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Four Combinatorial Probability Problems

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INTRODUCTION

In a first course in probability theory, a good portion of time is spent on combinatorial problems. While standard methods such as relating the problem to some random walk or writing and solving recursive formulae may be taught, finding solutions to new problems is nonetheless sometimes difficult. The challenge of a combinatorial problem often lies in linking up a technique which might have been applied successfully in some other context with the problem at hand.

The problems below were provided by the instructor as "hard" problems. Written solutions were requested from those who felt they had solved them. Several students independently solved each problem, often using entirely different techniques. Those who solved a problem one way found different solutions quite fascinating. Distinct solutions are provided for the problems below to demonstrate the richness of combinatorial techniques. Readers may want to try to solve the problems themselves before seeing our solutions.

THE PROBLEMS

1. *Coin Tossing*

   A symmetric coin is tossed n times. Find the probability $P(n)$ that no two heads occur in a row.

2. *Dice*

   k identical regular polyhedra each have their n sides painted n different colors. They are thrown like dice and the color of the face on which each lands is noted. If all k are thrown at once, how many results are distinguishable?

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[1]Most authors are graduate students in the Statistics department at the University of Pennsylvania. This paper was written while they were students in a probability course given by Geller in the fall semester of 1976.

3. *Ballot Problems*

In a ballot where c votes are cast, candidates A and B receive a and b number of votes, respectively, $a + b = c$. Assuming $a > b$, what is the probability that candidate A will always lead (at least by one vote) throughout the counting of the votes?

4. *Balls in Urns*

There are n urns labelled $1, 2, \cdots, n$, each containing α white and β black balls. A ball is chosen at random from urn 1 and put into urn 2. It is white. Next a ball is chosen from urn 2 and put into urn 3. This process continues. Find the probability $P(W_n)$ that the nth ball chosen is white. Find also $\lim_{n \to \infty} P(W_n)$.

THE SOLUTIONS

Solution 1a. After n tosses there will be 2^n possible outcomes. Let $I(n)$ be the number of outcomes which never have two heads in a row in n tosses. Then $P(n)$, the probability that no two heads occur in a row in n tosses, equals $I(n)/2^n$.

For $n = 1$, 2, and 3, it is easily seen that $I(n) = 2$, 3, and 5 respectively. Notice that $I(3) = I(2) + I(1)$.

In a sequence of n tosses, if two heads in a row do not occur, then either the sequence starts with one tail (there are $I(n - 1)$ of these) or the sequence starts with one head followed by one tail (there are $I(n - 2)$ of these). Thus $I(n) = I(n - 1) + I(n - 2)$. The $I(n)$ may be recognized as the sequence of Fibonacci numbers.

Solution 1b. Let $K(n)$ be the number of outcomes after n tosses that have two heads in a row. Then

$$P(n) = 1 - K(n)/2^n.$$

For $n > 2$, $K(n)$ can be determined by realizing that the $K(n - 1)$ outcomes that were favorable after $n - 1$ tosses are still favorable no matter what happens on the nth toss. There are $2\,K(n - 1)$ such outcomes. Secondly, of those outcomes not favorable after the $(n - 1)$st toss, only those outcomes with a head on the $(n - 1)$st toss can be favorable after the nth toss. There are $2^{n-3} - K(n - 3)$ such outcomes. Thus

$$K(n) = 2K(n - 1) + 2^{n-3} - K(n - 3) \text{ for } n > 2. \quad (1)$$

To solve this third order nonhomogeneous difference equation consider the characteristic equation for the homogeneous equation

$$K(n) - 2K(n - 1) + K(n - 3) = 0\,. \qquad (2)$$

The characteristic equation for (2) is $r^3 - 2r^2 + 1 = 0$ which has solutions: $r = 1, (1 \pm \sqrt{5})/2$.

It follows that the general solution to (1) is

$$K(n) = C_1 + C_2[(1 + \sqrt{5})/2]^n + C_3[(1 - \sqrt{5})/2]^n + C_4 2^n$$

for constants C_1, C_2, C_3, C_4. Solving for C_1, C_2, C_3, C_4 using $K(0) = K(1) = 0$ and $K(2) = 1$ we find

$$K(n) = [(-5 - 3\sqrt{5})/10][1 + \sqrt{5})/2]^n + [(-5 + 3\sqrt{5})/10][(1 - \sqrt{5})/2]^n + 2^n.$$

Now $P(n)$ can be calculated.

Solution 1c. Consider the process as a Markov chain which has three states: no heads in a row, one head in a row, and at least two heads in a row, with transition matrix

$$\pi = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}.$$

Initially the chain is in the state of "no heads in a row". Thus $(1\ 0\ 0)\pi^n$ is the probability that in n trials there are no heads in a row, one head in a row, and at least two heads in a row. It follows that $P(n) = 1 - (1\ 0\ 0)\pi^n(0\ 0\ 1)'$.

In Figure 1 $P(n)$ for $n = 1, 2, \cdots, 15$, calculated (to six decimal places) using APL functions:

Solution 2a. To enumerate the possibilities, first determine

n	$P(n)$
1	0
2	.75
3	.625
4	.5
5	.40625
6	.328125
7	.265625
8	.214844
9	.173828
10	.140625
11	.113770
12	.092041
13	.074463
14	.060242
15	.048737

Figure 1

the numbers of each color for possible color combinations (color need not be specified). Next, determine the number of distinguishable results for the color combinations state; and last, sum the number of distinguishable results for each color combination.

The possible number/color combinations can be described by occupancy numbers $m_1, m_2, \cdots m_n$, where m_i stands for the number of regular polyhedra resulting in the ith color. Every n-tuple of integers satisfying $m_1 + m_2 + \cdots + m_n = k$ describes a possible configuration of occupancy numbers and every n-tuple is considered unordered.

After all possible occupancy configurations are listed we calculate the number of distinguishable results for each configuration, which is

$$\frac{n!}{\prod_{\text{all distinct } m_i} \left(\begin{matrix} \text{\# of times any particular} \\ m_i \text{ is repeated} \end{matrix} \right)!}.$$

The total number of distinguishable results can be found by simple addition.

For example if $n = 3$ colors and $k = 5$ identical regular polyhedra the outcome (5, 0, 0) (all 5 outcomes are of one color) can occur $3!/1!2! = 3$ ways, whereas the outcome (3, 2, 0) (three results of one color and two of another) can occur in $3!/1!1!1! = 6$ ways. The possibilities here are: (5, 0, 0), (4, 1, 0), (3, 2, 0), (3, 1, 1), (2, 2, 1) and they can occur respectively in 3, 6, 6, 3, 3 ways. Thus there are 21 distinguishable results.

Solution 2b. This can be viewed as the problem of placing n indistinguishable particles into k distinguishable cells. Each particle represents on k-sided die and each cell represents one color. This reminds us of the Bose-Einstein statistics [2, p. 39ff], where what we count is how many particles are in each cell and not which particles are in each cell.

To obtain all possible distributions, arrange the cells in sequence in a straight line and then distribute the particles into the cells side-by-side along the same line. Then consider all possible permutations of both the particles and dividing walls of the cells. Since there are n particles and $k - 1$ dividing walls the number of such permutations is equal to the number of arrangements of $n + k - 1$ objects taken n at a time, that is, $\binom{n + k - 1}{n}$.

Solution 3. The first solution [3, p. 244] involves placing a As and b Bs in a circular arrangement. Recognize that going around the circle (perhaps several times) deleting successively adjacent pairs AB leaves $(a - b)$ proper starting positions for A to lead through the counting. Since there are $(a + b)$ starting positions altogether, the required probability is $(a - b)/(a + b)$. Karlin presents a second solution [3, p. 244ff] which uses exchangeable random variables and is too lengthy to describe here.

A third solution may be gotten via random walks. See Feller [1, p. 73].

A related problem is found in Gnedenko [2, pp. 43-44]:

> $2N$ people are lined up at a theater box office; N of them have only five dollar bills and the remaining N only ten-dollar bills. There is no cash in the box office when it opens, and each patron in turn is going to buy a single five-dollar ticket. What is the probability that nobody will be required to wait for change?

Gnedenko solves this problem using a random walk argument. However, restated, this problem asks for the probability that, with

$2n$ people, the number of people coming to the box office with five-dollar bills is always greater than or equal to the number of people with ten-dollar bills. This is equivalent to the probability that, with $2N + 1$ people, $N + 1$ with five dollars and N with ten dollars, the number of people with five-dollar bills is always greater than the number of people with ten-dollar bills, given the first person in line has a five-dollar bill. Thus considering the conditional case this ticket problem reduces to a ballot problem with $a = n + 1$ and $b = n$.

It follows that all three solutions to the ballot problem are now applicable.

Solution 4a. Let $P(B_n) = 1 - P(W_n)$. Since it is known that a white ball is drawn on the first trial, $P(W_1) = 1$. Also, $P(W_2) = (\alpha + 1)/(\alpha + \beta + 1)$. Let $P(W_n|W_{n-1})$ be the conditional probability that a white ball is drawn on the nth trial, given that white was drawn on the $(n - 1)$st trial. Similarly, define $P(W_n|B_{n-1})$.

Then

$$P(W_n) = P(W_n|W_{n-1}) \cdot P(W_{n-1}) + P(W_n|B_{n-1}) \cdot P(B_{n-1})$$

$$= [(\alpha + 1)/(\alpha + \beta + 1)] \cdot P(W_{n-1}) + [\alpha/(\alpha + \beta + 1)] \cdot [1 - P(W_{n-1})].$$

This equation can be solved recursively to obtain:

$$P(W_n) = \alpha/(\alpha + \beta) + \quad /[(\alpha + \beta)(\alpha + \beta + 1)^{n-1}].$$

From this it is clear that $\lim_{n\to\infty} P(W_n) = \alpha/(\alpha + \beta)$.

Solution 4b. Recognize the problem as a homogeneous Markov chain with transition probability matrix

$$\pi = \begin{vmatrix} (\alpha + 1)/(\alpha + \beta + 1) & \beta/(\alpha + \beta + 1) \\ \alpha/(\alpha + \beta + 1) & (\beta + 1)/(\alpha + \beta + 1) \end{vmatrix}.$$

Since the first ball drawn is white, $P(W_n) = (1\ 0)\pi^{n-1}(1\ 0)'$, and the results follow.

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A Logical Ring

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Early in his training, the mathematics student is exposed to rings in algebra and truth tables in logic. Our purpose is to describe truth tables via ring theory and to give a method for evaluating logical sentences numerically.

TRUTH TABLE RING

Consider the following truth table:

P	Q	$P * Q$
T	T	
F	T	
T	F	
F	F	

The column $P * Q$ assumes sixteen possible values depending on the meaning of $*$. For convenience, we list logical expressions for these columns: $\mathscr{T}$, P V Q, P → Q, Q, Q → P, P, P ↔ Q, P ∧ Q, ~ (P ∧ Q), ~ P ↔ Q, ~ P, ~(Q → P), ~ Q, P ∧ ~ Q, ~(P V Q), $\mathscr{F}$.

We denote $\mathscr{T}$ and $\mathscr{F}$ to be the columns with four T's and four F's respectively. Let L be the set of all sixteen elements $P * Q$ and let operations ↔ and V have their usual meanings.

THEOREM 1. *The system* (L, ↔, V) *is a Boolean ring.*

Proof: 1. *We first show that* (L, ↔, V) *is a Boolean ring.* group. Closure is obvious since all possible four-element columns are in L. Since ↔ is clearly abelian and $F \leftrightarrow T = F$, $T \leftrightarrow T = T$, then for any X in L, we have $X \leftrightarrow \mathscr{T} = X$ and $\mathscr{T}$ is the identity. Hence X is its own inverse since $F \leftrightarrow F = T = T \leftrightarrow T$. It remains to show that for X, Y, Z in L, that $(X \leftrightarrow Y) \leftrightarrow Z = X \leftrightarrow (Y \leftrightarrow Z)$. But this is obvious upon substitutions of T or F for X, Y, Z.

2. The system (L, V) is a semigroup. The arguments for closure an associativity of V are duals of those in 1.

3. The distinctive law follows by considering cases. We give one case:

$$T \vee (T \leftrightarrow F) = T \vee F = T = T \leftrightarrow T = (T \vee T) \leftrightarrow (T \vee F).$$

4. The ring $(L, \leftrightarrow, \vee)$ is Boolean. Since $T \vee T = T$ and $F \vee F = F$, then for column X in L, $X \vee X = X$.

