THE PENTAGON

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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Directions for Papers to be Presented at the Twenty-second Biennial Convention of Kappa Mu Epsilon

Pittsburg, Kansas
26-28 April 1979

A significant feature of this convention will be the presentation of papers by student members of **KME**. The mathematics topic which the student selects should be in his area of interest, and of such a scope that he can give it adequate treatment within the time allotted.

Who May Submit Papers? Any student member of **KME**, undergraduate or graduate, may submit a paper for use on the convention program. A paper may be co-authored; if selected for presentation at the convention it must be presented by one or more of the authors. Graduate students will not compete with undergraduates.

Subject: The material should be within the scope of the understanding of undergraduates, preferably those who have completed differential and integral calculus. The Selection Committee will naturally favor papers within this limitation, and which can be presented with reasonable completeness within the time limit.

Time Limit: The minimum length of a paper is 15 minutes; the maximum length is 25 minutes.

Form of Paper: *Four* copies of the paper to be presented, together with a description of the charts, models or other visual aids that are to be used in the presentation should be presented in typewritten form, following the normal techniques of term paper presentation. It should be presented in the *form* in which it will be presented, *including length*. (A long paper should not be submitted with the idea that it will be shortened for presentation.) Appropriate footnoting and bibliographical references are expected. A cover sheet should be prepared which will include the title of the paper, the student's name (which should *not* appear elsewhere in the paper), a designation of his classification in school (graduate or undergraduate), and a statement that the author is a member

of Kappa Mu psilon, duly attested to by the Corresponding Secretary of the student's Chapter.

Date Due: 22 January 1979.

Address to send Papers: Professor Ida Z. Arms
Department of Mathematics
Indiana University of Pennsylvania
Indiana, Pennsylvania 15701

Selection: The Selection Committee will choose about fifteen papers for presentation at the convention. All other papers will be listed by title and student's name on the convention program, and will be available as alternates. Following the Selection Committee's decision, all students submitting papers will be notified by the National Vice President of the status of their papers.

Criteria for selection and convention judging:

A. The Paper
   1. Originality in the choice of topic
   2. Appropriateness of the topic to the meeting and audience
   3. Organization of the material
   4. Depth and significance of the content
   5. Understanding of the material

B. The Presentation
   1. Style of presentation
   2. Maintenance of interest
   3. Use of audio-visual materials (if applicable)
   4. Enthusiasm for the topic
   5. Overall effect
   6. Adherence to the time limit

Prizes: The author of each paper presented at the convention will be given a two-year extension of his subscription to *The Pentagon*. Authors of the four best papers presented by undergraduates, based on the judgment of the Awards Committee, composed of faculty and students, will be awarded cash prizes of $60, $40, $30, and $20 respectively. If enough papers are presented by graduate students, then one or more prizes will be awarded to this group. Prize winning papers will be published in *The Pentagon*, after any necessary editing. All other submitted papers will be considered for publication, at the discretion of the Editor.

Piecewise Curve Fitting in an Engineering Application*

MICHELE L. HARTMAN
Student, Shippensburg State College

Mathematics has always been an intriguing discipline. Classroom work in itself can be enjoyable, but it is especially exciting to apply mathematics learned in the classroom to an actual on-the-job problem.

This past summer the author worked as a Student Engineer at AMP Incorporated in Harrisburg, Pennsylvania. AMP is one of the world's leading producers of electrical terminals and connectors. Much of the work involved the springs used inside these terminals to make an electrical connection with a wire and hold it in place. (Springs used for these purposes are not like a common coiled spring. These are extremely small strips of metal curved or bent so as to give in a certain way when a force is applied at some point on the spring.)

Electrical connectors are designed to be snapped together and apart numerous times, so it is vital to carefully analyze the strength of the springs involved. The stress and strain undergone by the material must be determined, along with the detection of any weakening in the spring from use.

Through experimentation, values of stress and the corresponding strain can be found for a particular material (Figure 1). This is an example of a typical stress-strain curve. Stress is the internal force applied to a sample of the material and strain is the resulting change in the dimensions of the sample.

Notice that over the line segment AB the relationship between stress and strain is linear or proportional. The material will be perfectly elastic in this range. If a force is applied to a sample of the material, it will be deflected, but it will return to its original position when the force is removed.

If the stress is increased beyond point B, however, the relation-

*A paper presented at the 1977 National Convention of KME and awarded first prize by the Awards Committee.

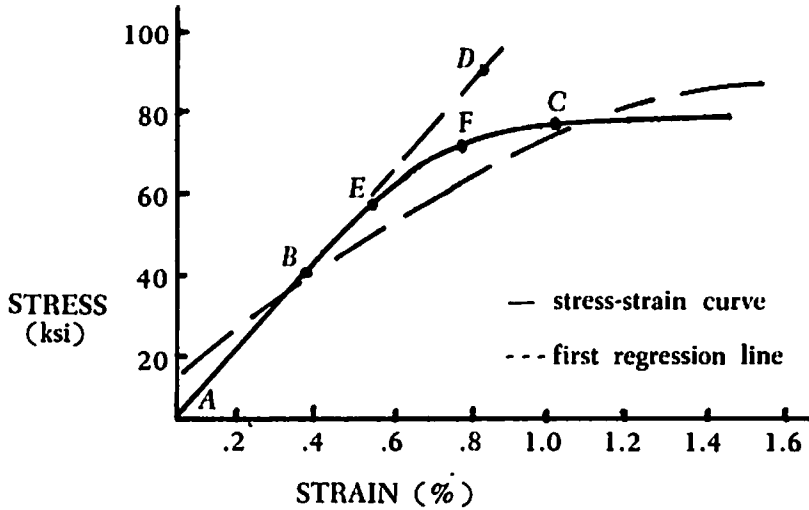

Figure 1

ship between stress and strain is no longer linear. When a force of this magnitude is applied to the material it becomes permanently weakened, and will not return to its original position when the force is removed. Eventually, the stress can remain almost constant—the force on the material does not have to be increased—and the material will continue to stretch.

As long as the stress is limited to the elastic range, an applied force and the resulting deflection of the spring will be proportional. The proportionality factor will equal the slope of the line. This factor is called the modulus of elasticity or simply, *E*. This quantity can be thought of as a measure of the stiffness of the spring.

As an example of spring analysis, consider the case of a simple cantilever beam (Figure 2a). The dimensions and material of the spring are known. Also, the maximum allowable stress in the spring has been predetermined. A force is applied to the spring (Figure 2b) until this maximum value of stress is reached. Equations (1) and (2) are then used to find the magnitude of the force and the resulting deflection. Notice that the modulus of elasticity is used in equation (2), and only one of the three quantities force, deflection, and stress is required for the other two to be calculated.

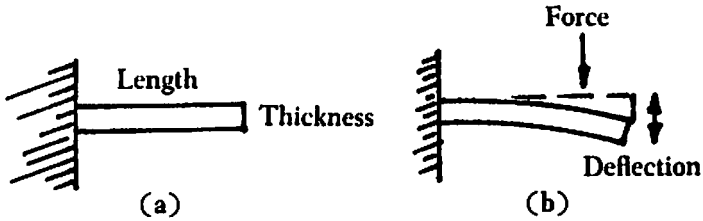

Figure 2

$$\text{Force} = \frac{\text{Stress} \times \text{Moment of Inertia}}{\text{Length} \times \frac{1}{2} \times \text{Thickness}} \quad (1)$$

$$\text{Deflection} = \frac{\text{Force} \times \text{Length}^3}{3 \times \text{Modulus of Elasticity} \times \text{Moment of Inertia}} \quad (2)$$

$$\text{Stress} = \frac{\text{Force} \times \text{Length} \times \frac{1}{2} \times \text{Thickness}}{\text{Moment of Inertia}} \quad (3)$$

In practice, it is not always necessary to use a perfectly elastic spring. Given that the stress does not exceed a specified limit, some distortion of the material is acceptable. For various production reasons, there is a significant cost savings in making springs which are not perfectly elastic. On the stress-strain curve, this means operating in the non-linear portion of the curve.

In the curved portion of the graph, however, there is no measurable modulus of elasticity. The slope of the tangent line to the curve can be found, but this value is not correct as a modulus of elasticity. As you will see, it does not describe the quantity we need. The value of E simply does not exist beyond the linear section of the stress-strain curve. If the analysis is continued, using the modulus obtained previously, the results will lie along the line AD in Figure 1, instead of being on the stress-strain curve BC. Hence, the analysis results will be inaccurate and invalid.

The problem, then, is how to perform the spring analysis when the stress-strain relationship is no longer linear. First, a value must be found for the modulus of elasticity. Over the linear portion of the curve, E was the slope of the linear segment. It is possible to use this idea by drawing the line segment AC from the origin to the point in interest (Figure 1). If the coordinates of point C are known, the slope of the segment AC can be calculated. While

this value is technically not the modulus of elasticity, it can be used in place of E in the formulas. Now the problem becomes one of fitting an equation to the curve so the coordinates of any point can be determined. Several curve fitting methods are available. I chose to work with least squares regression analysis.

The main idea of least squares linear regression analysis is to fit a regression line in the form $y = a + bx$ to the observed data points so that the sum of the squares of the vertical distances separating the points from the line is minimized. By calculating sums of the variables and applying various formulas, values for the coefficients a and b can be determined.

Correlation theory can then be applied to determine how well this regression line fits the original data points. Estimated values of y are found from the equation given by the regression analysis. The sums of the differences between these estimated values and the mean y of the observed points is the total variation. This total variation is equal to the explained variation plus the unexplained variation. When the explained variation is taken over the total variation, a ratio called the coefficient of determination is found. This quantity measures the degree to which the regression line fits the observed data points. Note that the value of this ratio will always lie between zero and one. A determination coefficient of one indicates perfect correlation; a coefficient of zero indicates no correlation.

These methods of analysis can be directly extended to curvilinear relationships. The equations of various types of curves can be put into a linear form by applying certain transformations to the variables and coefficients. The graph of the resulting equation is a straight line. A linear regression analysis can then be performed on this line.

For example, consider a series of data points appearing to be distributed exponentially. The general form

$$y = ab^x$$

describes this situation. The graph of this equation is an exponential curve.

Applying the log function to the general form we obtain

$$\log y = \log a + (\log b)x$$

Using the transformation, $z' = \log z$ we have the linear form

$$y' = a' + b'x'$$

Linear regression analysis is performed on the new equation, and values for a', b', and the coefficient of determination are calculated. Any necessary inverse transformations are then made on the coefficients a' and b', and the resulting values inserted into the equation for the exponential curve.

The resulting equation describes an approximating curve to the observed data points. There are many non-linear regression analysis. When this process is implemented on the computer, a number of transformations can be performed, determining which general type of curve best fits the data, and giving the equation of an approximating curve of this type.

Six types of equations were used in the computer program—the linear form, and five non-linear forms with the respective transformations shown below. The program performs a linear analysis on the input data points six times and chooses the most

GENERAL FORM	TRANSFORMED EQUATION
$y = a + bx$	none
$y = ae^{bx}$	$\log y = \log a + bx$
$y = ax^b$	$\log y = \log a + b \log x$
$y = a + \frac{b}{x}$	$y = a + b\left(\frac{1}{x}\right)$
$y = \frac{1}{(a + bx)}$	$\frac{1}{y} = a + bx$
$y = \frac{x}{(a + bx)}$	$\frac{1}{y} = b + a\left(\frac{1}{x}\right)$

successful method, selected by the highest coefficient of determination. The approximating curve found by this method is also returned as output. The curve fitting program was run using data points from the stress-strain curve in Figure 1.

The initial results were not very encouraging (Figure 1). The

regression analysis returned the curve giving the best possible fit to all the data points. But for the author's purposes, the curve was far too inaccurate.

The original curve was examined again, taking special note of the linear portion of the curve. Also, since very large stress values would render a material almost useless, and would therefore never be used, it was not necessary to fit an equation to the entire stress-strain curve. The curve fitting program was run again, but this time the curve was limited and broken into two sections *AB* and *BC*, the program was applied separately to each part.

