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CONTENTS

National Officers ---------- 66

Directions for Papers to Be Presented at the Twenty-first Biennial Convention ---------- 67

The Cantor Mapping ---------- 69

Euclid Today ---------- 75

The Leontief Input-Output Matrices ---------- 80

Geometry of Extrema ---------- 85

On Connected Subsets of the Real Line ---------- 89

The Mathematical Scrapbook ---------- 90

The Problem Corner ---------- 97

The Book Shelf ---------- 103

Kappa Mu Epsilon News ---------- 109

National Officers

Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Directions for Papers to be Presented at the Twenty-first Biennial Convention of Kappa Mu Epsilon

New Concord, Ohio

24-26 March 1977

A significant feature of this convention will be the presentation of papers by student members of KME. The mathematics topic which the student selects should be in his area of interest, and of such a scope that he can give it adequate treatment within the time allotted.

Who may submit papers: Any student KME member may submit a paper for use on the convention program. Papers may be submitted by graduates and undergraduates; however, graduate students will not compete with undergraduates. Papers which have been submitted for regional competition may also be submitted for this national competition.

Subject: The material should be within the scope of the understanding of undergraduates, preferably those who have completed differential and integral calculus. The Selection Committee will naturally favor papers within this limitation, and which can be presented with reasonable completeness within the time limit prescribed.

Time limit: The usual time limit is twenty minutes, but this may be changed on the recommendation of the Selection Committee if requested by the student.

Paper: The paper to be presented, together with a description of the charts, models, or other visual aids that are to be used in the presentation, should be presented in typewritten form, following the normal techniques of term paper presentation. It should be presented in the *form* in which it will be presented, *including length.* (A long paper should not be submitted with the idea that it will be shortened for presentation.) Appropriate footnoting and bibliographical references are expected. A cover sheet should be prepared which will include the title of the paper, the student's name (which should *not* appear elsewhere in the paper), a designation of his classification in school (graduate or undergraduate), and a statement that the author is a member of Kappa Mu Epsilon, duly attested to by the Corresponding Secretary of the student's chapter.

Date due: 21 January 1977.

Address to send papers: James E. Lightner
National Vice President, **KME**
Western Maryland College
Westminster, Maryland 21157

Selection: The Selection Committee will choose about fifteen papers for presentation at the convention. All other papers will be listed by title and student's name on the convention program, and will be available as alternates. Following the Selection Committee's decision, all students submitting papers will be notified by the National Vice President of the status of their papers.

Criteria for selection and convention judging:

A. The Paper
   1. Originality in the choice of topic
   2. Appropriateness of the topic to the meeting and audience
   3. Organization of the material
   4. Depth and significance of the content
   5. Understanding of the material

B. The Presentation
   1. Style of presentation
   2. Maintenance of interest
   3. Use of audio-visual materials (if applicable)
   4. Enthusiasm for the topic
   5. Overall effect
   6. Adherence to the time limit

Prizes: The author of each paper presented at the convention will be given a two-year extension of his subscription to *The Pentagon*. Authors of the four best papers presented by undergraduates, based on the judgment of the Awards Committee

Continued on page 84

The Cantor Mapping*

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The following article is an investigation into an aspect of transfinite numbers as they were applied to the real number system and Cartesian space by Georg Cantor. The investigation was prompted by Martin Gardner's article "Mathematical Games" in the March 1971 issue of *Scientific American*. All lines of inquiry centered around statements made in that article.

In the article Gardner describes the transfinite numbers, or alephs, $\aleph$, and gives a brief account of Cantor's attempt at distinguishing the dimensions by the different levels of infinity. Of course, $\aleph_0$ is the countable infinity or the number of integers. $\aleph_1$ was found to be the number of real numbers, or in a more useful sense here, the number of numbers, or points, on the line segment [0,1] (I). Cantor, in attempting to show the number of points in the square (I^2) was $\aleph_2$, instead found an ingenious method of pairing these points to the line segment I, thus showing the number of points was actually $\aleph_1$. Here's how it was done.

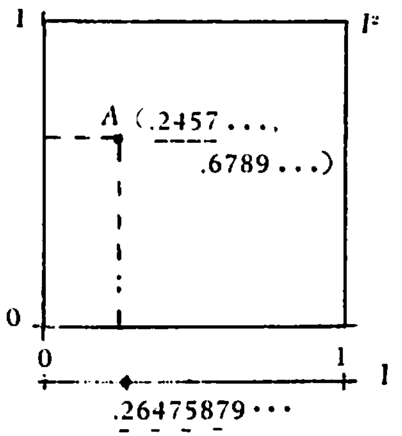

Figure 1

Assume we have point A with irrational coordinates as shown in

*A paper presented at the 1975 National Convention of KME, which received an honorable mention from the Awards Committee.

Figure 1. Starting with the x-coordinate, weave the digits in the two numbers. This results in one number, also irrational, which lies in I. The process is easily reversed resulting in the mapping of all the points in the square to the points on the line segment and vice versa. This is how Cantor counted $\aleph_1$ points in the square. It will be referred to as the Cantor mapping procedure.

In Martin Gardner's article he makes two statements about the mapping which will be discussed below:

1) "The matchings in the preceding paragraph are not continuous, that is, points close together on the line are not necessarily close on the square, and vice versa."

2) "The points of any square can be put into one-to-one correspondence with the points on any line segment."

The author doesn't completely agree with the first statement, for he has been able to find continuity for a certain set of points, and he disagrees completely with the second statement. However, the count of $\aleph_1$ points in the square can still be derived from this procedure.

It is important that the mapping procedure used below be precisely defined. Many variations can be used, so some rules must be established to avoid confusion. They are as follows:

1) All mappings will be between the closed interval $[0,1]$ and the closed square $[0,1] \times [0,1]$ (denoted by I and I^2 respectively.)

2) All terminating decimals must be represented by their nonterminating form when they are being mapped. For example, .5 now becomes $.4\overline{9}$ and .125 becomes $.124\overline{9}$.

3) When combining two coordinates of a point from I^2 to make a single number, always begin with the x-coordinate. Then all odd positions in the number will be formed by the x-coordinate and the even positions by the y-coordinate. The new number will be referred to by α. To form x and y from α the procedure is reversed.

Admittedly these rules are arbitrary, and perhaps the first may be a bit restrictive. For example, most of the following results can be extended to R^p space, and even to mappings from R^p to R^q, with certain relaxations and changes made in the third rule. But all the fundamentally important results can be seen most easily with these restrictions.

The mapping has some interesting properties. It is surprisingly well-behaved. First of all, the selection rules force the mapping to be functional in both directions. A point in I maps to only one point in I^2 and vice versa. Secondly the type of the number is basically unchanged. A pair of irrational numbers form an irrational number, and in the reverse mapping, an irrational number will form a pair containing at least one irrational number. Rationals are maintained in both directions, and a mixed pair in I^2 will form an irrational.

Another area where the mapping is well-behaved is in the area of convergent sequences.

THEOREM. *Suppose the sequence* $\{\overline{x_n}\}$ *in* I^2 *is related to the sequence* $\{\alpha_n\}$ *in* I *by the Cantor mapping where* $(x_n, y_n) \longleftrightarrow \alpha_n$. *Then, for* x, y, *and* α *non-terminating decimals, the sequence* $\{\overline{x_n}\}$ *converges in* I^2 *to a point* $(x, y) = (.a_1a_2a_3 \cdots, .b_1b_2b_3 \cdots)$ *iff the sequence* $\{\alpha_n\}$ *converges in* I *to* $\alpha = .a_1b_1a_2b_2a_3b_3 \cdots$.

Proof: Case I. Assume $\{\overline{x_n}\} \to (x,y) = (.a_1a_2a_3 \cdots, b_1b_2b_3 \cdots)$. Let $\{\alpha_n\}$ be the sequence in I generated by the sequence $\{\overline{x_n}\}$ and the Cantor mapping. Let $\epsilon > 0$ be given. If $\epsilon > 1$ take only its decimal portion. Let k be the negative integer such that $.1\epsilon \leqq 10^k < \epsilon$. Let $\delta(\epsilon) = 10^k$. By the convergence of $\{\overline{x_n}\}$ there is an N such that for $n \geqq N$, $|(x,y) - (x_n,y_n)| < \delta(\epsilon)$. Then, for n sufficiently large, (x_n,y_n) will agree with (x,y) to at least the k^{th} decimal place (i.e. $(x_n,y_n) = (.a_1a_2 \cdots a_kc_1c_2 \cdots, b_1b_2 \cdots b_kd_1d_2 \cdots))$ and $|\alpha_n - \alpha| < 10^{2k} < \epsilon$ (i.e. $\alpha_n = .a_1b_1a_2b_2 \cdots a_kb_kc_1d_1 \cdots$).

Case II. Assume $\{\alpha_n\} \to \alpha = a_1b_1a_2b_2 \cdots$. Let $\{\overline{x_n}\}$ be the sequence in I^2 generated by the sequence $\{\alpha_n\}$ and the Cantor mapping. Let $\epsilon > 0$ be given. Again if $\epsilon > 1$ take only its decimal portion. Let k be the negative integer such that $.1\epsilon \leqq 10^k < \epsilon$. Let $\delta(\epsilon) = 10^{2k}$. By the convergence of $\{\alpha_n\}$, there is an N such that if $n \geqq N$, $|\alpha_n - \alpha| < \delta(\epsilon)$. Then for n sufficiently large α_n will agree with α to at least the $2k^{th}$ decimal place and $|(x_n,y_n) - (x,y)| < 10^k < \epsilon$.

The problem with converging sequences whose limit can be expressed as a terminating decimal can be seen in the following example. Consider the following sequence in I: $\alpha_1 = .11$, $\alpha_2 = .101$,

$\alpha_3 = .1001, \cdots$. This sequence obviously converges to .1, and after selection rule two is applied we get: $\alpha_1 = .10\overline{9}$, $\alpha_2 = .100\overline{9}$, $\alpha_3 = .1000\overline{9}, \cdots$ which also converges to .1, or as we represent it $.0\overline{9}$. After applying the Cantor mapping we get a new sequence: $(x_1,y_1) = (.1\overline{9}, .0\overline{9})$, $(x_2,y_2) = (.10\overline{9}, .0\overline{9})$, $(x_3,y_3) = (.10\overline{9}, .00\overline{9})$, $(x_4,y_4) = (.100\overline{9}, .00\overline{9}), \cdots$. It is not difficult to see that this sequence will converge to the point (.1,0). But the original sequence converges to .1 (or $.0\overline{9}$) which after mapping becomes (.1,1). The limits of convergence are not the same in this case.

Now consider just the mapping from I to I^2. With this, along with Figure 2 to provide intuitive aid, we are now prepared to answer the question "Is this mapping continuous?" By considering the last example along with the Discontinuity Criterion it is obvious the mapping is discontinuous for at least the point .1 in I. And, by taking a similar approach it is quite easy to form a discontinuity at any rational point which can be expressed as a terminating decimal. However, what about the repeating decimals and the irrationals.

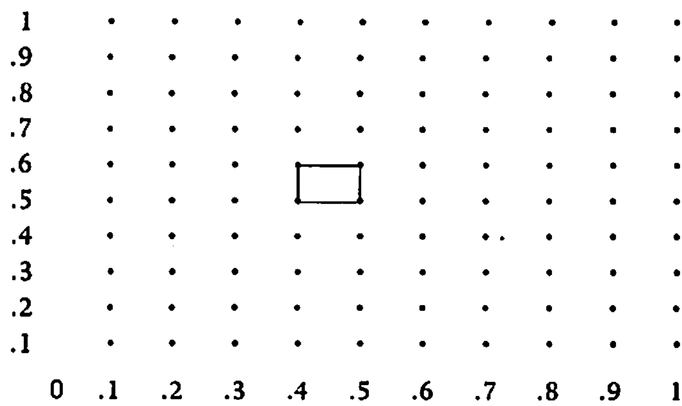

Figure 2

It has already been shown that any sequence converging in I to any point of this type will also converge in I^2 and the limit will correspond to the limit of the original sequence in I by the Cantor mapping. Since the mapping is functional we have satisfied all the requirements for the local continuity of the Cantor mapping at these points.

Thus there are precisely $\aleph_0$ discontinuities in the Cantor mapping in this direction and they occur at the terminating rationals. (An interesting question now arrises as to whether this is a necessary or merely a sufficient condition for any mapping between any two topologically different sets, each containing $\aleph_1$ points.)

