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CONTENTS

National Officers

Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

A Method Of Computing Coefficients For The Fourier Series

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In 1807 Jean Baptiste Joseph Fourier announced he was able to describe almost any function in terms of a trigonometric series. A trigonometric series is a series of the form

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \qquad (1)$$

where a_n and b_n are constants. In representing functions, the function, represented by a power series must possess derivatives of all orders since the power series does. The function represented by a trigonometric series does not have this infinite differentiability condition to satisfy; thus it has a definite practical advantage.

Now in (1) observe that each term repeats itself in intervals of 2π. Hence if (1) converges for all x, then its sum $f(x)$ must also have the property that $f(x + 2\pi) = f(x)$, and we say that f has period 2π.

DEFINITION. *A function* f *is said to be periodic and have period* p *if for all* x, $f(x + 2p) = f(x)$, *where* $p \neq 0$.

Some examples of periodic motion which occur naturally in nature include the motion of a pendulum, the oscillations of a spring, and the vibrations of a violin string. From J. B. J. Fourier's work, if we have a periodic function which satisfies certain general conditions, it is possible to write this function in the form of a trigonometric series. Consider, for example, the sound emitted from the violin. The $\frac{1}{2}a_0$ term is simply the neutral position of the strings. The fundamental tone is represented in the terms $a_1 \cos x + b_1 \sin x$, while the first overtone is $a_2 \cos 2x + b_2 \sin 2x$. Higher overtones can be written in a similar manner. The different sound produced by different musical instruments is due to the difference in the coefficients in the overtones.

We now examine some definitions which are needed for an understanding of the introductory theory of Fourier series.

DEFINITION. *A jump discontinuity of a function is a discontinuity in which the change of the function value through the discontinuity is of finite non-zero magnitude.*

DEFINITION. *A function* f *is said to be piecewise continuous on an interval* I *if there is a partition of* I *permitting the interval to be subdivided into a finite number of subintervals, inside each of which* f *is continuous and has finite limits at the left and right endpoints of the intervals.*

DEFINITION. *A function* f *is piecewise monotonic if it is possible to divide any finite domain into a finite number of intervals on which* f *is monotonically increasing or decreasing.*

DEFINITION. *A function* f *is piecewise smooth on an interval if it is piecewise continuous and has continuous first derivatives on the closed subintervals.*

DEFINITION. *A function* f *is piecewise very smooth on an interval if it is piecewise continuous and has piecewise continuous first and second derivatives there.*

DEFINITION. *Let* $\{S_n\}$ *be a sequence of functions on an interval* I. *Then* $\{S_n\}$ *converges uniformly to* S *if for every* $\epsilon > 0$ *there exists an* N, *where* N *is independent of a particular* x *in* I, *such that*

$$\left| S_n(x) - S(x) \right| < \epsilon \qquad \text{if } n > N \text{ for every } x \text{ in } I.$$

Some trigonometric identities which will be used in the discussion later are:

$$\int_{-\pi}^{\pi} \cos nx \quad \cos mx \, dx = \begin{cases} 0 & n \neq m \\ \pi & n = m \neq 0 \end{cases}$$

$$\int_{-\pi}^{\pi} \sin nx \quad \cos mx \, dx = \quad 0 \qquad\qquad \text{where } n, m \in I^+.$$

$$\int_{-\pi}^{\pi} \sin nx \quad \sin mx \, dx = \begin{cases} 0 & n \neq m \\ \pi & n = m \neq 0 \end{cases}$$

Now let f be a function which is the sum of a trigonometric series. Then we can write

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

At this point it is natural to ask: what is the relationship between f and the coefficients a_n and b_n? Assuming termwise integration of the series is permitted (i.e. the series converges uniformly), observe

$$\int_{-\pi}^{\pi} f(x)\cos mx\, dx = \int_{-\pi}^{\pi} \left[\frac{a_0}{2}\cos mx + \sum_{n=1}^{\infty} (a_2 \cos nx \cos mx + b_n \sin nx \cos mx)\right] dx$$

$$= \frac{a_0}{2} \int_{-\pi}^{\pi} \cos mx\, dx + \sum_{n=1}^{\infty} \left\{a_n \int_{-\pi}^{\pi} \cos nx \cos mx\, dx + b_n \int_{-\pi}^{\pi} \sin nx \cos mx\, dx\right\}.$$

If $m = 0$, then

$$\int_{-\pi}^{\pi} f(x)dx = \pi a_0$$

For $m \in I^+$, we have

$$\int_{-\pi}^{\pi} f(x) \cos mx\, dx = \pi a_m$$

Multiplying $f(x)$ by $\sin mx$, integrating from $-\pi$ to π, and then proceeding in the manner above, we get

$$\int_{-\pi}^{\pi} f(x) \sin mx\, dx = \pi b_m \quad \text{where } m = 1, 2, 3, \ldots$$

We conclude

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx \quad n = 0, 1, 2, \ldots$$
$$(2)$$
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx \quad n = 1, 2, 3, \ldots$$

We now formally state:

DEFINITION. *A Fourier series is a trigonometric series*

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

in which the coefficients a_n *and* b_n *are determined as* (2) *above.* A function can have its Fourier series coefficients computed in the manner presented, if it satisfies requirements which are known as the Dirichlet conditions. Basically these conditions are:

1) $\int_{-\pi}^{\pi} \left| f(x) \right| dx$ is finite

2) f is piecewise continuous and piecewise monotonic

From the definition the following theorem is derivable.

THEOREM. *Every uniformly convergent trigonometric series is a Fourier series.*

The proof can be completed by using the result from advanced calculus that the sum of a uniformly convergent series of continuous functions is continuous.

Let us now consider Fourier's theorem which describes the convergence of Fourier series and was proved by Dirichlet in 1829.

THEOREM (FOURIER). *Let* $f(x)$ *be piecewise very smooth in the interval* $[-\pi, \pi]$. *Then the Fourier series of* $f(x)$

$$\tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$

with coefficients

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \quad \textit{and} \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx,$$

(i) *converges to* $f(x)$ *wherever* $f(x)$ *is continuous inside* $[-\pi, \pi]$;

(ii) *converges to*

$$\tfrac{1}{2}\left[\lim_{x\to x_1^+} f(x) + \lim_{x\to x_1^-} f(x)\right]$$

at every point where a jump discontinuity of $f(x)$ *occurs and to the average of the right and left hand limits at* $x = \pm\pi$. *Furthermore, the convergence is uniform in every closed interval where no discontinuity occurs.*

The proof of this theorem may be found in Kaplan [7, p. 483-486].

Thus far we have restricted ourselves only to periodic functions in the interval $[-\pi, \pi]$. But if the series converges to $f(x)$ in $[-\pi, \pi]$, then it will converge outside the interval to a function which is the periodic extension of $f(x)$. Note that unless $f(\pi)=f(-\pi)$, the extension will introduce jump discontinuities at $x = \pm\pi + 2n\pi$ with n an integer. At these points the series will coverge to the arithmetic mean of the functional values of the extended function.

