THE PENTAGON

Volume XXIV | Spring, 1965 | Number 2

CONTENTS

National Officers

Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

A Non-Archimedean Ordered Ring*

George Poole
Student, Kansas State Teachers College, Emporia

The Archimedean property is stated most often as a theorem in reference to the real number system, but the concept is easily extended to other systems. Those systems, such as the rational integers and the field of rational numbers, to which this property can be applied have a stipulation that they are ordered (as defined below). By knowing what this definition of being ordered means, it would tend to lead us to believe that those systems which are ordered are necessarily Archimedean ordered. And this is true for those that we normally think of first.

After suitable definitions we will show with an example that this is not always the case. This will be done in a nonrigorous manner and the reader will keep in mind that these definitions given below are easily extended to other systems.

DEFINITION 1. A set S of elements is a *simply ordered* set under a relation $\sim$ if and only if the following hold:

(i) $a, b \in S, a \neq b \rightarrow a \sim b$ or $b \sim a$.

(ii) $a, b \in S, a \sim b \rightarrow a \neq b$.

(iii) $a, b, c \in S, a \sim b, b \sim c \rightarrow a \sim c$.

DEFINITION 2. An *ordered* ring R is a commutative ring whose elements are a simply ordered set and for which the ring operations satisfy the additional properties:

(i) $a \sim b \rightarrow a + c \sim b + c$; where $a,b,c \in R$.

(ii) $a \sim b,\ 0 \sim c \rightarrow a \bullet c \sim b \bullet c$; where 0 is the additive identity of R.

We note that a ring ordered by "$>$" can be ordered by "$<$" and will be considered as such.

The boldface + and • are used in Definition 2 to denote the ring operations which may or may not be the usual operations of multiplication and addition of numbers. In Definition 3, below,

*A paper presented at the KME Regional Convention at Kearney, Nebraska, April 4, 1964.

we will need the idea of a multiple of a ring element. Thus if $a \varepsilon R$, we use the symbol na to mean

$$na = a + a + a + \cdots + a$$

where n denotes the number of a's that have been combined by the ring operation $+$.

DEFINITION 3. An ordered ring R with the property that for every $a,b \varepsilon R$ such that $0 \sim a$ and $a \sim b$ there exists a multiple, na, of a for which $b \sim na$ is called an *Archimedean* ordered ring.

As an example, consider the set of rational integers with the relation "less than." Is this a simply ordered set?

(i) For any two distinct elements of the rational integers, certainly one element is less than the other by the usual definition of "less than." (ii) Certainly if of any two rational integers one is less than the other, then they are distinct, and (iii) the transitive law of "less than" for the rational integers holds.

Next, is the set of rational integers an ordered ring? Since the set of rational integers is an integral domain, certainly it is a commutative ring and it has been shown that the set of elements is a simply ordered set.

(i) If $a < b$, then after adding an equal quantity to both sides the sums are related in the same order; certainly true of the rational integers. (ii) If $a < b$ and the zero of the rational integers is less than c (or c is positive), then the products of a and b by c are related in the same order.

Thus the rational integers are an ordered ring and it can be readily seen to be Archimedean ordered. For if $a < b$ and a, b are natural numbers (positive elements of the rational integers), then we have

$$\begin{aligned} 0 < a \text{ and } a < b &\rightarrow 1 \leqq a \text{ and } 0 < b \\ &\rightarrow b \leqq ba \\ &\rightarrow b < ba + a \\ &\rightarrow b < (b + 1)a \end{aligned}$$

and $b + 1$ is a natural number.

Now, here is a ring which was ordered and was easily recognized as being Archimedean ordered. Is it true that all ordered rings are necessarily Archimedean ordered?

Consider the set of all polynomial functions over the rational integers with a variable x in the real numbers as given by the set P below.

$$P = \{ a_0x^0 + a_1x^1 + \cdots + a_nx^n \mid a_i \varepsilon I \ (i = 0, 1, \cdots, n), 0 \leqq n\}$$

We define addition and multiplication of polynomials in the usual fashion. If

$$f(x) = a_0x^0 + a_1x^1 + \cdots + a_nx^n$$

and

$$g(x) = b_0x^0 + b_1x^1 + \cdots + b_mx^m$$

then

$$f(x) + g(x) = (a_0 + b_0)x^0 + (a_1 + b_1)x^1 + (a_2 + b_2)x^2 + \cdots$$

and

$$f(x) \bullet g(x) = a_0b_0x^0 + (a_0b_1 + a_1b_0)x^1$$
$$+ (a_0b_2 + a_1b_1 + a_2b_0)x^2 + \cdots + a_nb_mx^{n+m}.$$

With these definitions of addition and multiplication, it can be verified by the reader that the set P is a comutative ring.

We will use the boldface letter **O** to denote the zero polynomial, the zero of the ring P, and the numeral 0 will of course denote the zero of the reals. First, the set of elements of P will be simply ordered by the relation "$<$" in which **O** $< f(x)$ if and only if there exists an $x_0 \varepsilon$ reals such that $0 < f(x_0)$ and for all $x_0 < x_j$, $0 < f(x_j)$. With this we will define the relation that

$$f(x) < g(x) \text{ if and only if } \mathbf{O} < g(x) - f(x).$$

For example, is $x - 1000$ a "positive" element of P, that is, does **O** $< x - 1000$? Yes, since if $x_0 = 1001$, we have $0 < 1$ and for all $1001 < x_j$, $0 < x_j - 1000$. Is $1000 - x$ "positive"? No, because whatever x_0 is chosen such that $0 < 1000 - x_0$, there exists an x_j such that $x_0 < x_j$ and $0 \nless 1000 - x_j$. Notice that any two elements can be compared or found to be "positive" or "negative" merely by observing the coefficient of the leading terms or term with the highest degree.

Using these definitions, any two distinct elements of P can be related. Under this relation, it can be verified that the set of elements of P is a simply ordered set.

Now, is P an ordered ring? If $f(x)$, $g(x)$, $c(x) \varepsilon P$, then, by definition,

$$f(x) < g(x)$$

implies that

$$O < g(x) - f(x).$$

But

$$g(x) - f(x) = [g(x) + c(x)] - [f(x) + c(x)]$$

and consequently

$$O < [g(x) + c(x)] - [f(x) + c(x)]$$

by substitution. Thus

$$f(x) + c(x) < g(x) + c(x)$$

satisfying (i) of Definition 2.

If $f(x) < g(x)$ and $O < c(x)$, then $O < g(x) - f(x)$ and $O < c(x)$. By definition, then, there exists an x_0, $x_1 \varepsilon$ reals such that

$$0 < g(x_0) - f(x_0), 0 < c(x_1)$$

and for all $x_0 < x_j$, $x_1 < x_k$ we have

$$0 < g(x_j) - f(x_j) \text{ and } 0 < c(x_k).$$

Consequently, choosing x_r to be the larger of x_0 or x_1, then

$$0 < c(x_r)\,[g(x_r) - f(x_r)]$$

which implies that

$$O < c(x) \bullet [g(x) - f(x)]$$

$$O < c(x) \bullet g(x) - c(x) \bullet f(x)$$

or

$$f(x) \bullet c(x) < g(x) \bullet c(x),$$

satisfying (ii) of Definition 2. Therefore P is an ordered ring.

Is it Archimedean ordered? To be Archimedean ordered, the Archimedean property must hold for every two "positive" elements, $f(x)$ and $g(x)$, of P for which $f(x) < g(x)$. Consider the two elements x^2 and x^4. It is easily recognized that these two elements are "positive" or that $0 < x^2$ and $0 < x^4$, and that $x^2 < x^4$ by definition.

The next question is "does there exist a large enough natural number n such that $x^4 < nx^2$?" In other words, does there exist a natural number n such that $0 < nx^2 - x^4$? However large an n is chosen and an $x_0 \,\varepsilon$ reals is chosen (notice that x_0 must be less than n) such that $0 < nx_0^2 - x_0^4$, there will be an x_j (namely $x_j = n$) such that $x_0 < x_j$ and $0 \nless nx_j^2 - x_j^4$, since $0 \nless n \cdot n^2 - n^4$. Therefore, P is an ordered ring which is non-Archimedean ordered.

An interesting sidelight is that the subring or rational integers of the ring P is an Archimedean ordered ring where P is not.

BIBLIOGRAPHY

[1] Hille, Einar, *Analytic Function Theory,* Ginn and Company, New York, 1955.

[2] Spencer, G. L. and Hall, D. W., *Elementary Topology,* John Wiley and Sons, New York, 1959.

Do mathematical truths reside in the external world, there to be dicovered by man, or are they man-made inventions? Does mathematical reality have an existence and a validity independent of the human species or is it merely a function of the human nervous system? Opinion has been and still is divided on this question.

—Leslie A. White

The Star Product*

YEUK-LAAN CHUI
Student, Anderson College, Indiana

The purpose of this paper is to establish some properties for a binary operation on sets called the "star product." Before doing so, we shall recall some basic definitions and operations for sets.

What is a set? A set is a well-defined collection of objects or elements. "Well-defined" means it is possible to determine whether or not an object is an element of the set.

For any two sets A and B, the union of A with B, denoted by $A \cup B$, is the set containing those elements that belong to at least one of the sets A and B while the intersection of A with B, $A \cap B$, is the set of elements that belong to both A and B. These are illustrated by the shaded parts in the diagrams of Figure 1.

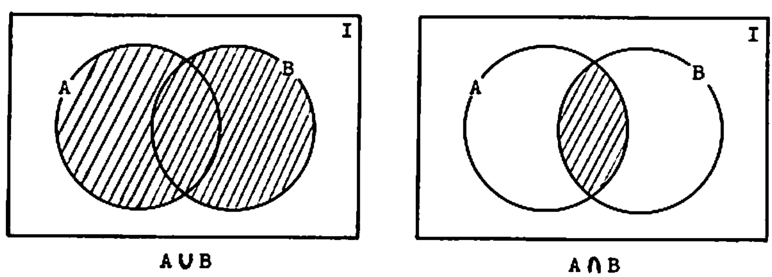


Figure 1.

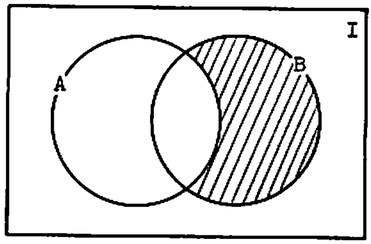
We shall let ϕ represent the null (or empty) set and I represent the universal set (the set of all objects under discussion). It is convenient to speak of the difference, $B - A$. This is the set of elements that are in B but not in A (see Figure 2). If B is replaced by I, we call

$$A^c = I - A$$

the complementary set of A. Thus the complementary set of A is the set of all objects under discussion that are not members of set A.

The following laws, which follow directly from the defini-

*A paper presented at the KME Regional Convention at Bowling Green, Ohio, April 10-11, 1964.



