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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

On Primitive Pythagorean Triples*

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A primitive Pythagorean triple is a set of positive integers $\{x, y, z\}$ whose greatest common divisor is 1 and which satisfy the equation $x^2 + y^2 = z^2$.

The problem of finding all primitive Pythagorean triples has been solved by others. In this article a solution is presented, which was found by using little more than inductive reasoning. While this method of solution may not be the shortest or most elegant, it is original with the author and has some interesting aspects.

First of all, in a primitive Pythagorean triple one of the numbers x and y must be even and the other odd. For if they are both even, then $x^2 + y^2$ is even and so is z. Hence the triple could not be primitive. On the other hand, if x and y are both odd, then there exist positive integers a and b such that $x = 2a - 1$ and $y = 2b - 1$. Now $z^2 = (2a - 1)^2 + (2b - 1)^2 = 4a^2 - 4a + 1 + 4b^2 - 4b + 1$ which is even but not divisible by 4. However, any perfect square which is even must be divisible by 4. Therefore, one of the numbers x or y is even and the other is odd; z must therefore always be odd.

From now on we let x be odd and y even. Let d denote the difference $z - y$ which is thus always odd. Let d' denote the difference $z - x$ which is of course even. Thus

$$x^2 = z^2 - y^2 = z^2 - (z - d)^2 = 2zd - d^2.$$

Therefore, every primitive Pythagorean triple $\{x, y, z\}$ is of the form $\{\sqrt{2zd - d^2}, z - d, z\}$.

In like manner if we express x, y, and z in terms of d', it can be shown that every primitive Pythagorean triple is of the form $\{z - d', \sqrt{2zd' - d'^2}, z\}$.

*A paper presented at the 1971 National Convention of KME.

However, not all triples in these two forms are primitive. In any triple with $d = 3$, all three members are divisible by 3. In fact, no triple where d is prime can be primitive. An inspection of a partial list of primitive triples with their associated d and d' might be helpful at this point:

x	y	z	d	d'
3	4	5	1	2
5	12	13	1	8
15	8	17	9	2
7	24	25	1	18
21	20	29	9	8
35	12	37	25	2
9	40	41	1	32
45	28	53	25	8
11	60	61	1	50
63	16	65	49	2
33	56	65	9	32

Notice that every d is the square of an odd integer and that every d' is twice the square of some integer. Our first aim is to prove that this conjecture is a theorem. The proof given here is based on the following lemma for which the proof is omitted.

LEMMA. *For positive integers a and b, if ab is a perfect square and a is not a perfect square, then a and b have a common factor c such that ac is a perfect square.*

THEOREM. *If $\{x, y, z\}$ is a primitive Pythagorean triple, y is even, $d = z - y$, and $d' = z - x$, then d is the square of an odd integer and d' is twice the square of an integer.*

Proof: Since $d(2z - d)$ is a perfect square, assume that d is not. Then d and $(2z - d)$ have a common factor c (according to the lemma) such that cd is a perfect square. Since c divides both $2z - d$ and d, c^2 divides $d(2z - d)$ and c divides $\sqrt{d(2z - d)}$ which is the first member of the triple. Also c divides $(2z - d) + d$. Thus c divides $2z$. Since d is odd, c is also odd and so is a factor of z. Thus c divides both d and z and consequently $z - d$. But c cannot be 1 because $1 \cdot d$ is not a perfect square. Therefore, c divides all members of the triple and c is not 1; that is, the triple is not a primitive triple. Hence, in order for the triple $\{\sqrt{2zd - d^2}, z - d, z\}$ to be primitive,

d must be a perfect square and since d is odd, it must be the square of an odd integer.

Assume that d' is not twice the square of an integer. Then $d'/2$ is not a perfect square. Since d' is even, we know that $d'/2$ is an integer. Since the triple has the form $\{z - d', \sqrt{2zd' - d'^2}, z\}$ when using the even difference d', we know that $2zd' - d'^2$ is a perfect square. Now $2zd' - d'^2$ is equal to the product of $d'/2$ and $(4z - 2d')$. By the lemma, therefore, $d'/2$ and $4z - 2d'$ have a common factor k such that $kd'/2$ is a perfect square. Let S be the set of all such k and let c' be the smallest number in S. Since c' divides $d'/2$, c' divides $2d'$ and therefore c' divides the sum $(4z - 2d') + 2d'$ which equals $4z$.

Now if 4 divides c', then 4 divides $c'd'/2$ and $(c'/4)(d'/2)$ is also a perfect square. This means that $c'/4$ is an element of S. This contradicts the fact that c' was chosen to be the smallest element of S. Therefore, 4 cannot divide c'.

Still under the assumption that $d'/2$ is not a perfect square, let us determine if 2 divides c'. We have just determined that 4 does not. If 2 does divide c' while 4 does not, then $c'/2$ is an odd integer.

But c' divides $(4z - 2d')$ and also $d'/2$ by the definition of c'. Thus c' divides $2d'$ and consequently also $4z$. Hence the integer $c'/2$ divides $2z$. But an odd integer dividing $2z$ must divide z itself. Hence $c'/2$ divides z as well as d'. This implies that it divides $z - d'$. It also implies that $c'/2$ divides $2z - d'$ and consequently both d' and $2z - d'$. Thus $c'/2$ divides $\sqrt{d'(2z - d')}$.

In short, $c'/2$ divides each of the integers $z - d'$, $\sqrt{2zd' - d'^2}$ and z. So if $c'/2$ is not the number 1, the given triple is not primitive. On the other hand, if $c'/2$ is the number 1, then by definition of c' we have that d' and $2z - d'$ are both perfect squares. They are both even since d' is even, and so are both multiples of 4. This implies that 4 divides $2z$ and thus that 2 divides z. This contradicts the fact that in a primitive Pythagorean triple z must be odd. Thus we conclude that 2 does not divide c'.

Still under the assumption that $d'/2$ is not a perfect square, and knowing that c' is not even, the only other possibility is that c' is odd. But in this case by the definition of c' it must divide d' and $4z - 2d'$. Hence, c' divides $4z$ and hence z. Also c' divides $z - d'$; since c' also divides both $2z - d'$ and d', it must divide $\sqrt{d'(2z - d')}$. It divides every number in the triple. By its definition $c' \neq 1$; yet the triple is not primitive unless $c' = 1$. Thus c' cannot be odd.

The assumption that $d'/2$ is not a perfect square is thus false, and we conclude that in truth d' is twice the square of some integer. Thus the theorem is proved.

So far two important results have been shown. First, d is the square of an odd integer; we denote this odd integer by Q. Second, d' is twice the square of an integer; we denote this integer by N. Thus, $d = Q^2$ and $d' = 2N^2$ and Q is odd.

Recall that the triple $\{x, y, z\}$, in this order, may be put in either the form $\{\sqrt{2zd - d^2}, z - d, z\}$ or the form $\{z - d', \sqrt{2zd' - d'^2}, z\}$. Thus, there are two expressions for x, and the resulting equation can be solved for z:

$$\begin{gathered} x = z - d' = \sqrt{2zd - d^2} \\ x = z - 2N^2 = \sqrt{2zQ^2 - Q^4} \\ z^2 - 4N^2z + 4N^4 = 2zQ^2 - Q^4 \\ z = 2N^2 + Q^2 \pm 2NQ \end{gathered}$$

Since x, y, z must all be positive, the expression with the minus sign does not hold. For assume that $z = 2N^2 + Q^2 - 2NQ$. Then since $x = z - d'$ and $y = z - d$, we have

$$\begin{aligned} x &= 2N^2 + Q^2 - 2NQ - 2N^2 = Q^2 - 2NQ = Q(Q - 2N) \\ y &= 2N^2 + Q^2 - 2NQ - Q^2 = 2N^2 - 2NQ = 2N(N - Q) \end{aligned}$$

Since y is positive, $N > Q$ and $2N > Q$, so x is negative and we have a contradiction.

Therefore, we have $\{Q^2 + 2NQ, 2N^2 + 2NQ, 2N^2 + Q^2 + 2NQ\}$ as a new form for the ordered triple $\{x, y, z\}$.

If N and Q have a common factor other than 1, we get a triple which is not primitive. We show now that only when N and Q have a common factor other than 1 will this happen.

Suppose that the triple $\{Q^2 + 2NQ, 2N^2 + 2NQ, 2N^2 + Q^2 + 2NQ\}$ is not primitive. Let f be the greatest common factor.

Since f divides the last two members, it must divide Q^2. Thus f is odd. Since f divides the first and last members, f must divide $2N^2$ and since f is odd, it must divide N^2.

Let g be a prime factor of f. Then g divides N^2 because f does so. Since g is prime and divides N^2, g divides N. Similarly g divides Q^2 because f does so. Since g is prime, g divides Q. Thus, N and Q have a common factor other than 1.

To summarize, $\{x, y, z\}$, where y is even, is a primitive Pythagorean triple if an only if there exist relatively prime positive integers N and Q, where Q is odd, such that $x = Q^2 + 2NQ$, $y = 2N^2 + 2NQ$, and $z = 2N^2 + Q^2 + 2NQ$.

A well known method for finding primitive Pythagorean triples states that $\{x, y, z\}$, with y even, is primitive if and only if there exist relatively prime integers u and v such that $x = u^2 - v^2$, $y = 2uv$, and $z = u^2 + v^2$. To derive these equations from those presented in this article, put $N + Q = u$ and $N = v$. Since N and Q are relatively prime, so are u and v and conversely. On making these substitutions we get:

$$x = Q^2 + 2NQ = N^2 + 2NQ + Q^2 - N^2 = u^2 - v^2$$
$$y = 2N^2 + 2NQ = 2N(N + Q) = 2uv$$
$$z = 2N^2 + Q^2 + 2NQ = N^2 + 2NQ + Q^2 + N^2 = u^2 + v^2.$$

The form for triples, which was derived in this article, has the distinction that it is based on the differences $2N^2$ and Q^2 between z and the other two members of the triple.

There are areas in mathematics which may be covered in textbooks for higher courses but which could be explored by the student himself before encountering these books. Making discoveries in these areas does not always call for much higher mathematics, although they may require certain assumptions by the student which he is not yet able to prove. This makes him look forward to the higher courses. The above method of finding primitive Pythagorean triples is a good example of this type of discovery.

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Expansive Flows

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If f is a homeomorphism of a metric space X (with metric d) onto itself, then f is said to be expansive with expansive constant $\delta > 0$ if $x, y \in X, x \neq y$ implies $d(f^n(x), f^n(y)) > \delta$ for some integer n. Expansive homeomorphisms have been widely studied. See [1], [2], [3], and [4], for instance.

It is natural to try to generalize the concept of expansiveness to more general families of homeomorphisms. In this note, we show that in certain cases, this is impossible.

Definition 1. *Let X be a metric space with metric d, and let $G \neq \{0\}$ be an additive subgroup of the reals. For each $t \in G$, let ϕ_t be a homeomorphism of X onto itself, and let ϕ_t be continuous from $X \times G$ to X. Finally let $\phi_s\phi_t = \phi_{s+t}$ for each $s, t \in G$. Then ϕ_t is said to be a flow on X.*

Definition 2. *If ϕ_t is a flow on X, then ϕ_t is expansive with expansive constant $\delta > 0$ if $x, y \in X, x \neq y$ implies $d(\phi_t(x), \phi_t(y)) > \delta$ for some $t \in G$.*

Clearly this definition generalizes that of an expansive homeomorphism; we merely let G be the group of integers and let $\phi_1 = f$.

Theorem. *Let X be compact and infinite, and let ϕ_t be an expansive flow on X. Then G is a cyclic group.*

Proof: Our proof is a modification of an unpublished result due to G. A. Hedlund. Professor Hedlund considered the case when G is the reals.