As an example, we will now write the Fourier series for a function which is often treated in introductory physics texts, namely the square wave function. This function is defined as follows:

$$f(x) = \begin{cases} -c & -\pi \leq x < 0 \\ c & 0 \leq x \leq \pi \end{cases}$$

Upon solving for the coefficients of the Fourier series of f, observe that

$$a_n = -\frac{c}{\pi}\int_{-\pi}^{0} \cos nx\,dx + \frac{c}{\pi}\int_{0}^{\pi} \cos nx\,dx = 0$$

where $n = 0, 1, 2, \ldots$

and

$$b_n = -\frac{c}{\pi}\int_{-\pi}^{0} \sin nx\,dx + \frac{c}{\pi}\int_{0}^{\pi} \sin nx\,dx$$

$$= \frac{2c}{n\pi}\left[1 + (-1)^{n+1}\right] \qquad \text{for } n = 1, 2, 3, \ldots$$

In particular, for $c = 1$, $x \in (-\pi, \pi)$, and $x \neq 0$ (where a jump discontinuity occurs),

$$f(x) = \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin 3x + \frac{4}{5\pi}\sin 5x + \ldots$$

$$= \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}$$

The first three partial sums of the series for $c = 1$ are

$$S_1 = \frac{4}{\pi}\sin x$$

$$S_2 = \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin 3x$$

$$S_3 = \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin 3x + \frac{4}{5\pi}\sin 5x$$

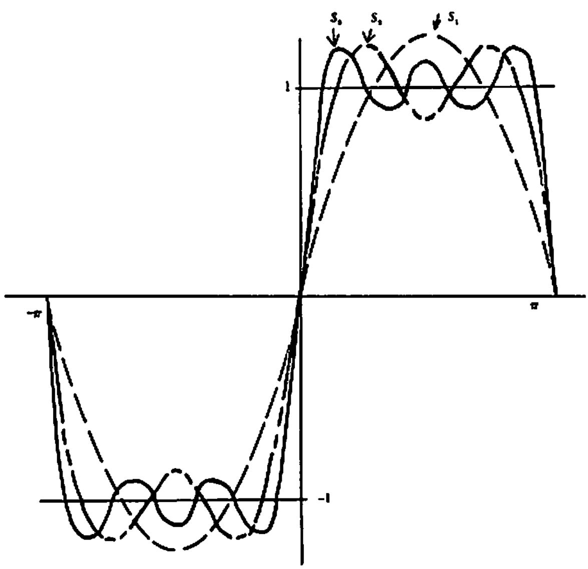

FIGURE 1

The graphs of these partial sums are shown in Figure 1. From these graphs the following observations can be made: 1) $f(x)$ is being approached in the limit except for $x = 0$, $x = \pm\pi$, 2) for $x = 0$ the series is converging to the mean value of the right and left hand limits at the jump, and 3) each partial sum gives the poorest approximation to $f(x)$ near the points of discontinuity.

From a practical point of view, we may not know $f(x)$ for all x, but only for a finite number of points, say $2N$. The case of an even number of points is treated in greater detail in Hamming [6], while Ralston [8] discusses the odd number of points. Now suppose we are fortunate enough to have these points equally spaced. Then it is possible to develop finite Fourier series in a manner similar to the continuous case considered previously.

The analogous trigonometric identities for the finite number of points which are needed in the derivation are:

$$\sum_{x=0}^{2N-1} \sin \frac{\pi}{N} kx \sin \frac{\pi}{N} mx = \begin{cases} 0 & k \neq m \\ N & k = m \neq 0 \end{cases}$$

$$\sum_{x=0}^{2N-1} \sin \frac{\pi}{N} kx \cos \frac{\pi}{N} mx = 0$$

$$\sum_{x=0}^{2N-1} \cos \frac{\pi}{N} kx \cos \frac{\pi}{N} mx = \begin{matrix} 0 & k \neq m \\ N & k = m \neq 0 \\ 2N & k = m = 0 \end{matrix}$$

Now assume that an arbitrary periodic function $f(x)$ can be written in the form

$$f(x) = \tfrac{1}{2}a_0 + \sum_{k=1}^{N-1} \left(a_k \cos \frac{\pi}{N} kx + b_k \sin \frac{\pi}{N} kx\right) + \frac{a_N}{2} \cos \pi x$$

Using the identities, we get the formulas for the coefficients:

$$\sum_{x=0}^{2N-1} f(x) \cos \frac{\pi}{N} mx = Na_m \qquad (1 \leq m \leq N-1)$$

$$\sum_{x=0}^{2N-1} f(x) = Na_0$$

$$\sum_{x=0}^{2N-1} f(x) \cos \pi x = Na_N$$

$$\sum_{x=0}^{2N-1} f(x) \sin \frac{\pi}{N} mx = Nb_m \qquad (1 \leq m \leq N-1)$$

From these result the general formulas for the coefficients:

$$a_k = \frac{1}{N} \sum_{x=0}^{2N-1} f(x) \cos \frac{\pi}{N} kx \qquad k = 0, 1, 2, \ldots, N$$

$$b_k = \frac{1}{N} \sum_{x=0}^{2N-1} f(x) \sin \frac{\pi}{N} kx \qquad k = 1, 2, 3, \ldots, N-1$$

On the computer these can be evaluated according to the algorithm developed by Goertzel and presented here in Hamming's notation [6, p. 72].

Let $U_0 = 0$

$U_1 = f(2N-1)$

For $m = 2, 3, \ldots, 2N-1$, do:

$$U_m = (2 \cos \frac{\pi}{N} k) U_{m-1} - U_{m-2} + f(2N-m), \text{ then}$$

$$Na_k = \sum_{x=0}^{2N-1} f(x) \cos \frac{\pi}{N} kx = (\cos \frac{\pi}{N} k) U_{2N-1} - U_{2N-2} + f(0)$$

$$Nb_k = \sum_{x=0}^{2N-1} f(x) \sin \frac{\pi}{N} kx = (\sin \frac{\pi}{N} k) U_{2N-1}$$

Now let us consider an example using this method. Suppose we are on the ground floor of a two story building that has an elevator which spends most of its time at rest (i.e. it travels infinitely fast in moving between the floors). Suppose further that we are able to look at the elevator only after a certain interval of time. Let us define the position of the elevator to have functional value 1 if the elevator is on the second floor and -1 if the elevator is on the ground floor.

After three intervals of time pass we have recorded the following observations: $f(0) = 1$, $f(1) = 1$, $f(2) = -1$, $f(3) = -1$. Thus we have four points and so $N = 2$. Now we write the finite Fourier series for this function (which describes the position of the elevator) according to the method we developed. Applying the algorithm, we find

$$U_0 = 0$$

$$U_1 = f(3) = -1$$

$$U_2 = (2\cos\frac{\pi}{2}k)(-1) - 0 + f(2)$$

$$= (-2\cos\frac{\pi}{2}k) - 1$$

$$U_3 = (2\cos\frac{\pi}{2}k)U_2 - U_1 + f(1)$$

Thus,

$$a_k = \tfrac{1}{2}\left[(\cos k)U_3 - U_2 + f(0)\right]$$

$$b_k = \tfrac{1}{2}\left[(\sin k)U_3\right]$$

so

$$a_0 = \tfrac{1}{2}\left[(1)(-4) - (-3) + 1\right] = 0$$

$$a_1 = \tfrac{1}{2}\left[(0)(2) - (-1) + 1\right] = 1$$

$$a_2 = \tfrac{1}{2}\left[(-1)(0) - (1) + 1\right] = 0$$

$$b_1 = \tfrac{1}{2}\left[(1)(2)\right] = 1$$

Finally we conclude

$$f(x) = \cos\frac{\pi}{2}x + \sin\frac{\pi}{2}x.$$

The elevator man tells us the elevator's position was defined as follows (the units are intervals of time):

$$f(x) = \begin{cases} 1 & 0 \le x < 1.5 \\ -1 & 1.5 \le x \le 3 \end{cases}$$

Observe how these functions compare on the graph in Figure 2. Note that the Fourier series agrees with the functional values we observed at times 0, 1, 2, and 3.

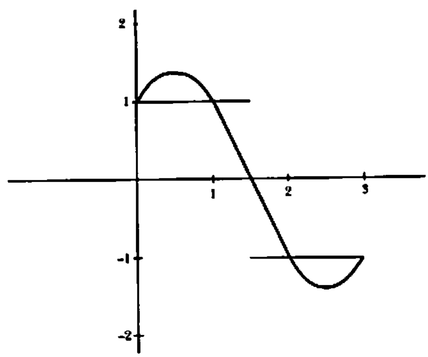

FIGURE 2

If it were possible to observe the elevator after shorter time intervals so that six observations ($N = 3$) are recorded in the same amount of time as four were, then the algorithm gives

$$f(x) = \tfrac{2}{3} \cos \frac{\pi}{4} + \frac{2\sqrt{3}}{3} \sin \frac{\pi}{4} + \tfrac{1}{3} \cos \pi x$$

which better approximates the actual function.

The development of continuous and discrete Fourier series has been presented. Fourier series, which does not require more mathematics than an advanced calculus course for basic understanding, allows us to approach certain physical problems from a point of view different from the Taylor series. Also note that the algorithm permits undergraduates to solve for the Fourier series coefficients on the computer.

