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CONTENTS

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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Nim by the Sprague-Grundy Theory*

CRYSTAL A. FRITZ
Student, Shippensburg State College

What is an impartial game? According to J. H. Conway it is a game that satisfies two conditions, the normal play convention and the finishing condition. By definition the normal play convention is that a player loses if and only if he is unable to move when it is his turn to do so. The finishing condition demands that there must not be an infinite sequence of legal moves [1, p. 419]. Knowing that an impartial game must satisfy these conditions, a theory has been developed to handle the solution of these games. One such theory is the Sprague-Grundy theory [2, p. 122].

The Sprague-Grundy theory and its relationship to impartial games in general give rise to a whole new way to look at the solution of impartial games. Developing the theory and applying it to games also yields many interesting properties involving impartial games. One such game is Nim [1, p. 427].

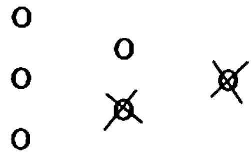
Nim is played with a number of counters arranged in any number of piles. An arrangement of piles for Nim might look like this:

```
O
      O
O           O
      O
O
```

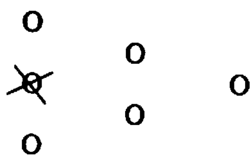
This grouping of 3, 2, 1 is very basic, but is still valid. The winning objective behind Nim is to be the last player to take counters from a pile, leaving the board clear. You may only take counters from one pile at a time and you may not remove counters from the center of any pile. Any number of counters can be removed from a pile at one time, starting at either end of the pile. For

*A paper presented at the 1977 National Convention of KME and awarded fourth prize by the Awards Committee.

example, taking the crossed off counters below is invalid because you are taking from two piles at one time.

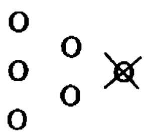


The following move is also invalid because a counter is removed from the center of the pile:

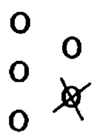


A sample Nim game might look like this:

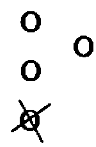
1. First player:



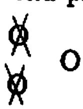
2. Second player:



3. First player:

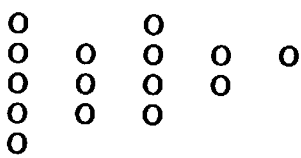


4. Second player:



5. First player:

The first player takes the last counter and therefore wins. This game of Nim is very basic and the best moves can be seen just by observing the combinations that exist. We can, however, look at a much more difficult example where best moves are not so obvious, such as



For cases such as this and even for the basic games a classical solution giving best moves was developed. This solution involves the conversion of the number of objects in the piles to binary numbers and then the addition of these numbers without carrying to determine if the position is even or odd. Using our first example, this addition would be

011	for the three counters in the first pile
010	for the two counters in the second pile
001	for the one counter in the first pile
000	

Thus the piles 3, 2, 1 yield all zeros which is an even position.

In Nim an even position consists of all zeros. Anything else is an odd position. All zeros is found to be a winning position. If a winning position does not exist, it is obtainable by changing one of the piles to reach this zero state [4]. For example, consider the following piles:

```
O
O   O   O
O   O
O
```

The addition of the piles using the binary approach yields

$$\begin{array}{r} 100 \\ 010 \\ \underline{001} \\ 111 \end{array}$$

To obtain a zero position from this point it is necessary to change all three 1's to zeros. To do this, it is necessary to change 100 to 011. This would give you the piles

```
O
     O
O         O
     O
O
```

Adding the binary representation of these piles we get

$$\begin{array}{r} 011\\ 010\\ 001\\ \hline 000 \end{array}$$

This is a winning position for Nim and now the second player can only change the piles to a position other than all zeros. The first player therefore wins because he can always change the piles back to zeros until he takes the last counter. We can play our original game over again using this binary strategy and using arbitrary moves:

$$\begin{array}{ccc} O & O & O\\ O & O & \\ O & & \\ & & \end{array} \qquad \begin{array}{r} 011\\ 010\\ 001\\ \hline 000. \end{array}$$

Initially we have a zero position, so the first player must move and cannot keep the value zero. Allowing the first player to move we get:

$$\begin{array}{ccc} O & \cancel{O} & O\\ O & \cancel{O} & \\ O & & \end{array} \qquad \begin{array}{r} 11\\ 01\\ \hline 10. \end{array}$$

The second player can change the pile of three to move to a zero position and have a winning position:

$$\begin{array}{cc} O & \\ \cancel{O} & O\\ \cancel{O} & \end{array} \qquad \begin{array}{r} 01\\ 01\\ \hline 00. \end{array}$$

The first player may now take either of the two remaining counters. Regardless of which counter he takes, he must leave one for the first player, thus allowing the first player to win.

The above game of Nim was somewhat trivial, but we can observe that the classical binary solution does work. There is, however, another solution for Nim that is not so well known. This solution involves the Sprague-Grundy theory of impartial games [2, p. 124]. R. P. Sprague and P. M. Grundy developed this theory independently of each other. Their theory analyzes games such as Nim and gives a method for determining the best strategies [1, p. 427].

Before tackling the theory, it is necessary to define the values *0, *1, *2, $\cdots$. These values, as defined by J. H. Conway, are assigned to Nim piles in the following manner [2, p. 122]:

A Nim pile of 0 elements has the value $*0$

A Nim pile of n elements has the value $*n$

These values are called Nim values or Nim numbers. From the Sprague-Grundy theory and using the values defined, we have four properties with which to manipulate the values $*0, *1, *2, \cdots$

(i) Every impartial game with only finitely many positions has one of the values $*0, *1, *2, \cdots$.

(ii) $(*a, *b, *c, \cdots) = *m$, where m is the least number from $0, 1, 2, \cdots$ that does not appear among $a, b, c, \cdots$. $*m$ is called the mex.

(iii) If $a, b, c, \cdots$ are distinct we have $*2^a + *2^b + *2^c + \cdots = *(2^a + 2^b + 2^c + \cdots)$.

(iv) $*n + *n = 0 (= *0)$.

These four properties allow us to perform arithmetical operations on the values $*0, *1, *2, \cdots$ in a manner similar to that of regular numbers [1, p. 427].

To illustrate how these properties work we can take the Nim piles 3, 4, 5.

```
O  O  O
O  O  O
O  O  O
   O  O
      O
```

Given these piles we have the values $*3$, $*4$, $*5$. Using (iii), we see that

$$\begin{aligned} *3 + *4 + *5 &= *2 + *1 + *2^2 + *2^2 + *1 \\ &= *(2 + 1 + 2^2 + 2^2 + 1) \end{aligned}$$

Using (iv), we have:

$$*(2 + 1 + 2^2 + 2^2 + 1) = *2 + (*2^2 + *2^2) + (*1 + *1) = *2.$$

Here $*2$ is the value of the game of Nim consisting of the piles 3, 4, 5.

To use the Sprague-Grundy theory on a game of Nim a winning position must be defined. From the Sprague-Grundy theory we get a $*0$ value for a winning position. This is the value you wish to pass to your opponent because it causes a second player win. A second player win is defined to be the position in which the first player cannot move or, stated another way, the second player takes the last counter and therefore wins. The second player is thought of as the first player to reduce the game to a zero value.

To reach a zero position in this example it is necessary that we change some pile so that $*2$ can go to $*0$. To do this we must change $*2 + *1$ to $*1$. That is, change the pile of 3 to 1:

$$\begin{array}{ccc} \cancel{O} & O & O \\ \cancel{O} & O & O \\ O & O & O \\ & O & O \\ & & O \end{array}$$

The value of these Nim piles is now $*1 + *4 + *5 =$ $*1 + *2^2 + *2^2 + *1 = *0$.

By using this method, Nim piles in a nonzero position can always be reduced to the winning strategy of $*0$.

If we now compare two games of Nim both starting the same, but one being solved by the binary method and the other by the Sprague-Grundy theory, we can observe the similarities.

Starting with piles of 4, 3, 1

$$\begin{array}{ccc} O & O & \\ O & O & O \\ O & O & \\ O & & \end{array}$$

and using the binary approach, we have

$$\begin{array}{c} 100 \\ 011 \\ 001 \\ \hline 110 \end{array}$$

To make this all zeros we must change 100 to 010 giving

$$\begin{array}{c} 010 \\ 011 \\ 001 \\ \hline 000. \end{array}$$

When we use the Sprague-Grundy theory we get:

$$*2^2 + *2 + *1 + *1 = *2^2 + *2$$

To reduce this to zero we must change the 4 to 2. This would give

$$*2 + *2 + *1 + *1 = *0$$

Both of these methods require the same moves, changing the piles to look like

```
O  O
O  O  O
   O
```

If we continue this process we can determine that the moves turn out the same, unless there is a choice to be made between two equally suitable moves. If two strategic choices are made differently, the winning result still remains the same. This game of Nim is somewhat trivial, but it still illustrates the solvability of Nim by the Sprague-Grundy theory [2, p. 128].

This theory for solving impartial games is only a small part of a much larger exposition by J. H. Conway in which he expands the known number system by using theories for solving games [1, p. 428]. Nim is only one of an entire class of games that this theory solves. Using the Sprague-Grundy theory, all of these are reducible to a form similar to Nim. This property allows the theory to solve many games and makes the Sprague-Grundy theory very powerful [2, pp. 122-135].

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One Dimensional Cryosurgical Simulation

JANET DANISON
Student, Muskingum College

"The essence of mathematics resides in its freedom." This statement made by Cantor makes no sense until the individual has actually experienced mathematical freedom. Often, a student of mathematics is presented with theory and that theory is not easily extended to practical applications (at least, in the student's mind). However, it has been, is being, and will be done.

In this article, a modified form of the heat equation is hypothesized to simulate cryosurgery. This writer has written a one dimensional, user-oriented computer program and makes the claim that this program can be used to simulate cryosurgery.

The research is based on an assertion of Dr. Alan Solomon, Oak Ridge National Laboratory, who claims that the soon to be explained method of solving certain partial differential equations can be used to simulate cryosurgery.

Problem Description

In untechnical terms, cryosurgery is a process in which tissue is killed by repeated freezing and thawing through the use of cold and warm probes of varying sizes and shapes [2, p. 871]. Pioneering cryosurgical work was done in the field of brain surgery, especially in the surgical management of Parkinson's disease. A dime-shaped opening is drilled in the skull to permit the entry of the freeze-probe (cannula) [2, p. 871]. The exact site of probe insertion is verified by x-ray. The freezing of the area of brain tissue forms a permanent lesion which usually relieves the patient's symptoms. Cryosurgery has also been used to relieve benign or malignant obstruction of the prostate gland in addition to relief of gastric ulcers. Cryosurgical procedures seem to have advantages over other types of surgery. For example, there is a reduction in pain and less blood loss.

