## THE PENTAGON

Volume XXXIII Fall 1973 Number 1
CONTENTS
National Officers ..... 2
A Proof of the Five-Color Theorem Utilizing Graph Theory By Kalhleen Shockley ..... 3
Log Jam in the Classroom
By Barbara Leamer ..... 10
On the Graphs of Groups
By Linda Grandstaff ..... 17
The Partition Function Revisited
By Daniel Q. Dye, Ir. ..... 20
Congruence Symmetry and Dimension
By Ali R. Amir-Moez ..... 22
The Mathematical Scrapbook ..... 29
Installation of New Chapters ..... 33
The Book Shelf ..... 34
The Problell Corner ..... 41
Kappa Mu Epsilon News ..... 46

## National Officers

William R. Smith . . . . . . . President Indiana University of Pennsylvania, Indiana, Pennsylvania<br>James E. Lightner . . . . . . Vice-President Western Maryland College, Westminster, Maryland<br>Elizabeth T. Wool.dridge . . . . . . Secietary Florence State University, Florence, Alabama<br>Eddie Robinson .<br>Treasurer<br>Southwest Missouri State College, Springficld, Missouri<br>Elsie Muller .<br>Historian<br>Morningside College, Sioux City, Iowa<br>George R. Mach<br>Past President<br>California State Polytechnic College,<br>San Luis Obispo, California

Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

# A Proof of the Five-Color Theorem Utilizing Graph Theory* 

Kathleen Shockley<br>Student, Central Missouri State University

Coloring any planar map consists of assigning colors to each region such that upon completion no two regions sharing a border are the same color. Any future references to maps in this article will imply planar maps. A map is $n$-colorable if it uses a minimum of $n$ colors. In 1840, A. F. Mobius mentioned the problem of colorability for $\boldsymbol{n}=4$. General attention was called to the conjecture ten years later when Francis Guthrie showed it to Augustus de Morgan. In 1878, Cayley stated he could not find a rigorous proof for the problem. At the present time a proof does not exist for colorability when $\boldsymbol{n}=4$. Neither has an example been found to disprove the conjecture. But it has been proved that for any map, five colors are always sufficient. This five-color theorem will be presented here.

Graph theory is a branch of mathematics particularly well suited to the study of maps in a plane, especially colorability. Graph theory has proved to be a very versatile discipline. It has been independently used many times in conjunction with physics, electronics, and chemistry (to name a few). Before giving the proof it is necessary to understand some basic definitions and concepts of this field.

The basic system behind graph theory is the graph G, defined as a finite nonempty set $X$ of $p$ points and a finite set $V$ of $q$ unordered pairs of distinct points. Each pair is represented diagrammatically as a line between the two points. It should be noted that the terms point and line are not standard in graph theory. Other common terms are node and arc, vertex and edge, and junction and branch, respectively.

Two points connected by a line are said to be adjacent while a line is said to be incident to each of its endpoints. Two lines which are incident with a common point are termed adjacent lines. The degree of a point is the number of lines incident with that point.

[^0]

FIGURE 1
In Figure $1, p_{2}$ and $p_{3}$ are adjacent points. Line $x$ is incident with $p_{a}$ and with $p_{s}$.

Lines $u$ and $w$ are adjacent with regard to $p_{3}$, but $x$ and $y$ are not adjacent. Though two lines cross in one representation of a graph, if the graph is planar the points may be rearranged so that lines only meet where they have a common point of incidence.

Points $p_{3}$ and $p_{4}$ are of degree 3 , while $p_{6}$ is of degree 0 .
A subgraph of $G$, say $G^{\prime}$, is identified by its two sets $X^{\prime}$ and $V^{\prime}$, both of which are subsets of $X$ and $V$, respectively. $G^{\prime}$ has all its points and lines in $G$. When $X^{\prime}=X, G^{\prime}$ is a spanning subgraph of $G$. That is, $G^{\prime}$ contains all the points of $G$. For any subset $S$ of $X$ in $G$, the induced subgraph $\langle S\rangle$ is the maximal subgraph of $G$ with point set $S$. That is, two points are adjacent in $\langle S\rangle$ if and only if they are adjacent in $G$.

Upon the removal of a point $v_{i}$ from $G$, the subgraph $G-v_{i}$ is left. $G-v_{i}$ contains all the points and lines of $G$ except point $v_{i}$ and any lines incident with $v_{i}$. On the other hand the subgraph $G-x_{i}$, where $x_{i}$ is some line in $G$, results in a spanning subgraph of $G$, there is no loss of points.

It is common to represent a graph with a diagram and refer to the diagram as the graph. As mentioned above, the drawing consists of $p$ points and $q$ lines, each line being determined by an ordered pair in $V$. The basic structure of a graph is defined in terms of the relationship between the points and lines of the graph. A walk is an
alternating sequence of points and lines, $v_{0}, x_{1}, v_{1}, x_{2}, \ldots v_{n-1}, x_{n}, v_{n}$, where each line is incident with the point preceding and following it. If all the points of a walk are distinct, it is a path. If the path of $n$ points, $n \geq 3$, is closed and the first and last points are the same, it is a cycle. A graph is connected if every pair of points is connected by a path. A connected subgraph of $G$ is a connected component or simply a component of $\mathbf{G}$.


FIGURE 2
Figure 2 illustrates some of the preceding definitions. Represented are the graph $G$, the subgraph $G-p_{s}$ and the subgraph $G-x_{11}$. The subgraph $G-p_{\text {s }}$ is an induced subgraph of $G$. It is maximal for $p_{i}$, $1 \leq i \leq 9, i \neq 5$. The subgraph $G-x_{11}$ is a spanning subgraph of $G$.

In G, $p_{1}, x_{s}, p_{s}, x_{8}, p_{g}, x_{\mathrm{k}}, p_{\mathrm{s}}, x_{\mathrm{B}}, p_{\mathrm{B}}$ is a walk but not a path. $p_{8}, p_{8}, p_{8}, p_{5}, p_{a}$ is a path. $p_{8}, p_{1}, p_{\mathrm{s}}, p_{4}$ is a closed path, in fact, a 9 -cycle path.
$G$ is a connected graph. $G-p_{\mathrm{s}}$ is a connected subgraph of $G$. $G-x_{12}$ is a disconnected subgraph of $G$, containing two components.

In considering the colorability of a map, the size and shape of the
individual regions are of no consequence, only the relations between the regions. Each region may be reduced to a single point. A line is drawn between points whose regions share a common border. The properties of the map are preserved, thereby establishing a direct parallel between adjacent points of the graph and adjacent regions of the map.

Since the maps under consideration are all planar, but all graphs are not, it is necessary to establish criteria for planar graphs. Through the work of Euler the following statement can be made; if $G$ is a ( $p, q$ ) plane graph in which every face is an $n$-cycle, then the number of lines in $G$ equals $n(p-2) /(n-2)$. Since the maximum number of lines in a plane graph occurs when each face is a $8 \cdot c y c l e$, substituting $n=3$ into $q=n(p-2) /(n-2)$ yields $q=3 p-6$. This equation represents the maximum number of lines for $p$ points. Since deletion of lines does not affect point count, a general formula can be derived for planar graphs: if $p \geq 3$, then $q \leq 3 p-6$.

Now we are equipped to prove a theorem for the planarity of $G$ given the degrees of all points $p$ in $G$.

Theorem: For any planar graph G for $\mathrm{p} \geq 3$, there exists at least one vertex of degree five or less.
Proof: Given $G$ is planar and $p \geq 3$, then $q \leq 3 p-6$ holds for all $G$. Assume all points $p$ in $G$ are of degree equal to six, then the number of lines $q$ in $G$ is represented by:

$$
q=\sum_{i} \operatorname{deg} p_{1} / 2=\frac{6 p}{2}=3 p
$$

Substituting this quantity for $q$ into the planarity relation $q \leq 3 p-6$ yields $3 p \leq 3 p-6$, but $0 \not \leq-6$. Therefore if $G$ is to be planar, six lines must be subtracted from G, always forcing at least one vertex to have a degree of 5 or less.

The proof of the five-color theorem presented here is due to the work of P. J. Heawood in 1890. The technique is mathematical induction.

Theorem: Every planar graph is 5-colorable.
Proof: For all graphs where $p \leq 5$, it is obvious that no more than five colors are required.

Assume all planar graphs of $p$ points are five-colorable. It remains only to prove that all graphs of $p+1$ points require no more than five colors. Let $G$ have $p+1$ points.

From the theorem on planarity we know there exists at least one vertex $v$ of degree 5 or less in G. By the inductive assumption, subgraph $G-v$ is five-colorable, so it remains only to assign a color to $v$. If $v$ is of degree 4 or less, assign $v$ to the color $c_{i}$ which was not utilized in coloring the points adjacent to $v$.

If $v$ is of degree 5 , two possibilities must be resolved. First, assign points adjacent to $v$ the names $p_{i}$, where $1 \leq i \leq 5$. The colors available will be $c_{i}$, where $1 \leq i \leq 5$. If the coloring of $G-v$ has required the use of four or fewer colors for the five points adjacent to $v$, then as in the case above, the unused color may be assigned to $v$ to produce a five-coloring.

Suppose that the coloring of G-v has used all five colors for the points adjacent to $v$. Let each point $p_{1}$ be colored $c_{i}$. Let $G_{13}$ be the subgraph of $G-v$ induced by those points having colors $c_{1}$ and $c_{3}$. If $p_{1}$ and $p_{i}$ belong to separate components of $G_{13}$ (Figure 3), then



FIGURE 3
the color scheme of the component containing $\boldsymbol{p}_{1}$ may be reversed. That is, exchange the colors $c_{1}$ and $c_{3}$. This results in $p_{1}$ now having color $c_{3}$ leaving $c_{1}$ free to color $v$.

If $p_{1}$ and $p_{3}$ belong to the same component of $G_{13}$, then there exists by definition a path between $p_{1}$ and $p_{3}$ (Figure 4). Adding $v$


FIGURE 4


FIGURE 5
to subgraph $G_{13}$ produces a cycle $v, p_{1}, \ldots$ the path between $p_{1}$ and $p_{3} \ldots p_{3}, v$ which encloses either $p_{2}$, or $p_{4}$ and $p_{3}$. Then considering the subgraph $G_{24}$ induced by points colored $c_{2}$ and $c_{4}$, there can be no path in $G_{24}$ between $p_{2}$ and $p_{4}$. Then by definition, $p_{2}$ and $p_{4}$ are in different components (Figure 5). In this case the colors of the component of $G_{24}$ containing $p_{2}$ can be interchanged. Now $p_{2}$ is colored $c_{4}$, leaving $v$ free to be colored $c_{2}$ thus producing the desired five-coloring.

Having satisfied all conditions for proof by induction, we may confidently state that for any planar graph, five colors will always be sufficient for coloring a planar graph. Since every map may be represented by a planar graph, every map is five-colorable.

## REFERENCES

1. Ball, Rouse W. W., revised Coxeter, H. S. M., Mathematical Recreations and Essays. New York: The Macmillan Company, 1960.

Harary, Frank, Graph Theory. Reading, Massachusetts: Addi-son-Wesley Publishing Company, 1969.
Ore, Oystein, Graphs and their Uses. New York: Random House, 1963.

# Log Jam in the Classroom* 

Barbàra Leamer<br>Student, Southwest Missouri State University

Imagine that you are teaching an attentive group of high school seniors. You have just finished a unit on the laws of exponents and logarithms and have begun an explanation of tables of logarithms and their uses. You finish explaining characteristics and mantissas and give an example:

$$
\log _{10} 248=2.3945
$$

One of the students says, "How do you know?"
So you start to repeat your explanation:

$$
\begin{aligned}
& \log _{10} 248=\log _{10} 100+\log _{10} 2.48 \\
& =2+\log _{10} 2.48=2+.3945=2.3945
\end{aligned}
$$

"That's fine" the questioner says, "up to the $\log _{10} 2.48=.3945$, or to state it differently, that $10.3945=2.48$. Prove itl"

You think to yourself, "What do you mean, 'Prove itl' It's in the table, what more do you want?"

Then, after a moment's comtemplation, you realize, "That's an excellent question. I wish I were the one asking instead of the one answering."

Being the quick thinker that you are, you realize that an approximation can be used for this problem which will result in a pretty straightforward solution.
"Let's approximate $\log 248$ as 2.4. We then want to show that $10^{2.4}=248$. Using our laws of exponents we can write $10^{2.4}=\left(1^{2}\right)$ $(10 \cdot 4)=100 \cdot 10 \cdot 4$. So in order to show $\log 248=2.4 \mathrm{it}$ is sufficient to show $10^{4}=2.48$. Okay?
"What will have to be true if $10.4=2.48$ ?
$\left(10^{.4}\right)^{2.5}=(2.48)^{2.5}$
$10^{1}=(2.48)^{2.8}$
"So we simply must show that $(2.48)(2.48)(2.48)^{\circ}=10 . "$

[^1]"Granted," the questioning student replies, "but what if we want to find 10 to a certain power, say $10^{.631}$. There must be a technique for doing this. How did the tables get there to begin with?"

This is the point at which you as a teacher use one of your standard "outs", probably, "We really don't have time to discuss this any further right now because we have a lot of material to cover, but I would be glad to look into the subject and we can discuss it a little later."

Being fresh out of college and well versed in higher mathematics, you recall that the irrational numbers which are of concern to us such as $\pi, c$, sines, cosines, and logarithms can be written as infinite series of products.

$$
\begin{aligned}
\pi & =\frac{2}{1} \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \frac{8}{9} \cdots \\
e & =\frac{1}{11}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\cdots \\
\sin x & =x-\frac{x^{3}}{3!}+\frac{x^{5}}{5!}+\frac{x^{7}}{7!}+\cdots
\end{aligned}
$$

$$
\log _{e} x=2 \frac{x-1}{x+1}+\frac{1}{3}\left(\frac{x-1}{x+1}\right)^{3}+\frac{1}{5}\left(\frac{x-1}{x+1}\right)^{s}+\cdots
$$

But the problem arises that your students have not studied series, especially of the infinite variety, and would probably be less likely to believe the results obtained using an infinite series than they would be to believe the $\log$ tables.
It seems like there should be a technique for calculating logs or for raising 10 to a given power based simply on the previous skills of the student.
(I wish you could sit down for a couple of hours and really come to appreciate the problem at hand, because the solution is not so elegant. It is obvious, but its beauty lies in the fact that it works in all cases and it will give you as high of a degree of accuracy as you desire.)

