THE PENTAGON

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CONTENTS

National Officers

Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Directions for Papers to be Presented at the Twenty-first Biennial Convention of Kappa Mu Epsilon

New Concord, Ohio

10-12 November 1977

A significant feature of this convention will be the presentation of papers by student members of KME. The mathematics topic which the student selects should be in his area of interest, and of such a scope that he can give it adequate treatment within the time allotted.

Who may submit papers: Any student KME member may submit a paper for use on the convention program. Papers may be submitted by graduates and undergraduates; however, graduate students will not compete with undergraduates. Papers which have been submitted for regional competition may also be submitted for this national competition.

Subject: The material should be within the scope of the understanding of undergraduates, preferably those who have completed differential and integral calculus. The Selection Committee will naturally favor papers within this limitation, and which can be presented with reasonable completeness within the time limit prescribed.

Time limit: The usual time limit is twenty minutes, but this may be changed on the recommendation of the Selection Committee if requested by the student.

Paper: Four copies of the paper to be presented, together with a description of the charts, models, or other visual aids that are to be used in the presentation, should be presented in typewritten form, following the normal techniques of term paper presentation. It should be presented in the *form* in which it will be presented, *including length*. (A long paper should not be submitted with the idea that it will be shortened for presentation.) Appropriate footnoting and bibliographical references are expected. A cover sheet should be prepared which will include the title of the paper, the student's name (which should *not* appear elsewhere in the paper), a designation of his classification in school (graduate or undergraduate), and a statement that the author is a member of Kappa Mu Epsilon, duly attested to by the Corresponding Secretary of the student's chapter.

Date due: 1 October 1977.

Address to send papers: Ida Z. Arms
National Vice President, KME
Indiana University of Pennsylvania
Indiana, Pennsylvania 15701

Selection: The Selection Committee will choose about fifteen papers for presentation at the convention. All other papers will be listed by title and student's name on the convention program, and will be available as alternates. Following the Selection Committee's decision, all students submitting papers will be notified by the National Vice President of the status of their papers.

Criteria for selection and convention judging:

A. The Paper
   1. Originality in the choice of topic
   2. Appropriateness of the topic to the meeting and audience
   3. Organization of the material
   4. Depth and significance of the content
   5. Understanding of the material

B. The Presentation
   1. Style of presentation
   2. Maintenance of interest
   3. Use of audio-visual materials (if applicable)
   4. Enthusiasm for the topic
   5. Overall effect
   6. Adherence to the time limit

Prizes: The author of each paper presented at the convention will be given a two-year extension of his subscription to *The Pentagon*. Authors of the four best papers presented by undergraduates, based on the judgment of the Awards Committee composed of faculty and students, will be awarded cash prizes of $60, $40, $30, and $20 respectively. If enough papers are presented by graduate students, then one or more prizes will be awarded to this group. Prize winning papers will be published in *The Pentagon*, after any necessary editing. All other submitted papers will be considered for publication, at the discretion of the Editor.

The Amazing Cycloid

John Williams
Student, Northeastern Oklahoma State University

The cycloid curve is one of the most interesting curves ever to have been studied in mathematics. A cycloid is the curve described by a point on a wheel as the wheel rolls along a horizontal straight line without slipping.

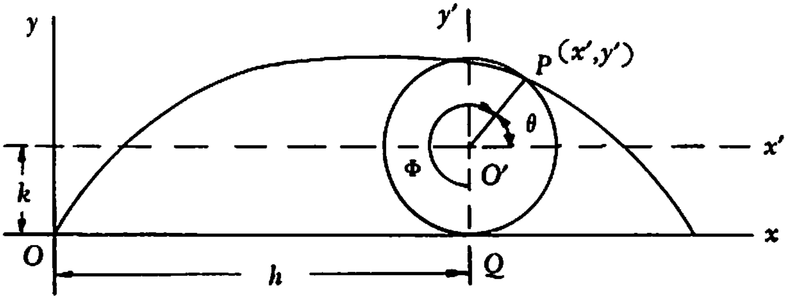

Figure 1

Allowing a wheel of radius r to begin rolling from the origin of the coordinate system (Figure 1) along the x-axis, point P on the circumference of the wheel will describe a cycloid curve. If the center of the wheel is O', parametric equations of the curve may be written using the angle Φ through which radius $O'P$ has rotated as parameter. Introducing $x'y'$-axes parallel to the xy-axes but having their origin at O', which has xy-coordinates (h,k), point P has coordinates:

$$\begin{aligned} x &= h + x' \\ y &= k + y' \end{aligned} \qquad (1)$$

Since the wheel has rolled without slipping, the arc length QP must equal the radius r multiplied by the angle Φ in radians through which the radius has rotated. Thus arc length $QP = r\Phi$. Also, the length of segment OQ must equal arc length $QP = r\Phi$. Thus the xy-coordinates of $O'(h,k)$ must be $h = OQ = r\Phi$ and $k = r$. Denoting the angle between $O'P$ and the x'-axis as θ we see that

$$x' = r \cos \theta$$

$$y' = r \sin \theta$$

and since $\theta = 3\pi/2 - \Phi$ we have

$$x' = -r \sin \Phi$$

$$y' = -r \cos \Phi$$

Substituting these results into the equations (1), we have

$$x = r(\Phi - \sin \Phi)$$

$$y = r(1 - \cos \Phi)$$

as the parametric equations of the cycloid [6].

This article will show that the cycloid curve possesses the following two interesting properties:

1. *Brachistochrone property*—among all smooth curves joining two given points, the cycloid is the curve along which a frictionless particle might slide between the two points, subject only to acceleration due to gravity, in the shortest time.
2. *Tautochrone property*—the time required for a frictionless particle, subject only to acceleration due to gravity, to slide to the bottom of the cycloid arc is constant no matter at what point along the arc the particle is released.

An easy way of thinking of this is to visualize the curve as a thin frictionless wire with a bead constrained to slide down the wire. We will show that the cycloid curve is the shape that the wire should take on if the time required for the bead to slide from one endpoint to the other is to be a minimum. Furthermore we will show that the time of descent to the bottom of the arc is the same no matter where the bead is released along the arc.

The Brachistochrone Problem

In the late seventeenth century two of the most prominent researchers in the calculus were the Swiss mathematician James Bernoulli and his younger brother John. John studied under James for a time at the University of Basle until about 1690 when he left Basle to travel and study mathematics in France. During this period a bitter rivalry sprang up between the two brothers. In June 1696, John Bernoulli proposed his now famous brachistochrone problem. He boasted of having a wonderful new solution and challenged the mathematicians of the world, especially his elder brother James, to try their skill on the problem [2]. The brachistochrone problem was proposed as follows:

> Imagine a frictionless particle constrained to slide along a curve joining a point *A* with a lower point *B*. If the particle begins at *A* with no initial velocity and is acted upon only by the force of gravity, along what curve will the time of descent be a minimum?

The term brachistochrone means "shortest time." At first one might think that a straight line path would minimize the time of descent. But if we begin to reflect upon the problem, intuitively we see that a somewhat circular arc might be faster because a path more vertically oriented at the beginning would allow a higher acceleration and might overcome the time lost in the lower more horizontal part of the arc. However we shall see that the actual solution curve is neither of these but is actually somewhere in between.

Those who responded to Bernoulli's challenge quickly began to see that this problem was different than the ordinary maximum-minimum problem solved by the differential calculus. The quantity to be minimized was no longer a function of one or more numerical variables, but rather was a function dependent upon the whole curve. This difference extended the problem beyond any methods known at that time.

The only discussions of the problem which were published in full in response to John Bernoulli's invitation were the solutions of the Bernoulli brothers themselves. Of the two, John's solution was the more ingenious and is the one which is discussed below.

John Bernoulli saw the similarity between his problem and the path of a ray of light governed by Fermat's principle in optics. Fermat had stated that light follows a path along which the time of travel is a minimum relative to any other path between the same two points. This principle closely paralleled the brachristochrone problem and Bernoulli was able to obtain a solution quickly.

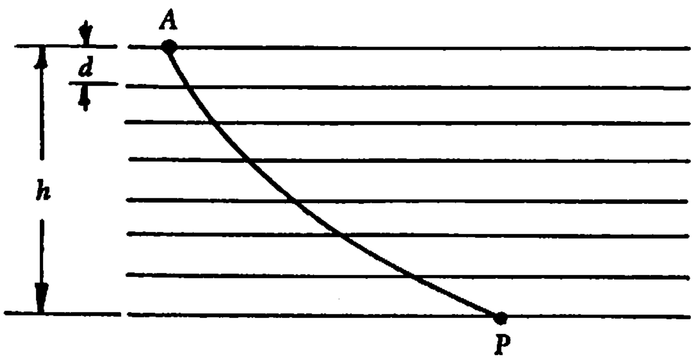

Figure 2

Bernoulli began with the fact from mechanics that a frictionless particle falling from rest at A along any curve will have a velocity at any point P that is proportional to $\sqrt{h}$ where h is the vertical distance between A and the point P. Then he dissected the space into n horizontal slabs each of thickness d as in Figure 2. He assumed that the velocity changed in "jumps" as the particle entered each new slab. Thus the velocity in the first slab would be $c\sqrt{d}$, in the second $c\sqrt{2d}$, and in the n^{th} slab $c\sqrt{nd}$ or $c\sqrt{h}$. This reduced the problem to a finite number of variables.

In each successive slab the path would be a straight line (Figure 3) and thus the solution would be a polygon. According to the law of refraction in optics the path must be such that the ratio of the sine of the angle α_i that the path makes with the vertical and the velocity along the path through a particular slab must equal the corresponding ratio in the next slab,

$$\frac{\sin \alpha_1}{\sqrt{d}} = \ldots = \frac{\sin \alpha_i}{\sqrt{id}} = \frac{\sin \alpha_{i+1}}{\sqrt{(i+1)d}} = \ldots = \frac{\sin \alpha_n}{\sqrt{h}}$$

Now Bernoulli imagined the slab widths d to become smaller and smaller, approaching zero as a limit, so that the solution of this approximate problem approached the solution of his brachistochrone problem. In approaching the limit Bernoulli found that the solution

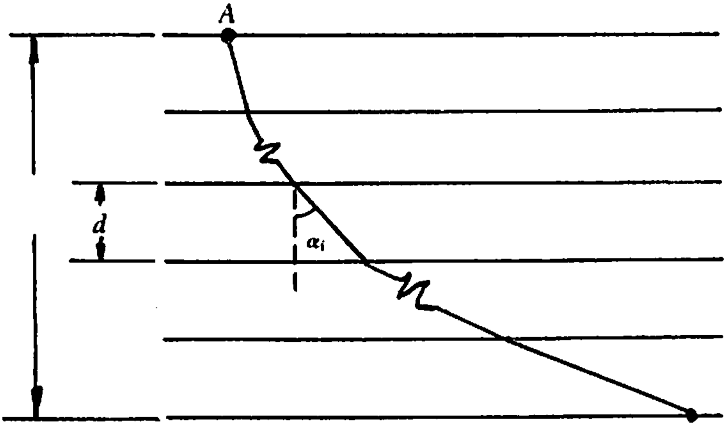

Figure 3

curve would have to have the property that $\frac{\sin \alpha}{\sqrt{h}}$ be constant all along the curve, where α is the angle between the tangent at a point P and the vertical, and h is the vertical distance from the point P to A.) It can be shown that this property describes the cycloid and thus Bernoulli had solved his problem [3].

In later years Euler and Lagrange discovered and developed more general methods for solving such extreme problems where the independent element was a whole curve or function or even system of functions. These developments have led to what is known today as the Calculus of Variations which can be applied to the brachistochrone problem giving us a much more rigorous proof than Bernoulli's "intuitive reasoning."