Use of the Heat Equation

The foundation of my computer program is a procedure for numerically solving a modified form of the heat equation. The problem is well posed. That is, there is a unique solution which depends continuously on the data of the problem. This data includes the initial time, and the conditions at the boundary of the region being studied. Starting with the simplest form of the heat equation which states that $T_t = \alpha T_{xx}$ where T_t is the first derivative of the function $T(x,t)$ with respect to x, and $\alpha = k/C\,\sigma$ is the physical constant representing thermal diffusivity with k the thermal conductivity, C the specific heat, and σ the density. Given an initial temperature $T = T_0$, and the position of the boundary, the heat equation can be solved. However, the position of the boundary moves and is not defined explicitly; additional conditions must be added. The first of these, $\sigma HX' = kT_x$ ($H =$ latent heat), describes heat flow in the system. The second, and last condition, states that when $T = T_{cr}$, the boundary location is $x = X(t)$.

The thermal diffusivity can be made a function of internal energy, E. We begin with an equation which describes the heat flow in the system, specifically $\sigma E_t + q_x = 0$, where $q = -kT$ is the flux. Thus, the following calculations yield E_t, the partial derivative of E with respect to t.

$$q = -kT = -k\,\frac{dT}{dE}\,E_x = \frac{-k}{C}\,E_x \text{ with } \frac{dE}{dT} = C$$

$$E_t = \frac{dE}{dT}\,T_t = CT_t$$

$$\sigma E = \left(\frac{k}{C}\,E_x\right)_x$$

$$E_t = \left(\frac{k}{C\sigma}\,E_x\right)_x$$

$E_t = (\alpha(E)E_x)_x$. The transformed heat equation in terms of internal energy is the result. We label this function $\alpha(E)$. Specifically, $\alpha(E)$ is defined as

$$\alpha(E) = \begin{cases} E \leq 0, \ \alpha_s \\ E \geq H, \ \alpha_l \\ 0 < E < H, \ 0 \text{ (but use small } \delta > 0 \text{ for program-} \end{cases}$$

ming purposes). Complications which arise in even the simplest case of cryosurgical simulation are numerous. Those complications resulting from two or more ice fronts are even more abundant.

Mathematical Procedures Used

The heat equation can be solved either numerically or analytically. The analytical method was used by this writer to check the results that were obtained numerically.

The computer program deals with two finite difference schemes. The first is an *explicit* scheme which states that for points $(j-1,n)$, (j,n), and $(j+1,n)$, the values of $(j,n+1)$ can be calculated in terms of the given points. Through a forward difference and central difference formula, E_j^{n+1} can be calculated.

$$E_j^{n+1} = E_j^n \left\{1 - \frac{\eta}{2}[\alpha(E_{j+1}^n) + 2\alpha(E_j^n) + \alpha(E_{j-1}^n)]\right\}$$
$$+ E_{j+1}^n \frac{\eta}{2}[\alpha(E_{j+1}^n) + \alpha(E_j^n)]$$
$$+ E_{j-1}^n \frac{\eta}{2}[\alpha(E_j^n) + \alpha(E_{j-1}^n)]$$
$$j = 1, \cdots, n-1, \ \eta = \frac{\Delta t}{\Delta x^2}$$

This method is fairly accurate, however it is time consuming and a stability condition exists such that $\alpha \frac{\Delta t}{\Delta x^2} < \frac{1}{2}$ where α is the larger of the two thermal diffusivities. If this condition is not met, the resulting calculations are not related to the physical problem. Thus, the author had to determine a better method for carrying out the computations.

An *implicit* method was tried which solves a tridiagonal system using Gauss elimination. A tridiagonal system consists of a matrix with nonzero values on the diagonal *and* the two adjacent diagonals, with zeros elsewhere. By a method similar to that of the *explicit* scheme stated above, values of E_j^n can be calculated.

$$E_j^n = -\eta\alpha(E_{j+\frac{1}{2}}^n)E_{j+1}^{n+1} + \{1 + \eta[\alpha(E_{j+\frac{1}{2}}^n) + \alpha(E_{j-\frac{1}{2}}^n)]\}E_j^{n+1} - \eta\,\alpha(E_{j-\frac{1}{2}}^n)E_{j-1}^{n+1}$$
$$j = 1, \cdots, n-1, \ A_1 = 0, \ C_{n-1} = 0$$

We have the system

$$A_jE_{j-1}^{n+1} + B_jE_j^{n+1} + C_jE_{j+1}^{n+1} = D_j$$

Gauss elimination can be used to solve the system. The next step in the research involved incorporating both the implicit and explicit schemes into a computer program.

Computer Program

The program itself is user-oriented. The operator may input the number of mesh points he/she wishes to use (the j's of the above), the size of the time step (that is, the $t + \Delta t$'s), and the length of the slab. The user may implant probes and thermocouples as desired. The thermocouples locate the points at which the temperature values will be printed as computer output. The output consists of these temperature values and a temperature gradient graph for each of the points.

In addition to the above features of the program which was written, the user may print out any scalar multiple of the time step desired, such that if he was trying to kill the tissue in a target area, he could place a thermocouple on the edge of the target area and ask for larger time steps until the thermocouple value approaches the temperature at which the probe is to be removed. Then he could take smaller time steps until the switching temperature is reached. The user may also terminate the program whenever he/she wishes.

We have calculated these values, but what do they mean? Are they accurate? Two methods were studied for analytically checking the numerical answers. The first is a polynomial approximation [4], and the second is an explicit similarity solution [1, pp. 281-289]. The percent difference between these two methods was determined by using

$$\text{percent difference} = \frac{|\,\text{analytical} - \text{calculated}\,|}{\min\,(\text{analytical, calculated})}$$

Results

In general, for the similarity solution, the distance values themselves of the explicit scheme were larger than those of the analytical scheme, however, those derived from the implicit method were smaller than from the analytical. The explicit scheme is more accurate than the implicit, but for larger time steps, the difference in accuracy is small for both methods. The polynomial approximation gives many of the same results.

Conclusions

In the future, optimal locations for one or more probes may be sought so that the tissue in the target area is killed, with the area of injured tissue being minimized. Changing probe temperatures, including the possible use of a warm probe could also be tested. A program could easily add flux conditions at the boundaries, simulating a major blood vessel, to see if this has any significant effect on the area of frozen tissue.

The program has been described and it does what is claimed. Having demonstrated this, one may see how the program could be used by a person modeling cryosurgery. As you see, mathematics is not all theory, and as Cantor said, the essence resides in its freedom.

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The Golden Ratio

MICHAEL KERNIN
Student, Niagara University

The Golden Ratio is a number denoted by the Greek letter Φ (phi), and is defined as follows. Let AB be a line segment of length L and let C be an internal point dividing AB into two segments of lengths A and B. Then the Golden Ratio is the ratio L/A or A/B, if they are equal (see figure 1).

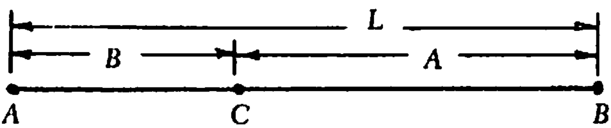

Figure 1

This number has been of interest to people for centuries. Euclid in Book II, 11, solved the problem of finding the Golden Section of a line. The Greek sculptor Phidias used the ratio and the Parthenon makes use of it. It was also used as the basis for a theory of beauty. Kepler investigated this ratio and termed it the "divine proportion". So it has a long history [2, p. 25].

A numerical value of phi may be found easily. Using the above diagram, let $B = 1$ and $A = X$. Then

$$\frac{x+1}{x} = \frac{x}{1} \text{ or } x^2 - x - 1 = 0$$

Solving for the positive root of X gives $\Phi = \dfrac{1 + \sqrt{5}}{2} =$ 1.61803, which is irrational. The negative root is Φ' and equals $-.61803$. These numbers are negative reciprocals of one another, namely $\Phi \cdot \Phi' = -1$. Also $\Phi + \Phi' = 1$, or $(\Phi^{-1}) + 1 = \Phi$. Phi is the only number whose reciprocal is 1 less than itself.

Another interesting property is found from the equation $\Phi^2 - \Phi - 1 = 0$ or $\Phi^2 = \Phi + 1$. This leads to $\Phi^3 = \Phi^2 + (\Phi) = 1 + 2 \cdot \Phi$. Continuing, one can construct an additive series whose values are determined by the formula $u_{n+1} = u_n + u_{n-1}$, with the first term $= 1$ and the second $= \Phi$, such that every term after the first corresponds to a power of Φ. Namely, 1, Φ, Φ^2, Φ^3,

$\cdots = 1, \Phi, 1 + \Phi, 1 + 2\Phi, \cdots$ The first is a geometric series of ratio Φ, the second an additive series. These series are equal and the series is called the Golden Series. It is the only series to be both additive and geometric.

Also of interest is the relation of Φ to the Fibonacci Series and continuing fractions. Taking the property that $\Phi = 1 + \dfrac{1}{\Phi}$ then $\Phi = 1 + \dfrac{1}{1 + \dfrac{1}{\Phi}}$. Forming successive continuing fractions yields that

$$\Phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$$

which is fascinating in itself, and was discovered by Girard in the 17th Century. By taking convergents of the continuing fraction for Φ, one gets the series (neglecting the 1) of 1, 1/2, 2/3, 3/5, 5/8, $\cdots$. Inspection shows these fractions oscillate about Φ^{-1}, and that the numerators and denominators of each fraction are successive Fibonacci Numbers. This suggests that the ratio of the n^{th} to the $(n - 1)^{\text{th}}$ Fibonacci Numbers is Φ as n approaches infinity, as was established by Binet [2, p. 148].

The Golden Ratio also presents several interesting geometric constructions and shows up in many places. The construction of a Golden Cut in a line segment is perhaps the simplest (see Figure 2). Given a line AB construct $BD = AB/2$ and $BD \perp AB$. Draw AD. With center D, radius DB, draw an arc cutting DA in E. With center A, radius AE, draw an arc cutting AB in C. Then C is the Golden Section of AB [2, p. 26]. The construction is proved by using the Pythagorean Theorem.

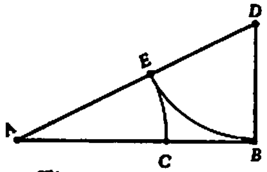

Figure 2

The line segment cut above lends itself to the construction of an angle of $\pi/5$ radians, of use in pentagon constructions. Using the same segment describe an arc with center C and radius CA (see Figure 3). Also cut an arc with the same radius and center B. The arcs meet at F and $\angle BAF = 36°$

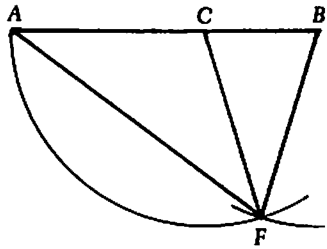

Figure 3

Further $\angle FCB = \angle FBC = 72°$ and $\triangle ACF$ is isosceles. A proof of the construction follows from either the law of cosines or law of sines.

Next there is the construction of a Golden Rectangle given a square. Let $ABCD$ be a square as in Figure 4. Bisect side AB at E. With center E and radius EC cut an arc intersecting AB extended at F. Drop a perpendicular from F to DC extended at point G.