B - A

Figure 2.

tions and are found in [1, pp. 447-454], will be needed for our proofs:

1. Associative laws:

   1a. $(A \cap B) \cap C = A \cap (B \cap C)$

   1b. $(A \cup B) \cup C = A \cup (B \cup C)$

2. Commutative laws:

   2a. $A \cap B = B \cap A$ 2b. $A \cup B = B \cup A$

3. Identity laws:

   3a. $I \cap A = A \cap I = A$ 3b. $\phi \cup A = A \cup \phi = A$

   3c. $\phi \cap A = A \cap \phi = \phi$ 3d. $I \cup A = A \cup I = I$

4. Complement laws:

   4a. $A \cap A^c = A^c \cap A = \phi$ 4b. $A \cup A^c = A^c \cup A = I$

   4c. $I^c = \phi$ 4d. $\phi^c = I$

5. Distributive laws:

   5a. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   5b. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

6. Idempotent laws:

   6a. $A \cap A = A$ 6b. $A \cup A = A$

7. De Morgan's laws:

   7a. $(A \cap B)^c = A^c \cup B^c$ 7b. $(A \cup B)^c = A^c \cap B^c$

8. Law of Involution: $(A^c)^c = A$

9. Laws of Absorption:

   9a. $A \cap (A \cup B) = A$ 9b. $A \cup (A \cap B) = A$

With these properties in mind, we are ready to define the star product of two sets.

DEFINITION 1. If A and B are sets, the *star product* of A and B, denoted by $A \star B$, is defined to be

$$A \star B = (A \cup B) \cap (A^c \cup B^c).$$

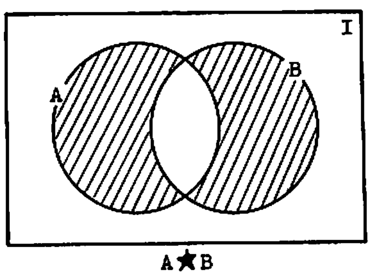


Figure 3.

The star product of A and B is illustrated by the shaded parts of the diagram in Figure 3. Thus we see that, while $A \cup B$ is the set of elements from the universal set that are either in A or in B or in both A and B, the star product, $A \star B$, denotes the set of elements from I that are either in A or in B but not in both. (Some writers call this the symmetric difference.)

It is easy to show that $A \star A = \phi$, $\phi \star A = A$, and $A \star B = B \star A$. It takes some manipulation to show that $A \star (B \star C) = (A \star B) \star C$, which completes the demonstration of:

THEOREM 1. The set of all subsets of a universal set I is a commutative group under the operation $\star$. The identity of the group is ϕ and each element is its own inverse.

There are other convenient representations of $A \star B$ as shown in the following two theorems.

THEOREM 2. $A \star B = (A \cup B) - (A \cap B)$.

Proof: $A \star B = (A \cup B) \cap (A^c \cup B^c)$ Definition 1.

$= (A \cup B) \cap (A \cap B)^c$ by 7a.

$= (A \cup B) - (A \cap B)$

since $C - D = C \cap D^c$ for any two set C and D.

THEOREM 3. $A \star B = (A \cap B^c) \cup (A^c \cap B)$.
Proof:

$A \star B = (A \cup B) \cap (A^c \cup B^c)$ Definition 1.

$= [(A \cup B) \cap A^c] \cup [(A \cup B) \cap B^c]$ 5a.

$= [(A \cap A^c) \cup (B \cap A^c)] \cup [(A \cap B^c) \cup (B \cap B^c)]$
2a and 5a.

$= [\phi \cup (B \cap A^c)] \cup [(A \cap B^c) \cup \phi]$ 4a.

$= (A^c \cap B) \cup (A \cap B^c)$ 3b and 2a.

$= (A \cap B^c) \cup (A^c \cap B)$ 2b.

The principal theorem of this paper is that the complement of the star product of n sets is equal to the star product of these sets with any odd number of the sets being replaced by their complements. In order to state this concisely, we define recursively

DEFINITION 2. $\mathop{\star}\limits_{i=1}^{0} A_i = \phi$

$$\mathop{\star}\limits_{i=1}^{n+1} A_i = (\mathop{\star}\limits_{i=1}^{n} A_i) \star A_{n+1}, \text{ for } n = 0, 1, 2, \cdots$$

In this symbolism our principal theorem can be stated as

THEOREM 4. (Theorem of complements for $\star$).

$$(\mathop{\star}\limits_{i=1}^{n} A_i)^c = (\mathop{\star}\limits_{i=1}^{2k-1} A_i^c) \star (\mathop{\star}\limits_{j=2k}^{n} A_j), \text{ for } 2k - 1 \leqq n.$$

In order to prove this theorem we will need the following lemmas.

LEMMA 1. $(A \star B)^c = A^c \star B = A \star B^c$.

Proof:

$$(A \star B)^c = [(A \cup B) \cap (A^c \cup B^c)]^c \qquad \text{Definition 1.}$$

$$= (A \cup B)^c \cup (A^c \cup B^c)^c \qquad 7a.$$

$$= (A^c \cap B^c) \cup (A \cap B) \qquad \text{7b and 8.}$$

$$= (A^c \cap B^c) \cup [(A^c)^c \cap B] \qquad 8.$$

$$= A^c \star B. \qquad \text{Theorem 3.}$$

Finally, due to the commutivaty of the star operation,

$$(A \star B)^c = (B \star A)^c = B^c \star A = A \star B^c.$$

LEMMA 2. $A \star B = A^c \star B^c$.

Proof: We use property 8 and two applications of Lemma 1.

$$A \star B = [(A \star B)^c]^c$$

$$= [A^c \star B]^c$$

$$= A^c \star B^c.$$

The third lemma is a special case of Theorem 4 and a generalization of Lemma 1 which we will state and prove as

LEMMA 3. $(\overset{n}{\underset{i=1}{\star}} A_i)^c = A_1^c \star A_2 \star \cdots \star A_n$

$$= A_1 \star A_2^c \star \cdots \star A_n$$

$$\cdots\cdots\cdots$$

$$= A_1 \star A_2 \star \cdots \star A_n^c.$$

In order to establish this lemma by finite induction we shall show that it is true for $n = 1$ and that it is true for the integer $n + 1$ whenever it is true for the positive integer n. We first note that the lemma is trivially true when $n = 1$. If we assume that it is true for the positive integer n, we have

$$(\mathop{\bigstar}_{i=1}^{n+1} A_i)^c = \left[(\mathop{\bigstar}_{i=1}^{n} A_i) \bigstar A_{n+1}\right]^c \qquad \text{Definition 2.}$$

$$= (\mathop{\bigstar}_{i=1}^{n} A_i)^c \bigstar A_{n+1} \qquad \text{Lemma 1.}$$

$$= (\mathop{\bigstar}_{i=1}^{n} A_i) \bigstar A^c_{n+1} \qquad \text{Lemma 1.}$$

and the proof of this lemma is completed.

Theorem 4 now follows from an application of the first form of Lemma 3 then $k - 1$ applications of Lemma 2 to A_2 and A_3, A_4 and $A_5, \cdots, A_{2k-2}$ and A_{2k-1}.

An immediate consequence of Theorem 4 and the Law of Involution (8) is

COROLLARY. $\mathop{\bigstar}_{i=1}^{n} A_i = (\mathop{\bigstar}_{i=1}^{2k} A_i^c) \bigstar (\mathop{\bigstar}_{j=2k+1}^{n} A_j)$,

for $2k \leqq n$. This could also be proved using k applications of Lemma 2.

The following two theorems illustrate applications of Theorem 4.

THEOREM 5. $A \bigstar A^c = I$.

Proof:

$$\begin{aligned} A \bigstar A^c &= (A \bigstar A)^c && \text{Theorem 4.} \\ &= \phi^c && \text{Theorem 1.} \\ &= I && \text{4d.} \end{aligned}$$

THEOREM 6. $I \bigstar A = A^c$.

Proof:

$$\begin{aligned} I \bigstar A &= \phi^c \bigstar A && \text{4d.} \\ &= (\phi \bigstar A)^c && \text{Theorem 4.} \\ &= A^c && \text{Theorem 1.} \end{aligned}$$

Theorems 7, 8, and 9 show some other interesting relationships among $A \bigstar B$, $A \cup B$, and $A \cap B$.

THEOREM 7. $(A \bigstar B) \cup (A \cap B) = A \cup B$.

Proof:

$$(A \bigstar B) \cup (A \cap B) =$$
$$[(A \cup B) \cap (A \cap B)^c] \cup (A \cap B) \qquad \text{Def. 1 and 7a.}$$

$= [(A \cup B) \cup (A \cap B)] \cap [(A \cap B)^c \cup (A \cap B)]$ 2b, 5b.

$= \{A \cup [B \cup (A \cap B)]\} \cap I$ 1b, 4b.

$= A \cup B$ 3a, 9b.

THEOREM 8. $(A \star B) \cap (A \cup B) = A \star B$.

Proof:

$(A \star B) \cap (A \cup B) = [(A \cup B) \cap (A^c \cup B^c)] \cap (A \cup B)$ Definition 1.

$= (A \cup B) \cap (A^c \cup B^c)$ 1a, 2a, and 6.

$= A \star B$ Definition 1.

THEOREM 9. $(A \cup B) \star (A \cap B) = A \star B$.

Proof:

$(A \cup B) \star (A \cap B) =$

$[(A \cup B) \cup (A \cap B)] - [(A \cup B) \cap (A \cap B)]$ Theorem 2.

$= \{A \cup [B \cup (A \cap B)]\} - \{[(A \cup B) \cap A] \cap B\}$ 1b, 1a.

$= (A \cup B) - (A \cap B)$ 9b, 9a.

$= A \star B$ Theorem 2.

We close with some special cases of the relationship betweeen A and B. They are stated as Theorems 10, 11, and 12. The proofs will be left to the reader.

THEOREM 10. $(A \cap B = \phi) \longleftrightarrow (A \star B = A \cup B)$.

THEOREM 11. $(A \cup B = I) \longleftrightarrow (A \star B = A^c \cup B^c)$.

THEOREM 12. $(A \cap B = A) \longleftrightarrow (A \star B = B \cap A^c)$.

BIBLIOGRAPHY

[1] Allendoerfer, Carl B. and Oakley, Cletus O., *Principles of Mathematics,* McGraw-Hill Book Co., 1963.

The Second Order Linear Differential Equation With Constant Coefficients And The Corresponding Riccati Equations*

JANET DORMAN
State College of Iowa
and
F. MAX STEIN
Colorado State University

1. Introduction. It is well known that the second order linear differential equation with constant coefficients,

$$(1) \qquad y'' + 2by' + ay = 0, \ a \text{ and } b \text{ real},$$

has solutions in one of the three forms given below, depending on the coefficients, a and b, see [3, 4, 6]. It is also well known that (1) can be transformed into the corresponding Riccati equation,

$$(2) \qquad u' = a - 2bu + u^2,$$

by the substitution $y = \exp[-\int u dx]$ in (1), see [2, 3, 5]. The Riccati equation (2) is a first order, nonlinear differential equation which also has constant coefficients with the coefficient of u^2 being 1.

In this paper we propose to compare the forms of the corresponding solutions of (1) and (2) and to examine a second transformation given by Sugai in [7] that also leads to a Riccati equation.

