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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Binary Permutation Groups

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Define the following permutations:

$$F_0 = \begin{pmatrix} 1 & 2 & 3 & \ldots & x & \ldots n \\ 1 & 3 & 5 & \ldots & 2x\text{-}1 & \ldots 2n\text{-}1 \end{pmatrix} \quad \text{where } F_0(x) \equiv 2x-1 (\text{mod } n)$$

$$F_1 = \begin{pmatrix} 1 & 2 & 3 & \ldots & x \ldots n \\ 2 & 4 & 6 & \ldots & 2x \ldots n \end{pmatrix} \quad \text{where } F_1(x) \equiv 2x (\text{mod } n)$$

and where n is a positive odd integer. Let $\langle F_0, F_1 \rangle = \mathfrak{F}$

Any element of $\mathfrak{F}$ must be a finite product of the elements F_1 and F_0. Remember that a permutation group generated by members of S_n can be no larger than S_n. Every such product can be characterized by a sequence of zeros and ones. We claim that the binary number suggested by such notation contains information which reveals where a particular permutation in $\mathfrak{F}$ will send 1. For example, $F_1F_0F_1F_0F_1F_1(1)$ is associated with the sequence 110101. Notice that the convention here is composition from right to left and that the sequence is written in reverse. The binary number the sequence defines is 53. We claim that $F_1F_0F_1F_0F_1F_1(1) \equiv 1+53$ (mod n). If n is 9, then 1 is sent to 9. If $n=7$, then 1 is sent to 5, since $54 \equiv 5$ (mod 7). In general, if in some permutation F_0 is applied first, the sequence will not start with 0 but with the first 1 from left to right.

The proof of the above claim will be done by induction. $F_0(1)$ is associated with 0. Hence by definition $F_0(1)=1 \equiv 1+0(\text{mod } n)$. $F_1(1)=2 \equiv 1+1$ (mod n). Two cases arise; the sequence may end in either 1 or in 0. Suppose our claim works in the latter case. Let $1\ldots0$ be such a binary type sequence derived from the element $F_0 \ldots F_1(1)=y-1$. We show that our claim works for the sequence $1\ldots1$. We assume both sequences are identical except for the last term. When the last F_0 and F_1 are applied to the same number x the result is $y-1 \equiv 2x-1$ and $2x$ respectively. The difference between $2x-1$ and $2x$ is 1. Furthermore, the difference between the binary numbers in the two sequences is also 1; so we are done for this case.

Suppose our claim works for a sequence ending with a 1. Suppose there are k ones from right to left before a zero is encountered. As our inductive step we assume that the function associated with the sequence ending in 1 will send 1 to the number $1+l \pmod n$, where l is the number represented by the binary sequence. We wish to show that the function associated with $l+1$ will send 1 to the number $1+l+1 \pmod n$. For example $F_1F_1F_1F_0F_0F_1(1)=39+1 \equiv 4 \pmod 9$. In this case, we would want to show that the function associated with $100111+1 = 101000$ (i.e., $F_0F_0F_0F_1F_0F_1$ will send 1 to $40+1=5 \pmod 9$)).

The difference between the function associated with the sequence ending in 1 and the function associated with the sequence ending in 0 is the last $k+1$ terms from right to left. The first function will end in k "F_1"s and one "F_0" (from left to right) and the second will end in k "F_0"s and one "F_1" (from left to right).

The two functions will each send 1 to x by their first terms in common from right to left. Then x will be sent as follows:

$$\begin{aligned} F_0{}^kF_1(x) = F_0{}^k(2x) &= 2(\ldots(2(2(2(2x)-1)-1)-1)\ldots)-1 \\ &= x2^{k+1} - \sum_{j=0}^{k-1} 2^j \\ &= x2^{k+1} - (2^k-1) \\ &= x2^{k+1} - 2^k + 1 \end{aligned}$$

Also

$$F_1{}^k F_0(x) = 2^k(2x-1) = x2^{k+1} - 2^k.$$

Since the binary notation (l and $l+1$) differ by 1 and since the result of mapping 1 differs by one, our result is proved.

In summary, if we wish to determine where a member of $\mathfrak{F}$ sends one, just add one to its binary representation. Conversely, if $f \epsilon \mathfrak{F}$ sends 1 to x then its binary representation must be congruent to $x-1 \pmod n$. This must be true, since if $f(1)=x$ and the binary representation was not congruent to $x-1 \pmod n$ then it would be congruent to $y-1 \pmod n$ for some $y \neq x$. That implies that $f(1)=y$ by our proven result. But then $y=x$, a contradiction. Therefore, *$f(1)=x$ if and only if the binary representation is congruent to $x-1$ (mod n).*

It is clear that to send 1 to x there must exist $f\epsilon\ \mathfrak{F}$ so that $f(1)=x$. Call the f which sends 1 to x with the least number of F_0 and F_1 terms L_x. For example, for $n=9$ and $x=6$, we write $L_x = F_1F_0F_1$ since $101=5$ and $5+1 \equiv 6 \pmod 9$. Note that $1110 \equiv 5 \pmod 9$, but we choose $F_1F_0F_1$ instead of $F_0F_1F_1F_1$ because the former has the fewest terms. In other words represent x in binary (mod n) such that no reduction (mod n) is necessary.

Now $L_y(1)=y$ and since $L_y \epsilon\ \mathfrak{F}$ there exists L_y^{-1} such that $L_y^{-1}(y) = 1$. Thus for arbitrary x and y there is a function that sends x to y, namely, $L_y\ L_x^{-1}$. Therefore we have displayed the group to be *transitive.*

Let H_1 be the stabilizer of 1. This is the set of all $f\epsilon\ \mathfrak{F}$ such that $f(1)=1$. The stabilizer of 1 clearly forms a subgroup of $\mathfrak{F}$. The composition of two permutations that fix 1 separately will fix 1 together. The identity fixes 1 and the inverse of every element that fixes 1 also fixes 1.

We claim that H_1 (or H_x for that matter since they are isomorphic) forms a subgroup of index n (the number of numbers in the set we are permuting). Form the cosets of H_1. All the elements in a coset are of the form t_xh for some $h\epsilon H_1$. Hence all members of a given coset send 1 to the same number. Since there n possible numbers to be sent to by mapping 1 (due to transitivity), it must be we have n cosets.

More exactly, if $t_1h_1(1)=y$ and $t_2h_2(1)=y$, then $t_1h_1(1)=t_2h_2(1)$ or equivalently $t_2^{-1}t_1h_1(1)=h_2(1)$. Thus $t_2^{-1}t_1(1)=1$ and $t_2^{-1}t_1\ \epsilon\ H_1$, which is equivalent to $t_2H_1=t_1H_1$ or that t_1 and t_2 are in the same coset. If t_1 and t_2 are in the same coset then $t_1=t_1h_1$ and $t_2=t_1h_2$, but $t_2(1)=t_1(h_2(1)\ )=t_1(1)$. Thus $t_1(1)=t_2(1)$.

We now know that $|\ \mathfrak{F}\ |=n|H_1|$, where n is an odd integer, which gives us the size of the set our permutation operates on. We claim that $H_1=\langle\ F_0\rangle$. That is, H_1 is the cyclic subgroup generated by F_0.

Suppose $h\epsilon\ \mathfrak{F}$ and $h(1)=1$. Then $h\epsilon\ H_1$. We wish to show that $h\epsilon\ \langle F_0\rangle$. Since h fixes 1 its binary representation must be a number $1-1\equiv 0 \pmod n$. If zeros are dropped from left to right in the sequence little will be changed since that implies that F_0 is being applied first (if we can show that members of H_1, with F_0 removed until the first F_1 is reached from right to left, are members of $\langle F_0\rangle$ surely the truncation by F_0 strings will not affect anything). Let $h = \ldots F_1$. The associated binary sequence is $1 \ldots$. Where does h send x? $h(x) = \ldots F_1(x) = \ldots F_1(L_x(1))$.

Now L_x is unique since we stipulated that its binary sequence be the smallest. Let the binary sequence be $1 \cdots$ for $x-1$. Then $h(x) = 1 + 1 \cdots 1 \ldots \pmod{n}$. Notice that the two binary sequences join to form a new sequence; the sequence represents in binary a new number with important information. That number can be resolved into two other numbers. One is the original number represented by the dotted sequence on the right. It is congruent to 0 (mod n) because the function it represents can send one to one ($h(1)=1$). The sequence on the left with the dashes can be thought of as the same sequence followed by zeros wherever the sequence on the right had entries. This second number is $x-1$ followed by k zeros on its right. The sum of the first and second is our new number.

$$h(x) \equiv 1 + 2^k(x-1) + 0 \pmod{n}$$

$$h(x) \equiv 2^k(x-1) + 1 \pmod{n}$$

Any truncated member of H_1 sends x to $2^k(x-1) \pmod{n}$. But

$$\begin{aligned} F_0^k(x) &\equiv 2(\ldots(2(2(2x-1)-1)-1)\ldots)-1 \\ &\equiv x2^k - \sum_{j=0}^{k-1} 2^j \\ &\equiv x2^k - (2^k - 1) \\ &\equiv x2^k - 2^k + 1 \\ &\equiv 2^k(x-1)+1 \pmod{n} \end{aligned}$$

Thus any element of H_1 belongs to $\langle F_0 \rangle$. Conversely if $f \in \langle F_0 \rangle$, then $f(1) = F_0^k(1) = 1$. Thus, f must belong to H_1. Hence $\langle F_0 \rangle = H_1$ which yields an important result: $|\mathfrak{F}| = n|H_1| = n|\langle F_0 \rangle|$.

$|\langle F_0 \rangle|$ can easily be calculated by breaking up F_0 into disjoint cycles and taking the least common multiple of their lengths. If $n=p$, a prime, and if 2 is a primitive root of p, $|\mathfrak{F}| = p(p-1)$, which eliminates the work of decomposition into disjoint cycles. This follows, since we have $F_0^k(x) \equiv 2^k(x-1)+1 \pmod{n}$ and when $x=2$, $F_1^k(2) \equiv 2^k(1)+1 \pmod{n}$. If $n=p$, $2^{\phi(p)}+1 \equiv 2 \pmod{p}$, letting $k=\phi(p)$. Now $2^k \equiv 1$ first when $k=\phi(p)$. But then $F_0^k(2)=2$ and no sooner, so it is clear that $\phi(p) = (p-1) = |\langle F_0 \rangle|$.

How can each element of the group $\mathfrak{F}$ be represented? Since

$\langle F_0 \rangle$ is the stabilizer of 1 every element can be written as $L_x F_0^k$ choosing L_x as the representative of a coset for a given x, where $k=1, \ldots, |\langle F_0 \rangle|$. So if $f \in \mathfrak{F}$ then $f = L_x F_0^k$ for some $x=1, \ldots, n$ and $k=1, \ldots, |\langle F_0 \rangle|$. We have thus a *unique way of representing* $f \in \mathfrak{F}$ *in the form* $L_x F_0^k$. By specifying x and k, f is completely determined.

Actually there are n different ways to display the $n\ |\langle F_0 \rangle|$ permutations in $\mathfrak{F}$. We know that all the members of $\mathfrak{F}$ can be exhibited in the form $L_x F_0^k$. Translating each one of those $n |\langle F_0 \rangle|$ permutations by L_y^{-1}, for a given y, results in an isomorphism. Because there are n values of y we get n different isomorphisms. Hence every element of $\mathfrak{F}$ can be represented in the form $L_x F_0^k L_y^{-1}$, for a given y. We have now extracted the information that the elements of $\mathfrak{F}$ that send y to x are all given by $L_x F_0^k L_y^{-1}$ since, for that specific y, the form can represent all $f \in \mathfrak{F}$. Table 1 shows $|\mathfrak{F}|$ for values of n.