When the specific points on the line segment corresponding to the tenths, hundredths, thousandths, etc. are mapped, the square is subdivided into continually smaller rectangles. This shows the mapping to be fairly well ordered since all the points say between .45 and .46 will be in the small outlined square. In other words, points close on the line do tend to be close on the square.

The second statement can now be analyzed, which says that by this procedure a one-to-one correspondence can be established between the points on a square and the points on a line segment. This is an important statement because it is precisely what allowed Cantor to state there were $\aleph_1$ points in the square.

However, as shown earlier, there are inconsistencies in the mapping which lead me to believe this statement is in error. More precisely, this procedure does not create a one-to-one correspondence between the points on a square and the points in a line segment because it fails to be surjective. For instance, consider the point $.129\overline{09}$ in I. What point in I^2 is mapped to this point? In mapping to I^2 this point maps to the point $(.1\overline{9}, .2)$, or just the point $(.2,.2)$, in I^2. But, when mapping back to I, the point $(.2,.2)$, or $(.1\overline{9}, .1\overline{9})$, maps to the point $.11\overline{9}$ and can map to no other. Then what point from I_2 does map to $.129\overline{09}$. The answer is there is none.

Consider the problem from another point of view. Suppose we are given the point $(.325,.62)$ and we wish to map it to I. Since both coordinates are terminating decimals they can each take two different forms:

$$.325 \quad \text{or} \quad .324\overline{9}$$
$$.62 \quad \text{or} \quad .61\overline{9}$$

This results in four separate numbers: $.36225$, $.362159\overline{09}$, $.36224\overline{09}$, and $.36214\overline{9}$. Only the last is allowed by the selection rules. What maps to the other three numbers? The first, rewritten as $.36224\bar{9}$, is mapped to by the point $(.324\bar{9}, .62\bar{9})$, or $(.325, .63)$. However, the middle two remain unmapped. Thus each pair of rational points in I_2 "creates" two unpaired rational numbers, and therefore two holes in the line segment. Since there are $\aleph_0$ pairs of rational numbers in I^2 there must be $\aleph_0$ holes in I created by them. In fact, by considering mixed pairs of terminating rationals and irrationals together, where one unpaired irrational is created, it can be shown that there are actually $\aleph_1$ holes in the line segment. This would tend to make the square smaller in its number of points than the line segment. Obviously this can't be true, but it does show the mapping from I^2 to I is not surjective. Also, in the example above, the last three numbers will all map to the same point in the square, $(.325, .62)$, so the mapping from I to I_2 fails to be injective. Thus a bijection, or one-to-one correspondence, cannot exist in either direction.

Now we must consider whether the count will change. Clearly it cannot. The line segment has no more than $\aleph_1$ points and every point in the square does map to one distinct point in the line, so the square must have either $\aleph_0$ or $\aleph_1$ points. Since line segments are contained in the square it must have $\aleph_1$ points.

This article has attempted to map certain sets of points in one dimension to sets of points in a second dimension. The points themselves are uniquely determined by their geometric definition. However, these points are represented by decimal expansions which may not be uniquely represented. No problem is encountered when dealing with irrationals or repeating decimals, but selection rule two does not really solve all the problems with rationals which can be represented by terminating decimals. Inconsistencies, such as the examples given still arise and turn out to be quite important when applied to the two statements listed earlier.

Euclid Today*

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The topic of this paper is Euclid; not the mathematician, but an intriguing game based upon the Euclidean algorithm from which its name is derived. The beauty of this game is that it can be enjoyed by almost anyone. To be sure, the average person is only interested in how it is played, caring little about the strategy and theory involved, which so interests the mathematician and others of inquisitive natures. Euclid offers much for everyone.

To begin with, let us first review Euclid's algorithm. This algorithm suggests a procedure for computing the greatest common divisor (g.c.d.) of two given integers. It states that given non-zero p and q with $p > q$ (let us consider p and q to be positive in this discussion, since the g.c.d. is always positive regardless of the sign of the integers in question and the use of negative integers will change only the signs, not the numerical values.):

(1) Divide p by q, yielding remainder, r_0, $p = a_0 q + r_0$; $0 \leqq r_0 < q$,

(2) Divide q by r_0, yielding remainder, r_1, where $0 \leqq r_1 < r_0$, $q = a_1 r_0 + r_1$,

(3) Divide r_0 by r_1 which yields the remainder r_2; $r_0 = a_2 r_1 + r_2$; $0 \leqq r_2 < r_1$.

All r_i are obtained in the same fashion by dividing the preceding divisor by the remainder, yielding non-negative integers; decreasing step by step, so that finally at some r_i, $r_i = 0$; $r_k = a_n r_m + r_n$ and $r_m = a_p r_n + 0$. The last positive remainder, r_n, can be readily proven to be the g.c.d. of p and q. This result is written as $(p, q) = r_n$ [3, p. 105].

Now that we have a general background, we may proceed to Euclid. The game itself is quite simple. The players are given a set of two numbers, (p, q), and each player may only move once per turn. A move consists of replacing the larger of the two numbers, p, by any non-negative number obtained by subtracting a

*A paper presented at the 1975 National Convention of KME.

positive multiple of the smaller from the larger, writing the larger number first. The game thus continues until a player obtains zero for the new smaller number. This player is then declared winner. [1]. Let us examine a particular game where A denotes the player to move first; B the second player. Given $p = 72$, $q = 41$, there are at least two possible combinations of moves, each resulting in a different outcome:

	(72, 41)	or		(72, 41)	
A	(41, 31)		A	(41, 31)	
B	(31, 10)		B	(31, 10)	
A	(11, 10)		A	(10, 1)	
B	(10, 1)		B	(1, 0)	B wins
A	(1, 0)	A wins			

This naturally leads to the questions "Is there a way to force a win? Given a choice of moves, which will be my best?" Before answering these questions, it is necessary to first take a closer look at the game itself. After examining the structure of Euclid, we find the following to be true [1, p. 355]:

(1) There are two players who alternatively move;

(2) The game will terminate in a finite number of moves, always resulting in a win for one of the players;

(3) Each move consists of a choice from the set of permissible moves;

(4) Both players know the current state of the game and what moves are allowed.

Because of the aforementioned characteristics of Euclid, we may be assured that a winning strategy exists [5, chapter 2]. This strategy as we shall see, can best be summarized by stating that A has a winning strategy if and only if $q < \frac{1}{2}(\sqrt{5} - 1)p$ for a given p and q.

Before proving this theorem, let us pause to make the following statements and definitions. Let $c = \frac{1}{2}(\sqrt{5} - 1)$, which is approximately equal to 0.618. This is commonly called the Golden Section. Let (p, q), where p is greater than q, be a pair occurring in the game of Euclid. Let us define (p, q) as a safe position if q/p is more than c. Otherwise (p, q) is an unsafe position.

Proposition 1. A player moving from an unsafe position is always capable of moving to a safe position.

Proposition 2. A player moving from a safe position can make just one move and that move will always be to an unsafe position.

From the above we see that a player who once is given an unsafe position from which to move can ensure that in all future moves he will be able to move to a safe position, thus always forcing the other player into an unsafe position. It is also clear that at some point in the game, since p steadily decreases and p is not equal to q at the beginning of the game, one player must necessarily pass through the unsafe position (kq, q), where k is a positive integer greater than one. The object is to force your opponent into this position so that on your next turn you can make the winning move. Now having summarized the strategy, it still remains to be proven [4].

The proofs are as follows [1]:

LEMMA 1. *Given that* p *and* q *are positive integers with* $\frac{1}{2}p < q < p$, *either* $q < cp$, *which implies* $p - q > cq$; *or* $q > cp$, *which implies* $p - q < cq$.

Proof: Suppose that $q < cp = \frac{1}{2}(\sqrt{5} - 1)p$. This implies that $\frac{1}{2}(\sqrt{5} + 1)q < p$ and $p - q > \frac{1}{2}(\sqrt{5} + 1)q - q$. Therefore $p - q > \frac{1}{2}(\sqrt{5} - 1)q$.

Now suppose that $q > \frac{1}{2}(\sqrt{5} - 1)p$. This implies that $\frac{1}{2}(\sqrt{5} + 1)q > p$ and $p - q < \frac{1}{2}(\sqrt{5} + 1)q - q$. Therefore $p - q < \frac{1}{2}(\sqrt{5} - 1)q$. Thus Lemma 1 is proven.

LEMMA 2. *Let* $2q < p$ *and* p/q *is not an integer. Let* n *denote the largest integer such that* $nq < p$. *This being established, either* $p - nq > \frac{1}{2}(\sqrt{5} - 1)q$ *or* $q > \frac{1}{2}(\sqrt{5} - 1) \times (p - (n - 1)q)$.

Proof: Suppose that $q < \frac{1}{2}(\sqrt{5} - 1) \times (p - (n - 1)q)$. Then $\frac{1}{2}(\sqrt{5} - 1)q < p - (n - 1)q$, which implies that $\frac{1}{2}(\sqrt{5} - 1)q < p - nq$. Thus Lemma 2 is proven.

THEOREM. *Let* $p > q$; *there is a winning combination for* A *if and only if* $q < \frac{1}{2}(\sqrt{5} - 1)p$.

Proof: The proof is by induction. For $p = 2$, the result is trivial, since the only possible value for q is one. For $p = 3$, there are two possible games, (3, 1) and (3, 2). The first game proves to be a winning one for A; the second for B.

Now suppose the theorem is true for all $p < P$ and all the corresponding permissible values of q. Let us consider the case (P, Q) where $0 < Q < P$.

If $P = nQ$ for some integer n more than zero, A will win in one move.

Suppose that P is not equal to nQ. This implies if $\frac{1}{2}P < Q < P$, A has only one move which is to reduce (P, Q) to $(Q, P - Q)$. Then by the inductive assumption and Lemma 1, B will win if and only if $Q > \frac{1}{2}(\sqrt{5} - 1)P$. Therefore A wins if and only if $Q < \frac{1}{2}(\sqrt{5} - 1)P$.

Now suppose that $0 < Q < \frac{1}{2}P$, and n is the largest integer such that $nQ < P$. Now A can choose either $(P - (n - 1)Q, Q)$ or $(Q, P - nQ)$. By the inductive assumption and Lemma 2, one of these moves is a losing move for B.

COROLLARY. *If* q $< \frac{1}{2}(\sqrt{5} - 1)$p, *then a winning strategy for* A *consists of always making the move as described above so that the resulting* (p′, q′) *satisfies the condition* q′ $>$p′ $\times \frac{1}{2}(\sqrt{5} - 1)$. *If* q $> \frac{1}{2}(\sqrt{5} - 1)$p, *then* A *has only one move, which will allow* B *to win.*

These proofs make it quite clear that in Euclid, there are only 2 types of positions of (p, q) at which the player has a choice of moves. They are when:

(1) p/q is an integer more than or equal to two;

(2) $p > 2q$ and p/q is not an integer.

Now that the strategy has been outlined and there exists a winning strategy for one player at any given place during the game, the question may arise concerning which player has the best chance of having a winning strategy available. In other words, what is the probability of an arbitrarily chosen position at the beginning being unsafe? Let us assume that the following suggests an "arbitrarily chosen starting position". Both numbers are to be chosen from a set, Z, of positive integers with a limiting value of n, where

n is the largest integer belonging to set Z. The first number chosen can be considered as drawn with uniform probability from Z; and the second number is chosen with uniform probability from $Z - \{x\}$, where x stands for the first number chosen. After the two numbers are chosen, they are labeled p and q, with p being the larger of the two.

By using standard probability arguments, it can be shown that q can be considered as chosen with uniform probability from the set $\{1, 2, 3, \cdots, p - 1\}$. Given p, we know that $P(q/p < c) = P(q < cp) = [cp]/(p - 1)$ where $[cp]$ represents the fractional part [4]. So for a large p, this fraction approaches c, and thus for a large n, the probability of an arbitrarily chosen starting position being unsafe approaches c. All of this implies that the first player to move, in this case, A, has the better chance of having a winning strategy available.

Although the bulk of this paper has emphasized the winning strategy and the probability theories behind it, Euclid has many other fascinating relations to other phenomena in mathematics, among which is its resemblance to the Fibonacci sequence. This is not surprising as the relationship between the Fibonacci sequence, Euclid's algorithm, and the Golden Section is well known [2, pp. 80-82]. In conclusion Euclid is a perfect example of a mathematical recreation which proves that learning a new mathematical concept need not be drudgery, but can offer a leisure activity.