2. The Second Order Linear Differential Equation. We use the usual method to solve (1). Assume a solution of the form $y = e^{mx}$, m to be determined, and obtain the auxiliary algebraic equation,

$$(3) \qquad m^2 + 2bm + a = 0.$$

Upon solving (3) for m, we find that $m = -b + \sqrt{d}$, where $d = b^2 - a$. This suggests the consideration of three cases, $d > 0$, $d = 0$, and $d < 0$.

---

*Prepared in a National Science Foundation Undergraduate Science Education Program in Mathematics at Colorado State University by Miss Dorman under the direction of Professor Stein.

Table 1

	Linear Equation	Riccati Equation
General Form	$y'' + 2by' + ay = 0$	$u' = a - 2bu + u^2$
Case 1 $d > 0$	$y = c_1e^{(-b+\sqrt{d})x} + c_2e^{(-b-\sqrt{d})x}$	$u = \dfrac{b + \sqrt{d} + (-b + \sqrt{d})e^{(2\sqrt{d}x + c)}}{1 - e^{(2\sqrt{d}x + c)}}$
Case 2 $d = 0$	$y = c_1e^{-bx} + c_2xe^{-bx}$	$u = \dfrac{bx + bc - 1}{x + c}$
Case 3 $-\Delta = d < 0$	$y = e^{-bx}(c_1\sin\sqrt{\Delta}x + c_2\cos\sqrt{\Delta}x)$	$u = \sqrt{\Delta}\tan(\sqrt{\Delta}x + c) + b$

Case 1. $d > 0$. If $a \neq 0$ and $b \neq 0$, the solution of (1) is

$$(4) \qquad y = c_1 e^{(-b+\sqrt{d})x} + c_2 e^{(-b-\sqrt{d})x}.$$

Both a and b cannot equal zero at the same time since $d > 0$ by assumption. The following special cases result if $a = 0$ or $b = 0$. If $a = 0$ we obtain $y = c_1 + c_2 e^{-2bx}$, while if $b = 0$ (with $a < 0$, since $d > 0$) we obtain $y = c_1 e^{\sqrt{d}x} + c_2 e^{-\sqrt{d}x}$, where $d = -a$.

Case 2. $d = 0$. If $d = 0$ with $a \neq 0$ and $b \neq 0$, the general solution for (1) is

$$(5) \qquad y = c_1 e^{-bx} + c_2 x e^{-bx},$$

the case of repeated roots.

Now if $a = 0$, then $b = 0$ as well, and vice versa; hence (1) reduces to $y'' = 0$, having the solution $y = c_1 + c_2 x$. This is the only special case of (5).

Case 3. $d < 0$. To avoid confusion we shall define $\Delta = -d$ in this case. Then the resulting general solution of (1) when $a \neq 0$ and $b \neq 0$ is

$$(6) \qquad y = e^{-bx}(c_1 \sin\sqrt{\Delta}x + c_2 \cos\sqrt{\Delta}x)$$

The only possible subcase occurs when $b = 0$ and $a > 0$; this yields the solution

$$(7) \qquad y = c_1 \sin\sqrt{a}x + c_2 \cos\sqrt{a}x.$$

3. The Riccati Equation. Associated with the linear equation (1) is the Riccati equation with constant coefficients (2), obtained by making the substitution $y = \exp[-\int u\,dx]$. Each Riccati equation may be solved either directly (notice that the variables are separable) or by using the solutions obtained from the related linear equations. The resulting solutions in the three cases are summarized in Table I.

4. Transformations. In order to obtain the Riccati equation corresponding to (1) we used the transformation, $y = \exp[-\int u\,dx]$, the standard transformation. However, there is another transformation, $y = \exp[\int \frac{1}{v}\,dx]$, discussed by Sugai in [7], which may be used, provided $a \neq 0$. Using the Sugai transformation we obtain the Riccati equation,

$$(8) \qquad v' = 1 + 2bv + av^2,$$

associated with (1) in which the constant term is 1 rather than a.

The solutions for (8) given in Table 2 appear to be very closely related to those obtained for (2) given in Table 1. Indeed they are, for (2) and its solutions may be transformed to (8) and its solutions if we let $u = -\frac{1}{v}$.

An examination of Table 2 shows why it is necessary to restrict the use of the second transformation to linear equations in which $a \neq 0$. Otherwise, the solutions obtained would be undefined. Also (8) would not be a Riccati equation if $a = 0$, a most unhappy state of affairs for us.

5. Observations. Upon examination of the solutions given in Tables 1 and 2, we observe that the solution to the second order linear differential equation contains two constants, while the

Table 2

	Second Riccati Equation
General Form	$v' = 1 + 2bv + av^2$
Case 1 $d > 0$	$v = \dfrac{-b + \sqrt{d} + (b + \sqrt{d})e^{(2\sqrt{d}x + c)}}{a - ae^{(2\sqrt{d}x + c)}}$
Case 2 $d = 0$	$v = \dfrac{-(bx + bc + 1)}{ax + ac}$
Case 3 $-\Delta = d < 0$	$v = \dfrac{\sqrt{\Delta}\tan\sqrt{\Delta}x + c) - b}{a}$

solutions to the Riccati equations contain only one. The student may wonder, how is this possible, have we made an error? To answer this equation let us consider again Case 1, $d > 0$.

Since we used the transformation $y = \exp[-\int u dx]$ to obtain (2) from (1), we can use the inverse transformation $u = -y'/y$, to obtain the solution for (2). We proceed as follows:

$$u = \frac{-y'}{y} = \frac{-(-b+\sqrt{d})c_1e^{(-b+\sqrt{d})x} - (-b-\sqrt{d})c_2e^{(-b-\sqrt{d})x}}{c_1e^{(-b+\sqrt{d})x} + c_2e^{(-b-\sqrt{d})x}}$$

Multiplying through by $\frac{e^{(b+\sqrt{d})}}{e^{(b+\sqrt{d})}}$ yields

$$\begin{aligned} u &= \frac{(b-\sqrt{d})c_1e^{2\sqrt{d}x} + (b+\sqrt{d})c_2}{c_1e^{2\sqrt{d}x} + c_2} \\ &= \frac{(b+\sqrt{d}) + (b-\sqrt{d})\,(c_1/c_2)\,e^{2\sqrt{d}x}}{1 + (c_1/c_2)\,e^{2\sqrt{d}x}} \\ &= \frac{b+\sqrt{d} + (-b+\sqrt{d})e^{2\sqrt{d}x+c}}{1 - e^{2\sqrt{d}x+c}}, \end{aligned}$$

where $e^c = -c_1/c_2$. This is precisely the solution which resulted when we solved for u directly, see Table 1.

Now the student may wonder if the constant c which appears in the solutions of the Riccati equations can always be expressed in terms of a ratio of the constants c_1 and c_2, which appear in the linear equation. Indeed, this is the case, see Table 3. These results may be verified by the student if a procedure similar to that used in the example is followed. Notice that the inverse transformation for (8) is $v = y/y'$.

Another way of looking at this situation may help to remove some of the mystery concerning the number of arbitrary constants. The given linear equation (1) is of order two and one expects its general solution to contain two arbitrary constants. Both of the associated Riccati equations (2) and (8) are of order one hence their solutions should involve one arbitrary constant. However, as far as the original variable y is concerned, the relation $u = \phi(x)$, the general solution for (2), still involves a derivative of order one in y since $u = -y'/y$. Thus a solution for y requires one more integration which results in the second arbitrary constant. Similar remarks can be made concerning the solution of (8) and its relation to the solution of (1). Thus the number of arbitrary constants is consistent with the general rule.

Table 3

	Constants For Riccati Equation (2)	Constants For Riccati Equation (8)
Case 1 $d > 0$	$-e^c = \frac{c_1}{c_2}$	$\frac{e^c(\sqrt{d} + b)}{(\sqrt{d} - b)} = \frac{c_1}{c_2}$
Case 2 $d = 0$	$c = \frac{c_1}{c_2}$	$c + \frac{1}{b} = \frac{c_1}{c_2}$
Case 3 $-\Delta = d < 0$	$-\tan c = \frac{c_1}{c_2}$	$\tan c = \frac{b + (c_1/c_2)\sqrt{\Delta}}{\sqrt{\Delta} - b\, c_1/c_2}$

BIBLIOGRAPHY

[1] Cohen, Abraham, *An Elementary Treatise on Differential Equations,* D. C. Heath and Company, New York, 1933.

[2] Ince, Edward L., *Ordinary Differential Equations,* Longmans, Green and Company, New York, 1927.

[3] Kibbey, Donald E., and Reddick, Harry W., *Differential Equations,* John Wiley and Sons, London, 1956.

[4] Murphy, George M., *Ordinary Differential Equations and Their Solutions,* D. Van Nostrand Company New Jersey 1960.

[5] Rainville, Earl D., *Intermediate Differential Equations,* John Wiley and Sons, New York, 1959.

[6] Spiegel, Murray R., *Applied Differential Equations,* Prentice-Hall, New Jersey, 1962.

[7] Sugai, Iwav, "Riccati's Nonlinear Differential Equations," *American Mathematical Monthly,* vol. 67 (1960), pp. 134-139.

An Elementary Problem On Numbers*

Hsin Chu and P. A. Lucas
Research Institute, University of Alabama

1. Introduction.

In this note we consider the following problem: "Let T_0 be an unknown number of objects such that no object can be divided into a fractional part. If p/r parts of T_0 plus s/r of one object are removed from T_0, where $0 < s < r$ and $0 < p < r$, the remainder, T_1, is an integral number. If the process is continued n times so that p/r parts of T_k, plus s/r of one object are removed from T_k, leaving a remainder, T_{k+1} $(k = 0, 1, 2, \cdots, n - 1)$, which has no fractional parts, the last remainder, T_n, will be zero. Can one determine how many objects there were in the beginning?" The answer is quite elegant and simple, namely:

(a) If sr^i is not divisible by $(r - p)^{i+1}$ for all $i = 0,1,2,\cdots, n - 1$, the problem has no solution.

(b) If sr^i is divisible by $(r - p)^{i+1}$ for *all* $i = 0,1,\cdots, n - 1$, then

$$T_{(0,n)} = \frac{s}{r-p}\left[\frac{\left(\dfrac{r}{r-p}\right)^n - 1}{\dfrac{r}{r-p} - 1}\right].$$

2. Lemmas.

It is necessary to establish the following notation in order to clarify later formulas.

Notation: We denote $T_{(j,n)}$ as the remainder after j divisions $(j = 0,1,2,3,\cdots, n)$ where n is the total number of divisions to be performed.

As immediate consequence, we have the following lemmas:

LEMMA 1: $T_{(0,n)}$ = the total number of objects before any divisions.

LEMMA 2: $p/r\,[T_{(j-1,n)}] + \dfrac{s}{r}$ = the number of objects discarded on the j division.

*This research work was partially supported by Contract NsG-381 with the National Aeronautics and Space Administration.

LEMMA 3: $T_{(j,n)} = \frac{r-p}{r}[T_{(j-1,n)}] - \frac{s}{r}$.

