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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

An Introduction to Game Theory And The Simplex Method*

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Almost everyone has heard of parlor games such as checkers, chess and poker, and knows that certain strategies and chances are involved. The term "game" in game theory might suggest that the subject is narrow and unimportant, but this is certainly not the case. John von Neumann and Oskar Morgenstern [1, p. viii] felt that "the typical problems of economic behavior become strictly identical with the mathematical notions of suitable games of strategy." Today game theory has opened a large new field of research. The applications are not restricted to parlor games. Actually, game theory was originally created to provide a new approach to economic problems. It has also been applied to political science, pure mathematics, psychology, sociology, marketing, finance, and warfare.

The term "game" means many different things to different people, but most games are made up of similar elements. They involve players, their actions or decisions, and as a result of these, a definite outcome – a gain or loss for the players. A player can be one person in competition with another person, or it can be a team, a corporation, or even a nation. In this paper any group of individuals who have a common interest will be regarded as a single player.

A game is a model of real life and one model could hardly reflect all activities of life. Therefore, game theory is actually made up of many theories. Generally, the fewer the players, the simpler the game. As one progresses from simple games to more complex ones, the theories become less and less applicable. In other words, "the greater the significance of a game – that is, the more applications it has to real problems – the more difficult it is to treat analytically," as stated by Morton D. Davis in *Game Theory* [1, p. x]. In more complex games, a player is faced with forces that he cannot control. The less control that a person has on a situation or on the final outcome, the more difficult that game is to analyze.

* A paper presented at the 1973 biennium convention of KME at Sioux City, Iowa.

There are several kinds of games. One person games and games against nature are the simplest. Two person games are much more complex. This article will discuss only one person games and two person zero-sum games.

One person games are so simple they are often not even considered games at all. There are three types of one person games:

1) Games in which nature has no role at all. In this type of game there is a connection between choice and strategy. For example, a man in an elevator makes the choice of which button to push and that automatically determines what floor he will end up on — what the outcome will be.

2) Games which incorporate laws of chance. Decisions are made on a basis of information. For instance, a man must decide what crops to plant. He will not know exactly what the weather will be like, but through past experience he can make his decision.

3) Games with no advance information about how nature will play. For example, a manufacturer is offered the rights to a new but imperfect product and he must decide whether to accept or reject it. He has no idea whether it will sell or not — no advance information — but he must make his decision.

One person games are the simplest extrema problems. The best solution to a one person game is the one which brings a predetermined gain. The game-theorist can make little or no contribution. The decision whether to gamble and which strategy to use lies entirely up to the player. One person games are relatively unimportant. The essential point is that a player must have decided on his goals before he can find a course of action.

In two person zero-sum games, the sum of the winnings is naturally zero. In other words, what Player *A* wins, Player *B* loses. The essential elements of two person zero-sum games are:

1) There are two players.

2) They have opposite interests with respect to the outcome of the game.

3) There are a finite number of alternatives. That is, the game is ended after a finite number of moves.

There are two types of zero-sum games – that of perfect information and that of imperfect information. In games of perfect information the players are completely informed of all possible alternatives. Parlor games such as chess, checkers and tick-tack-toe are examples of games of perfect information. In games of imperfect information, strategies are played simultaneously so that neither player knows what the other is going to do, thus, imperfect information. An example of an imperfect information game is "Stone, Paper, Scissors."

Games of perfect information have what is called an "optional pure strategy". In other words, there is a sequence of moves such that the player using it will have the safest strategy possible, regardless of what his opponent does. If he has a winning strategy, he can win no matter what his opponent does; in fact, he can announce his strategy in advance. If there is sensible play, then the outcome is predetermined or "strictly determined". In 1912, Ernest Zermelo, in what was probably the earliest paper on game theory, proved that games of perfect information are strictly determined [1, p. 13].

In games of imperfect information the situation is more complicated. Players must choose their strategies simultaneously, neither knowing what the other is going to do. The player's principle concern is keeping his strategy from being discovered. Consider an imperfect information problem in which two manufacturers are competing for a given consumer market, and each is considering three different sales strategies. This is symbolized in the given matrix. The number in each entry indicates the pay-off for A over

	$B1$	$B2$	$B3$
$A1$	2	1	4
$A2$	2	3	2
$A3$	2	−1	1

B. We can see by inspection that the worst possible pay-off that Player A can expect in row one is 1, in row two it is 2 and in row three it is −1. Since he wants the maximum payment, Player A would naturally choose row 2. Player *B* sees that the most he has to pay to *A* in column one is 2, in column two it is 3, and in column three it is 4. He naturally wants to pay the minimum, which is located in column 1. These choices would result in the value 2. This is called the minimax value of the game.

The Minimax Theorem by John von Neumann [1, p. 38], states that one can assign to every finite, two person, zero-sum game a value z, which is the average amount that Player *A* can expect to win from Player *B* if both players act sensibly. This predicted outcome is plausible for three reasons:

1) There is a strategy for Player *A* that will protect this return. Against this strategy nothing that Player *B* can do will prevent Player *A* from getting an average return of z.

2) There is a strategy for Player *B* that will guarantee that he will lose no more than an average value of z – that is, Player *A* will be prevented from getting any more than z.

3) By assumption, the game is zero-sum. What Player *A* gains, Player *B* must lose. Since Player *B* wishes to minimize his losses, Player *B* is motivated to limit Player *A*'s average return to z.

Because of the Minimax Theorem, we can now treat all two person zero-sum games as though they had equilibrium points. The game has a clear value, and either player can enforce this value by selecting the appropriate strategy. The virtue of the Minimax Theorem is security.

Let us examine a second matrix.

	*B*1	*B*2	*B*3
*A*1	4	1	1
*A*2	0	3	1
*A*3	0	0	2

With inspection we can see that there is no one simple best strategy. If A chooses strategy $A1$, Player B can limit his profit to one unit by using strategy $B2$ or $B3$. If A chooses strategy $A2$ or $A3$, Player B can deprive him of any profit by choosing strategy $B1$. Therefore, it should be obvious that each player should use a combination of strategies in order to gain the most profit. We can use what is called the Simplex Method to solve this problem.

Let (p,q,r) represent Player A's strategy. We know that

$$p \geq 0 \qquad q \geq 0 \qquad r \geq 0$$

$$p + q + r = 1$$

Then A's payoffs against B's strategies can be represented by

$$\begin{aligned} &4p + 0q + 0r \\ &1p + 3q + 0r \\ &1p + 1q + 2r \end{aligned}$$

Let g denote the largest of the three payoffs. Then

$$\begin{aligned} 4p + 0q + 0r &\leq g \\ 1p + 3q + 0r &\leq g \\ 1p + 1q + 2r &\leq g \end{aligned}$$

$$\frac{p}{g} + \frac{q}{g} + \frac{r}{g} = \frac{1}{g}$$

If we divide through by g we can see that this will not change the inequalities.

$$\frac{4p}{g} + \frac{0q}{g} + \frac{0r}{g} \leq 1$$

$$\frac{1p}{g} + \frac{3q}{g} + \frac{0r}{g} \leq 1$$

$$\frac{1p}{g} + \frac{1q}{g} + \frac{2r}{g} \leq 1$$

Now let us simplify these inequalities by letting

$$x = \frac{p}{g} \qquad y = \frac{q}{g} \qquad z = \frac{r}{g}$$

Since g is a maximum, then $\frac{1}{g} = m$ is a minimum. Therefore

$$x + y + z = m \text{ or } m - x - y - z = 0.$$

Now our inequalities look like this:

$$\begin{aligned}
4x + 0y + 0z &\leq 1 \\
1x + 3y + 0z &\leq 1 \\
0x + 1y + 2z &\leq 1 \\
m - x - y - z &= 0
\end{aligned}$$

We can change these to equalities by adding new variables.

$$\begin{array}{lllllll}
4x + 0y + 0z & + a & & & & = & 1 \\
1x + 3y + 0z & & + b & & & = & 1 \\
1x + 1y + 2z & & & + c & & = & 1 \\
-x - y - z & & & & + m & = & 0
\end{array}$$

We can now set up the matrix.

$$\begin{array}{ccccccc|c}
x & y & z & a & b & c & m & \\
4 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 2 & 0 & 0 & 1 & 0 & 1 \\
\hline
-1 & -1 & -1 & 0 & 0 & 0 & 1 & 0
\end{array}$$

Using row operations we obtain the new matrix:

$$\begin{array}{ccccccc|c}
x & y & z & a & b & c & m & \\
1 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} = x \\
0 & 1 & 0 & -\frac{1}{12} & \frac{1}{3} & 0 & 0 & \frac{1}{4} = y \\
0 & 0 & 1 & -\frac{1}{12} & -\frac{1}{6} & \frac{1}{2} & 0 & \frac{1}{4} = z \\
\hline
0 & 0 & 0 & \frac{1}{12} & \frac{1}{6} & \frac{1}{2} & 1 & \frac{3}{4} = m
\end{array}$$

Thus $m = \frac{1}{g} = 3/4$, so $g = 4/3$. Also $xg = p$, $yg = q$, and $zg = r$.

So we have

$$p = xg = \tfrac{1}{4}\ (4/3) = 1/3$$
$$q = yg = \tfrac{1}{4}\ (4/3) = 1/3$$
$$r = zg = \tfrac{1}{4}\ (4/3) = 1/3$$

Referring back to the original problem of the two manufacturers, A's optional strategy is (1/3, 1/3, 1/3) with his expected payoff being 4/3.

We can use the same method to find Player B's optional strategy. We find that B's best strategy is (1/9, 2/9, 2/3), with his expected payoff being 4/3.

Using Game Theory we can solve a game for the best strategy to win the most. But we must understand that the value of the theory is not to solve games. In most games we have only an approximation of real life. Game Theory offers specific solutions to highly simplified situations, but in reality we can only approximate the solutions. Problem solving is not the intrinsic value of the game. The true value of game theory is that it shows to us the different kinds of conflicts. Anatol Rapoport [4, p. 310] says, "In bringing techniques of logical and mathematical analysis to bear on problems involving conflicts of interest, game theory gives men an opportunity to bring conflicts up from the level of fights, where the intellect is beclouded by passions, to the level of games, where the intellect has a chance to operate." This is definitely the most important achievement of game theory.

REFERENCES

1. Davis, Morton D. *Game Theory — A Non-technical Introduction.* New York: Basic Books, Inc., 1970.

2. Glicksman, S. M. *An Introduction to Linear Programming and the Theory of Games.* New York: John Wiley and Son's, 1963.

3. Owen, Guillermo. *Game Theory.* Philadelphia: W. B. Saunders Company, 1968.

4. Morgenstern, Oskar, "The Theory of Games", *Mathematics in the Modern World.* Scientific American, Inc., 1968, pp. 300-303.

5. Rapoport, Anatol, "The Use and Misuse of Game Theory", *Mathematics in the Modern World.* Scientific American, Inc., 1968, pp. 304-312.

6. von Neumann, John and Morgenstern, Oskar. *Theory of Games and Economic Behavior.* Princeton, N.J.: Princeton University Press, 1947.

The Problem of Apollonius*

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Apollonius of Perga, a well known geometer who studied and taught in Alexandria about 225 B.C., became famous through his work with conic sections. In fact, Apollonius gave the ellipse, parabola, and hyperbola their present names. In considering a special case of one of these conic sections, Apollonius developed what has since become one of the most famous of the classical construction problems. The problem of Apollonius, as it is commonly called, can be stated:

> Given three circles in the same plane. Construct all possible circles which are tangent to all three given circles and are coplanar with the given circles.

Included in this are the cases where one or more of the given circles have degenerated into a point or a straight line. A point represents a "circle" with radius zero and a line represents a "circle" with infinite radius.

A variation in the given circles will produce a change in the number of distinct solutions. For example, if the three given circles were point circles, we would be searching for all the circles that contain the three given points. Since there is only one circle through three points, we would have only one possible solution in this case. If the three given circles are concentric, the problem has no real solution. In fact, the number of solutions varies from zero to a finite number as the given circles vary.

The finite number of solutions occurs when the given circles are not specialized. Now, we must determine the number. Let the given circles be denoted by C_1, C_2, and C_3. Since two circles could be tangent either by external contact or by internal contact, let us denote a circle which touches C_1 externally by + and one which touches C_1 internally by − . Then a combination such as $++-$ describes a circle which touches C_1 externally, C_2 externally, and C_3 internally. The possible combinations of signs are: $+++$, $---$,

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$++-$, $--+$, $+-+$, $-+-$, $-++$, $+--$. If there exists a unique circle which corresponds to each of these triplets of symbols, then eight possible circles or solutions to the problem exist in the general case where no specialization occurs in the given.