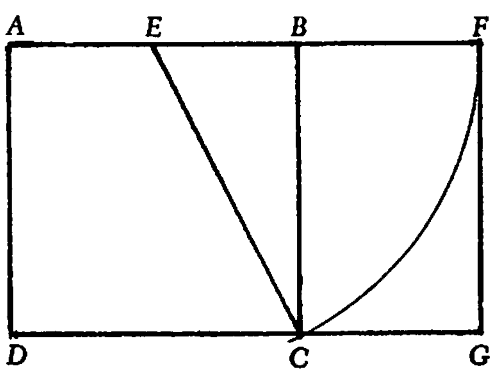

Figure 4

Then $AFGD$ is a Golden Rectangle, since it is easy to show that $\frac{AF}{FG} = \frac{1+\sqrt{5}}{2} = \Phi$ [2, p. 61]. Further $\frac{BC}{BF} = \Phi$, so also rectangle $BFGC$ is a Golden Rectangle. A spiral of Golden Rectangles can be made by subdividing each smaller rectangle into a square and a further Golden Rectangle. This spiral has interesting properties [1, p. 17].

The Golden Ratio also shows up in problems where it is not expected. Consider the problem of inscribing a triangle within a given rectangle so that the three triangles thus formed are of equal area. Let $ABCD$ be the rectangle and APQ the triangle. Then the problem is to find the positions of P and Q. Consider Figure 5.

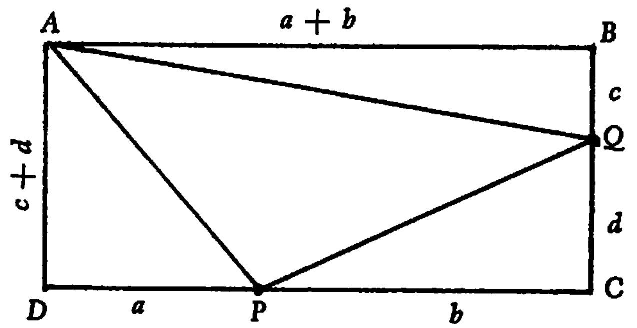

Figure 5

Since the 3 triangles are of equal area, $c(a+b) = bd = a(c+d)$ So that $bc = da$. Therefore

$$a = \frac{bc}{d}$$

And $\frac{bd}{c+d} = \frac{bc}{d}$ or $d^2 - cd - c^2 = 0$ or

$$\left(\frac{d}{c}\right)^2 - \left(\frac{d}{c}\right) - 1 = 0 \iff \frac{d}{c} = \Phi.$$

Since $\frac{d}{c} = \frac{b}{a}$, then $\frac{b}{a} = \Phi$

So the sides must be in the above ratios for the solution [2, pp.

93-94].

Finally, the pentagon and the related pentagram star, the symbol of the Pythagorean Brotherhood, are rich in the appearance of Φ. Let $P_1P_2P_3P_4P_5$ be a regular pentagon inscribed in a circle of radius OP (see Figure 6). Then the diagonals $P_1P_4 = P_2P_4 = P_3P_5$, and the angles such as $\angle P_4P_5P_1 = 108°$. The central angles formed by successive radii are $72°$ $(360° \div 5)$. Considering $\triangle OP_2P_3$, $OP_2 = OP_3 = r$ and $\angle P_2OP_3 = 72°$ so $\angle OP_2P_3 = \angle OP_3P_2 = 54°$, and similarly for the other 4 triangles congruent to $\triangle OP_2P_3$. Considering $\triangle P_1P_2P_3$, $\angle P_1P_2P_3 = 108°$ and $P_1P_2 = P_2P_3$ so $\angle P_2P_1P_3 = \angle P_2P_3P_1 = 36°$, and similarly for the four other triangles congruent to $\triangle P_1P_2P_3$.

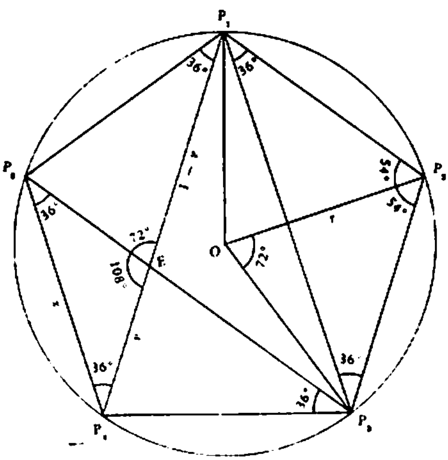

Figure 6

Next consider the diagonals P_1P_4 and P_3P_5 intersecting at E. As previously shown, $\angle P_5P_4P_1 = \angle P_5P_1P_4 = 36°$, and $\angle P_4P_3P_5 = \angle P_4P_5P_3 = 36°$. Then $\angle P_4EP_5 = 108°$. So

$\triangle P_4EP_5 \approx \triangle P_4P_5P_1$. Let $P_4P_1 = 1$, $EP_4 = v$, and $P_4P_5 = x$. Then $EP_1 = 1 - v$. By similar triangles, $\frac{v}{x} = \frac{x}{1}$. Since $\angle P_5EP_1 = 72°$ and $\angle P_5P_1E = 36°$, $\angle EP_5P_1 = 72°$ and $P_5P_1 = EP_1$. But $EP_1 = 1 - v$ and $P_5P_1 = x$, so that $1 - v = x$. Substituting in the first equation gives $x^2 + x - 1 = 0$. Then $x = \Phi$, so the ratio of a diagonal to a side of the pentagon is Φ. Also the point of intersection of two diagonals (such as E) divides each diagonal in the Golden Ratio [3, p. 93].

The pentagram star, constructed by drawing the diagonals of the regular pentagon, also has an abundance of usages of Φ. Each diagonal is divided into three segments which yield three Golden Ratios. Thus the diagonal and its segments form a geometric progression. Other examples of Φ can be shown by examining the internal regular pentagon formed by the five diagonals, its circumscribed circle, and various line segments.

We have shown just a few of the appearances of Φ. So the Golden Ratio is very useful in many areas of mathematics.

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How to Build a Magic Square

RICHARD A. SMITH
Student, Morningside College

Magic squares are mathematical curiosities which have fascinated man from the time of the ancients. The first known record of a magic square (Figure 1) comes from China; this magic square is purported to have been brought to man on the back of a turtle from the River Lo [1, p. 219]. Interest in magic squares in the West was incited by the fifteenth century Italian mathematician Luca Pacioli (1445-1514), whose influence on the great German

4	9	2
3	5	7
8	1	6

Figure 1

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Figure 2

artist Albrecht Durer may be seen in the magic square in Figure 2, found in Durer's celebrated 1514 engraving entitled *Melancholia* [1, p. 326]. Although magic squares may be considered fascinating merely for their curiosity value, they have played a serious role in the development of mathematics, for the concern for such patterns led the Chinese to develop a method to solve systems of simultaneous linear equations by, of all things, column operations on matrices, almost 250 years before the birth of Christ [1, p. 219].

A magic square is defined to be a square array, or matrix, of integers, the sums in each column and each row being the same. For example, the sums in each column of the magic square in Figure 1 are each 15, as are the sums in each row. The order of a magic square is the number of rows, or equivalently, the number of columns, of the array. The magic square in Figure 1 is of order 3. There exists a relatively simple method for constructing a magic square of any odd order; that is, relatively simple to perform, but nontrivial to confirm for a general case. I shall demonstrate the method for a specific magic square of order 5 and outline a proof of the general algorithm [2, pp. 32-33].

Consider the square array of twenty-five empty squares in Figure 3. The columns are numbered 1,2,3,4,5 from left to right: the rows are similarly numbered frcm top to bottom. Let {c,r} denote the small square in the *c*th column and *r*th row. Choose one of the small squares, say {2,3}, and in this square place the integer 1. Now choose two integers *h* and *v* as counting parameters, *h* representing the number of squares counted in the horizontal direction from {2,3}, and *v* representing the number of squares counted in the vertical direction *after* *h* squares have been counted horizontally (the choices of *h* and *v* are not entirely arbitrary, as will be shown later); for this example, $h = 1$ and $v = 2$ have been chosen. Counting one square horizontally from {2,3} and two squares vertically, one reaches square {3,1}; place the integer 2 in this square. Continuing in this manner to locate the square in which the integer 3 is to be placed will take one outside the square; in this case, counting is performed as if the array were bent into a cylinder, so that 3 will be located in square {4,4}. Similarly,

	1	2	3	4	5
1					
2					
3					
4					
5					

Figure 3

	1	2	3	4	5
1			2		
2					4
3		1			
4				3	
5	5				

Figure 4

4 will be located in {5,2}, and 5 will be in {1,5}. The array will now appear as in Figure 4.

One may see that counting h and v from {1,5} will return one to square {2,3}, which is already occupied by 1. Hence, introduce two new counting parameters h' and v' (again, not entirely arbitrary) so that 6 may be placed in an unoccupied square; here $h' = 1$ and $v' = 1$ have been chosen. Counting one square horizontally and one square vertically from {2,3} will place 6 in {3,2}. Now revert to the original h and v parameters, so that 7 will be placed in square {4,5}, 8 in {5,3}, 9 in {1,1}, and 10 in {2,4}. Again, counting one square horizontally and two squares vertically will place 11 in the square already occupied by 6, so that one must

	1	2	3	4	5
1	9	18	2	11	25
2	13	22	6	20	4
3	17	1	15	24	8
4	21	10	19	3	12
5	5	14	23	7	16

Figure 5

use h' and v' to place 11 in square {4,1}. Continuing in this manner until the entire square is filled will produce the array in Figure 5.

It is a simple matter to verify that each column sums to 65, as well as each row. Hence the array in Figure 5 is a magic square. Note also that

$$65 = \frac{5(5^2 + 1)}{2}$$

One proceeds in an analogous manner for the general case. Begin with a square array of n^2 small squares, with the columns numbered

$1, 2, 3, \cdots, n$ from left to right, and the rows similarly numbered from top to bottom. Choose a small square $\{a,b\}$, and write 1 in this square. Then choose two counting parameters h and v such that $(h,n) = (v,n) = 1$, where the notation (c,d) denotes the greatest common divisor of c and d. Count h columns to the right and v rows up from square $\{a,b\}$; write a 2 in this square (if counting takes you outside the array, count as if the array were bent into a cylinder, so that 2 will be located in one of $\{a + h,\ b - v\}$, $\{a + h,\ b - v + n\}$, $\{a + h - n,\ b - v\}$, or $\{a + h - n,\ b - v + n\}$). Continue in this manner, writing $3, 4, \cdots, n$ in their appropriate squares.

