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Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Square Trigonometry*

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Square trigonometry is based upon a square instead of a unit circle as in regular trigonometry. A square with dimensions two by two is centered at the origin so that the sides are parallel to the coordinate axes (Figure 1):

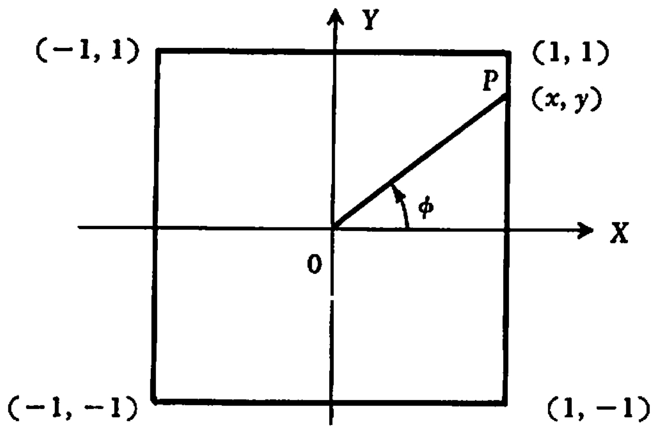


FIGURE 1

Positive angles are measured in a counter-clockwise manner as in regular trigonometry. The coordinates of the point of intersection of the terminal side ($\overline{OP}$ in the figure) of any angle ϕ with the square are used to define the following functions†:

$$\operatorname{san}\phi = x + y \qquad \operatorname{nas}\phi = \frac{1}{x+y}$$

$$\operatorname{cus}\phi = x - y \qquad \operatorname{suc}\phi = \frac{1}{x-y}$$

$$\operatorname{tin}\phi = x^2 - y^2 \qquad \operatorname{nit}\phi = \frac{1}{x^2 - y^2}$$

* A paper presented at the 1971 National Convention of KME and awarded first place by the Awards Committee.

† The suggestion for developing these functions was given in the following article: John C. Biddle, "The Square Function: An Abstract System for Trigonometry," The Mathematics Teacher, LX (February 1967), 121-23.

(Another system which more closely parallels regular trigonometry could be developed if we let tin $\phi = \dfrac{\text{san } \phi}{\text{cus } \phi}$). The spelling of the functions is significant and helpful since functions which are multiplicative inverses of each other are spelled with the same letters in reverse order.

Many identities can be developed with these functions. The most obvious ones, of course, are the inverse identities:

$$(\text{san } \phi)(\text{nas } \phi) = \frac{(x+y)}{1} \cdot \frac{1}{(x+y)} = 1 \qquad (1)$$

$$(\text{cus } \phi)(\text{suc } \phi) = \frac{(x-y)}{1} \cdot \frac{1}{(x-y)} = 1 \qquad (2)$$

$$(\text{tin } \phi)(\text{nit } \phi) = \frac{(x^2-y^2)}{1} \cdot \frac{1}{(x^2-y^2)} = 1 \qquad (3)$$

Other identities can be developed by using algebraic operations. For example, (san ϕ)(cus ϕ) $= (x+y)(x-y) = x^2 - y^2$ $=$ tin ϕ. The following identities exemplify the many possibilities for the development of identities:

$$(\text{nas } \phi)(\text{suc } \phi) = \frac{1}{(x+y)(x-y)} = \frac{1}{x^2-y^2} = \text{nit } \phi \quad (4)$$

$$(\text{cus } \phi)(\text{ nit } \phi) = \frac{(x-y)}{1} \cdot \frac{1}{x^2-y^2} = \frac{x-y}{(x-y)(x+y)}$$

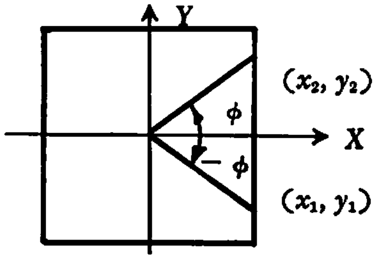
$$= \frac{1}{x+y} = \text{nas } \phi \qquad (5)$$

$$(\text{cus } \phi)(\text{tin } \phi) = (x-y)(x^2-y^2) = (x-y)(x-y)(x+y)$$
$$= (x-y)^2(x+y) = (\text{san } \phi)(\text{cus}^2 \phi) \qquad (6)$$

$$\text{san}^2 \phi - \text{cus}^2 \phi = (x+y)^2 - (x-y)^2 = x^2 + 2xy + y^2$$
$$- (x^2 - 2xy + y^2) = 4xy \qquad (7)$$

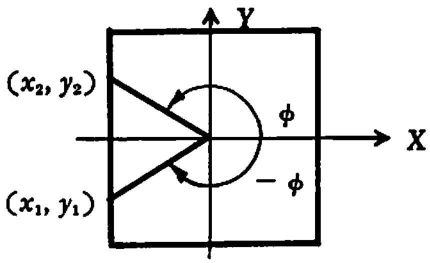
Still other identities can be proved intuitively by using the symmetry of the square. For example, consider the identity san $(-\phi)$ = cus ϕ (negative angles are measured in a clockwise direction using the positive half of the x-axis as the initial side):

Case 1:



When ϕ is any angle in the first quadrant, $-\phi$ is in the fourth quadrant. Then, san $(-\phi) = x_1 + y_1$ and cus $\phi = x_2 - y_2$. By symmetry, $x_1 = x_2$ and $y_1 = -y_2$. Hence, $x_1 + y_1 = x_2 - y_2$ by substitution.

Case 2:



When ϕ is any angle in the second quadrant, $-\phi$ is in the third quadrant. Then, san $(-\phi) = x_1 + y_1$ and cus $\phi = x_2 - y_2$. By symmetry, $x_1 = x_2$ and $y_1 = -y_2$. Hence, $x_1 + y_1 = x_2 - y_2$ by substitution.

Similarly, one can verify the identity for any angle ϕ whose terminal side lies in the third or the fourth quadrant.

We have considered all angles between 0 and 2π radians except those whose terminal sides coincide with axes. When $\phi = 0$, $-\phi = 0$. Then, san $(-\phi) = 1 + 0 = 1$ and cus $\phi = 1 - 0 = 1$. When $\phi = \pi/2$, san $(-\pi/2) = 0 + (-1) = -1$ and cus $(\pi/2) = 0 - 1 = -1$. When $\phi = \pi$, san $(-\pi) = -1 + 0 = -1$ and cus $\pi = -1 + (-0) = -1$. Finally, when $\phi = 3\pi/2$, san $(-3\pi/2) = 0 + 1 = 1$ and cus $(3\pi/2) = 0 - (-1) = 1$. Hence, since we have considered all the possibilities, we have the identity:

$$\text{san}\ (-\phi) = \text{cus}\ \phi. \qquad (8)$$

The following two identities can be proved in a similar manner:

$$\text{cus}\ (-\phi) = \text{san}\ \phi \tag{9}$$

$$\text{tin}\ (-\phi) = \text{tin}\ \phi \tag{10}$$

The domains of these six functions are fairly easy to find. The method involves finding points of discontinuity for functions of two variables. Consider san $\phi = x + y$. Since $x + y$ is defined for all values of x and y, san ϕ is defined for any angle. The situation is slightly more difficult when one considers nit $\phi = 1/(x^2 - y^2)$. First, we note that the expression $1/(x^2 - y^2)$ is undefined when $|x| = |y|$. Hence, nit ϕ is defined for any angle except $\pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$, since the terminal sides of these angles belong to the line $x = y$ or the line $x = -y$. Certainly, all angles which yield reference angles equivalent to these values are excluded also. By analyzing all six functions, one arrives at the following table:

FUNCTION	DOMAIN $(0 \leq \phi \leq 2\pi)$
san $\phi = x + y$	all ϕ
cus $\phi = x - y$	all ϕ
tin $\phi = x^2 - y^2$	all ϕ
nas $\phi = 1/(x + y)$	all ϕ except $3\pi/4$, $7\pi/4$
suc $\phi = 1/(x - y)$	all ϕ except $\pi/4$, $5\pi/4$
nit $\phi = 1/(x^2 - y^2)$	all ϕ except $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$

When one considers the ranges of these functions, many patterns come to light because of the manner in which the square was drawn. For instance, when ϕ is between 0 and $\pi/4$ radians inclusive, x always equals one. Furthermore, when ϕ is between $\pi/4$ and $\pi/2$ radians inclusive, y always equals one. Continuing this process, one arrives at the following table:

ϕ IN RADIANS	CONSTANT VALUES
ϕ between 0 and $\pi/4$	$x = 1$
ϕ between $\pi/4$ and $3\pi/4$	$y = 1$
ϕ between $3\pi/4$ and $5\pi/4$	$x = -1$
ϕ between $5\pi/4$ and $7\pi/4$	$y = -1$
ϕ between $7\pi/4$ and 2π	$x = 1$

To find other values of x and y one can utilize the regular trigonometric relationship, $\tan \phi = y/x$. When ϕ is between 0 and $\pi/4$, $x = 1$. Hence, when ϕ is between 0 and $\pi/4$ radians inclusive,

$y = \tan \phi$. Now, the symmetry of the square permits one to generalize. Consider Figure 2:

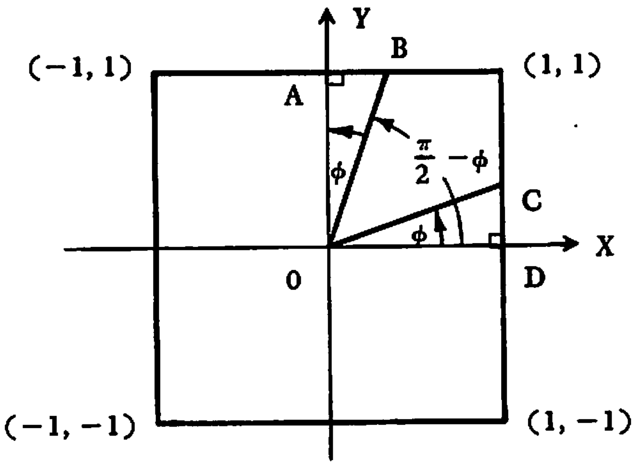


FIGURE 2

First, consider an arbitrary ϕ, $0 \leqq \phi \leqq \pi/4$. By ASA, $\triangle OAB \cong \triangle ODC$. Hence, $AB = CD$. Therefore, the y value corresponding to any ϕ between 0 and $\pi/4$ radians (inclusive) is equal to the x value for the angle $(\pi/2 - \phi)$.

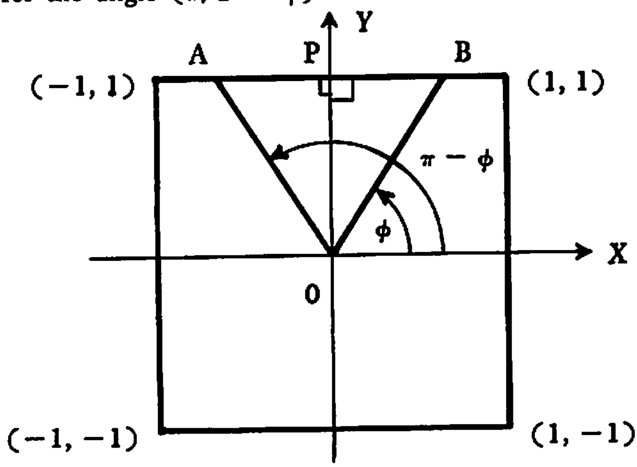


FIGURE 3

Even further generalization is possible. For instance, consider Figure 3. Again, by ASA, $\triangle OPA \cong \triangle OPB$ and $AP = PB$. Since we must consider the quadrant in which we are working, it is

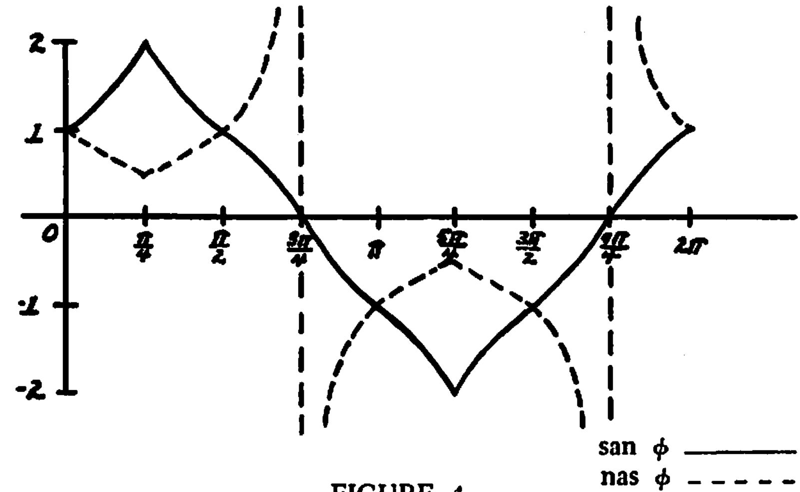


FIGURE 4

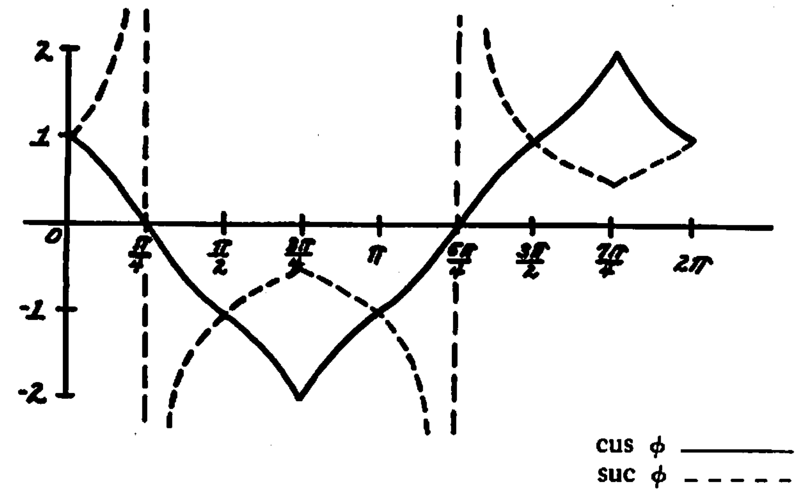


FIGURE 5

obvious that the x value corresponding to ϕ between $\pi/4$ and $\pi/2$ radians (inclusive) is the additive inverse of the x value corresponding to $(\pi - \phi)$.

Using the above results, tables of values for the functions can be made using an interval for ϕ of, say $\pi/36$. Based on these tables the graphs of the functions can be drawn (Figures 4, 5, 6). An interesting feature of these graphs should be mentioned. The graph of san ϕ is certainly more "abrupt" and pointed than is the graph of the sin ϕ function in regular trigonometry. As a matter of fact, cusps seem to exist in the graph of the san ϕ function when ϕ is $\pi/4$ and $5\pi/4$. One can utilize limits to prove that these are in fact cusps.