We must build on that which the student is already capable of doing (or that which the student accepts as possible even if he does not possess the behavior himself). We are talking about $10^{n}=x$ where $0<n<1$ and $1<x<10$. What are the values of $10^{n}$ for
$0<n<\mathrm{I}$ which the student has the knowledge to find? He can raise $10^{.5}$ (i.e., take the square root) and $10^{.25}, 10^{125}$, or in general $10^{n}$ where $n=(.5)^{m}$. That is, the student can (in theory) take any number of square roots.

Having made this assumption we must convince the student that any number between 0 and 1 can be written as the sum of the appropriate powers of $\underset{\varepsilon}{l}$. This is nothing new to the student who has studied bases. He made this assumption when he tried to write a base ten decimal fraction as a decimal fraction in base two, because the places in base two to the right of the decimal point are $\frac{1}{2}$, $\left(\frac{1}{2}\right)^{2},\left(\frac{1}{2}\right)^{3}$, etc.

We are now set to find $10^{n}$. For an example, let's try $10^{-931}$ as our quick-thinking student suggested. We want to write .631 as the sum of powers of $\frac{1}{2}$ and then, since $a^{x+y}=a^{x \cdot} a^{y}$, we can write $10^{.831}$ as a product of 10 to the various powers of $\frac{1}{2}$. We know that we can find 10 to any power of $\frac{1}{2}$, so we can solve the problem. Before we actually do this, let's make a table of $10^{n}$ for $n=.5,(.5)^{2}$, etc.

| $m$ | $(.5)^{m}$ | $10 c(. s)^{m}$ |
| ---: | :--- | ---: |
| 1 | .5 | 3.162278 |
| 2 | .25 | 1.778279 |
| 3 | .125 | 1.333521 |
| 4 | .0625 | 1.154782 |
| 5 | .03125 | 1.074609 |
| 6 | .015625 | 1.036639 |
| 7 | .0078125 | 1.018152 |
| 8 | .00990625 | 1.009035 |
| 9 | .001953125 | 1.004507 |
| 10 | .0009765625 | 1.002251 |
| 11 | .00048828125 | 1.001125 |
| 12 | .000244140625 | 1.000562 |
| 13 | .0001220703125 | 1.000281 |
| 14 | .00006103515625 | 1.000141 |
| 15 | .000030517578125 | 1.000070 |
| 16 | .0000152587890625 | 1.000035 |
| 17 | .00000762989453125 | 1.000018 |
| 18 | .000003814697265625 | 1.000009 |
| 19 | .0000019073486328125 | 1.000004 |
| 20 | .00000095367431640625 | 1.000002 |
| 21 | .000000476897158203125 | 1.000001 |

Let's write $\mathbf{. 6 3 1}$ as a sum:

$$
\begin{aligned}
.631 & =.5+.131 \\
& =.5+.125+.006 \\
& =.5+.125+.00390625+.00209375 \\
& =.5+.125+.00390625+.001953125+.000140625
\end{aligned}
$$

So to within three significant figures we can write:

$$
.631=.5+.125+.00390625+.001953125
$$

## Thus

$$
\begin{aligned}
10^{.831} & =\left(10^{.5}\right)\left(10^{.5^{3}}\right)\left(10.5^{8}\right)\left(10.5^{8}\right) \\
& =(3.162278)(1.393521)(1.009035)(1.004507) \\
& =4.274
\end{aligned}
$$

This is accurate to three decimal places where the thousandth's place is correct only within a couple of digits ( $\pm .002$ or so). If we want greater accuracy we must add another power of (.5).

So

$$
\begin{aligned}
10.68100 & =(10 \cdot 3)\left(10 \cdot 5^{3}\right)\left(10.5^{8}\right)\left(10 \cdot 5^{2}\right)\left(10 \cdot .^{23}\right)\left(10 . .^{18}\right)\left(10 . .^{10}\right) \\
& =4.27561
\end{aligned}
$$

As was true before, the last decimal place is only an approximation.
We can use this technique in reverse to find logarithms. For example, $\log _{1 .} 7.4052$. This must be the product of a combination of $10^{n}$ where $n=(.5)^{m}$ for certain $m^{\prime}$ 's. We can find the factors of 7.4052 . Since we know the logarithms of those factors, we can add the logarithms to get our solution. $(\log x y=\log x+\log y)$. Just as we could express any number between 0 and $l$ as the sum of the appropriate powers of $\frac{1}{2}$, any number between 1 and 10 can be expressed as the product of 10 to the appropriate powers of $\frac{1}{2}$.

Let's look at our example: $\log 7.4052$

$$
\begin{aligned}
7.4052 & =(3.162278)(2.341730) \\
& =(10.5)(1.778279)(1.316852) \\
& =(10.5)\left(10.5^{2}\right)(1.154782)(1.140347) \\
& =(10.5)\left(10.5^{2}\right)\left(10.5^{4}\right)(1.074608)(1.061174) \\
& =(10.5)\left(10.5^{2}\right)\left(10.5^{4}\right)\left(10.5^{5}\right)(1.036633)(1.023674) \\
& =(10.5) \ldots\left(10.5^{6}\right)(1.018152)(1.005424) \\
& =(10.5) \ldots\left(10.5^{5}\right)(1.004507)(1.000913) \\
& =(10.5) \ldots\left(10.5^{9}\right)(1.000562)(1.000350)
\end{aligned}
$$

Thus $\log 7.4052=.5+.5^{2}+.5^{4}+.5^{3}+.5^{6}+.5^{7}+.5^{8}+.5^{12}$ $=.8694$

If we want greater accuracy, we find that

$$
\begin{aligned}
\log 7.4052 & =.5+.5^{2}+\ldots+.5^{12}+.5^{13}+.5^{16}+.5^{17} \\
& =.86953
\end{aligned}
$$

Having developed your own method for finding logarithms or antilogarithms your interest and curiosity is stimulated and you begin to wonder how the "discoverers" of logarithms calculated the values for their tables - or for that matter, how they developed their theory to begin with.

As is often true in mathematics or any of its sister sciences, much of the original thought is lost by the time an idea reaches us. Concepts are revised and fitted into the mathematical pattern so that by the time the student of mathematics learns the idea, much resemblence to the historical concept has been lost. Logarithms are no exception to this rule.

Logarithms are usually developed as the inverse function of exponents and are defined in terms of exponential expressions. But
"It is one of the greatest curiosities of the history of science that Napier constructed logarithms before exponents were used." ${ }^{1}$ Not being concerned with exponents, he was likewise not concerned about the base of his logarithms (For what is the base but the number which must be raised to the $\log x$ in order to get $x$.) Not being worried about a base, Napier's logarithms did not have a base in the present sense of the word. So the natural or Naperian logarithms with base e were not those invented by Napier.

If logarithms did not develop from the concept of exponents, what did stimulate their development? We might hazard a guess to say that Napier was interested in prosthaphaeresis - the process of replacing the operations of multiplication and division by addition and subtraction. Trigonometry, being of much concern at the time, made this process not untamiliar: $\sin a \cdot \sin b=\frac{1}{2} \cos (a-b)-\frac{1}{2} \cos (a+b)$. Thus when John Napier published his table of logarithms in 1614 they succeeded remarkably in accomplishing what he intended simplifying complicated calculations. "It is no exaggeration to say with Laplace that the invention of logarithms 'by shortening the labours doubled the life of the astronomer.' "?

Logarithm was the term which Napier used to describe his artificial numbers. It comes from the Greek and means "ratio number" a suitable description for Napier's system because it, like logarithms which we know today, was a relationship or mapping between a geometric sequence and an arithmetic sequence.
$\log 1=0$
$\log 10=1$
$\log 100=2$
$\log 1000=3$
$\log 10000=4$

During the seventeenth and eighteenth centuries many men spent time calculating log tables, each with wider ranges or more places. There were natural or Napierian logarithms (with base e), common or Briggian logarithms (with base 10), and logarithms of trigonometric functions. One of the most impressive feats of logarithm calculation must be the table of natural logarithms constructed by Wolfram, a Dutch lieutenant of artillary, who calculated the natural logarithms for 1 to 10,000 to 48 places! ${ }^{3}$

Later mathematicians such as Gregory St. Vincent, Issac Newton, and Nicolaus Mercator discovered that logarithms could be written as infinite series and that doing so simplified the calculation of logarithms. ${ }^{4}$

It may seem of little value to dig up the pieces of the history of mathematics and look at them when we have such a much nicer total picture of logarithms now. What difference does it make how or why Napier developed his logarithms? Little, perhaps, for our appreciation and understanding of logarithms, but maybe it will help to clear up a misconception which most students have about the development of mathematics (or science in general.) A perfected concept is rarely, if ever, the work of one individual. The seeds may come to one person; he may discuss his idea with a collegue and publish it, but the process of the incorporation of an idea into the overall mathematical scheme is one involving many people. So you don't have to be a Euclid, a Newton, or an Einstein to make a contribution to the development of science. A quick-thinking, inquiring student like the one who posed the question at the beginning of this article would be a likely candidate for an expander of mathematical horizons.

## FOOTNOTES

1. Florian Cajori, A History of Elementary Mathematics (New York, 1905), p. 157.
2. Ibid., p. 155.
3. Ibid., p. 166.
4. Ibid., p. 167.

## REFERENCES

Cajori, Florian. A History of Elementary Mathematics. New York: The Macmillian Company, 1905.
Eves, Howard. An Introduction to the History of Mathematics. New York: Holt, Rinehart and Winston, 1969.
"Logarithms." The Encyclopedia Britannica, 1911, XI, pp. 869-878.
Sanford, Vera. A Short History of Mathematics. Cambridge, Mass.: Houghton Mifflin Company, 1958.
Smith, David Eugene. A Source Book in Mathematics, Vol. II. New York: Dover Publications, Inc., 1959.

# On the Graphs of Groups* 

Linda Grandstaff<br>Student, Washburn University

In "A note on the Graphs of Groups, I," Dr. Billy E. Milner states that with respect to groups of order sixteen there exists nonisomorphic groups that have isomorphic graphs. It is of interest to note that the non-Abelian groups $G_{3}$ and $G_{4}$ (as defined in the article) both have nontrivial, cyclic centers and are the only non-Abelian groups of order sixteen possessing this characteristic. Hence, this would suggest that perhaps it would be possible to obtain an isomorphism between an Abelian group and a non-Abelian group (having order greater than or equal to sixteen) whenever the center of the non-Abelian group is nontrivial and cyclic. However, consideration of Sym: $\times Z_{1}$, a non-Abelian group of order eighteen having a nontrivial, cyclic center, shows that this conjecture is not the case. Now looking back at the original isomorphism example, one finds that the order of $Z\left(G_{i}\right)=4$ and the order of $Z\left(G_{4}\right)=4$, while the order of $Z\left(S y m_{:} \times Z_{z}\right)=3$. Hence, perhaps the desired isomorphism could still be obtained if one restricts himself to an examination of non-Abelian groups having nontrivial, cyclic centers of nonprime order. This conjecture is not workable since $D_{*} \times Z_{3}$ is a non-Abelian group of order twenty-four with a nontrivial, cyclic center of order six and yet the graph of this group is not isomorphic to the graph of any Abelian group of order twenty-four. Once again, let us return to the original isomorphism to see if any other peculiarities with respect to the centers of the groups may be found. Indeed, one linds that the order of the centers of $G_{i}$ and $G_{i}$ are powers of a prime. Let us then take as a conjecture that the desired isomorphism can be defined if one restricts himself to consideration of non-Abelian groups having nontrivial, cyclic centers of order $p^{k}$ where $p$ is a prime and $k>1$. Again, a counterexample to the conjecture can be found in the group $;=<a, b: a^{8}=b^{3}=c, b a=a b^{2}>$. This group is of order twenty-four and has a cyclic center of order $2^{2}$. The graph of the group, however, is not isomorphic to the graph of any Abelian group of order twenty-four.

[^2]While the previous conjectures were not as profitable as one would desire, they did provide information concerning characteristics of groups and relationships between different groups and their graphs. Indeed, it was while working with these conjectures that the following theorem was obtained.

Definition. The graph of a group G is isomorphic to the graph of a group H if and only if there exists a one-to-one mapping $\phi$ of $\mathbf{G}$ onto H such that $\phi\left(\mathrm{g}^{2}\right)=[\phi(\mathrm{g})]^{2}$ for each g in $\mathbf{G}$.

Theorem. If G is a finite, non-Abelian group having order p : where $p$ is an odd prime and having the defining relations $x^{\prime \prime}=$ $y^{\mathrm{p}}=\mathrm{e}, \mathrm{y}^{-1} \mathrm{xy}=\mathrm{x}^{1+\mathrm{p}}$, then the graph of $\mathrm{C}_{\mathrm{p}}{ }^{2} \times \mathrm{C}_{\mathrm{p}}$ is isomorphic to the graph of G .

Before presenting the proof of the theorem, some additional information (as contained in the three lemmas which follow) is helpful. It should also be understood that henceforth $G$ will be taken to indicate a finite, non-Abelian group having order $p^{3}$ where $p$ is an odd prime and having the defining relations $x^{p}=y^{p}=e$, $y^{-1} x y={ }^{1+\rho}$.

Lemma 1: If x , y are elements of a group $G$, then $\mathrm{yx}^{\mathrm{m}}=\mathrm{x}^{\mathrm{m}(1-\mathrm{m})} \mathrm{y}$ for m in N .