BIBLIOGRAPHY

1. Adams, L. J. *Applied Calculus.* New York: Wiley and Sons, 1963, p. 236-240.
2. Carslaw, H. S. *An Introduction to the Theory of Fourier's Series and Integrals, 3rd rev. ed.* New York; Dover, 1930, p. 1-19, 215-249.
3. Courant, R. and Hilbert, D. *Methods of Mathematical Physics.* New York: Interscience, 1953, p. 48-76.
4. Davis, Harry *Fourier Series and Orthogonal Functions.* Boston: Allyn and Bacon, 1963, p. 87-188.
5. Fulks, Watson *Advanced Calculus, 2nd ed.* New York: Wiley and Sons, 1969, p. 523-528.
6. Hamming, Richard *Numerical Methods for Scientists and Engineers.* New York: McGraw-Hill, 1962, p. 67-78.
7. Kaplan, Wilfred *Advanced Calculus, 2nd ed.* Reading, Mass.: Addison-Wesley, 1973, p. 461-478.
8. Ralston, Anthony *A First Course in Numerical Analysis.* New York: McGraw-Hill, 1965, p. 254-260.
9. Rogosinski, Werner *Fourier Series.* New York: Chelsea Publ. Co., 1950, p. 1-8.
10. Stuart, R. D. *Fourier Analysis.* London: Metheun and Co., Ltd., 1966, p. 9-22.

ERRATA

1.) p. 11, Fall 1973 issue (vol. 33, no. 1). The first set-off formula on the page should begin "$\frac{\pi}{2} =$ " .

2.) p. 94, Spring 1974 issue (vol. 33, no. 2). The proof of the third conjecture is valid only if $p \nmid a$. If $p|a$, the result is trivially true.

The editor thanks the readers who wrote concerning these errors.

Directions for Papers to be Presented at the Twentieth Biennial Convention of Kappa Mu Epsilon

MILWAUKEE, WISCONSIN

17-19 April 1975

A significant feature of this convention will be the presentation of papers by student members of KME. The mathematics topic which the student selects should be in his area of interest, and of such a scope that he can give it adequate treatment within the time alloted.

Who may submit papers: Any student KME member may submit a paper for use on the convention program. Papers may be submitted by graduates and undergraduates; however, graduate students will not compete with undergraduates.

Subject: The material should be within the scope of the understanding of undergraduates, preferably those who have completed differential and integral calculus. The Selection Committee will naturally favor papers within this limitation, and which can be presented with reasonable completeness within the time limit prescribed.

Time limit: The usual time limit is twenty minutes, but this may be changed on the recommendation of the Selection Committee if requested by the student.

Paper: The paper to be presented, together with a description of the charts, models, or other visual aids that are to be used in the presentation, should be presented in typewritten form, following the normal techniques of term paper presentation. Appropriate footnoting and bibliographical references are expected. In addition, a statement that the author is a member of KME and a designation of his classification in school (graduate or undergraduate) should accompany the paper.

Date due: 17 January 1975

Address to send papers: James E. Lightner
National Vice President, KME
Western Maryland College
Westminster, Maryland 21157

Selection: The Selection Committee will choose about ten to twelve papers for presentation at the convention. All other papers will be listed by title and student's name on the convention program, and will be available as alternates. Following the Selection Committee's decision all students submitting papers will be notified by the National Vice President of the status of their papers.

Prizes: The author of each paper presented at the convention will be given a two-year extension of his subscription to *The Pentagon*. Authors of the three best papers presented by undergraduates, based on the judgment of the Awards Committee composed of faculty and students, will be awarded cash prizes of $50, $30, and $20 respectively. If enough papers are presented by graduate students, then one or more prizes will be awarded to this group. Prize-winning papers as well as other selected papers will be published in *The Pentagon*, provided they are in appropriate form.

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgment will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

In the usual elementary number theory course, the student studies Fermat's theorem and notes that in general its converse is not true. If the converse were true for all integers $m > 1$, we would have a convenient test for testing the primality of m. Unfortunately, this is not the case. We will now examine the situation for which the converse is true and a test for primality of any integer $m > 1$ will be devised.

We recall that the Euler ϕ-Function, $\phi(m)$, is the number of positive integers not exceeding m which are relatively prime to m.

We first state for review Fermat's theorem and a generalization called the Euler-Fermat theorem.

THEOREM 1. (Fermat's theorem) If p is a prime and $(a,p) = 1$, then $a^{p-1} \equiv 1 \pmod{p}$.

THEOREM 2. (Euler-Fermat theorem) If $(a,m) = 1$, then $a^{\phi(m)} \equiv 1 \pmod{m}$.

We will also need the following definition and the following two theorems which will be useful in the later development of this article.

DEFINITION 1. Let m denote a positive integer and let a be any integer such that $(a,m) = 1$. Let h denote the smallest positive integer such that $a^h \equiv 1 \pmod{m}$. We say that a belongs to the exponent h modulo m.

THEOREM 3. If a belongs to exponent h modulo m and if $a^k \equiv 1 \pmod{m}$, then $h \mid k$.

THEOREM 4. If a belongs to h modulo m, then $h \mid \phi(m)$.

As an immediate consequence of Fermat's theorem, we have its contrapositive which provides a useful method of determining whether an integer $m > 1$ is composite. We state it now as a theorem for later reference.

THEOREM 5. If $(a,m) = 1$ and $a^{m-1} \not\equiv 1 \pmod{m}$, then m is composite.

Another useful consequence of Fermat's theorem would be its converse which will now be stated as a conjecture.

Conjecture 1. If $a^{m-1} \equiv 1 \pmod{m}$ and $(a,m) = 1$, then m is a prime.

The above conjecture was believed to be true for many years. The ancient Chinese considered the congruence $2^{m-1} \equiv 1 \pmod{m}$ and believed that its existence for an integer $m > 1$ implied that m was a prime number. Their belief passed unchallenged among them for many years. But if one considers the following counterexample he will find that the conjecture is indeed false.

Example 1. Consider $m = 341$ and note that $2^{341-1} \equiv 2^{340} \equiv 1$ (mod 341). But $341 = 11 \cdot 31$ and hence 341 is not a prime. Therefore, the conjecture is false. Note however that $2^{10} \equiv 1$ (mod 341) and hence $(2^{10})^{34} \equiv 1$ (mod 341).

From the above example arises the following theorem which is a converse of Fermat's theorem with an extended hypothesis.

THEOREM 6. If $(a,m) = 1$ and a belongs to the exponent $m - 1$ modulo m $(m > 1)$, then m is a prime.

Proof: Suppose $(a,m) = 1$ and a belongs to the exponent $m-1$ modulo m. Then by Theorem 4, $(m-1) \mid \phi(m)$, where $\phi(m)$ denotes the number of integers in a reduced residue system modulo m. Hence $\phi(m) = (m-1) \cdot r$, where r is a positive integer. But $\phi(m) \leq m - 1$ and so r must equal 1. Hence $\phi(m) = m - 1$ and m must be a prime.

As a consequence of the above theorem one would note that if we can find *just one* integer a where $(a,m) = 1$ and a belongs to the exponent $m - 1$, then the integer m is a prime. We are guaranteed the existence of at least one such a if m is prime by the theory of primitive roots.

DEFINITION 2. If p is a prime and if a belongs to the exponent $p - 1$ modulo p, then a is called a primitive root modulo p.

THEOREM 7. Every prime p has $\phi(p-1)$ primitive roots.

Proof: Since p is a prime and $p - 1$ is a divisor of $p - 1$, it follows that there are $\phi(p-1)$ residue classes belonging to $p-1$ modulo p and, according to our definition of a primitive root, each of these classes is a primitive root modulo p.

Another interesting conjecture might be the following.

Conjecture 2. If $a^{m-1} \equiv 1 \pmod{m}$ for all integers a such that $(a,m) = 1$, then m is a prime.

This conjecture can be shown to be false by considering the counter-example below.

Example 2. Let $m = 561 = 3 \cdot 11 \cdot 17$. Let a be any integer such that $(a,3) = (a,11) = (a,17) = 1$. By Fermat's theorem we have $a^2 \equiv 1 \pmod{3}$, $a^{10} \equiv 1 \pmod{11}$, and $a^{16} \equiv 1 \pmod{17}$. Here $m - 1 = 560$ and we note that $2 \mid 560$, $10 \mid 560$, and $16 \mid 560$. Hence $a^{560} \equiv 1$ for each of the moduli 3, 11, and 17. Therefore, $a^{560} \equiv 1 \pmod{561}$. Hence the conjecture is false.