Can these circles be constructed using only a ruler and a compass? Given C_1 with center P_1 and radius r_1, and C with center P and radius r. The two circles can be tangent in each of the three ways shown in Figure 1. Therefore, C and C_1 are tangent if and only if $\overline{PP_1}^2 = (r \pm r_1)^2$.

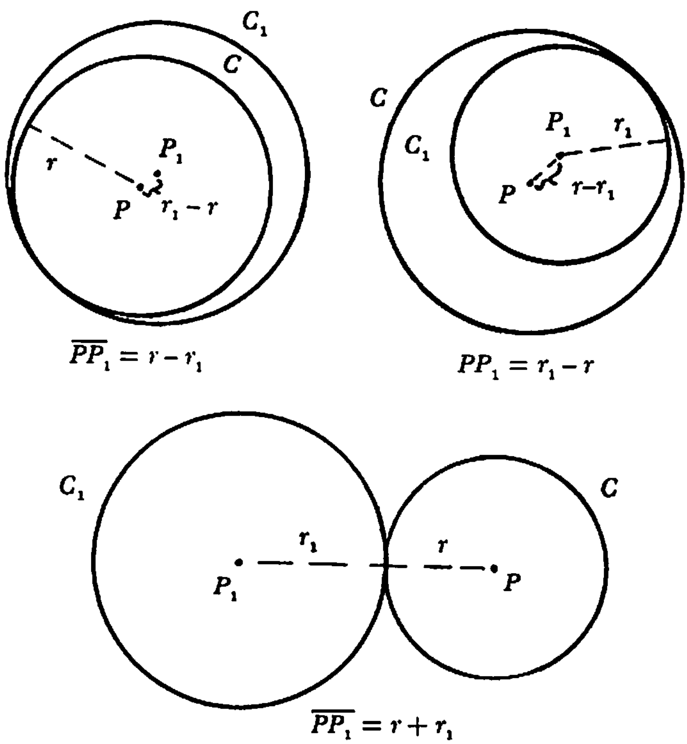

FIGURE 1

Let the centers of the three given circles have coordinates (a_1,b_1), (a_2,b_2), (a_3,b_3), respectively, with radii r_1, r_2, and r_3. Represent the center and radius of the required tangent circle by (a,b) and r. Remember that the required circle is tangent to the three given circles if the distance between the centers of two tangent circles is equal to the sum or difference of the radii. These equations follow.

$$(a - a_1)^2 + (b - b_1)^2 = (r \pm r_1)^2 \qquad (1)$$

$$(a - a_2)^2 + (b - b_2)^2 = (r \pm r_2)^2 \qquad (2)$$

$$(a - a_3)^2 + (b - b_3)^2 = (r \pm r_3)^2 \qquad (3)$$

Through simple multiplication, equation (1) becomes

$a^2 + b^2 - r^2 - 2a_1a - 2b_1b \pm 2r_1r = r_1^2 - a_1^2 - b_1^2$. This can be written in the form:

$$a^2 + b^2 - r^2 + A_1a + B_1b + C_1r + D_1 = 0 \qquad (4)$$

where the coefficients are numbers that can be plotted. In the same manner, equations (2) and (3) can be written:

$$a^2 + b^2 - r^2 + A_2a + B_2b + C_2r + D_2 = 0 \qquad (5)$$

$$a^2 + b^2 - r^2 + A_3a + B_3b + C_3r + D_3 = 0 \qquad (6)$$

By subtracting (4) from (5) and (6), we get two linear equations:

$$E_2a + F_2b + G_2r + H_2 = 0 \qquad (7)$$

$$E_3a + F_3b + G_3r + H_3 = 0 \qquad (8)$$

Since the coefficients are differences of plottable numbers, the new coefficients can be plotted. Solving (7) and (8) for a and b in terms of r then substituting in (1), we get a quadratic equation in r, which can be solved through rational operations and the extraction of a square root. Only one positive solution will result. After obtaining r in this manner, a and b can be found by substitution into the linear equations. We have now obtained a circle with center (a, b) and radius r that is tangent to the three given circles. Since this circle was determined strictly through algebraic means and only valid maneuvers such as square root extractions and rational operations were employed, it follows that a, b, and r can be constructed by ruler and compass alone [1, p. 125-126; 4, p. 225-227].

Now that we know it is possible to construct a circle tangent to three given circles, let's examine how this could be accomplished. First, let us consider this special case [3, p. 94]:

> Construct a circle tangent to each of three given circles having equal radii.
>
> Let each of the circles with centers at A, B, and C (see Figure 2) have radius r. Let O be the point of intersection of perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$. Then O is the center of the circle through points A, B, and C. If O is outside each given circle, the circle with center O and radius $OA - r$ is externally tangent to each given circle. The circle with center O and radius $OA + r$ is internally tangent to each given circle.

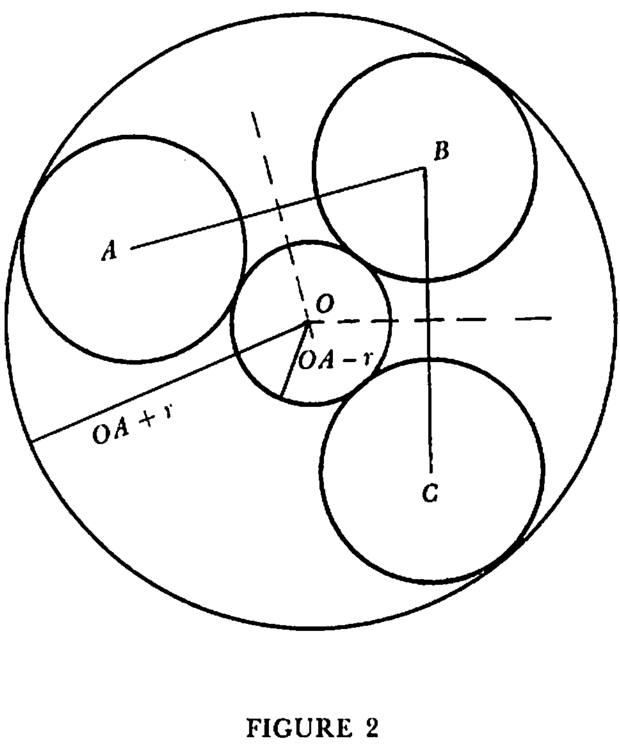

FIGURE 2

Now, let's look at a general solution to the problem of Apollonius [3, p. 96-97].

> Construct a circle tangent to three given circles.
>
> Let circles with centers A, B, and C and with radii a, b, and c be given (see Figure 3). Assume that circle A has the smallest radius. Draw the circle with center at B and with radius $b - a$, and draw the circle with center at C and radius $c - a$. Next, construct a circle through A tangent externally to the two new circles. If the last circle has center R and radius r, the circle with center R and radius $r - a$ must be externally tangent to each of the three given circles A, B, and C.

Modifications can be made to allow for internal tangency. The principles of inversion may also be used to construct the required tangent circle. Thus, the problem of Apollonius has several solutions, each of which may be constructed with a ruler and compass.

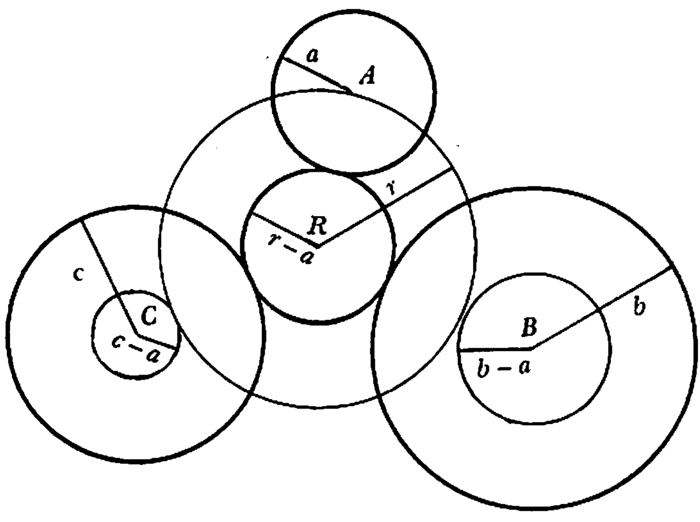

FIGURE 3

REFERENCES

1. Courant, Richard and Robbins, Herbert *What is Mathematics?* London: Oxford University Press, 1948.
2. Johnson, Roger A. *Modern Geometry*. Boston: Houghton Mifflin Company, 1929.
3. Miller, Leslie H. *College Geometry*. New York: Appleton-Century-Crafts, Inc., 1957.
4. Moise, Edwin E. *Elementary Geometry From An Advanced Standpoint*. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1963.
5. Pedoe, D. *A Course Of Geometry For Colleges And Universities*. Cambridge: Cambridge University Press, 1970.
6. Taylor, E. H. and Bartoo, G. C. *An Introduction To College Geometry*. New York: The Macmillan Company, 1962.

Three Theorems*

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Last semester one of my teachers proposed the following:

Theorem. *If* N *is the set of non-negative integers and if* n $\in$ N, *then any (4*n+*1)st power of any integer* K *will terminate in the same digit as does* K.

That is, prove that any integer and its $(4n+1)$st power will terminate in the same digit. In the process of proving this statement, I began to consider what the situation would be if the base of the numeration system used were something other than 10. These considerations led to certain conjectures, three of which I have proved. Thus, this paper is really about three theorems concerning the question: given a base, when does some fixed power of every integer terminate in the same digit as the integer itself.

Before proving the assertion originally made where the base was assumed to be 10, we need this lemma and corollary.

Lemma. *The only possible last digit of the product of any two positive integers is the last digit of the product of the last digits.*

Corollary. *The ending digit of* x^n *will be the product of the last digit of* x *and the last digit of* x^{n-1}.

Since there are only 10 possible terminating digits, let us investigate what happens when they are raised to various powers. The table below gives the resulting terminating digits. This indicates that any integer x^{4n+1} will terminate in the same digit as x was asserted. This seems obvious since the sequence below repeats with a period of 4. To prove that this is actually the case we use math induction.

Proof: Let M be the set of positive integers for which the assertion is true.

(1) $1 \in M$: $x^{4+1} = x^5$ ends in the same digit as x. There are only 10 last digits possible, each of which raised to any power is the

* A paper presented at the 1973 biennium convention of KME at Sioux City, Iowa.

product of itself a given number of times. Any given sequence repeats at least after the fourth power. The preceding table verifies this. Thus $x^{4(1)+1}$ ends in the same digit as x and $1 \epsilon M$.

(2) Assume $k \epsilon M$, then x^{4k+1} ends in the same digit as x. $x^{4(k+1)+1} = x^{4k+1} \cdot x^4$ and x^{4k+1} ends in the same digit as x, so the ending digit of $x^{4(k+1)+1}$ will be the same as the last digit of $x \cdot x^4$ (which is x^{4+1}). Furthermore, it is known that x^{4+1} ends in the same digit as x, which means that $k + 1 \epsilon M$.

(3) Since $1 \epsilon M$ and $k \epsilon M$ implies $(k+1) \epsilon M$, the relationship "x^{4n+1} ends in the same digit as x" must hold for any n in the set of natural numbers.

Of course it is trivial that x^{4n+1} terminates in the same digit as x when n = 0. So the theorem is proved.

Digit \ Power	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	4	8	6	2	4	8	6	2
3	9	7	1	3	9	7	1	3
4	6	4	6	4	6	4	6	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	9	3	1	7	9	3	1	7
8	4	2	6	8	4	2	6	8
9	1	9	1	9	1	9	1	9

We want to investigate now the cases where the base is not 10. We list the tables of terminating digits for various bases. A study of the tables presented yields these results. For bases 2, 3, 5, 7, and 11 the first repetition of all last digits occurs when the power is equal to the base. For bases 4, 8, 9, and 12 repetition of all last digits never occurs. For bases 6 and 10, repetition occurs but not in the same pattern as for the prime bases. We can make the following four conjectures:

BASE 2

Digit \ Power	2	3	4
0	0	0	0
1	1	1	1

BASE 3

Digit \ Power	2	3	4	5
0	0	0	0	0
1	1	1	1	1
2	1	2	1	2

BASE 4

Digit \ Power	2	3	4	5
0	0	0	0	0
1	1	1	1	1
2	0	0	0	0
3	1	3	1	3

BASE 6

Digit \ Power	2	3	4	5
0	0	0	0	0
1	1	1	1	1
2	4	2	4	2
3	3	3	3	3
4	4	4	4	4
5	1	5	1	5

BASE 5

Power / Digit	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	4	3	1	2	4	3	1	2
3	4	2	1	3	4	2	1	3
4	1	4	1	4	1	4	1	4

BASE 7

Power / Digit	2	3	4	5	6	7	8	9	10	11	12	13
0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1
2	4	1	2	4	1	2	4	1	2	4	1	2
3	2	6	4	5	1	3	2	6	4	5	1	3
4	2	1	4	2	1	4	2	1	4	2	1	4
5	4	6	2	3	1	5	4	6	2	3	1	5
6	1	6	1	6	1	6	1	6	1	6	1	6

BASE 8

Digit \ Power	2	3	4
0	0	0	0
1	1	1	1
2	4	0	0
3	1	3	1
4	0	0	0
5	1	5	1
6	4	0	0
7	1	7	1

BASE 9

Digit \ Power	2	3	4
0	0	0	0
1	1	1	1
2	4	8	7
3	0	0	0
4	7	1	4
5	7	8	4
6	0	0	0
7	4	1	7
8	1	8	1

Conjecture 1: Let p be a prime number and k be an integer larger than 1. If p^k is the base of a numeration system, there is no exponent $n \neq 1$ such that each integer x will terminate with the same digit as does the integer x^n.