Proof: The proof of the theorem will proceed by mathematical induction on $m$. Let $x, y$ be elements of a group G. Assume $m=1$. The claim is that $y x=x^{1-p} y$. Now since $x^{p}$ is an element of $Z(G)$, $x^{-p} y=x y x^{-\mu}=y y^{-1} x y x^{-p}=y x^{1+p} x^{-p}=y x$. Hence, the claim is true. Now assume that for $m=k, y x^{k}=x^{k(1-p)} y$. It must be shown that $y x^{k+1}=x^{(k+1)(1-p)} y$. Notice that $y x^{k+1}=y x^{k} x=x^{k(1-p)} y x=$ $x^{k(1-m} x^{1-n} y=x^{(k+1)(1-p)} y$. Hence, for any $m$ in $N, y x^{m}=x^{m(1-p)} y$.

Lemma 2. If $x$,y are elements of a group $G$, then $y^{n} x^{m}=x^{m-m n p y n}$ for $\mathrm{m}, \mathrm{n}$ in N .

Proof: The proof of the theorem will proceed by mathematical induction on $n$. Let $x, y$ be elements of a group G. Assume $n=1$. The claim is that $y x^{m}=x^{m-m p} y$. This statement follows from Lemma 1. Now assume that for $n=k, y^{k} x^{m}=x^{m-k m p} y^{k}$. It must be shown that $y^{k+1} x^{m t}=x^{m-(k+1) m p} y^{k+1}$. Notice that $y^{k+1} x^{m}=y y^{k} x^{m}=$ $y x^{m-k m p} y^{k}=x^{(m-k m p)(2-p)} y y^{k}=x^{(m-k m p)(2-p)} y^{k+1}=x^{m-(k+1) m p} y^{k+1}$. Hence, for any $m, n$ in $N, y^{n} x^{m}=x^{m-m n p} y^{n}$.

Lemma 3. If $\mathrm{x}, \mathrm{y}$ are elements of a group G, then $\left(\mathrm{x}^{\mathrm{m}} \mathrm{y}^{\mathrm{n}}\right)^{\mathbf{2}}=$ $\mathbf{x}^{2 \mathrm{~m}-\mathrm{mnD}} \mathrm{y}^{2 \mathrm{n}}$ for $\mathrm{m}, \mathrm{n}$ in N .

Proof: Let $x, y$ be elements of a group G. Notice that $\left(x^{m} y^{n}\right)^{2}=$ $\left(x^{m} y^{n}\right)\left(x^{m} y^{n}\right)=x^{m} x^{m-m n p} y^{n} y^{n}$ by Lemma 2. Hence, $\left(x^{m} y^{n}\right)^{2}=$ $x^{2 m-m n p} y^{2 n}$ for $m, n$ in $N$.

Proof of Theorem: Define a mapping $\phi: C_{p}{ }^{2} \times C_{p} \rightarrow G$ by $\phi\left(a^{m} b^{n}\right)=x^{r} y^{s}$ where $r=m-m n p$ and $s=2 n$. It is clear that $\phi$ is a one-to-one, onto mapping. Hence, it need only be shown that $\phi$ is square preserving, that is, $\phi\left(\left(a^{m} b^{n}\right)^{2}\right)=\left[\phi\left(a^{n} b^{n}\right)\right]^{2}$. Now $\phi\left(\left(a^{m} b^{n}\right)^{2}\right)=\phi\left(a^{2 m} b^{2 n}\right)=x^{2 m-4 m n p} y^{+n}$. Notice that $\left[\phi\left(a^{m} b^{n}\right)\right]^{2}=$ $\left[x^{m-m n p} y^{2 n}\right]^{2}=x^{2 m-2 m n p-(m-n i n p)(2 n)(p)} y^{4 n}=x^{2 m i-4 m n p} y^{n}$. Thus $\phi$ is square preserving. Hence, the graph of $C_{p}=\times C_{p}$ is isomorphic to the graph of $G$.

Note that $C_{p^{2}} \times C_{p}$ is certainly not isomorphic to $G$. Hence, this theorem enables one to obtain examples of two groups which are not isomorphic but which have isomorphic graphs.

Although there clearly is not a one-to-one correspondence between groups and graphs of groups (which is a desirable condition), the theory being developed in the specified field is worthwhile since the exploration that can be done within this area enables a student to gain invaluable experience in mathematical research. Indeed, there are many more topics with regard to the graphs of groups which are open to investigation. Attempts to characterize the graphs of groups having prime orders, to examine the relationship (if any) between the Frattini subgroup of a given group and the graph of the group, or to deal with graphs of infinite groups should prove to be challenging endeavors for individuals interested in, this area of study.

## REFERENCES

Burnside, William. Theory of Groups of Finite Order, second edition. New York: Dover Publications, Inc., 1911.
Milner, Billy E. "A Note on the Graphs of Groups, I." Mathematics Magazine, 45, No. 1 (January 1972), 45.
Polites, George W. An Introduction to the Theory of Groups. Scranton: International Textbook Company, 1968.

# A Partition Function Revisited 

Daniel Q. Dye Jr.<br>Alumnus, Colorado State University, Ft. Gollins

This note offers an improved program for the partition function, $p(n)$, offered by Migliozzi in the Spring 1973 issue of THE PENTAGON. Several minor errors have been rectified, and unnecessary repeated calculations have been eliminated. Note that the values of $\frac{1}{2}\left(3 j^{2}-j\right)$ are computed once and stored in a table, reducing the frequent calculations of subscripts to simple subtractions. (Values of $\frac{1}{\underline{2}}\left(3 j^{2}+j\right)$ need not be stored, since $\frac{1}{2}\left(3 j^{2}+j\right)=\underline{p}\left(3 j^{2}-j\right)+j$ ) The upper bound for $j$ is easily determined, given $n \leq 100$, since $j$ $=8$ yields values of 92 and 100 for the expression $\ell\left(3 j^{2} \pm j\right)$. The sign-changing expression $(-1)^{j+1}$ has been replaced by a variable with values $\pm 1$ as needed.

DIMENSION IP(100), IJ (8)
DO $20 \mathrm{~J}=1,8,1$
20
$\mathrm{I}(\mathrm{J})=\left(3^{*}\left(\mathrm{~J}^{* * 2}\right)-\mathrm{J}\right) / 2$
DO $60 \mathrm{~N}=1,100,1$
$\operatorname{IP}(N)=0$
ISGN = 1
DO $50 \mathrm{~J}=1,8,1$
$\mathrm{L}=\mathrm{N}-\mathrm{IJ}(\mathrm{J})$
IF(L) 55,31,32
$31 \quad \operatorname{IP}(\mathrm{~N})=\operatorname{IP}(\mathrm{N})+$ ISGN
GO TO 55
32

$$
\begin{aligned}
& \operatorname{IP}(N)=\operatorname{IP}(N)+\operatorname{ISGN}{ }^{*} \operatorname{IP}(\mathrm{~L}) \\
& M=\mathrm{L}-\mathrm{J} \\
& \operatorname{IF}(\mathrm{M}) 55,41,42 \\
& \mathrm{IP}(\mathrm{~N})=\mathrm{IP}(\mathrm{~N})+\mathrm{ISGN}
\end{aligned}
$$

GO TO 55
$42 \quad \operatorname{IP}(\mathrm{~N})=\mathrm{IP}(\mathrm{N})+\operatorname{ISGN}{ }^{*} \mathrm{IP}(\mathrm{M})$
50 $\quad$ ISGN $=-$ ISGN
55 WRITE (5,56) N,IP(N)
56 FORMAT (IX,I3,2X,I9)
60 CONTINUE
STOP
END

## REFERENCE

Migliozzi, Nicholas V., "A Note On The Partition Function p(n)," The Pentagon, Spring 1973, pp. 85-86.

# Congruence, Symmetry and Dimension 

Ali R. Amir-Moez<br>Faculty, Texas Tech University

In a Euclidean plane, it is customary to define: "Two configurations are congruent if one can be superposed on the other". In the prools of many theorems concerning congruence, one may convince students appealing to their intuitions. When the tool of analytic geometry is introduced, then one talks about a rigid motion which is a combination of a translation and a rotation. But still a rigid motion in the plane is not sufficient for superposition of one configuration upon its congruent. Either one has to change the definition of congruence or embed the plane in a three dimensional Euclidean space. In this article we shall study these ideas and some generalizations of them.

In what follows we shall use notations of elementary linear algebra. All coordinate systems are taken to be rectangular (orthonormal).

1. Symmetry with respect to a line: Let $d$ be a line in the plane (Figure 1). Consider the triangle $A B C$. Let $A^{\prime}, B^{\prime}$, and $C^{\prime}$, be respectively symmetricals of $A, B$, and $C$ with respect to $d$. It is clear that triangles $A B C$ and $A^{\prime} B^{\prime} C^{\prime}$ are congruent. But in order to superpose $A^{\prime} B^{\prime} C^{\prime}$ on $A B C$ we must, for example, rotate $A^{\prime} B^{\prime} C^{\prime}$ about $d$ through an angle $\pi$. This way we must make use of the third dimension.


FIGURE 1

Let us study this algebraically. Let $d$ be the $y$-axis. Then the matrix of the symmetry will be


Let us consider $A \Longleftrightarrow\left(x_{1}, y_{1}\right)$. Then $A^{\prime} \Leftrightarrow\left(x_{1}, y_{1}\right)$ such that

$$
\left(x_{1}, y_{1}\right)=\left(x_{1}, y_{2}\right)\left[\begin{array}{cc}
-1 & \overline{0} \\
0 & 1
\end{array}\right]=\left(-x_{1}, y_{1}\right)
$$

Let $B \Longleftrightarrow\left(x_{2}, y_{2}\right)$ and $C \Leftrightarrow\left(x_{3}, y_{3}\right)$. Then $B^{\prime} \Leftrightarrow\left(-x_{2}, y_{2}\right)$ and $C^{\prime} \Leftrightarrow$ $\left(-x_{3}, y_{3}\right)$. This way one can show that there is no rotation and translation in the plane which superposes $A^{\prime} B^{\prime} C^{\prime}$ on $A B C$. Indeed, the inverse of

$$
A=\left[\begin{array}{cc}
-1 & 0 \\
0 & 1
\end{array}\right]
$$

is itself. One easily notes that $A$ is not the matrix of a rotation since the matrix of a rotation is of the form


It is easy to carry out the algebra of a translation which puts one of the vertices of the triangle $A^{\prime} B^{\prime} C^{\prime}$ on the corresponding one of the triangle $A B C$. Without loss of generality let us put $A^{\prime}$ on $A$ and translate the coordinate system to the position, where $A$ is on the origin (Figure 2). In this case let $B \Longleftrightarrow(p, q)$ and $C \Longleftrightarrow(r, s)$. Then $B^{\prime} \Longleftrightarrow(-p, q)$ and $C^{\prime} \Leftrightarrow(-r, s)$. Now let $\alpha$ be the angle $C^{\prime} A C$. Then the rotation of the plane through $\alpha$ put $C^{\prime}$ on $C$, but $B^{\prime}$ will not go to $B$. Here one can easily carry out the algebra. Using inner product one may obtain

$$
\cos \alpha=\frac{-r^{2}+s^{2}}{r^{2}+s^{2}}
$$

$$
\sin \alpha=-\sqrt{1-\cos ^{2} \alpha}=\frac{-2 r s}{r^{2}+s^{2}}
$$



FIGURE 2

Thus the matrix of this rotation is

$$
\frac{1}{r^{2}+s^{2}}\left[\begin{array}{cc}
-r^{2}+s^{2} & -2 r s \\
2 r s & -r^{2}+s^{2}
\end{array}\right]=[A]
$$

We note that

$$
(-r s)\left[\begin{array}{cc}
\frac{-r^{2}+s^{2}}{r^{2}+s^{2}} & \frac{-2 r s}{r^{2}+s^{2}} \\
\frac{2 r s}{r^{2}+s^{2}} & \frac{-r^{2}+s^{2}}{r^{2}+s^{2}}
\end{array}\right]=(r s)
$$

but

$$
(-p q)[A] \neq(p q)
$$

The reader may perform the algebra.
One can also study the rotation which take $B^{\prime}$ to $B$ and show that their rotation does not carry $C^{\prime}$ to $C$. We shall leave the algebra of it to the reader.

Now we shall refer to Figure 1 again. Let us embed the plane in a three-dimensional Euclidean space. This way the matrix of the symmetry with respect to the $y$-axis will be

$$
S=\left[\begin{array}{ccc}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{array}\right]
$$

But $A \Leftrightarrow\left(x_{1}, y_{1}, 0\right), B \Leftrightarrow\left(x_{3}, y_{3}, 0\right)$ and $C \Leftrightarrow\left(x_{3}, y_{3}, 0\right)$. The matrix of the rotation about the $y$-axis through $\pi$ is also

$$
S=\left[\begin{array}{ccc}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{array}\right]
$$

Since $S^{2}=I$, the identity matrix, $S$ is the rotation which superposes $A^{\prime} B^{\prime} C^{\prime}$ on $A B C$.
2. Symmetry with respect to plane: Here again we study the same idea for a three-dimensional space. Let $\mathcal{P}$ be a plane and $A B C D$ be a tetrahedron (Figure 3). Let $A^{\prime}, B^{\prime}, C^{\prime}$ and $D^{\prime}$ be respectively symmetricals of $A, B, C$, and $D$ with respect to $\mathscr{P}$. Indeed the tetrahedrons $A B C D$ and $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ are congruent. But there is no


FIGURE 3
rigid motion of the space which superposes $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ on $A B C D$. We shall treat the problem algebraically. Let $P$ be the $x y$-plane. Then the matrix of the symmetry with respect to $P$ is

$$
A=\left[\begin{array}{rrr}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{array}\right]
$$

Indeed, the inverse of $A$ is $A$ itself, and one can see easily that $A$ is not a rotation. This will be explained more thoroughly in Section 3.

Now we embed the space in a four-dimensional space in such a way that $w$-axis is perpendicular to the $x y z$-space. In this case the matrix of the symmetry with respect to the $x y$-plane is

$$
S=\left[\begin{array}{rrrr}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{array}\right]
$$

In Section 3 we shall explain with more details, that this is the matrix of a rotation and one may call it the rotation about the xyplane through an angle $\pi$. Since $S^{2}=I$, one takes $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ to the four-dimensional Euclidean space and rotate it about the $x y$-plane through an angle $\pi$ in order to superpose it on $A B C D$.
3. Rotations about subspaces: In the plane, the only rotation possible is about a fixed point. In the Euclidean three-space rotations leave a direction invariant. One may choose the coordinate system such that the matrix of the rotation would be


Here we have set the $x$-axis to be the axis of the rotation. The reader may write the matrix of a rotation whose axis is the $y$-axis or the $z$-axis. For example, the rotation described in Section 1 is about the $y$-axis through $\pi$.