Remark. It follows that each element $P * Q$ in L may then be expressed in terms of elements $\mathscr{F}$, P, Q and the operations $\leftrightarrow$ and $\vee$.

We now turn to another ring. Let p and q be independent variables and let $Z_2 = \{0, 1\}$, the integers modulo 2. Let $(R, +, \cdot)$ be the Boolean ring generated by $\{1, p, q\}$ over the ring Z_2, where $R = \{a + bp + cq + dpq : a, b, c, d \text{ in } Z_2\}$. We have the following: The ring $(L, \leftrightarrow, \vee)$ is isomorphic to the ring $(R, +, \cdot)$. Both rings L and R are Boolean of order 16 and hence isomorphic; but what is the isomorphism? First, some preliminaries.

Definition: An element x is an *atom* in a Boolean Ring if and only if xy equals either x or the additive identity in the ring, for any y in the ring. Atom is sometimes called "point" [1, p. 296]. The atoms of a Boolean ring generate ideals of order two and the ring, if finite, is the direct sum of these ideals [3, pp. 17, 25]. We wish to find the four atoms of L and R respectively. Any one-to-one mapping of atoms will induce an isomorphism.

THEOREM 2. *The atoms of* L *are*: $P \vee Q$, $P \rightarrow Q$, $Q \rightarrow P$, $\sim P \wedge Q$. *The atoms of* R *are*: pq, $pq + p$, $pq + q$, $pq + p + q + 1$.

Proof: Let

$$Y = \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \text{ be an arbitrary element in } L.$$

Then

$$(P \vee Q) \vee Y = \begin{pmatrix} T \\ T \\ T \\ F \end{pmatrix} \vee \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} T \\ T \\ T \\ F \end{pmatrix} \text{ or } \begin{pmatrix} T \\ T \\ T \\ T \end{pmatrix}$$

That is, $(P \vee Q) \vee Y$ equals $(P \vee Q)$ or $\mathscr{T}$, depending on whether D is F or T, respectively. Likewise, the other elements with only one F and three T values per column are shown to be atoms of L.

Let $y = a + bp + cq + dpq$ be an arbitrary element in R. We have the following:

1) $(pq)y = (a + b + c + d)pq = pq$ or 0.
2) $(pq + p)y = (a + b)(pq + p) = pq + p$ or 0.
3) $(pq + q)y = (a + c)(pq + q) = pq + q$ or 0.
4) $(pq + p + q + 1)y = a(pq + p + q + 1) =$ $pq + p + q + 1$ or 0.

We can now construct an isomorphism Φ from L to R. We map $P \vee Q$, $P \rightarrow Q$, $Q \rightarrow P$, $\sim(P \rightarrow Q)$ to pq, $pq + q$, $pq + p$, $pq + p + q + 1$, respectively. We complete Φ by mapping sums $(\leftrightarrow)$ of atoms in L to the corresponding sums $(+)$ of atoms in R, as the following cases illustrate.

1) Since $\sim P = \mathscr{T} \leftrightarrow P$, then $\Phi(\sim P) = 1 + p$.
2) Since $\Phi(P \rightarrow Q) = pq + q$ and $\Phi(Q \rightarrow P) = pq + p$, then $\Phi(P \leftrightarrow Q) = \Phi[(P \rightarrow Q) \leftrightarrow (Q \rightarrow P)] =$ $(pq + q) + (pq + p) = p + q$.
3) Since $P \quad Q = (P \vee Q) \rightarrow (P \rightarrow Q)$, then $\Phi(P \quad Q) = pq + p + q$.

Examples. We give two examples illustrating the use of the isomorphism Φ to prove tautologies of the predicate calculus.

1) *Contrapositive.* Show that $(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)$ is a tautology. Map, by Φ, $[(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)]$ to $(pq + q) + (1 + p)(1 + q) + (1 + p) = 0$. Since Φ is an isomorphism, then $\Phi^{-1}(0) = \mathscr{T}$.

2) *De Morgan's Laws.* We have

$\Phi[\sim(P \vee Q) \leftrightarrow (\sim P \wedge \sim Q)] = 1 + pq +$
$[(1 + p)(1 + q) + (1 + p) + (1 + q)] = 0$ and
$\Phi[\sim(P \wedge Q) \leftrightarrow (\sim P \vee \sim Q)] = (1 + pq + p + q) +$ $(1 + p)(1 + q) = 0$. But in both cases the inverse of 0 is $\mathscr{T}$.

LARGER FINITE RINGS

We now turn to logical statements involving more than two variables. We first construct the Boolean ring. $(R_n +, \cdot)$.

Let $S = \{1, x_1, \cdots, x_n\}$ where $1 \in Z_2$, and the x_i are independent variables. Let $(R_n +, \cdot)$ be the Boolean ring generated by S, where R_n consists of all finite sums of finite products of elements in S. We write $R_n = [1, x_1, \cdots, x_n]$. Considered as a vector space over Z_2, R_n has dimension 2^n and basis $B = \{1, x_{i_1} x_{i_2} \cdots x_{i_k} : i_1 < i_2 < \cdots < i_k$ and $k = 1, 2, \cdots, n\}$. We now find the atoms of R_n.

Theorem 3. *The atoms of* $(R_n + \cdot)$ *is the set* $\mathscr{A} = \{a_1 a_2 \cdots a_n : a_i = x_i$ or $x_i + 1\}$. *The order of* $\mathscr{A}$ *is* 2^n.

Proof: 1. Now $x_i a_i = a_i$ or zero. If $b = x_{i_1} \cdots x_{i_k}$ in B, then $b(a_1 \cdots a_n) = a_1 \cdots a_n$ or zero. If element $\Sigma c_i b_i$ is arbitrary in R_n, where $c_i = 0$ or 1 and $b_i \in B$, then $(\Sigma c_i b_i)(a_1 \cdots a_n) = \Sigma c_i (b_i a_1 \cdots a_n) = a_1 \cdots a_n$ or zero.

2. The ring R_n, being a vector space of dimension 2^n, is the direct sum of 2^n ideals, each isomorphic to Z_2. Thus R_n has 2^n atoms. We wish to show that $\mathscr{A}$ lists all 2^n atoms. Let $a_1 \cdots a_n = a'_1 \cdots a'_n$ where each a_i and a'_i is x_i or $x_i + 1$. Suppose that $a_j = x_j$ and $a'_j = x_j + 1$ for some j. Then $a_1 \cdots a_n = x_j(a_1 \cdots a_n) = x_j(a'_1 \cdots a'_n) = 0$, a contradiction. Therefore, the order of $\mathscr{A}$ is 2^n.

The Boolean Ring $(L_n, \leftrightarrow, V)$. We now construct truth tables with columns $\{x_1, x_2, \cdots, x_n, *\}$ each of length 2^n. Each column x_i has period 2^i, where each period begins with 2^{i-1} T's followed by 2^{i-1} F's. The star column represents logical sentences in the X_i's. The number of distinct possible columns under * is $2^{(2^n)}$. Let addition $(\rightarrow)$ and multiplication (V) be defined as usual, and let $\mathscr{F}$ and $\mathscr{T}$ represent columns with all F's and T's respectively. The set of all finite sums of all finite products of elements in $\{\mathscr{F}, X_1, \cdots, X_n\}$ is written $L_n = [\mathscr{F}, X_1, \cdots X_n]$.

Theorem 4. *The system* $(L_n, \leftrightarrow, V)$ *is a Boolean Ring of order* $2^{(2^n)}$.

Proof: 1. As with the truth table ring, looking at truth table columns, it is clear that the two operations are commutative and associative. They are also closed by the definition of the set L_n. It is clear that the operation $\vee$ is distributive over $\leftrightarrow$. Also, for Y in L_n, $Y \leftrightarrow \mathscr{T} = Y = Y \vee Y = Y \vee \mathscr{F}$ and $Y \leftrightarrow Y = \mathscr{T}$. Thus $(L_n, \leftrightarrow, \vee)$ is a Boolean Ring with identity $\mathscr{F}$.

2. We still must prove that the order of L_n is $2^{(2^n)}$. Consider L_n as a vector space over $\{\mathscr{F}, \mathscr{T}\}$. We will show that its basis is $B = \{\mathscr{F}, X_{i_1} \vee X_{i_2} \vee \cdots \vee X_{i_k} : i_1 < i_2 < \cdots < i_k$ and $k = 1, 2, \cdots, n\}$. First, $\{\mathscr{F}\}$ is an independent set. Considering $\mathscr{F}$ as X_0, assume elements in B with subscripts less than s to be independent. Suppose S is a sum $(\leftrightarrow)$ of distinct elements in B with subscripts less than or equal to s and that $S = \mathscr{T}$. We will show a contradiction. Write $S = S' \leftrightarrow (S'' \vee X_s)$ where S' and S'' are sums of elements in B with subscripts less than s. Since S' is not identically $\mathscr{T}$, its column has an F in it. In fact, it has an F among the first 2^{s-1} terms and also among the next 2^{s-1} terms. But the first 2^{s-1} terms of X_s are all T followed by 2^{s-1} F's. Therefore, $S' \leftrightarrow (S'' \vee X_s) = S$ cannot be identically $\mathscr{T}$. Finite induction completes the argument for independence. The order of B then is $1 + \sum_{k=1}^{n} \binom{n}{k} = 1 + (2^n - 1) = 2^n$ and the order of L_n is $2^{(2^n)}$.

Remark. Since the number of possible columns under * is $2^{(2^n)}$ and the order of L_n is $2^{(2^n)}$, it follows that $(L_n, \leftrightarrow, \vee)$ represents all logical sentences involving the X_i's.

The ring $(L_n \leftrightarrow, \vee)$ is isomorphic to $(R_n, +, \cdot)$. Let $\Phi(\mathscr{F}) = 1$ and for other elements in B let $\Phi(x_{i_1} \vee \cdots \vee x_{i_k}) = x_{i_1} \cdots x_{i_k}$. Extend Φ by sums to all of L_n. Since the order of L_n equals the order of R_n, Φ is a vector space isomorphism. Also, Φ preserves products of basis elements and hence is a ring isomorphism.

THEOREM 5. *The atoms of* $(L_n \leftrightarrow, V)$ *is the set* $\{A_1 V \cdots VA_n : A_i = X_i \text{ or } X_i \leftrightarrow \mathscr{F}\}$ *of order* 2^n.

Proof: This follows from the isomorphism above and from Theorem 3. However, it is instructive to show directly that L_n has 2^n atoms. Let $A_1 V \cdots VA_n = A'_1 V \cdots VA'_n$ where each A_i and A'_i is X_i or $(X_i \leftrightarrow \mathscr{F}\cdot)$. Suppose $A_i = X_i$ and $A_i = X_i \leftrightarrow \mathscr{F}$ for some i. Then $\mathscr{T} \neq A_1 V \cdots VA_n \; X_i(A_1 V \cdots VA_n) = X_i(A'_1 V \cdots VA'_n) = \mathscr{T}$ which is false.

Example. We will show that $(X \rightarrow Y) \vee (X \rightarrow Z) \leftrightarrow (X \rightarrow Y \vee Z)$ is a tautology. Our method, though cumbersome, is logically convincing and lends itself to solution by computer.

Since, for variables P and Q, the meaning of $P \rightarrow Q$ is $(P \vee Q) \leftrightarrow Q$, the image of $P \rightarrow Q$ is the image of $(P \vee Q) \leftrightarrow Q$. In our example, treating X, Y, Z like X_1, X_2, X_3 respectively in L_n the image of $X \rightarrow Y$ is $xy + y$ and the image of the whole statement is $(xy + y)(xz + z) + xyz + yz$. Now, if $x = 1$, we have $(2y)(2z) + 2y$ which is zero. If $x = 0$, we have $yz + yz$ which is again zero. Our statement is therefore a tautology.

A GENERAL RING.

Consider the countably infinite set $\{X_1, X_2, \cdots\}$. A truth table for these variables may be constructed as follows. The columns are named in order: $X_1, X_2, \cdots, *$. The column under each X_i begins with a subcolumn of length 2^i consisting of 2^{i-1} consecutive T's followed by 2^{i-1} consecutive F's. This subcolumn, for each i, is repeated periodically ad infinitum. Let $\mathscr{F}$ and $\mathscr{T}$ be the columns with all F's and T's, respectively. Let $L_n = [\mathscr{F}, X_1, \cdots, X_n]$ be the set of all finite sums of finite products of elements in $\{\mathscr{F}, X_1, \cdots, X_n\}$. Then $(L_n \rightarrow, V)$ is a Boolean ring with identity $\mathscr{F}$ and is isomorphic to the ring of the same designation in the previous section. If R_n is formed as before, we have L_n isomorphic to R_n In a natural manner, we can form $\mathscr{L} = \bigcup_1^\infty L_n$ and $\mathscr{R} = \bigcup_1^\infty R_n$. We have the following.

