilon > 0$, there exists a $\delta > 0$, such that, if $0 < |\Delta_{a_1}| < \delta$, then $|\Delta_r| < \varepsilon$, where Δa_1 is the change in the coefficient of x^{19}, and Δ_r is the change in the root. The problem that arises is that although the existence of δ is guaranteed, its size is unrestricted. As a result, to find certain roots of the equation to an accuracy of .001, the coefficient of x^{19} must be known to an accuracy of approximately 10^{-12}, which is totally unrealistic in most applied problems.

This approximation of δ comes from the partial derivatives of a root of the polynomial with respect to the coefficients of the polynomial. For a root z_j and a coefficient a_k, $\frac{\delta z_j}{\delta a_k}$ provides an approximation to the rate of change of z_j with respect to a_k, and thus $\frac{\delta z_j}{\delta a_k} \Delta a_k$ provides an approximation to the accuracy to which z_j can be determined.

To evaluate $\frac{\delta z_j}{\delta a_k}$, let us start with the equation

$$a_0 z_j^n + a_1 z_j^{n-1} + \ldots + a_{n-1} z_j + a_n = 0,$$

and differentiate implicitly with respect to a_k. We obtain

$$na_0 z_j^{n-1} \frac{\delta z_j}{\delta a_k} + \ldots + (n-k)a_k z_j^{n-k-1} \frac{\delta z_j}{\delta a_k} + z_j^{n-k} + \ldots + a_{n-1} \frac{\delta z_j}{\delta a_k} = 0$$

$$[na_0 z_j^{n-1} + (n-1)a_1 z_j^{n-2} + \ldots + a_{n-1}]\frac{\delta z_j}{\delta a_k} = -z_j^{n-k}$$

and $$\frac{\delta z_j}{\delta a_k} = -\frac{z_j^{n-k}}{p'(z_j)}$$

From this we see that there are two ways in which $\frac{\delta z_j}{\delta a_k}$ can become large. The first is for the denominator, $p'(z_j)$, to be very small. It can be proven that if r is simultaneously a root of a polynomial and its derivative, then r is a double root of the polynomial. Thus, if z_j were a double root, $p'(z_j)$ would be zero and $\frac{\delta z_j}{\delta a_k}$ would be undefined. If z_j were not a double root, but were close to another root, $p'(z_j)$ would be non-zero, but could be very small in absolute value, making $\frac{\delta z_j}{\delta a_k}$ very large. However, it can be shown that $p'(z_j) = a_0(z_j-z_1)\ldots(z_j-z_{j-1})(z_j-z_{j+1})\ldots(z_j-z_n)$. In Wilkinson's problem, $a_0 = 1$ and all the roots are separated by a distance of at least 1. Thus $|p'(z_j)|$ will be greater than or equal to one, and this will not cause $\frac{\delta z_j}{\delta a_k}$ to be large.

The second situation which can cause the partial derivatives to become large occurs when high-degree polynomials possess roots widely separated from the origin. This situation occurs in Wilkinson's example, and results in the partial derivatives shown in Table II.

Table II

j	$\frac{\delta z_j}{\delta a_1}$	$\frac{\delta z_j}{\delta a_1} \Delta a_i$
1	8.2×10^{-18}	-9.8×10^{-25}
2	-8.2×10^{-11}	9.8×10^{-18}
3	1.6×10^{-6}	-1.9×10^{-13}
4	-2.2×10^{-3}	2.6×10^{-10}
5	6.1×10^{-1}	-7.3×10^{-8}
6	-5.8×10^{1}	7.0×10^{-6}
7	2.5×10^{3}	-3.0×10^{-4}
8	-6.0×10^{4}	7.2×10^{-3}
9	8.4×10^{5}	-1.0×10^{-2}
10	-7.6×10^{6}	9.1×10^{-1}
11	4.6×10^{7}	-5.5×10^{0}
12	-2.0×10^{8}	2.4×10^{1}
13	6.1×10^{8}	-7.3×10^{1}
14	-1.3×10^{9}	1.6×10^{2}
15	2.1×10^{9}	-2.5×10^{2}
16	-2.4×10^{9}	2.9×10^{2}
17	1.9×10^{9}	-2.3×10^{2}
18	-1.0×10^{9}	1.2×10^{2}
19	3.1×10^{8}	-3.7×10^{1}
20	-4.3×10^{7}	5.2×10^{0}

These figures agree with Wilkinson's results in that the first nine roots have experienced small changes, and the remaining eleven have partial derivatives which are so large that it is impossible to tell which new root each of these is mapped into.

Conclusions

Clearly, the complete solution of such a problem would require data far more accurate than is commonly attainable. Fortunately, the difficulties encountered are peculiar to polynomials of high degree, and do not affect the more commonly encountered low-degree polynomials. For a comparison, let us consider the twelfth and fifth degree polynomials analogous to p in Wilkinson's example. While the absolute values of the partial derivatives were bounded by 2½ billion in the twentieth degree case, suitable bounds in the twelfth and fifth degree cases are 150,000 and fifty, respectively. Thus polynomials of low degree will not present this particular type of difficulty, and will conform to our intuition much better than did Wilkinson's example.

The problem remains, though, of how to handle a situation similar to Wilkinson's. The answer to this question is dependent upon a precise definition of what is acceptable as a solution. For example, if the determination of a single root of the equation constitutes a satisfactory solution, it may be possible to find a root which is small in absolute value and relatively unaffected by inaccuracies in the coefficients. If this were the case in Wilkinson's problem, one could find the root $z_1 = 1$ to three digits accuracy even if any one of the coefficients of p differed from its true value by as much as 10^{14}! (This is because for the root $z_1 = 1$ and any coefficient a_k, $\frac{\delta a_1}{\delta a_k} = 8.2 \times 10^{-18}$.)

If the solution consists of finding a particular root (one in a specified interval), the above procedure could be applied if the experiment could be modified in such a way that all measurements were made from a point of reference which was close to the desired root (for example, the midpoint of the interval.) Mathematically, this corresponds to the linear change of variables

$$x = y - c,$$

where c is a real constant. This transformation will change the polynomial (1) to a polynomial in y, all of whose roots are displaced by a distance c. However, if this transformation were performed numerically, the calculation of the new coefficients would be severely affected by roundoff error. This difficulty, combined with the existing sensitivity of the equation, would lead to highly unreliable results and would not constitute a satisfactory treatment of the problem. It would be necessary in this case to perform the experiment again with the new frame of reference to determine the new coefficients.

If several widely separated roots are desired, the problem becomes more difficult. If the experiment which produces the coefficients can be repeated easily, it may be possible to perform the experiment repeatedly with different frames of reference in order to obtain a set of local approximations to the polynomial, each of which may be solved for roots in the interval to which it applies.

Problems which cannot be handled by any of these methods will be very difficult indeed. Solutions of such problems would probably require certain ad hoc techniques, possibly including the alteration of the basic experiment. Without such changes, the numerical solution of the problem to a reasonable degree of accuracy would be quite unrealistic.

References

[1] Forsythe, G. E. "Pitfalls in Computation, or Why a Math Book Isn't Enough, American Mathematical Monthly, 77 (1970), 931-944.

[2] Marden, Morris, Geometry of Polynomials. Providence, Rhode Island, 1966.

THE SEARCH FOR π: RATIONAL OR IRRATIONAL?

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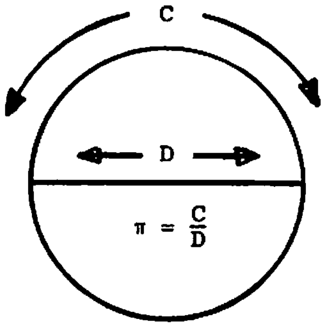
The number π has long been of interest. What is there about π that has fascinated mathematicians and non-mathematicians for thousands of years? Why calculate π to 100,000 decimal places? Although these questions may not be answered completely in these few pages, it is hoped that the presentation of some history, some formulas, and some trivia may promote some interest in and understanding of π.

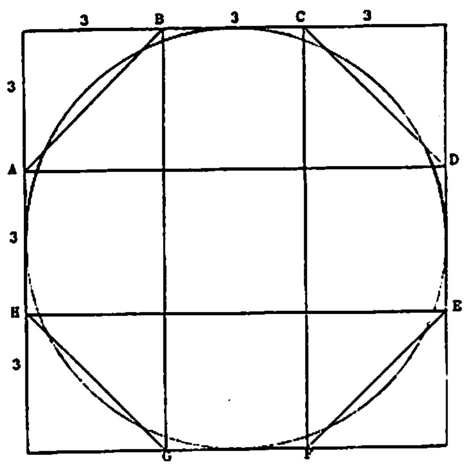
What is π? π is a ratio of the circumference of a circle to the diameter. It is a number without end. π is equal to 3.141592654.... or approximately 22/7. One of the fundamental uses of π is in finding the area of a circle: Area=$\pi \cdot (\text{radius})^2$



When considering π, there are three distinct periods characterized by fundamental differences in method, immediate aims, and available mathematical tools. The first period, called the geometrical period, extended from the earliest determinations of the ratio of the circumference of a circle to its diameter to the invention of calculus about the middle of the seventeenth century. The main effort was directed toward the approximation of this ratio by the calculation of perimeters or areas of regular inscribed and circumscribed polygons. The second period began in the middle of the seventeenth century and lasted for more than 100 years. During this time the methods of calculus were employed in the development of an expression for π in the form of infinite series, products, and continued fractions. The third period, which extended from the middle of the nineteenth century, was devoted to the studies of the nature of the number π.

People of early history were fascinated by circles and other geometric shapes. Many basic principles and foundations of mathematics were discovered during the geometric period. These early people calculated approximations of π using methods involving circles. I would like to discuss two of the methods used at this time: The Egyptian method and the Archimedian method.

The Egyptian scribe Ahmes proposed the Egyptian method of calculating π. The area of a circular field with a diameter of nine units is the same as the area of a square with a side of eight units. An irregular octagon is formed by trisecting the sides of a square with the length nine units and cutting off the corner triangles. The area of ABCDEFGH does not differ much

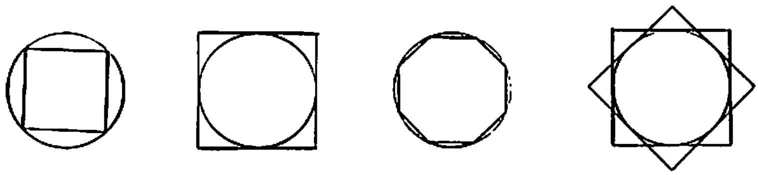


from the area of the circle and equals the area of the five shaded squares plus the four triangles of four and one-half square units each. The area of the circle with a diameter of nine approximately equals sixty four square units(the area of a square with side eight units in length.) Ahmes cheated twice, once in setting the area of the octagon equal to the circle and again setting sixty three approximately equal to sixty four. These two approximations partially compensated for each other. Using $A = \pi R^2$, this yields $\pi(9/2)^2 = 8^2$ and the Egyptian value of π was

$$\pi \approx 4 \times (8/9)^2 = 3.16049...$$

Archimedes, two hundred years before Christ, used inscribed and circumscribed polygons to find a value of π. His concept used polygons to approximate the circumference. This method is based on the fact that the perimeter of a regular polygon of n sides inscribed in a circle is smaller than the circumference of the circle, whereas the perimeter of a similar polygon circumscribed about the circle is greater than its circumference. By increasing n, the two perimeters will approach the circumference, one from above, the other from below. Archimedes found the value of π to be between 3 1/7 and 3 10/71.