Next we shall talk about rotations about a plane in a four-dimensioal space. In this case one may choose the coordinate system in such a way that the $x y$-plane remains fixed and for the matrix of the rotation one obtains.
$\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos t & \sin t \\ 0 & 0 & -\sin t & \cos t\end{array}\right]$

Now one can easily see that $S$ in Section 2 is the matrix of the rotation about the $x y$-plane through $\pi$.
4. A generalization: We may generalize the idea of Section 3. Let the basis in an $n$-dimensional Euclidean space be chosen such that the matrix of a linear transformation is

$$
A=\left[\begin{array}{llll}
I_{n-2} & & 0 & \\
0 & {\left[\begin{array}{lll}
\cos t & \sin t \\
-\sin t & \cos t
\end{array}\right]}
\end{array}\right]
$$

where $I_{n-2}$ is the identity matrix of order $n-2$. Let the chosen orthonormal basis be $\left\{\alpha_{1}, \ldots, \alpha_{n}\right\}$. Then we say that $A$ is the rotation about the subspace $\left[\alpha_{1}, \ldots, \alpha_{n-2}\right]$ through an angle $t$.
5. Symmetries and Rotations: Let $E_{n}$ be a Euclidean space of dimension $n$. Let $\left\{\alpha_{1}, \ldots, \alpha_{n}\right\}$ be an orthonormal basis in $E_{n}$. Let $A$ be the symmetry with respect to the subspace $\left[\alpha_{1}, \ldots, \alpha_{n-1}\right]$. Then the matrix of $A$ is

$$
A=\left[\begin{array}{ll}
I_{n-1} & 0 \\
0 & -1
\end{array}\right]
$$

where $I_{n-1}$ is the identity matrix of order $n-1$. It is clear that $A^{-1}=A$. Let $\alpha_{n+1}$ be a unit vector orthogonal to $E_{n}$; thus $\left\{\alpha_{1}, \ldots, \alpha_{n}, \alpha_{n+1}\right\}$ generates an $(n+1)$-dimensional Euclidean space. Then for the symmetry with respect to $\left[\alpha_{1}, \ldots, \alpha_{n-1}\right]$, we obtain the matrix

$$
S=\left[\begin{array}{llll}
I_{n-1} & & 0 & \\
& {\left[\begin{array}{ccc}
-1 & & 0 \\
0 & & -1
\end{array}\right]}
\end{array}\right]
$$

which is also the matrix of a rotation about the subspace $\left[\alpha_{1}, \ldots, \alpha_{n-3}\right.$ ] through an angle $\pi$, and clearly $S^{-1}=S$.

Whatever has been said so far points out that:
The symmetrical of a configuration $T$ with respect to an ( $n-1$ ) -dimensional subspace of an $n$-dimensional Euclidean space $E_{n}$ is congruent to $T$. The superposition is possible when we embed the space in a Euclidean space of dimension $n+1$.

## The Mathematical Scraphook

Emted by Richard Lee Barlow

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgment will be made in The Pentacion. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard I. Barlow, Kearney State College, Kearncy, Nebraska 68847.

Mathematical puzzles have long interested the general public as well as the Mathematician. Riddles or puzzles originated with the ancients and have been passed down through the centuries. Their history entails nothing short of the development of the thinking of man. Many mathematical puzzles have lead to important developments in the theory of numbers and have provided long hours of thought for the researcher. A couple of interesting mathematical puzzles are the following:

## Puzzle: I. Circling the Squares

The object of this puzzle is to place a different natural number in each of the ten squares (Figure 1) such that the sum of the squares of the numbers in any two adjacent squares shall be equal


FIGURE 1
to the sum of the squares of the numbers in the two squares diametrically opposite them. For example, four numbers have been placed in squares $A, B, F$, and $G$, and must remain in these positions. The reader will note that

$$
16^{2}+2^{2}=256+4=260
$$

and

$$
8^{2}+14^{2}=64+196=260
$$

and hence the conditions of the puzzle are satisfied.
To complete the required distribution of the remaining six numbers, $B$ and $C$ should correspond to $G$ and $H$ (the sum will not necessarily be $\mathbf{2 6 0}$ ), C and D to H and I , etc.

At first this puzzle might appear quite difficult and its solution a "matter of luck." However, upon closer examination, it is noted that the squares of the numbers that are diametrically opposite have a common difference.
For example,

$$
14^{2}-2^{2}=196-4=192
$$

and

$$
16^{2}-8^{2}=256-64=192
$$

It can be proven that this must be so in every case. From the theory of numbers one might recall that the difference of the squares of any two consecutive numbers is always twice the smaller of the two numbers plus one, and furthermore that the difference of the squares of any two numbers (not necessarily consecutive) can always be expressed as the difference of the two numbers multiplied by the sum of the two numbers. Hence, as an illustration of the above two principles,

$$
5^{2}-4^{2}=9=(4 \times 2)+1
$$

and

$$
7^{2}-3^{2}=40=(7-3) \times(7+3)
$$

Now consider 192 which represented the differences of the squares of the given numbers. Decomposing 192 into products of pairs of even natural numbers we obtain

$$
\begin{aligned}
192 & =2 \times 96 \\
& =4 \times 48 \\
& =6 \times 32 \\
& =8 \times 24 \\
& =12 \times 16
\end{aligned}
$$

Dividing each factor by 2 we obtain $1 \times 48,2 \times 24,3 \times 16,4 \times 12$, and $6 \times 8$. The difference and the sum of each of these number pairs respectively are 47 and 49,22 and 26,13 and 19,8 and 16 , and 2 and 14. These five number pairs represent the required ten numbers needed to solve Figure 1. Can you now place the numbers in the correct squares?

Upon further study of the circling the squares problem, it can also be noted that other number combinations are possible, with an infinite number of solutions possible if fractions are allowed. The number of squares in a circle of this type must always be of the form $4 n+6$, where $n$ is a non-negative integer. Can you show this to be the case?

## Puzzle II. The Frog Puzzle

The six highly educated frogs in Figure 2 are trained so as to reverse their order so that their present order of $1,2,3,4,5,6$ becomes $6,5,4,3,2$, 1 , with the blank square remaining in its present position. A frog can jump to the next square (if vacant) or leap over onc adjacent frog to the next square (if vacant) and can move forwards or backwards at pleasure. The problem is to find the pattern the frogs must use to perform this feat in the fewest possible moves.


FIGURE 2

Upon closer examination of the problem one finds that it is not terribly difficult. The following order of movement satisfies the puzzle's conditions: $2,4,6,5,3,1,2,4,6,5,3.1,2,4,6,5,3,1,2,4$, 6. You will note that it took twenty-one moves to complete the solution - the fewest possible moves.
For a more general case with $n$ frogs and $n+1$ spaces, we have the following results:

If $n$ is even, we require $\frac{n^{2}+n}{2}$ moves of which $\frac{n^{2}-n}{2}$ will be
leaps and $n$ simple moves.
If $n$ is odd we shall need $\frac{\left(n^{2}+3 n\right)}{2}-4$ moves of which $\frac{n^{2}-n}{2}$
will be leaps and $2 n-4$ will be simple moves.
For $n$ even, write for the moves all the even numbers in ascending order and then all the odd numbers in descending order. Repeat this series of moves $\frac{n}{2}$ times and follow it by the even numbers in ascending order once. Hence, if you had twelve frogs, the order of moves would be $2,4,6,8,10,12,11,9,7,5,3,1$ repeated six times and then followed by $2,4,6,8,10,12$, or seventy-eight moves.

For $n$ odd, write for the moves all the even numbers in ascending order and then all the odd numbers in descending order and repeat this series $\frac{(n-1)}{2}$ times to be followed by the even integers written in ascending order (cmitting $\boldsymbol{n}-1$ ), the odd numbers in descending order (omitting 1) and finally followed by all the numbers (both even and odd) written in their natural order (omitting 1 and $n$ ). Hence for $n=11$ we would have 2, 4, 6, 8, 10, 11, 9, 7, 5, 3, 1 repeated five times followed by $2,4,6,8,11,9,7,5,3,2,3,4,5,6,7$, $8,9,10$. You will note that this sequence of moves required seventythree total moves. Can you perform the required sequence of moves for 15 or 16 frogs?

# Installation of New Chapters 

Edited by Loretta K. Smith

Information for the department should be sent to Mrs. Loretta K. Smith, 829 Hillerest Road, Orange, Connecticut 06477.

## SOUTH CAROLINA GAMMA CHAPTER <br> Winthrop College, Rock Hill, South Carolina

The South Carolina Gamma Chapter was installed at Winthrop College on 3 November 1972. Lora McCormick from the Tennessee Gamma Chapter at East Tennessee State University performed the installation. There were thirty-eight initiates. The officers installed were Vivian Moore, President; Harriette Parker, Vice President; Gayle Gibson, Secretary; Susan Keating, Treasurer; Dr. Edward Guettler, Faculty Sponsor; and Donald Aplin, Corresponding Secretary. A banquet was held in one of the College dining halls. The installation followed and the program ended with Mrs. McCormick telling about THE PENTAGON and about the National Convention.

## OKLAHOMA GAMMA CHAPTER

Southwestern Statc College, Weatherford, Oklahoma
On 1 May 1973, the Oklahoma Gamma Chapter was installed by Professor Mike Reagan, Corresponding Secretary and Faculty Sponsor of the Oklahoma Alpha Chapter. Professor Reagan explained many of the goals and objectives of Kappa Mu Epsilon and then proceeded with the initiation of the twenty new members. The following is a list of officers and members:

| Eugene Nikkel | President |
| :---: | :---: |
| Bryan Vogt | Vice President |
| Mary Harper - | Recording Secretary |
| Patricia Bieger | Treasurer |
| Wayne Hayes | Corresponding Secretary |
| Don Prock | Faculty Sponsor |
| Debbie Allison | Kenneth Schmidt |
| Ronald Baker | Roy 1. Short |
| Bill Drury | Desmond Stack |
| Kenneth Hahn | Jack Stefanick |
| Raymond McKellips | Dale Teeter |
| David Ogletree | Marcia Wanek |
| Lucille Penner | Dan Williams |
| Patty Potter |  |

## The Book Sheif

Edited by Oscar Beck


#### Abstract

This department of The Pentagon brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. Oscar Beck, Department of Mathematics, Florence State University, Florence, Alabama 35630.


Introduction to Computer Programming, 2nd Ed., Donald I. Cutler, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1972, 313 pp., $\$ 11.00$.

The reviewer began this task with an unusual approach. Rather than to read the text in a short span of time, he chose to read it over a period of weeks somewhat comparable to the length of a course for which students might be using the book. This provided time for considerable attention to each chapter in turn.

The book is very well written, has an abundance of illustrative examples which are well chosen and has excellent problems for the students to attempt. The first four chapters are typical of any intro ductory text providing the background for programming. Chapter IV on Flow Diagramming is particularly well done. It is very detailed and brings out the necessary concepts one-by-one.

From this point on the reviewer was somewhat puzzled by the approach of the text. It considers a mythical computer, a good idea, and introduces a language for its operation. This language appeared to be very similar to what one would have expected to find ten years ago. Realizing that this was the second edition with no reference given as to the date of the first, the reviewer obtained a copy of the first edition which has a copyright date, 1964. Examinations of parallel passages then explained the feeling that this was about ten years old. It is! The major significant change in the text from the first to second edition is the complete revision of the first four sections of chapter ten which deals with SMILEY, a mythical modern giant computer. Other new ideas and a few new problems are inserted as the author obviously felt appropriate from his experience with the first edition. Probably the most important change for the student is that the second edition has the complete solution to all problems except a few which call for a discussion or essay type answer. These solutions appear to be quite thorough and thus should give the student a great deal of security.

This text would be most appropriate for use at institutions where a computer is not available. However, even when a computer is available, the general approach of this text could be suitably used together with a manual for the given computer language to be used. It is suitable for any introductory class in programming and does not assume a background of mathematics beyond elementary algebra.

Professor Robert A. Estes<br>University of Maine, Portland-Gorham

Mathematics for Liberal Arts Students, Gloria Olive, The Macmillan Company, New York, 1973, 288 pp., $\$ 8.95$.

Except as noted below, I found this book a pleasure to read with its informal and intuitive style and its selection of exercises ranging from routine to thought-provoking. I feel that the liberal arts student would be able to read this book and learn from it.

The first five chapters deal with Abstract Mathematical Systems as a central theme in mathematics. The subjects of Logic and Sets are adequately presented as well as the interconnection of the two - a topic often omitted from books of this category. There is a discussion of intuitionism and formalism which seems aimed more at the instructor than the student. I cannot resist the remark that the work of Gödel is not mentioned at all in the discussion of these two schools of thought.

Chapters 6 through 9 deal with probability, statistics, linear algebra and game theory respectively. The material on probability is held to a minimum; for example, nothing is said about conditional probability. For students of the intended audience, this seems to be a good idea. The all too short chapter on statistics is excellently written and deals almost exclusively with the normal curve. Linear Algebra seems out of place here but is used as a source of examples in the chapter on game theory. The student never deals with a matrix larger than a $2 \times 2$ even in the section on linear programming. The chapter on game theory builds on those dealing with probability and linear algebra. The central ideas are developed in a few elementary but well motivated and well written examples. The connection between Game Theory and Linear Programming is noted as is the Von Neumann "minimax" theorem for mixed strategies.

Those, like myself, who have taught calculus for the past 10 years using a variety of "baby real-analysis" textbooks may have trouble gearing their presentation to the intuitive and informal Chapter 10 on calculus. No mention is made of existence of limits, mean value theorem, continuous functions or non-differentiable functions; no differentiation or integration of anything beyond polynomials is demanded. However, the logarithmic and exponential functions are developed in a problem set. It would be very easy for a course to get bogged down in this chapter, so caution should be observed throughout.