THEOREM 6. *The system* $(\mathscr{L}, \leftrightarrow, \vee)$ *and* $(\mathscr{R}, +, \cdot)$ *are isomorphic Boolean rings.*

Here, however, $\mathscr{L}$ differs somewhat from L_n. First, $\mathscr{L}$ has no atoms. An atom should have only one *F* in its column. But, any element of $\mathscr{L}$ with one *F*, being periodic, contains an infinite number of *F*'s. Secondly, $\mathscr{L}$, as a Boolean ring, by the Stone Representation Theorem [1, p. 297] is a subset of the complete direct sum of an infinite number of copies of Z_2, but is not the direct sum of 2-element ideals.

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The Relationship of Pascal's Triangle and Perfect Numbers

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A positive integer is called a perfect number if it is equal to the sum of all its positive divisors other than itself. A perfect number will be designated by the Greek letter π. Showing the fact that all even perfect numbers are on the third diagonal of Pascal's triangle is not arduous. Each number of Pascal's triangle can be put into the form $\binom{n}{j}$, where $\binom{n}{j} = \frac{n!}{j!(n-j)!}$ is the binomial coefficient nj. Using this factorial notation, 0! is defined to be one.

Designating the horizontal rows as n rows and the diagonal rows as j rows, the triangle can now be formed with the binomial coefficient nj by taking the intersection of n and j.

$$\begin{array}{lcccccccl}
n=0 \rightarrow & & & & 1 & & & & \swarrow j=0 \\
n=1 \rightarrow & & & 1 & & 1 & & & \swarrow j=1 \\
n=2 \rightarrow & & 1 & & 2 & & 1 & & \swarrow j=2 \\
n=3 \rightarrow & 1 & & 3 & & 3 & & 1 & \swarrow j=3
\end{array}$$

integrally

$$\begin{array}{ccccccc}
 & & & \binom{0}{0} & & & \\
 & & \binom{1}{0} & & \binom{1}{1} & & \\
 & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\
\binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3}
\end{array}$$

binomial coefficient nj

Figure 1

Euler proved a satisfying theorem that allows us to determine all even perfect numbers. An even integer is a perfect number if it is of the form $2^{p-1}(2^p - 1)$, where $2^p - 1$ is prime. In order for this to be true, p must be what is known as a Mersenne prime. An example of a Mersenne prime is 3 because $2^3 - 1 = 7$ is prime.

*This article was written while the student attended Lincoln High School, Des Moines, Iowa.

Since each number of Pascal's triangle can be put in the form $\binom{n}{j}$ and if certain restrictions are placed on n and j, then all even perfect numbers can be shown to be in Pascal's triangle.

The restriction for n is that it must equal a Mersenne prime power of 2. Since 3 is a Mersenne prime, 2^3 is a Mersenne prime power of 2. The restriction for j is that it must equal 2 since the third diagonal of the triangle is $j = 2$ (see Figure 1).

With these restrictions, it follows that

$$\binom{n}{j} = \frac{2^p!}{2!(2^p - 2)!} = \frac{2^p(2^p - 1)(2^p - 2)!}{2!(2^p - 2)!} = \frac{2^p(2^p - 1)}{2!} = 2^{p-1}(2^p - 1)$$

which is an even perfect number π.

Since each number on the triangle is the sum of all the integers on the preceding diagonal and since all even perfect numbers are on the third diagonal, then the second diagonal would form the even perfect numbers. The second diagonal is the natural numbers. Therefore, every even perfect number is the sum of the first $2^p - 1$ numbers. That is, any even perfect number $\pi = \sum_{i=1}^{2^p - 1} i$. This is an arithmetic sequence with one as the first term and $2^p - 1$ as the number of terms. Some examples of even perfect numbers are: 6, 28, 496, 8128, $\cdots$.

Now that we know an even perfect number is the sum of the first $2^p - 1$ positive integers, we can derive a new formula for an even perfect number π. The sum S_n of an arithmetic sequence is $S_n = (n/2)(a_1 + a_n)$. After substituting the appropriate values, we find upon evaluation that all perfect numbers π can be expressed as $2^{2p-1} - 2^{p-1}$.

We now have several different ways of computing perfect numbers. We must first compute Mersenne primes p. Knowing the Mersenne primes, we can;

(1) sum up the first $2^p - 1$ positive integers,

(2) using Euler's formula, compute $\pi = (2^{p-1})(2^p - 1)$,

(3) using the newly derived formula, compute $\pi =$
$\frac{2^p - 1}{2} (1 + (2^p - 1))$
$= 2^{2p-1} - 2^{p-1}$, or

(4) with the given restrictions on n and j, compute $\pi =$
$\binom{n}{j}$.

It is from (4) that we find the fascinating fact that all even perfect numbers are on the third diagonal of Pascal's triangle.

On the Composition of the Iterates of a Function

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In many areas of analysis and topology, it is useful to consider the iterates of a given function which maps a set into itself. It is clear that if m and n are non-negative integers, $f^n(f^m) = f^{n+m}$ and $(f^n)^m = f^{nm}$. In this note, we investigate the validity of these expressions for arbitrary integers m and n.

In order to simplify matters somewhat, let f map a set X *onto* itself. In the usual way, f and f^{-1} can be viewed as set mappings, i.e. for $A \subseteq X$, $f(A) = \{f(x): x \in A\}$ and $f^{-1}(A) = \{x \in X: f(x) \in A\}$. (If f were not onto, then $f^{-1}(A)$ would be empty whenever A failed to intersect the range of f.) Define the iterates of f inductively, i.e. let f^0 be the identity function on X, let $f^n = f(f^{n-1})$ for $n > 1$, and let $f^n = f^{-1}(f^{n+1})$ for $n \leq -1$. It is clear that for $n < 0$ and $A \subseteq X$, $f^n(A) = \{x \in X: f^{-1}(x) \in A\}$, and for $m, n \geq 0$, $f^n f^m = f^{n+m}$ and $(f^n)^m = f^{nm}$. In general, we have the following.

THEOREM. *If* f *is a function which maps a set* X *onto itself, then for each* $x \in X$,

(a) $m, n \leq 0$ *implies* $f^n(f^m(x)) = f^{n+m}(x)$.

(b) $n \geq 0$ *implies* $f^n(f^m(x)) = f^{n+m}(x)$ *for each* m.

(c) $f^n(f^m(x)) \supseteq f^{n+m}(x)$ *for each* m *and* n.

(d) $n > 0$ *implies* $(f^n)^m(x) = f^{nm}(x)$ *for each* m.

(For the purpose of the notation, we have made no distinction between x and $\{x\}$.)

Proof: For part (a), $y \in f^n(f^m(x))$ if and only if $f^{-n}(y) \in f^m(x)$, which is true if and only *if* $f^{-m-n}(y) = x$, which is equivalent to $y \in f^{m+n}(x)$, i.e., $y \in f^n(f^m(x))$ if and only if $y \in f^{m+n}(x)$.

For part (b), if $m \geq 0$, the conclusion is obvious. Assume $m < 0$. If $m + n < 0$, part (a) implies $f^n(f^m(x) =$ $f^n(f^{-n}(f^{m+n}(x))) = f^{m+n}(x)$. If $m + n > 0$, $f^n(f^m(x)) =$ $f^{n+m}f^{-m}(f^m(x)) = f^{n+m}(x)$.

For part (c), we may assume that $n < 0$, since the case $n \geq 0$ is covered by part (b). Then $y \varepsilon f^{n+m}(x)$ implies $f^{-n}(y) \varepsilon f^{-n}(f^{n+m}(x)) = f^m(x)$, by part (b). Hence, $y \varepsilon f^n A f^m(x))$, and $f^{n+m}(x) \subseteq f^n(f^m(x))$.

For part (d), the conclusion is obvious if $m \geqq 0$. If $m < 0$, $(f^n)^m(x) = \{y\colon x = (f^n)^{-m}(y)\} = \{y\colon x = f^{-nm}(y)\} = \{y\colon y \varepsilon f^{nm}(x)\} = f^{nm}(x)$.

Part (b) is not valid if f is not onto. For if x is not in the range of f and if $m < 0$, then $f^{-m}f^m(x) = f^{-m}(\Phi) \neq \Phi = x = f^0(x)$. However, onto-ness implies $f^{-m}f^m(x) = x$ for $m < 0$, which yields the last equality in the proof of part (b).

Part (c) cannot be strengthened to an equality, as the following example shows. Let X be the positive integers, and let $f(1) = 1$, $f(n) = n - 1$ for $n > 2$. Then f maps X onto itself, but $f^{-1}(f(1)) = f^{-1}(1) = \{1,2\} \neq \{1\} = f^0(1)$.

In part (d), we required $n \geqq 0$ since only the iterates of *functions* have been defined. If $n < 0$ and f is not one-to-one, f^n is not a function.

A Functional Equation Approach To e

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The elementary properties of the log function and particularly the number e have long been topics of mathematical interest. This paper considers a slightly different approach to the natural log function. The derivitive of the log function and the resulting functional equation are considered. For the function $f(x) = \log_b x$ for $x > 0$ and $b > 1$ satisfying $b^{f(x)} = x$, two properties are assumed:

(a) $f(x^y) = y \cdot f(x)$ for $x > 0$ and all y, and

(b) $f'(x)$ exists for $x > 0$ and is continuous at $x = 1$.

Using these two assumptions $f(x^2) = 2f(x)$ and the basic functional equation

$$f'(x) = f'(x^2) \cdot x \tag{1}$$

for $x > 0$ follows.

LEMMA 1. $f'(x)$ *satisfies* (1) *iff* $f'(x) = \frac{g(x)}{x}$ *where* $g(x) = g(x^2)$.

LEMMA 2. *If* $g(x) = g(x^2)$ *for* $x > 0$, *then* $g(x^{2^{-n}}) = g(x)$ *for* $n = 1, 2, \cdots$.

The proofs of these two lemmas are direct and omitted. (Throughout a^{b^c} denotes as usual $a^{(b^c)}$ and NOT $(a^b)^c$.)

To solve $g(x) = g(x^2)$ the constant function is not the only solution. Any function $h(x)$ defined on $[\sqrt{2}, 2]$ so $h(\sqrt{2}) = h(2)$ can be extended to $(1, \infty)$ by successively applying $h(x) = h(x^2)$. To extend $h(x)$ to $(0, 1)$ define $h(x) = h(x^{-1})$ for $0 < x < 1$. If $h(1)$ is then arbitrarily defined, $h(x) = h(x^2)$ for $x > 0$. However, assuming continuity at $x = 1$ gives:

THEOREM 1. *Assuming* $f(x) = \log_b x$ *for* $x > 0$ *and* $b > 1$ *has a derivitive for* $x > 0$ *and a continuous derivitive at* $x = 1$, *then*

$$f'(x) = \frac{c}{x} \tag{2}$$

for $x > 0$ *and* c *a constant.*

Proof: By Lemma 1, $f'(x) = \dfrac{g(x)}{x}$ where $g(x) = g(x^2)$. By continuity at $x = 1$, $g(x)$ is continuous at $x = 1$. Let $\varepsilon > 0$. There is a $\delta > 0$ so $|x - 1)| < \delta$ implies $|g(x) - g(1)| < \varepsilon$. For $y > 0$, $\lim\limits_{n \to \infty} y^{2^{-n}} = 1$ so choose n so $|y^{2^{-n}} - 1| < \delta$.

By Lemma 2 and continuity $|g(y) - g(1)| = |g(y^{2^{-n}}) - g(1)| < \varepsilon$. Since ε is arbitrary $g(y) = g(1)$ or $g(y) = c$, a constant, for $y > 0$.

The constant c is a function of b and denote this by $c(b)$.

LEMMA 3. *For* $b > 1$, $c(b) > 0$.