Consider the graph of the san ϕ when ϕ is $5\pi/4$ radians. Let $f(\phi) = \text{san } \phi = x + y$. One may compute the left-handed derivative of f at $5\pi/4$ as follows:

$$f'_{-}(5\pi/4) = \lim_{h \to 0^-} \frac{\text{san}(5\pi/4 + h) - \text{san } 5\pi/4}{h}$$

$$= \lim_{h \to 0^-} \frac{\text{san}(5\pi/4 + h) - (-2)}{h}.$$

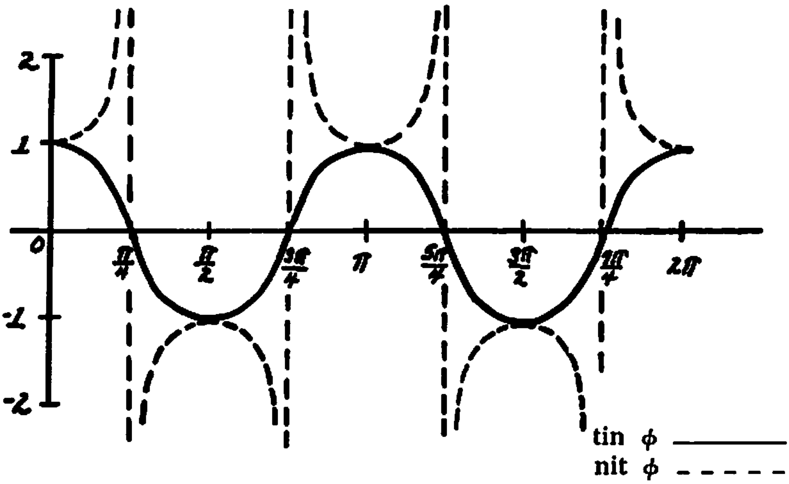


FIGURE 6

Now, san $(5\pi/4 + h) = x' + y'$ for some x', y'. Since $h < 0$, it is evident that $x' = -1$. One is interested in expressing the value y' in terms of the variable h. Consider Figure 7:

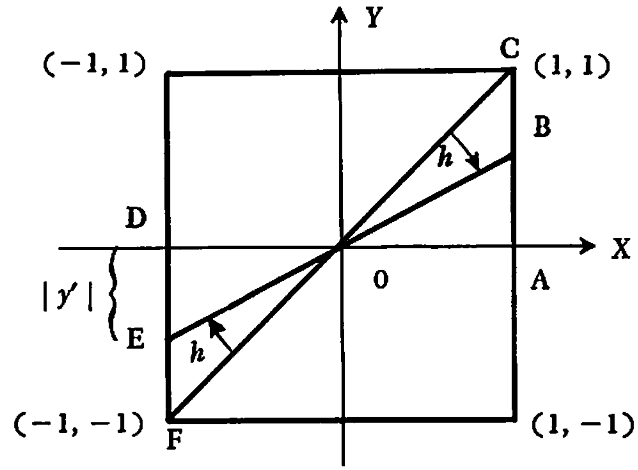


FIGURE 7

It is clear that $ED = BA$. Now, $m\angle BOA = \pi/4 + h$ (since h is negative). Hence, by regular trigonometry, $BA = OA \cdot \tan(\pi/4 + h) = 1 \cdot \tan(\pi/4 + h) = \tan(\pi/4 + h)$. Thus, $ED = |y'| = \tan(\pi/4 + h)$. Since, y' is negative, $y' = -\tan(\pi/4 + h)$. Therefore, san $(5\pi/4 + h) = x' + y' = -1 - \tan(\pi/4 + h)$. So we have

$$\lim_{h \to 0^-} \frac{\text{san}\,(5\pi/4 + h) - (-2)}{h} = \lim_{h \to 0^-} \frac{(-1 - \tan(\pi/4 + h)) - (-2)}{h} = \lim_{h \to 0^-} \frac{1 - \tan(\pi/4 + h).}{h}$$

This is an indeterminant form (0/0). Hence, one can apply L'Hospital's Rule:

$$\lim_{h \to 0^-} \frac{1 - \tan(\pi/4 + h)}{h} = \lim_{h \to 0^-} \frac{-\sec^2(\pi/4 + h)}{1} = -\sec^2(\pi/4) = -2\,.$$

Now, consider the right-handed limit of f at $5\pi/4$:

$$f'(5\pi/4) = \lim_{h \to 0^+} \frac{\text{san}\,(5\pi/4 + h) - \text{san}\,5\pi/4}{h}$$

$$= \lim_{h \to 0^+} \frac{\text{san}\,(5\pi/4 + h) + 2}{h}.$$

Now, san $(5\pi/4 + h) = x'' + y''$ for some x'', y'' (Figure 8). As $h \to 0^+$, one knows that $y'' = -1$. Also, h is positive. One is interested in expressing the value x'' in terms of the variable h. By considering Figure 8 and by using a similar argument as above, we find that $EF = |x''| = \tan(\pi/4 - h)$ or $x'' = -\tan(\pi/4 - h)$. Therefore, san $(5\pi/4 + h) = x'' + y'' = -\tan(\pi/4 - h) - 1$. Hence,

$$\lim_{h \to 0^+} \frac{\text{san}\,(5\pi/4 + h) + 2}{h}$$

$$= \lim_{h \to 0^+} \frac{-\tan(\pi/4 - h) - 1 + 2}{h}$$

$$= \lim_{h \to 0^+} \frac{-\tan(\pi/4 - h) + 1}{h}.$$

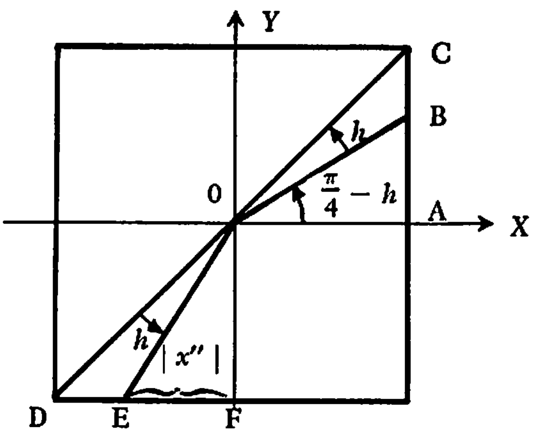


FIGURE 8

Again, this is an indeterminant form (0/0). Hence, one can apply L'Hospital's Rule:

$$\lim_{h \to 0^+} \frac{-\tan(\pi/4 - h) + 1}{h}$$
$$= \lim_{h \to 0^+} \frac{(-\sec^2(\pi/4 - h))(-1)}{1}$$
$$= \lim_{h \to 0^+} \sec^2(\pi/4 - h)$$
$$= 2\,.$$

In review one finds that $f'_-(5\pi/4) = -2$ and that $f'_+(5\pi/4) = +2$ where f is the san ϕ function. Since these limits are not equal, the san ϕ function does not have a derivative at $\phi = 5\pi/4$ radians, indicating that the graph does indeed come to a cusp at $\phi = 5\pi/4$. This method can also be utilized to prove that another cusp exists in the graph of the san ϕ when $\phi = \pi/4$ (cusps exist in the graph of the nas ϕ when ϕ equals these same values) and that both the graph of the cus ϕ and the graph of the suc ϕ contain cusps at the points where $\phi = 3\pi/4$ and $7\pi/4$ radians.

Conversely, the graph of the tin ϕ function appears to be smoother than the graph of the san ϕ. For instance, the derivative of tin ϕ at $\phi = \pi/2$ appears to exist and to equal 0. Hence, it is not necessary to consider one-sided derivatives since tin $\phi = x^2 - y^2$, $y' = y'' = 1$, and $x'' = -x'$, indicating that tin $(\pi/2 + h)$ will be the same regardless if whether h is positive or negative (Figure 9).

To begin, let $f(\phi) = \text{tin}\,\phi$. Then,

$$f'(\pi/2) = \lim_{h \to 0} \frac{f(\pi/2 + h) - f(\pi/2)}{h}$$
$$= \lim_{h \to 0} \frac{\text{tin}(\pi/2 + h) - \text{tin}(\pi/2)}{h}$$
$$= \lim_{h \to 0} \frac{\text{tin}(\pi/2 + h) + 1}{h}\,.$$

But, $\text{tin}(\pi/2 + h) = (x')^2 - (y')^2 = (x'')^2 - (y'')^2$ where $y' = y'' = 1$ and $x'' = -x'$. Now, $x' = \tan|h|$. Hence, $\text{tin}(\pi/2 + h) = \tan^2|h| - 1^2 = \tan^2 h - 1$, since $\tan^2(-h) = \tan^2 h$. Hence,

$$\lim_{h \to 0} \frac{\operatorname{tin}(\pi/2 + h) + 1}{h} = \lim_{h \to 0} \frac{\tan^2 h - 1 + 1}{h}$$
$$= \lim_{h \to 0} \frac{\tan^2 h}{h} .$$

This again is an indeterminant form (0/0). Hence, by applying L'Hospital's Rule, we obtain

$$\lim_{h \to 0} \frac{\tan^2 h}{h} = \lim_{h \to 0} \frac{2(\tan h)(\sec^2 h)}{1}$$
$$= 0 .$$

Hence, the derivative of tin ϕ at $\phi = \pi/2$ does exist and does indeed equal zero.

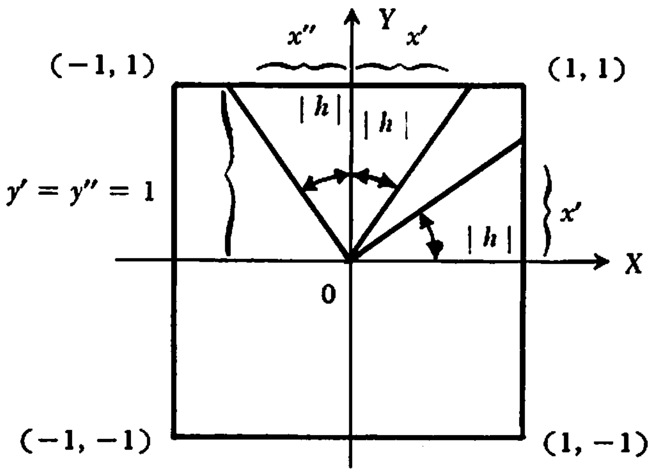


FIGURE 9

It should be evident that a trigonometric system could be developed based upon almost any geometric figure. Because of the symmetry of the square, this system is probably one of the more mathematically convenient ones. If the reader would like to develop a different system, he should try one based upon a two by two square which is centered at the origin so that its vertices lie on the axes. Of course, the appropriate values for x and y would then be more difficult to calculate.

The Probability That a Determinant of Order n Will be Even*

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Influencing almost every aspect of man's life, the theory of probability has been continually refined into an important mathematical tool. With its countless applications, this decision-making subject occurs in many everyday events.

Today students study the results of past mathematical experiments and look for new models; they sometimes find surprising results. For example, the probability that a determinant of order two has an even value is surprisingly not one-half. In this paper, we will investigate the likelihood that a determinant of order n has an even value and what its limiting value is as n increases. The procedure will develop from a few discrete examples to a Fortran program and then finally to a generalization for the determinant of order n.

For the case when n equals two, let us consider the following determinant where each entry a_{ij} is an integer and therefore has a probability of being even one-half of the time.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \tag{1}$$

The value of this determinant is defined in terms of the difference of two products. The event that the product of two numbers is even (E) or odd (O) is shown in Table 1. We note that

TABLE 1

•	E	O
E	E	E
O	E	O

TABLE 2

—	E	O
E	E	O
O	O	E

* A paper presented at the 1971 National Convention of KME and awarded second place by the Awards Committee.

three of the four products are even. Therefore, in the evaluation of the determinant, the products $(a_{11}a_{22})$ and $(a_{12}a_{21})$ each have a probability of three-fourth of being even.

Since the evaluation of the determinant also involves a difference, we have Table 2 for the even outcomes when two numbers are subtracted (or added).

We will use the notation $P(a:X)$ for the probability that the quantity a assumes an X value, where X can be E for even and O for odd. Thus, $P(a_{11}a_{22} - a_{12}a_{21}:E)$ represents the probability that the quantity $a_{11}a_{22} - a_{12}a_{21}$, the value of a determinant of order 2, has an even value. Since, from Table 2, the difference of two numbers is even only when the numbers are both even or both odd, then $a_{11}a_{22}$ and $a_{12}a_{21}$ must be both even or both odd for the determinant to have an even value. Because of the independency of the products, we have

$$P(a_{11}a_{22} - a_{12}a_{21}:E) = P(a_{11}a_{22}:E) \cdot P(a_{12}a_{21}:E) + P(a_{11}a_{22}:O) \cdot P(a_{12}a_{21}:O) \qquad (2)$$

Using the results of Table 1 each product is even ¾ of the time and, by complementary events, odd ¼ of the time. Thus

$$P(a_{11}a_{22}:E) = P(a_{12}a_{21}:E) = 3/4 \qquad (3)$$

and

$$P(a_{11}a_{22}:O) = P(a_{12}a_{21}:O) = 1/4 \qquad (4)$$

Thus, using (2), (3), and (4)

$$\begin{aligned} P(a_{11}a_{22} - a_{12}a_{21}:E) &= P(a_{11}a_{22}:E) \cdot P(a_{12}a_{21}:E) + P(a_{11}a_{22}:O) \cdot P(a_{12}a_{21}:O) \\ &= (3/4)(3/4) + (1/4)(1/4) \\ &= 5/8 \qquad (5) \end{aligned}$$

Before investigating higher order determinants, we will introduce the notation which will be used when considering the probability of a determinant of order n being even or odd. The expression $P(D^n_x)$ represents the probability (P) of a determinant (D) of order n assuming the value x, where x can be E (for even) and O for odd). Then (5) can be rewritten as follows:

$$P(a_{11}a_{22} - a_{12}a_{21}:E) = P(D^2_E) = 5/8 \qquad (6)$$

and the complementary event:

$$P(a_{11}a_{22} - a_{12}a_{21}:O) = P(D_O^2) = 1 - P(D_E^2) = 3/8 \qquad (7)$$

Another way of finding $P(D_O^2)$ is to use Tables 1 and 2 again. Then we find the following equation similar to (2):

$$\begin{aligned} P(D_O^2) &= P(a_{11}a_{22} - a_{12}a_{21}:O) \\ &= P(a_{11}a_{22}:O) \cdot P(a_{12}a_{21}:E) + P(a_{11}a_{22}:E) \cdot P(a_{12}a_{21}:O) \\ &= (1/4)(3/4) + (3/4)(1/4) \\ &= 3/8 \end{aligned}$$

This checks with the result of (7). The idea of complementary events gives a much simpler method for finding the odd probability given the even and vice versa. This concept will be useful in the exploration of higher order determinants.

Next, let us consider the determinant of order 3.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \qquad (8)$$

This determinant involves three products where one component of each product is a determinant of order two. Each of these determinants of order two is the minor of the other factor of the product. To simplify (8), we will let A_{ij} represent the minor associated with the corresponding entry a_{ij}. Thus, in the case of order 3,

$$A_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \qquad A_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \qquad A_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

The probability that a determinant of order three being even is thus symbolized by $P(a_{11}A_{11} - a_{12}A_{12} + a_{13}A_{13}:E)$. The determinant is even only when there are three even products or two odd and one even product in the evaluation of the determinant; i.e.,

$$\begin{aligned} &E - E + E \rightarrow E \\ &E - O + O \rightarrow E \\ &O - E + O \rightarrow E \\ &O - O + E \rightarrow E \end{aligned}$$

All other combinations of three numbers yield odd outcomes. Since each product has the same probability of assuming an even or odd value, the subscripts can be eliminated. Thus

$$\begin{aligned} P(D_E^3) &= P(a_{11}A_{11} - a_{12}A_{12} + a_{13}A_{13} : E) \\ &= (P(aA : E))^3 + 3(P(aA : E))(P(aA : O))^2 \qquad (9) \end{aligned}$$

Looking at the general form of a product in Table 1, we find three ways for the product aA to be even and only one way (both factors are odd) for it to be odd. Since finding the probability that aA is odd is easier, we will do this first then, using the concept of complementary events, subtract that probability from one to find the even probability.

Since $P(a : O)$ is independent of $P(A : O)$, then $P(aA : O)$ can be expressed as follows:

$$P(aA : O) = P(a : O) \cdot P(A : O)$$

and since A here represents a determinant of order 2,

$$P(aA : O) = P(a : O) \cdot P(A : O) = P(a : O) \cdot P(D_O^2) .$$

From (7)

$$P(aA : O) = P(a : O) \cdot P(D_O^2) = (1/2)(3/8) = 3/16$$

and

$$P(aA : E) = 1 - P(aA : O) = 1 - (3/16) = 13/16$$

Returning to (9) and substituting the above values, we find $P(D_E^3)$:

$$\begin{aligned} P(D_E^3) &= (P(aA : E))^3 + 3(P(aA : E))(P(aA : O))^2 \\ &= (13/16)^3 + 3(13/16)(3/16)^2 \\ &= 637/1024 \end{aligned}$$

As n increases, the evaluation of a determinant becomes more time consuming, involving the addition and subtraction of n products where each product consists of a single number and a determinant of $(n - 1)$ order. To find any one probability for a certain determinant involves the consideration of the n terms involved and ways in which they can be combined to yield an even value. The probability of each combination must be found then the final result is the sum of these probabilities.