Suppose that 0 is not isolated in G. Choose a sequence of positive terms in G, say $\{t_n\}$, such that $t_n \to 0$. Let $H = \{0\} \cup \{t_n : n = 1, 2, \ldots\}$. Since H is compact, ϕ_t is uniformly continuous on $X \times H$. Let $\delta > 0$. Choose $\eta > 0$ such that $x \in X$, $t \in H \cap [0, \eta)$ implies $d(\phi_t(x), x) = d(\phi_t(x), \phi_0(x)) < \delta$.

Case 1. Suppose there exists $x \in X$ and $t \in H \cap [0, \eta)$ such that $\phi_t(x) \neq x$. Then for each $s \in G$, $d(\phi_s \phi_t(x), \phi_s(x)) = d(\phi_t \phi_s(x), \phi_s(x)) < \delta$. Since δ was arbitrary, this contradicts the expansiveness of ϕ_t.

Case 2. Suppose $\phi_t(x) = x$ for each $x \in X$ and each $t \in H \cap [0, \eta)$ Choose $x \neq y$ such that $d(x, y) < \delta$. Let $s \in G$. Choose a sequence of elements of $H \cap [0, \eta)$, say $\{s_n\}$, and a sequence of integers, say $\{k_n\}$, such that $k_n s_n \to s$. Then $\phi_{k_n s_n}(x) \to \phi_s(x)$ and $\phi_{k_n s_n}(y) \to \phi_s(y)$. But $\phi_{k_n s_n}(x) = x$ and $\phi_{k_n s_n}(y) = y$, so $d(\phi_s(x), \phi_s(y)) = d(x, y) < \delta$, again contradicting the expansiveness of ϕ_t.

Thus 0 is isolated in G. Since G contains the difference of any two of its elements, it cannot contain arbitrarily close elements. Thus G contains a smallest positive element a. Clearly a generates G and the proof is complete.

The theorem above shows that if ϕ_t is expansive on X, then each ϕ_t is of the form ϕ_{na}, i.e. each is an iterate of ϕ_a. Hence the theory of expansive flows on compact infinite metric spaces reduces to the theory of expansive homeomorphisms. Of course, if X is finite, the flow defined by $\phi_t(x) = x$ for each $x \in X$ and each $t \in G$ is expansive for any group G.

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Even Perfect Numbers Modulo 12

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For several centuries students of mathematics have been fascinated by the system of positive integers and some of the remarkable properties that it possesses. Among the positive integers which have received special investigation are perfect numbers.

A positive integer n is a perfect number if and only if the sum of its positive integer divisors is $2n$. For example, 28 is a perfect number since its positive integer divisors are 1, 2, 4, 7, 14, and 28 and

$$1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 \times 28.$$

The first few perfect numbers are 6; 28; 496; 8,128; 33,550,336; and 8,589,869,056. Notice that each of these perfect numbers is even. We do not know whether there are any odd perfect numbers. However, it has been established by Bryant Tuckerman of IBM's Watson Research Center that if an odd perfect number exists it must be greater than 10^{36}. The greatest of the perfect numbers presently known is the number $2^{19936}(2^{19937} - 1)$ which has 12,003 digits (in the base 10). There are 24 known perfect numbers at this time. The question of whether there are infinitely many perfect numbers has not been answered.

Euclid showed that if n is a positive integer of the form $2^{p-1}(2^p - 1)$ where $2^p - 1$ is a prime then n is a perfect number. Later Euler established that every even perfect number is of the Euclid type. A necessary condition that $2^p - 1$ be a prime number is that p be a prime. Thus all even perfect numbers have the form $2^{p-1}(2^p - 1)$ where p is a prime number.

Each of the perfect numbers given above, except 6, is congruent to 1 modulo 9. This property was mentioned by Tartaglia way back in the sixteenth century [1]. For a proof of the fact that every even perfect number greater than 6 is congruent to 1 modulo 9, see [2].

Another look at the even perfect numbers listed earlier leads us to conjecture:

THEOREM. *If n is an even perfect number greater than* 6, *then* $n \equiv 4(mod\ 12)$.

Proof: By hypothesis n is an even perfect number. Thus $n = 2^{p-1}(2^p - 1)$ where p is a prime number. $n > 6$ implies that $p > 2$. Hence $p \geq 3$ is an odd number and $p - 1 \geq 2$ is an even number. From $p - 1 \geq 2$, it follows that $2^2 = 4$ divides 2^{p-1}. Thus 4 divides $2^{p-1}(2^p - 1)$. Since 4 divides n and 4 divides itself, 4 divides $n - 4$. $2 \equiv -1(\text{mod } 3)$ implies that $2^{p-1} \equiv 1(\text{mod } 3)$.

Hence $2^{p-1} \equiv 3k + 1$ for some positive integer k. Multiplying by 2, $2^p = 6k + 2$ or $2^p - 1 = 6k + 1$. Thus $n - 4 = 2^{p-1}(2^p - 1) - 4 = (3k + 1)(6k + 1) - 4 = 18k^2 + 9k - 3 = 3(6k^2 + 3k - 1)$, and 3 divides $n - 4$. 4 divides $n - 4$, 3 divides $n - 4$, and 4 and 3 relatively prime implies that 12 divides $n - 4$. Therefore $n \equiv 4\ (\text{mod } 12)$.

COROLLARY 1. *If n is an even perfect number greater than* 6, *then* 4 *divides n.*

Proof: This fact is established in the first part of the proof of the preceding theorem.

COROLLARY 2. 6 *is the only squarefree perfect number.*

Proof: Clearly 6 is a squarefree perfect number. From Corollary 1, every even perfect number greater than 6 has 4 as a factor and hence is not squarefree. Euler proved that any odd perfect number must be of the form $r^{4i+1}p^2$, where r is a prime of the form $4t + 1$ [3]. Thus no odd perfect number (if indeed such a number exists) can be squarefree. Therefore 6 is the only squarefree perfect number [4].

Can you find any additional interesting properties which perfect numbers have?

(concluded on p. 112)

A Method To Construct Convex, Connected Venn Diagrams for Any Finite Number of Sets

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Given a finite class of sets $A_1, A_2, \ldots, A_n$, a Venn diagram for the class will consist of 2^n regions, each representing a distinct set formed by intersection of the sets and their complements. For example, a typical Venn diagram for three sets A_1, A_2, and A_3 will consist of the eight regions: $A_1 - A_2 - A_3$, $A_2 - A_1 - A_3$, $A_3 - A_1 - A_2$, $A_1 \cap (A_2 - A_3)$, $A_1 \cap (A_3 - A_2)$, $A_2 \cap (A_3 - A_1)$, $A_1 \cap A_2 \cap A_3$, $-A_1 - A_2 - A_3$. The Venn diagram is shown in Figure 1, where A_1 is the upper circle, A_2 is the lower left circle, and A_3 is the lower right circle.

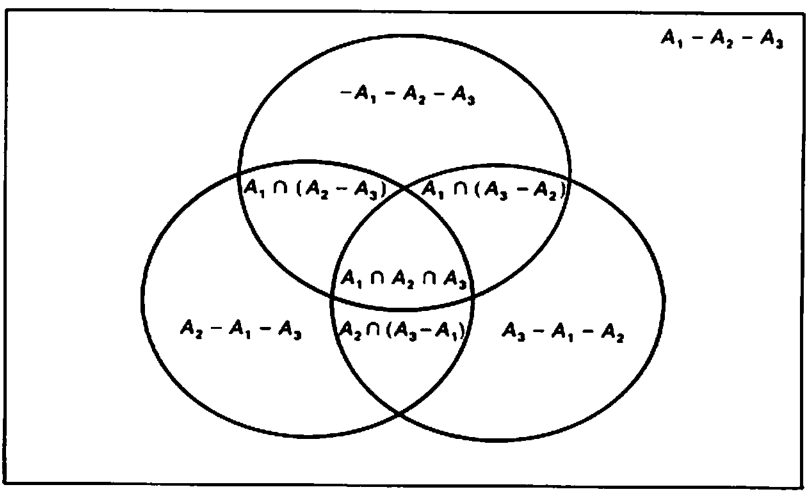

FIGURE 1

Notice that the sets A_1, A_2, and A_3 in Figure 1 are all convex. We can make this a requirement in the construction of a Venn diagram for an arbitrary finite number of sets.

Starting with a square as the universal set, let the upper half be the first set, the right half be the second set, and a circle centered inside the square be the third set. For the rest of the construction, we can concentrate on the upper right quadrant since the same will be done to every quadrant of the square. Divide the right angle formed by the sides of the first and the second sets into 2^{n-2} equal parts. We label the rays $k/2^{n-2}$, $k = 0, 1, 2, \ldots, 2^{n-2}$ in the counter-clockwise manner (for $n = 5$ see Figure 2). Let us call the intersection of the $k/2^{n-2}$-ray

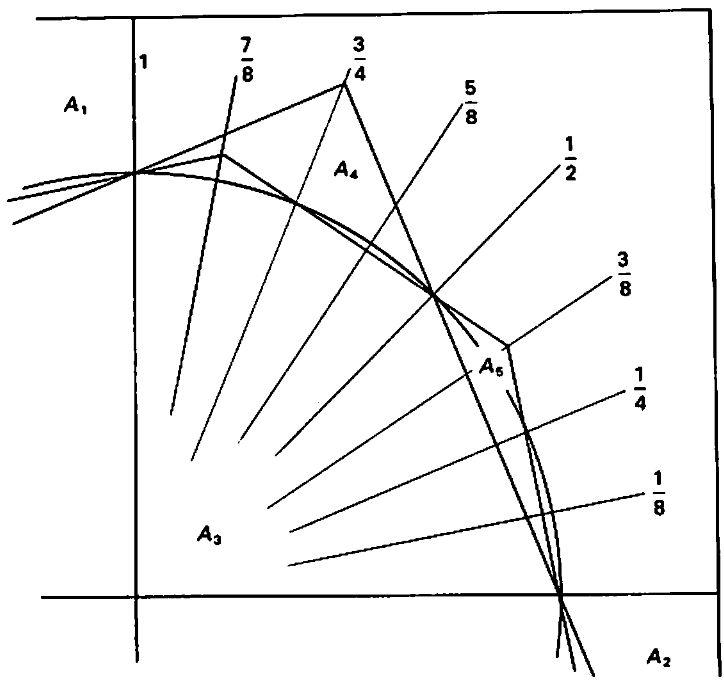

FIGURE 2

and the circle the $k/2^{n-2}$-point for short. The fourth set, A_4, is constructed by drawing a straight line from the 0-point through the 1/2-point, meeting the 3/4-ray at a point, and then joining that point with the 1-point by a straight line that continues into the second quadrant. Notice that the fourth set is a square with one-fourth of it contained in each quadrant of the universal set. The fifth set, A_5, is then an octagon with vertices on the 3/8 and 7/8-rays and sides going through the 0, 1/4, 1/2, 3/4, and 1-points in the upper right quarter.

In general, the set A_k, $k \geq 4$, is a 2^{k-2}-gon, with vertices on the $3/2^{k-2}, 3 + 4/2^{k-2}, \ldots$, and $2^{k-2} - 1/2^{k-2}$, and 1-points. It is easy to see that the set A_k cuts into all the previous regions. Continuing in this way until the set A_n is constructed, we obtain the desired Venn diagram.

To fully appreciate the construction, we conclude with Figure 3, showing the complete construction through A_5.