Proof: Since $b > 1$, $1 < b < b^2$ and $f(b) = 1 < 2 = f(b^2)$. Since $f'(x) = \dfrac{c}{x}$ is always 0, always positive, or always negative for fixed b, $f(x)$ is either constant, strictly increasing or strictly decreasing. $f(b) < f(b^2)$ implies $f(x)$ is strictly increasing so $f'(x) > 0$ or equivalently $c(b) > 0$.

LEMMA 4. *For* b *and* d *greater than* 1, $c(b) = \dfrac{c(d)}{\log_d b}$.

Proof: Let $y = \log_b x$ or $b^y = x$. Take the $\log_d$ of the second equality. Using $f(x^y) = y \cdot f(x)$ and differentiating gives the result.

LEMMA 5. *There is a number* $e > 1$ *so* $c(e) = 1$.

Proof: Fix d in Lemma 4. By Theorem 1 and Lemma 3, $\log_d b$ is strictly increasing in $b(b > 1)$. Thus by Lemma 4, $c(d)$ is strictly decreasing and continuous in b. Setting $b = d^n$ in Lemma 4, yields $c(d^n) = \dfrac{c(d)}{n}$. Since $c(d) > 0$, n can be chosen so

$c(d^n) < 1$. Setting $b = d^{n^{-1}}$ in Lemma 4, yields $c(d^{n^{-1}}) = n \cdot c(d)$. Here n can be chosen so $c(d^{n^{-1}}) > 1$. Thus there are values of b so $c(b) < 1$ and values of b so $c(b) > 1$. By continuity there is a number $e > 1$ so $c(e) = 1$. This number is unique as $c(b)$ is strictly decreasing in b.

Next the traditional limit for e is obtained.

LEMMA 6. *For* b > 1 *and* x > 0 *and* n *a positive integer,*

$$\frac{1}{n(b^{n^{-1}} - 1)} < c(b) < \frac{1}{n(1 - b^{-n^{-1}})}. \qquad (3)$$

Proof: By Theorem 1 and Lemma 3, $f''(x) = -\frac{c(b)}{x^2} < 0$ for $x > 0$. Thus $f(x)$ is concave down. Since $b^{n^{-1}}$ decreases to 1 as n goes to ∞, concavity gives that the chord slopes satisfy

$$f'(1) = c(b) > \frac{f(b^{n^{-1}}) - f(1)}{b^{n^{-1}} - 1} = \frac{1}{n(b^{n^{-1}} - 1)}$$

for $n = 1, 2, \cdots$. The other half of the inequality follows from concavity and the fact that $b^{-n^{-1}}$ increases to 1 as n goes to ∞.

LEMMA 7. *For* $b > 1$, $n(1 - b^{-n^{-1}}) < \log_e b < n(b^{n^{-1}} - 1)$ *for* $n = 1, 2, \cdots$.

Proof: Set $d = e$ in Lemma 4 and substitute into equation (3). The desired inequalities follow directly since the left side of (3) is positive.

THEOREM 2. $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$.

Proof: Set $b = \left(1 + \frac{1}{n}\right)^n$ in Lemma 7. Since $b > 1$, this gives

$$\left(1 + \frac{1}{n}\right)^{-1} < \log_e \left(1 + \frac{1}{n}\right)^n < 1$$

for $n = 1, 2, \cdots$. Since $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{-1} = 1$, the

$\lim_{n \to \infty} \log_e \left(1 + \frac{1}{n}\right)^n$ exists and the limit is 1. To show $\left(1 + \frac{1}{n}\right)^n$ has a limit, write $x_n = \left(1 + \frac{1}{n}\right)^n$ and $f(x_n) = \log_e x_n$. There exists a number B so $|f(x_n)| < B$ for all n since $\lim_{n \to \infty} f(x_n)$ exists. The sequence $|x_n|$ is also bounded since $f(x)$ is a strictly increasing continuous function. Let U be a bound for $|x_n|$. Each x_n is greater than 1 so $1 < x_n < U$ and $1 > \frac{1}{x_n} \geqq \frac{1}{U}$ for all n. For any n and m, the mean value theorem gives that there is a $\Theta_{n,m}$ between x_n and x_m so

$$f(x_n) - f(x_m) = f'(\Theta_{n,m}) \cdot (x_n - x_m). \tag{4}$$

Using Lemma 5, Theorem 1, and the fact that $\Theta_{n,m}$ is between x_n and x_m yields that $f'(\Theta_{n,m}) = \frac{1}{\Theta_{n,m}} \geqq \frac{1}{U}$. Taking absolute values in equation (4) gives that for all n and m

$$|x_n - x_m| \leq U \cdot |f(x_n) - f(x_m)|.$$

As the $\lim_{n \to \infty} f(x_n)$ exists, the $\lim_{n \to \infty} x_n$ then also exists. The continuity of $f(x)$ gives that

$$1 = \lim_{n \to \infty} \left[\log_e \left(1 + \frac{1}{n}\right)^n\right]$$

$$= \log_e \left[\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n\right].$$

Since $f(x) = \log_e x$ is strictly increasing, $x = e$ is the only number so $f(x) = 1$. Thus $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

Corollary 1. *For* $c > 0$, $\lim_{n \to \infty} \left(1 + \frac{c}{n}\right)^n = e^c$.

Proof: As $c > 0$, setting $b = \left(1 + \frac{c}{n}\right)^n$ gives $b > 1$.

Lemma 7 yields $\dfrac{c}{1+\dfrac{c}{n}} < \log_e\left(1+\dfrac{c}{n}\right)^n < c$

and thus $\lim\limits_{n\to\infty}\left[\log_e\left(1+\dfrac{c}{n}\right)^n\right] = c$. The proof continues as Theorem 2.

COROLLARY 2. *For* c > 0, $\lim\limits_{n\to\infty}\left(1-\dfrac{c}{n}\right)^n = e^{-c}$.

Proof: As $c > 0$, setting $b = \left(1-\dfrac{c}{n}\right)^{-n}$ gives $b > 1$. Lemma 7 yields

$$c < \log_e\left(1-\frac{c}{n}\right)^{-n} < \frac{c}{1-\dfrac{c}{n}}.$$ Using

$\log_e\left(1-\dfrac{c}{n}\right)^{-n} = -\log_e\left(1-\dfrac{c}{n}\right)^n$ gives $\lim\limits_{n\to\infty}\left[\log_e\left(1-\dfrac{c}{n}\right)^n\right] = -c$. The proof continues as Theorem 2.

The Problem Corner

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1978. The best solutions submitted by students will be published in the Fall 1978 issue of **The Pentagon**, with credit being for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

297. *Proposed by Charles W. Trigg, San Diego, California.*

$R\ E\ T\ I\ R\ E$ is a square number in the decimal system with $R\ E + T\ I = R\ E$. Each letter represents a different digit and the sum of three digits equals the fourth. What is this square number?

298. *Proposed by H. Laurence Ridge, Toronto, Ontario, Canada.*

It is well known that all primitive pythagorean triangles (PPT) are generated by the formulae $x = 2ab$, $y = a^2 - b^2$ and $z = a^2 + b^2$ where a and b are positive integers of opposite parity and $(a,b) = 1$. Let N be an arbitrary positive integer. If N is the leg (or hypotenuse) of a PPT, it is possible to determine the PPT's of which N is a leg (or hypotenuse). What are the necessary and sufficient conditions for N to be a leg (or hypotenuse) of exactly one PPT?

299. *Proposed by the editor.*

Devise a method for dividing a 17° angle into seventeen equal parts.

300. *Proposed by the editor.*

A and B play a game according to the following rules:
A selects a positive integer. B then must determine the number chosen by A by asking A not more than thirty questions, each of which can be answered by only no or yes. What is the largest number which A can choose which

can be determined by B in thirty questions? Generalize to n questions.

301. *Proposed by the editor.*

If 65% of the populace has kidney trouble, 70% have diabetes, 85% have respiratory problems, and 90% have athlete's foot, what is the smallest portion of the populace who are afflicted with all four maladies?

SOLUTIONS

287. *Proposed by Randall J. Covill, West Newbury, Massachusetts.*

Solve the following alphametic in which each letter represents a unique digit in base 14 and $E \neq 0$.

$$\begin{array}{r} SUBTEND \\ +\ ADDEND \\ \hline ANSWERS \end{array}$$

Solution by Charles W. Trigg, San Diego, California.

Let the larger "digits" in base fourteen be $10 = a$, $11 = b$, $12 = c$, and $13 = d$. Working from the right, the successive columns establish the following equations:

$$2D = S + 10k, \text{ so } S \text{ is even.} \quad (1)$$
$$2N + k = R + 10, \text{ so } N > 6. \quad (2)$$
$$2E + 1 = E + 10, \text{ so } E = d. \quad (3)$$
$$T + D + 1 = W + 10. \quad (4)$$
$$B + D + 1 = S + 10k, \text{ so } B + 1 = D. \quad (5)$$
$$U + A + k = N + 10, \text{ so } A > 7. \quad (6)$$
$$S + 1 = A, \quad \text{so } S > 6. \quad (7)$$

Within the established restrictions, working with the equations in the order (1), (7), (5), (6), (2), and (4), values of the letters are determined to be as follows:

k	D	S	A	B	U	N	R	T	W
0	4	8	9	3	c	7	0	a	1
0	4	8	9	3	c	7	0	b	2
0	5	a	b	4	c	9	4	duplicate	
1	b	8	9	a	c	8	duplicate		
1	c	a	b	b	duplicate				

Thus, there are 2 solutions:

$$\begin{array}{r} 8\,c\,3\,a\,d\,7\,4 \\ 9\,4\,4\,d\,7\,4 \\ \hline 9\,7\,8\,1\,d\,0\,8 \end{array} \quad \text{and} \quad \begin{array}{r} 8\,c\,3\,b\,d\,7\,4 \\ 9\,4\,4\,d\,7\,4 \\ \hline 9\,7\,8\,2\,d\,0\,8 \end{array}$$

The cryptarithm has no solution in the decimal system.

Also solved by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas; Johnnie C. Roberts, University of Missouri-Rolla, Rolla, Missouri; and the proposer.

288. *Proposed by Charles Trigg, San Diego, California.*

Factor $6x^5 - 15x^4 + 20x^3 - 15x^2 + 6x - 1$

Solution by John Molini, Columbia, Missouri.

Let $P(x) = 6x^5 - 15x^4 + 20x^3 - 15x^2 + 6x - 1$.
Then $P(x) = -x^6 + (x-1)^6 = (-1)(x^3)^2 - ((x-1)^3)^2$

$= (3x^2 - 3x + 1)(2x^3 - 3x^2 + 3x - 1)$

Testing the rational roots of the cubic factor yields

$(2x^3 - 3x^2 + 3x - 1) = (2x - 1)(x^2 - x + 1)$.

Hence $P(x) = (2x - 1)(x^2 - x + 1)(3x^2 - 3x + 1)$ in the field of real numbers. Applying the quadratic formula to the quadratic factors yields the complex factorization

$$P(x) = (2x-1)\left(x - \left(\frac{1+i\sqrt{3}}{2}\right)\right)\left(x - \left(\frac{1-i\sqrt{3}}{2}\right)\right)\left(x - \left(\frac{3+i\sqrt{3}}{6}\right)\right)\left(x - \left(\frac{3-i\sqrt{3}}{6}\right)\right)$$

Also solved by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas; Terri L. O'Dell, Missouri Southern State College, Joplin, Missouri; Johnnie C. Roberts, University of Missouri-Rolla, Rolla, Missouri; Leo Sauve, Algonquin College, Ottawa, Ontario, Canada; Harland H. Shoemaker, Jr., Bloomsburg State College, Bloomsburg, Pennsylvania; and the proposer.

289. *Proposed by Charles Trigg, San Diego, California.*

Each of the three consecutive integers 4, 5, and 6 terminates its own cube. That is $4^3 = 64$, $5^3 = 125$, and $6^3 = 216$. Find four pairs of larger consecutive integers in which each integer terminates its own cube.

Solution by Kathleen A. Carlson, Indiana University of Pennsylvania, Indiana, Pennsylvania.

An integer x terminates its own cube whenever

$$x^3 \equiv x(\text{mod } 10^k) \text{ or } (x - 1)(x)(x + 1) \equiv 0(\text{mod } 10^k) \quad (1)$$

where k is a positive integer.

By inspection, 7, 8 and 9 are not solutions so $x \geq 10$.

Considering the range $10 \leq x < 100$ and taking $k = 2$ in (1) we have $(x - 1)(x)(x + 1) \equiv 0(\text{mod } 100)$.