Conjecture 2: Let the base of a number system be a product of primes not all distinct. Then there is no exponent $n \neq 1$ such that each integer x will terminate with the same digit as does the integer x^n.

Conjecture 3: Let a prime number p be the base of a number system. Then the last digit of every integer x will be the last digit of the integer $x^{(p-1)n+1}$ for all non-negative integers n.

BASE 11

Digit \ Power	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1
2	4	8	5	T	9	7	3	6	1	2	4
3	9	5	4	1	3	9	5	4	1	3	9
4	5	9	3	1	4	5	9	3	1	4	5
5	3	4	9	1	5	3	4	9	1	5	3
6	3	7	9	T	5	8	4	2	1	6	3
7	5	2	3	T	4	6	9	8	1	7	5
8	9	6	4	T	3	2	5	7	1	8	9
9	4	3	5	1	9	4	3	5	1	9	4
T	1	T	1	T	1	T	1	T	1	T	1

Conjecture 4: If the base is a product of distinct primes, the last digit of any integer x will be the same as the last digit of $x^{(p-1)n+1}$ where p is the largest prime factor of the base.

The remainder of this paper is concerned with the verification that the first three conjectures listed here are actually theorems.

Consider first the conjecture where the base is a prime power. That is, the base is $b = p^k$ where p is a prime number and k is an integer larger than 1.

BASE 12

Digit \ Power	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1
2	4	8	4	8	4	8	4	8	4	8	4
3	9	3	9	3	9	3	9	3	9	3	9
4	4	4	4	4	4	4	4	4	4	4	4
5	1	5	1	5	1	5	1	5	1	5	1
6	0	0	0	0	0	0	0	0	0	0	0
7	1	7	1	7	1	7	1	7	1	7	1
8	4	8	4	8	4	8	4	8	4	8	4
9	9	9	9	9	9	9	9	9	9	9	9
T	4	4	4	4	4	4	4	4	4	4	4
E	1	E	1	E	1	E	1	E	1	E	1

1. Now p is a 1-digit number in this system since $p < b$.
2. Let x_0 be an integer terminating in the digit p.
3. Then x_0^k terminates in the digit 0 (since p^k terminates in 0).
4. Hence by the corollary, x_0^n must terminate with 0 for all $n > k$.

5. Hence $x_0{}^n$ cannot terminate with the digit p if $n > k$.

6. If $x_0{}^n$ terminates with the digit p for some n, $1 < n < k$, then a period of repetition would have been established, and by the corollary, $x_0{}^n$ could never terminate with 0 for any n.

7. This contradicts step 3.

8. Hence there is no $n \neq 1$ such every integer x will terminate with the same digit as does the integer x^n.

Consider next the conjecture where the base is a product of primes not all distinct. That is, $b = p_1{}^{k_1} p_2{}^{k_2} p_3{}^{k_3} \dots p_m{}^{k_m}$ where p_1, p_2, p_3, $\dots$, p_m are distinct primes and at least one k_i is greater than 1.

1. Let x_0 terminate with the last digit of the integer $p_1\, p_2 \dots p_m$.

2. Let k be the largest k_i.

3. $x_0{}^k$ terminates in zero since $(p_1\, p_2 \dots p_m)^k = b(p_1{}^{k-k_1}\, p_2{}^{k-k_2} \dots p_m{}^{k-k_m})$.

4. Thus $x_0{}^n$ cannot terminate in the same digit as x for $n \geq k$.

5. $x_0{}^n$ cannot terminate in the same digit as x for $1 < n < k$, since the period of repetition so established would contradict the fact that x^k must terminate with 0.

6. The integer x_0 cannot terminate with the same digit as the integer $x_0{}^k$ for any $k > 1$.

7. Hence there is no $n \neq 1$ such that every integer x will terminate with the same digit as does the integer x^n.

To prove the third conjecture, we need this theorem from number theory: for any integer a, if p is a prime, then $a^p = a \pmod{p}$. Next let us derive another congruence from this theorem. Suppose p is a prime. Then

$$
\begin{aligned}
a^p &\equiv a \pmod{p} \\
a^{(p-1+1)} &\equiv a \pmod{p} \\
a^{p-1} &\equiv 1 \pmod{p} \\
[a^{(p-1)}]^k &\equiv \pmod{p}, \text{ where } k \text{ is any non-negative integer.} \\
a^{(p-1)k} &\equiv 1 \pmod{p} \\
a^{(p-1)k+1} &\equiv a \pmod{p}
\end{aligned}
$$

With this last result the verification of our third conjecture is immediate. Let the base be a prime number p and x be any integer. Then $x^{(p-1)k+1} = x \pmod{p}$ for all non-negative integers k. Hence, x and $x^{(p-1)k+1}$ must have the same remainder when divided by p. That is, x and $x^{(p-1)k+1}$ both terminate with the same digit.

Thus, the three conjectures are proved as theorems.

REFERENCES

Griffin, Harriet *Elementary Theory of Numbers.* New York: McGraw-Hill Book Company, 1954.

An Unsolved Trigonometric Equation

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As far as I know the problem remains unsolved of finding all the solutions of the equation

$$N \arctan 1/x = \arctan 1/y \tag{1}$$

where N, x, and y are positive integers, $N > 1$. Moreover, I believe the only known solutions of (1) are $(N, x, y) = (M, 1, 1)$, where $M \equiv 1 \pmod 4$. In this article I will present some elementary results concerning the problem, among them a very simple proof that if N is an even integer, (1) has no solutions at all.

To begin let us discuss the case of $x = 1$. Then (1) becomes equivalent to

$$\operatorname{Tan} N\pi/4 = 1/y.$$

It is easily checked that the only possible values of $\operatorname{Tan} N\pi/4$ are -1, 1, and 0. Thus $y \equiv 1$, which occurs if and only if $N = 1 \pmod 4$.

Before going on, we must find some way of simplifying N arctan $1/x$ for values of N greater than 1. To do this we use the well known formula for the tangent of the sum of two angles. If for some positive integer N, N arctan $1/x$ = arctan $F_n(x)/G_n(x)$ and $G_n(x) \neq 0$, then $(N-1)$ arctan $1/x$ = arctan $F_{n-1}(x)/G_{n-1}(x)$, where

$$F_{n-1}(x) = x\ F_n(x) - G_n(x)$$
and
$$G_{n-1}(x) = x\ G_n(x) - F_n(x).$$

In our case $F_1(x) = 1$ and $G_1(x) = x$. To be sure that we can use the above formulas for all N and x, both integers greater than 1, we only need to show that $G_n(x)$ is never 0. We will make use of a result due to J.M.H. Olmsted* which states that the only rational values of Tan πr (r a rational number) are 0 and $\pm$ 1. Thus clearly N arctan $1/x$ is not of the form πr and N arctan $1/x \neq$ arctan $F_n(x)/0$ since arctan $F_n(x)$ is always of the form πr.

THEOREM: *If* N *is an even positive integer, then (1) has no solutions with the given conditions.*

Proof: In (1), if for given values of N and x greater than 1, y is an integer, then $G_n(x)/F_n(x)$ is also an integer. For all even N it is easily verified that $G_n(x) \equiv 1 \pmod{x}$ and $F_n(x) \equiv 0 \pmod{x}$. Unless $x = 1$, $G_n(x)/F_n(x)$ cannot be an integer. Thus the result follows.

To complete the solution of the problem we only need to find all possible values of x, for which $G_n(x)/F_n(x)$ is a positive integer for some odd integer n greater than one. In particular if $x = 2$ there seems to be no solution with N smaller than 50. It may be difficult to show that none exist.

* J.M.H. Olmstead, "Rational Values of Trigonometic Functions," American Mathematical Monthly, 52 (1945), 507-508.

The Area Of A Rectangle

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For a great many subsets of the plane the techniques of the integral calculus provide us with a number which is in some sense a measure of the two-dimensional-size of the set in question. In the case of the elementary figures of plane geometry (triangles, rectangles, polygons, circles, etc.) this number coincides with the number which we usually refer to as area. Thus, extending our terminology, it is not unreasonable to designate the number assigned to the non-elementary figures also by "area".

In reality then, we are considering a set-function whose domain is a certain collection of subsets of the plane (but not all possible subsets) and whose range is a subset of the non-negative real numbers. In many instances the value of this function can be easily obtained by the use of the techniques of the integral calculus.

Most books deal with the geometric motivation for the definition of the integral in terms of extending the idea of area from the elementary figures of plane geometry to more complicated figures and generally base the treatment along the following lines.

Some elementary figure is usually chosen as basic and its "area" is assumed known or a value for the area is hypothesized. For example, the most common choice is that of a rectangle, and the area is assumed to be the product of the linear dimensions of the rectangle. Using the rectangle as a base and the area-function as indicated, the usual areas for the remaining elementary figures can be derived. This choice also leads to the standard definition of "area under a curve" in terms of a Riemann integral.

In this context we have tacitly assumed that the area of a rectangle depends solely on the rectangle under consideration and is independent of the position of the rectangle in the plane. More precisely, the value of the area-function remains constant if the rectangle is rotated and translated in any prescribed manner. The area we associate with the more complicated figures is also independent of position in the plane.

Let us denote the area-function by $A(R)$ where R denotes the subset of the plane we wish to consider. If R is a rectangle with linear dimension x and y we have $A(R) = xy$. In this special case, we will write $A(R)$ as $(A(x,y)$, so that $A(x,y) = xy$.

If it seems reasonable that the area or "two-dimensional-size" of a rectangle should in some way depend on the linear dimensions of the rectangle, one might wonder if there are other choices possible for the function $A(x,y)$ and, even if there are or not, what motivated the choice $A(x,y) = xy$?

Let us consider the latter question first. We will do this in the form of a thought experiment. Suppose that we are given a supply of square tiles of dimension one metre by one metre, and that we are to completely cover the floor of a rectangular room with dimensions 34 metres by 79 metres. How many tiles will be needed to complete the job?

We begin at one end of the room placing tiles so that the sides are parallel to the sides of the room and that the first tile is placed in the corner of the room parallel to the walls and against two of the walls. The second tile will be placed along one wall so that it has one side in common with the first tile and one side in common with the wall. We continue this process until it is no longer possible to place a tile with one side in common with the previously placed tile and one side in common with the wall. In fact, the last placed tile will have two sides in common with the walls of the room (we continue until we reach one of the other corners).

We now have a row of tiles placed along one wall of the room and the number of tiles used (assuming we worked along one of the shorter walls) is 34. There is no question of computing area at this stage for all that we are comparing here is one linear dimension of the room with one of the linear dimensions of the 1 by 34 rectangle that we have constructed.

Next, forgetting about the row of tiles already placed, we are now faced with a rectangular room of dimension 34 by 78. Proceeding just as in the first step we may place a row of tiles along one of the shorter walls and use exactly 34 tiles.

We continue this process until the room is completely tiled.

This process will terminate when we have laid out 79 rows of tiles. Again, there is no question of computing an area here as we are again merely comparing one of the linear dimensions of the room (79 in this case) with one of the linear dimensions of the several rows of tiles (1 unit for each row) that we have placed. As it happens, each row of tiles fits exactly from one side of the room to the other. However, we are concerned with how many rows fit into the room if each row has one of its linear dimensions equal to 1 and the room has one of its linear dimensions equal to 79. The answer is, of course, 79 rows.

Now, we apply a familiar counting principle: we have 79 rows of objects of some sort and each row contains 34 objects; the total number of objects, or in our case tiles, is (79) (34).

We could, with a minimal amount of additional justification, extend the counting principle to the following situation. Suppose that the dimensions of the room are x and y where x is a positive integer and y is a positive rational number. It would not be difficult to show that the number of tiles (again, of size one by one) used is xy. (Of course, to complete a row in step one for example, it will be necessary to use only a portion of one of the unit tiles and so too for each succeeding row.)

Thus with additional justification we can compute the number of objects involved in an array of y objects in x rows if the objects are of the sort that a fractional part of one of them is meaningful.