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The Leontief Input-Output Matrices*

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The Leontief input-output model was developed by Wassily Leontief to study production and flow of goods in a system. The model is useful as a guide in solving practical economic problems. Matrices and inversion of matrices are used in the model. The method of inversion by the use of a power series will be a major consideration of this article.

When all aspects of an economy are considered in the use of the Leontief model, the model can become quite complex. Therefore, we will not consider all aspects, instead we will use a very simplified model employing just the essentials in this paper to understand some of the principles of the Leontief model.

There are many goods and processes in our economic system. The demand for these goods is a great concern to our industries. It influences how much of a good or product an industry should make, or in other words at what intensity the industrial process should be operating to meet the demand. In order for an industry to produce output of a good it must have input of that same good or other goods. This input-output relationship can be represented in matrix form.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ a_{n1} & a_{n2} & \cdot & \cdot & \cdot & a_{nn} \end{bmatrix}$$

The matrix A is called the technology input-output matrix for the system, or simply the technology of the system [2].

Each coefficient a_{ij} is the amount of the i^{th} good used to produce a unit of j^{th} product. It is assumed that $a_{ij} \geqq 0$, and at least one $a_{ij} > 0$ exists in each column of the matrix. This assumption can be made since we can not produce something from nothing.

*A paper presented at the 1975 National Convention of KME.

We see then that the j^{th} column indicates the amounts of various goods needed to produce one unit of good, G_j. The i^{th} row indicates how the good G_i is distributed in each process.

By our definition matrix A represents what goes into one unit of a good. We, however, are concerned in our economy with producing many units, x_j, of a good. This number x_j is called the *intensity* of the process. We make the assumption that the ratio of input to output is a constant, so that if we want 100 units of a good we would use 100 times more input than we would for one unit of the good. With the assumption of a fixed ratio of input to output we find that the sum

$$u_i = \sum_{j=1}^{n} a_{ij}x_j$$

gives the amount of G_i that is used for production when the system is operating to produce x_j units of G_j for $j = 1, 2, 3, \cdots, n$ [2]. These sums can be represented in the form

$$U = \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{bmatrix}$$

where U indicates the amounts used in the system when operating at a certain intensity.

Two other vectors

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} d_1 \\ d_2 \\ \cdot \\ \cdot \\ \cdot \\ d_n \end{bmatrix}$$

must be defined in order to proceed. The vector X represents the intensity or the gross outputs, and the vector D represents the demand or the amount of each good available for finel use.

From the above definitions we have the equation

$$D' = X - U$$

which can also be written as

$$X = U + D$$

or substituting AX for U

$$X = AX + D.$$

With the above equation we can now find at what intensity an industry should operate at in order to fulfill a known demand D, where $D \geqq 0$, by solving for X,

$$X - AX = D$$

$$(I - A)X = D.$$

If the inverse of $I - A$ exists, then multiplying both sides by $(I - A)^{-1}$ we get

$$X = (I - A)^{-1}D.$$

DEFINITION: *The matrix* $(I - A)^{-1}$ *is called a Leontief inverse if* $(I - A)^{-1}$ *exists and* $(I - A)^{-1} \geqq 0$. *Matrix* A *is then defined to be a Leontief matrix.*

It is convenient to have ways of recognizing if a matrix is a Leontief matrix. The above definition provides one method of doing this.

The *norm* of a matrix is found by summing the columns. The largest sum is considered the norm of the matrix. The notation used for the norm is $N(A)$, which is read the norm of matrix A.

A monetary unit or a physical unit can be used in applying the Leontief model. In using a monetary unit we speak of a dollar's worth of a good, and then can assume that $N(A) < 1$. This assumption can be made since any column j gives the amounts of other goods (in monetary units) needed to produce one dollar's worth of the good G_j. If the result when summing any column is greater than or equal to 1, the good G_j is being produced at a loss for the industrial process, which is not realistic since making a profit is the purpose of any industry or process.

Finding X or the intensity an industry should be operating at is equivalent to showing matrix A is a Leontief matrix. Matrix inversion is involved in this process. Matrix inversion can be quite a problem when a large matrix is necessary to describe the input-output relationship, and large errors often result during the inverting of matrices by a direct method such as the Gauss-Doolittle process.

A method for approximating the inverse of $I - A$ to a pre-assigned degree of accuracy involves the use of the power series [3]:

$$I + A + A^2 + \cdots + A^n + \cdots .$$

We can use this power series to approximate the inverse since

$$(I - A)(I + A + A^2 + \cdots + A^n)$$
$$= I - A^{n+1}$$

and

$$I - A^{n+1} \to I$$

This conclusion is arrived at from the theorem that follows.

THEOREM: *If* $N(A) < 1$, *then* $a_{ij}^k \leq [N(A)]^k$ *for all* k *and* $\lim_{k\to\infty} A^k = 0$.

Proof (by induction): For $k = 1$ we can assume $a_{ij} \leq N(A)$ since no element of a matrix can be larger than the norm of the matrix. We assume the statement is true for $k = n$ so that

$$a_{ij}^n \leq [N(A)]^n.$$

We now show that it is true for $k = n + 1$.

$$a_{ij}^{n+1} = \sum_{k=1}^{n} a_{ik}^n a_{kj}$$

substituting

$$\begin{aligned} a_{ij}^{n+1} &\leq [N(A)]^n \ a_{kj} \\ &\leq [N(A)]^n \cdot N(A) \\ &= [N(A)]^{n+1} \end{aligned}$$

We can conclude from the theorem that since $a_{ij} \leq N(A)$, then $a_{ij} < 1$. It follows from this that each successive term is smaller than the preceding one so that

$$a_{ij}^{k+1} < a_{ij}^k$$

and

$$a_{ij}^k \to 0 \text{ as } k \to \infty .$$

Since every term a_{ij}^k of the matrix A approaches 0 it follows that $A^k \to 0$ as $k \to +\infty$.

Going back then we have

$$(I - A)(I + A + A^2 + \cdots + A^n) = I - A^{n+1} \to I$$

and the power series

$$(I + A + A^2 + \cdots + A^n)$$

can be used to approximate the inverse of $(I - A)$.

From our conclusions we observe that we have the following

THEOREM: *If* $A \geqq 0$ *and* $N(A) < 1$, *then* A *is Leontief.*

This method of approximating the inverse of a Leontief matrix by a power series is of real advantage since a computer can do the necessary multiplications in a matter of minutes. This method though cannot always be employed because the power series does not converge for all matrices.

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Concluded from page 68

composed of faculty and students, will be awarded cash prizes of \$60, \$40, \$30, and \$20 respectively. If enough papers are presented by graduate students, then one or more prizes will be awarded to this group. Prize winning papers will be published in *The Pentagon*, after any necessary editing. All other submitted papers will be considered for publication, at the discretion of the Editor.

Geometry of Extrema

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Sometimes geometric observations simplify problems of maxima and minima. In fact, whenever possible, one should take a look at some of these problems from the point view of geometry before using calculus. In what follows we shall study a few examples.

1. *Some basic ideas:* We shall accept a few well-known propositions of algebra and geometry, such as *the triangle inequality.* We may use some of these propositions without proof. All objects will be in a Euclidean plane.

2. *Proposition:* Let a, b, and x stand for the lengths of line segments such that $a > b > x$, where x is variable. Then $\sqrt{a^2 - x^2} - \sqrt{b^2 - x^2}$ is minimum at $x = 0$.

Proof: Construct a right triangle ABC, where B is the vertex of the right angle, $AB = x$, and $AC =$ b (Figure 1). We obtain D on the line BC such that $AD = a$ and C is between B and D. In the triangle ACD we may write $DC \geqq a - b$. But $DC = \sqrt{a^2 - x^2} - \sqrt{b^2 - x^2}$ which implies $\sqrt{a^2 - x^2} - \sqrt{b^2 - x^2} \geqq a - b$, which proves the proposition.

The reader may supply an algebraic proof or try the problem with techniques of derivatives.

2. *Problems:* We shall suggest a few problems for the reader.

(i) Let (C) be a circle of center C and radius r. Let P be a point inside the circle. A line through P intersects the circle at A and B. Obtain positions of AB for which AB is maximum or minimum.

(ii) Consider the hypotheses of (i). Obtain AB when $(PA)^2 + (PB)^2$ is maximum or minimum.

The reader may use techniques of derivatives and compare them with a geometric approach.

(iii) Consider the hypotheses of (i). Obtain maximum and minimum of PA.

(iv) Move P outside (C) and ask the same questions as in (i), (ii), and (iii).

3. *A maximin problem*: Let (C) be a circle of center C and radius r and let d be a line which does not intersect (C). Consider the line segment AB, where A is on (C) and B on d. Obtain the maximum of all the minimums of AB as B slides on d and AB rotates about C.

Solution: Let CK be perpendicular to d at K (Figure 2). We draw CH perpendicular to AB. Let $CH = x$ and $KB = y$. Then the problem is to obtain $\text{Max}_x[\text{min}_y(AB)]$.

Let AB intersect (C) in D such that D is between A and B. Let $CK = a$ then

$$AH = \sqrt{r^2 - x^2},\ HB = \sqrt{(CB)^2 - x^2},\ (CB)^2 = a^2 + y^2.$$

Thus

$$AB = \sqrt{r^2 - x^2} + \sqrt{a^2 + y^2 - x^2}\ .$$

Since $\sqrt{a^2 + y^2 - x^2} \geqq \sqrt{a^2 - x^2}$ it follows that

$$\text{Min}_y(AB) = \sqrt{r^2 - x^2} + \sqrt{a^2 - x^2}\ .$$

Therefore

$$\text{Max}_x[\text{Min}_y(AB)] = r + a.$$

4. *Problems:* Here are a few variations on §3.

(i) Obtain the minimum of AB.

(ii) Show that AB has no maximum.

(iii) Study $\text{min}_y[\text{max}_x\ AB]$.

(iv) Study the case that d intersects (C).

5. *A Problem of Ibn Haytham:* Let d be the line of the stream, A the point at which a cowboy is standing, and B be his home. He wants to go to the stream, water his horse and then go home. Where should he water the horse at the river in order to take the shortest distance? Usually the problem is solved by the use of symmetry with respect to d. We shall leave the solution to the reader [1, p. 26].

6. *A generalization:* Let p be the edge of a pasture, d the line of a stream, A where the cowboy is, and B his home. Again the

minimum path is desired in order to get to the edge of the pasture, water the horse, and go home.

Are there other generalizations? In §5 and §6 discuss all cases.

7. *A maximum angle:* Let d be a straight line and A, B be two points on one side of d (Figure 3). Let P be a variable point on d. Obtain P such that the angle APB is maximum.

Solution: Consider only the angles whose vertices are on one side of the line AB. Let (C) be a circle through A, B, and tangent to d. Suppose M is the point of tangency. Then $\angle AMB = \text{Max}_p(\angle APB)$. To prove this one may consider $P \neq M$. The line AP intersects (C) at K. Clearly $\angle AMB = \angle AKB > \angle APB$.

8. *Problems:* We suggest some variations on §7.

(i) Usually in §7 there are two tangent circles. Study the other one. Learn the construction of these circles. See [1, p. 39].

(ii) In §7 replace the line by a circle.

(iii) Study all cases in (i) and (ii); discuss all possibilities.

There are many interesting extremum problems which can be treated geometrically. The reader may ask all sorts of questions and answer them.