Without working through the derivation, the probability that a determinant of order 4 has an even value is as follows:

$$\begin{aligned}
P(D_E^4) &= (P(aA:E))^4 + 6(P(aA:E))^2(P(aA:O))^2 \\
&\qquad + (P(aA:O))^4 \\
&= (1 - P(aA:O))^4 + 6(1 - P(aA:O))^2(P(aA:O))^2 \\
&\qquad + (P(aA:O))^4 \\
&= (1 - (1/2)(387/1024))^4 \\
&\qquad + 6(1 - (1/2)(387/1024))^2((1/2)(387/1021))^2 \\
&\qquad + ((1/2)(387/1024))^4 \\
&= .57487\ldots
\end{aligned}$$

Each A in the above expression is a determinant of order three. Taking $(P(aA:O))^4$ to analyze, we find that this represents the combination of the addition and subtraction of four odd products, i.e.

$$O - O + O - O \rightarrow E$$

We can generalize by saying that a determinant of order n is even only when it is a combination of all even products or an even number of odd products alone or together with even products. In notation

$$\sum_{i=1}^{n} E_i \rightarrow E \qquad \text{or} \qquad \sum_{i=1}^{n} O_i \rightarrow E$$

where n is even or

$$\sum_{i=1}^{j} E_i + \sum_{k=j+1}^{n} O_k \rightarrow E$$

where $(n - j)$ is even.

The preceding ideas were the basis used to write the Fortran program which is at the end of this paper. The program generates the probabilities that determinants of orders three to forty assume even values. Although not proven, it is evident that the probability approaches one-half as n increases. The general recursive formula is given below.

$$P(D_O^2) = 3/8 \text{ and } P(D_O^n) = 1 - P(D_E^n)$$

then

$$P(D_E^n) = \sum_{k=0}^{m} \binom{n}{2k} (1 - (1/2)P(D_O^{n-1}))^{n-2k}((1/2)P(D_O^{n-1}))^{2k}$$

for $n \geqq 3$, where m is the greatest integer in $n/2$.
Let us again consider a determinant of order 3 using this formula to find the probability for an even value.

$$\begin{aligned} P(D_E^3) &= \binom{3}{0}(1 - (1/2)(3/8))^3((1/2)(3/8))^0 \\ &\qquad + \binom{3}{2}(1 - (1/2)(3/8))^1((1/2)(3/8))^2 \\ &= (1 - (3/16))^3 + 3(1 - (3/16))(3/16)^2 \\ &= (13/16)^3 + 3(13/16)(3/16)^2 \\ &= 637/1024 \end{aligned}$$

We will now explain the formula for the case of a determinant of order n. The summation takes care of adding the probabilities for all cases that result in even-valued determinants. When $k = 0$, it is the probability that each of the n products involved are even. When $k = 1$, we have two odd products considered and all the others are even. The combination $\binom{n}{2k}$ allows for the ways in which the even and odd products can be arranged (ordered) to yield even-valued determinants. To illustrate this, we again look at the following combinations for a determinant of order three:

$$E - O + O \rightarrow E$$
$$O - E + O \rightarrow E$$
$$O - O + E \rightarrow E$$

This shows the three ways $\binom{3}{2}$ of arranging two odd and one even product, all of which produce an even outcome.

This process continues similarly for the values of k in the general formula. If n is odd, the last term of the sum contains the probability for one even product and the rest (an even number) odd products. If n is even, the last term represents the probability for all odd products.

This formula is a special application of the binomial distribution that uses alternate terms of the binomial expansion.

As a consideration of the probability that a determinant of order n has an even value, this paper illustrates the theory behind the derivation of the general formula. This formula was then used in writing the computer program which provides the empirical data for the conclusion that as n increases, the limiting value of this probability is one-half.

TABLE 3

PROBABILITIES FOR EVEN-VALUED DETERMINANTS

ORDER	PROBABILITY	ORDER	PROBABILITY
3	0.6220703125000	22	0.5000001192111
4	0.5767860217894	23	0.5000000596050
5	0.5319185856234	24	0.5000000298024
6	0.5117359274635	25	0.5000000149012
7	0.5045950618810	26	0.5000000074506
8	0.5021510286968	27	0.5000000037253
9	0.5010150307863	28	0.5000000018626
10	0.5005024899098	29	0.5000000009313
11	0.5002468531482	30	0.5000000004657
12	0.5001231213707	31	0.5000000002328
13	0.5000612308282	32	0.5000000001164
14	0.5000305941140	33	0.5000000000582
15	0.5000152727999	34	0.5000000000291
16	0.5000076348695	35	0.5000000000146
17	0.5000038156876	36	0.5000000000073
18	0.5000019077343	37	0.5000000000036
19	0.5000009537435	38	0.5000000000018
20	0.5000004768640	39	0.5000000000009
21	0.5000002384234	40	0.5000000000005

FORTRAN PROGRAM TO FIND PROBABILITIES

```
C-----------------------------------------------------------------------
C   ALL STATEMENTS STARTING WITH A  C  ARE EXPLANATORY COMMENTS
C  FORTRAN PROGRAM TO FIND PROBABILITIES FOR EVEN-VALUED DETERMINANTS OF ORDERS
C  3 TO 40
C  OPROB REPRESENTS ODD PROBABILITY
C  EPROB REPRESENTS EVEN PROBABILITY
C  COMB(I,J) IS A FUNCTION SUBPROGRAM USED TO FIND A COMBINATION OF I THINGS
C  TAKEN J AT A TIME
C  DOUBLE PRECISION IS USED IN ORDER TO CARRY MORE DECIMAL PLACES
C-----------------------------------------------------------------------
      DOUBLE PRECISION OPROB, EPROB, COMB
C-----------------------------------------------------------------------
C  OPROB HERE IS THE PROBABILITY THAT A DETERMINANT OF ORDER 2 IS ODD
C-----------------------------------------------------------------------
      OPROB = 3.0/8.0
C-----------------------------------------------------------------------
C  I STANDS FOR THE ORDER OF THE DETERMINANT (FROM 3 TO 40)
C-----------------------------------------------------------------------
C-----------------------------------------------------------------------
C  PUTS OUTPUT ON NEW PAGE
C-----------------------------------------------------------------------
      WRITE(3,300)
  300 FORMAT ('1')
      DO 2 I = 3, 40
C-----------------------------------------------------------------------
C  INITIALIZES EPROB (SUM) AND THIS IS THE TERM FOR THE SUM OF  I  EVEN PRODUCTS
C-----------------------------------------------------------------------
      EPROB = (1.0 - 0.5 * OPROB)**I
C-----------------------------------------------------------------------
C  INCREMENTS J BY 2 FOR COMBINATIONS AND SETS LIMIT
C-----------------------------------------------------------------------
      DO 1 J = 2, I, 2
C-----------------------------------------------------------------------
C  FIND SUM FOR PROBABILITY
C-----------------------------------------------------------------------
      EPROB = EPROB + COMB(I,J)*(1.0-0.5*OPROB)**(I-J)*(0.5*OPROB)**J
    1 CONTINUE
C-----------------------------------------------------------------------
C  OUTPUT
C-----------------------------------------------------------------------
      WRITE(3,200) I, EPROB
  200 FORMAT (20X,'PROBABILITY THAT A DETERMINANT OF ORDER ',I3,' HAS AN
     * EVEN VALUE IS ',D21.13,//)
C-----------------------------------------------------------------------
C  CHANGES VALUE OF OPROB TO THAT FOR NEXT HIGHER ORDER
C-----------------------------------------------------------------------
      OPROB = 1 - EPROB
    2 CONTINUE
      STOP
      END

C-----------------------------------------------------------------------
C  FUNCTION SUBPROGRAM FOR FINDING COMBINATION OF I THINGS TAKEN J AT A TIME
C-----------------------------------------------------------------------
      FUNCTION COMB(I,J)
      DOUBLE PRECISION PROD, DIV, COMB
      JK = J + 1
      PROD = 1.0
      DO 1 L = JK, I
      X = L
      PROD = PROD * X
    1 CONTINUE
      N = I - JK + 1
      DIV = 1.0
      IF ( N.EQ.1) GO TO 3
      DO 2 K = 2, N
      Y = K
      DIV = DIV * Y
    2 CONTINUE
    3 COMB = PROD/DIV
      RETURN
      END
```

An Instance of Intuition and Lengths of Limiting Curves*

NANCY EDWARDS

Student, Kansas State Teachers College

The family of curves $S_n = \{ (x, y) \mid y = \frac{1}{n} \sin n^2x, 0 \leq x \leq \pi \}$, where n is a positive integer, presents a novel state of affairs to anyone interested in comparing what seems to be with what actually is.

For example, the limit curve as n increases without bound is clearly the line segment on the x-axis from 0 to π. This is an immediate consequence of the fact that the amplitudes approach zero as n increases. However, it is not obvious whether the lengths of the curves get closer to the length of this limit curve as n increases.

Consider the accurate sketches of the curves S_1, S_2 and S_3, together with the limit curve denoted as S_L, in Figure 1. From these figures S_1 seems to be shorter than S_2 which in turn seems to be shorter than S_3. Yet all three seem to be longer than S_L. The question is: are these curves actually getting longer as n increases or do they begin to get closer to the length of S_L?

We note first a lesson from geometry that the concept of infinitude of a curve does not imply unboundedness of the curve. Hence the fact that all the curves under consideration are bounded (the circle of radius 7 with center at the origin incloses all members of the family) tells us nothing about whether they are all finite in extent.

We note a second lesson from our first encounter with the notion of infinite sets that what is true about every finite set of n elements may not hold for a set of infinitely many elements. The properties shared by every member of an infinite sequence need not be shared by the limit of the sequence.

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* A paper presented at the 1971 National Convention of KME and awarded third place by the Awards Committee.

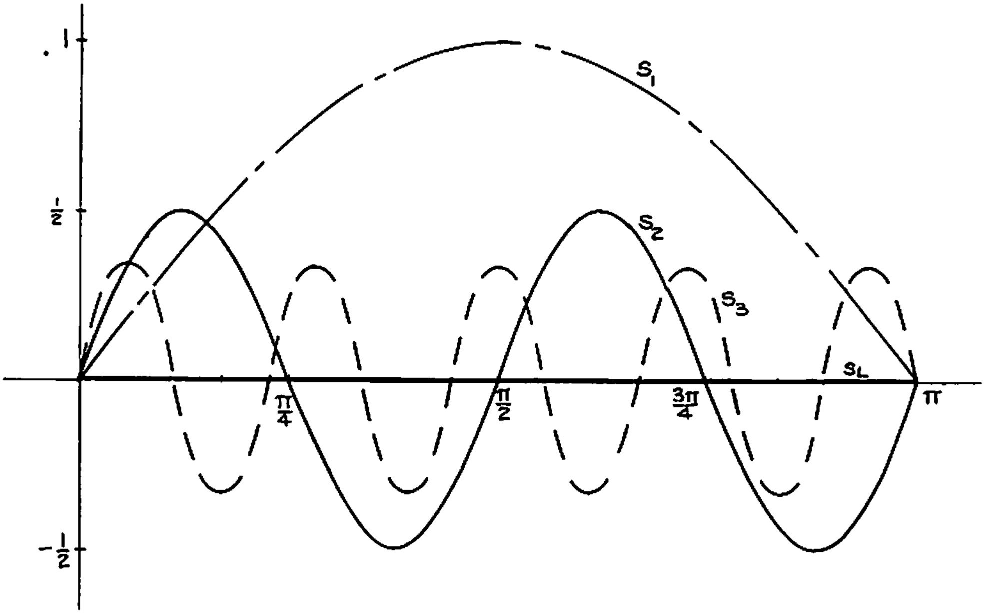

FIGURE 1

Thus, if the limit of the lengths of the curves S_n is the length π of the limit curve S_L, this must be proved. If the limit of the lengths is some other number or does not exist, then this must be proved. The problem can not be resolved on the basis of what seems to be the case from an inspection of the graphs or on the basis of some intuitive feeling.

To attack the problem the first step should be to find the length of each curve. The length of any curve $y = f(x)$ between $x = a$ and $x = b$ is found by evaluating the integral

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx\,.$$

Thus the length of the nth curve, S_n, would be found by evaluating the integral $\int_0^\pi \sqrt{1 + (n \cos n^2 x)^2}\, dx$. However, we cannot easily evaluate this integral even with the aid of standard mathematics tables.

This apparent dead-end turns out to be merely a minor handicap. For a third lesson from mathematics is that the lengths of two objects can be compared sometimes without knowing the lengths of either. For example, one can tell which of two people is taller without measuring either.

Since we want only to compare the lengths of the different curves, we do not need to compute their actual lengths. Fair estimates would probably suffice. Since the drawings above seem to indicate that the length of S_1 is less than that of S_2, perhaps we can find a way to prove, without evaluation, that

$$\int_0^{\pi} \sqrt{1 + \cos^2 x}\, dx < \int_0^{\pi} \sqrt{1 + 4\cos^2 4x}\, dx$$

If we can find functions $f(x)$ and $g(x)$ such that $\sqrt{1 + \cos^2 x} \leq f(x)$, $\int_0^{\pi} f(x)\, dx < \int_0^{\pi} g(x)\, dx$, and $g(x) < \sqrt{1 + 4\cos^2 4x}$, then clearly we can conclude that $\int_0^{\pi} \sqrt{1 + \cos^2 x}\, dx < \int_0^{\pi} \sqrt{1 + 4\cos^2 4x}\, dx$. Thus, it would be established that the length of S_1 is less than that of S_2.

Upon investigation we submit that the functions $f(x) = \sqrt{1 + |\cos x|}$ and $g(x) = 2\,|\cos 4x|$ suffice for this purpose. Our procedure is as follows.

$$|\cos x| \leq 1$$
$$|\cos x|^2 \leq |\cos x|$$
$$\cos^2 x \leq |\cos x|$$
$$1 + \cos^2 x \leq 1 + |\cos x|$$
$$\sqrt{1 + \cos^2 x} \leq \sqrt{1 + |\cos x|}$$
$$\int_0^{\pi} \sqrt{1 + \cos^2 x}\, dx \leq \int_0^{\pi} \sqrt{1 + |\cos x|}\, dx$$

Now the integrand $1 + |\cos x|$ is the same as the function $1 + \cos x$ on the interval from 0 to $\pi/2$. Furthermore, $1 + |\cos x|$ has a period of π, a maximum value at $x = 0$, and symmetry about the line $x = \pi/2$. It follows that

$$\int_0^{\pi} \sqrt{1 + |\cos x|}\, dx = 2 \int_0^{\pi/2} \sqrt{1 + \cos x}\, dx$$

By use of standard tables this latter integral has a value of 4. Hence the length of S_1 is given by $\int_0^\pi \sqrt{1 + \cos^2 x}\, dx$ and is less than or equal to 4.

On the other hand, regarding the length of S_2:

$$1 + (2 \cos 4x)^2 > (2 \cos 4x)^2$$

$$\sqrt{1 + (2 \cos 4x)^2} > \sqrt{(2 \cos 4x)^2} = | 2 \cos 4x | = 2 | \cos 4x |$$

$$\int_0^\pi \sqrt{1 + (2 \cos 4x)^2}\, dx > \int_0^\pi 2 | \cos 4x | \, dx$$

But the function $| \cos 4x |$ has the same values on the interval from 0 to $\pi/8$ as the function $\cos 4x$. Also, $| \cos 4x |$ has a period of $\pi/4$, a maximum at $x = 0$, and symmetry about the line $x = \pi/8$. Hence $\int_0^\pi 2 | \cos 4x | \, dx = 2 \cdot 8 \int_0^{\pi/8} | \cos 4x | \, dx = 2 \cdot 8 \int_0^{\pi/8} \cos 4x \, dx = 4$. Therefore, $\int_0^\pi \sqrt{1 + (2 \cos 4x)^2}\, dx > 4$, and so it is proven that the length of S_1 is less than the length of S_2.

Moreover, the method of proof used here in comparing the lengths of the curves S_1 and S_2 seems equally applicable to other curves of the family. That is, it seems that this same method could be used to prove that the length of S_n is greater than $2n$ in much the same way that it has shown the length of S_2 is greater than 4.