LEMMA 4: $T_{(n,n)} = 0$ since no objects remain after n divisions.

LEMMA 5: $T_{(j-1,n-1)} = T_{(j,n)}$ where $j = 1,2,\cdots,n$. (1)

Lemmas 1 through 4 are immediately obvious from the conditions of the problem and the definition of $T_{(j,n)}$. We now give a proof for Lemma 5.

(a) Consider the case where $j = n$. By Lemma 4, $T_{(n,n)} = 0$. Thus,

$$T_{(n-1,n-1)} = T_{(n,n)}.$$

(b) Consider the general case where $j = k$. Assume $T_{(k-1,n-1)} = T_{(k,n)}$. Show that $T_{(k-2,n-1)} = T_{(k-1,n)}$.

By Lemma 3,

$$T_{(k-1,n-1)} = \frac{r-p}{r}[T_{(k-2,n-1)}] - \frac{s}{r}$$

and

$$T_{(k,n)} = \frac{r-p}{r}[T_{(k-1,n)}] - \frac{s}{r}$$

Therefore,

$$\frac{r-p}{r}[T_{(k-2,n-1)}] - \frac{s}{r} = \frac{r-p}{r}[T_{(k-1,n)}] - \frac{s}{r}.$$

Thus we have $T_{(k-2,n-1)} = T_{(k-1,n)}$ (by induction the lemma is proved).

3. Proof of the Theorem.

In deriving the formula for determining $T_{(0,n)}$ ($n = 1,2,3,\cdots$), we consider the following cases:

Case I. First we consider the case where no objects remain after one division.

From Lemma 4,

$$T_{(1,1)} = 0$$

and from Lemma 3,

$$T_{(1,1)} = \frac{r-p}{r} T_{(0,1)} - \frac{s}{r}.$$

Thus

$$\frac{r-p}{r} T_{(0,1)} - \frac{s}{r} = 0$$

and

$$T_{(0,1)} = \frac{s}{r-p}. \tag{2}$$

From Case I we establish the necessity sr^i being divisible by $(r - p)^{i+1}$ when $i = 0$. Since s must always be divisible by $(r - p)$, we shall denote s by $(r - p)s'$, i.e., $s = (r - p)s'$ where s' is a positive integer. Thus (2) becomes

$$T_{(0,1)} = s'. \tag{2.1}$$

Case II: No objects remain after two divisions. From (1) and (2.1) we have that $T_{(1,2)} = T_{(0,1)}$ and $T_{(1,2)} = s'$.

Using Lemma 3, $T_{(1,2)} = \frac{r-p}{r}[T_{(0,2)} - s']$

$$s' = \frac{r-p}{r}[T_{(0,2)} - s']$$

$$T_{(0,2)} = \frac{s'r}{r-p} + s' = \frac{sr}{(r-p)^2} + \frac{s}{r-p} \tag{3}$$

This shows that sr^i must be divisible by $(r - p)^{i+1}$ when $i = 0,1$.

Notice that formulas (2.1) and (3) can be rewritten in the following manner:

$$T_{(0,1)} = s' \left[\frac{\left(\frac{r}{r-p}\right)^1 - 1}{\frac{r}{r-p} - 1} \right] \tag{2.2}$$

$$T_{(0,2)} = s'\left[\frac{\left(\dfrac{r}{r-p}\right)^2 - 1}{\dfrac{r}{r-p} - 1}\right] \tag{3.1}$$

Case III: No objects remain after n divisions. From (2.2) and (3.1) we may conjecture that

$$T_{(0,n)} = s'\left[\frac{\left(\dfrac{r}{r-p}\right)^n - 1}{\dfrac{r}{r-p} - 1}\right], \tag{4}$$

and that sr^i must be divisible by $(r-p)^{i+1}$ for all $i = 0,1,\cdots, n-1$. We now show that this formula is true for n equal to any positive integer. The proof is by induction. For $n = 1,2$, the formula has been established. We will assume the formula is true $n = k$ and show that it is true for $n = k+1$. By (1), $T_{(0,k)} = T_{(1,k+1)}$ and from the inductive assumption,

$$T_{(0,k)} = s'\left[\frac{\left(\dfrac{r}{r-p}\right)^k - 1}{\dfrac{r}{r-p} - 1}\right]$$

where sr^i must be divisible by $(r-p)^{i+1}$, for all $i = 0,1,\cdots, k-1$.

Using Lemma 3,

$$T_{(1,k+1)} = \frac{r-p}{r}\,[T_{(0,k+1)} - s'].$$

Thus

$$s'\left[\frac{\left(\dfrac{r}{r-p}\right)^k - 1}{\dfrac{r}{r-p} - 1}\right] = \frac{r-p}{r}\,[T_{(0,k+1)} - s']$$

$$T_{(0,k+1)} = \frac{rs'}{r-p}\left[\frac{\left(\frac{r}{r-p}\right)^k - 1}{\frac{r}{r-p} - 1}\right] + s'$$

$$T_{(0,k+1)} = s'\left[\frac{\left(\frac{r}{r-p}\right)^{k+1} - \frac{r}{r-p} + \frac{r}{r-p} - 1}{\frac{r}{r-p} - 1}\right]$$

$$T_{(0,k+1)} = s'\left[\frac{\left(\frac{r}{r-p}\right)^{k+1} - 1}{\frac{r}{r-p} - 1}\right]$$

$$= \frac{s}{r-p}\left[\left(\frac{r}{r-p}\right)^k + \left(\frac{r}{r-p}\right)^{k-1} + \cdots + \frac{r}{r-p} + 1\right]$$

$$= \frac{s\,r^k}{(r-p)^{k+1}} + \frac{s\,r^{k-1}}{(r-p)^k} + \cdots + \frac{sr}{(r-p)^2} + \frac{s}{r-p}.$$

But, by Lemma 5,

$$T_{(0,k)} = T_{(1,k+1)}.$$

From the inductive assumption we know that $\frac{sr^i}{(r-p)^{i+1}}$, where $i = 0,1,2, \cdots, k-1$, are all positive integers. Consequently, $\frac{sr^k}{(r-p)^{k+1}}$ must be a positive integer also. This completes the proof by the induction principle.

COROLLARY: Under the same assumption of the theorem if any one of the following statements is true:

(1) s is divisible by $(r-p)^n$

(2) s and r are both divisible by $r-p$

(3) $p = r - 1$

then, $T_{(0,n)}$ always exists.

To illustrate the application of the formula in several different cases, we consider the following examples:

Example 1: $(n = 2, r = 4, s = 18, p = 1, r - p = 3)$. In this example, s is divisible by $(r - p)^n$.

$$T_{(0,2)} = \frac{18}{3}\left[\frac{\left(\frac{4}{3}\right)^2 - 1}{\frac{4}{3} - 1}\right]$$

$$T_{(0,2)} = 6\left[\frac{4}{3} + 1\right]$$

$$T_{(0,2)} = 14.$$

Example 2: Both s and r are divisible by $(r - p)$. $n = 3$, $r = 12$, $s = 4$, $p = 10$, $r - p = 2$.

$$T_{(0,3)} = \frac{4}{2}\left[\frac{\left(\frac{12}{2}\right)^3 - 1}{\frac{12}{2} - 1}\right]$$

$$T_{(0,3)} = 2\left[\left(\frac{12}{2}\right)^2 + \frac{12}{2} + 1\right]$$

$$T_{(0,3)} = 2\,[36 + 6 + 1]$$

$$T_{(0,3)} = 86$$

Example 3: When $p = r - 1$, the formula simplifies as follows: $n = 6$, $r = 3$, $s = 10$, $p = 2$, $r - p = 1$.

$$T_{(0,6)} = 10\left[\frac{\left(\frac{3}{1}\right)^6 - 1}{\frac{3}{1} - 1}\right]$$

$$T_{(0,6)} = 10\left[\left(\frac{3}{1}\right)^5 + \left(\frac{3}{1}\right)^4 + \left(\frac{3}{1}\right)^3 + \left(\frac{3}{1}\right)^2 + \left(\frac{3}{1}\right)^1 + 1\right]$$

$$T_{(0,6)} = 10\,[243 + 81 + 27 + 9 + 3 + 1]$$

$$T_{(0,6)} = 3{,}640$$

Finally we consider two examples which have no solution.

Example 4: In some cases sr^i is divisible by $(r - p)^{i+1}$ for *some* values of $i = 0,1,2,\cdots, n - 1$ but not for *all* values of $i = 0,1,\cdots,n - 1$.

(a) $i = 0$, $n = 1$, $r = 3$, $s = 2$, $p = 1$, $r - p = 2$.

$$T_{(0,1)} = 1\left[\frac{\frac{3}{2} - 1}{\frac{3}{2} - 1}\right]$$

$$T_{(0,1)} = 1. \qquad \text{The formula holds.}$$

(b) But if $i = 1$, $n = 2$, $r = 3$, $s = 2$, $p = 1$, $r - p = 2$, we have

$$T_{(0,2)} = 1\left[\frac{\left(\frac{3}{2}\right)^2 - 1}{\frac{3}{2} - 1}\right]$$

$$T_{(0,2)} = \frac{3}{2} + \frac{2}{2}$$

$$T_{(0,2)} = \frac{5}{2}.$$

Example 5: If sr^i is not divisible by $(r - p)^{i+1}$ for *any* value of $i = 0,1,2,\cdots, n - 1$, the formula fails at the first division. $i = 0$, $n = 1$, $r = 4$, $s = 1$, $p = 2$, $r - p = 2$.

$$T_{(0,1)} = \frac{1}{2}\left[\frac{\frac{4}{2} - 1}{\frac{4}{2} - 1}\right]$$

$$T_{(0,1)} = \frac{1}{2}.$$

Mathematics Teachers and The Library

CAROLYN KACENA
Student, State College of Iowa

The combination of library science and mathematics has always struck others as strange, but I never thought much about it until I read the National Education Association report "The Secondary School Teacher and Library Services."

According to this 1958 publication, there are three groups of teachers when classified according to library use: the major users, the minor users, and the potential users. Mathematics teachers are prime examples of grade A non-users. Most of them feel libraries are nice, but there is no place for library use in mathematics courses. A specific breakdown of the figures given in the NEA survey shows mathematics teachers giving the following responses: Library use is

Essential	Important	Of limited importance	Unimportant	Don't know
4.1	15.7	63.6	13.8	2.8[1]

In many cases, they complained about inadequate materials while at the same time the librarian might receive one title a year for suggested acquisition from the whole mathematics department. Most of the mathematics teachers didn't feel they knew the librarian well enough to judge her general competency or her knowledge of mathematics materials.

A surprising reversal occurs when questioned about the use of professional materials. Minor users as a whole use the professional collections just as frequently as the major users. Unfortunately, no figures on mathematicss teachers in specific are given. The assumption must be that they follow the general trend.

The crux of the problem seems to be the view that teaching mathematics is one and the same as assigning problems from the textbook and proving the answer in the key to be right. A very self-contained classroom results and one must admit that there is really no justification for library use in such a situation.