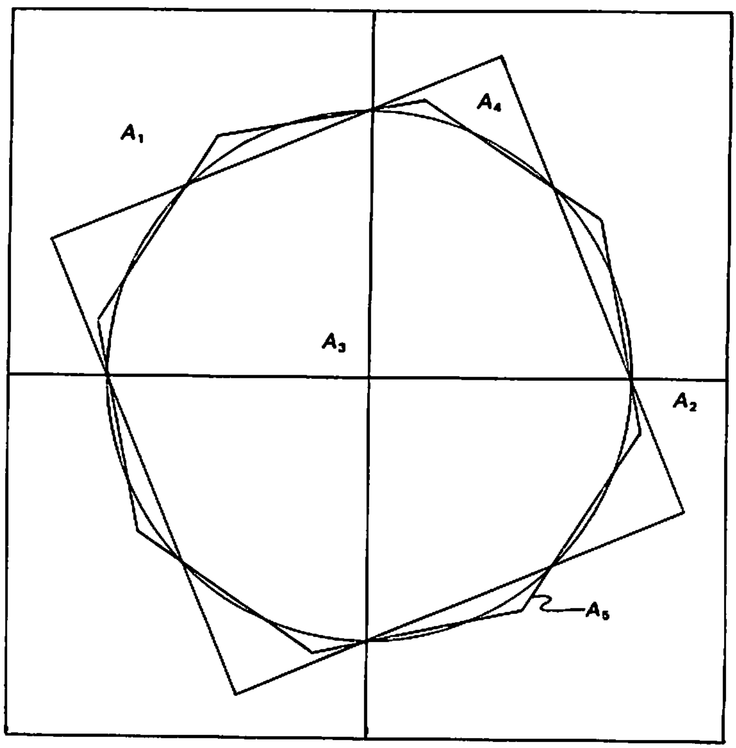

FIGURE 3

How a Flexible Tetrahedral Ring Became a Sphinxx

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Frasier and Gingery, Inc., Denver, Colorado

A regular tetrahedron consists of four plane equilateral triangles arranged symmetrically about a center. Regular tetrahedrons cannot be stacked together to perfectly fill three dimensional Euclidean space. However, there are other types of tetrahedrons which can perfectly fill three dimensional Euclidean space.

One of these is a tetrahedron which is built up of four congruent isosceles triangles and has two opposite dihedral angles equal to 90° and four dihedral angles equal to 60°. The tetrahedron can be made from the pattern shown in Figure 1. This basic tetrahedron will be called a *link* in this article. The edge of a link

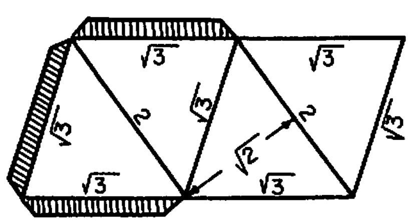

FIGURE 1

at which a dihedral angle of 90° is formed will be called a *hinge axis*. In Figure 2 two links are shown taped together at a hinge axis. They are free to rotate about their hinge axes 180° with respect to one another.

The rest of the discussion will be devoted to describing how the links may be used to create different flexible chains and rings. It will then attempt to describe how one of these rings

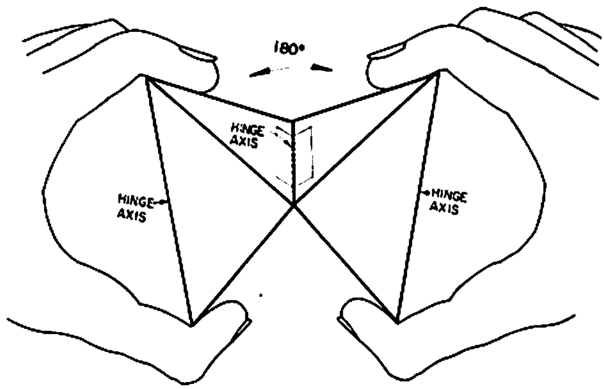

FIGURE 2

can interlock to form solids of rather exotic character which will perfectly stack together to fill three dimensional Euclidean space.

The hexaflexagons discovered in 1939 by Arthur H. Stone were the original motivation for the construction of the solid *flexahedrons* in this article. Martin Gardner's article in *Scientific American* [1] gives an excellent description of some of the hexaflexagons.

All of the results beyond the link in this discussion were originally arrived at by trial and error. There are no straight forward mathematical methods of doing it as far as is known. What was discovered was arrived at by following a random trail.

If eight links are taped together at their hinge axes, they form an eight link *chain*. This chain may now be connected into a *ring* by using the remaining two hinge axes at the head and tail ends of the chain.

It is not possible to give the ends of the eight link chain a full twist before connecting into a ring. It is possible to continuously turn the eight link ring in the same direction about a circular axis formed by a circle passing through the points of the centers of the eight links.

When 16 links are connected into a chain, a 360° twist can be given to the chain before connecting it into a ring. When 24 links are connected into a chain, two 360° twists are possible before connecting it into a ring. Both the twisted ring of 16 links and the twisted ring of 24 links will undergo a cycle of flections so that they appear to turn inside out. In one cycle of flections, a ring returns to its starting position.

In order to effect a single flection, a set of links of a ring must be rotated with respect to the rest of the ring. This rotation may be of 90° or 180° and always occurs about two hinge axes which are in line.

The ring does not really turn inside out if you try to think of it as a solid enclosing a three dimensional space. An imaginary axis can be drawn connecting the centers of all the links in the ring called the ring axis. A ring is *flexible* if the links of the ring can be made to rotate continuously in the same direction about the ring axis.

A chain of 32 links can be given three full twists before connecting into a ring. It is also flexible as defined above. A chain of 40 links can be given four full twists before connecting into a ring. It is not flexible in three dimensional Euclidean space. This author has conjectured that it would be flexible in a four dimensional space. It can be forced to complete a cycle of flections if one is careful not to tear it apart.

We will now return to the ring of 24 links with two 360° twists. Figure 3 shows this ring undergoing a complete cycle of flections. Nine separate rotations of various parts of the ring are required before the cycle is complete. Because of the twists in the ring only certain motions are allowed. The twists appear to restrict the freedom of the ring. A flexible ring can best be flexed by taking symmetrical sets of opposed parts for each rotation. The twist given to the ring makes it asymetrical as a whole in three dimensional Euclidean space.

Consider position 1 or 9 in Figure 3 for the 24 link ring. In this position all the links have one of their faces adjacent to a face of another link. This is called the *closed angle position.* As far as is known no other flexible ring has a closed angle position.

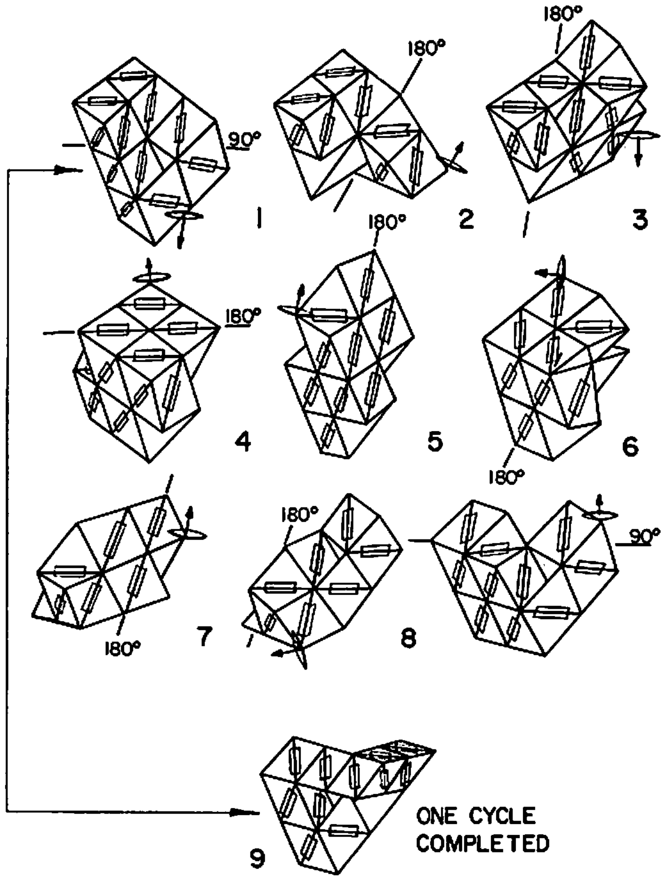

FIGURE 3

A wood rod that has an equilateral triangle for a cross section may be cut into congruent pieces as shown in Figure 4. Each of these pieces is equivalent to a chain of six links with a 180° twist. Figure 5 shows how four of these rods may be glued together to form a solid which duplicates position 9 of the 24 link ring shown in Figure 3. In this position, the ring has two triangular holes which allow two of the solid rings made of wood to interlock in various ways.

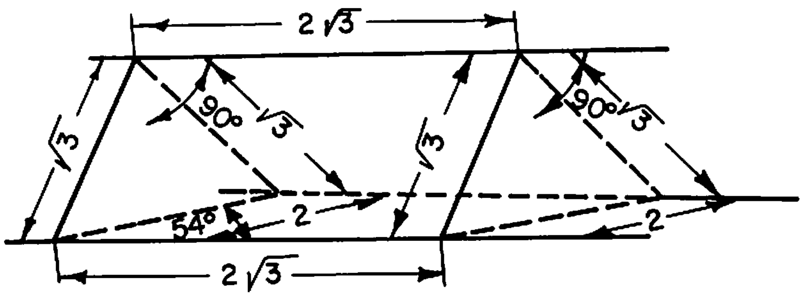

FIGURE 4

In what follows the 24 link ring in its closed angle position will be called a *sphinxx*.* The sphinxx should be considered as a new solid derived from the 24 link ring. The author named it a sphinxx because of its puzzling property of forming larger rings.

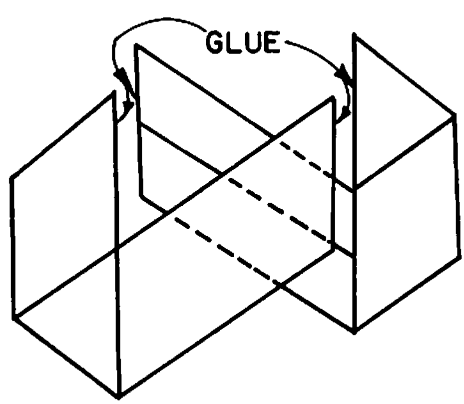

FIGURE 5

*It is spelled with two x's to distinguish it from the legendary Sphinx.

Two sphinxx may interlock in four different ways as shown in Figure 6. One of the two-sphinxx chains is actually a ring (number 4 in Figure 6). This two sphinxx ring has, among other properties, the ability to perfectly fill three dimensional Euclidean space in a number of different ways. Figure 7 illustrates how a "plane" of these rings can be stacked together. These "planes" may then be stacked one atop the other to completely fill three space.

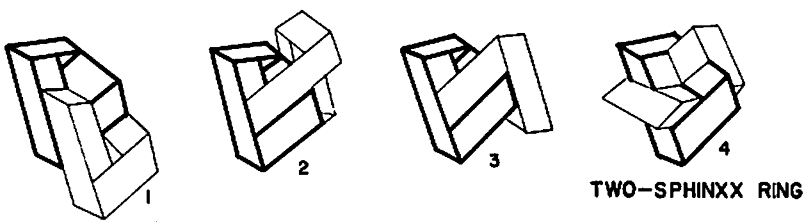

FIGURE 6

The alert reader will now begin to conjecture about forming more complicated structures by connecting a greater number of sphinxxs into rings. Six sphinxx will combine together in three different ways to form rings. This is shown in Figures 8, 9, and 10. Two of these contain a 360° twist. This can be shown by making a ring out of rubber links and then cutting it apart at one link and straightening it into a chain. The other (Figure 10) has a zero twist. All three can perfectly fill three dimensional Euclidean space by stacking the rings into a row as shown in Figures 8, 9, and 10. Rows of rings may then be placed side by side to form a continuous "plane" of rings. These "planes" can be stacked as before to completely fill three space.