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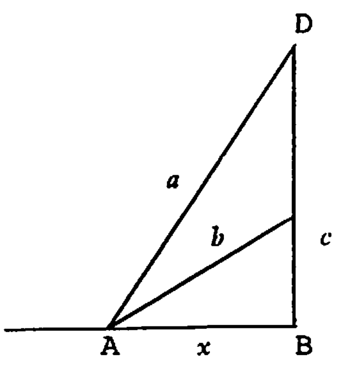

Figure 1

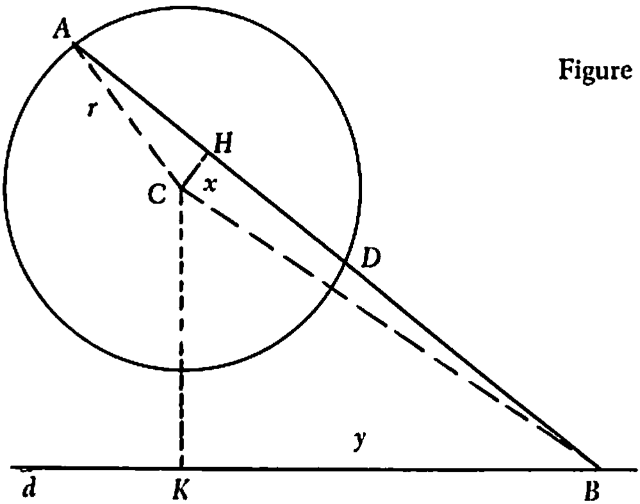

Figure 2

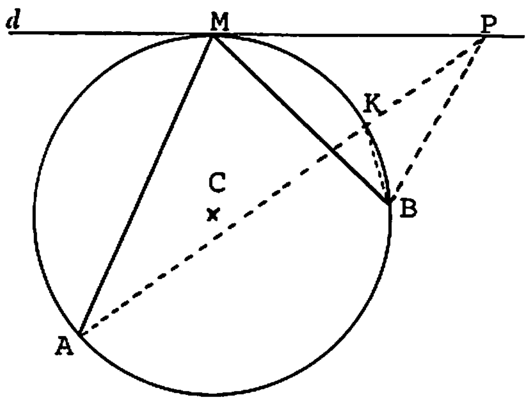

Figure 3

On Connected Subsets of the Real Line

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One of the most elementary facts about the concept of connectedness in Topology is that the real line is connected. It follows from this that any interval, bounded or unbounded, open, closed, or half-open and half-closed, is also connected. Conversely, any connected set in the reals must be an interval.

These facts are usually proved as soon as one considers the idea of connectedness. The proof that connected sets are intervals is particularly easy. The purpose of this note is to give a different proof of the connectedness of the reals. This proof is an easy consequence of a well-known theorem on open sets of real numbers which is usually proved quite early in most works on either Real Analysis or Topology. For completeness, it will also be shown how the connectedness of the reals implies the connectedness of any interval.

We now state the theorem which will be used to prove the main result.

Theorem 1: *Each non-empty open set of real numbers can be written as a (countable) union of disjoint open intervals.* (See [1], p. 62.)

Theorem 2: *The real line is connected.*

Proof: Assume the line is not connected, i.e. $R^1 = G \cup H$, where G and H are non-empty, disjoint, open sets. By Theorem 1, G and H are unions of disjoint open intervals. (The countability will not be needed here.) Let (a,b) be one of these intervals. Then $a \notin G$ and $a \notin H$, since all of the intervals are open and disjoint. Thus $a \notin G \cup H = R^1$, a contradiction.

Theorem 3: *Any interval is connected.*

Proof: Since any interval is a continuous image of the reals, and since connectedness is a continuous invariant, any interval is connected.

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1. Simmons, George F., *Introduction to Topology and Modern Analysis*. New York, McGraw-Hill: 1963.

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgement will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

When one thinks of the irrational number π, one normally thinks of its irrational value of 3.14159 26535 89793 ··· or one of its familiar rational approximations of $\frac{22}{7}$ or 3.1416. The value of π, however, can be represented in many rather surprising fashions, some of which follow.

One of the more widely known products involving π is:

$$\frac{\pi}{2} = \frac{2}{1} \times \frac{2}{3} \times \frac{4}{3} \times \frac{4}{5} \times \frac{6}{5} \times \frac{6}{7} \times \frac{8}{7} \times \frac{8}{9} \times \frac{10}{9} \times \frac{10}{11} \times \frac{12}{11} \times \cdots,$$

which was discovered by John Wallis (1616-1703). Others are:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17} - \frac{1}{19} + \cdots$$

which was developed by Gottfried Leibniz, and $\frac{\pi}{4} =$

$$4\left(\frac{1}{5} - \frac{1}{3 \cdot 5^3} + \frac{1}{5 \cdot 5^5} - \frac{1}{7 \cdot 5^7} + \frac{1}{9 \cdot 5^9} - \cdots\right) - \left(\frac{1}{239} - \frac{1}{3 \cdot 239^3} + \frac{1}{5 \cdot 239^5} - \frac{1}{7 \cdot 239^7} + \cdots\right)$$

which was discovered by John Machin (1680-1752). Still another expression was developed by Lord Brouncker (1620-1684) as:

$$\pi = \cfrac{4}{1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cfrac{9^2}{\cdots}}}}}}$$

The transcendental number $e = 2.71828182845904523 \cdots$ is usually thought of as the base of the natural logarithms, as the value of the infinite series:

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots, \text{ or as the}$$

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

Euler developed the symbol e and calculated its value to 23 decimal places. He also discovered that e could also be represented in the following two forms:

$$e = 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{2}{3 + \cfrac{3}{4 + \cfrac{4}{5 + \cfrac{5}{6 + \cfrac{6}{\cdots}}}}}}},$$

and

$$\sqrt{e} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{\cdots}}}}}}}}}$$

– △ – △ –

Two interesting limit problems are the following. First, draw a circle with a radius equal to one. Now inscribe an equilateral triangle inside the circle. In the equilateral triangle inscribe another circle, and then inscribe in this second circle a square. Continuing this process, now inscribe a circle in the square. Then inscribe a regular pentagon in the circle. Repeat this process, alternating the inscribing of circles and regular polygons, always increasing the number of sides of the regular polygon by one each time.

Upon first consideration, one would probably conclude that the radii of the inscribed circles approach zero as a limit. But upon inspecting the actual drawing, as in Figure 1, one notes that the radii appear to be approaching some non-zero constant which can be shown to be:

$$R = \text{Cos}\,\frac{\pi}{3} \times \text{Cos}\,\frac{\pi}{4} \times \text{Cos}\,\frac{\pi}{5} \times \cdots \times \text{Cos}\,\frac{\pi}{n-2},$$

which is about 1/12th the original radius.

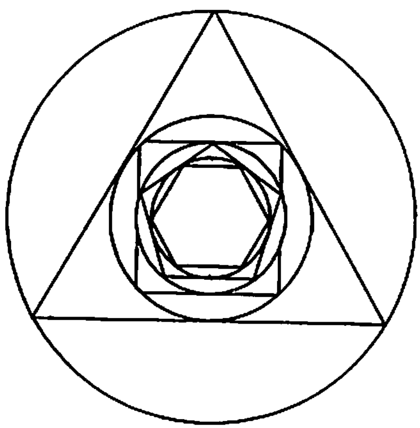

Figure 1

One would also note that the inscribing process itself is approaching a limit as the circles and inscribed polygons become approximately equal.

Another closely related problem is the one shown in Figure 2 below.

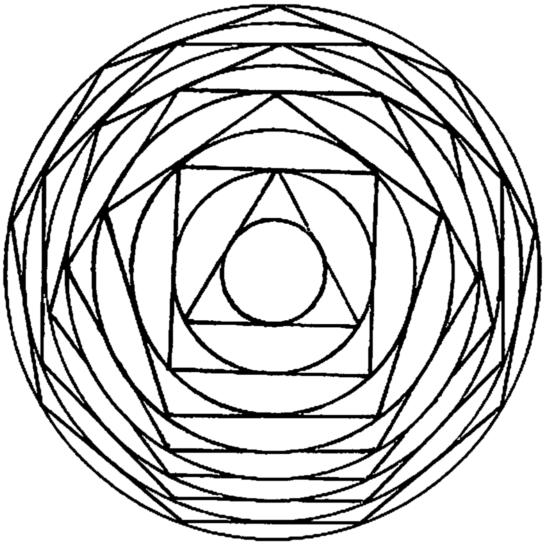

Figure 2

Here we alternately circumscribe the regular polygons and circles about the starting circle of unit radius. It would appear that in this case our radii of the circumscribed circle will increase without bound. But upon carefully inspecting our figure, we discover that the radii approach a finite number S whereby

$$S = \frac{1}{\operatorname{Cos}\frac{\pi}{3} \times \operatorname{Cos}\frac{\pi}{4} \times \operatorname{Cos}\frac{\pi}{5} \times \operatorname{Cos}\frac{\pi}{6} \times \cdots \times \operatorname{Cos}\frac{\pi}{n-2}}$$

where S is approximately 12 times the original unit radius. One will also note that R and S are reciprocals.

– △ – △ –

Some cube roots can be found to be equal to the sum of its digits. Five such cube roots are:

$$\sqrt[3]{512} = 5 + 1 + 2 = 8$$
$$\sqrt[3]{4913} = 4 + 9 + 1 + 3 = 17$$
$$\sqrt[3]{5832} = 5 + 8 + 3 + 2 = 18$$
$$\sqrt[3]{17576} = 1 + 7 + 5 + 7 + 6 = 26$$
$$\sqrt[3]{19683} = 1 + 9 + 6 + 8 + 3 = 27.$$

Still some other cube roots are equal to the reversal of the sum of their digits. Five such cases are:

CUBE ROOT	SUM OF DIGITS
$\sqrt[3]{148\ 877} = 53$	$1 + 4 + 8 + 8 + 7 + 7 = 35$
$\sqrt[3]{238\ 328} = 62$	$2 + 3 + 8 + 3 + 2 + 8 = 26$
$\sqrt[3]{373\ 248} = 72$	$3 + 7 + 3 + 2 + 4 + 8 = 27$
$\sqrt[3]{531\ 441} = 81$	$5 + 3 + 1 + 4 + 4 + 1 = 18$
$\sqrt[3]{551\ 368} = 82.$	$5 + 5 + 1 + 3 + 6 + 8 = 28.$

Some cube roots can be evaluated by multiplying their first and last digits. Three such cases are:

$$\sqrt[3]{125} = 1 \times 5 = 5$$
$$\sqrt[3]{3375} = 3 \times 5 = 15$$
$$\sqrt[3]{91\ 125} = 9 \times 5 = 45$$

– △ – △ –

Suppose now we have four ones. What is the largest possible number obtainable using these four ones. At first glance, one would probably say 1111. But considering the possible addition combinations we get

$$1 + 1 + 1 + 1 = 4$$
$$11 + 11 = 22$$
$$111 + 1 = 112$$
$$1111 = 1111,$$

the largest of which is 1111. Now consider the products

$$1 \cdot 1 \cdot 1 \cdot 1 = 1 = 1$$
$$11 \cdot 1 \cdot 1 = 11$$
$$11 \cdot 11 = 121$$
$$111 \cdot 1 = 111,$$

the largest of which is 121. Considering powers we obtain

$$1^{111} = 1$$
$$11^{11} = 285{,}311{,}670{,}611$$
$$111^{1} = 111.$$

Hence the largest combination using four ones yields

$$285{,}311{,}670{,}611.$$

Now consider the largest possible number obtainable using only three twos. We get for addition:

$$222 = 222$$
$$22 + 2 = 24$$
$$2 + 2 + 2 = 6$$

for multiplication:

$$22 \cdot 2 = 44$$
$$2 \cdot 2 \cdot 2 = 8$$

and for powers:

$$22^{2} = 484$$
$$2^{22} = 4{,}194{,}304$$
$$2^{2^{2}} = 2^{4} = 16.$$

Hence the largest combination yields 4,194,304.

Can you obtain the largest combination using only three three's? Three fours? Three nines?

– △ – △ –

Palindromes are defined to be words, sentences, numbers, etc., which read the same forwards as backwards. An example of such a sentence is "ABLE WAS I ERE I SAW ELBA." A numerical example is 424.

Many palindromes may be found by adding a two-digit number to its reversal. For example,

$$16 + 61 = 77$$
$$24 + 42 = 66$$
$$35 + 53 = 88$$
$$38 + 83 = 121$$
$$506 + 605 = 1111$$

There exists 99 positive integer palindromes less than 1000. Of these 99 palindromes, 16 are primes. They are:

11, 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, 197, 919, 929.

Several palindromes are perfect squares. Some are:

$11^2 = 121$	$111^2 = 12321$
$22^2 = 484$	$121^2 = 14641$
$26^2 = 676$	$202^2 = 40804$
$101^2 = 10201$	$212^2 = 44944$

Some are the differences of two squares. For example:

$$86^2 - 68^2 = 2772$$
$$83^2 - 38^2 = 5445$$
$$80^2 - 08^2 = 6336$$

Other palindromes are cubes as are $343 = 7^3$, $1331 = 11^3$, and $1030301 = 101^3$. A fourth power palindrome is $14641 = 11^4$. One would also note that 11, 121, 1331, and 14641 are all palindromes and that

$$11 = 11^1$$
$$121 = 11^2$$
$$1331 = 11^3$$
$$14641 = 11^4$$

$161051 = 11^5$ but 161051 is not a palindrome.