TABLE 1

n	$\lvert\langle F_0 \rangle\rvert$	$\lvert\mathfrak{F}\rvert$	NOTES
1	1	1	the trivial group
3	2	6	isomorphic to D_3
5	4	20	2 is a primitive root of 5
7	3	21	the only non-abelian group of size 21
9	6	54	
11	10	110	2 primitive root of 11
13	12	156	2 primitive root of 19
15	4	60	not simple
17	8	136	
19	18	342	2 primitive root
21	6	126	
29	28	812	2 primitive root
37	36	1332	2 primitive root
51	8	408	
53	52	2756	2 primitive root
59	58	3422	2 primitive root
61	60	3660	2 primitive root
67	66	4422	2 primitive root
83	82	6806	2 primitive root

Figure 1 is an array of the elements in $\mathfrak{F}$ for $n=5$. Under each permutation is a circled number which gives the order of the permutation. Below the circled number is the permutation's representation in the form $L_xF_0^k$. For $n=5$, $\mathfrak{F}$ is clearly non-abelian and not isomorphic to D_{10} which has an element of order 10.

$$
\begin{array}{cccc}
\begin{pmatrix}1&2&3&4&5\\1&3&5&2&4\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\1&5&4&3&2\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\1&4&2&5&3\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\1&2&3&4&5\end{pmatrix} \\
4 & 2 & 4 & 1 \\
F_0 & F_0^2 & F_0^3 & F_0^4=i \\[1em]
\begin{pmatrix}1&2&3&4&5\\2&1&5&4&3\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\2&5&3&1&4\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\2&3&4&1&5\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\2&4&1&3&5\end{pmatrix} \\
2 & 4 & 5 & 4 \\
F_1F_0 & F_1F_0^2 & F_1F_0^3 & F_1F_0^4=F_1 \\[1em]
\begin{pmatrix}1&2&3&4&5\\3&1&4&2&5\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\3&4&5&1&2\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\3&5&2&4&1\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\3&2&1&5&4\end{pmatrix} \\
4 & 5 & 4 & 2 \\
F_0F_1F_0 & F_0F_1F_0^2 & F_0F_1F_0^3 & F_0F_1 \\[1em]
\begin{pmatrix}1&2&3&4&5\\4&2&5&3&1\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\4&5&1&2&3\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\4&1&3&5&2\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\4&3&2&1&5\end{pmatrix} \\
4 & 5 & 4 & 2 \\
F_1F_1F_0 & F_1F_1F_0^2 & F_1F_1F_0^3 & F_1F_1 \\[1em]
\begin{pmatrix}1&2&3&4&5\\5&1&2&3&4\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\5&2&4&1&3\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\5&4&3&2&1\end{pmatrix} & \begin{pmatrix}1&2&3&4&5\\5&3&1&4&2\end{pmatrix} \\
5 & 4 & 2 & 4 \\
F_0F_0F_1F_0 & F_0F_0F_1F_0^2 & F_0F_0F_1F_0^3 & F_0F_0F_1
\end{array}
$$

FIGURE 1

We next show that 1) the rotation permutation R is a member of $\mathfrak{F}$, 2) R and F_0 generate $\mathfrak{F}$, 3) members of $\mathfrak{F}$ can be written in the form $R^kF_0^m$, and 4) the cyclic subgroup generated by R is normal in $\mathfrak{F}$ so $\mathfrak{F}$ *can never be simple.*

First, we have

$$L_n(x)=L_nL_x(1) \equiv 1+1\ldots\underbrace{1\cdots}_{k\text{ places}}$$

where the notation represents a number in binary form. The first

1 from the left with the dots is related to L_r and the second 1 with the dashes is related to L_n. Now

$$L_n(x) \equiv 1+2^k(x-1)+n-1 \equiv 2^k(x-1) \pmod{n}.$$

Choose j so that $j+k \equiv 0 (\text{mod } |\langle F_0 \rangle|)$.

Then $F_1{}^j L_n(x) \equiv 2^j 2^k(x-1) \equiv 2^{j+k}(x-1) \equiv 1(x-1) \pmod{n}$.

Thus $F_1{}^j L_n(x) = R^{-1}(x)$. Since $R^{-1} \epsilon\ \mathfrak{F}$, $R \epsilon\ \mathfrak{F}$.

Thus $\langle R \rangle \subset \mathfrak{F}$ and $\mathfrak{F} = \langle F_1, F_0 \rangle$. For a given x, $F_1(x)=2x$ and $F_0(x)=2x-1$. Now $R^{-1}F_1(x)=2x-1$, so $F_0(x)=R^{-1}F_1(x)$. Thus $\mathfrak{F} = \langle F_0, R \rangle$.

Taking R^k as a representative of the coset containing L_k, we find that any element can be given in the form $R^kF_0{}^m$ for $k=1,\ldots,n$ and $m=1,\ldots, |\langle F_0 \rangle|$.

By displaying $\langle R \rangle$ to be a proper normal subgroup of $\mathfrak{F}$ it can be shown that $\mathfrak{F}$ is never simple except for the trivial group arising when $n=1$.

Let $g \epsilon\ \mathfrak{F}$, then $g=R^kF_0{}^m$. If $r \epsilon \langle R \rangle$, then $r=R^s$ for some s, $1 \leq s \leq n$. Now $R=(123\ldots n)$, so

$$R^s = (\ 1 \quad 1+s \quad 1+2s \ldots)\ (\ldots)\ldots$$

$$gR^sg^{-1} = g(1)\ g(1+s) \quad g(1+2s)\ldots)\ (\ldots)\ldots$$

$$=(\ R^kF_0{}^m(1) \quad R^kF_0{}^m(1+s) \quad R^kF_0{}^m(1+2s)\ \ldots)\ (\ldots)\ldots$$

with all entries reduced mod n.

If the difference between the adjacent entries in a given disjoint cycle is a constant c, then the entire permutation is a member of $\langle R \rangle$ and is precisely R^c, We have

$$R^kF_0{}^m(\ 1+(x+1)s\ ) \ - \ R^kF_0{}^m(\ 1+xs\ )$$

$$= k+2^m(\ 1+(x+1)s-1)+1 \ - (\ k+2^m(1+xs-1)+1\ )$$

$$= 2^m s = c$$

Thus $g\ R^s\ g^{-1} = R^{(2^m s)} \epsilon \langle R \rangle$, which tells us that $\langle R \rangle \lhd \mathfrak{F}$.

Since $\langle R \rangle$ is always a normal subgroup the only way $\mathfrak{F}$ can be simple is if $\langle R \rangle$ is not a proper subgroup. This occurs only for

$n=1$ which is the trivial group of size one.

Up to now we have been working with n as a prime or at best an odd integer. It is easily seen that the permutation $F_1(x) \equiv 2x \pmod{10}$ is not well defined, since $F_1(3) \equiv 6 \equiv F_1(8), \pmod{10}$. How can we insure that $kx \pmod{n}$ is well defined?

Suppose $kx=ky \pmod{n}$. We are working in a ring so $(kx-ky) \equiv 0 \pmod{n}$. Thus $k\,(x-y)=0$, which implies that $x=y$ or that k is a divisor of zero. Since we want $x=y$ to be our conclusion, we insist that k is not a divisor of zero, or equivalently, that k and n are relatively prime.

If n is a prime then any k will do. If $k=2$ any odd integer n will suffice. For $n=10$, k could be either 1, 3, 7, or 9 for these are the integers that are relatively prime to 10. Using $k=3$ the cycle structure is (3971) (2684) (5) (10) and the least common multiple of 1 and 4 is 4. Notice that the cycles are of different lengths. This is because though $\langle 3 \rangle$ forms a group of numbers under multiplication, it is not a subgroup of a multiplicative group involved here. We have

$$|\mathfrak{F}_3| = |\langle F_0, F_1, F_2 \rangle| = (10)(4)=40$$

For $k=9$, we have (91) (28) (37) (46) (5) (10) and

$$|\mathfrak{F}_9| = |\langle F_0, F_1, \ldots, F_8 \rangle| = (10)(2)=20$$

Finally, information can be extracted (as to where $f \in \mathfrak{F}_k$ sends the element 1) from the base k sequence that arises when products are formed in $\mathfrak{F}_k$. We have already shown this for $k=2$. It follows by induction for all positive n and $2 \leq k \leq n-1$ where k and n are relatively prime. For example with $k=3$ and $n=11$, $F_1F_2F_0F_1(1) \equiv F_1F_2F_0(2) \equiv F_1F_2(4) \equiv F_1(12) \equiv F_1(1) \equiv 2$. But $1+(1021) \equiv 1+(1+6+0+27) \equiv 1+34 \equiv 1+1 \equiv 2 \pmod{11}$.

Summary of the Results

1) $\mathfrak{F} = \langle F_0, F_1 \rangle$ is transitive.

2) $|\mathfrak{F}| = n|\langle F_0 \rangle|$, where n is an odd integer and $|\langle F_0 \rangle| = |H_1|$ can always be calculated.

3) For all $f \in \mathfrak{F}$, $f=L_xF_0^k$ for $x=1, 2, 3, \ldots, n$ and $k=1, 2, 3, \ldots, |\langle F_0 \rangle|$.

4) If $f(x)=y$, then $f=L_yF_0{}^kL_x{}^{-1}$ for some $k=1, 2, 3, \ldots, |\langle F_0 \rangle|$.

5) For $n>1$, $\mathfrak{F}$ is never simple.

6) If $f\in\mathfrak{F}$, $f=R^kF_0{}^m$ for $k=1, 2, 3, \ldots, n$ and $m=1, 2, 3, \ldots, |\langle F_0 \rangle|$.

7) $\langle R \rangle$ is normal in $\mathfrak{F}$.

8) If $n=p$, a prime, then $|\mathfrak{F}|=p(p-1)$ provided that two is a primitive root of p.

The Sentential Calculus via Algebra*

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Logic plays a significant role in today's world. Many problems in business and industry are solved by the use of logic. Corporations have found symbolic logic very helpful in analyzing their contracts. Inconsistencies and loopholes are more easily found with the help of symbolic logic. The field of engineering also uses logic to analyze electrical circuits. One of its most significant uses is in the design of large-scale electronic calculating machines [Pfeiffer 1968].

In sentential logic, we consider arbitrary variables, p and q, which we assume range over $\{0,1\}$. These variables reflect atomic sentences which assume a true or false value.

We can make compound sentences, which are reflected in the sentential forms of logic, from atomic sentences by the use of connectives. There are five connectives used for this purpose in the sentential calculus. The first one is "$\wedge$" and it is used to form the conjunction, $p\wedge q$, read as "p and q". The second, "$\vee$", is used to form a disjunction, $p\vee q$, read as "p or q." The next compound statement, the conditional, $p\Longrightarrow q$, is read "p implies q," "if p then q," or "p only if q." The fourth, called a biconditional, is a conditional in both directions. The connective used is "$\Longleftrightarrow$" and $p\Longleftrightarrow q$ is read "p implies and is implied by q", or "p if and only if q." The last is the denial, p', which is read "not p."

Once we have established the various compound statements, we can consider how the connectives take the range values of the atomic sentences and translate them to one value which can be associated with the compound statement. An easy way to organize the various values of the atomic and compound statements is to build a truth table. We assign 1 to a statement which is true and 0 to a statement which is false. To build the truth table, we first take all the combinations of truth values for our two variables p and q. We can see that there are four such possibilities; and knowing these values we can complete the entire truth table. Since there are only two values

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which a variable p may take, the denial, p', takes the values contrary to that of p. The statement $p \wedge q$ will be true when both atomic sentences represented by the variables p and q are true. The compound sentence will be false every other time. The disjunction, $p \vee q$, will be true when either p or q is true or both are true. The conditional, $p \Rightarrow q$, is false only when p is true and q is false, otherwise it is true. The biconditional, $p \Leftrightarrow q$, is true only when p implies q and q implies p; this occurs when p and q have the same truth value.

TABLE 1

p	q	p'	$p \wedge q$	$p \vee q$	$p \Rightarrow q$	$p \Leftrightarrow q$
1	1	0	1	1	1	1
1	0	0	0	1	0	0
0	1	1	0	1	1	0
0	0	1	0	0	1	1

In this way our truth table (Table 1) is built and the values of each of the compound sentences is known. These values are based on their agreement with common language usage.

There are certain compound sentences which will have a truth value that is always 1 no matter what values are given to its atomic sentence components. Statements of this type are called tautologies. Tautologies play a significant role in logic, because by their very nature, they allow one form to be substituted for another. For example, the propositional form $(p \Leftrightarrow q) \Leftrightarrow (p' \Leftrightarrow q')$ is a tautology. This means that the left-hand side of the biconditional may be substituted for the right-hand side or vice versa in a logical argument. This manner of substitution can be very useful and may help lead to a desired conclusion in a specific argument.

Upon observing the truth table, it looks like a table of functions. The variables p and q have values 0 or 1. The connectives are like operations on the variables p and q. The connectives, interpreted as functions, have domain $\{0,1\} \times \{0,1\}$ and range $\{0,1\}$. They take $\{0,1\} \times \{0,1\} \rightarrow \{0,1\}$. All of the functions are binary operations except p' which is a unary operation. The denial takes $\{0,1\} \rightarrow \{0,1\}$, so it too can be considered as a function.

Each of the connective functions will now be expressed in terms of algebraic formulas. These algebraic formulas which represent the connective functions are known in the field of logic but do not often appear in texts. [Fine 1965, p. 30-31 and Tarski 1969, p 426]. For this reason, the explanation which follows might be of interest to students in their studies in logic.

The denial, p', takes the value contrary to that of p. When p is 1, p' is 0; when p is 0, p' is 1. This can be transferred into the algebraic formula $p' = (1-p)$.

By just looking at the truth table we can see that our function $p.\ q$ is equal to $p \cdot q$. In each instance the value of p multiplied by the value of q leads to the desired value of our conjunctive function $p\ \ q$.