With this the discussion is complete. The mystery of why the sphinxx form complicated rings which happen to fill three space remains. Part of the mystery may be connected with a property discussed in a previous paper about a twisted ring made of eight sphinxx [2]. Another part of the mystery deals with something very difficult to illustrate in a picture. It is a generalized solid made of sphinxx. The solid itself may be taken apart into a set of complete sphinxx rings. It is therefore really composed of a set of interknotted sphinxx rings.

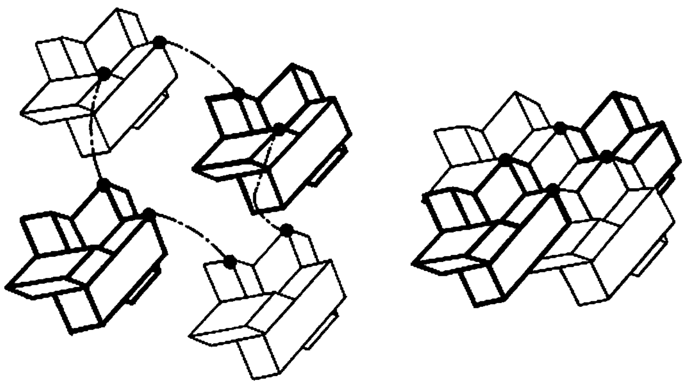

FIGURE 7

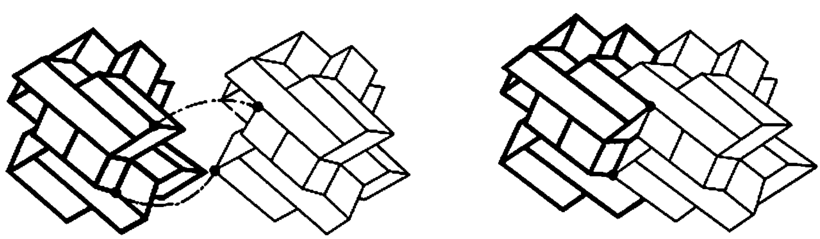

FIGURE 8

This solid will probably never be drawn or illustrated schematically because there is no need to. Several of them have been constructed by the author. This generalized super-sphinxx can

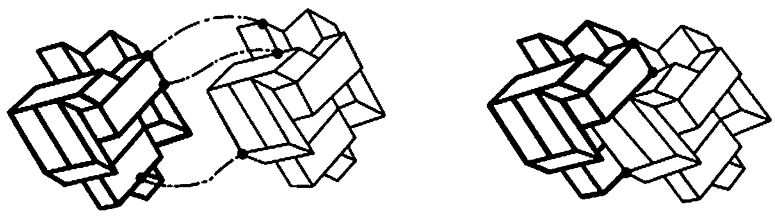

FIGURE 9

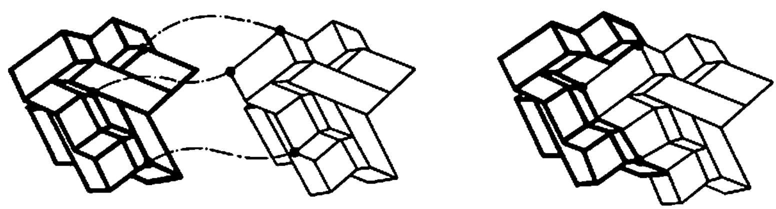

FIGURE 10

be described mathematically with a set of simple equations containing three variables. The rings which make up a super-sphinxx and their mutual relationships appear to be closely allied to number theory (prime numbers, factored numbers, and congruences.)

Perhaps in a future discussion, the properties of the super-sphinxx can be developed. The sphinxx may then in its own way have added to the interest of the delightful field of number theory.

REFERENCES

1. Martin Gardner, "Mathematical Puzzles and Pastimes", *Scientific American*, 202 (May 1958), 122-23.
2. D. A. Engel, "Can Space be Overtwisted?", *The Mathematics Teacher*, 61 (October 1968), 571-574.

The Problem Corner

EDITED BY ROBERT L. POE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before September 1, 1972. The best solutions submitted by students will be published in the Fall 1972 issue of *The Pentagon*, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Professor Robert L. Poe, Department of Mathematics, Berry College, Mount Berry, Georgia 30149.

PROPOSED PROBLEMS

250. *Proposed by the Editor.*

During the depression of the early '70's, students at BIG University (Bountiful Institutional Grants University) had an exceptionally hard time finding employment to help defray their expenses. Dr. Grant Ghettar, the president of BIGU, had a terrible time securing research grants from government agencies and private corporations these lean years. As a result students no longer occupied good paying, soft jobs in campus offices supported by the overhead money portions of research grants. Actually the female students had the worst of it, since male students could obtain useful employment in the commercial and industrial areas of nearby Crisis City if they got haircuts, shaved off their beards, and wore shoes and socks.

Miss Brunhilde Hibernackle, an enterprising sophomore, solved the problem for many of the BIG girls. She formed an organization called BIG Protesters, Inc. The purpose of this group was to demonstrate for the Women's Lib, ethnical, racial, parents opposing school busing, hippies, ecological, hard hats, and any other organizations that wanted to stage protests, have sit-ins, or promote riots in Crisis City. The

BIG Protesters, Inc. was a cinch to make money since for a fee the active members of an organization could stay safely away from their demonstration, thusly avoiding exertion and arrest. By skillful application of make-up and false whiskers, donning of appropriate clothing, and creation of suitable placards these BIG girls focused attention on many social ills and caused worthy social strife in Crisis. Besides, they enjoyed the work and looked good on television newscasts.

Not even the Pentagon spies were ever able to determine the number of girls working for BIG Protesters, Inc. Miss Hibernackle, as the coordinator of BIG Protesters, Inc., always had a problem determining how many girls actually worked in a demonstration and how much each should be paid since at every function the number of BIG girls participating varied. For example at the protest march demanding equal opportunity employment for men with waist measurements larger than 44 inches as telephone linemen sponsored by the CCCC (Crisis Civil Crisis Club) she forgot to count the girls on the job. All she could remember when she returned to campus was that there had been exactly enough BIG girls present so that when lined to march in ranks of three abreast, they had two girls left over; in fives, four BIG girls extra; in sevens, six too many; and in elevens, ten in excess. If BIG Protesters, Inc. was paid $3000.00 for the protest march and Brunhilde paid herself $10.00, how much should each professional BIG girl protester be paid?

251. *Proposed by Stephen C. Hennagin, University of California, Davis, California.*

Find $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n+k}$.

252. *Proposed by Frank R. Dangello, Shippensburg State College, Shippensberg, Pennsylvania.*

Prove that $\int_0^x \left[\int_0^{x_1} f(t)dt\right] dx_1 = \int_0^x (x - x_1) f(x_1) dx_1$ for $x > 0$, $0 \le x_1 \le x$, and f continuous on $[0, x]$.

253. *Proposed by Stephen C. Hennagin, University of California, Davis, California.*

Show that $\lim\limits_{x \to \infty} \dfrac{x}{1 - e^{x^2}} \displaystyle\int_0^x e^{t^2}\, dt = -1.$

254. *Proposed by the editor.*

Four neighbors, Jones, Smith, Williams, and Harris regularly play table tennis. Smith was once with the State Department and is a champion table tennis player. The probability that Smith will win from Jones is 11/12; from Williams is 9/10; and from Harris 15/16. In a long series of games, Smith first plays Williams, then plays a game with Jones, and next plays Harris. If the neighbors continue playing in this sequence until Smith loses a game, what is the probability that either Harris or Jones is the first to defeat Smith?

SOLUTIONS

246. *Proposed by Kenneth Rosen, University of Michigan, Ann Arbor, Michigan.*

Let x and y be positive real numbers such that $x + y = 1$. Prove that $x \ln x + y \ln y \geqq -\ln 2$ and discuss where equality holds.

Solution by Stephen C. Hennagin, University of California, Davis, California.

Since x, y are both positive reals, $0 < x < 1$ and $0 < y < 1$. Let $f(t) = t \log t + (1 - t) \log (1 - t) + \log 2 = t \log\left(\dfrac{t}{1 - t}\right) + \log (1 - t) + \log 2$. Thus, f is differentiable for $0 < t < 1$, and $f'(t) = \log\left(\dfrac{t}{1 - t}\right)$, and $f''(t) = \dfrac{1}{t(1 - t)}$. Now, $f'(t) = 0$ iff $\dfrac{t}{1 - t} = 1$. But then $t = 1/2$ and $f''(1/2) > 0$. Hence, $f(1/2)$ is a local minimum for $f(t)$. It is clear that $f(1/2)$ is also the absolute minimum of $f(t)$ on $0 < t < 1$. Therefore, $f(t) \geq f(1/2)$, or

$t \log t + (1 - t) \log (1 - t) + \log 2 \geq 0$ and $x \log x + y \log y \geq - \log 2$ for $x + y = 1$. Equality holds iff $x = y = 1/2$.

Also solved by Kenneth M. Wilke, Topeka, Kansas.

247. *Proposed by R. S. Luthar, University of Wisconsin, Waukesha, Wisconsin.*

Find all integral solutions of the equation $x^4 + 3y^2 + 4z = 26$.

Solution by Christopher Thompson, Ward Melville High School, Stony Brook, New York.

26 is an even number that is not divisible by 4. In order to get such a number from the sum of three numbers, the following must be true: When each is divided by 4, the sum of the remainders must be equal to 2 or $4n + 2$. In the case of three numbers the only other possibility is 6. This can be accomplished in one of five ways:

(1) Remainders of 1,1,0,
(2) Remainders of 1,2,3,
(3) Remainders of 2,0,0,
(4) Remainders of 2,2,2, or
(5) Remainders of 3,3,0.

Since $4z$ must be divisible by 4, only (1), (3), and (5) need be considered.

Case (1): x^4 and $3y^2$ are both odd with remainders of 1 when divided by 4. If x^4 and $3y^2$ are odd, then x and y must also be odd. When an odd number of the form $2n + 1$ (n in the set of integers) is squared, the result is $4n^2 + 4n + 1 = 4(n^2 + n) + 1$. Substituting in for y, $2n + 1$, $3[4(n^2 + n) + 1]$ is the new second term. Distributing the 3, $4[3(n^2 + n)] + 3$, contradicts the original premise that dividing through by 4 will leave a remainder of 1.

Case (5): x^4 and $3y^2$ are both odd with remainders of 3 when divided by 4. Same as Case (1) until substitution. Substitute $2n + 1$ for x^2 (If x is odd x^2 must also be odd), $(2n + 1)^2 = 4n^2 + 4n + 1 = 4(n^2 + n) + 1$. This contradicts the original premise that dividing through by 4 will leave a remainder of 3.

Case (3): x^4 and $3y^2$ are both even, one divisible by 4, one not divisible by 4. If x^4 and $3y^2$ are even, x and y must also be even. The square of an even number of the form $2n$ (n an integer) is $4n^2$. Substituting $2n$ for x^2, you get that $x^4 = 4n^2$, which is divisible by 4. Since x^4 must be divisible by 4, $3y^2$ must be even but not divisible by 4. Substituting $2n$ for y, $3y^2 = 3(2n)^2 = 3(4n^2) = 4(3n^2)$. Since both x^4 and $3y^2$ must both be divisible by 4, this case yields no solutions. Since none of the five cases can be satisfied the answer is the empty set.

Also solved by Kenneth M. Wilke, Topeka, Kansas.

248. *Proposed by R. S. Luthar, University of Wisconsin, Waukesha, Wisconsin.*

Solve the equation $2(5x - 1)^2 + 7 = y^4$ over the domain of positive integers.

Solution by Kenneth M. Wilke, Topeka, Kansas.