The following are palindromes as a whole:

$$3 \times 51 = 153$$
$$6 \times 21 = 126$$
$$8 \times 86 = 688$$

The smallest three-digit palindrome is 101, a prime number. Can you determine the largest power of 101 which yields a palindrome power?

The Problem Corner

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 February 1977. The best solutions submitted by students will be published in the Spring 1977 issue of The Pentagon, with credit being for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

282. *Proposed by the editor.*

A farmer has a circular plot of radius 50 feet. At a point on the circumference of the plot he places a stake to which a goat is connected by a rope. How long is the rope if the goat can graze on exactly one-half of the area of the plot?

283. *Proposed by the editor.*

On Professor Knowitall's College Algebra exam, the following question appeared: Which is larger $\sqrt[6]{4}$ or $\sqrt[7]{5}$? Find the solution without using tables. Young Percival Whizkid solved the problem easily. How did he do it?

284. *Proposed by the editor.*

Given two equal sides of an isosceles triangle, what is the length of the third side which produces the maximum area?

285. *Proposed by Randall J. Covill, Indian Hill, West Newbury, Massachusetts.*

If O is an octagonal number $= n(3n - 2)$ and P is a pentagonal number $= m(3m - 1)/2$ and $m = n =$ a positive integer, then P and O are the complements of each other. It can be easily shown by algebraic manipulation of the formulas for P and O that to every difference between an octagonal number and its complementary pentagonal number there corresponds a multiple of 3 that is a unique positive integer. Show that for at least one multiple of 3 that is a positive integer there is not any corresponding difference between an octagonal number and its complementary pentagonal number.

286. *Proposed by the editor.*

In Alcatraz Prison an eccentric jailer decided to effect a "selective release" of the prisoners. The cells are numbered consecutively beginning with the number 1. First he unlocked all the cells. Then after returning to the place of beginning, he turned the key in the lock of every second cell. Next he repeated the process by returning to the place of beginning and turning the key in the lock of every third cell. The jailer repeats this process and on the i^{th} trip he turns the key in every i^{th} cell after returning to the place of beginning at cell number 1. Assuming that Alcatraz has 200 cells and that no prisoner escapes during the process, how many prisoners are released and what cells did they occupy?

SOLUTIONS

272. *Proposed by Charles W. Trigg, San Diego, California.*

A partition of a positive integer n is a representation of n as a sum of positive integers,

$$n = a_1 + a_2 + a_3 + \cdots + a_k.$$

If the order of the parts is considered insignificant, the partition is called unordered. Thus, $5 = 3 + 2$ and $5 = 2 + 3$ are the same unordered partition of 5.

Let $P_k(n)$ be the number of partitions of n into k unordered parts. Show that $[P_2(2n)][P_2(2n + 1]$ is a perfect square.

Solution by the proposer.

$2n = a_1 + a_2$. Then a_1 can range from 1 up to and including n, with a_2 remaining greater than or equal to a_1, so $P_2(2n) = n$.

$2n + 1 = a_3 + a_4$. Then a_3 can range from 1 only up to and including n if a_4 is to remain greater than a_3, so $P_2(2n + 1) = n$.

Hence, $[P_2(2n)][P_2 + 1)] = n^2$.

273. *Proposed by Gary Schmidt, Washburn University, Topeka, Kansas.*

Show that $\frac{d^2y}{dx^2} = \frac{-\,d^2x/dy^2}{(dx/dy)^3}$ is an identity. Assume functions are real, single valued, continuous, and have continuous derivatives.

Solution by the proposer.

Let $\frac{dy}{dx} = p$. Then $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dp}{dx} = p\,\frac{dp}{dy}$

$$\text{Now } \frac{-\,(d^2x/dy^2)}{(dx/dy)^3} = \frac{-\,\frac{d}{dy}\left(\frac{dx}{dy}\right)}{(dx/dy)^3} = \frac{-\,\frac{d}{dy}\left(\frac{1}{p}\right)}{(1/p)^3} =$$

$$\frac{+\left(\frac{1}{p}\right)^2\frac{dp}{dy}}{(1/p)^3} = +\,p\,\frac{dp}{dy} = +\,\frac{dp}{dx} = \frac{d^2y}{dx^2}$$

274. *Proposed by R. S. Luthar, University of Wisconsin, Janesville, Wisconsin.*

Show that the greatest integer contained in $(2 + \sqrt{2}\,)^n$ is odd, where n is any positive integer.

Solution by the proposer.

Let $[(2 + \sqrt{2}\,)^n] = i$ for some integer i. Then $(2 + \sqrt{2}\,)^n = i + q$ where $0 < q < 1$. Now since $0 < (2 - \sqrt{2}) < 1$ then $0 < (2 - \sqrt{2})^n < 1$ for any positive integer n. Let $(2 - \sqrt{2})^n = j$. Then $(2 + \sqrt{2})^n + (2 - \sqrt{2})^n = i + q + j = 2k$ where k is an integer. Thus $i + q + j$ is an even integer. Then since $0 < q < 1$ and $0 < j < 1$, we must have $q + j = 1$. Hence $i = [(2 + \sqrt{2})^n] = 2k - 1$, an odd intege:.

Editor's comment. To show that $(2 + \sqrt{2})^n + (2 - \sqrt{2})^n$ is an even integer, use the binomial theorem as follows:

$$(2 + \sqrt{2})^n = 2^n + \binom{n}{1} 2^{n-1}\sqrt{2} + \binom{n}{2} 2^{n-2}(\sqrt{2})^2$$

$$+\binom{n}{3} 2^{n-3}(\sqrt{2})^3 + \cdots = r\ (2 - \sqrt{2})^n =$$

$$2^n - \binom{n}{1} 2^{n-1}\sqrt{2} + \binom{n}{2} 2^{n-2}(\sqrt{2})^2 +$$

$$\binom{n}{3} 2^{n-3}(\sqrt{2})^3 + \cdots = s$$

Hence $(2 + \sqrt{2})^n + (2 - \sqrt{2})^n = 2 \cdot 2^n + 2 \cdot \binom{n}{2}$ $2^{n-2}(\sqrt{2})^2 + 2(\cdots) = 2k$ for some integer k, because all terms involving radicals either become integers or cancel out in the sum $(2 + \sqrt{2})^n + (2 - \sqrt{2})^n$.

275. *Proposed by the editor.*

In Professor Hy Potenuse's Geometry class he proved that any two triangles are similar if all their corresponding angles are equal and two triangles are congruent if they have two sides and their included angle respectively equal. One bright student observed to the professor's amazement, that two similar triangles can be drawn which are not congruent even though two sides of one triangle are equal to two sides of the second triangle. How did he do it and what relationship is necessary for this to occur?

Solution by the proposer.

Since the triangles are similar but not congruent, the angles included between the two sides common to both triangles must be different. Let a, b, c, d form two triangles having common sides a and b.

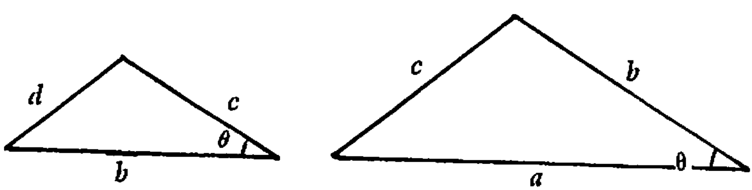

Figure 1

Then $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = r$ for some non-negative number r. Then $c = rd$, $b = cr = r^2d$ and $a = br = r^3d$. WLOG we can take $r > 1$. Then $a < b + c$ and $b < c + d$ (the triangle inequality) each imply $r^2d < rd + d$ or $r^2 - r - 1 < 0$ which implies $r < \frac{1 + \sqrt{5}}{2}$ Hence if d is the smallest side in either triangle, then $1 < r < \frac{1 + \sqrt{5}}{2}$ generates triangles which are similar but not congruent. Thus $d = 8$ and $r = 1.5$ generates triangles having sides (8, 12, 18) and (12, 18, 27) respectively. The geometric construction becomes elementary once the sides of both triangles are known.

276. *Proposed by the editor.*

A class of school children were to run an unusual race. In the school yard there were two flagpoles, one located 60 feet due south of the wall of the building and the other located 90 feet due southeast from the first pole. Each child starts at the first pole, runs to any point in the wall, makes a chalk mark on the wall, and then runs to the other pole. One child's time was much better than any other's. Assuming that all the children are equally fast, what path did the winner take?

Solution by Bob Sjoberg, Southern Methodist University, Dallas, Texas.

Distance $a = 63.6$ feet since pole #2 is 90 feet due southeast of pole # 1. Distance $b = 63.6$ feet since the triangle formed by pole #1, pole #2 and point Z is an isosceles right triangle with the right angle at point Z. Distance $c = 60$ feet $+ a$ $= 123.6$ feet.

Let x denote the distance from point Y at which the sum $d_1 + d_2$ is minimized where d_1 and d_2 denote portions of the path taken by the winner. By the Pythagorean theorem

$$d_1 = (x^2 + 3600)^{1/2} \text{ and } d_2 = [(63.6 - x)^2 + 123.6^2]^{1/2} = (19321.9 - 127.2x + x^2)^{1/2}.$$

Hence $d_1 + d_2$

$$= (x^2 + 3600)^{1/2} + (19321.9 - 127.2x + x^2)^{1/2}.$$

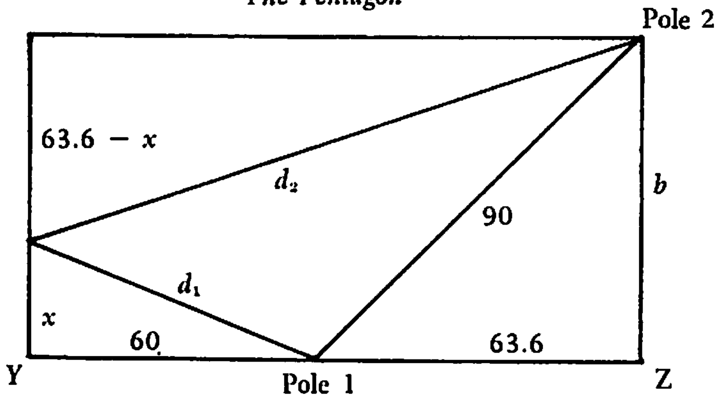

Figure 2

Differentiating with respect to x and setting the derivative equal to zero, we obtain

$$0 = \frac{d(d_1 + d_2)}{dx} = \frac{2x}{(x^2 + 3600)^{1/2}} + \frac{2x - 127.2}{(x^2 - 127.2x + 19321.9)^{1/2}}$$

which leads to the equation

$$0 = x(x^2 - 127.2x + 19321.9)^{1/2} + (x - 63.6)(x^2 + 3600)^{1/2}.$$

Simplifying we obtain $11676.9x^2 + 457920x - 14562000 = 0$ or $x^2 - 39.2x - 1247.0 = 0$ which has the positive root $x = 20.8$ so that $63.6 - x = 42.8$. Hence $d_1 + d_2 \doteq 194.3$ feet for the shortest possible path.

Editor's comment: Another approach involves realizing that the length of the path is the same as if the wall were a mirror and one could run directly from pole #2 to the "reflected image" of pole #1 (distance d) as shown in Figure 3 with the distances and angle θ as shown.

Then $\tan\theta = \dfrac{45\sqrt{2}}{2.60 + 45\sqrt{2}} = 9 - 6\sqrt{2} = .5148$ so

$\theta = 27°14'$ and $d = \dfrac{45\sqrt{2}}{\sin\theta} = 194.36$ feet.

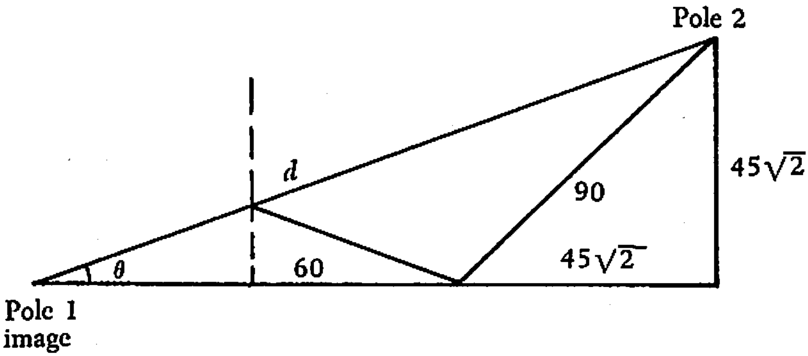

Figure 3

The Book Shelf

Edited by O. Oscar Beck

This department of The Pentagon brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, University of North Alabama, Florence, Alabama 35630.