From mechanics we know that after falling a distance y, a particle has a velocity $\frac{dy}{dt} = \sqrt{2gy}$ where g is acceleration due to gravity. From the calculus we know that the element of arc length ds that the point has traveled may be expressed as $\sqrt{1 + (y')^2}\, dx$. Thus the time necessary for the point to travel an incremental arc length ds is:

$$\frac{ds}{v} = dt = \frac{\sqrt{1 + (y')^2}\, dx}{\sqrt{2gy}}$$

Therefore the total time of descent along a curve must equal

$$T = \frac{1}{\sqrt{2g}} \int_0^x \frac{\sqrt{1 + (y')^2}\, dx}{\sqrt{y}}$$

Since the brachistochrone problem asks for the arc that gives the shortest time descent, we are looking for the curve along which the integral above will be a minimum.

In the differential calculus the necessary condition for a value of a function to be an extreme point is that the derivative $f'(x)$ be equal to zero at that point. The Calculus of Variations sets up an analogous necessary condition for a functional describing the curve of descent to be a minimized. In general the functional may be represented by $T(y) = \int_{x_1}^{x_2} F(y,y')dx$ where the function F depends on two arguments (y,y'). We assume that F has continuous second derivatives with respect to y' and y within a domain containing x_1 and x_2. Thus we determine that the class of admissible functions must satisfy the following two requirements:

1. $y(x)$ must be continuously differentiable on $[x_1,x_2]$
2. $y(x_1) = y_1$ and $y(x_2) = y_2$.

There are no other restrictions on the possible curves that we must consider. We will assume that the minimum of the integral exists and call it $y(x)$.

Now consider the family of functions dependent on parameter α

$$\overline{y}(x) = y(x) + \alpha\eta(x).$$

In order for $\overline{y}(x)$ to be an admissible function for any value of α, $\eta(x)$ must be continuously differentiable and $\eta(x_1) = \eta(x_2) = 0$. Now the integral $T(\overline{y}) = \int_{x_1}^{x_2} F(y + \alpha\eta,\ y' + \alpha\eta')dx$ may be defined as $\Phi(\alpha)$.

Since $y(x)$ is the minimum value of the integral, $\Phi(\alpha)$ must have a minimum value for $\alpha = 0$. When $\Phi(\alpha)$ is a minimum its derivative at that point must vanish. Thus, differentiating $\Phi(\alpha)$ with respect to α and then setting $\alpha = 0$, we have

$$\Phi'(0) = \int_{x_1}^{x_2} (F_y\eta + F_{y'}\eta')dx \qquad (2)$$

where F_y and $F_{y'}$ are the partial derivatives of $F(y,y')$. Using integration by parts,

$$\int_{x_1}^{x_2} F_{y'}\eta' dx = F_{y'}\eta \Big|_{x_1}^{x_2} - \int_{x_1}^{x_2} \eta \, (dF_{y'}/dx)dx$$

Now $\quad F_{y'}\eta \; dx \Big|_{x_1}^{x_2} = 0$ because $\eta(x_1) = \eta(x_2) = 0$.

Therefore

$$\int_{x_1}^{x_2} F_{y'}\eta' \; dx = - \int_{x_1}^{x_2} \eta \, (dF_{y'}/dx)dx$$

Substituting this result into equation (2) we have

$$\Phi'(0) = \int_{x_1}^{x_2} \eta(F_y - dF_{y'}/dx) \; dx = 0.$$

Now $\eta(x)$ is an arbitrary continuously differentiable function. This means that $F_y - dF_{y'}/dx = 0$, for otherwise we could select some function $\eta(x)$ such that the integral would not be zero. Thus if a function minimizes the integral $T(y)$, it must satisfy this differential equation

$$F_y - dF_{y'}/dx = 0$$

which is called Euler's differential equation. This condition is analagous to setting the first derivative equal to zero in the differential calculus [5].

The Euler equation is of the second order and its general solution will contain two arbitrary constants

$$y = \Phi(x, C_1, C_2).$$

The solution of this boundary value problem can only supply a suspected minimum. Since in our case the Euler equation has only one solution that satisfies the given boundary conditions, and we have only one admissible curve that is a suspected minimum, we may be sure that it is the actual solution.

In our problem

$$T(y) = \int_0^{x_2} \frac{\sqrt{1 + (y')^2}}{\sqrt{y}}\, dx$$

with boundary conditions

$$y(0) = 0,\ y(x_2) = y_2.$$

Thus to use Euler's equation we let

$$F = \frac{\sqrt{1 + y'^2}}{\sqrt{y}}$$

After expansion and some manipulation the Euler equation takes the form

$$\frac{2y''}{1 + (y')^2} = \frac{-1}{y}.$$

Multiplying both sides by y' and integrating, we get

$$\ln\,(1 + y')^2) = -\ln y + \ln C_1$$

or $$(y')^2 = \frac{C_1}{y} - 1$$

Thus $$dx = \frac{\sqrt{y}}{\sqrt{C_1 - y}}dy$$

We introduce a new variable Φ defined by the equation

$$y = \frac{C_1}{2}\,(1 - \cos\Phi),$$

After substituting and simplifying

$$dx = \frac{C_1}{2}(1 - \cos\Phi)\, d\Phi$$

from which by integrating, we get

$$x = \frac{C_1}{2}(\Phi - \sin\Phi) + C_2$$

Since the curve passes through (0,0), C_2 must equal zero. Thus we see that the solution curve has the parametric equations

$$y = \frac{C_1}{2}(1 - \cos\Phi)$$

$$x = \frac{C_1}{2}(\Phi - \sin\Phi)$$

where the constant $\frac{C_1}{2}$ will depend on the position of the end points of the curve [5]. These are the equations derived earlier that describe the cycloid curve.

The Tautochrone Property

The cycloid has another remarkable property called the tautochrone property which was actually discovered earlier than the brachistochrone property. The term tautochone means "same time" and the cycloid gained this title after Huygens had discovered early in the seventeenth century that the time of descent for a particle sliding down a cycloid is independent of the starting point.

As before the functional describing the time of descent is

$$T = \int_{x_1}^{x_2} \frac{\sqrt{1 + (y')^2}}{\sqrt{2gy}}\,dx$$

Now we must show that the time required for the particle to slide to the low point of the arc $(\pi r, 2r)$ is the same regardless of the release point (x_0,y_0). After being released at (x_0,y_0) the velocity at any point $P(x,y)$ along the curve will be $2g(y - y_0)$. Substituting this into the time equation we have

$$T = \int_{\Phi_0}^{\pi} \frac{\sqrt{r^2(2 - 2\cos\Phi)}}{\sqrt{2gr(\cos\Phi_0 - \cos\Phi)}}\,d\Phi$$

which after integration becomes

$$T = 2\frac{\sqrt{r}}{\sqrt{g}}\left(-\sin^{-1}\frac{\Phi/2}{\Phi_0/2}\right)\Bigg|_{\Phi_0}^{\pi}$$

$$T = 2\frac{\sqrt{r}}{\sqrt{g}}(-\sin^{-1}0 + \sin^{-1}1) = \pi\frac{\sqrt{r}}{\sqrt{g}}$$

This answer is independent of the value of Φ_0 or (x_0,y_0) and thus it follows that the time of descent will be the same no matter at what point along the curve the particle is released from rest [6].

Huygens was able to use this discovery to set up a cycloidal pendulum in which the period of oscillation was the same no matter how great or how small the amplitude of the oscillation was. The principle of the cycloidal pendulum is based on the fact that the evolute of a cycloid is a cycloid. This was an amazing discovery for the clockmakers of Huygen's day.

Summary

The cycloid curve had just begun to be widely studied in the seventeenth century and one can imagine the mathematicians' amazement upon discovering the brachistochrone and tautochrone properties possessed by this "magical curve." Perhaps John Bernoulli said it best when he stated [2, p.54]

> With justice we admire Huygens because he first discovered that a heavy particle falls on a cycloid in the same time always, no matter what the starting point may be. But you will be petrified with astonishment when I say that exactly this same cycloid, the tautochrone of Huygens, is the brachistochrone which we are seeking.

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On Cones and Conic Sections

DAVID CALVIS

Student, Case Western Reserve University

Apollonius of Perga was one of the most famous of the ancient Greek Mathematicians. His treatise on the conic sections earned him the title, "The Great Geometer". Unfortunately, most modern works on conic sections contain very little reference to an actual cone. Thus, in this article we shall derive the equations of the ellipse, parabola, and hyperbola in terms of the angle formed by the intercepting plane and the axis of the cone and the generating angle of the cone.

Let us consider one nappe of a cone $\mathfrak{C}$ with a vertex at A and C a point on its axis [Figure 1]. Let B and D be two points on the cone such that B, C, and D are collinear and BD is perpendicular to AC. Let plane p be perpendicular to plane BAD and have point F in common with AD, intersecting the axis at I. Further, let H be on AC such that $HF \perp AC$. Finally, let $\overline{AH} = h$, the generating angle of the cone, $\angle CAD$, $= \theta (0 < \theta < \pi/2)$, and $\angle HFI = \alpha$, where $0 < \alpha < \theta + \pi/2$. (Note that there is no loss of generality in this restriction.) We now consider three cases.

CASE 1. $\alpha < \pi/2 - \theta$

The curve defined by these conditions is an ellipse. This can be proved from the classical focus-directrix definition of an ellipse. We shall find constants a and b such that $x^2/a^2 + y^2/b^2 = 1$ is the equation of the ellipse.

First, let the intersection of plane p and AB be E and extend FH to intersect AB at C [Figure 1]. Now, since a is equal to the length of the semi-major axis, which is equal to $\frac{1}{2}(\overline{EF})$, we get $\angle EGF = \theta + \pi/2$ and $\angle GEF = \pi/2 - (\theta + \alpha)$. By the law of sines, we have $a = \frac{1}{2}EF = h \sin \theta / \cos(\theta + \alpha)$.

To find b, we let M be the midpoint of the major axis EF and N the point on AC such that MN is perpendicular to AC [Figure 1]. Also we let P be a point on $\mathfrak{C}$ such that P,M, and N are collinear. By the law of sines,

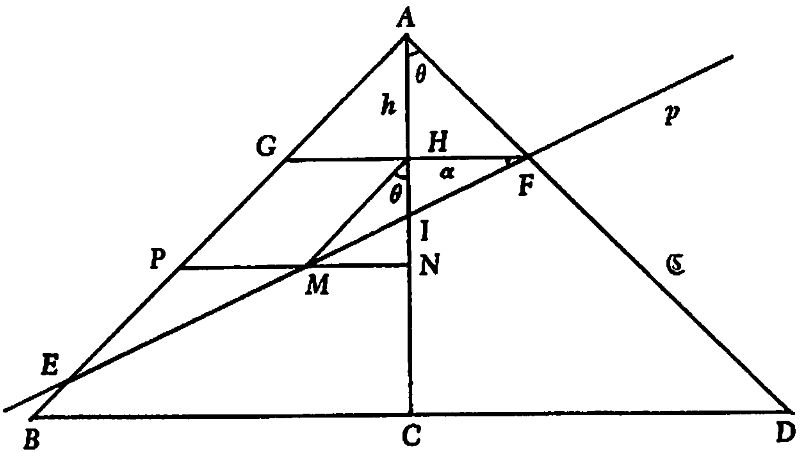

Figure 1

$$\tfrac{1}{2}GE = \frac{h \tan\theta \sin\theta}{\cos(\theta+\alpha)} = HM.$$

Further, HM is parallel to AB, which yields $\angle MHN = \theta$. Finally,

$$MN = HM \sin\theta = \frac{h \tan\theta \sin\alpha \sin\theta}{\cos(\theta+\alpha)}.$$

and $PM = GH$, so

$$PN = \frac{h \sin\theta \cos\alpha}{\cos(\theta+\alpha)}.$$

Now consider the circle formed by the intersection of a plane with ℭ such that the plane meets AC at N and is perpendicular to AC [Figure 2]. Let R be an endpoint of the minor axis. (R is, of course, on the circle.) Then, by the Pythagorean Theorem, $RM^2 = RN^2 - MN^2$. Also, $RN = PN$. Thus

$$b = \frac{h \sin\theta(\cos^2\alpha - \sin^2\alpha \tan^2\theta)^{\frac{1}{2}}}{\cos(\theta+\alpha)}$$