The given equation is equivalent to $50x^2 - 20x + 9 = y^4$ or $10(5x^2 - 2x) = y^4 + 1$. Hence $y^4 + 1$ is divisible by 10. Let $y = 10k + a$ where $a = 0,1,2,3,4,5,6,7,8,9$. Then a^4 ends in 0,1,6,1,6,5,6,1,6,1 and $y^4 + 1$ ends in 1,2,7,2,7,6,7,2,7,2 correspondingly, so that $y^4 + 1$ is not divisible by 10. Hence the given equation has no solutions in integers.

Also solved by Christopher Thompson, Ward Melville High School, Stony Brook, New York.

249 *Proposed by R. S. Luthar, University of Wisconsin, Waukesha, Wisconsin.*

From the vertex A of the curve $x^3 + y^3 = 3axy$, a line is drawn to cut the curve in two other points, P and Q. Prove that OP and OQ are perpendicular to each other where O is the origin of reference.

(concluded on p. 114)

The Book Shelf

EDITED BY ELIZABETH T. WOOLDRIDGE

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. Elizabeth T. Wooldridge, Department of Mathematics, Florence State University, Florence, Alabama 35630.

Mathematics, A Human Endeavor, Harold R. Jacobs, W. H. Freeman and Company, San Francisco, California, 1970, 646 pp., $8.50.

The textbook covers standard topics for Liberal Arts students, or, as the author says, "A Textbook for those who think they don't like the subject." The chapter titles are: 1. The Mathematical Way of Thinking, 2. Number Sequences, 3. Functions and Their Graphs, 4. Large Numbers and Logarithms, 5. Regular Polygons, 6. Mathematical Curves, 7. Some Methods of Counting, 8. The Mathematics of Chance, 9. An Introduction to Statistics, 10. Some Topics in Topology.

What makes this not "just another text" is the author's imagination and far-ranging knowledge. The book is filled with diagrams, pictures, cartoons, and comic strips. The text and problem sets are full of interesting applications, which should make the material fun for both the student and the teacher. The teacher's guide contains many suggestions for conducting the class so as to enhance the enthusiasm the text generates.

The material is not difficult, and should be accessible to any college freshman, whether or not he has taken mathematics in high school. This book proves that easy need not mean boring.

E. R. Deal
Colorado State University

Calculus Two: Linear and Nonlinear Functions, Francis J. Flanigan and Jerry L. Kozdan, Prentice-Hall, Englewood Cliffs, N.J., 1971, 443 pp., $10.95.

The authors state that "this book is intended for the—possibly mythical—'average' student." The text is a study of the calculus of functions of several variables, using linear algebra, by attempting to parallel the calculus of functions of a single variable. The text is appropriate for technical schools, junior colleges, and four-year undergraduate colleges seeking a highly motivated approach to multivariate calculus. The scope is not restricted to analysis alone, but includes linear algebra and matrix concepts as well.

The first three chapters deal with vector algebra and vector geometry as well as linear functions. Vector algebra, including the study of addition, subtraction, and the solution of equations in vectors, is considered in the first chapter. In Chapter Two, vector geometry in Euclidean 2-space, 3-space, and n-space is considered, including the concept of distance between vectors. Linear functions and the more general "affine" functions are studied in Chapter Three, including generalizations in higher dimensional spaces. In Chapter Three, linear algebra and matrix theory are also introduced.

The next two chapters treat non-linear functions in two or more dimensions, including limits and continuity, and the construction of a tangent plane to the graph (a surface) of $y = f(x_1, x_2, \ldots, x_n)$ above a typical point $(x'_1, x'_2, \ldots, x'_n)$ which motivates differentiable functions in n-space.

Chapters Six, Seven, and Eight deal with the standard questions about $y = f(x_1, x_2, \ldots, x_n)$: maxima and minima, rates of change, and approximate values.

Chapters Nine and Ten consider integration in n-space including standard topics such as estimation of integrals, computing integrals, the theory of integration, line integrals, and Stokes' Theorem.

The exercises selected cover a range of abilities and attempt to provide for adequate practice of concepts and skills. Answers are provided for some problems.

The most impressive feature of the text is the use of many drawings depicting the graphs of functions in 3-dimensions which motivate students to the parallel concept in higher dimensions.

Ronald D. Dettmers
University of Wisconsin
at Whitewater

Mathematics for Technicians, Edward M. Tronaas, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1971, 414 pp., $10.95.

This text is written for first-year vocational students interested in drafting, automechanics, electronics, or similar technical areas. The text is written in a simple, readable style with many solved example problems. Many practice problems are provided with answers to odd numbered problems in the back of the text.

The only prerequisite assumed is a knowledge of whole numbers. The selection of topics and the treatment seems suitable for the intended students.

Explicitly the selection of topics is as follows: operations with fractions, per cent problems, operations with approximate numbers, scientific notation, use of slide rules, interpolation in tables, fundamentals of algebra, geometry, and trigonometry. There is also an introduction to logarithms. Of great value to the student should be the 120 pages of tables which are included.

Ben F. Plybon
Miami University

Geometry, An Intuitive Approach, Meridon V. Garner and B. G. Nunley, Goodyear Publishing Company, Inc., Pacific Palisades, California, 1971, 186 pp., $3.95.

In the Preface the authors state, ". . . we found that there is no textbook available that is both mathematically correct and fulfills the needed activities and applications for elementary teachers." I agree with this statement but this book does not fill the gap. The authors also assert, "Sufficient materials are provided to adequately prepare an elementary teacher for the classroom." I firmly disagree! Indeed the book is essentially a collection of definitions.

There is no discussion of similar triangles. The Pythagorean theorem is only obliquely mentioned (page 174) as it pertains to the distance formula in the last chapter. Two figures are defined to be congruent if one figure exactly "covers" the other—there is no mention of congruence criteria (SAS, SSS, etc.) or transformations. There is a treatment of area and perimeter of a triangle, but a complete omission of interesting properties of triangles. The amount of compass and straight-edge constructions in this book is substantially less than one finds in most fourth or fifth grade textbooks.

In my opinion, the greatest failing of the book is an almost complete lack of deductive reasoning even at this intuitive level. Following the definition of vertical angles appears the sentence, "Vertical angles are also congruent angles." Certainly it is reasonable to ask for, indeed demand, an explanation or intuitive proof of this statement as well as other statements of comparable difficulty.

Among the inaccuracies and ambiguities in the book we find the statement A = {A, E, I, O, U}, where A is a set and an element of the same set. The statement, "The pairs of angles which have a common side are called *adjacent* angles" contradicts Definition 2.12. In the definition of vertical angles, it appears that every pair of coplanar lines intersect. The discussion of "continuous sets" is confusing and there is the implication that only discrete sets have a measure. The method of finding the measure of the sum of two adjacent angles is incorrect if the sum is greater than 180°.

While my overall impression of the book is negative there are some strong points. The authors have succeeded in writing at the level that many prospective elementary teachers desire and comprehend. In this regard the book is more desirable than most of the pseudo-rigorous and often overwhelming competitors. The wealth of exercises (Problem and Activity Sets) are for the most part very good. The third chapter nicely shows the analogy between linear and angular measure. Section 5.5 on the sphere contains a good exposition of parallels of latitude, meridians, and time zones on the earth.

We would be performing a disservice to elementary school teachers if we limit their geometric knowledge to this book. Of course, the students of these teachers have the most to lose. For use with a class, the book needs to be supplemented by the instructor to a large degree so that elementary school teachers will not consider geometry simply as a long and tiring list of definitions.

Alan R. Hoffer
University of Oregon

Essentials of Trigonometry, Irving Drooyan, Walter Hadel, and Charles C. Carico, Macmillan, New York, 1971, 352 pp., $8.95.

The authors state that "this text is intended for use in a one-semester or one-quarter course in trigonometry." The reviewer believes that the text can be used with either a high school or college population and that this is sufficient material for a one-semester college course or one-year high school course in trigonometry. The text is traditional in the sense that it contains the topics we would expect to find in an elementary trigonometry course. At the same time it is modern in the sense that it contains precise language which is carefully used. It develops proofs in a manner which can be followed by the student.

An abundant number of applied problems and numerical examples help to give the concepts clearer meaning. A few well chosen proofs are given as exercises, but these are well developed in order to lead the student through the proof. The proof of Heron's Formula is such an exercise which is given at the end of a sequence of exercises leading up to it.

The text has pleasant use of normal and bold face print as well as the use of color in some printing and in the parts of figures under discussion. The chapter summaries and review exercises provide good opportunities to tie together the concepts developed in the chapter.

Logarithms are covered in an appendix along with their application to the solution of triangle problems. With this approach logarithms can be used throughout the course, at the end, or not at all depending on the aims of a particular course. The reviewer could find no use of series to compute function values, nor of any mention of using slide rules, calculators, or small or large computers to assist students with problems.

All things considered, this text should rank high on any list of books usable for a course in trigonometry. It would be usable from both the student's and the teacher's point of view.

Robert A. Estes
University of Maine at
Portland-Gorham

Intermediate Algebra, Ward D. Bouwsma, The Macmillan Company, New York, 1971, 359 pp., $8.50.

The stated goal of the author was to write an algebra course for junior colleges and four year colleges which would train students to perform standard manipulations and also prepare them for courses in statistics and analytic geometry. The prerequisite assumed by the author is a year of high school algebra.

The text covers basic concepts of algebra including a treatment of fields; linear equations and inequalities; systems of linear equations and inequalities; quadratic equations; complex numbers; functions; polynomial, exponential, and logarithmic functions; and finite number systems including an intuitive discussion of finite differences.

The text is very easy to read and throughout the text the examples are set off in blue rectangles for greater emphasis. The exercises appear reasonable and rather traditional. Answers are provided to most of the odd-numbered problems and a few even-numbered problems. A check of the answers of a small sample revealed no incorrect answers.

The most impressive features of the text were the use of discovery oriented discussions in an attempt to provide students with creative approaches to problem solving, the use of examples from various fields as applications of mathematics, and historical remarks where appropriate.

A few minor criticisms might be offered; namely, there is no summary of concepts for any of the chapters, nor a list of properties and/or theorems covered in the chapter, and no chapter review. It would be helpful to have a set of exercises for comprehensive review spaced from time to time throughout the text.

The overall impression was that the text should be considered seriously for a course in intermediate algebra from the modern viewpoint for colleges and junior colleges.

Ronald D. Dettmers
Wisconsin State University
at Whitewater

Elementary Statistics, 3rd Ed., Paul G. Hoel, John Wiley and Sons, Inc., New York, 1971, 319 pp., $9.50.

This text is designed for a one semester general introductory statistics course for those students whose background is limited to algebra. This book differs from the other two editions principally in organization and exposition.

The topics covered in the first nine chapters are quite usual for texts of this kind. They are: descriptive statistics, probability and probability distributions, sampling, estimation, testing hypothesis, correlation, and regression. In a section labeled Special Topics, the reader finds statistical decision, chi-square, ANOVA, and non-parametrics. There is a good problem set at the end of each chapter. The problems are ample in number and in general illustrate applications to various disciplines. The discrete probability problems, however, mainly concern cards, dice, and urns. This reviewer would prefer more "real" applications of discrete probability. Answers to odd-numbered exercises are in the text with the answers to even-numbered exercises available from the publisher. The appendix has the usual tables of square root, random numbers, combinations, and percentiles of various probability distributions. This reviewer considers the set of tables adequate for a text of this type.

The author does not use any set notation although a sample space is defined in the usual manner as a set of outcomes. The probability of combinations of two or more events is denoted by using "and" and "or" rather than "$\cup$" and "$\cap$". Descriptive statistics is given only minor attention while considerable space in the text is devoted to probability, probability distributions, and sampling. This reviewer thought the level in these sections appropriate for the mathematical preparation of students who would use such a text. There are many well-chosen figures used in illustrating the various concepts.

Professor Hoel is a distinguished and respected author in the field of probability and statistics. This text is well written with very few errors. The mode of presentation is similar to that of many other texts written in the area. It is a text that should be considered by teachers of a non-calculus introductory course in statistics.