Recall that the length of a general curve S_n in the given family is given by the integral $\int_0^\pi \sqrt{1 + (n \cos n^2x)^2}\, dx$. Now

$$1 + (n \cos n^2x)^2 > (n \cos n^2x)^2$$

$$\sqrt{1 + (n \cos n^2x)^2} > \sqrt{(n \cos n^2x)^2} = n | \cos n^2x |$$

Therefore,

$$\int_0^\pi \sqrt{1 + (n \cos n^2x)^2}\, dx > \int_0^\pi n | \cos n^2x | \, dx$$

(concluded on p. 45)

Fortieth Anniversary – Kappa Mu Epsilon*

EDDIE W. ROBINSON
National Historian, Kappa Mu Epsilon

Imagine with me a day in history. This day that we are imagining was a Saturday. The weather was fair throughout most of the country. The temperature in New York rose to 77 degrees. Herbert Hoover was President of the United States, and he went fishing on this day. At his fishing lodge in the Blue Ridge Mountains of Virginia, he caught his limit of twenty trout. On this day, Jimmy Walker was Mayor of New York City, and he was in political trouble. On this day in Gatlinsburg, Tennessee, the Deans of Men in American Colleges and Universities were in convention. Among their considerations was the topic that airplanes were giving rise to new problems among students. Barnard College held its Greek games on this day. It was reported that 400 freshmen and sophomore girls participated in the games and 4000 men were spectators. It was announced on this day that twenty million Catholics lived in the country and that the total number of priests in the United States was 27,854. The State of Oklahoma collected the inheritance tax of seven million dollars on the estate of the oil man T. B. Slick which overjoyed the citizens of Oklahoma since the state had a deficit in the treasury of six million dollars. It was announced on this day that building of houses in the country was at its lowest point in ten years. On this day, there was an earthquake in Vancouver, British Columbia. Copper was selling for 9½¢ a pound, and the average income of a doctor of medicine was $5,059. Noel Coward and Gertrude Lawrence were the toasts of Broadway in a play called "Private Lives." The New York Yankees beat the Red Sox 5 to 4 in the 15th inning of a ball game, and on this day in the tiny Cherokee Village of Tahlequah, Oklahoma, Kappa Mu Epsilon was founded. The day was April 18, 1931. The college was Northeastern State College and was a continuation of the Male and Female Seminaries of the Cherokee Nation. The establishment of these seminaries was authorized by the Cherokee Nation on May 6, 1846.

The need for a national mathematics fraternity which would

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* A speech delivered at the 1971 National Convention of KME at Indiana University of Pennsylvania.

appeal essentially to the undergraduate was recognized by both the instructors and students of mathematics. Dr. Emily Kathryn Wyant is considered the founder of Kappa Mu Epsilon, which was organized to fill this need. Dr. Wyant was a graduate of the University of Missouri and was a member of Pi Mu Epsilon. In the fall of 1930 she went to Northeastern Oklahoma State Teachers College as a professor of mathematics. She went to work with vigor and enthusiasm to transform the mathematics club there, which had been in existence since 1927, into the first chapter of a national fraternity. Professor L. P. Woods, who was head of the Department of Mathematics and Dean of Men, was a valuable co-worker in working out the many details pertaining to the project. He was largely responsible for the completed rituals used for the initiation of members and installation of officers.

Since the first serious group of students of mathematics to be organized into a fraternity was the Society of Pythagoras, it was decided that the emblems of Kappa Mu Epsilon would be those of the Pythagoreans as nearly as possible. The emblems chosen for the new fraternity were the five-pointed star and the pentagon. Since $\rho = a \sin 5\theta$ (a five-leaved rose) fits into the pentagon, the wild rose which usually has five petals was chosen as the fraternity flower. The pink of the wild rose and the silver of the star were chosen for the colors. In making the crest, it seemed advisable that the sciences using mathematics should be recognized, so five emblems were selected for these and placed around the star on the shield. The motto, translated into English is "Develop an appreciation for the beauty of mathematics." The objective of the organization since its inception has been the fulfillment of this motto.

Dr. Wyant wrote to Dr. Walter Miller, Professor of Greek at the University of Missouri asking him to suggest a motto. He made several suggestions on the idea "Mathematics disciplines the mind." Dr. Wyant did not like these suggestions, so she sent some motto suggestions to him, asking that they be translated into Greek. From these translations, the Greek for "Develop an appreciation for the beauty of mathematics" was chosen for the motto. The key words are Ektamasin, Kaloos and Mathematicon—words which have Greek initial letters, Epsilon, Kappa and Mu, so the name of this organization came from a rearrangement of the letters Epsilon, Kappa and Mu.

Dr. Wyant and Prof. L. P. Woods along with 22 other faculty and students became charter members of Oklahoma Alpha, Northeastern Oklahoma State Teachers College, Tahlequah, April 18, 1931, thereby making the dream for the fraternity a reality. On the same day, the national organization elected the following officers: President Pythagoras, Dr. Kathryn Wyant; Vice-president Euclid, Professor Ira A. Condit; Secretary Diophantus, Miss Lorena Davis; Treasurer Newton, Professor L. P. Woods; Historian Hypatia, Miss Bethel DeLay. At a later date, the names of specific mathematicians were omitted from the names of national officers.

Dr. Wyant stated in one of her letters that she was aware of different fraternities where the national officers had titles like Director General and Grand High Mogul. Since so much symbolism had been based on the Pythagoreans, it was decided to name the titles of the National Officers after early mathematicians, with Pythagoras as the name for the President. Zeno was a follower of Pythagoras, so the Past President was named Zeno. Three different branches of mathematics furnished the appellations for Vice President, Euclid; Secretary, Diophantes; and Treasurer, Newton. Dr. Wyant felt that women had to be represented so the Historian was named Hypatia; and until 1947, it was stipulated that this office was to be held by a woman. These officers made the center and the five points of the Kappa Mu Epsilon star.

The following newspaper account was given to the transformation of the Mathematics Club at Tahlequah into Kappa Mu Epsilon:

> "The king is dead, long live the king." This may be applied to the "Mathematics Club" of Northeastern. As "The Pentagon" the club is dead; as Kappa Mu Epsilon, it lives.
>
> Kappa Mu Epsilon had its Founders' Day Banquet last Saturday evening, April 18 at the Hotel Thompson. At that time, 24 people took the pledge and signed the constitution, thus becoming charter members of the Oklahoma Alpha chapter. The banquet room was decorated in pink and white. Wild flowers and red buds were in the corners of the room. Tall white candles and floor lamps gave a soft light. Rose nut cups and hand painted place cards added to the color of the room. The menu as written inside the place cards told of paraboloids, ruled surfaces, and even parallel lines that were to be eaten at the mathematical table.
>
> Paul Lewis was the Radical Axis (toastmaster) of the evening. The program consisted of the fraternity song and the following talks: Parabolas (parables) by "Bus" Layton; Comic (conic)

sections by Dr. Kathryn Wyant; Lipstick (elliptic) conditions by Clara Green and Transformations by Dean L. P. Woods.

The formal transformation from the Pentagon into Kappa Mu Epsilon was directed by Mr. Woods.

Dr. Wyant had carried on an extensive correspondence with faculty members at other colleges in regard to the founding of a national fraternity such as this. Among those with whom she corresponded were Dr. Ira A. Condit of Iowa State Teachers College, Cedar Falls, and Dr. J. A. G. Shirk of Kansas State Teachers College, Pittsburg. Dr. Condit participated in the preliminary negotiations for the founding of the fraternity and indicated such interest that he was elected first Vice-president. The enthusiasm for this organization spread on his own campus with the result that the second chapter of Kappa Mu Epsilon, Iowa Alpha, was installed at Iowa State Teachers College, Cedar Falls, May 27, 1931. Kansas Alpha, the third chapter, was installed January 30, 1932, at Kansas State Teachers College, Pittsburg. Next was Missouri Alpha, Southwest Missouri State College, Springfield, May 20, 1932.

The first convention was held at Northeastern State Teachers College, Tahlequah, Oklahoma, April 21 and 22, 1933. By this time, there were eight chapters on the roll. It was decided at this convention that each chapter should pay to the National Treasurer fifty cents for each resident member on the chapter roll on December 1 of each year. For each member initiated between December 1 and June 1 of each year, twenty-five cents was to be paid to the National Treasurer. Actually, there were nine chapters on the rolls. Oklahoma Beta chapter had been installed on January 20, 1933 at Oklahoma State College for Women in Chickasha, Oklahoma. By 1935, this chapter was completely defunct—so completely that my office has no records for this chapter. It seems that the philosophy in the early years was to ignore any chapter which became inactive, even to the point of destroying records and reusing the name. It is unfortunate that Oklahoma Beta has been used for the name of two chapters, and I sincerely hope that the present Oklahoma Beta chapter at Tulsa University does not mind too much.

Much of the early success of Kappa Mu Epsilon is attributed to the dynamic and inspiring leadership of Dr. J. A. G. Shirk. He succeeded Dr. Wyant as National President in 1935 and served in that capacity until 1939. "The Early Years of Kappa Mu Epsilon," an article which appeared in the Spring, 1942, issue of THE

PENTAGON, was written by Dr. Shirk. Dr. Ira S. Condit helped formulate the policies of the organization and set up the first conventions. Dr. Wyant, Dr. Condit, Dr. Shirk, and Mr. Woods are now dead. However, they will always be remembered in Kappa Mu Epsilon for their many contributions in the development of the fraternity.

Another early leader in Kappa Mu Epsilon and one who has made great contributions to the organization over a long period of time has been Miss E. Marie Hove. From 1933 to 1937 she served as National Historian and she served as National Secretary from 1937 to 1955. Miss Laura Z. Greene, Washburn University of Topeka has served as Secretary since 1955. In the interval from 1941 to 1947, no conventions were held due to conditions caused by World War II. However, during this time, the National Council met to handle the business of the fraternity. At a meeting in 1947, the Council voted to have the ritual revised, since the old ritual did not express the depth of purpose that many members felt it should. Dr. C. V. Newsom, who was then at Oberlin College, Oberlin, Ohio, was asked to prepare a draft of a revised ritual for the consideration of the Council. Using the draft prepared by Dr. Newsom, the Council revised the initiation ritual in 1948.

In 1947, the stipulation that a woman had to hold the office of Historian was dropped and National President Van Engen wrote to Professor Richtmeyer, the newly elected Historian, "I do think there are lots of extra little frills and fads in the Historian's job which might be reviewed very carefully with the view of omitting some." This review did take place and it was decided that the National Historian no longer would have to make scrapbooks. Incidentally, the cost of running the Historian's office for the 1947-49 biennium was $1.95. The National Council revised the initiation and installation ceremonies in February, 1970.

The first issue of THE PENTAGON appeared the fall of 1941; an official journal for Kappa Mu Epsilon had been authorized at the fifth biennial convention April 18 and 19, 1941. The task of planning the journal and formulating its editorial policy was entrusted to Dr. C. V. Newsom who was at the University of New Mexico, Albuquerque. Dr. Newsom served as editor until his resignation in 1943. Dr. O. J. Peterson, who was then National President, said, "The publication of such a journal is probably the most significant project ever undertaken by the fraternity." This magazine was to cater to the needs of the college students of mathematics and serve as a medium through which outstanding student papers could be published. Dr. Harold D. Larsen, University of New Mexico, followed Dr. Newsom as its editor. The first issue under his editorship appeared in the fall of 1943. The Spring, 1953, issue of THE PENTAGON had as its editor Dr. Carl V. Fronabarger, Southwest Missouri State College, Springfield. The next two editors were Dr. Fred W. Lott and Dr. Helen Kriegsman.

Since the first chapter was installed in 1931 with twenty-four members, the organization has grown to a membership of over 29,000. Dr. J. A. G. Shirk has aptly said, "History renders the ultimate verdict as to the value of any movement, and the growth and the influence of Kappa Mu Epsilon . . . give a portent of its greater contributions in the decades yet to come." This statement is true. Forty years of history has rendered the verdict that Kappa Mu Epsilon has been a valuable movement. For forty years, we have fulfilled the objectives.

For forty years we have furthered the interest of mathematics in those schools which plan their primary emphasis on the undergraduate program.

For forty years we have helped the undergraduate realize the important role that mathematics has played in the development of western civilization.

For forty years we have developed an appreciation of the power and beauty possessed by mathematics.

For forty years we have provided a society for the recognition of outstanding achievement in the study of mathematics.

For forty years we have made this an organization of excellence and honor and for forty years we have made Kappa Mu Epsilon something of value.

Installation of New Chapters

EDITED BY LORETTA K. SMITH

PENNSYLVANIA KAPPA CHAPTER

Holy Family College, Torresdale, Philadelphia, Pennsylvania

The installation and initiation ceremonies of the Pennsylvania Kappa Chapter were held at Holy Family College on Saturday, January 23, 1971. Mr. William R. Smith, National Vice President of Kappa Mu Epsilon conducted the ceremonies. The following were inducted into Kappa Mu Epsilon:

Sister Mary Grace Kuzawa, Department Chairman

Mr. Allan Becker, Faculty Sponsor and Corresponding Secretary

Mr. Louis Hoezle, Instructor of Physics

Katherine Klusek, Chapter President and Treasurer

Violet Cali, Chapter Vice President and Recording Secretary

Lois Fowler

Marilyn Gontkof

Lucia Ho

Mary Ann McNulty

Francis Mulherin

Mary Jane Pacos

Maureen Robinson

After the ceremonies, Mr. Smith gave a talk entitled "How to Choose a Wife" which was both interesting and entertaining. Society members and their guests enjoyed a delicious buffet luncheon.

COLORADO BETA CHAPTER

Colorado School of Mines, Golden, Colorado

The Colorado Beta Chapter was installed on March 4, 1971, by Professor Walter C. Butler, Kappa Mu Epsilon's Treasurer. The following are charter members:

Raul E. Alvarado
William R. App
William R. Astel
Charles Baer
Ronald H. Bissett
Ardel J. Boes
G. George Cochell
John C. Darrow
M. Russell Frisinger
C. Brent Hirschman
Robert L. Johnson
Edward C. Karg, Jr.
Joseph R. Lee
Edward P. Milker
Anthony L. Morroni
Raymond K. Mueller
Jack W. Musser
Byron A. Palmer

Lynn Ray Patten
Ronald R. Smith
Paul T. Snowden, Jr.
Robert P. Snowden
Hunter Swanson
Brent Troutman
William E. Westbrook
Robert W. Wiley

Professor Butler presented some historical facts about Kappa Mu Epsilon and offered a few suggestions on the operation of a successful chapter.

In the evening the members of the Colorado Beta Chapter met at the Old Heidelberg Inn for a banquet. Guests at the banquet were Professor Walter C. Butler and Dr. Walter Whitman of the Colorado School of Mines Mathematics Department. Dr. Whitman delivered a speech on the unusual personalities and abilities of certain mathematicians.

TENNESSEE DELTA CHAPTER

Carson-Newman College, Jefferson City, Tennessee

The Tennessee Delta Chapter was installed on May 15, 1971. The installation ceremony, which was preceded by a banquet, was held at the Holiday Inn of Morristown, Tennessee. Professor Lora D. McCormick of East Tennessee State University served as installing officer. Five faculty members and seventeen students are charter members:

Faculty Members:

Denver R. Childress
Howard Chitwood
Verner T. Hansen
Carey R. Herring
Sherman B. Vanaman

Students:

Daniel W. Barker
Shelley I. Butler
Susan K. Cooper
Josephine Cridlin
Henry D. Dickson
Judy E. Edwards
Kathy A. Eichenbrenner
David F. Fraley
Rebecca J. Jessee
Christine J. Manbeck
John W. McPherson
Joyce Moore
Jenny A. Rives
Richard A. Rosenberger
Douglas S. Shafer
Roy L. Vice
David E. Wheeler

Tennessee Delta Chapter's officers are:

President: Douglas S. Shafer
Vice President: Roy L. Vice
Recording Secretary: Daniel W. Barker
Treasurer: Rebecca J. Jessee
Corresponding Secretary: Denver R. Childress
Faculty Sponsor: Howard Chitwood

NEW YORK IOTA CHAPTER

Wagner College, Staten Island, New York

The New York Iota Chapter was installed on Wednesday, May 19, 1971, at Wagner College by Professor William R. Smith, National Vice President of Kappa Mu Epsilon. Professor Smith was accompanied by Professor Woodard from the Department of Mathematics, Indiana University of Pennsylvania.