Let $\{x_k, y_k\}$ denote the square in which the integer k has been written, $1 \leq k \leq n$. To locate k, a total of $h(k - 1)$ squares have been counted horizontally to reach column x_k; and a total of $v(k - 1)$ squares have been counted vertically to reach row y_k. Since counting was performed as if the array were bent into a cylinder, $1 \leq x_k \leq n$ and $1 \leq y_k \leq n$, and it must be the case that

$$\begin{aligned} x_k &\equiv a + h(k - 1) \pmod{n} \\ y_k &\equiv b - v(k - 1) \pmod{n} \end{aligned}$$

where $x \equiv y \pmod{z}$ means that $x - y$ is divisible by z.

Suppose that two of these squares, say $\{x_i,y_i\}$ and $\{x_j,y_j\}$, are the same, where $i \neq j$. Then

$$\begin{aligned} a + h(i - 1) &\equiv a + h(j - 1) \pmod{n}, \text{ and} \\ b - v(i - 1) &\equiv b - v(j - 1) \pmod{n}, \end{aligned}$$

Simplifying the above congruences, using the fact that $(h,n) = (v,n) = 1$, one obtains

$$i \equiv j \pmod{n}$$

But $1 \leq i \leq n$ and $1 \leq j \leq n$, and the above congruence is clearly impossible. Hence each of the integers $1, 2, 3, \cdots, n$ have been placed in a unique square.

Note that if one continues the same h,v counting process from $\{x_n,y_n\}$, $n + 1$ would be located in $\{x,y\}$ where

$$\begin{aligned} x &\equiv a + h(n + 1 - 1) \equiv a + hn \quad a \equiv \pmod{n} \\ y &\equiv b - v(n + 1 - 1) \equiv b - vn \quad b \equiv \pmod{n} \end{aligned}$$

But $\{x,y\} = \{a,b\}$ is already occupied by the integer 1. Hence, introduce two new counting parameters h' and v' such that $(h',n) = (v',n) = 1$, and $(hh' - vv',n) = 1$. Count h' to the right and v' up from $\{a,b\}$, and write $n + 1$ in this square. Then revert to the original h,v counting process to locate $n + 2, n + 3, \cdots, 2n$. The parameters h' and v' should be used only to locate $n + 1$, $2n + 1, 3n + 1, \cdots$, and $(n - 1)n + 1$. When n^2 has been inserted, the counting process is complete.

For $1 \leq k \leq n^2$, k can be shown to be in $\{x_k,y_k\}$, where

$$x_k \equiv a + h(k - 1) + h' \left[\frac{k - 1}{n}\right] \pmod n$$

$$y_k \equiv b - v(k - 1) - v' \cdot \left[\frac{k - 1}{n}\right] \pmod n$$

and $1 \leq x_k \leq n$, $1 \leq y_k \leq n$. Here $\left[\frac{k - 1}{n}\right]$ denotes the greatest integer $z \leq \frac{k - 1}{n}$. Also, since $(hh' - vv', n) = 1$, it can be shown that each square contains one and only one integer.

Write $k - 1 = qn + s$, where $0 \leq s \leq n - 1$. Then the entries in the cth column are just the $k = qn + s + 1$ for which $0 \leq q \leq n - 1$, and as $\equiv c - a - h'q \pmod n$. If K_c denotes the sum of k in the cth column,

$$K_c = \sum_{q=0}^{n-1} qn + \sum_{s=0}^{n-1} s + n = \frac{n(n^2 + 1)}{2}$$

The same can be shown for the sum in each row. Hence the array just constructed is a magic square.

Note that if n is even, h,v,h', and v' must each be odd to satisfy the conditions $(h,n) = (v,n) = (h',n) = (v',n) = 1$. But then $hh' - vv'$ is even, and $(hh' - vv', n) > 1$. Hence the method is not applicable for magic squares of even order. If n is odd, however, there usually exists a wide range of suitable values for h, v, h', and v' that will yield a magic square.

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2. Niven, Ivan and Zuckerman, Herbert S. *An Introduction to the Theory of Numbers.* 3rd ed. New York, 1972.

The Problem Corner

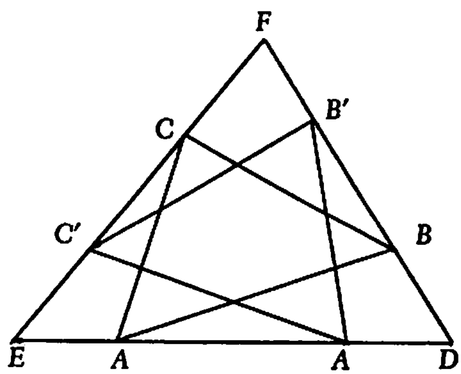
EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solusions of the following problems should be submitted on separate sheets before 1 August 1979. The solutions will be published in the Fall 1979 issue of **The Pentagon**, with credit being given for other solutions received. Preference will be given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

307. *Proposed by Fred A. Miller, Elkins, West Virginia*

Let A, B, C denote the vertices of a triangle which lie on sides DE, EF, and FD respectively of triangle DEF. Let $A'B'C'$ be a second triangle whose vertices lie on the sides of triangle DEF in such a way that A and A' are equidistant from the midpoint of DF, and B and B' are equidistant from the midpoint of DE, and C and C' are equidistant from the midpoint of EF as shown in the figure below. Prove that triangles ABC and $A'B'C'$ have equal areas.



308. *Proposed by John A. Winterink, Albuquerque Technical-Vocational Institute, Albuquerque, New Mexico*

On the sides of quadrilateral $ABCD$, isosceles right triangles ABP, BCQ, CDR, and DAS are constructed. Show that $PR = QS$ and PR is perpendicular to QS.

309. *Proposed by Richard A. Gibbs, Fort Lewis College, Durango, Colorado*

Once upon a time in a far away kingdom there lived many married couples. It came to the attention of the King (himself unmarried) that there were some unfaithful wives in his kingdom and he issued the following decree:

> "It has come to my attention that there are unfaithful wives in my kingdom. If a husband discovers that his wife is unfaithful he may slay her without punishment provided he does so on the day of the discovery."

Now, it so happens that if a man's wife were unfaithful he would be the only husband not to know it. Further, husbands never talked among themselves about the fidelity of their wives and an unfaithful wife was clever enough not to be caught by her husband.

Well, following the King's decree a month passed without incident. Then, on the 40th day, 40 unfaithful wives were slain; all that were in the kingdom.

The King was amazed! He summoned his Math Wizard for consultation and told him what had happened. The Wizard said, "That's not at all amazing." Prove that the Wizard knew that all unfaithful wives in the kingdom would be slain on the same day.

310. *Proposed by the editor.*

Consider the sequence of numbers 10001, 100010001, 1000100010001, etc. Are there any primes in this sequence?

311. *Proposed by the editor.*

A teacher of mathematics propounded the following addition problem: two numbers are selected at random and each succeeding number equals the sum of the two preceding numbers until a list of ten numbers is reached; e.g., starting with 365 and 142 the list to be added by the class was $365 + 142$

$+ 507 + 649 + 1156 + 1805 + 2961 + 4766 + 7727 + 12493$. Just as the teacher told the class to add these numbers, young Leslie Morely announced the sum to be 32571. Astounded, the teacher verified the correctness of Leslie's answer with a pocket calculator. Assuming that Leslie performed this feat mentally, how did he do it?

SOLUTIONS

The solution for problem 297 will appear in the next issue due to the printing error in the original presentation of the problem.

298. *Proposed by H. Laurence Ridge, Toronto, Ontario, Canada*

It is well known that all primitive pythagorean triangles (PPT) are generated by the formulae $x = 2ab$, $y = a^2 - b^2$ and $z = a^2 + b^2$ where a and b are positive integers of opposite parity and $(a,b) = 1$. Let N be an arbitrary positive integer. If N is the leg (or hypotenuse) of a PPT, it is possible to determine the PPT's of which N is a leg (or hypotenuse). What are the necessary and sufficient conditions for N to be a leg (or hypotenuse) of exactly one PPT?

Solution by the proposer.

Let N be an arbitrary positive integer. Then there are three cases to consider depending upon whether N is the odd leg, the even leg, or the hypotenuse of the PPT.

Case I. Let N be the odd leg. Then $N = a^2 + b^2$ for some positive integers a and b of opposite parity such that $(a,b) = 1$. Then $N = (a + b)(a - b)$ and $(a + b, a - b) = 1$ results from $(a,b) = 1$ and the opposite parity of a and b. For N to be the odd leg of a PPT it is necessary that $a - b = 1$ and $a + b = p^k$ for some odd prime p where k is a positive integer, otherwise N has at least one other representation as the difference of two squares. Knowing that $a + b = p^k$, the PPT can be determined easily. That this condition is also sufficient follows from the fact that for any odd prime p, p^k can be expressed in only one way as the difference of two squares, $a^2 - b^2$ with $(a,b) = 1$, because p^k splits into the product of two relatively prime factors in only one way.

Case II. Let N be the even leg. Then $N = 2ab$ where a and

b are subject to the same conditions as before. Then the odd member of the pair (a,b) must be 1 for otherwise ab could be split into the product of an odd integer and an even integer in the two ways $(a)(b)$ and $(ab)(1)$. W.L.O.G. let $b = 1$. Then if a contains an odd factor we must repeat the preceding argument. Hence $a = 2^k$ for some positive integer k. Thus for N to be the even leg of exactly one PPT it is necessary for $a = 2^k$ and $b = 1$ where $k > 0$. That this condition is also sufficient follows from the fact that if $N = 2^{k+1}$ for some positive integer k, then N has only one odd divisor, namely 1, so that N can be expressed in only one way in the form $2ab$ where $(a,b) = 1$ and a and b have opposite parity.

Case III. N is the hypotenuse of a PPT. Then $N = a^2 + b^2$ where a and b are subject to the same conditions as before. Now by [1, p. 270], for $(a,b) = 1$, N can have no divisors of the form $4m + 3$. Next suppose that $u = A^2 + B^2$ and $v = C^2 + D^2$ where $(u,v) = 1$. Then

$$uv = (AC \pm BD)^2 + (AD \mp BC)^2 \qquad (1)$$

This identity implies that N can be expressed as the sum of two squares corresponding to each factorization of N into relatively prime factors. But N has only one such representation because N is the hypotenuse of exactly one PPT. Hence it is necessary for $N = p^e$ where p is a prime of the form $4m + 1$ and e is a positive integer, for N to be the hypotenuse of exactly one PPT.

To show that this condition is also sufficient, note that by [1, p. 270] any prime p of the form $4m + 1$ has a unique expression as the sum of two squares, say $p = a^2 + b^2$. It follows easily that $(a,b) = 1$ and that a and b have opposite parity. We proceed by induction to show that p^e has exactly one representation in the form $R^2 + S^2$ with $(R,S) = 1$. Since equation (1) is valid without the restriction $(u,v) = 1$, we can suppose that $p^{e-1} = c^2 + d^2$ with $(c,d) = 1$ for otherwise $p|(c,d)$ so that p would be a factor of each of the squares in the representation of p^e. Then by equation (1),

$$\begin{aligned} p^e &= (a^2 + b^2)(c^2 + d^2) \\ &= (ac \pm bd)^2 + (ad \mp bc)^2 \qquad (2) \end{aligned}$$

Then since $(c,d) = 1$ either $c \equiv a \pmod p$ and $b \equiv d$

mod p) or vice versa. By symmetry assume that the conditions stated are true. Then $ac + bd \equiv a^2 + b^2 \equiv 0 \pmod p$ and $ad - bc \equiv ab - ab \equiv 0 \pmod p$ so we must have $(ac - bd, ad + bc) = 1$.