Wilbur Waggoner
Central Michigan University

Computers and Their Uses, Second Edition, William H. Desmonde, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1971, 296 pp., $8.95

"Encyclopedic" is the word to describe this book. It contains material ranging from solid state physics to sociology. Notable by its absence, however, is any detailed description of the usual higher-level programming languages. Hence this is not a book for those desiring proficiency in such a language, nor does it purport to be. Nevertheless it should be read by any serious student of programming.

Dr. Desmonde has taken great pains to present, in simplest possible terms, descriptions of such intricacies as representation of information, machine logic, mchine language programming, and programming techniques. But following this material there are several of the most interesting chapters in the book. Drawing on what must clearly be a wide experience in the computer field, Dr. Desmonde describes many of the "real world" applications of computer technology. Included are applications to physics, geology, engineering, biology, and business. Perhaps the latter are the most provocative, since they include descriptions of commonly encountered things: for example, the SABRE airline reservation system, medical records processing, SAGE data processing network for air defense, and commercial data processing such as keeping the records for checking accounts.

The technical quality of the book is good. Diagrams are clear and strategically placed. Misprints are few and then only misspellings. The reader should not overlook the review questions for each chapter, which all come after the last chapter of the book. Those considering this as a text for a course should likewise examine the review questions.

As stated earlier, any serious student of programming would do well by himself to read this book. His preparation mathematically need be only that usually require for college entrance, though some "mathematical maturity" may be needed in the chapters on representation of information, machine logic, and algebra of automata. Some familiarity with computer programming would surely be an asset to understanding this book, but there is a good deal to be gained by the individual who is just starting out and is looking for a general introduction to the field of computers and computer programming.

James E. McKenna
State University College at
Fredonia, New York

Nineteenth Biennial Convention

April 5-7, 1973

The ninteenth biennial convention of Kappa Mu Epsilon will be hosted by the Iowa Gamma chapter and will be held on the campus of Morningside College on April 5-7, 1973. Students are encouraged to prepare and submit papers for presentation at the convention. Complete directions for the submission of papers are found on page 113 of this issue of *The Pentagon*.

All chapters are encouraged to plan early for as large a delegation of students and faculty as possible.

George R. Mach
National President

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgment will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Sciapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

Many puzzles have been invented which fascinate both young and old. Possibly in your childhood you came across a puzzle which consisted of fifteen square blocks, sometimes numbered from 1 to 15, placed in a shallow square tray large enough to hold sixteen blocks. One of the sixteen positions is an empty space as shown in Figure 1.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

FIGURE 1

The tray as well as the blocks are manufactured in a tongue and groove fashion so that the blocks will easily slide within the tray and cannot be removed. Usually, certain arrangements of the blocks are stated to be possible while others are given as impossible. After purchasing the puzzle, one usually tries the possible as well as the impossible cases, hoping to prove that some of the impossible cases are indeed possible. Have you ever wondered why some configurations are known to be impossible?

In considering this puzzle, we shall consider the empty space as though it were any one of the other fifteen blocks, giving it no special properties. We first must consider what we mean by a "move". A "move" will consist of sliding a block adjacent to the empty space into the empty space as shown in Figure 2. A "move" can only be made horizontally or vertically. If one were to consider the empty space to be block 16, then the move in Figure 2 is an interchange of blocks 3 and 16.

2	3	4		2		4
6		8	→	6	3	8
13	14	1		13	14	1

FIGURE 2

Let the blocks be in numerical order as in Figure 1 and the empty space be block 16. We can also consider the sixteen positions on the tray to form a checker board of *E*'s and *O*'s as shown in Figure 3.

E	*O*	*E*	*O*
O	*E*	*O*	*E*
E	*O*	*E*	*O*
O	*E*	*O*	*E*

FIGURE 3

You will note that the odd numbered blocks in Figure 1 are not necessarily *O*'s in Figure 3 and, similarly, the even numbered blocks in Figure 1 are not necessarily E's. Block 16 (the empty space) is originally in an E space. But after one move, it must be in an O space (either interchanged with block 15 or 12). After a second move, it will again return to an E space. With each additional move, block 16 will always move from an E space to an O space or visa versa, since it must move either vertically or horizontally as shown in Figure 4.

E	*O*	*E*
O	16	*O*
E	*O*	*E*

→

O	*E*	*O*
E	16	*E*
O	*E*	*O*

FIGURE 4

Hence, after an even number of moves, the empty space (block 16 must be an E space and after an odd number of moves it must be an O space. Therefore, if you begin with the puzzle as represented in Figure 1, and wish to conclude the puzzle so that the blank space is an E space, this must be done using an even number of moves. Some of the conceivable configurations of the blocks 1 through 16 cannot ever be achieved by an even number of interchanges of two blocks at a time. If the configuration you wish to achieve is of this type, it will be an impossible case.

Similarly, if you start with the puzzle as represented in Figure 1 and wish to conclude the puzzle so that the blank space is an O space, this must be done using an odd number of moves. Some of the conceivable configurations of blocks 1 through 16 will never be achieved by an odd number of interchanges of two blocks at a time. If your desired configuration is of this type, it is an impossible case.

To consider the impossible cases more fully, we need the following definition and theorem:

DEFINITION. *A transposition is the interchange of a pair of numbers (blocks).*

THEOREM. *Any rearrangement of the numbers 1 through 16 can be accomplished by some series of transpositions. If a desired rearrangement of the numbers 1 through 16 can be accomplished by an even number of transpositions, then every series of transpositions which produces that rearrangement will be achieved by an even number of transpositions. If a desired rearrangement can be accomplished by an odd number of transpositions, then every series of transpositions which produces that rearrangement will consist of an odd number of transpositions.*

More than a hundred proofs are currently known for this theorem. Most of them are quite heavily involved with minute details. Hence, I will now state that this proof is "clearly" true.

From the theorem, it follows that all possible rearrangements of the numbers 1 through 16 can be placed in one of two categories: even rearrangements or odd rearrangements depending upon the number of transpositions necessary. The theorem further implies that each possible rearrangement is either even or odd but not both.

To determine whether a desired rearrangement is even or odd, we shall work the problem in reverse. That is, we shall begin with the desired rearrangement and apply the various transpositions necessary until we have transformed it to the normal listing of the numbers 1 through 16. The same sequence of transpositions applied in reverse order will convert the normal listing of numbers into the desired rearrangement.

As an example, let the desired rearrangement of the numbers 1 through 16 be:

13 10 2 6 12 5 16 4 1 3 9 15 14 11 7 8

To put 1 in its proper position, we must first transpose 1 and 4, then 1 and 16, 1 and 5, 1 and 12, 1 and 6, 1 and 2, 1 and 10, and finally 1 and 13. You will note that 1 was transposed with each number to the left of it which is greater than 1. To place the number 2 in its proper position, we transpose it first with the number 10 and then with 13—the only two numbers to the left of 2 which are greater than 2. Similarly, to place

13 10 2 6 12 5 16 4 1 3 9 15 14 11 7 8

1 13 10 2 6 12 5 16 4 3 9 15 14 11 7 8

1 2 13 10 6 12 5 16 4 3 9 15 14 11 7 8

1 2 3 13 10 6 12 5 16 4 9 15 14 11 7 8

1 2 3 4 13 10 6 12 5 16 9 15 14 11 7 8

1 2 3 4 5 13 10 6 12 16 9 15 14 11 7 8

1 2 3 4 5 6 13 10 12 16 9 15 14 11 7 8

1 2 3 4 5 6 7 13 10 12 16 9 15 14 11 8

1 2 3 4 5 6 7 8 13 10 12 16 9 15 14 11

1 2 3 4 5 6 7 8 9 13 10 12 16 15 14 11

1 2 3 4 5 6 7 8 9 10 13 12 16 15 14 11

1 2 3 4 5 6 7 8 9 10 11 13 12 16 15 14

1 2 3 4 5 6 7 8 9 10 11 12 13 16 15 14

1 2 3 4 5 6 7 8 9 10 11 12 13 14 16 15

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

FIGURE 5.

the number 3 in its proper position, we transpose it first with 4, then with 16, 5, 12, 6, 10, and finally 13. Again, each of these seven numbers is to the left of 3 and greater than 3. The rest of the numbers are similarly transposed as shown in Figure 5.

From Figure 5 the following table can be derived:

Number	Transpositions Necessary to be in Proper Position
1	8
2	2
3	7
4	6
5	4
6	2
7	8
8	8
9	4
10	1
11	5
12	1
13	0
14	2
15	1
16	0
	59 = Total Number of Transpositions

Since 59 is odd, it follows from the theorem that it will always require an odd number of transpositions to obtain the desired arrangement. But block 16 in the desired arrangement is in an *E* space which requires an even number of moves. Hence, the desired configuration is an impossible one. To be a possible configuration, the parity of the number of transpositions must match the parity of the position of block 16 in the desired rearrangement.

From the previous example, you may have noticed a shortcut method for determining the number of transpositions or "switches" necessary to place a number in its proper position. In general, to determine the number of transpositions necessary to move a number into its proper position, we count the number of times a larger number appears to the left of a smaller number in the desired rearrangement. For example, for the number 7, there were eight numbers to the left of 7 which were greater than 7 (namely 13, 10, 12, 16, 9, 15, 14, and 11). Thus eight transpositions were necessary as shown in the table.

Can you determine whether the following arrangement is possible?

1	2	3	4
8	6	13	14
10	5	7	15
16	11	12	9

< — — — >

After studying number bases, one many times gets the feeling that all number systems use the digits 0, 1, 2, ... However, an unusual number system is one which allows us to construct all integers using only the three symbols 0, +, and −. This can be done by writing:

Number	*New System*	*Number*	*New System*
.	.	3	+ 0
.	.	4	+ +
.	.	5	+ − −
−6	− + 0	6	+ − 0
−5	− + +	7	+ − +
−4	− −	8	+ 0 −
−3	− 0	9	+ 0 0
−2	− +	10	+ 0 +
−1	−	.	.
0	0	.	.
1	+	.	.
2	+ −		

Upon carefully considering the above system, you will find that each space in the system corresponds to a power of 3. That is, the spaces from right to left correspond to 3^0, 3^1, 3^2, ... The power of 3 is added if the space has a +, subtracted if it has a −, and neglected if it has a 0. Hence for + 0 − we have $3^2 + 0 + (-1) = 8$ and for − + 0 we have $-3^2 + 3 + 0 = -6$.

Using the system indicated above, one can define the usual operations of addition, subtraction, etc. Can you combine by addition + + 0 − 0 + and − + + −?

(continued from p. 79)

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Directions for Papers to Be Presented At the Nineteenth Biennial Kappa Mu Epsilon Convention

Sioux City, Iowa

April 5-7, 1973

A significant feature of this convention will be the presentation of papers by student members of KME. The mathematics topic which the student selects should be in his area of interest and of such a scope that he can give it adequate treatment within the time allotted.

WHO MAY SUBMIT PAPERS: Any student KME member may submit a paper for use on the convention program. Papers may be submitted by graduates and undergraduates; however, graduates will not compete with undergraduates.

SUBJECT: The material should be within the scope of the understanding of undergraduates, preferably those who have completed differential and integral calculus. The Selection Committee will naturally favor papers within this limitation and which can be presented with reasonable completeness within the time limit prescribed.

TIME LIMIT: The usual time limit is twenty minutes, but this may be changed on the recommendation of the Selection Committee if requested by the student.

PAPER: Four copies of the paper to be presented, together with a description of the charts, models, or other visual aids that are to be used in the presentation, should be presented to the Selection Committee. A bibliography of source materials, together with a statement that the author of the paper is a member of KME, and his official classification in school, undergraduate or graduate, should accompany his paper.