The Archimedian method:



It is interesting that there are ways to find the value of π without using integral calculus. Now let's examine the more "modern" methods used in determining π.

The great majority of the calculations of π to many decimal places have been based upon the power series

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \dots, \quad -1 \leq x \leq 1$$

which was discovered by James Gregory in 1671. From here it was a simple step to substitute one for x. Since the arctan of 1 = $\pi/4$, we now have

$$\pi = 4(1 - 1/3 + 1/5 - 1/7 + 1/9 - \dots)$$

which was the first infinite series ever found for π.

Most computations of π in modern times have used Gregory's series in conjunction with certain arctangent relations. Only nine of these relations have been employed to any extent in such computations. The following are arranged to increasing precision of approximations computed by their use.

I. $\pi/4 = 5 \arctan 1/7 + 2 \arctan 3/79$

Euler in 1755 calculated π to twenty decimal places in one hour. In 1794, Baron Georg von Vega evaluated π to 140 decimal places, of which 136 were correct.

II. $\pi/4 = 4 \arctan 1/5 - \arctan 1/70 + \arctan 1/99$

Euler published this formula in 1764. It was used by William Rutherford in 1841 to compute π to 208 places (152 correct).

III. $\pi/4 = \arctan 1/2 + \arctan 1/5 + \arctan 1/8$

Zacharias Dahse in a period of two months in 1844 evaluated π correct to 200 places.

IV. $\pi/4 = \arctan 1/2 + \arctan 1/3$

Charles Hutton published this series in 1776. W. Lehman used it to calculate π to 261 places in 1853. Tseng Chi-hung in 1877 evaluated π to 100 decimals in a little over a month.

V. $\pi/4 = 2 \arctan 1/3 + \arctan 1/7$

Hutton published this series in 1776 and also independently in 1779 by Euler. Vega computed π to 143 decimals (126 correct) in 1789. Thomas Clausen extended the calculation to 248 correct decimals in 1847 and Lehman to 261 decimals in 1853.

VI. $\pi/4 = 3 \arctan 1/4 + \arctan 1/20 + \arctan 1/1985$

Published by S. L. Loney in 1893 and Carl Störmer in 1896, this series was rediscovered by R.W. Morris in 1944. In September of 1947, a man by the name of Ferguson used a desk calculator to approximate π to 808 places.

VII. $\pi/4 = 8 \arctan 1/10 - \arctan 1/239 - 4 \arctan 1/515$

Discovered by S. Klingenstierna in 1730, this equation was used by C. C. Camp in 1926 to evaluate $\pi/4$ to 56 places.

VIII. $\pi/4 = 12 \arctan 1/18 + 8 \arctan 1/57 - 5 \arctan 1/239$

The approximations computed from the formulas VII and VIII agreed within three units of the 10,021 st. decimal place. G. E. Fenton calculated VII on a Pegasus computer on March 31, 1957 and VIII on March 1, 1958.

IX. $\pi/4 = 4 \arctan 1/5 - \arctan 1/239$

This is the most celebrated of all the relations discovered by John Machin and computed to 100 places in conjunction with Gregory's series in 1706. William

Shanks published two papers containing approximations of π beyond 600 places. This formula was one of those used to compute as well as check values of π computed on various computers at various times.

A computation of π to 100,265 decimal places was done on July 29, 1961 by Dr. Daniel Shanks and Dr. John Wrench on an IBM 7090 system requiring 7 hours and 43 minutes.

The first computation used the formula

$$\pi = 24 \tan^{-1} 1/8 + 8 \tan^{-1} 1/57 + 4 \tan^{-1} 1/239$$

which was published by Carl Störmer in 1896. The check calculation was based on Gauss's formula

$$\pi = 48 \tan^{-1} 1/18 + 32 \tan^{-1} 1/57 - 20 \tan^{-1} 1/239$$

We see there are many variations that can be used to find an approximation of π. We have seen how man has progressed through history in this endless search. In the beginning man used circles to achieve the goal of an approximation. He found π to be equal to 3.16049 and between 3 1/7 and 3 10/71. Man progressed and invented new methods. Using calculus, man calculated π to more and more accuracy, and using the computer found π to 100,265 decimal places. Today's value of 3.141592654... is representative of many, many hours of work and many calculations that have taken place throughout the years.

Another method of finding π involves Buffon's problem. This method ties the approximation of π with probability theory. Suppose we let a needle of length L be thrown at random onto a horizontal plane ruled with parallel straight lines spaced at a distance D (which is greater than L) from each other. The question is, 'what is the probability that the needle will intersect one of the lines?'

The answer is

$$P = 2L/\pi D$$

Pierre Laplace, 1749-1827,one of the greatest French mathematicians of his time saw a new light in Buffon's problem: a new way to evaluate π by rearranging the formula to state:

$$\pi = 2L/DP$$

The length of the needle L and spacing between the lines D are known and usually are made to be equal. The probability P can be measured by throwing a needle onto the ruled paper a very large number of times, recording the fraction of throws resulting in an intersection of the needle with a line. This can then be used to calculate the first decimal places of π. This method is very inefficient as far as the numerical computation of π is concerned. For example, the probability of obtaining π correct to five decimal places in 3,400 throws of a needle is less than 1.5%, which is very poor.

So far we have seen many ways of finding π, some questionable as well as some involving extensive time and effort. But how far can we go? π has been evaluated, checked and rechecked to as many as 100,265 decimal places. Euler's evaluation of π gave a complete answer to the question of its numerical value. But what kind of number is π? Is π rational or irrational, algebraic or transcendental?

With each new decimal digit discovered the hope that π might be rational faded, for no pattern could be found in the digits. In the beginning there was no proof, but most investigators sensed that π was irrational. Finally the Swiss mathematician Johann Heinrich Lambert proved π irrational in 1767. Adrien-Marie Legendre in 1794 proved π^2 was irrational,dashing hopes that π might have been the square root of a rational number.

We now know that π is an irrational number. But is π transcendental? A transcendental number is an irrational number that satisfies no algebraic equation. The existence of transcendental numbers was proven in 1840 by Joseph Liouville. In 1882 the German mathematician Ferdinand von Lindeman proved that π was a transcendental number.

The history of π is only a small part of the history of mathematics. It is an interesting phenomenon that the number π has been approximated to hundreds of decimal places. There seems to be something magical about the number π that fascinates people. Uncounted hours of human toil have been spent over the past two centuries in computing hundreds of its decimal digits and attempting to find some pattern in them that would prove it had a definite value. No pattern has been found.

The hundred thousand decimal calculation took 8 hours, 43 minutes of computer time, working at an average speed of over 100,000 additions and subtractions per second. At a desk calculator, the same job would take 30,000 years! Why compute π to 100,000 decimals places? Certainly not for any practical purpose. Ten decimal places are sufficient to compute the circumference of the earth to better than an inch. Thirty decimals would suffice to specify the circumference of the visible universe to an error too small for the most powerful microscope to detect.

Why compute π? We want to know the next decimal because "it is there," because it adds one tiny fraction to man's knowledge. Or else we have the hope of discovering some sort of regularity that would throw light on the nature of π.

The motivation of modern calculations of π to many decimal places was conjectured by Professor P. S. Jones in 1950 as being attributable to "intellectual curiosity and the challenge of an unchecked and long untouched calculation."

In this paper, we have examined many methods of approximating π, computed π from six to 100,265 decimal places by hand and by the computer, have tied π to probability, and questioned why anyone would want to calculate π. We have seen that π is an irrational number. But the hardest question of all may just be the one that asks if the search for π is irrational. This "simple" yes-no answer I leave to the reader to decide.

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ON PRIME POWERS WHICH ARE CONSECUTIVE INTEGERS

DAVE PREDMORE
Student, Emporia State University

The set of numbers {7,8,9}, besides being a set of consecutive positive integers, is also a set of prime powers $\{7^1, 2^3, 3^2\}$. This paper is a study of such sets and is aimed at answering the question as to how many such sets exist.

We begin by considering pairs of prime powers which are consecutive positive integers--that is, numbers of the form p^n, q^m such that p and q are prime numbers, n and m are natural numbers, and $p^n - q^m = 1$.

We consider three distinct possibilities: 1) both exponents m and n are equal to 1; 2) neither exponent m or n is equal to 1; 3) exactly one of the exponents m or n is equal to 1.

Case I. $n = m = 1$.

Here p and q must be primes such that $p - q = 1$. Since no pair of odd integers can have a difference of 1, and since there exists only one even prime number, the only solution to this equation is the ordered pair (3,2). Thus, the only two prime numbers which are consecutive integers are the numbers 2 and 3.

Case II. Neither n or m is 1.

Two facts should be noted immediately:

1) an odd number raised to any power is odd;
2) the only prime number which is not odd is the number 2.

Thus, we can see that p and q cannot both be odd primes. For, raised to any power, they would result in odd numbers and the difference between two odd numbers can never be 1. Hence either p or q must be the number 2.

Part A. Let $q = 2$, and consider the problem in the form $p^n - 1 = 2^m$. Now $p^n - 1$ may be factored and expressed as $(p - 1)(p^{n-1} + p^{n-2} + \ldots + p + 1)$. This product being equal to 2^m is therefore an even number. Assume now that n is odd. Then there are an odd number of terms in the factor $p^{n-1} + p^{n-2} + \ldots + p + 1$, an even number involving p and the additional term 1. This odd number when multiplied by the factor p-1 must equal a power of 2. Since this would imply that a power of two has an odd divisor, this is clearly impossible. Thus, the assumption that n is odd is false. We conclude that if $p^n - 1 = 2^m$, then n must be an even number. Let p^n be written as p^{2k} which in turn is $(p^k)^2$. Now p odd implies that p^k is odd and so can be written as $(2x + 1)$ where x is a non-negative integer. $p^n - 1$ becomes expressible then as $(2x + 1)^2 - 1$. This equals $4x^2 + 4x$. And so the equation $p^n - 1 = 2^m$ is equivalent to the equation $4x^2 + 4x = 2^m$. On factoring, $4x(x + 1) = 2^m$. This equation has as one solution $x = 1$ since by inspection $4(2) = 2^3$. Let x be any other positive odd integer. Then a power of 2 is divisible by this odd integer, an impossibility. Let x be any other positive even integer. Then $(x + 1)$ is an odd factor of a power of 2. Again an impossibility. Thus, the only solution is $x = 1$. Hence, $p^k = 3$. And finally $p^n = 3^2$. We conclude that the only values of n and m and p to satisfy $p^n - 1 = 2^m$ are $p = 3$, $n = 2$, and $m = 3$. That is, if $p^n - 2^m = 1$ and $n \neq 1$, and $m \neq 1$, and p is a prime, then we must have $3^2 - 2^3 = 1$. A different proof of this fact appeared in the Jan., 1978 issue of the American Mathematical Monthly.