An interesting theorem results when we pick a particular integer a.

THEOREM 8. For every integer $a > 1$, there is an infinity of composite integers m satisfying $a^{m-1} \equiv 1 \pmod{m}$.

Proof: Let p be any odd prime which does not divide $a \cdot (a^2 - 1)$. Let

$$m = \frac{a^{2p}-1}{a^2-1} = \left(\frac{a^p-1}{a-1}\right) \cdot \left(\frac{a^p+1}{a+1}\right).$$

One will note that $a^p - 1$ and $a^p + 1$ can be factored into a product of factors with integer coefficients such that $a - 1$ and $a + 1$ are factors in their respective products and so m is clearly composite. Now

$$\begin{aligned}
(a^2-1)(m-1) &= (a^2-1)\left(\frac{a^{2p}-1}{a^2-1} - 1\right) \\
&= (a^2-1)\left(\frac{a^{2p}-1-a^2+1}{a^2-1}\right) \\
&= a^{2p} - a^2 \\
&= a(a^{2p-1} - a) \\
&= a(a^{p-1} - 1)(a^p + a).
\end{aligned}$$

Always a and a^p are of the same parity and so $2 \mid (a^p + a)$. But $a^{p-1} - 1$ is divisible by p by Fermat's theorem and is also divisible by $a^2 - 1$ since $p - 1$ is even. By our choice of p, $p \nmid (a^2 - 1)$ and so $(p, a^2 - 1) = 1$ which implies $p(a^2 - 1) \mid (a^{p-1} - 1)$. Hence $2p(a^2 - 1) \mid (a^2 - 1)(m - 1)$, since $2p(a^2 - 1) \mid a(a^{p-1} - 1)(a^p + a)$. Thus $2p \mid (m-1)$ so $m - 1 = 2pu$ or $m = 2pu + 1$ for some integer u. Then since $a^{2p} - 1 = m(a^2 + 1)$ we have $a^{2p} \equiv 1 + m(a^2 + 1) \equiv 1 \pmod{m}$ and hence $a^{m-1} \equiv a^{2pu} = (a^{2p})^u \equiv 1 \pmod{m}$. Since we have a different value of m for every odd prime p which does not divide $a \cdot (a^2 - 1)$, we will thus have an infinity of composite m for each integer a. Hence the theorem is proven.

In summary, one will note that we can conclude that for $(a,m) = 1$,

(1) if $a^{m-1} \not\equiv 1 \pmod{m}$, then m is composite;

(2) if $a^{m-1} \equiv 1 \pmod{m}$ and a belongs to the exponent m $-$ 1 modulo m, then m is a prime; and

(3) if $a^{m-1} \equiv 1 \pmod{m}$ and a does not belong to the exponent $m - 1$, then m could be either prime or composite; that is, in this case the test is inconclusive.

To devise a simple test for primality for the cases (2) and (3), we have the following theorem.

THEOREM 9. (A test for primality for integer m) If $a^{m-1} \equiv 1 \pmod{m}$ and if $m - 1$ has just k distinct prime factors (that is, if $m - 1 = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \ldots p_k^{\alpha_k}$), then to test for the primality of the integer m, determine whether $a^{(m-1/p_i} \equiv 1 \pmod{m}$ for any one of the k primes $p_1, p_2, \ldots, p_k$. Then,

(1) if there exists a prime p_i such that $a^{(m-1)/p_i} = 1 \pmod{m}$ we can reach no conclusion as to the primality of the integer m using this integer a. We would in this case choose another value for a, say a', such that $(a', m) = 1$ and $a \not\equiv a' \pmod{m}$ and then re-examine a^{m-1} modulo m for the new value of a.

(2) if $a^{(m-1)/p_i} \not\equiv 1 \pmod{m}$ for all primes p_i, then m is a prime number.

Proof: We are given that $a^{m-1} \equiv 1 \pmod{m}$. We wish to determine whether a belongs to the exponent $m - 1$ modulo m. If a does not belong to $m - 1$ modulo m then there exists an integer k such that $1 \leq k < m - 1$ and $a^k \equiv 1 \pmod{m}$. But by Theorem 3, we must have $k \mid (m - 1)$. If $m - 1 = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$, then the only primes which divide $m - 1$ are $p_1, p_2, \ldots, p_k$ and hence k is some combination of the various powers of the primes in the decomposition of $m-1$. Clearly, if $a^k \equiv 1 \pmod m$ and $1 \leq k < m-1$ then one of $a^{(m-1)/p_1}, a^{(m-1)/p_2}, \ldots, a^{(m-1)/p_k}$ must be congruent to 1 modulo m. For if none of these are congruent to 1 modulo m and $a^{m-1} \equiv a^{[(m-1)/p_i] \cdot p_i} \equiv 1 \pmod m$, then a must belong to the exponent $m-1$ modulo m, a contradiction. Hence if a does not belong to the integer $m-1$ modulo m then one of $a^{(m-1)/p_1}$, $a^{(m-1)p_2}, \ldots, a^{(m-1)/p_k}$ will be congruent to one modulo m. Hence for this integer a, we cannot reach a conclusion as to the primality of m. We would thus have to choose another value for a, say a', such that $(a',m) = 1$ and $a \not\equiv a' \pmod m$ and then re-examine a^{m-1} modulo m for the new value of a.

If none of $a^{(m-1)/p_1}, a^{(m-1/p_2}, \ldots, a^{(m-1)/p_k}$ are congruent to one modulo m then a belongs to the exponent $m-1$ modulo m and hence by Theorem 6, m is a prime. This concludes the proof of the theorem.

One will note that after one knows that $a^{m-1} \equiv 1 \pmod m$ where $(a,m) = 1$, the above theorem requires a maximum of k-tests to determine whether a belongs to the exponent $m-1$ modulo m. Therefore, this technique is a valuable time saver in computer calculations for the primality of an integer m.

A flow chart for the complete test for the primality of an integer $m > 1$ is indicated below.

In conclusion we examine the following two examples illustrating the results of computer calculations using this method.

Example 3. Consider $m = 511$. Since $(2,511) = 1$, we will use 2 as our value of a. We first test $2^{511-1} \equiv 2^{510} \equiv 64 \not\equiv 1 \pmod{511}$ Hence $2^{510} \not\equiv 1 \pmod{511}$ and so 511 is not a prime number.

Example 4. Consider $m = 601$. Since $(2,601) = 1$, we will let $a = 2$. We now test $2^{601-1} \equiv 2^{600}$ modulo 601. We find by computer calculation that $2^{600} \equiv 1 \pmod{601}$. Hence 601 could be either a prime or a composite number. We now wish to find an integer $a > 1$ such that $a^{600} \equiv 1 \pmod{601}$ and a belongs to the exponent 600 modulo 601. By computer calculation, the smallest integer greater than 1 belonging to 600 modulo 601 is the integer 7. Hence 601 is a prime.

Can you now write a computer program as indicated by the flow chart and test 4709 for primality?

FLOW CHART

Start   Read m,a $I = 1$ $I = I + 1$ $a = a'$

Is   where $m-1 = p_1^{\alpha_1}\ p_2^{\alpha_2} \ldots p_k^{\alpha_k}$

$a^{m-1} = 1 \pmod{m}$

?

No   Yes   Yes

Print   No   No

m is composite   Is

Choose another $\phi(m) - I$

a, say a', $\ni$ > 0

$(a',m)=1$ and   ?

$a' \neq a \pmod{m}$ Print

Yes.   Print

Is m is a prime

$a^{(m-1)/p_i}$ m is composite

$\equiv 1 \pmod{m}$ End

for any prime p_i,

Installation of New Chapters

EDITED BY LORETTA K. SMITH

Information for the department should be sent to Mrs. Loretta K. Smith, 829 Hillcrest Road, Orange, Connecticut 06477.