DATE AND PLACE DUE: The papers must be submitted no later than January 12, 1973, to the office of the National Vice-President.

SELECTION: The Selection Committee will choose ten to twelve papers for presentation at the convention. All other papers will be listed by title and student's name on the convention program and will be available as alternates.

William R. Smith
National Vice-President, Kappa Mu Epsilon
Department of Mathematics
Indiana University of Pennsylvania
Indiana, Pennsylvania 15701

(continued from p. 95)

Solution by the proposer, R. S. Luthar, University of Wisconsin, Waukesha, Wisconsin.

The vertex A is the point $(3a/2, 3a/2)$. Any line through the vertex can be written as $y - 3a/2 = m(x - 3a/2)$. Eliminating a between the above equation and the equation of the given curve, we obtain the equation, $3(1 - m)(x^3 + y^3) = 2xy(y - mx)$, which shows three lines joining the origin O with the points A, P, and Z. The last equation can be re-written as $(x - y)[x^2(1 - m) + xy(1 + m) + y^2(m - 1)] = 0$. Since $x - y = 0$ represents OA, the other factor represents the other two lines. Clearly the product of the slopes of these lines is -1, implying that these lines are perpenducular to each other.

Kappa Mu Epsilon News

EDITED BY ELSIE MULLER, *Historian*

News of Chapter activities and other noteworthy KME events should be sent to Elsie Muller, Historian, Kappa Mu Epsilon, Department of Mathematics, Morningside College, Sioux City, Iowa 51106.

CHAPTER NEWS

Arkansas Alpha, Arkansas State College

Chapter President—Terry Short

11 members

The chapter meets once a month with a lecture by a member of the faculty. Topics have included the Jordan Curve Theorem for Polygons and the Fibonacci Numbers. Recently, they have been corresponding with the Henderson State College Mathematics Club of Arkadelphia, Arkansas in order to assist them in affiliating with KME.

California Gamma, California State Polytechnic College, San Luis Obispo

Chapter President—Hal D. Norman

50 student members and 20 faculty members

The chapter meets monthly and features both student and faculty speakers. As for activities the members assist the mathematics department in an annual mathematics contest involving about 500 high school students. The actives also staff a mathematics laboratory which offers free tutoring to all mathematics students at the college. Two initiation banquets feature outside speakers.

California Delta, California State Polytechnic College, Pomona

Chapter President—Eddy W. Hartenstein

12 actives—15 pledges

The chapter meets biweekly at which time it sponsors guest speakers and seminars. Free tutoring is offered daily.

Colorado Alpha, Colorado State University

Chapter President—Galen Carter
30 members—12 pledges
As a special project the chapter has sponsored an Alumni Seminar on Employment Opportunities in the Mathematical Sciences. The all-day program featured CSU Mathematics Alumni who spoke about job opportunities in areas related to mathematics as found in industry, education, and computer science. Meetings are held in the homes of mathematics professors. Topics have included "Favorite Mathematical Puzzles," "Linear Programming," "Combinatorics," and "Biology and Mathematics."

Florida Alpha, Stetson University

Chapter President—Michael Camp
9 members—11 pledges
The chapter is becoming much more active. It sponsored a county-wide two-day mathematics contest in February. Also, it is conducting a weekly mathematics quiz in the campus newspaper. The past president is now at the University of Chicago where he is a special student.

Illinois Alpha, Illinois State University, Normal

Chapter President—John Lally
23 members—30 pledges
The chapter holds 8 meetings during the year with a picnic in May.

Illinois Gamma, Chicago State University

Chapter President—Timothy O'Donnell
3 members—20 pledges
Students who were interested in joining Kappa Mu Epsilon met with Dr. Hardy, chairman of the department of mathematics, and expressed concern about the inactivity of the chapter. Mr. Don Bunt, assistant professor of mathematics, agreed to act as corresponding secretary and faculty sponsor for the chapter. Mr. Bunt, Mr. O'Donnell, and Mr. Jim Griffin met to draft a letter to national headquarters in order to get help.

Illinois Zeta, Rosary College

Chapter President—Sherry Treston
15 members

Judith Yocins, Margaret Sweda, Judith Schwalbe, Marie Negri, and Anne Hayes are recent new members. At the December meeting Terri Delke presented a report of her research in programming the game of Monopoly. Assignments for tutoring are handled by the chapter president.

Illinois Eta, Western Illinois University

Chapter President—Dorothy Anderson
15 members—7 pledges

Regular monthly meetings are held at which time the students present papers and conduct discussions. The annual banquet was held on 23 February; Professor J. E. Wetzel of the University of Illinois was the speaker. Picnics are held both in the spring and fall. The chapter offers free tutoring in the dormitories.

Indiana Delta, University of Evansville

Chapter President—Lana Turner
78 members

The chapter publishes a monthly newsletter, *The Straight Line*, which carries announcements of KME activities, student and faculty reports, reviews of mathematics books, challenging problems, and other news articles related to mathematics. Members have convinced the University Bookstore to stock paperbacks of some mathematical content. An article, "Emmy Noether: Greatest Woman Mathematician" by Professor Clark Kimberling appeared in the February issue of the *American Mathematical Monthly*.

At one of the meetings Dr. Donald Koehler from Miami University of Oxford, Ohio discussed in a lively way some basic counting problems in the serial composition of music. At another time one of the students, Mr. Ernest Nolan, spoke on data screening with the use of flowcharts. In November another student, Robert Thompson, presented an analysis of collegiate football scores. By using regression analysis and the SSP package for IBM 360, he was able to present the 1970 college football scores in several unusual and informative ways. In January a student, Samuel Lawrence, spoke on the motion of a doorbell clapper, using calculus and differential equations. In February, Professor V. C. Bailey discussed the theory of curve tracing and the creation of new curves.

Dr. Traver C. Sutton, head of the Science Department at Adirondack-Southern School, St. Petersburg, Florida, was awarded the Humanitarian Award of the University of Evansville last December. The citation read, in part, "We are grateful for his unselfish devotion to his students, his care for their well-being and his zeal for learning and for life. Through his example and his occasional exhortation, Dr. Sutton's students have profited in unmeasurable ways, as have all who have known him."

Iowa Alpha, University of Northern Iowa

Chapter President—Cathy Cable Porter
25 members

Monthly meetings are held with students presenting papers. The initiation banquet was held in January with a paper given by a new initiate. Other officers are: Ronald Lamb, vice-president; Charles Doss, secretary and treasurer; John Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University

Chapter President—Lonnie Yoder
13 members—8 pledges

Other officers are: Tom Brackett, vice-president; Joel Hanusa, secretary; Sandra McChesney, treasurer; Professor Wayne Woodworth, corresponding secretary; Professor Alexander Kleiner, Jr., faculty sponsor.

Iowa Gamma, Morningside College

Chapter President—Sue Nelson
28 members

A guest lecturer in December was Professor W. A. Thompson of the University of Missouri at Columbia. At the initiation dinner in March Professor Warren Loud of the University of Minnesota gave a lecture on noninvertible matrices, a new topic which he has worked into a lecture, and the Iowa Gamma chapter was the first to hear it.

In April the chapter hosted a mathematics seminar on *Mathematical Models and Their Applications* for faculty and students in Colleges of Mid-America, Inc., an association of eleven liberal arts colleges in the area. The luncheon featured the relationship of mathematics to other disciplines in the liberal arts.

Kansas Alpha, Kansas State College of Pittsburg

Chapter President—Mark Davis

35 members

Recipient of the annual Robert Miller Mendenhall Award for scholastic achievement was John Thornton. He received a KME pin in recognition of this achievement. Programs have been presented by Kerry Ryman and Terry Viets. Kerry discussed properties associated with zero and Terry presented an approximation for pi.

Kansas Beta, Kansas State Teachers College of Emporia

Chapter President—Leslie Kinsler

40 members

Programs have dealt with binomial distributions, generation of integration tables, and the [ ] function. Other officers are: Bryan Schurle, vice-president; Barbara Kukuk, secretary; Philip Amburn, treasurer; Charles Tucker, corresponding secretary; Thomas Bonner, faculty sponsor.

Kansas Gamma, Benedictine College

Chapter President—Ann Baumann

10 members plus 4 faculty, 9 pledges

Plans are being made for an invitational high school mathematics contest to be held at Benedictine College for area schools. This will be the fifth such event sponsored by Kansas Gamma in the even-numbered years. Sister Malachy Kennedy, charter member of Kansas Gamma and moderator last year, died 15 October 1971 after a long illness. Sister Malachy continued to be active as associate professor of mathematics at Mt. St. Scholastica until less than a year before her death. During the current academic year Sister Helen Sullivan is an academic counselor for American students abroad in Vienna, Austria with the Institute of European Studies. Five program meetings were held during the first semester with topics ranging from the Königsberg bridge problem and topological notions to student teaching experiences.

Kansas Delta, Washburn University

Chapter President—Michael Wellman

Other officers are: Frank Fusseneger, vice-president; Carol Marie Klein, secretary; Marry Murrow, treasurer; Margaret Martinson, corresponding secretary; Harlan J. Koch and Robert H. Thompson, faculty sponsors.

Kansas Epsilon, Fort Hays Kansas State College

Chapter President—William Paget

18 members

The fall meetings have featured programs on equivalent infinite sets, flexagons, and a computer demonstration. The lecture at the spring banquet consisted of music anecdotes of the life of Hilbert. Other officers are: Thomas Lonnon, vice-president; Rita Mills, secretary and treasurer; Eugene Etter, corresponding secretary; Dr. Elton Beougher, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland

Chapter President—Michele Mules

9 members, 4 pledges

The chapter celebrated the 500th anniversary of Albrect Durer's birthday in December. Jeanette Gilmore and Sister Marie Augustine led a team discussion of his art and his contribution to geometry. In November, Michele Mules delivered a paper on Markov chains.

Maryland Beta, Western Maryland College

Chapter President—Bonnie Green

28 members

Dr. Rosenzweig, a new staff member, delivered the talk at the November initiation on the topic, "Classic Impossible Geometric Construction—Why." In January Dr. Mario Borelli of Notre

Dame presented two talks to the chapter. The initiation was held in March and the picnic in May.

Maryland Gamma, St. Joseph College

Chapter President—Linda Raudenbush
7 members, 5 pledges

On 12 February, there was a campus-wide "Women's Day." The members and pledges presented "Women in Mathematics" and "Opportunities for Today's Women in Mathematics." At the initiation Sister Marie Augustine Dowling, the faculty sponsor of the Maryland Alpha chapter, lectured on "The Super Egg." The chapter planned to actively participate in the proposed spring regional convention. The chapter bulletin board displays mathematical recreations and encourages both students and faculty to indulge.

Michigan Beta, Central Michigan University

Chapter President—Pam Gilbert
66 members

The chapter does tutoring for the elementary classes and sponsors a freshman mathematics examination on campus. The group planned to attend the region II convention in Milwaukee. Other officers are: Neal Eichler, vice-president; Kathryn Bryson, secretary; Steve Caffrey, treasurer; Dean Hinshaw, corresponding secretary and faculty sponsor.

Mississippi Alpha, Mississippi State College for Women

Chapter President—Deborah Ann Moss
16 members

The chapter holds monthly meetings, one of which dealt with professional opportunities in mathematics. Other officers are: Betty Sue Johnson, vice-president; Kathryn E. White, secretary and treasurer; Donald A. King, corresponding secretary and faculty sponsor.

Mississippi Delta, William Carey College

Chapter President—Fred Hawg
11 members, 3 pledges
A social was held for all KME members at the faculty sponsor's home just before Christmas vacation. Other officers are: Charles Ernest, vice-president; Susan Langston, secretary; W. N. Fellabaum, corresponding secretary; Dr. Gaston Smith, faculty sponsor.