Chapter 11, "Computers," as compared with the others does not give the student a good intuitive and informal excursion into that subject. I am afraid that such an attempted excursion is doomed to failure when limited to one chapter. To be of value to the student, this chapter would need to be supplemented by the instructor, perhaps to the point of mutual exhaustion.

James E. McKenna
State University College, Fredonia
Development of Modern Mathematics, J. M. Dubbey, Crane, Russak \& Company, Inc., New York, 1972, 145 pp., \$6.75.
"Can there be any value at all for mathematics in the study of its history?" asks John M. Dubbey in his introductory chapter. This slim volume, a well written, concise, and often penetrating exposition of "the development of mathematics from earliest times to the present day," is Dr. Dubbey's eloquent affirmative answer. In fact, his aim in writing this book was to produce "a work which tries to reach an undergraduate level, and bring to the student's notice something of the great possibilities inherent in the study of the history of mathematics." Such a study, argues the author, allows both the mathematics student and the mathematician to better appreciate not only the accomplishments of mathematics but also its limitations, nature, unity, and societal status as well.

The plan of the book is to treat early mathematics rather briefly, in this way building up the necessary background for the final three chapters where "emphasis is placed on the very rapid growth of the subject in the last 150 years and its increasing influence on modern life." Thus, after the introductory first chapter, the remaining seven are titled: From Pre-History to the Middle Ages, The Rebirth of Mathematics, The Invention of Calculus, The Eighteenth

Century, The Origins of Modern Mathematics, Foundations, The Twentieth Century. This modern emphasis is particularly welcome since few mathematical histories properly treat twentieth century mathematics. Even the latest edition of E. T. Bell's Development of Mathematics, whose title apparently inspired Dubbey's, is obviously inadequate concerning developments of the past 30 years because it was published in 1945.

The years 1820 to 1840 are taken as the founding period of modern mathematics, since significant advances in geometry, algebra, and analysis were occurring at that time. In A. L. Cauchy's books during the 1820's the limit concept was given a relatively modern rendering and taken as fundamental in defining the derivative, integral, and other concepts of the calculus. Contributions to amalysis during this period were also being made by B. Bolzano, C. F. Gauss, C. G. J. Jacobi, and others. In 1824 young H. Abel proved that the general duintic equation could not be solved in terms of radicals. The ill fated E. Galois (1811-1832) made highly original discoveries in the theory of efpuations during his brief lifetime. G. Peacock's Treatise on Algebra of 1830 suggested that algebra should be considered more abstractly than as simply a generalization of arithmetic, a sentiment which had been earlier expressed in C. Babbage's The Philosophy of Analysis in 1821, an unpublished work which Dr. Dubbey has recently discovered in the British Museum. W. R. Hamilton's famous quaternions of 1843 served to illustrate the greater levels of abstraction of which algebra is capable. Most important for the birth of modern mathematics, however, was the independent and nearly simultaneous development of hyperbolic geometry by N. I. Lobachevsky (1829) and J. Bolyai (1832). This shocking new geometry - in which, for example, the sum of the angles of a triangle is always less than $180^{\circ}$ - more than any other development established the nature of modern mathematics as dependent upon arbitrarily chosen sets of axioms independent of how the axioms might or might not be related to the physical world. This divorce of mathematical from physical truth or falsity has led to the ever greater abstraction and generality so characteristic of twentieth century mathematics, whose advancements are depicted in the book's concluding chapter. Here Dubbey discusses modern applied mathematics, with an interesting section on computers. He also treats accomplishments in analysis, algebra, and topology, noting that often it is difficult to distinguish any one of these three important fields from the other two.

To the mathematics student interested in a brief but informative history of mathematics that perceptively deals with modern developments, I would recommend J. M. Dubbey's book. Further reading suggestions are given by the author in Chapter 1, and the list of a dozen and a half references at the end of the book is also useful. The following corrections should be noted: page 9 line 22 for " 1920 " read " 1858 ," page 16 line 25 for "nine" read "five," page 49 line 4 for "surface" read "volume," page 54 line 2 for " 1668 " read "1691-1693," page 105 last line for " $\epsilon>\mathrm{K}-\epsilon$ " read " $x>K-\epsilon$."

> Randall Longcore
> Atkinson College
> York University

A Short Calculus: An Applied Approach, Daniel Saltz, Goodyear Publishing Company, Inc., Pacific Palisades, California, 1973, 403 pp., \$11.95.
From the Introduction, "This book is meant to introduce some of the important concepts and methods found in calculus, and to indicate how these concepts can be used in constructing mathematical models in such areas as economics, the life sciences, and psychology. ... attention is restricted to those functions that are obtained by taking powers and roots of ratios of polynomial functions, logether with exponential and logarithmic functions. ... The principal audience addressed is the nonmathematician, $\ldots$ there are no involved formal proofs. ... the basic properties of limits are given essentially as an axiom."

The idea behind the book is good. There is a difference, however, between rigor and preciseness. If the student is expected to learn to say exactly what he means and mean exactly what he says mathematically, he must have a text which does likewise. For example ( $p .40$ ), "the function $f$ is continuous at a if $\lim f(x)=$

$$
\mathbf{x} \rightarrow \mathbf{a}
$$

[(a)". What if $f(a)$ does not exist? Again, just what is a sufficiently close distance" (p. 2\%): or how does one succeed in "Rationalizing Square Roots" (p. 315)? The axes are not labeled in many of the illustrations. In the Appendix (p. 290) the definition of a subset is incorrect. The book has its share of typographical errors as well.

The format, with the left three-eighths of each page largely blank, and with blanks left to be filled in by the student in many
examples does not lend itself to textbook use unless all students buy new books.

If the objective is only to learn some of the applications of the Calculus to economics, the life sciences, and psychology the book is good. There are many excellent examples and problems for this purpose. As a textbook in mathematics it cannot be recommended, however.

F. Virginia Rohde<br>Mississippi State University

Elementary Matrix Algebra (Third Edition), Franz E. Hohn, The Macmillan Company, New York, 1973, 538 pp., $\$ 11.95$.
The book under review is the third edition of a standard work on matrix algebra that first appeared in 1958. The book (as were the earlier cditions) is intended for use by juniors, seniors, and graduates whose interests lie in numerical analysis, the physical or social sciences, and pure mathematics.

In addition to the usual topics of matrix algebra, the author gives an introduction to abstract vector spaces and linear transformations. This new edition incorporates two major changes. First, considerable geometric material has been added in order to motivate the idea of vector space. Second, the chapter on determinants is postponed until it really is needed. As a consequence of these changes, the author now defines the rank of a matrix in terms of its row and column space instead of doing it with determinants. This is a definite improvement over earlier editions.

Overall, the book is very well written and contains many detailed examples and a large number of interesting exercises. A student should be able to read this book on his own with a minimum of help from an instructor.

Richard Dowds<br>State University College, Fredonia

Algorithmic Combinatorics, Shimon Even, The Macmillan Company, New York, 1973, 272 pp., \$13.95.

Combinatorics may be defined as the study of families of finite subsets. Thus among its branches are graph theory, finite geometry, enumeration, and coding theory. It has important applications to
statistical physics, genetics, information retrieval, and networks. It includes both profound results and an illustrious unsolved problem - that of the four-color map. No single book can cover more than a small part of such a subject.

Algorithmic Combinatorics is motivated by the increasing importance of combinatorics in computer science. While computer implementations of combinatorial techniques have been important in many fields, combinatorics also provides the basis for such significant aspects of computer science as data structures, switching theory, logical design, and automata theory. The topics treated are those the author has found useful in computer science and are mostly from graph theory - e.g. trees, paths in graphs, and flow in graphs. Chapters are also included on permutations, combinations, and enumeration problems.

The book is distinguished by its presentation of computationally effective algorithms for realizing the principle results. The reader who finds a needed subject included here will not experience the frustration of finding only existence proofs or such unwieldy constructions that the goal sought remains out of reach in practice. There are especially helpful references to the Algorithms of the Comm. A.C.M., and as would be expected, Knuth's work is frequently cited.

The book is accessible to the reader acquainted with the basics of modern algebra. It is largely self-contained, proofs are given, and no programming languages are used to state the algorithms.

O. R. Plummer<br>University of Missouri-Rolla

# The Problem Corner 

Edited by Robert L. Poe

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old oncs of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 March 1974. The best solutions submitted by students will be published in the Spring 1974 issue of The Pentagon, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all commanications to Kenneth M. Wilke, c/o Department of Mathematics, Washburn University, Topeka, Kansas 66621.

It is with great sadness and feeling of loss that I must report the death of Robert L. Poe earlier this year on 4 May 1973. His effective handling of this department of THE PENTAGON will be greatly missed. He became associate editor in Fall 1968.

## PROPOSED PROBLEMS

260. Proposed by Charles W. Trigg, San Diego, Califomin.

| 157 | 13 | 211 |
| ---: | ---: | ---: |
| 131 | 127 | 73 |
| 43 | 241 | 97 |

is a magic square with prime elements. Form another magic square with nine prime elements, seven of which are the same as those of the given square.
261. Proposed by Charles W. Trigg, San Diego, California. In the scale of eight there is just one integer $n$ such that $n$ and $n^{2}$ together contain the eight digits once each. Find $n$ and show it to be unique.

## SOLUTIONS

[^3]Show that $\operatorname{limit}_{x \rightarrow 0} \frac{x}{1-e^{2}} \int_{0}^{e^{2}} d t=-1$.
Solution by Gary Kolks, St. Francis College, Brooklyn, New York.

As $x \rightarrow 0$ we have a fraction whose numerator and denominator goes to zero. Use L'Hopital's Rule to evaluate the limit.
$\operatorname{limit}_{x \rightarrow 0} \quad \frac{x \int_{0}^{x} e^{t^{2}} d t}{1-e^{x^{2}}}=\operatorname{limit}_{x \rightarrow 0} \frac{x e^{x^{2}}+\int_{0}^{x} e^{t} d t}{-2 x e^{2}}$
Letting $x \rightarrow 0$ in the right hand member, we again get a fraction of the form \%. Therefore apply L'Hopital's Rule again:

$$
\begin{aligned}
\operatorname{limit}_{x \rightarrow 0} \frac{x e^{2}+\int_{\mathrm{n}}^{e^{2}} d t}{-2 x e^{2}} & =\operatorname{limit}_{x \rightarrow 0} \frac{2 x^{2} e^{x^{2}}+e^{2}+e^{2}}{-4 x^{2} e^{x^{2}}-2 e^{x^{2}}} \\
& =\operatorname{limit}_{x \rightarrow 0} \frac{2 x^{2}+2}{-4 x^{2}-2} \\
& =-1 .
\end{aligned}
$$

Also solved by Bob Sellinger, North Park College, Chicago, Illinois.
256. Proposed by Charles W. Trigg, San Diego, California.

Before the football game the student representatives met to select the best cheer for the rooters to use. Falling back on tradition, they decided to let

$$
I T / B E=. R A H R A H R A H \ldots
$$

In the lucky base of numeration, eleven, each letter of this cryptarithm uniquely represents a digit. When reordered, the
digits involved are consecutive. Find the sole solution.
Solution by Larry R. Byrd, William Jewell College, Liberty, Missouri.
Let $X$ represent the largest digit in base 11, and let $y$ equal the fraction $I T / B E$. Then, doing all work in base 11 ,

$$
\begin{aligned}
1000 y & =R A H \cdot R A H R A H \cdots \\
y & =(R A H R A H \cdots
\end{aligned}
$$

for some integral f. Factoring $X X X$, we find that $B E$ could be 13, 18, 32, 35, 64, or 87 . Now $I<B$, so $B>1$, and $B E$ is 32, 35,64 , or 87 , with corresponding values for $f$ of $35,32,18$, and 13. Then by trial and error, choose $I T<B E$, multiply $I T$ by the $f$ corresponding to $B E$, and check that all digits used are distinct and consecutive. It is not difficult to find the solution: $42 / 87=.536536 \ldots$
Also solved by Kemneth M. Wilke, Topeka, Kansas, and David M. Tobolowsky, Southern Methodist University, Dallas, Texas.
257. Proposed by Charles IF. Trigg, San Diego, California.

In the decimal system, solve the cryptarithm

$$
(E I N)^{2}+(C O T)^{2}=(C S C)^{2}
$$

wherein $C, S, T$ in some order are consecutive digits.
Solution by Lary R. Byrd, William Jewell College, Liberty, Missowi.

It is well known that at least one side of a Pythagorean triangle must be a multiple of 5 . So one of $N, T$, and $C$ must be either 5 or 0 . Obviously, $C \neq 0$. Suppose $T=0$. Then $C$ and $S$ are 1 and 2, in some order. So $E I N>300$ but $C S C<300$, and we cannot have a triangle. Suppose $N=0$. Then (COT) and (CSC): end in the same two digits. An examination of a squares table from 1 to 100 reveals that there are no solutions satisfying all conditions. So we find that $N, T$, and $C$ are not 0 .
Therefore one of $N, T$, and $C$ is 5 . Suppose $C=5$. We can then choose $S, T$, and $O$ in 24 different ways, none of which allows a solution (although we do find the Pythagorean tri-
angle 184, 513, 545). Suppose $T=5$. Using the fact that one leg must be divisible by 4 , we find that $C$ is odd, and thus reduce the possible choices for $C, S$, and $O$ to 8 cases. None yield a solution.

Therefore $C=5$, by elimination. A little work using known facts reduces the possibilities for $S, T$, and $O$ to 21 cases. One of these yields the proper solution: $(295)^{2}+(708)^{2}+(767)^{2}$.

Also solved by David M. Tobolowsky, Southern Methodist University, Dallas, Texas, and Kenneth M. Wilke, Topeka, Kansas.
258. Proposed by Charles W. Trigg, San Diego, California.

What is the smallest positive integer that can be expressed in exactly four different ways as the sum of consecutive positive integers?