It is only when the teacher goes beyond the textbook and

[1]E.S. Bianchi, "Study on the Secondary School Library and the Classroom Teacher," National Association of Secondary School Principals Bulletin, 43:124 (November 1959).

inspires his students to go deeper into the subject that libraries become necessary—even essential. Albert E. Meder, Jr., in discussing library use in mathematics says, "The goals should be nothing less than the development of mathematical insight, power, and understanding to the fullest extent possible . . . The only way to live in the mathematical world of today is to gain insight, power, and understanding as well as problem solving." [2] The minimum along this line is the teacher's use of the library as a resource for models to be used in class and for renewal of his background.

A second problem is developing lines of communication between the two departments once the need for supplementary material is felt. While it is the librarian's responsibility to set up the basic collection for each department, she cannot be an expert in all fields. Any development beyond the essentials must have some initiation from the experts in the field—the teachers who assign the work and who know what their students need and want. No librarian will build up a mathematics collection without having some indication of future use. When the North Central Association set up library standards they listed approximate percentages of collections according to use and needs. Mathematics shares 10% of the collection with science, a department classified as a major user in the NEA report. These suggested norms are often rigidly followed. Unless mathematics teachers express their needs they will get the small end of the 10%. Those who complain about inadequate collections should look first at their teaching requirements and the requests for new materials they have made, and then complain if they have not been given the aid they requested.

A third facet of the problem involves the materials themselves. There are a few bibliographies available in mathematics but many times they prove to be rearrangements of previous lists. What is judged as good by one compiler is likely to be judged the same by the next one because of lack of competition. This may actually be an extension of the supply-and-demand situation of problem two. An endless cycle can be set up: No library materials because of no need shown in requests, no requests because nothing especially suitable has shown up in new publications, no new publications because of lack of demand, and so on ad infinitum. The place to break this chain must be in the mathematics departments. It might

[2] A. E. Meder, Jr., "Using the Library in High School Mathematics," School Libraries, 8:10 (March 1959).

take quite a revolution in the mathematics departments across the country to have a publisher risk the expense of a mathematics book, but it could be done.

Although the necessity to do more in teaching than solve problems has been proclaimed by the Dean of Rutgers University, how and why the library can and must be used can be shown more concretely. Returning to Mr. Meder, we find seven needs for libraries in mathematics.

> (1) The teacher needs the library to bring his own understanding of mathematics up-to-date.
>
> (2) The teacher needs the library to enable him to find supplemental materials with which to adapt his instruction to individual needs.
>
> (3) The teacher needs the library to assign supplemental material to the bright student who wants to know more.
>
> (4) The teacher needs the library for reading material to assign to students to secure coverage of topics that for lack of time, or some other reason, cannot be covered in class.
>
> (5) The teacher needs the library to enable him to kindle mathematical interests of able students by suggesting independent reading for their own enjoyment.
>
> (6) The teacher needs the library to enable him to teach mathematics as part of the cultural heritage of mankind, not merely as a tool or a language.
>
> (7) The teacher needs the library to stimulate original thinking, the research attitude, and the solving of problems.[3]

Complementary suggestions come from Jack N. Sparks, Research Fellow at the University of Iowa and Kenneth L. Taylor, Librarian at West Leyden High School in Franklin Park, Illinois. They suggest using the mathematics library for:

> (1) creating interest in mathematics through recreational activities such as mathematics clubs and/or daily problems boards; (2) selecting projects for mathematics students in class and out which would lead into independent research; (3) using library materials to supplement texts for students in all classes to interest them in greater depth and to aid them in gaining the necessary insights; (4) providing materials for the gifted so that they might either go ahead of the class or enrich the regular assignments; (5) providing for professional growth and (6) providing helpful procedures for the improvement of instruction.[4]

---

[3]Ibid, 10-13.

[4]Sparks and Taylor, "The Secondary School Mathematics Library: Its Collection and Use," *National Association of Secondary School Principals Bulletin*, 43:138-152 (November 1959).

Many other similar suggestions have been made for the enrichment of programs for the gifted students. The need for such programs and the usefulness of the library have been seen by many teachers. A little consideration for the interested average student who might even need the extra readings to fully comprehend what was given in the textbook, and the slightly below average student who should use his training right away in some project to show its practical use should be given. In the latter case, practical outside models would be especially helpful, especially in the trade or craft for which the student is training.

The tradition of problem solving is very strong, and I am as familiar as anyone with the problems of limited time and maximum number of new concepts to present. In fact, as a mathematics student, I'd probably groan loudly at the thought of doing a mathematics project or extra study on my own on top of the assigned problems. After all, I've been taught by the problem solving method and 'what was good enough for me, is good enough for my students'. I am also aware of the few choices of good materials from the librarian's point of view. But if a cry was raised for more and better mathematics libraries for the total range of student abilities, a step would be taken in the right direction toward the more creative mathematics the modern age requires.

BIBLIOGRAPHY

[1] Bianchi, E. S. "Study on the Secondary School Library and the Classroom Teacher," *National Association of Secondary School Principals Bulletin,* 43 (November 1959), pp. 123-129.

[2] Gordon, Garford. *Providing for Outstanding Science and Mathematics Students.* Los Angeles: University of Southern California Press, 1955.

[3] Hankin, Aaron. *Meaningful Mathematics Teaching.* Valley Stream, Long Island, New York: Teachers Practical Press, Inc., 1961.

[4] Meder, A. E., Jr. "Using the Library in High School Mathematics," School Libraries, 8 (March 1959), pp. 10-13.

[5] National Council of Teachers of Mathematics. *Enrichment Mathematics for High School.* 28th Yearbook. Washington, D.C.: National Council of Teachers of Mathematics, 1963.

(Bibliography continued on page 98)

Inverse Functions

W. M. Perel
Faculty, Charlotte College, North Carolina

In modernizing our teaching of mathematics, too much emphasis has been placed on the ordered pair definition of function and sight has been lost of the fact that the older rule or correspondence definition is still needed and is sometimes more illuminating. In particular, look at the problem of finding or even defining the inverse of a given function.

It is quite easy to find the inverse of some functions from the ordered pair definition. For example if f is defined by

$$f = \{(1,7), (2,9), (3,8), (4,10), (5,6)\},$$

then we have

$$f^{-1} = \{(6,5), (7,1), (8,3), (9,2), (10,4)\}.$$

Reversing the pairs as is required in the ordered pair definition is easily done.

However, most useful functions are not of this type. Let us examine so simple a function as the function f defined by $f(x) = 3x + 5$. We find in many books which have defined a function as a set of ordered pairs, the following instructions: "Set $y = f(x)$. Solve for x in terms of y. Replace all the y's by x's. The result is $f^{-1}(x)$." Somehow, this set of instructions does not seem very modern, nor does it have anything at all to do with a function as a set of ordered pairs.

Suppose we think of a function as a machine which takes a number, which we might call the "input" and performs certain transformations on this number and so produces another number which we might call the "output." Then, we may define the inverse function as the machine which reverses the roles of the input and output numbers.

For example, let f be defined by $f(x) = \dfrac{2x + 5}{3}$. Thus f is a machine which takes an imput number, multiplies it by 3, adds 5 and divides this result by 3. The inverse function should, therefore, take an input number, multiply it by 3, subtract 5 and divide this result by 2. Thus $f^{-1}(x) = \frac{3x - 5}{2}$.

As another example, let f be defined by

$$f(x) = \frac{5 \log(3x + 5) + 7}{4}.$$

In this case, f is a machine which takes a number, multiplies it by 3, adds 5, takes the log of this result, multiplies it by 5, then adds 7 and finally divides by 4. The inverse function ought to take a number x, multiply it by 4, giving $4x$, subtract 7, giving $4x - 7$, divide this result by 5, giving $\frac{4x - 7}{5}$, take the exponential of this result, giving $\exp \frac{4x - 7}{5}$. The next steps are to subtract 5 and divide by 3. The result is

$$f^{-1}(x) = \frac{1}{3}[\exp\frac{4x - 7}{5} - 5].$$

I claim that with a little practice, a student can write the inverse of such a function as fast as he can write. Of course, he needs to know that exp is the inverse of log before beginning.

It is not contended that the above method finds inverses for all functions which have inverses, nor that the method is particularly useful in showing that a particular function fails to have an inverse. However, it is superior to the method of "setting equal to y and solving for x" in that it is quicker and more meaningful.

Consider a slightly more general case. Let F be an affine function from R^n to R^n defined by

$$FX = A[cX + K] + H,$$

where A is a real n by n matrix, K and H are n-dimensional column vectors, and X is input column vector. The real number c is a nonzero scalar. Let us assume that the student knows that F will not have an inverse unless A is non-singular and also that he knows how to find the inverse of a non-singular matrix.

Now F is a machine which multiplies the vector X by the scalar c, adds the vector K, multiplies the matrix A times this result, and adds the vector H. To find the inverse function, we seek a machine to "undo" this result. In other words we expect the function F^{-1} to take a vector X, subtract the vector H, and multiply this result by the matrix A^{-1}, producing $A^{-1}[X - H]$. Then we want to subtract the vector K and multiply the result by $1/c$. Thus we have

$$F^{-1}X = \frac{1}{c}(A^{-1}[X - H] - K).$$

The method described above has been used in teaching inverse functions at various levels with considerable success.

(Continued from page 95)

[6] National Education Association. Research Division. *The Secondary School Teacher and Library Services.* Washington, D.C.: National Education Association, 1958.

[7] North Central Association of Colleges and Secondary Schools. *Policies and Criteria for the Approval of Secondary Schools.* The Association, 1962.

[8] Sparks, Jack N. and Kenneth L. Taylor. "The Secondary School Mathematics Library: Its Collection and Use," *National Association of Secondary School Principals Bulletin,* 43 (November 1959), pp. 138-152.

As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited. But when these sciences joined company, they drew from each other fresh vitality and thenceforward marched on at a rapid pace toward perfection.

—Joseph Louis LaGrange

The Problem Corner

EDITED BY F. MAX STEIN

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before October 1, 1965. The best solutions submitted by students will be published in the Fall 1965 issue of *The Pentagon*, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Professor F. Max Stein, Colorado State University, Fort Collins, Colorada.

PROPOSED PROBLEMS

181. *Proposed by George W. Norton, III, Marietta College, Marietta, Ohio.*

Suppose a shack, 10 feet by 10 feet, stands next to a tree 100 feet tall. If the tree breaks at B, the top A falls down (rotating about the point B) and meets the ground at C. This fallen part BC just touches the shack at D. How high from the ground is point B?

182. *Proposed by J. Frederick Leetch, Bowling Green State University, Bowling Green, Ohio.*

If x is irrational, what is the nature of $x + h$ and $x - h$?