To establish this, let $(ac - bd, ad + bc) = \varepsilon$. Then either $\varepsilon = 1$ or $p|\varepsilon$ by equation (2). Now let $c = Kp + a$ and $d = Lp + b$ for some integers K and L. Thus $ad + bc = p(aL + bK) + 2ab$. Then $p|\varepsilon|\ (ad + bc)$ requires that $p|2ab$. But this requires either $p|a$ or $p|b$, both of which are impossible because $p = a^2 + b^2$. Hence $\varepsilon = 1$ and that the necessary condition is also sufficient follows by mathematical induction.

REFERENCE

1. Ore, Oystein, *Number Theory And Its History*, McGraw-Hill, 1948.

299. *Proposed by the editor.*

Devise a method for dividing a 17° angle into seventeen equal parts.

Solution by Frederick Cripe, Manchester College, North Manchester, Indiana.

Since a 17° angle is the sum of seventeen 1° angles, it suffices to construct a 1° angle. To do so, construct an 85° angle (= five 17° angles) and subtract it from a right angle. Use the resulting 5° angle to construct a 15° angle and subtract the 15° angle from a 17° angle. The resulting 2° angle is bisected to obtain the desired 1° angle.

Solution by Leo Sauve, Algonquin College, Ottawa, Ontario, Canada.

Bisect twice a constructible angle of 60° to obtain a 15° angle and proceed as in the preceding solution.

Solution by Richard A. Gibbs, Fort Lewis College, Durango, Colorado.

Since a 60° angle can be constructed, the solution follows by observing that $2 \times 60 - 7 \times 17 = 1$.

Editor's Comment: This geometric problem can be solved in many ways by using arithmetic considerations similar to those featured above. Other possibilities include

$$8 \times 17 - 3 \times 45 = 1$$
$$4 \times 17 - 60 = 8$$
$$2 \times 17 - 30 = 4$$
$$(72/4) - 17 = 1$$

where a power of 2 in a denominator or on the right side of the equation indicates the necessity of an appropriate number of bisections.

Editor's Comment: This problem was suggested by problem 96 which appeared in *Eureka* Volume I No. 10 (December 1975) at page 97. *Eureka* is now published under the name *Crux Mathematicorium*.

300. *Proposed by the editor.*

A and B play a game according to the following rules:

A selects a positive integer. B then must determine the number chosen by A by asking A not more than thirty questions, each of which can be answered by only no or yes. What is the largest number which A can choose which can be determined by B in thirty questions? Generalize to n questions.

Solution by: *Michael W. Ecker, City University of New York, New York, New York.*

It will be shown that for n questions, A may select any integer up to and including $2^n - 1$ if zero is a permissible choice and 2^n otherwise. Let N be the chosen number and consider the representation of N in base 2; i.e., $N = a_{n-1}2^{n-1} + a_{n-2}2^{n-2} + \cdots + a_2 2^2 + a_1 2^1 + a_0$ where each a_k is either 0 or 1. The kth question might be: "Is a_{k-1} equal to 1?" for $k = 1, 2, \cdots, n$. This procedure determines the binary representation for N from which N can be determined. Hence the largest possible choice of N for the given problem is $N = 2^{30} - 1$ if zero is a permissible choice and $N = 2^{30}$ otherwise. An alternate set of questions might be: "Is the number larger than _____?" B first uses 2^{n-1} to fill in the blank, and depending upon whether the answer is yes or no,

B next uses $2^{n-1} + 2^{n-2}$ or $2^{n-1} - 2^{n-2}$ respectively.

One incorrect solution was received.

301. *Proposed by the editor.*

If 65% of the populace has kidney trouble, 70% have diabetes, 85% have respiratory problems, and 90% have athlete's foot, what is the smallest portion of the populace who are afflicted with all four maladies?

Solution by Frederick Cripe, Manchester College, North Manchester, Indiana.

Let the population with kidney trouble bet set A, the population with diabetes be set B, those with respiratory problems be set C, and the population with athlete's foot be set D. Then, the population with all four maladies is:

$$A \cap B \cap C \cap D \quad \text{or} \quad (A' \cup B' \cup C' \cup D')'$$

To find the smallest portion of the population with all four diseases, we need to find the largest part of the population not afflicted by all four, which is $A' \cup B' \cup C' \cup D'$. This will be largest when A', B', C' and D' are disjoint.

$$35\% + 30\% + 15\% + 10\% = 90\%$$

so 90% of the population is not afflicted with all four diseases and 10% has all four maladies.

Also solved by Richard A. Gibbs, Fort Lewis College, Durango, Colorado.

The Book Shelf

EDITED BY O. OSCAR BECK

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, University of North Alabama, Florence, Alabama 35630.

Computer Methods for Mathematical Computations, G. Forsythe, M. Malcolm, and C. Moler, Prentice Hall, Inc., Englewood Cliffs, N.J., 1977, 271 pages, $15.95.

For those acquainted with the work of the late Professor Forsythe, this introductory text in numerical methods will be well appreciated. For those who have not had the opportunity to study Professor Forsythe's work this will provide an outstanding introduction to the spirit of numerical computation. The text provides an excellent supplement for a numerical analysis course and a good text in its own right for a numerical methods course.

The book begins with an excellent discussion of roundoff error and floating point computation. The other topics covered are linear systems of equations, interpolation, numerical integration, ordinary differential equations, nonlinear equations, optimization, least squares, random number generators, and Monte Carlo methods. There are ten FORTRAN sub-routines designed to represent state of the art of scientific computing and to be used in solving the extensive problems in the book. They are intended to be portable and useful outside the classroom as well. I think you will find all these goals are met.

The text is accessible to students who have studied a standard 3 semester calculus course and some linear algebra. An elementary knowledge of FORTRAN is also assumed. It is quite useful as a vehicle for self study. The extensive list of references gives the reader direction for further study. I strongly recommend this book to serious students of numerical computation.

James E. McKenna
SUNY College at Fredonia

Trigonometry, A New Approach, C. L. Johnson, Prentice Hall, Inc., Englewood Cliffs, N.J., 1978, 365 pages, $11.95.

The words "A New Approach" in the title refer to the book's treatment of the trigonometric functions one at a time rather than collectively. The author states that this method of presentation alleviates the confusion faced by a student when all six functions are presented at once.

Each of the trigonometric functions is given a careful and concise development. The unit circle is used extensively in defining the functions and in developing formulas and identities.

The book includes an abundance of illustrative examples and numerous figures. Each chapter concludes with a set of review exercises and a diagnostic test. The material is presented in a relaxed, easy-to-read fashion, though it seems that exactness is occasionally sacrificed for the purpose of clarity. Some—but not all—of the definitions are boxed off for emphasis.

The author has incorporated numerous step-by-step examples of how to use hand-held calculators in solving trigonometric problems. Particular attention is given to the accuracy involved in rounding off entries from both the calculator and from the tables in the book.

This reviewer agrees with the author's claim that the first five chapters provide all the necessary trigonometry for students enrolled in a two-year curriculum such as drafting, mechanical engineering, or electronics. The entire book should provide an excellent text for a basic pre-calculus trigonometry course.

Jack C. Sharp
Floyd Junior College (Georgia)

Foundations of Applied Mathematics, Michael D. Greenberg, Prentice- Hall, Inc., Englewood Cliffs, N.J., 1978, 656 pages, $18.95.

A text written primarily for first year graduate students or seniors in engineering, the prerequisites are stated as "the usual undergraduate four-semester sequence in calculus and ordinary differential equations, together with the general maturity and background of a senior or beginning graduate student. Knowledge of computer programming is not required." The "general maturity and background . . ." is the real prerequisite. Unless the reader has a more or less working knowledge of the material, there is just too much there to progress at any reasonable pace. Also, the author has a habit of saying something like "as we will see in chapter so and so", which is fine if one knows something about the subject but not so fine otherwise. For those who have not seen some of the topics before, there are excellent references, however.

From the beginning pages in which functions and functionals are introduced and a good foundation in real analysis (advanced calculus) given to the final sections on finite difference methods applied to partial differential equations, just about all the mathematics an engineering or science student could conceivably need is there, including the more modern aspects. For instance, concerning differential equations, "It has become more fashionable to discuss existence/uniqueness in terms of contraction mappings and the fixed point theorem."

The author writes as he might lecture. This makes for interesting reading, although occasionally it does make for slight problems, such as whether one has an exclamation point or a factorial notation at the end of an equation such as $x^2 - \sin x = 0!$ There are other minor incidents in the printing, and occasionally a statement, such as the "converse" part of the Fundamental Theorem of the Integral Calculus on page 10, which leaves something to be desired. But these hardly detract from the overall value of the work. Of course it is to be hoped that not all copies will have pages 509-540 missing and pages 541-572 duplicated as is true of the examination copy.

From the Preface, "I try to emphasize not only how the mathematics permits us to clarify and understand the physics but also how our physical insight can support the mathematics and to provide the key to finding the appropriate mathematical line of approach. Practical and numerical aspects are emphasized as well." The effort is highly successful. This reviewer recommends the work to all who teach mathematics for engineering or science students as well as to all such students whether as a text or a reference work. It should also be a valuable study for graduate mathematics majors who want to know more about applications.

F. Virginia Rohde
Mississippi State University

Mathematical Methods in the Physical Sciences, Merle C. Potter, Prentice-Hall, Inc., Englewood Cliffs, N.J., 478 pages, $18.95.

In this text we have a collection of advanced mathematical methods for students or workers in the physical sciences. This includes ordinary and partial differential equations, vector analysis, matrices, numerical methods, and complex variables. The author assumes a mathematical background which includes some ordinary differential equations.

The text is well written in a clear readable style. The format of the text is good.The emphasis is on methods rather than theory. Some FORTRAN computer programs are included in the text.

Problem sets are lengthy with a good selection of problems. Answers to selected problems are given.

The text should be a good textbook for a Junior or Senior level methods course. It would be a useful addition to the library of any student.

Ben F. Plybon
Miami University (Ohio)

Intermediate Algebra, Second Edition, John H. Minnick, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1978, 416 pages, $13.95.

This text is intended for college students who have studied algebra before but are not ready for trigonometry. As such, it starts with a three chapter review of beginning algebra. The review is structured so that the student is given new, more advanced material at the same time. Next is a chapter on equations and inequalities in two variables which treats parabolas with horizontal or vertical axes and central ellipses and hyperbolas. The fifth chapter deals with linear systems in two and three variables, including a discussion of row reduction of a matrix. Chapter six contains material on functions, progressions and the binomial expansion. Chapter seven presents material on logarithms and exponentials from both a table and a calculator approach. The book concludes with a brief discussion of complex numbers.