As expected, the program fit a linear regression line to the segment *AB*. Although a curve was fit to the second section of the graph, the results still were not accurate enough. These springs are extremely small—the dimensions of most springs require four decimal places. It was absolutely necessary for the results to be precise.

Continuing with the same idea, the curve was broken into four sections (Figure 1), *AB*, *BE*, *EF*, and *FC*. This time all the coefficients of determination were greater than .9, which satisfied accuracy requirements.

Solving the equations two at a time, coordinates of the tangent points of the lines were calculated. A program was developed to trace the curve using each equation until the tangent point was reached. It would then shift to the next equation and continue in this manner until the curve was completed, accurately matching the original curve. Any stress-strain curve could be described with a series of equations and the tangent points of these equations. Desired values for the coefficients of determination could be obtained by breaking the curve into more or fewer sections.

Since the coordinates of any point could be found on the curve by using the correct equation, it became possible to obtain the slope of the secant line from the origin to that point. This slope is then taken as a value for *E*, the modulus of elasticity.

Expanding this idea, it was now possible to move by increments along the stress-strain curve until the limit of allowable stress was reached. At each increment point, the stress and modulus of elasticity could be used in the formulas (1), (2), and (3) to cal-

culate the force and deflection at that point. These force and deflection values are then graphed (Figure 3).

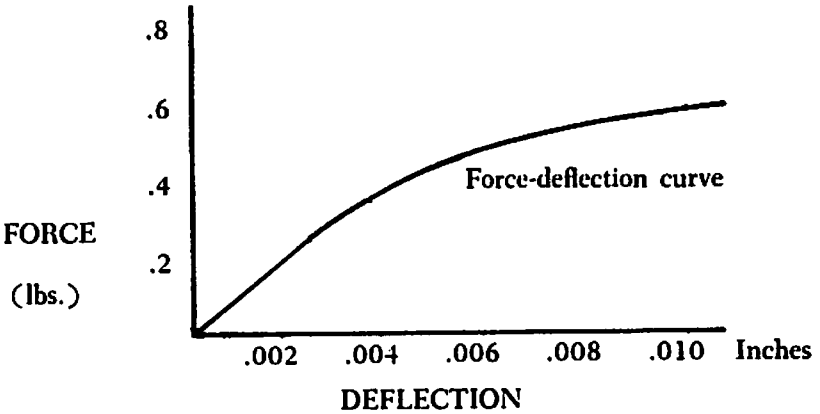

Figure 3

What is the end result? Instead of specifying the data for one particular point on the linear portion of the curve, it is now possible to determine a whole range of forces and the resulting deflections. In effect, we can trace the movement of a spring as the applied force is increased, even when the spring is no longer perfectly elastic. A static, limited analysis has been expanded into a dynamic one. This result illustrates how it was possible, and, in fact, necessary, to combine mathematics with physics and computer science to solve a vital production application.

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Where Is My Hat*

ROBERT KELLENBERGER

Student, Bloomsburg State College

The problem of the forgetful hatcheck girl is one of those problems which appears to be a simple probability exercise at first glance, but a more difficult one upon further inspection. However, with a little work, it is indeed an easy problem, with a very interesting answer.

The problem is this: Six men walk into a restaurant and each man checks in his hat before the meal. After the meal, as they leave, the hatcheck girl forgets whose hat is whose. If she hands them out at random, what is the probability that no man gets his proper hat?

In order to solve the problem, it is necessary to first state some basic principles of derangements. A series of elements placed in an assigned order is called an *arrangement.* If this arrangement is permuted so that no element is mapped to itself, then this permutation is called a derangement. For example, (b, c, a) is a *derangement* of (a, b, c) whereas (b, a, c) is not since the element c is mapped to itself.

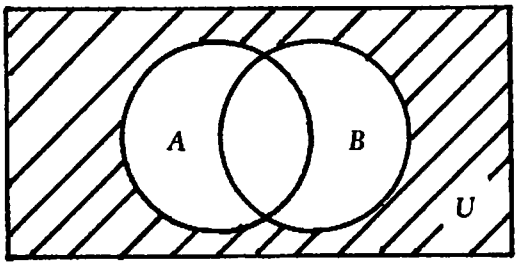
Before derangements are covered, it is necessary to digress and discuss the principle of inclusion-exclusion. If we have a set U, with two subsets A and B, and we wish to find the number of elements in the universal set, U, but in neither A nor B, we use the following formula:

$$|U| - |A| - |B| + |A \cap B| = |A' \cup B'|$$

To understand this it is only necessary to look at the Venn diagram below. After taking the total number of elements and subtracting those elements in A and then those in B, the elements in their intersection must be added on to make up for subtracting them twice.

After looking at the formula for counting the number of elements

*A paper presented at the 1977 National Convention of KME and awarded second prize by the Awards Committee.



not any one of three subsets of the universal set, it is easy to generalize for any number of subsets. The formula for three subsets is:

$$|U| - |A| - |B| - |C| + |A \cap B| + |A \cap C| + |B \cap C| - |A \cap B \cap C|$$

In order to generalize let us consider a set of n elements with various properties. Denoting these as $a_1, a_2, a_3, \cdots, a_n$, using the notation $N(a_i)$ to represent the number of elements with the property a_i, and keeping in mind that the properties are not mutually exclusive, the general formula is

$$\begin{aligned} N(a_1', a_2', a_3', \cdots, a_n') = n &- N(a_1) - N(a_2) - N(a_3) - \\ &\cdots - N(a_n) + N(a_1 a_2) + N(a_1 a_3) \\ &+ \cdots + N(a_{n-1} a_n) - N(a_1 a_2 a_3) \\ &\cdots - N(a_{n-2} a_{n-1} a_n) \\ &\vdots \\ &+ (-1)^n N(a_1 a_2 a_3 a_4 \cdots a_n) \end{aligned}$$

This formula can quite easily be proved correct by induction. Since this paper is not intended to discuss the inclusion-exclusion principle the proof is left for the reader.

Using this principle, it is now possible to determine the number of derangements (permutations where no element is mapped to itself). Leaving $N(a_i)$ equal the number of permutations where element i is mapped to itself we wish to find $N(a_1' a_2' a_3' \cdots a_n')$

In a set of n elements there are $n!$ possible permutations so the universal set is $n!$. If we count the number of permutations where the i^{th} element is mapped to itself we have $\binom{n}{1}$ ways of choosing the element and $(n-1)!$ permutations where at least one element is mapped to itself. Using the same rationale there are $\binom{n}{2}(n-2)!$ ways where at least two elements are mapped to themselves.

Now using the principle of inclusion-exclusion, we have the total number of derangements. That is,

$$n! - \binom{n}{1}(n-1)! + \binom{n}{2}(n-2)! - \binom{n}{3}(n-3)! + \cdots + (-1)^n \binom{n}{n}(n-n)!$$

Examining the individual terms it is easy to see that an $n!$ can be factored out. That is:

$$\binom{n}{1}(n-1)! = \frac{n!(n-1)!}{(n-1)!1!},$$

$$\binom{n}{2}(n-2)! = \frac{n!(n-2)!}{(n-2)!2!}, \cdots$$

Therefore the number of derangements is:

$$N(a'_1 a'_2 a'_3 \cdots a'_n) = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right)$$

To solve the problem it is only necessary to let $n = 6$ and grind out the answer. This proves to be quite a cumbersome task and lengthy computation should n be large. But if one looks at the expansion of e^x a correlation is obvious. Since

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

and letting $x = -1$,

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!} + \cdots$$

Therefore

$$N(a'_1 a'_2 a'_3 \cdots a'_n) = n!\frac{1}{e} + R$$

where R is a small fraction whose value is between

$$\frac{1}{n+1} \text{ and } \frac{1}{n+2}$$

Thus the number of derangements is the integer nearest $n!/e$.

Thus the probability of none of the six men receiving their own hat is approximately $\dfrac{n!\frac{1}{e}}{n!} = \dfrac{1}{e} \approx .367879$. The correct value is $.3680\overline{5}$. So indeed the estimate is rather accurate, especially for larger numbers. Interestingly enough, the probability does not vary to any large extent for any value of n. At first inspection this seems illogical, but as demonstrated here, the only change is to get asymptotically closer to e^{-1}.

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Finite State Machines*

JEFF TOWNE
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The purpose of this paper is to introduce a mathematical model of a *finite state machine* and to present an algorithm which can be used to minimize the number of *states* required to perform the function of any completely defined finite state machine. A *finite state machine* (or just *machine*) is defined as a 5-tuple $\{A, S, Z, \mu, \eta\}$. A is a set of input symbols for the machine. For the examples used here A will consist of the binary digits (which are the basic input symbols used by digital computers). Thus, we have $A = \{0,1\}$. Z is a set of output symbols. For the machines defined in this paper we will also use $Z = \{0,1\}$. $S = \{s_1, s_2, s_3, \cdots, s_n\}$ is a set of internal stable states of the machine. μ is a function $S \times A \rightarrow S$ which determines what the next state of the machine will be. μ depends on both the present state of the machine and the input symbol. η, a function from $S \times A$ into Z, determines what output symbol will be generated.

A *completely defined machine* (that is, one in which $\mu(s, a)$ and $\eta(s, a)$ are specified for all possible arguments) can be represented by a *state diagram* or a *state table* or both. For example, let a machine M be defined graphically by the state diagram in Figure 1. Each node of the graph represents a *state* of the machine.

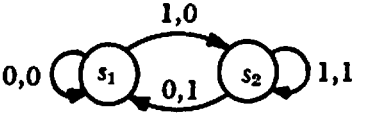


Figure 1—A State Diagram

*A paper presented at the 1977 National Convention of KME and awarded third prize by the Awards Committee.

Each directed arc is labeled with an ordered pair (a, z) where a represents the input symbol causing the machine to act, z represents the output symbol to be generated, and the arrow indicates the direction of the transition to be made.
The same machine can also be represented by the *state table* in Figure 2. We can see from either the diagram or the table that if the machine is in state s_2 when it receives an input of $a = 0$,

M	Next State μ		Output η	
Present State \ Input	0	1	0	1
s_1	s_1	s_2	0	0
s_2	s_1	s_2	1	1

Figure 2—A State Table

it will generate an output of 1 and go to state s_1. Thus $\eta(s_2,0) = 1$ and $\mu(s_2,0) = s_1$.

While the initial state must be uniquely specified in both μ and η, the input can consist of any number of input symbols. A series or string of input symbols will be denoted by $\alpha = a^0a^1a^2 \cdots a^{n-1}$, $\alpha \in A^n$. Similarly, an output string of length n can be denoted by $Z = z^0z^1z^2 \cdots z^{n-1}$, $Z \in Z^n$. In order to be able to handle input of the form $\alpha \in A^n$ we need to define two new functions, $\mu_n : S \times A^n \to S^n$ and $\eta_n : S \times A^n \to Z^n$. These new functions can be defined as follows. Given an input string $\alpha = a^0a^1a^2 \cdots a^{n-1}$, let $z^j = \eta(s^j, a^j)$ for $j = 0, 1, 2, \cdots, n-1$ where s^0 is given and s^j can be found by $s^j = (s^{j-1}, a^{j-1})$ for $j = 1, 2, 3, \cdots, n-1$. Letting $\alpha = 10010$ and $s^0 = s_1$, we have $\mu_5(s^0, \alpha) = \mu_5(s_1, 10010) = s_1s_2s_1s_1s_2$ and $\eta_5(s^0, \alpha) = \eta_5(s_1, 10010) = 01001$ for the machine in Figure 1 or 2.

In the design and construction of a finite state machine, it is desirable to minimize the number of internal states used to perform a given function. The reasons for this are mostly practical—the cost is directly proportional to and the reliability is inversely pro-

portional to the number of states in the machine. Given any completely defined machine, M, it is reasonable to ask if there is any other machine, $\overline{M}$, with an equal or smaller number of internal states that will produce the same output as M for any input string. Any such $\overline{M}$ is said to *cover* M. Obviously, every machine *covers* itself. A machine M is said to be a *minimal state machine* if there exists no machine which covers M and has fewer internal states than M. We can see, then, that in order to determine the optimum design for any completely defined machine, we need only to find a minimal state machine that covers the original machine. Before an algorithm for finding such a minimal state machine can be presented, however, some additional terminology and notation must be introduced.