183. *Proposed by the Editor.*

Sammy Sophomore couldn't perform the integration $\int \frac{dx}{x}$, so he multiplied the numerator and denominator of the integrand by x. He then integrated by parts as follows:

$$\int \frac{dx}{x} = \int \frac{xdx}{x^2} = -\frac{x}{x} + \int \frac{dx}{x}.$$

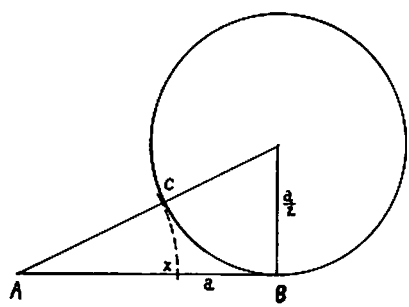
He then concluded that $-1 = 0$. Find the fallacy in his reasoning (if there is one).

184. *Proposed by Joseph D. E. Konhauser, HRB-Singer, Inc., State* College, Pennsylvania.

Let P be any point on an ellipse with semi-major axis a and semi-minor axis b. The circle with center P and radius b intersects the line containing the major axis in two points. Let A denote the point of intersection which is farthest from the center of the ellipse. The circle with center P and radius a intersects the line containing the minor axis in two points. Let B denote the point of intersection which is farthest from the center of the ellipse. Prove that the points P, A, and B are collinear.

185. *Proposed by Howard Frisinger, Colorado State University, Fort Collins, Colorado.*

Show that $\frac{Ax}{a} = \frac{\sqrt{5} - 1}{2}$ in the figure below.



SOLUTIONS

176. *Proposed by Joseph D. E. Konhauser, HRB-Singer, Inc., State College, Pennsylvania.*

Discuss the sequence of integers $[n + \sqrt{n} + \frac{1}{2}]$, $n = 1, 2, \cdots$, where $[x]$ is the greatest integer not exceeding x.

Solution by John L. Lebbert, Washburn University, Topeka, Kansas.

For any integer n, let us denote the above sequence by u_n. From the table below we can clearly see that the sequence yields all of the positive integers with a few exceptions. What I propose to do is find these exceptions (a close look at the integers not appearing under u_n, suggests what these exceptions may be).

n	u_n	n	u_n
1	2	17	21
2	3	18	22
3	5	19	23
4	6	20	24
5	7	21	26
6	8	22	27
7	10	23	28
8	11	24	29
9	12	25	30
10	13	26	31
11	14	27	32
12	15	28	33
13	17	29	34
14	18	30	35
15	19	31	37
16	20	32	38

Consider the two consecutive perfect squares n^2 and $(n + 1)^2$. The integers $n^2 + 1$, $n^2 + 2, \cdots$ are such that $u_{n^2+a} = u_{n^2} + a$ up to some number $n^2 + x$ for which we have $u_{n^2+x} = u_{n^2} + x + 1$. This occurs when $\sqrt{n^2 + x} \geqq n + \frac{1}{2}$. This inequality simplifies to $x > n + \frac{1}{4}$. Since x must be an integer, the jumps in the above table occur for the numbers $n^2 + n + 1$, where n may assume the value of any positive integer. We now have:

$$u_{n^2+n} = u_{n^2} + n = (n^2 + n) + n = n^2 + 2n.$$

$$\begin{aligned} u_{n^2+n+1} &= u_{n^2} + (n + 2) = (n^2 + n) + (n + 2) \\ &= n^2 + 2n + 2. \end{aligned}$$

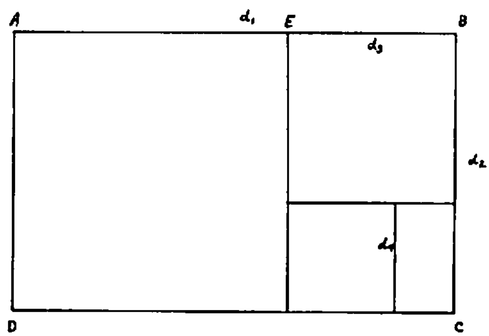
Thus numbers on the form $n^2 + 2n + 1 = (n + 1)^2$ do not appear for u_n where n may be any positive integer. Since the integer 1 does not appear for u_n we have the following result: u_n consists of all positive integers which are not perfect squares.

Also solved by Thomas P. Dence, Bowling Green State University, Bowling Green, Ohio.

177. *Proposed by Howard Frisinger, Colorado State University, Fort Collins, Colorado.*

Given a rectangle of length d_1 and width d_2, $d_1 > d_2$. If a square of side d_2 is removed from the rectangle, the remaining rectangle has length d_2 and width d_3, $d_2 > d_3$. If this process is continued, find the number r where $r = d_{i+1} / d_i$, $i = 1, 2, 3, \cdots$.

Solution by Thomas P. Dence, Bowling Green State University, Bowling Green, Ohio.



$$r = \frac{d_2}{d_1} = \frac{d_3}{d_2}$$

(1) $$r = \frac{BC}{AB} = \frac{EB}{BC}$$

(2) Let AB have length x; thus

(3) $$r = \frac{BC}{AB}, \; BC = xr.$$

$$EB = AB - AE = AB - BC$$

(4) $$EB = x - xr.$$

Substituting in (1) the values from (2), (3), and (4) we get:

$$\frac{xr}{x} = \frac{x - xr}{xr},$$

$$r^2 + r - 1 = 0,$$

$$r = \frac{\sqrt{5} - 1}{2}$$

Also solved by John L. Lebbert, Washburn University, Topeka, Kansas, and George W. Norton, III, Marietta College, Marietta, Ohio.

178. *Proposed by Douglas A. Engel, Hays, Kansas.*

Prove that the following formula is true:

$$n! = (n-1)(n-1)! + (n-2)(n-2)! + \cdots + 2(2!) + 1(1!) + 1(0!).$$

Solution by Paul M. Flynn, Kansas State College of Pittsburg, Pittsburg, Kansas.

The proof is by finite induction. For $n = 1$

$$1(0!) = 1 = 1!$$

Assume the proposition true for $n = k$.

$$1(0!) + (1)(1!) + 2(2!) + \cdots + (k-2)(k-2)! + (k-1)(k-1)! = k!$$

Then we have for $n = k + 1$

$$\begin{aligned} &1(0!) + (1)(1!) + \cdots + (k-1)(k-1)! + [(k+1) - 1][(k+1) - 1]! \\ &\quad = k! + [(k+1) - 1][(k+1) - 1]! \\ &\quad = k! + k(k!) = (k+1)k! = (k+1)! \end{aligned}$$

Since the proposition is true for $n = 1$ and is true for $n = k + 1$ when it is true for $n = k$, the proposition is true for all positive integers.

Also solved by Harold Darby, Florence State College, Florence, Alabama, John L. Lebbert, Washburn University, Topeka, Kansas, LeRoy Simmons, Washburn University, Topeka, Kansas, and the proposer.

179. *Proposed by Leigh R. Janes, Houston, Texas.*

Using the base eight or nine, determine mappings from digits into letters that will make the following addition correct:

$$\begin{array}{r} W\ R\ O\ N\ G \\ \underline{W\ R\ O\ N\ G} \\ R\ I\ G\ H\ T \end{array}$$

Solutions by John L. Lebbert and LeRoy Simmons, Washburn University, Topeka, Kansas.

Base eight

$$\begin{array}{r} 12634 \\ \underline{12634} \\ 25470 \end{array} \quad \text{and} \quad \begin{array}{r} 25706 \\ \underline{25706} \\ 53614 \end{array}$$

They also gave solutions if the base nine is used.

$$\begin{array}{r} 24173 \\ \underline{24173} \\ 48356 \end{array} \qquad \begin{array}{r} 12746 \\ \underline{12746} \\ 25603 \end{array} \qquad \begin{array}{r} 37541 \\ \underline{37541} \\ 76182 \end{array}$$

Also solved by Thomas P. Dence, Bowling Green State University, Bowling Green, Ohio (one solution to base 8 and two to base 9), Barbara McLaughlin, Immaculata College, Immaculata, Pennsylvania (one solution to base 8), and the proposer (one solution to base 8 and two to base 9).

180. *Proposed by Fred W. Lott, Jr., State College of Iowa, Cedar Falls, Iowa.*

My house is on a road where the numbers run 1, 2, 3, $\cdots$ consecutively. My number is a three digit one, and, by a curious coincidence, the sum of all house numbers less than mine is the same as the sum of all house numbers greater than mine. What is my number, and how many houses are there on my road?

Solution by Thomas P. Dence, Bowling Green State University, Bowling Green, Ohio.

Letting n equal the number of houses on the road, and letting

$k + 1$ equal the unknown house number, we can set up the equation:

$$\sum_{x=1}^{k} x = \sum_{x=k+2}^{n} x.$$

This yields

$$\tfrac{1}{2}k(k+1) = \tfrac{1}{2}n(n+1) - \tfrac{1}{2}(k+1)(k+2)$$

or

$$(k+1)^2 = \tfrac{1}{2}n(n+1).$$

With the restriction $100 < k + 1 < 999$, this means that

$$141 < n < 1412.$$

Since $k + 1$ is an integer, then $\frac{n(n+1)}{2}$ must be a perfect square. Since we have a product of two consecutive integers, it can be proved that either of the two factors $n \cdot \frac{(n+1)}{2}$ or $\frac{n}{2} \cdot (n+1)$ must be a product of two perfect squares. In other words, either n and $\frac{n+1}{2}$ or $\frac{n}{2}$ and $n + 1$ must each be a perfect square. Setting both n and $n + 1$ equal to the squares of 12 through 37 ($12^2, 13^2, \cdots, 37^2$) we test to see if the other factor, $\frac{n+1}{2}$ or $\frac{n}{2}$, turns out to be a perfect square. Thus when $n + 1 = 17^2$, $\frac{n}{2} = 12^2$. Therefore there are 228 houses on the road, and the unknown number is 204.

Also solved by Leigh R. Janes, Houston, Texas, John L. Lebbert, Washburn University, Topeka, Kansas, and Richard M. Parker, University of Tulsa, Tulsa, Oklahoma.

The Mathematical Scrapbook

EDITED BY J. M. SACHS

Mathematics is a science. It is the most exact, the most elegant, and the most advanced of the sciences and therefore it has been called the *Queen of Sciences*. Nothing, not even the modern miracles of applied science and technology, gives a better idea of the apparently unlimited capacity of the human mind than higher mathematics.

—H. M. DADOURIAN

=△=

The beginnings of the modern theory of probability were directly related to the cult of games of chance and the rise of insurance. . . . Card games became a fashion in European courts in the fourteenth century A.D. The manufacture of cards was probably one of the first commercial uses found for printing from wood blocks, before books were produced from moveable type. The first serious contribution to the mathematical theory of probability is contained in a correspondence between two French mathematicians, Fermat and Pascal, about wagers in a game of chance. . . . Today it may seem a far cry from the card table to the insurance corporation. It is still more surprising to see the astrologer in the background of the picture. . . . The astrologer Kurz, who used the horoscope to prophesy the prices of pepper, ginger, and saffron a fortnight in advance, was "surrounded with work as a man in the ocean with water". (Kurz-15th Century)

—L. HOGBEN

=△=

We could express a lot of our dialogue in mathematical form. Thus:—

"How is your grandmother's health?"

"Oh, it depends a good deal on the weather and her digestion, but I am afraid she always fusses about herself: today she's about fifty-fifty".

