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National Officers

Kappa Mu epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

A STOCHASTIC PROCESS FOR COMPOSING MUSIC*

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Music is one of my hobbies and I have often wanted to compose music. Since Paul Simon is one of my favorite composers, I decided to try to develop a procedure that would compose music, based upon some of the patterns found in his music. Paul Simon, of course, was part of the team of Simon and Garfunkel, and has written such hit songs as "The Sound of Silence" and "Bridge Over Troubled Water". Since he has had such success, I thought that there might be some patterns in his music that could be used as a basis for a music-composing process.

First, we'll discuss the basic terminology needed. We want the final result of this procedure to be a finite sequence of notes: $N_1,N_2 \ldots,N_k$. Each note of this sequence can be defined by two characteristics: its duration and its pitch. The duration can assume any one of five distinct values--Whole, Half, Quarter, Eighth, or Sixteenth. We will define the set $D = \{W,H,Q,E,S\}$ to be the set of all possible values for the duration of a note. The pitch can assume any one of the thirty-six values from C below Middle C to B above C above Middle C. This three-octave range is wide enough to ensure the inclusion of all notes used in Paul Simon's songs. The note can also have no pitch at all, in which case it is known as a rest. We will define the 37-member set

$$P = \{C,C\#,D,D\#,E,F,F\#,G,G\#,A,A\#,B,C',\ldots,C'',\ldots,B'',R\}$$

to be the set of all possible values for the pitch of a note. Thus our final sequence of notes $N_1,N_2 \ldots,N_k$ can be characterized as a sequence of ordered pairs (d_1,p_1), $(d_2,p_2) \ldots,(d_k,p_k)$ where each $d_i \in D$ and each $p_i \in P$.

We don't want this sequence of notes to be just any sequence of notes, but one that will conform to to several conditions:

(a) It should start with some arbitrary note of our choice;

(b) It should end with some arbitrary note of our choice;

(c) The sequence must in some way reflect the patterns found in songs written by Paul Simon; and

(d) The sound of the sequence should be reasonably pleasing to the ear.

*A paper presented at the 1979 National Convention of KME and awarded first place by the Awards Committee.

It is not hard to develop a procedure that will produce a sequence of notes that satisfies the first two conditions. To satisfy the third condition, we need to examine Paul Simon's songs and discern some of the patterns found in them.

To do this, I wrote a computer program that, given a sequence of notes (as in the tune of a song), examines each adjacent pair of notes in this sequence and determines the statistical probability that, if note N_i has duration d_i, then note N_{i+1} has any duration $d_{i+1} \in D$. Since there are five elements in D. this means that there are twenty-five different possible pairs of durations (each of the durations can be followed by any of the elements of D). The program computes these probabilities by setting up a 5 x 5 matrix, whose rows and columns are labeled with the elements of D. It then examines the sequence of notes and determines the number of times that each of the five durations was followed by any of the five durations. For example, if it is observed that a note with duration of H was followed by a note with a duration of Q, then 1 was added to the entry of the matrix that has the H row and Q column coordinates. After examining the entire sequence of notes, the matrix might look like the one pictured in Figure 1. The twenty-five entries of this matrix represent the number of times that each possible pair of durations occurred in the sequence. If we normalize every row, by dividing each entry by the sum total of all the entries of its row, then the entries become the one-step transition probabilities for the set D. That is, the mth row, nth column entry represents the probabiltity that if note N_i has duration d_m, then note N_{i+1} has duration d_n. The matrix containing these one-step transition probabilities is known as the one-step transition matrix, the terminology being taken from Markov Chain Theory [4]. We will denote this matrix as matrix R (Figure 2). The same procedure can be used to find a 37 x 37 transition matrix for the set P. We will denote this matrix as matrix S. The matrix in Figure 2 is the actual transition matrix produced by the program after examining several songs written by Paul Simon.

We need to note here some considerations about the input used for this program. The input used was the melody line of the main verse of ten songs written by Paul Simon in the key of C. Only the melody line was used because any introductions, endings, and choruses might have different patterns of notes that would affect these transition probabilities. Also, songs in different keys will defintely have

	W	H	Q	E	S
W	25	10	20	35	10
H	14	7	9	21	15
Q	18	22	7	25	23
E	19	31	12	27	21
S	4	11	21	13	27

Figure 1

$$R = \begin{matrix} .21 & .36 & .36 & .07 & .00 \\ .05 & .21 & .43 & .31 & .00 \\ .00 & .06 & .48 & .44 & .01 \\ .02 & .06 & .31 & .55 & .06 \\ .04 & .06 & .10 & .35 & .44 \end{matrix}$$

Figure 2

different transtiton probabilities and should not be mixed together.

Let us see what sequence of notes we can produce using these transition matrices. We will select the starting note for our sequence to be $N_1 = (Q,C')$. Since we are in the key of C, it is a logical choice and also one that Paul Simon used often to start his songs. We will end the sequence on the first note after the 70th note that has a pitch of C′. The average length of the tunes used as input was approximately eighty notes, therefore the use of seventy as a cutoff point. Examining the transition matrices, we see that we have the highest probability that, given $d_1 = Q$ and $p_1 = C'$, then the next note has duration $d_2 = Q$ and $p_2 = C'$. So let's make $N_2 = (Q,C')$. Again examining the transition matrices, we see that given $N_2 = (Q,C')$, then N_3 should also be (Q,C'). In fact, the rest of the notes will all be (Q,C'). This sequence of notes is not the desired result, because, of course, a monotone sequence strongly violates condition (d).

One problem is that we are producing the next note by examining only the previous note. It seems reasonable, and even probable, that a composer uses several of the previous notes to pick the next note. Let us try to include this in our procedure by using the last five notes to pick the next note. We will also alter condition (a) to read "...start with an arbitrary sequence of notes of our choice".

To be able to include this in the procedure, we need to find the two-step, three-step, four-step, and five-step transition probabilities that, if we have any note N_i with duration d_i, then it is followed two, three, four, or five notes later by a note N_j with any duration $d_j \in D$. We also need the same information about the pitch of the notes. It can be shown that if we compute the matrix $R^2 = R \cdot R$, then the resulting martix is the two-step transition matrix which contains the two-step transition probabilities for D [4,p. 268]. In general, we can find the n-step transition matrix for D or P by finding the nth multiple of their original transition matrix.

We now examine the sequence of notes we can produce using this new information. First we select five notes to start the tune with, say

$$N_1,N_2,N_3,N_4,N_5 = (Q,C'), (E,E'), (H,G'), (Q,F'), (Q,E').$$

From R^5 we have the different probabilities that, given $d_1=Q$, then $d_{1+5}=d_6$ is W,H,Q,E, or S. We have the same for d_6 from d_2 and R^4, d_3 and R^3, etc. If we add the appropriate rows together (using vector addition), we get a cumulative vector C = (row Q of R^5) + (row E of R^4) + (row H of R^3)+ (row Q of R^2) + (row Q of R). If we normalize this vector, the entries represent the probability that d_6 is W,H,Q,E, or S. We also need to go through the same procedure for the pitch of the notes. If we again take the duration and pitch with the highest probabilities to be d_6 and p_6, then we find that $N_6=(E,G')$. Going through the procedure again and using what are now the last five notes (N_2-N_6), then we find that $N_7=(E,G')$. Continuing with the procedure, we find that all of the rest of the notes are also (E,G'). As you can see, this violates condition (b), not to mention condition (d). There is still some element missing that will give a more interesting song.

To improve upon the results of our model, we need to introduce into the procedure an element of randomness. Music does, in fact, have an element of randomness to it, so we are justified in adding it to the procedure. After we have the five cumulative probabilities for the duration and the thirty-seven cumulative probabilities for the pitch, we will let the computer select two random numbers between 0 and 1. We will let one of them correspond to a duration and let the other correspond to a pitch. The ones selected will then be used as the duration and the pitch of the next note. The duration and pitch with the highest probabilities will be more likely to be picked, but they will not necessarily be the ones picked. This permits some structure to the tune, while still allowing the randomness to play a role in the selection process. Using the random factor, we can now start the sequence with the single note $N_1=(Q,C')$. We will use N_1 to pick note N_2; N_1 and N_2 to pick N_3 and so on, until we have the five notes to use to pick N_6. Subsequently, to pick notes N_7-N_k, we will use the five preceding notes of the sequence. The tune produced in this way is the 92-note sequence pictured in Figure 3.

Figure 3

This tune has parts which are pleasant, but, taken as a whole, it still does not, in my opinion, satisfy condition (d). One problem is that each of the previous five notes plays an equal part in the selection of the next note of the sequence. It seems more reasonable that the notes nearer the one being selected should play a greater role in the selection process than the earlier notes. Thus, what we need is some type of weighting system. We will introduce here a weight vector $W=(w_1,w_2,w_3,w_4,w_5)$, which contains the weights for the five notes being used to select the next note of the sequence. w_1 is the weight for the previous note, w_2 the weight for the note before that, and so on up to w_5, which is the weight for the fifth note back. When we are adding the vectors taken from the transition matrices into the total vector, we'll first multiply them by their appropriate weight. Several vectors were tried, and one which produced some

Figure 4

interesting results is the vector $W = (10,7,4,2,1)$. When we add this to the procedure, we get the tune pictured in Figure 4. This tune, I believe, does satisfy condition (d). Thus, we have been able to produce a tune using a procedure that fulfills the specified conditions:

(a) It starts on an arbitrary sequence of note(s) of our choice;
(b) It ends on an arbitrary note of our choice;
(d) It depends on patterns found in Paul Simon's compositions--namely, the probabilistic way in which certain notes follow other notes; and
(e) It is relatively pleasing to the ear.

Contrary to what I thought (and hoped) during the development of this procedure, I recently found out that methods similar to this one have already been developed and used for many years. Music generated in this fashion is known as stochastic music. References which were available to me give the impression that there has not been much success in producing pleasant tunes using this method. However, none of the references mentioned the use of a weighting system in their procedure. The use of a weighting system made a significant difference in the tunes produced by this procedure, and I believe it would help produce positive results, if included in other processes of this type.

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A CHEMICAL APPLICATION OF GRAPH THEORY*

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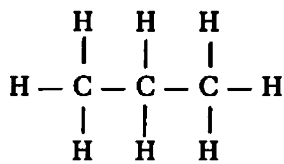
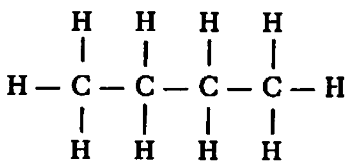
Chemists have long been beleaguered by the following problem: how to count the number of isomers of a particular chemical compound. This paper will deal with compounds composed of carbon and hydrogen with molecular formulas C_nH_{2n+2} and $C_nH_{2n-1}X$. These compounds are known as alkanes and mono-substituted alkanes respectively. The X group in the mono-substituted alkanes replaces an H_3 group forming a bond or bonds with a single carbon atom (e.g. $-OH$, $-OOH$).

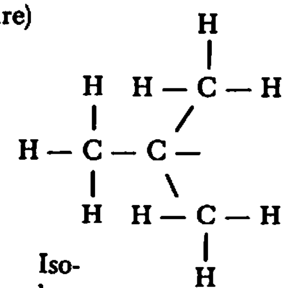
Structural formulas (the diagrams seen in elementary chemistry books) are used to illustrate the way the atoms are bonded within



C_3H_8
(only one structure)



Butane



Iso-
butane

C_4H_{10}
(two structures)

FIGURE 1

*A paper presented at the 1979 National Convention of KME and awarded second place by the Awards Committee.

the molecule, almost like a picture of the compound's physical appearance. Note that once the number of carbons is greater than three, various arrangements are possible for the carbon atoms, in terms of the way they bond, as is shown in Figure 1. If two compounds have the same molecular formula, but different structural formulas they are known as constitutional isomers. The representations of butane (C_4H_{10}) and iso-butane (also C_4H_{10}) in Figure 1 show that they are constitutional isomers.

The mathematical analog to the structural formulas of the chemical compounds considered here is a tree, which is defined, in the language of graph theory as a connected acyclic graph [1, p. 32]. Simply put, a tree is a graph in which, starting at any point s you cannot return to point s by following an alternating sequence of points and lines [1, p. 13]. It can easily be seen that every point in these "chemical trees" will be of degree four or one, that is each point has four lines or one line incident to it.

Whether a given point is of degree four or one depends on whether it represents a carbon or hydrogen atom, carbon corresponding to degree four and hydrogen to degree one, as shown in Figure 2. It is this analog that sets up the foundation for this paper, which will deal with the enumeration of isomers utilizing the mathematical model of trees.

In order to count these isomers, planted identity 4-trees may be used. 4-trees are those in which each point can be of degree one, two, three, or four. These trees correspond to removing the hydrogen atoms from the structural formula. This can be done since they are not necessary for the stated counting, and yet, the original diagram can be uniquely reconstructed. Planted trees are trees that have one of their endpoints distinguished. Using planted trees allows for the mono-substituted compounds discussed earlier since they would be planted at the point of substitution. An identity tree is one that has no geometric symmetries, as seen in Figure 3.

Much of the early work in analysis of this type was in terms of a one-variable counting series, P, where the general formula is given by

$$P(x) = \sum_{n=1}^{\infty} a_n x^n \qquad (1)$$

where a_n is the number of isomers corresponding to n carbons. The coefficients of this power series in x, then, are exactly the answers we're looking for. Unfortunately, this type of analysis was fairly limited in its scope, and while it was of much value in formulating ideas about the structure of chemical compounds, much remained to be done.

In a recent paper by L. Quintas and J. Yarmish [2] counting series in three variables were obtained. Here the general form for the power series was the following:

$$P(x,y,z) = \sum_{\substack{i,j=0 \\ n=1}}^{\infty} P_{nij}\, x^n y^i z^j \qquad (2)$$

The two new variables introduced into the equation, y and z correspond to types of carbons: y corresponds to the number of carbons bonded to three other carbons (called tertiary carbons) and z corresponds to the number of carbons bonded to four other carbons (quaternary carbons) within the molecule. $P(x,y,z)$ then, is a counting series where the coefficient P_{nij} of the term $x^n y^i z^j$ gives you the number of planted identity 4-trees of that type, i.e. with n carbons, i of which are tertiary, and j of which are quaternary.

$P(x,y,z)$ itself is derived using the recursive formula:

$$P(x,y,z,) = x^2 + xP(x,y,z) + yZ(A_2\text{-}S_2;P(x,y,z)) + \tfrac{z}{x}Z(A_3\text{-}S_3;P(x,y,z)) \qquad (3)$$

where the third and fourth terms indicate an application of an enumeration theorem by Polya. For a discussion of Polya's theorem see [1, p. 189]. For a derivation of (3) see [2, p. 180]. The formula for the non-mono-substituted alkanes is derived explicitly in terms of $P(x,y,z)$. Even though the above formula seems complicated, it does reduce into a calculable form which involves much computation.