We say that two states s_i and s_j are *r-equivalent* if and only if $\eta_r(s_i, \alpha) = \eta_r(s_j, \alpha)$ for all $\alpha \in A^r$. That is, s_i and s_j are r-equivalent if and only if, given any input string of length r, the machine produces the same output when it is started in state s_i as when it is started in state s_j. R-equivalence will be denoted by $s_i E_r s_j$ or $(s_i, s_j) \in G(E_r)$. If two states s_m and s_n are not r-equivalent, we write $s_m E'_r s_n$ or $(s_m, s_n) \in G(E'_r)$. If $s_i E_r s_j$ for all r, we write $s_i E s_j$ or $(s_i, s_j) \in G(E)$ and say that s_i and s_j are equivalent states. Observe that for any i and any r, $(s_i, s_i) \in G(E_r)$ (that is, $s_i E_r s_i$) since E_r is an equivalence relation (a relation in which the reflexive symetric, and transitive properties hold). For the same reason, if $(s_i, s_j) \in G(E_r)$, then $(s_j, s_i) \in G(E_r)$ for any i, j, and r. When listing the elements of $G(E_r)$, we will include either (s_i, s_j) or (s_j, s_i) (but not both) and omit (s_i, s_i) for brevity. Note that $G(E_r) \cup G(E'_r)$ is always equal to the set of all possible pairs of states in S. Thus, if we can find $G(E'_r)$, we can also find $G(E_r)$.

We can determine whether or not two states are 1-equivalent simply by looking at a state table. For example, consider the machine defined by the state table in Figure 3. We can see that

$$G(E_1) = \{(s_1, s_4), (s_1, s_5), (s_4, s_5), s_2, s_3)\} \quad \text{and}$$

$$G(E'_1) = \{(s_1, s_2), (s_1, s_3), (s_2, s_4), (s_2, s_5), (s_3, s_4), (s_3, s_5)\}.$$

M	Next State μ		Output η	
Present State \ Input	0	1	0	1
s_1	s_1	s_4	1	0
s_2	s_2	s_4	0	1
s_3	s_3	s_2	0	1
s_4	s_4	s_5	1	0
s_5	s_4	s_1	1	0

Figure 3

We now present without proof some results that provide the basis for our algorithm for finding a minimal state machine $\overline{M}$ which covers any given machine M. First, note that, for all $k \geqq r$, if $(s_i, s_j) \in G(E'_r)$ then $(s_i, s_j) \in G(E'_k)$, and if $(s_i, s_j) \in G(E_k)$ then $(s_i, s_j) \in G(E_r)$. This should be obvious from the definition of r-equivalence. Next, consider two states s_i and s_j which are $(r-1)$-equivalent: $(s_i, s_j) \in G(E_{r-1})$. If s_i and s_j are *not* equivalent for some input string of length r we conclude that there must be some (single) input symbol $a \in A$ that takes (s_i, s_j) into some (s_m, s_n) such that s_m and s_n are $(r-2)$-equivalent but not $(r-1)$-equivalent: $\mu(s_i, a) = s_m$, $\mu(s_j, a) = s_n$, $(s_m, s_n) \in G(E'_{r-1}) - G(E'_{r-2})$. These results tell us that in order to determine $G(E_r)$, we need only look at the elements of $G(E_{r-1})$ and see which pairs $(s_i, s_j) \in G(E_{r-1})$ are taken by some input symbol into some other pair $(s_m, s_n) \in G(E'_{r-1}) - G(E'_{r-2})$. All such

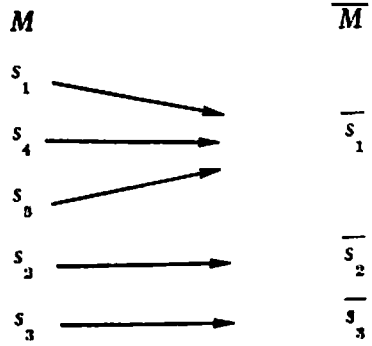
pairs (s_i, s_j) are removed from $G(E_{r-1})$ and added to $G(E'_{r-1})$ to obtain $G(E_r)$ and $G(E'_r)$, respectively.

The next important result follows from those already presented and provides the stopping criterion for the algorithm: If no new pairs of states are added to $G(E'_{r-1})$ to obtain $G(E'_r)$, then $G(E'_r) = G(E')$ and $G(E_r) = G(E)$.

Our algorithm consists of two basic steps—we will find, using the results presented above, all sets of equivalent states in M. We will then merge each set of equivalent states in M into a single state in $\overline{M}$. Since $\overline{M}$ will cover M and contain no equivalent states that could be merged, it will be the minimal state machine we seek. To demonstrate the algorithm, consider again the machine defined by the state table in Figure 3.

We have already determined $G(E_1)$ and $G(E'_1)$. To find $G(E_2)$ and $G(E'_2)$, we examine each $(s_i, s_j) \in G(E_1)$. We must compute $s_m = \mu(s_i, a)$ and $s_n = \mu(s_j, a)$ and determine whether or not (s_m, s_n) is in $G(E'_1)$ for any $a \in A$. If it is, then we know that (s_i, s_j) will be in $G(E'_2)$; if $(s_m, s_n) \in G(E_1)$ for all $a \in A$ then (s_i, s_j) will be in $G(E_2)$. Of course, all elements of $G(E'_1)$ will be in $G(E'_2)$. Thus we find $G(E_2) = \{(s_1, s_4), (s_1, s_5), (s_4, s_5)\}$ and $G(E'_2) = G(E'_1) + \{(s_2, s_3)\}$. To find $G(E_3)$ and $G(E'_3)$, we compute (s_m, s_n) as above for each element of $G(E_2)$, and see if it is in $G(E'_2) - G(E'_1)$ for any $a \in A$. If it is, then (s_i, s_j) will be in $G(E'_3)$. Observing that $G(E'_2) - G(E'_1) = \{(s_2, s_3)\}$ and performing this step of the algorithm, we obtain $G(E_3) = \{(s_1, s_4), (s_1, s_5), (s_4, s_5)\}$ and $G(E'_3) = G(E_2) + \emptyset$. Since $G(E'_3) = G(E'_2)$, we have $G(E'_3) = G(E')$ and $G(E_3) = G(E)$. (If this were not the case, the previous step of the algorithm would be repeated until $G(E'_{r-1}) = G(E'_r)$ for some

r.) By examining $G(E)$, we can see that s_1, s_4, and s_5 are equivalent states, and so will be merged into one state in the minimal state machine $\overline{M}$. The states of $\overline{M}$ can be described in terms of the states of M:



M can also be defined by a state table, as in Figure 4.

$\overline{M}$	Next State μ		Output η	
Present State \ Input	0	1	0	1
$\overline{s}_1$	$\overline{s}_1$	$\overline{s}_1$	1	0
$\overline{s}_2$	$\overline{s}_2$	$\overline{s}_1$	0	1
$\overline{s}_3$	$\overline{s}_3$	$\overline{s}_2$	0	1

Figure 4—A State Table for M

Since no two states of M are equivalent (each set of equivalent states in M was combined to form a single state in $\overline{M}$), and $\overline{M}$ covers M (for each state in M, there is a state in $\overline{M}$ that performs the same function), by replacing M with $\overline{M}$ we will accomplish our objective of minimizing the number of states required to perform the function of M.

REFERENCE

Birkhoff, Garret and Bartee, Thomas C. *Modern Applied Algebra.* New York: McGraw-Hill, 1970, pp. 63-80.

The Problem Corner

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 February 1979. The best solutions submitted by students will be published in the Spring 1979 issue of **The Pentagon**, with credit being for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

297. Correction of Error. The equation should read $-RE + TI = RE$. Solutions to 297 corrected can be sent until 1 February 1979. Solutions will be printed with those for the proposed problems below.

302. *Proposed by Randall J. Covill, Newburyport, Massachusetts.*
Consider the following digital display problem. A character is a set of parallel and/or perpendicular non-intersecting line segments of constant length. If a character has height, the height is equal to a constant whole number of line segments. If a character has width, the width is equal to a different constant whole number of line segments. If any segment or subset of segments can be either displayed or not displayed, what is the minimum number of segments necessary to represent all ten digits 0 to 9?

303. *Proposed by Charles W. Trigg, San Diego, California.*
Show that the ratio of the volume of a sphere to the volume of its inscribed regular octahedron is π.

304. *Proposed by Charles W. Trigg, San Diego, California.*
Does any three-digit number, N, equal 11 times the sum of the squares of its digits?

305. *Proposed by John A. Winterink, Albuquerque Technical Vocational Institute, Albuquerque, New Mexico.*
If $(x - h)^2 + (y - g)^2 = r^2$ represents a circle tangent

to three given circles, then (h, g, r) is called an Apollonian triple. Given the three circles

$$(x + 3)^2 + (y - 3)^2 = 6^2$$
$$(x - 1)^2 + (y + 5)^2 = 2^2$$
$$(x - 2)^2 + (y + 2)^2 = 1^2$$

find all Apollonian triples (h, g, r) for the given circles such that h, g, and r are rational and such that $r > 0$.

306. *Proposed by the editor.*

Let $F(n) = \dfrac{(2n + 1)(3n^2 + 3n - 1)}{15}$ be a function whose domain is the positive integers. If n is a positive integer selected at random, what is the probability that $F(n)$ is an integer?

SOLUTIONS

292. *Proposed by Leo Sauve, Algonquin College, Ottawa, Ontario, Canada*

The number 9,x29, 50y, zt7 is known to be divisible by 73 and 137. Determine the digits x, y, z, t and thereby identify the number.

Solution by R. Lynn Witmer, Manchester College, North Manchester, Indiana

Since the number 9,x29,50y,zt7 is divisible by both 73 and 137, it is also divisible by $73 \times 137 = 10{,}001$. The largest (smallest) the number could possibly be would occur if all the unknowns were equal to 9 (0), making the number 9,929,509,997 (9,029,500,007). Let B be the other divisor of the number. Then

$$\begin{array}{rcrcrcl} 9{,}029{,}500{,}007 & \leq & 10{,}001 & B & \leq & 9{,}929{,}509{,}997 & \text{or} \\ 902{,}859 & < & & B & < & 992{,}582 & \end{array}$$

Then since 10,001 ends in one, both the number and B end in 7, or

$$902{,}857 < B < 992{,}857.$$

Now the similarities in the boundaries of B are evident. Changing only the dissimilar fifth digit, one quickly finds

$B = 932{,}857$ so that the number is 9,320,502,857 and therefore $x = 3$, $y = 2$, $z = 8$ and $t = 5$.

Solution by Charles W. Trigg, San Diego, California.

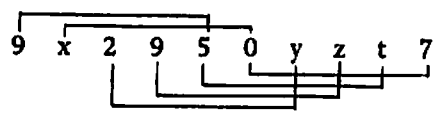
The number $N = 9{,}x29{,}50y{,}zt7$ is a multiple of 73×137 or 10001. Now

$$\begin{array}{rcccccccccc} & 9 & x & 2 & 9 & 5 & 0 & y & z & t & 7 \\ - & 9 & 0 & 2 & 8 & 9 & 7 & 2 & 8 & 0 & 7 \\ \hline & & x & 0 & 0 & 5 & 3 & y\text{-}2 & z\text{-}8 & t & 0 \end{array}$$

Obviously the subtrahend is a multiple of 10001 [10001×902807 *ed.*], so the remainder is also. It follows that $x = 3$, $t = 5$, $y = 2$, $z = 8$ and $N = 9{,}329{,}502{,}857$.

Also solved by: Gregory Hayward, Emporia State University, Emporia, Kansas: and the proposer.