This text impresses me for several erasons. The reasonable physical size of the book is a pleasant relief from the usual atlas sized texts recently favored by publishers. Even with the small size, the pages have an open uncluttered look. The author has chosen material that should make the text more attractive for a precalculus course than some other recent algebra texts; for example, the inclusion of conic sections. This reviewer is especially impressed by the author's inclusion of word problems in as many of the exercise sets as possible. This consistent return to the subject of converting a verbal expression into a mathematical one should be an invaluable learning experience for the student. Finally, the review exercises for each chapter assure that the student has a sufficient number of practice problems available.

In conclusion, I would adopt this text over any of those I am currently familiar with.

R. M. Bullock
Miami University (Ohio)

Installation of New Chapters

EDITED BY LORETTA K. SMITH

Information for this department should be sent to Mrs. Loretta K. Smith, Hillcrest Road, Orange, Connecticut 06477.

MARYLAND DELTA CHAPTER

Frostburg State College, Frostburg, Maryland

The installation of the Maryland Delta Chapter of Kappa Mu Epsilon was held on 17 September 1978. Dr. James E. Lightner, National President of Kappa Mu Epsilon, conducted the installation ceremony and also gave a short talk on the history of honor societies. The following nineteen people were initiated into the society as charter members.

Dr. John H. Biggs, Associate Professor of Mathematics
Dr. John P. Jones, Department Chairman
Mrs. Agnes B. Yount, Instructor of Mathematics
C. Michael Dicken—President
Debra E. Boyd—Vice President
Janet G. Jessup—Secretary
Karen D. Appler—Treasurer

Susan A. Bratten
Peter K. Clardy
John D. Coleman
Michael T. Dominick
W. Michael Groves
Michael F. Jefferson
Leslie A. Johnson
Reinaldo M. Machado
William J. Milligan
Lynne E. Moskowitz
Dawn S. Razaewski
Kathleen D. Young

Mathematics faculty members, Dr. Walter Rissler and Mrs. Roberta White, who were already members of Kappa Mu Epsilon, will serve as Faculty Sponsor and Corresponding Secretary respectively.

A social hour followed the ceremonies at which time guests were invited to join in welcoming the new members of the Maryland Delta Chapter.

WISCONSIN GAMMA CHAPTER

University of Wisconsin—Eau Claire, Eau Claire, Wisconsin

The Wisconsin Gamma Chapter was installed 4 February 1978 by Dr. Wilbur Waggoner, National Treasurer. The ceremony was held at the University Student Center with the Chancellor of the University, Dr. Leonard Haas, and the Vice Chancellor for Student Affairs, Dr. Ormsby Harry, in attendance. The charter group consisted of fifty-one initiates and the dinner was attended by eighty-three initiates and guests. The charter members are:

Kristi Amdall
Debra K. Block
Victoria Boerner
DuWayne Boettcher
Pamela Breden
Joyce Breitweiser
Deborah Cummings
Lori Deitte
Alison Elwood
James Fowler
Jane Gasper
Kenneth Goranson
Mark S. Hall
Paul F. Hansen
Alan Hapke
Colleen Hilber
Kimberly Hill
Daniel Hsu
Edward Jessen
Linda Jezek
Diane Karloske
Linda Leupold
Kristine Ludvigson
Cheryl Lueders
Maureen Lynch
Mary Mihalyi
Gregory Miller
Thomas Paige
Frances Parrulli
Susan Pederson
Kathleen Perkins
Richard Petri
Jean M. Pluke
Gerald V. Post
Michael Rattle
Katharine Rohr
Catherine Ruff
Donald Sandman
Donald Santoski
Donna Schvetz
Robert Sindelar
Jennifer Sirek
Dr. Billie Earl Sparks
Daniel Splitt
Samuel Stagliano
Linda Swan
Lynn Vandrell
Susal Vandoorn
Paul Volkert
Dr. Lawrence Wahlstrom
Dr. Marshall Wick

The program consisted of the installation and initiation ceremony, a banquet, musical selection, and talks by the Chancellor and the installing officer. The officers of Wisconsin Gamma Chapter are:

President—Gerald Post
Vice President—Donald Sandman
Recording Secretary—Diane Karloske
Treasurer—Jean Pluke
Corresponding Secretary—Alvin Rolland
Faculty Sponsor—Alvin Rolland

Kappa Mu Epsilon News

EDITED BY SISTER JO ANN FELLIN, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Sister Jo Ann Fellin, Historian, Kappa Mu Epsilon, Benedictine College, North Campus Box 43, Atchison, Kansas 66002.

REPORT ON THE 1978 REGION I CONVENTION

PA Theta hosted the Region I meeting at Susquehanna University in Selinsgrove, PA on 29 April 1978. Carol Harrison, corresponding secretary, and Rick Erdman, president, coordinated the program. Four chapters were represented at the meeting—MD Alpha, MD Beta, PA Theta, and PA Lambda.

After registration and a welcome by Dr. Tyler, chairman of the mathematics department, five student papers were presented. The twenty-four participants then divided into two groups to discuss the purpose of KME. Following the luncheon Marsha Lehman, alumna of Susquehanna University and employee of Eastman Kodak in Rochester, gave a practical talk on career planning.

Jeff Towne, PA Theta, Susquehanna University, took first place with his paper on "Einstein and Intuition: Which Way Did He Go?" Second place went to Jeff Gates, MD Beta, Western Maryland College, for his presentation on "A Climbing Class of Critical Points." Other papers given included: "Arbitrarily Long Sequences of Consecutive Integers Which Are Not Sums of Two Squares" by Dave Eberly, PA Lambda, Bloomsburg State College;. "Counting the Gaussian Primes" by Cherie Sperling, PA Lambda, Bloomsburg State College; and "They Had Zero First" by Fran Pittelli, MD Alpha, College of Notre Dame of Maryland.

After the awards were made James E. Lightner, KME President, urged the students to begin work soon on papers for the April 1979 national meeting.

REPORT ON THE 1978 REGION IV CONVENTION

Nine Region VI chapters were present with approximately seventy-five in attendance at the regional meeting hosted by MO Iota at Missouri Southern State College in Joplin, MO on 15 April 1978. Chapters represented included IA Gamma, KS Alpha, KS Gamma, KS Delta, MO Alpha, MO Beta, MO Zeta, MO Theta, and MO Iota.

Mary Elick, corresponding secretary of the host chapter, coordinated the meeting. Terri O'Dell, MO Iota president, and Robert Dampier presided over the morning and afternoon sessions, respectively. Dr. Floyd Belk, Academic Dean, welcomed the group to the campus.

Ron Wasserstein, KS Delta, Washburn University, received the first place award of $25 for his presentation of "A Baseball Scheduling Model." The second place award of a KME white gold pin went to Gary Houle, MO Beta, Central Missouri State College, for his paper on "The Mathematics of Being Too Hot to Handle." The third place award of a KME white gold key was given to Gary Cobb, MO Zeta, University of Missouri-Rolla for "A Study of Sequences Convergent to Zeros of Polynomials."

Other undergraduate presentations were: "What's My Line?" by Richard Smith, IA Gamma, Morningside College; "Where Do You Live?" by Tom Smith, MO Iota, Missouri Southern State College; and "Fun and Games in Mathematics" by Laura Cowan, MO Beta, Central Missouri State University. In addition, Jim Carlson and Thea Barrett, KS Alpha graduate students, gave the featured paper on "The Golden Search and Multidimensional Optimization Applications."

Following the buffet luncheon Dr. Harold L. Thomas, Region IV Director, presented the awards and encouraged students to prepare for the national meeting to be hosted by KS Alpha on 26-28 April 1979.

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

Chapter President—Judy Muse
40 actives

One of the most interesting programs during the semester involved the game, New Elusis, recently described in *Scientific American.* The annual spring picnic brought old and new members together in a social setting. The climax of the year was the initiation of twenty-one new members followed by Dr. Eddy Joe Brackin's report of the fall national convention. Other 1978-79 officers: Wade

Auten, vice-president; Mary Ann Stratford, secretary and treasurer; Jean T. Parker, corresponding secretary; Oscar Beck, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President—Jeff Jones
58 actives, 40 pledges

Over 100 students participated in a math field day sponsored by the chapter for all the Junior Highs in the county. Chapter members assisted the mathematics department faculty in the annual open house, *Poly Royal,* during which time over 600 high school students participated in the mathematics contest. The chapter sponsored a Career Day with six speakers from business and industry. Dr. John Todd of Cal Tech spoke at the spring banquet following the initiation of forty new members into the chapter. Other 1978-79 officers: Cindy Brophy, vice-president; Jane Hansen, secretary; Carolyn Bircher, treasurer; George R. Mach, corresponding secretary; Adelaide Harmon-Elliott, faculty sponsor.

California Delta, California State Polytechnic University, Pomona

Chapter President—Karen Britt
20 pledges

Chapter members set up math displays for *Poly Vue* and offered free tutoring during the semester. The chapter continues to give $50 book scholarships, one per quarter. Two faculty-student get-togethers were held. Other 1978-79 officers: Suzanne Crawford, vice-president; Jeffrey Eakins, secretary and treasurer; Richard Robertson, corresponding secretary; Cameron C. Bogue, faculty sponsor.

Florida Beta, Florida Southern College, Lakeland

Chapter President—Lynne Gardner
25 actives

Other officers: Nelia Miller, vice-president; Diane McClelland, secretary and treasurer; Henry Hartji, corresponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton

Chapter President—Wade Gibbs
21 actives, 8 pledges

For the second straight year GA Alpha had a 100 percent response from those students invited to membership. The 15 May ceremony was followed by a reception in honor of the eight new initiates. During the spring quarter chapter members prepared a display table for Math Day, a special day for high school students to visit the West Georgia mathematics department. Other 1978-79 officers: Teresa Stamps, vice-president; Phyllis Walker, secretary; Brenda Dale Jones, treasurer; Thomas J. Sharp, corresponding secretary and faculty sponsor.

Illinois Alpha, Illinois State University, Normal

Faculty sponsor—Orlyn Edge

Illinois Eta, Western Illinois University, Macomb

Chapter President—James Harold Lutzow
14 actives

The spring of '78 was the beginning of new KME involvement on the campus of Western Illinois University. Through the energetic desires of new officers and sponsors the breath of new life arose in the chapter and brought about a closer relationship between the mathematics faculty and students. The chapter inaugurated the first annual Math Appreciation Week. Activities scheduled throughout the week included a bake sale, a student conducted colloquium, a presentation on career opportunities for students of mathematics, a fun day in all mathematics classes, and a happy hour for students and faculty. The week's activities were climaxed by the hosting of the regional ICTM conference at Western Illinois. Illinois. As a service to the mathematics department IL Eta sponsored two film showings during the semester and plans to bring noted speakers to the campus next fall. The chapter honored the sponsors and presented certificates to new members at the honors dinner. Other 1978-79 officers: Gina Griffith, vice-president; Terri Weishar, secretary and treasurer; Kent Harris, corresponding secretary; Larry Morley, faculty sponsor.