We cannot further extend our results to a room for which both linear dimensions are rational as the counting principle no longer applies. While in some cases it may be possible to have a noninteger number of objects in each row, it does not make much sense to ask for a noninteger number of rows. In a given array, a subset either constitutes one row of y objects or it does not and thus the number of rows is a non-negative integer.

By the same sort of argument, the counting principle will not help us if the linear dimensions of the room are irrational numbers.

Who would justify that he had exactly one dozen eggs by invoking the counting principle after attempting to arrange them into $\sqrt{3}$ rows of $\sqrt{48}$ eggs each? Nevertheless, it is an intuitive leap of just this sort that would lead us without any pangs of conscience to assert that a rectangular room with these linear dimensions would have area A $(\sqrt{3},\sqrt{48}) = \sqrt{3}\sqrt{48} = 12$, i.e. that exactly twelve tiles would be needed to cover the room.

Let us now consider the properties that we require of any "area function" (whether it is $A(x,y) = xy$ or some other). One of these we already mentioned above.

I. The area of a rectangular depends only on the linear dimensions of the rectangle.

If F represents any potential area function and R any rectangle we may then write $F(R) = F(x,y)$ if the linear dimensions of R are x and y. It follows that $F(x,y)$ is independent of the position of the rectangle R, i.e. rotations and translations of R will not affect the value of $F(x,y)$.

In order to select a *unit of area* let us us agree that a rectangle with both linear dimensions equal to one unit of length will be counted as one unit of area. This may be expressed in terms of F as our second axiom.

II. $F(1,1) = 1$

It would also seem reasonable to assert that it does matter which of the two linear dimensions is measured first.

III. If the rectangle R has linear dimensions x,y then $F(x,y) = F(y,x)$.

Given two rectangles R and S with linear dimensions x,y and x,z respectively we can, by suitable rotation and translation, construct a new rectangle T with linear dimensions x and $y+z$. Our next axiom asserts that whatever the value we assign to R and to S we will assign to T the sum of these two values. This may also be expressed in terms of the function F.

IV. $F(x,y+z) = F(x,y) + F(x,z)$

Because of Axiom III, we also have the following lemma.

LEMMA. $F(y+z,x) = F(y,x) + F(z,x)$

Proof:
$$\begin{aligned} F(y+z,x) &= F(x,y+z) \\ &= F(x,y) + F(x,z) \\ &= F(y,x) + F(z,x) \end{aligned}$$

Finally, our intuitive concept of area leads us to require that "small" changes in the linear dimensions of the rectangle should result in "small" changes in the area that we assign to these rectangles. More precisely, we require that the function F be a continuous function of each of the linear dimensions of the rectangle.

V. $F(x,y)$ is continuous in x and in y.

Let us now consider properties of functions F which satisfy I - V and determine whether or not choices other than $A(x,y)$ for the area-function are possible.

THEOREM 1. *If* n *is a positive integer,*
$$F(nx,y) = nF(x,y) = F(x,ny).$$

Proof: (By induction) The result is trivially true for $n = 1$. Assume that the result holds for some integer k.

$$\begin{aligned} F((k+1)x,y) &= F(kx+x,y) \\ &= F(kx,y) + F(x,y) \\ &= kF(x,y) + F(x,y) \\ &= (k+1)F(x,y) \end{aligned}$$

$F(x,(k+1)y) = (k+1)F(x,y)$ is proved in exactly the same fashion.

THEOREM 2. *If* $r = p/q$ *is any positive rational number (and without loss of generality we may assume that* r *is in "lowest terms") then*

$$F(rx,y) = rF(x,y) = F(x,ry).$$

Proof: $F(rx,y) = F(p(x/q),y) = pF(x/q,y)$ by Theorem 1, since p is a positive integer.

Also, $qF(x/q,y) = F(qx/q,y) = F(x,y)$ by Theorem 1, since q is a positive integer.

Thus $F(x/q,y) = (1/q)F(x,y)$. Finally,

$$\begin{aligned} F(rx,y) &= pF(x/q,y) \\ &= p[(1/q)F(x,y)] \\ &= rF(x,y). \end{aligned}$$

Analogously, we can show that $F(x,ry) = rF(x,y)$.

THEOREM 3. *If* r *is any non-negative real number, then*

$$F(rx,y) = rF(x,y) = F(x,ry).$$

Proof: Let $\{r_i\}$ be any sequence of positive rational numbers such that $\lim_{i\to\infty} r_i = r$.

Our argument depends on Theorem 2 and Axiom V.

$$\begin{aligned} F(rx,y) &= F(x\cdot\lim_{i\to\infty} r_i, y) \\ &= F(\lim_{i\to\infty} xr_i, y) \\ &= \lim_{i\to\infty} F(xr_i, y) \\ &= \lim_{i\to\infty} r_i F(x,y) \\ &= (\lim_{i\to\infty} r_i) F(x,y) \\ &= rF(x,y) \end{aligned}$$

Similarly we have $F(x,ry) = rF(x,y)$.

Intuitively, if one of the linear dimensions of the rectangle is zero we would expect that the area also be zero.

COROLLARY. $F(0,x) = 0 = F(x,0)$.

Proof: Choosing $r = 0$ in Theorem 3, we have

$$\begin{aligned} F(0,x) &= F(0\cdot 1,x) \\ &= 0F(1,x) \\ &= 0 \end{aligned}$$

Likewise, $F(x,0) = 0$.

COROLLARY. $F(x,y) = A(x,y)$.

Proof: Recall that $A(x,y) = xy$. Using Theorem 3 and Axiom II we have

$$\begin{aligned} F(x,y) &= F(x \cdot 1, y \cdot 1) \\ &= xF(1, y \cdot 1) \\ &= xyF(1,1) \\ &= xy \\ &= A(x,y) \end{aligned}$$

Thus with Axioms I – V, we have no choice for the area of a rectangle and thus no choice for the area of the elementary figures of plane geometry or for areas obtained using the theory of Riemann integrals based on the area of a rectangle. At the same time, we have justified the intuitive leap regarding the number of tiles needed to cover a rectangular room with irrational linear dimensions, i.e. we have justified the extension of the familiar counting principle mentioned above.

The following obvious modifications of the five axioms provide us with analogous results for n-dimensional space, i.e. the n-dimensional volume of a box in n-space is equal to the product of the n linear dimensions.

I′. The n-volume depends only on the linear dimensions of the box.

II′. $F(1,1,\ldots,1) = 1$

III′. If a box R has linear dimensions $x_1, x_2, \ldots, x_n$ then
$F(\ldots, x_i, \ldots, x_j, \ldots,) = F(\ldots, x_j, \ldots, x_i, \ldots)$
for any choice of i and j with $1 \leq i, j \leq n$.

IV′. $F(\ldots, x_i + y_i, \ldots) = F(\ldots, x_i, \ldots) + F(\ldots, y_i, \ldots)$
for any i with $1 \leq i \leq n$.

V′. $F(x_1, \ldots, x_n)$ is continuous in x_i for each $i = 1, 2, \ldots, n$.

We can define a box in n-space as follows.

DEFINITION. *A subset* R *of Euclidean* n-*space* E^n *is said to be a box (or* n-*box) if there exist* 2n *numbers* $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n$ *with* $a_i \leq b_i$ for each $i = 1, 2, \ldots, n$, *and if* $R = \{x \epsilon E^n : a_i \leq x_i \leq b_i, i = 1, 2, \ldots, n\}$ *where* x_i *denotes the* i-*coordinate of the point* x. *The linear dimensions of the box are then* $b_i - a_i$ *for each* $i = 1, 2, \ldots, n$.

Consecutive Prime Power Numbers

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Over the ages, from ancient times to the present, one of the most fascinating (and sometimes most frustrating) pastimes known to man has been the investigation of the positive integers and their properties. The study of prime numbers has been particularly enticing to both amateur and professional mathematicians. Even with all of this attention the subject still has numerous areas where little light has been shed. There are many very innocent sounding conjectures which have been made that even the greatest of mathematicians thus far have been unable to resolve.

To illustrate the preceding remarks suppose we take a short look at one aspect of prime number theory. Sierpinski [1, p. 108] proposes the problem:

> P^2_{36}. Do there exist infinitely many natural numbers n for which each of the numbers n and $n + 1$ have only one prime divisor?

He then states "We know only twenty-six such numbers thus far. The smaller of them are $n = 2,3,4,7,8,16,31,127,246$, and the largest known: $n = 2^{4423}-1$. We can prove that of any three successive natural numbers greater than seven at least one has more than one prime divisor."

In this article we will look in detail at the comments made above. We shall call positive integers of the form p^k prime power numbers, where p is a prime number and k is a positive integer. The examples given by Sierpinski lead us to conjecture that the pair 8,9 is the only pair of consecutive prime power numbers where the odd number in the pair is not prime. In proving this conjecture, we first establish two lemmas.

LEMMA 1. *If* $q \geq 3$ *is an odd integer and* $x > 1$ *is a positive integer, then* q^x+1 *has at least one odd prime factor.*

Proof: Case 1. x is odd. Then $\dfrac{q^x+1}{q+1} = q^{x-1}-q^{x-2}+ \ldots -q+1$. $q^{x-1}-q^{x-2}+ \ldots -q+1$ is odd because it contains an odd number of odd addends.

Case 2. x is even. Then $x = 2k$. Since q is an odd number, $q \equiv 1 \pmod 4$ or $q \equiv 3 \pmod 4$. In either case $q^2 \equiv 1 \pmod 4$, from which it follows that $q^{2k} \equiv 1 \pmod 4$. Thus $q^x = 4m+1$ and $q^x+1 = 4m+2$ which implies that $\frac{q^x+1}{2} = 2m+1$. Hence $\frac{q^x+1}{2}$ is odd.

Therefore in each case q^x+1 has at least one odd prime factor.

LEMMA 2. *If* $p \geq 3$ *is an odd integer and* $x > 2$ *is a positive integer, then* p^x-1 *has at least one odd prime factor.*

Proof: $\frac{p^x-1}{p-1} = p^{x-1}+p^{x-2}+ \ldots +p+1$.

Case 1. x is odd. Then $p^{x-1} + p^{x-2} + p^{x-3} + \ldots + p+1$ is odd because it contains an odd number of odd addends.

Case 2. x is even. Then $x = 2j$. Thus $p^x-1 = p^{2j}-1 = (p^j+1)(p^j-1)$, since $x>2$, $j>1$. Thus p^j+1 has at least one odd prime factor by Lemma 1.

Therefore in each case p^x-1 has at least one odd prime factor.

We have two corollaries, one to Lemma 1 and one to Lemma 2.

COROLLARY 1. $\{(x,y)$: x *and* y *are positive integers and* $3^x-2^y = -1\} = \{(1,2)\}$.

Proof: $3^x-2^y = -1$ if and only if $3^x+1 = 2^y$. Hence by Lemma 1, $x = 1$.

COROLLARY 2. $\{(x,y)$: x *and* y *are positive integers and* $3^x-2^y = 1\} = \{(1,1),(2,3)\}$.

Proof: $3^x-2^y = 1$ if and only if $3^x-1 = 2^y$. Hence by Lemma 2, $x = 1$ or $x = 2$.

The reader should have little difficulty in establishing several corollaries of a similar nature.

THEOREM 1. *The pair* 8,9 *is the only pair of consecutive prime power numbers where the odd number in the pair is not prime.*

Proof: The pair must be given by p^k, p^k+1 or p^k-1, p^k where p is an odd prime number.

Case 1. The pair is of the form p^k, p^k+1 where p is an odd prime. Then $p^k+1 = 2^j$. Hence by Lemma 1, $k = 1$. But this contradicts the hypothesis that the odd number in the pair is not prime.

Case 2. The pair is of the form p^k-1, p^k where p is an odd prime. Then $p^k-1 = 2^t$. Hence by Lemma 2, $k = 1$ or $k = 2$. If $k = 1$, we have a contradiction to the hypothesis that the odd number in the pair is not prime. If $k = 2$, then $2^t = p^2-1 = (p-1)(p+1)$ from which it follows that both $p-1$ and $p+1$ must be powers of two. Thus $p+1$ and $p-1$ are powers of two which differ by two. Hence $p+1 = 4$ and $p-1 = 2$, whence $p = 3$ so the pair is 8,9.

It is apparent that 2,3,4; 3,4,5; and 7,8,9 are triples of successive positive integers which are prime power numbers. The following theorem will establish that these are the only such triples.

THEOREM 2. *The triples* 2,3,4; 3,4,5; *and* 7,8,9 *are the only triples of successive positive integers which are prime power numbers.*

Proof: Suppose there are integers n, $n+1$, $n+2$ such that $n = q^k$, $n+1 = p^j$, and $n+2 = r^s$ where q, p, r are prime numbers and k, j, s are positive integers.