But $100 - 2^2 \cdot 5^2$ so that one of the integers $x - 1$, x, $x + 1$ is divisible by 25 while the product of the other two is divisible by 4. Testing the possible triples of consecutive integers (23, 24, 25) (24, 25, 26), (48, 49, 50) (50, 51, 52)(74, 75, 76) and (75, 76, 77) yields the two pairs ($24^3 = 13824$, $25^3 = 15625$) and ($75^3 = 421875$, $76^3 = 438976$). Considering the range $100 < x < 1000$ and taking $k = 3$ in (1) we have $(x - 1)(x)(x + 1) = 0(\text{mod } 1000)$.

Since $1000 = 2^3 \cdot 5^3$ we consider the multiples of 125. Testing these multiples yields the two pairs ($375^3 = 52734375$, $376^3 = 53157376$) and ($624^3 = 242970624$, $625^3 = 244140625$). Thus four pairs of integers satisfying the conditions of the problem are: (24, 25) (75, 76), (375, 376) and (624, 625).

Also solved by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas; Leigh James, Rocky Hill, Connecticut; Terri L. O'Dell, Missouri Southern State College, Joplin, Missouri; Johnnie C. Roberts, University of Missouri-Rolla, Rolla, Missouri; Lynn Wessel, Eastern Illinois University, Charleston, Illinois; and the proposer.

Editor's Comment: In this comment, pairs of positive integers $(r_n, r_n + 1)$ are such that $r_n^3 \equiv r_n \pmod{10^n}$ and $(r_n + 1)^3 \equiv r_n + 1 \pmod{10^n}$ simultaneously are called Trigg numbers. Trigg numbers are closely related to automorphic numbers. Automorphic numbers are defined to be those positive integers which terminate their own squares and all higher powers; i.e. positive integers x such that $x^2 \equiv x \pmod{10^n}$ where n is a positive integer [1].

Then $(r_n + 1)^3 \equiv r_n + 1 \pmod{10^n}$ and $r_n^3 \equiv r_n \pmod{10^n}$ imply

$$r_n^2 + r_n \equiv 0 \pmod{10^n} \text{ or } r_n + 1)^2 \equiv r_n + 1 \pmod{10^n}. \qquad (1)$$

Hence the larger member of a pair of Trigg numbers is automorphic.

It is well-known [2] that if x is an automorphic number containing n digits, then $10^n + 1 - x$ is automorphic also. Trigg numbers possess a similar property. Let r_n be the smaller member of a pair of Trigg numbers each containing n digits and define

$$s_n = 10^n - 1 - r_n. \qquad (2)$$

Then $s_n^3 \equiv (10^n - 1 - r_n)^3 \equiv -(r_n + 1)^3 \equiv 10^n - 1 - r_n \pmod{10^n}$. Similarly $(s_n + 1)^3 \equiv 10^n - 1 -(r_n + 1)^3 \equiv 10^n - r_n \equiv s_n + 1 \pmod{10^n}$. Thus $(s_n, s_n + 1)$ are a pair of Trigg numbers also which are complementary to the pair $(r_n, r_n + 1)$.

To generate an infinite number of integers r_n satisfying the conditions $r_n^3 \equiv r_n \pmod{10^n}$ and $(r_n + 1)^3 \equiv r_n + 1 \pmod{10^n}$ simultaneously, let $r_{n+1} = 10^n t_n + r_n$ and use relation (1) (i.e. $r_n + 1$ is an automorphic number). Then t_n is defined by the congruence

$$(10^n t_n + r_n + 1)^2 \equiv 10^n t_n + r_n + 1 \pmod{10^{n+1}} \qquad (3)$$

which reduces to

$$t_n(2r_n + 1) \equiv \frac{-(r_n)(r_n + 1)}{10^n} \pmod{10}. \qquad (4)$$

Then since $r_n \equiv r_1 \pmod{10}$, t_n (when it exists) is uniquely determined by

$$t_n(2r_1 + 1) \equiv \frac{-(r_n)(r_n + 1)}{10^n} \pmod{10}. \qquad (5)$$

For example, take $r_1 = 5$ in congruence (5). Then $t_1 \equiv \frac{-(5)(6)}{10} \equiv 7 \pmod{10}$. Thus $r_2 = 75$.

Then $t_2 \equiv \frac{-(75)(76)}{100} \equiv 3 \pmod{10}$.

Thus $r_3 = 375$ and the process can be repeated to generate arbitrarily large pairs of Trigg numbers. Complementary pairs of Trigg numbers containing ten digits are (1,787,109,375; 1,787,109,376) and (8,212,890,624; 8,212,890,625).

NOTES

1. M. Kraitchik, *Mathematical Recreations,* 2nd ed., Dover (1942), pp. 77-78.
2. Problem 701, *Mathematics Magazine,* October 1968 issue, p. 212.

290. *Proposed by Charles Trigg, San Diego, California.*

"Come on in, Bob," said Dan, "only small stakes tonight." "That's good," replied Bob, "I haven't quite three dollars in nickels, dimes, and quarters." "I haven't any pennies either," said Dan, "but I have the same number of coins that you have. That includes twice as many dimes as you have," "Correct," replied Bob, "but my number of nickels is twice yours. It also equals the number of all our quarters combined. The total value of your change is the same as mine." "O.K., let's go," said Dan, "It appears that my lucky half-dollar is the largest coin on the table."

How many coins of each type did Bob and Dan have?

Solution by Jeffry D. Hart, Southern Methodist University, Dallas, Texas.

Let x, y, z, and w be chosen as follows:

	Bob's Coins	Dan's Coins
Dimes	x	$2x$
Nickels	$2y$	y
Quarters	z	w
Half-Dollars		1

Then since Bob's number of nickels equals the number of all quarters combined:

$$z + w = 2y \text{ or } w = 2y - z.$$

Since the boys have the same number of coins,

$$x + 2y + z = 2x + y + (2y - z) + 1 \text{ or}$$
$$x + y - 2z = -1.$$

Since the value of their change is the same,

$$10x + 10y + 25z = 20x + 5y + 25(2y - z) + 50$$
$$\text{or } 2x + 9y - 10z = -10$$

Thus, we have the system of equations

$$x + y - 2z = -1$$
$$2x + 9y - 10z = -10$$

Allowing z to be a parameter, we obtain

$$x = \frac{1}{7}(8z + 1) \qquad y = \frac{1}{7}(6z - 8)$$

Now x, y, z must be restricted to positive integer values since they represent the number of coins each boy possesses. Thus $z = 6$ is the first positive integer which yields x and y as positive integers. However, any subsequent choices of a positive integer z which yield x and y as positive integers also implies that the total value of each boy's change exceeds \$3.00. Thus, z must be 6.

$$x = 7,\ y = 4 \text{ and } w = 2$$

Therefore, Bob has 7 dimes, 8 nickels, and 6 quarters, while Dan has 14 dimes, 4 nickels, 2 quarters, and 1 half-dollar.

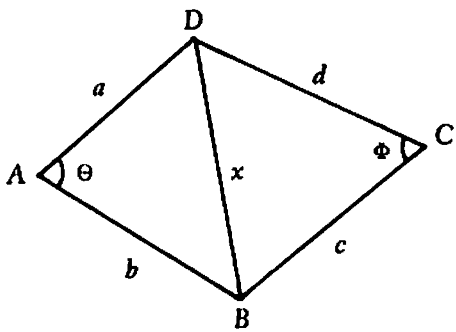
Also solved by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas and the proposer.

291. *Proposed by Leigh James, Rocky Hill, Connecticut.*

Prove that the quadrilateral having sides a, b, c, and d has maximum area when the quadrilateral is cyclic; i.e., the sum of opposite interior angles is 180°.

Composite of Solutions by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas and the proposer.

Let the opposite interior angles Θ and Φ and diagonal x of the quadrilateral be as described in the figure below. Then



the area of the quadrilateral Q is given by

$$Q = \frac{1}{2} ab \sin \Theta + \frac{1}{2} cd \sin \Phi \quad \text{so that}$$

$$\frac{dQ}{dx} = \frac{1}{2} ab \cos \Theta \frac{d\Theta}{dx} + \frac{1}{2} cd \cos \Phi \frac{d\Phi}{dx} \qquad (1).$$

By the Law of Cosines, $\cos \Theta = \dfrac{a^2 + b^2 - x^2}{2ab}$ so that

$$\frac{d\Theta}{dx} = \frac{x}{ab \sin \Theta} \qquad (2).$$

Similarly $\dfrac{d\Phi}{dx} = \dfrac{x}{cd \sin \Phi}$ (3).

Substituting (2) and (3) and setting $\dfrac{dQ}{dx} = 0$, we obtain

$$\frac{x \cos(\Theta + \Phi)}{\sin \Theta \sin \Phi} = 0$$

Hence $\Phi + \Theta = 180°$ and the quadrilateral of maximum area having sides a, b, c, and d is cyclic.

Solution by Leo Sauve, Algonquin College, Ottawa, Ontario, Canada.

If $s = \frac{1}{2}(a + b + c + d)$, a well known formula for the area K of the quadrilateral is

$$K^2 = (s - a)(s - b)(s - c)(s - d) - abcd \cos^2 \frac{A + C}{2}$$

where A and C are a pair of opposite interior angles. Thus the maximum value of K requires $\cos \frac{A + C}{2} = 0$ or $A + C = 180°$, and so the quadrilateral must be cyclic.

Also solved by Charles W. Trigg, San Diego, California (two solutions).

Editor's Comment: Mr. Trigg points out that the formula used in Leo Sauve's solution was known to Brahmagupta (circa 728 A.D.) and that this problem appeared as problem 6 in the *Pentagon* in 1948.

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgement will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

Suppose one selects two unequal positive integers a and b at random. One might ask what is the probability that a and b are relatively prime; that is, $(a,b) = 1$. Our first impression of this problem might be that the problem's magnitude is so vast that we might not be able to answer such a problem. After all, the set of positive integers is an infinite set and picking at random two unequal elements of this set is in itself creating an infinite sample space.

The first major problem is the selection procedure for selecting the two distinct positive integers from the set of all positive integers. We cannot place the set of all positive integers in a hat, shuffle, and then draw two numbers from the hat. One might consider various methods for doing such a random selection process but one of the more convenient is to perform the experiment for finite subsets of the set of all positive integers and note the limit toward which the desired probability is approaching. Fortunately, the procedure will yield interesting results which, after a few trials, will converge toward a definite limit.

Before we can proceed any further, we should first note the following definitions and rules of probability theory.

Let A be an event. Then the probability for the successful completion of event A, denoted by $P(A)$, is equal to the number of ways we can obtain a successful outcome of A divited by the total number of possible outcomes of any kind. Hence,

$$P(A) = \frac{\text{number of successes}}{\text{total number of possible outcomes}} .$$

Since an experiment must either result in success or failure, we must have $p + q = 1$, where p is the probability of success of event A and q is the probability of failure of event A.

We say two events A and B are independent when the occurrence of event A does not affect the probability of event B occurring or visa versa. If two events A and B are independent, then $P(A \text{ and } B) = P(A) \cdot P(B)$. This rule can be extended for N independent events; whereby the probability of the occurrence of all N independent events is simply the product of their individual probabilities.

One last item we need to be familiar with before we attack the problem is the following identity:

$$\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right) \cdot$$
$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdot \left(1 - \frac{1}{5^2}\right) \cdot$$
$$\left(1 - \frac{1}{7^2}\right) \cdot \left(1 - \frac{1}{11^2}\right) \cdot \left(1 - \frac{1}{13^2}\right) \cdots = 1.$$

One will note that the first parentheses contains the infinite series whose terms are the reciprocals of the squares of all the positive integers. Using either Fourier series or the "integral test" learned in calculus, one can show that the first parentheses converges to $\pi^2/6$.