Part B. Let $p = 2$, and consider the problem in the form $2^n - q^m = 1$ with $n \neq 1$ and $m \neq 1$.

$$2^n = q^m + 1$$

Assume that m is odd. Then

$$2^n = (q + 1)(q^{m-1} - q^{m-2} + \dots -q+1).$$

The second factor being the sum of an odd number of odd terms is odd. Hence 2^n has an odd factor. This contradiction implies that m cannot be odd.

Assume that m is even. Then

$$2^n = q^m + 1$$
$$q^m = q^{2k} = (q^k)^2$$

But q being odd implies that $q^k = 2x + 1$.

Therefore
$$2^n = (2x + 1)^2 + 1$$
$$= 4x^2 + 4x + 2$$
$$2^n - 2 = 4x(x+1)$$
$$2^{n-1} - 1 = 2x(x+1)$$

The left number is odd and the right number is even. This contradiction implies that m cannot be even. Hence, there is no $n \neq 1 \neq m$ such that $2^n - q^m = 1$. Thus, there exist no positive integers $m \neq 1$ and $n \neq 1$ and prime number q such that $2^n - q^m = 1$ has a solution.

Part A and Part B together imply that the equation $p^n - q^m = 1$ and p and q primes and neither n nor m equal to 1 has only the one solution $p = 3$, $n = 2$, $p = 2$, $m = 3$.

Case III. In the last class we have those primes p and q such that $p^n - q^m = 1$ and exactly one of the exponents n or m is 1.

Thus we have $p - 2^m = 1$ and $2 - q^m = 1$ and $p^n - 2 = 1$ and $2^n - q = 1$ as the four possibilities, where in each case the indicated exponent is not 1. The middle two cases are not possible. We are left with $p = 2^m + 1$ or $q = 2^n - 1$. These can be combined so that we are seeking all primes p and exponents k not 1 satisfying $p = 2^k \pm 1$.

There are many examples of such numbers: $3 = 2^2 - 1$, $5 = 2^2 + 1$, $7 = 2^3 - 1$, $17 = 2^4 + 1$, $31 = 2^5 - 1$. I have not been able to prove whether there are infinitely many or not.*

I conclude then that the number of ordered pairs of primes falling into the first class mentioned is one, the number in the second class is one, and the number in the third class is not one. It remains to determine how many there are in this class.

We now turn attention to the probability of more than two prime powers being consecutive integers.

If there can be four such elements, two of them must be even numbers. As prime powers, these two would have to be of the form 2^k and 2^{k+1}. Being in the set of consecutive integers, they must also satisfy the condition that $2^{k+1} = 2^k + 2$.

Thus, $2^{k+1} = 2(2^{k-1} + 1)$ and this last factor must be a power of 2.

This is true if and only if $k = 1$, in which case the two even prime powers in the set must be 2 and 4.

We conclude that the only set of four consecutive integers that are prime powers is the set {2,3,4,5}. Hence there does not exist a set of five consecutive integers all of which are prime powers.

Furthermore, we note that there are several sets of three consecutive integers all of which are prime powers. These are the sets {2,3,4} and {3,4,5} which are subsets of the set discussed above, the set {7,8,9}

*This is a long-standing unsolved problem.

derived from case II of the first part of this paper, and any other set consisting of two twin primes separated by a power of 2.

Finally, note that in any set of three consecutive integers, one number is divisible by 3. Now, except for the case {2,3,4}, the way for the numbers to be powers of primes is $\{q^m, 2^n, p^k\}$. Therefore, either p or q is 3, and as noted in Case II, it must be p, with n = 3, and k = 2. The conclusion: There are only three sets of three consecutive powers of primes, namely {2,3,4}, {3,4,5}, and {7,8,9}.

THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 February, 1981. The solutions will be published in the Spring 1981 issue of *The Pentagon*, with credit being given for other solutions received. Preference will be given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 322. Proposed by John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico.

In ΔABC, $AB=25$, $AC=56$, points D and E lie on BC and AC respectively. Also $AE=8$, $BC=3BD$ and $DE=26$. Calculate the length of BC.

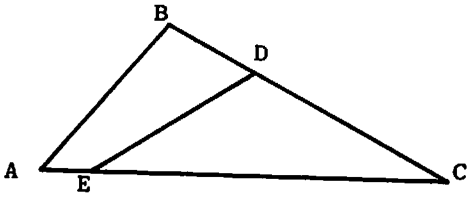

FIGURE 1. Problem 322

Problem 323. Proposed by Michael W. Ecker, Pennsylvania State University, Worthington-Scranton Campus, Scranton, Pennsylvania.

Let N be the set of natural numbers and define a function d(n) as follows:

$$d(n) = \det \begin{vmatrix} 1 & 2 & \cdots & n \\ n+1 & n+2 & \cdots & 2n \\ \cdots & & & n^2 \end{vmatrix}$$

the determinant of the nxn matrix shown. Find a formula for d(n).

Problem 324. Proposed by Michael W. Ecker, Pennsylvania State University, Worthington-Scranton Campus, Scranton, Pennsylvania.

Let $x_1, x_2, \ldots, x_n$ be positive numbers whose sum is 1. What is the smallest possible value of the sum of the reciprocals $\sum_{i=1}^{n} \frac{1}{x_i}$?

Problem 325. Proposed by the editor.

In the prison sits a prisoner who is sentenced to die. Fortunately the warden, an eccentric, offers the prisoner a chance to live. The warden gives the prisoner 12 black balls and 12 white balls. Next the warden gives the prisoner two boxes and tells the prisoner that tomorrow the executioner will draw one ball at random from one of the boxes. If a white ball is drawn, the prisoner will be freed; if a black ball is drawn, the sentence will be carried out. How should the prisoner arrange the balls in the boxes so as to maximize his chance for freedom?

Problem 326. Proposed by the editor.