Editor's comment: Another method of solution results from observing that $N = 9{,}x29{,}50y{,}zt7$ is a multiple of 10001 and pairing the first and fifth digits, etc. as shown below:



Now starting from the right as in the division process, we obtain the following equations:

$$7 + x = 10 \quad \text{or} \quad x = 3$$
$$t + 1 + 9 = 15 \quad \text{or} \quad t = 5$$
$$z = 8$$
$$y = 2$$

Hence $N = 9\ 3\ 2\ 9\ 5\ 0\ 2\ 9\ 5\ 7$.

293. *Proposed by the editor.*

On a trigonometry test, one question asked for the largest angle of the triangle having sides 21, 41, and 50. L.A.Z. Thinker, a student, obtained the answer as follows: let C denote the desired angle, then $\sin C = \frac{50}{41} = 1.2195$. But $\sin 90° = 1$ and $.2195 = \sin 12° 40' 48''$. Therefore $C = 90° + 12° 40' 48'' = 102° 40' 48''$ which is correct. Find

another triangle having this property which is not similar to the given triangle.

Solution by Gregory Hayward, Emporia State University, Emporia, Kansas.

Let $c > b > a$ be the sides of the triangle. Then by the Law of Cosines

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \qquad (1).$$

Now considering Thinker's method

$$\frac{c}{b} - 1 = \sin (C - 90) \qquad (2).$$

Now by using the identity $\sin (x - 90) = -\cos x$, equations (1) and (2) imply

$$\frac{c - b}{b} = -\frac{a^2 + b^2 - c^2}{2ab} \text{ which simplifies to}$$

$$2a(c - b) = c^2 - b^2 - a^2 \qquad (3).$$

Thus, in any triangle whose sides satisfy equation (3), the largest angle C can be found by Thinker's method. To find a triangle with integral sides, treat equation (3) as a quadratic equation in c. The quadratic formula yields

$c = \frac{1}{2}(2a \pm \sqrt{4a^2 + 4(a - b)^2})$. Then since the quantity $4a^2 + 4(a - b)^2$ must be a perfect square, take $a = 4$, $b = 7$ which gives $c = 9$ since c is positive. Thus the triangle whose sides are 4, 7, and 9 has the same property as Thinker's triangle having sides 21, 41, and 50 and is not similar to Thinker's triangle. The verification is simple.

$\sin C = \frac{9}{7} = 1.2857$, $\sin 90° = 1$ and $.2857 = \sin 16.6°$. Thus $C = 90° + 16.6° = 106.6°$ which is correct.

Also solved by Leigh Janes, Rocky Hill, Conn.; Charles W. Trigg, San Diego, California; and R. Lynn Witmer, Manchester College, North Manchester, Indiana.

Editor's comment: Equation (3) in Hayward's solution can

be rewritten in the form:

$$(b - a)^2 = (c - a)^2 - a^2 \qquad (4)$$

which is the familiar Pythagorean relationship. Only primitive right triangles need to be considered. Since all primitive right triangles are given by $x = m^2 - n^2$, $y = 2mn$ and $z = m^2 + n^2$ where m and n are integers of opposite parity such that $m > n$ and $(m,n) = 1$, substitution of these formulas in (4) yields two families of solutions:

$$\begin{array}{lll} a = 2mn & & a = m^2 - n^2 \\ b = (m + n)^2 - 2n^2 & \text{and} & b = (m + n)^2 - 2n^2 \\ c = (m + n)^2 & & c = 2m^2 \end{array}$$

Variations of this problem have appeared:

1. E. A. Maxwell, *Fallacies in Mathematics*, Cambridge University Press, (1963), pp. 88-90.
2. Problem 229, *Eureka* Vol. 3 #3 (March, 1977), p. 66 (solution, Vol. 3 #8 (October, 1977), p. 231-32).

294. *Proposed by Leo Sauve, Algonquin College, Ottawa, Ontario, Canada.*

Two cars leave at the same time from two towns A and B, going towards each other. When the faster car reaches the midpoint, M, between A and B, the distance between them is 96 miles. They meet 45 minutes later. Finally, when the slower car reaches M, they are 160 miles apart. Find (a) the speed of each car and (b) the distance between the two towns.

Solution by Kathleen A. Carlson, Indiana University of PA.

Let the distance between towns A and B be $2d$. I will denote as A' the car leaving town A and as B', the car leaving town B. WLOG, let the faster car be A'. Let t denote the time it takes for A' to reach M. Then after t hours, A' has traveled a distance d miles at a rate of d/t hours; correspondingly B' has traveled a distance $d - 96$ miles at a rate of $\dfrac{d - 96}{t}$ miles per hour.

When both cars meet, each has traveled for $t + 3/4$ hours

which yields the equation

$$2d = \frac{d(4t + 3)}{4t} + \frac{(d - 96)(4t + 3)}{4t} \qquad (1).$$

When B' reaches M, A' has traveled a distance of $d + 160$ miles in $\frac{t}{d}(d + 160)$ hours while B' has traveled a distance of d miles in $dt/(d - 96)$ hours which yields the equation

$$\frac{t(d + 160)}{d} = \frac{dt}{d - 96} \qquad (2).$$

Since $t > 0$, equation (2) becomes $(d + 160)(d - 96) = d^2$ which yields $d = 240$ miles. Hence A and B are 480 miles apart.

Substituting $d = 240$ into equation (1) and simplifying yields $t = 3$ hours. Hence car A' traveled at 80 miles per hour and car B' traveled at 48 miles per hour.

Also solved by Gregory Hayward, Emporia State University, Emporia, Kansas; Charles W. Trigg, San Diego, California; R. Lynn Witmer, Manchester College, North Manchester, Indiana; and the proposer.

295. *Proposed by the editor.*

Let $S_k = 1^k + 2^k + \cdots + n^k$ where n is an arbitrary positive integer and k is an odd positive integer. Under what conditions is S_k divisible by $S_1 = \frac{n(n + 1)}{2}$ for all positive integers n?

Solution by Charles W. Trigg, San Diego, California.

Let S_k denote the sum of the k^{th} powers of the first n positive integers. Then

$$S_k = 1^k + 2^k + \cdots + (n - 1)^k + n^k$$
$$S_k = n^k + (n - 1)^k + \cdots + 2^k + 1^k$$

$$2S_k = \{n^k + 1^k\} + \{(n - 1)^k + 2^k\} + \cdots + \{(n - 1)^k + 2^k\} + \{n^k + 1^k\}$$

Now if k is odd, then $(a + b) \mid (a^k + b^k)$. Hence, each term of the symmetrical right side of the last equation is divisible by $n + 1$. Furthermore, if the first term on the right

side of the first equation is combined with the second term on the right side of the second equation, and so on [i.e. $2S_k = 1^k + (n-1)^k + 2^k + (n-2)^k + \cdots + (n-1)^k + 1^k + 2n^k$ *ed.*] it becomes evident that this simplified form of the sum $2S_k$ contains no constant term for odd k. It follows that S_k is divisible by $\frac{n(n+1)}{2}$ for all n and odd k.

Editor's Comment: Trigg points out that when S_k is expressed in terms of n, S_k contains the factor $\left(\frac{n(n+1)}{2}\right)^2$ whenever $k > 1$ is odd and S_k contains the factor $\frac{n(n+1)(2n+1)}{6}$ whenever k is even.

296. *Proposed by Charles W. Trigg, San Diego, California.*

Using three consecutive digits repeated, form an arithmetic progression of three three-digit primes in the decimal system.

Solution by the proposer.

A three-digit integer composed of consecutive digits is divisible by 3. An integer of the form *aaa* is divisible by *a*. Hence, each prime must contain a digit repeated once. The digits in each prime cannot differ by more than 2. Furthermore, the units' digits of the three primes must be like.

From a table of primes we extract the following:

101
113 233 313
131 211 311 331
353 433 443
557 577 677 757
787 797 877 887 977 997

The only arithmetic progression of three primes in these sets is 797, 887, 997 in which the first and last terms contain the same digits.

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgement will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

One of the most popular mathematical puzzles which interests both the general public and the serious mathematical enthusiast is the magic square. However, few people have discovered the equally fascinating magic stars.

First, we will consider the usual five-pointed star polygon as shown in Figure 1. If one were to place circles at the intersections

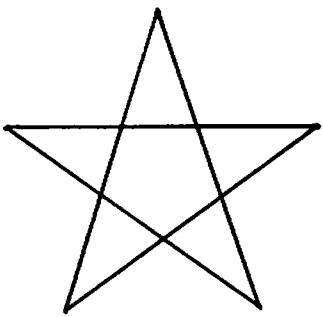


Figure 1

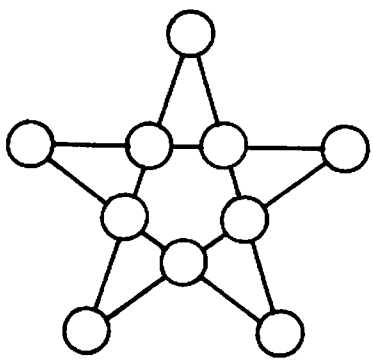


Figure 2

of any two lines, one would obtain the star with the indicated ten circles as shown in Figure 2. An obvious question would be whether one can place the integers 1 to 10 in those ten circles such that each line (containing four circles) will have the same sum? If such a star is possible, one might also ask what its "magic sum" should be.

For a solution to this problem, one would first note that each circle (and hence each integer) lies on exactly two lines (and hence in exactly two sums). The sum of the integers 1 to 10 is 55 and we have exactly five lines (sums). Hence, the sum of all five lines must equal 2×55 or 110. Recalling that each of the five sums

must be equal, we must therefore have each line have a sum of $110 \div 5$ or 22 if a magic star exists. Upon attempting to find the required sum, one will note that such a magic five-pointed star is impossible. Can you prove this?

A slight variation of the above requirements will allow us to find a five-pointed magic star if we relax the condition of requiring the use of the consecutive integers 1 to 10. If we allow the substitution of the integer 7 by 12, we can arrive at a solution. Since the sum of the resulting ten integers 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, is 60, our "magic sum" would be $(60 \times 2) \div 5$ or 24 for each line of four circles each. Our resulting solution is as shown in Figure 3. Can you determine another such magic star alternative?

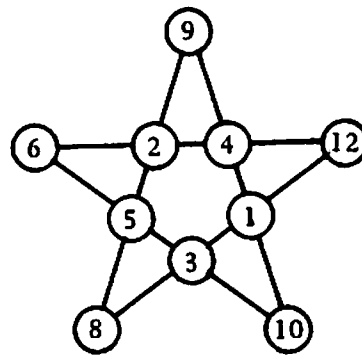


Figure 3

Consider now the case of a six-pointed star which is many times referred to as the Star of David, as shown in Figure 4. Here we have six lines with the lines intersecting in pairs resulting in twelve vertices. If we again require the placement of only the integers 1 to 12 in those twelve circles, can a magic star result? We would first note that the magic sum here must be 26 since the sum of the integers 1 to 12 is 78 and hence each sum would be $(2 \times 78) \div 6$ or 26. As shown in Figure 4, such a six-pointed magic star does indeed exist. Is this solution unique?

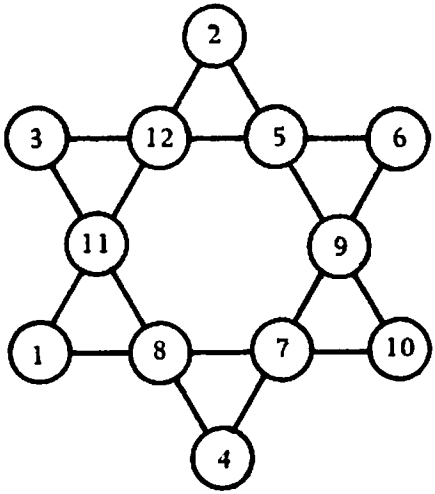


Figure 4

The septagram or seven-pointed star is similarly found to be a magic star in 56 different possible solutions by placing the integers 1 to 14 in the 14 circles. One such solution is given in Figure 5. Can you find another distinct solution?