The formula for the disjunctive function cannot be readily derived by just analyzing the truth table. It is much easier if we change the form by using a well-known tautology known as DeMorgan's Law. It states that

$$(p \vee q)' \Longleftrightarrow (p' \wedge q') \qquad (1)$$

We know from the truth tables that

$$(r \Longleftrightarrow s) \Longleftrightarrow (r' \Longleftrightarrow s') \qquad (2)$$

Now using (2), we can negate both sides of (1) and evolve a new tautology in the form we desired:

$$(p \vee q) \Longleftrightarrow (p' \wedge q')'$$

By using the denial and conjunctive functions, we can alter the right hand side of this tautology and arrive at the arithmetic statement

$$(p \vee q) = (p' \wedge q')' = 1-((1-p) \cdot (1-q)) = p+q-pq$$

With the conditional function, it is easier to change its form, by use of a tautology, to an equivalent one:

$$(p \Longrightarrow q) \Longleftrightarrow (p' \vee q)$$

With this form we can use our previously derived functions to obtain the arithmetic formula:

$$\begin{aligned} p' \vee q = p'+q-p'q &= (1-p)+q-(1-p)q \\ &= (1-p)+q-q+pq \\ &= 1-p+pq \end{aligned}$$

Thus, $(p \Longrightarrow q) = (1-p+pq)$.

Our last function, the biconditional, takes the conditional in both directions. Therefore

$$(p \Longleftrightarrow q) \Longleftrightarrow [(p \Longleftrightarrow q) \wedge (q \Longrightarrow p)]$$

Again by using the previous formulas, we can find the arithmetic replacement for the biconditional

$$\begin{aligned}(p \Longleftrightarrow q) &= (1-p+pq)\cdot(1-q+qp)\\ &= 1-q+qp-p+pq-p^2q+pq-pq^2+p^2q^2\end{aligned}$$

Since p takes only the values 1 or 0, p^2 has the same value as p, so we may simplify the algebraic statement above to the following form:

$$\begin{aligned}(p \Longleftrightarrow q) &= 1-q+qp-p+pq-pq+pq-pq+pq =\\ &= 1-q-p+2pq\\ &= 1-(q^2-2pq+p^2)\\ &= 1-(p-q)^2\end{aligned}$$

By checking the result of the algebraic computations with those on the truth table, it is seen that the results do agree.

By considering the sentential connectives as functions, defined by the algebraic formulas above, the process of determining the truth value of a propositional form is reduced to simple algebraic calculations.

We conclude this article with some examples of how these functions are used to verify that a sentential form is a tautology. The technique in the first example is to show that the algebraic expression for the sentential form reduces to 1. The first example is known as the law of double negative, $(p')' \Longleftrightarrow p$. We use the denial and biconditional functions:

$$\begin{aligned}(p')' &\Longleftrightarrow p\\ (1-p)' &\Longleftrightarrow p\\ 1-(1-p) &\Longleftrightarrow p\\ p &\Longleftrightarrow p\\ 1-(p-p)^2 &= 1\end{aligned}$$

Therefore, $(p')' \Longleftrightarrow p$ is a tautology.

The next example is one of DeMorgan's Laws:

$$(p \wedge q)' \Longleftrightarrow (p' \vee q')$$

The technique here is to work with the right side, reducing it to its algebraic expression, then by using the functions in reverse to derive the left side. If we can do this then the sentential form is a tautology because the truth value of the right side equals the truth value of the left side which is a necessary and sufficient condition for the biconditional to be true.

We begin with the right side:

$$p' \vee q' = (1-p) \vee (1-q)$$

Using the disjunctive function, we have

$$\begin{aligned} p' \vee q' &= (1-p) + (1-q) - (1-p)(1-q) \\ &= (1-p) + (1-q) - 1 + p + q - pq \\ &= 1 - pq \\ &= 1 - (p \wedge q) \\ &= (p \wedge q)' \end{aligned}$$

Now we have shown that the left side of the sentential form is the same as the right side. Therefore the left and right sides have the same truth values, and because of the truth value convertions for the biconditional this formula is always true. Therefore it is a tautology.

The next example is the rule of detachment which says if p is true and $p \Longrightarrow q$ is true, then q is true. We begin by using conditional and conjunctive functions.

$$\begin{aligned} [p \wedge (p \Longrightarrow q)] &\Longrightarrow q \\ [p \cdot (1-p + pq)] &\Longrightarrow q \\ [p(1-p + pq)] &\Longrightarrow q \\ [p - p^2 + p^2q] &\Longrightarrow q \\ [p - p + pq] &\Longrightarrow q \\ pq &\Longrightarrow q \end{aligned}$$

Now by using the conditional function this becomes

$$1 - (pq) + (pq)q$$

$$1 - pq + pq = 1$$

Therefore, $[p \wedge (p \Rightarrow q)] \Rightarrow q$ is a tautology.

We will now examine what happens in the algebraic representation when the corresponding form is not a tautology. The following example will show that the conditional is not an associative operation. Consider:

$$[(p \Rightarrow q) \Rightarrow r] \Leftrightarrow [p \Rightarrow (q \Rightarrow r)]$$

Using the conditional function this becomes

$$[1 - (1-p + pq) + r(1-p+pq)] \Leftrightarrow [1-p + p(1-q + qr)]$$

$$[p-pq + r-rp + rpq] \Leftrightarrow [1-pq + rpq].$$

Using the biconditional function this becomes

$$1 - [p - pq + r - rp + rpq - (1-pq + rpq)]^2,$$

$$1 - [p + r - rp - 1]^2.$$

This can be simplified further, but instead of doing this immediately some observations will be made at this point:

1) The truth value is independent of the truth value of q, since q does not occur in the final form.

2) The functional form enables us to easily pick out cases where it assumes the value 0. When $r = 0$ and $p = 0$, then the functional value is 0, thus the form is not a tautology.

3) Because $1-[p + r-rp-1]^2 = 1-[p-1-r(p-1)]^2 = [p' \Leftrightarrow (r \wedge p')]$, the original form is equivalent to $[p' \Leftrightarrow (r \wedge p')]$. This can also be derived by using the algebraic schemes in reverse.

In this article we have shown and explained how one can use an algebraic scheme to verify that a propositional form is a tautology. Through the examples we have clarified this scheme and have shown the usefulness of verifying tautologies in this manner. We hope that we have given the reader another tool with which to prove that a sentential form is a tautology.

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A Note On The Partition Function P(N)

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Any positive integer n can be written as a sum of positive integers in a finite number of ways. For example, 4 can be written as 4, $3+1$, $2+2$, $2+1+1$, and $1+1+1+1$. Each sum, including 4 itself, is called a *partition of* 4. Two partitions are equal if they differ only in the order of the addends. That is, $2+1+1$ and $1+2+1$ are the same partition of 4. We define $p(n)$ to be the number of partitions of the positive integer n. Thus $p(4) = 5$. Further $p(0)$ is defined to be 1 for later convenience.

The function $(1-x^n)^{-1}$ can be used to show that the coefficient of x^n in $\prod_{n=1}^{\infty} (1-x^n)^{-1}$ is $p(n)$. Thus

$$\sum_{n=0}^{\infty} p(n)x^n = \prod_{n=1}^{\infty} (1-x^n)^{-1}$$

The Euler formula for $\prod_{n=1}^{\infty} (1-x^n)$ can be utilized to prove the following theorem [1, pp. 229-35].

THEOREM. *If n is a positive integer then*

$$\begin{aligned} p(n) &= p(n-1) + p(n-2) - p(n-5) - p(n-7) + p(n-12) + \\ &\qquad p(n-15) - \dots \\ &= \sum_{j>0} (-1)^{j+1} p(n - \tfrac{1}{2}(3j^2 \pm j)) \end{aligned}$$

where the arguments for the partition function are non-negative.

This note offers a computer program for $p(n)$ based on the above theorem. The following program evaluates $p(n)$ for $1 \leq n \leq 100$.

This note offers a computer program for $\underline{p}(\underline{n})$ based on the above theorem. The following program evaluates $\underline{p}(\underline{n})$ for $1 \leq \underline{n} \leq 100$.

```
      DIMENSION IP(100)
      IO=5
      DO 22 N=1,100
      IP(N)=0
      DO 3 J=1,50
      IJN=0.5*(3*(J**2)-J)
      IF(N-IJN)2,15,10
   15 IP(N)=IP(N)+((-1)**(J+1))
      GO TO 2
   10 L=N-IJN
      IP(N)=IP(N)+((-1)**(J+1))*IP(L)
      IJP=0.5*(3*(J**2)+J)
      IF(N-IJP)2,16,11
   16 IP(N)=IP(N)+((-1)**(J+1))
      GO TO 18
   11 M=N-IJP
      IP(N)=IP(N)+((-1)**(J+1))*IP(M)
   18 IF(J-50)3,79,79
    3 CONTINUE
    2 WRITE(5,5)N,IP(N)
    5 FORMAT(5XI5,10XI22)
   22 CONTINUE
   79 CALL EXIT
      END
```

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On Conversion Of Bases In Natural Numbers

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While teaching different number bases for natural numbers, one frequently encounters the problem of conversion of bases. Most textbooks on this subject only show conversion from base 10 to base X and vice versa [1, pp. 118-125]. Some textbooks contain problems of conversion from base X to base Y; however, following the methods in the text, the reader would have to convert from base X to base 10, then from base 10 to base Y [2, pp. 35-36]. In this note, we give a method of conversion from base X to base Y directly using techniques from the elements of vector spaces and matrix multiplication.

Let $N = (a_n a_{n-1} \ldots a_1 a_0)_x$ be a natural number written in base X, its base 10 representation is then $a_0 \cdot 1 + a_1 X + a_2 X^2 + \ldots + a_{n-1} X^{n-1} + a_n X^n$, where $0 \leq a_i < X$. Writing $1, X, X^2, \ldots$ in terms of Y, we have

$$
\begin{aligned}
1 &= 1 + 0 \cdot Y + 0 \cdot Y^2 + \ldots 0 \cdot Y^k \\
X &= C_{10} + C_{11} Y + C_{12} Y^2 + \ldots + C_{1k} Y^k \\
X^2 &= C_{20} + C_{21} Y + C_{22} Y^2 + \ldots + C_{2k} Y^k \qquad (1) \\
&\cdot \\
&\cdot \\
&\cdot \\
X^n &= C_{n0} + C_{n1} Y + C_{n2} Y^2 + \ldots + C_{nk} Y^k
\end{aligned}
$$

where $0 \leq C_{ij} < Y$.

Call the matrix $[C_{ij}]$ the XY – conversion matrix. (1) can be rewritten as a matrix product

$$
\begin{bmatrix} 1 \\ X \\ X^2 \\ \cdot \\ \cdot \\ \cdot \\ X^n \end{bmatrix} = [C_{ij}] \begin{bmatrix} 1 \\ Y \\ Y^2 \\ \cdot \\ \cdot \\ \cdot \\ Y^k \end{bmatrix}
$$

Now think of the number N as the vector product

$$[a_0\ a_1\ a_2\ \ldots\ a_n]\begin{bmatrix}1\\X\\X^2\\\cdot\\\cdot\\\cdot\\X^n\end{bmatrix}$$

We call $[a_0\ a_1\ \ldots\ a_n]$ the vector associated with N for base X. Thus we have

$$[a_0\ a_1\ a_2\ \ldots\ a_n]\begin{bmatrix}1\\X\\X^2\\\cdot\\\cdot\\\cdot\\X^n\end{bmatrix} = [a_0\ a_1\ a_2\ \ldots\ a_n]\ [C_{ij}]\begin{bmatrix}1\\Y\\Y^2\\\cdot\\\cdot\\\cdot\\Y^k\end{bmatrix}$$

and $[a_0\ a_1\ a_2\ \ldots\ a_n]\ [C_{ij}] = [b_0\ b_1\ \ldots\ b_k]$ is the vector associated with N for base Y, except for converting those b_i that are greater than Y.

As an example we convert $(1201)_3$ to base 2. The 32-conversion matrix is formed from the equations

$$\begin{aligned}
1 &= 1\\
3 &= 1 + 1\cdot 2\\
3^2 &= 1 + 0\cdot 2 + 0\cdot 2^2 + 1\cdot 2^3\\
3^3 &= 1 + 1\cdot 2 + 0\cdot 2^2 + 1\cdot 2^3 + 1\cdot 2^4\\
3^4 &= 1 + 0\cdot 2 + 0\cdot 2^2 + 0\cdot 2^3 + 1\cdot 2^4 + 0\cdot 2^5 + 1\cdot 2^6\\
&\cdot\\
&\cdot\\
&\cdot
\end{aligned}$$

Then

$$[C_{ij}] = \begin{bmatrix}1&0&0&0&0&0&0\\1&1&0&0&0&0&0\\1&0&0&1&0&0&0\\1&1&0&1&0&0&0\\1&0&0&0&1&0&1\\&&&\cdot&&&\\&&&\cdot&&&\\&&&\cdot&&&\end{bmatrix}$$

and $(1201)_3$ has the associated vector [1021] and [1021] $[C_{ij}]$ = [41031] = [011101] upon converting into base 7. Thus $(1201)_3$ = $(101110)_2$.