Even though the equations are straightforward in theory, in practice the calculation presents some difficulty since the coefficients expand rather rapidly. As a result, a third discipline had to be brought in, that of computer science. By programming the counting series (3) on a computer, many hours of hand computation could be saved. Unfortunately, this avenue also had its difficulties. First, very little work has been done in three-variable power series. Even the now-famous Cauchy product for power series is only seen in its one-dimensional form. Second, even the computer takes time with power series having many terms of high degree. As a result, efficiency in the program design was of paramount importance, both in terms of the time it took for the calculations as well as the amount of data to be stored.

To solve the first problem, since many multiplications were required, a three-variable Cauchy product formula had to be devised. Given $\Sigma a_{ijk} x^i y^j z^k$ as one power series in x, y, and z and $\Sigma b_{ijk} x^i y^j z^k$ as the other, the solution for c_{ijk} in the following equation:

$$(\Sigma a_{ijk} x^i y^j z^k)\,(\Sigma b_{ijk} x^i y^j z^k) = \Sigma\, c_{ijk} x^i y^j z^k \qquad (4)$$

can be shown to be:

$$c_{ijk} = \sum_{p=0}^{k} \left[\sum_{m=0}^{j} \left(\sum_{n=0}^{i} (a_{n,m,p})\,(b_{i-n,j-m,k-p}) \right)\right] \qquad (5)$$

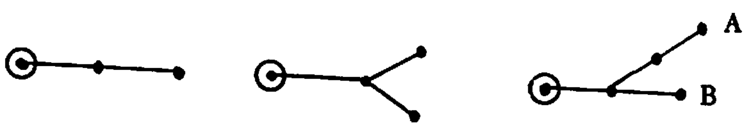
Other algorithms for substitution of powers of x, y, and z in $P(x,y,z)$ also had to be developed.

With respect to data storage, a three-dimensional matrix, $N(i,j,k)$ was used. A given entry in the matrix, $n(i,j,k)$, corresponds to the coefficient $P(n,i,j)$ in (2). Since the coefficients expand so rapidly, and the calculations take much time, even in a streamlined form, certain limitations are imposed on the program. First, there are theoretical limits on the number of tertiary and quaternary carbons for a given number of carbons, so that irrelevant data was not calculated. Second, since (3) is recursive, you only need calculate up to the specific n,i,j you are looking for. Terms appearing after that term in the power series will not enter into the calculations for that term. This also cuts down on the number of irrelevant calculations.

Even considering all these difficulties, though, the results are detailed enough to give valence isomers, stereoisomers, and finally, a glimpse of the actual structure of the molecules. The first step was to use only the number of carbons as a parameter in the counting series. Now more restrictions, such as tertiary and quaternary variables, allow for even greater insight into molecular structure. These enumerations are fundamental in the analysis of unknown chemical compounds, and this is but one example of such an enumeration.

C_4H_{10} (Butane) C_4H_{10} (Iso-butane)

FIGURE 2



Planted tree

Non-identity planted tree since A and B are identical

Planted Identity Tree

FIGURE 3

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PRODUCING MOTION PICTURES OF COMPUTER SIMULATED FLUID FLOW*

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The increasing use of nuclear reactors has sparked an intense interest in their operation, especially with regard to safety. Physical experiments involving reactor analysis may be quite expensive, potentially dangerous, and difficult or impossible to perform. As a result, computer modeling codes to simulate various situation are being developed. This modeling codes are constantly becoming more sophisticated; the complexity of the problems studied is growing and the amount of detail in the results is increasing. A side effect of these advances in computer modeling is a tremendous increase in the amount of output produced. To make such large amounts of information comprehensible, a suitable form of presentation must be used. One convenient way to present large amounts of data compactly is through pictures; when the data is time-dependent, movies can be used to reflect this added dimension.

GEN/PLT-2P3D is a pair of FORTRAN programs which uses data generated by COMMIX† to produce movies which illustrate two-phase fluid flow in three dimensions. While the primary product of GEN/PLT is a 16mm movie, single frames may be plotted on 16mm or 35mm film, or at an online graphics terminal. The function performed by GEN/PLT has been divided into two distinct processes: GEN is responsible for the computations involving the COMMIX data, while PLT is dedicated to plotting the data generated by GEN. A macro flowchart indicating the files required by and produced by GEN and PLT is shown in Figure 1.

* A paper presented at the 1979 National Convention of *KME* and awarded third place by the Awards Committee.

† COMMIX is a two-phase, three-dimensional modeling code under development in the Analytical Modeling Section of the Components Technology Division at Argonne National Laboratory.

The work discribed in this paper was performed by the author between 9/5/78 and 12/20/78 as a part of the Fall 1978 Undergraduate Research Participation Program at Argonne National Laboratory, a contract laboratory of the United States Department of Energy; the program is coordinated by the Argonne Center for Educational Affairs. The work was done under the supervision of Mr. Robert C. Schmitt in the Analytical Modeling Section of the Components Technology Division.

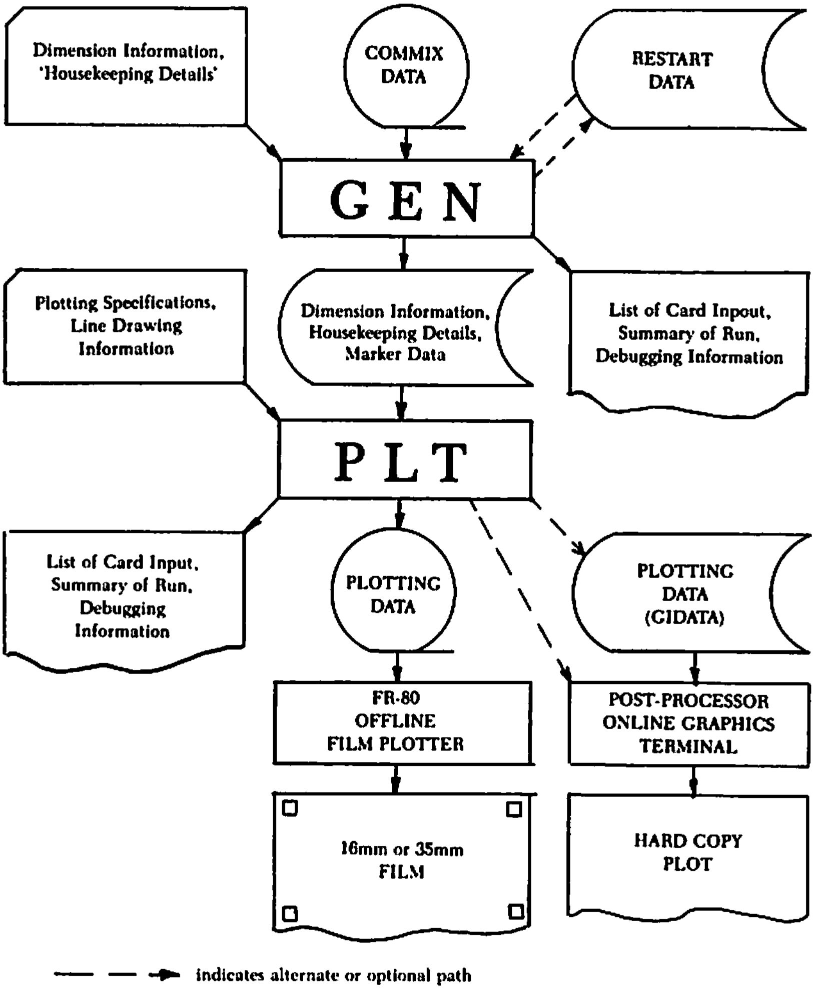

FIGURE 1. Macro-Flowchart of GEN/PLT

The geometry used by GEN/PLT is a three dimensional mesh system defined in cartesian coordinates. A rectangular solid is divided into rectangular cells; the geometry of a particular problem (including the number of cells, cell sizes, and boundary cells) is specified by the user at run time. Symbols or markers are used to represent matter; these markers are "pushed" through the mesh by velocity fields to simulate fluid flow. Different types of markers are used to represent different types of matter (gas or liquid).

The Staggered Mesh Geometry

The geometry used in GEN/PLT is called a *staggered mesh* because the physical characteristics associated with each cell are defined at different points within the cell. For example, the temperature of the fluid in a cell is evaluated at the center of the cell, while the components of the fluid velocity are evaluated at the centers of the cell surfaces.

One of the most important capabilities of GEN/PLT is its ability to maintain a distinction between the liquid and gaseous phases of the fluid. This is accomplished through the use of *void fractions*. Each cell has associated with it a void fraction which indicates what fraction of the matter in that cell is liquid and what fraction is gas.

The reasons for using a staggered mesh are related to the characteristics of the COMMIX modeling code and the physical calculations involved, and need not concern us here. Different geometries can be, and are, used in other modeling codes.

The MS Structure

In order to define to the computer the geomtry of the particular problem under study several arrays are used; one of the most important of these is the MS array. MS is a three-dimensional transformation array which provides indirect addressing to the arrays containing information about the cells in the mesh. For example, the void fraction for cell (I,J,K) can be found in VOID(MS(I,J,K)).

The MS structure was implemented primarily as a space saving measure. Depending on the geometry of the problem being studied, some of the cells in the mesh will never be used. These cells are *undefined*, and space is not reserved for them in the arrays addressed through MS. For a problem with a simple rectangular solid geometry, the cells touching edges of the solid are undefined. When the problem under consideration uses a cylindrical or spherical geometry the amount of space saved by using MS and undefined cells can be quite significant.

Boundary Cells

Another array used to define the geometry of the problem under consideration is the IFIELD array; it is used to determine the characteristics of defined cells in the mesh. Interior cells are designated by positive values of IFIELD, while boundary cells are designated by negative values.

There are two kinds of boundary cells: *flow-boundary* cells and *wall-boundary* cells. Wall-boundary cells represent physical walls or structures. When fluid hits a wall, it is deflected; so that the model will accurately reflect this situation, any markers that are pushed into wall cells must be moved back into nearby interior cells. Flow-boundary cells on the hand, represent inlets and outlets. When fluid enters a flow-boundary cell, it physically leaves the system. So that the model will accurately reflect this situation, markers that enter these cells are destroyed and are no longer considered a part of the system.

The Calculations

In order to produce movies which adaquately illustrate fluid flow, data which accurately describes the flow must be provided; GEN/PLT has been designed to use data generated by the COMMIX modeling code. The first part of the COMMIX data consists of dimension specifications and values for the MS and IFIELD arrays, to specify the geometry of the system analyzed. The bulk of the information provided by COMMIX is in the form of *timestep* data (data which applies to a specified time period). For each timestep COMMIX supplies 1) the timestep number, 2) the time, 3) values of the void fraction for each cell, and 4) temperatures and values of the x, y, and z velocity components for each cell, for both gas and liquid phases of the fluid. The void fraction, temperature, and velocity component arrays are all addressed through MS.

It is not unusual to have hundreds or even thousands of markers in the mesh for a particular problem; keeping track of all these markers as they move through the mesh requires some rather extensive record-keeping. Two elements of GEN/PLT are used to perform this function: the linked list and the marker counting arrays.

The linked list is used to keep track of the type, temperature, and exact position of each marker in the mesh; this is accomplished with six arrays and two pointers. LINK, the linked list array is divided into two parts: the valid or active list, and the available list. LACTIV and LAVAIL point to the first entries in the active and available lists, respectively. Each active entry points to the next active entry, while each available entry points to the next available entry. For every element of the linked list there are corresponding entries in the type, temperature, and x-, y-, and z-coordinate arrays; these corresponding entries provide complete information about the individual markers in the mesh.

To keep track of the number of markers of each type in each cell, GEN/PLT uses four marker counting arrays: MAXMRK, NVOID, NMRK, and NTYPE; all of these arrays are addressed through MS. The elements of MAXMRK indicate the total number of markers that each cell must contain; they are determined by the cell sizes and computed only once. The values in NVOID are calculated once for each timestep using the values of the corresponding elements in MAXMRK and VOID; the elements of NVOID determine the number of markers of each type that each cell must contain. The elements of NMRK and NTYPE indicate the total number of markers in each cell and the number of markers of each type in each cell, respectively; the values in these two arrays are updated whenever necessary to reflect the current situation.

For each frame of the movie, marker accounting in GEN/PLT is done in a three-step process. First, the markers are "pushed" by the velocity fields, as discussed below. Markers which are moved into flow-boundry cells are destroyed, and those which are pushed into wall-boundary cells are moved back inside. Next, excess markers in each cell are destroyed, or their phases are changed, so that the values in NMRK and NTYPE do not exceed the corresponding values in MAXMRK and NVOID. Finally, new markers are created where necessary so that the values in NMRK and NTYPE match those in MAXMRK and NVOID, respectively.

As noted above, the velocities provided by COMMIX are defined at the centers of the cell surfaces. Each of the three velocity components for each marker must be determined for each frame of the movie. Either an 8-point or a 2-point linear interpolation scheme can be used; both methods are illustrated for the x component in Figure 2. (The y and z components can be determined similarly.) The 2-point scheme uses only the x coordinate of the marker M and the velocities at C and G to determine the x component of the velocity of M. The 8-point scheme uses all three coordinates of the marker and all 8 of the velocities shown to determine the x component of the velocity of M. After all three velocity components for a marker have been computed, each component is multiplied by dt and the product is added to the appropriate coordinate of the marker.

The computation of marker velocities is one of the most expensive of GEN/PLT in terms of computer time. While using the 8-point interpolation will significantly increase the amount of computing time, it will also improve the smoothness of the flow shown in the resulting movie. For some problems, especially those with unusual geometries, the 8-point scheme may be unsuitable, and the 2-point interpolation must be used.

Plotting the Results

At first glance, converting arrays of numbers in a computer into movies of three-dimensional objects in motion seems like a formidable task. Fortunately, the powerful tools of elementary linear algebra provide a simple and elegant solution to the problem. Through the use of transformation matricies, the coordinates of the points to be plotted can be translated, scaled, and rotated in three dimensions, and then projected onto a plane for plotting.

A three-dimensional transformation matrix can be written as

$$T = \left[\begin{array}{c|c} & 0 \\ R & 0 \\ & 0 \\ \hline S & 1 \end{array}\right]$$

where R is a 3×3 matrix and S is a 1×3 matrix. The elements in the fourth column of T could have values other than those shown, in which case a perspective and/or global scaling transformation would result. Perspective transformations are not required for the problem at hand, and global scaling is achieved in another way. For our purposes, the values in the fourth column will always be those shown above, and will not be discussed further.