Solve the dual cryptarithm and give the retired Cockney sailor a hand.

```
  FIX       FIX
+ ME      - ME
 BOAT      'ELM
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SOLUTIONS

312. Proposed by John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico.

Let L_1 and L_2 be the axes of a plane coordinate system which cut off line segments $a_ib_i (i = 1,2,3,4)$ of the sides (extended if necessary) of a quadrilateral ABCD in such a manner that each point a_i lies on L_1 and each point b_i lies on L_2. Let K denote the intersection of L_1 and L_2. Now if similar triangles $a_ib_ic_i$ are drawn on each line segment a_ib_i such that each angle with its vertex at c_i is equal to the angle formed by L_1 and L_2, then show that the vertices c_i and the intersection K of the axes are collinear.

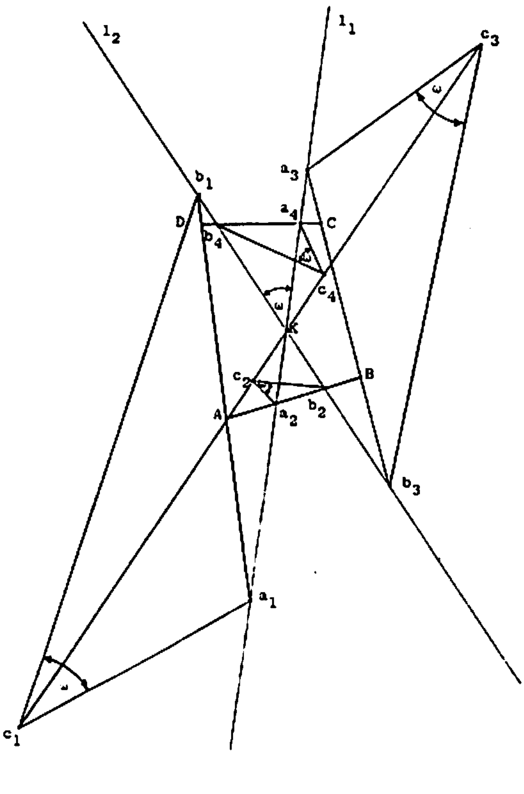


FIGURE 2
Problem 312

Solution by the proposer.

Proof:
Use the line segments $\overline{a_ib_i}$ as chords of the four circles determined by the points a_i, K, and b_i. The construction of the similar triangles $a_ib_ic_i$ lead us to the conclusion that there are four cyclic quadrilaterals $a_ib_iKc_i$. Since each $a_ib_ic_i$ is an inscribed angle which intercepts an arc c_ia_i, and since the four angles $a_ib_ic_i$ are equal by construction, the measures of the four arcs c_ia_i are equal. The four points a_i are on L_1 by choice, and each of the inscribed angles c_iKa_i intercepts an arc c_ia_i, therefore points c_i and K are collinear. See Figure 3.

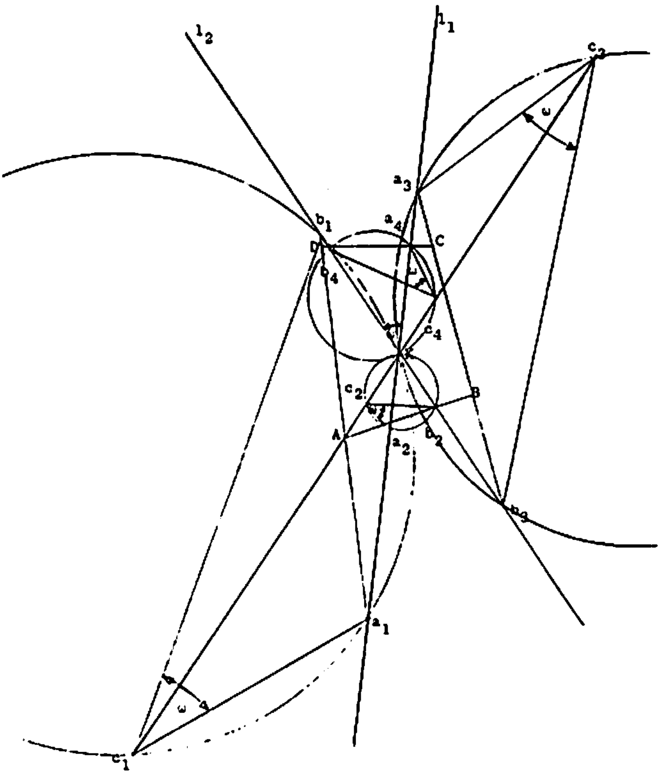


FIGURE 3

313. Proposed by Michael W. Ecker, Scranton, Pennsylvania.

Joe and Moe plan to meet for lunch at the pizza parlor between noon and 1:00 P.M., but they can't decide what time to meet. Joe suggested that whoever arrives first should wait 10 minutes for the other before leaving. Moe likes Joe's suggestion but he wonders if a 10 minute wait will guarantee that they will have at least an even chance of meeting for lunch. Assuming that each of Joe's and Moe's times of arrival is random, what is the minimum time the first to arrive must wait to guarantee that their probability of having lunch together is at least ½?

Solution by Ferrell Wheeler, Forest Park High School, Beaumont, Texas.

Let t be the number of units of time in the time interval that Joe and Moe plan on meeting one another. Let w be the number of units of time that one should wait for the other, $w \in [0,t]$. Let a be the probability that Joe and Moe meet, in this case for lunch. Let x and y be the amount of time that Joe and Moe, respectively, arrive after the initial time in the time interval they chose. Therefore, x and y are randomly chosen points in time, where x and y are in the interval $[0,t]$. Now, a is the probability that the point (x,y) in the square below has the property $|x-y| \le w$, i.e., $y-x \le w$ and $x-y \le w$. All points in the square with this property are located between the lines $y-x=w$ and $x-y=w$, i.e., the shaded region below. Therefore, a is the ratio between the area of the shaded region and the area of the square.

$$a = (t^2-(t-w)^2/t^2$$

$$a = 1-(t-w)^2/t^2$$

$$w^2-2wt+at^2 = 0$$

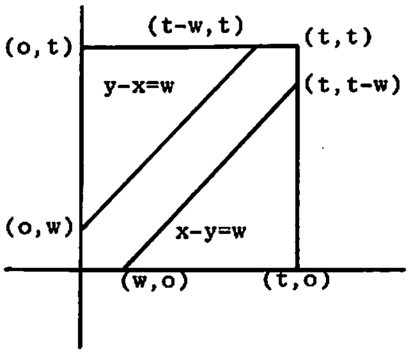


FIGURE 4
Problem 313 Solution

In the given problem, t = 60 min. and for w=10, a=11/36≐.31 which does not guarantee they will have an even chance of meeting for lunch. The minimum w that will guarantee $a=\frac{1}{2}$ is for $w=60-30\sqrt{2}\doteq 17.57$ min. In general, for $a=\frac{1}{2}$, $w=t-\frac{1}{2}t\sqrt{2}$. I would like to see this problem generalized to n people.

Also solve by the proposer.

314. Proposed by the editor.

One day John met Bill on the street.
"Hi Bill."
"Hi John. How's your family?"
"Fine," replied John.
"Tell me," asked Bill. "How old are your three children?"
"Well," said John, "The product of their ages is 36 and the sum is one less than the address of the white building across the street."
Bill noted the number on the building and went home.
The next day Bill called John and complained that he wasn't given enough information.
John said "My eldest child is a girl," whereupon Bill immediately gave the correct ages.
What were the ages?

Composite of solutions submitted by Mike Hewitt, Kansas University, Lawrence, Kansas, and Matt Maggio, Elon College, Burlington, North Carolina.

Starting with the only obvious clue, the three children must have one of the following combinations of ages:(1,1,36),(1,2,18), (1,3,12),(1,4,9),(1,6,6), (2,2,9), (2,3,6), (3,3,4). The sums of the elements in each of these combinations respectively is: 38, 21, 16, 14, 13, 13, 11, 10. Since Bill knows the address of the white building, then he should have enough information to determine the children's ages - except in one case. That being if the address is 14. This is because there are two combinations of ages whose elements sum to 13. Since Bill does not have enough information, then the address must be 14 and we are left to decide between (1,6,6) and (2,2,9). John's answer the next day implies a unique eldest child, which tells us that the correct ages are 2, 2 and 9.

Also solved by: Stephen T. Lowe, Colorado School of Mines, Golden, Colorado; Charles W. Trigg, San Diego, California; Ferrell Wheeler, Forest Park High School, Beaumont, Texas; John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico; Robert Granville, Hofstra University, Hempstead, New York; and Michael Harvilla, Pace University, Pleasantville, New York

315. Proposed by the editor.

Four married couples meet for dinner.
There is some shaking of hands. No one shakes hands more than once with the same person. Spouses do not shake hands.
When the hand shaking is finished one husband asks all of the other people how many times they shook hands.
Everyone gives a different answer.
How many times did the questioner's wife shake hands?

Solution by Ferrell Wheeler, Forest Park High School, Beaumont, Texas.

Obviously, the 7 different answers given to the questioner must be 0,1,...,6. Putting the problem in a graph theory setting, one first needs to find q, the number of people the questioner shook hands with, such that the degree sequence 6,5,4,3,2,1,q is graphical. Since there must be an even number of odd degrees, q must be 1,3, or 5. Using a test first discovered by Havel, it can be determined which of the following three degree sequences is graphical.

(1) 6,5,4,3,2,1,1 is graphical iff 4,3,2,1 is graphical, but this is ridiculous because there are only 4 points and one of the points would have to be adjacent to 4 distinct points.

(2) 6, 5, 4, 3, 3, 2, 1 is graphical iff 4, 3, 2, 2, 1 is graphical. This is graphical iff 2,1,1 is graphical.Since 2,1,1 is obviously graphical,the original degree sequence is graphical.

(3) 6, 5, 5, 4, 3, 2, 1 is graphical iff 4, 4, 3, 2, 1 is graphical. This is graphical iff 3, 2, 1 is graphical,but this seems to be impossible by using the same argument as in (1).

Thus, the only graphical degree sequence satisfying the criteria is 6,5,4,3,3,2,1. Therefore, the questioner shook hands with q=3 people. The unique graph with this degree sequence and satisfying the problem is: (c_n is a member of the nth couple)

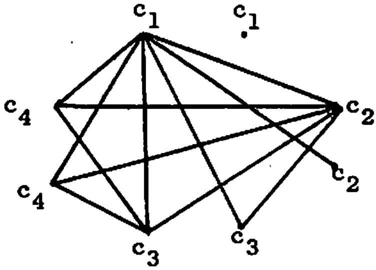


FIGURE 5
Problem 315 Solution

From this, one sees that the questioner's wife shook hands with 3 people.

Solution by Charles W. Trigg, San Diego, California.

The maximum number of handshakes possible per person is 6, so the seven different answers must have been 0,1,2,3,4,5, and 6 handshakes. Then the total of the reported handshakes was 21. But the overall number of handshakes must have been even. Hence, the questioner shook an odd number of hands, thus duplicating the count of one other person. The spouse of the person shaking 6 hands, had to be the person who shook none. The spouse of the person who shook 5 hands,must have shaken only 1. The spouse of the person shaking 4 hands,could only have shaken 2. Of the remaining couple, each shook 3 hands, a duplication.One of these had to be the questioner, and the other his wife. This reasoning may be clarified by the following sequential tabulation in which upper and lower cases of the same letter represent spouses.

Person	Shook Hands With			Hands Shaken
A	BbCcDd			6
a				0
B	A	CcDd		5
b	A			1
C	A	B	Dd	4
c	A	B		2
D	A	B	C	3
d	A	B	C	3

In general, if k married couples go through the same routine of handshaking, questioning, and distinct responses, then the questioner and his wife will each shake k-1 hands.

Also solved by: Michael W. Ecker, The Pennsylvania University, Worthington-Scranton Campus, Scranton, Pennsylvania, Mike Hewitt, Kansas University, Lawrence, Kansas, and Robert Granville, Hofstra University, Hempstead, New York.

316. Proposed by Randall J. Covill, Mansfield, Massachusetts.

A repunit is an integer in the decimal system whose representation consists of a finite string of ones; e.g., the numbers 11, 11111, 1111111 are all repunits. A Fermat number has the form $2^{2^k} + 1$ for any integer $k > 0$. Are any Fermat numbers also repunits?

Solution by Robert Granville, Hofstra University, Hempstead, New York.

Any number which can be written as 2^i where i is an integer greater than 0 must end in either 2,4,6, or 8. Since numbers which can be written as 2^{2^k} where k is an integer greater than 0 are a subset of the numbers which can be written as 2^i where i is an integer greater than 0, these numbers must also end in either 2,4,6, or 8. Fermat numbers, numbers of the form $2^{2^k} + 1$, must therefore end in either 3,5,7, or 9 and therefore cannot be repunits. There are no Fermat numbers which are also repunits.

Solution by Sally R. Irvin, Student, Fort Hays State University, Hays, Kansas.

The answer is no. It is enough to show that no integer of the form 2^{2^k} can be a multiple of 10 since then no Fermat number can end with a 1, making a repunit impossible. Let us look at $2^{2^k} \equiv 0 \pmod{10}$ For this to be true 10 must divide 2^{2^k} which implies that 5 divides 2^{2^k}. But this is impossible since there is no 5 in the prime factorization of 2^{2^k}. Thus no integer of the form 2^{2^k} can be a multiple of 10 and therefore no Fermat number can be a repunit.

Also solved by: Michael W. Ecker, Scranton, Pennsylvania, Mike Hewitt, Kansas University, Lawrence, Kansas, Matt Maggio, Elon College, Burlington, North Carolina, Ferrell Wheeler, Forest High School, Beaumont, Texas, Charles W. Trigg, San Diego, California, Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin, and the proposer.

THE MATHEMATICAL SCRAPBOOK

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material directly to the Scrapbook editor. Material will be used where possible and acknowledgement will be made in THE PENTAGON. Address all materials to Dr. Richard Lee Barlow, Department of Mathematics, Kearney State College, Kearney, NE 68847

Editor's Note: The following was submitted by Thomas Salyard, Student, Kearney State College.

There exists a number of special functions which arise in applied problems. One such function, which has many statistics and physics applications, is the so-called gamma function. This and a related function, the beta function, are of interest.

By definition, the gamma function is $\Gamma(p) = \int_0^\infty x^{p-1}e^{-x}dx$, where $p > 0$. Integrals of this form arise frequently, so the evaluation of the general integral is important. It follows that $\Gamma(p+1) = \int_0^\infty x^p e^{-x}dx$ from the definition of the gamma function. If we integrate this integral by parts, calling $x^p=u$, $e^{-x}dx=dv$, $px^{p-1}dx=du$ and $-e^{-x}=dv$, then

$$\Gamma(p+1)=-e^{-x}x^p\Big|_0^\infty - \int_0^\infty -e^{-x}px^{p-1}dx = 0+p\int_0^\infty e^{-x}x^{p-1}dx=p\Gamma(p).$$