Solution by Kenneth M. Wilke, Topeka, Kansas.
Let $S=m+(m+1)+\ldots+(m+k-1)$ where $S$ is a positive integer and $m, k$ are positive integers determining $k$ consecutive integers whose sum is $S$.
Hence $S=k m+\frac{k(k-1)}{2}=\frac{k(2 m+k-1)}{2}$ and either $k$ is odd and $2 m+k-1$ is even or vice versa. It now follows that there is a unique set of $k$ consecutive integers whose sum is $S$ corresponding to each odd divisor of $S$. Excluding the trivial solution $S=m$ corresponding to $k=1, S$ must have four additional odd divisors. Hence $S$ has exactly 5 odd divisors and the smallest such $S$ is $\mathbf{3}^{*}=81$. Hence the desired representations are:
$k=2 m=4040+41 \quad=81$.
$k=3 m=2626+27+28 \quad=81$.
$k=6 m=1111+12+13+14+15+16=81$.
$k=9 m=5 \quad 5+6+7+8+9+10+11+12+13=81$.
If the trivial solution $m=S$ is counted as one of the four representations then $S=15=7+8=4+5+6=1+2+$ $3+4+5$ is the desired solution.

Also solved by David M. Tobolowsky, Southern Methodist University, Dallas, Texas.
259. Proposed by Chnrles W. Trigg, San Diego, California.

In the triangular array 3
14
256
the row sums are $3,5,13$ (all prime); or $6,9,6$ (all composite): or $2,6,13$, depending upon which side is taken as the base of reference. Can the digits be rearranged in a triangular array so that all nine "sums" are (a) prime or (b) composite?

Sohution by David M. Tobolowsky. Sowhern Methodist University. Dallas, Texas.
(a). No solution possible: The one-member row sums, i.e. the corners must all be prime.

```
                3
2 - 5
```

Replacing any of the hyphens by 1 will cause the sum of a iltree-member row to be 6,9 , or 8 , all composite.
(b). No solution possible: The comers here must be composite, hence the primes must be placed on the inside of the array.

$$
23
$$

- 5 -

However, the sums of the two-member rows are 5, 7 , and 8 , two of which are prime.
Also solied by Kathyn Kuiathouski, Collegr of Notre Dame of Mayland, Ballimone. Maryland: Lany R. Byrd. William lewell College, Liberty, Missonti: May Ellen Beam, Waync: burg College, Waynesbugg, Pennsyliania: Kemeth M. Wilke, Topeka, Kansas.

# Kappa Mu Epsilon News 

## Edited by Elsie Muller, Historian

News of Chapter activities and other noteworthy KME events should be sent to Elsie Muiler, Historian, Kappa Mu Epsilon. Department of Mathematics, Morningside College, Sioux City, Iowa 51106 .

The Nineteenth Biennial Convention of Kappa Mu Epsilon convened Thursday, 5 April 1973 on the Morningside College Campus, with Iowa Gamma Chapter as host chapter.

Following the registration in the Commons there was a mixer in the Randolph Room of the Commons, and at the same time a meeting of the National Council.
The first general session met in Klinger-Neal Theatre with the National President, George R. Mach, presiding. Dr. Raymond S. Nelson, Dean of Academic Affairs and Faculty, Morningside College welcomed the group. William R. Smith, National Vice-President, responded for the society. The roll call of the chapters was made by Laura Z. Greene, National Secretary.

Professor William R. Smith presided during the presentation of the following papers:

1. Log Jam in the Classroom, Barbara Leamer, Missouri Alpha, Southwest Missouri State University.
2. The Golden Rectangle, Ronald Stair, Kansas Beta, Kansas State Teachers College of Emporia.
3. Getting the Most Out of Random Number Generators, J. Kenneth Haygood, California Gamma, California State Polytechnic College.
4. Cryptography, Kathy Drew, lowa Alpha, University of Northern Iowa.

After lunch and the taking of the group picture, the regional directors presided at meetings of the members present from the six regions.

The convention reconvened at $2: 15 \mathrm{p} . \mathrm{m}$. and the following student papers were presented.
5. An Investigation of the Fourth Dimension, Duane Schriemann, Missouri Beta, Central Missouri State University.
6. Introduction to Game Theory and the Simplex Method, Christine Goldsmith, Wisconsin Beta, University of Wisconsin at River Falls.
7. On the Graphs of Groups, Linda Grandstaff, Kansas Delta, Washburn University of Topeka.
8. Problem of Apollonius, Wanda Garner, Kansas Alpha, Kansas State College of Pittsburg.
9. Three Theorems, Gregg Stair, Kansas Beta, Kansas State Teachers College of Emporia.

Steve Bolka, President of Iowa Gamma Chapter was master of ceremonies for the traditional banquet Friday evening, 6 April, in the Randolph Room of the Commons. The guest speaker, Professor Robert V. Hogg of the University of Iowa gave the address, Adaptive Statistical Inference: An Introduction for Mathematicians.

The music department of Morningside College furnished a delightful program in which they presented both Professor Lawrence Graham, pianist, and the Concert Choir. The choir was directed by Dr. James Wood.

The convention resumed on Saturday morning with the presentation of the remainder of the student papers:
10. A Proof of the 5-Color Theorem Utilizing Graph Theory, Kathleen Shockley, Missouri Beta, Central Missouri State University.
11. Countability, Harlan Hullinger, Lowa Gamma, Graduate Student at University of Iowa.
12. On the Nature of Natural Numbers, Don Mertz, Graduate Student at Kansas State College of Pittsburg.
The following papers were listed by title as alternates:

1. Why Students Have Unfavorable Attitudes Toward Arithmetic, Ruth Ann Bruny, Colorado Alpha, Colorado State University.
2. The Golden Rectangle, Linda Funk, Kansas Alpha, Kansas State College at Pittsburg.
3. Pi, A Significant Number in Geometry, Anna Lee Wiley, Kansas Alpha, Kansas State College at Pittsburg.

Dr. Fred Lott, of Iowa Alpha, chairman of the nominating committee reported. The following officers were clected for the biennium, 1973-1975.


Professor Glen Bernet, of Missouri Theta, chairman of the awards committee announced the winners for the papers presented by undergraduates at the convention.


Dr. Harold Thomas, Kansas Alpha, reported for the resolutions committee. The following resolutions were adopted:

Whereas the Nineteenth Biennial Convention on this beautiful campus of Morningside College has been a very enjoyable and profitable conference, be it resolved that we express our appreciation and gratitude to:

1. The host chapter, lowa Gamma, its faculty sponsor and corresponding secretary, Elsie Muller, and Morningside College of Sioux City, Iowa, for their hospitality and efficient organization of all major and minor details that contributed so well to the success of the convention.
2. Each National officer: President George R. Mach, Vice-Presi-
dent William R. Smith, Secretary Laura Z. Greene, Treasurer Eddie W. Robinson, and Historian Elsie Muller for their efficient assistance and performance in their respective offices during the past biennium.
3. James K. Bidwell, the editor of THE PENTAGON, Wilbur J. Waggoner, the Business Manager of THE PENTAGON, and the associate editors of THE PENTAGON, Elizabeth T. Wooldridge, Loretta K. Smith, Elsie Muller, Richard L. Barlow, and Robert L. Poe who have so satisfactorily maintained the high quality of our magazine.
4. Professor Robert V. Hogg of the University of Iowa, who provided the challenging and entertaining program at the convention banquet.
5. Dr. James Wood and the Morningside Choir and Professor Lawrence Graham who provided the enjoyable musical portion of the banquet program.
6. The twelve students who have prepared and presented the excellent papers which form an integral part of the convention program; and also the three students who prepared alternate papers.

Be it further resolved that we extend special recognition and gratitude to:

1. Fred Lott, who completes his tenure on the National Council. Professor Lott has faithfully served Kappa Mu Epsilon as editor of THE PENTAGON, 1959-65, Vice-President, 1965-66, President, 1966-69, and Past President, 1969-73.
2. Laura Greene who resigned this year after serving eighteen years as National Secretary. In addition, Miss Greene was National Historian, 1951-55 and an associate editor of THE PENTAGON in 1949.

We are very appreciative to both Professor Lott and Professor Greene for the years of dedicated service they have given to our society.

## REPORT OF THE NATIONAL PRESIDENT

Kappa Mu Epsilon's growth and improvement have continued during the past biennium. Four new chapters have been installed. They are: Tennessee Delta at Carson-Newman College, New York

Iota at Wagner College, South Carolina Gamma at Winthrop College, and Iowa Delta at Wartburg College (installed at our banquet last night). The Society now has 95 active chapters in 29 states. By vote of this convention, two more chapters will soon be installed.

The regional organization for the Society was announced in my biennial report two years ago. After that convention the regional directors were appointed and they are:

| Region 1-James E. Lightner | Region 4-Harold L. Thomas |
| :--- | :--- |
| Region 2-Dean O. Hinshaw | Region 5-Mike Reagall |
| Region 3-Jack D. Munn | Region 6-Joyce Curry |

Three regional conventions were held in 1972, the greatest number ever held in one year. Wisconsin Alpha hosted one on 24-25 March at Mount Mary College, Missouri Gamma hosted one on 15 April at William Jewell College, and Pennsylvania Iota hosted one on $3-4$ November at Shippensburg State College. At this biennial convention, for the first time, regional meetings were held and were led by the regional directors.

During the past biennium, Miss Greene and I have represented the Society at two annual meetings of the Association of College Honor Societies. The meeting this year on 22-24 February was hosted by Kappa Mu Epsilon at Washburn University of Topeka. Chapter handbooks have been completed and distributed, largely through Miss Greene's efforts. Insignia are now purchased in quantity and stocked in the National Secretary's office with resulting savings and better service for our members.

I have enjoyed my four years as your National President. The National Council members with whom I have served have been willing and helpful. All are worthy of tribute, but I especially want to mention Fred Lott, who is retiring after six years as editor of THE PENTAGON and eight years as National Vice-President, National President, and National Past President and Laura $Z$. Greene, who is retiring after two years as associate editor of THE PENTAGON, four years as National Historian, and eighteen years as National Secretary.

George R. Mach

## REPORT OF THE VICE PRESIDENT

During my second term as Vice President of Kappa Mu Epsilon there have been fewer meetings to attend, but this term has been as interesting as the first.

Shortly alter the last convention I officiated at the installation of New York Iota Chapter at Wagner College on Staten Island. These installations continue to be rewarding experiences. In fact, meeting the many members and faculty sponsors is the best part of the job.

Last November I was invited to give the principal address at the Regional Convention hosted by Pennsylvania Iota at Shippensburg State College. The meetings were well attended and those responsible for the event did an outstanding job.

The chairmanship of the Selection Committee has occupied my time since the beginning of this year. I would like to thank Professors Denver Childress of Carson-Newman College, Gene Bennett of Evansville, and James Lightner of Western Maryland, for a job well done.

William R. Smith

## REPORT OF THE NATIONAL SECRETARY

Kappa Mu Epsilon is a growing organization. Twenty years ago there were fifty chapters in twenty-three states with 1,142 new members initiated during the biennium and a total membership of $\mathbf{9 , 7 2 5}$. Today there are ninety-five chapters in twenty-nine states, with a total membership of 31,500 . During the last two years 2,649 members were initiated.

Kansas Alpha is the largest chapter with 1,163 members, with New Mexico Alpha in second place with 1,113 members, and Oklahoma Alpha is in third place with 1,024 members.

The total registration at the convention today is two hundred thirty-three representing thirty-six chapters from nineteen states.

It has been my privilege to work with many excellent corresponding secretaries, some are present today. Would you stand as I call your name and remain standing until all of you are recognized? Your careful work makes the work of the national secretary easy. I have appreciated your help. I want express my appreciation to the National Officers for their cooperation and support. It has
been a real pleasure to work with an organization such as Kappa Mu Epsilon, The Mathematics Honor Society.

Laura Z. Greenc

## REPORT OF THE NATIONAL HISTORIAN

During the past biennium a new steel file has been purchased which means that all the records, programs, and documents pertaining to the chapters can be well preserved. All correspondence with the chapters and national officers have been filed in appropriate folders. As new chapters have been installed, folders have been made for them.

As in the past I have continued to request news items semiannually for the KME News section of THE PENTAGON from each chapter. During the past two years 73 chapters have responded at one time or another. For greater interaction between the chapters it is highly desirable that each corresponding secretary respond at least once during the year. The following chapters replied to all four questionnaires: Colorado Alpha, Illinois Eta, Iowa Gamma, Kansas Gamma, Maryland Beta, Missouri Beta, Nebraska Alpha, New Mexico Alpha, Pennsylvania Zeta, Pennsylvania Iota, and Wisconsin Alpha. All the news submitted to me has been printed. In case the report arrived after the deadline the material was printed in the next issue of THE PENTAGON.

Since we now have a regional organization, folders have been made for each of the regions. In the future more emphasis might be placed in collecting information with respect to activities within each region.

For excellent cooperation I wish to express my deepest appreciation to the editor, James Bidwell, to all the national officers, and to the corresponding secretaries of each chapter.