The elements of the 1x3 matrix S produce translation in three dimensions. For example, a translation of 2 units in the x direction and -1 and unit in the y direction with no translation in the z direction can be accomplished by letting $S = [2\ {-1}\ 0]$ (and $R = I_3$). The elements on the main diagonal of R produce local scaling in each direction; global scaling can be obtained through equal local scaling in all three directions. For example, if

$$R=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$$

the y coordinates will be doubled and the z coordinates will be tripled, while if

$$R=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{2}\end{bmatrix}$$

the size of the entire figure will be reduced by half. Projections onto the $x=0$, $y=0$, and $z=0$ planes can be achieved by filling the appropriate column of T with zeroes; other kinds of projections are certainly possible, but need not concern us here.

All nine elements of R are involved in producing three-dimensional rotations. While rotations about an arbitrary axis are possible, rotations about the three coordinate axes are all that is required to solve the problem at hand. Rotations about the x axis are obtained by matricies of the form $T=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & \sin\theta & 0\\ 0 & -\sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$,

matricies of the form $T=\begin{bmatrix}\cos\theta & 0 & -\sin\theta & 0\\ 0 & 1 & 0 & 0\\ \sin\theta & 0 & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$ and $T=\begin{bmatrix}\cos\theta & \sin\theta & 0 & 0\\ -\sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$

will result in rotations about the y and z axes, respectively.

The movies produced by PLT show an outline of the physical object involved and the markers flowing through it. So that all sides of the object can be seen clearly, the entire figure rotates as the markers flow through it. The period, phase angle, axis of rotation, and orientation of the axis of rotation can all be specified by the user. The angle of elevation of the viewpoint above a plane normal to the the axis of rotation can also be determined by the user.

In order to properly orient the figure for plotting, up to nine elementary transformation must be used. First, the figure is translated so that it will be centered at the origin. Next, it is scaled so that it will fit on the page. Third, the figure is rotated so that the axis of rotation is the y-axis (the vertical axis); this may require two separate rotations, or it may be unnecessary, depending on the particular problem. Fourth, the figure is rotated about the y-axis through some angle θ which is determined by the time, the phase angle, and the period of rotation. Fifth, another matrix rotates the figure about the x-axis through an angle of elevation of the viewpoint. Next, if the user requests it, the figure is rotated so that the axis of ratation is horizontal instead of vertical. Finally the figure is translated away from the origin so that it will be centered on the page. The resulting figure is then projected onto the $z=0$ plane for plotting.

As mentioned above, there may be hundreds or thousands of markers to be plotted. The coordinates of each marker must be transformed by as many as nine transformation matricies for each frame of the movie; this is obviously a rather expensive approach to the problem in terms of computer time. Fortunately, however, matrix multiplication is associative. This means that the nine transformation matricies can be multiplied to form one total transformation matrix, and this total transformation matrix can be applied to the coordinates of the markers.

There are, of course, only three coordinates for each marker; the total transformation matrix is, however, a 4×4 matrix. So that the transformation matrix can operate on the marker coordinates, each marker is given a fourth coordinate. For more general three-dimensional transformation, this fourth coordinate plays an important role; for our purposes, it serves only to make the matrix multiplication possible and will always have a value of 1, both before and after transformation.

Once the coordinates of the markers and the points in the outline of the figure have been transformed by T, plotting them is a very straightforward task. The different types of markers representing the liquid and gaseous phases of the fluid can be plotted in different shapes and/or colors so that they will be easily distinguishable. The temperature of the fluid at different points in the mesh can also be used to determine the color of the markers.

Hopefully it can be seen how some of the tools provided by mathematics can be combined with a computer to provide solutions to contemporary problems in physics and engineering.

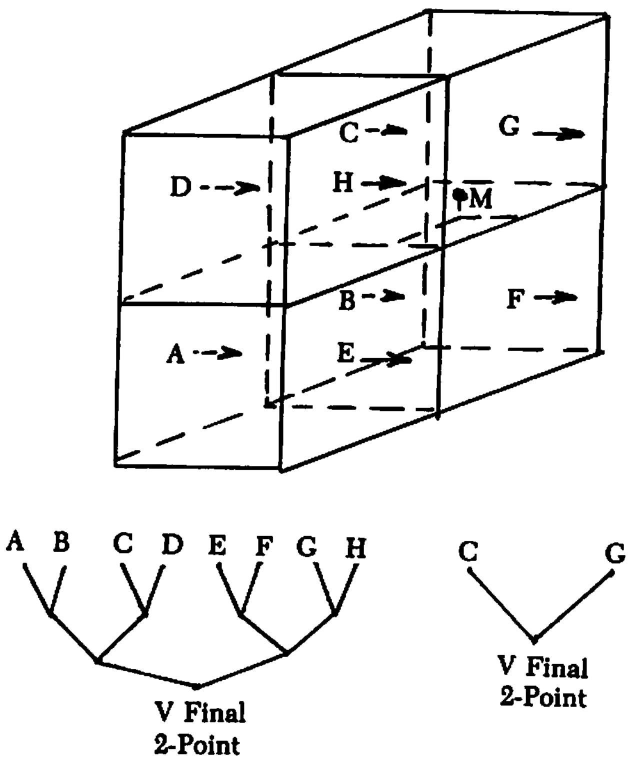

FIGURE 2. Velocity Interpolation

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THREE BASIC GEOMETRICAL TRANSFORMATIONS SOME APPLICABLE LINEAR ALGEBRA*

JOHN GRAHAM
Student, Pittsburg State University

This paper will present three basic geometrical transformations of Euclidean three-space which are useful in such diverse areas as an alternative to Gaussian elimination and the description of the motion of a rotating, rigid body. †

To be specific, the paper will consider: projections onto lines and hyperplanes in three-space, reflections in a hyperplane, and rotations about a line in three-space. While many undergraduate texts in linear algebra, geometry, and vector analysis discuss rotation of three-space, it is surprisingly difficult to find a discussion of rotations about a line. A. S. Householder was the first mathematician to use reflection in hyperplanes to develop a numerically stable alternative to Gaussian elimination [1].

Projections onto lines and hyperplanes in three-space

A *line through the origin* will be denoted in non-zero parametric form, $x = ta$, where a is a vector lying in the line and t is a real number,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

The hyperplane H through the origin with normal C is the set H = $\{x \varepsilon R^3 | c^T x = 0\} = \{(x_1, x_2, x_3) | c_1 x_1 + c_2 x_2 + c_3 x_3 = 0\}$.

* This paper was presented at the 1979 National Convention of *KME* and awarded fourth place by the Awards Committee.

† The author first became interested in the topics presented in this paper while taking a course in Vector Analysis from Dr. McGrath, (Pittsburg State University) to whom I am very grateful for arousing my interest and helping me meet my goals in these subjects.

First we consider projection onto a line through the origin. The *projection* of b *onto* a is defined as follows

$$b = \text{proj}_a b + c$$

where (1) the vector $\text{proj}_a b = ta$ (t real) is parallel to a and

(2) the vector c is orthogonal to a (the dot product $a^T c = 0$).

An explicit expression for $\text{proj}_a b = ta$ can easily be derived. Since $b = \text{proj}_a b + c$ and $b = ta + c$, we have $a^T b = ta^T a + a^T c$. Since $a^T c = 0$, $t = a^T b / a^T a$. This gives us the expression $\text{proj}_a b = (a^T b / a^T a) a$.

It is known from linear algebra that an orthogonal projection of three-space onto itself is a linear transformation whose equations relative to a fixed basis (coordinate system) are given by the matrix P with the defining properties: $P^2 = P$ and $P^T = P$ [2,3]. The matrix P for a projection of three-space onto a line can be easily obtained from our expression for $\text{proj}_a b$.

In fact, $\text{proj}_a b = ta = at = a(a^T b / a^T a) = (aa^T / a^T a) b = Pb$. Thus the matrix equations for $\text{proj}_a b$ are given by: $\text{proj}_a b = Pb$ where $P = (aa^T / a^T a)$. For example, consider the line through the origin containing the non-zero vector $a = (1,2,-3)$. We have

$$P = \frac{\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} [1\ 2\ -3]}{[1\ 2\ -3] \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}} = 1/14 \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -3 & -6 & 9 \end{bmatrix}$$

To find $\text{proj}_a b$, one simply multiplies P by b.

Second we consider projections onto a hyperplane in three-space. Let $H = \{x | a^T x = 0\}$. We define $\text{proj}_H b$ by: $b = \text{proj}_H b + \text{proj}_a b$.

Thus $\text{proj}_H b = b - \text{proj}_a b = Ib - Pb = (I - P)b = (I - aa^T / a^T a) b$.

A straightforward calculation shows that if $P^2 = P$, then $(I - P)^2 = I - P$ and $(I - P)^T = I - P$. Thus the projection of three-space onto a hyperplane as defined above is an orthogonal projection.

As an example, let $H = \{(x_1,x_2,x_3)|x_1 + 2x_2 - 3x_3 = 0\}$. The P matrix can be obtained as in the first example.

$$I - P = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} - 1/14\begin{bmatrix}1 & 2 & -3\\2 & 4 & -6\\-3 & -6 & 9\end{bmatrix} = 1/14\begin{bmatrix}13 & -2 & 3\\-2 & 10 & 6\\3 & 6 & 5\end{bmatrix}$$

To find $\mathrm{proj}_H b$, one simply multiplies $I - P$ by b.

Reflections in a hyperplane (Householder Transformations)

Let $H = \{x|a^Tx = 0\}$ be a hyperplane through the origin containing the vector a. The reflection of three-space in H is defined by:

$$Rb = b - 2\,\mathrm{proj}_a b$$

An expression for R can be obtained as follows:

$$Rb = Ib - 2Pb = (I - 2P)b$$

Thus R is expressed as:

$$R = I - 2\,aa^T/a^Ta$$

Such reflections are often called Householder Transformations. If $a = (a_1,a_2,a_3)$, then a straightforward calculation gives an explicit expression for R.

$$R = \frac{1}{a_1^2 + a_2^2 + a_3^2}\begin{bmatrix}-a_1^2 + a_2^2 + a_3^2 & -2a_1a_2 & -2a_1a_3\\ -2a_2a_1 & a_1^2 - a_2^2 + a_3^2 & -2a_2a_3\\ -2a_3a_1 & -2a_3a_2 & a_1^2 + a_2^2 - a_3^2\end{bmatrix}$$

Let $H = \{(x_1,x_2,x_3|x_1 + 2x_2 - 3x_3 = 0\}$ as in the second example. The matrix R which reflects three-space in the hyperplane is given by

$$R = 1/14\begin{bmatrix}12 & -4 & -6\\-4 & 6 & -12\\-6 & -12 & -4\end{bmatrix}$$

We will now show how Householder was able to use such reflections to develop a numerically stable alternative to Gaussian elimination. The basic idea in Gaussian elimination is to replace an initially given set of linear equations $Ax = b$ by a set of linear equations $Ux = c$ where (1) U is an upper triangular matrix and (2) the set of solutions to $Ux = c$ coincides with the set of solutions to $Ax = b$. Since U is an upper triangular matrix, the linear equations $Ux = c$ can be solved by backward substitution.

Given the set

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ 3x_1 + 4x_2 &= 3 \\ 2x_1 + 10x_2 + 4x_3 &= 10 \end{aligned}$$

We have

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 0 \\ 2 & 10 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 10 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & -3 \\ 0 & 0 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ -14 \end{bmatrix}$$

Backward substitution yields $x_3 = 2$, $x_2 = 0$, $x_1 = 1$.

In order to replace the equations $Ax = b$ by an equivalent set of equations $Ux = c$, Guass used his well-known method of elimination to produce zeros below the diagonal.

Householder noted that by an appropriate reflection in a hyperplane any vector could be reflected onto a vector of the same length which lies on one of the coordinate axes and thus has all coordinates, save one, equal to zero.

For example if $b = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, there is a reflection R in a hyperplane in three-space such that $Rb = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$. R is constructed as follows. Given b, compute the length L of b and consider a vector c along the first coordinate axis whose length is L. Now reflect b in the hyperplane H, with normal $c - b$, so as to obtain $Rb = c$.

The above discussion provides the background for the following theorem.

Theorem: *Let* $R = I - 2\,aa^T/a^Ta$ *be a Householder transformation defined in Euclidean* n*-space. Then*

1. $R^2 = I$
2. $R^T = R$
3. $R^{-1} = R^T$
4. $Ra = -a$
5. *If* $a^Tw = 0$, *then* $Rw = w$.
6. *For* $u^Tu = v^Tv \neq 0$, $u \neq v$, *and* $a = v - u$, $Ru = v$.

Proof

(1) Recall that $P = aa^T/a^Ta$ is an orthogonal projection onto the line through the origin containing the vector a. Thus $P^2 = P$ and $P^T = P$. Now $R = I - 2P$, so $R^2 = (I - 2P)^2 = I - 4P + 4P^2 = I - 4P + 4P = I$.

(2) $R^T = R$, since $R = I - 2P$ and $P^T = P$.

(3) By assertion (1) $R^2 = I$. Therefore $R^{-1} = R$. But $R = R^T$ by assertion (2). Thus $R^{-1} = R = R^T$.

(4) $Ra = (I - 2\,aa^T/a^Ta)a = a - 2\,a(a^Ta/a^Ta) = a - 2a = -a$

(5) Let $a^Tw = 0$. $Rw = (I - 2(aa^T/a^Ta))w =$
$w - 2(aa^Tw/a^Ta) = w - 0 = w$.

(6) $Ru = (I - 2(aa^T/a^Ta))u$, where $a = v - u$ and $v^Tv = u^Tu$. Thus $Ru = u - 2\,(v - u)\,(v - u)^Tu/(v - u)^T(v - u)$

If it could be shown that $(v - u)^T\,(v - u) = -2(v-u)^Tu$, then $Ru = u + (v - u) = v$. But $(v - u)^T\,(v - u) = v^Tv - 2v^Tu + u^Tu)$ $= (u^Tu - 2v^Tu + u^Tu) = 2u^Tu - 2v^Tu = 2(u - v)^Tu = -2(v - u)^Tu$. Thus $Ru = v$ as was to be shown.