NEW YORK KAPPA CHAPTER

Pace University, New York, New York

The New York Kappa Chapter was installed at Pace University by Dr. William R. Smith, National President of Kappa Mu Epsilon, on 24 April 1974. Dr. Melvin Woodward, Chairman of the Mathematics Department, Indiana University of Pennsylvania, assisted Dr. Smith in initiating faculty, alumni, and students into Kappa Mu Epsilon. The faculty members initiated are: Dr. Joseph Houle, Dean of the School of Arts and Sciences; Dr. Louis Quintas, Chairman of the Mathematics Department; Dr. Bernkopf, Professor of Mathematics; Mr. William Adams, Assistant Professor of Mathematics; and Ms. Sandra Pulver, Instructor of Mathematics. Mr. John Ogle, Adjunct Assistant Professor of Mathematics, Mr. Isidore Dressler, Adjunct Assistant Professor of Education, and Mr. Gabriel Rosenberg, Adjunct Assistant Professor of Mathematics, also became members. Ms. Rochelle Rosenfeld, Instructor of Mathematics, is a member of the Hofstra University Chapter.

The following alumni and students are also charter members:

Berger, Kenneth
Buckley, Fred
Casey, Margaret
Costello, Jeanne Rybaczyk
Cuda, Rosa Leston
Cuda, Tony
Dedonato, Pamela
DeLuca, Tom
Demme, Lorraine
Emmon, Mary Anne
Fenster, Tom
Girard, Richard
Harvey, Jean
Hoefer, Peter
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Last, Walter
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Silber, Pearl
Thobin, Maria
Vittorio, Salvatore

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This index is a minimal one and consisting of an author index, a title index, and an index of installation of KME chapters. Since the list of articles is relatively short no subject index was attempted. Reports of national officers, including treasurer reports, are published in the Fall issue after every biennial convention beginning in 1941 (volume 1). Information about chapters maybe found in the KME Chapter News section of every issue.

This index was prepared by the current editor. He regrets any omission and mispelling that may have occurred in names. He also hopes that the index will help researchers discover valuable information in older issues of THE PENTAGON. The index is bound in the middle of this issue for easy removal.

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At the dinner held for the installation were several guests of honor. They were Dr. Edward Mortola, President of Pace University, Dean Houle, and Ms. Julie Thompson, Assistant Dean of the School of Arts and Sciences.

The officers of the New York Kappa Chapter are: President, Richard Girard; Vice President, Tom DeLuca; Secretary, Larry Jermyn; and Treasurer, Salvatore Vittorio.

+ + +

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Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President – Barbara Junghans
5 actives

At the March meeting each initiate gave a talk: Linda Starr on "Permutations and Combinations", Nancy Bernards on "The Magic of the Number Nine", and Sister Adrienne on "Instant Insanity and Related Topics". Other officers: Mary Lou Meyers, vice-president; Linda Starr, secretary; Karen Loesl, treasurer; Sister Mary Petronia, corresponding secretary and faculty sponsor.

The Book Shelf

Edited by O. Oscar Beck

This department of The Pentagon brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, University of North Alabama, Florence, Alabama 35630.

A Short Course in Differential Equations (5th Ed.) and *Elementary Differential Equations* (5th Ed.), Earl Rainville and Phillip Bedient, Macmillan Publishing Company, New York, 1974, 332 pp. and 525 pp., $10.95 each.

The two books are new editions of the books with the same titles written by Professor Earl D. Rainville with minor changes and one major addition in the chapter on systems of equations made by Professor P. E. Bedient.

The first book contains the first 16 chapters of the second book. It covers the usual topics very well, with good examples, exercises and applications.

The second book is suitable for a two-semester course in differential equations. Besides additional material on ordinary differential equations – such as power series solutions, singular points, hypergeometric equations and numerical methods – it contains chapters on partial differential equations, orthogonal sets, Fourier series, and boundry value problems.

Both books have been very popular with students; this popularity is well deserved, because the explanations are clear, the material is useful and basic, and the examples are well chosen. The second book is especially valuable for those who wish to acquire a good knowledge of the basic theory and the fundamental techniques for solving differential equations.

Juan C. Aramburu
University of North Alabama

Graphs, Models, and Finite Mathematics, Joseph Malkevitch and Walter Meyer, Prentice Hall, Englewood Cliffs, N.J., 1974, 525 pp., $10.95.

This book is designed for courses in finite mathematics or mathematics for the liberal arts student. Unusual features are its explicit use of the notion of mathematical modeling as unifying theme and the attention devoted to graph theory and its applications.

Dispensing with the usual chapters on set theory and logic, the authors immediately consider four "practical" problems and proceed to their solution by constructing mathematical models, thereby translating the problems into questions involving such graph theoretic concepts as Euler circuits, Hamilton circuits, and graph coloring. These and other topics from graph theory are then studied and illustrated with a variety of applications from such areas as chemical bonding, food webs, scheduling, and the critical path method. After an introduction to computers and flow charts, the authors discuss elementary matrix theory and apply it to graphs, giving an interesting application of graphs and their adjacency matrices to the problem of contamination in a food web.

Leaving graph theory, the authors turn to more traditional material, considering functions and their graphs, linear programming, statistics and probability with applications to genetics, the theory of games and decisions, and an interesting discussion of the theory of elections. The book closes on a less traditional note with a discussion of difference equations and their application to population growth and equilibrium conditions for populations of competitive species.

For the most part, the book is very well written. Concepts are well motivated and amply illustrated with interesting examples, and there are many helpful drawings. A large collection of exercises is provided, ranging from routine drill and applications to occasional proofs or development of new concepts. Praiseworthy features are the inclusion of pictures and biographical sketches of mathematicians whose contributions are discussed, and the chapter bibliographies recommending books or articles for further study.

The book has occasional typographical errors and incorrect answers to exercises, but no more than would be expected for a first printing. On rare occasions there is evidence of mathematical slopiness, as in the discussion of Euler circuits in which a graph with isolated vertices provides counterexamples to the theorems on pages 83 and 89. Occasional notational carelessness also intrudes, e.g., the reference (p. 79) to "cut point" instead of the previously defined "cut vertex", or the notation in the illustration of matrix multiplication (p. 149ff) which gives the impression that the product of a row matrix and a column matrix is a number instead of a 1 x 1 matrix. Such flaws are relatively infrequent, however, and they are more than balanced by a generally clear and readable style.

A minimal mathematical background is assumed — probably a year each of high school algebra and geometry would suffice. Consequently the level of the book might be unsuitable for many courses in finite mathematics. However, its expository style and the variety and relevance of its applications should make it an excellent text for the liberal arts student seeking an introduction to the spirit of mathematics and its applications.

Edward Z. Andalafte,
University of Missouri — St. Louis

Constructive Linear Algebra, A. Gewirtz, H. Sitomer, and A. Tucker, Prentice-Hall, Englewood Cliffs, N.J., 1974, 504 pp., $13.95.

This book is written at the freshman level and requires only high school mathematics, and it is quite different from other linear algebra books in circulation. Its objective is to stimulate interest and to develop mechanical techniques for applications in a way that is adaptable to use of a computer. Students will find it easy to read and more interesting than most. The authors take an algorithmic approach, motivating and developing techniques with numerous examples, followed by good lists of exercises. The pace is slow, but clarity of presentation should produce good results in the limited list of topics covered. The book should be considered for courses aimed at majors in biology, business administration, or the social sciences, and majors who are more mathematically inclined will find it useful if taken early.

The main consideration throughout is the treatment of systems of linear equations. Chapter 1 presents the simple ideas of linear programming to stimulate interest and to initiate an innovative mechanical technique which is used throughout the book. This technique, developed in Chapter 2, is a manipulation of a "tableau" as a means of changing the roles of basic and nonbasic variables in a mapping represented by a system of linear equations. Chapter 3 motivates and develops the algebra of matrices, and Chapter 4 discusses the solving of systems of equations by the "pivotal exchange" method (developed in Chapter 2) and by Gaussian elimination. In Chapter 5, properties of the vector space R^n are developed, and applications to affine geometry are considered in Chapter 6. Linear mappings from R^m to R^n are discussed in Chapter 7. Chapter 8 gives a constructive definition of determinants and discusses some uses.

C. J. Pipes
Southern Methodist University

MINIREVIEWS

Mathematics for the Biological Sciences, Stanley I. Grossman and James E. Turner, Macmillan Publishing Company, New York, 1974. 526 pp., $10.95.