Computational Methods in Linear Algebra, R. J. Goult, R. F. Hoskins, J. A. Milner, M. J. Pratt, Halsted Press, New York, 1975, 211 pp., $11.95.

In the authors' own words, "The book is designed for engineers and scientists who are engaged in work which requires the use of digital computers in problems involving matrices and sets of linear equations. The object is to provide a reasonably comprehensive survey of the numerical methods available, . . . "

This reviewer feels that they have accomplished their purpose admirably. The first chapter contains a fairly comprehensive look at the general eigenvalue problem with worked examples. The second chapter is on error analysis in general. Errors in the basic arithmetic operations as well as in the forming of matrix products are discussed, together with the ideas of the correspondence between these errors and equivalent initial perturbations of the data. The concepts of conditioning and backward analysis are explained, again with examples.

After these more or less general chapters, the two main problems for which the book was designed are attacked, namely, the solution of sets of linear equations and the finding of eigenvalues and eigenvectors of matrices. The third chapter discusses the solution of linear equations by elimination and decomposition methods and the fourth chapter the solution by iterative methods. This is followed by an excellent discussion of errors and their effects together with a section on iterative improvement of solutions.

The sixth chapter, on computation of eigenvalues and eigenvectors, considers both the symmetric and the non-symmetric problem as well as the generalized problem. The relative advantages as well as disadvantages of the methods given are pointed out. The final chapter concerns errors in eigenvalues and eigenvectors, including discussions of bounds for eigenvalues, perturbations of eigenvectors, condition numbers, and error bounds.

Except for the chapters on error analyses, there are good problem sets at the end of each chapter. References are used frequently, and these are also listed at the ends of the chapters in which they occur.

There is an appendix which contains an excellent "Survey of Essential Results from Linear Algebra", including vector spaces, transformations, vector and matrix norms, and convergence of matrices. According to the authors, "This is intended partly to amplify some of the discussion in the main text, and partly to serve as a bridge between this and more advanced works in the field." As for the main body of material in the book, they say that they "assume little more than a knowledge of elementary matrix algebra and of the basic theory of simultaneous linear equations." Here this reviewer disagrees. While it could be done by the better student, over 30 years of teaching experience says that if the book is to be used as a text book the main objective, namely "Computational Methods . . . " would best be achieved if some knowledge of most of the material in both the first chapter and the appendix had already been acquired in an advanced undergraduate level course in matrices and linear algebra. These portions would then give an excellent review and serve to "fill in the gaps" as needed to continue.

There are not too many good books in this field that are suited for the level this one is. It is a most welcome addition, either as a reference work or as a text. It is unfortunate, however, that such a good work in other respects should not have been proofread more carefully. Some three dozen errors were found. There are surely more. Most, it is true, are typographical and do not affect the mathematics. But there are some that could make the results totally incorrect. A prime example is on page 44 when Gauss' method of row reduction is used on an augmented matrix and it states that "The process proceeds similarly, terminating when the last row contains only one non-zero element." Or on page 115 where it states, "The elements of A^{-1} are given by $M_{ij}/|A|$, where M_{ij} is a minor of A." Hopefully another printing will correct these things.

F. Virginia Rohde
Mississippi State University

College Mathematics With Business Applications (2nd Ed.), John E. Freund, Prentice-Hall, Englewood Cliffs, N.J., 1975, 679 pp., $13.95.

The author's stated goal was to write a text designed for students of business and economics which not only provides a general understanding and appreciation for mathematics in management science but also provides an adequate foundation for more advanced work in statistics, operations research, econometrics, etc. He has assumed some high school algebra as prerequisite mathematical background for this text, however, it would seem to the reviewer that a course in college algebra would be necessary to master the material without difficulty.

The first three chapters deal with the introductory concepts of mathematical models, numbers and numerals, and the idea of functions. Everything dealing with linear mathematics is grouped together in the next four chapters, beginning with linear functions and systems of linear equations and inequalities, then a chapter on matrix algebra and finally two chapters on linear programming—the second dealing with the Simplex Method. The material on non-linear functions (quadratic functions, exponential functions, etc.) is taken up immediately before the calculus chapters. The last part of the book contains material on probability (including a chapter on probability distributions with a brief introduction to the normal distribution), decision making, and simulation.

The feature that most impressed this reviewer was the presentation of optional topics in the exercises with detailed explanations. This made it possible to introduce special topics, special applications, extra details, and more advanced material without cluttering up the main body of the text. For example, non-linear depreciation is explained among the exercises on exponential functions, optional material on analytical geometry is explained among the exercises on linear and quadratic functions, the Method of Least Squares is explained among the exercises on partial differentiation, and aspects of network theory is included among the exercises on matrices.

The book is well designed and very easy to read. The exercises appear reasonable and answers are provided to most of the odd-numbered problems. A small sample revealed no incorrect answers. Most theorems are set off either in bold type or shaded rectangular boxes.

The overall impression was that the text should be given serious consideration for a course in business mathematics from the modern viewpoint for college freshman and/or sophomores.

Ronald D. Dettmers
University of Wisconsin—Whitewater

Completeness, Compactness, and Undeciqbility, Alfred B. Manaster, Prentice-Hall, Englewood Cliffs, N.J., 1975, 160 pp., $8.50.

This book provides a self-contained introduction to mathematical logic based on a Gentzen-type system of natural deducation and closes with a discussion of decidable and undecidable theories, based on the notion of Turing computability.

The title, "Completeness, Compactness, and Undecidability" accurately describes the thrust of the book. After a brief chapter on preliminaries the author turns to parallel developments of the calculus of propositions and the first order predicate calculus in Chapters 1 and 2, respectively. Each of these theories is considered from the model-theoretic and the proof-theoretic points of view, and the goals in each chapter are the Completeness Theorem and the Compactness Theorem for the respective theory. Chapter 3 introduces the notion of computability by means of Turing machines and proceeds to a proof of the undecidability of the predicate calculus and of arithmetic. A brief section relates these undecidability results to the Gödel incompleteness theorem, which is not proved in this book. Each chapter concludes with a set of exercises which range from routine problems to extensions of the theory and from (in Chapter 1) applications of the compactness theorem to well known problems in combinatorial theory.

In some parts of the book, for example the introduction of Turing machines, the exposition is quite good. However, the pace is often too rushed to permit adequate motivation of concepts. Proofs are frequently sketched, leaving many details to be filled in by the reader. There are relatively few typographical errors, but the reader could be confused in the two or three instances in which such errors occur in the statements of theorems.

The author assumes of the reader a mathematical preparation equivalent to the first two or three years of an undergraduate mathematics major, and presumes no previous experience with formal logic. However, the rapid pace of the book, the early introduction of a high level of abstraction, and the lack of examples or motivation in many instances would appear to make this difficult reading for any but the most mature student. For such a student, however, this book provides a compact development of some of the most important results from twentieth century logic.

Edward Z. Andalafte
University of Missouri—St. Louis

Mathematics For College Students, Elementary Concepts, (2nd Ed.), A. William Gray and Otis M. Ulm, Glencoe Press, 1975, 361 pp., $7.95.

The second edition of the text has several announced changes and features which should make the book attractive to a wide mathematical audience. Most notable changes are in the area of exercises which were expanded for algebra and real numbers.

The text should be useful to institutions looking for a book that could be used in general education courses as well as to departments in need of a treatment of algebra and related topics prior to an applied course for business majors.

The brief historical introductions to geometry and computers were very well done and provide interesting reading to all students.

Ramon L. Avila
Ball State University

Introduction to Probability Theory with Computing, J. Laurie Snell, Prentice-Hall, Englewood Cliffs, N.J., 1975, 302 pp., $9.95 (paper).

The most striking feature of this textbook is its use of simple computer programs (written in the language BASIC) to aid in the student's understanding of an appreciation for the major concepts in elementary probability. The inclusion of the material on computing has enabled the author to present far more interesting examples and exercises than are normally found in textbooks. The author should be commended for his efforts.

The author has presented an excellent mathematical treatment (carefully planned and lucidly written) of finite probability spaces including an excellent treatment of Markov Chains. I found no substantial errors in the text. There are many minor (but annoying) typographical errors, probably due to the fact that the text was photographed from a computer printout. I suspect that not many computers could pass a course in English composition.

Although the core material is accessible to bright freshmen and sophomores, the book is most appropriate for a course at the junior-senior level. Matrix algebra, some exposure to calculus, and familiarity with the elements of BASIC are required for a complete understanding of the core material.

I have not yet taught a course from this book although I am directing an independent study project based on Chapter IV, Markov Chains. The student, a senior mathematics major, is experiencing no difficulty in reading the material.

My overall impression is that the use of this text would be an inspiring and rewarding experience for both the instructor and the students.

Edward M. Bolger
Miami University (Ohio)

Kappa Mu Epsilon News

EDITED BY SISTER JO ANN FELLIN, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Sister Jo Ann Fellin, Historian, Kappa Mu Epsilon, Benedictine College, North Campus Box 43, Atchison, Kansas 66002.

ANNOUNCEMENT

The President's Commission on White House Fellowships has announced the start of the thirteenth nationwide search for outstanding young men and women to serve as White House Fellows.

Established in 1964, the White House Fellowship program is designed to give rising leaders one year of firsthand, high-level experience with the workings of the Federal Government and to increase their sense of participation in national affairs. The program is open to U.S. citizens from all fields who are not less than 23 and not more than 35 years of age. Employees of the Federal Government are not eligible, with the exception of career armed services personnel.

In addition to their educational assignments with the Vice President, Cabinet officers or principal members of the White House staff, the Fellows participate in an extensive seminar program, typically consisting of some 300 off-record sessions with top government officials, scholars, journalists, and leaders from the private sector. The young men and women who have, to date, been selected as White House Fellows have included lawyers, scientists, engineers, corporate business entrepreneurs, scholars and academic administrators, writers and journalists, medical doctors, social workers, architects, and local public officials.

The Fellowship is designed to be a one-year sabbatical in public service. Fellows are expected to return to their professional careers at the end of their experience in government, with their perspectives of national issues broadened and their qualifications for significant service in their chosen careers and to their communities permanently enriched.

Leadership, intellectual and professional ability, high motivation, and a commitment to community and nation are the broad criteria employed in the selection process.

Requests for applications for next year's program must be postmarked not later than November 1, 1976. Application forms and additional information can be obtained by sending a postal card to the President's Commission on White House Fellowships, Washington, D.C. 20415.

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

Chapter President—Anthony Eckl

38 actives

For the first meeting of the year Anthony Eckl presented magic and card tricks. The next two programs concerned the works of M. C. Escher. A film on his works was shown and Dr. John Locker gave an illustrated lecture on the subject. As a result students became interested in tessellations of the plane and in creating original tessellations. The annual Christmas party was well attended. Other officers: Alan Vines, vice-president; Nancy Ray, secretary and treasurer; Jean Parker, corresponding secretary; Oscar Beck, faculty sponsor

Arkansas Alpha, Arkansas State University, State College

Chapter President—Conrad Cunningham

14 actives, 1 pledge

Arkansas Alpha met monthly with business meetings and short talks by faculty members. A committee was set up to study possible long-term projects to be undertaken by the chapter. These include the solicitation of donations for an endowed scholarship fund for winners of the annual math contest for high school seniors, the creation of a job information service for math majors and graduates, and the establishment of a cooperative work-study program with industry. The chapter Christmas party was held at the home of Dr. Hal McCloud. Other officers: Eugene Morris, vice-president; Judy Russell, secretary; Maria Malham, treasurer; Jerry Linnsteadter, corresponding secretary; Robert Rossa and Robert Johnson, faculty sponsors.

sale to raise funds. That same afternoon the chapter sponsored a picnic to acquaint the members with underclassmen who were potential math majors. Alumnae also attended the picnic. The chapter hosted a square dance for hospitalized veterans on 23 November. Other officers: Ann Strangarone, vice-president; Rose Onlewski, secretary; Thomas Kovrim, treasurer; Mordechai S. Goodman, corresponding secretary and faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President—Cheryl A. Ross
35 actives

Richard Kroeger, sophomore, presented his paper on the Calculus of Variations at the September meeting. The annual homecoming breakfast, held 25 October at the home of Dr. and Mrs. E. W. Hamilton, was well attended by local members, faculty, and alumni —the most successful homecoming in recent years. Senior Chris VanDeventer spoke on "Drills of Basic Arithmetical Operations" at the November meeting. Iowa Alpha initiated member number 600 on 4 December. New initiate Mark McCarville gave his paper on applications of mathematics to business at the initiation banquet. The annual Christmas party was held 12 December at the home of Professor Ina Mae Silvey. Other officers: Richard A. Kroeger, vice-president; Christy C. VanDeventer, secretary and treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines

Chapter President—Mary Bauer
12 actives, 6 pledges

Chapter activity in the fall semester began with the annual fall picnic. Programs at the regular meetings featured student and faculty presentations. John Diehl and David Trautman discussed the probability that a quadratic equation has complex roots, Mary Bauer spoke on agenda ordering, and Dr. Alex Kleiner talked about fair division schemes. Other officers: Carol Behrens, vice-president; Alan Carpenter, secretary; David Trautman, treasurer; Christina Bahl, corresponding secretary; Alex Kleiner, faculty sponsor.