The reflection of three-space in a hyperplane that will reflect u onto v can be computed. By using such reflections, it is possible to develop an alternative to Gaussian elimination. Suppose we have

$u = (2,1,-2)$, $v = (3,0,0)$, and $a = v—u\,(1,-1,2)$. Then

$R = I - 2\,aa^T/a^Ta =$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 2/6 \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{bmatrix} = 1/3 \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & 2 \\ -2 & 2 & -1 \end{bmatrix}$$

As a check, we compute

$$Ru = 1/3 \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & 2 \\ -2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = v$$

Rotations of three-space about a line

In this section, the Euclidean length of $a = (a_1, a_2, a_3)$ is defined in the usual fashion:

$$\|a\| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}.$$

First consider the cross product as a linear transformation by writing $a \times b$ as Tb where T is a 3 x 3 matrix. $a \times b = (- a_3 b_2 + a_2 b_3, a_3 b_1 - a_1 b_3, - a_2 b_1 + a_1 b_2)$, thus

$$a \times b = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = Tb$$

Second, we wish to write the parametric equation of a circle in three-space given that the circle has its center at the origin, lies in the hyperplane $H = \{x | a^T x = 0\}$, and passes through the vector b.

By using the cross product, a positively oriented orthogonal basis (e_1, e_2, e_3) can be found such that (1) $e_1 = b$, (2) $\|e_2\| = \|e_1\|$, and (3) $e_3 = a / \|a\|$. To do this let $e_2 = e_3 \times e_1 = a/\|a\| \times b = Sb$, where

$$S = 1/\sqrt{a_1^2 + a_2^2 + a_3^2} \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & a_1 \\ -a_2 & -a_1 & 0 \end{bmatrix}$$

The parametric equations of the circle can now be written as $x = (\cos t)\, e_1 + (\sin t) e_2$.

We are now in a position to rotate three-space about the line through the origin containing the vector a. We have

$R_t b = Pb + (\cos t) e_1 + (\sin t) e_2$

$R_t b = Pb + (\cos t)(I - P) b + (\sin t) S (I - P) b$

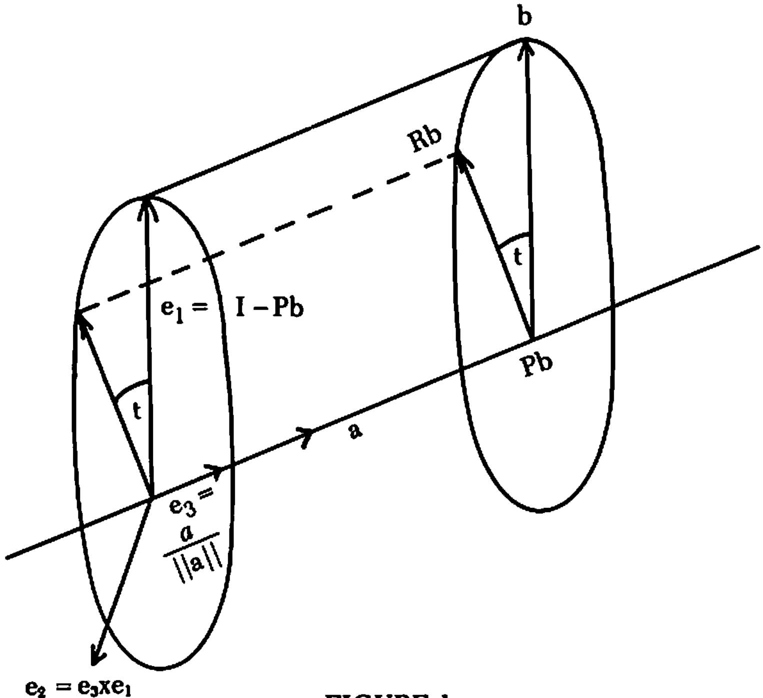

FIGURE 1

Consider Figure 1. By a straightforward calculation, one can verify that $SP = 0$. This is intuitively clear since SPb computes $a/\|a\|$ x $\text{proj}_a b$. But the cross product of parallel vectors is always the zero vector. Simplifying we have: $R_t b = Pb + (\cos t)(I - P)b + (\sin t)Sb = (P + (\cos t)(I - P) + (\sin t)s)b$. Thus $R_t = P + (\cos t)(I - P) + (\sin t)S$, where $P = aa^T/a^Ta$ and $S =$

$$1/\|a\| \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

This formula can easily be programmed. It can also be shown that $X = R_t b$ is a solution of the matrix differential equation

$$d/dt\, X = S\, X = a \times X$$

with initial condition $X(0) = b$. Written in terms of coordinates the matrix equation becomes:

$$\begin{aligned}
\dot{x}_1 &= 0 - a_3x_2 + a_2x_3, & x_1(0) &= b_1\\
\dot{x}_2 &= a_3x_1 + 0 - a_1x_2, & x_2(0) &= b_2\\
\dot{x}_3 &= -a_2x_1 + a_1x_2 + 0, & x_3(0) &= b_3
\end{aligned}$$

As a last example, rotate three-space about the line containing $a = (1,2,-3)$ through the angle t.

$$R_t = 1/14 \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -3 & -6 & 9 \end{bmatrix} + (\cos t)/14 \begin{bmatrix} 13 & -2 & 3 \\ -2 & 10 & 6 \\ 3 & 6 & 5 \end{bmatrix}$$

$$+ (\sin t)/14 \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

To rotate b about the line through the origin containing a, one simply multiplies R_t by b.

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4. Young, D. M. and Gregory, R. T., A Survey of Numerical Mathematics, Vol. II, Addison Wesley (1972).

THE PROBLEM CORNER

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1980. The solutions will be published in the Fall 1980 issue of The Pentagon, with credit being given for other solutions received. Preference will be given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 317. **Proposed by John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico.**
In the figure, arc ABC has a measure of $\frac{5\pi}{6}$ radians (or 150°) and its center at D. If $AB = 3$ and $BC = 2$, what is the value of cot $(\frac{1}{2}\angle BDC)$?

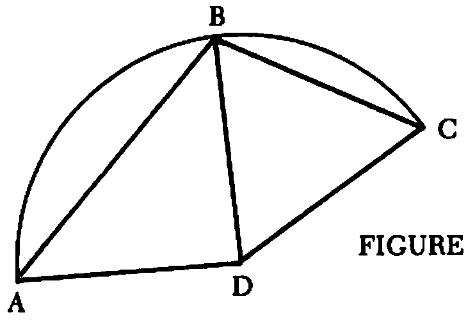


FIGURE 1. Problem 317

Problem 318. **Proposed by Charles W. Trigg, San Diego, California.**
The series of positive integers is divided into the groups 1; (2,3,4,5); (6,7,8,9,10,11,12); (13,14,15,16,17,18,19,20,21, 22); . . . . Find the sum of the integers in the nth group.

Problem 319. **Proposed by Charles W. Trigg, San Diego, California.**
Use the basic nine-digit third order magic square to generate eight other third order magic squares that have a common magic constant. Each new square is to have nine distinct elements, and at least three elements are to be prime in five of the new squares.

Problem 320. **Proposed by Michael W. Ecker, Pennsylvania State University/Worthington Scranton Campus, Scranton, Pennsylvania.**

Define a permuted repunit pair (PRP) to be a pair of positive integers x,y with $x > y$ such that

(a) the decimal digits of x and y are permutations of one another; and

(b) $x + y =$ a repunit (i.e., a decimal integer consisting solely of ones such as 111111.)

If n is the number of ones in a given repunit, for which values of n do corresponding PRP's exist?

For a given integer n for which PRP's exist, find the PRP (x,y) such that the product xy is a maximum.

Problem 321. **Proposed by Fred A. Miller, Elkins, West Virginia.**

In a circle whose center is at *O*, radii *OA* and *OB* are drawn. Construct a chord which will be trisected by radii *OA* and *OB*.

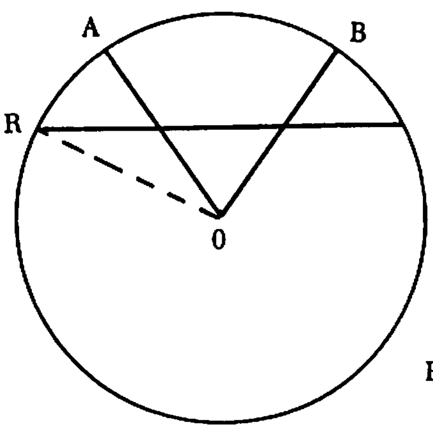


FIGURE 2. Problem 321

SOLUTIONS

307. **Proposed by Fred A. Miller, Elkins, West Virginia.**

Let *A*, *B*, *C* denote the vertices of a triangle which lie on sides *DE*, *DF* and *EF* respectively of triangle *DEF*. Let *A'B'C'* be a second triangle whose vertices lie on the sides of triangle *DEF* in such a way that *A* and *A'* are equidistant from the midpoint of *DE*, and B and B' are equidistant from the midpoint of *DF*, and C and C' are equidistant from the midpoint of EF as shown in the figure below. Prove that triangles ABC and $A'B'C'$ have equal areas.

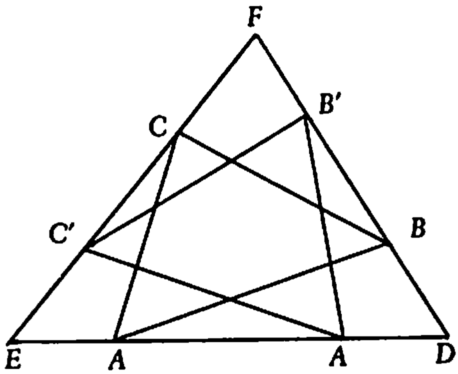


FIGURE 3.
Problem 307

Solution by Charles W. Trigg, San Diego, California.

Let the sides opposite D, E, and F, be $2d$, $2e$, and $2f$, respectively. Also, let $AA' = 2x$, $BB' = 2y$, $CC' = 2z$, and the area of triangle DEF be Δ .

If two triangles have an angle of one equal to an angle of the other, their areas are to each other as the products of the sides including the equal angles. So the area of triangle DAB is $(f + x)(e - y)\Delta/4ef$. It follows that the combined area of the triangles adjacent to triangle ABC equals

$(\Delta/4)[(f+x)(e-y)/ef + (d+z)(f-x)/df + (e+y)(d-z)/ed]$.

Similarly, the combined area of the three triangles adjacent to triangle $A'B'C'$ equals

$(\Delta/4)[(f+x)(d-z)/fd + (d+z)(e-y)/de + (e+y)(f-x)/ef]$.

Each of these expressions simplifies to

$$\Delta(3def - dyx - exz - fzy)/4def.$$

Since the areas of the two adjacent trios are equal, it follows that triangle ABC and $A'B'C'$ have the same area.

Also solved by D. Moody Bailey, Princeton, West Virginia, and the proposer.

Editor's Comment. Trigg also points out that this problem is "a hardy perennial" and that some of the more recent appearances of solutions are:

1. Roger A. Johnson, Modern Geometry, Houghton Mifflin Co., Cambridge, Massachusetts, 1929, p. 80.
2. C.W. Trigg, Solution of Problem 1642, School Science and Mathematics, 40 (April 1940), pp. 387-388. (Here it is also shown that the triangle pairs exemplified by CBA' and $C'B'A$ have equal areas, and that the vertices of the triangles may lie on the sides of DEF or the sides produced).
3. (Proposed by Fred A. Miller.) Mangho Ahuja, Solution of Problem 3707, School Science and Mathematics, 78 (December 1978), pp. 715-716.
4. Clayton W. Dodge, Solution of Problem 320, Crux Mathematicorum, 4 (October 1978), pp. 238-239.

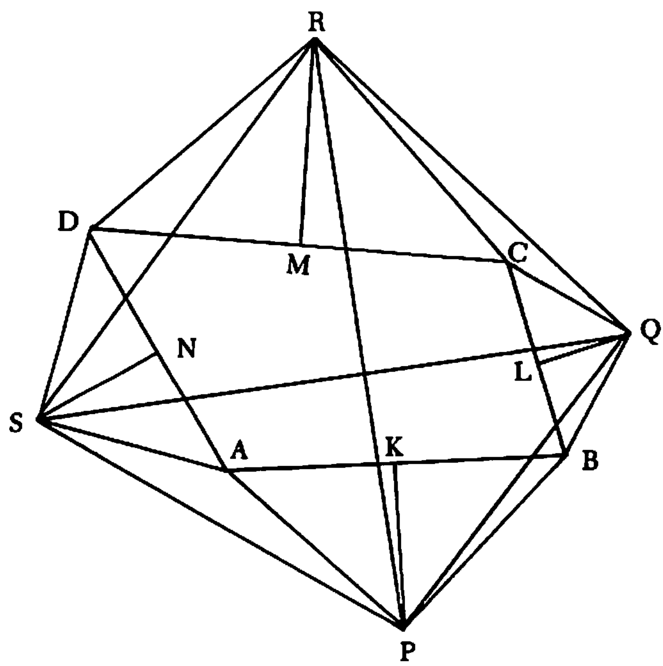


FIGURE 4
Problem 308

308. **Proposed by John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico.**

On the sides of quadrilateral *ABCD*, isosceles right triangles *ABP*, *BCQ*, *CDR*, and *DAS* are constructed. Show that *PR* = *QS* and *PR* is perpendicular to *QS*.

Solution by the proposer.

Let *ABC* be a quadrilateral with isosceles right triangles *ABP*, *BCQ*, *CDR* and *DAS* constructed on sides *AB*, *BC*, *CD* and *DA* respectively. Draw *RS*, *SP*, *PQ*, *QR*, *QS* and *PR* as shown in the figure below.

We use the concept of directed line segments in the complex plane. Thus if p denotes a directed line segment pi denotes the same line segment rotated counterclockwise through an angle of 90°; hence p and pi are perpendicular. Let a, b, c and d represent the directed line segments *AB*, *BC*, *CD* and *DA* respectively. Let *K*, *L*, *M*, *N* denote the midpoints of *AB*, *BC*, *CD* and *DA* respectively. Note that the directed sum of segments forming a closed circuit is zero; i.e., $a+b+c+d=0$.

Note also that $\overrightarrow{AB} = -\overrightarrow{BA}$

Now $\overrightarrow{SP} = \overrightarrow{AP} + \overrightarrow{SA} = \frac{a}{2} - \frac{ai}{2} + \frac{d}{2} \quad \frac{di}{2}$

$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ} = \frac{a}{2} - \frac{ai}{2} + \frac{b}{2} - \frac{bi}{2}$

$\overrightarrow{QR} = \overrightarrow{QC} + \overrightarrow{CR} = \frac{b}{2} + \frac{bi}{2} + \frac{c}{2} - \frac{ci}{2}$

But $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \frac{a+c}{2} + b + \frac{i}{2}(a + c)$

Thus $\overrightarrow{PRi} = \frac{c-a}{2} + \frac{i}{2}\,\underline{(a+c+2b)}$

But $\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS} = \frac{-2a-b-d}{2} + \frac{i(b-d)}{2}$

and $-\overrightarrow{QS} + \overrightarrow{PRi} = \frac{(a+b+c+d)}{2} + \frac{i(a+b+c+d)}{2} = 0$

because $a+b+c+d = 0$

Hence $QS = PR$ and $QS \perp PR$.