Recognizing the rapid rise in the application of varied mathematical tools to many kinds of biological phenomena, the authors present this work with two aims – to make the relevant mathematics accessible in a reasonable time and to develop the ability of students to relate mathematics to problems in biology and medicine. The included topics are probability, vectors and matrices, linear programming, Markov chains, game theory, difference equations, and differential equations. The prerequisites are minimal; only high school algebra is assumed in the first six chapters, while the remaining three chapters require some of the basics from calculus. (Two of the appendices were designed to serve as a one-term course in calculus if so needed.) The authors suggest several paths one might follow in designing a course from this text. Throughout the book the emphasis is on the biological motivation at each step.

Algebra Text: Elementary and *Algebra Text: Intermediate*, Robert Alwin, Robert Hackworth, and Joseph Howland, Prentice-Hall, Englewood Cliffs, N. J., 1974, 714 pp. and 666 pp., $8.95 each (paper).

These two paper-bound works have excellent potential for students needing first-time or remedial exposure to algebra. Each is a self-contained instructional text designed for use either as a self-study guide or as a text in a more traditional class. Topics are contained in "Learning Modules" which are further subdivided into "units" and "sections." Learning objectives and progress tests are included in addition to the usual exercise sets at the end of each unit. The *Elementary* text includes the topics of sets, counting numbers, rational numbers, equations with two variables, polynomials, factoring, fractions, fractional equations, and quadratic equations. Included in the *Intermediate* text are systems of numbers (rational, real, complex), relations and functions, inequalities, absolute value, graphing, exponents, factoring, and logarithms.

Finite Mathematics, Hugh Campbell and Robert Spencer, Macmillan Publishing Company, New York, 1974, 336 pp., $9.95.

The authors of this book treat the standard topics of finite mathematics, such as algebraic systems, logic, sets, probability, matrices, linear programming, and game theory. Additionally, they have made a good effort to provide the reader with some application of each topic to indicate its usefulness; many applications are referenced to permit further investigation. Many of the applications and examples come from the areas of social, management, and life sciences. The book has been organized so that various options are available for designing a course from it. The prerequisites for each chapter are stated at the beginning of the chapter. Chapter 1 requires only high school algebra.

Mathematical Games and Puzzles, Trevor Rice, St. Martin's Press, New York, 1974, 95 pp., $5.95.

This book is a collection of 43 puzzles and games by a British author. Old and new items are included, but mostly traditional games are presented. Geometric puzzles include Tangrams, pentominoes, and the Steinhaus cube. Constructible models include pyramid rings, hexa-hexaflexagons, and the pantograph. Standard "magic" tricks with numbers and vanishing figures are included. Games such as Lucas' problem, the 15 puzzle, hex, and nine men's morris (or mill) complete the book.

The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before March 15, 1975. The best solutions submitted by students will be published in the Spring 1975 issue of The Pentagon, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

267. *Proposed by Charles W. Trigg, San Diego, California.*

In what triangles do an altitude, median and angle bisector from one vertex of the triangle divide the angle at the vertex into four equal parts?

268. *Proposed by Charles W. Trigg, San Diego, California.*

Find the unique four-digit integer in the decimal system which can be converted into its equivalent in the septenary system by moving its left hand digit to the far right.

269. *Proposed by Leigh James, Rocky Hill, Connecticut.*

Find a formula for $\sum_{k=0}^{n-1} (-1)^k(n-k)^2$ where n and k are integers.

270. *Proposed by Charles W. Trigg, San Diego, California.*

The expression $abcd$ is capable of two interpretations: (1) the product of the four quantities $a,b,c,$ and d; (2) an integer with digits in position; i.e., the sum of the four digits a,b,c and d each multiplied by an appropriate power of the base in which the integer is expressed. Does any integer exist in any system of notation for which the two interpretations lead to the same final value?

271. *Proposed by the editor.*

A regular polygon is cut from a piece of cardboard. A pin is put through the center to serve as an axis about which the polygon can rotate. Find the least number of sides which the polygon can have in order that a rotation through an angle of $27\frac{1}{2}$ degrees will put the polygon in coincidence with its original positon.

SOLUTIONS

260. *Proposed by Charles W. Trigg, San Diego, California.*

157	13	211
131	127	73
43	241	97

is a magic square with prime elements. Form another magic square with nine prime elements, seven of which are the same as those of the given square.

Solution by the editor.

In order for the given square to be magic, the number 131 must be replaced with 181 which has been done in this solution. Since each of these primes has the form $6k + 1$, the given square is equivalent to

26	2	35
30	31	12
7	40	16

which can be rearranged as

2	7	12
16	21	26
30	35	40

The numbers in the second figure have the property that they form three arithmetic progressions both horizontally and vertically. The horizontal progressions all have the same constant difference and similarly for the vertical progressions. It is well known that numbers having these properties can be used to construct 3 x 3 magic squares. (See Kraitchik's *Mathematical Recreations* or Andrew's *Magic Square and Cubes,* pp. 129-133.)

To solve our problem we need to replace exactly two elements in the second figure above without disturbing the arithmetic progression properties. This can be done by picking elements on opposite corners (this is tantamount to shifting the top row to the right (left) and the bottom row to the left (right) one position in each case). Then check to see if the "new" elements generate primes of the form $6k + 1$. We obtain two new arrays which yield the respective magic squares:

−3	2	7
16	21	26
35	40	45

and

7	12	17
16	21	26
25	30	35

157	−17	241
211	127	43
13	271	97

and

157	43	181
151	127	103
73	211	97

261. *Proposed by Charles W. Trigg, San Diego, California.*

In the scale of eight there is just one integer n such that n and n^2 together contain the eight digits once each. Find n and show it to be unique.

Solution by the editor.

Since $8^k - 1$ is a multiple of 7 for all nonnegative integers k, any integer (base 8) of the form $a\ b\ c\ \ldots$ becomes a multiple of 7 upon subtracting $a + b + c + \ldots$. (This is equivalent to saying that $a\ b\ c\ \ldots \equiv a + b + c + \ldots \pmod 7$.) Then since n^2 and n utilize each digit exactly once, then $n^2 + n - (0 + 1 + 2 + 3 + 4 + 5 + 6 + 7) = n^2 + n - 28$ is a multiple of 7. Hence $n^2 + n$ is also a multiple of 7 which implies that either n or $n + 1$ must be a multiple of 7 and n must have a digital sum of 6 or 7 (note the similarity to "casting out nines" in the decimal system). Then since $76543 \geq n^2 \geq 20134$, $263 \geq n \geq 133$ and the possible choices for n are: 257, 256, 250, 247, 241, 240, 232, 231, 223, 222, 214, 213, 205, 204, 176, 175, 167, 166, 160, 157, 151, 150, 142, 141, 133. Now all squares in base eight end in 0, 1 or 4. Hence we have:

1. n cannot contain the digit 1 if n is odd,
2. even n cannot end in 0 and contain the digit 4 or end in 4 and contain the digit 0 or end in 2 or 6 and contain the digit 4,
3. n cannot contain repeated digits.

Checking the possibilities, only these need testing: 257, 256, 247, 214, 205, and 176. Of these only $n = 256$ yields a solution, which is $n = 256$ and $n^2 = 73104$.

Kappa Mu Epsilon News

EDITED BY ELSIE MULLER, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Elsie Muller, Historian, Kappa Mu Epsilon, Department of Mathematics, Morningside College, Sioux City, Iowa 51106.

CHAPTER NEWS

Alabama Beta, Florence State University, Florence

Chapter President — Vann Bush
35 actives

The year was concluded with an initiation banquet on 23 April 1974 at which twenty new members were initiated. Other officers: Dan Copeland, vice-president; Carolyn Malone, secretary and treasurer; Jean T. Parker, corresponding secretary; and Dr. Eddy Joe Brackin, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President — Terry L. Shell
44 student members, 19 faculty members

Monthly meetings include chapter business, student speakers, faculty speakers, or special visiting lecturers. During the year there is one banquet and two or three initiations. In the mathematics laboratory, the chapter makes available a tutorial service for all students on the campus. Displays are made for *Poly Royal*, the annual all-campus open house. Members also assist the mathematics faculty in the annual mathematics contest for more than 500 high school students. Other officers: Douglas R. Morgan, vice-president; Sandra L. McKaig, secretary; Karen L. Barks, treasurer; Dr. George Mach, corresponding secretary; Dr. Ralph M. Warten, faculty sponsor.