Next we note that the other parentheses after the first are of the type $1 - \dfrac{1}{p^2}$ where p is a prime. Hence, to show that the above equation is indeed an identity, we merely multiply the left-hand side noting patterns as they develop. That is, consider

$$\begin{array}{cccccccccccc}
1 & + & \frac{1}{2^2} & + & \frac{1}{3^2} & + & \frac{1}{4^2} & + & \frac{1}{5^2} & + & \frac{1}{6^2} & + \cdots \\
1 & - & \frac{1}{2^2} & & & & & & & & & \\
\hline
1 & + & \frac{1}{2^2} & + & \frac{1}{3^2} & + & \frac{1}{4^2} & + & \frac{1}{5^2} & + & \frac{1}{6^2} & + \cdots \\
 & - & \frac{1}{2^2} & & & - & \frac{1}{4^2} & & & - & \frac{1}{6^2} & - \cdots \\
\hline
1 & & & + & \frac{1}{3^2} & & & + & \frac{1}{5^2} & & & + \cdots
\end{array}$$

One will readily note that all the multiples of $\frac{1}{2^2}$ have been eliminated from the final result. Next one would multiply this result by $1 - \frac{1}{3^2}$ and note that all multiples of $\frac{1}{3^2}$ are now also eliminated from the result. Hence now all multiples of $\frac{1}{2^2}$ $\frac{1}{3^2}$, $\frac{1}{4^2}$, $\frac{1}{6^2}$, etc., are eliminated from our resulting sum. Continuing this process, one will note that after multiplying by all the products $1 - \frac{1}{p^2}$, we will find our resultant product is 1 as claimed. Hence one will note that

$$\left(1 - \frac{1}{2^2}\right) \cdot \left(1 - \frac{1}{3^2}\right) \cdot \left(1 - \frac{1}{5^2}\right) \cdots = \frac{1}{\pi^2/6} = \frac{6}{\pi^2}.$$

We will now reconsider our original problem. Let a and b be any two distinct positive integers selected at random from the set of *all* positive integers. Let p be any prime. Here $\frac{1}{p}$ of all the positive integers are divisible by p since p divides every p-th one; that is, every multiple of p. Therefore, the probability that p divides a is $\frac{1}{p}$. Therefore, the probability that p divides both a and b is $\frac{1}{p} \cdot \frac{1}{p} = \frac{1}{p^2}$. Hence the probability that p does not divide both a and b is $1 - \frac{1}{p^2}$. To have a and b relatively prime, we must have no prime dividing *both* a and b. Hence our desired probability will be $\prod \left(1 - \frac{1}{p_i^2}\right)$ for all primes p_i; that is, our desired probability will be

$$\left(1 - \frac{1}{2^2}\right) \cdot \left(1 - \frac{1}{3^2}\right) \cdot \left(1 - \frac{1}{5^2}\right) \cdot \left(1 - \frac{1}{7^2}\right) \cdots = \frac{6}{\pi^2},$$

or .6079 approximately.

As previously indicated, this experiment could be conducted using finite sets of positive integers from 1 to N and computing the probabilities of selecting a and b relatively prime. If one performs this experiment for finite sets of positive integers 1 to N, he will find that as N increases, our probabilities converge to .6079.

Now suppose we choose one positive integer a from the set of all positive integers. Can you determine the probability that a and 3 are relatively prime?

The Book Shelf

EDITED BY O. OSCAR BECK

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, University of North Alabama, Florence, Alabama 35630.

Applied Linear Algebra, (2nd ed.), Ben Noble and James W. Daniel, Prentice-Hall, Inc. Englewood Cliffs, N.J., 1977, 493 pp., $15.95.

The authors suggest that the book can be used for a variety of courses such as a classical theoretical linear algebra course, an introductory applied linear algebra course, or an intermediate applied linear algebra course. Also, "Several other types of applied courses can be created . . . statistics and the social and behavioral sciences . . . engineering . . .". Suggested sections to be used for these various purposes are given in the Preface.

The standard topics, matrix algebra, vectors and vector spaces, linear transformations, solutions of simultaneous equations, linear programming, eigenvalues and eigenvectors, transformations, and quadratic forms, are included. The second chapter contains some excellent simple applications of matrices. There are other applications as well. ". . . the presentation throughout the book hinges on the use of elementary row operations . . ." The discussion of row operations, however, seems rather cumbersome. Those who like to introduce many ideas from the standpoint of determinants and the solution of sets of linear equations are slated for disappointment, as determinants are not mentioned until the sixth chapter and then primarily "as a conceptual or descriptive device for representing solutions and inverses."

There are many exercises, numbered consecutively in each chapter and scattered throughout the text. Theorems are likewise numbered. Because of the format it is sometimes a little difficult to find one. Theorem 8.4, for example, is in section 8.3. Certain theorems deemed to be of special importance are labeled as "key theorems."

Taken as a whole, the material in the book is excellent. It would seem to this reviewer, however, that it is best suited for an intermediate level course in which a reasonable amount of material could be assigned to be read as review. To include "brief presentations of the main results" of chapters and "a small amount . . . selected" is usually very difficult with an elementary class.

F. Virginia Rohde
Mississippi State University

Arithmetic: A Practical Approach, G. J. Shugar, R. Shugar Bauman, R. A. Shugar, and L. Bauman, Glencoe Press, Beverly Hills, 1977, 195 pp., $8.95.

This book contains twelve chapters covering whole numbers, addition, subtraction, multiplication, division, fractions, decimals, percent, ratio and proportions, measurement, weight, and temperature.

Each chapter begins by telling what is to be learned. After each item is introduced there are practice exercises. The chapter ends with a review.

The last three chapters present the conversion from the English system to the metric system.

This reviewer could find no indication of the grade level for the book. The material presented seems to be for grades 4 through 6 or perhaps on through 8, but it is not all for one grade level.

The materials and procedures given should be very easily understood by the elementary student.

The book is a paperback so perhaps will not stand up as well as a hardback would with the daily use it would have.

An answer booklet accompanies the book.

Naulda Welke
Ewing, Nebraska

An Introduction to Matrices, Vectors, and Linear Programming, (2nd ed.), Hugh G. Campbell, Prentice Hall, Inc., Englewood Cliff, N.J., 1977, 332 pp., $10.95.

This book is an elementary and even-handed introduction to those topics usually studied in linear algebra courses for nonmathematics majors. The focus is on matrices, and, to a lesser extent, on linear programming. The treatments of vectors, determinants, systems of equations, and the eigenvalue problem are not slighted. They are woven into the continuing development of matrix concepts. This helps tie these topics together.

The book has many good features. The occasional sections on history and applications add much interest. The chapter on algebraic structures (Chapter 3) brings into one chapter many of the issues about operations that other books sometime scatter through the text and problems. The optional sections on groups, rings, and fields though brief are adequate for those courses which require an introduction to algebraic structures. The two chapters on linear programming give a good introduction to the simplex method and the primal-dual approach without going into all of the details.

The writing is clear and direct. Definitions and notation are carefully established. Simple examples follow every discussion. Proofs are written in what some call a T-format: steps on the left, and reasons on the right. They are carefully done and even when not covered in class should help the interested reader. A few of the most awkward proofs are omitted. An unfortunate use of unnecessary terms, such as "premultiply" and "conformable" for addition, did mar the exposition occasionally.

The reviewer admits that he prefers not to follow the matrix road through linear algebra. This and the deliberately slow start of the text would prompt him to omit, downplay, or scurry through much of the first four chapters. On the other hand clearly there are situations when matrices should be the topic of discussion, and where a book with the matrix multiplication section on page 150, or beyond, is clearly inappropriate. In such a case this book has much to recommend it.

Charles S. Holmes
Miami University (Ohio)

Basic Mathematics: What Every College Student Should Know, Shirley O. Hockett, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1977, 527 pp., $10.75 (paper).

The goal expressed by the author was to write a text designed to aid the incoming college freshman to acquire or review basic competencies in arithmetic and algebra. In addition, the text was written to be read and used by the student in a number of situations: as the text used in a normal classroom situation, as a text for a self-paced system of instruction, and as a workbook to be used by a student who is studying independently in an attempt to review familiar mathematics or to learn new mathematics.

The first two chapters deal extensively with the arithmetic of whole numbers and signed numbers. Some basic concepts of set theory are then introduced, followed by chapters on fractions and decimal numbers. A section on the metric system is also included in the chapter dealing with decimals. The last five chapters deal with the basic concepts of algebra including the addition, subtraction, and multiplication of polynomials; some work on exponents and radicals; algebraic fractions and manipulation with formulas; solving linear equations and inequalities; quadratic equations; and finally some work with functions and their graphs. The last part of the book consists of eight appendices which are included for enrichment. Some of the topics covered are bases other than ten, scientific notation, square roots, division of polynomials, factoring the sum and difference of two cubes, equations involving radicals, and the complex numbers.

The feature that most impressed this reviewer was the organization of each chapter. Every chapter begins with a pretest which allows students to determine how much of a chapter should be studied. The questions of the pretest are keyed to specific sections of the chapter so students can tell on which topics of the chapter they should concentrate. Each chapter has a set of review exercises which are keyed to specific sections of the chapter along with answers, hints, and in some cases quite detailed explanations of specific problems. Finally, each chapter ends with two chapter tests. Test A with answers gives students the opportunity to check their readiness to take the exam and Test B with answers in the teacher's manual allows the teacher an opportunity to test achievement.

The text is very easy to read and exercises appear reasonable with each set of exercises followed immediately by the answers. A small sample revealed no incorrect answers. The important concepts of each section are framed in rectangular boxes to attract the attention of students.

One criticism that might be offered is that the reviewer feels the author is somewhat ambitious in attempting to write a text which can be used as a classroom text, used in self-paced instruction, and also can be used for studying in an independent manner. It would seem that a very detailed student manual would be necessary for the student involved in independent study.

The overall impression is that the text should be given serious consideration for a course in arithmetic and algebra for freshmen in community and junior colleges and also for a review (or no-credit course) in arithmetic and algebra in colleges and universities with an open-admission policy.

Ronald D. Dettmers
University of Wisconsin—Whitewater

Installation of New Chapters

EDITED BY LORETTA K. SMITH

Information for the department should be sent to Mrs. Loretta K. Smith, 829 Hillcrest Road, Orange, Connecticut 06477.

FLORIDA BETA CHAPTER

Florida Southern College, Lakeland, Florida

The Florida Beta Chapter was formally installed at Florida Southern College on 31 October 1976, by the National Secretary, Dr. Elizabeth T. Wooldridge, who spoke on the history and goals of Kappa Mu Epsilon. The charter officers were Carol Barth, president; David Schreck, vice-president; and Donna Gibson, recording secretary and treasurer. Dr. Henry Hartje, Jr. was selected as adviser and corresponding secretary. Faculty and student charter members included:

Carol Barth
Barbara Cavanah
Carolyn Clark
Cindy Clarke
George Cusson, Faculty
Timothy Forrest
Lynne Gardner
Donna Gibson
Lane Goodson, Faculty
Susan Harmon
Dr. Henry Hartje, Jr., Faculty
Elizabeth Kennedy
Robert Rowley, Faculty
David Schreck
Dr. Gerald Smith, Faculty
Lynn Tilsdale
Melinda Tucker
Kevin Yarnell

Kappa Mu Epsilon News

EDITED BY SISTER JO ANN FELLIN, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Sister Jo Ann Fellin, Historian, Kappa Mu Epsilon, Benedictine College, North Campus Box 43, Atchison, Kansas 66002.

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

Chapter President—Diane Bruce
35 actives

Dr. Elizabeth Woodridge, national secretary during the last biennium, presented an interesting program on the history of Kappa Mu Epsilon. Another lecture and visual presentation was given at the planetarium. The climax of the year was the initiation banquet, at which time 17 new members were added to the chapter roll and Jim Diehl, president at the time, was honored as a Hughes fellow to UCLA. Other 1977-78 officers: Bobby Ray Wells, vice-president; Judy Muse, secretary and treasurer; Jean Parker, corresponding secretary; Oscar Beck, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President—Michael Skora
50 actives, 39 pledges

Chapter members conducted a Junior High Math Field Day for over 100 students and made demonstrations and displays for the mathematics department portion of Poly Royal (campus wide open house). The chapter held a pledge ceremony in April and sponsored a Career Day with ten speakers from business, industry, and teaching for the benefit of all students majoring in mathematics, computer science, and statistics. The spring banquet in May featured Dr. Stephen Kenton as speaker. Other 1977-17 officers: Matthew Tedone, vice-president; Mary Danborn, secretary; Cheryl Antaki, treasurer; George R. Mach, corresponding secretary; Adelaide Harmon-Elliott, faculty sponsor.