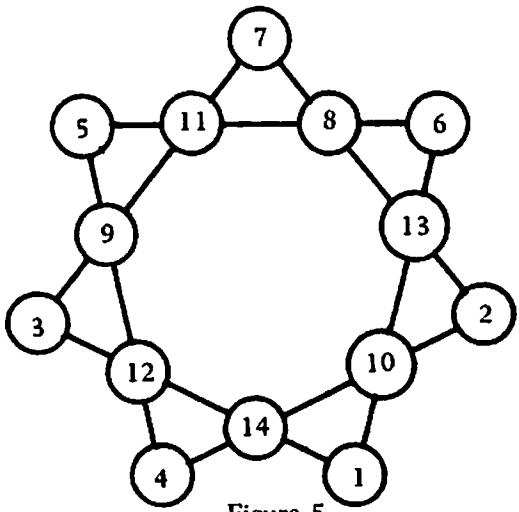


Figure 5

Obviously the solution of magic star problems is endless. In each case, one should first determine the magic constant and then proceed to find an appropriate solution. Can you find a magic star solution for an eight-pointed star?

Another problem which would at first glance appear to be completely unrelated to the above discussion of magic stars is the following. Can one label the edges of the octahedron shown in Figure 6 so that the sum of the four edges intersecting at each corner is the same?

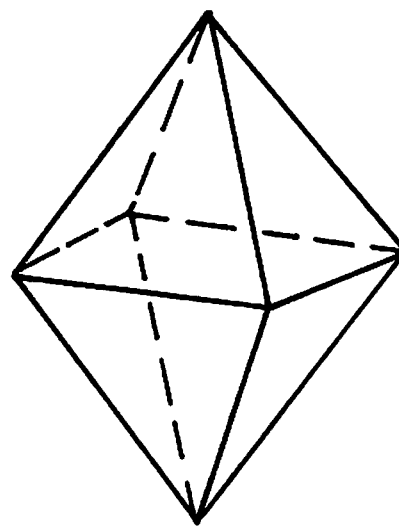


Figure 6

Upon closer examination of the problem, one will discover that its solution is the same as that of one of the magic star problems namely that of the six-pointed star of Figure 4. One needs only to interchange the vertices along any one line of the magic star with the edges intersecting at each vertex of the octahedron. The result is as shown in Figure 7.

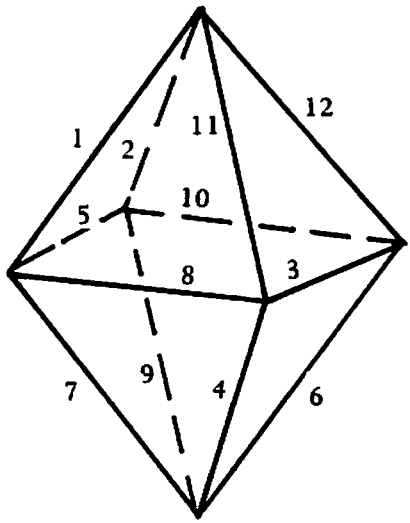


Figure 7

Can you determine a three dimensional figure for which the septagram solution would apply?

ERRATA

Robert W. Prielipp has pointed out an error in the Fall 1977 issue. One page 16 of that issue the term *Mersenne prime* is given a non-standard definition. Normally a Mersenne prime is *of the* term $2^p - 1$. We regret this error.

The Book Shelf

EDITED BY O. OSCAR BECK

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, University of North Alabama, Florence, Alabama 35630.

Patterns of Symmetry, Edited by Marjorie Senechal and George Fleck, University of Massachusetts Press, Amherst, 1977, 160 pages, $12.00.

This book evolved from a Symmetry Festival held at Smith College, Northampton, Massachusetts in 1973.

Ideas of symmetry are investigated in many fields including color, ringing of bells, print type, poetry, dance, molecular structure, crystallography, plane translation, packing in space, and plant structure. There is history of the study of symmetry and some mathematics of symmetry.

The reader can delve into each of these topics to the depth he wishes. A quick reading will give him an introduction into ideas of symmetry new to him. A student of any one of the topics will find a detailed approach and challenging ideas.

For a novice in any area, the author has covered the subject well. The reviewer was intrigued by the change ringing of a set of bells in a tower. The mechanics of ringing the bells was discussed as well as the symmetric patterns of ringing.

This book will interest many people including mathematicians. The idea of symmetry is so universal that this book would be a good addition to the library. The problem will be not to hide it under one category in the library.

Mary Lindblom
Central Michigan University

Kappa Mu Epsilon News

EDITED BY SISTER JO ANN FELLIN, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Sister Jo Ann Fellin, Historian, Kappa Mu Epsilon, Benedictine College, North Campus Box 43, Atchison, Kansas 66002.

The Twenty-first Biennial Convention of Kappa Mu Epsilon was held November 10-12, 1977 on the campus of Muskingum College, New Concord, Ohio, with Ohio Zeta as the host chapter.

On Thursday evening, November 10, following registration in the lobby of the Science Center, a square dance was held in John Glenn Gymnasium, computer games were played in the Science Center, and the National Council met in Cambridge Hall.

On Friday morning, November 11, the Regional Directors met with Ida Z. Arms of Pennsylvania Zeta, National Vice-President, for breakfast in the Patton Hall private dining room. Registration continued in the lobby of the Science Center.

The first general session (business meeting) was held on Friday morning, November 11, in the Science Center with James E. Lightner of Maryland Beta, National President, presiding. Dr. J. Anthony Brown, President of Muskingum College, gave the address of welcome and Ida Z. Arms of Pennsylvania Zeta, National Vice-President, responded for the society. The roll call of the chapters was made by George R. Mach of California Gamma, National Secretary. 28 chapters and about 160 members were in attendance. There were visitors from Miami University and Mount Union College. The representatives of Missouri Iota were given special greetings as the chapter was installed since the last biennial convention.

Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

1. "Projections Onto Subsets of E^n", Donald M. Hayman, Missouri Zeta, University of Missouri—Rolla.
2. "One Dimensional Cryrosurgical Simulation", Janet Danison, Ohio Zeta, Muskingum College.
3. "Where Is My Hat?", Robert Kellenberger, Pennsylvania Lambda, Bloomsburg State College.

4. "Groups Isomorphic to C_{12}", Ann Bremehr—author, Patricia McDonald—presenter, Kansas Gamma, Benedictine College.

At noon, a group picture was taken on the steps of John Glenn Gymnasium. After lunch the student section met in the Science Center with Rebecca Tucker, Ohio Zeta President, presiding and the faculty section met in Cambridge Hall with James E. Lightner of Maryland Beta, National President, presiding. The convention reconvened at 2:30 p.m. in the Science Center and was entertained by the John Glenn High School "Glenn Tones".

Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

5. "Many Valued Logic Systems", Elizabeth A. Birks, Pennsylvania, Iota, Shippensburg State College.
6. "How to Build a Magic Square", Richard A. Smith, Iowa Gamma, Morningside College.
7. "A Mathematical Treatment of Traffic Flow", Donald Ray Evatt, Tennessee Beta, East Tennesse State University.
8. "The Golden Ratio ($\varnothing$)", Michael Kernin, New York Eta, Niagara University.
9. "Nim by the Sprague-Grundy Theory", Crystal A. Fritz, Pennsylvania Iota, Shippensburg State College.

Rebecca Tucker, President of Ohio Zeta, reported for the student section meeting and James E. Lightner of Maryland Beta, National President, reported for the faculty section meeting.

The convention banquet was held on Friday evening, November 11, in the Kelly Hall Dining Room with Rebecca Tucker, President of Ohio Zeta, as master of ceremonies. Musical entertainment was provided by Paul Couch. The guest speaker, Lottie E. Brown of the NASA Goddard Space Research Center, gave the address, "What Makes Ducks Fly?"

The convention resumed on Saturday morning, November 12, in the Science Center. Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presided during the presentation of the following student papers:

10. "Distance—What Is Is?", Harland Shoemaker, Pennsylvania Lambda, Bloomsburg State College.
11. "Piecewise Curve Fitting in an Engineering Application",

Michele Hartman, Pennsylvania Iota, Shippensburg State College.

12. "Finite State Machines", Jeff Towne, Pennsylvania Theta, Susquehanna University.
13. "Structure and Sequence", Kim L. Rescorla, Pennsylvania Lambda, Bloomsburg State College.
14. "Sorting Algorithms and Their Computerization", Thomas J. Kourim, Illinois Zeta, Rosary College.

The following student papers were listed as alternates:

1. "Inversive Geometry", Connie Strelow, Iowa Alpha, University of Northern Iowa. (This paper was selected for presentation but it was not presented due to illness.)
2. "Updating the Old Gambler's Rule For Blackjack and Poker", Thomas A. LeKostaj, Illinois Zeta, Rosary College.
3. "Could This Be Magic?", Nancy Jean Wyshinski, Pennsylvania Lambda, Bloomsburg State College.
4. "How Infinite Is Infinity?", Beverly Marcy, Pennsylvania Lambda, Bloomsburg State College.
5. "Solving the Cubic and Quartic Equation", David Hall, Iowa Gamma, Morningside College.
6. "A Determination of the Number of Regions Determined By the Diagonals of an Irregular Convex Polygon If No Three Diagonals Are Concurrent", Dennis Whaley, Kansas Beta, Emporia State University.

The second general session (business meeting) was held on Saturday morning, November 12, in the Science Center with James E. Lightner of Maryland Beta, National President, presiding. The following national officers presented reports (copies attached), which were distributed to all of the delegates:

Wilbur J. Waggoner—Buriness Manager, THE PENTAGON
Michigan Beta
(Douglas Nance, Michigan Beta, the new Business Manager, was introduced to the convention.)

James K. Bidwell—Editor, THE PENTAGON
Michigan Beta

Sister JoAnn Fellin—National Historian
Kansas Gamma

Wilbur J. Waggoner—National Treasurer
Michigan Beta

George R. Mach—National Secretary
California Gamma

Ida Z. Arms—National Vice-President
Pennsylvania Zeta

James E. Lightner—National President
Maryland Beta

A convention questionnaire, prepared by the host Ohio Zeta Chapter, was distributed to the delegates as a means of evaluating the convention.

The National Council met at noon in the Patton Hall Private dining room.

The third general session (business meeting) was held on Saturday afternoon, November, 12 in the Science Center with James E. Lightner of Maryland Beta, National President, presiding.

Elsie Muller of Iowa Gamma reported for the auditing committee that the treasurer's books and accounts were examined and verified.

Sister Mary Petronia of Wisconsin Alpha reported for the resolutions committee. The following resolutions were adopted:

Resolved: That the Twenty-first Biennial Convention of Kappa Mu Epsilon express its appreciation:

1. To Wilbur Waggoner who has served as Business Manager of THE PENTAGON for twenty years and
2. To Elizabeth Woolridge who has served as National Secretary of Kappa Mu Epsilon for four years, both of whom have given so generously of their time and talents.

Resolved: That the 21st Biennial Convention of Kappa Mu Epsilon express its appreciation:

3. To James L. Smith and the members of Ohio Zeta for their work in the expeditious planning of this convention.
4. To Dr. J. Anthony Brown, President of Muskingum College, for the gracious hospitality and

the many services rendered the chapters and officers of the Convention.

5. To Lottie E. Brown for her interesting talk.
6. To the Selection Committee and the Awards Committee who gave so unselfishly of their time to the primary activity of Kappa Mu Epsilon.
7. To the students who prepared and presented papers at the Convention.
8. To the National Officers of Kappa Mu Epsilon for their diligent service during and preceding the Biennial Convention.
9. To Ms. Helen Rickey, Paul Couch and the John Glenn High School "Glenn Tones" who provided entertainment for us during the Convention.

Resolved: That the National Council of Kappa Mu Epsilon—

1. Consider compiling a set of guidelines of the host chapter for the Biennial Convention.

Invitations to host the Twenty-Second Biennial Convention were issued by Kansas Alpha, Pittsburg State University, and Pennsylvania Theta, Susquehanna University.