Iowa Gamma, Morningside College, Sioux City

Chapter President—Raejean Furne
29 actives

On the morning of 18 October the chapter held a homecoming breakfast for returning KME alumni—an annual event which is always well attended. Two off-campus professors spoke at chapter meetings. Dr. Percy of Iowa State University lectured on the topic "The Place of Science in a Liberal Education" and Dr. Henry Walker of Grinnell College talked on "Math for Spies—Cryptanalysis before 1920". Another meeting featured Dr. Franklin Terry of the local religion department who spoke on the relationship between science and religion. Other officers: Penney Plumb, vice-president; Kim Helmbrecht, secretary; Bob Barke, treasurer; Elsie Muller, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg College, Waverly

Chapter President—Gary Wipperman

27 actives

Student Paul Koch spoke at the September meeting on his summer experiences in the actuarial department of CNA Insurance Company in Chicago. Dr. Warren Boe of the University of Iowa talked about the uses of mathematics in business and economics at a joint meeting of the math and business clubs on 21 October. As part of an all-college week in November emphasizing one's "Berufung" (German for "calling), Iowa Delta members heard a panel composed of math graduates who talked about their math-related jobs. Social activities of the semester included a card party and a Christmas dinner party. Other officers: Jennifer Zelle, vice-president; Deborah Ehlers, secretary; Paul Koch, treasurer; Glenn C. Fenneman, corresponding secretary and faculty sponsor.

Kansas Alpha, Kansas State College of Pittsburg, Pittsburg

Chapter President—Laura Spain

50 actives

Program meetings included the following: mathematical games and puzzles conducted by Laura Spain and Roy Bryant; a demonstration of hand-held calculators by Roy Bryant; an explanation of placement officer procedure as well as employment opportunities in mathematics by Dr. L. L. Tracy, KSCP placement officer director; and a slide program presented by Dr. Robert Backes of the KSCP physics department concerning energy needs and problems that face the country. Six new members were initiated during the October

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President—Sandra McKaig
46 actives, 11 pledges

Tutorial service has again been provided for students this year. Surplus books donated by faculty members were sold to students to raise funds for the chapter. Monthly meetings featured faculty speakers. On 4 December the Christmas social included the pledge ceremony. The chapter sponsored a student-faculty coffee hour on 29 January. Other officers: Robert Kernaghan, vice-president; Marguerite Liem, secretary; Robert Watanabe, treasurer; George R. Mach, corresponding secretary; Adelaide T. Harmon, faculty sponsor.

California Delta, California State Polytechnic University, Pomona

Chapter President—Cristel Sautter
20 actives

The chapter meets monthly. Tutoring is made available free of charge. Other officers: Melissa Stinson, vice-president; Tim Cleary, secretary and treasurer; Samuel Gendelman, corresponding secretary; Joseph Kachun, faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins

Chapter President—Debra Meacham
25 actives, 2 pledges

Weather modification, lasars, and knot theory were among the topics covered at bi-weekly meetings. The main activity sponsored by the chapter—the Alumni Seminar on Employment Opportunities in the Mathematical Sciences—occurred on 8 November. Speakers from a variety of businesses and professions included representatives from Mountain Bell, IBM, and the National Center for Atmospheric Research. Other officers: Jane Darling, vice-president; Vicki Lusk, secretary; Brad Gubser, treasurer; Duan Clow, corresponding secretary; Bennet Manvel, faculty sponsor.

Colorado Beta, Colorado School of Mines, Golden

Chapter President—John Turner
25 actives

The most notable activity developed into the "Problem of the

Month" section of the school paper with a $5 cash prize for each monthly winner. Two programs were held. John Turner, student, reported on the National Convention. Professors Fausett and Boes presented some selections from "Euclid and His Modern Rivals" by Lewis Carroll. Other officers: Kendrick Killian, vice-president; Mark McCuen, secretary; Leonard Witkowski, treasurer; A. J. Boes, corresponding secretary; D. W. Fausett, faculty sponsor.

Connecticut Alpha, Southern Connecticut State College, New Haven

Chapter President—Robert Elia
6 actives, 4 pledges

Other officers: Marianne Lonardo, vice-president; Kathleen Hendricks, secretary; Alberta Zimbardi, treasurer; E. R. Sparks, corresponding secretary and faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton

Chapter President—Karen Furr Stephens
14 actives

The chapter planned an active recruitment drive since many of their members graduated after the installation in late spring, 1975. Other officers: George D. Bagwell, vice-president; Vivian G. Dunn, secretary; Rebecca Allen, treasurer; Thomas J. Sharp, corresponding secretary and faculty sponsor.

Illinois Alpha, Illinois State University, Normal

Chapter President—James Murdock
20 actives, 10 pledges

Chapter members heard speakers from Illinois State University and from the Caterpillar Company. A hayrack ride was held in November. Historian and Social Chairperson for the year are Mike Hogan and Paula Rogers, respectively. Other officers: Rita King, vice-president; Marcia Heinz, secretary; Colleen Kirby, treasurer; Orlyn Edge, faculty sponsor.

Illinois Zeta, Rosary College, River Forest

Chapter President—Anita Koziol
19 actives

On the morning of 13 October chapter members held a plant meeting. A special Christmas meeting was held at the home of Dr. Helen Kriegsman, chairperson of the mathematics department. Other officers: Roy Bryant, vice-president; Deanne Anderson, secretary; Darla Hedrick, treasurer; Harold Thomas, corresponding secretary; J. Bryan Sperry, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison

Chapter President—Le Ann Fischer
11 actives

A calculator raffle proved to be a good fund-raiser as well as profitable for the student winner from the business department. Another activity involving both members and other mathematics students was a house painting party in October at the home of Richard Farrell, chairman of the department. The all day affair resulted in quite a transformation apparent to anyone passing the corner of 6th and Riley. A new column—KME's Korner by Professor Kal Kulus—has appeared during the semester in the student newspaper, *The Circuit.* The Professor (really KME member Keith Langel) challenges readers to solve his brain puzzlers. The winner (determined by a drawing of those who submit correct solutions) receives a geometric poster. Sister Helen Sullivan, in Atchison at the time of the Wassail Christmas Party, enjoyed the traditional gathering with Kansas Gamma members at the home of faculty member Jim Ewbank. Other officers: Bruce Fisher, vice-president; Cathy Molini, secretary and treasurer; Sister Jo Ann Fellin, corresponding secretary; Jim Ewbank, faculty sponsor.

Kansas Delta, Washburn University, Topeka

Chapter President—Debbie Evans
26 actives, 5 pledges

At the November meeting Janet Guyer presented her paper entitled "A Discussion of the Division Algorithm in Integral Domains other than Z" for which she received honorable mention at the last national convention. The chapter is making plans for the annual initiation dinner and the annual Math Day for area high school juniors and seniors. Other officers: Wayne Hamilton, vice-president; Beth Artzer, secretary; Jim Rinne, treasurer; Robert Thompson, corresponding secretary; A. Allan Riveland and Emmuel Calys, faculty sponsors.

Kansas Epsilon, Fort Hays Kansas State College, Hays

Chapter President—Carol Hilt

18 actives

Dr. Montgomery of the University of Kansas presented a lecture to the chapter members on 8 September. On 29 September student member Steve Alston spoke to the group on his investigation of Fourier's series and its application. A demonstration of the Moog Synthesizer was given by representatives of the Hays Music Company in early October. Other officers: Larry Hornbaker, vice-president; Michael Moyers, secretary and treasurer; Eugene Etter, corresponding secretary; Charles Votaw, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President—Karen Pichler

9 actives, 4 pledges

Meetings this academic year are being devoted to discussion of career options for those majoring in mathematics. Maryland Alpha met jointly with Maryland Beta for the December meeting at Western Maryland College. At this meeting Karen Pichler spoke on "Game Theory and Decision Making". Other officers: Colleen Baum, vice-president and treasurer, Frances Pittelli, secretary; Sister Marie Augustine Dowling, corresponding secretary and faculty sponsor.

Maryland Beta, Western Maryland College, Westminster

Chapter President—Barry Watson

21 actives

At the Fall Convocation in September Deborah Simmons was awarded the KME-funded Clyde Spicer Mathematics Award for the most outstanding prospective mathematics major in the class of 1977. During that month the chapter held a picnic for all mathematics majors. Featured speaker at the October initiation meeting was student Tony Sager who discussed "Transformation Geometry". The fund-raising project for the semester was a November bakesale. Also during November Maryland Beta members attended a Mathematics Department sponsored lecture on groups given by a Gettysburg College professor. Maryland Beta President, Barry Watson, talked on "The One-dimensional Wave Equation" at the joint meeting in December with Maryland Alpha. Historian for the year is

Carol Rouzer. Other officers: Michael Kline, vice-president; Deborah Simmons, secretary; Virginia Bevans, treasurer; James Lightner, corresponding secretary; Robert Boner, faculty sponsor.

Michigan Beta, Central Michigan University, Mount Pleasant

Chapter President—Patricia West
36 actives, 24 pledges

During Homecoming the chapter held a coffee hour for alumni. Chapter members have offered tutoring services through evening Help Sessions. Program meetings have included topics such as "Computer Science as a Profession" and "Jobs in Mathematics". Ray Nacin, F.S.A., of Mutual Life Insurance of Southfield, Michigan was guest speaker at the Central Michigan University Mathematics Colloquium. Michigan Beta initiated 24 new members on 15 October. Other officers: Hans Heikel, vice-president; Joann Ostrowski, secretary; Susan Goode, treasurer; Edward H. Whitmore, corresponding secretary and faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg

Chapter President—Larry Lok
50 actives

Eighteen new members were initiated at the annual fall cookout and initiation on 3 October. Social Chairperson for the year is Donny McClesky. Other officers: David Presser, vice-president; Susan Horlock, secretary; Jack D. Munn, corresponding secretary; Alice Essary, faculty sponsor.

Mississippi Delta, William Carey College, Hattiesburg

Chapter President—Ann Banes
12 actives

On 13 November Mississippi Delta initiated Ronald Wagster and Ken Kin Chen. Initiates of last year not yet published include: Anna Landon, Donald Wilburn, Shiao Wang, George Satcher, Mike Reynolds, and Betty Jones. Graduate student Grenda Short shared her teaching experiences with the chapter at one of their meetings. The Mississippi Delta group is working on a project concerning the history of mathematics in the United States from colonial days to the present. This Bicentennial program will be presented at a joint meeting with Mississippi Gamma. Other officers: Anna Landon, vice-president; Betty Jones, secretary; Gaston Smith, corresponding secretary and faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield

Chapter President—Pamela Grassle
40 actives

Highlighting the semester for the Missouri Alpha chapter was speaker Gary Juhl, representative of Southwestern Bell Telephone Company. He explained the opportunities for the use of mathematics in management. At another meeting, student Ruell Chappell discussed the ways in which social and religious movements have historically affected the learning process. An October tea honored thirteen new members initiated at that time. The chapter closed its semester activities with an end-of-the-semester volleyball game. Other officers: Marcel Wehrman, vice-president; Karen Lefler, secretary; Ruell Chappell, treasurer; Eddie W. Robinson, corresponding secretary; Tom Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg

Chapter President—Dale Bratzler
41 actives, 26 pledges

The chapter continues to be active with meetings and social gatherings. Eight members attended the national convention in Milwaukee. Other officers: Charlotte Statter, vice-president; Marilyn Carlson, secretary; Ken Jones, treasurer; Homer F. Hampton, corresponding secretary and faculty sponsor.