Finally, we point out that the product of an $XY-$conversion matrix and the corresponding $YX-$conversion matrix is a "quasi-identity" matrix for base X, in the sense that it becomes the identity matrix upon converting the row rectors into base X as done above. Similarly, the reversed product yields a "quasi-identity" matrix for base Y.

For example, the 75-conversion matrix $[C_{ij}]$ and the 57-conversion matrix $[D_{ij}]$ are, respectively

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \\ 2 & 1 & 0 & 0 & \cdots \\ 4 & 4 & 1 & 0 & \\ 3 & 3 & 3 & 2 & \\ & \cdot & & & \\ & \cdot & & & \\ & \cdot & & & \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 & \\ 5 & 0 & 0 & 0 & \cdots \\ 4 & 3 & 0 & 0 & \\ 6 & 3 & 2 & 0 & \\ & \cdot & & & \\ & \cdot & & & \\ & \cdot & & & \end{bmatrix}$$

The product

$$[D_{ij}]\cdot[C_{ij}] = \begin{bmatrix} 1 & 0 & 0 & 0 & \\ 5 & 0 & 0 & 0 & \cdots \\ 10 & 3 & 0 & 0 & \\ 20 & 11 & 2 & 0 & \\ & \cdot & & & \\ & \cdot & & & \\ & \cdot & & & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \\ & \cdot & & & \\ & \cdot & & & \\ & \cdot & & & \end{bmatrix}$$

upon converting the rows into base 5. Similarly the product

$$[C_{ij}]\cdot[D_{ij}] = \begin{bmatrix} 1 & 0 & 0 & 0 & \\ 7 & 0 & 0 & 0 & \cdots \\ 28 & 3 & 0 & 0 & \\ 42 & 15 & 4 & 0 & \\ & \cdot & & & \\ & \cdot & & & \\ & \cdot & & & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \\ & \cdot & & & \\ & \cdot & & & \\ & \cdot & & & \end{bmatrix}$$

upon converting into base 7.

To do one or two problems of this type, this method has novelty, but does not save any time. However, if one is to do many of such conversions, after he computed the conversion matrix, the rest would be a snap. Students could be asked to compute several of the matrices and keep them for reference. Then this method is quite similar to the method of indices in number theory.

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Uses Of The Definite Integral Formula For Arc Length

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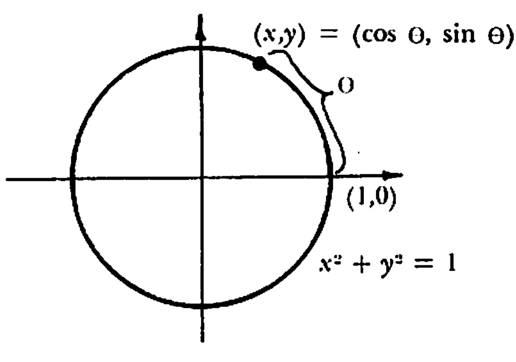
The definite integral formula $\int_a^b \sqrt{1 + (f'(x))^2}\, dx$, which gives the arc length of the curve $y = f(x)$ from $(a,f(a))$ to $(b,f(b))$, is often one of the topics presented in calculus texts. The examples and problems presented must be selected with great care as the integrand, $\sqrt{1 + (f'(x))^2}$, is often difficult to integrate by elementary methods. Because of this, the topic of arc length is not very exciting for students.

The purpose of this paper is to show that there are other uses of this formula. It can be used to find the derivative of the cosine and sine functions, the derivative of the arc cosine and arc sine functions, the familiar limit of $\sin \Theta/\Theta$ as $\Theta \to 0$, and the limit of the sequence (a_n), where $a_n = \sum_{i=0}^{n-1} \frac{1}{\sqrt{n^2 - i^2}}$. For simplicity throughout the paper, only points in the first quadrant will be considered.

THEOREM 1. $\frac{d}{d\Theta} \cos \Theta = \sin \Theta$.

Proof: Let (x,y) be a point on $x^2 + y^2 = 1$ and let Θ be the arc length between $(1,0)$ and (x,y) so that $x = \cos \Theta$ and $y = \sin \Theta$.



By the definite integral formula for arc length,

$$\Theta = \int_{\cos\Theta}^{1} \sqrt{+ \left(\frac{d}{dx}\sqrt{1-x^2}\,\right)^2}\; dx,$$

$$= -\int_{1}^{\cos\Theta} \frac{1}{\sqrt{1-x^2}}\, dx. \tag{1}$$

Differentiating both sides of (1) by the fundamental theorem of calculus and the chain rule (assuming that $\frac{d}{d\Theta}\cos\Theta$ exists),

$$1 = \frac{1}{\sqrt{1-\cos^2\Theta}} \cdot \frac{d}{d\Theta}\cos\Theta,$$

so that $\frac{d}{d\Theta}\cos\Theta = -\sqrt{1-\cos^2\Theta} = -\sin\Theta.$

THEOREM 2. $\frac{d}{d\Theta}\mathit{sin}\,\Theta = \mathit{cos}\,\Theta.$

Proof: As in Theorem 1, the definite integral formula for arc length yields

$$\Theta = \int_{0}^{\sin\Theta} \sqrt{1 + \left(\frac{d}{dy}\sqrt{1-y^2}\,\right)^2}\; dy,$$

$$= \int_{0}^{\sin\Theta} \frac{1}{\sqrt{1-y^2}}\, dy. \tag{2}$$

Differentiating both sides of (2) by the fundamental theorem of calculus and the chain rule (assuming that $\frac{d}{d\Theta}\sin\Theta$ exists),

$$1 = \frac{1}{\sqrt{1-\sin^2\Theta}} \cdot \frac{d}{d\Theta}\sin\Theta$$

so that $\frac{d}{d\Theta}\sin\Theta = \sqrt{1-\sin^2\Theta} = \cos\Theta.$

Other methods are often used to find the derivative of the cosine and sine functions. Many of these depend upon $\lim_{\Theta\to 0} \frac{\sin \Theta}{\Theta} = 1$. This result can also be obtained from the definite integral formula for arc length as seen in the next theorem.

THEOREM 3. $\lim_{\Theta\to 0} \frac{\sin \Theta}{\Theta} = 1.$

Proof: Applying the mean value theorem for the definite integral to statement (2).

$$\Theta = \frac{1}{\sqrt{1 - \sin^2 u}} (\sin \Theta - 0)$$

where $0 \leqq \sin u \leqq \sin \Theta$, for $0 \leqq u \leqq \Theta$. Thus, $\cos u = \frac{\sin \Theta}{\Theta}$.

As $\Theta \to 0$, then $u \to 0$, so that

$$\lim_{\Theta\to 0} \frac{\sin \Theta}{\Theta} = \lim_{u\to 0} \cos u = 1.$$

THEOREM 4. $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1 - x^2}}$.

Proof: From (1) we have

$$\Theta = -\int_1^{\cos \Theta} \frac{1}{\sqrt{1 - t^2}}\, dt.$$

If $x = \cos \Theta$, then $\cos^{-1} x = \Theta$ and conversely, so that

$$\cos^{-1} x = -\int_0^x \frac{1}{\sqrt{1 - t^2}} dt.$$

By the fundamental theorem of calculus,

$$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1 - x^2}}.$$

THEOREM 5. $\dfrac{d}{dy}\, sin^{-1}y = \dfrac{1}{\sqrt{1-y^2}}$.

Proof: From (2) we have

$$\theta = \int_0^{\sin\theta} \frac{1}{\sqrt{1-t^2}}\,dt.$$

If $y = \sin\theta$, then $\sin^{-1}y = \theta$ and conversely, so that

$$\sin^{-1}y = \int_0^{y} \frac{1}{\sqrt{1-t^2}}\,dt.$$

By the fundamental theorem of calculus,

$$\frac{d}{dy}\sin^{-1}y = \frac{1}{\sqrt{1-y^2}}\,.$$

THEOREM 6. *For the sequence* (a_n), *where* $a_n = \sum_{i=0}^{n-1} \dfrac{1}{\sqrt{n^2-i^2}}$, $a_n \to \pi/2$ *as* $n \to \infty$.

Proof: On the circle $x^2 + y^2 = 1$,

$$\pi/2 = \int_0^1 \sqrt{1 + \left(\frac{d}{dx}\sqrt{1-x^2}\right)^2}\,dx = \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx,$$

where $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{1}{\sqrt{1-(i/n)^2}} \cdot \left(\frac{1}{n}\right)$, the limit of a Riemann Sum.

Hence, $\displaystyle\pi/2 = \lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{1}{\sqrt{n^2-i^2}}$.

An Alternate Proof Of The Chinese Remainder Theorem

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This note describes a constructive proof of the Chinese Remainder Theorem by means of mixed radix representation. This approach can be used to find the unique solution with a minimum amount of computation, particularly if the moduli, m_i, are ordered so that $m_i < m_j$ whenever $i < j$.

Theorem. (*mixed radix representation*) *Let* $m_1, m_2, \ldots, m_n$ *be any natural numbers, where for each i,* $m_i \geqslant 2$. *Every natural number* $x < M = \prod_{i=1}^{n} m_i$, *has a unique representation of the form:*
$x = a_1 + a_2 m_1 + a_3 m_1 m_2 + \ldots + a_n m_1 m_2 \ldots m_{n-1}$ *where* $0 \leq a_i < m_i$ *for each i.*

Proof: (by induction on n) If $n = 1$ the theorem is true, since $x_1 = a_1 < m_1$. Suppose true for $n = k-1$. Let $a_1, a_2, \ldots, a_k$ be any natural numbers with $m_i \geqslant 2$ for $i = 1, \ldots, k$. Suppose that $x < m_1 m_2 \ldots m_k$. We know there exists unique numbers q and r such that $x = qm_1 + r$ and $0 \leq r < m_1$. Now $q = \dfrac{x-r}{m_1} < \dfrac{\prod m_i}{m_1} = m_2 m_3 \ldots m_k$. By assumption, q has the unique form $q = a_2 + a_3 m_2 + \ldots + a_k m_2 m_3 \ldots m_{k-1}$. Thus $x = m_1 q + r = r + a_2 m_1 + \ldots + a_k m_1 m_2 \ldots m_{k-1}$. Now let $a_1 = r$ and we have the proper form for x, and the a_i's are unique.

Theorem. (*Chinese Remainder Theorem*) *Let* $\{m_1, m_2, \ldots, m_n\}$ *be a set of relatively prime pair wise natural numbers, with* $m_i \geqslant 2$ *for each i. Let* $r_1, r_2, \ldots, r_n$ *be any natural numbers. There exists a solution of the system of congruences,* $x \equiv r_i \pmod{m_i}$, $i = 1, \ldots, n$. *Moreover, the solution is unique, mod* $M = \prod_{i=1}^{n} m_i$.

Proof: By the previous theorem, any $x < M$ has the unique form $y_1 + y_2 m_1 + \ldots + y_n m_1 m_2 \ldots m_{n-1}$, $0 \leq y_i < m_i$. Consequently, the congruence system is equivalent to

$$r_1 \equiv y_1 \pmod{m_1}$$

$$r_2 \equiv y_1 + y_2 m_1 \pmod{m_2}$$

$$\vdots$$

$$r_n \equiv y_1 + y_2 m_1 + \ldots + y_n m_1 \ldots m_{n-1} \pmod{m_n}$$

There is a unique solution $y_1 = a_1$, $a_1 < m_1$ for the first congruence. The second congruence becomes $r_2 - a_1 \equiv y_2 m_1 \pmod{m_2}$. Since $(m_1, m_2) = 1$, there is a unique solution $y_2 = a_2$, $a_2 < m_2$. Iteratively, we can uniquely determine solutions $y_i = a_i$, $a_i < m_i$ for each i. We have constructed a unique $x < M$ that solves the system. Hence $x = a_1 + a_2 m_1 + \ldots + a_n m_1 \ldots m_{n-1}$ is the unique solution to the system, mod M.

To illustrate the usefulness of this construction consider the system of congruences.

$$x \equiv 3 \pmod{2}$$

$$x \equiv 5 \pmod{5}$$

$$x \equiv 4 \pmod{7}$$

$$x \equiv 2 \pmod{27}$$

We seek x in the form $y_1 + 2y_2 + 10y_3 + 70y_4$. First $y_1 \equiv 3 \pmod{2}$ has the solution $a_1 = 1$. Second, $1 + 2y_2 \equiv 5 \pmod{5}$ has the solution $a_2 = 2$. Third, $1 + 2(2) + 10y_3 \equiv 4 \pmod{7}$ has the solution $a_3 = 2$. Fourth, $1 + 2(2) + 10(2) + 70y_4 \equiv 2 \pmod{27}$ has the solution $a_4 = 7$. So the unique solution (mod 1890) is $x = 1 + 2(2) + 70(7) = 515$. Notice that the last (and largest) modulus, 27, does not enter into the computation of the solution. It is only used in solving the last congruence.