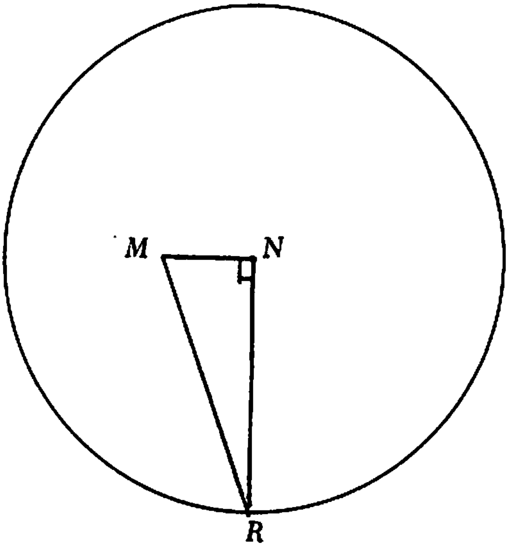

Figure 2

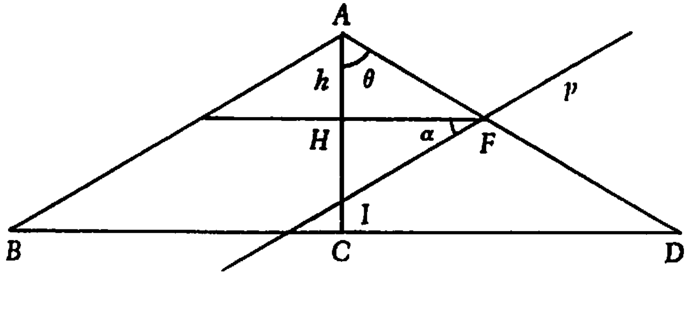

Figure 3

CASE 2. $\alpha = \pi/2 - \theta$

The parabola represented here may be put in the form $y = kx^2$, and we shall find k. First, note that $FI = AF = h \sec \theta$ [Figure 3]. Then $FI = h \sec \theta = k(2h \tan \theta)^2$, where $2h \tan \theta$ is the radius of the cone at I. Thus

$$k = \csc \theta \cot \theta / 4h.$$

CASE 3. $\alpha > \pi/2 - \theta$

Since this curve is a hyperbola, it will involve the other nappe of $\mathfrak{C}$ [Figure 4]. With all points as defined above, we let the vertex other than F of the hyperbola be V. Since the equation of the curve can be written as $x^2/a^2 - y^2/b^2 = 1$, where the foci are on the x-axis and equidistant from the origin, it may be seen that $a = \frac{1}{2}VF$. Now $\angle VAF = \pi - 2\theta$, and since $\angle AFH = \pi/2 - \theta$, $\angle AFV = \pi/2 + \theta - \alpha$, we get $\angle AVF = \theta + \alpha - \pi/2$. Thus, by the law of sines, $a = -h \sin \theta / \cos(\theta + \alpha)$.

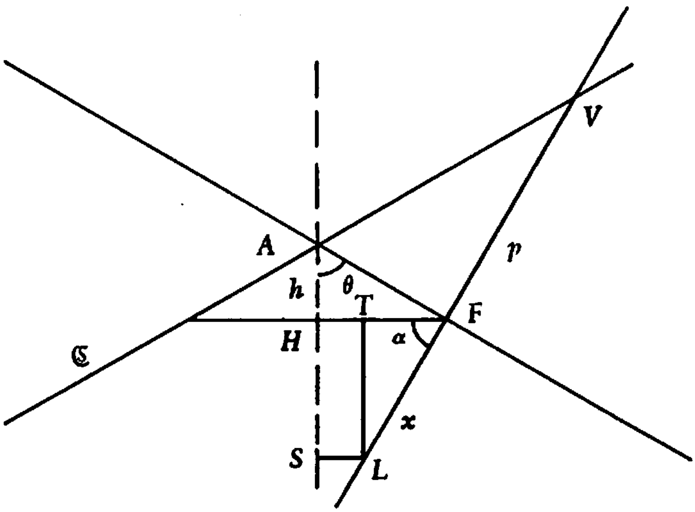

Figure 4

To find b, consider a point L on VF with $LF = x$. Let T be on HF such that LT is perpendicular to HF. Then $TF = x \cos \alpha$, $TL = x \sin \alpha$. Next, let S be on AH such that $AS = AH + TL = h + x \sin \alpha$. Thus, $AS \tan \theta$ is, loosely speaking, the radius of the cone at S. Now, as in Case 1, let us consider the circle of this radius whose plane is perpendicular to AS which has center at S. If we take a point Q on the circle such that VL is perpendicular to LS, it is seen that $SL = h \tan \theta - x \cos \alpha$, since $SL = HT$. Since QS is a radius, $QS = (h + \sin \alpha)\tan \theta$. Thus, by the Pythagorean Theorem,

$$QL = [x^2(\sin^2\alpha \tan^2\theta - \cos^2\alpha) + 2hx \tan \theta(\tan \theta \sin \alpha + \cos \alpha)]^{\frac{1}{2}}.$$

Now $b/a = \lim_{x \to \infty} (QL/x) = (\sin^2\alpha \tan^2\theta - \cos^2\alpha)^{\frac{1}{2}}$, and thus

$$b = \frac{h \sin \theta(\sin^2\alpha \tan^2\theta - \cos^2\alpha)^{\frac{1}{2}}}{\cos(\theta + \alpha)}$$

We may compare these values of a and b with those found for the ellipse.

The Tangent Function and A Ruled Surface

F. MAX STEIN

Faculty, Colorado State University

1. *Introduction.* The inquisitive undergraduate mathematics student may often think that there are no more mathematical worlds to conquer; all of the problems and ideas he is able to develop with his current mathematical training have long ago been exhausted. This paper illustrates that this is not necessarily the case; even a function as simple as the tangent function leads to interesting interpretations that the undergraduate student with only a background in trigonometry and analytic geometry can develop. To fully appreciate the ideas of ruled surfaces mentioned below, more advanced training is required, however. This training is not necessary for the discussion presented here.

2. *The tangent function.* The tangent function for any angle θ is defined as the quotient of the ordinate and the abscissa of any point (x,y) on the radius vector for the angle θ in the xy-plane, or simply

$$\tan \theta = \frac{y}{x}. \tag{1}$$

Now for any values of x (except possibly $x = 0$) and y we get a value of $\tan \theta$, which we choose to call z. That is, we look at the problem in three space with

$$z = \frac{y}{x}, \quad \text{or} \quad xz = y. \tag{2}$$

3. *Quadric surfaces.* The equation $xz = y$ is a special case of the quadric equation

$$Ax^2 + By^2 + Cz^2 + Dyz + Ezx + Fxy + Gx + Hy + Iz + J = 0 \tag{3}$$

where A through J are constants. The surfaces represented by this equation for particular choices of the coefficients include planes, ellipsoids (of which spheres are special cases), elliptic and hyperbolic paraboloids, hyperboloids of one and two sheets, quadric cylinders and quadric cones. (See any text on solid analytic geometry, [1], for instance.)

In our case, $xz = y$, with all coefficients in (3) equal to zero except $E = 1$ and $H = -1$, we have a hyperbolic paraboloid. The planes $y = c$ intersect the surface $xz = y$ in hyperbolas parallel to the xz-plane. For instance, when $y = 3$, $xz = 3$ is a hyperbola in the $y = 3$ plane.

Similarly the plane $x + z = 1$ intersects the surface $xz = y$ in a parabola. This can be seen better after $z = 1 - x$ is substituted in $xz = y$ to give $y = x - x^2$; i.e., the plane $x + z = 1$ intersects the parabolic cylinder $y = x - x^2$ in a parabola.

The two standard ways of representing curves in 3-space are by the intersection of two surfaces $F(x,y,z) = 0$ and $G(x,y,z) = 0$ considered simultaneously and by three parametric equations $x = f(t)$, $y = g(t)$, $z = h(t)$. Since lines are special cases of curves, they can be represented by the intersection of two planes or by linear parametric equations.

4. *Ruled surfaces.* We now need the definition of a ruled surface. If a surface is such that through any point on the surface a line can be drawn which lies entirely on the surface, then the surface is said to be a *ruled surface.* Examples of ruled surfaces are planes, cylinders, and cones. Slightly more complicated ruled surfaces are hyperboloids of one sheet and hyperbolic paraboloids.

The hyperbolic paraboloid $xz = y$ arising from the tangent function is a ruled surface. The plane $x = a$ intersects the surface $xz = y$ in a line; that is, the planes $x = a$ and $y = az$ intersect in a line. Also the plane $z = b$ intersects the surface $xz = y$ in another line; that is, the planes $z = b$ and $y = bx$ also intersect in a line. Thus we have *two* lines through each point (a, ab, b) on the surface. See Figure 1.

If there are lines from *two* families that pass through each point on the surface, the surface is said to be *doubly ruled*; thus our hyperbolic paraboloid $y = xz$ is a doubly ruled surface. It can be shown that only quadric surfaces can be doubly ruled, see [2], but of course not all quadric surfaces are doubly ruled, or even ruled; e.g., the sphere is not a ruled surface.

5. *Observations.* We now make two important observations, both of which are actually quite obvious. First, a model of a ruled surface can be constructed by the use of a frame and strings. For

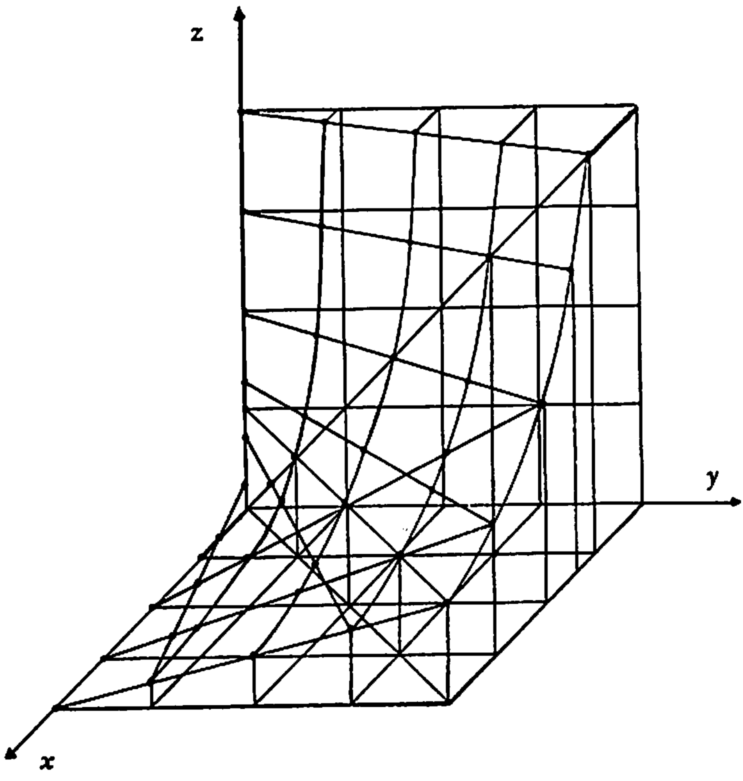

Figure 1

a model of our hyperbolic paraboloid for the first octant, say, a frame is constructed of wood or plastic with holes drilled at strategic locations. Through these holes strings are laced to show the rulings;

different colored strings to show the two rulings is effective. Next a third set of colored strings can be attached at the intersections of the two rulings and dropped perpendicular to the xy-plane. Finally a fourth set of strings can be placed under the rulings in such a manner as to outline the hyperbolas in planes parallel to the xz-plane. It should be obvious from Figure 1 how the model of this ruled surface can be constructed.

From the third set of strings described above we have the second observation—the important observation of this article—the length of the projection from any point on the surface to the xy-plane is numerically equal to the tangent of the angle in the xy-plane measured from the x-axis to the base of the projection of the point. This follows since

$$z = \tan \theta = \frac{y}{x} .$$

Notice in Figure 1, for instance, where each vertical line has length 1. Thus, although we were introduced in an elementary trigonometry course to the tangent function as in (1), we can now look at this function in a different light as in (2). This interpretation leads us in turn to another topic—the subject of ruled surfaces.