Elsie Muller

| KAPPA MU EPSILON |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FINANCIAL REPORT OF THE NATIONAL TREASURER For the period March 22, 1971 to April 4, 1973 |  |  |  |  |
|  |  |  |  |  |
| 1. Cash on hand March 22, 1971 |  | \$ 5,979.81 |  |  |
| Receirts |  |  |  |  |
| 2. Receipts from Chapters |  |  |  |  |
| Initiates 2,649 | \$18,623.50 |  |  |  |
| Supplies, Jewelry, Installations | 1,964.33 |  |  |  |
| Replacement certificates | 11.25 |  |  |  |
| Total receipts from chapters |  | \$20,599.08 |  |  |
| 3. Miscellaneous |  |  |  |  |
| Interest on Savings | \$ 2,307.21 |  |  |  |
| Pentagon | 52.91 |  |  |  |
| From Savings to Checking Account | 7,000.00 |  |  |  |
| Total miscellancous |  | 9,360.12 |  |  |
| 4. Total receipts |  |  |  |  |
| 5. Total receipts plus cash on hand - - $\quad$ \$35,939.01 |  |  |  |  |
| Expenditures |  |  |  |  |
| 6. National convention 1971 |  |  |  |  |
| Paid to chapter delegates | * 2,636.59 |  |  |  |
| Officers Expenses | 1,209.69 |  |  |  |
| Speaker | 280.00 |  |  |  |
| Host Chapter and Holiday Inn | 401.46 |  |  |  |
| Prizes | 100.00 |  |  |  |
| Printing | 57.92 |  |  |  |
| Total cost of convention |  | \$ 4,685.66 |  |  |
| 7. Balfour Company |  | 3.122 .29 |  |  |
| 8. Blake Printery |  | 2,420.2: |  |  |
| 9. Pentagon |  | 8,674.50 |  |  |
| 10. Installations |  | 158.83 |  |  |
| 11. National Offices Expenses |  | 1,773.97 |  |  |
| 12. Regional Conventions |  | 282.69 |  |  |
| 13. Miscellanemus Expenditures |  |  |  |  |
| Assoc. of College Honor Societies | \$ 186.70 |  |  |  |
| Refunds to chapters | 78.07 |  |  |  |
| Short Checks | 14.00 |  |  |  |
| Saving Accounts | 9,402.58 |  |  |  |
| Security Bond | 156.00 |  |  |  |
| Total misc. expenditures |  | 9,837.35 |  |  |
| 14. Total Expenditures |  |  | \$30,955.54 |  |
| 15. Cash on hand April 4, 1973 |  |  | 4,983.47 |  |
| 16. Total expenditures plus cash on hand |  |  |  |  |
| 1. Total Assets - April 4, 1973 | \$19,386.05 |  |  |  |
| 18. Total Assets - March 22, 1971 | 18.658.24 |  |  |  |
| 9. Net Gain for period | \$727.81 |  |  |  |
|  | Red | spectfully die W. Ro | submitted <br> binson |  |

## REPORT OF THE EDITOR OF THE PENTAGON

During the past biennium there has been (thankfully) only one change in THE PENTAGON staff. Dr. Elizabeth Wooldridge of Florence State University has been the Book Shelf Editor for the past two years. The other editors are Loretta Smith, Elsic Muller, Richard Barlow, and Robert Poe. I most gratefully thank them for their efficient and accurate editorial work. Wilbur Waggoner, the Business Manager, has continued to make my job an easier onc. I would especially like to thank my colleagues who have willingly reviewed manuscripts. They are Robert Chaffer, Willianı Iakey, and David McDowell.

THE PENTAGON is designed to serve the members of KME. It can only do this as long as the members of each chapter read and contribute to the journal. In particular, we need continuing contributions in the form of manuscripts for articles, problems and solutions, contributions to the scrapbook, and KME news items. I encourage each of you to contribute something to the future issues of THE PENTAGON.

You have undoubtedly noticed the erratic arrival dates of your past journals. We have been faced with printing delays and difficulties for the past two years. We have recently changed printers. Hopefully future issues of THE PENTAGON will again be delivered to you in late December and May. We thank you for your patience in this matter.

Since the last biennium there have been three issues of THE PENTAGON and the fourth is in press. In these four issues there are thirteen articles by students (not all were members of KME) and nine by faculty and other non-students. I would like the ratio to change in favor of student articles in future issues.

In order to maximize the usefulness of THE PENTAGON for KME members, 1 would like each of you present to fill out the Interest Poll form that is available and return it to me personally at this meeting or by mail. You may take additional copies of the poll to your chapter if you wish. A copy of this poll will be in the Spring 1973 issue for all members to use. Our aim is to better serve each of you.

James K. Bidwell

## REPORT OF THE BUSINESS MANAGER of the PENTAGON

This is the eighth biennial report on the office of Business Manager of the journal of Kappa Mu Epsilon that I have presented to our society. During that period, there has been a considerable change in the number of PENTAGONS printed and in the cost of printing and mailing. In 1956 there were two thousand journals printed at a printing cost of nine hundred dollars and it cost eight cents to mail a single issuc. Today an issue runs over thirty-one hundred copies at a printing cost of over twenty-three hundred dollars and it costs twenty cents to mail a single issue.

Some PENTAGONS serve our society twice. When a journal is returned to my office because the postal service is unable to deliver it because ol an insufficient or incorrect address, I have no alternative but to remove the subscribers' address cards from the file of current subscribers. No more PENTAGONS are mailed to this incorrect address. I then send these returned PENTAGONS to new subscribers. Since our journal is not a "current events" journal and is published semi-annually, I try to mail a journal to each new subscriber for a period of two or three months after a publication date. This action prevents new subscribers from waiting as long as six months to receive their first issuc of THE PENTAGON.

During this biemnium, I searched for and with the approval of the national council secured a new printer for our journal. Hopefully when our current printer, Enterprise Printers of Mt. Pleasant, Michigan, becomes familiar with the rather complex task of printing a mathematics magazine, your journal will be mailed by the established publication dates of 15 May and 15 December. I apologize for the slow delivery of your magazines during the past two years.

During the past biennium over twelve thousand five hundred PENTAGONS were mailed. They were addressed to subscribers in cach of the fifty states and to many foreign countries including India, Russia, Yugoslavia, Belgium, Sweden, and England. Libraries in many high schools, community colleges, colleges, and universities subscribe to our journal. In my first biennial report I noted that the states with the most subscribers were Kansas, California, Illinois, Texas, and New York in descending order. Today the most PENTAGONS go to Pennsylvania followed by Illinois, Ohio, Kansas, and New York in numerical order. Over one-third of all PENTAGONS mailed are mailed to the five states just cited.

Complimentary copies of THE PENTAGON are sent to the library of each college or university with an active chapter of Kappa Mu Epsilon. Subscriptions are automatically extended for two years for each student speaker at this convention. Authors of articles in THE PENTAGON receive complimentary copies of the issue in which their article appears.

I have appreciated the privilege of working with our editors Drs. Kriegsman and Bidwell. My thanks to chapter corresponding secretaries, and to our national secretary, Miss Laura Greene. Your cooperation makes the duties of my office pleasant and rewarding.

Wilbur Waggoner

## CHAPTER NEWS

Alabama Beta, Florence State University, Florence
Chapter President - Robert O'Connor
39 actives
A student paper was presented by Robert O'Connor on the LaPlace Transform. He has received a graduate assistantship to Auburn University. James Bowers was granted one from the University of Alabama. Two faculty members, Dr. Juan Aramburu and David Cope, presented a program on mathematical applications to industry. Other officers: Jane Bickel, vice-president; Teressa Rowland, secretary and treasurer; Mrs. Jean T. Parker, corresponding secretary; Dr. E. J. Brackin, faculty sponsor.

## Alabama Gamma, University of Montevallo, Montevallo <br> Chapter President - Becky Golden <br> 15 actives -3 pledges

The chapter meets monthly and begins with a picnic in October. Other officers: Tommy Griswold, vice-president; Susan Martin, secretary and treasurer; Dr. Angela Hernandez, corresponding secretary; Dr. D. R. McMillan, faculty sponsor.

## California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President - Dave Bullard
40 actives, 19 faculty
The monthly meetings include chapter business and student or
faculty speakers. Two initiations and one banquet are held during the year. Tutorial service is available for all students on campus. The chapter furnishes mathematical displays at Poly Royal, an all campus open house, and assists the mathematics faculty in an annual mathematics contest which involves over 500 high school students. Other officers: Andy Harrington, vice-president; Sue Genung, secretary; De Ette Nelson, treasurer; Dr. George Mach, corresponding secretary: Dr. Warten, faculty sponsor.
California Delta, California State Polytechnic College, Kellogg
Voorhis Campus, Pomona
Chapter President - Steven Mitchell
20 actives, 10 pledges
Meetings are held once each month, and we provide a tutoring service in mathematics for both college and high school students. The chapter furnishes the display for the Polyvue-Open House at the university. Other officers: David Durham, vice-president; Christopher Weber, secretary and treasurer.

## Colorado Beta, Colorado School of Mines, Golden

Chapter President - Robert Johnson
25 actives, 26 pledges
Clyde Kerns, Regional Geophysicist of the Exporation and Producing Division of Mobil Oil Company addressed the chapter on mathematical problems in geophysical exploration and oil production. In early 1973, the chapter initiated a project to design and cast an aluminum KME key at the foundry in the CSM metallurgy department. The pattern was carved from wood by the president of CSM, Dr. McBride, and the foundry labor was supplied by the chapter members under the supervision of Dr. Olson of the metalIurgy deparment. When the pattern board is perfected, the pledges will be required to make the sand molds for their own keys. Other officers: Richard Martin, vice-president: Carol Packard, secretary: Lynn Patten, treasurer: Dr. A. J. Boes, corresponding secretary: Dr. J. O. Kork, faculty sponsor.

## Illinois Beta, Eastern Illinois University, Charleston

Chapter President - Larry Dowling
56 actives
Chapter members win four of the five scholarships given by the
mathematics department. In the spring of 1973 a KME member won the all school Heller Award. Other officers: Eric Wingler, vice-president; Debra Ziegle, secretary and treasurer; Ruth Queary, corresponding secretary and faculty sponsor.

## Illinos Delta, College of St. Francis, Joliet

Chapter President - Cathy Weisenburger
11 actives
In February 1974 the chapter will hold its third annual mathematics contest for high school students. About 30 attended the first contest and 80 attended the second. One hundred juniors and seniors are expected in 1974. Mary Joe Molitor, a chapter member and a senior, won third place at the Mathematics Symposium of the Associated Colleges of the Chicago Area, held at Elmhurst College in May 1973. Other officers: Mary Joe Molitor, vice-president; Louise Becker, secretary; Kay Shimanski, treasurer; Dr. Arnold Good, corresponding secretary and faculty sponsor.

## Illinois Zeta, Rosary College, River Forest

Chapter President - Chris Biggio

## 17 actives

At the monthly meetings members presented research problems. Members also manned the Mathematics-Statistics Learning Center under the direction of Sister M. Philip Steele where they gave assistance to students, faculty, and staff members. Money was raised in a taffy-apple sale to send representatives to the national convention. Six new members were inducted on 18 March when Mr. Dan Ferguson and Mrs. Mollie D'Esposite of the First National Bank of Chicago were the speakers. Other officers: Mary Bartik, vice-president; Christine Stephens, secretary; Ruth Ann Matzger, treasurer; Sister Mary Therese O'Malley, corresponding secretary and faculty sponsor.

## Indiana Gamma, Anderson College, Anderson <br> Chapter President - William Dorff <br> 12 actives

At the meeting on 25 March, nine students were initiated. Other officers: Timothy Sieka, vice-president; Carol Miller, secretary and
treasurer; Mr. Paul Saltzmann, corresponding secretary; Dr. Stanley L. Stephens, faculty sponsor.

Indiana Delta, University of Evansville, Evansville
Chapter President - Eugene Bettag
88 actives
Programs have covered many interesting topics: Angle Trisection, Fields of Maximum Area, Inequalities for Means, Variations on the Towers of Hanoi, Knitting Mathematics, An Euler's Triangle, Intuitive Topology, Directions in Topology, Symmetries of a Cube, and Uses of Statistics. Carol Jones received a $\$ 25$ bond for her Pythagorean Theorem project. Michael Fisk was the winner of the Freshman Mathematics Achievement Award. From 8-14 April the chapter sponsored jointly with the Association for Computing Science a symposium on The World, Mathematics, and Related Topics. Financial assistance was received from the Informal Learning Sequence of the University of Evansville. Other officers: Michael Lachance, vice-president; Carol Smith, secretary; Dr. Gene Bennett, treasurer and corresponding secretary; Mr. Kenneth Stofflet, faculty sponsor.

## Iowa Alpha, University of Northern Iowa, Cedar Falls

## Chapter President - Kathleen M. Drew

32 actives, 28 faculty
The annual KME Homecoming Breakfast was scheduled for 29 September 1973 at the home of Professor and Mrs. Carl Wehner. At the national convention at Morningside, Iowa Alpha presented Dr. Fred W. Lott with a plaque which honored him for his 22 years of distinguished service to Kappa Mu Epsilon. Dr. Lott continues his active support of local KME activities. Other officers: Anna M. Studt, vice-president; Cheryl R. Wagner, secretary and treasurer; John S. Cross, corresponding secretary and faculty sponsor.

## Iowa Gamma, Morningside College, Sioux City

Chapter President - Rodney Powles
25 actives
The fall semester began with an "Open House" for the freshmen and sophomores enrolled in mathematics classes. Slides of the
national convention were shown and mathematical games played. The chapter is sponsoring tutoring sessions for mathematics students. Barb Okonoski, Mark Fegan, and Larry Boyd received the book awards for this semester. The Homecoming Breaklast was held on 20 October. Other officers: Barb Okonoski, vice-president; Deb Hanson and Gary Pardekooper, secretaries; Dallas Courtney, treasurer; Elsie Muller, corresponding secretary and faculty sponsor.

## Iowa Delta, Wartburg College, Waverly

Chapter President - Terry Ackman
26 actives, 26 pledges
The chapter became a reality on 6 April 1973 when 12 members were initiated at the national convention at Morningside College. Since that time they have initiated another 14 members and are looking forward to their first full academic year of existence and the development of a strong and active chapter. Other officers: Bruce Foster, vice-president; Patricia Flebbe, secretary; Pamela Snyder, treasurer; Marvin J. Ott, corresponding secretary and faculty sponsor.

## Kansas Alpha, Kansas State College of Pittsburg, Pittsburg

Chapter President - Randy Timi
50 actives
Programs have been given by Don Mertz, "On the Nature of Natural Numbers", by Randy Timi, "Principles and Applications of Contraction Mapping", and by Anna Wiley, "Pi: A Significant Number in Geometry". The annual Robert M. Mendenhall Awards for scholastic achievement were presented at the April meeting. Recipients were Nancy Campbell, Wanda Garner, Anna Wiley, and Stephen Wolf. Each received a KME pin in recognition of this achievement. Other officers: Linda Funk, vice-president; Gail Schindler, secretary; Beth Gray, treasurer; Dr. Harold Thomas, corresponding secretary; Prof. J. Bryan Sperry, faculty sponsor.