Colorado Beta, Colorado School of Mines, Golden

Chapter President – Richard Fahlsing
20 actives – 36 pledges

A most enjoyable meeting of the year was a field trip to the National Bureau of Standards. The chapter provides a tutorial program for the lower level courses and prepares an exhibit for *Engineer's Day*. Other officers: Lloyd Scheidt, vice-president; Jim Baylis, secretary; Melinda Smith, treasurer; Ardel Boes, corresponding secretary; John Kork, faculty sponsor.

Illinois Alpha, Illinois State University, Normal

Chapter President – Bruce Iverson
32 actives – 6 pledges

The Thursday night monthly meetings include a *Get-to-Know* faculty night, lectures on job opportunities by alumni, as well as other guest lecturers who speak on interesting mathematical topics. New projects for the coming school year include special study sessions to review for the fall actuarial examination and possibly a baby-sitting service. Other officers: Rose Danelczuk, vice-president; Rita King, secretary; Carol Born, treasurer; Marci Traurnicht, corresponding secretary; Dr. Oryln Edge, faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston

Chapter President – Bill Taber
48 actives

This year the annual spring KME and Mathematics Club trip for members, pledges, and guests was taken to St. Louis, Missouri for a week-end with 13 members and 8 guests participating. Four of the five scholarship awards ranging from $25 to $500 from the mathematics department go to KME members. These awards are presented and the new initiates are honored at a banquet each spring sponsored jointly by the department and Illinois Beta. Other officers: Kevin Settle, vice-president; Connie Kutosky, secretary and treasurer; Ruth Queary, corresponding secretary; Larry Williams, faculty sponsor.

Illinois Delta, College of St. Francis, Joliet

Chapter President — Gary Bebar
10 actives — 4 pledges

Other officers: Angy Speca, vice-president; Dr. Arnie Good, faculty sponsor.

Indiana Gamma, Anderson College, Anderson

Chapter President — Carol Buehler Miller
13 actives

The initiation was held on 31 March 1974 when 7 students were inducted into the chapter. Other officers: Timothy A. Sipka, vice-president; Deborah Ware, secretary; Stanley L. Stephens, corresponding secretary.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President — Patricia Johnson Fox
22 actives

The annual KME — mathematics faculty picnic was quite successful. The chapter is again sponsoring a homecoming breakfast, hosted by Dr. Fred Lott, for mathematics faculty and KME alumni. Monthly chapter meetings feature presentation of one or more student papers. Other officers: Shirlee S. Huisinga, vice-president; Jo Ann Wenthold, secretary and treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Delta, Wartburg, College, Waverly

Chapter President — Robert Basham
40 actives

Other officers: David Neve, vice-president; Pamela Snyder Egts, secretary; Laurel Kuntz, treasurer; August W. Waltmann, corresponding secretary and faculty sponsor.

Iowa Gamma, Morningside College, Sioux City

Chapter President — Marc Burkhart
23 actives

In the fall the chapter sponsors a Freshman-Sophomore Orientation to introduce KME to the new students in mathematics as well as to discuss careers related to mathematics. Professors James Cornette and Arlington Fink of Iowa State University were guest lecturers during the year. At Homecoming the members entertain the alumni at a breakfast. Three students attended the regional convention. In the spring the members cooperated with the mathematics faculty in hosting an *Open House* for high school students in the area. Other officers: Mark Fegan, vice-president; Raejean Pacholke, secretary; Dale Lenderts, treasurer; Michael Sorn, secretary; Elsie Muller, corresponding secretary and faculty sponsor.

Kansas Alpha, Kansas State College of Pittsburg, Pittsburg

Chapter President — Gary Morella
45 actives

Programs were given by Jeannett Quigley, "Numbers, Bases, And Computers"; Dr. Gary McGrath, "Logic Circuits"; Jim Wylie, "Simulations Modeling In Macroeconomic Analysis"; Laura Spain and Margie Davis, "A Report On The Regional Convention". At the regional convention in Rolla, Missouri Linda Funk presented a paper, *Plane Patterns* for which she was awarded second place. In May the recipients of the annual Robert M. Mendenhall Awards for scholastic achievement were Beth Gray, Randall Timi, and Steve Scott. Each received a KME pin. Other officers: Laura Spain, vice-president; Judy Wilson, secretary; Roy Bryant, treasurer; Dr. Harold Thomas, corresponding secretary; Prof. J. Bryan Sperry, faculty sponsor.

Kansas Beta, Emporia Kansas State College, Emporia

Chapter President — Patti Shirley
4 pledges

The name of the college has been changed from Kansas State

Teachers College to Emporia Kansas State College. Other officers: Beth Ann Ridenour, vice-president; Jane Nietfeld, secretary; Gregg Stair, treasurer; Dr. Donald Bruyr, corresponding secretary; Dr. Tom Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison

Chapter President – Unknown
9 student members – 3 faculty members

An interesting program developed this past semester because recently several students became interested in actuarial science. The chapter hosted a guest speaker from Kansas City Life Insurance Company who described a typical day in the firm as an actuary. Some of the students then took the spring actuarial examinations. The senior mathematics majors were honored at a steak cookout on the lawn of a faculty member's home. The Sister Helen Sullivan Scholarship was awarded to Mary Kay Stewart, a senior mathematics major from Durango, Colorado. The corresponding secretary is Sister Jo Ann Fellin.

Kansas Epsilon, Fort Hays Kansas State College, Hays

Chapter President – Jean Ingersoll
18 actives

Other officers: Brenda Mauck, vice-president; Camellia Tuttle, secretary; Marilyn Horinek, treasurer; Eugene Etter, corresponding secretary; Dr. Charles Votaw, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President – M. Claire Wagner
79 actives – 3 pledges

During the past year Maryland Alpha and Maryland Beta had two joint meetings where, in each case, three student papers were presented. The plans are to continue with such joint meetings. For the local chapter three meetings were devoted to papers by Kathryn

Kwiatkowski, *Six Started A Society;* Julie Kennedy, *Where Do I Go From Here;* and Mini Schultes, *Trigonometric Analysis Of Musical Sounds.* Five members of the chapter attended the Region 1 meeting in April at LaSalle College in Philadelphia. Other officers: Cathy Brown, vice-president and treasurer; Coleen Baum, secretary; Sister Marie Augustine Dowling, corresponding secretary.

Maryland Beta, Western Maryland College, Westminster

Chapter President – Mark Miller
11 actives

The following programs are tentatively planned for the coming year: A talk on operations research by Dr. W. Edward Cushen, an alumnus and trustee of WMC; a picnic for mathematics majors; a car wash; 2 joint meetings with Maryland Alpha; careers night; a film; and a banquet at the induction of new members. Other officers: Bette Gemma, vice-president; Carol Zynel, secretary; Douglas Bitz, treasurer; James E. Lightner, corresponding secretary; Robert P. Boner, faculty sponsor; Susan Steinhilber, historian.

Michigan Alpha, Albion College, Albion

The chapter has become inactive.

Michigan Beta, Central Michigan University, Mt. Pleasant

Chapter President – Don Palmer
40 actives

The programs at the monthly meetings are usually provided by the members on mathematical topics. Activities include the freshman mathematics exams, student reports, and a volley ball game with the faculty. Other officers: Sharon Phare and Elaine LaChapelle, vice-presidents; Pat Carls, secretary; Vicki Hagley, treasurer; Dean O. Hinshaw, corresponding secretary and faculty sponsor.