6. *Extensions.* Hopefully this short discussion will inspire readers to look for extensions of the ideas presented. What does the ruled surface $xz = y$ look like in other octants? How can parabolas be outlined in this model? Can the lengths of the projections to the other coordinate planes lead to interesting interpretations? Can other trigonometric functions be treated in a manner similar to that discussed above? Do other elementary non-trigonometric functions lead to interesting models?

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Pascal's Pyramid

Mike Alfonso,

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Paul Hartung,

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Pascal's Triangle represents the coefficients of the terms in the expansion of $(a + b)^n$, where n is a whole number. The triangle is formed by adding two numbers to get a third, placed below the preceding two, as illustrated here.

$$
\begin{array}{ccccccccccccc}
 & & & & & a^0 & & b^0 & & & & & \\
 & & & & & & 1 & & & & & & \\
 & & & & a^1 & & & & b^1 & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & a^2 & & & & & & b^2 & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & a^3 & & & & & & & & b^3 & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & a^4 & & & & & & & & & & b^4 & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
a^5 & & & & & & & & & & & & b^5 \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
 & & & & & & \cdot & & & & & & \\
 & & & & & & \cdot & & & & & & \\
 & & & & & & \cdot & & & & & &
\end{array}
$$

As one moves along the diagonals from right to left the powers of a increase, while from left to right b increases. A number at the intersection of two diagonals is the coefficient of the product of the corresponding powers of a and b. The fourth row of 1, 3, 3, 1 represents $(a + b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$.

In a similar fashion, a Pascal Pyramid can be formed which would illustrate the coefficients of $(a + b + c)^n$, n being a whole number. The first layer is 1.

The second layer is

$$\begin{array}{ccc} & 1 & \\ 1 & & 1 \end{array}$$

the third layer is

$$\begin{array}{ccccc} & & 1 & & \\ & 2 & & 2 & \\ 1 & & 2 & & 1 \end{array}$$

while the fourth layer is

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 3 & & 3 & & \\ & 3 & & 6 & & 3 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

Note that numbers in the third layer are added to get numbers in the fourth layer. For example, $1 + 2 = 3$ and $2 + 2 + 2 = 6$.

The numbers in the fourth layer are the coefficients of $(a + b + c)^3$. A similar method for knowing the literal terms of the expansion is formed by starting at the apex of the layer and increasing the powers of a until the next vertex, starting there increasing the powers of b likewise and finally increasing the powers of c. A number at the intersection of the three diagonals is the coefficient of the product of the corresponding powers of a, b, and c. The method outlined above is illustrated for the fourth layer:

$$\begin{array}{cccccccc} & & a^0 & & & & & \\ & & & 1 & & & & c^3 \\ & a^1 & & & & & & \\ & & 3 & & 3 & & & c^2 \\ a^2 & & & & & & & \\ & 3 & & 6 & & 3 & & c^1 \\ a^3 & & & & & & & \\ 1 & & 3 & & 3 & & 1 & c^0 \\ b^0 & & b^1 & & b^2 & & b^3 & \end{array}$$

Hence $(a + b + c)^3 = a^3 + 3ac^2 + 3bc^2 + 3a^2c + 6abc + 3b^2c + b^3 + 3a^2b + 3ab^2 + c^3$.

The Pascal Triangle is used in binomial probability to find, for example, the probability of getting exactly three heads in a toss of five coins. The number of ways of getting three heads on five coins is 10, an entry in the sixth row of the Pascal Triangle. The total number of ways five coins can come up is the sum of the entries in the sixth row, 32, which is 2^5. So the probability of getting exactly three heads on five dice is 10/32.

The Pascal Pyramid can be used in the same way in trinomial probability. Imagine we have four triangular prisms, each of them capable of landing on side *A*, *B*, or *C*. The number of ways we can get two *A*'s, one *B*, and one *C* in a toss of the four prisms is 12, an entry in the fifth layer of the pyramid. The total number of ways the four prisms can land is the sum of all the entries in the fifth layer, 81, or 3^4. So the probability of two *A*'s, one *B*, and one *C* is 12/81.

The probability of one *A* and three *B*'s on a toss of four prisms is 4/81. The probability of one *A*, one *B*, and one *C* on a toss of three prisms is 6/27. The probability of two *A*'s, two *B*'s and one *C* on a toss of five prisms is 30/243.

Just as each row of the Pascal Triangle can be written with combination symbols, so can each layer of the Pyramid. The form is illustrated with row five and layer five. The fifth row of the Triangle can be written:

$$\binom{4}{0}\binom{4}{1}\binom{4}{2}\binom{4}{3}\binom{4}{4}, \text{ where } \binom{4}{k} = \frac{4!}{k!(4-k)!}.$$

In a similar fashion, the fifth layer of the Pyramid is

$$\begin{array}{c}
\binom{4}{4\ 0\ 0} \\
\binom{4}{3\ 1\ 0}\binom{4}{3\ 0\ 1} \\
\binom{4}{2\ 2\ 0}\binom{4}{2\ 1\ 1}\binom{4}{2\ 0\ 2} \\
\binom{4}{1\ 3\ 0}\binom{4}{1\ 2\ 1}\binom{4}{1\ 1\ 2}\binom{4}{1\ 0\ 3} \\
\binom{4}{0\ 4\ 0}\binom{4}{0\ 3\ 1}\binom{4}{0\ 2\ 2}\binom{4}{0\ 1\ 3}\binom{4}{0\ 0\ 4}
\end{array}$$

where $\binom{4}{a\ b\ c} = \frac{4!}{a!\ b!\ c!}$ and $a + b + c = 4$. Just as we can observe $\binom{3}{k-1} + \binom{3}{k} = \binom{4}{k}$, by adding two elements of the fourth row of the Pascal Triangle to get the proper number of the fifth row, we can see that

$$\binom{3}{m\ \ n\ \ k-1} + \binom{3}{m\ \ n-1\ \ k} + \binom{3}{m-1\ \ n\ \ k}$$
$$= \binom{4}{m\ \ n\ \ k}, \text{ where } m + n + k = 4.$$

In general if $m + n + k = a + 1$, we have

$$\binom{a}{m\ \ n\ \ k-1} + \binom{a}{m\ \ n-1\ \ k} + \binom{a}{m-1\ \ n\ \ k} =$$
$$\frac{a!}{m!n!(k-1)!} + \frac{a!}{m!(n-1)!k!} + \frac{a!}{(m-1)!n!k!}$$
$$= \frac{a!k + a!n + a!m}{m!\ n!\ k!}$$
$$= \frac{a!(a+1)}{m!\ n!\ k!}$$
$$= \binom{a+1}{m\ \ n\ \ k}.$$

The pyramid and triangle have many similar characteristics. Both are formed by adding numbers in a previous row or layer. The numbers are coefficients of $(a + b)^n$ or $(a + b + c)^n$, where n is a whole number. The sum of a row is 2^n, while a layer's sum is 3^n. Also both exhibit symmetric properties and the side of a layer in the pyramid is the corresponding row of the triangle. Both can be applied to probability problems.

The Problem Corner

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 February 1978. The best solutions submitted by students will be published in the Spring 1978 issue of The Pentagon, with credit being for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

ERRATA: In the Fall 1976 issue two problems numbered 289 appeared. The second of these is problem 290. Problem 290 in that issue is Problem 291.

292. *Proposed by Leo Sauve, Algonquin College, Ottawa, Ontario, Canada*

The number 9,x29,50y,zt7 is known to be divisible by 73 and 137. Determine the digits x, y, z, t and thereby identify the number.

293. *Proposed by the editor.*

On a trigonometry test, one question asked for the largest angle of the triangle having sides 21, 41, and 50. L.A.Z. Thinker, a student, obtained the answer as follows: let C denote the desired angle, then $\sin C = \frac{50}{41} = 1.2195$. But $\sin 90° = 1$ and $.2195 = \sin 12° \, 40' \, 48''$. Therefore $C = 90° + 12° \, 40' \, 48'' = 102° \, 40' \, 48''$ which is correct. Find another triangle having this property which is not similar to the given triangle.

294. *Proposed by Leo Sauve, Algonquin College, Ottawa, Ontario, Canada*

Two cars leave at the same time from two towns A and B, going towards each other. When the faster car reaches the midpoint, M, between A and B, the distance between them is 96 miles. They meet 45 minutes later. Finally, when the slower car reaches M, they are 160 miles apart. Find (a) the speed of each car and (b) the distance between the two towns.

295\. *Proposed by the editor.*

Let $S_k = 1^k + 2^k + \ldots + n^k$ where n is an arbitrary positive integer and k is an odd positive integer. Under what conditions is S_k divisible by $S_1 = \frac{n(n+1)}{2}$ for all positive integers n?

296\. *Proposed by Charles W. Trigg, San Diego, California*

Using three consecutive digits repeated, form an arithmetic progression of three three-digit primes in the decimal system.

SOLUTIONS

282\. *Proposed by the editor.*

A farmer has a circular plot of radius 50 feet. At a point on the circumference of the plot he places a stake to which a goat is connected by a rope. How long is the rope if the goat can graze on exactly one-half of the area of the plot?

Solution by Leigh James, Rocky Hill, Connecticut

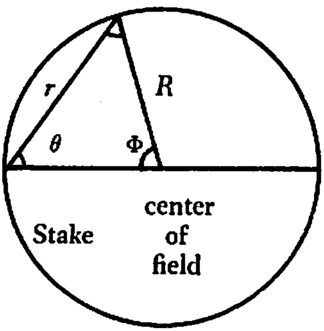

Figure 1.

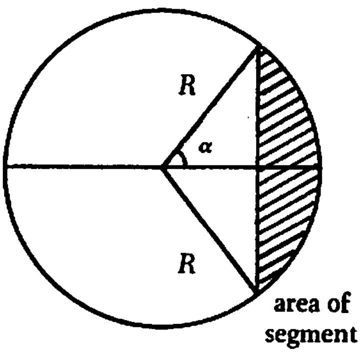

Figure 2

Let r be the length of the rope and R be the radius of the field as shown in Figure 1. Now by using polar coordinates as shown in Figure 2, the area of a segment of a circle of radius R is given by

$$A_{\text{segment}} = \left(\pi R^2 \cdot \frac{2\alpha}{2\pi}\right) - R^2 \sin\alpha \cos\alpha \quad = R^2(\alpha - \tfrac{1}{2}\sin 2\alpha)$$

Hence the area grazed is

$$r^2 \left(\theta - \tfrac{1}{2} \sin 2\theta\right) + R^2 \left(\Phi - \tfrac{1}{2} \sin 2\Phi\right) = \tfrac{1}{2} \pi R^2$$

which is one-half the area of the field. Note that $r = 2R\cos\theta$. Hence $4 R^2 \cos^2 \theta(\theta - \frac{1}{2}\sin 2\theta) + R^2(\Phi - \frac{1}{2}\sin 2\Phi) = \frac{1}{2}\pi R^2$ or taking $x = 2\theta$ and $\Phi = \pi - x$,

$$4 \left(\frac{1 + \cos x}{2}\right)\left(\frac{x}{2} - \frac{\sin x}{2}\right) +$$

$$\left((\pi - x) + \frac{\sin 2x}{2}\right) = \frac{\pi}{2}$$

which reduces to

$$\frac{\pi}{2} - \sin x + x \cos x = 0.$$

Solving this equation by successive approximations gives $x = 1.9056$ radians so that $\theta = .9528$ radians and $r = 1.158 R = 57.94$ feet since $R = 50$ feet.

Also solved by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas and Leo Sauve, Algonquin College, Ottawa, Ontario, Canada.

283. *Proposed by the editor.*

On Professor Knowitall's College Algebra exam, the following question appeared: Which is larger $\sqrt[6]{4}$ or $\sqrt[7]{5}$? Find the solution without using tables. Young Percival Whizkid solved the problem easily. How did he do it?

Solution by Leo Sauve, Algonquin College, Ottawa, Ontario, Canada.

Since $\sqrt[6]{4} = \sqrt[3]{2}$, it follows that $(\sqrt[3]{2})^{21} = 128$ while $(\sqrt[7]{5})^{21} = 125$. Then since $128 > 125$, $\sqrt[6]{4} > \sqrt[7]{5}$.