Missouri Alpha, Southwest Missouri State College

Chapter President—Peggy Stuckmeyer
40 members
Meetings are held on the second Tuesday of each month. In November Mr. James Downing reported on three intriguing problems. In December Dr. Tom Shiflett gave the chapter history. Ten new members were initiated.

Missouri Beta, Central Missouri State College

Chapter President—Paul Bowman
31 members, 6 pledges
In addition to eight meetings and two initiations per year the chapter conducts a field trip and holds a spring banquet.

Missouri Gamma, William Jewell College

Chapter President—Ken Kruse
19 members
Mr. Bob Sherrick, statistician for Hallmark Cards at Kansas City, Missouri gave a program. The chapter hosted the biennial KME convention for the midwest region IV on 15 April 1972. A significant feature of the convention was the presentation of eight papers, limited to 20 minutes, by student members of KME.

Missouri Epsilon, Central Methodist College

Chapter President—Mary Jane Thornton
9 members, 2 pledges
Dr. Roy Utz of the University of Missouri was a guest speaker. Other officers are: Ed Knight, vice-president; Laurie Muns, secretary and treasurer; Dr. William McIntosh, corresponding secretary and faculty sponsor.

Missouri Zeta, University of Missouri—Rolla

Chapter President—Leonard Laskowski
25 members, 15 pledges

The main service project that has been set up was the conducting of help sessions for the three classes in the calculus series. Also, KME has been transcribing books and lessons onto tapes as well as debugging programs for a blind student at UMR who is majoring in computer science. In October Mr. Raider and Mr. Blazer from Bell Telephone Laboratories gave a talk on job opportunities in the Bell system. Dr. Hatfield, professor of mathematics at UMR, was the after dinner speaker on the topic, "Visual and Abstract Thinking," at the largest KME initiation banquet ever held at UMR. Other officers are: Dana Nau, vice-president; Curt Killinger, treasurer; Debbi Fuggitt, secretary; Peggy Shackles, historian; Dr. Rakestraw, corresponding secretary; and Dr. Joiner, faculty advisor.

Nebraska Alpha, Wayne State College

Chapter President—Dale Ruehling
16 members, 16 pledges

In April a committee constructed and administered an examination which determined the outstanding freshman mathematics major at Wayne State. Students who take the examination are nominated by the faculty of the mathematics department. Lyle Nelson of Wayne, Nebraska was selected outstanding freshman mathematics major for 1970-1971. The award includes honorary membership in KME with initiation fees being paid by the club, the placement of his name on the permanent plaque, and public announcement of the award at the spring Honors Convocation.

Nebraska Beta, Kearney State College

Chapter President—Dennis Fisher
34 members, 14 pledges

Two faculty auctions have been held to raise money for the scholarship fund. Two were awarded to KME members. As for programs, a lecturer whose specialty is actuarial science visited the chapter, video-tape programs have been shown, and a lecture on degrees in computer science was delivered. Help sessions are held twice a week for other students. Other officers are: David Hays, vice-president; Peg Walters, secretary; Dan Mowrey, treasurer; Richard Barlow, corresponding secretary; Dr. Randall Heckman, faculty sponsor.

Nebraska Gamma, Chadron State College

Chapter President—Bruce Morley
32 members, 3 pledges
Programs at the chapter meetings are short talks given by any student who volunteers. Some topics have been Fibonacci numbers, major magic squares, and others. During the first two days of the semester the chapter held a book exchange as a service. Other officers are: Terry Welke, vice-president; Dana Lyn Foster, secretary; Rick Nowak, treasurer; Lenora Briggs, corresponding secretary; and Dr. Olson, Faculty sponsor.

New Mexico Alpha, University of New Mexico

Chapter President—Jacquelyn Krohn
70 members
The chapter reports that they are rather inactive. Other officers are: John Stark, vice-president; Dawn Tinsley, secretary; Timothy Burns, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

New York Eta, Niagara University

Chapter President—Mike White
17 members
The emphasis in the chapter meetings is on student papers. The guest speaker in the fall was Dr. Gerald Rising of the State University of New York at Buffalo on "Mathematical Trivialities." The chapter planned to attend the regional convention in March at Mount Mary College in Milwaukee, Wisconsin.

New York Theta, St. Francis College

Chapter President—Michael Polito
41 members
The chapter has accomplished the following:

1. *Freshman Calculus Workshop.* KME members present selected topics in calculus to the freshman students.

2. *Film a Month.* An advanced MAA film is shown once a month.

3. *Chapter Magazine.* For the second year the chapter is publishing a *Proceedings.*

4. *Mathematics Bowl.* A mathematics contest is held between St. Francis and other neighboring institutions. On 8 December the other contestant was Molly College.

5. *Field Trip.* On 4 October the chapter journeyed to IBM, Poughkeepsie.

6. *Student Papers.* Three have been given this term.

7. *Faculty Speaker.* Professor Joseph Layzara presented a talk on numerical analysis.

Ohio Alpha, Bowling Green State University

Chapter President—Janice Csokmay

37 members, 30 pledges

The theme for the year in the chapter meetings is "Opportunities in Mathematics." The program, "Is Graduate School for You," featured a panel of three, a graduate student, a new Ph.D. mathematics professor, and the dean of the graduate school. A speaker from the placement office talked on job opportunities for mathematicians in government and industry. A copy of the chapter booklet, *Origin,* was completed in early February. A KME mathematics prize examination, a test with problems of varying degree of difficulty has been developed by the faculty and is given in February. The members hosted a "Meet the Profs" party for their first meeting.

Ohio Gamma, Baldwin-Wallace College

Chapter President—Ida Mae Olin

9 members

The chapter is in the process of organizing a mathematics club for students not eligible for KME. Speakers at meetings have talked on the relationship of mathematics to other disciplines and one on graduate school opportunities. In addition to a field trip to Chi Corporation Computer Center there are also the spring and fall picnics. Other officers are: Robert Gregory, vice-president; Ann Drapos, secretary; Sharon Sante, treasurer; Professor Robert Schlea, corresponding secretary and faculty sponsor.

Oklahoma Alpha, Northeastern State College

Chapter President—Joe Morris
38 members
Raymond Carpenter, Ed.D., professor of mathematics at Northeastern State, Tahlequah, Oklahoma, retired on 1 December 1971. He taught at Northeastern from January 1946, until his retirement. He served as sponsor of the Oklahoma Alpha chapter from 1946 until 1964. Since 1964, he has served as corresponding secretary for the chapter. Other officers are: James McFarland, vice-president; Mrs. Derry Venters, secretary; Ronald Smith, treasurer; Mr. Mike Reagan, corresponding secretary and faculty sponsor.

Pennsylvania Alpha, Westminister College

Chapter President—Donald J. Dawson
40 members
Programs have stressed career opportunities both in occupations and graduate schools. Other officers are: John Giesmann, vice-president; Charlene Holt, secretary; Sandy Larsen, treasurer; J. Miller Peck, corresponding secretary; and Dr. Thomas Nealeigh, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania

Chapter President—John Nelson
59 members, 3 pledges
Three of the programs have featured the following speakers, Dr. Joseph Angelo who spoke on "Formalism in Mathematics," Dr. John Droughton on "Lebesgue Integration," and Dr. John Hoyt on "Fibonacci Numbers."

Pennsylvania Theta, Susquehanna University

Chapter President—Doreen Bolton
One of the interesting meetings consisted of playing "Hollywood Squares" by using mathematics questions and featuring nine professors of the mathematics department as celebrities. Alyce Zim-

mer received the Stine Mathematics Award, which is given to the junior with the highest mathematics average.

Pennsylvania Iota, Shippensburg State College

Chapter President—Leroy Jones

33 members, 4 pledges

At the joint spring picnic for KME and the mathematics club, Dr. Dangello, the treasurer of the chapter, served as an auctioneer of mathematics books and assorted specialities of the mathematics professors. At the fall annual chapter banquet Dr. William McArthur and Dr. Howard Bell gave a talk on the applications of mathematics, particularly in the caging of a lion in the Sahara Desert. The constitutional revision committee has been busy to prepare for the change from term to semester plan. The chapter is awarding a first prize of $25 for the best undergraduate paper in the field of mathematics.

Pennsylvania Kappa, Holy Family College

Chapter President—Kathy Stehr

4 members

Mrs. Kathleen Kuiz is in the process of preparing a paper on "Cardinality" which she will present to the Beta Chi Mathematics Club. The chapter, in conjunction with the mathematics club, did sponsor a speaker, Dr. Peter Hagis of Temple University, who spoke on *Prime Numbers*. In February, three new members, Maria Herczeg, Brenda Nadyika, and Kathleen Newton, were initiated. At the chapter meetings articles from *The Mathematics Teacher*, the *American Mathematical Monthly*, and the *Pentagon* are discussed.

Tennessee Beta, East Tennessee State University

Chapter President—Beverly Ann Price

30 members

The chapter is now a member of the new East Tennessee State University Honor Societies Council whose purpose is to promote cooperation among and recognition of the national honor societies on the campus. Other officers are: Linda Gail Perry, vice-president; Donna Bowler, secretary; Ken Oster, treasurer; Lora D. McCormick, corresponding secretary; Sallie P. Carson and T. H. Jablonski, faculty sponsors.

Tennessee Gamma, Union University

Chapter President—Becky Whik
14 members
Meetings are held monthly. Other officers are: Dwayne Jennings, vice-president; Martha Woffard, secretary; Morrelle Sartain, treasurer; Richard Dehn, corresponding secretary; Dr. Joe Tucker, faculty sponsor.

Texas Gamma, Texas Women's University

Chapter President—Tonya Hancock
3 members
Meetings have been held to plan a money-making project and to plan the spring initiation. Other officers are Connie Cayard, vice-president; Glenna Pettit, secretary and treasurer; Mrs. Turner Hogan, corresponding secretary, and Turner Hogan, faculty sponsor.

Texas Zeta, Tarleton State College

Chapter President—Bruce McNellie
13 members, 5 pledges
Topics for programs were the following: "The Properties of Infinity," "Distance and Imagination," "What's Zit?," "Techniques of Integration not Taught at TSC," "Let's Teach Guessing." Other officers are: Mark Eakin, vice-president; Rhonda Gillum, secretary; Suzan Smith, treasurer; Timothy Flinn, corresponding secretary; Conley Jenkins, faculty sponsor.

Wisconsin Alpha, Mount Mary College

Chapter President—Betty Witt
10 members, 5 pledges
Because this is the twenty-fifth anniversary year of the installation of Kappa Mu Epsilon on the Mount Mary campus, the Wisconsin Alpha chapter hosted the regional meeting on 24–25 March. The annual mathematics contest for high school students

was held on 18 March. Other officers are: Linda Hilgendorf Daute, vice-president; Patricia Rass, secretary; Barbara Pazorski, treasurer; Sister Mary Petronia, corresponding secretary and faculty sponsor.

Wisconsin Beta, Wisconsin State University, River Falls

Chapter President—George Hansen
40 members

Monthly meetings have featured the following speakers: Louis Goertzen from 3M, Joe Herman from Control Data Corporation, and Ed Lundgren with Doug Mountain from Normandale Jr. College. Other officers are: Sandra Anderson, vice-president; Sherry Bohlinger, secretary; Joseph Goldsmith, treasurer; Sherry Bohlinger, corresponding secretary; Dr. Edward Mealy, faculty sponsor.

REGIONAL DIRECTORS

The following have been appointed as regional directors and were in charge of the 1972 regional conventions:

James Lightner	— Maryland Beta	— Region 1
Dean O. Hinshaw	— Michigan Beta	— Region 2
Jack D. Munn	— Mississippi Gamma	— Region 3
Harold L. Thomas	— Kansas Alpha	— Region 4
Mike Reagan	— Oklahoma Alpha	— Region 5
Joyce R. Curry	— California Gamma	— Region 6