Ida Z. Arms of Pennsylvania Zeta, National Vice-President, presented certificates to all students who presented papers at the convention. James Pomfret of Pennsylvania Lambda reported for the awards committee and announced the following student paper awards:

Michele L. Hartman Pennsylvania Iota	—	First Place
Robert Kellenberger Pennsylvania Lambda	—	Second Place
Jeff Towne Pennsylvania Theta	—	Third Place
Crystal A. Fritz Pennsylvania Iota	—	Fourth Place
Janet Danison Ohio Zeta	—	Honorable Mention
Richard Smith Iowa Gamma	—	Honorable Mention

Michael Kernin — Honorable Mention
New York Eta

Travel allowances were paid to the delegates by Wilbur J. Waggoner of Michigan Beta, National Treasurer. Kappa Mu Epsilon brochures were distributed by George R. Mach of California Gamma, National Secretary. The convention adjourned at 1:30 p.m.

REPORT OF THE NATIONAL PRESIDENT

During the last biennium there has been a good deal of activity, some of it rather troubling but most of it very positive. The Society has grown with the addition of five new chapters: Hardin-Simmons University, Abilene, Texas (installed by Regional Director Mike Reagan); Missouri Southern State College, Joplin, Missouri (installed by Eddie Robinson); West Georgia College, Carrolton, Georgia (installed by Elizabeth Wooldridge); Florida Southern College, Lakeland, Florida (installed by Elizabeth Wooldridge); and Bethany College, Bethany, West Virginia (which I installed). At its November 1975 meeting the National Council declared five chapters inactive (because of no recent inductions of members), later reversing the status for Michigan Alpha Chapter which has since reactivated. So we stand at the moment with 96 active chapters, and with several inquiries and one petition now being voted on by the chapters. It is gratifying to see this continued interest when we are continually told that mathematics students and interest are on the wane.

To assist us in getting the Kappa Mu Epsilon story to eligible colleges and interested students, I am pleased to announce that a new edition of our brochure has just been published and is available to take with you today (or to be ordered from the National Secretary as needed for your local chapter use). Our thanks to Sister John Frances Gilman, New York Eta Chapter, for overseeing the production of this revised brochure.

As you heard from the Vice President, the regions continue to function well; indeed, more activity is seen each biennium. My hope is that, during the next biennium, we will see *more* regional activity which I think is very useful in bringing the national organ-

ization closer to the local chapters. At the beginning of the last biennium several new regional directors were appointed to fill existing vacancies; our regional directors are (I) Sister Marie Augustine, Maryland Alpha; (II) James Smith, Ohio Zeta; (III) Eddy Brackin, Alabama Beta; (IV) Harold Thomas, Kansas Alpha; (V) Mike Reagan, Oklahoma, Alpha; and (VI) George Mach, California Gamma. Our thanks to those who have served in this capacity in the past and to those who serve us now. It is with sincere regret that I note the death by cancer on 3 January 1977 of Jack Munn, who served for several years as director for Region III.

During the past two years, as you are aware, we were forced to raise the national dues to $10.00, in order to accommodate for the rising costs of the printing of *The Pentagon* and to provide for an increase in the mileage allowance to 10¢ per mile. As a result of a resolution presented at the last convention, the Council unanimously voted to invite student representatives to attend the Council meetings at Conventions and this occurred last evening (Thursday) when the presidents of the host chapters of the present and preceding conventions sat with us in our deliberations. Some more careful ground rules were also established for the presentation of convention papers, and these are in effect at this convention, under the direction of the Vice President. We hope that these guidelines have made the submission, selection, and judging of papers easier and fairer for all.

One of the functions of the President is to appoint the convention committees and I would like to thank all those who agreed to serve on these committees for their efforts in behalf of the Society. In order to assist in the creation of these committees for the *next* convention in 1979, I would ask that anyone interested in serving indicate this by signing your name on the appropriate list at the stage at the close of this session. While we cannot guarantee appointments because of the geographical distribution we try to maintain, your interest will help us greatly in the initial planning for the 22nd Biennial Convention.

A year ago this month, I learned of the irregularities which had been reported in our treasury. In the difficult months which followed, in which our March convention had to be cancelled due to the instability of the treasury while the books were in the hands

of the auditors, I can state to you quite candidly that the integrity of the Society was maintained through the yeoman efforts of then-president, William Smith. By keeping a cool head and seeking legal advice throughout all our deliberations, he led us to a resolution of our financial difficulties. I know you join me in expressing our sincere thanks to him for a job well done, not only during this troubled time for KME, but throughout the entire four years of his tenure as President.

Before moving on to the remaining few items, I want to report to you that, by mid-summer, 1977, our treasury was back in "good working order". We had paid all of our outstanding bills, had deposited some funds in a savings account to accrue interest, and had generally put all financial matters on a firmer, more secure, basis than had seemed necessary in the past. We are still in the in the process of trying to recover some of our costs in getting our treasury back in functioning order; to do this we have again engaged legal counsel. If reports of the financial situation have seemed rather legalistic and less than completely candid, it is only because our lawyers have suggested this posture to avoid any accusatory statements which might prejudice our recovery process. Your cooperation, understanding, and faith in the Society and its leadership (especially during the election-by-mail which our constitution fortunately provided for) have been greatly appreciated by us all.

Relative to the Constitution, there are periodic needs to re-examine the document which governs the Society; recently, some questions have been raised about the wording of certain articles and the Council has decided to appoint an *ad hoc* Constitutional Revision Study Committee, which has already been at work during this convention, determining those areas, if any, which need further study. Past-President William Smith serves as chairman, and the members are Harold Thomas, Sister John Frances Gilman, Becky Tucker, and Kathleen Tandetzte (the last two being the student representatives to the Council meeting).

Before closing, I would be remiss if I did not recognize one other faithful member, Dr. Elizabeth Wooldridge, who until May served the Society generously as National Secretary for the past four years. Our best wishes to her for the future. She has been succeeded by Past-President George Mach.

My thanks to all the new officers who "rallied around" in May when we met in Kansas City to assess our situation and to plan for the future. I am sure I speak for them when I say that we pledge our very best service to you and the Society during the next biennium. I know that the future will see Kappa Mu Epsilon even stronger and more viable as we approach our fiftieth anniversary in 1981.

James E. Lightner

REPORT OF THE NATIONAL VICE PRESIDENT

This report covers the activities of this office since the last biennial Convention held in April, 1975.

On 21 May 1975 your Vice President, Dr. James Lightner, installed West Virginia Alpha Chapter at Bethany College.

As Coordinator of the Regional Organization the Vice President encouraged the Regional Directors to hold Conventions whenever feasible. In the Spring of 1976 three Regional Conventions were held. Region 1 (Sister Marie Augustine, Director) held its Convention at Bloomsburg State College (Pennsylvania Lambda Chapter) in April, 1976. Region 2 (Dr. James Smith, Director) held a Convention at Niagara University (New York Eta Chapter) in March, 1976. Region 4 (Dr. Harold Thomas, Director) held a Convention at Benedictine College (Kansas Gamma Chapter) in April, 1976. Also, Region 1 held a Convention in March, 1977 at Shippensburg State College (Pennsylvania Iota Chapter). Dr. Lightner was the guest speaker.

After the election of National officers in the Spring of 1977 my first official act as Vice President was issuing a call for student papers to be presented at the Convention announced for November 10-12 to be held at Muskingum College. Also, attendance at the meeting of the National Council in Kansas City, Missouri, on May 18 and 19 immediately got me involved in the activities of this office; plans for the Convention were discussed, along with many other items.

As it is the Vice President's responsibility to make arrangements for presentation of student papers at the Convention, I am pleased

to report that twenty students, representing twelve chapters and eight states, submitted papers. Fourteen of these papers are to be presented at the Convention. My special thanks to the Paper Selection Committee who read and ranked all the papers: Dr. Helen Kriegsman (Kansas Alpha Chapter), Dr. Eddy J. Brackin (Alabama Beta Chapter), and Dr. Carl Kerr (Pennsylvania Iota Chapter). I am particularly grateful to the twenty students who prepared and submitted papers under rather stringent time limitations caused by the rescheduling of the convention. These papers are very important in helping to make a Convention of Kappa Mu Epsilon a success.

Ida Z. Arms.

REPORT OF THE NATIONAL SECRETARY

During the past biennium five new chapters of Kappa Mu Epsilon were installed. They are: Texas Eta at Hardin-Simmons University, Missouri Iota at Missouri Southern State College, Georgia Alpha at West Georgia College, West Virginia Alpha at Bethany College, and Florida Beta at Florida Southern College.

Four chapters were declared inactive by the National Council. They are: Alabama Epsilon at Athens College, Illinois Gamma at Chicago State University, New Jersey Alpha at Upsala College, and New York Zeta at Colgate University.

The Society now has 96 active chapters in 31 states with a combined membership of 33,523 and 20 inactive chapters with a combined membership of 4,213, making the total membership of Kappa Mu Epsilon 37,736 at the end of the biennium on 10 May 1977.

The National Secretary retains a permanent record card for each member, files reports of all chapter initiations, orders engraved membership certificates for all new members, and stocks all supplies including forms, invitations, and jewelry. I attempt to assist corresponding secretaries in any way possible.

George R. Mach

REPORT OF THE NATIONAL HISTORIAN

The files of the National Historian are being maintained and continually updated with the records received from the chapters about their events and activities.

News items have been solicited from the corresponding secretaries semi-annually and have been edited for publication in the chapter news section of *The Pentagon.* A new time schedule for these requests was established in January, 1976. News is solicited in January and May for publication in the spring and fall issues, respectively.

During the past biennium 75 percent of the chapters responded at least once to the chapter news request. Special mention goes to the following 19 chapters for their cooperation in responding to all four inquiries: AL Beta, CA Gamma, CO Alpha, IL Alpha, IA Alpha, IA Gamma, IA Delta, KS Alpha, KS Gamma, KS Epsilon, MD Alpha, MD Beta, MI Beta, MO Beta, NE Beta, NM Alpha, OH Zeta, PA Epsilon, PA Zeta.

My thanks to all with whom I have corresponded relative ot this office during the past and the current biennium—the national officers, the regional directors, all the chapter corresponding secretaries, and James Bidwell, editor of *The Pentagon.*

Sister Jo Ann Fellin

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON

This is the tenth and *final* report I will give to a biennial convention of Kappa Mu Epsilon concerning the activities and duties of the Business Manager of THE PENTAGON. Some comparative statistics for this 20 year period might be of interest to you. The cost of a single issue of THE PENTAGON has doubled from fifty cents to one dollar. The cost of mailing a single issue has tripled from eight cents to twenty-four cents. The number of Pentagons printed has varied from a low of 2000 in 1957 to a high of 3450 in 1969. Our last printing, Spring, 1977, had 3050 copies. In my first report the states receiving the most copies of THE PENTAGON were, in descending order, Kansas, California, Illinois, Texas, and

New York. For the past biennium the states receiving the most copies of THE PENTAGON were Pennsylvania, Missouri, Illinois, Kansas, and Ohio. Approximately forty percent of all PENTAGONS mailed go to subscribers in the preceding five states.

PENTAGONS have been mailed during this biennium to every state except Vermont. Our journal also goes to many foreign addresses in Europe, Asia, and South America.

Many PENTAGONS are returned to the office of the Business Manager by the postal service as undeliverable due to incorrect addresses. Please inform your chapter members that to receive thir journal they must keep a *current* address on file with the Business Manager. If a subscriber has any problem with receiving his PENTAGON, he should contact the Office of the Business Manager.

Since our journal is not a "current events" journal and is published semi-annually, it is the policy of this office to mail a current journal to each new subscriber for a period of two to three months after the printing of an issue. This policy prevents new subscribers from waiting as long as six months to receive their first issue of THE PENTAGON.

Complimentary copies are sent to the library of each college or university with an *active* chapter of Kappa Mu Epsilon. Also, complimentary copies are sent to authors of articles in THE PENTAGON. Speakers of this convention will automatically have their subscriptions extended for two years.

I have received much cooperation from our editor, James Bidwell, from our past national secretary, Elizabeth Wooldridge, and from the chapter corresponding secretaries. This cooperation is gratefully acknowledged. I have appreciated the opportunity to serve Kappa Mu Epsilon as Business Manager of our journal during the past twenty years. In my new position as National Treasurer I will continue to enjoy my association with the national honor society in mathematics.