Case 1. n is even. Then $n = 2^k$. Also $n + 2$ is even. Thus $n + 2 = 2^s$ where $s > k$. Hence $2^s = n + 2 = 2^k + 2$ which implies $2^{s-1} = 2^{k-1} + 1$. Therefore $k = 1$ and $n = 2$. This yields the triple 2,3,4.

Case 2. n is odd. Then $n + 1$ is even and $n + 1 = 2^j$ where $j \geq 2$, so $n = 2^j - 1$. If j is even, say $j = 2w$, then $q^k = n = 2^{2w} - 1 = (2^w-1)(2^w+1)$. Thus $2^w-1 = q^u$ for some nonnegative integer u and $2^w + 1 = q^v$ for some positive integer v. Hence $q^u(q^{v-u}-1) = 2$ so $q^u = 1$ because q^u is odd. Therefore $w = 1$ and $n = 3$. This yields the triple 3,4,5. If j is odd, 3 divides $2^j + 1$. Also $r^s = n+2 = 2^j + 1$ where r is a prime number. Thus $r = 3$ and $2^j + 1 = 3^s$, so $3^s - 2^j = 1$. Hence by the Corollary 2, either $s = 1$ and $j = 1$ or $s = 2$ and $j = 3$. Since 1 is not a prime power number, $j \neq 1$. Therefore $j = 3$ and $n = 7$. This yields the triple 7,8,9.

Therefore 2,3,4; 3,4,5; and 7,8,9 are the only triples of successive positive integers which are prime power numbers.

COROLLARY 3. 2,3,4,5 *is the only qradruple of successive prime power numbers.*

REFERENCES

1. Sierpinski, Waclaw *A Selection of Problems in the Theory of Numbers.* New York: Pergamon Press, 1964.
2. Niven, Ivan and Zuckerman, Herbert S. *An Introduction to the Theory of Numbers, Second Edition.* New York: John Wiley and Sons, Inc., 1966.

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

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In number theory one studies the integers and their unusual properties. The so-called number "magic" frequently involves the products of numbers which result in rather peculiar and often times quite unexpected results. From the ancients to the present time these properties have amazed many students of mathematics and numerologists. Often times the number involved is the number nine or a multiple of nine as shown in Table 1. One will note that the result in Table 1 appears reasonable if the left-hand side for the

TABLE 1.

$$\begin{aligned}
1 \cdot 9 &+ 2 = 11\\
12 \cdot 9 &+ 3 = 111\\
123 \cdot 9 &+ 4 = 1111\\
1234 \cdot 9 &+ 5 = 11111\\
12345 \cdot 9 &+ 6 = 111111\\
123456 \cdot 9 &+ 7 = 1111111\\
1234567 \cdot 9 &+ 8 = 11111111\\
12345678 \cdot 9 &+ 9 = 111111111\\
123456789 \cdot 9 &+ 10 = 1111111111
\end{aligned}$$

n-th expression is written in the form $[10^{n-1} + 2 \cdot 10^{n-2} + 3 \cdot 10^{n-3} + \ldots + (n-1) \cdot 10 + n] \cdot (10-1) + (n+1)$
When expanded and simplified this becomes

$$10^n + 10^{n-1} + 10^{n-2} + 10^{n-3} + \ldots + 10 + 1 = (10^{n+1} - 1)/9$$

since the left-hand side represents the sum of a geometric series. One then notes that $(10^{n+1} - 1)/9$ becomes unity repeated $n + 1$ times.

An unexpected result also occurs when the digits 1 2 3 4 5 6 7 8 9 are written in reverse order, multiplied by nine and then finally added to an appropriate integer as shown in Table 2. By writing

TABLE 2.

9	·	9	+	7	=	88
98	·	9	+	6	=	888
987	·	9	+	5	=	8888
9876	·	9	+	4	=	88888
98765	·	9	+	3	=	888888
987654	·	9	+	2	=	8888888
9876543	·	9	+	1	=	88888888
98765432	·	9	+	0	=	888888888

the n–th expression on the left-hand side in a general form one obtains

$$[9 \cdot 10^{n-1} + 8 \cdot 10^{n-2} + 7 \cdot 10^{n-3} + \ldots + S \cdot 10^{n-10+s} + \ldots + (10-n)] \cdot (10-1) + (8-n)$$

When expanded and regrouped this becomes

$9 \cdot 10^{n} - (10^{n-1} + 10^{n-2} + 10^{n-3} + \ldots + 10 + 1) - 1 = 8(10^{n+1} - 1)/9$ since the quantity in parenthesis is a sum of a geometric series. But $(10^{n+1} - 1)/9$ is merely unity repeated $n + 1$ times; hence, one has the desired result after multiplication by eight.

A somewhat different result occurs if one replaces the number nine with eight and a suitable addend is substituted in Table 1. The result is Table 3. As before, if one writes the left-hand side in gen-

TABLE 3.

1	·	8	+	1	=	9
12	·	8	+	2	=	98
123	·	8	+	3	=	987
1234	·	8	+	4	=	9876
12345	·	8	+	5	=	98765
123456	·	8	+	6	=	987654
1234567	·	8	+	7	=	9876543
12345678	·	8	+	8	=	98765432
123456789	·	8	+	9	=	987654321

eral form for the n–th term one obtains

$$[10^{n-1} + 2 \cdot 10^{n-2} + 3 \cdot 10^{n-3} + \ldots + (n-1) \cdot 10 + n] \cdot (10-2) + n$$

When expanded and regrouped this becomes

$$10^n - [0 \cdot 10^{n-1} + 1 \cdot 10^{n-2} + 2 \cdot 10^{n-3} + \ldots + (n-3) \cdot 10^2 + (n-2) \cdot 10 + n]$$

The subtraction yields the required result.

If one were to use all the digits in descending order and multiply by multiples of nine one would obtain Table 4. This result is immediate upon writing the n–th term of the left-hand side in general

TABLE 4.

987654321	•	9	=	8888888889
987654321	•	18	=	17777777778
987654321	•	27	=	26666666667
987654321	•	36	=	35555555556
987654321	•	45	=	44444444445
987654321	•	54	=	53333333334
987654321	•	63	=	62222222223
987654321	•	72	=	71111111112
987654321	•	81	=	80000000001

notation and simplifying as follows:

$$(9 \cdot 10^8 + 8 \cdot 10^7 + 7 \cdot 10^6 + 6 \cdot 10^5 + 5 \cdot 10^4 + 4 \cdot 10^3 + 3 \cdot 10^2 + 2 \cdot 10 + 1) \cdot 9K$$
$$= 8 \cdot 10^9 + 8 \cdot 10^8 + 8 \cdot 10^7 + \ldots + 8 \cdot 10^2 + 8 \cdot 10 + 9) \cdot K$$
$$= 8888888889 \cdot K.$$

The number 12345679 = $(10^9 - 1)/81$ (Can you prove this?). If this number (note that the digit 8 is omitted) is multiplied by a multiple of 9, one has $9K(10^9 - 1)/81 = K(10^9 - 1)/9 = K \cdot (111111111)$. Hence one can form Table 5.

Table 6 is an interesting one which dates back to a German-American algebra dated 1837. Its origin is clearly based upon the decimal expansion of the rational number 1/7. Three other tables of interest are listed below. Can you prove the results of Tables 6 through 9?

TABLE 5.

$$
\begin{aligned}
12345679 \cdot 9 &= 111111111 \\
12345679 \cdot 18 &= 222222222 \\
12345679 \cdot 27 &= 333333333 \\
12345679 \cdot 36 &= 444444444 \\
12345679 \cdot 45 &= 555555555 \\
12345679 \cdot 54 &= 666666666 \\
12345679 \cdot 63 &= 777777777 \\
12345679 \cdot 72 &= 888888888 \\
12345679 \cdot 81 &= 999999999
\end{aligned}
$$

TABLE 6.

$$
\begin{aligned}
1 \cdot 7 + 3 &= 10 \\
14 \cdot 7 + 2 &= 100 \\
142 \cdot 7 + 6 &= 1000 \\
1428 \cdot 7 + 4 &= 10000 \\
14285 \cdot 7 + 5 &= 100000 \\
142857 \cdot 7 + 1 &= 1000000 \\
1428571 \cdot 7 + 3 &= 10000000 \\
14285714 \cdot 7 + 2 &= 100000000 \\
142857142 \cdot 7 + 6 &= 1000000000 \\
1428571428 \cdot 7 + 4 &= 10000000000 \\
14285714285 \cdot 7 + 5 &= 100000000000 \\
142857142857 \cdot 7 + 1 &= 1000000000000
\end{aligned}
$$

TABLE 7.

$$
\begin{aligned}
1 \cdot 1 &= 1 \\
11 \cdot 11 &= 121 \\
111 \cdot 111 &= 12321 \\
1111 \cdot 1111 &= 1234321 \\
11111 \cdot 11111 &= 123454321 \\
111111 \cdot 111111 &= 12345654321 \\
1111111 \cdot 1111111 &= 1234567654321 \\
11111111 \cdot 11111111 &= 123456787654321 \\
111111111 \cdot 111111111 &= 12345678987654321
\end{aligned}
$$

TABLE 8.

7 · 7	=	49
67 · 67	=	4489
667 · 667	=	444889
6667 · 6667	=	44448889
66667 · 66667	=	4444488889
666667 · 666667	=	444444888889
6666667 · 6666667	=	44444448888889
66666667 · 66666667	=	4444444488888889

TABLE 9.

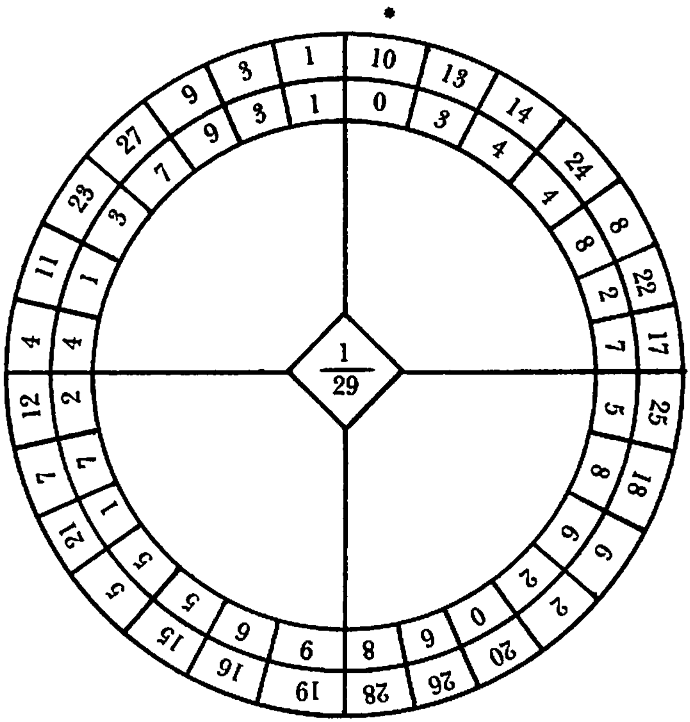
7 · 9	=	63
77 · 99	=	7623
777 · 999	=	776223
7777 · 9999	=	77762223
77777 · 99999	=	7777622223
777777 · 999999	=	777776222223
7777777 · 9999999	=	77777762222223

--- △ ---

A repeating decimal can be represented as a circle of its digits in the following fashion. When dividing the numerator by its denominator, one notes the quotient and the remainder obtained in each step. Then one constructs a circle having two rings: the inner ring gives the quotients while the outer ring gives the remainders. For the repeating decimal corresponding to the fraction 1/29, one would obtain the following circle proceding clockwise and starting at the asterisk.

It is clear from the above circle that $1/29 = 0.03448275\ldots$ Also the decimal equivalent repeats in a cycle of 28 digits. One will also note that diametrically opposite digits in the quotient circle sum to 9 and that the remainders in the remainder circle sum to 29.

To find the decimal equivalent for any fraction $n/29$, where $1 \leq n \leq 28$, one may use the above circle. For example, if one wishes to determine the decimal equivalent for the fraction 17/29, one must first find 17 in the remainder circle and note that its corresponding quotient is 7. Disregarding this quotient of 7, one then starts with the next quotient clockwise from 7, namely 5, and continues clockwise around the circle to obtain the desired decimal for 17/29.



Hence, 17/29 = 0.5862068. . . Using the above circle, can you find the decimal equivalent for 38/29?

--- △ ---

The properties of squares are of interest to many a student of mathematics. This is evidenced by the considerable study which has been devoted to Pythagorean triples. Here is one interesting problem concerning squares: "How many squares exist which contain all of the nine digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 with no digit repeated?" The solution to this problem almost certainly requires the use of a digital computer. It can be shown that 83 such numbers exist. A list of the smallest 20 appears in Table 10. Can you write a computer program and find the remaining 63 square numbers?