Editor's Comment: Leo Sauve is the editor of a Canadian Journal entitled EUREKA. Problem solvers should find this publication an interesting source of a wide selection of problems and other articles on mathematics. Those interested should contact: Leo Sauve, Editor, EUREKA, Algonquin College, 281 Echo Drive, Ottawa, Ontario K1S, 1N4, Canada. *Also solved by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas; David K. Yee, California Polytechnic State University, San Luis Obispo, California; and Charles W. Trigg, San Diego, California.*

284. *Proposed by the editor.*

Given two equal sides of an isosceles triangle, what is the length of the third side which produces the maximum area?

Solution by David K. Yee, California Polytechnic State University, San Luis Obispo, California.

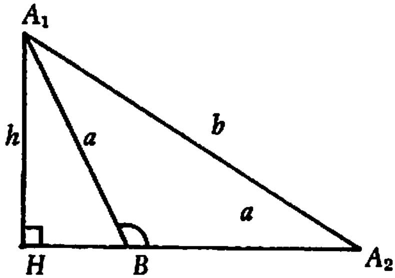

Figure 3.

Let a be the base of the isosceles triangle $A_1\ B\ A_2$ as shown in Figure 3 above. Then since $h = a \sin(180 - \theta)$ the area of $\triangle A_1BA_2$ is $\frac{1}{2}a^2 \sin(180 - \theta) = \frac{1}{2}a^2 \sin \theta$. Then since $\sin \theta$ reaches a maximum value of 1 when $\theta = 90°$, the maximum area is $\frac{1}{2}a^2$. Then from the law of cosines $b = a\sqrt{2}$.

Solution by Charles W. Trigg, San Diego, California.

The isosceles triangle will have a maximum area if the right triangle with one of the equal sides, a, as hypotenuse has a maximum area. In a semicircle with a as diameter, the inscribed right triangle with greatest altitude and hence maximum area is the isosceles right triangle, with side $a/\sqrt{2}$. Hence for maximum area, the original triangle should have a third side of $2(a/\sqrt{2})$ or $a\sqrt{2}$.

Editor's Comment: Charles W. Trigg also points out that problem also can be solved by differentiation in the usual manner.

Also solved by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas.

285. *Proposed by Randall J. Covill, Indian Hill, West Newbury, Massachusetts.*

If O is an octagonal number $= n\ (3n - 2)$ and P is a pentagonal numb r $= m(3m - 1)/2$ and $m = n =$ a positive integer then P and O are the complements of each other. It can be easily shown by algebraic manipulation of the formulas for P and O that to every difference between an octagonal number and its complementary pentagonal number there corresponds a multiple of 3 that is a unique positive integer. Show that for at least one multiple of 3 that is a positive integer there is not any corresponding difference between an octagonal number and its complementary pentagonal number.

Solution by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas.

The n^{th} octagonal number O is given by $n(3n - 2)$ while the m^{th} pentagonal number P is given by $\frac{m(3m - 1)}{2}$. Since P and O are complements, $m = n$ and $O - P = n(3n - 2) - \frac{n(3n - 1)}{2} = \frac{3n(n - 1)}{2}$. Then n is odd and $n - 1$ is even (or vice versa), $\frac{n(n - 1)}{2}$ is an integer, and $O - P$ is a multiple of 3. To find a multiple of 3 which is not the difference of an octagonal number and its corresponding pentagonal number, note that the sequence $0, 3, 9, 18, \ldots, \frac{3n(n - 1)}{2}$ skips 6. Then since the equation $\frac{3n(n - 1)}{2} = 6$ has no integral solutions, 6 satisfies the conditions of the problem.

Also solved by Charles W. Trigg, San Diego, California.

286. *Proposed by the editor.*

In Alcatraz Prison an eccentric jailer decided to effect a "selective release" of the prisoners. The cells are numbered consecutively beginning with the number 1. First he unlocked all the cells. Then after returning to the place of beginning, he turned the key in the lock of every second cell. Next he repeated the process by returning to the place of beginning and turning the key in the lock of every third cell. The jailer repeats this process and on the i^{th} trip he turns to key in every i^{th} cell after returning to the place of beginning at cell number 1. Assuming that Alcatraz has 200 cells and that no prisoner escapes during the process, how many prisoners are released and what cells did they occupy?

Solution by Gregory Hayward, Emporia Kansas State College, Emporia, Kansas.

From the statement of the problem, all cells are locked initially. Then for a cell to be left open, the key must have been turned in the lock an odd number of times. Then since the key in cell c is turned once for each divisor of c, we seek those numbers which have an odd number of divisors. For example, the key is turned in the lock of cell number 36 those numbers which have an odd number of divisors. For on each of the trips numbered 1, 2, 3, 4, 6, 9, 12, 18 and 36 corresponding to the divisors of 36.

In general divisors occur in pairs so that for any integer n, $n = a\ b$ for some integers a, b. a and b are distinct unless $a = b$ which implies $n = a^2$ so that only squares of integers have an odd number of divisors. Hence 14 prisoners occupying cells numbered 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, and 196 were released.

Solution by Charles W. Trigg, San Diego, California

The number of times, n, that the key was turned in the q^{th} cell is equal to the number of divisors of q. Thus, if

$$q = p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}, \text{ then } n = (a_1 + 1)(a_2 + 1) \ldots (a_k + 1).$$

(Continued on page 104)

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgement will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

When one studies motion, one many times obtains some rather strange results. Mathematical puzzles and paradoxes concerning motion often illustrate how false our previous concepts dealing with motion might be; especially when we consider the paths of moving objects.

First consider the following problem dealing with two identical coins. If one places the two coins next to each other as shown below in Figure 1, and then rolls the coin (an upright head) at the left

Starting Position — Fixed Coin — Final Position

Figure 1

along half the circumference of the other coin in the direction indicated by the arrow, one would probably expect the final position at the right (as indicated) to be an inverted head. This suspected result would appear quite logical since after the rotation of the coin through a semicircle, the upright head on the face of the coin would be rotated half its circumference to the final position, and hence should be inverted. If, however, this rotation is performed, one finds that the final position is instead an upright head ,exactly what we would have believed to be the case if the coin had been rotated completely about the stationary coin's circumference. To explain this paradox, one need only note that the starting and final positions where the coin touches the stationary coin are at opposite ends of a diameter.

Now consider another absurdity. The circle below in Figure 2 is rotated one complete revolution from A to B.

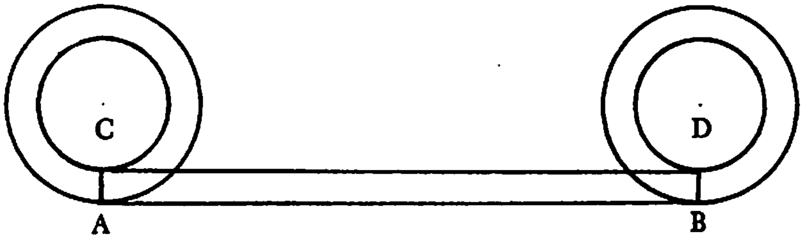

Figure 2

Hence distance AB equals one circumference of the larger circle. The smaller circle inside has also made one complete revolution in traveling from C to D. But AB = CD and here CD should equal one circumference of the smaller circle. Therefore, the circumference of the smaller circle equals the circumference of the larger circle.

To explain the apparent irregularities above, one recalls from calculus the cycloid as shown in Figure 3 below.

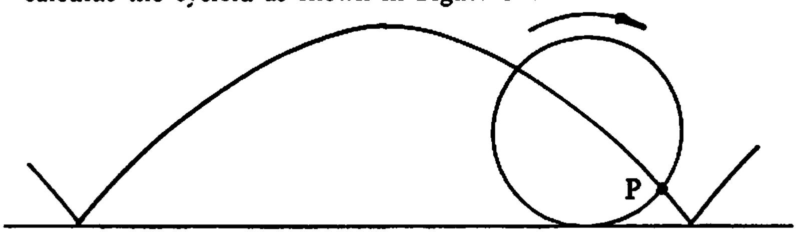

Figure 3

The cycloid is the path traced by a fixed point P on the circumference of a wheel as it rolls (without slipping) on a straight line. Now consider Figure 4 with the indicated circle having A and B as endpoints of a diameter, with initial positions A′ and B′.

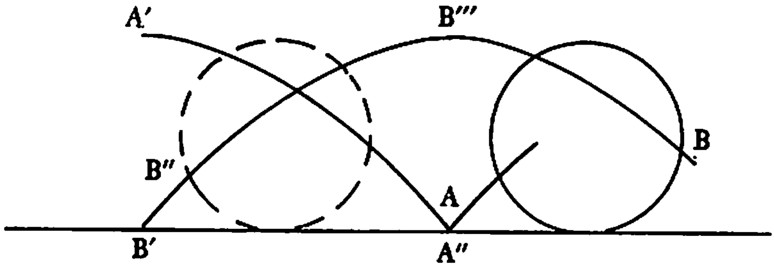

Figure 4

When the circle has completed half of a complete revolution, the point A is resting at point A‴ and point B is at B‴. At this time, points A and B have traveled the same distance but have they traveled at the same speed? To answer this question, let's consider an intermediate position. If we consider now the intermediate points A″ and B″, which are the points of location of points A and B after the circle has completed a quarter revolution, one notes that point A has traveled a distance much greater than that of point B in this time interval. This distance lag of point B is then counterbalanced in the second quarter turn when point B travels from B″ to B‴, a much greater distance than that traveled by point A when it travels from A″ to A‴. One notes that the distance traveled by B from B″ to B‴ equals the distance traveled by point A when it traveled from A′ to A″. Hence, in one half a revolution, points A and B have traveled exactly the same distance but at intermediate times their traversed distances are not necessarily equal.

The cycloid and its peculiar behavior help to logically explain the previous paradox. One notes that the part of the circle furthest from the line at any given time actually moves along the horizontal faster than the part of the circle in contact with the straight line. It is obvious that when the point on the circle touching the horizontal line starts moving upward, its horizontal speed increases until it reaches a maximum when its position is furthest from the horizontal line.

For the concentric circle problem, a simple extention of the cycloid is needed called the prolate cycloid. An interior point (on an inner concentric circle) of a circle which rolls on a straight line describes a prolate cycloid as shown in Figure 5.

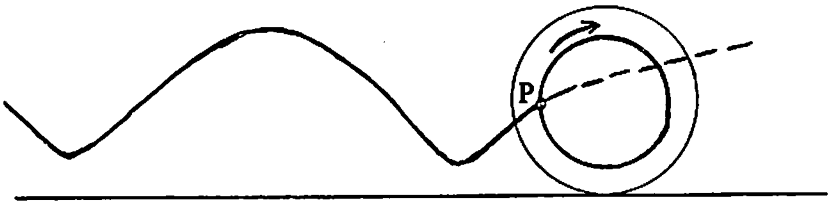

Figure 5

The small circle in Figure 2 makes only one revolution in moving from point C to point D and a point on the circumference of the inner circle describes a prolate cycloid. But upon comparing the prolate cycloid to the cycloid, one notes that the small circle does not cover the distance CD as a point on the larger circle does. A part of the distance is covered by the circle as it rolls, but simultaneously, it is being carried forward by the large circle as it moves from A to B.

Another interesting concept dealing with cycloids is that dealing with points outside the circumference of a circular wheel rolling on a straight line. A point on the circumference of a circular wheel rolling on a straight line. A point on the circumference has a cycloid path, a point in the interior has a prolate cycloid path, while a point outside the circumference of a circle has a path called a curtate cycloid. An example of such a point would be an outermost point of a flanged wheel such as those used on railroad cars as shown in Figure 6.

A point on the wheel such as point P is not in contact with the rail upon which the wheel revolves. The curve a point P generates as the wheel revolves along a horizontal rail is called the curtate cycoid as shown in Figure 6. A paradox which exists here is at any given instant, a train never moves entirely in the direction in which the engine is pulling. There will always be parts of the train which will be traveling in the opposite direction. Can you determine which points of the flanged railroad wheel are moving backward as the train moves forward and visa versa?