The officers of the newly-formed chapter are:

President: Robert Sager
Vice President: Lois Bredholt
Treasurer: Karen Dybing
Recording Secretary: Beverly Fraser
Corresponding Secretary: Mary Petras
Faculty Sponsor: Myles McConnon

The chapter members are:

Donald Berkebile
Susan Biederman
Linda Chacon
Salvatore Delassandro
Corinne Dieli
William Horn
Judith Jacobson
Brian Manske
Shirley Merrell
Barbara Olsen
Raymond Traub
Sydney Welton
Linda Wolfsohn

Following the installation, Professor Smith gave an amusing mathematically-oriented talk entitled "How to Choose a Wife." After his talk, refreshments were served.

The Problem Corner

EDITED BY ROBERT L. POE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before February 1, 1972. The best solutions submitted by students will be published in the Spring 1972 issue of *The Pentagon,* with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Professor Robert L. Poe, Department of Mathematics, Berry College, Mount Berry, Georgia 30149.

PROPOSED PROBLEMS

246. *Proposed by Kenneth Rosen, University of Michigan, Ann Arbor, Michigan.*

Let x and y be positive real numbers such that $x + y = 1$. Prove that $x \ln x + y \ln y \geqq - \ln 2$ and discuss where equality holds.

247. *Proposed by R. S. Luthar, University of Wisconsin, Waukesha, Wisconsin.*

Find all integral solutions of the equation $x^4 + 3y^2 + 4z = 26$.

248. *Proposed by R. S. Luthar, University of Wisconsin, Waukesha, Wisconsin.*

Solve the equation $2(5x - 1)^2 + 7 = y^4$ over the domain of positive integers.

249. *Proposed by R. S. Luthar, University of Wisconsin, Waukesha, Wisconsin.*

From the vertex A of the curve $x^3 + y^3 = 3axy$, a line is drawn to cut the curve in two other points, P and Q. Prove that OP and OQ are perpendicular to each other where O is the origin of reference.

SOLUTIONS

241. *Proposed by the Editor.*

Prove or disprove that $f(x) = \sin x^2$, for x any nonnegative real number, is uniformly continuous.

Solution. No solution was received. The hints listed below are provided by the Editor.

Pick $\varepsilon = \frac{1}{2}$ and $\delta > 0$. Now there exists a positive integer n such that $\sqrt{\frac{\pi}{32n}} < \delta$ and $\left| \cos \frac{\pi}{32n} \right| > \frac{1}{2}$.

Take $x_1 = \sqrt{2n\pi}$ and $x_2 = \sqrt{2n\pi} + \sqrt{\frac{\pi}{32n}}$ to negate the definition of uniform continuity of function.

242. *Proposed by the Editor.*

1,804,229,351 is the fifth power of a positive integer. Find the integer without extracting roots or using logarithms.

Solution by Carol J. Worthy, Central Missouri State College, Warrensburg, Missouri.

$1 + 8 + 0 + 4 + 2 + 2 + 9 + 3 + 5 + 1 = 35$
The integer we are seeking is a 2 digit prime greater than 35. $3 + 5 = 8$. The sum of the digits of the prime number is 8. The 2 digit primes greater than 35 are: 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, & 97. $7 + 1 = 8$. Therefore, the prime being sought is 71.

Also solved by Vance L. Johnson, Western Illinois University, Macomb, Illinois; Barbara Learner, Southwest Missouri State College, Springfield, Missouri; Robert Thompson, University of Evansville, Evansville, Indiana; Kenneth M. Wilke, Topeka, Kansas.

243. *Proposed by the Editor.*

Find all three-digit numbers each of which is the sum of all possible permutations of its three digits taken two at a time.

Solution by Jacque Ogden, Northeast Missouri State College, Kirksville, Missouri.

If we represent one of the required 3 digit numbers of htu then the permutations of its 3 digits taken 2 at a time are ht, th, hu, uh, tu, ut and since the number is equal to the sum of the permutations of its digits taken 2 at a time we have:

$$\begin{aligned} 100h + 10t + u &= (10h + t) + (10t + h) + (10h + u) \\ &\quad + (10u + h) + (10t + u) + (10u + t) \\ &= 22h + 22t + 22u \end{aligned}$$

Thus

$$78h - 12t - 21u = 0$$
$$26h - 4t - 7u = 0 \qquad (1)$$

where $1 \leq h \leq 9$, $0 \leq t \leq 9$, $0 \leq u \leq 9$. Rewriting (1) as $26h - (4t + 7u) = 0$ we see that the largest value of $(4t + 7u)$ is 99, and therefore h must be less than 4. We find the first required 3 digit number by letting $h = 1$ in equation (1), giving us a solution 132. The other two solutions are 264 and 396.

Also solved by Vance L. Johnson, Western Illinois University, Macomb, Illinois; Marcia Kay Sowles, Manchester College, North Manchester, Indiana; Robert Thompson, University of Evansville, Evansville, Indiana; Kenneth M. Wilke, Topeka, Kansas.

244. *Proposed by the Editor.*

A man's advice concerning women's fashions had better add up. Check the advice below by addition by replacing each letter with a digit. (The same letter for the same digit throughout.)

$$\begin{array}{r} W\ E\ A\ R \\ A \\ M\ I\ N\ I \\ \hline H\ O\ N\ E\ Y \end{array}$$

Further, if there is not a unique solution avoid the maxi completely and the midi if possible; that is, look for the mini.

Solution by Kenneth M. Wilke, Topeka, Kansas.

Let c_i denote the addition carry-over from the ith column to the $i + 1$st column, counting from right to left. Clearly $H = c_4 = 1$. Our problem indicates the following relations:

$$R + A + I = 10c_1 + Y \qquad (1)$$
$$c_1 + A + N = 10c_2 + E \qquad (2)$$
$$c_2 + E + I = 10c_3 + N \qquad (3)$$
$$c_3 + M + W = 10 + O \qquad (4)$$

Relations (2) and (3) combine to yield

$$A + I + c_1 + c_2 = 10(c_2 + c_3) \qquad (5)$$

Now relation (5) yields four cases upon examination:

I. $A + I + c_1 + c_2 = 20$ whence $c_1 = 2$, $c_2 = c_3 = 1$ and $A + I = 17$.

II. $A + I + c_1 + c_2 = 10$ whence $c_2 = 0$, $c_1 = c_3 = 1$ and $A + I = 9$.

III. $A + I + c_1 + c_2 = 10$ whence $c_1 = c_3 = 0$, $c_2 = 1$ and $A + I = 9$.

IV. $A + I + c_1 + c_2 = 10$ whence $c_1 = c_2 = 1$, $c_3 = 0$ and $A + I = 8$.

Case I. $A + I + c_1 + c_2 = 20$, $c_1 = 2$, $c_2 = c_3 = 1$ and $A + I = 17$. Hence $A, I = (8, 9)$ in some order. If $A = 8, I = 9$ relation (3) implies $10 + E = 10 + N$ which is impossible. Hence $A = 9$ and $I = 8$. Relation (3) implies $E = N + 1$. Relation (1) implies $R = 3 + Y$; whence (R, Y) has the possible values $(3, 0)$, $(5, 2)$, $(6, 3)$, $(7,4)$. Suppose $R = 6, Y = 3$. The remaining digits are 0, 2, 4, 5, 7 whence $E = 5$, $N = 4$ and thus $O = 0$, $M = 2$ and $W = 7$ and we have the following solution

$$\begin{array}{r} 7\,5\,9\,6 \\ 9 \\ 2\,8\,4\,8 \\ \hline 1\,0\,4\,5\,3 \end{array}$$

(Here as in all solutions M and W are interchangeable but M is taken so that $M < W$ in order to minimize $MINI$.)

Examining the other possibilities for (R, Y) we find that $(5, 2)$ and $(7, 4)$ lead only to contradictions. $(R, Y) = (3, 0)$ which leads to $E = 6$, $N = 5$ and the solution $W = 7, M = 4, O = 2$.

$$\begin{array}{r} 7\,6\,9\,3 \\ 9 \\ 4\,8\,5\,8 \\ \hline 1\,2\,5\,6\,0 \end{array}$$

Case II. $A + I + c_1 + c_2 = 10$ and $c_2 = 0$, $c_1 = c_3 = 1$ and $A + I = 9$.

Relation (1) implies $R - 1 + Y$ and relation (2) requires $A + N = E \leqq 8$. Furthermore, of the possible choices for A; i.e. 0, 2, 3, 4, 5, 6, 9, 7 we can exclude $A = 9$ immediately. $A = 7$ implies $N = 0$, $E = 8$ and no suitable values for R, Y can be found. Similar examination of the other possible choices of A yield the solutions: $A = 0$ yields:

$$\begin{array}{r} 6\,4\,0\,8 \\ 0 \\ 5\,9\,3\,9 \\ \hline 1\,2\,3\,4\,7 \end{array} \quad \begin{array}{r} 8\,5\,0\,7 \\ 0 \\ 3\,9\,4\,9 \\ \hline 1\,2\,4\,5\,6 \end{array} \quad \begin{array}{r} 6\,8\,0\,4 \\ 0 \\ 5\,9\,7\,9 \\ \hline 1\,2\,7\,8\,3 \end{array} \quad \begin{array}{r} 8\,7\,0\,3 \\ 0 \\ 5\,9\,6\,9 \\ \hline 1\,4\,6\,7\,2 \end{array}$$

$A = 2$ yields: $\qquad$ $A = 6$ yields:

$$\begin{array}{r} 4\,6\,2\,9 \\ 2 \\ 5\,7\,3\,7 \\ \hline 1\,0\,3\,6\,8 \end{array} \qquad \begin{array}{r} 5\,9\,6\,8 \\ 6 \\ 4\,3\,2\,3 \\ \hline 1\,0\,2\,9\,7 \end{array}$$

and $A = 4$ yields:

$$\begin{array}{r} 6\,7\,4\,9 \\ 4 \\ 3\,5\,2\,5 \\ \hline 1\,0\,2\,7\,8 \end{array}$$

while $A = 3$ or 5 yield no solutions.

Case III. $A + I + c_1 + c_2 = 10$ and $c_1 = c_3 = 0$, $c_2 = 1$ and $A + I = 9$.

Here relation (1) implies $R + A + I = Y \leqq 9$ whence $R = 0$ and $Y = 9$. Relation (3) implies $1 + E + I = N \leqq 8$. Now the possible choices for I are 2, 3, 4, 5, 6, 7. $I = 7$ implies $E = 0$ which is impossible. $I = 6$ implies $A = 3$, $E = 0$, $N = 7$ which is also impossible. $I = 5$ implies $A = 4$, $E = 2$, $N = 8$ whence $W = 7$, $M = 6$, $O = 3$ and our solution is:

$$\begin{array}{r} 7\,2\,4\,0 \\ 4 \\ 6\,5\,8\,5 \\ \hline 1\,3\,8\,2\,9 \end{array}.$$

Similarly $I = 3$, $A = 6$, $E = 4$, $N = 8$ yields:

$$\begin{array}{r} 7\,4\,6\,0 \\ 6 \\ 5\,3\,8\,3 \\ \hline 1\,2\,8\,4\,9 \end{array}.$$

Similarly $I = 5$, $A = 4$, $E = 2$, $N = 8$ yields:

$$\begin{array}{r} 7\ 2\ 4\ 0 \\ 4 \\ 6\ 5\ 8\ 5 \\ \hline 1\ 3\ 8\ 2\ 9 \end{array}$$

$I = 4$ or 2 lead to no solutions.

Case IV. $A + I + c_1 + c_2 = 10$ and $c_1 = c_2 = 1, c_3 = 0$ and $A + I = 8$.

Relation (5) indicates $A + I = 8$; hence relation (1) becomes $R + 8 = 10 + Y$ or $R = Y + 2$. Relation (3) yields $E + I + 1 = N \leqq 9$. Hence $E + I \leqq 8$. The choices for A are 0, 2, 3, 5, 6, 8. These and relation (3) cut down the number of trials necessary. $A = 0, I = 8$ implies $E = 0$ which is impossible. $A = 2, I = 6$ implies $E + 7 = N \leqq 9$ whence $E = 0, N = 7, R = 5, Y = 3$ and no consistent values can be found for W, M, O. $A = 3, I = 5$ implies $6 + E = N \leqq 9$ whence $(E, N) = (2, 8)$ or $(0, 6)$. Of these $E = 0, N = 6$ leads to a solution which is

$$\begin{array}{r} 9\ 0\ 3\ 4 \\ 3 \\ 8\ 5\ 6\ 5 \\ \hline 1\ 7\ 6\ 0\ 2 \end{array}$$

while $E = 2, N = 8$ leads to

$$\begin{array}{r} 6\ 2\ 3\ 9 \\ 3 \\ 4\ 5\ 8\ 5 \\ \hline 1\ 0\ 8\ 2\ 7 \end{array}.$$

$A = 5, I = 3$ implies $4 + E = N \leqq 9$ whence $(E, N) = (0, 4)$, $(4, 8)$ or $(2, 6)$. Of these only $E = 4, N = 8$ leads to a solution which is

$$\begin{array}{r} 9\ 4\ 5\ 2 \\ 5 \\ 7\ 3\ 8\ 3 \\ \hline 1\ 6\ 8\ 4\ 0 \end{array}.$$

$A = 8, I = 0$ implies $E + 1 = N \leqq 9$ but no solutions are found.

$A = 6, I = 2$ implies $E + 3 = N \leqq 9$ whence $(E, N) = (0, 3)$, $(4, 7)$ or $(5, 8)$, none of which lead to any solutions where $H = 1$. If solutions are permitted wherein $H = 0$ then we get

$$\begin{array}{r} 3\,5\,6\,9 \\ 6 \\ 1\,2\,8\,2 \\ \hline 0\,4\,8\,5\,7 \end{array} \quad \text{and} \quad \begin{array}{r} 8\,4\,6\,5 \\ 6 \\ 1\,2\,7\,2 \\ \hline 0\,9\,7\,4\,3 \end{array}.$$

Hence if the last sentence requires that *HONEY* be minimized, the solution is:

$$\begin{array}{r} 6\,7\,4\,9 \\ 4 \\ 3\,5\,2\,5 \\ \hline 1\,0\,2\,7\,8 \end{array}$$

which also minimizes *MINI*.

Also solved by Robert Thompson, University of Evansville, Evansville, Indiana.

245. *Proposed by the Editor.*

Find the smallest positive integer which ends with the digit 9 such that if this 9 is moved from the last place to the first place the number formed is three times as large as the original.

Solution by Robert Thompson, University of Evansville, Evansville, Indiana.

The answer is of the form:

$$(9\_\,\_\,x\_\,\_\,\_) = 3(\_\,\_\,y\_\,\_\,\_\,9)$$

where y is the number being sought. First, start by multiplying $9 \times 3 = 27$, then, place the 7 before the 9 in y with 7 being the last digit in x. Next multiply 7×3 plus 2 for the carry digit above, which is 23 and place the 3 in front of both x and y. Continue similarly until you multiply 3×3 with no carry digit, to obtain:

$$\begin{array}{rr} y = & 3103448275862068965517241379 \\ & 3 \\ \hline x = & 9310344827586206896551724137 \end{array}$$

The smallest integer is:

$$3103448275862068965517241379\,.$$

Also solved by Vance L. Johnson, Western Illinois University, Macomb, Illinois; Barbara Learner, Southwest Missouri State College, Springfield, Missouri; Jacque Ogden, Northeast Missouri State College, Kirksville, Missouri; Kenneth M. Wilke, Topeka, Kansas.

The Mathematical Scrapbook

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material to the Scrapbook editor. Material will be used where possible and acknowledgment will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Richard L. Barlow, Kearney State College, Kearney, Nebraska 68847.

During the college football season, almost every sports fan would like to forecast the outcomes of the various games each weekend. The method of prediction could be a "hunch" or possibly could be based on some mathematical procedure. One such mathematical method is based upon matrices, in particular, the sociometric matrices.

A sociometric matrix is a square matrix having only the entries 0 and 1. They have been used quite extensively to analyze the structure of the dominance patterns in groups of subjects.

We shall use the notation $A_1 >> A_2$ to indicate that subject A_1 dominates subject A_2. Hence, in the case of football teams, $A_1 >> A_2$ means that team A_1 beats team A_2.

As a more formal definition, we shall define the relation $>>$ as a dominance relation if it satisfies the following properties:

(i) It is false that $A_i >> A_i$. That is, no individual can dominate himself.

(ii) For each pair of individuals as A_1 and A_2, either $A_1 >> A_2$ or $A_2 >> A_1$, but not both. That is, in every pair of individuals, there is exactly one who is dominant.