TABLE 10.

SQUARE NUMBERS CONTAINING 123456789 NON-REPEATED

$11826^2 = 139854276$	$19629^2 = 385297641$
$12363^2 = 152843769$	$20316^2 = 412739856$
$12543^2 = 157326849$	$22887^2 = 523814769$
$14676^2 = 215384976$	$23019^2 = 529874361$
$15681^2 = 245893761$	$23178^2 = 537219684$
$15963^2 = 254817369$	$23439^2 = 549386721$
$18072^2 = 326597184$	$24237^2 = 587432169$
$19023^2 = 361874529$	$24276^2 = 589324176$
$19377^2 = 375468129$	$24441^2 = 597362481$
$19569^2 = 382945761$	$24807^2 = 615387249$

One can consider the same problem but allow the digit 0 and find all square numbers containing all ten digits non-repeated. Some 87 numbers of this type exist. Ten of the numbers are given in Table 11. Can you find the remaining 77 square numbers?

TABLE 11

SQUARE NUMBERS CONTAINING 0123456789 NON-REPEATED

$32043^2 = 1026753849$	$45624^2 = 2081549376$
$32286^2 = 1042385796$	$55446^2 = 3074258916$
$33144^2 = 1098524736$	$68763^2 = 4728350169$
$35172^2 = 1237069584$	$83919^2 = 7042398561$
$39147^2 = 1532487609$	$99066^2 = 9814072356$

The Problem Corner

EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before September 15, 1974. The best solutions submitted by students will be published in the Fall 1974 issue of **The Pentagon**, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

Due to circumstances which existed when I became problem editor, I have received only one proposed problem. If those who submitted proposed problems will resubmit them, they will be considered for future publication.

Generally, I will follow the format established by my predecessor. However, I plan to implement a few minor changes. In the future, if it becomes feasible, problems proposed in one issue will have their solutions published in the following issue. To promote problem solving, if a proposed problem can be solved easily by the use of "proper insight", a solution using such insight will be supplied by the editor if space permits.

Finally, I would appreciate all comments and suggestions concerning the types of problems which you, the reader, would like to see in this column. As a guide, please answer the questions on page 137 and return the form to me.

Kenneth M. Wilke
Problem Editor

PROPOSED PROBLEMS

262. *Proposed by the editor.*

A rope hangs over a pulley. On one end of the rope hangs a weight. On the other end of the rope hangs a monkey equal in weight to the weight. The combined ages of the monkey and its mother are four years, and the rope weighs four ounces per foot. The monkey's weight in pounds equals the mother's age in years. The mother is twice as old as the monkey was when the mother was one-half as old as the monkey will be when the monkey is three times as old as the mother was when the mother was three times as old as the monkey. The weight of the rope plus the weight of the weight is one-half again as much as the difference between the weight of the weight and the weight of the weight and the monkey. How long is the rope?

263. *Proposed by the editor.*

Suppose a hole is drilled through the center of a sphere of radius r such that all the sides of the hole are 12 inches long. How much volume remains? How do you interpret your answer?

264. *Proposed by the editor.*

Professor Hy Potenuse noticed that an algebra student solved the equation $(4-x)(x-6) = -3$ as follows: either $4 - x = -3$ and $x = 7$ or $x - 6 = -3$ and $x = 3$. The answers are correct. The professor would like to find the quadratic equations which can be "solved" by the same method. Can you help him?

265. *Proposed by the editor.*

Two flagpoles, each 40 feet tall, are 40 feet apart. One piece of rope 120 feet long is fastened to the top of each flagpole and pulled taut by an iron ring in the ground. Assuming that the rope loses no length in being fastened to the flagpoles and assuming the rope remains in the same plane as the flagpoles, how far is the iron ring from the nearer flagpole?

266. *Proposed by the editor.*

Let *abcd* denote a four digit square in the decimal system. If *abcd* is broken up into two two-digit numbers *ab* and *cd*, then the square has the property that $abcd = (ab + cd)^2$. Find all squares *abcd* in the decimal system having this property provided that $a > 0$.

The Book Shelf

EDITED BY O. OSCAR BECK

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, Florence State University, Florence, Alabama 35630.

Elementary Probability and Statistics, A. William Gray and Otis M. Ulm, Glencoe Press, Beverly Hills, California, 1973, 286 pp., $10.95.

The novel feature of this textbook on elementary probability and statistics is a beginning chapter on the computer language BASIC and the inclusion of programs for computing statistical quantities. The inclusion of this material is helpful for those who have some knowledge of BASIC.

Some sections are quite confusing. In sections 3.8 and 3.9, the authors attempt to introduce the "product rule" for evaluating $P(A \cap B)$ without (explicit mention of) the notion of conditional probability. In a later section, the authors state: "This distribution (referring here to the F-distribution) provides a procedure to determine if the variance of sample means is greater than could be expected from chance alone. If so, we can conclude that the population means differ significantly." However, in the next paragraph, the F-distribution is introduced as the usual ratio of sample variances used to test the equality of two population variances.

The discussion of the sample correlation coefficient is too short and is poorly written. For instance: "On the other hand, a value of r that is near zero indicates that the independent variable is capable of explaining very little of the variation in the dependent variable and we have a weak relationship." The authors neglect to point out that they are referring here only to linear relationships.

The book contains a large number of minor errors and annoying misprints including the following five:

1) "The true proportion lies in the interval $.8412 < P < .9588$ with probability .90" (page 178)

2) $\sigma = \sqrt{\frac{pq}{n}}$ where $p = \frac{x.}{n}$(page 177)

3) SEE $= \sqrt{\frac{\Sigma(y_i - \overline{Y}_i)^2}{n-1}}$ (where $\overline{Y}_i$ is the estimated value of Y corresponding to x_i.) (page 223)

4) $z = \frac{\frac{\overline{x} - \mu}{s}}{\sqrt{n}}$ (page 154)

5) There is an error in the figure on page 198.

Some readers may wish that the concept of probability had been introduced before page 51. In my opinion, there is too much emphasis on counting formulas.

In order to find selected areas under the standard normal curve, the reader is referred to a table in the appendix. Unfortunately, the copy I received must have undergone an appendectomy.

Since several excellent textbooks on elementary probability and statistics have recently been published, I would find it difficult to recommend this book for classroom use.

Edward M. Bolger
Miami University (Ohio)

Introductory Linear Algebra, M. A. Akivis and V. V. Goldberg (Translated by R. A. Silverman), Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1972, 160 pp., $9.00.

This text by two well known Russian authors is primarily concerned with discussion of vector spaces of dimension three or less over the real or complex number fields.

The restrictive nature of these spaces is one reason for the brevity of the text which covers only one hundred sixty pages. There is no need for a discussion of determinants, since none have order greater than three. For the same reasons, discussion of systems of equations become unnecessary or, at worse, trivial.

Another reason for the brevity of the text is because many important concepts are relegated to the exercise sets. Many teachers believe this is the proper approach for students to attain maximum understanding of these concepts. Although sympathetic to this attitude, the reviewer believes that the process has been carried too far when such things as the dimension theorem for subspaces and the fact that rank plus nullity equals dimension are found in the exercise sets of an undergraduate text.

The book does have some features which are not usually found in a text at this level. One of these is the use of the Einstein summation notation from very early in the text. Students will find this notation difficult in the beginning, but they will soon appreciate the benefit of avoiding double, triple, or worse, sigma summation notation.

Another exceptional feature of the book is the coverage given to tensors. It is here that greatest benefit is derived from working in three space and using the Einstein notation. The tensor product is defined and discussion given to symmetric and antisymmetric tensors through the associated linear forms.

This reviewer believes that if one is seeking a linear algebra text which of necessity must give an introduction to the theory of tensors, then the Akivis-Goldberg text would be a good choice. If coverage of tensors is not desired, then there are many other texts which give a more complete and beneficial introduction to undergraduate linear algebra.

Eddy Joe Brackin
Florence State University

Calculus, A First Course, Louis J. De Luca and James T. Sedlock, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1973, 413 pp., $10.95.

This relatively small book is for beginning calculus courses. A student may use it to gain a thorough grasp of the fundamentals of modern analytic geometry and calculus. Coverage is, however, insufficient for engineering and science students, who would have to follow it by a course or two including trigonometric functions and such topics as multiple integration, infinite series, etc. This might involve some difficulty in finding a suitable textbook as a sequel. Perhaps the authors have in mind a second volume especially for such cases.

The book begins with an introduction to the real number system with the usual emphasis upon inequalities and absolute values. Then follows Chapter 1, which is in essence a short course in analytic geometry through various types of curves including conic sections, although ellipses and hyperbolas are covered in the exercises rather than in the text material. Chapter 2 is an excellent treatment of functions, and it is followed by an equally well written chapter on limits and continuity. Chapter 4 covers the derivatives for algebraic functions and the usual applications, and ends with an introduction to the notion of antiderivatives. Chapter 5 is a typical study of integration of algebraic functions with geometric, economics and physical applications. Logarithmic and exponential functions, along with some applications of these, are taken up in Chapter 6. Chapter 7 is a short study of algebraic functions of two variables. A two page common logarithm table followed by "answers to selected problems" completes the book.

This book has interesting features which would appear to make it a very "teachable" text. There are lots of well-worked-out examples and drawings. Many students will enjoy the benefits of the historical accounts appropriately spaced. The fact that no knowledge of trigonometry is needed may be of value unless the student is about to become involved with studies in engineering or applied science. The preface clearly and properly states the book is intended for those who seek an "*initial* exposure" to the calculus.

There are many excellent exercises and these are generally well graded. Although there may be other slight flaws in the first printing, this reviewer questioned one in particular, number 16, page 183, where it is believed the last word "greatest" should be replaced by "smallest" unless the student would be expected to recognize that a maximum would occur only if the whole wire formed a circle. This is the familiar problem whereby a wire is cut into two pieces, one a circle, the other a square, to form a minimum area sum.

This reviewer feels that the book could be used successfully in a terminal college course whose topics concur with those covered. It also should be a "must" for consideration as a text for high school seniors who are above average in mathematical maturity, as it could, thus, serve those who plan no further mathematics as well as those who intend to proceed to the more extensive mathematics required of applied mathematicians, engineers and scientists.

D. H. Erkiletian, Jr.
University of Missouri-Rolla

Statistics and Calculus: A First Course, James Murtha and Earl Willard, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1973, 592 pp., $12.00.

This text is a welcome addition to the number of publications in the field of Statistics. It has been written for students who plan to study the social, biological, or management sciences; it provides them with a sound introduction to statistical thinking and methods both from the discrete and continuous point of view. Calculus is introduced where necessary in order to enable the student to handle and appreciate the problems presented by the continuous random variable.

While studying from this text, students will be exposed to the concepts and techniques of descriptive, discrete, and continuous statistics, basic elements of probability and associated counting procedures, correlation, regression, and non-parametric problems. Each section contains clear and concise explanations of the topic considered as well as numerous examples which not only illustrate the technique involved but give one a feeling for the many and varied areas of application. In general the examples are not the traditional and artificial ones; they are creative and contemporary which makes them more interesting. Emphasis is not placed on the proofs of theorems but on the understanding of the fundamentals and their place in problem solving. After a brief summary, ample exercises are included; solutions to selected exercises are given. Each chapter concludes with a set of supplementary exercises. A Teacher's Manual is available to the instructor.

Although not essential to the text, computer techniques are included for those acquainted with or who have access to the use of a computer. The Appendix contains the tables necessary for the proper use of the textual material making the text self-contained.

Statistics and Calculus is recommended as a text for those who have occasion to teach or study (independently or not) an introductory course in statistics for use in the social, life, management and behavioral sciences.

Sister Marie Augustine Dowling
College of Notre Dame of Maryland

Trigonometry: An Analytic Approach (Second edition), Irving Drooyan, Walter Hadel and Charles C. Carico, The Macmillan Company, New York, 1973, 407 pp., $8.95.

The authors state that the organization of the material and the large number of exercises allow the use of the text in a wide range of trigonometry courses that differ in the amount of time available and the objectives of the course. The prerequisites assumed by the authors are previous work in algebra and geometry.

The text covers the standard topics of trigonometry including a treatment of the real number system and the function concept; circular functions; the graphs of the circular functions and their inverses; trigonometric identities; the solution of trigonometric equations; and the trigonometry of the right triangle. In addition, extensive work with vectors and complex numbers is included. Vectors are introduced both as geometric vectors and as ordered pairs of real numbers along with vector applications (Note: Portions of the chapter on vectors could be omitted without loss of continuity). The text gives three useful ways to represent a complex number, namely: (1) as an ordered pair of real numbers (a, b), (2) in the form $a + bi$, and (3) in trigonometric form. In addition, the text discusses the field properties of the complex number system along with DeMoivre's theorem and its applications to powers and roots of complex numbers.