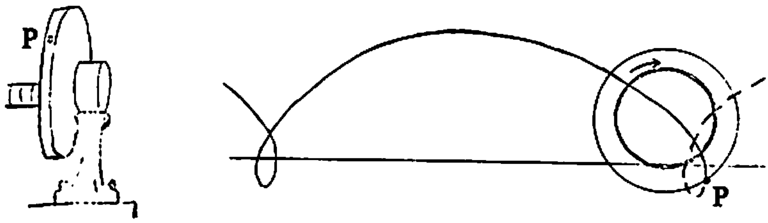

Figure 6

Still another coin puzzle is to take a triangular arrangement of 10 identical coins (as shown in Figure 7) and to require one to reverse this triangle (turn it upside down) by sliding one coin at a time to a new position in which it touches two other pennies. On each move, only a single coin can be slid to a new position so as to touch two other coins that rigidly determine its new position.

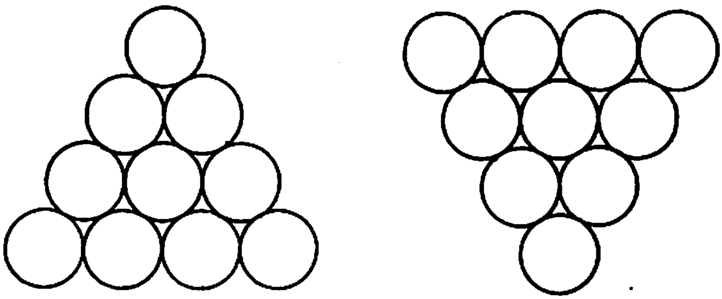

Figure 7

This problem is the famous "tetractys" problem of the ancient Pythagoreans. Can you solve this problem in three moves?

To generalize this situation one will note that a triangle of three coins can be inverted by moving one coin, a triangle of six coins can be inverted by moving two coins, and a triangle of ten coins can be inverted by moving three coins. Can a triangle of fifteen coins be inverted by moving four coins?

Upon careful consideration of the above problem, one will find that it will take a minimum number of five moves to invert a fifteen coin triangle. Can you develop a method of determining the minimum number of coins that must be moved to invert a triangle of n coins?

As a final puzzle consider the rhombus consisting of six identical coins closely arranged as shown in Figure 8.

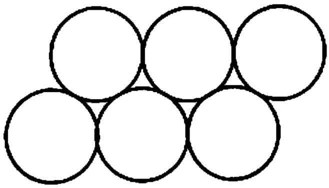

Figure 8

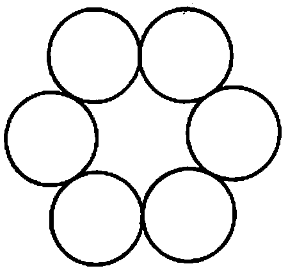

Figure 9

The problem is to try to form a circular pattern as shown in Figure 9 so that if we were to place a seventh coin in the circle's center, the six coins would be closely packed around it. Can you perform this feat in three moves?

(Concluded from page 98)

If any a_i is odd, n is even, and the corresponding cell eventually remained locked. If all the a_i are even, q is a square number, n is odd, and that "square" cell remained open. Now, $14^2 = 196 < 200 < 225 = 15^2$, so the 14 cells, 1, 9, 16, . . . , 196, remained open at Alcatraz. If there was exactly 1 prisoner in each "square" cell, 14 prisoners were released.

The Book Shelf

EDITED BY O. OSCAR BECK

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, University of North Alabama, Florence, Alabama 35630.

Technical Calculus, D. Ewen and M. A. Topper, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1977, 384 pp., $12.95.

This text is designed for students in engineering technology programs and for a one semester or two quarter course. Assuming a background of algebra and trigonometry, the first two chapters cover the usual topics in analytic geometry; a nice algebra review is provided here concerning solving simultaneous equations. Limits are introduced on page 58 after a good eight page motivation using motion and instantaneous speed; the treatment is very intuitive with no epsilons and deltas; the limit theorems are stated but not proved. The derivative is motivated using the slope of a tangent line to a curve; the definition uses delta y over delta x and there is a section using this limit definition to calculate derivatives. The derivative formulas are proved in a section at the end of this chapter. The usual applications of the derivative are given. Chapter six on integration, starting on page 109, begins with anti-derivatives; in section five area under a curve is treated using Riemann sums. The usual intuitive proof of the fundamental theorem is given using the area function. Applications of integration include work, insulation resistance of a shielded cable, and current in electric circuits. However, the variety of applications is not great. After chapter eight on transcendental functions come the methods of integration. Here there are careful and well written sections on integration by partial fractions, parts, and trigonometric substitutions; there are many examples and problems. Chapter ten on series is weak on theory but perhaps adequate for the audience; it includes a section on Fourier series. The chapter on numerical methods of approximation includes a section on data along a straight line as well as Newton's method and the usual numerical methods of integration. The last two chapters treat first and second order differential equations; they include nice sections on first order applications, simple harmonic motion, and the Laplace transform. At the end are tables for weights and measures, mass, lengths, areas, volume, and other useful conversion factors; not much use of these tables is made in the text, however. There are also adequate tables for logs, exponentials, trigonometry, and integration. Answers are provided for the odd numbered problems.

This reviewer found the text well written. A nice feature was that objectives were stated at the beginning of each chapter. The format was excellent; points were quickly made without verbosity. The print was large and the pages not crowded. There were plenty of meaningful examples and exercises. There was nice geometric motivation. The level of exposition is right for the student audience although there is a deficiency in challenging problems.

My only concern is the number of applications concerning engineering examples and problems. There are some but there could be more variety and number. There is also no attempt to use calculators and computing for motivation; no special sections or problems have been designated for computing.

This text provides an excellent intuitive introduction to calculus but engineering applications beyond a bare minimum will have to be supplied by the instructor or a later course.

Milton D. Cox
Miami University (Ohio)

Calculus and Its Applications, L. J. Goldstein, D. C. Lay, D. I. Schneider, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1977, 530 pages, $13.95.

This text is directed at students in the biological, social, and business sciences in that the extremely large number of applications discussed are in these areas. The large number of applications treated is an impressive feature of this text, but what makes it especially unusual is the excellent job the authors have done in integrating the applications into the exercise sets. This is a rare feature indeed. The wealth of applications together with the completely intuitive development of the calculus make the book well-suited to a course in the techniques of calculus. A comfortable four-semester-hour course could be taught from this text as well as a six-hour year-long course.

The book starts immediately with the derivative and proceeds expiditiously through a complete treatment of differential and integral calculus of polynomial and exponential functions including optimization in business and economics, compound interest, exponential growth, and several interesting applications of the integral. Only at this point are the product, quotient, and chain rules introduced, a very unusual feature. Following this order, a one-semester course could conceivably fail to cover these topics.

The text also contains chapters on functions of several variables, the logarithm, trigonometric functions, techniques of integration, and differential equations. The coverage of the several-variable chapter includes Lagrange multipliers and the method of least squares. The chapter on trigonometric functions is routine and contains only one unusual application. The chapter on differential equations is interesting in that it contains a long section on qualitative theory. In particular, techniques for sketching solution curves to a single first order autonomous equation are given. This chapter concludes with a discussion of the Lokta-Volterra equations including a proof of the periodicity of solutions.

This book would be an excellent choice for a techniques course taught to biological, social, and business majors. The concepts are well motivated and developed in sufficient algebraic detail to appeal to students with a minimal mathematical background. Students should also obtain a good appreciation for the utility of calculus using this text. However, the complete absence of theory may make this text inappropriate for students who plan to take additional mathematics courses beyond the calculus.

R. M. Bullock
Miami University (Ohio)

Topics in Mathematics, Donald R. Burleson, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1977, 528 pp., $12.95.

The author says this is a "two-semester college freshmen-level . . .

conceptual course sequence appropriate for liberal arts or education majors." It includes three chapters (Chapters 1, 2, and 3; 90 pages) on sets and logic; two chapters (Chapters 5 and 6; 90 pages) on numbers—integers to the reals; five chapters (Chapters 4, 7, 8, 9, and 10; 200 pages) on mathematical systems, finite geometry, semigroups, groups, rings, fields, and matrices; and two chapters (Chapters 11 and 12; 100 pages) on probability and statistics.

The book is well written, and should be easy to teach out of and easy for the students to read. Each section has a good selection of the sort of somewhat routine problems one would like to assign, plus a selection of "things to ponder," a set of more theoretical and harder problems. Theorems, definitions, examples, etc. are well displayed, and it is easy to sort the rigor from the heuristic.

In spite of the fact that the author says ". . . a number of different courses are possible with this book, . . .", I don't really see one I would choose to teach. For the liberal arts student, I don't believe a course aimed at getting a smattering of abstract algebra is the optimum—perhaps that's just a matter of taste. For the abstract algebra course for the prospective teacher, I'd like something written for a student with a little more mathematical maturity and going a little deeper into the topics. But if you want to teach logic, axiomatics, and abstract algebra with perhaps a little statistics thrown in, and you want to teach it to freshmen with possibly poor backgrounds, you ought to consider this book.

E. R. Deal
Colorado State University

Precalculus Mathematics, David G. Crowdis, Brandon W. Wheeler, Glencoe Press, Beverly Hills, 1976, 434 pp., $12.95.

Precalculus mathematics texts have become replacements for the more traditional separate courses in trigonometry, college algebra, and analytic geometry. The text by Crowdis and Wheeler is another book in this vein. The topics familiar to the separate courses are found in the table of contents and in addition the authors have included optional material on computer mathematics in each of the chapters.

In the preface the authors suggest that the book could be taught in a five-semester-hour course or for a whole year meeting three times a week if one left out the computer sections. The pace suggested would probably be ambitious for college students. If the text were used at the high school elvel it could very easily be the senior course devoted to two semesters.

The text has an abundance of exercises and the answers to the odd numbered problems as well as complete answers to the review exercises.

It should be noted that the book has minimal material in analytical geometry and is lacking in three dimensional topics. The usual material considered formerly as college algebra and trigonometry is treated completely and is very well done.

The text warrants consideration by high school mathematics teachers responsible for the college preparatory mathematics program as well as colleges and universities offering courses in the precalculus series.

Ramon Avila
Ball State University

Elementary Mathematical Analysis, Iain T. Adamson, Longman, Inc., New York, 1976, 226 pp., $9.00 (paper).

The first chapter deals with the real numbers and real-valued functions.

The real numbers are introduced via the field and completeness axioms. The discussion is lucid and concise. Functions are introduced as mappings from one set to another. This approach conveys the meaning of a function much better, in the opinion of this reviewer, than the usual approach of reviewing a function as a set of pairs.

The discussion of functions, including an elementary introduction to trigonometric functions, is illuminating.

The chapter on limits and continuity starts with the definition and elementary properties of continuous functions. The concept of limit is introduced later in the chapter, and the way the author defines it is a good indication of the strength of the book. He does it carefully, but in a simple and concise manner, avoiding both the pitfalls of intuition and the complications of premature rigor.

Derivatives are introduced in Chapter 4. The discussion concerning the usual notation, and the one used by the author, is another good feature of the book. The classical notations fail to indicate, among other things, that differentiation is a process that makes one function into another function.

The distinction between a power series as a series of numbers and as a series of functions is clearly made (p. 104).

The trigonometric functions are introduced in the usual intuitive way (on the unit circle) and put on a sound analytical basis in a later chapter. That is, the author lets the student develop a need for mathematical rigor, rather than engaging in fastidious reasoning before the student is ready.

His definition of π (p. 133) is unusual, but clear and well motivated.

The author leaves out the proofs of some of the more difficult theorems. Those of us who teach calculus and who are forced to skip proofs in order not to bore the students to death know that difficult proofs at this stage are rather premature.

The integral is introduced using upper and lower sums. The concept of primitive, or antiderivative, is introduced via the fundamental theorem.

The last chapter is an introduction to the study of real functions of several variables.

This is a book for students who want or need a solid, logically sound, first course on calculus. In the opinion of this reviewer, it compares favorably with most of the textbooks now in use, as far as the mathematics is concerned. However, it is not a viable textbook because it does not contain all the material the students need to learn, and because it contains no applications. It should be a valuable supplementary book for mathematically inclined students.