The more general theorem, which relaxes the condition that the moduli must be relatively prime pairwise, can also be proved by means of mixed radix representation.

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgment will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

From the early beginnings of the development of mathematics, solving equations has occupied much of the thought and effort of mathematicians. Solving equations of degree greater than or equal to three presented problems to the ancient mathematician. The following geometric procedure was developed by the ancients and used their knowledge of the conics.

Suppose you wish to solve the cubic equation $x^3 + ax^2 + bx + c = 0$. By substituting $y = x^2$ in this equation we get $xy + ay + bx + c = 0$. This equation represents a hyperbola whose asymptotes are parallel to the x and y-axes. The equation $y = x^2$ obviously represents a parabola. The desired roots of $x^3 + ax^2 + bx + c = 0$ will be the abscissas of the three points of intersection of the parabola and the hyperbola as shown in Figure 1.

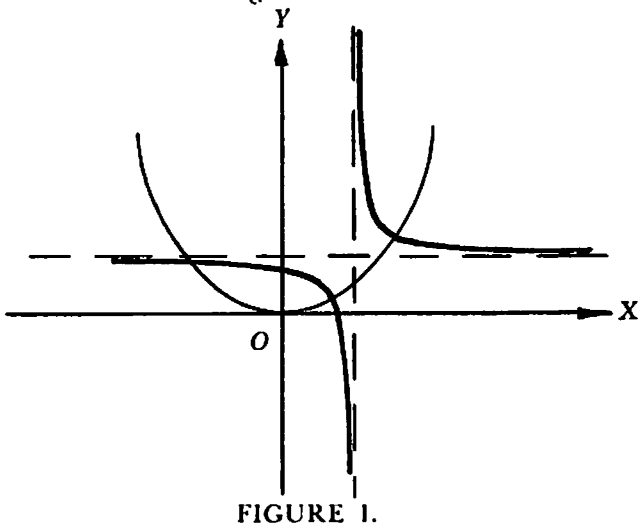


FIGURE 1.

Now suppose we wish to solve the biquadratic equation $x^4 + ax^3 + bx^2 + cx + d = 0$. Again letting $y = x^2$ and substituting we obtain the equation $y^2 + axy + by + cx + d = 0$. This equation when graphed represents a hyperbola having one asymptote parallel to the x-axis and the other asymptote parallel to neither axis. The desired four roots of the original equation are the abscissas of the four points of intersection of the parabola and the hyperbola as shown in Figure 2.

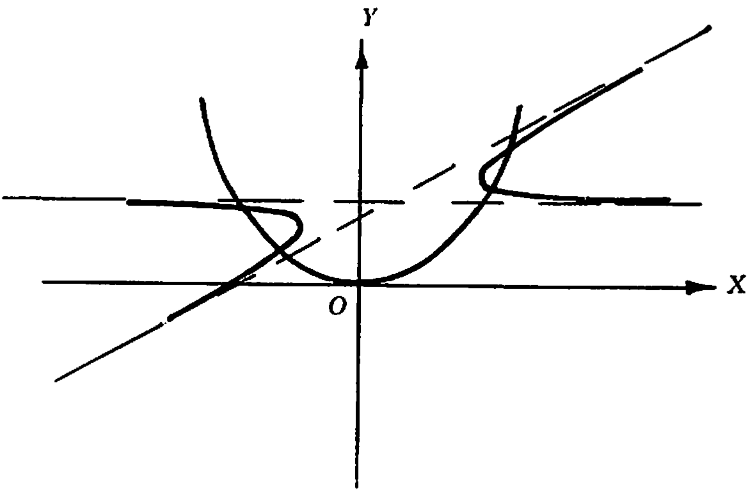


FIGURE 2.

Can you use the above geometric procedure to solve $x^4 + 2x^3 + x^2 + x + 4 = 0$?

— — △ — — △ — —

An unusual trigonometric formula for sin x can be derived using repeated applications of the formula $\sin x = 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}$ as follows:

$$\sin x = 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}$$

$$= 2(2 \sin \frac{x}{4} \cos \frac{x}{4}) \cos \frac{x}{2}$$

$$= 2^2 \sin \frac{x}{4} \cos \frac{x}{4} \cos \frac{x}{2}$$

$$= 2^3 \sin \frac{x}{8} \cos \frac{x}{8} \cos \frac{x}{4} \cos \frac{x}{2}$$

$$= 2^4 \sin \frac{x}{16} \cos \frac{x}{16} \cos \frac{x}{8} \cos \frac{x}{4} \cos \frac{x}{2}$$

$$\vdots$$

$$= 2^n \sin \frac{x}{2^n} \prod_{k=1}^{n} \cos \frac{x}{2^k}.$$

But from the calculus we recall that $\lim_{\Theta \to 0} \frac{\sin \Theta}{\Theta} = 1$ and hence

$$\lim_{n \to +\infty} \frac{\sin \frac{x}{2^n}}{\frac{x}{2^n}} = 1.$$

But

$$\lim_{n \to +\infty} \frac{\sin \frac{x}{2^n}}{\frac{x}{2^n}} = \lim_{n \to +\infty} \frac{2^n}{x} \cdot \sin \frac{x}{2^n}$$

$$= \frac{1}{x}\left[\lim_{n \to +\infty} 2^n \cdot \left(\sin \frac{x}{2^n}\right)\right]$$

$$= 1$$

and so

$$\lim_{n \to +\infty} 2^n \cdot \left(\sin \frac{x}{2^n}\right) = x.$$

From the two results above we have

$$\frac{\sin x}{x} = \frac{2^n \sin \frac{x}{2^n} \prod_{k=1}^{+\infty} \cos \frac{x}{2^n}}{\lim\limits_{n \to +\infty} \left(2^n \sin \frac{x}{2^n}\right)} = \prod_{k=1}^{+\infty} \cos \frac{x}{2^k} \cdot$$

If $x = \pi/2$, then $\frac{\sin \pi/2}{\pi/2} = \frac{1}{\pi/2} = \frac{2}{\pi}$. Thus

$$\frac{2}{\pi} = \prod_{k=1}^{+\infty} \cos \frac{\pi/2}{2^k}$$

$$= \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{8} \cdot \cos \frac{\pi}{16} \cdots$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots,$$

a formula known as Vieta's formula.

Can you derive the expression $\frac{\sin x}{x}$ when $x = \cdot \frac{3\pi}{2}$?

– – Δ – – Δ – –

Let r be a real number with $0 \leq r \leq 1$. Then r can be uniquely written in the form $\frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots$, where each a_i is either 0 or 1. The form is unique if we define the terminating expansions of this form so that all the a_i from a certain point are all 0 rather than all 1. Suppose we wish to expand $\frac{7}{8}$ in this form. We would define

$$\frac{7}{8} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{0}{2^4} + \frac{0}{2^5} + \cdots$$

rather than

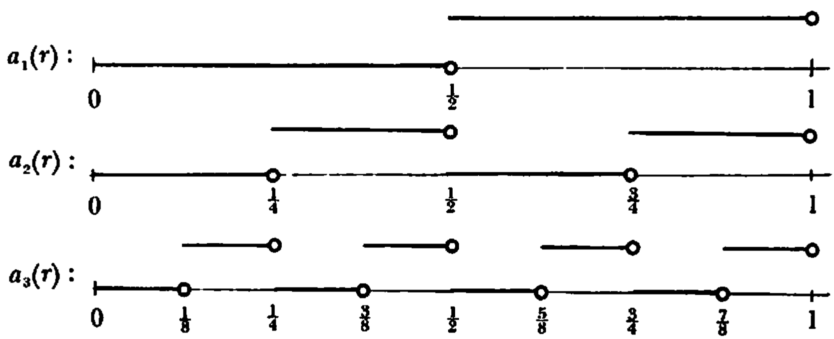
$$\frac{7}{8} = \frac{1}{2} + \frac{1}{2^2} + \frac{0}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \cdots$$

As you have probably already noticed, the expansions above are merely the usual binary expansions of the number r.

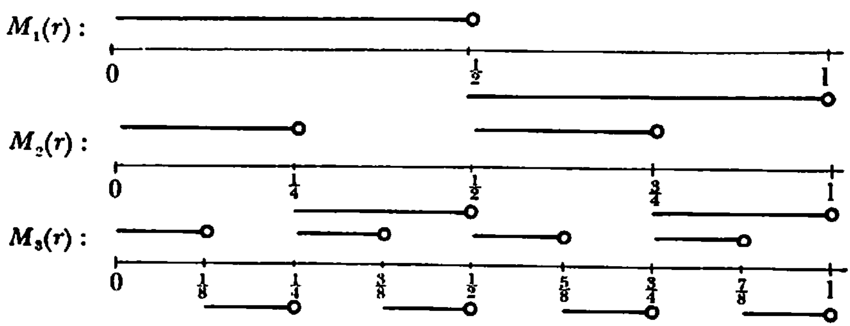
As the previous example indicates, the values of a_i are functions of the number r and hence we can write

$$r = \frac{a_1(r)}{2} + \frac{a_2(r)}{2^2} + \frac{a_3(r)}{2^3} + \cdots.$$

Their graphs are:



Now a new function $M_k(r)$ can be defined as $M_k(r) = 1-2a_k(r)$, for each $k = 1, 2, 3, \ldots$. The graphs of the $M_k(r)$ functions are:



The function $M_k(r)$ is the usual Rademacher function. We note that

$$1-2r = 1-\left(2\frac{a_1(r)}{2} + \frac{a_2(r)}{2^2} + \frac{a_3(r)}{2^3} + \ldots\right)$$

$$= 1 - a_1(r) - \frac{a_2(r)}{2} - \frac{a_3(r)}{2^2} - \ldots$$

$$= \sum_{k=1}^{+\infty} \frac{M_k(r)}{2^k}\,.$$

An interesting result follows from the previous work if one notes that

$$\int_0^1 \exp\left(ix\frac{M_k(r)}{2^k}\right)dr = \cos\frac{x}{2^k}$$

and

$$\int_0^1 \exp(ix(1-2r))\,dr = \frac{\sin x}{x}.$$

Using the above results we obtain

$$\frac{\sin x}{x} = \int_0^1 \exp(ix(1-2r))\,dr$$

$$= \int_0^1 \exp\left(ix\sum_{k=1}^{+\infty}\frac{M_k(r)}{2^k}\right) dr$$

$$= \prod_{k=1}^{+\infty}\cos\frac{x}{2^k}$$

$$= \prod_{k=1}^{+\infty}\int_0^1 \exp\left(ix\frac{M_k(r)}{2^k}\right) dr.$$

Hence

$$\int_0^1 \prod_{k=1}^{+\infty}\exp\ ix\left(\frac{M_k(r)}{2^k}\right)dr = \prod_{k=1}^{+\infty}\int_0^1 \exp\left(ix\frac{M_k(r)}{2^k}\right)dr.$$

We therefore conclude that in this case the integral of a product equals the product of the integrals. Can you recall another case where this type of result holds in analysis?

The Problem Corner

EDITED BY ROBERT L. POE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before September 15, 1973. The best solutions submitted by students will be published in the Fall 1973 issue of The Pentagon, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Professor Robert L. Poe, Department of Mathematics, Berry College, Mount Berry, Georgia 30149.

PROPOSED PROBLEMS

255. *Proposed by Stephen C. Hennagin, University of California, Davis, California.*

Show that $\lim_{x \to 0} \dfrac{x}{1-e^{x^2}} \displaystyle\int_0^x e^{t^2}\,dt = -1.$

256. *Proposed by Charles W. Trigg, San Diego, California.*

Before the football game the student representatives met to select the best cheer for the rooters to use. Falling back on tradition, they decided to let

$$IT/BE = .RAHRAHRAH\ldots$$

In the lucky base of numeration, eleven, each letter of this cryptarithm uniquely represents a digit. When reordered, the digits involved are consecutive. Find the sole solution.

257. *Proposed by Charles W. Trigg, San Diego, California.*

In the decimal system, solve the cryptarithm

$$(EIN)^2+(COT)^2 = (CSC)^2$$

wherein C, S, T in some order are consecutive digits.

258. *Proposed by Charles W. Trigg, San Diego, California.*

What is the smallest positive integer that can be expressed in exactly four different ways as the sum of consecutive positive integers?

259. *Proposed by Charles W. Trigg, San Diego, California.*

In the triangular array

$$\begin{array}{ccccc} & & 3 & & \\ & 1 & & 4 & \\ 2 & & 5 & & 6 \end{array}$$

the row sums are 3, 5, 13 (all prime); or 6, 9, 6 (all composite): or 2, 6, 13, depending upon which side is taken as the base of reference. Can the digits be rearranged in a triangular array so that all nine "sums" are (a) prime or (b) composite?