From the students' standpoint the text is a very readable text. The exercises selected cover a range of abilities and attempt to provide for adequate practice of concepts and skills. Answers (including graphs) are provided to odd numbered exercises in each section, except that exercises for which there may be several alternate solutions are generally not shown.

The most impressive features of the text were the use of examples from various fields as applications of mathematics, the use of chapter summaries and review exercises at the end of each chapter, the use of numerous worked out example problems in conjunction with each exercise set, and the relegation to the appendices of some material which is no longer considered an integral part of a course in trigonometry, for example, the topic of logarithmic functions and their properties along with the use of logarithms in the solution of a triangle.

The overall impression was that the text should seriously be considered for a course in trigonometry from the modern view point for talented high school students, junior colleges and colleges.

Ronald D. Dettmers
University of Wisconsin-Whitewater

Kappa Mu Epsilon News

EDITED BY ELSIE MULLER, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Elsie Muller, Historian, Kappa Mu Epsilon, Department of Mathematics, Morningside College, Sioux City, Iowa 51106.

CHAPTER NEWS

Alabama Beta, Florence State University, Florence

Chapter President — Jeanne Burnett
27 members

Florence State has a planetarium and observatory where the chapter held one of its meetings to learn about mathematical aspects of astronomy. At another time the president, Jeanne Burnett, told about her summer work at TVA. Dr. Henry Miller of the University of Alabama visited the chapter to lecture on topics from Non-Euclidean geometry and to discuss graduate school opportunities and financial assistance. The group continues to maintain a tutoring service for both high school and college students. Many former members were welcomed back at the annual Homecoming Coffee Hour. Other officers are: Vann Bush, vice-president; Janet Peters, secretary and treasurer; Jean Parker, corresponding secretary; Eddy Joe Brackin, faculty sponsor.

Arkansas Alpha, Arkansas State University, State University

Chapter President — Robert Rahrle
20 members — 10 pledges

At the meetings students have given talks on "Golden Mean" and "Cryptanalysis." Faculty members have given talks on "Differential Geometry" and "Integral Equations." The chapter assisted the department of mathematics in administration of a mathematics contest for high school students. Other officers are: Judy Kay Fetters, vice-president; Nancy Kramer, secretary; Sandra Distretti, treasurer; Dr. J. L. Linnstaedter, corresponding secretary; Dr. A. C. Shilepsky, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President — Dave Bullard
40 student members and 19 faculty members

As for activities the chapter provides tutorial service in a mathematics laboratory which is available to all students on the campus, provides mathematical displays at Poly Royal (annual all-campus open house), and assists the department of mathematics in an annual mathematics contest on campus which involves over 500 high school students. Both students and faculty provide talks at the chapter meetings. Other officers are: Andy Harrington, vice-president; Sue Genung, secretary; De Ette Nelson, treasurer; Dr. George Mach, corresponding secretary; and Dr. Warten, faculty sponsor.

California Delta, California State Polytechnic University, Pomona

Chapter President — Steven Mitchell
20 actives, 10 pledges

Free tutoring in mathematics is provided twice a week. Usually two meetings are held per quarter for which the chapter arranges guest speakers. Other officers are: David Durham, vice-president; Christopher Weber, secretary and treasurer; Samuel Gendelman, corresponding secretary; Joseph Kachun, faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins

Chapter President — Mark Horney
31 actives — 12 pledges

In November the second Alumni Seminar on *Employment Opportunities in the Mathematical Sciences* was held. The day-long program consisted of three panel discussions on employment opportunities in the areas of teaching, government, and industry. Chapter meetings are held twice a month in the homes of the department of mathematics faculty. Other officers are: Jacquie Ostrom, vice-president; Marge Yoder, secretary; Ruth Ann Bruny, treasurer; Dr. Howard Frisinger, corresponding secretary; and Dr. Kenneth Klopfenstein, faculty sponsor.

Colorado Beta, Colorado School of Mines, Golden

Chapter President – Richard Fahlsing
30 members

In addition to the usual business meetings, field trips have been scheduled to the National Bureau of Standards and the National Center for Atmospheric Research. A display of items tracing the historic development of man-made devices used for calculation is being planned for the Colorado School of Mines Engineering Day. Other officers are: Lloyd Scheidt, vice-president; Jim Baylis, secretary; Melinda Smith, treasurer; Dr. A. J. Boes, corresponding secretary; Dr. J. O. Kork, faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston

Chapter President – Larry Dowling
56 members

Illinois Beta awards the KME Calculus Prize in memory of Professor Van Deventer each year. Students who received a grade of A or B in the last course of the calculus sequence are invited to participate in the competition. The winner will be that eligible student who scores highest on a Calculus Honors test. The winner receives a nominal cash prize, and his name is engraved on the plaque which is displayed in the mathematics department area. Other officers are: Eric Wingler, vice-president; Debra Ziegle, secretary and treasurer; Ruth Queary, corresponding secretary and faculty sponsor.

Illinois Delta, College of St. Francis, Joliet

Chapter President – Nancy Wicnienski
11 actives – 4 pledges

On 30 March a mathematics contest was held for high school students. About 100 students participated in 1973. Other officers are: Mars Sportiello, vice-president; Dr. Arnie Good, corresponding secretary and faculty sponsor.

Illinois Zeta, Rosary College, River Forest

Chapter President – Christine Biggio
16 actives – 6 pledges

A recent guest speaker was Mrs. Blakeslee, an employee of the

First National Bank of Chicago and a graduate of Rosary College. She spoke on career opportunities in the field of banking. To raise money a bake sale was held in February. Other officers are: Mary Bartik, vice-president; Christine Stephens, secretary; Ruth Ann Metzger, treasurer; Mordechai Goodman, corresponding secretary.

Illinois Eta, Western Illinois University, Macomb

Chapter President – Tom Glenn
20 members

The chapter is presently in the middle of a membership drive and in the process of restructuring the organization to provide for more student participation. At the programs the presentations in the past have included topics such as "Horizontal Chords," "Finite Geometry," and "A New Approach to Graphing." Several members attended the Illinois Convention of Mathematics Teachers held last November. Other officers are: Janet Tsukamoto, vice-president; Denise Andreas, secretary; Sandy Kammermann, treasurer; Kent Harris, corresponding secretary; James Calhoun, faculty sponsor.

Indiana Delta, University of Evansville, Evansville

Chapter President – Eugene Bettag
60 members

In October Mr. Eugene Bettag spoke on the topic "Getting the Most Out of Least Squares." In November Dr. Milton Cox of Miami University at Oxford, Ohio, discussed "A Geometry Problem Requiring a Calculus Solution." In February Mr. Kenneth Stofflet presented "A Heart for Applied Mathematics." A problem solving group was organized under the leadership of Michael Lachance. Each Friday during the year a problem sheet appears on a problem bulletin board. Solutions are accepted, checked, and usually posted with the names of the problem solvers recorded. From time to time discussion meetings are held. The chapter sponsored an award for the best mathematics related exhibit in the tri-state science fair, 6 April 1974. This is the third year for the award, a $25 bond. Other officers are: Michael Lachance, vice-president; Carol Smith, secretary; Dr. Gene Bennett, treasurer and corresponding secretary; Mr. Kenneth Stofflet, faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President – Kathleen Drew
37 members

The monthly meetings feature the presentation of a paper by a student member. The spring initiation and banquet was scheduled for 2 May 1974. The annual KME and mathematics faculty picnic was scheduled for Sunday, 5 May 1974 at Black Hawk Park. Other officers are: Anna Studt, vice-president; Cheryl Wagner, secretary and treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Gamma, Morningside College, Sioux City

Chapter President – Rodney Powles
41 members

The chapter has been fortunate to have two speakers from Iowa State University this year. Professor A. M. Fink spoke on the topics "The Fair Division Problem" and "Secrets of a Mississippi River Boat Gambler." Professor James Cornette spoke on "An Introduction to Population Genetics" and "The Hodgkin-Huxley Model of a Nerve Axon." At other monthly meetings students have presented papers. A carload of members plan to attend the regional convention at the University of Missouri in Rolla. Other officers are: Barbara Okonoski, vice-president; Deborah Hanson and Gary Pardekooper, secretaries; Dallas Courtney, treasurer; Elsie Muller, faculty sponsor.

Iowa Delta, Wartburg College, Waverly

Chapter President – Terry Ackman
44 members

At the September meeting there was a talk on actuarial work by one of the members who spent the summer working with the Continental Assurance Company. In October members gave presentations of recreational mathematics. In January Professor A. M. Fink of Iowa State University spoke on "The Fair Division Problem" and "Variations on Geometric Mean – Arithmetic Mean Inequality." On 26 March there was an initiation dinner for 28 students. Other officers are: Bruce Foster, vice-president; Patricia Flebbe, secretary; Pamela Snyder, treasurer; Marvin Ott, corresponding secretary and faculty sponsor.

Kansas Alpha, Kansas State College of Pittsburg, Pittsburg

Chapter President — Linda Funk
40 members

The following presented the programs at the monthly meetings: Beth Gray — "Riemann Geometry," Jim Wylie — "Mathematical Games and Demonstration of the Monroe 1880 Calculator," Susan Monsour — "A Study of the Mathematical Implications of Escher's Graphic Art," and Linda Funk — "Plane Patterns." Nine new members were initiated at the October meeting and thirteen in February. Other officers are: Jim Wylie, vice-president; Gail Schindler, secretary; Beth Gray, treasurer; Dr. Harold Thomas, corresponding secretary; Professor J. Bryan Sperry, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison

Chapter President — Mary Kay Stewart
16 members

Kansas Gamma sponsored three guest speakers this year. The first spoke on career opportunities for the mathematics major. The other two were in the areas of linear algebra and graph theory. "Caroms" and "Symmetries of the Cube" were included in the showings on two film nights. The chapter hosted its 6th biennial invitational mathematics tournament for high school students in the surrounding area on 30 March 1974. Other officers are: David Dover, vice-president; Ann Barmann, secretary; Tom Metzger, treasurer; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

Kansas Delta, Washburn University, Topeka

Chapter President — Teresa Thomas
19 members

On 21 March the chapter sponsored an area high school mathematics day which was called *Math-O-Rama*. Other officers are: Sandra Peer, vice-president; Janet Guyer, secretary; Lonnie Stauffer, treasurer; Margaret Martinson, corresponding secretary; Robert Thompson and Billy Nilner, faculty sponsors.

Kansas Epsilon, Fort Hays Kansas State College, Hays

Chapter President — Steve Kaufman
21 members

One of the programs included a "Mathematical Squares" game with the faculty answering questions. At another a student, Kent Juffman, discussed some of the geometrical models he has constructed. The chapter has prepared instructional aids to be used by the mathematics faculty. The following eight new members have been initiated: Charles Deuth, Dora Gross, Marilyn Horinek, Jean Ingersoll, Brena Mauck, Camellia Tuttle, Allen Wegele, and Dan Ziegler. Other officers are: Tabetha Eichman, vice-president; Craig McClellan, secretary and treasurer; Eugene Etter, corresponding secretary; Charles Votaw, faculty sponsor.

Maryland Beta, Western Maryland College, Westminister

Chapter President — Gerard Kurek
29 members

In September Sister Marie Augustine of Maryland Alpha spoke on "The Creative Intellect." In October James Hopkins, '72, was the guest lecturer on "National Security Administration, The Job-Getting Process." A joint meeting with Maryland Alpha was held in November and another was planned for April. A field trip to Washington and the Smithsonian was a possibility for late April. Other officers are: Thomas Yates, vice-president; Sandra Stokes, secretary; Nancy Fishpaugh, treasurer; James Lightner, corresponding secretary; Robert Boner, faculty sponsor.

Michigan Alpha, Albion College, Albion

The chapter has become inactive.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg

Chapter President - Cheryl Goodman
20 members

Professor John Thomas of New Mexico State University spoke at the March meeting on "Mathematical Toys." Other officers are: Rory McDowell, vice-president; Julie Faust, secretary and treasurer; Jack D. Munn, corresponding secretary; Ed Oxford, faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield

Chapter President — Julie Otto
51 members

Dr. Alex Cramer spoke at the March meeting on "Nonstandard Analysis." As for activities the chapter members both tutor and proctor, grade for the mathematics relays, and assist with the High School Mathematics Contest. Other officers are: Dan Wilson, vice-president; Esther Key, secretary; Greg Darnaby, treasurer; Eddie W. Robinson, corresponding secretary; Dr. L. T. Sheflett, faculty sponsor.