This recursive relation is useful in the evaluation of the gamma function. Consider $\Gamma(p)$, where p is a positive integer. From repeated application of this recursive relationship, it is seen that

$\Gamma(p)=(p-1)(p-2)\ldots(3)(2)(1)\Gamma(1) = (p-1)!\Gamma(1)$. However, $\Gamma(1) = \int_0^\infty x^0 e^{-x} dx = 1$, so $\Gamma(p) = (p-1)!$.

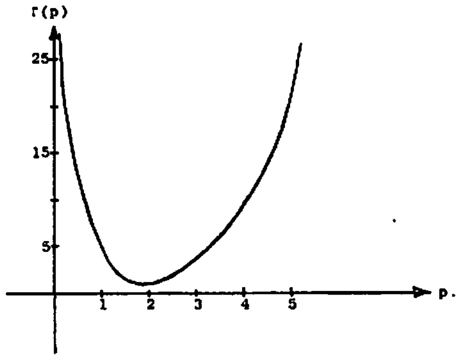
Tabulated values exist for the gamma function for values of p between 1 and 2, making it possible to evaluate the gamma function for any $p > 0$. As an example, consider $\Gamma(4.3)$. $\Gamma(4.3) = (3.3)(2.3)(1.3)\Gamma(1.3)$. $\Gamma(1.3) = 0.8975$ from standard tables, making $\Gamma(4.3) = 8.8556$ to four decimal places.

Actually, $\Gamma(p)$ may be found for any real p which is not a negative integer or zero. Solving the recursive relation for $\Gamma(p)$ yields

$$\Gamma(p) = (1/p)\Gamma(p+1),$$

which defines $\Gamma(p)$ for $p < 0$. Again, for example, $\Gamma(-0.5)=(1/-0.5)\Gamma(0.5)$, $\Gamma(-1.5) = (1/-1.5)(1/-0.5)\Gamma(0.5)$, and so on. It is easily seen that the gamma function would be undefined at zero and at all negative integers.

It is now possible to construct a graph of the gamma function as:



One important value of the gamma function is $\Gamma(1/2)$. To calculate this, let's return to the definition, obtaining

$$\Gamma(1/2) = \int_0^\infty t^{-1/2}e^{-t}dt.$$

(Since we are evaluating a definite integral, it does not matter what we call the variable of integration.) If we let $t = y^2$, then $dt = 2ydy$ and $\Gamma(1/2)$ becomes $\int_0^\infty (1/y)e^{-y^2}2ydy = 2\int_0^\infty e^{-y^2}dy$. Since it is a definite integral, we could also say that $\Gamma(1/2)$ is equal to $2\int_0^\infty e^{-x^2}dx$. Multiplying these two integrals together, and writing the result as a multiple integral, we have

$$(\Gamma(1/2))^2 = 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy.$$

This integral is more easily evaluated in terms of polar coordinates. If the transformation $x=r\cos\theta$ and $y=r\sin\theta$ is made, the integral reduces to

$$(\Gamma(1/2))^2 = 4\int_0^{\pi/2}\int_0^\infty e^{-r^2}\, r\, drd\theta = 4(\pi/2)\, e^{-r^2}/-2=\pi,$$

hence $\Gamma(1/2) = \sqrt{\pi}$.

If p is large, it is obvious that it could be difficult to find $\Gamma(p)$ directly. This difficulty can be avoided. By definition,

$$\Gamma(p+1) = \int_0^\infty x^pe^{-x}dx = \int_0^\infty e^{p\ln x-x}dx.$$

If we now make the substitution $x = p+y$, we obtain

$$\Gamma(p+1)=e^{-p}\int_{-n}^\infty e^{p\ln(p+y)-y}dy = e^{-p}\int_{-n}^\infty e^{p\ln p+p\ln(1+y/p)-y}dy,$$

which is equal to

$$p^p e^{-p} \int_{-n}^{\infty} e^{p\ln(1+y/p)-y} dy.$$

For large p, ln(1+y/p) may be expanded as the power series $w-w^2/2+w^3/3-\ldots$ where $w=y/p$. Letting $y=pv$, we obtain

$$\Gamma(p+1) = p^p e^{-p} \int_{-n}^{\infty} e^{-y^2/2p+y^3/3-\ldots} dy$$

$$= p^p e^{-p}\sqrt{p} \int_{-\sqrt{n}}^{\infty} e^{-v^2/2+v^3/3-\ldots} dv.$$

When p is large, all terms after the first are ignored in the expansion, and

$$\Gamma(p+1) = p^p e^{-p}\sqrt{p} \int_{-\infty}^{\infty} e^{-v^2/2} dv = \sqrt{2\pi p}\; p^p e^{-p}.$$

This approximation to the gamma function is known as Stirling's formula.

One use of the gamma function is the evaluation of integrals of the form $\int_0^{\infty} x^p e^{-x} dx$, or an alternate form $\int_0^1 x^p(\ln x)^q dx$, where $x=e^{-u}$. Such integrals arise in statistics, as, for example, in the case of the Chi-square and Student-t probability distributions. Gamma functions also arise in the physical sciences, as in the case of the Maxwell-Boltzmann distribution of molecular velocities.

Related to the gamma function is the beta function which is also useful in evaluating a number of general integral forms. The beta function is defined as

$B(p,q) = \int_0^1 x^p(1-x)^{q-1}dx$, where $p > 0$, $q > 0$. Evaluation of the beta function will be considered momentarily.

Any number of integral forms may be solved as beta integrals with several transformations. To begin, the range of integration for the beta integral may be changed by letting $x=y/a$. When $x=1$, then $y=a$, and when $x=0$, then $y=0$. The integral becomes

$$B(p,q) = \int_0^a (y/a)^{p-1}(1-y/a)^{q-1}dy/a$$

$$= (1/a)^{p+q-1}\int_0^a y^{p-1}(a-y)^{q-1}dy.$$

The trigonometric form of the beta integral may be created by making the substitution $x=\sin^2\theta$. Considering the limits of integration, when $x=1$, then $\theta=\pi/2$, and when $x=0$, then $\theta=0$, one obtains $(1-x)=1-\sin^2\theta = \cos^2\theta$. Finally, $dx=2\sin\theta\cos\theta d\theta$. The beta integral now becomes

$$B(p,q) = \int_0^{\pi/2}(\sin^2\theta)^{p-1}(\cos\theta)^{q-1}2\sin\theta\cos\theta d\theta$$

$$= 2\int_0^{\pi/2} (\sin\theta)^{2p-1}(\cos\theta)^{2q-1}d\theta.$$

Another possible substitution can be made with $x=y/(y+1)$. The integral can then be shown to reduce to

$$B(p,q) = \int_0^\infty (y^{p-1}dy)/(1+y)^{p+q}.$$

The beta function is not tabulated as is the gamma function. Since the beta function can be written in terms of the gamma function, its table is unnecessary. What we wish to prove is that

$$B(p,q)=\Gamma(p)\Gamma(q)/\Gamma(p+q).$$

Letting $z=x^2$, then $\Gamma(p)=\int_0^\infty z^{p-1}e^{-z}dz = 2\int_0^\infty x^{2p-1}e^{-x^2}dx$. Also $\Gamma(q) = 2\int_0^\infty y^{2q-1}e^{-y^2}dy$. Thus,

$$\Gamma(p)\Gamma(q) = 4(\int_0^\infty x^{2p-1}e^{-x^2}dx)(\int_0^\infty y^{2q-1}e^{-y^2}dy)$$

$$= 4\int_0^\infty \int_0^\infty x^{2p-2}y^{2q-1}e^{-(x^2+y^2)}dxdy.$$

If a transformation to polar coordinates is made ($x=r\cos\theta$, $y=r\sin\theta$), then

$$\Gamma(p)\Gamma(q) = 4\int_0^{\pi/2}\int_0^\infty r^{2(p+q)-1}e^{-r^2}\cos^{2p-1}\theta\sin^{2q-1}\theta\, dr d\theta$$

$$= 4(\int_0^\infty r^{2(p+q)-1}e^{-r^2}dr)(\int_0^{\pi/2}\cos^{2p-1}\theta\sin^{2q-1}\theta\, d\theta)$$

$$= 2\Gamma(p+q)\int_0^{\pi/2}\cos^{2p-1}\theta\sin^{2q-1}\theta\, d\theta$$

$$= \Gamma(p+q)B(p,q).$$

Hence,

$$B(p,q) = \Gamma(p)\Gamma(q)/\Gamma(p+1).$$

Mathematically speaking, the gamma and beta functions are not in themselves particularly significant.It is their frequent occurrence in topics of applied mathematics that makes these two special functions of interest and of importance.

THE BOOK SHELF

Contributed by IRATNALAK JARI

This department of THE PENTAGON brings to the attention of its readers, recently published books (textbooks and tradebooks) and articles which are of interest to students and teachers of mathematics. Readers are encouraged to contribute to this section by presenting a glimpse into an interesting book or article they have read.

What is the name of this book?
The Riddle of Dracula and other logical puzzles:

Raymond Smullyan, Prentice-Hall, Inc.
Englewood Cliffs, New Jersey, 1978, 241 pp., $4.95

I was visiting a friend in San Diego and it was raining hard. I mean really hard. It was one of those rain storms that you see in the old detective movies. It always seemed tranquil and purposeful to me to be walking purposelessly on a hard rainy night with a trench coat and a rimmed hat; the collar pulled up of course. But I had never experienced the stroll. You see, every rain storm I had been in,had also been windy and cold.It is hard to feel tranquil if you are shivering. I had begun to doubt if what I saw in the old movies was true until that night in San Diego.

I had been in San Diego for three days. It had been raining every minute of my stay. In the morning of that day, I met with Jeff in his office to do business; we did mathematics all morning. About noon we decided to take a break and we visited this small Italian restaurant located in the general campus area. It was a pretty little restaurant with red checkerboard tablecloths and a big jar of parmesan cheese on every table. We took a small table and while waiting for our pepperoni and mushroom pizza, we talked about curious puzzles and problems we had heard lately. The pizza was great but I had a hard time dealing with the way the waitress changed her mood the moment she heard us discuss whether a certain function had a compact domain or not. I didn't like that at all. You probably know how it is when people think you are from a different planet when you talk mathematics, don't you? Anyhow, I decided not to be bothered by her attitude and ordered a dish of spumoni.

In the afternoon, we did business again, I mean we talked mathematics. This time we were at the coffee room of the mathematics department at the university and anybody who was eavesdropping was much more enthusiastic than Karen the waitress. I sipped at coffee all afternoon and enjoyed listening to Jeff's sharp techniques and new topics he had discovered. It seemed we had brewed a new subject to investigate.

The boulevard visible from Jeff's office seemed tired and deserted; it was wet and the rain was pounding on it more. The sun was setting in the distant waters. It looked like the sun was sinking in the ocean. A question came to me; at what point of the sunset is the area of the visible part of the sun equal to the area of the hidden part of it? I tried to solve it mentally with no success and since the scenery was captivating I decided to file the problem away until I got a pencil, some paper and a fresh cup of coffee sometime. It was that long look at the boulevard full of old California cars with no rust spots,turning their dim lights on which caused an echo of old movies in my mind and made me desire to walk in the rain.

It was about six thirty. I asked Jeff to drop me in a neighborhood where I could get a decent warm meal and catch a movie. I could catch a bus to my quarters later. I found I was looking forward to the walking I had to do between the events. I got dropped off at the corner of an intersection; a car honked at me and continued driving through the rain. I looked around and saw a flickering neon light of a restaurant a few blocks north. The waitress was a smiling one. She looked as if she enjoyed her job. I always have respect for those who like their jobs. She brought me a Studioburger with bacon and a tall glass of cola. The food was good and the old fashioned juke box was playing a cut from the new album of Olivia Newton John; she sings with warmth.

My mood was changing, the kafka-ish feeling that had come upon me from looking down into the boulevard was gone. The warm atmosphere, the warm food and the warm music had me all determined to enjoy my walk and a movie. I left her a 20% tip, pulled the collar up on my trench coat and walked into the street. I counted eighteen intersections as I crossed them but I found no movies. I was tired of looking and almost wet. Suddenly, across the street, hidden behind the leaving bus, I found a bookstore. Like a flash my mind was made up. I was to go and find myself a good mystery novel, get back to my quarters and drown myself in the plot, not to forget about a thermos of hot coffee. It was a modern bookstore with lights all over the many parallel racks. A man with a straw hat was reading a book on pottery. There was no one else in the store except the storekeeper; fortyish, she was sitting on a stool behind the counter and was reading a paperback. Her bifocals sat on the edge of her slim nose but the chain around her neck and tied to the handles promised security. As she finished her page she looked up and smiled at me. I nodded and looked around. Across from the best seller's rack,next to the revolving rack of cards, I found the mystery rack. I loosened my raincoat's belt and started a search for a promising whodunit.