Colorado Beta, Colorado School of Mines, Golden

Chapter President—Gregory Golike
20 actives

Member Debbie Bell presented a talk on "Applications of Group Theory: It's All Done with Mirrors" on 20 January. The February and March meetings featured films—*Infinite Acres and Unsolved Problems/Three Dimensions* and *Gottingen and New York*, respectively. On 19 April the initiation of 36 new members took place in the Ted Adams Reading Room. Dr. Fausett gave a brief address to the initiates and refreshments were served. Tom Lovelace won $10 for his solution to the E-Day Prize Problem. Other 1977-78 officers: Stephen Hamburg, vice-president; Kent Peaslee, secretary; T. David Burleigh, treasurer; Ardel J. Boes, corresponding secretary; Donald W. Fausett, faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton

Chapter President—Patsy C. Fields
23 actives, 13 pledges

The Georgia Alpha Chapter continues to grow at West Georgia College. The chapter held its third annual initiation banquet on 18 May. All 13 students who had been invited to join were initiated that evening. The 100% response is indicative of the honor felt by those invited to membership in Kappa Mu Epsilon. Earlier in the quarter several members assisted with the annual Math Day proceedings at the college. This participation involved exhibits, lectures, mathematics puzzles, and various other events for high school mathematics students in the surrounding area. Other 1977-78 officers: Robert Ogletree, vice-president; Art Kirchoffer, secretary; Teresa C. Stamps, treasurer; Thomas J. Sharp, corresponding secretary and faculty sponsor.

Illinois Alpha, Illinois State University, Normal

Chapter President—Cathy Galkowski
8 actives, 1 pledge

The monthly meetings of the chapter included pledge papers and pledge ceremony, initiation, mathematical films, and a panel in which student teachers related their teachings experiences. Illinois Alpha members staffed a tutoring program during the year for students in lower level mathematics courses. Approximately 60 mathematics majors, faculty, and guests attended the May "Spring Fling" picnic organized by the chapter. Other 1977-78 officers: Jennifer Sward, vice-president; Jeanette Kokosinski, secretary; Shirley King, treasurer; Albert Otto, corresponding secretary; Orlyn Edge, faculty sponsor.

Illinois Delta, College of St. Francis, Joliet

Chapter President—Kevin Risney
11 actives

The chapter held a mathematics contest for local high school students. About 80 students participated in the event. Other 1977-78 officers: Sonia Marcum, vice-president; Arnold Good, corresponding secretary.

Illinois Zeta, Rosary College, River Forest

Chapter President—Tom LeKostaj
15 actives, 2 pledges

Members again offered tutoring services to mathematics students of the college. Nancy Beahan, Tom LeKostaj, Tom Kourim, and Joanne Trotti presented problems at monthly meetings. Ann Stangarone, Tom LeKostaj, Tom Kourim, and Sister Nona Mary Allard attended the Region I Convention at Shippensberg State College. They won the distinction of being the delegation from the greatest distance. Tom Kourin's paper on "Linear Programming Methods of Obtaining Estimates of Regression Coefficients" received honorable mention at this region meeting. Reception of new members took place on 31 March and new officers were installed on 11 May. Other 1977-78 officers: Mary Rose Grano, vice-president; Nancy Wagner, secretary; Tom Graham, treasurer; Sister Nona Mary Allard, corresponding secretary and faculty sponsor.

Illinois Eta, Western Illinois University, Macomb

Chapter President—Sharon Govekar

During the spring quarter Wally Hill gave a talk on computer languages to the chapter group. The chapter held its annual spring picnic at Lake Argyle State Park on 20 May. Kent Harris, corresponding secretary; James Calhoun, faculty sponsor.

Indiana Gamma, Anderson College, Anderson

Chapter President—Christopher Linamen

Other 1977-78 officers: Dwight Stewart, vice-president; Jay Collins, secretary and treasurer; Stanley Stephens, corresponding secretary and faculty sponsor.

Indiana Delta, University of Evansville, Evansville

Chapter President—Damon Thompson
50 actives, 25 pledges

Melba Patberg, corresponding secretary; Dennis Hop, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President—Diana S. Dickinson
47 actives

The February and March meetings featured two student papers: "Theory of Games" by John Schreck and "Affine Planes" by Betty Oswald. For the April meeting Dr. Fred W. Lott, past national president of Kappa Mu Epsilon, related the national and local history of the society with illustrations containing historical memorabilia from the files of the Iowa Alpha Chapter. New initiate Connie K. Strelow presented her paper on "Inversion Geometry" at the initiation banquet on 5 May. Following the cancellation of the national convention, Iowa Alpha members made plans for a field trip to Chicago. Eight students, two faculty, and four chapter alumni toured the University of Chicago, the Museum of Science and Industry, and portions of the Chicago Loop area. On 1 May ideal weather resulted in an attendance of approximately 80 at the annual picnic. Other 1977-78 officers: John A. Schreck, vice-president; Bonita L. Marlett, secretary; Denese K. Smith, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines

Chapter President—Darien Hall
13 actives, 2 pledges

Iowa Beta held a non-traditional spring banquet on 1 May. The chapter officers purchased and prepared a charcoal-broiled steak dinner. The three students with the best papers in the annual mathematics contest were guests. Initiation ceremonies were held following an invited address by Dr. B. E. Gillam. Other 1977-78 officers: Rick Lang, vice-president; Jon Wells, secretary; Bob Southerland, treasurer; Christina Bahl, corresponding secretary; Alex Kleiner, faculty sponsor.

Iowa Gamma, Morningside College, Sioux City

Chapter President—Richard Smith
31 actives

The chapter earned money by selling T-shirts to the entire student body. On 22 March Professor A. M. Fink of Iowa State University at Ames, Iowa presented a series of three lectures on the following topics: "Polynomials Are All We Have to Integrate," "Counting Intersections of Convex Sets," and "Roundheads Have Higher IQ's Than Squares." In April the chapter cooperated with the science department in hosting an open house of approximately 600 area high school students. Other 1977-78 officers: Robert Tesch, vice-president; Bruce Cook, secretary; John Steele, treasurer; Elsie Muller, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg College, Waverly

Chapter President—Kent R. Floy
18 actives, 15 pledges

Iowa Delta revised its constitution and by-laws at its January meeting. The program that evening centered on recreational and geometrical puzzles constructed by Dr. William Cutler of the Wartburg mathematics department. At the February meeting visiting lecturer Dr. Peter Colwell spoke on "Some Problems Whose Correct Solution Isn't Right.' This lecture was open to the public and jointly sponsored by the chapter and the mathematics department. Guest speaker for the 15 March initiation and election banquet was Dr. Fred Lott, former national president. Dr. Lott spoke on "The History of KME" citing exemplary quotes from handwritten correspondence dealing with the formation of the society. The talk was appreciated by the current membership as well as by the 15 new pledges received that evening. Iowa Delta held its last function of the year, the annual picnic, at the Cedar Bend Conservation Park on 27 April. Other 1977-78 officers: Gregory J. Diercks, vice-president; Kay D.

Bixbee, secretary; Mark H. Behle, treasurer; August Waltmann, corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg

Chapter President—Terri Wilson
55 actives

Kansas Alpha reports that the name of the school was officially changed to Pittsburg State University on 21 April 1977. The chapter held monthly meetings. Seventeen new members were initiated into the chapter at the February meeting which was preceded by a banquet at a local restaurant. Following the initiation ceremony Ronald Stockstill presented a paper on "Coding." The March program was given by Mark Rabuse on "The Rise of Non-Euclidean Geometry." Dr. Will Self also introduced a game called "Hex" at that meeting. In April Tim Cohle talked about magic circles and at the May meeting Dr. Elwyn Davis reported on the book *Proof and Refutations* by Imrie Lakatos. Darla Hedrick, recipient of the annual Robert M. Mendenhall Award for scholastic achievement, was awarded a **KME** pin in recognition of this achievement. Other 1977-78 officers: Ronald Stockstill, vice-president; Nonetta Thomas, secretary; Christy Stine, treasurer; Harold L. Thomas, corresponding secretary; J. Bryan Sperry, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison

Chapter President—Joe Gress
16 actives, 13 pledges

John Benage, Dan Kaiser, Kristi Schmidt, and Steve Vogel were initiated into Kansas Gamma on 14 March. That same evening 13 freshmen received local membership cards establishing them as formally related to Kansas Gamma. Alumnus Dr. J. C. Kelly, now teaching at the University of Missouri in Columbia, returned to the campus to speak to the group on "Tuning Your Guitar." Social activities in the spring semester included a volleyball game on 28 March and a steak cookout on 8 May honoring the mathematics graduates. At the college spring honors banquet Ellen Champ and Ann Bremehr received certificates as corecipients of the Sister Helen Sullivan Scholarship Award. Other 1977-78 officers: Ann Bremehr, vice-president; Dan Kaiser, secretary; John Kohler, treasurer; Sister Jo Ann Fellin, corresponding secretary; Jim Ewbank, faculty sponsor.

Kansas Delta, Washburn University, Topeka

17 actives, 9 pledges

Over 200 high school students participated in the math day sponsored by Kansas Delta on 31 March. Robert Thompson, corresponding secretary; Billy Milner and Gary Schmidt, faculty sponsors.

Kansas Epsilon, Fort Hays State University, Hays

Chapter President—Reggie Romine
20 actives

The chapter sponsored a spring picnic for the mathematics department which included faculty, students, staff and their families. About 60 enjoyed the evening meal and games. Other 1977-78 officers: Ramona Weigel, vice-president; Teresa Willis, secretary; Darlene Sawyer, treasurer; Eugene Etter, corresponding secretary; Charles Votaw, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President—Frances Pittelli
9 actives, 3 pledges

Seniors Helene Murtha and Mary Beth Baker and alumnae Kathy Kwiatkowski Bankard and Pat Creel Robinson presented a panel on teaching as a profession for one of the chapter meetings. At another meeting Helene Murphy and Kathy O'Grady gave papers on practical probability. Maryland Alpha hosted a joint meeting with Maryland Beta in April at which time the two chapters celebrated the 200th birthday of Gauss with a symposium on Gauss and his contributions to mathematics. At the May initiation meeting three new members were inducted. The buffet dinner was followed by a presentation in the campus planetarium by Dr. Joseph Di Rienzi on "An Evening in the Planetarium." Other 1977-78 officers: Ann Shaughnessy, vice-president and treasurer; Laura Nesbitt, secretary; Sister Marie Augustine Dowling, corresponding secretary and faculty sponsor.

Michigan Beta, Central Michigan University, Mount Pleasant

Chapter President—Barbara Bierlein
32 actives, 12 pledges

March activities included a presentation on "Student Teaching in England" and the second annual banquet. In April the chapter members assisted with the 13th annual Beginning Teachers Conference while in May they helped with the Mathematics Contest for Junior High School Students sponsored by the Michigan Council of Teachers of Mathematics. The annual picnic was held in May. Other 1977-78 officers: Steve Lawyer, vice-president; Stephanie Hill, secretary; Linda Le Sage, treasurer; E. H. Whitmore, corresponding secretary and faculty sponsor.

Mississippi Alpha, Mississippi University for Women, Columbus

Chapter President—Ann Marie Tomberlin
21 actives, 17 pledges

The following were initiated at the annual initiation held this year on 17 February: B. Bowen, J. Chan, H. Denham, C. Hall, K. Hearn, S. Ki, C. King, K. Mak, C. Merritt, L. Schroyer, P. Stewart, H. Stickney, M. Troyofuku, L. Wasson, L. Williams, H. Yeung, and N. Young. The guest speaker for the occasion was Dr. Eldon Miller, acting chairman of the mathematics department at the University of Mississippi. New officers were elected at the spring picnic held on 21 April at Carrier Lodge on the Tombigbee River. Other 1977-78 officers: Cindy Wells, vice-president; June Chappell, secretary and treasurer; Jean Ann Parra, corresponding secretary and faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg

Chapter President—Lucille Lisenbee
47 actives

On 29 April seventeen new members were initiated at the spring barbecue supper. A book of mathematical tables was awarded to Blair Driskell, outstanding freshman mathematics student. Other 1977-78 officers: Nolann Nelson, vice-president; Ken Kays, secretary; Alice W. Essary, treasurer and corresponding secretary; Nancy Dunigan, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg

Chapter President—Mary Beth Snodgrass
29 actives, 24 pledges

The chapter meetings during the past year included: two initiations, a Christmas party, a spring Honors Banquet, and the first of the William Klingenberg Lecture Series to be held annually. Other 1977-78 officers: Rik Drummond, vice-president; Laura Cowan, secretary; Robin Langford, treasurer; Homer F. Hampton, corresponding secretary and faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne

Chapter President—Kay Pankratz
17 actives

Throughout second semester eight members served as tutors for students enrolled in lower level mathematics courses. Seven new members were installed at the January meeting of the chapter. In April Kirk Knapp, Gordon Cook, Bob Nissen, and Dick Buntgen competed as the KME team in the annual College Bowl. The annual KME-LDL spring banquet was held at the Wagon Wheel Steak House in Laurel, Nebraska on 22 April. Two awards were presented at the banquet. Dr. Russell Rasmussen of the chemistry department was named outstanding professor in the Mathematics and Science Division. Trudy Barbeck of Clarkson, Nebraska was chosen the outstanding freshman student majoring in mathematics. Her selection was based on a competitive examination administered by the chapter to freshmen mathematics majors. Trudy's name was placed on a permanent plaque of the chapter. Other 1977-78 officers: Karen Doeshot, vice-president; Lois McKinze, secretary and treasurer; Gordon Cook, historian, Fred Webber, corresponding secretary; James Paige and Hilbert Johs, faculty sponsors.