Juan C. Aramburu
University of North Alabama

Vector and Tensor Methods, Frank Chorlton, Halsted Press, New York, 1976, 336 pp., $22.50.

This book provides a very complete account of classical vector analysis, at a level suitable for most undergraduate students of mathematics or the physical sciences. Numerous examples are provided which illustrate both theory and applications of vector analysis to geometry and physical problems.

The text includes a lengthy discussion of potential theory followed by chapters on hydrodynamics and electromagnetic problems. These chapters follow the mathematical development of Vector Analysis which contains miscellaneous scattered applications to geometry and mechanics. A chapter on particle mechanics would be desirable, with perhaps less discussion of potential theory.

A brief account of Tensor Analysis is included in which only cartesian tensors are studied. Some applications of cartesian tensor analysis to continuum mechanics are provided.

The text overall is not written in a rigorous mathematical form and there are some small technical errors. The author, on page 43, makes the remark that any equation of the form $f(x,y,z) = 0$ can always be solved to give z explicitly in the form $z = \Phi(x,y)$ where Φ may be single-valued or multi-valued. Most mathematics students, if well-trained, should flinch at this statement. But statements like this do not provide any serious logical fault in the mathematical development as a whole.

Ben F. Plybon
Miami University (Ohio)

Dots and Lines, Richard J. Trudeau, The Kent State University Press, 1976, 199 pp., cloth $11.00; paper $6.50.

Dots and Lines is different. By the author's admission, he is writing for the following types of readers: a) the mathematically traumatized, b) the mathematical hobbyist, and c) the serious student of mathematics. Certainly graph theory (which is what the book is about) is an appropriate vehicle for such a mission.

The author assumes only "a year or so of high school algebra," [p. ix] as previous mathematical training by the reader. Consequently, many arguments (proofs?) are intuitive. This would seem to appeal to readers of type a and b. A difficulty associated with the prerequisite assumption is that some other mathematical content needs to be introduced. Mathematical induction is one such example. The terse, intuitive presentation of such material might have a negative effect on readers of type a.

Mr. Trudeau seems to have effectively reached readers of type b. Concepts are succinctly presented with ample illustrations. He also includes elaborate problem sets. These should be examined in their entirety because some subsequent work depends on prev ious problems. One negative aspect of the problem sets is that some definitions are included there. In particular, wheel, component, connectivity, bridge, and even-odd vertices are relegated to being defined in a problem. Many readers might miss these because of the length of the problem sets (Chapter Two contains forty problems on seven pages).

Suggested readings at the end of each chapter are excellent. The reader wishing more detailed development has an immediate reference. There are also references to related fields. For example, Chapters Two and Three list suggested reading for sets, paradoxes, Bertrand Russell, logic, mathematical discovery, mathematical jargon, and impossibility.

The last three chapters (six, seven, and eight) will bother some readers. Chapter Six discusses coloring, but what has been a classic unsolved coloring problem is now solved. That is, the Four-Color *conjecture* is now a *theorem*. However, because of the recent publication of this proof, most readers will tolerate this error.

A potentially more serious problem exists with the sequence of presentation of materials in Chapters Seven and Eight. Chapter Eight contains interesting, basic intuitive concepts such as Eulerian and Hamiltonian walks. Any introductory book on graph theory should contain these. The problem is that Chapter Seven is quite difficult and readers might get discouraged enough to not read the last chapter.

Summarizing, *Dots and Lines* is an enjoyable, relatively easy to read introduction to graph theory. It is probably most appropriate for the mathematical hobbyist or the reader wishing to have a quick but not terribly rigorous introduction to the subject.

Douglas W. Nance
Central Michigan University

Kappa Mu Epsilon News

EDITED BY SISTER JO ANN FELLIN, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Sister Jo Ann Fellin, Historian, Kappa Mu Epsilon, Benedictine College, North Campus Box 43, Atchison, Kansas 66002.

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

Chapter President—Jim Diehl
39 actives

Old members and new initiates were welcomed back to school with a picnic and recreational activities at the lake. Programs during the semester were diverse. Dr. Elizabeth Wooldridge, National Secretary, presented the history of Kappa Mu Epsilon. Interest was generated in the field of astrophysics by Dr. David Curott, who explained some of his work in that area. Chapter President, Jim Diehl, presented his paper which was to be submitted for presentation at the national convention. It concerned his original work on magic pentagons.

Alabama Gamma, University of Montevallo, Montevallo

Chapter President—Sandra K. Hayes
12 actives, 2 pledges

In addition to monthly meetings, the chapter sponsored two special programs. An award was given each month for the best solution to a mathematical problem posted on the mathematics floor. As a service project chapter members participated in a tutoring program through the counseling center. New officers in the chapter since the last publication: Connie Reynolds, secretary; Cathy Zahumensky, treasurer.

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President—Robert Watanabe
50 actives, 39 pledges

New activities of the chapter included the institution of a KME alumni newsletter with Liz Smith as editor and the sponsorship of a job placement conference. Several workshop sessions were held to write all the tests for a county-wide Junior High Math Field Day. Other activities were monthly meetings featuring faculty speakers, a faculty-student picnic, a faculty-student coffee hour, and a Christmas social and pledge ceremony.

California Delta, California State Polytechnic University, Pomona

Chapter President—Robert Beauchamp
19 actives, 10 pledges

The chapter conducted tutoring sessions open to all mathematics students at the university. A book scholarship award of $50 was given to a deserving student. A November picnic held at a local park was well attended by both students and faculty together with their families.

Colorado Alpha, Colorado State University, Fort Collins

Chapter President—Jane Darling
15 actives

Solar energy, solar heating, and tricks with math were a few of the topics presented at meetings during the semester by engineering and mathematics professors. Other activities included a halloween party and a pot-luck dinner. Prospective new members were invited to the latter.

Colorado Beta, Colorado School of Mines, Golden

Chapter President—Greg Golike
20 actives

During the semester Colorado Beta entertained an illustrious group of speakers. Professor Robert S. Fisk talked on "Painless Derivatives," Professor Donald W. Fausett presented "The Interstellar Hotel," and Professor William S. Dorn, under the auspices of the Visiting Lectureship Program of the Society for Industrial and Applied Mathematics, spoke on the subject "Mathematical Modelling and Computing in the Social and Behavioral Sciences." Other officers: Steve Hamburg, vice-president; Kent Peaslee, secretary; David Burleigh, treasurer; Ardel Boes, corresponding secretary; Don Fausett, faculty sponsor.

Florida Beta, Florida Southern College, Lakeland

Chapter President—Carol Barth
17 actives

Florida Beta Chapter was formally installed at Florida Southern College on 31 October 1976. Dr. Elizabeth T. Wooldridge, National Secretary, visited the campus to initiate the eighteen charter members. An additional five members, who were unable to make the October ceremony were initiated on 9 December 1976. Other officers: David Schreck, vice-president; Donna Gibson, secretary and treasurer; Dr. Henry Hartje, corresponding secretary and faculty sponsor.

Illinois Alpha, Illinois State University, Normal

Chapter President—Lynne McKinty
16 actives, 8 pledges

Members and pledges volunteered their time to tutor fellow university students in calculus and pre-calculus mathematics classes. Speakers at the monthly meetings included Anthony Jones, a mathematics faculty member on exchange from England, and Dr. Albert Otto, chairman of the mathematics department at Illinois State. Dr. Otto is now serving as corresponding secretary for the chapter. Members and pledges enjoyed a Christmas get-together in early December.

Illinois Zeta, Rosary College, River Forest

Chapter President—Ann Stangarone
15 actives

Chapter members are again offering their services as tutors. Representatives of KME assisted at an open house for prospective students on 26 September. On 30 September the chapter hosted a picnic to which all mathematics students were invited. Senior Tom Kourim presented a computer program for the solution of the coconut problem at the meeting on 21 October. Sister Nona Mary Allard is now serving as corresponding secretary and faculty sponsor.

Indiana Gamma, Anderson College, Anderson

Chapter President—Christopher Linamen
12 actives, 6 pledges

Other officers: Dwight Stewart, vice-president; Jay Collins, secretary and treasurer; Stanley L. Stephens, corresponding secretary and faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President—John Scott Daup
41 actives

The annual Homecoming breakfast was held in October at the home of Dr. and Mrs. E. W. Hamilton. This event has enjoyed increasing popularity. This year, in addition to the local members, there were thirteen alums and guests in attendance. The following students presented papers at chapter meetings: Denise Smith on "Graph Theory," Howard Batt on "The Nine Point Circle," and Bonnie Marlett on "The Shortest Path Through a Network." Constance M. Martin was elected vice-president when John Scott Daup assumed the duties of president.

Iowa Gamma, Morningside College, Sioux City

Chapter President—Kim Helmbrecht
26 actives

A Homecoming breakfast served at the home of Dr. Elsie Muller by the actives to returning alumni and their spouses drew a good attendance. Chapter members made themselves available for tutoring during the semester. A film prepared by the Nebraska Actuaries Club was shown at one of the chapter programs on possible careers.

Iowa Delta, Wartburg College, Waverly

Chapter President—David A. Zelle
19 actives, 14 pledges

Two students, who participated in the college's extended field experience course during the May term of 1976, presented programs during the fall semester about their work. On 20 September, President David Zelle described his computer science experience at the State University of New York in Albany and at the 22 November meeting Vice-president Deborah Ehlers told about her actuary science project at Continental Assurance Company in Chicago. The chapter participated in the college's Berufung Week. A Christmas dinner meeting was held 8 December at which Dr. William H. Cutler of the mathematics department of Wartburg talked about his design and construction of wooden puzzles. At the 26 January meeting, after revisions of the chapter's constitution and by-laws were made to reflect the increase in national dues, the members attempted to solve some of Dr. Cutler's puzzles. The 15 February meeting featured a lecture by Dr. Peter Colwell on "Some Problems Whose Correct Solution Isn't Right." Dr. Colwell's visit was part of the Iowa Section of MAA's visiting lecture program.

Kansas Alpha, Kansas State College of Pittsburg, Pittsburg

Chapter President—Theresa Audley
50 actives

Tim Cohle was elected vice-president of the chapter to fill that office left vacant when Theresa Audley was elected president to replace Polly Mertz who did not enroll in the fall semester. In September the chapter hosted a picnic for all mathematics and physics students. At the October meeting Dr. Richard Hay from the Kelce Center for Business and Economic Development discussed mathematics in economics. Dr. Bruce Daniel from the physics department gave a demonstration in the planetarium for the November program. Five new members were also initiated at the November meeting. A special Christmas meeting was held at the home of Dr. Helen Kriegsman, chairperson of the mathematics department. Gale Russell presented a biography of Alan M. Turing for the program at that time.

Kansas Gamma, Benedictine College, Atchison

Chapter President—Le Ann Fischer
13 actives

Kansas Gamma members welcomed the new year and all interested freshmen students at a picnic at the home of faculty sponsor Jim Ewbank. During the semester two student papers were presented. The first, "Instant Insanity," was given by Le Ann Fischer on 5 October. Michael Hannon talked about soap bubbles and minimal surfaces on 2 November. That same night the film titled "Dihedral Kaleidoscope" was shown. On 18 October after having dinner with chapter members, John Hutchinson, alum of the college and presently chairman of the mathematics department at Wichita State University, spoke to the group on "Linear Input-Output Models." The traditional Christmas Wassail party, held at the home of Sister Jo Ann Fellin culminated the semester activities.

Kansas Delta, Washburn University, Topeka

Chapter President—Ronald Wasserstein
25 actives, 8 pledges

Monthly meetings included programs and planning sessions for the math day for high school students scheduled for 31 March.