It looked as if I was not going to leave the store with my hand securely holding a book in my long pocket. I was disappointed,I had read them all. Then, as I was examining my courage for a further pursuit of a moviehouse, I saw this book on the other rack on the back wall; it was the rack titled 'General'. The book had a catchy name, especially for a logician. Besides I had heard about it before. By this time you are probably curious and ask what is the name of this book? Yes I answer "*What Is The Name of This Book?*" by Raymond Smullyan. Before we fall into an Abbott-Costello routine, let me assure you that the phrase with the quotation marks above is the name of the book which is published by Prentice-Hall in 241 pages (paperback) and sells for $4.95. It is not a whodunit I said to myself and began to examine it right where I start every book like this, the Table of Contents. The list includes 'Logical Recreations', 'Portia's Caskets and Other Mysteries','Weird Tales' and 'Logic is a Many-Splendored Thing'.

I was curious and felt there was a chance for a good reading for the rest of the night. I opened to page 3 and read

Was I Fooled?
My introduction to logic was at the age of six. It happened this way: On April 1, 1925, I was sick in bed with grippe, or flu, or something. In the morning my brother Emile (ten years my senior) came into my bedroom and said: "Well, Raymond, today is April Fool's Day, and I will fool you as you have never been fooled before!" I waited all day long for him to fool me, but he didn't. Late that night, my mother asked me, "Why don't you go to sleep?" I replied, "I'm waiting for Emile to fool me." My mother turned to Emile and said, "Emile, will you please fool the child!" Emile then turned to me, and the following dialogue ensued:

Emile/ So, you expected me to fool you, didn't you?
Raymond/ Yes.
Emile/ But I didn't, did I?
Raymond/ No.
Emile/ But you expected me to, didn't you?
Raymond/ Yes.
Emile/ So I fooled you, didn't I!

Well, I recall lying in bed long after the lights were turned out wondering whether or not I had really been fooled. On the one hand, if I wasn't fooled, then I did not get what I expected, hence I was fooled. (This was Emile's argument.) But with equal reason it can be said that if I was fooled, then I did get what I expected, so then, in what sense was I fooled. So, was I fooled or wasn't I?

I smiled. I moved on and came to page 9 where I found a whodunit.

Who Was the Murderer?
This story concerns a caravan going through the Sahara desert. One night they pitched tents. Our three principle characters are A, B, and C. A hated C and decided to murder him by putting poison in the water of his canteen (this would be C's only

water supply). Quite independently of this, B also decided to mumder C, so (without realizing that C's water was already poisoned) he drilled a tiny hole in C's canteen so that the water would slowly leak out. As a result, several days later C died of thirst. The question is, who was the murderer, A or B? According to one argument, B was the murderer, since C never did drink the poison put in by A; hence he would have died even if A hadn't poisoned the water. According to the opposite argument, A was the real murderer, since B's actions had absolutely no effect on the outcome; once A poisoned the water, C was doomed, hence C would have died even if B had not drilled the hole.

Which argument is correct?

I was amused. I thought to myself this is as good as a mystery novel. It could keep me occupied for the rest of the night. I paged forward and found the Files of Inspector Craig: page 68.

The Case of the Identical Twins.
In this more interesting case, the robbery occurred in London. Three well-known criminals A,B,C were rounded up for questioning. Now, A and C happened to be identical twins and few people could tell them apart. All three suspects had elaborate records, and a good deal was known about their personalities and habits. In particular, the twins were quite timid, and neither one ever dared to pull a job without an accomplice. B, on the other hand, was quite bold and despised ever using an accomplice. Also several witnesses testified that at the time of the robbery, one of the two twins was seen drinking at a bar in Dover, but it was not known which twin.

Again, assuming that no one other than A,B,C was involved in the robbery, which ones are innocent and which ones guilty?

I was submerged in the book until the sudden ring of the bell attached to the door of the store announced exit of the man with straw hat. I walked to the clerk and smiled. I knew I had found something to read, in fact I knew I was hooked. I paid for the book and avoided a long chit-chat with the clerk; I was anxious to read on in Smullyan's book.

I walked out of the store into the rain;it was California warm. I thought about catching a bus but decided to walk. It was only a couple of miles to my quarters! I started to analyze the puzzles I had read in Smullyan's book while I headed to my quarters ...

I got home three hours later. I walked and stopped in three different coffee shops on my way. In each cafe, I read more of the book. I read puzzles about "knights" who always tell the truth and "knaves" who always lie. I read puzzles about Alice in the Forest of Forgetfulness, and Portia's quest for an intelligent husband. I learned about how to defend myself in court and how to marry a king's daughter. I visited the strange island of Baal which no person is known to have found his way to. Then there was the island of Zombies where any question whose answer is <u>yes</u> or <u>no</u> is answered by the natives by "Bal" or "Da"; which is which,no one knows but everyone loves to figure out. Finally, before I finished my seventh cup of coffee I even read about Dracula in Transylvania where humans and vampires are physically indistinguishable but a reader can solve puzzles to determine which is which.

I took off my raincoat;it was wet, inside and out. I decided on a hot shower. When I was ready to retire for the night, I thought about the whole day; not bad at all. The mathematics Jeff and I did in the day was promising, the food was good everywhere and the coffee not intolerable even at its worst. And what a delicious book I had found for entertainment. Smullyan has done a superb job I said to myself. I had seen and read a lot of books in this format but I recalled no other book keeping me so fascinated for so long. It still was not over. I laid in bed,ready to examine my schedule for the next day. My mind wandered off and fixed on many amusing puzzles I had read that night. I couldn't resist it, I had to read the last chapter. What would I do the next evening? Well, I simply had to find a movie to go to! Besides next night was not there yet.

The last chapter is on logic. Smullyan is a logician, I am a logician. He amused me with popular characterizations of "logic"; page 184.

> *Another Characterization of Logic.* A friend of mine--an ex-police officer--when he heard I was a logician, said: "let me tell you how I see logic. The other day my wife and I were at a party. The hostess offered us some cake. On the platter were just two pieces, one larger than the other. I thought for a while, and then I decided to take the larger piece. Here is how I reasoned: I know my wife likes cake and I know she knows that I like cake. I also know she loves me and wants me to be happy, therefore she would want me to have the larger piece. Therefore I took the larger piece."

I went on. On page 200 he shows how to prove anything! I said to myself. I have to hide this chapter away from my students! I might become obsolete!

I could not stop. I read the entire book and ended up skimming through a last time before I pulled the chain on the small lamp next to my bed. As I was falling asleep, I thought to myself: I am glad people write books; this is one good book every library should have. Every student who enjoys recreational logic and ingenious puzzles with solutions, would find this book humorous and a great supply of topics for conversation; to go with good coffee of course.

KAPPA MU EPSILON NEWS

EDITED BY HAROLD L. THOMAS, HISTORIAN

News of chapter activities and other noteworthy KME events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department, Pittsburg State University, Pittsburg, Kansas 66762.

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence
Chapter President-Deborah Thigpen
46 actives, 21 pledges

The chapter held a "get acquainted" meeting with two new members of the mathematics faculty. For one of the regular fall meetings a report was given on opportunities in computer related fields. An initiation banquet was also held to induct 21 new members. Other officers for 1979-80 not published in the fall '79 issue include Sherry Stratford, historian.

Alabama Gamma, University of Montevallo, Montevallo
Chapter President-Susan E. Mays
9 actives, 12 pledges

An initiation for new members was held January 22, 1980. Other officers not previously published: James Rickey, vice-president; Danny Blackerby, treasurer; Joseph Cardone, corresponding secretary; Angela Hernandez, faculty sponsor.

Arkansas Alpha, Arkansas State University, State University
Chapter President-Samuel D. Young
12 actives

New officers were installed October 9, 1979. These include Susan Boyd, vice-president; Greg Hester, secretary; Jody Carreiro, treasurer. Committees were formed and future programs were discussed at this time.

California Gamma, California Polytechnic State University, San Luis Obispo
Chapter President-Dan Moczarny
30 actives, 24 pledges

Fall activities began with a faculty-student picnic. Monthly chapter meetings featured a faculty speaker, a representative from Bell Labs, and a placement office speaker. Workshops were held to assemble the tests to be used for the county-wide Junior High Math Field Day to be held in the spring. The quarter concluded with a Christmas social and a pledge ceremony. Other officers remain as published in the fall issue.

California Delta, California State Polytechnic University, Pomona

Chapter President-Jeffrey Eakins

6 actives, 5 pledges

Fall programs included a lecture given by Dr. Marcus on "Simple Solutions to Hard Questions." Other officers remain as published in the fall issue.

Colorado Beta, Colorado School of Mines, Golden

Chapter President-Shelby Switzer

20 actives

The chapter met twice during the fall semester. Dr. D.C.B. Marsh gave a special talk at one of these meetings on "Trap-door Methods in Cryptography." Other officers for 1979-80 remain as published in the spring '79 issue.

Georgia Alpha, West Georgia College, Carrollton

Chapter President-Brenda Dale Jones

20 actives

The fall quarter meeting was held jointly with the Physics Club on November 20, 1979. The program included a brief physics demonstration, a film entitled "Mercury: Exploration of a Planet," and a question and answer session. Refreshments were also served. Other officers remain as published in the fall issue.

Illinois Beta, Eastern Illinois University, Charleston

Chapter President-Ellen Newberg

26 actives

Other officers for 1979-80: Jeff Bivin, vice-president; Laurie Saviano, secretary; Lee Anne Eubanks, treasurer; Patricia Ryan, corresponding secretary; Dr. J. Nanda, faculty sponsor.

Illinois Zeta, Rosary College, River Forest

Chapter President-Mark Siwek

14 actives

Chapter members amended their by-laws to conform with provisions of the new constitution of KME. Plans for service activities were discussed and plans for a reunion with alumni members were made. Three new members were initiated on November 14. Other officers remain as published in the fall issue.

Illinois Eta, Western Illinois University, Macomb

Chapter President-Debbie Oganovich

8 actives, 10 pledges

The chapter held four meetings and two student-faculty social events during the fall semester. Both the October picnic and softball game at Lake Argyle and the December Christmas party were well attended. Six new members were initiated, including one faculty member. Illinois Eta chapter was chosen to host the Region IV convention in the spring. Preparations for this event have begun. The chapter was also pleased to learn that their corresponding secretary, Dr. Kent Harris, was selected as the new PENTAGON editor. Dr. Larry Morley will take over corresponding secretary responsibilities, and Dr. Iraj Kalantari will become faculty sponsor.