Editor's Comment. The result proved in this problem was first established by the German mathematician Van Aubel. The method utilized in the proposer's solution is simple, elegant and also powerful. For further information concerning this method, consult Jack Garfunkel's article entitled "Solving Problems in Geometry By Using Complex Numbers" which appeared in the November 1967 issue of *Mathematics Teacher*.

309. **Proposed by Richard A. Gibbs, Fort Lewis College, Durango, Colorado.**

Once upon a time in a far away kingdom there lived many married couples. It came to the attention of the King (himself unmarried) that there were some unfaithful wives in his kingdom and he issued the following decree:

> "It has come to my attention that there are unfaithful wives in my kingdom. If a husband discovers that his wife is unfaithful he may slay her without punishment provided he does so on the day of the discovery."

Now, it so happens that if a man's wife were unfaithful he would be the only husband not to know it. Further, husbands never talked among themselves about the fidelity of their wives and an unfaithful wife was clever enough not to be caught by her husband.

Well, following the King's decree a month passed without incident. Then, on the 40th day, 40 unfaithful wives were slain; all that were in the kingdom.

The King was amazed! He summoned his Math Wizard for consultation and told him what had happened. The Wizard said, "That's not at all amazing." Prove that the Wizard knew that all unfaithful wives in the kingdom would be slain on the same day.

Solution by the prosper.

Suppose there were only one unfaithful wife. Then the whole kingdom except this wife's husband would know this. However, the husband, knowing of no cases of unfaithfulness and believing his wife to be faithful would, upon reading the decree, deduce that his wife was the unfaithful one and kill her the first night. Now suppose there are two unfaithful wifes w_1 and w_2 with husbands h_1 and h_2 respectively. Now on the day of the decree, h_1 knows w_2 is unfaithful but since h_2 did not kill w_2 on the day of the decree, h_1 deduces his wife w_1 is also unfaithful. By the same argument h_2 deduces his wife w_2 is unfaithful and two unfaithful wives were killed on the second day.

We proceed by induction. Suppose there are n unfaithful wives $w_1, w_2, \ldots, w_n$ and respective husbands $h_1, h_2, \ldots, h_n$. Let the induction hypothesis be that each husband h_i *knows the identity of exactly* $n-1$ unfaithful wives distinct from w_i. *Now each of the* n husbands, believing that his own wife is faithful, expects that the $n-1$ unfaithful wives will be killed on the $n-1$st day according to the terms of the decree. but when all of the unfaithful wives are alive on the nth day, each of the n husbands realizes his own wife is also unfaithful. Hence our induction is complete and all n of the unfaithful wives are executed on the nth day.

310. **Proposed by the editor.**

Consider the sequence of numbers 10001, 100010001, 1000100010001, etc. Are there any primes in this sequence?

Solution by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

There are no prime numbers in the given sequence. This can be seen by observing the following:

$10001 = 10^4 + 1 = 73 \cdot 137$

$100010001 = 10^8 + 10^4 + 1 = (10^4 + 10^2 + 1)(10^4 - 10^2 + 1)$

$1000100010001 = 10^{12} + 10^8 + 10^4 + 1 = (10^4 + 1)(10^8 + 1)$

$10001000100010001 = 10^{16} + 10^{12} + 10^8 + 10^4 + 1 =$
$(10^8 + 10^6 + 10^4 + 10^2 + 1)(10^8 - 10^6 + 10^4 - 10^2 + 1)$

$100010001000100010001 = 10^{20} + 10^{16} + 10^{12} + 10^8 + 10^4 + 1$
$= (10^4 + 1)(10^{16} + 10^8 + 1)$

$$1000100010001000100010001 = 10^{24} + 10^{20} + 10^{16} + 10^{12} + 10^{8} + 10^{4} + 1 = (10^{12} + 10^{10} + 10^{8} + 10^{6} + 10^{4} + 10^{2} + 1) \cdot (10^{12} - 10^{10} + 10^{8} - 10^{6} + 10^{4} - 10^{2} + 1)$$

In general if the highest power of 10 in a given term is $8k+4$ the term is the $2k+1$st and

$\sum_{i=0}^{2k+1} 10^{4i} = (10^4+1)\left(\sum_{i=0}^{k} 10^{8i}\right)$ which is composite for all $k>0$.

If the highest power of 10 in a given term is $8k$, the term is the $2k$th and

$\sum_{i=0}^{2k} 10^{4i} = \left(\sum_{i=0}^{2k} 10^{2i}\right)\left(\sum_{i=0}^{2k} (-1)^i (10^{2i})\right)$ which is composite for all $k>0$.

Each of these general formulas are easily established by induction on k.

Solution by the proposer.

The nth term of the sequence is $10^{4n} + 10^{4(n-1)} + \cdots + 10^4 + 1$

$\frac{10^{4(n-1)}-1}{10^4-1}$ which is an integer. It suffices to show that each term is composite.

Now if $n+1 = 2r$ for some integer $r \geqslant 1$, then

$$\frac{10^{4(n+1)}-1}{10^4-1} = \frac{10^{4r}-1}{10^4-1}(10^{4r}+1).$$

Now if $r = 1$, $10^4+1 = 73 \cdot 137$

and if $r>1$, $\frac{10^{4r}-1}{10^4-1}$ is an integer >1. Hence if $n+1$ is even, each term is composite.

If $n+1$ is odd, then $n=2r$ for some integer $r \geqslant 1$ and

$$\frac{10^{4(n+1)}-1}{10^4-1} = \frac{10^{4(2r+1)}-1}{10^4-1} =$$

$$\left(\frac{10^{2(2r+1)}+1}{10^2+1}\right)\left(\frac{10^{2(2r+1)}-1}{10^2-1}\right).$$

Now since $2r+1$ is odd, $(10^2+1) \mid (10^{2(2r+1)}+1)$ and $(10^2-1) \mid (10^{2(2r+1)}-1)$ so that each term of this type is composite. Hence there are no primes in the given sequence.

Editor's Comment. This is problem 63 from the reference: Shklarsky, Chentzov, Yaglom *The USSR Olympiad Problem Book*, Freeman & Co., pp18,149.

311. **Proposed by the editor.**

A teacher of mathematics propounded the following addition problem: two numbers are selected at random and each succeeding number equals the sum of the two preceding numbers until a list of ten numbers is reached; e.g., starting with 365 and 142 the list to be added by the class 365 + 142 + 507 + 649 + 1156 + 1805 + 2961 + 4766 + 7727 + 12493. Just as the teacher told the class to add these numbers, young Leslie Morely announced the sum to be 32571. Astounded, the teacher verified the correctness of Leslie's answer with a pocket calculator. Assuming that Leslie performed this feat mentally, how did he do it?

Solution by Michael W. Ecker, Pennsylvania State University-Worthington Scranton Campus, Dunmore, Pennsylvania.

If the initial two numbers are a and b, then we are adding:

$$S = a+b+(a+b)+(2a+2b)+(2a+3b)+(3a+5b)+(5a+8b)$$

$$+(8a+13b)+(13a+21b)+(21a+34b).$$

Note that the coefficients of a and b are all Fibonacci numbers. The sum of the first k consecutive Fibonacci numbers is well known; specifically, if $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all integers $n > 2$, the sum of the first k Fibonacci numbers is given by $F_0 + \cdots + F_k = F_{k+2} - 1$ which can be verified readily by induction on k.

Hence $S = a + (55-1)\,a + (89-1)\,b = 55a + 88b = 11\,(5a+8b)$ or 11 times the seventh term in the series to be added. Here $a = 365$ and $b = 142$ gives the result $S = 32571 = 11 \cdot (2961)$.

Also solved by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin and the proposer.

KAPPA MU EPSILON NEWS

EDITED BY HAROLD L. THOMAS, HISTORIAN

News of chapter activities and other noteworthy *KME* events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department, Pittsburg State University, Pittsburg, Kansas 66762.

The Twenty-second Biennial Convention of Kappa Mu Epsilon was held April 26-28, 1979 on the campus of Pittsburg State University, Pittsburg, Kansas, with Kansas Alpha as the host chapter.

On Thursday evening, April 26, following registration in the lobby of Yates Hall, games and get-acquainted activities were held in Yates Hall and the National council met in the Conference Room of Yates Hall.

On Friday morning, April 27, registration continued in the lobby of Yates Hall. The first general session (business meeting) was held in Yates Hall 102 commencing at 8:45 a.m. with James E. Lightner of Maryland Beta, National President, presiding. Dr. James B. Appleberry, President of Pittsburg State University, gave the address of welcome and Ida Z. Arms of Pennsylvania Zeta, National Vice-President responded for the Society. The roll call of the chapters was made by George R. Mach of California Gamma, National Scretary. 32 chapters and about 200 members were in attendance. Delegate certification forms and travel vouchers were filed and delegate voting cards were issued. The representatives of Maryland Delta and Wisconsin Gamma were given special greetings as those chapters were installed since the last biennial convention.

The report of the Constitution and Bylaws Revision Committee was presented by William R. Smith of Pennsylvania Zeta. He distributed draft revisions including changes made by the National Council.

Petitions for new chapters at Illinois Benedictine College, Lisle, IL and Saint Francis College, Loretto, PA were presented and approved by the delegates.

Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

1. "Existence of a Relatively Prime Element in Sets of Consecutive Integers," Gregory Lee Hayward, Kansas Beta, Emporia State University.

2. "Burnside's Theorem in Counting Symmetries," Howard Thompson, Kansas Alpha, Pittsburg State University.
3. "The Search for Pi: Rational or Irrational," Jim Davies, Iowa Alpha, University of Northern Iowa.
4. "Tessellations," Donald D. Sandman, Wisconsin Gamma, University of Wisconsin-Eau Claire.

At noon, a group picture was taken on the steps of the Student Union. Convention committees and the National Council met during lunch.

The convention reconvened at 1:30 p.m. in Yates Hall. Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

5. "The Diamond Functions," Diane Dahlgren, California Gamma, California Polytechnic State University.
6. "The Sensitivity of Polynomial Equations to Perturbations in Data," Gary Kent Cobb, Missouri Zeta, University of Missouri-Rolla.
7. "The Essence of Galois Theory: Extraordinary, Yet Simple," Maureen Swierczynski, New York Eta, Niagara University.

At 2:45 p.m. the student section met in Yates Hall with Howard Thompson, Kansas Alpha President, presiding and the faculty section met in the Student Union with James E. Lightner of Maryland Beta, National President, presiding.

The convention reconvened at 3:45 p.m. in Yates Hall. Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

8. "An Isomorphism Question," Bruce D. Jones, Ohio Zeta, Muskingum College.
9. "GEN/PLT-2P3D Fortran Code for Producing Motion Pictures of Computer Simulated Two-Phase Three Dimensional Fluid Flow," Jeffrey R. Towne, Pennsylvania Theta, Susquehanna University.
10. "A Chemical Application of Graph Theory," Mark Stehlik, New York Kappa, Pace University.

Howard Thompson, President of Kansas Alpha, reported for the student section meeting and James E. Lightner of Maryland Beta, National President, reported for the faculty section meeting.

The convention banquet was held on Friday evening, April 27, in the Student Union with Howard Thompson, President of Kansas Alpha, as master of ceremonies. Musical entertainment was provided by the Girard High School "Reflections." Guest speaker, Dr. Walter Stromquist, gave the address, "The Four Color Theorem."

The convention reconvened at 8:30 a.m. on Saturday, April 28, in Yates Hall. Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

11. "Three Basic Geometrical Transformations: Some Applicable Linear Algebra," John Graham, Kansas Alpha, Pittsburg State University.
12. "Patterns in Problem Solving," Laura A. Cowan, Missouri Beta, Central Missouri State University.
13. "A Stochastic Process For Composing Music," Michael S. Hewett, Kansas Delta, Washburn University.
14. "On Prime Powers Which Are Consecutive Integers," Dave Predmore, Kansas Beta, Emporia State University.

The proposed Constitution and Bylaws revision was again discussed. The delegates voted to amend the proposed revision of the constitution V, 1, to provide that additional names may be added to the ballot at the convention if the nomination is made and seconded by certified delegates from different chapters and the proposed candidate indicates a willingness to serve.

The delegates voted (48 yes - 0 no - 0 abstain) that in accordance with the constitution VIII, the convention approve the proposed amendments (revisions) of the constitution and bylaws, with changes proposed by the National Council and the convention delegates, and that the amendments be submitted to the chapters for ratification with response by November 1, 1979.

The following national officers presented reports (copies attached), which were distributed to all of the delegates:

Douglas Nance	-	Business Manager, THE PENTAGON Michigan Beta
James K. Bidwell	-	Editor, THE PENTAGON Michigan Beta

Sister Jo Ann Fellin	-	National Historian Kansas Gamma
Wilbur J. Waggoner	-	National Treasurer Michigan Beta
George R. Mach	-	National Secretary California Gamma
Ida Z. Arms	-	National Vice-President Pennsylvania Zeta
James E. Lightner	-	National President Maryland Beta

Carol Harrison of Pennsylvania Theta reported for the auditing committee that the treasurer's books and accounts were examined and verified.

Robert Bailey of New York Eta reported for the resolutions committee. The following resolutions were adopted:

Resolved: That the Twenty-second Biennial Convention of Kappa Mu Epsilon express its gratitude to Sister Jo Ann Fellin who has served as Historian of Kappa Mu Epsilon and who has given so generously of her time and talents.

Resolved: That the Twenty-second Biennial Convention of Kappa Mu Epsilon express its appreciation:

1. To Howard Thompson and the members of Kansas Alpha for their work in the expeditious planning of this convention.
2. To Dr. James B. Appleberry, President of Pittsburg State Univesity, for the gracious hospitality and the many services rendered the chapters and officers of the convention.
3. To Dr. Walter Stromquist for his entertaining and colorful banquet talk.
4. To the Selection, Auditing, Nominating, Constitutional Revision and Awards Committees who gave so unselfishly of their time to the primary activity of Kappa Mu Epsilon.
5. To the students who prepared and submitted papers for the convention.

6. To the National Officers of Kappa Mu Epsilon for their diligent service during and preceding the Biennial Convention.
7. To Ms. Janice De Chicchio and the Girard High School " Reflections" who provided excellent entertainment for us during the convention banquet.