Missouri Gamma, William Jewell College, Liberty

Chapter President — Tom Lehman

At the monthly meetings students have given talks on the following topics: "Infinity of Diophantine Solutions to an Equation with Ascending Exponents," "Polish Notation," "A Non-Euclidean Metric," and "Calculus of Finite Differences." The chapter is starting a mathematics club which includes underclass mathematics students. Activities include a tour of the computer center at the United States Marine Corps Automated Service Center in Kansas City. Other officers are: Lynn Doss, vice-president; Esther Edwards, secretary; Debbie Gilbert, treasurer; Mr. Sherman Sherrick, corresponding secretary; Mr. Truett Mathis, faculty sponsor.

Missouri Theta, Evangel College, Springfield

Chapter President — Keith Sorbo
5 members — 5 pledges

The chapter attended the regional convention at Rolla, Missouri. Other officers are: Phyllis Ogletree, vice-president; Sandra Butler, secretary and treasurer; Don Tosh, corresponding secretary and faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne

Chapter President — Max Conneally
33 members — 13 pledges

The monthly meetings are followed by entertainment in the form of mathematical puzzles, problems, or novelties. Presentations are made by two members who challenge the remaining members for solutions. This year members are tutoring students in lower level mathematics courses. Eight members planned to attend the regional convention in Rolla, Missouri.

The outstanding freshman majoring in mathematics selected this year is Dave Lashier of Farragut, Iowa. The award includes honorary membership in KME with initiation fees paid by the club, the placement of his name on the permanent plaque, and public announcement of the award at the spring Honors Convocation. He is selected by competitive examination from a group recommended by the mathematics faculty.

Other officers are: Kim Bolte, vice-president; Verna Rosener, secretary and treasurer; Fred Webber, corresponding secretary; Jim Paige, faculty sponsor; Ellen Funk, historian and reporter.

Nebraska Gamma, Chadron State College, Chadron

Chapter President — Stephanie Larsen
16 members — 11 pledges

The chapter helps sponsor Scholastic Day and enjoys a spring picnic with the mathematics faculty. Other officers are: Dave Wolfe, vice-president; Lynne Lux, secretary; Rod Cain, treasurer; Jim Kaus, corresponding secretary and faculty sponsor.

New York Alpha, Hofstra University, Hempstead

Chapter President — Diane Goldman
14 members — 25 pledges

In September the chapter sponsored a faculty-student get-together during the lunch hours with refreshments and an atmosphere in which faculty and students could become better acquainted. In November two faculty members, Dr. Hastings and Dr. S. Althoen, lectured on topology. In March there was a discussion of job opportunities and graduate schools. Graduating seniors shared their experiences and a representative from the Placement Office was there. Other officers are: Lillian Nilsen, vice-president; Carol Kaye, secretary; Alan Blayne, treasurer; Prof. Stanley Kertzner, corresponding secretary and faculty sponsor.

New York Gamma, State University of New York College at Oswego, Oswego

Chapter President – Sherryl Johnson
21 members

The main emphasis of meetings has been the promotion of interaction between the mathematics faculty and students outside the classroom. The chapter sponsored a series of mathematics-related films and discussed various problems of interest to members. Other officers are: David Gibbs, vice-president; Mary Prisco, secretary; Stephen Kistler, treasurer; Steven Reyner, corresponding secretary and faculty sponsor.

New York Eta, Niagara University, Niagara University

Chapter President – Becky Smith
28 members – 6 pledges

In December Alyth Coupar, a graduate student, presented a paper on "Amicable Numbers and Some Related Topics in Number Theory." The February meeting consisted of a demonstration-lecture of a computer program for Tic-Tac-Toe by John Milleville. A pizza sale provided money for attendance at the regional convention at La Salle College in Philadelphia. Other officers are: Tom Jones, vice-president; Judy Miller, secretary; John Mancini, treasurer; Robert Bailey, corresponding secretary; Sr. John Frances Gilman, faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green

Chapter President – Rita Biesiot
60 members – 30 pledges

The year began with a "Meet the Prof's" night, which was followed by a meeting on curriculum as well as several coffee hours. Within the group is an active problem solvers club. In the spring there is the annual initiation banquet and the student-faculty picnic. At the banquet the recipient of the annual *Award for Excellence in Teaching Mathematics* is announced. Professor William Kirby was the recipient of last year's award. Other officers are: Sandi Mueller, vice-president; Aggie Gorup, secretary; Craig Cooper, treasurer; Waldemar C. Weber, corresponding secretary; V. Frederick Rickey and L. David Sabbagh, faculty sponsors.

Ohio Zeta, Muskingum College, New Concord

Chapter President — Sue Syroski

On 1-2 March the chapter hosted a KME convention for Region 2 with the assistance of Ohio Epsilon at Marietta College. Fifty people attended the convention from Michigan, Ohio, and Pennsylvania. Professor Dean Hinshaw, Region 2 Director, presided at an information session which was followed by a talk by Professor Stephan Kublank of Muskingum. Highlights of the two-day meeting included seven excellent student papers and a post-banquet square dance party. Other officers are: Emily Henderson, vice-president; Alan Hurst, secretary and treasurer; Dr. James L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern State College, Weatherford

Chapter President — Bryan Vogt
16 members — 19 pledges

The chapter held a fall picnic and has had two special speakers, Dr. Robert Morris and Mr. Stan Compton. A drawing for a portable calculator was held. Other officers are: Kenneth Hahn, vice-president; Mary Harper, secretary; Patricia Bieger, treasurer; Dr. Wayne Hayes, corresponding secretary; Dr. Don Prock, faculty sponsor.

Pennsylvania Alpha, Westminister College, New Wilmington

Chapter President — Elaine Beattie
31 members

At a Career Seminar recent Westminister graduates were invited to discuss career possibilities for mathematics majors. The fields represented were banking, computer programming, actuary, teaching, and underwriting. In April at the activation dinner new initiates must assume the role of a famous mathematician while giving a speech. Four members attended the KME regional convention at Muskingum. Other officers are: Clyde Goldbach, vice-president; Barbara Schreiber, secretary; Richard Buckman, treasurer; J. Miller Peck, corresponding secretary; Dr. T. R. Nealeigh, faculty sponsor.

Pennsylvania Beta, LaSalle College, Philadelphia

Chapter President – Frances Parrotto
7 members – 7 pledges

In September the department of mathematics in conjunction with KME initiated a series of colloquia. The topics of the fall semester included: "The Parabola Revisited" by Brother Hugh Albright, "A proof of the Irrationality of π and e" by Dr. John Baker, "Latin Squares and Galois Theory" by Mr. Stephen Leonard, "Coin Tossing" by Mr. Joseph Troxell, and "An Introduction to Control Theory and Optimal Control" by Dr. John O'Neill.

The chapter issues a weekly news-letter (KMEWS-letter). KMEWS is one means of announcing social events, department announcements, and special notices. The KMEWS-letter is definitely an indication of the creativity and sense of humor of the members.

Under the direction of Sister John Frances Gilman, Dr. Samuel Wiley, and the La Salle chapter of KME, the second spring conference of Region I was scheduled to be held at La Salle College on 6 April 1974. Other officers are: Frank Dziedzic, vice-president; Mathew Coleman, secretary; Debbie Wissman, treasuer; Brother Damien Connelly, corresponding secretary and faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

Chapter President – John Tauber
20 members

Other officers are: Jane Follweiler, vice-president; Lucille Greiss, secretary; Lucille Trittenbach, treasurer; Irving Hollingshead, corresponding secretary; Edward Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President – Gaylen Hauze
47 members – 11 pledges

At a business meeting Mr. Crooks discussed multinomial expansions. In addition to weekly mathematics help sessions where tutoring services are provided, eight members of the chapter attended the Region 2 convention at Muskingum. Professor William Smith, president of KME, sustained a back injury two weeks before the convention and could not entertain as speaker of the banquet. However, he is out of the hospital and doing well again at I.U.P.

Other officers are: Wayne Wendt, vice-president; Deborah Lahoski, secretary; Nancy Schwetz, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City

Chapter President — Dan McWhertor
35 members — 20 pledges

Other officers are: Karen Williams, vice-president; Carolyn Lape, secretary; Betsy Guffey, treasurer; Marvin C. Henry, corresponding secretary; Cameron C. Barr, faculty sponsor.

Pennsylvania Iota, Shippensburg State College, Shippensburg

Chapter President — Peggy J. Benfer
40 members — 9 pledges

At the monthly meetings members gave short mathematical talks. All pledges are required to give either a talk or a paper to be eligible for membership. Several members presented papers at the KME Regional Conference held at La Salle College in Philadelphia. Other officers are: Rodney D. Hart, vice-president; Billie D. Belles, secretary; Howard T. Bell, treasurer; John S. Mowbray, corresponding secretary; James L. Sieber, faculty sponsor.

South Carolina Gamma, Winthrop College, Rock Hill

Chapter President — Lynn Greene
42 members — 7 pledges

Guest lecturers have been Dr. Brawley of Clemson University with the topic, "Probability," and Dr. Huff of Winthrop College with the topic, "Number Theory." Other officers are: Shirley Bleasdale, vice-president; Deborah Johnston, secretary; Janice Tucker, treasurer; Donald Aplin, corresponding secretary; Edward Guettler, faculty sponsor.

Tennessee Alpha, Tennessee Polytechnic Institute, Cookeville

Chapter President — Dan Stancil
60 members

Other officers are: Lester Jones, vice-president; Linda Phillips, secretary; Randy Mehlon, treasurer; Evelyn Brown, corresponding secretary; Dan Buck and F. R. Toline, faculty sponsors.

Tennessee Gamma, Union University, Jackson

Chapter President – David Stephan
18 members

The annual initiation banquet was held on 19 March when Mr. Spence Dupree of Lambuth College was the featured speaker. Other officers are: Mike Jordan, vice-president; Carol Boggs, secretary; Kay Brown, treasurer; Prof. Richard Dehn, corresponding secretary; Prof. Joseph Tucker, faculty sponsor.

Texas Beta, Southern Methodist University, Dallas

Chapter President – Roslyn Slapper
45 members – 25 pledges

Guest speakers have been Dr. Robert Davis with the topic, Irrationality of π" and Dr. David Anderson on "Compartment Analysis of Physiological Systems." Mr. David M. Tobolowsky is the first recipient of the *John David Brown Mathematics Award*, a $50 award given by the chapter to an outstanding junior or senior mathematics major in memory of Dr. Brown, a former **KME** faculty sponsor. Other officers are: Morey Silverman and Marion Scott, vice-presidents, Patricia O'Rourke, secretary and treasurer; Raymond V. Morgan, Jr., corresponding secretary; Richard K. Williams, faculty sponsor.

Virginia Beta, Radford College, Radford

Chapter President – Nancy Watson

During the December meeting new members presented their research projects. Other events have included a picnic for **KME** and Tri-M, and an interesting discussion led by Dr. Susan Milton concerning random models for epidemics.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President – Barbara Junghans
10 members – 3 pledges

One of the programs consisted of film loops, *Calculus in Action*. Another consisted of records and film strips on the metric system.

Members have prepared a bulletin board on the metric system for the whole college. At the December meeting student teachers told about their experiences as student teachers. Virginia Klecha, economist for Rexnord, Inc., spoke to the chapter on opportunities other than teaching for the mathematics major. Wisconsin Alpha will host the national convention 17-19 April 1975. Other officers are: Sue Walczak, vice-president; Karen Loesl, secretary; Mary Lou Meyers, treasurer; Sister Mary Petronia, corresponding secretary and faculty sponsor.

Wisconsin Beta, Wisconsin State University — River Falls, River Falls

Chapter President — Dave Hetrick
43 members

Guest speakers included a former KME member who talked about his experiences at graduate school and a professor of mathematics who demonstrated computer generated movies. To promote enthusiasm for mathematics, college bowls have been held between members and with neighboring universities. As a special activity, the chapter sponsored a field trip to Argonne National Laboratory and Batavia Nuclear Accelerator in Chicago. The latest project is to establish a faculty-student lounge in the mathematics department. Other officers are: Barbara Heldke, vice-president; Dian Mortensen, secretary; Craig Emerson, treasurer; Lyle Oleson, corresponding secretary; Edward Mealy, faculty sponsor.

PROBLEM CORNER POLL

1. I would like to see problems taken from: algebra ____, geometry ____, trigonometry ____, calculus ____, elementary number theory ____, elementary probability ____________, other ____________. Check three.

2. Generally the problems have been
   a. too easy ________
   b. easy ________
   c. about right ________
   d. difficult ________
   e. much too difficult for me ________

   Check one.

3. The solutions to problems presented in the past issues have been understandable ________ difficult to understand ________

   Check one.

4. Concerning solutions to proposed problems. I would like to see:
   a. solutions presented as is now done ________
   b. more than one solution presented ________
   c. one solution with comments about other modes of solution ________ Check one.

5. I think the Problem Corner would be more valuable to me if

____

____

____

____

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