Missouri Gamma, William Jewell College, Liberty

Chapter President—Howard Hays

The chapter meets regularly on the first Wednesday of each month. For these meetings emphasis is placed on the presentation of papers by student members. KME coordinates some of its activities with the physics honorary society on campus, Sigma Pi Sigma, since there is a significant overlapping of membership in the two groups. Other officers: Howard Brooks, vice-president; Susan Radke, secretary; Karen Wagenknecht, treasurer; Sherman Sherrick, corresponding secretary; Truett Mathis, faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville

Chapter President—Don Hackmann
23 actives

During the fall semester senior members of KME presented talks on various mathematical topics at the monthly meetings. An initiation for new members was held in October. In November the chapter met with math majors of Northwest Missouri State University, Maryville. A film, two interesting talks on mathematics, and a demonstration of a mini-computer were presented by the faculty and math majors of NWMSU. Other officers: Amy Barrow, vice-president; Debbie Reinker, secretary; Doyle Taylor, treasurer; Sam Lesseig, corresponding secretary; John Erhart, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne

Chapter President—Deanna Fey
13 actives, 7 pledges

Chapter members have set up a free tutoring service for students in lower level mathematics courses. Regular meetings are held with mathematical entertainment following the meetings. Members aid the mathematics department in administering the annual mathematics contest, a competition for area high school students. Each year the membership, with the aid of the mathematics faculty, selects the outstanding freshman mathematics student through a competitive examination. Public announcement of the award occurs at the spring Honors Convocation and the recipient's name is placed on a permanent plaque. Receiving this award for the 1974-75 school year was Bill Pieper of Lyons, Nebraska. Each year the national science honorary and KME select by secret ballot the outstanding professor in the Department of Mathematics and Science. Dr. James Paige of the mathematics department was selected for 1974-75. Members Alan Anderson, Deborah Dubs, Craig Hellwege, Jean Herfordt, and Fred Webber attended the last biennial convention in Milwaukee. Sue Wragge is the chapter historian. Other officers: Dave Lashier, vice-president; Lavern Lodl, secretary and treasurer; Fred Webber, corresponding secretary; James Paige, faculty sponsor.

Nebraska Beta, Kearney State College, Kearney

Chapter President—Donna Chramosta
6 pledges

Math help sessions were re-established this last fall. A faculty auction of items donated by downtown businesses netted $120 for the chapter. A halloween party was held in October and the Christmas party was combined with initiation during December. Some members helped with Senior Day and others attended the Natural and Social Sciences banquet. Other officers: Linda Williams, secretary; Julie Mackey, treasurer; Charles Pickens, corresponding secretary; Randall Heckman, faculty sponsor.

Nebraska Gamma, Chadron State College, Chadron

Chapter President—Duane Fritz
12 actives, 8 pledges

The chapter hosted the following activities: fall picnic, homecoming float, initiation, calculator raffle, and Christmas carolling. The head of the mathematics department was the guest speaker at the December meeting. Other officers: Lynn Grantham, vice-president; Joyce Serbousek, secretary; Kristy Brost, treasurer; James A. Kaus, corresponding secretary and faculty sponsor.

New Jersey Beta, Montclair State College, Upper Montclair

Chapter President—Debra Hartley
35 actives, 35 pledges

On 4 November new staff members presented talks to the chapter members. An October picnic and a December Christmas party were the social events. The chapter sponsors a math contest. Historian for the year is Daniel Gualtieri. Other officers: Sharon Gall, vice-president; Elaine Vitale, secretary; Michael Jankowsky, treasurer; Carl Bredlau, corresponding secretary; Helen Marcus Roberts, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President—Beverly Riese
40 actives

During the fall of 1975 the chapter sponsored an open house in the mathematics department for high school students visiting the campus as part of the Student-for-a-day program. Other officers: Turner Laquer, vice-president; Bonnie Zimmermann, secretary; Gerald McNerney, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Kappa, Pace University, New York

Chapter President—Larry Jermyn
40 actives

The second annual induction and dinner of the chapter was held on 7 May 1975. At that time ten new members were initiated. Two inductees were faculty members of the mathematics department—Dr. John Markey and Dr. Marcel Stein. Students inducted were: Jennifer Costas, Dolores Freund, Susie Lee, Linda Mango, Lynn Rosbeck, Mary Ann Sucic, Christine Szalay, and James Touwsma. Other officers: Salvatore Vittorio, vice-president; Jennifer Costas, secretary; Christine Szalay, treasurer; Sandra M. Pulver, corresponding secretary and faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green

Chapter President—Julie Osmon
45 actives

The chapter sponsored two October events—"Meet the Prof" night on 14 October and a volleyball game on 18 October. On 17 November the chapter gathered for a panel discussion on graduate school. Other officers: Debra Bird, vice-president; Diane Sparks, secretary; Jane Kline, treasurer; Waldemar Weber, corresponding secretary; Thomas A. Hern and W. Charles Holland, faculty sponsors.

Ohio Epsilon, Marietta College, Marietta

Chapter President—Richard Talcott
15 actives, 3 pledges

Fall initiation was held for three students. Richard Talcott presented a talk entitled "Puzzle Solving by Induction". The chapter members have organized help sessions for the freshmen in the Calculus sequence courses. Other officers: Richard Dempsey, vice-president; Charles Eyermann, secretary; Jim Patterson, treasurer; Robert Fraser, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord

Chapter President—Warren Brown
31 actives

Student members Debbie Gutridge and Warren Brown presented reports on the applications of mathematics to Chemistry and Physics, respectively. The new members gave short talks at the initiation banquet in October. The chapter sponsored two guest speakers from Miami Univrsity—Dr. D. Koehler and Dr. F. Gass. A Christmas and mathematical games party was held at the J. R. Smith home. Other officers: Debra Gutridge, vice-president; Sandra Mumaw, secretary and treasurer; James L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford

Chapter President—Robert Whittenberg

Oklahoma Gamma initiated new members on 2 December. New officers elected at that time were installed at the January meeting. Other current officers are: Richard Cowan, vice-president; Linda Ommen, secretary; Pablo Reyes, treasurer; Wayne Hayes, corresponding secretary.

Pennsylvania Alpha, Westminster College, New Wilmington

Chapter President—David Schneckenburger
54 actives

Highlighting the fall program was a "Fall Careers Night" planned by the chapter president and faculty sponsor in conjunction with the Director for Career Planning and Placement, Mr. Sternbergh. The event took place 18 November. The program, including two trial interviews, proved to be a very educational experience. Plans for spring consist of another careers night, initiation, and a final spring party get-together. Other officers: Jim Goldbach, vice-president; Gail Boberg, secretary; Karen Evans, treasurer; Miller Peck, corresponding secretary; Thomas Nealeigh, faculty sponsor.

Pennsylvania Beta, LaSalle College, Philadelphia

Chapter President—Matt Coleman
22 actives, 5 pledges

Bi-weekly meetings are held by the chapter. A tutoring program is organized for fellow students. Social activities include skiing, intramural sports, and picnics. Other officers: Steve Gauder, vice-president; Pat Nepps, secretary; Joe Rakszawski, treasurer; Brother Damian Connelly, corresponding secretary; Sam Wiley, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President—James Risko

30 actives, 5 pledges

The chapter sponsored a college-wide mathematical games night for students and faculty. Other officers: Jeffrey Angstadt, vice-president; Roseann Eroh, secretary; Rita Borillo, treasurer; Irving Hollingshead, corresponding secretary; Edward Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President—David Elko

37 actives

Dr. Gerald Buick, member of the mathematics faculty, spoke on "Riemann Surfaces" at the October meeting. At the November meeting three student members, April Cassel, Nancy Neral, and David Elko, shared their experiences of last summer related to their internships in mathematics. In December mathematics faculty member Doyle McBride spoke on "Fallacious Proofs" at a meeting of the chapter. Other officers: Kristine Mangone, vice-president; Sharon Evans, secretary; Teresa Pavlekovsky, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City

Chapter President—Milo Dorr

35 actives, 20 pledges

Pennsylvania Eta has prepared an exam to administer to freshmen math majors for the purpose of selecting the outstanding freshman mathematics student. The first winner of the award was presented with a copy of the CRC Handbook at an awards program held during "Parents Day" at Grove City College. The chapter dropped its research paper requirement as a condition for membership. Other officers: Cynthia Loveland, vice-president; Linda Amon, secretary; Sherry DuCarme, treasurer; Marvin C. Henry, corresponding secretary; John H. Ellison, faculty sponsor.

Pennsylvania Iota, Shippensburg State College, Shippensburg

Chapter President—Warren Becker

37 actives

Monthly meetings were held by the chapter with both outside speakers and pledge talks. Well attended were the roller skating party and the basketball game. Fall initiation was conducted at the home of Dr. James Sieber, chairman of the mathematics department. Other officers: Keith Livingston, vice-president; Lynn Dotter, secretary; Howard T. Bell, treasurer; John S. Mowbray, corresponding secretary; James L. Sieber, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia

Chapter President—Kathleen Britt

6 actives

During the fall semester the chapter held regular meetings to learn linear programming under Louis Hoelzle. A free tutoring program was set up with each member giving one to two hours time per week to the project. On 19 November the chapter co-sponsored a lecture with Beta Chi. Dr. Samuel Kotz of Temple University spoke on "Probability and Statistics in the West and East". The 50-50 raffle took place at refreshment time following the lecture. On 17 March 1976 three new members were inducted at the Holy Family College Honors Convocation—Barbara Bader, Susan Capozio, and Janice DiGirolamo. Other officers: Guy Giardine, vice-president; Louise Wallowicz, secretary; Robert Papsun, treasurer; Louis F. Hoelzle, corresponding secretary and faculty sponsor.

Pennsylvania Lambda, Bloomsburg State College, Bloomsburg

Chapter President—Dave Espe

30 actives, 14 pledges

Monthly meetings were held by the chapter. Socially, the members enjoyed a recreation night in the college gymnasium and a Christmas banquet. They also assisted with the activities of the annual math day. Other officers: Rich Stver, vice-president; Diane Gilroy, secretary; Bill Bachman, treasurer; James P. Pomfret, corresponding secretary; Joseph Mueller, faculty sponsor.

Tennessee Beta, East Tennessee State University, Johnson City

Chapter President—Lucia Jan Trotter
28 actives

A successful social event for chapter members took place at the Kingsport University Center. Other officers: June Rampy, vice-president; Jamie Davis, secretary; Tim Stecker, treasurer; Lyndell Kerley, corresponding secretary; Sallie Pat Carson, faculty sponsor.

Virginia Alpha, Virginia State College, Petersburg

Chapter President—Patricia A. Johnson
16 faculty and 13 students

The following papers were delivered at chapter meetings: "Mathematical Games" by Dr. Loretta Braxton, "Game Theory" by Jean Coleman, and "Pythagorean Triplets" by Claude Newman. Other officers: Louis Richards, vice-president; Lisa Barge, secretary; Martina Lewis, treasurer; LaVerne Goodridge, corresponding secretary; Vidya Bakhshi, faculty sponsor.

Virginia Beta, Radford College, Radford

Chapter President—Jo Ann Wright
28 actives

New members presented papers at the fall semester meetings. The chapter is actively involved in a tutoring systm. Other officers: Patricia Moore, vice-president; Pam Snider, secretary; Glenda Wright, treasurer; J. S. Milton, corresponding secretary; Dr. Hansard, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President—Jane Reinartz
5 actives, 4 pledges

Wisconsin Alpha members participated in Octoberfest on campus with a games booth. The chapter viewed the film loops, "Calculus in Motion", at one of their meetings. In December a salesman from Texas Instruments demonstrated calculators to the group. Other officers: Joy Rademacher, vice-president; Cathy Borchert, secretary; Nancy Bernards, treasurer; Sister Mary Petronia, corresponding secretary and faculty sponsor.

Wisconsin Beta, Wisconsin State University, River Falls

Chapter President—Jeff Gray
30 actives, 10 pledges

Nine of the ten pledges were able to attend the initiation ceremony on 29 January. Other officers: Terry Awes, vice-president; Jacqueline Haines, secretary; Thomas Lehmann, treasurer; Lyle Oleson, corresponding secretary; Edward Mealy, faculty sponsor.