Indiana Gamma, Anderson College, Anderson

Chapter President—Dwight Stewart
15 actives, 6 pledges

The 16 April initiation ceremony included a dinner sponsored by the Anderson College mathematics department. Other 1978-79 officers: Jay L. Collins, vice-president; James LeMay, secretary and treasurer; Stanley L. Stephens, corresponding secretary and faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President—Steven Hild
37 actives

The following students presented papers at the monthly meetings: Jill Roesch on "The Orchard Problem," Kaye Dickinson on "Computerized Tic-Tac-Toe" and Dan Mohr on "Where is the Point—An Investigation in Non-Euclidean Geometry." The December Christmas party which was canceled due to blowing and drifting snow was rescheduled as a Valentines' party in February at the home of Professor Ina Mae Silvey. Professors Ina Mae Silvey and E. W. Hamilton, both long time supporters of IA Alpha, retired at the end of the spring 1978 semester. IA Alpha participated in a reception held in their honor. Other 1978-79 officers: Bonnie Marlett, vice-president; Kaye Dickinson, secretary; Patricia Lange, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines

Chapter President—Sharon Carlson
8 actives, 3 pledges

The activities for the semester began with a party at the home of Professor Alex Kleiner. During the regular meetings, Sharon Carlson talked about Emmy Noether and other women in mathemativs, Mitch Adams discussed infinite products, and Mark Sand presented topics in number theory related to perfect numbers. As usual, the chapter co-sponsored, along with the mathematics department, the Freshman Math Contest and participated in the Drake Relays Street Painting Contest. At the annual banquet new members were initiated, officers for the 1978-79 academic year were elected, and the film "Adventures in Perception" was shown. Other 1978-79 officers: Mark Sand, vice-president; Jon Wells, secretary; Mitch Adams, treasurer; Basil Gillam, corresponding secretary; Joseph Hoffert, faculty sponsor.

Iowa Gamma, Morningside College, Sioux City

Chapter President—John Steele
27 actives

IA Gamma member Richard Smith presented a paper entitled "What's My Line" at the regional convention at Missouri Southern State College in Joplin, MO. Thirteen new members were initiated at the April meeting. Professor Alexander Mehaffey, Jr., guest speaker from the University of South Dakota, talked about the life and contributions of Karl Friedrich Gauss. At the Science Open House on 5 April IA Gamma sponsored five rooms with the following titles: mathematical careers, games and puzzles, and paradox box, computer science, and statistics. At the last meeting the chapter presented a plaque to Dr. Elsie Muller for her services as corresponding secretary of IA Gamma since the time of its installation on campus. The year closed with a panic on 30 April. Other 1978-79 officers: Janet Liibbe, vice-president; Julie Movall, secretary; Roger Bobolz, treasurer; Tom Trevathan, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg College, Waverly

Chapter President—Mark Behle
21 actives, 12 pledges

In addition to the regular monthly meetings the chapter sponsored a math field day for high school students. This activity was undertaken jointly with the Wartburg College mathematics faculty on 4 March. Other 1978-79 officers: John Tanner, vice-president; Mark Reinhardt, secretary; Susan Stockdale, treasurer; Christopher K. Schmidt, corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg

Chapter President—Howard Thompson
50 actives

KS Alpha began the spring semester with a banquet and initiation of twelve new members. At this February initiation meeting Theresa Audley, who received her degree in mathematics from Pittsburg State in 1977, explained her work with Southwestern Bell Telephone Company. Nonetta Thomas gave the March program on "25-point Geometry." At the April meeting Tom Pope reviewed the book *Flatlands*. In addition, Jim Carlson and Thea Barrett gave a trial run of the featured paper for the Region IV convention entitled "The Golden Search and Multidimensional Optimization Applications." Six students and six faculty members attended this regional meeting hosted by MO Iota on 15 April. Chapter members made a large contribution to the success of the Math Relays held on 25 April. They assisted the mathematics department faculty in the administering and the grading of the tests given at this annual event. Ronald Stockstill presented the May program on "Fermat's Last Theorem." At this meeting Ronald Stockstill and Diane Inloes received the annual Robert M. Mendenhall Award for scholastic achievement. Each received a KME pin from the chapter in recognition of this achievement. Other 1978-79 officers: Tom Pope, vice-president; Kay Conklin, secretary; Douglas Johnston, treasurer; Harold L. Thomas, corresponding secretary; J. Bryan Sperry, faculty sponsor.

Kansas Beta, Emporia State University, Emporia

Chapter President—Greg Hayward
35 actives, 14 pledges

Other 1978-79 officers: Karin Schroeder, vice-president; Gail Flippo, secretary; Lyla Brinkley, treasurer; Don Bruyr, corresponding secretary; Tom Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison

Chapter President—Dan Kaiser
24 actives, 5 pledges

April was an active month for KS Gamma. It began with a schoolwide computer dance sponsored by the chapter jointly with the biology club on 1 April. On 15 April students Connie Beal and Monica Sittenauer attended the regional meeting in Joplin, MO with Sister Jo Ann Fellin. That same day many of the students along with faculty Jim Ewbank and George Blodig handled the 8th biennial Math Tournament held at Benedictine for area high school students. Paul Coleman assumed the leadership role in coordinating the event. On 6 April Dr. Simone of the University of Missouri at Kansas City spoke on topological concepts. Alum J. C. Kelly of the University of Missouri at Columbia described his work with computer graphics in medical image processing at the 27 April meeting. KS Gamma initiated fifteen new members during the semester. Patricia McDonald is the recipient of the 14th annual Sister Helen Sullivan Scholarship. Sister Jo Ann Fellin will spend her 1978-79 sabbatical leave from Benedictine at the University of Ilinois in Champaign-Urbana. Other 1978-79 officers: Kristine Schmidt, vice-president; Joyce Heideman, secretary; Barbara Miller, treasurer; George Blodig, corresponding secretary; Jim Ewbank, faculty sponsor.

Kansas Delta, Washburn University, Topeka

Chapter President—Susan Glotzbach
20 actives, 8 pledges

The chapter sponsored a regional math day for high school students. Participating in the 16 March event were 280 area students. In early April Ron Wasserstein did a trial run for the chapter of the paper he prepared for the regional meeting. At the May meeting the chapter rejoiced with him on his success in presenting the first place paper at that meeting. Other 1978-79 officers: Douglas Stacken, vice-president; Brian Broadbent, secretary; Al Ross, treasurer; Robert Thompson, corresponding secretary; Allan Riveland and Ann Ukena, faculty sponsors.

Kansas Epsilon, Fort Hays State University, Hays

Chapter President—Kenneth Eichman
20 actives, 13 new members

Thirteen new members were initiated on 1 May. Other 1978-79 officers: Dan Cress, vice-president; Terri Hooper, secretary and treasurer; Eugene Etter, corresponding secretary; Charles Votaw, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President—Laura Nesbitt
7 actives, 4 pledges

Fran Pitteli spoke on "Soap Bubble Geometry" at the March meeting. The April presentation entitled "Four Color Theorem Updated" was given by Cathy Bidini. Famous Solved and Unsolved Problems of Mathematics comprised the theme for the joint meeting of MD Alpha and MD Beta held on 25 April. Annette Pyzik ('80) and Maura Kelly ('81) discussed the angle trisection problem; Jeanette Filmore, faculty member of the College of Notre Dame of Maryland, presented the donkey puzzle; and Jeff Gates of MD Beta spoke on the last theorem of Fermat. At the regional conference at Susquehanna University Fran Pittelli ('79) presented a paper on Mayan mathematics called "They Had Zero First." Four new members, initiated on 9 May, were welcomed at an informal dinner after which Dr. Robert Hanson of Towson State University spoke to the group on "A Practical Application of the Problem of Appollonius." Other 1978-79 officers: Maura Kelly, vice-president and treasurer; Nancy Callanan, secretary; Sister Marie Augustine Dowling, corresponding secretary; Sister Delia Mary Dowling, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster

Chapter President—Jeffrey Gates
18 actives

In April MD Beta held a joint meeting with MD Alpha at Western Maryland. It consisted of two student presentations and a social hour. Two new members were initiated in April at the annual banquet. The chapter ran a booth in the annual May Day Carnival to raise funds for the Putnam and Spicer Awards. Jeff Gates received the Putnam Award for obtaining the highest local score on the Putnam exam. The Spicer Award is given each fall to the outstanding rising junior. The last recipient was Nancy Maitland. Other 1978-79 officers: Brenda Eccard, vice-president; Nancy Maitland, secretary; Barry Whiteley, treasurer; James E. Lightner, corresponding secretary; Robert Boner, faculty sponsor; William Link, historian.

Mississippi Alpha, Mississippi University for Women, Columbus

Chapter President—Lynn Williams
15 actives, 12 lpedges

Other 1978-79 officers: Linda Larger, vice-president; Pam Rowe, secretary and treasurer; Jean Ann Parra, corresponding secretary and faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg

Chapter President—Myra Lea Thornton
25 actives

Initiation of nine new members was held in conjunction with the spring cook-out on 21 April. The KME Freshman Math Award was presented to Terry Martin. Billie Parrott, Director of Cooperative Education at the University of Southern Mississippi, spoke on opportunities for the mathematics major in the Co-op program. Other 1978-79 officers: Gary Applegate, vice-president; David A. Green, secretary; Alice W. Essary, corresponding secretary; Ed Oxford, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield

Chapter President—Charles Armstrong
34 actives

The faculty of the mathematics department contributed the following presentations at the monthly meetings this past semester: "Combinations of Indistinct Objects" by Dr. Melvin Foster in February, "What is Topology" by Dr. Edward Huffman in March, and "Metric Spaces: A Non-archimedean Metric" by Dr. David Lehmann in April. The annual picnic was held in May. Bruce Campbell received the KME Merit Award. He was recognized as the member "who has contributed the most to achieving the goals of KME" during the past year. Other 1978-79 officers: Donna Newton, vice-president; Rita Scroggins, secretary; Melanie Thornhill, treasurer; John B. Prater, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg

Chapter President—Kimberly Owen
24 members

The chapter took part in two professional meetings and the annual William Klingenberg Lecture. Four awards were given at the spring honors banquet. Two students were recipients of the Brown-Hemphill Scholarship. The outstanding freshman mathematics student and the outstanding senior mathematics student were each recognized for their achievements. Chapter members participated in the Math Relays. Other 1978-79 officers: Richard Komer, vice-president; Barbara Vogel, secretary; Rebecca Montgomery, treasurer; Homer F. Hampton, corresponding secretary; Alvin Tinsley, faculty sponsor.

Missouri Epsilon, Central College, Fayette

Chapter President—Janet Doll
5 actives, 4 pledges

Other 1978-79 officers: Arthur Sherman, vice-president; Barbara Hoover, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

Nebraska Beta, Kearney State College, Kearney

Chapter President—Douglas Dummel
20 actives, 4 pledges

Chapter meetings took place twice monthly, initiation was held 4 April and the spring picnic on 29 April. Other 1978-79 officers: Mike Sunderman, vice-president; Cynthia Niedfeldt, secretary; Jane Samp, treasurer; Charles J. Pickens, corresponding secretary; Randall Heckman, faculty sponsor.