SOLUTIONS

250. *Proposed by the Editor.*

During the depression of the early '70's, students at BIG University (Bountiful Institutional Grants University) had an exceptionally hard time finding employment to help defray their expenses. Dr. Grant Ghetlar, the president of BIGU, had a terrible time securing research grants from government agencies and private corporations these lean years. As a result students no longer occupied good paying, soft jobs in the campus offices supported by the overhead money portions of research grants. Actually the female students had the worst of it, since male students could obtain useful employment in the commercial and industrial areas of near by Crisis City if they got haircuts, shaved off their beards, and wore shoes and socks. Miss Brunhilde Hibernackle, an enterprising sophomore, solved the problem for many of the BIG girls. She formed an organization called BIG Protesters, Inc. The purpose of this group was to demonstrate for the Women's Lib, ethnical, racial, parents opposing school busing, hippies, ecological, hard hats, and any other organizations that wanted to stage protests, have sit-ins, or promote riots in Crisis City. The BIG Protesters, Inc. was a cinch to make money since for a fee the active members of an organization could stay safely away from their demonstration thus avoiding exertion and arrest. By skillful application of make-up and false whiskers, donning of appropriate clothing, and creation of suitable placards these BIG girls focused attention on many social ills and caused worthy social strife in Crisis. Besides, they enjoyed the work and looked good on television news casts.

Not even the Pentagon spies were able to determine the number of girls working for BIG Protesters, Inc. Miss Hibernackle, as the coordinator of BIG Protesters, Inc., always had a problem determining how many girls actually worked in a demonstration and how much each should be paid since at every function the number of BIG girls participating varied. For example at the protest march demanding equal opportunity employment for men with waist measurements larger than 44 inches as telephone linemen sponsored by the CCCC (Crisis Civil Crisis Club) she forgot to count the girls on the job. All she could remember when she returned to campus was that there had been exactly enough BIG girls present so that when lined to march in ranks of three abreast, they had two girls left over; in fives, four BIG girls extra; in sevens, six too many; and in elevens ten in excess. If BIG Protesters, Inc. was paid $3000 for the protest march and Brunhilde paid herself $10.00 how much should each professional BIG girl protester be paid?

A solution was presented by Theresa Radziemski, Assumption College, Worcester, Massachusetts. However, the following solution is published due to its brevity and simplicity.

N, the number of BIGU girls protesting, lacks of one of being divisible by 3, 5, 7, and 11. Therefore, $N+1=3\text{x}5\text{x}7\text{x}11=1155$ or $N=1154$. Now $2990.00 ÷ 1154=$2.59.

251. *Proposed by Stephen C. Hennagin, University of California, Davis, California.*

Find $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{1}{n+k}$.

Solution by the proposer.

Note that $\sum\limits_{k=1}^{n} \dfrac{1}{n+k} = \sum\limits_{k=1}^{n} \left(\dfrac{1}{1+k/n}\right)\left(\dfrac{1}{n}\right)$ and that if we let $f(x) = \dfrac{1}{1+x}$, then $f(x_i)\Delta x_i = f(i/n)\ (1/n)$ over $[0,1]$. Hence

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n+k} = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\ \Delta x_k = \lim_{n\to\infty}\sum_{k=1}^{n} f(k/n)\ (1/n) =$$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{1+k/n}\right)\left(\frac{1}{n}\right) = \int_0^1 \frac{dx}{1+x} = \log 2, \text{ as a Riemann sum.}$$

252. *Proposed by Frank R. Dangello, Shippensburg State College, Shippensberg, Pennsylvania.*

Prove that $\displaystyle\int_0^x \left[\int_0^{x_1} f(t)dt\right] dx_1 = \int_0^x (x - x_1)\ f(x_1)dx_1$ for

$x > 0$, $0 \leq x_1 \leq x$, and f continuous on $[0, x]$.

Solution by Stephen C. Hennagin, University of California, Davis, California.

Since f is continuous on $[0,x]$, $\displaystyle\int_0^{x_1} f(t)dt$ is differentiable on $(0,x)$.

Thus, in order to prove the assertion, the technique of integration by parts is applicable. Let

$$u(x_1) = \int_0^{x_1} f(t)dt \text{ and } v(x_1) = x_1, \text{ then}$$

$$\int_0^x \left[\int_0^{x_1} f(t)dt\right] dx_1 = \int_0^x u(x_1)d(v(x_1))$$

$$= u(x_1)v(x_1)\Big|_0^x - \int_0^x v(x_1)d(u(x_1))$$

$$= x\int_0^x f(x_1)dx_1 - \int_0^x x_1 f(x_1)dx_1$$

$$= \int_0^x (x-x_1)f(x_1)dx_1.$$

253. *Proposed by Stephen C. Hennagin, University of California, Davis, California.*

Show that $\lim_{x \to \infty} \dfrac{x}{1 - e^{x^2}} \displaystyle\int_0^x e^{t^2}\, dt = -1.$

Solution by the proposer.

Originally this problem was stated incorrectly, that is, rather than $x \to \infty$ it should have been $x \to 0$. However, the problem as stated is very simply solved by L'Hopital's rule. In other words, a solution to

$$\lim_{x \to \infty} \frac{x}{1 - e^{x^2}} \int_0^x e^{t^2}\, dt$$

can be obtained.

254. *Proposed by the editor.*

Four neighbors, Jones, Smith, Williams, and Harris regularly play table tennis. Smith was once with the State Department and is a champion table tennis player. The probability that Smith will win from Jones is 11/12; from Williams is 9/10; and from Harris 15/16. In a long series of games, Smith first plays Williams, then plays a game with Jones, and next plays Harris. If the neighbors continue playing in this sequence until Smith loses a game, what is the probability that either Harris or Jones is the first to defeat Smith?

Solution.

P(Harris defeats Smith first) $= (9/10 \times 11/12 \times 1/16) + (9/10 \times 11/12 \times 15/16)(9/10 \times 11/12 \times 1/16) + (9/10 \times 11/12 \times 15/16)^2 (9/10 \times 11/12 \times 1/16) + \ldots = 33/640 + (33/640)(99/128) + (33/640)(99/128)^2 + (33/640)(99/128)^3 + \ldots$ and P(Jones defeats Smith first) $= (9/10 \times 1/12) + (9/10 \times 11/12 \times 15/16)(9/10 \times 1/12) + (9/10 \times 1/12)(9/10 \times 11/12 \times 15/16)^2 + \ldots = 3/40 + (3/40)(99/128) + (3/40)(99/128)^2 + (3/40)(99/128)^3 + \ldots$. Both of these are geometric progressions with $|r| = 99/128 < 1$ where $a_1 = 33/640$

in the first and $a_1 = 3/40$ in the second. Therefore, P(Either Harris or Jones defeats Smith first) = P(Harris defeats Smith first) + P(Jones defeats Smith first) =

$$\frac{33/640}{1-99/128} + \frac{3/40}{1-99/128} = \frac{33}{145} + \frac{48}{145} = \frac{81}{145}.$$

The Book Shelf

Edited by Elizabeth T. Wooldridge

This department of The Pentagon brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. Elizabeth T. Wooldridge, Department of Mathematics, Florence State University, Florence, Alabama 35630.

Elementary Functions: Backdrop for the Calculus, Melcher P. Fobes, The Macmillan Company, New York, 1973, 520 pp.

The author of this textbook states in his preface that "Part of what I wish my students knew before they begin calculus, together with the way in which I wish they knew it, make up this book." If it were only possible for his wish to be granted, the teaching of calculus would be a pleasure! The book cannot be covered in a single course of normal length. The author, in fact, suggests several possible designs for courses from his text. He considers, as a basic to each of these plans, the first five chapters which cover such topics as introductory comments, sets and systems of real numbers, relations and functions, the analytic geometry of lines, and linear and quadratic functions. The remainder of each course concentrates on particular types of functions chosen from the following: polynomials, other algebraic functions, exponential, inverse, logarithmic, trigonometric, and inverse trigonometric functions. A brief chapter on polar coordinates is also included.

A student whose true need is for "remedial algebra" will find little assistance here. Instead, this book is aimed at that student whose background includes, according to the author's prerequisites, "a successful completion of some three semesters of algebra, including a touch of exponents and radicals, and a few of the primary theorems of plane geometry." A fair amount of maturity and sincere interest in mathematics should also be included in this list. The author is not content to present facts for memorization; he makes an excellent attempt to instill in the reader the desire (as referred to in the title of an introductory section) "To Think Like a Mathematician." For those who accept this challenge, several chapters contain bonus exercise sets of "Things to Think About" which augment, re-examine, explain, or extend the topics of the chapters.

The author's style of presentation is excellent. His explanations are clear and understandable. New ideas are well-motivated. There is an ample supply of routine drill problems with answers provided in the Appendix for the odd-numbered problems. The Appendix also contains tables of values for the exponential and logarithmic functions as well as trigonometric functions of numbers and of angles measured in degrees.

This reviewer feels that this book is, perhaps, too ambitious as a pre-calculus text for "average" students. However, it appears to be an excellent book for those students with a true interest in mathematics who wish to add to their background before taking calculus.

O. Oscar Beck
Florence State University

Mathematics I and II, Max A. Sobel and Evan M. Maletsky, Ginn and Co., Boston, 1971, 459 pp. and 461 pp., $5.60 each.

Book I is designed for seventh grade students and Book II for the eighth grade. Both were written to present contemporary ideas and approaches in the junior high school mathematics program. Much is presented in Computer Mathematics.

Book I includes the following topics: Sets and Numbers, Number Sentences, Rational Numbers as Fractions and Decimals, Ratios and Per Cent, Measurement, Area and Volume, Coordinates and Real Numbers, Probability and Statistics.

Book II presents: Integers and Rational Numbers, Decimals and Real Numbers, Constructions and Congruency, Equations and Inequalities, Formulas and Functions, Geometry, Trigonometric Ratios.

Each chapter has a chapter test and "Test Your Skill." Sections entitled "Excursions in Mathematics" and "Library and Laboratory Excursions" are given near the end of each chapter. These should give much challenge to the student.

Vaulda Welke
Superior, Nebraska

An Introduction to Mathematical Reasoning, Boris Inglewicz and Judith Stoyle, The Macmillan Company, New York, 1973, 239 pp., $4.95.

The authors state in the preface "Many otherwise intelligent students will make mathematical statements that are divorced from even a remote logical basis." They contend that a course separate from the usual content sequences needs to be offered that is "devoted solely to the methods and logical basis of the mathematical proof." This text is a basis for such a course.

This brief paperback book contains thirteen chapters. Two are devoted to an introduction and elementary symbolic logic. Two chapters review essential background mathematics including algebraic manipulation, functions, graphing, logarithms, trigonometry, and sigma notation. The rest of this book is devoted to proof.

After a brief introduction to the concept of proof the authors put together an excellent development on types of proof including induction, direct proof, indirect proof, existence proof, counterexample, and geometric proof. The wealth of examples and problems are all within the grasp of good high school students or beginning college mathematics students. Most of the examples involve number theory, standard formulae, or sigma notation. However, many different problems are given. Examples of these are the following:

> For any positive number a, show that $(a + 1/a)/2 \geq 1$ (p. 105)
>
> Show that a number written in decimal form which does not have a repeated pattern of decimals must be irrational. (p. 140)
>
> There exists a rational number between any two distinct irrational numbers. (p. 150)
>
> Contradict by counter example "If p_i $i = 1, 2, 3, \ldots, n$, are n prime numbers, the $p_1, p_2, p_3, \ldots, p_n + 1$ is a prime number. (p. 161)
>
> Let $f(x)$ have the property $f(ax) = a^2f(x)$. (a) Which function would you use for an analogy with $f(x)$? (b) Obtain the general form of $f(x)$. (p. 175)

The book contains a final chapter of which the authors say "If (the reader) finds this chapter reasonably simple and has little difficulty with the problems at the end of the chapter, then he has learned the basic techniques for doing proofs and he is ready for more advanced study." This reviewer agrees with this statement. The whole book does an excellent job of leading the student to the concept of proof and problem analysis. The student who takes the time to read with understanding this book will find his later savings in mathematics study more than worth the "lost" time.

James K. Bidwell
Central Michigan University

Mathematics With Understanding, Book 2, Harold Fletcher and Arnold A. Howell, Pergamon Press, New York, 1972, 258 pp., $9.75.

This book is designed for prospective elementary teachers. Although the author regards it as "a guide to the new ideas in primary mathematics for students in colleges of education," this reviewer believes that it contains enough mathematical content to be suitable for a course in mathematics for prospective teachers. If the students are required to take two content courses in mathematics, then this book could well serve as the text for the second course.

The main contents of the chapters are (in order), number systems, fractions and rational numbers, integers, algebraic relations, shape, measurement, modular arithmetic and groups, and probability. There are also two appendixes dealing with assignment cards and check cards. Mathematics content is carefully integrated with what children can *do* in order to learn the same content.