Indiana Delta, University of Evansville, Evansville
Chapter President-Mark Steeber
18 actives

Monthly meetings with topics from areas of applied mathematics were held as usual. In addition, KME and the Mathematics Department are sponsoring a university-wide mathematics problem solving competition. Corresponding secretary for Indiana Delta is now Dr. Duane Broline and faculty sponsor is Dr. Gene Bennett.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President-Jill Roesch
32 actives

The Annual Homecoming Breakfast, held October 6, 1979 at the home of Dr. and Mrs. Fred W. Lott, was well attended by students, faculty, and KME alumni. Students who presented papers at KME meetings during the fall semester were: Nancy Bennett, "The Mathematics Behind a Cost Function," Eileen Myers, "The Quadratic Equation in Chemistry," and John Christensen, "Life or Death: It Matters to an Actuary." The KME Christmas party was held December 14, 1979 at the home of Dr. and Mrs. Greg Dotseth. Other officers remain as published in the fall issue.

Iowa Beta, Drake University, Des Moines
Chapter President-Mark Sand
10 actives, 10 pledges
Other officers remain as published in the fall issue.

Iowa Gamma, Morningside College, Sioux City
Chapter President-Roger Bobolz
20 actives

The chapter welcomed a new faculty sponsor in the fall of 1979, Dr. Carol White. Fall activities centered around getting acquainted with Dr. White and also participation in a Math-Science Symposium held for about 500 high school students from Sioux City and surrounding communities. Iowa Gamma is looking forward to several new members to be initiated in the spring semester. Other officers remain as published in the fall issue.

Iowa Delta, Wartberg College, Waverly
Chapter President-Mark Reinhardt
27 actives

The fall semester included a variety of interesting meetings. A games meeting was highlighted by Al Brunner's presentation of Lew Carroll's mathematical croquet. Another meeting focused on a microcomputer demonstration by Dr. Walter Beck. In November, student papers included: Zahif Rohim, "Large and Interesting Numbers,"Al Brunner, "Graph Theory," and Dan Guetzlaff, "Reverse Polish Notation." Other officers remain as published in the fall issue.

Kansas Alpha, Pittsburg State University, Pittsburg
Chapter President-Kevin Sperry
40 actives

The chapter hosted a fall picnic for all mathematics and physics students. Five new members were initiated in October at which time Kevin Sperry, Kay Conklin, and Pat Keating discussed interview techniques. Book reviews were also given by Dr. Helen Kriegsman and Prof. J. Bryan Sperry. In place of a formal November meeting, the chapter toured the computing facilities at the local McNally Corporation. Members gathered at the home of Dr. Helen Kriegsman, mathematics department chairman, for the special Christmas meeting in December. At that time Penny Lane presented "Death-day and Birthday: An Unexpected Connection." Howard Thompson also related his experiences in taking the Putnam examination. Other officers remain as published in the fall issue.

Kansas Beta, Emporia State University, Emporia
Chapter President-Angie Miller
17 actives, 9 pledges

Various speakers presented talks at the fall meetings and at initiation. One of the most enlightening was Mr. Maurice Clark, KG & E Quality Assurance manager of the Wolf Creek Nuclear Plant. Student talks covered various aspects of random problems. Mary Beth Fry was nominated for Who's Who among College Students. The chapter also held special Christmas and Halloween parties. Other 1979-80 officers not published in the fall issue are: Peg Schultz, vice-president and LeAnn Copeland, treasurer.

Kansas Gamma, Benedictine College, Atchison
Chapter President-Patricia McDonald
22 actives, 9 pledges

During the fall semester the usual activities-fall picnic, volleyball game, and Christmas Wassail party-were once again enjoyed by Kansas Gamma members. At an early meeting several students shared with the group information concerning their summer employment in the areas of mathematics and computer science. In November, two alums now teaching at Pittsburg State University in Pittsburg, Kansas returned to campus for the day. Dr. Gary McGrath spoke on "Least Square Techniques," and Dr. Felix Dreher talked on "The Mathematical Foundations of Relational Data Bases." Other officers for 1979-80 not published in the fall issue include Joyce Heideman, historian.

Kansas Epsilon, Fort Hays State University, Fort Hays
Chapter President-Terri Hooper
15 actives
Fall activities included helping with the departmental picnic in September and holding a Halloween Party the latter part of October. Other officers remain as published in the fall issue.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore
Chapter President-Maura Kelly
10 actives, 5 pledges
Fall chapter activities included an introduction to the TRS-80 microcomputers now used on campus and a career-oriented presentation by a representative of the Metropolitan Life Insurance Company. Members also enjoyed their annual field trip to the Smithsonian Institution in Washington, D.C. Other officers remain as published in the fall issue.

Maryland Beta, Western Maryland College, Westminster
Chapter President-Terry Reider
14 actives
Two new members were initiated in November. The chapter held a film festival in the January term to raise money. It is hoped that several students can attend the regional meeting at Shippensburg State College in April 1980 and also the National KME convention in April 1981. Other officers remain as published in the fall issue.

Maryland Delta, Frostburg State College, Frostburg
Chapter President-Reinaldo Machado
12 actives
Other officers for 1979-80 are: Janet Jessup, vice-president; James Martens, secretary; Peter Crickman, treasurer; Agnes Yount, corresponding secretary; John

Jones, faculty sponsor.

Michigan Beta, Central Michigan University,Mt. Pleasant
Chapter President-Laurie Cooper
92 actives

Fall activities began with a picnic in September. A member of the campus placement office spoke on job opportunities for mathematics students at the October meeting. Twenty-nine new members were initiated at the November meeting. The initiation speaker was Jan Novasad, a 1977 graduate of Central Michigan University and a very active KME member. She spoke of her job activities as a member of the mathematics section of Dow Chemical Company. Several of her projects involved statistics, linear programming, and operations research. The semester concluded with a Christmas party in December. KME members are active throughout the semester in providing tutorial assistance to students in freshman and sophomore level courses. Other officers for 1979-80 are: Amy Blanzy, vice-president; Kay Meeks, secretary; Donna Leslie, treasurer; Arnold Hammel, corresponding secretary and faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President-Donna Pearce
30 actives

Seven new members were initiated in the fall of 1979. Programs presented at regular meetings included Dr. Temple Fay, "Curve Tracing," and Dr.Douglas McCain, "Group Theory in Chemistry." The semester concluded with a Christmas party in December. Other officers remain as published in the fall issue.

Mississippi Delta, William Carey College, Hattiesburg
Chapter President-Pam Sellers
5 actives

Other officers for 1979-80 are: Frank Gasparovic, vice-president, Deborah Farrior,secretary and treasurer; Gaston Smith, corresponding secretary; Maury Shurlds, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President-Rita Scroggins
45 actives, 8 pledges

The chapter held three monthly meetings and a joint December meeting with Missouri Iota chapter. Other officers remain as published in the fall issue.

Missouri Beta, Central Missouri State University, Warrensburg

Chapter President-Robert Penniston

35 actives, 8 pledges

Fall semester activities included three regular meetings, one initiation, and a Christmas party. Officer changes from the fall issue are: Lynn Hill, vice-president;Cindy Kemna,secretary;Gina Dunhum, treasurer; Al Tinsley, faculty sponsor.

Missouri Gamma, William Jewell College, Liberty

Chapter President-Susan Stewart

20 actives

Programs for the fall semester included a film, a student presentation, and a discussion of "dragon curves" by a member of the faculty. The annual banquet and initiation will be held March 25, 1980. Other 1979-80 officers are: Wayne Grooms, vice-president; Ruth Carter, secretary; Jim Ginn, treasurer; Sherman Sherrick, corresponding secretary; Truett Mathis, faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette

Chapter President-Janet Doll

5 actives

Other officers remain as published in the fall issue.

Missouri Zeta, University of Missouri-Rolla, Rolla

Chapter President-Barbara Horter

20 actives, 9 pledges

Chapter activities have included a fall picnic, regular monthly meetings with speakers, and a new members banquet and initiation. The members also offer tutorial help sessions for other mathematics students. Other officers for spring 1980 are: Allen Crider, vice-president; Patti Donovan, secretary, Kathy Fitzgerel, treasurer; Tim Wright, corresponding secretary; Jim Joiner, faculty sponsor.

Missouri Eta, Northeast Missouri State University,Kirksville

Chapter President-Theresa McGuire

23 actives, 8 pledges

The chapter sponsored a picnic for all mathematics students and a get acquainted party for all students in any mathematics class. Regular monthly meetings are held with talks given by members. All seniors must present a paper. The semester was concluded with a Christmas party for mathematics faculty, KME members and guests. Other officers remain as published in the fall issue.

Nebraska Alpha, Wayne State College, Wayne
Chapter President-Lois Bright
13 actives, 11 pledges

Throughout the fall semester, the chapter provided a free tutorial service to pre-calculus and calculus students. The tutors help individual students and conduct review sessions for student groups. KME members were also involved in several homecoming activities. Karen Reestman and Rod Bubke were queen and king candidates, respectively. The members also built a school spirit billboard and sponsored a free polka and square dance for homecoming participants. Monthly meetings are held and students present mathematical novelties. The chapter also maintains a bulletin board of mathematical puzzles. Other officers for 1979-80 are Karen Reestman, vice-president; Kay Nickelson, secretary-treasurer; Rod Bubke, historian; Fred Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President-Mark Allen
75 actives

Other officers remain as published in the fall issue.

New York Eta, Niagara University, Niagara
Chapter President-Mary Heneghan
18 actives, 4 pledges

The chapter met to discuss projects for raising funds and to plan a field trip for the spring semester. The chapter hopes to attend the regional convention in the spring. Other chapter officers for 1979-80 are: Janice Layo, vice-president and secretary; John Michalek, treasurer; Robert Bailey, corresponding secretary; Sr. John Frances Gilman, faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green
Chapter President-Curtis Lambert

Fall programs included a "Careers Night." A panel consisting of a faculty member from Applied Statistics, a member of the Placement Office, and two interviewers from Allstate Insurance discussed mathematical careers. Other officers for 1979-80 are: Albert Copper, vice-president; Suzanne Gregg, secretary; David Webster, treasurer; Waldeman Weber, corresponding secretary; William Kirby, faculty sponsor.