Nebraska Beta, Kearney State College, Kearney

Chapter President—Diane Meyer
20 actives, 5 pledges

Other 1977-78 officers: Mike Walsh, vice-president; Sue Micek, secretary; Jane Herfkins, treasurer; Charles Pickens, corresponding secretary; Randall Heckman, faculty sponsor.

New Jersey Beta, Montclair State College, Upper Montclair

Chapter President—David D'Andrea
47 actives, 26 pledges

Outgoing faculty advisor Dr. Helen Marcus Roberts and her husband, Dr. Fred Roberts, became the parents of a baby girl on 22 February. On 12 March the chapter held its fifth annual mathematics competition for high school students. Thirty New Jersey high schools entered three-person teams in the contest. Guest speaker Dr. Max Sobel spoke on "The Magic of Mathematics" at the induction banquet. The chapter ended their semester activities with a spring picnic. Other 1977-78 officers: Judi Gruber, vice-president; Donna Mazzei, secretary; Jim Matthews, treasurer; Judy LaSardo, historian; Carl Bredlau, corresponding secretary; Edward Boyno, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President—Don Poulsen
35 actives

Other 1977-78 officers: Charlotte Harrison, vice-president; Bill Ricker, secretary; Gary Mayhew, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara University

Chapter President—Joanne Esposito
21 actives, 12 pledges

The highlight of the semester for the chapter was the regional KME convention held at Shippensburg State College in Shippensburg, Pennsylvania. John Schaefer, Jeff Lucia, and Robert Bailey composed the New York Eta delegation. John Schaefer presented a paper at the meeting entitled "A Short Study of the Columnar Transportation System of Encoding and Decoding Messages." Other 1977-78 officers: Dianne Fraher, vice-president; Maureen Swiercznski, secretary; Brian Covney, treasurer; Robert Bailey, corresponding secretary and faculty sponsor.

New York Theta, St. Francis College, Brooklyn

Chapter President—Richard Imperato
6 actives, 5 pledges

New York Theta members and math club members at St. Francis College are involved in the four math contests each year between St. John's Molloy and Adelphi. Other 1977-78 officers: Daniel Piselli, vice-president; Dr. Guaraldo, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord

Chapter President—Rebecca Tucker
32 actives

Dr. Elliott Jacobs spoke on "Nonstandard Numbers" at the February meeting. Eleven new members were inducted at the initiation ceremony on 15 March. After the initiation dinner the new members each presented a short talk on a mathematician. In April Doug Harms talked to the chapter group on microprocessing and described the work he did while at Westminster College during the January term. The combined departmental and KME spring banquet was held at Baker's Motel on 5 May. The following awards were presented: freshman mathematics award to Eric Ribble and senior computer science awards to John Stewart and Jim Wingerter. Other 1977-78 officers: Janet Danison, vice-president; Barbara Bauer, secretary; Jeffrey Russell, treasurer; James L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford

Chapter President—Steve Thomas
25 actives, 17 pledges

Other 1977-78 officers: Jeanette Mack, vice-president; Kim Roof, secretary; Danny Basler, treasurer; Wayne Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington

Chapter President—Mike Dzuricky
45 actives, 17 pledges

The chapter initiated new members at the initiation banquet on 2 May and held a picnic for all members on 20 May. Other 1977-78 officers: Jim Yahner, vice-president; Gary Wood, secretary; Mary De Carbo, treasurer; J. Miller Peck, corresponding secretary; Thomas R. Nealeigh, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President—Debra Kistler
20 actives, 6 pledges

Other 1977-78 officers: Tim Buchman, vice-president; Cynthia Matthews, secretary; Janise Lohin, treasurer; Irving Hollingshead, corresponding secretary; Edward Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President—James Blauser
30 actives

"The History of Pi" was the title of the talk given at the initiation meeting in February by Dr. Donald McKelvey of the chemistry department. In March William Nelson and Mike Melcher, both alumni of the University and employees of a local bank, spoke about banking operations. An area of applications was explored at the April meeting when Dr. John Dinkel of Pennsylvania State University talked about "Applied Mathematics, Operations Research, and the Real World." At the annual banquet in May Eija Inberg, an exchange student from Finland, discussed the Youth for Understanding Exchange Program and presented a rhythmic gymnastic demonstration. Other 1977-78 officers: Kathy Carlson, vice-president; Barbara Howe, secretary; Kathy Buczek, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City

Chapter President—Christopher Cox
32 actives, 12 pledges

During the spring semester the Pennsylvania Eta officers prepared and administered a competitive examination in mathematics to select the outstanding freshman mathematics student. This student received a copy of the CRC Handbook of Math Tables and was recognized at an award ceremony on Parents' Day at Grove City College. The annual KME spring picnic was held on 5 May at the Grove City Country Club. Other 1977-78 officers: Jeanne Nailor, vice-president; Barbara Guarnieri, secretary; Julie Albrecht, treasurer; Marvin C. Henry, corresponding secretary; John Ellison, faculty sponsor.

Pennsylvania Theta, Susquehanna University, Selinsgrove

Chapter President—Ricky Erdman
8 actives

The chapter has begun plans for an active program for the 1977-78 academic year. Other 1977-78 officers: Rebecca Botts, vice-president; Richard Olson, secretary and treasurer; Carol Harrison, corresponding secretary and faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia

Chapter President—Susan Capozio
8 actives, 3 pledges

Cynthia Bodziak, Linda Mahon, Glen Ritter, and Judith Washburn were initiated into Pennsylvania appa at the Honors Convocation Ceremony on 16 March. A reception followed the ceremony. Sister Mary Grace lectured on the metric system on 2 May. Her talk included things that won't change and some long range effects of conversion to the metric system. Other 1977-78 officers: Cynthia Bodziak, vice-president and treasurer; Glenn Ritter, secretary; Sister M. Grace, CSFN, corresponding secretary and faculty sponsor.

Tennessee Beta, East Tennessee State University, Johnson City

25 actives, 17 pledges

Seventeen students were initiated on 6 May at the initiation ceremony conducted in the D. P. Culp University Center on the campus. Dr. Gary Walters of the physics department gave a demonstration of an environment energy simulator. Lyndell M. Kerley, corresponding secretary; Lyndell M. Kerley and Sallie Carson, faculty sponsors.

Texas Alpha, Texas Tech University, Lubbock

Chapter President—Suzanne Tooker
10 actives, 28 pledges

At the February meeting Professor Dick Saeks of the electrical engineering department spoke to the chapter. The talk given at the March meeting was delivered by Professor Peter Kelemen of the mathematics department. A special pledge project was held to raise money for a scholarship. In April the chapter had a party as well as the initiation banquet. At the banquet the new officers were installed and awards were presented. Other 1977-78 officers: Mark Brown, vice-president; Donna Terral, secretary; Tim Mayberry, treasurer; Rhonda Luxton, pledge trainer; L. R. Hunt, corresponding secretary and faculty sponsor.

Texas Gamma, Texas Woman's University, Denton

Chapter President—Karen McDowell
9 actives, 2 pledges

Two new members were initiated. Dr. Tom Kehler of the mathematics department spoke to the group on "The Computer and Liguistics." Another talk sponsored by the chapter and open to the public was Dr. Barry Horwitz' discussion of "Catastrophe Theory.' Dr. Horwitz teaches in the physics department. Other 1977-78 officers: Patti Padgett, vice-president; Deborah Odom, secretary; D. T. Hogan, corresponding secretary and faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene

Chapter President—Cynthia Young
25 actives, 5 pledges

The five new members inducted into Texas Eta on 16 April are: Cynthia Young, Rick Foser, Nora Black, David Gitelman, and Mike Lindsay. Other 1977-78 officers: Cymbe Alford, vice-president; David Simmons, secretary and treasurer; Anne B. Bentley, corresponding secretary; Charles D. Robinson and Edwin Hewett, faculty sponsors.

West Virginia Alpha, Bethany College, Bethany

Chapter President—Eugene Turley
22 actives, 11 pledges

Initiation was held 13 April. Other 1977-78 officers: Eric Magyar, vice-president; David Horton, secretary; Douglas Pfendler, treasurer; David T. Brown, corresponding secretary and faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President—Kathy Tandetzke
11 actives

Before February initiation each of the initiates gave a talk or a project as part of initiation. "Egyptian Mathematics," "Roman Numerals and the Roman Abacus," and "The Trachtenberg Speed Method of Basic Mathematics" were presentations given by Mary Grzechowiak, Debbie Schultz, and Beth Jacobs, respectively. Jeri Wenner's project on "Paper Folding in Relation to a Triangle" entailed folding the triangle to find perpendicular bisectors of the sides, bisectors of angles, medians, and altitudes. Eileen Korenic gave an explanation and demonstration on "The Geometry of Soap Film and Soap Bubbles." Formal initiation of the students named above was held on 15 February. KME graduates joined the local chapter for dinner following the initiation. On 12 April Wisconsin Alpha conducted its annual mathematics contest for high school students. Other 1977-78 officers: Jeri Wenner, vice-president; Eileen Korenic, secretary; Debbie Schultz, treasurer; Sister Mary Petronia Van Straten, corresponding secretary and faculty sponsor.

REPORT ON THE 1977 REGION I CONVENTION

On 25-26 March 1977 the Fourth Conference for Region I was held at Shippensburg State College, Shippensburg, Pennsylvania. Chapters of Region II were invited to participate in the meeting. A total of 60 student and 16 faculty delegates attended from eight Region I chapters and two Region II chapters. An additional 11 alumni of the host chapter, PA Iota, also participated in the event. Dr. John Mowbray, corresponding secretary, and Dale Meyers, president, of PA Iota directed arrangements.

On Friday evening, after a smorgasbord buffet, the PA Iota chapter initiated twelve new members. Dr. James Lightner, now National President of KME, gave an entertaining and thought-provoking talk entitled "Mathematicians Are Human Too—But Not Always Serious." The evening's activities closed with an informal party at the home of Dr. Jim Sieber.

Nine student papers were presented Saturday morning. Pat Schmieg, PA Iota, Shippensburg State College, took first place with her paper on the "Solution to Instant Insanity." Second place went to Tony Sager, MD Beta, Western Maryland College, for his presentation of "Inversion Geometry and Its Applications." Richard Styer, PA Lambda, Bloomsburg State College, received the third place award for "Is the Identity Function Continuous?"

Other presentations included: "The Fibonacci Method of Search Optimization" by Jan Gipe, PA Iota, Shippensburg State College; "Regression Coefficients and Linear Programming" by Thomas J. Kourim, IL Zeta, Rosary College; "A Short Study of the Columnar Transposition System of Encoding and Decoding Messages" by John Schaefer, NY Eta, Niagara University; "Everything You Wanted to Ask About Computers But Were Afraid to Ask" by Annastasia Maliaros, PA Epsilon, Kutztown State College; "How to Grab a Line" by Dale Myers, PA Iota, Shippensburg State College; and "Mathematics in Weather Forecasting" by Timothy Buchamn, PA Iota, Shippensburg State College. Honorable mention was given to two of these presentations—the student from IL Zeta and one of the students from PA Epsilon.

Prior to the mid-day luncheon, three concurrent sessions were held: a meeting for student officers, a meeting for faculty officers, and an address by Dr. Irving Hollingshead, corresponding secretary of PA Epsilon, entitled "Solar Energy—How to Stay Warm on a Cold Day." Following the luncheon the meeting closed with the presentation of awards by the Director of Region I and corresponding secretary of MD Alpha, Sister Marie Augustine Dowling.