Each concept is introduced in a variety of ways. Subtraction of fractions is shown in six ways: physical cut-outs, number line, renaming mixed numbers, equivalent fractions, inverse operations. The chapter on algebraic relations introduces cartesian product, relation line graphs, and curve graphs. All of these topics have many illustrations useful in the elementary school which also clearly reinforce the basic content under discussion.

The chapters on shape and measurement are essentially informal geometry and cover topics such as tessellations, polyominoes, tangrams, and the Pythagorean theorem. The chapter on groups utilizes clock arithmetic to first generate basic group axioms, then other models are introduced: 90° rotation of a nail and flipping a rectangle. The probability chapter is also informal and introduces simple sample spaces, probability trees, and classroom experiments.

Throughout the book current ideas on how children learn mathematics are generously given. Since the authors are British, there is a distinctive and pleasant flavor to the book, both in content and in suggested methods. This reviewer believes the book to be most useful and a viable alternative to the more formal texts for elementary teachers which are frequently "good" mathematics but intelligible to only a minority of the elementary education students. Any deficiency in mathematics that an instructor believes exists can be remedied by a few lectures.

James K. Bidwell
Central Michigan University

MINIREVIEWS

Mathematical Statistical Mechanics, Colin J. Thompson, Macmillan Company, New York, 287 pp., $12.75.

This book is designed as an introductory text for mathematics students interested in applying their knowledge to some relatively new and exciting branches of physical mathematics, or for physics students with a mathematical bent. Topics included are Kinetic Theory, Thermodynamics, the Gibbs Ensembles, Phase Transitions, both to algebraic approach and some application to biology of the Ising model. Some advanced material is included as appendices. The book should be interesting and challenging to mathematicians and to physicists.

Calculus for Business, Biology, and the Social Sciences, David G. Crowdis, Susanne M. Shelley, Brandon W. Wheeler, Glencoe Press, Beverly Hills, California, 1972, 559 pp., $10.60.

This book is intended to be a mathematics text, and deals primarily with the basic concepts of a first course in calculus. The approach is intuitive, rather than formal. The first six chapters parallel the introductory calculus courses described in the CUPM reports. The remaining chapters introduce concepts of multiple-variable calculus and infinite series. Applications are taken from social, management, and biological sciences, rather than from the usual physics and engineering.

Fundamentals of Mathematics, Fourth Edition, Leonard T. Richardson, The Macmillan Company, New York, 1973, 608 pp., $10.95.

"This book was conceived to provide a survey course in mathematics – either a terminal course for students of the liberal arts and social sciences, or an introductory course for those students . . . who decide to go on with the study of mathematics." This fourth edition is a modernization of the previous editions in that functions are introduced earlier and used to provide a clearer understanding of basic algebra. The topics presented are interesting and varied and are so arranged that the instructor can vary the course to suit the needs of his class. It enables the bright student to study what the average student can easily omit. It is readable, interesting, and can help a student to "understand the open-ended nature of mathematics."

Basic Mathematics with Electronics Applications, Julius L. Smith, Jr. and David S. Burton, The Macmillan Company, New York, 1972, 698 pp., $13.95.

"The purpose of this text is to present to a student of electronics fundamentals a comprehensive study of basic mathematics with electronic applications." It is intended for students in community colleges, technical institutes, trade or vocational schools, and industrial training programs. The material is presented in a manner that is within the grasp of a student with the usual mathematical background. The subject matter begins with the basic concepts of arithmetic and proceeds in a step-by-step manner through the fundamental elements of binary arithmetic and Boolean algebra. First the mathematical principles are explained in detail, then illustrated by numerous examples, followed by exercise problems for the student.

Kappa Mu Epsilon News

EDITED BY ELSIE MULLER, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Elsie Muller, Historian, Kappa Mu Epsilon, Department of Mathematics, Morningside College, Sioux City, Iowa 51106.

REGIONAL NEWS

Region I Meeting, 3-4 November 1972
Director: James E. Lightner of Maryland Beta

Over sixty KME members, representing six chapters, attended one or more of the sessions, which were very well planned by the Pennsylvania Iota chapter at Shippensburg. On Friday evening Mr. William Smith, National KME Vice-President, presented a most interesting talk, "The Dance Problem." Saturday morning was devoted to student presentations of papers, two invited addresses by graduates followed by six undergraduate ones. The judges ultimately awarded a dual first prize to Theresa Meterchick, Pennsylvania Iota, for her paper, *Sentential Logic via Algebra,* and to Ronald Jemmerson, Maryland Beta, for his paper, *Applications of Mathematics to Chemical Enzyme Reactions.* After visiting the Franklin Science Center for a planetarium demonstration, there was a delicious lunch followed by a delightful and humorous dialogue on "How to Catch a Lion using Mathematics" given by two Shippensburg professors.

CHAPTER NEWS

Alabama Beta, Florence State University, Florence

Chapter President — Robert O'Connor
34 actives

At the October meeting Jane Bickel, a senior student presented a paper, *Equivalence Relations and Complex Numbers.* At the 1972 Homecoming of FSU, the chapter sponsored a coffee hour for all alumni members. Larry R. Smith, a 1970 initiate, received a graduate assistantship to the University of Alabama. Euel Cutshall, 1969 initiate, received a graduate assistantship to the University of Florida.

Alabama Gamma, University of Montevallo, Montevallo

Chapter President — Anne Tishler
15 actives — 3 pledges

The chapter meets monthly and holds an annual picnic in October. Other officers: Randy Dunlap, vice-president; Ida Smith, secretary and treasurer; Becky Golden, social chairman; Dr. Angela Hernandez, corresponding secretary; Dr. D. R. McMillan, faculty sponsor.

Arkansas Alpha, Arkansas State University, State University

Chapter President — Susan Duff
22 actives

Topics at the monthly meeting have included the following: "Fixed Endpoint Problem in the Plane," "Differential Equations," "Game Theory," and "Importance of the Space Center." Other officers: Gary Roberts, vice-president; Judy Kay Fetters, secretary; Sandra Distretti, treasurer; Dr. Jerry Linnstaedter, corresponding secretary; Dr. Robert P. Smith, faculty sponsor.

California Gamma, California Polytechnic State University, San Luis Obispo

Chapter President — Jim Pearce
22 student members, 19 faculty, 6 pledges

The monthly meetings feature either student or faculty speakers. Usually there are two or three initiations per year with at least one banquet per year.

Colorado Alpha, Colorado State University, Fort Collins

Chapter President — Kate Legge
22 actives — 3 pledges

The chapter had a booth at the annual CSU activities night which depicted the work of the organization throughout the year. Also, they sponsored a book drive to collect textbooks for Wilkes-Barre College to help rebuild their library which was destroyed last summer in the hurricane. A representative of Career Services on campus gave a talk on job placement and interviewing techniques. A faculty member of the statistics department conducted a tour of the statistics laboratory and demonstrated the equipment.

Florida Alpha, Stetson University, Deland

Chapter President — Gayla Bierley
18 actives — 15 pledges

Professor Jack McCabe of the department of mathematics took a group of seniors to the University of Leicester, Leicester, England, to study philosophy and logic of mathematics for four weeks. Other officers: Dan Stephenson, vice-president; Elaine Bradley, secretary and treasurer; Mr. Emmett Ashcraft, corresponding secretary; Dr. Gene Medlin, faculty sponsor.

Illinois Alpha, Illinois State University, Normal

Chapter President — Frank Hirsch
18 actives — 28 pledges

In November Mr. Fasking, an actuary from State Farm, was the speaker. In January Dr. Calgaro presented the program at the planetarium. During February and March there were speakers from the economics and music departments who showed the relationship between their respective fields and mathematics. In April a panel of student teachers who are KME members summarized their teaching experiences with the supervisor present. A field trip to the University of Illinois is being planned to observe their computer science department.

Illinois Beta, Eastern Illinois University, Charleston

Chapter President — Ted Sanders
49 actives

Each year, during October and April, the chapter has a series consisting of the following: a business meeting for the approval of a list of prospective pledges, a pledge meeting, a pledge-active mixer, and a formal initiation of the new members. After the spring initiation there is a banquet to honor all the initiates of the academic year, to announce the official Honors, and to present the departmental scholarship awards. The latter includes a freshman award to the student who makes the highest grade on a pre-calculus test and a Van Deventer Memorial Award to the one who makes the highest grade on a calculus test. Parents of the recipients of these awards are guests at the banquet. Other officers: James Davito, vice-president; Marilyn O'Brien, secretary and treasurer; Ruth Queary, corresponding secretary and faculty sponsor.

Illinos Delta, College of St. Francis, Joliet

Chapter President — Cathy Weisenburger
9 actives — 5 pledges

To raise money the chapter has conducted a raffle. Members have also attended meetings of N.C.T.M. in Chicago and the I.C.T.M. at Dekalb. It is hoped that thirty students from the high schools in the Joliet area will participate in a mathematics contest. Other officers: Louise Becker, secretary; Arnold Good, faculty sponsor.

Illinois Eta, Western Illinois University, Macomb

Chapter President — Dale Myers
15 actives

The Chapter sponsored its first problem-solving contest in the fall and is considering holding such contests each quarter with books and cash rewards as prizes. Mike Schaubroeck, the winner of the first problem solving contest, presented the program at the monthly meeting in January. Other officers: Russell Merdian, vice-president; Joyce Effa, secretary and treasurer; Professor Larry Morley, corresponding secretary; Professors Jim Calhoun and Scott Harrod, faculty sponsors.

Indiana Alpha, Manchester College, North Manchester

Chapter President — Helen Taylor
13 actives

Monthly meetings are held jointly with the campus mathematics club. Programs are given by students and other speakers which include professors, recent graduates, and guests.

Iowa Alpha, University of Northern Iowa, Cedar Falls

Chapter President — Brian B. Hogue
40 actives — 15 faculty

Past KME National President, Fred W. Lott, has accepted a position as Assistant Vice-President for Academic Affairs at U.N.I. Dr. Lott continues to be active in his support of KME with the Iowa Alpha chapter. The homecoming breakfast in October was well attended.

Iowa Gamma, Morningside College, Sioux City

Chapter President — Stephen Bolks
31 actives

At the initiation meeting during the first semester Dr. David Wooten of Briar Cliff College gave a talk, "Applications of Mathematics in Careers." Two members, Stephen Bolks and Paul Franken, participated in the William Lowell Putnam Competition. In February Rodney Powles, Deborah Hanson, and Cheryl Hogeboom presented papers at the Colleges of Mid-America Colloquium on applications. Professor Walter Mientka of the University of Nebraska was the guest lecturer. The chapter was very busy preparing for the national convention on 5-7 April.

Kansas Alpha, Kansas State College of Pittsburg, Pittsburg

Chapter President — Donna Geisler
50 actives

One of the programs featured the vice-president, Walter Parrish, who presented a talk, "A Finite Look at an Infinite Fibonacci Sequence." At another meeting August Aplitter discussed the topic, "Construction of a Hyperbolic Paraboloid." Seven new members were initiated at the October meeting. Other officers: Walter Parrish, vice-president; Linda Funk, secretary; Randy Timi, treasurer; Dr. Harold L. Thomas, corresponding secretary; Professor J. Bryan Sperry, faculty sponsor.

Kansas Beta, Kansas State Teachers College, Emporia

Chapter President – Ron Stair
62 actives

Program topics have included the programmable desk calculator and a presentation on the Fibonacci Numbers. A special treat this year was an old-fashioned Christmas party.

Kansas Gamma, Benedictine College, Atchison

Chapter President – Bob Croll
11 actives – 3 pledges

In addition to student and faculty presentations the meetings included two films and one guest speaker. The films shown were *Projective Generation of Conics* and *Isometries,* both from the College Geometry series. Dr. J. C. Kelley, a former graduate of one of the parent institutions of Benedictine College, St. Benedicts, and currently a member of the faculty at Missouri University, spoke to the group on problems in the calculus of variations dealing with minimal surface area. The chapter awarded the Sister Helen Sullivan Scholarship to Julia Croghan for the 1972-73 academic year.

Kansas Epsilon, Fort Hays Kansas State College, Hays

Chapter President – Roger Kaufman
18 actives

At the September meeting Dr. Beougher gave a lecture on the topic, "Infinite Sets." The film, *Mathematical Induction,* constituted the program in October. During the November meeting the time was spent in building mathematical models for the mathematics department. Other officers: Rodney Burgett, vice-president; Rita Pekarek, secretary and treasurer; Eugene Etter, corresponding secretary; Dr. Elton Beougher, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President – Marilyn Radziminski
71 actives – 4 pledges

In November Kathleen Drenga presented a paper, *The Mathe-*

matics of Braids. Other officers: Mary C. Resop, vice-president; Kathy Kwiatkowski, secretary; Mary C. Resop, treasurer; Sister Marie Augustine, corresponding secretary; Jeanette Gilmore, faculty sponsor.