Nebraska Gamma, Chadron State College, Chadron

Chapter President—Cindy Roffers
10 actives, 4 pledges

Regular meetings were held every first and third Thursdays of each month. On 14 April chapter members monitored and graded tests for the annual High School Scholastic Contest held at Chadron State College. Other 1978-79 officers: Bill Ferguson, vice-president; Carol Lukkes, secretary; Ginger Evans, treasurer; James A. Kaus, corresponding secretary and faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President—Chris Ashley
40 actives

Other 1978-79 officers: Tina Sibbitt, vice-president; Chuck Paine, secretary; Mike Wester, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara

Chapter President—Brian Covney
23 actives, 8 pledges

Eight new members were inducted at the March initiation banquet at which time Professor John J. Moore spoke on "How to Lose Your Shirt at the Horses." In April the chapter had a car wash to raise funds for expenses for the next biennial convention. Other 1978-79 officers: Maureen Swierczynski, vice-president; Denise Goodman, secretary; Nancy Heier, treasurer; Robert L. Bailey, corresponding secretary and faculty sponsor.

New York Theta, St. Francis College, Brooklyn

Chapter President—Bryan Black
8 actives, 5 pledges

Other 1978-79 officers: Anne Brennan, vice-president; Mary Lovaglio, secretary; Rosalind Guaraldo, corresponding secretary and faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green

Chapter President—Greg Poth

50 actives, 17 pledges

On 20 January several speakers shared information about their jobs in industry as part of Careers Night. The chapter received seventeen new initiates at the annual initiation banquet on 25 April. Maureen Bardwell gave a demonstration and talk on "Campanology: Mathematics and Bell-Ringing." Musical entertainment for the occasion was provided by "The Logarhythms," a barbershop quartet of mathematics professors. Chapter activities culminated with a picnic and softball game on 25 May. Other 1978-79 officers: Karen Blakemore, vice-president; Mike Kudlac, secretary; Evanne Webb, treasurer; Waldemar C. Weber, corresponding secretary; Dean A. Neumann, faculty sponsor.

Ohio Gamma, Baldwin-Wallace College, Berea

Chapter President—Kevin Cullen

25 actives, 15 pledges

Two field trips were organized by the chapter. The group visited the United States Steel Research Lab at Monroeville in April and toured the Cedar Point Computer Center in May. Other 1978-79 officers: Bill Peterjohn, vice-president; Joan Lowder, secretary; Gary Monda, treasurer; Robert E. Schlea, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord

Chapter President—Barbara Bauer

34 actives

At the February meeting Tom Tykodi reported on the European School Mathematics January interim program in which he participated. He described the different approaches to teaching mathematics K-12 in the countries that the group visited. New members presented talks at the initiation banquet on 30 March. Student talks were featured also at the 20 April meeting. The following Mathematics and Computing Science and KME awards were presented at the 3 May banquet: Freshman Mathematics Award to

Mark Hurst; Graduate School Award to Janet Danison; and Outstanding Senior Award to Becky Tucker. Other 1978-79 officers: Jeffrey Russell, vice-president; Mary Torchia, secretary; J. Bradley Sharp, treasurer; James L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Alpha, Northeastern Oklahoma State University, Tahlequah

Chapter President—Ray First
29 actives

OK Alpha sponsored a mathematics contest for high school students. Twenty-nine high schools were represented with 474 contestants participating in the event. Other 1978-79 officers: Jane Starr, vice-president; Stewart Ramsey, secretary and treasurer; Mike Reagan, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford

Chapter President—Maryetta Unruh
30 actives, 13 pledges

Chapter members assisted the mathematics department in hosting the annual Southwestern Interscholastic Meet on 15 April by grading tests. A bake sale was held that day to raise funds for chapter activities. Thirteen pledges were initiated on 20 April. In conjunction with the initiation the chapter organized a reception honoring Dr. Don Prock on his twentieth year of teaching in the mathematics department. OK Gamma decided to establish the tradition of honoring mathematics department faculty on their twentieth year of teaching. The year's activities closed with the spring picnic at Rader Park on 4 May. Other 1978-79 officers: Cindy Robertson, vice-president; Ray Moreau, secretary; Sherry Caton, treasurer; Wayne F. Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington

Chapter President—John Stafford
46 actives, 16 pledges

Several members attended the MAA district convention on 15 April in Pittsburgh, PA. The annual banquet was held at Troggio's on 26 April at which time sixteen new members were initiated.

The spring picnic on 12 May was on the campus. Other 1978-79 officers: John Robinson, vice-president; Alyce Marcotuli, secretary; Lauri Sassaman, treasurer; J. Miller Peck, corresponding secretary; Barbara Faires, faculty sponsor.

Pennsylvania Beta, La Salle College, Philadelphia

Chapter President—Karen Heist
7 actives, 16 pledges

Brother Damian Connelly, responsible for the establishment of the PA Beta chapter in 1953, died in May of 1977. Brother Connelly served as corresponding secretary and the guiding spirit of the chapter for these many years. The department at La Salle College has developed into a mathematical sciences department with a substantial computer science component. Chapter activities during the spring semester included the film "Regular Homotopy of Curves" and a lecture on game theory by guest speaker John McCleary. In May PA Beta initiated sixteen students and two faculty members, the largest number initiated in any year of the chapter's history. Other 1978-79 officers: Joseph Waldron, vice-president; Joseph Dibiase, secretary; Margaret Umberger, treasurer; Hugh N. Albright, corresponding secretary; Carl McCarty, faculty sponsor.

Pennsylvania Delta, Marywood College, Scranton

Sister M. Robert Ann von Ahnen, IHM, reports plans to interest students in joining PA Delta during the coming year. Dr. Marie Loftus continues as corresponding secretary.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President—Stephen Fox
20 actives

Activities included regular monthly meetings and a March initiation banquet. Other 1978-79 officers: Gary Rollman, vice-president; Joan Schifano and Meegan Davis, secretaries; Sharon Mazar, treasurer; William Jones, corresponding secretary.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President—Terry Gillis
26 actives

At the February initiation of six new members faculty adviser Ida Z. Arms spoke on "Women Mathematicians." Other talks during the semester included those of two other mathematics department faculty members. Doyle McBride and Merle Stilwell spoke on "Symbolic Logic" and "The Cycloid", respectively. Dr. Samuel Wieand, charter member of PA Zeta and currently a professor at the University of Pittsburgh, presented the featured address on "What Do Statisticians Do?" at the annual banquet in May. Other 1978-79 officers: Judith Brandt, vice-president; Cynthia Gurgacz, secretary; Lucille Ligas, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia

Chapter President—Jan Buzydlowski
6 actives, 3 pledges

Jan Buzydlowski was initiated into PA Kappa on 15 March at the Honors Convocation Ceremony. Several members have again contributed free tutoring opportunities to Holy Family College students. A Math Clinic Center is planned for the coming year. At the last meeting of the semester members played the game "Calcuball" which they purchased for the mathematics department. This will be one of the aids used to help Calculus students in the Clinic. Other 1978-79 officers: Cynthia Bodziak, vice-president and treasurer; Glenn Ritter, secretary; Sister M. Grace Kuzawa, corresponding secretary and faculty sponsor.

South Carolina Gamma, Winthrop College, Rock Hill

Chapter President—Cheryl R. Elrod
20 actives, 2 pledges

Activities included a talk by the Director of Placement and Career Planning on job opportunities in mathematics and a picnic honoring the graduating senior mathematics students. Other 1978-79 officers: Carol Susan Stephenson, vice-president; Charlotte Anne Ledford, secretary; Jovada Lynn Sims, treasurer; Donald Aplin, corresponding secretary; Edward Guettler, faculty sponsor.

Tennessee Beta, East Tennessee State University, Johnson City

Chapter President—Margie McGee
32 actives, 18 pledges

Following the banquet meal and address by Dr. Al Tirman, the initiation ceremony was conducted for eighteen new initiates. A KME award was presented to Melita Feathers as the outstanding senior mathematics major. Sallie Pat Carson, who retired in June of 1978, was recognized for her nineteen years of service to TN Beta. Other 1978-79 officers: Donald Evatt, vice-president; Lynda Ledford, secretary; Lyndell Kerley, corresponding secretary.

Texas Alpha, Texas Tech University, Lubbock

Chapter President—Margaret Street

20 actives, 21 pledges

Two Texas Tech professors spoke to the group during the semester. David Lutzer presented "Is There a Largest Set?" and Larry Whitlock talked on "Computer Security." TX Alpha provided help for the Texas Academy of Science Meeting at Texas Tech in March. The chapter held both a faculty-student party and an initiation banquet in April. Other 1978-79 officers: Donna Terrall, secretary; Jeff Clampitt, treasurer; John White, corresponding secretary and faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene

Chapter President—Cynthia Young

31 actives, 6 pledges

The following new members were initiated on 13 April: Bill Lacewell, Julie North, Karla Smith, Marcus Whitmore, Laurel Williamson, and faculty member Roland Steele. Other 1978-79 officers: Cymbe Walston, vice-president; David Simmons, secretary and treasurer; Anne B. Bentley, corresponding secretary; Charles Robinson and Edwin Hewett, faculty sponsors.

West Virginia Alpha, Bethany College, Bethany

Chapter President—Eugene Turley

14 actives, 7 pledges

New members were initiated on 11 May. Other 1978-79 officers: Perry Gaughan, vice-president; Elizabeth Penfield, secretary and treasurer; David T. Brown, corresponding secretary and faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President—Jane Simeth

10 actives, 2 pledges

The two new initiates to WI Alpha presented talks to the chapter before the formal initiation ceremony and dinner. Jane Simeth talked on "Calculations of the Circumerence of the Earth and the

Distance to the Moon by the Early Greeks" and Laurie Woodruff told about "Some Phases of Recreational Mathematics." On 8 April the chapter conducted a mathematics contest for young women. Sixteen high schools from the Wisconsin area participated with a total of sixty-five contestants. Awards were presented to the highest ranking individuals and a trophy was given to the highest ranking school. Other 1978-79 officers: Eileen Korenic, vice-president; Mary Grzechowiak, secretary; Debbie Schultz, treasurer; Sister Mary Petronia Van Straten, corresponding secretary and faculty sponsor.

Wisconsin Gamma, University of Wisconsin—Eau Claire, Eau Claire

Chapter President—Diane Karloske

Activities included hosting a college Math Bowl in April, participating in the University Honors Week, student papers at two monthly meetings, and a spring picnic. Other 1978-79 officers: Jean Pluke, vice-president; Frances Parrulli, secretary; Don Sandman, treasurer; Alvin Rolland, corresponding secretary; Tom Wineinger, faculty sponsor.