Wilbur W. Waggoner

KAPPA MU EPSILON

FINANCIAL REPORT OF THE NATIONAL TREASURER

For the Period 21 May 1999 to 1 November 1977

RECEIPTS

1.	Cash on Hand 21 May 1977			$ 000.00
2.	Transferred from previous Treasurer		$ 775.93	
3.	Receipts from Chapters			
	Initiates (979)	$ 9,592.00		
	Jewelry	294.67		
	Supplies	78.30		
			9,964.97	
4.	Miscellaneous			
	Interest	$ 000.00		
	From Bonding Company	13,030.00		
	Installation	195.00		
	Reactivation Fee	10.00		
			13,235.00	
5.	Total Receipts			$23,975.90

EXPENDITURES

6.	National Officers Expense		$2,484.49	
7.	Regional Directors Expense		182.40	
8.	Balfour Company		000.00	
9.	Blake Printing		3,065.12	
10.	Pentagon		9,092.80	
11.	Miscellaneous			
	ACHS	$ 376.25		
	Audit	2,135.00		
	Uncollectible check	7.77		
	Savings	5,000.00		
			7,519.02	
12.	Total Expenditures			$22,344.43
13.	Cash on Hand, 1 November 1977			1,631.47
14.	Total Expenditures plus Cash on Hand			$23,975.90
15.	Total Assets 1 November 1977			
	Cash on Hand		$ 1,631.47	
	Savings		5,000.00	
	Interest		45.03	
			$ 6,676.50	

Wilbur J. Waggoner

REPORT OF THE EDITOR OF THE PENTAGON

THE PENTAGON is the journal of **KME**. It should serve the needs of the members. However, more direct participation of the membership is needed. I urge each of you to contribute to the journal by submitting articles, problems and solutions, contributions to the Scrapbook, and **KME** News items. In particular, THE PENTAGON needs to publish more student papers.

During the last biennium, eight faculty articles were published and only seven student articles. That trend needs to be changed. May I suggest that winning papers at each regional convention be submitted automatically for publication, if they are not held for the next national biennium. Further, I urge each of you to consider writing potential articles for THE PENTAGON. I will continue to publish non-student articles of interest to you, but I would like the number to decrease.

In the Fall, 1976 issue, the lead article was an introduction to graph theory. That article was written by Douglas Nance at my request. It was designed to be used by **KME** members or chapters as a study guide to a topic they may not otherwise study as an undergraduate. I would like to know if any of you have made use of it in this way. If you find it valuable, I will continue to print other such articles as they are submitted.

This is my third biennium report. Since the last biennium convention there have been two changes in THE PENTAGON staff. Since 1975, Sister JoAnn Fellin has served as associate editor for the **KME** *News* section, replacing Elsie Muller. Recently, Douglas Nance has become our new Business Manager, replacing Wilbur Waggoner who was Business Manager for 20 years. I wish to extend my special thanks to both of them for their valuable service to **KME** and THE PENTAGON. I am sure their replacements will continue their good work.

The production of THE PENTAGON has now stabilized, although publication time is still late. Neither James Orrison, our typesetter, nor Enterprise Printers handle any other mathematical work. This puts an increased burden on them. I thank them for their continued efforts on our behalf.

As editor, my task is made much easier by the able work of all the associate editors. The others not mentioned above are Oscar Beck, Loretta Smith, Richard Barlow, and Kenneth Wilke. I sincerely thank them for their efforts over the last two years. I hope all of them continue on the job. To help me read manuscripts, I rely on members of the Mathematics Department at Central Michigan University. I particularly wish to thank Robert Chaffer, Robert DeBruin, David McDowell, Richard St. Andre, and Douglas Smith.

James K. Bidwell

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

Chapter President—Diane Bruce
36 actives

AL Beta was recognized and honored for its winning exhibit in the homecoming activities which contrasted mathematics of long ago with mathematics of the future. Two of the interesting programs this past semester centered on mathematical entertainment. Student Bobby Wells shared problems in fun and logic with the group and Dr. John Locker presented puzzles and mathematical games.

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President—Mike Skora
58 actives, 40 pledges

The chapter has continued to be active in several ways. Social activities included co-sponsoring a faculty-student picnic with the Math Club and a Christmas social and pledge ceremony. The chapter sponsored a job placement conference and coffee hour as well as hosted a program given by an IBM representative on the co-op program with IBM. Members participated in several workshop sessions to write the tests for the county-wide Junior High Math Day being held this spring. Other activities included the

mailing of an alumni newsletter and assisting the department faculty in hosting a joint MAA-SIAM meeting. Secretary for the chapter is Mary Danbom (spelling correction to name published in last issue.

California Delta, California State Polytechnic University, Pomona

Chapter President—Claudia Tuttle
15 actives, 13 pledges

The chapter presents a $50 book scholarship to a deserving student and throughout the semester chapter members provide tutoring services to mathematics students. The members help in preparations for the mathematics display in the spring campus-wide fair called *Poly Vue*. Other 1977-78 officers: Richard Wylie, vice-president; Brian Guthrie, secretary and treasurer; Richard A. Robertson, corresponding secretary; Joseph Kachun, faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins

Chapter President—Joe Tinucci
30 actives, 12 pledges

Other 1977-78 officers: Jim Peterson, vice-president; Doug Dalman, secretary; Brenda Drake, treasurer; Duane Clowe, corresponding secretary; Mr. Deal, faculty sponsor.

Colorado Beta, Colorado School of Mines, Golden

Chapter President—T. David Burleigh
20 actives, 70 pledges

At the October meeting the talk presented by Dr. Ardel Boes on "The Cantor Set" was followed by a brief business meeting and the serving of refreshments. The November meeting was activity oriented with each member bringing to the meeting a mathematical puzzle. The group assembled then uncovered the tricks. Other new officers: Shelby Swizter, vice-president; Ted Huston, secretary; Mark Braeco, treasurer.

Illinois Alpha, Illinois State University, Normal

Faculty sponsor—Orlyn Edge

Illinois Beta, Eastern Illinois University, Charleston

Chapter President—Patricia Ryan
56 actives

Chapter members Pat Ryan, Janice Brown, Jim Harrich, and Mike Zwilling attended the national meeting at Muskingum College in November. The chapter is conducting the annual spring initiation ceremony and awards banquet as well as a spring trip to St. Louis. Other 1977-78 officers: Kent Haake, vice-president; Lynn Wessel, secretary; Steve Hooser, treasurer; Michael Zwilling, corresponding secretary and faculty sponsor.

Illinois Zeta, Rosary College, River Forest

Chapter President—Tom Le Kostaj
8 actives

Problems presented by the members continue to be a feature of the regular chapter meetings. In September chapter members participated in the college Activities Fair which acquaints students with the purposes, requirements, and activities of the various campus organizations. The chapter hosted a picnic in October for mathematics students, faculty, and recent graduates. President Tom LeKostaj and alumnus member Tom Kourim represented IL Zeta at the national convention in New Concord, Ohio.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President—Diana Dickinson
38 actives

The September and October meetings featured presentations by two students: "On-the-Job Programming Experience" by John Schreck and "Perfect Numbers" by Steven Wild. The annual homecoming breakfast, held in October at the home of Dr. and Mrs. E. W. Hamilton, attracted 14 **KME** alumni including a charter member of IA Alpha from the class of 1932. At the November meeting Pat Lange presented her paper on "Solving Polynomial Equations." New initiate Jim Davies presented "The Search of Pi: Rational or Irrational" at the initiation banquet on 1 December. Nine foot snow drifts made travel off campus impossible and caused cancellation of the December Christmas party. This marked the

first time in 17 years that IA Alpha did not have its December meeting at the home of Professor Ina Mae Silvey. The meeting was rescheduled as a Valentine's Party.

Iowa Beta, Drake University, Des Moines

Chapter President—Darien Hall
8 actives, 10 pledges

Fall semester activities began with a picnic to which prospective members were invited. The purpose and activities of the chapter were explained to the guests. Professor Joseph Hoffert discussed aspects of the life and works of Carl Friedrich Gauss at a chapter meeting in observance of the bicentennial of Gauss' birthday. At another meeting Frederic Lang talked on the calculus of variations.

Iowa Gamma, Morningside College, Sioux City

Chapter President—Richard Smith
18 actives

Several alumni attended a homecoming breakfast at the home of Dr. Elsie Muller on 8 October. The chapter sold used books to obtain funds to attend the national convention. Three students were initiated at the December meeting.

Iowa Delta, Wartburg College, Waverly

Chapter President—Kent R. Floy
16 actives, 17 pledges

In an effort to better acquaint the members with recreational mathematics the chapter featured mathematical games, puzzles, quizzes, and humorous stories at each of its monthly meetings. Members Greg Diereks, Kay Bisbee, Kent Floy, and Mark Behle took an active part in the presentations. Much of first semester activity revolved around preparations for the Math Field Day held on 4 March for over 70 high school students and their team advisors. Members served on six committees for this event which was co-hosted by IA Delta and the Wartburg mathematics department. The games committee entertained with some of the field day games at the Christmas banquet party.

Kansas Alpha, Pittsburg State University, Pittsburg

Chapter President—Terri Wilson
50 actives

Following the September picnic for all mathematics and physics students, the chapter held regular monthly meetings. Thea Barrett presented the program on the role of mathematics in industry at the October initiation meeting at which time three new members were received into KS Alpha. The seven students and three faculty who had attended the national convention in New Concord, Ohio shared their experiences at the November meeting. Ronald Stockstill spoke on "The Knapsack Problem" at a special Christmas meeting held at the home of Dr. Helen Kriegsman, chairperson of the mathematics department.

Kansas Beta, Emporiia State University, Emporia

Chapter President—Sherry A. Anderson

The chapter held a business meeting on 19 September and fall initiation on 28 September. During October members had a halloween party and helped with Emporia State's Math Day. Dennis Whaley's paper appeared as an alternate on the program of KME's 21st biennial convention. Student papers were delivered at the meeting on 1 December. Other 1977-78 officers: Greg Hayward, vice-president; Gail Flippo, secretary; Mary Williams, treasurer; Don Bruyr, corresponding secretary; Tom Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison

Chapter President—Joe Gress
11 actives, 13 pledges

Social activities for the semester included the fall picnic at Warnock Lake and the traditional Christmas Wassail at the home of Sister Jo Ann Fellin. KS Gamma vice-president Ann Bremehr, associate member Patricia McDonald, and faculty members Jim Ewbank and Sister Jo Ann Fellin represented the chapter at the national meeting in November.

Kansas Epsilon, Fort Hays State University, Hays

Chapter President—Reggie Romine
20 actives

Other officers: Romona Weigel, vice-president; Teresa Willis, secretary and treasurer.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President—Frances Pittelli
5 actives, 4 pledges

Other officers: Ann Shaughnessy, vice-president and treasurer; Laura Nesbitt, secretary.

Maryland Beta, Western Maryland College, Westminster

Chapter President—Sherry Sheckler
16 actives

The annual fall picnic was sponsored by the chapter for incoming mathematics majors. Four new members were inducted at the October meeting. Michael Foster, guest speaker and chapter alumnus of the class of 1973, talked on the value of mathematics as a preparation for his position as a C.P.A. employed now by Coopers and Lybrand in Baltimore. Tony Sager of the class of 1977, another chapter alumnus, was guest speaker in November. He told of his experiences as a summer intern in statistics at Aberdeen Proving Ground and of his job hunt which led to his present position as a mathematician with the National Security Agency in Fort Meade, Maryland. Other 1977-78 officers: Mark Katz, vice-president; Leslie Mylin, secretary; Jeffrey Gates, treasurer; James Lightner, corresponding secretary; Robert Boner, faculty sponsor.

Michigan Beta, Central Michigan University, Mount Pleasant

Chapter President—Steve A. Lawyer
48 actives

During homecoming weekend the chapter sponsored a tea for alumni. Guest speakers during the semester spoke on a variety of topics related to this year's theme—opportunities in mathematics. Chapter members continued the tutoring program and helped out

with the local career day activities. Vice-president Steve A. Lawyer assumed the duties of the office of president. Sandy Nielsen was elected vice-president to replace him.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg

Chapter President—Lucille Lisenbee
30 actives

Seven new members were initiated at the fall initiation supper on 28 October. Chapter members assist as tutors in the Math Learning Center at the college.