Maryland Beta, Western Maryland College, Westminister

Chapter President – Ronald R. W. Jemmerson
16 actives

Several interesting programs have been presented. In October Dr. Harry Rosenzweig gave a talk on topology. In November Mrs. Janice Sharper Almquist, an alumnus of WMC, submitted information on mathematical careers in health statistics and government positions. During the same month another guest speaker was Dr. David Gosslee of Oak Ridge National Laboratory who spoke on the uses of statistics and applications. The president, Ronald Jemmerson, won first prize on his paper, *Applications of Mathematics to Chemical Enzyme Reactions* at the Region I meeting.

Michigan Beta, Central Michigan University, Mt. Pleasant

Chapter President – Bob Elenbaas
30 actives – 4 pledges

The most recent major task undertaken by the chapter was the revision of the local by-laws. Under the new by-laws, members will be pledged to participate in some meaningful way in their local chapter through one of the following ways: 1. research and presentations, 2. bringing guests and speakers of note before the group so all may share in mathematical applications, 3. tutoring students in the academic and/or surrounding community. The chapter has also started a regional newsletter with other chapters of Kappa Mu Epsilon in an effort to keep in touch and know what others are doing. Other officers: Neal Eichler, vice-president; Debbie Kontas, secretary; Marie Sadowski, treasurer; Mr. Dean O. Hinshaw, corresponding secretary and faculty sponsor.

Mississippi Alpha, Mississippi State College for Women, Columbus

Chapter President – Janet McPhail
21 actives – 15 pledges

Mr. Gil Harris, Chief Engineer of Mitchell Engineering Company, and Dr. Billy F. Bryant, Chairman of the Mathematics Department of Vanderbilt University, presented very interesting programs. Other officers: Patricia Weathersby, vice-president; Patricia Warren, secretary and treasurer; Dr. Donald A. King, corresponding secretary and faculty sponsor.

Missouri Alpha, Southwest Missouri State University, Springfield

Chapter President — Alan Washburn
45 actives

One of the programs was a talk, "Etymology of Hyperbola, Parabola, and Ellipse" by Eddie W. Robinson. Each member has purchased a sweatshirt with the KME insignia.

Nelda Burton and Peggy Stuckmeyer received the KME merit award.

Missouri Beta, Central Missouri State College, Warrensburg

Chapter President — Shana McCann
49 actives — 35 pledges

The chapter holds eight meetings per year with a Christmas party and one field trip.

Missouri Gamma, William Jewell College, Liberty

Chapter President — Debbie Miles
16 actives

Other officers: Tom Lehman, vice-president; Vicki Hullender, secretary; Clella Ross Goodwin, treasurer; Sherman Sherrick, corresponding secretary; Joseph T. Mathis, faculty sponsor.

Missouri Zeta, University of Missouri at Rolla, Rolla

Chapter President — Dana Nau
20 actives — 22 pledges

Dr. Seldon Trimble spoke at the smoker on "How to Put a Bigger Cube inside a Cube." At the initiation banquet Dr. John Hanblen, chairman of the computer science department, spoke on the evolution of computer languages. The chapter voted to give an award annually at the regional science fair held each spring in Rolla. Help sessions for all mathematics courses below and including differential equations were held twice each week. The chapter sponsored a fall outing which was enjoyed by KME members, graduate students, and professors. The outing ended with a rousing football game.

Missouri Eta, Northeast Missouri State University, Kirksville

Chapter President – Ann C. Roemerman
33 actives

The programs have included presentations on Lolography by Dr. R. R. Nothdurft from the Physics department, on three valued logic system, on magic Greek square, and on p-adic numbers by senior members. The chapter has started a mathematics club for freshmen and other undergraduates who are interested in mathematics but have not yet qualified for KME. Other activities include a mathematical treasure hunt, Christmas caroling, volleyball games with the mathematics club, fall and spring picnics, and a spring initiation. Twenty-one members attended the regional meeting at William Jewell College. Other officers: Rick Barker, vice-president; Ellen Martin, secretary; Jennifer Sawyer, treasurer; Mr. Samuel Lesseig, corresponding secretary; Ms. Mary Sue Beersman, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne, Nebraska

Chapter President – Dick Mays
17 actives – 10 pledges

The monthly business meetings are followed by entertainment in the form of mathematical puzzles, problems, or novelties. Presentations are usually made by two members who challenge the remaining students for solutions. In March a committee developed and administered a mathematics examination which was used to identify the outstanding freshman mathematics major. The mathematics faculty selects three students to take the examination. Jim Meyer of Wayne, Nebraska was selected as the outstanding freshman. The award includes honorary membership in KME with initiation fees paid by the club, the placement of his name on the permanent plaque, and public announcement of the award at the spring Honors Convocation. Other officers: Terry Hamer, vice-president; Yuvonne Brinks, secretary and treasurer; Fred Webber, corresponding secretary; Jim Paige, faculty sponsor; Donna Von Glan, historian and reporter.

New Jersey Beta, Montclair State College, Upper Montclair

Chapter President – Arlene Albano
55 actives

The meetings are held the first Monday of each month. In addition to the business meeting there is a guest speaker, usually a member of the faculty. Plans are being made to sponsor a tutoring service to majors in the mathematics department and to have a book sale. In April the chapter sponsored its first annual high school mathematics competition for the local public and parochial schools in cooperation with the mathematics department. Other officers: Joyce Vitale, vice-president; Deborah De'Alesio, secretary; Joyce Scalzitti, treasurer; Dr. Carl Bredlav, corresponding secretary; Mr. Gus Muttel, faculty sponsor; Maria Dell'Osso, historian.

New Mexico Alpha, University of New Mexico, Albuquerque

Chapter President – Pat Russell
50 actives

The chapter has been doing the usual routine things.

New York Alpha, Hofstra University, Hempstead

Chapter President – Eileen Bennett
31 actives

At the banquet Professor Inman was the speaker on "Humans, Machines, Intelligence." Other officers: Diane Friedman, vice-president; Dennis Weygard, secretary; Karen McCarthy, treasurer; Alexander Weiner, faculty sponsor.

New York Eta, Niagara University, Niagara University

Chapter President — Walter Mazurowski
17 actives

Programs are held in conjunction with the campus mathematics club in order to attract the largest possible audience. In the fall semester a series of programs describing job opportunities for the mathematician in the fields of banking and actuarial work was presented by representatives from local businesses. An annual faculty-student Christmas party is held in order that students may meet with their instructors on an informal basis. Other officers: Becky Smith, vice-president; Judy Helmbrecht, secretary; Carol Kosinski, treasurer; Robert Bailey, corresponding secretary and faculty sponsor.

New York Iota, Wagner College, Staten Island

Chapter President — Brian Manske
19 actives

During the fall semester the chapter held a faculty-student mathematics bowl, the third such contest during the past year. There were three teams of students, each with a faculty advisor. The contest was a huge success and resulted in plans for more in the future.

On Sunday evening, 15 October, the chapter held induction ceremonies, open to all mathematics majors, for five new members. A brief talk by Professor Raymond Traub followed the ceremony. Also, the chapter has recently published a bulletin containing general information on the mathematics department.

During the spring semester, the chapter plans to start a seminar system to be conducted on a bimonthly basis. Members of KME will give the talks on topics of their own choice but approved beforehand by the faculty sponsor.

Ohio Gamma, Baldwin — Wallace College, Berea

Chapter President — Sandra Sikorski
15 actives

One of the programs featured a meeting with Ken Hoyt, Director of Placement Services, to discuss various opportunities and occupations for mathematicians. The chapter sponsored help sessions for mathematics students during the fall of 1972. Other officers: Christine Raineri, vice-president; Robert Graham, secretary; Jose Pagan, treasurer; Robert E. Schlea, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord

Chapter President — Grace Vellenga
18 actives

At the September meeting Emily Henderson spoke on Egyptian mathematics. At the initiation banquet the new pledges presented short talks on mathematicians of former years. In November James Brink presented work he had done in implementing the computer with the college alumni files. At the Christmas party at the home of Dr. James Smith, the highlight of the evening was the viewing of the slides of the International Congress at the mathematician's meeting in Oslo, Norway. Other officers: Richard Martinelli, vice-president; Susan Syroski, secretary and treasurer; Dr. James Smith, corresponding secretary and faculty sponsor.

Pennsylvania Alpha, Westminister College, New Wilmington

Chapter President — Gabert D. Molnar, Jr.
36 actives

Other officers Jay D. Myers, vice-president; Barbara Mitchell, secretary; Thomas Ritchey, treasurer; Dr. I. R. Nealeigh, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

Chapter President — John Nelson
55 actives — 1 pledge

Eighteen new members were initiated in October when Dr. Jack Shepler and Dr. Ronald McBride spoke on "Mastery Learning." At the November meeting Dr. James Reber spoke on the topic, "In the

Beginning Was the Real Number Line." At the December meeting he gave the lecture, "Use of the Computer in Calculus Courses." Other officers: John Schutte, vice-president; Arleen Leslie, secretary; Arlene Hlasnick, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City

Chapter President — David Durkee
5 pledges

Monthly meetings are held in the homes of faculty members where a paper is presented by a student. To generate more interest in the organization there was a colloquium at the January meeting at which three papers by KME members were read. The meeting was open to all interested mathematics students.

Pennsylvania Iota, Shippensburg State College, Shippensburg

Chapter President — Brenda Csencsits
40 actives

The chapter hosted the first Region 1 convention of KME. Members of the chapter who won awards on their papers were Theresa Meterchick and Elaine Stevens. To make money the members sold computer-made calendars at a Christmas Shoppe.

Pennsylvania Kappa, Holy Family College, Philadelphia

Chapter President — Maria Herczeg
2 actives — 4 pledges

At the meetings the members solve problems from *The Pentagon* and past Putnam exams. There is also discussion of topics not covered in the regular sequence of undergraduate courses. At one of the meetings, Mr. Louis Hoelzle, assistant professor of mathematics at Bucks County Community College spoke on the topic, "Simple Parlor Games."

Tennessee Gamma, Union University, Jackson

Chapter President – Clyde Dennis
15 actives

Meetings are held monthly and plans were made for the annual spring initiation. Other officers: Martha Wofford, vice-president; Martha Goodrum, secretary; Wayne Wofford, treasurer; Richard Dehn, corresponding secretary; Dr. Joe Tucker, faculty sponsor; Doug Rogers, reporter and historian.

Texas Zeta, Tarleton State College, Stephenville

Chapter President – Harold Harrison
5 actives – 10 pledges

At one of the meetings Dr. Joe Cude spoke on the topic, "Job Opportunities in Mathematics Fields." Plans are being made for spring initiation and for a mathematics contest to be followed by a banquet at which the awards will be presented. Other officers: José Villanueva, vice-president; Eddie Garner, secretary; Debbie Turner, treasurer; Timothy Flinn, corresponding secretary; Conley Jenkins, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee

Chapter President – Geri-lynn O'Boyle
10 actives – 6 pledges

At the Region 2 meeting it was decided that each chapter would prepare a newsletter to send to the other chapters in the region. At the December meeting Dr. Deshpande from Marquette University spoke on the topic, "Unsolved and Unsolvable Arithmetic Problems." One of the activities for the semester was a student teacher panel. Five girls in mathematics education participated. Attendance at this meeting was the largest it had been all year. One of the upcoming projects is the formation of a human computer which is to be molded after the one presented by Mr. Doug Kenny.

Wisconsin Beta, Wisconsin State University — River Falls, River Falls

Chapter President — Steven Hesprich
35 actives — 20 pledges

In February a guest lecturer was Duane Houston from NASA. He discussed the benefits of space exploration and the skylab project. Other departments and neighboring high schools were invited. The chapter has become involved in many activities. A film committee has been formed to film interesting parts of the department to send to high schools all over the state to encourage high school students to come to River Falls. The chapter has purchased a mathematics award to be presented each year to the most deserving mathematics student. An evaluation committee is working on a series of items to be made into a test to evaluate faculty performance. To raise money for sending delegates to the national convention the chapter held a raffle in which the prize was a portable television.

PENTAGON INTEREST POLL

1) I am a ____ freshmen ____ junior ____ graduate student

____ sophomore ____ senior ____ faculty

other (name) ______________________________

2) I read the following sections of *The Pentagon* (In order of preference: 1, 2, 3 etc.)

________ articles by students ________ *Scrapbook*

________ articles by faculty ________ *Book Shelf*

________ *Installation of New Chapters* ________ *KME News*

________ *Problem Corner*

3) The articles are generally

____ much too easy ____ easy ____ about right ____ hard

____ much too hard for me to read.

4) I prefer to read in the areas of

____ analysis ____ algebra ____ geometry ____ number theory ____ applied mathematics ____ computer science

____ mathematics education

other (please name) ______________________________

5) I really read and enjoyed the article entitled ______________

by ______________ in ______________issue.

6) I think *The Pentagon* would be more valuable to me if

____

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____

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