L. Thomas Shifflett of Missouri Alpha reported for the nominating committee. Nominations were requested from the floor. There being none, nominations were declared closed and ballots were distributed.

Invitations to host the Twenty-third Biennial Convention were extended by: Missouri Alpha, Southwest Missouri State University, and Missouri Theta, Evangel College (joint invitation); Nebraska Beta, Kearney State College; California Gamma, California Polytechnic State University; and Wisconsin Beta, University of Wisconsin - River Falls and Wisconsin Gamma, University of Wisconsin - Eau Claire (joint invitation).

Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presented certificates to all students who had presented papers at the convention. Gary Mulkey of Missouri Iota reported for the awards committee and annnounced the following student paper awards:

First Place	Michael S. Hewitt, Kansas Delta
Second Place	Mark Stehlik, New York Kappa
Third Place	Jeffrey R. Towne, Pennsylvania Theta
Fourth Place	John Graham, Kansas Alpha

The election ballot tally was announced. The following officers were elected for the next biennium, 1979-81, and they were installed by William R. Smith of Pennsylvania Zeta, Past President:

President	James E. Lightner Maryland Beta
Vice-President	Ida Z. Arms Pennsylvania Zeta
Secretary	George R. Mach California Gamma
Treasurer	Wilbur J. Waggoner Michigan Beta
Historian	Harold Thomas Kansas Alpha

Travel allowances were paid to the delegates by Wilbur J. Waggoner of Michigan Beta, National Treasurer. Convention evaluation forms were collected by the host chapter. The convention adjourned at 11:56 p.m.

REPORT OF THE NATIONAL PRESIDENT

During the last biennium there has been a good deal of activity within the Society and among the membership. The Society has grown with the addition of two new chapters: Wisconsin Gamma at University of Wisconsin - Eau Claire, installed by Wilbur Waggoner, and Maryland Delta at Frostburg State College which I installed. The National Council has also received and voted favorably on two petitions and have submitted these to the membership at this Convention for their vote. Two other mathematics clubs are presently preparing petitions for submission, and a number of inquiries for information have been received and responded to. It is gratifying to see this continued and growing interest in mathematics and honor societies.

As you heard from the Vice President, the regions continue to function, some more actively than others. The even-year meeting cycle was partially thrown off last year by our postponed 21st Biennial Convention. But now that we are back in our normal cycle, I hope that during the next biennium we will see more activity. Indeed, it would be a major contribution to the strength of the Society if *every* region were to hold a conference next Spring to maintain the interest developed here at this national convention. In accordance with our policy, directors for regions II, IV and VI will be appointed at this convention for a four year term.

At the last convention I announced the appointment of an *ad hoc* Constitutional Revision Study Committee to examine our present constitution and suggest possible changes. The committee, which was chaired by Past-President William Smith, began its work at that convention and has continued to deliberate over the past eighteen months. The Committee's suggested revisions have been sent to all chapters, we have considered the proposals at this convention, and the final revision will be submitted to all the chapters for approval in the immediate future. I want to thank Chairman Smith and his committee members, Harold Thomas, Sister John Frances Gilman, Becky Tucker, and Kathy Tandetzte, for all their efforts on this major project for the benefit of the Society.

At the last convention, I also reported that the National Council was in the process of trying to recover the costs of getting our treasury back in order following the irregularities which were discovered in November 1976. With the help of legal counsel we finally agreed to a settlement which was received this past fall and which covered the cost of the 1977 audit, finance charges we were forced to pay on overdue bills, and interest which should have accrued in savings accounts. We now consider the case closed. As you can see from the Treasurer's Report, our financial picture is sound, although inflation is taking its toll and an increase in the initiation fee is being seriously considered in order that the Society may continue to remain viable and solvent.

I want to express my thanks to all those members who agreed to serve on the various convention committees. Much of their work goes on behind the scenes, either before or during the convention, so that things will go smoothly and we can accomplish our purposes. I also want to thank all my colleagues on the Council and the Regional Directors for their support and help during the last biennium. Especially do I want to recognize Sister Jo Ann Fellin, who completes today her second and last term as National Historian. She has performed her duties with dispatch and care, and we all appreciate her efforts in our behalf and wish her well for the future.

In conclusion, I think we can consider the past biennium a successful one, during which we fully got "back to normal" and moved forward. I know that the future will see Kappa Mu Epsilon even stronger as we approach our fiftieth anniversary in April 1981. Your Council has begun to plan for this special convention which should mark a high moment in our history.

James E. Lightner

REPORT OF THE VICE PRESIDENT

In the Spring of 1978 two Regional Conventions were held. Region 1 (Sister Marie Augustine Dowling, Director) held its Convention at Susquehanna University (Pennsylvania Theta Chapter). Region 4 (Dr. Harold Thomas, Director held its Convention at Missouri Southern State College (Missouri Iota Chapter). The Vice President serves as Coordinator of the Regional Directors.

It is the Vice President's responsibility to make arrangements for presentation of student papers at the Biennial Convention. I am pleased to report that seventeen students, representing thirteen Chapters and nine states, submitted papers. Fourteen of these papers are to be presented at the Convention. My special thanks to the members of the Paper Selection Committee who read and ranked the papers: Dr. Ralph McBride (Indiana Alpha), Sister Nona Mary Allard (Illinois Zeta) and Professor Lyle Oleson (Wisconsin Beta). I am particularly grateful to the seventeen students who prepared and submitted papers. These papers are very important in helping to make a Convention of Kappa Mu Epsilon a success.

Ida Z. Arms

REPORT OF THE NATIONAL HISTORIAN

The files of the National Historian are being maintained and continually updated with the records received from the chapters about their events and activities.

News items have been solicited from the corresponding secretaries semi-annually, in January and in May, and have been edited for publication in the chapter news section of The Pentagon.

During the past biennium 74 per cent of the chapters responded at least once to the chapter news request. Special mention goes to the following nineteen chapters for their cooperation in responding to all four inquiries: AL Beta, CA Gamma, IL Alpha, IA Alpha, IA Delta, KS Alpha, KS Gamma, KS Epsilon, MD Alpha, MS Gamma, MO Beta, NM Alpha, NY Eta, OH Zeta, OK Gamma, PA Alpha, PA Epsilon, PA Zeta, WI Alpha.

My thanks to all with whom I have corresponded relative to this office -- the national officers, the regional directors, all the chapter corresponding secretaries, and the editor of The Pentagon.

Sister Jo Ann Fellin

REPORT OF THE NATIONAL SECRETARY

During the past biennium two new chapters of Kappa Mu Epsilon were installed. They are: Wisconsin Gamma at the University of Wisconsin-Eau Claire, installed on February 4, 1978, and Maryland Delta at Frostburg State College, installed on September 17, 1978. The Society now has 98 active chapters in 31 states.

During the last biennium, 1,799 members were initiated. The 98 active chapters have a combined membership of 35,322 and the 20 inactive chapters have combined membership of 4,213 making the total membership of Kappa Mu Epsilon 39,535 at the end of the last biennium on April 7, 1979.

As National Secretary, I maintain permanent chapter files, including reports of all chapter initiations. I order membership certificates for all new members and I stock all supplies including forms, invitations, and jewelry. I attempt to assist corresponding secretaries in any ways that I can and I take minutes of National Council meetings and conventions.

George R. Mach

FINANCIAL REPORT OF THE NATIONAL TREASURER

For the Period November 1, 1977 to March 31, 1979

	Receipts			
1.	Cash on Hand November 1, 1977			$ 1,631.47
2.	Receipts from Chapters			
	Initiates (1531)	$15,400.00		
	Jewelry	687.81		
	Supplies	124.80		
			$16,212.61	
3.	Miscellaneous			
	Interest	45.03		
	From Previous Treasurer	5,169.90		
	From Savings	5,454.97		
	Installations(2)	257.88		

	Replacement Certificates	3.50		
	University Microfilms (Royalties)	31.68		
			10,962.96	
4.	Total Receipts			27,175.57
5.	Receipts plus cash on hand			28,807.04
	Expenditures			
6.	National Officers Expense		$ 1,911.73	
7.	Regional Directors Expense		432.57	
8.	Balfour Company		854.09	
9.	Blake Printery & Herff-Jones		2,663.38	
10.	Pentagon (2 issues)		5,685.69	
11.	Biennial Convention		4,012.96	
12.	Miscellaneous			
	ACHS (two conventions and Dues)	$ 821.48		
	Refunds	75.86		
	KME Brochures	420.00		
	Savings	9,000.00		
			10,317.34	
13.	Total Expenditures			25,877.76
14.	Cash on Hand, March 31, 1979			2,929.28
15.	Total Expenditures plus Cash on Hand			$28,807.04
16.	Total Assets, March 31, 1979			
	Cash on Hand		$ 2,929.28	
	Savings		8,545.03	
	Interest		264.74	
			$11,739.05	

REPORT OF THE EDITOR OF THE PENTAGON

THE PENTAGON should serve the needs of the student members. More direct participation of the membership is needed and can be achieved by submitting articles, problems and solutions, contributions to The Scrapbook, and KME News items. In particular, THE

PENTAGON still needs to publish more student papers.

During the last biennium, 8 faculty articles were published and 11 student articles. I want all winning papers at each regional convention submitted automatically for possible publication, if they are not held for the next national bienniem. I urge each of you to consider writing potential articles for THE PENTAGON. I will continue to publish non-student articles of interest to you, but I would like the number to decrease.

We have had no editorial changes in the last biennium. I wish to thank my associates, without whom my work load would be greatly multiplied. Richard Barlow, Oscar Beck, Sister Jo Ann Fellin, Loretta Smith, and Kenneth Wilke all deserve our special thanks. Also appreciated are the services of Douglas Nance, Business Manager.

The production of THE PENTAGON is still in a state of change. Douglas Nance and I continue to struggle with schedule and price changes. The Spring 1979 issue is the first to be set by a computer-assisted technique. For various reasons it was necessary to abandon the former cold type style of the journal. I regret this change, but it could not be avoided. Hopefully, the change will provide a more timely delivery of THE PENTAGON.

This is the last convention I will attend as Editor of THE PENTAGON. I wish to extend my thanks to the National Council for the opportunity to serve KME since 1968 in the production of THE PENTAGON. I have met many people and have many new friends because of this association, and I am very grateful for it. I know that the journal will move into good hands and that you will continue to receive a journal worthy of KME.

James K. Bidwell

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON

It is a pleasure to make my first official Business Manager's report to a biennial convention of Kappa Mu Epsilon. During the past biennium we have mailed an average of 2800 Pentagons per issue. The mailing list includes subscribers in forty-four states and fifteen foreign countries. States receiving the most copies of THE PENTAGON are, in descending order, Pennsylvania, Missouri, Tennessee, Ohio, California, and Kansas.

During each semi-annual mailing, approximately 50 PENTAGONS are returned to the office of the Business Manager by the postal service as undeliverable due to incorrect address. Please inform your chapter members that to receive their journal they must keep a current address on file with the Business Manager. If a subscriber has any problem with receiving his PENTAGON, he should contact the office of the Business Manager.

New members should not normally have to wait more than three months to receive their first issue of THE PENTAGON. At each printing, an additional 300 issues are printed so that one bulk mailing can be made to new members two to three months after the initial mailing. This bulk mailing procedure is necessitated by the current postal rate of forty cents for each individually mailed PENTAGON.

Complimentary copies are sent to the library of each college or university with an active chapter of Kappa Mu Epsilon. Also, complimentary copies are sent to authors of articles in THE PENTAGON. Speakers of this convention will automatically have their subscriptions extended for two years.

During this past biennium, I have received cooperation and support from former Business Manager Wilbur Waggoner, Editor James Bidwell, and National Secretary George Mach. This cooperation is gratefully acknowledged.

Douglas W. Nance

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

Chapter President - Deborah Thigpen

40 actives

Spring meeting programs were given by Dr. Michael Moeller of the UNA chemistry faculty and Dr. Juan C. Arambura of the UNA mathematics faculty. Dr. Moeller demonstrated and discussed the BASIC programming language and Dr. Arambura presented mathematical topics. A special program was held to honor Dr. Elizabeth Wooldridge on her retirement from the UNA mathematics faculty. Dr. Wooldridge was former National Secretary of KME. The semester closed with the annual picnic. Other 1979-80 officers: Susan Coburn, vice president; Beth Mullaney, secretary and treasurer; Oscar Beck, corresponding secretary; David Cope, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President - Dan Moczarny

50 activies, 32 pledges

Approximately 150 students participated in a math field day sponsored by the chapter for all the Junior Highs in the county. Several seminars were held featuring speakers from the placement office. The chapter sponsored a Career Day with speakers from business and industry. Other 1979-80 officers: Roberto Fontillas, vice president; Nonnetto Mello, secretary; Marie Grossi, treasurer; George R. Mach, corresponding; Adelaide Harmon-Elliott, faculty sponsor.

California Delta, California State Polytechnic University, Pomona

Chapter President - Jeffrey Eakins

18 actives

The chapter set up and ran Poly-Vue, for the mathematics department. Members also provide free tutoring service to students in various mathematics classes. Other 1979-80 officers: Karen Britt, vice president; Karen Head, secretary and treasurer; Richard Robertson, corresponding secretary; Cameron C. Bogue, faculty sponsor.

Colorado Alpha, Colarado State University, Fort Collins

Chapter President - Andrew G. Bennett

Other officers for 1979-80: Karen Femmer, vice president; Fred

Hansen, secretary; Ted Puls, treasurer; E. R. Deal, corresponding secretary and faculty sponsor.

Colorado Beta, Colorado School of Mines, Golden
Chapter President - Shelby Switzer
30 actives, 27 pledges
Spring semester programs included the film, "Mr. Simplex Saves the Aspidistra" and Paul Treece of the CSM computing center who presented "Computer Science at CSM." The chapter offered $5 for the "best" solution of each of the five problems and $5 for the "best" set of solutions posed for the Prindle, Weber, & Schmidt undergraduate mathematics competition III. Twenty-seven new members were initiated in April. Other officers for 1979-80 remain as published in the spring issue.

Florida Beta, Florida Southern College, Lakeland
Chapter President - Karen Ring
6 actives, 14 pledges
Other officers for 1979-80: Georgie Mason, vice president; Suzanne Hardy, secretary and treasurer; Henry Hartje, corresponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Brenda Dale Jones
20 actives, 7 pledges

The spring quarter was very active for Georgia Alpha. Several members assisted with the 1979 MATH DAY activities. The chapter sent its first-ever delegation to the national convention at Pittsburg State University. Seven new members were initiated in May. The new members were honored with a reception following the initiation and were also recognized at the Honors Day convocation. The newly elected officers are enthusiastic and look forward to a very active year in 1979-80. Other officers for 1979-80: Phyllis Walker, vice president; Lisa Yates, secretary; Patricia Ingram, treasurer; Thomas J. Sharp, corresponding secretary and faculty sponsor.