## Kansas Gamma, Benedictine College, Atchison

Chapter President - Mary Kay Stewart
Second semester meetings were highlighted by a film showing of "The Definite Integral as a Limit" and a lecture on "Intuition and the Scientist" presented by Sister Helen Sullivan. The Sister Helen

Sullivan Scholarship was awarded to Mary Kay Stewart for 1978.74 academic year.

## Kansas Epsilon, Fort Hays Kansas State College, Hays

## Chapter President - Steve Kaufman <br> 15 actives

One of the meetings was spent in building mathematical models that will be used in mathematics classes. Another meeting dealt with recreational mathematics. At the spring initiation and banquet Dr. Bernhart of the University of Oklahoma spoke on the fourcolor problem. The chapter is considering organizing a mathematics club for interested students who are not yet eligible for membership in Kappa Mu Epsilon. Other officers: Tabetha Eichman, vicepresident; (iraig McClellan, secretary and treasurcr: Eugene Etter, corresponding secretary: Dr. Charles Votaw, faculty sponsor.

## Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President - Marilyn Radziminski
74 actives, 5 pledges
The following people have presented papers at meetings: Marilyn Radziminski, "Linear Programming and Business Applications"; Kathleen Fahey, "Mathematics and Musical Scales": and Frances Hoffler, "The Great Pyramid of Cheops". At the initiation Sister John Frances Gilman of Maryland Gamma spoke on the topic, "The Queen's Counting House". Other officers: Mary Resop, vice-president and treasurer: Kathy Kwiatkowski, secretary; Sister Marie Augustine, corresponding secretary; Jeanette Gilmore, faculty sponsor.

## Maryland Beta, Western Maryland College, Westminister

Chapter President - Gerard Kurek
22 actives
In the fall Janice Sharper Almquist spoke on her work in Health Statistics with HEW and gave lots of advice on job hunting and governmental work. The chapter sponsored a booth in the May Day Carnival to raise money for a mathematics anard. Mark Milleı received the Clyde Specer Award for 1973 as the most promising mathematics major. Other officers: Thomas Yates, vice-president;

Sandra Stokes, secretary; Nancy Fishpaugh, treasurer; James Lightner, corresponding secretary; Robert Bones, faculty sponsor.

## Massachusetts Alpha, Assumption College, Worcester

## Chapter President - Gary Riess

15 actives
Eleven new members were initiated on 7 May 1973. Following a dinner in honor of the new members, Thomas Blodgett spoke on the topic, "Axiom Systems and Consistency". Other officers: Thomas Blodgett, secretary; Charles Brusard, corresponding secretary: Rev. Richard P. Brunelle, faculty sponsor.

## Michigan Alpha, Albion College, Albion

The chapter has been inactive for about two years, but there are signs that some students may be interested in re-activating this year. W. Keith Moore is the corresponding secretary.

## Mississippi Gamma, University of Southern Mississippi, Hattiesburg

Chapter President - Cheryl Goodman
32 actives
In May the main event was a steak cook-out and in July there was an informal meeting which included a barbecue. The plan is to have some outside speakers for this year. Other officers: Rory MacDowell, vice-president; Julie Faust, secretary; Jack D. Munn, corresponding secretary; Ed Oxford, faculty sponsor.

## Missouri Alpha, Southwest Missouri State University, Springfield

Chapter President - Julie Otto
30 actives
Barbara Leamer presented a paper, "Log Jam in the Classroom". The chapter provided proctors and guides for the SMSS Mathematics Relays. At the annual picnic an awards ceremony was held where Alan Washburn received the KME merit award. Other officers: Dan Wilson, vice-president; Esther Key, secretary; Greg Darnaby, treasurer; Eddie W. Robinson, corresponding secretary; L. T. Shiflett, faculty sponsor.

## Missouri Beta, Central Missouri State College, Warrensburg

Chapter President - Kathleen Shockley
31 actives
Kathleen Shockley received the first place award of $\$ 50$ for her paper at the national convention at Morningside College. During the year the chapter did make a field trip to Midwest Research Institute and the Linda Hall Technical Library. Other officers: Ned Doelling, vice-president; Deborah McRay, secretary; Lu Rae Sampson, treasurer; Judith Bruce, historian; H. Keith Stumpff, corresponding secretary and faculty sponsor.

## Missouri Gamma, William Jewell College, Liberty

Chapter President - Tom Lehman
21 actives
Other officers: Alice Preston, vice-president; Ester Edwards, secretary; Clella Goodwin, treasurer; Sherman Sherrick, corresponding secretary; Prof. Mathis, faculty sponsor.

## Missouri Zeta, University of Missouri at Rolla, Rolla

Chapter President - Deborah Fugitt
27 actives
Dr. Eddie Robinson, the national treasurer, spoke at the initiation banquet in March. At other meetings Dr. Arlan Dekock spoke on "Computers in the Future", Dr. Ted Lewis on "Palindromes", and Dr. Howard Pyron on "Matrices in Numerical Analysis". A Problem of the Month program has been set up and the person with the best solution is awarded a five dollar prize. Integral and derivative tables have been written up and given to the mathematics classes. The chapter has issued an invitation for the regional convention to be held at Rolla in the spring of 1974. Other officers: Paul Horstmann, vice-president; Pat Long, secretary; Larry Lancaster, treasurer; Peter G. Sawtelle, corresponding secretary; James W. Joiner, faculty sponsor.

## Missouri Theta, Evangel College, Springfield

Chapter President - Keith Sorbo
7 actives

Program topics have included "Linear Alegbra and Some Applications", "Number System Constructions", and "Number Theory". Other officers: Phyllis Ogletree, vice-president; Sandra Butler, secretary and treasurer; Don Tosh, corresponding secretary and faculty sponsor.

## Nebraska Gamma, Chadron State College, Chadron

Chapter President - Cheri Landrey
20 actives
Meetings are held the first and third Thursday of each month. The chapter tabulates scores for the Inter-high Scholastic Day and presents a slide rule to the outstanding freshman mathematics student. Annually, there is a fall picnic. Other officers: Gerald Strauch, vice-president; Laurel Gorfield, secretary; Steve Bruce, treasurer; James Kaus, corresponding secretary and faculty sponsor.

New York Iota, Wagner College, Staten Island
Chapter President - Maryann Gisonda
17 actives
The chapter sponsored a series of four talks by guest professors from various universities. There was also a series of seminars given by students and faculty. Brian Manske gave a talk on Peano's Axioms while Donald Berkibile spoke on Polish notation. "Markov Chain Systems" was the topic for the talks by Nick Phillips and Raymond Traub. Lois Bredholt received a $\$ 25$ savings bond from the chapter for her outstanding work in mathematics. The chapter sponsored two mathematics bowls. "Mathematics Monthly" is a new bulletin begun by New York Iota. Not only does it contain the monthly department news but also challenging games and riddles. The co-editors are Brian Manske and Nick Phillips. Other officers: Jacqueline Potter, vice-president; Nick Phillips, secretary; Brian Manske, treasurer; Mary Petras, corresponding secretary; Raymond Traub, faculty sponsor.

## Ohio Gamma, Baldwin - Wallace College, Berea

Chapter President - Diane De Joy
20 actives, 11 pledges
A meeting was held with the director of placement services or
employment opportunities. The chapter also holds help sessions for students in the low level mathematics classes. Other officers: Chris Raineri, vice-president: Kathi Hock, secretary; Wilbur Van Dyke, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.

## Ohio Epsilon, Marietta College, Marietta

Chapter President - Joe Cowdery
30 actives
There will be several guest speakers throughout the school year. Some will be from Marietta College while others will be visiting mathematicians. In addition, the chapter provides a tutoring service for students enrolled in any of the college mathematics courses. Other officers: Terry Fry, vice-president: Nancy Kavula, secretary; Judy Terla, treasurer: George Trickey, corresponding secretary and faculty sponsor.

## Ohio Zeta, Muskingum College, New Concord

Chapter President - Sue Syroski
24 actives
In February Rich Martinelli spoke on "Game Theory and the Batte of Waterloo". There was the initiation of new members in March. In April John Caspole spoke on "Mathematics and Economics". There was also a discussion of the convention by Sue Syroski, Mary Galani, Pat Cross, and Doug Martin. In May the chapter awards a prize to the freshman mathematics student with the highest score on a special mathematics test for which the prerequisite is a grade of A in Calculus I. Other officers: Emily Henderson, vice-president: Alan Hurst, secretary and treasurer; James L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Gamma, Southwestern State College, Weatherford
Chapter President - Eugene Nikkel
22 actives
Other officers: Bryan Vogt, vice-president; Mary Harper, secretary; Pat Rieger, treasurer; Dr. Wayne Hayes, corresponding secretary; Dr. Don Prock, faculty sponsor.

## Pennsylvania Beta, LaSalle College, Philadelphia

Chapter President -- Frances Parrotto
30 members, 8 pledges
The chapter meets monthly and this year hosted two ice skating trips, sponsored exhibits at La Salle's yearly Open House, attended the Region I conference at Shippensburg State College, helped to tutor students in mathematics courses wherever possible, and held its yearly picnic. They have also sponsored several talks: Actuary Work as a Profession, Monte Carlo Methods, Calculus of Variations, Embedding Graphs in Squashed Cubes, and the Addressing Problem for Loop Switching. Another activity is the publishing of a biweekly newsletter with department and club news as well as a KMEdians Korner. Other officers: Francis Dziedzic, vice-president; Matthew Coleman, secretary; Deborah Wissman, treasurer; Brother Damian Connelly, corresponding secretary and faculty sponsor.

## Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

## Chapter President - Richard Murtha

58 actives
In February Mr. Doyle McBride gave a talk on "Logic in Mathematics". In March Dr. John Baum, a visiting MAA lecturer from Oberlin College presented two talks during his visit, "New Light on the Quadratic Formula - Investigations of Finite Felds" and "Why There are Only Five Regular Polyhedra". In April Susan Kinol, Marcia Bromyard, and William Ekey reported on the national convention activities. The annual banquet was held in May where Dr. Arlo Davis presented a talk, "The Family Tree of Mathematics". Other officers: Rebecca Cimini, vice-president; Debra Horbiak, secretary; Lynnette Cox, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

## Pennsylvania Eta, Grove City College, Grove City

Chapter President - Stanley Hetrick
20 actives, 10 pledges
Other officers: Kathy Morris, vice-president; Marth Kuttesch, secretary; Kevin Kasweck, treasurer; Marvin C. Henry, corresponding secretary; Cameron C. Barr, faculty sponsor.

Pennsylvania Iota, Shippensburg State College, Shippensburg
Chapter President - Patricia Harrison
40 actives, 11 pledges
On 28 April the chapter held its annual mathematics contest to which all surrounding high schools were invited to send delegates to participate in the test and compete for prizes. The annual picnic was held at Scotland Park with entertainment provided by Dr. Weller. Other officers: Peggy Benfer, vice-president; Billie Belles, secretary; Dr. Howard Bell, treasurer; Dr. John Mowbray, corresponding secretary; Dr. James Sieber, faculty sponsor.

## Pennsylvania Kappa, Holy Family College, Philadelphia

Chapter President - Marguerite Leicht
4 actives
Meetings are held four times during a semester to discuss PENTAGON problems and problems from old Putnam exams. In March Brother David Pendergast of La Salle College spoke to the chapter on general computer topics. Other officers: Margaret Jankowski, vice-president; Allan Becker, corresponding secretary.

## Tennessee Alpha, Tennessee Polytechnic Institute, Cookeville

## Chapter President - Daniel Stoncil

75 actives
The past year the chapter built and erected a KME emblem in front of the Physics-mathematics building. It looks very nice and has helped to publicize KME. It has also cooperated with the mathematics club and the mathematics department in establishing the R. H. Moorman Memorial Scholarship Fund. Other officers: Lester Jones, vice-president; Mary Baisky, secretary; Randall Mehlon, treasurer; Evelyn Brown, corresponding secretary; Francis R. Toline and Daniel Buck, faculty sponsors.

## Texas Beta, Southern Methodist University, Dallas

Chapter President - Roslyn Ann Slapper 60 actives

The chapter is pleased to announce a cash award to be given in honor of the late Dr. John David Brown, a former faculty spon-
sor, to the outstanding senior mathematics student. The award is made possible by an endowment given by P. J. Brown, Cather of Dr. Brown. Other officers: Marion Scoll, vice-president; Morey Silverman, vice-president; Patricia O'Rourke, secretary and treasurer; Raymond V. Morgan, Jr., corresponding secretary; Richard Williams, faculty sponsor.

## Wisconsin Alpha, Mount Mary College, Milwaukec

Chapter President - Barbara Junghans 10 actives, 27 pledges
Six new members were initiated on 20 March 1973 followed by a dinner which KME alumnae also attended. About 200 students from 25 different high schools participated in the annual mathematics contest for high school students on 31 March 1973. Other officers: Sue Walczak, vice-president; Karen Loesl, secretary; Mary Lou Meyers, treasurer; Sister Mary Petronia, corresponding secretary and faculty sponsor.

## Wisconsin Beta, Wisconsin State University - River Falls, River Falls

Chapter President - David Hetrick
34 actives
Meetings, which usually feature a speaker or a film, are held once a month. The chapter has begun a mathematics tutoring program in which members volunteer to help students who need it. Also, the members assist the freshmen during registration in planning a good schedule. Money making projects included a book sale where old ones from the mathematics department were sold. On 1 February 1973 Mr. Duane Huston of the National Aeronautics and Space Administration gave a lecture and demonstration on the coming Skylab missions. Jed Simpson received the Mathematics Recognition Award. Other officers: Barb Heldke, vice-president; Dian Mortensen, secretary; Craig Emerson, treasurer; Dr. Lyle Oleson, corresponding secretary; Dr. Edward Mealy, faculty sponsor.


## Kappa Mu Epsilon Convention, 5-7 April 1973 <br> Morningside College, Sioux City, lowa


[^0]:    - A paper presented at the 1973 National Convention of KME and awarded first place by the Awards Committee.

[^1]:    - A paper presented at the 1973 National Convention of KME and awarded second place by the Awards Committee.

[^2]:    -A paper presented at the 1973 National Convention of KME and awarded third place by the Awards Committec.

[^3]:    255. Proposed by Stephen C. Hennagin, University of California. Davis, California.