You may have noticed that we did not assume that if $A_1 >> A_2$ and $A_2 >> A_3$, then $A_1 >> A_3$. This is the so-called transitive law, and if you think about it, you will notice that it is not necessarily true. Take for example football teams. If team A_1 beats A_2 and team A_2 beats A_3, you cannot assume that A_1 will beat A_3. In every football season there are several examples where "upsets" occur.

Can one use this relation to predict the winner of a game if given the results of previous games? If so, to what degree of accuracy?

Dominance relations can be exhibited by means of matrices, called dominance matrices, having only zeros and ones as entries. An example is:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \end{array} .$$

An entry of 1 in the row of individual A_i and the column of individual A_j means that individual A_i dominates A_j; that is $A_i >> A_j$. Similarly a 0 entry there means that A_i does not dominate A_j. Here, $A_1 >> A_3$, $A_2 >> A_1$, and $A_3 >> A_2$. This matrix satisfies the requirements of the definition of a dominance relation.

Since a dominance matrix, called D, is a square matrix, we can compute the powers of the matrix D: D^2, D^3, etc. Let $M = D^2$ and consider the entry m_{ij} in the ith row and jth column of M. We have

$$m_{ij} = d_{i1}d_{1j} + d_{i2}d_{2j} + \ldots + d_{in}d_{nj} .$$

Now, a term of the form $d_{ik}d_{kj}$ can be nonzero only if both factors are nonzero; that is, only if both factors are equal to 1. But if $d_{ik} = 1$, then $A_i >> A_k$ and if $d_{kj} = 1$, then $A_k >> A_j$. This implies that $A_i >> A_k >> A_j$. We shall call this kind of dominance a two-stage dominance. ($A_i >> A_j$ is called a one-stage dominance to avoid confusion). Entry m_{ij} hence gives the total number of two-stage dominance paths which exist between A_i and A_j (in that direction).

For example, let D be the matrix:

$$D = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} .$$

Then $M = D^2$ is the matrix

$$D^2 = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} .$$

Therefore, A_1 has one two-stage dominance path with A_2 and two two-stage dominance paths with A_3. Similar results hold for A_2, A_3, and A_4 as noted by the matrix D^2. As an illustration, we note that for A_1 the two stage dominance paths are

$$A_1 >> A_4 >> A_2$$
$$A_1 >> A_2 >> A_3$$
$$A_1 >> A_4 >> A_3 .$$

Then define matrix S as:

$$S = D + D^2 = \begin{pmatrix} 0 & 2 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 0 \end{pmatrix}.$$

This gives us the power of an individual; that is, the total number of one-stage and two-stage dominances which he can exert. Since the total number of one-stage dominances exerted by A_i is the sum of the entries in row i of the matrix D, and the total number of two-stage dominances exerted by A_i is the sum of the entries in row i of the matrix D^2; therefore, we can see that the power of A_i can be expressed as the sum of the entries in row i of the matrix S.

This gives us for the above matrix:

The power of A_1 is 5
The power of A_2 is 2
The power of A_3 is 3
The power of A_4 is 4

It can be shown that the individual or individuals having the largest row sum in the matrix $S = D + D^2$ can dominate everyone else in one or two stages. Also, the individuals having the largest column sum in S can be dominated by all other individuals in one or two stages.

When one uses the method of sociometric matrices on baseball or basketball games giving a 1 for a win and 0 for a loss to a team, then one does not run into the problem of tie games which are possible in football. Slight variations are necessary to account for tie football games. Should you give both teams 1, ½, or 0 for a tie game? You could argue that both teams should receive 0 since neither team dominated the other, but this rather cheats both teams on computing their powers and hence maybe giving both ½ or 1 might be better. A good example of this problem would have occurred last year if you wished to predict the outcome of the bowl games.

Both Nebraska and USC were in bowl games and early in the season these two teams had played each other to a tie score. How should this have been considered? By actual practice with results from previous seasons, it appears that in general you will be better off if you give both teams a zero for a tie game. The accuracy in predicting the game's outcome is better.

Another problem occurs when a team beats another team by a large margin. Again, by experimentation, you will find that using simply a 1 for the win is best.

Computational problems arise when you compute the matrices D^2 and S when the number of teams involved is large. For ease in computation, a computer is recommended. To best predict the results of each bowl game, you will need to consider all the common teams the two playing teams have previously met. This can involve several teams, and this method gives better results when they have played several common teams.

By the above method of sociometric matrices, can you predict the outcome of the next game involving your college team?

(Continued from p. 25)

Now the function $\cos n^2x$ has the same values as the function $|\cos n^2x|$ over the interval $[0, \pi/2n^2]$. The period of $|\cos n^2x|$ is π/n^2, it has symmetry about the line $x = \pi/2n^2$, and a maximum occurs at $x = 0$. Hence

$$n \int_0^{\pi} |\cos n^2x|\, dx = n \cdot 2n^2 \int_0^{\pi/2n^2} \cos n^2 x\, dx$$

$$= n \cdot 2n^2 \left(\frac{1}{n^2} \sin n^2x\right)\Big|_0^{\pi/2n^2} = 2n\,.$$

Thus, $\int_0^{\pi} 1 + (n \cos n^2x)^2\, dx > 2n\,.$

Since the length of the nth curve is always greater than $2n$, the length of S_n increases without bound as n increases without bound. But the length of the limit curve S_L (the geometric point set that S_n approaches as n increases) is quite definitely π. Therefore we arrive at the unavoidable conclusion that the length of the limit curve is not the limit of the lengths of the curves.

In summary, the set of curves defined by $y = 1/n \sin n^2x$, $0 \leq x \leq \pi$, is a sequence of geometric curves which converges to the line segment on the x-axis from 0 to π, but the lengths of these curves form a sequence of positive numbers which increases without bound.

The Book Shelf

EDITED BY ELIZABETH T. WOOLDRIDGE

This department of THE PENTAGON brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. Elizabeth T. Wooldridge, Department of Mathematics, Florence State University, Florence, Alabama 35630.

Mathematical Analysis for Management Decisions. A. K. McAdams, The Macmillan Company, New York, 1970, 363 pp., $10.95.

This is essentially a Calculus text written for business students. There is also an introduction to linear programming and a chapter on vectors, matrices, and determinants. The material is presented in a clear readable style illustrated by business management examples.

The author begins with a discussion of sets and functions followed by a chapter on limits and continuity with much motivation and illustration of these concepts. Limit theorems are stated without proofs. Most of the theory of the calculus is presented without proofs. Proofs are given for differentiation rules and the Fundamental Theorem of the Calculus, however. Partial derivatives are discussed with many applications to maximum-minimum problems in several variables. The author emphasizes optimization problems in business management. The discussion of linear programming focuses on this problem. Many examples are provided. A discussion of sensitivity analysis is included with a brief introduction to parametric programming. The final chapter consists of a survey of concepts and methods for vectors and matrices. There are few examples or applications here, but a clear discussion of basic concepts suitable for business students is given.

A large number of exercises is included for student practice, but one fault of this text may be the failure to include answers. This lowers the value of the text for student self-study and drill.

Ben F. Plybon
Miami University

Principles of Arithmetic and Geometry, Carl B. Allendoerfer, The Macmillan Company, New York, 1971, 672 pp., $9.95.

Textbooks for use in preparation of elementary teachers in the area of mathematics which reflected the recommendations of

CUPM and other groups vitally interested in updating the preparation were very scarce a few short years ago. Now we find many books on the market which satisfy these needs, one of which is the contribution of Carl B. Allendoerfer.

The book, although somewhat ambitious for the stated purpose of a year course which meets three times a week, has all the ingredients needed to supply appropriate mathematics in the preservice aspects of elementary teacher training. Logical development is not sacrificed at the expense of pedagogy. The "skeleton" of mathematics needed for elementary school programs is built in a sophisticated manner which in turn is ready for the "meat" to be supplied by a good elementary teacher reflecting sound pedagogy.

The development of the book is unique in that it combines the standard format of a textbook with a programmed supplement which enables immediate feedback to the student.

The chapters devoted to geometry are exceptionally strong in the intuitive development of both plane and solid geometry concepts.

The textbook should be a welcome addition to the list of books considered for content courses in mathematics for elementary majors.

Ramon L. Avila
Ball State University

Nonparametric Statistical Inference, Jean D. Gibbons, McGraw-Hill Book Co., New York, 1971, 306 pp., $11.95.

This is an excellent addition to the growing list of textbooks in nonparametric statistics intended for an introductory course for graduate students. It assumes knowledge of probability and statistical inference at the first-year graduate level.

Its business is to derive probability distributions, to validate such standard tests as the Kolmogorov-Smirnov test for goodness of fit of empirical distributions, and to teach methods for handling rank-order and distribution-free problems in inference.

Applications as such are not discussed in the text, but many examples of uses are suggested in the large variety of problems for the reader's attention.

Professor Gibbons' book is well done and can be highly recommended.

Paul D. Minton
Southern Methodist University

Arithmetic for Teachers, Second Edition, Wilbur H. Dutton, Colin C. Petrie, and L. J. Adams, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1970, 318 pp., $7.95.

This book represents still another attempt at defining what is appropriate preservice mathematics education for prospective elementary teachers. This reviewer does believe that a valuable contribution has been made to that end with this book.

The authors attempt to make extensive use of current research studies which bear on mathematics education is perhaps the book's strongest feature. The book contains a chapter on problem-solving which is very complete and helpful in explaining the higher mental processes which are involved therein. Another impressive chapter concerns evaluation. A serious study and application of the ideas included in this chapter would go a long way toward the restructuring of many elementary mathematics courses—a restructuring which so often seems so necessary to this reviewer.

Though this reviewer was generally impressed with the book there were a few shortcomings which must be commented on. The chapters seemed to vary in quality from one to the next. Chapter 11 is an introduction to algebra which seems to be carefully written with some good suggested teaching strategies included. The next chapter is an introduction to geometry which by contrast is simply a continuous recital of definitions and resulting conclusions. Informal geometry certainly deserves better treatment than this.

In the introduction to plane geometry the reviewer found some statements which were a bit disconcerting. As an example, the "measure" of an angle is determined by the "amount of turning" between the rays that form the angle. Then an angle of 1° is defined as an angle with an opening which is 1/180 of a straight angle. That is an unnecessary inconsistency. In another instance the following statements are made. A point on a line separates the line into two parts called *rays.* A *ray* is a subset of a line having one endpoint. These two statements are not compatible. These deficiencies could admittedly be patched up fairly easily.

Despite the criticisms given above this reviewer does believe that the book has many positive features and would recommend its consideration in any preservice mathematics course for prospective elementary teachers.

Alton T. Olson
The University of Alberta

Calculus with Analytic Geometry, Nathan O. Niles and George E. Haborak, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1971, 601 pp., $11.95.

Topics covered by this book are: (1) an introduction including preliminary remarks concerning sets, functions, limits, and continuity; (2) the derivative and some applications, geometric and physical, for simple algebraic functions, particularly polynomials, and Newton's method of finding roots; (3) the integral, the trapezoidal rule (Simpson's rule being deferred until later), areas of summation, the definite integral; (4) applications of the definite integral, areas, moments, volumes and work where mostly simple algebraic expressions and polynomials are involved; (5) more of the same, except that more advanced algebraic equations are treated; (6) logarithmic and exponential functions and differentiation and integration of these; (7) trigonometric and inverse trigonometric functions; (8) more integration methods (parts, partial fractions, etc.) and use of integral tables; (9) conics; (10) polar coordinates; (11) differential equations; (12) functions of two variables and partial differentiation; (13) series.

The book is generally well written, comparing favorably with previous elementary college texts written by U.S. Naval Academy mathematicians. As might be expected, the book is slanted toward practicality, although sufficient theoretical aspects are present giving a good balance. At the end of each chapter appears a list of "important words and concepts" for the chapter, an innovation which may be useful for student review.

One would question whether the part of the first chapter devoted to introductory set theory is worthwhile, since no use of it as such appears later in the book. The book contains material on the Laplace transform and also on Fourier Series, this being something of a departure from tradition except in some of the recent large, two volume treatments of the calculus. It is doubtful, at least to this reviewer, that many mathematics departments would have need for these topics in their beginning calculus courses. On the other hand, even a brief study of them could afford the student a wealth of information.

The most striking omission would seem to be that of triple integrals. The very good treatment of double integrals might be followed to advantage by at least a short discussion of triple integrals.

There are many, many problems. Answers are given for the odd ones.

An excellent set of useful tables appears at the back of the book. There is also a short appendix which takes up three topics: completing the square, method of least squares, and vectors.

Generally, any shortcomings the book may have are far outweighed by its good points. It should be quite "teachable," in so far as such a decision may be made without having actual classroom experience with it.

D. H. Erkiletian, Jr.
University of Missouri-Rolla

MINIREVIEWS

Perspectives on Secondary Mathematics Education, Henry A. McIntosh (Editor), Prentice-Hall, Inc., Englewood Cliffs, N.J., 1971, 269 pp., $4.95 (paper).

A collection of recent (1958-1969) articles from a wide assortment of periodicals. Sections of book are titled: Evolution of Mathematics Education, Methods, Curriculum, Research. Certain "classics" are included by Stone, Bruner, and Allendoerfer. Excellent source if used before it is out of date.

Finite Mathematics with Applications, A. W. Goodman and J. S. Ratti, The Macmillan Company, New York, 1971, 506 pp., $10.95.

Quoting from the preface, "The guiding principle is to present the reader a slice of mathematics that is interesting, meaningful, and useful and at the same time does not involve the calculus." Topics are covered from logic, probability, vectors, theory of games, induction, and others. The book uses brown and gold ink!

Intermediate Algebra: Structure and Use, R. A. Barnett, McGraw-Hill Book Company, New York, 1971, 411 pp.

The first six chapters are a review of elementary algebra with some extensions. The last four chapters are independent of each other and are titled: Exponential and Logarithmic Functions, Sequences and Series, Polynomial Functions of Higher Degree, and Matrices and Determinants. Odd-numbered problems have answers in appendix. Book uses black and blue ink on gray paper.

Kappa Mu Epsilon News

EDITED BY ELSIE MULLER, *Historian*

The Eighteenth Biennial Convention of Kappa Mu Epsilon met at Indiana University of Pennsylvania, Indiana, Pennsylvania on April 1-3, 1971. Much credit is due the Pennsylvania Zeta chapter and its corresponding secretary, Ida Z. Arms, for a profitable and enjoyable convention.

After the registration in the lobby of the Holiday Inn there was a mixer in Rooms B and C during the evening of April 1. The National Council also held a meeting at that time.

In the morning registration was continued in the lobby of Pratt Hall on the campus of Indiana University of Pennsylvania. At the opening session in the auditorium of Pratt Hall with the National President, George R. Mach of California Gamma, presiding, Dr. Bernard T. Gillis, Dean of Academic Affairs and Faculty, Indiana University of Pennsylvania gave the address of welcome with Past President Fred Lott, Jr. of Iowa Alpha responding. The National Secretary, Laura Z. Greene of Kansas Delta, conducted the roll call of the chapters.

The National Vice President, William R. Smith of Pennsylvania Zeta, presided during the presentation of the following student papers:

1. *Taxi Analytic Geometry* by Sherry Stradley, Ohio Zeta, Muskingum College
2. *Quasi-Trigonometric Functions* by Marcellus Graham, Alabama Beta, Florence State College
3. *Square Trigonometry* by William Georgou, Iowa Alpha, University of Northern Iowa
4. *An Introduction to Gaussian Primes* by Roberta Perry, Pennsylvania Zeta, Indiana University of Pennsylvania

During the noon hour a group picture was taken in front of Pratt Hall. After lunch, rap sessions were held with Ida Z. Arms of Pennsylvania Zeta presiding over the faculty section and Diane Mularz of Pennsylvania Zeta presiding over the student section. After the convention reconvened, the recorders, Allan Becher of Pennsylvania Kappa and Linda Hilgendorf of Wisconsin Alpha reported from the two sections.