Illinois Alpha, Illinois State Univesity, Normal
Faculty sponsor - Orlyn Edge

Illinois Zeta, Rosary College, River Forest
Chapter President - Mark Siwek
7 actives, 8 pledges
Sharon Holder, elementary education student, spoke on "Chisan-

bop, the Korean method of hand reckoning," at the March meeting. Eight new members were received at this time. Other officers for 1979-80: Kathleen Tracy, vice president; Barbara Spaniak, secretary; Mary Skryp, treasurer; Sister Nona Mary Allard, corresponding secretary and faculty sponsor.

Illinois Eta, Western Illinois University, Macomb
Chapter President - Deborah Oganovich
The chapter hosted visiting lecturer, Dr. Allen V. Butterworth, chairman of the Department of Mathematics, General Motors Research Laboratories, Warren, Michigan. Dr. Butterworth lectured on "Policy Analysis - Making the Most of Alternatives." In addition, he discussed the role of the mathematician at General Motors. Another major activity for the chapter was the Math Appreciation Week. This included 10 undergraduate student presentations, a graduate student paper, a department contest, and a social gathering. Terri Weishar and Debbie Oganovich represented the chapter at the 22nd Biennial Convention at Pittsburg State University. Other officers for 1979-80: Greg Shippee, vice president; Susan Dufek, secretary and treasurer; Kent Harris, corresponding secretary; Larry Morley, faculty sponsor.

Illinois Theta, Illinois Benedictine College, Lisle
Chapter President - Mary Vrtis
26 actives
The installation ceremony for the chapter was held May 18, 1979. Sister JoAnn Fellin, former National Historian KME, presided. Sister Nona Mary Allard, corresponding secretary for Illinois Zeta chapter assisted as conductor. Twenty-five charter members form the chapter as well as Dean Marvin Comburn who was already a member of KME. Other officers for 1979-80: Lisa Kozloski, vice president; Patricia Gramme, secretary; Mahnaz Shahidi, treasurer; Phyllis Kittel, corresponding secretary and faculty sponsor.

Indiana Gamma, Anderson College, Anderson
Chapter President - Jay Lee Collins
9 actives, 6 pledges
The chapter held a dinner in April to honor new members. Regular business meetings were held through the semester. Other officers for 1979-80: Kevin Flick, vice president; Cynthia Leach, secretary and treasurer; Stanley L. Stephens, corresponding secretary and faculty sponsor.

Indiana Delta, University of Evansville, Evansville
Chapter President - J. Mark Steber
14 actives, 11 pledges
Other officers for 1979-80: George Brunemann, vice president; Mark E. Simpson, secretary; Melba Patberg, treasurer and corresponding secretary; Dennis Hopkinson, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
Chapter President - Jill Roesch
35 actives
The following students presented papers at meetings during the spring semester 1979: Jo Ann Vannini on "Perfect Numbers," Mary Ann Johnson on "Catastrophe Theory," Sue Quirk on "Computer Security and Protection," and new initiate Virginia Chizek spoke on "Plane Curves" at the initiation banquet in May. Seven students and one faculty member attended the 22nd biennial convention in Pittsburg, KS. Jim Davies presented his paper on "Pi" at the convention. The KME-faculty picnic was held the first Sunday in May. Other 1979-80 officers: Anita Doehrmann (Fall '79) and Kyle Stravers (Spring '80), vice president; Brenda Walker, secretary; Jo Ann Vannini, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President - Mark Sand
15 actives, 4 pledges
Student papers presented at the monthly meetings were unusually good this past spring. The annual initiation banquet was held in April. Other 1979-80 officers: Mitch Adams, vice president; Jim Holsapple, secretary; Mr. Brian Jones, treasurer; Wayne Woodworth, corresponding secretary; Mark Hopkins, faculty sponsor.

Iowa Gamma, Morningside College, Sioux City
Chapter President - Roger Bobolz
8 actives, 2 pledges
Other 1979-80 officers: Karlee Schempf, vice president; Mary Rock, secretary; Bill Barke, treasurer; Carol White, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President - Mark E. Reinhardt
39 actives

The chapter together with the mathematics and computer science faculty sponsored the 2nd annual math field day on March 3. John Tanner, past vice president, gave an overview of his microcomputer assembly project for one of the monthly meetings. Other 1979-80 officers: John W. Beck, vice president; Allan Brunner, secretary; Tami D. Worby, treasurer; Lynn J. Olson, corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg

Chapter President - Howard Thompson

50 actives

Kansas Alpha began the spring semester with a banquet and initiation for the February meeting. Nine new members were initiated at that time. Howard Thompson gave the program on "Burnside's Theorem in Counting Symmetries." The March program was presented by John Graham on "Three Basic Geometrical Transformations: Some Applicable Linear Algebra." April was a busy month for Kansas Alpha. The regular meeting program was given by Tom Pope on "Geometrical Disections." The chapter then assisted the mathematics department faculty in administering and grading tests given at the annual Math Relays, April 24, 1979. The final activity for April was that of hosting the 22nd biennial convention of *KME*. The chapter was pleased to host the convention which was attended by approximately 33 chapters and 200 students and faculty. The final meeting of the spring semester was highlighted by election of officers for the 1979-80 school year. In addition, the annual Robert M. Mendenhall award for scholastic achievement was presented at the May meeting to Mary Huning and Howard Thompson. They each received a *KME* pin in recognition of this achievement. Howard also presented the program on "Isometry." Other 1979-80 officers: Doug Johnston, vice president; Kay Conklin, secretary; Nancy Tanner, treasurer; Harold L. Thomas, corresponding secretary; J. Bryan Sperry, faculty sponsor.

Kansas Beta, Emporia State University, Emporia

Chapter President - Angie Miller

21 actives, 4 pledges

The chapter had various money raising projects to send eight students and two faculty to the national convention. A banquet was held when new initiates were received. The annual spring picnic was held which seems to coincide with the annual spring rain. Kansas Beta was saddened by the death of their beloved and faithful corresponding secretary, Donald L. Bruyr, on June 13, 1979. Dr. Bruyr received his bachelors and masters degree from Pittsburg State University and his doctorate from Oklahoma State University. He was very active in *KME*, civic, and religious affairs. He will be sorely missed by friends and colleagues. Other 1979-80 officers: Carol Whitney, secretary; John Gerriets, corresponding secretary; Tom Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison
Chapter President - Patti McDonald
28 actives, 11 pledges
The chapter held monthly meetings as well as the annual picnic on May 7, 1979. The seniors were presented pins at this picnic. The annual Math Contest was held March 31 with fourteen area schools attending. Other 1979-80 officers: Rick Strecker, vice president; Sue Voda, secretary; Janice Jones, treasurer; Sister JoAnn Fellin, corresponding secretary; James Ewbank, faculty sponsor.

Kansas Delta, Washburn University, Topeka
Chapter President - Randy Leimer
25 actives, 6 pledges
Washburn University Math Day was sponsored by Kansas Delta in cooperation with the mathematics department. Four students attended the national convention, including the first place winner in papers presented. Other 1979-80 officers: Darla Brockmeyer, vice president; Betty Pinter, secretary; Susan Glotzbach, treasurer; Robert H. Thompson, corresponding secretary; Gary Schmidt, faculty sponsor.

Kansas Epsilon, Fort Hays State University, Hays
Chapter President - Terri Hooper
15 actives, 11 pledges
Other 1979-80 officers: Sally Irvin, vice president; Geralyn Kraus, secretary and treasurer; Eugene Etter, corresponding secretary; Charles Votaw faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore
Chapter President - Maura Kelly
7 actives, 5 pledges
Students presenting papers in March included Nancy Callanan, "History of Mathematical Symbols," and Lizann Doskos, "Cryptography." Initiation of five new members followed a pot luck dinner on May 8, 1979. Five KME alumnae from Maryland Alpha com-

prised a Career Panel after the initiation. Discussing their respective jobs were Barbara Tipton, '66, and Rhette Martha, '77, from IBM; Debbie Manger Corscke, '72, with C & P Telephone Co.; Helene Murtha, '77, and Fran Pittelli, '78 both in high school teaching. Other 1979-80 officers: Lizann Doskos, vice president and treasurer; Rita McCardell, secretary; Sister Marie A. Dowling, corresponding secretary; Sister Delia Dowling, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster
Chapter President - Terry Reider
19 actives

The annual KME banquet and induction of new members was held in March. The chapter sponsored a booth at the college's annual May Day Carnival in May. A pizza party was also held to close the school year. Other 1979-80 officers: Mike O'Laughlin, vice president; Rebecca Weller, secretary; Bill Spring, treasurer; James Lightner, corresponding secretary; Robert Boner, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President - Reinaldo M. Machado
18 actives

The spring semester of 1979 was a productive one for the Maryland Delta Chapter of Kappa Mu Epsilon. On February 8, 1979, the chapter held its first initiation ceremony at which time six new members were inducted into the chapter. During the semester, the Maryland Delta Chapter hosted two guest speakers from the local community college at two of its programs. A presentation by student member, John Coleman, was also featured at one meeting. In addition to such formal programs, the chapter provided service for students who need help in lower level mathematics courses, prepared displays for the departmental showcase, helped in sponsoring the annual symposium on mathematics education presented by the department, sponsored a pie eating contest at the Bluegrass festival conducted by the student association, and held a spring picnic at which time retiring faculty member and faculty sponsor, Dr. Walter J. Rissler, was honored. Plans are now being made for activities for the fall semester. Other 1979-80 officers: Janet Jessup, vice president; James Martens, secretary; Thomas J. Podles, treasurer; Roberta L. White, corresponding secretary; John Jones, faculty sponsor.

Mississippi Alpha, Mississippi University for Women, Columbus

Chapter President - Barbara Gene James
7 actives, 11 pledges
Other 1979-80 officers: Bobbie Janine Richmond, vice president, Laura Alana Lee, secretary and treasurer; Jean Ann Parra, corresponding secretary and faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg
Chapter President - Donna Pearce
30 actives
A spring cookout was held at Johnson State Park on April 27, 1979. Fifteen new members were initiated this semester. Other 1979-80 officers: Susie Robertson, first vice president; Melissa Harris, second vice president; Martha Holloway, secretary; Alice Essary, corresponding secretary; Steve Doblin, faculty sponsor; Issac Traxler, publicity chairman.

Missouri Alpha, Southwest Missouri State University, Springfield
Chapter President - Rita Scroggins
30 actives
Programs for monthly meetings this past semester included Allan MacDougall, SMSU placement office, "Employment Opportunities for Mathematicians," Dr. Donald H. McInnis, SMSU geography-geology department, "Simple Questions with Tough Solutions," Dr. Melvin Foster, SMSU mathematics department, "Problem E2696 -*MAA Journal*." The chapter also viewed the film, "Albert Einstein -The Education of a Genius." The *KME* Merit Award, given annually to the member of the chapter who has made the greatest contribution to the organization during the academic year, was awarded to Charles Armstrong for his work as Chapter President during 1978-79. Other 1979-80 officers: Eileen Wilson, vice president; Janet Burrough, secretary; Terri Gist, treasurer; Michael Awad, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg
Chapter President - Bob Penniston
26 actives, 22 pledges
The chapter held four meetings including an initiation, an honors banquet and the William Klingenberg Lecture. Other 1979-80 officers: Vicki Goodin, vice president; Lynn Hill, secretary; Teresa Rau, treasurer; Homer F. Hampton, corresponding secretary and faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette
Chapter President - Janet Doll
5 actives, 1 pledge
Other 1979-80 officers: Arthur Sherman, vice president; Martha Tennyson, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

Missouri Zeta, University of Missouri-Rolla, Rolla
Chapter President - Deborah Carleton
15 actives, 6 pledges
Other 1979-80 officers: John Reed, vice president; Barbara Barkey, secretary; Kathy Pendergrass, treasurer; Tim Wright, corresponding secretary; Jim Joiner, faculty sponsor.

Missouri Eta, Northeast Missouri State, Kirksville
Chapter President - Theresa Roark
35 actives, 5 pledges
The chapter hosted mathematics contests for outstanding students in Northeast Missouri, Southeast Iowa, and Western Illinois. 655 students attended. Dr. Alex Mahaffey of South Dakota University was this year's invited speaker. Missouri Eta also assisted in the promotion and running of the special Olympics on the college campus. Seven members attended the national convention in Pittsburg, Kansas. The tenth annual spring picnic for KME members and other mathematics students was held in May. Other 1979-80 officers: Leslie Lisko, vice president; Jeanette Brown, secretary; Debra Brockschmidt, treasurer; Sam Lesseig, corresponding secretary; May Sue Beersman, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
Chapter President - Lois Bright
18 actives
Throughout the semester, nine KME members provided a free tutoring service for students in calculus and pre-calculus courses. The local KME team won the annual Wayne State College Bowl which was held April 10 and 11. Team members included Kirk Knapp, Lois Bright, Rod Bubke, Kay Nickelson, Steve Erwin, Barb Thomsen, Joe Painter, and Dave Sindelar. Sixteen teams competed. Four members attended the national convention in Pittsburg April 27-28, 1979. The local chapter participated in the annual Juvenile Diabetes Drive auction by auctioning off car washes. Proceeds were contributed to the fund. The outstanding freshman in mathematics

selected this year is Rod Koch of Wayne, Nebraska. The selection is based upon results of a competitive mathematics examination administered to freshmen enrolled in mathematics courses and recommended by the mathmatics faculty. The annual banquet was held in Laurel, Nebraska with speaker, Professor Carroll Moore of Nebraska Wesleyan University. He spoke on eclipses of the sun. Other officers remain as published in the spring issue.

Nebraska Beta, Kearney State College, Kearney
Chapter President - Tom Salyard
20 actives
Other 1979-80 officers: Brad Mattox, vice president; Bob Rathe, secretary; Scott Blunk, treasurer; Charles Pickens, corresponding secretary.

Nebraska Gamma, Chadron State College, Chadron
Chapter President - Shirley Essam
12 actives, 5 pledges
Regular meetings were held every first and third Thursday of each month. Initiation was held February 15 with new officers elected at that time. Members helped at the High School Scholastic Contest held at the college on March 30, 1979. The chapter closed the year with a party May 3. Other 1979-80 officers: Kelvin Sharp, vice president; Jamalee Bussinger, secretary; Laurie Lessman, treasurer; James Kaus, corresponding secretary and faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque
Chapter President - Mark Allen
50 actives
Other 1979-80 officers: Ron Everett, vice president; Bob Craven, secretary; Mike Wester, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Theta, St. Francis College, Brooklyn
Chapter President - Thomas Corrigan
8 actives, 4 pledges
Other 1979-80 officers: Janet Kerrigan, vice president; Carmen Archetto, secretary; Katherine Noethig, treasurer; Rosalind Guaraldo, corresponding secretary and faculty sponsor.