Ohio Gamma, Baldwin-Wallace College, Berea
Chapter President-Lynn Jones
24 actives

The chapter took a field trip to Republic Steel Research Center in Cleveland, Ohio to learn about the mathematics services department and the computer center. Another field trip to the computer center at Cedar Point is planned for spring. The members are raising money by selling T-shirts. Other officers for 1979-80 remain as published in the fall issue.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford

Chapter President-Troy Harden

20 actives, 8 pledges

Chapter activities for the fall semester began with a picnic. Eight new members were initiated. Dr. Leroy Folks from Oklahoma State University provided the program for one of the regular meetings. Other officers remain as published in the fall issue.

Pennsylvania Alpha, Westminster College, New Wilmington

Chapter President-Laurie Sassaman

46 actives, 3 pledges

Fall activities began with a picnic for all mathematics and computer science majors. The chapter made a field trip to Buhl Planetarium for a lyceum show and train exhibit. Chapter members also assisted with the high school mathematics competition. Other officers remain as published in the fall issue.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President-Rhonda Cranage

13 actives, 13 pledges

The chapter had a very active fall semester. Members attended a picnic sponsored by the mathematics department in late semester. During the semester, for regular meetings, guest speakers gave talks on "Chisanbop," "Measurements in Space," and "Solar Energy." The members hosted the annual mathematical games night which was open to all Kutztown students. Applications were approved for thirteen pledges who will be formally initiated in March. Other officers for 1979-80 are Jann Eline, vice-president (fall semester); Bonnie Bailey, vice-president (spring semester); William Folk, secretary; George Malafarina, treasurer; William E. Jones, Jr., corresponding secretary; Irving Hollingshead, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President-Debra Mentch

24 actives

Regular meetings were held in October, November, and

December. New members were initiated at the October meeting. Corresponding secretary, Ida Z. Arms, showed slides of her summer, 1979 trip to China. The November meeting was held in conjunction with Career Information Night for mathematics majors. Three IUP mathematics graduates discussed the type of work they have been doing since graduation. Also three executives from Sharon Steel Company told about job opportunities for math and computer science majors. They also discussed the characteristics they look for in a prospective employee. For the December meeting Amy Sitzler discussed her internship she had during the summer of 1979 with Sharon Steel Company. New officers were elected in December to serve from January 1980 through December 1980. They are:Terry Sheaffer, vice-president;Loretta Kachline,secretary, Michele Stelma,treasurer;Ida Z.Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President-Joan Jewell
30 actives, 13 pledges

The initiation of new members was held in the home of Dr. Ed Daggit on November 5, 1979. The members have also formed plans for providing free tutorial sessions for students in Calculus II. Other officers for 1979-80 remain as published in the fall issue.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President-Suzanne Moriat
6 actives, 4 pledges

KME members provide tutorial services in the Math Center. The November meeting was devoted to the discussion of Magic Squares. Enthusiasm from this led to a Magic Square Contest sponsored by KME for all students of Holy Family College. Linda Czojka was the winner of this contest. The chapter made plans to attend the NCTM convention in March at Cherry Hill, New Jersey. Jan Buzydlowski is now vice-president and secretary. Other officers remain as published in the fall issue.

Pennsylvania Lambda, Bloomsburg State College, Bloomsburg
Chapter President-Cathy Folk
24 actives, 8 pledges

Fall initiation for new members was held at the home of Dr. James Pomfret on October 14, 1979. Eight new members joined at that time. Spring initiation was held February 10, 1980 at the Faculty Dining Hall of the Scranton Commons. Nine new members were received this time. In March, the chapter will sponsor the first annual B.S.C. mathematics contest for High School students. This college bowl contest will consist of oral competition among approximately thirty high school teams. Several members plan to present papers at the regional convention at Shippensburg State College in April. Throughout the year, KME and the Math Club join together for picnics, dinners, and other recreational activities. Other officers for 1979-80 are: Kris Perkins, vice-president; Anne Sobeck, secretary; Kim Hellerman, treasurer; Dr. James Pomfret, corresponding secretary; Mr. Joseph Mueller, faculty sponsor.

Pennsylvania Mu, Saint Francis College, Loretto
Chapter President-Jay Flaherty
20 actives
Pennsylvania Mu chapter was installed September 14, 1979. Hence, this is the first chapter news report re-received from them. Ida Arms, National vice-president from Pennsylvania Zeta was the installing officer. Following her presentation on the history of KME, the new chapter held a dinner jointly with the Math Club and school officials. Other fall activities included co-sponsoring a table at parents weekend, sale of computer-pictured-calendars, and sponsoring a lecture by Dr. Cashing entitled "Why Democracy Can't Work." Other officers for 1979-80 are Judy Hemmerle, vice-president; Marsha Oakes, secretary and treasurer; Rev. John Kudrick, T.O.R., corresponding secretary; Dr.Adrian Baylock, faculty sponsor.

South Carolina Beta, South Carolina State College, Orangeburg
Chapter President-Jacquelyn D. Nelson
10 actives, 13 pledges
Other officers for 1979-80 are: James E. Roberts, vice-president; Eleanor J. Garvin, secretary and treasurer; Frank M. Staley, Jr., corresponding secretary; C. Allen Jones, faculty sponsor.

Tennessee Alpha, Tennessee Technological University, Cookeville
Chapter President-Teresa Johnson
100 actives
Other officers remain as published in the fall issue.

Tennessee Beta, East Tennessee State University, Johnson City
Chapter President-Eric Bowman
16 actives
Several KME members visited the University of Tennessee to learn about opportunities in mathematics.

Mike Bell, KME president for 1978-79, participated in the Putnam mathematics competition. Other officers for 1979-80 are: Karen Pierce, vice-president; Cheryl Ponder, secretary; Dr. Lyndell Kerley, corresponding secretary and faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President-Jana Davis
47 actives
Other officers for 1979-80 are: Anita Meeker, vice-president; Karla Smith, secretary and treasurer; Anne B. Bentley, corresponding secretary; Dr. Charles D. Robinson and Dr. Edwin Hewett, faculty sponsors.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President-Jane Simeth
7 actives, 4 pledges
At regular chapter meetings, the members worked together on problems that had been used for last year's mathematics contest. Jane Simeth gave a report on the National Convention she attended in Pittsburg, Kansas last spring. At the December meeting, the members generated Christmas spirit by playing a divisibility game involving the words "Merry" and "Christmas." The chapter also held a donut sale to raise money to apply to expenses to future conventions. Officer changes from those published in the fall issue include Eileen Knowles, vice-president and Eileen Korenic, treasurer.

Wisconsin Beta, University of Wisconsin, River Falls
Chapter President-Mark Hoffman
36 actives, 4 pledges
The chapter has had several alumni come back and relate their work experiences. Speakers from the 3M Company have also provided programs. Other officers remain as published in the fall issue.

Directions for Papers to be Presented at the Twenty-Third Biennial Convention of Kappa Mu Epsilon

Springfield, Missouri
2-4 April, 1981

A significant feature of this convention will be the presentation of papers by student members of KME. The mathematics topic which the student selects should be in his area of interest, and of such a scope that he can give it adequate treatment within the time allotted.

Who May Submit Papers? Any student member of KME, undergraduate or graduate, may submit a paper for use on the convention program. A paper may be co-authored; if selected for presentation at the convention it must be presented by one or more of the authors. Graduate students will not compete with undergraduates.

Subject: The material should be within the scope of the understanding of undergraduates, preferably those who have completed differential and integral calculus. The Selection Committee will naturally favor papers within this limitation, and which can be presented with reasonable completeness within the time limit.

Time Limit: The minimum length of a paper is 15 minutes; the maximum length is 25 minutes.

Form of Paper: Four copies of the paper to be presented, together with a description of the charts, models or other visual aids that are to be used in the presentation should be presented in typewritten form, following the normal techniques of term paper presentation. It should be presented in the form in which it will be presented, including length. (A long paper should not be submitted with the idea that it will be shortened for presentation.) Appropriate footnoting and bibliographical references are expected. A cover sheet should be prepared which will include the title of the paper, the student's name (which should not appear elsewhere in the paper), a designation of his classification in school (graduate or undergraduate), and a statement that the author is a member of Kappa Mu Epsilon, duly attested to by the Corresponding Secretary of the student's Chapter.

Date Due: January 21, 1981

Address to send Papers: Professor Ida Z. Arms
Department of Mathematics
Indiana University
of Pennsylvania
Indiana, Pennsylvania 15705

Selection: The Selection Committee will choose about fifteen papers for presentation at the convention. All other papers will be listed by title and student's name on the convention program, and will be available as alternates. Following the Selection Committee's decision, all students submitting papers will be notified by the National Vice President of the status of their papers.

Criteria for selection and convention judging:

A. The Paper

1. Originality in the choice of topic
2. Appropriateness of the topic to the meeting and audience
3. Organization of the material
4. Depth and significance of the content
5. Understanding of the material

B. The Presentation

1. Style of presentation
2. Maintenance of interest
3. Use of audio-visual materials (if applicable)
4. Enthusiasm for the topic
5. Overall effect
6. Adherence to the time limit

Prizes: The author of each paper presented at the convention will be given a two-year extension of his subscription to THE PENTAGON. Authors of the four best papers presented by undergraduates, based on the judgment of the Awards Committee, composed of faculty and students, will be awarded cash prizes of $60, $40, $30, and $20 respectively. If enough papers are presented by graduate students, then one or more prizes will be awarded to this group. Prize winning papers will be published in THE PENTAGON, after any necessary editing. All other submitted papers will be considered for publication, at the discretion of the Editor.

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3) The articles are generally

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