Kansas Epsilon, Fort Hays Kansas State College, Hays

Chapter President—Deanna Bowman
16 actives

Kentucky Alpha, Eastern Kentucky University, Richmond

Chapter President—Lois Coulter
13 actives, 10 pledges

During the fall semester the chapter met twice a month for business meetings and programs which included a trip to South Central Bell Telephone Plant and a demonstration of a music synthesizer. A Christmas party was given for all mathematics majors, minors, and faculty. Other activities included preparing bulletin boards and serving as tour guides for Math Day for high school students, sponsoring weekly tutoring sessions, and publishing brain teasers in the school newspaper. Alvin McGlasson is now acting as faculty sponsor for the chapter along with Glynn Creamer.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President—Kate Wildberger
7 actives, 2 pledges

The chapter began the year in September with an orientation toward careers by means of a tape-filmstrip presentation of "Why Study Mathematics?" Student papers during the semester were given by Bonnie Stingl on "How to Narrow the Communications Gap with Mathematics" and Rhette Murtha on "Envelopes." At the fall joint meeting with Maryland Beta, Kate Wildberger spoke on "Polyominoes." The November meeting was devoted to a career-oriented talk by alum Barbara Tipton (graduate of 1966) who has been an IBM employee for ten years.

Maryland Beta, Western Maryland College, Westminster

Chapter President—Deborah Simmons
25 actives, 3 pledges

In September the chapter sponsored a picnic for all mathematics majors. Induction of three new members took place in October. Speaker for the event was a 1972 Western Maryland College graduate, Robert Chapman, who is now with the National Bureau of Standards. He spoke about "Statistical Solutions to the Lead Paint Problem." At the joint meeting with Maryland Alpha in December, Debbie Simmons talked about graph theory.

Michigan Alpha, Albion College, Albion

Word was received from Dr. Brian J. Winkel of Albion College that members are in the process of reactivating the Michigan Alpha chapter at Albion College and plan a March initiation. Students and faculty planning to become members of Kappa Mu Epsilon have been actively involved in a new quarterly devoted to all aspects of cryptology. Dr. Winkel is one of the editors of this new journal called *Cryptologia*. Some of the areas to be discussed in the new journal are mathematical, computational, literary, historical, political, military, mechanical, and archeological aspects of cryptology.

Michigan Beta, Central Michigan University, Mount Pleasant

Chapter President—Pamela Moore
40 actives

In mid-October 19 new members were initiated. Speaker for the occasion was the president of Central Michigan University, Dr. Harold Abel. Other meetings included a talk on the relevance of physics and mathematics by Dr. Calman Levich, head of the physics department; solutions to Putnam problems by chapter vice-president, John Schantz; a faculty vs. students volleyball game; and a Christmas party at the home of Dr. and Mrs. E. H. Whitmore. Other officers: John Schantz, vice-president; Barbara Borthwick, secretary; Jan Novasad, treasurer; E. H. Whitmore, corresponding secretary and faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg

Chapter President—Andrew Fortenberry
55 actives

On 1 October at the fall cookout 23 new members were initiated. The chapter was saddened by the death of Jack D. Munn on 1 January 1977. Jack Munn, former associate professor of mathematics at the University of Southern Mississippi, had served as corresponding secretary for the Mississippi Gamma chapter from 1954 until June, 1976 when he retired from the university due to ill health. He also served Kappa Mu Epsilon as Director of Region III.

Missouri Beta, Central Missouri State University, Warrensburg

Chapter President—Beverly Parnell
27 actives, 19 pledges

During the semester Missouri Beta held six meetings with visiting speakers and a Christmas party. Homer Hampton serves as corresponding secretary and faculty sponsor for the chapter.

Missouri Eta, Northeast Missouri State University, Kirksville

Chapter President—Malia Monday
21 actives, 13 pledges

Missouri Eta entertained the Mathematics Club of Northeast Missouri State University for a weekend in the fall with activities from volleyball and a mathematical treasure hunt to a talk on the last one hundred years in mathematics by Mr. Jamison, age 93. The chapter held a Christmas party for all mathematics majors and faculty and a caroling party to faculty homes. Other officers: Amy Barrow, vice-president; Pat McDonald, secretary; Debbie Reinker, treasurer; Samuel Lesseig, corresponding secretary; John Erhart, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne

Chapter President—Kay Pankratz
14 actives, 7 pledges

The primary project for the fall semester was a tutoring program sponsored by the chapter. Members offered their services free to students needing help in pre-calculus and general education mathematics courses. Eight chapter members are continuing the service during the spring semester. Other officers: Karen Doeschot, vice-president; Lois McKenzie, secretary and treasurer; Gordon Cook, historian; Fred Webber, corresponding secretary; James Paige, faculty sponsor.

Nebraska Beta, Kearney State College, Kearney

Chapter President—Kenneth Hutton
12 actives, 7 pledges

Seven pledges were initiated into the chapter on 14 December.

Nebraska Gamma, Chadron State College, Chadron

Chapter President—Mark Wieting
25 actives, 5 pledges

Regular meetings were held every first and third Thursdays of the month. On 11 December the chapter held a Christmas party with caroling and roller skating. Treasurer for the chapter is Cyndi Moravek.

New Jersey Beta, Montclair State College, Upper Montclair

Chapter President—Jeffrey Smith
47 actives

The Career Night program on 18 November highlighted the fall semester activities. Six Montclair State College alumni spoke about their present jobs, opportunities in their fields, and how best to prepare for these positions. Areas covered included teaching statistics, computing, and banking. A film on the actuarial profession was also shown that evening. Social activities for the semester included a picnic and a Christmas party. Chapter members are preparing for the annual mathematics contest for about 30 local high schools. Other officers: Mary Ann Spina, vice-president; Elizabeth Pratt, secretary; Theresa Vander Zee, treasurer; Elvira Kohlhammer, historian; Carl Bredlau, corresponding secretary; Helen Roberts, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President—H. Turner Laquer

New York Eta, Niagara University, Niagara

Chapter President—John Schaefer
27 actives, 13 pledges
John Schaefer, senior, prepared a paper on cryptanalysis.

New York Theta, St. Francis College, Brooklyn

Chapter President—Steve Virgadamo
4 actives, 5 pledges

Other officers: Sharon Cimakasky, vice-president; Ann Moran, secretary; Joseph Rosalie, treasurer; Dr. Guaraldo, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord

Chapter President—Becky Tucker
24 actives

In September two student presentations were made at chapter meetings—"Mathematical Curiosities" by Deb Gutridge and "All You Ever Wanted to Know About Linear Programming but Were Afraid to Ask" by Becky Tucker. These talks were also given that same month at the fall meeting of Pi Mu Epsilon held at Miami University in Oxford, Ohio. Talks by new members and the initiation banquet were held in October. During November the chapter had two visiting speakers from Miami University—Dr. Edward Bolger and Dr. Milton Cox. A Christmas party ended the semester activities.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford

Chapter President—Steve Thomas
35 actives, 9 pledges

Oklahoma Gamma initiated nine new members on 7 December. Other newly elected officers: Jeanette Mack, vice-president; Kimberly Roof, secretary; Danny Basler, treasurer; Wayne F. Hayes, corresponding secretary; Robert Morris, faculty sponsor.

Pennsylvania Beta, LaSalle College, Philadelphia

Chapter President—Pat Nepps
6 actives, 4 pledges

Chapter members participated in a campus-wide Open House program by creating a mathematics laboratory for secondary education and a metric system display which emphasized the integration of the system into daily living. The chapter sponsored a lecture on catastrophe theory and provided free tutoring esrvices to mathematics students. Other officers: Joe Rakszawski, vice-president; Kevin Burns, treasurer; Brother Damian Connelly, corresponding secretary; Samual Wiley, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President—Rebecca Lykens
25 actives, 10 pledges

The chapter held regular meetings with brief student presentations, participated in the biennial mathematics conference held at Kutztown State College, and sponsored the annual games night with math games, puzzles, and refreshments.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President—James Blauser
30 actives

At the October meeting Wallace Morrell of the mathematics department gave a talk entitled "A Change of Pace." The talk involved some interesting and challenging mathematics problems. Faculty sponsor and national KME President, William R. Smith, spoke on "How to Choose a Wife" at the December meeting. Other officers: Kathy Carlson, vice-president; Barbara Howe, escretary; Kathy Buczek, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Iota, Shippensburg State College, Shippensburg

Chapter President—Dale Myers
39 actives, 7 pledges

Initiation of the fall pledge class took place at the home of faculty sponsor James Sieber. Regular meetings were held as well as a combined math club and KME picnic along with a field trip to a nuclear power plant. The chapter was host for the Region I meeting held at Shippensburg State College on 25-26 March.

Pennsylvania Kappa, Holy Family College, Philadelphia

Chapter President—Louise Wallowicz
5 actives, 3 pledges

Chapter members participated in a series of seminars on topics relating to Statistics. The chapter, along with the mathematics club Beta Chi, sponsored a lecture on the metric system during the fall semester and plan another lecture on the same topic for the spring semester. With the money gained from a 50-50 raffle the chapter purchased a minicalculator for the mathematics department. Members have contributed about 100 hours of free time to tutoring mathematics students. Other officers: Janice Di Girolamo, vice-president; Susan Capozio, secretary and treasurer; Sister Mary Grace, corresponding secretary and faculty sponsor.

Pennsylvania Lambda, Bloomsburg State College, Bloomsburg

Chapter President—Richard Styer
25 actives, 8 pledges

The chapter has initiated the Reardin Memorial Award for outstanding mathematics achievement in memory of former department chairman Charles Reardin. Dr. Frank Bernhart presented a series of talks on "The Four Color Problem." The chapter sponsored a mathematics day for high school students and had a Christmas banquet for all members.

Tennessee Alpha, Tennessee Technological University, Cookeville

Chapter President—Mary Anne Koltowich
100 actives

William Jones is serving as faculty sponsor with Donald Ramsey. Evelyn Brown continues as corresponding secretary for the chapter.

Tennessee Beta, East Tennessee State University, Johnson City

Chapter President—Jamie Davis
31 actives, 15 pledges

Other officers: Robin Ferrell, vice-president; Melita Feathers, secretary; James Teague, treasurer; Lyndell Kerley, corresponding secretary; Sallie Pat Carson, faculty sponsor.

Texas Alpha, Texas Tech University, Lubbock

Chapter President—Cheryl Harper
15 actives, 30 pledges

The chapter presents the following awards: faculty recognition award, an undergraduate mathematics scholarship, and outstanding pledge recognition. Activities include the spring initiation and the faculty-student fall and spring parties. Other officers: Pam Boyle, vice-president; Joan Tiede, secretary; Paul Sauer, treasurer; L. R. Hunt, corresponding secretary and faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene

Chapter President—Susan Porter
25 actives

The fallowing seven new members were inducted on 10 April 1976: Cymbe Alford, David Simmons, Taras Hetzel, Carol Bierdeman, Carol Evans, Sharon Grimes, and Dr. Jimmis Purser. Several members entered the Putnam Contest this year. In 1975 four members entered and one student placed in the top five hundred. Other officers elected at the meeting on 25 October: Cymbe Alford, vice-president; Taras Hetzel, secretary and treasurer; Anne Bentley, corresponding secretary; Charles Robinson and Edwin Hewett, faculty sponsors.

Virginia Beta, Radford College, Radford

Chapter President—Vicki Burnette
15 actives

Other officers: Judy Sabo, vice-president; Carolyn Gilliam, secretary; Sue Esslinger, treasurer; J. S. Milton, corresponding secretary; J. D. Hansard, faculty sponsor.

West Virginia Alpha, Bethany College, Bethany

Chapter President—Mary Ann Mercer
16 actives, 20 pledges

Other officers: Howard Verbofsky, vice-president; Rich Gurich, secretary; Mark Schmidt, treasurer; Mike Rommel, historian; Tim Hamilton, publicity; David T. Brown, corresponding secretary and faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President—Kathleen Tandetzke
8 actives, 5 pledges

One meeting was devoted to solving challenging problems and making discoveries. A student presentation was given at another meeting on "Plato's World of Forms" by Kathleen Tandetzke. Jim Margenau from Nicolet High School presented a talk on "Infinitesimal Calculus." He gave a history of the calculus and then explained the difference between infinitesimal calculus and standard calculus. Other activities of the chapter included the viewing of the film "Ascent of Man—Music of the Spheres" and preparing displays on the metric system for the official school bulletin boards.