The National Vice President, William R. Smith, continued to preside over the presentation of the following student papers:

5. *An Instance of Intuition and Lengths of Limiting Curves* by Nancy Edwards, Kansas Beta, Kansas State College, Emporia
6. *Gergonne's Pile Problem* by Larry D. Snider, Texas Zeta, Tarleton State College
7. *Elementary Difference Equations and Their Solutions* by Harold J. Rouster, Ohio Zeta, Muskingum College
8. *Basic Properties of Diabolic Magic Squares* by Margaret Hoehl, Kansas Gamma, Mount Saint Scholastica College

A most enjoyable banquet was held at the Holiday Inn during the evening of April 2 with Dr. Merle Stilwell of Pennsylvania Zeta as master of ceremonies. The I-Uppers of Indiana University of Pennsylvania, with Leonard De Fabo as director, furnished the delightful musical entertainment. The featured speaker of the evening was Professor Ernst Snapper of Dartmouth College with the address, *Foundations of Geometry.*

The convention resumed on Saturday with the presentation of the remainder of the student papers:

9. *Dedekind's Definition of Real Numbers* by Sylvia Schorn, Kansas Gamma, Mount Saint Scholastica College
10. *The Probability that a determinant of Order n Will be Even* by Frances Coledo, Pennsylvania Zeta, Indiana University of Pennsylvania
11. *On Primitive Pythagorean Triples* by Michael Brandley of Kansas Beta, Kansas State College, Emporia
12. *Student Creativity and Beta Functions* by Charles L. Breindel (graduate student) of Pennsylvania Zeta, Indiana University of Pennsylvania

The following papers were listed as alternates:

1. *Peano Curves* by Barbara Shappard of Kansas Delta, Washburn University
2. *Introduction to Biomathematics: A Model in Bio Assay* by Janice Sharper of Maryland Beta, Western Maryland College
3. *Hyperbolic Trigonometry, an Introduction: Absolute Unit*

of Length and an Important Figure by A. Jean Hall of Kansas Gamma, Mount Saint Scholastica College

4. *Fundamentals of Graph Theory* by Kathleen Davidek of Pennsylvania Zeta, Indiana University of Pennsylvania
5. *An Introduction to the Theory of Qualitative Linear Systems* by Susan Jarchow of Kansas Gamma, Mount Saint Scholastica College
6. *Perfect Numbers* by Barbara Shappard of Kansas Delta, Washburn University

A high point of the convention occurred when Eddie Robinson, National Historian, gave an address on the occasion of the fortieth anniversary of KME. He also distributed a booklet on the history of KME.

Margaret E. Martinson of Kansas Delta, chairman of the nominating committee, reported. The following officers were elected for 1971-1973:

President - - - -	Dr. George R. Mach California State Polytechnic College
Vice President - -	Professor William R. Smith Indiana University of Pennsylvania
Secretary - - -	Professor Laura Z. Greene Washburn University of Topeka
Treasurer - -	Professor Eddie W. Robinson Southwest Missouri State College
Historian - - - - -	Dr. Elsie Muller Morningside College

The Awards Committee announced the following winners for papers presented by undergraduates at the convention:

First Place - - -	William Georgou, Iowa Alpha
Second Place -	Frances Coledo, Pennsylvania Zeta
Third Place - - -	Nancy Edwards, Kansas Beta

REPORT OF THE NATIONAL PRESIDENT

Kappa Mu Epsilon has experienced growth and improvement during the past biennium. Eleven chapters have been installed and this is the largest number of new chapters in any biennium. They are: Illinois Eta at Western Illinois University; Ohio Zeta at

Muskingum College; Pennsylvania Theta at Susquehanna University; New York Theta at St. Frances College; Pennsylvania Iota at Shippensburg State College; Maryland Gamma at Saint Joseph College; Mississippi Delta at William Carey College; Missouri Theta at Evangel College; Pennsylvania Kappa at Holy Family College; Colorado Beta at Colorado School of Mines; and Kentucky Alpha at Eastern Kentucky University. There are now ninety-one active chapters in 29 states.

There were two regional conventions in 1970. Michigan Beta Chapter hosted one on April 17-18 at Central Michigan University with over 50 in attendance. Missouri Beta Chapter hosted another on April 25 at Central Missouri State College with about 125 in attendance.

During the past biennium, the National Council has undertaken a number of new programs, some not yet completed, which will improve the Society's operations and better serve the members. The National Council met in Kansas City in January of 1970 and worked through an agenda of over 40 items. Many of our new developments are the results of that meeting.

The Constitution and Bylaws have been revised and reprinted. The Initiation and Installation Ceremonies have been revised and reprinted. The chapter charter has been revised and reprinted. The membership certificate was revised and new plates were engraved. The establishment of cash awards of $50, $30, and $20 for student papers at this convention is a new feature. The policy of having the nominating committee present two candidates for each office (except an incumbent secretary or treasurer) was adopted this year. A chapter handbook is now being developed. It is anticipated that this handbook will clarify and simplify many of the functions of the faculty sponsor and the corresponding secretary. This year, for the first time, the reports of the national officers were submitted in advance so that you may have a copy of them before you leave the convention.

The National Council has approved a regional organization for the Society and it is being announced for the first time now. One of the purposes of this organization is the encouragement of more regional conventions, which have been held very successfully for many years in some "regions" and not at all in others. The regions and their active chapters are:

Region 1—

Massachusetts Alpha	New York Theta	Pennsylvania Theta
Connecticut Alpha	New Jersey Alpha	Pennsylvania Iota
New York Alpha	New Jersey Beta	Pennsylvania Kappa
New York Gamma	Pennsylvania Beta	Maryland Alpha
New York Delta	Pennsylvania Delta	Maryland Beta
New York Epsilon	Pennsylvania Epsilon	Maryland Gamma
New York Zeta		

Region 2—

New York Eta	Ohio Epsilon	Wisconsin Alpha
Pennsylvania Alpha	Ohio Zeta	Illinois Alpha
Pennsylvania Gamma	Michigan Alpha	Illinois Beta
Pennsylvania Zeta	Michigan Beta	Illinois Gamma
Pennsylvania Eta	Indiana Alpha	Illinois Delta
Ohio Alpha	Indiana Beta	Illinois Epsilon
Ohio Gamma	Indiana Gamma	Illinois Zeta

Region 3—

Indiana Delta	Tennessee Gamma	Alabama Beta
Kentucky Alpha	North Carolina Alpha	Alabama Gamma
Virginia Alpha	Mississippi Alpha	Alabama Epsilon
Virginia Beta	Mississippi Beta	South Carolina Alpha
Tennessee Alpha	Mississippi Gamma	South Carolina Beta
Tennessee Beta	Mississippi Delta	Florida Alpha

Region 4—

Wisconsin Beta	Colorado Alpha	Missouri Alpha
Iowa Alpha	Colorado Beta	Missouri Beta
Iowa Beta	Kansas Alpha	Missouri Gamma
Iowa Gamma	Kansas Beta	Missouri Epsilon
Illinois Eta	Kansas Gamma	Missouri Zeta
Nebraska Alpha	Kansas Delta	Missouri Eta
Nebraska Beta	Kansas Epsilon	Missouri Theta
Nebraska Gamma		

Region 5—

Oklahoma Alpha	New Mexico Alpha	Texas Gamma
Oklahoma Beta	Texas Alpha	Texas Epsilon
Arkansas Alpha	Texas Beta	Texas Zeta

Region 6—

California Gamma	California Delta

The next step will be to appoint a director for each region. The concept is not fully developed, but it is anticipated that the directors might eventually perform such functions as: promote the general interests of Kappa Mu Epsilon in the region, take the initiative in the establishment of regional conventions in the even-numbered years, represent the society on various occasions, etc.

Throughout the biennium I have had the pleasure of contacts with many willing and helpful corresponding secretaries. I have had the honor of serving with an enthusiastic and dedicated National Council. All are worthy of tribute, but I especially want to mention Helen Kriegsman, who is retiring after six years as editor of THE PENTAGON, and Walter Butler, who is retiring after twelve years as National Treasurer.

George R. Mach

REPORT OF THE VICE-PRESIDENT

It has been an interesting and different two years experience serving as Vice-President of Kappa Mu Epsilon.

My first chance to represent the Society was at the meeting of the Association of College Honor Societies at Auburn University in February, 1969. A copy of the minutes of that meeting are in the hands of our secretary who also attended the meeting. Copies may be obtained from my office if you would like one. The session dealt with The Honor Society Movement in the Junior College, Long Range Financing for ACHS, and The Creative Role of the Honor Society Outside of the Classroom.

The National Council met in Kansas City for two days of intensive work on many topics under the direction of President Mach.

I was honored to install the chapters at Susquehanna University, Holy Family College, and Eastern Kentucky University. I was scheduled to officiate at Saint Joseph College, but snow cancelled my trip. I would like to express my appreciation to Dr. James Lightner who filled in for me on very short notice.

Plans for the convention, particularly the selection of the papers, has been my major activity recently. The selection committee, Professors John Cross, James Smith, and Harold Thomas did a fine job and they deserve the thanks of all of us.

William R. Smith

REPORT OF THE NATIONAL SECRETARY

Kappa Mu Epsilon initiated approximately twenty-four hundred members this past biennium. The present membership is almost 29,400.

The secretary has the responsibility of maintaining the permanent records of the organization. Information for these records is taken from the chapter reports. Prompt, accurate reports from the corresponding secretaries are most important. All orders for supplies and jewelry are sent to the secretary. The jewelry orders are sent to the L. G. Balfour Company for shipment to you. This process usually takes four to six weeks.

The membership certificates are now being printed at the Blake Printery in San Luis Obispo, California. This service has been more satisfactory.

The National Officers are preparing a handbook which will be sent to each chapter. We hope to have this ready in the fall of 1971.

I appreciate the cooperation of all of the corresponding secretaries and the excellent support of the National Officers. Each of you has been most generous with your time and energy. Without the help of all of you, Kappa Mu Epsilon could not continue.

Laura Z. Greene

REPORT OF THE NATIONAL HISTORIAN

The primary objective of the office of the National Historian is to be a depository of records and documents relative to the activities, programs, and events of the chapters. I have continued the practice of past historians of soliciting news items semi-annually from each of the chapters for the ΚΜΕ news section of THE PENTAGON. Each semester, I requested a list of officers and news items covering the previous semester. These lists and news items have been filed in the folder of each chapter.

Since 1971 is the 40th Anniversary of the founding of Kappa Mu Epsilon, I have compiled a booklet of history and information to be distributed to each chapter. This booklet contains a statement of history and lists of presidents, vice presidents, secretaries, treasurers, historians, editors of the PENTAGON, business managers of the PENTAGON, locations of conventions, and number of initiates for each chapter. This booklet is not as extensive as I would have liked, but the activities of my office were restricted by a two-month absence on my part.

I wish to thank all the national officers for their help during my tenure of office and for their personal help and concern during my illness and absence from this office. As an associate editor of the PENTAGON, I wish to thank Helen Kriegsman, the Editor of the PENTAGON, for her consideration and help in the preparation of the KME News. The duties of this office are impossible without the cooperation of the Corresponding Secretaries, and to them I give my thanks.

The highlight of my term in office was the privilege of installing a chapter, Missouri Theta. I wish to officially thank Dr. Carl Fronabarger, Past President, for his assistance in this installation.

Eddie W. Robinson

REPORT OF THE EDITOR OF THE PENTAGON

During the past biennium only two changes have occurred in the editorial staff of THE PENTAGON. When George R. Mach became National President of KME, Richard L. Barlow of Kearney State College replaced him as Mathematical Scrapbook Editor. In the fall of 1970, Sister Helen Sullivan was granted a leave of absence to be on the staff at the University of Galway in Ireland, and Loretta Smith of Southern Connecticut State College assumed responsibility for the section on Installation of Chapters. As National Historian, Eddie Robinson has continued to record the Kappa Mu Epsilon News. James Bidwell, Book Shelf Editor, and Robert L. Poe, Problem Corner Editor, have fulfilled their duties in their usual admirable manner.

The valuable services of Business Manager, Wilbur Waggoner, and the National Secretary, Laura Z. Greene, have contributed substantially. I also wish to commend Bruce Eldred, Manager of the University Press at Central Michigan University, and his staff for their cooperation in printing the publication.

The editorial staff recognizes the worthwhile effort of all those who have contributed articles and material for the various sections. The support of advisors, corresponding secretaries, and other interested faculty members is also acknowledged.

Three issues of the publication have been completed since the last convention and the fourth issue is in process. The eighteen articles have been prepared by twelve students, five faculty members, and one alumnus.

Since this report is my last one as editor of THE PENTAGON, I wish to express my sincere appreciation to all the National Officers, the entire editorial staff, all contributors, and others who have had a part in making these past six years a most memorable experience. My only request is that my successor be given this same fine cooperation, and please keep his mailbox full of manuscripts.

Helen Kriegsman

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON

This is the seventh report of the activities and duties of the Business Manager of THE PENTAGON that I have presented to a biennial convention of Kappa Mu Epsilon. It becomes increasingly difficult to find something to report that I have not previously reported. Briefly my responsibility to Kappa Mu Epsilon is to see that each subscriber to our official journal receives a copy of THE PENTAGON as it is published. To do this requires that I have a current address for each subscriber. Chapter sponsors and delegates are urged to encourage chapter members to keep an address on file to which THE PENTAGON can be delivered. When a PENTAGON is returned to my office, because of an incorrect address, I remove the subscriber's card from the file of current subscribers. No more PENTAGONS are mailed to this incorrect address.

Since THE PENTAGON is not a "current events" journal and since it is published semi-annually, I try to mail PENTAGONS to each new subscriber for a period of approximately three months after a publication date. This action prevents new initiates from waiting as long as six months, after becoming members of Kappa Mu Epsilon, to receive their first copy of THE PENTAGON.

During the past biennium about thirteen thousand PENTAGONS were printed. Some of these were mailed twice as they were returned for incorrect addresses or were unclaimed by the addressee. A small stock of PENTAGONS is kept to fill requests for back issues. During the biennium copies of our journal went to all fifty states and several foreign countries. Libraries in many high schools, community colleges, colleges, and universities subscribe to THE PENTAGON. Almost five thousand PENTAGONS were mailed to just five states. These states in order of number of journals mailed to addresses within the state are Pennsylvania, Illinois, Texas, New York, and Missouri.

Complimentary copies of our magazines are sent to the library of each college or university with an *active* chapter of Kappa Mu Epsilon. Speakers at this convention will have their subscription extended for a period of two years. Authors of articles in THE PENTAGON receive complimentary copies of the issue in which their article appears.

My thanks to chapter corresponding secretaries, to Miss Laura Greene, our National Secretary, and to Dr. Helen Kriegsman, our editor whose cooperation makes my work for Kappa Mu Epsilon a pleasant and rewarding task.

Wilbur Waggoner

(The Report of the National Treasurer is on p. 67)

CHAPTER NEWS

California Delta, California State Polytechnic College, Pomona

12 actives, 10 pledges

The chapter sponsors free monthly seminars at which the lecturer is usually a local professor. All members tutor at least one hour per week on a free basis. All mathematics courses in the curriculum are covered in this way. Officers: Eddy Hartenstein, president; Cathy Ohl, secretary and treasurer; Professor Konigsberg, corresponding secretary; and Professor Kachun, faculty sponsor.

Connecticut Alpha, Southern Connecticut State College, New Haven

25 actives

Special programs during the year consisted of a talk on probability by Dr. B. R. Gulati, a talk on algebra by Dr. M. R. Heck, and a discussion on mathematics education led by Dr. R. M. Washburn and Tad Day of SEED. Officers: Gary Orlando, president; Judy Gallagher, vice-president; Cecilia Lowe, secretary; Stephen Comkowyez, treasurer; Erwin Sparks, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston

60 actives

The chapter monitored a calculus test for an award in honor of Professor Van Deventer, a deceased former faculty member. The chapter also entertains each pledge class at an informal Coke hour.

In the spring 30 members and 5 faculty sponsors took an educational and entertaining field trip via bus to St. Louis, Missouri. Officers: Bill Standerfer, president; Joan Wilson, vice-president; Kathy Gentile, secretary; Ruth Queary, corresponding secretary; Larry Williams, faculty sponsor.

Indiana Alpha, Manchester College, North Manchester

23 actives, 7 pledges

The chapter holds joint meetings twice a month with the local mathematics club, Chi Psi. The annual banquet featured Dr. David Wells of Oakland Schools in Michigan. Officers: David Warrick, president; Julia Gingrich, vice-president; Ann Swartz, secretary; Patricia Brewer, treasurer; Dr. David Neuhouser, corresponding secretary; Dr. Ralph McBride, faculty sponsor.