Mississippi Alpha, Mississippi University for Women, Columbus

Chapter President – Gail Brownlee
18 actives

Twelve new members were initiated on 25 February 1974. Programs for the year included the following:

Dr. Jean Parra – THE HISTORY OF PROJECTIVE GEOMETRY

Dr. James Johnson – MATHEMATICAL STRATEGY GAMES

Mr. Gil Harris – USE OF COMPUTERS IN INDUSTRY

Miss Shirley Killebrew – APPLIED MATHEMATICS

Other officers: Sue Ellen Hange, vice-president; Sandra Oliver, secretary and treasurer; Dr. Jean Ann Parra, corresponding secretary and faculty sponsor.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg

Chapter President – Harry Dole
20 actives

Initiation of new members was held at the spring quarter cookout in May. During the winter quarter there were 2 meetings with invited speakers. Other officers: Sherry Forten Berry, vice-president; Frances Weber, secretary; Jack Munn, corresponding secretary; Mrs. Alice Essary, faculty sponsor.

Missouri Beta, Central Missouri State College, Warrensburg

Chapter President – Stephen Lacy
25 actives – 23 pledges

Seven meetings were held during the past year for which programs were prepared each time. Other officers: Robert Stuckmeyer, vice-president; Debbie West, secretary; Donna Lay, treasurer; Dr. Homer F. Hampton, corresponding secretary and faculty sponsor.

Missouri Eta, Northeast Missouri State University, Kirksville

Chapter President — Leila Barge
21 actives — 11 pledges

One of the programs was a treasure hunt with mathematical clues. The chapter also conducted a high school mathematics contest. Eighteen members attended the Region 4 meeting at Rolla. Other officers: Ken Eccher, vice-president; Pam Keller, secretary; Janet Sunstrom, treasurer; Sam Lesseig, corresponding secretary; Mrs. Beersman, faculty sponsor.

Nebraska Gamma, Chadron State College, Chadron

Chapter President — Stephanie Larsen
24 actives — 2 pledges

Each year KME sponsors a high school band in the Homecoming parade. Each fall and each spring there is a picnic with the mathematics faculty. At Christmas time there is a caroling party with the other mathematics and science organizations on campus. The chapter assists with *Scholastic Day,* the time when neighboring high schools gather at Chadron for inter-school tests. Duff McCafferty, the outstanding freshman mathematics student, received the award, a slide rule, on *Ivy Day.* Other officers: Dave Wolff, vice-president; Lynne Lux, secretary; Rod Cain, treasurer; James Kaus, corresponding secretary and faculty sponsor.

New Jersey Beta, Montclair State College, Upper Montclair

Chapter President — Gail D. Hofmann
24 actives — 30 pledges

In April, the club acted as hosts for visiting high school seniors on Math Day and also sponsored a mathematics contest in which local schools competed. In addition, many members attended the NCTM convention in Atlantic City on 18 April. Thirty-three new members were welcomed in the induction ceremony held on 15 May at the Robin Hood Inn. On the lighter side the club sponsored a Christmas party for members and faculty, a volley ball game, and a picnic for the entire mathematics department. Other officers: Janet Lynn Herman, vice-president; Janice Nagy, secretary; Robert Brau, treasurer; Elaine Telewski, historian; Dr. Philip Zipse, corresponding secretary and faculty sponsor; Dr. Samuel Heft, faculty sponsor.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President – David Billingsley
30 actives

Other officers: Mike Huffman, first vice-president; Patrick Garcia, second vice-president; Beverly Riese, secretary; John Starrer, treasurer; Merle Nutchell, faculty sponsor.

New York Gamma, Suny College at Oswego, Oswego

The chapter has become inactive.

New York Zeta, Colgate University, Hamilton

The chapter has become inactive.

New York Eta, Niagara University, Niagara

Chapter President – Mike Gallagher
23 actives

At the regional convention at LaSalle College in Philadelphia Carol Schwab presented her paper, *Examination Of Semigroups: An Isomorphic Approach,* for which she received first prize in her division. Other officers: Peter Fallon, vice-president; Wai Ming Tom, secretary; Mary Kroviak, treasurer; Robert Bailey, corresponding secretary; Sr. John Frances Gilman, faculty sponsor.

New York Iota, Wagner College, Staten Island

Chapter President – Diane Morse
15 actives – 12 pledges

The chapter continues to publish the *Mathematics Monthly,* edited by Nick Phillips and Brian Manske. It is sent to the local high schools and the mathematics majors at Wagner. Professors Welton and McCoy were the speakers at the two induction ceremonies. Other officers: Ellen Lepowsky, vice-president; Anita Globbie, secretary; Anthony Tropeano, treasurer; Professor Mary Petras, corresponding secretary; Dr. Raymond Traub, faculty sponsor.

Ohio Zeta, Muskingum College, New Concord

Chapter President — Daniel Ledsome
29 actives

The chapter hosted the Region 3 convention on 1-2 March 1974 at which the following members gave papers: Al Hill, Mary Pritchard, Sue Syroski, Al Hurst, and Emily Henderson. At the May meeting Dr. Robert Wilkins of Ohio University spoke on the topic, "Urban Traffic Simulator." At the spring banquet Kenneth Fouts received the freshman mathematics award. Other officers: Mary Galani, vice-president; Debi Lewis, secretary and treasurer; Dr. J. L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern State College, Weatherford

Chapter President — Marton Powell
31 actives — 18 pledges

The chapter participated in two fund raising projects and sponsored an end-of-the-school-year picnic for the mathematics department. Other officers: Debra Coody, vice-president; Debra Miranda, secretary; Rex McAnally, treasurer; Dr. Wayne Hayes, corresponding secretary; Dr. Don Prock, faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington

Chapter President — David Carothers
43 actives

Other officers: Richard Buckman, vice-president; Nancy Cooper, secretary; James Goldbach, treasurer; Mr. J. M. Peck, corresponding secretary; Dr. J. R. Nealeigh, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President – Maxine Cranage
10 actives

Other officers: Kathi Baver, vice-president; Barbara Diehl, secretary; Diane Green, treasurer; Irving Hollingshead, corresponding secretary; Edward Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President – Gaylen Hauze
60 actives

Guest lecturers during the past year have been Dr. John Broughton with the talk, "Some Intersection Properties of Convex Sets", and Dr. Joseph Angelo on "Notes off the Cuff of a Mathematics Professor." Other officers: Wayne Wendt, vice-president; Theresa Krizay, secretary; Nancy Schwetz, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsyvania Eta, Grove City College, Grove City

Chapter President – Dan McWhertor
30 actives

Each applicant for membership in KME must submit a paper on some mathematical topic which has been approved by a faculty member. Other officers: Karen Williams, vice-president; Carolyn Lape, secretary; Betsy Guffey, treasurer; Marvin C. Henry, corresponding secretary; Cameron C. Barr, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia

Chapter President – Marguerite Leicht
6 actives

During the past year weekly problem sessions were held at which time old and new topics in mathematics were discussed. It is planned to set up tutoring sessions for non-mathematics majors who are experiencing difficulties in their mathematics courses. In March the chapter sponsored a guest speaker, Dr. Peter Hagis of Temple University, who talked about perfect numbers. At the initiation of new members in April, Dr. James Crawford of Lafayette College presented "Building Probability Models or How Not to Solve It". Other officers: JoAnn Dellavalle, vice-president; Kathleen Britt, secretary; Robert Papsun, treasurer; Margaret Jankowski, corresponding secretary; and Mr. Louis F. Hoelzle, faculty sponsor.

Pennsylvania Lambda, Bloomsburg State College, Bloomsburg

Chapter President — Janet Fiora
25 actives

Other officers: Mary Lepley, vice-president; Nancy Kulp, secretary; Jean Clemo, treasurer; Dr. James Pomfret, corresponding secretary; Joseph Mueller, faculty sponsor.

Tennessee Delta, Carson-Newman College, Jefferson City

Chapter President — Candace Hammer
20 actives

The members initiated at the 6 April banquet numbered seventeen, the largest group taken into the chapter since the admittance of the 22 charter members in 1971. Other officers: Steven Hansen, vice-president; Jane Rohrer, secretary; John C. Fraley, treasurer; Denver R. Childress, corresponding secretary; Howard Chitwood, faculty sponsor.

Texas Beta, Southern Methodist University, Dallas

Chapter President — Roslyn Slapper
23 actives — 28 pledges

David M. Tobolowsky was the first recipient of the John David Brown mathematics award last spring. Other officers: Morey Silverman and Brooks Haney, vice-presidents; Connie Smith, secretary and treasurer; R. V. Morgan, corresponding secretary; Richard K. Williams, faculty sponsor.

(Continued on page 47)