New York Kappa, Pace University, New York
Chapter President - JoAnn Kealy
30 actives

The sixth induction dinner was held on April 4, 1979. Congratulations and welcome were extended to the following nine inductees: Irene Driggs, Lisa Fauci, Michael Harvilla, William Martino, Deborah L. McCaffrey, David Panza, Therese Parente, Michael Schiano, Ira Wolf. The guest speaker for the induction dinner was Dr. Alan Konheim of IBM, who spoke on "Cryptographic Methods for Data Protection." New York Kappa is proud to announce that one of its members, Mark Stehlik, presented his paper and was awarded the second prize at the 22nd Biennial Convention. Other 1979-80 officers: Ralph Trombetta, vice president; Lorraine Fu, corresponding secretary; Martin Kotler, faculty sponsor.

Ohio Gamma, Baldwin-Wallace College, Berea
Chapter President - Lynn Jones
25 actives, 12 pledges
Dr. Tony Lauria of Union Carbide gave a lecture on an application of linear programming at Union Carbide. The annual spring picnic was held the last day of classees. Other 1979-80 officers: Debbie Killian, vice president; Chris Grever, secretary; Dan Whittaker, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President - Martin Cook
42 actives
Bruce Jones presented his paper, "An Isomorphism Question," at the January 18, 1979 meeting. He also gave the paper at the national convention at Pittsburg State University in April. The chapter sponsored two talks by Weldon Lodwick from Michigan State University. These talks were "Computers related to Natural Resource Problems," and "Mathematical Modelling." Nine new members were initiated in March. Each presented a short initiatory talk on selected mathematicians. The chapter helped sponsor the annual Mathematics and Computing Science Department awards banquet in April. Five members attended the national convention in Pittsburg, Kansas. Other 1979-80 officers: Mary Torchia, vice president; Lee Bowman, secretary; Kevin McCaffrey, treasurer; Larry J. Zettel, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
Chapter President - Troy Harden

20 actives, 13 pledges

Robert Langmacher from Western Electric was guest speaker on April 19, 1979. He spoke on "Operations Research." The chapter honored Dr. Raymond McKellips, chairman of the mathematics department, for completing 20 years of service to the department. Thirteen new members were initiated on April 23, 1979. Other 1979-80 officers: Jack McNeely, vice president; Jimmie Abbott, secretary and treasurer; Wayne F. Hayes, corresponding secretary; Robert O. Morris, faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington

Chapter President - Laurie Sassaman

78 actives

The initiation banquet was held March 28 at the Oak tree Country Club. The program was provided by the initiates. The chapter enmjoyed its spring picnic on May 13. A tour of the computer controlled works of Babcock and Wilcox Steel Mill was taken by Pa. Alpha. Theresa Presecan attended the National Convention at Pittsburg, Kansas. Other 1979-80 officers: Linda Wright, vice president; Becky Wheat, secretary; Sue Gotkiewicz, treasurer; Miller Peck, corresponding secretary; Barbara Faires, faculty sponsor.

Pennsylvania Delta, Marywood College, Scranton

5 actives, 2 pledges

Sr. Robert Ann vonAhnen, IHM, reports that the chapter is reviving after dormancy over the past two years. They initiated two new members at a dinner at the close of the 1979 spring semester. Dr. Marie Loftus continues as corresponding secretary.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President - Luann Murtiff

30 actives

At the February initiation of nine new members, Dr. John Broughton, Mathematics Dedpartment faculty, spoke on "A Linear Programming Problem." In March, Mr. Doyle McBride, Mathematics Department faculty, spoke on "Probability of Cases-Specifically Bayes' Theorem." Dr. Melvin Woodard, Chairman of the Mathematics Department, presented the feature address on "Mathematical Oddities" at the annual banquet in May. In addition, students who had attended the National Convention in Pittsburg, Kansas, reported on the activities of the conventiion. Other 1979-80 officers remain as published in the spring issue. William R.

Smith is faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President - Joan Jewell
20 actives, 11 pledges
During the spring semester, the chapter continued a program begun in the fall of providing free tutoring for calculus students. Plans are to provide the same service next year. The annual spring picnic was held May 7 at the Grove City Country Club. Other 1979-80 officer: Rose Bevilacqua, vice president; Susan Snavely, secretary; Rob Mueller, treasurer; Marvin C. Henry, corresponding secretary; John Ellison, faculty sponsor.

Pennsylvania Iota, Shippensburg State College, Shippensburg
Chapter President - Nelson Fernandez
51 actives, 8 pledges
The chapter helped coordinate and carry out math-computer day for high school students. The annual initiation banquet was held on April 8, 1979 at the University Lodge. Activities for initiation included dinner, presentations, ceremony, and a talk. Election of officers was held April 24, 1979. Other 1979-80 offciers: Tim Davidheiser, vice president; Patricia Ann Estock, secretary; Howard Bell, treasurer; John Mowbray, corresponding secretary; Winston Crawley, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President - Suzanne Moaut
7 actives, 5 pledges
Ralph Ciarroco and Suzanne Moaut were initiated on February 13, 1979. Former KME members, JoAnn Dellavalle, '70, with Prudential Insurance Company, and Susan Capozio, '78, with Environmental Agency, spoke to the chapter about mathematics they use in their work. They were especially grateful to KME because they worked on special assignments and problems found in **The Pentagon.** The chapter also sponsored free tutorial service this past year. Other 1979-80 officers: Cynthia Bodziak Simone, vice president and treasurer; Glenn Pelter, secretary; Sister M. Grace, corresponding secretary and faculty sponsor.

Tennessee Alpha, Tennessee Technological University, Cookeville
Chapter President - Teresa Johnson
100 actives, 57 pledges
Six major activities included initiation of new members,

cooperating with the mathematics club to provide refreshments, and aid in paper grading for the high school mathematics contest. Other 1979-80 officers: Jeff Baines, vice president; Angela Howard, secretary; Stan Harper, treasurer; Evelyn Brown, corresponding secretary; Donald Ramsey and Bryan O'Conner, faculty sponsors.

Tennessee Beta, East Tennessee State University, Johnson City
Chapter President - Mike Bell
12 actives, 9 pledges
Following the banquet meal and address by Dr. Jack Higgs of the English Department, the initiation ceremony was conducted for nine new members. Regina Wice received the annual KME mathematics award. Other officers remain as published in the spring issue.

Tennessee Gamma, Union University, Jackson
Chapter President - John Brown
19 actives, 8 pledges
The annual initiation banquet was held on April 21, 1979. Eight new members were initiated into KME. Professor Charles Baldwin presented an interesting program on solar energy. Other 1979-80 officers: Joel Brown, vice president; Kathy McClatchey, secretary; Rose Steed, treasurer; Nancy Blass, reporter-historian; Richard Dehn, corresponding secretary; Kyle Hathcox, faculty sponsor.

Texas Eta, Hardin - Simmons University, Abilene
Chapter President - Cynthia Young
37 actives, 10 pledges
Ten new members were initiated at the annual spring banquet on March 30, 1979. Guest speaker for the banquet, Dr. Bill Dulin, chairman of the Mathematics Department at McMurry College, Abilene spoke on "Mathematical Odds and Ends." New members are Terry Biggs, Nancy Chege, Austin Cooper, Jeff Dart, Anita Meeker, LeRoy Michaelson, Randal Phillips, Phil Terrall, Dr. John Peslak. Officers will be elected in the fall 1979 semester except for Karla Smith, secretary. Anne Bentley continues as corresponding secretary.

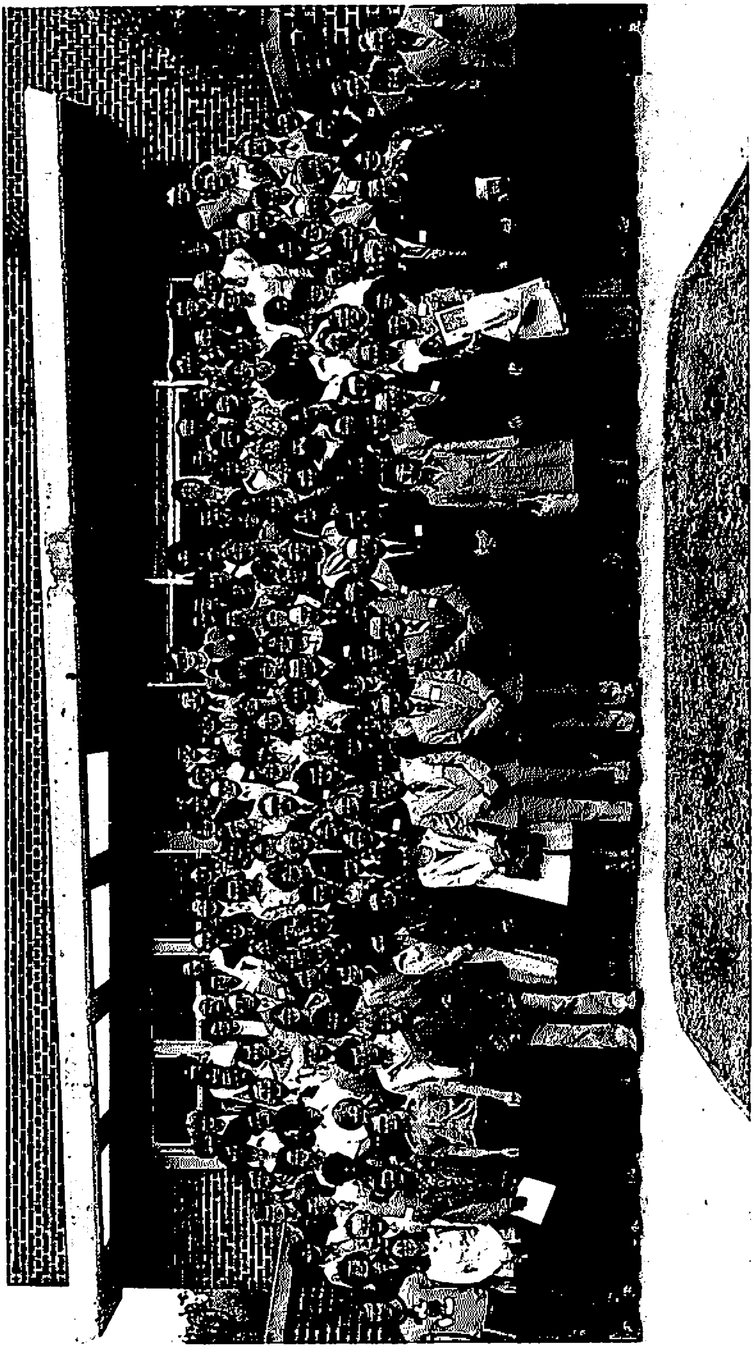
Virginia Beta, Radford College, Radford
Chapter President - June Crawford Howell
20 actives, 6 pledges
The chapter built flower boxes for the front of the mathematics building and planted them with red geraniums. They also sponsored

a film festival and held a potluck supper in honor of new members. The semi-annual math majors picnic was co-sponsored by Virginia Beta. Other officers remain as published in the spring issue.

West Virginia Alpha, Bethany College, Bethany
Chapter President - David Hayes
15 actives, 13 pledges
The initiation of new members was held on May 9, 1979. Other 1979-80 officers: Dan Martich, vice president; Terryl Beriman, secretary and treasurer; David T. Brown, corresponding secretary and faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
Chapter President - Jane Simeth
10 actives, 2 pledges
The chapter held a doughnut sale on March 6 to raise money to send a student to the National Convention. Two new members were initiated into the chapter on March 9. Several former KME members returned for the initiation ceremony. Following the initiation, the group went out for dinner. Other 1979-80 officers: Eileen Korenic, vice president; Mary Pat Ganzer, secretary and treasurer; Sister Mary Petronia Van Straten, corresponding secretary and faculty sponsor.

Wisconsin Beta, University of Wisconsin, River Falls
Chapter President - Mark Hof
12 actives, 19 pledges
The chapter held meetings twice a month with guest speakers or professors at the university giving talks. The members serve as tutors to elementary mathematics students. Wisconsin Beta participated in a college bowl with Universities of Wisconsin from Eau Claire, Stout, and Superior. The chapter has plans for a picnic this fall and making Christmas decorations. Other 1979-80 officers: Jeff Teeters, vice president; Gail Norderhaug, secretary; James Griffin, treasurer; Lyle Oleson, corresponding secretary; Ed Mealy, faculty sponsor.



Kappa Mu Epsilon Convention, April 26-28, 1979
Pittsburg State University, Pittsburg, Kansas

INSTALLATION OF NEW CHAPTERS

EDITED BY LORETTA K. SMITH

Information for the department should be sent to Mrs. Loretta K. Smith, 829 Hillcrest Road, Orange, Connecticut 06477.

ILLINOIS THETA CHAPTER

Illinois Benedictine, Lisle, Illinois

The Illinois Theta Chapter of Kappa Mu Epsilon was installed at Illinois Benedictine College in Lisle, Illinois on 18 May, 1979 by the immediate past Historian, Sister Jo Ann Fellin. Sister Nona Mary Allard of Illinois Zeta, as conductor, introduced the following students for acceptance as charter members:

Alan Burnham	Kristi Keeler
Michael Davern	Lisa Kozloski
John Doherty	Maureen Maney
Steven Earle	Judith Nocek
Robert Gallee	Susan Rudzinski
Gregory Gorski	Mahnaz Shahidi
Patricia Gramme	David Spokas
James Haidu	Chung Cheng Tai
Denis Harkin	Kerry Vrabel
Joseph Jirka	Mary Vrtis

The following faculty members were initiated as charter members of Illinois Theta: Rose Carney, Eileen Clark, Phyllis Kittel, Father Richard Shonka, and Father Paul Tsi. Faculty member Marvin Camburn, already a member of Kappa Mu Epsilon, will join in Illinois Theta activities.

Assuming the responsibilities of officers of Illinois Theta are: Mary Vrtis, president; Lisa Kozloski, vice-president; Patricia Gramme, secretary; Mahnaz Shahidi, treasurer; Phyllis Kittel, corresponding secretary and faculty sponsor.

Following the installation ceremony Sister Jo Ann Fellin gave a slide presentation which emphasized the values of learning and fellowship as seen in the lives of men and women mathematicians.

A reception completed the installation festivities.

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