Mathematically this is a function of two variables and a constant, and reads:—

$$f(\text{W.D.} + \text{fuss}) = \tfrac{1}{2}.$$

The preceding is an excerpt from *Funny Pieces* by Stephen Leacock. It is included in the hopes that some readers unfamiliar with Leacock will read this chapter on the invasion of human thought by mathematical symbols. It is both serious and comic and well worth reading. The editor also recommends highly Leacock's version of Lord Ullin's Daughter as a problem in trigonometry from Moonbeams from the Larger Lunacy.

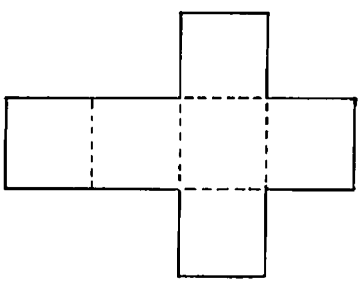
=△=

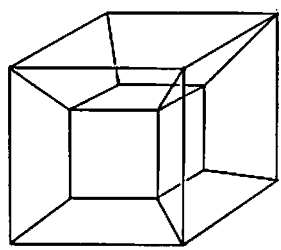
. . . but when the calculation is one of no constant and several capricious variables, guesswork, personal bias, and pecuniary interests, come in so strongly that those who began by ignorantly imagining that statistics cannot lie end by imagining, equally ignorantly, that they never do anything else.

—G. B. Shaw

=△=

The tesseract or four-dimensional cube can be illustrated by using a cube within a cube in three-space. This is the result of trying to assemble the six cubes one can build on the six faces of the three-dimensional cube. Consider the construction of a cube from a sheet of cardboard. The cube can be assembled by bending along





the dotted lines and taping edges together. Instead of doing this, consider building a cube on each of the squares first. If we agree that we are willing to distort distance and angle within reasonable limits and if the material we are using is sufficiently flexible, we can assemble these six "cubes" into a three-space model of the four dimensional cube. What happens if we try to do this with the other regular solids or with any solid which can be assembled from a plane model by bending and taping? The tetrahedron seems fairly simple. Can you visualize the result? Can you draw it or make it as a model? What ideas do you have for visualizing or drawing the octahedron? What about the dodecahedron and the icosahedron? The editor of the Scrapbook would welcome drawings and models.

=△=

He (the teacher) must bridge the intellectual chasm which lies between the developing mind of the pupil and the crystallized thoughts of the writer as expressed in the text. Also the teacher must ever keep in mind that, at the commencement of a pupil's career in any new field of thought, a principle or demonstration is not received and understood by the pupil as quickly as it is explained by himself; again that a pupil does not, and cannot of himself, generalize at all — *he must be taught to do so;* and furthermore, that to the average pupil a general demonstration often affords no conviction whatever. It is too abstract, and not having learned how to reason, he cannot institute comparisons and declare deductions. DeMorgan wisely said, "It is as necessary to *learn to reason* before we can expect to be able to reason, as it is to learn to swim or fence, in order to obtain either of these arts".

—E. S. Loomis

(Your editor is in general agreement with the advice given above, but there are some parts which make him uneasy. Do you agree wholeheartedly, in part or not at all? If you disagree in part what is the source of your disagreement?)

=△=

Let (a, b, c) be a primitive Pythagorean Triple of positive integers. The word *primitive* indicates that the greatest common divisor of a, b, and c is unity. The hypotenuse of the triple is c, i.e., $c^2 = a^2 + b^2$. Obviously a, b, and c cannot all be even. Why not? Could two of the three be even? Can you argue that if any two of these were even all three would have to be even? Could all three be odd? Can you argue that this is impossible? If you can make these arguments, what cases are left? Do you agree that the only primitive triples must have two odds and one even? Is it possible for c to be the even? Can you make the argument that the even must be a or b? If b is the even then $(c + b)$ and $(c - b)$ must have greatest common divisor unity. This follows from the fact that $(c + b)$ and $(c - b)$ must be odd and a common divisor d must divide their sum and difference, $2c$ and $2b$. If d is greater than unity it will be an odd divisor of c and b, as well as of $(c + b)$ and $(c - b)$. This

d will also divide a since $a^2 = (c + b)(c - b)$. This approach to Pythagorean Triples can be found in the writings of Leonardo of Pisa, also known as Fibonacci.

=Δ=

. . . the real function of art is to increase our self-consciousness; to make us more aware of what we are, and therefore of what the universe in which we live really is. And since mathematics, in its own way, also performs this function, it is not only aesthetically charming but profoundly significant. It is an art, and a great art. It is on this besides its usefulness in practical life that its claim to esteem must be based.

—J. W. N. SULLIVAN

=Δ=

Thales of Miletus armed intuition with a brain, and out of the nebulous mist emerged mathematics. But neither Thales nor his followers armed the thinker with an organ of speech which would fittingly express his thoughts, subtle yet precise, or describe the countless forms which his imagination could conjure up. Greek mathematics had to depend upon common speech, a medium replete with ambiguities yet inflexible; open to inconsistencies which it could not detect; where an interchange of words could jeopardize meaning, and where emphasis could be attained only through intonation. These were the handicaps under which Greek mathematics laboured throughout the thousands of years of its existence.

And then, as though by magic, mathematics was freed from the vagaries of human speech and presented with a language all its own. I use the word *magic* advisedly, for, the most striking feature of the event was the spontaneity and rapidity of this transition from the old mathematics to the new. It began at the threshold of the century (17th), and, by 1650 the new medium had already infiltrated into every field of mathematics, pure or applied.

—T. DANTZIG

=Δ=

I have no controversy about your conclusions, but only about your logic and method; how you demonstrate? what objects you are conversant with, and whether you conceive them clearly? what principles you proceed upon; how sound they may be; and how you apply them?

—BISHOP BERKELEY

The Book Shelf

EDITED BY H. E. TINNAPPEL

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of *The Pentagon*. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Harold E. Tinnappel, Bowling Green State University, Bowling Green, Ohio.

Random Essays on Mathematics, Education and Computers, John G. Kemeny, Prentice-Hall, Inc., 1964, 163 pp., $4.95.

This book contains sixteen essays grouped into three parts as indicated in the title. If one wishes to avoid some of the randomness of this organization he may regroup many of the author's ideas into the following sets: suggestions for the improvement of our colleges and secondary schools, some convictions about a liberal education, and future uses of computers. This triple of sets is not pairwise disjoint.

Suggestions for improvements in our schools are addressed to administrators, teachers, and curriculum planners. "The Secondary School Curriculum" proposes a pattern for grades seven through twelve which contains a maximum number of core courses for those who *might* go to college. "The 3 × 3 System" describes Dartmouth's program of three courses in each of three terms of a school year. Four of the essays would probably be related to the notion of "modern mathematics" if this notion can ever be satisfactorily defined.

Teachers will find some suggestions for encouraging creativity in "Rigor vs. Intuition in Mathematics." College deans may be interested in two novel proposals for foundation supported teacher training and post-doctoral teaching and research programs.

"A Library for 2000 A.D." is the longest essay and provides the most unusual suggestion of the book. Proponents of special sectioning for special students will find support in two essays. Even the college admissions office is accorded due consideration in "The Well-rounded Man vs. the Egghead."

Several essays present specific ideas related to the concept of a liberal education. One of these would include knowledge of computers as part of this concept. A detailed outline of a practical means to achieving a liberal education in an undergraduate college is contained in "Education of the Well-rounded Man."

Non-numerical uses of computers comprise the suggestions for future use of these machines. The two essays specifically dealing with this subject could well provide enrichment material for secondary schools.

Professor Kemeny's style is straightforward and may provoke a contrary response in some readers. His sometimes inconsistent interchange of "mathematics" and the colloquial "math" and his coining of the verb form "inputted" support his criticism of the difficulties of the English language.

—J. F. LEETCH
Bowling Green State University

Combinatorial Mathematics, Carus Monograph No. 14, Herbert John Ryser, The Mathematical Association of America, 1963, 154 pp., $4.00.

This book in a brief 141 pages of exposition presents more than an introduction to combinatorial mathematics. Proceeding from elementary theorems the reader is led in the final chapters to the limits of known results in some areas of mathematics. Suitable for the undergraduate or graduate student, as a source of reference for workers in this area, or as an introduction to combinations, the information, examples, and selected references found here mark this as a book welcome to this area of mathematics.

The first chapter discusses basic selection problems. It should be noted that the emphasis in the book is on questions of existence of mathematical phenomena although there are counting discussions. In Chapter Two the importance of combinatorial theorems in other areas is strongly emphasized by several illustrations in number theory of the inclusion and exclusion formula. Additional applications are presented from the classical derangements question and a discussion of permanents follows based on this theorem. A discussion of the "problems of ménages" begun in Chapter Two is continued in Chapter Three. The historical interest of and also problems in which this theory is important are discussed.

Chapter Four gives a proof of Ramsey's Theorem and illustrates its importance in geometry (convex polygons) and in the theory of (0, 1)-matrices, i.e. matrices whose entries are either zero or one. The latter matrices are important in questions of incidence in geometry and intersection in set theory and algebra.

Latin rectangles and squares are introduced in Chapters Four and Five, of great importance in statistics, they are also in algebra. Orthogonal Latin squares are discussed and it is shown that some questions in finite geometries are directly related to and solvable from corresponding properties of orthogonal Latin squares. Combinatorial designs as generalizations of the theory of finite projective planes and their importance to the design of experiments in statistics, to questions in algebra (linear), to deeper questions in finite geometry and number theory (perfect difference sets) are discussed in the final chapters.

Each chapter is followed by suggested references with extensive references in the later chapters for the interested reader desiring to go deeper into the field.

The above resume of the book shows the importance of this subject to a wide number of diverse fields in mathematics. Professor Ryser has added to the distinguished Carus series an outstanding work in *Combinatorial Mathematics,* noteworthy for its fine exposition and coverage, and sure to introduce many to the study of combinatorial mathematics.

—ARCHIE K. LYTLE, III
Central Michigan University

Mathematical Logic and the Foundations of Mathematics, G. T. Kneebone, D. Van Nostrand Company Ltd., London, 1963, 435 pp., $12.50.

The author addresses his work to two main classes of readers. The first comprises honors undergraduates in mathematics, high school mathematics teachers and postgraduates whose special interest is some other branch of mathematics. "All these require a book which covers the whole field, is informative, seriously written and substantial but not overloaded with technicalities". The second group includes postgraduate students who want some introduction to mathematical logic. They desire a philosophy of mathematics that will orient them correctly, give them a survey of the entire subject and its literature and will enable them to work from primary sources.

The contents of the book fall into three major sections. The first two cover the whole field of mathematical logic and the foundations of mathematics; the third deals with the philosophy of mathematics and attempts to establish the epistemological status of mathematics.

The opening chapters treat of mathematical logic. After a brief review of traditional logic the author discusses three distinct types of mathematical logic. These increase in complexity as the discussion develops. The first type is the familiar "propositional calculus" where p, q, . . . represent propositions and the five logical operators [$\sim$ (negation), $\vee$ (or), $\wedge$ (and), $\rightarrow$ (implication), $\leftarrow\rightarrow$ (equivalence)] are used to combine propositions. The initial treatment of this subject is fairly intuitive after which the author reduces this calculus to an axiomatic form and then discusses its metalogical characteristics.