Mississippi Delta, William Carey College, Hattiesburg

Chapter President—Wayne Broadwater
5 actives

In October the group met to discuss chapter business. MS Delta plans an initiation during the spring semester and a joint program with MS Gamma at the University of Southern Mississippi. Other 1977-78 officers: Betty Jones, vice-president; Larry Burt, secretary; Gaston Smith, corresponding secretary; Maury Shurlds, faculty sponsor.

Missouri Alpha, Southwest Missouri State College, Springfield

Chapter President—Bruce Campbell

The film "Donald in Mathmagicland" was shown at the initiation meeting on 20 October. Two speakers were featured during the fall semester. Dr. John Hatcher, professor of mathematics at the University, spoke on "How to Integrate by Parts." Lt. Marsha Cruz, U. S. Navy Officer, presented a program on "The Nuclear Power Program." Other 1977-78 officers: David Mentis, vice-president; Donna Newton, secretary; Eloise Brockschmidt, treasurer; John B. Prater, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg

Chapter President—Mary Beth Snodgrass
18 actives, 8 pledges

The chapter held regular meetings during the fall semester including initiation and a Christmas party.

Missouri Gamma, William Jewell College, Liberty

Chapter President—Jim Lamb
16 actives, 10 pledges

Regular meetings were held with student presentations. The real world application of classroom learning was the focus of a talk given by a guest speaker for one of the fall meetings. The speaker told how he used basic linear algebra to maximize the efficiency of truck routes used by Hallmark Cards. Other 1977-78 officers: Randy Lewis, vice-president; Dennis Harlan, secretary; Tuyen Nguyen, treasurer; Sherman Sherrick, corresponding secretary; Truitt Mathis, faculty sponsor.

Missouri Epsilon, Central College, Fayette

Chapter President—Lexa Bond
6 actives

Professor Emeritus W. H. Ehrich, corresponding secretary of MO Epsilon for many years, died suddenly last August. Other 1977-78 officers: John Zoellner, vice-president; Sharon McCall, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

Missouri Zeta, University of Missouri—Rolla, Rolla

Chapter President—Kent Cobb
45 actives

Four students and two faculty members of MO Zeta attended the biennial convention in New Concord. Chapter members have been holding math help sessions twice a week. At one of the chapter meetings Don Hayman presented a paper entitled "Projections onto Subsets of Euclidean n-space." Other 1977-78 officers: Cliff Klein, vice-president; Steve Guntley, secretary; Harry Laswell, treasurer; Johnnie Roberts, historian; Farroll T. Wright, corresponding secretary; James Joiner, faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville

Chapter President—Robert Gaw
17 actives, 10 pledges

MO Eta had three guest speakers in the fall semester and a November initiation. Some of the members spent a weekend with the Pi Mu Epsilon chapter at Maryville, Missouri. Preparations for the spring math contest for area high school students engaged the time of chapter members. Other 1977-78 officers: Rita Bax, vice-president; Debbie Sportsman, secretary; Steve Bowser, treasurer; Samuel Lesseig, corresponding secretary; John Erhart, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne

Chapter President—Kay Pankratz
16 actives

The chapter holds monthly meetings and provides free tutoring service to students in calculus and pre-calculus courses. The tutors are on call to help students individually. They also conduct review sessions prior to examinations. Seven new members were initiated at the January meeting. David Sindelar of Silver Creek, Nebraska was selected outstanding freshman mathematics student for the 1977-78 academic year. The selection was based on a competitive test developed by the chapter some years ago. The award includes the placement of the honoree's name on the permanent plaque and honorary membership in the local chapter.

Nebraska Gamma, Chadron State College, Chadron

Chapter President—Cindy Roffers
11 actives, 2 pledges

Activities hosted by the chapter included initiation, a homecoming float, and a fall picnic. Brian Wells and Jeff Walz presented papers at the November meetings. The new initiates prepared and executed bulletin board displays in the Math and Science Building during the semester. Other 1977-78 officers: Brian Wells, vice-president; Reva Carl, secretary; Dirl Steffe, Jr., treasurer; James A. Kaus, corresponding secretary and faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President—Don K. Poulsen
45 actives

Other 1977-78 officers: Charlotte J. Harrison, vice-president; William A. Ricker, secretary; Gary Mayhew, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara University

Chapter President—Joanne Esposito
27 actives, 8 pledges

Senior student Michael Kernin presented a paper entitled "The Golden Ratio" at the national convention in New Concord, Ohio. The delegation from NY Eta consisted of two student and two faculty members.

Ohio Alpha, Bowling Green State University, Bowling Green

Chapter President—Karen Thomas
50 actives

The chapter provided two special programs for the membership. The first on 18 October called "Graduate Information Night" covered aspects of graduate education with professors and graduate students available for a question-answer session. At "Careers Night" on 20 February recent graduates Debbie Bird of the Federal Reserve Bank of Cleveland, Debbie Kindelberger of Ford, and Aggie Stanonik of Marathon Oil shared their experiences and gave advice on career preparation. Other 1977-78 officers: Dawn Shaw, vice-president; Paula Neargarder and John Patrick, secretaries; Gary Welch, treasurer; Waldemar Weber, corresponding secretary; Thomas Hern and Dean Neuman, faculty sponsors.

Ohio Gamma, Baldwin Wallace College, Berea

Chapter President—Kevin Cullen
25 actives, 10 pledges

Other 1977-78 officers: Bill Peterjohn, vice-president; Joan Lowder, secretary; Gary Monda, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord

Chapter President—Rebecca Tucker
35 actives

At the September meeting Janet Danison discussed her summer research at the Oak Ridge National Laboratories on the topic of cryosurgery. In October the following new members were initiated: Nancy Bronson, Andrew Clark, John Dutro, Christian Hilty, Bruce Jones, Gregory Morrison, John Sharp, Rebecca Sorg, and Mary Torchia. OH Zeta had the privilege of hosting the national convention and felt it was worth all the effort put forth by the membership. They wish to thank **KME** for coming to Muskingum. The fall semester activities concluded with a Christmas and game party.

Oklaroma Gamma, Southwestern Oklahoma State University, Weatherford

Chapter President—Marietta Unruh
40 actives, 13 pledges

The chapter members participated in a bake sale to help finance the trip of five members to the national convention last fall. The December meeting featured a speaker from the Department of the Navy. Other new officers: Cindy Robertson, vice-president; Ray Moreau, secretary; Sherry Caton, treasurer.

Pennsylvania Alpha, Westminster College, New Wilmington

Chapter President—Michael J. Dzuricky
44 actives

The chapter held several meetings in the fall semester. At the picnic in late September plans for the year's activities were discussed. A field trip to Pittsburgh to visit several industries and the Buehl Panetarium was one of the spring semester activities.

Pennsylvania Gamma, Waynesburg College, Waynesburg

Chapter President—Grant Koher
16 actives

Other 1977-78 officers: Linda Loughman, vice-president; Dean

Christian, secretary and treasurer; Rosalie B. Jackson, corresponding secretary; David S. Tucker, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President—Debra Kistler
25 actives, 8 pledges

Everyone in attendance at the annual games night held in the mathematics laboratory enjoyed playing mathematical games. At several meetings throughout the semester student speakers addressed the group.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President—Terry Gillis
22 actives

Dr. Joseph Angelo, mathematics department faculty member, presented "Math is a four letter word" at the October initiation meeting. Students reported their impressions of the national meeting at Muskingum at the November meeting. At the December gathering of the chapter, Professor William R. Smith spoke on "The Dance Problem." Other new officers: Judy Brandt, vice-president; Cynthia Gurgacz, secretary; Lucille Ligas, treasurer.

Pennsylvaina Eta, Grove City College, Grove City

Chapter President—Christopher L. Cox
25 actives

Faculty members Ralph Carlson and John Ellison hosted the two fall meetings in their homes. At the first meeting students Chris Cox and Karen Johnson discussed a statistical analysis they had completed on the testing of incoming freshmen. The test is designed to distinguish between students capable of studying calculus and those needing remedial algebraic skills. At the second meeting future activities and the initiation of new members were planned.

Chapter President—Michele Hartman

44 actives, 9 pledges

Five students and one faculty member represented the chapter at the national meeting at Muskingum. Michele Hartman and Crystal Fritz took first place and fourth place respectively with their presentations. Betsy Birks from PA Iota also presented a paper at the convention. Nine students were initiated on 15 November at the home of faculty sponsor Dr. James Sieber. At the last meeting of the first semester vice-president Crystal Fritz was elected president to undertake the duties of that office vacated by the mid-year graduation of Michele Hartman. Russell Huff was elected to the then vacated vice-president position to assume the duties of vice-president for the spring semester. Other 1977-78 officers: Lisa Rickrode, secretary; Howard Bell, treasurer; John Mowbray, corresponding secretary; James Sieber, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia

Chapter President—Susan Capozio
5 actives, 3 pledges

Sister Mary Grace, chairperson of the mathematics department, spoke on "Biorhythm and the Sine Curve" at a meeting co-sponsored by the chapter and Beta Chi, the math club on campus. Members of PA Kappa attended the National Council of Teachers of Mathematics meeting held in Philadelphia on 13-15 October.

Pennsylvania Lambda, Bloomsburg State College, Bloomsburg

Chapter President—Nancy Jean Wyshinski
25 actives, 5 pledges

Nine chapter members participated in a meeting held with the PA Theta chapter at Susquehanna University. Three of the nine students from Bloomsburg who attended the national meeting presented papers there. Bob Kellenberger took second place with his paper on "Where is my Hat?" Besides the usual initiation ceremony chapter members enjoyed a picnic at Dr. Hartung's resort, co-sponsored a math day for local high school students with the math club, and held a Christmas banquet. Other 1977-78 officers: Robert Kellenberger, vice-president; Barb Cooper, secretary; David Follett, treasurer; James C. Pomfret, corresponding secretary; Joseph Mueller, faculty sponsor.

Tennessee Delta, Carson Newman College, Jefferson City

Chapter-President—Paul Collier
23 actives

Semester programs included two film viewings and a presentation on computer programming. The chapter sponsored a hike to the Great Smoky Mountain National Park. Other 1977-78 officers: Monica Parker, vice-president; Ruth Ziegler, secretary; Faye Harris, treasurer; Denver R. Childress, corresponding secretary; Carey R. Herring, faculty sponsor.

Texas Eta, Hardin Simmons University, Abilene

Chapter President—Cynthia Young
31 actives

Cymbe Walston (name correction to TX Eta news of last issue) is currently vice-president of the chapter.

Virginia Beta, Radford College, Radford

Chapter President—Carolyn Gilliam
11 actives

VA Beta sponsors a tutoring service for college students having difficulties with their mathematics classes. Other 1977-78 officers: Debbie Horton, vice-president; Kathy Sellars, secretary; Martha Ellen Peake, treasurer; Janet Susan Milton, corresponding secretary; J. D. Hansard, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President—Kathleen Tandetzke
10 actives, 2 pledges

The chapter had a doughnut sale to raise funds for delegates attending the national meeting in New Concord. The students who accompanied faculty sponsor Sister Mary Petronia were: Kathleen Tandetzke, Judy Gesell, and Laurie Woodruff.

Wisconsin Gamma, University of Wisconsin—Eau Claire, Eau Claire

Chapter President—Gerald Post

The WI Gamma chapter was installed 4 February 1978 by national treasurer Wilbur Waggoner. The charter group consisted of 51 initiates. Approximately 30 guests attended the installation ceremony along with the new initiates. Other officers: Donald Sandman, vice-president; Diane Karloske, recording secretary; Jean Pluke, treasurer; Alvin Rolland, corresponding secretary, Thomas Wineinger, faculty sponsor.

Kappa Mu Epsilon Convention, 10-12 November 1977 Muskingum College, New Concord, Ohio