Iowa Gamma, Morningside College, Sioux City

30 actives

Four students and one faculty member attended the national convention at Indiana University. Douglas Dawson ranked in the top one-fourth of those who participated in the William Lowell Putnam Competition. He will be an assistant in the department of mathematics at Arizona State for the coming year. Officers: Sue Nelson, president; Stephen Bolks, vice-president; Sandra Popenhagen, secretary; Donald Hintz, treasurer; Dennis Watkins, corresponding secretary; Dr. Elsie Muller, faculty sponsor.

Kansas Alpha, Kansas State College of Pittsburg

60 actives

Susan Rogers presented a special proof of $\lim_{n \to a} n^k x^n = 0$ if $0 \leq |x| < 1$ at the February meeting. At the March meeting James Harlin spoke on the roots of a polynomial. Pat Kuhel and Karolyn Burgan reported on their experiences at the national convention during the final meeting. Recipient of the annual Robert Miller Mendenhall Award for scholastic achievement was John Thornton. He also received a KME pin in recognition of his achievement. Officers: Mark Davis, president; Donna Grisler, vice-president; Jeannie Spigarelli, secretary; Kerry Ryman, treasurer; Dr. Harold L. Thomas, corresponding secretary; Professor J. Bryan Spury, faculty sponsor.

Maryland Beta, Western Maryland College, Westminster

25 actives

Maryland Alpha and Maryland Beta held a joint meeting to listen to a representative from the American Statistical Society. Maryland Beta also attended the installation of Maryland Gamma with Dr. Lightner as the installing officer. In April Dr. Boner presented a talk on the Kayeka problem. Officers: Bonnie Green, president; Robert Chapman, vice-president; Sarah Tarr, secretary; Ronald Jemmerson, treasurer; James Lightner, corresponding secretary.

Maryland Gamma, Saint Joseph College, Emmitsburg

9 actives

In April Margaret F. McKiever spoke on the topic, "Reflections of a Mathematician" at the induction ceremonies. On April 26 Maryland Alpha, Beta, and Gamma were invited to hear the national vice-president, William R. Smith, present the topic, "How to Select a Wife." Officers: Linda Randenbush, president; Gloria Knott, vice-president; Maureen Hinke, secretary; Barbara Herbig, treasurer; Sister John Frances Gilman; corresponding secretary; Donald F. Shriner, faculty sponsor.

Massachusetts Alpha, Assumption College, Worcester

7 actives, 5 pledges

After the initiation dinner on April 28, the following papers were presented: "Finite Differences" by Robert Dupuis and "Fourier Series" by Philip Pelosi. Officers: Robert Dupuis, president; Philip Pelosi, vice-president; Paul Tessier, secretary; Charles Brussard, corresponding secretary; Rev. Richard P. Brunelle, A.A., faculty sponsor.

Mississippi Alpha, Mississippi State College for Women, Columbus

15 actives, 16 pledges

The program highlight was a talk by Mr. Gil Harris, Chief Engineer of Mitchell Engineering Company and a subsequent visit to their computer center. Officers: Martha Pope, president; Shirley McLemore, vice-president; Jeanne King, secretary and treasurer; Dr. Donald A. King, corresponding secretary and faculty sponsor.

Missouri Alpha, Southwest Missouri State College, Springfield

40 actives

Valerie Bottom received the KME Merit Award from the Missouri Alpha chapter. At the annual picnic the chapter presented Outstanding Freshmen Awards. Officers: Peggy Stuckmeyer, president; James Springton, vice-president; Nelda Burton, secretary; Hedy Chen, treasurer; Eddie Robinson, corresponding secretary; Dr. L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri, Warrensburg

41 actives, 29 pledges

The chapter had six formal meetings, a Christmas party, and a spring banquet during the year. Officers: Paul Bowman, president; Stanley Scott, vice-president; Janet Barnes, secretary; Barbara Alford, treasurer; Velma Birkhead, corresponding secretary; Shana McCann, historian; and Homer Hampton, faculty sponsor.

Missouri Zeta, University of Missouri—Rolla

25 actives

The chapter each year gives a first and second prize to the exhibits picked by their own judges as being the best mathematics exhibits at the Science Fair. Both prizes were given to the same exhibit, *The Theory of Braids as Related to Permutations and Combinations,* by Nancy Zaiser from West Plains. At the Spring Smoker, Professor Luffel of UMR presented a talk on "The History of Mathematics." At the spring initiation Dr. Penico, Professor of Mathematics at UMR, spoke on "The Humane Use of Mathematics." Officers: Richard Karhuse, president; Chuck Fuller, vice-president; Leonard Laskowski, treasurer; Sherida Dugan, secretary and historian; Dr. Roy Rahestraw, corresponding secretary; Dr. James Joiner, faculty sponsor.

Missouri Eta, Northeast Missouri State College, Kirksville

32 actives

Senior papers are presented at each meeting. The chapter continued its tutoring programs and began problem-solving sessions. Several attended the Missouri Council of Teachers of Mathematics convention in Jefferson City. Other activities have included the annual spring picnic and playing mathematical games at the home of Professor Mary Sue Beersman. Officers: John Grice, president;

Greg Klokkenga, vice-president; Jacque Ogden, secretary; Rose Ann Wommack, treasurer; Sam Lesseig, corresponding secretary; Mary Sue Beersman, faculty sponsor.

New York Eta, Niagara University, New York

19 actives, 6 pledges

This spring the chapter held the annual student-faculty picnic and softball game. Three of the meetings this spring included discussions of career opportunities in the fields of data processing, actuarial science, and applied statistics. Officers: Mike White, president; Walter Mazurowski, vice-president; Kathleen Sermak, secretary; Dave Miller, treasurer; Robert L. Bailey, corresponding secretary and faculty sponsor.

New York Theta, St. Francis College, Brooklyn

19 actives

Films shown were "IBM Question Tree," "MAA Theory of Limits, Part I," and "IBM Small Miracles." Students spoke on "The Direct Reflecting Principle of Loudspeakers Design," "A Problem From the Pentagon," and "Groups and Their Graphs." The chapter has commenced the publication of its own mathematics magazine. In this way the chapter can inform the alumni of the proceedings of the past year. The magazine contains student papers, alumni news, student news, faculty news, and a problem corner. Officers: Michael Polito, president; Charles Traina, vice-president; John Javaruski, secretary; Timothy Marco, treasurer; Professor Donald R. Coscia, corresponding secretary and faculty sponsor.

Ohio Alpha, Bowling Green State University, Bowling Green

37 actives, 30 pledges

Programs have consisted of monthly seminars at which university mathematics professors lecture. The chapter has established a mathematics contest with prizes awarded to the outstanding freshman mathematics student and other undergraduate mathematics students who attain the highest score; also, it sponsors and selects a faculty member who meets the qualifications as the "Outstanding Mathematics Professor." Members act as assistants to professors, tutor students, and maintain the display. They also plan to publish *Origin*, a mathematics booklet which includes the activities of the chapter and biographies of faculty and members. Officers: Janice M. Csokmay, president; David Stewart, vice-president; Doris Osterloh, secretary; Barbara Parrish, treasurer; Dr. Long and Dr. Rickey, faculty sponsors.

Ohio Zeta, Muskingum College, New Concord

35 actives

Programs have included the following topics: "Analytic Geometry for Geometric Progressions and Trigonometric Functions," "Minicomputer and Belgian School Mathematics," "Management Sciences Division Research with A.T.&T.," and "Mathematicians." Dr. Jim Smith will be on sabbatical leave at University of New Hampshire from August 1971 to August 1972. Officers: Grace Vellenga, president; Linda Wassum, vice-president; Elizabeth Crawford, secretary and treasurer; Dr. Larry Zettel, corresponding secretary and faculty sponsor.

Pennsylvania Gamma, Waynesburg College, Waynesburg

10 actives, 2 pledges

The chapter sponsors seminars with other science honoraries, tutoring sessions for freshmen mathematics courses, and a monthly publication for the mathematics department. Dr. Lester T. Moston was honored by the annual KME banquet after 37 years of teaching and the dedication of the *Lester T. Moston Mathematics Reading Room*. Officers: Robert Roos, president; Ted Dacko, vice-president; Linda Yanusiewski, secretary and treasurer; Gabriel Basil, corresponding secretary and faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown

8 actives

Ten regular meetings were held during the past year featuring 8 students and 2 outside speakers. Officers: Joyce Noecker, president; Ray Biery, vice-president; Doris Troxell, secretary; Irwin Sonon, treasurer; Irving Hollingshed, corresponding secretary; Edward Evans, faculty sponsor.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

45 actives, 10 pledges

The majority of the meetings were directed toward planning for the national biennial convention. The last meeting of the year consisted of an evaluation of the convention activities and ideas and suggestions resulting from conversations with members from other chapters. Officers: John J. Smith, president; Diane Mularz, vice-president; Linda Bernacchi, secretary; Adrienne Toth, treasurer; Professor Ida Z. Arms, corresponding secretary; Professor William R. Smith, faculty sponsor.

Pennsylvania Iota, Shippensburg State College, Shippensburg

Each pledge was required to present a talk on some mathematical topic. Nine members attended the national convention. Officers: Leroy Jones, president; Dennis Walker, vice-president; Verna Walmer, recording secretary; Kathy Fischer, historian; Markwood Reid, program chairman; Dr. Frank Dangello, treasurer; Dr. John Mowbray, corresponding secretary; Dr. James L. Suber, faculty sponsor.

Tennessee Beta, Tennessee State University, Johnson City

25 actives

Members attend the monthly colloquial meetings of the mathematics department. They also assisted with the annual convention of the Tennessee Mathematics Teachers' Association which was held on the campus in May. Mrs. Nancy F. Forrester, Miss Shannon E. Whitehead, and Mr. John R. Drake received mathematics awards. Officers: Linda Perry, president; Beverly Price, vice-president; Donna Bowler, secretary; Ken Oster, treasurer; Mrs. Lora McCormick, corresponding secretary; David Phillips, historian; Sallie Carson and Henry Jablonski, Jr., faculty sponsors.

The first rule of discovery is to have brains and good luck. The second rule of discovery is to sit tight and wait till you get a bright idea. —George Polya

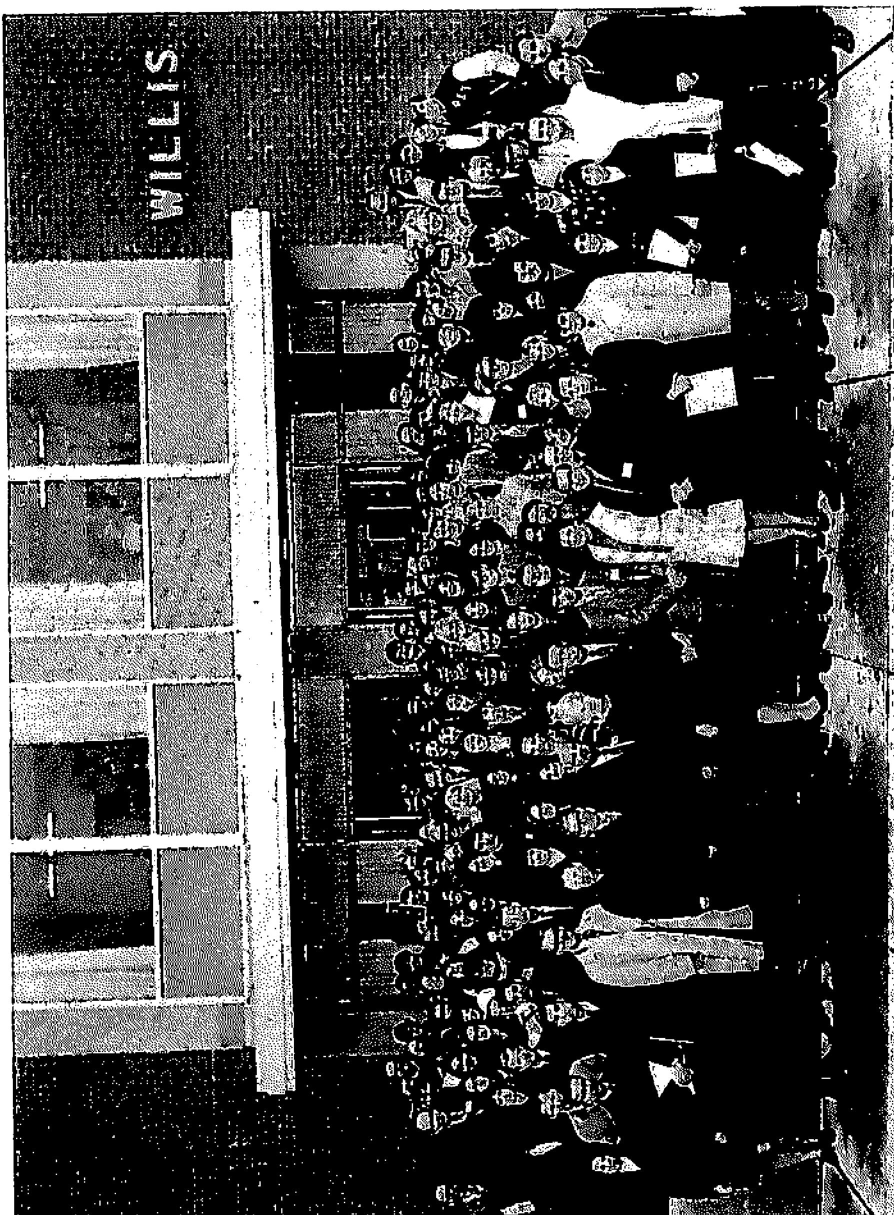
KAPPA MU EPSILON

FIANACIAL REPORT OF THE NATIONAL TREASURER

For the period April 22, 1969 to March 22, 1971

1. CASH ON HAND APRIL 22, 1969			$ 5,367.57	
RECEIPTS				
2. RECEIPTS FROM CHAPTERS				
Initiates (2618), 917 at $5.00 less $4.00 1701 at $7.00	$16,488.00			
Supplies, Jewelry, Installations	2,334.82			
Total Receipts from Chapters		$18,822.82		
3. MISCELLANEOUS RECEIPTS				
Interest on Bonds and Savings	661.27			
Pentagon (Surplus)	289.68			
U.S. Treasury Bond	1,000.00			
Short Checks and $10 overpayment	199.00			
From Savings to Checking Account	2,027.60			
Total Miscellaneous Receipts		4,177.55		
4. TOTAL RECEIPTS			$23,000.37	
5. TOTAL RECEIPTS PLUS CASH ON HAND				$28,367.94
EXPENDITURES				
6. NATIONAL CONVENTION, 1969				
Paid to Chapter Delegates	2,057.80			
Officers Expenses	614.87			
Speaker	100.00			
Host Chapter	263.52			
Total Cost of National Convention		3,036.19		
7. BALFOUR COMPANY (Stationery, memberships)		2,813.92		
BLAKE PRINTERY		1,943.87		
8. PENTAGON (Printing, Mailing of 4 Issues)		7,250.50		
9. INSTALLATION EXPENSES		55.65		
10. NATIONAL OFFICE EXPENSES (K. C. Meeting), Post.		829.04		
Two Regional Conventions		198.36		
11. MISCELLANEOUS EXPENDITURES				
Association of College Honor Societies (dues 70.00)		779.14		
Refunds		10.00		
Short Checks		189.00		
National Sec'y Expenses, Supplies, postage, etc.		902.46		
Deposit to Savings		4,380.00		
12. TOTAL EXPENSES			22,388.13	
13. CASH BALANCE ON HAND MARCH 22, 1971			5,979.81	
14. TOTAL EXPENDITURES PLUS CASH ON HAND				$28,367.94
15. C. D. Savings Certificates March 22, 1971	$12,000.00			
Plus interest to collect in April 1971	678.43			
		12,678.43		
16. TOTAL ASSETS AS OF MARCH 22, 1971			$18,658.24	
17. TOTAL ASSETS AS OF APRIL 22, 1969			15,900.17	
18. NET GAIN FOR PERIOD				$2,758.07

Respectfully submitted Walter C. Butler
National Treasurer for the past 12 years



Kappa Mu Epsilon Convention April 1-2 1971