The second type of mathematical logic is called "The Restricted Calculus of Predicates". It is a much wider system than the preceding, and, in addition to the concepts introduced there, it makes use of propositions of two or more arguments; it employs universal and existential quantifiers ("all" and "some"); it determines the universe of discourse as well as the domain of individuals. It is a rather complete and easy-to-follow treatment. Besides the elements previously listed we find a definition for a "well-formed formula"; seven rules of derivation; rules for substitution, inference, quantifiers, and the re-labeling of bound variables. The remainder of this section proves certain theorems notably Skolem's, Godel's, and the Deduction Theorem.

"The Extended Calculus of Predicated" is the name given to the third type of mathematical logic. It includes, in addition to the matter discussed in the preceding type, the relation of identity, methods for formalizing definite and indefinite description, the characteristic function of a formula, and the notion of a class or set. The most essential difference between this type and its immediate predecessor, is the fact that predicate variables can be handled with the same freedom as individual variables. To complete the axiomatization in this section the author introduces the axiom of choice and that of extensionality.

The second major section of the book is concerned chiefly with the foundations of mathematics. In opening his discussion of "the critical movement in mathematics in the nineteenth century" the author states that "this modern logic owes much more to mathematicians than it does to pure logicians for it originated largely as a by-product of investigations into the logic of mathematics" (p. 133). Heretofore, mathematics had been regarded as a set of absolute and immutable truths, exempt from all criticism and dispute once these have been proved. Today "mathematics must be looked upon as an activity of thinking, not as a totality of facts of some special kind" (p. 134). The critical movement of examining and strengthening the foundations of mathematics was initiated by Descartes. Other innovators who carried on this work were Newton, Leibniz, Frege, Legendre, Gauss, Cauchy, and Weierstrass. It is said that Frege "devoted his life's work to the task of making arithmetic so rigorous that it would surpass even Euclid's geometry in this respect" (p. 138). The prevalent mode of thought was one of critical examination and "the work of tightening up concepts and proofs went on all through the nineteenth century" (p. 139). Besides the efforts of mathematicians to rigorize the subject matter itself, there were those who concerned themselves with the purely logical side of mathematics (its form) and here the names of Dedekind and Peano come readily to mind. The axiomatic approach to mathematics as we know it today found its origins in the work of these two men. Both Dedekind and Peano rejected common sense as an adequate basis for mathematics and so the stage was set for the appearance of Russell. According to Russell "The only primitive concepts that were necessary belonged already to logic and all mathematical concepts were definable and all mathematical theorems provable within the logical system" (p. 157). The remainder of this section is given over to a very complete and detailed discussion of Russell's logistic theory, Cantor's work in classes, and Frege's logical analysis.

The other portions of this second major section are devoted to Hilbert's formalistic approach to the foundations of mathematics as set forth in his famous *Grundlagen der Geometrie;* Godel's metamathematical method; Brouwer's intuitionism; Heyting's intuitionist logical calculus. Each of these topics is so extensive and so detailed as to defy adequate treatment here.

A brief word should be made regarding recursive arithmetic as a formal system. The names of Church and Kleene are associated with this type of investigation. In the final chapter of this second major section the author discusses the axiomatic theory of sets. Pure mathematics is regarded as an extension of the theory of sets and mention is made of the work of Bourbaki, Zermelo, von Neumann, and Bernays in this area.

The third major section is called "Philosophy of Mathematics". In this the author evaluates the theories proposed in the three schools of thought. Likewise he explores the application of mathematics to the natural world as we know it today.

The book is well written and easy to follow. In the opinion of the reviewer one of its chief merits is the inclusion of supplementary notes for each chapter. These are quite extensive and contain much very valuable material. A case in point is those appended to Chapter IV in which a collection of the vicious-circle paradoxes is presented. Many readers will find the bibliography a tremendous asset.

Dr. G. T. Kneebone is Lecturer in Mathematics at Bedford College in the University of London.

—SISTER HELEN SULLIVAN, O.S.B.
Mount St. Scholastica College

The Elements of Real Analysis, Robert G. Bartle, John Wiley & Sons, Inc., New York, 1964. 447 pp.

In his preface, the author states, "Most of the topics generally associated with courses in 'advanced calculus' are treated here in a reasonably sophisticated way." Together with the title, we have the clue to the book — a text in "advanced calculus" for the teacher who feels such a course should be more than an extension of the first course in calculus.

The book starts with chapters on set theory, an axiomatic treatment of complete ordered fields, and topology in Euclidean p-space, at a level which should be well within the grasp of Junior-Senior students. Then follow chapters on sequences, continuous functions, differentiation, integration, and series. The table of contents for these chapters reads much like many other advanced calculus books — one finds the Mean Value Theorem, interchange of order of differentiation, Lagrange's Method, First and Second Mean Value Theorems for Integrals, tests for convergence of improper integrals, Root Test, Ratio Test, Raabe's Test, and so on. But there are also discussions of the Arzela-Ascoli Theorem, the Riesz Representation Theorem, Tauber's Theorem, etc., which one might have expected to be delayed to a later course.

The main difference between this text and one entitled "Advanced Calculus" is the spirit of the material. The author has incorporated topological and functional analytic ideas with the spirit of a real variable course and the topics and level-of-difficulty of an advanced calculus course. A course from this book would be an excellent transition from elementary calculus to graduate courses in analysis.

There are exercises at the end of the twenty-eight main sections, and often there are also "projects" which are sequences of problems leading the student through the development of some result. The author has combined definitions, theorems, proofs, examples, and discussion in an easily read, continuous discourse. At the same time, these things are all clearly labeled, so that the student knows when a theorem is being stated, when proved, and when discussed. This book should become a very popular text.

—E. R. DEAL
Colorado State University

A Survey of Numerical Analysis, Edited by John Todd, McGraw-Hill Book Company (330 West 42nd Street) New York 36: 1962, 584 pp., $12.50.

This book consists of seventeen chapters written by fourteen well-known mathematicians surveying the field of numerical analysis. As the title indicates, the coverage is quite broad; but in most cases it is also of considerable depth. As in most books of this type there is a small amount of overlap in the material. However, each author's approach to the subject is sufficiently distinct that it only tends to clarify certain topics.

Nearly all of the material presented in the seventeen chapters was presented in the form of lectures or mimeographed notes to the participants in National Science Foundation training programs in numerical analysis for senior university staff members at the National Bureau of Standards, Washington, D.C. This reviewer was fortunate enough to be a participant in one of these programs in 1959.

The references at the end of each chapter are extensive and provide the reader with sources where he may dig deeper into the subject. There are problems only at the end of Chapters Two and Three, which limit its possibility as a textbook in numerical analysis especially as a first course.

Everyone with a sincere interest in the field of numerical analysis will want to have this book, edited by John Todd, available for reference.

—RALPH E. LEE
The University of Missouri at Rolla

Statistical Management of Inventory Systems, Harvey M. Wagner, John Wiley & Sons, Inc. (440 Park Avenue South) New York 16, 1962, 230 pp., $8.95.

Inventory theory, involving mathematical methods including probability and statistics, is a growing state of development and use. A frequently-used policy is the (s, S) rule by which, when inventory falls below a quantity s, and order is placed to bring the total quantity up to S units. Demand is often assumed to follow some probability distribution, and this distribution determines to a large measure the characteristics in practice of the ordering rule. There will be probabilities related to quantities in inventory and on order, average amount of purchases, amounts of shortages, etc.

In this book, the author investigates the effects of various control policies on inventory. By this is meant that, top management having determined an inventory policy, an (s, S) policy in this case, what methods are effective in insuring that the policy is followed by lower management? Such control policies as quota schemes and barometer schemes, based on aggregate indices, are studied for consistency and other properties. A quota policy applies penalties if an aggregate index quota is exceeded; a barometer scheme penalizes lower management according to the amount by which targets are missed as measured by the aggregate index. A consistent scheme is one that encourages lower management to adhere to recommended policies.

The book considers various situations, such as knowledge, or lack of knowledge, of demand functions, demand generated by personnel, the determination of demand distributions generated by known stockage policies, and lack of knowledge of the (s, S) policy being followed or of the demand distribution, and asks in each case whether consistent controls can be devised.

Mathematically, the exposition makes use of standard statistical theory of probability distributions, sufficient statistics, etc., and of Markov chain theory for the stationary or steady-state situation. Some use is made of Monte Carlo calculations.

—Paul D. Minton
Southern Methodist University

Studies in Modern Algebra, Studies in Mathematics, Vol. 2, A. A. Albert, Editor, The Mathematical Association of America, 1963, 190 pp., $4.00.

This book consists of six articles covering various aspects of algebra. The first two articles, by Saunders MacLane, are survey articles. These articles are concerned primarily with associative algebras and among the topics discussed are valuations, finite groups, local rings, homological algebra, and modules and tensor products, to name only a few. Of the many topics treated none are covered in great detail.

The third article, by R. H. Bruck, relates loops to elementary algebraic topics. This paper begins with groupoids and discusses latin squares, geometric nets, Steiner triple systems and Room's designs. There is also a section devoted to nonassociative integers. An appendix to this paper gives a proof of the existence of a Steiner triple system of order n for each integer $n \geqq 3$ such that $n \equiv 1$ or 3 modulo 6. The author also provides the reader with a few exercises.

The fourth article is by Charles W. Curtis and discusses non-associative algebras and division algebras. The main result in this paper is to give a solution to Hurwitz's problem, that is, to prove that the only normed algebras over the real field are the complex numbers, the quaternions, and the Cayley numbers. The author also states that the same result holds for any field with characteristic different from 2.

The fifth article is a characterization of the Cayley numbers by Erwin Kleinfield. This paper gives some elementary properties of division rings related to the concepts of the nucleus and the center. The main result is a constructive proof that alternative division rings of characteristic not two are either Cayley-Dickson division algebras or associative division rings.

The final article is by Lowell J. Paige and is a development of of theory of Jordan Algebras. The concepts of radical, semi-simplicity and simplicity of algebras are discussed as well as the derivation of Jordan and Lie Algebras from associative algebras.

Each of the six articles is followed by a bibliography listing many sources where the interested reader will find more extensive treatments of the discussed topics. The reader should find that a generous background in algebra is convenient but not necessary for profitable reading.

—James S. Biddle
Ohio State University

Statistics, an Intuitive Approach, George H. Weinberg and John A. Schumaker, Wadsworth Publishing Company, Inc., Belmont, California, 1962, xii + 338 pp., $6.50.

As the title implies, *Statistics, an Intuitive Approach* offers the reader an understanding of the basic aspects of symbols and notation. It is designed primarily for students in the fields of psychology, education, and the social sciences who have little knowledge of mathematics. There is no question but that there are many college students who fit this description today. Hence, assuming the need for an "intuitive" approach which minimizes mathematical symbolism, this book probably comes as near as is possible to bridging the gap between background and understanding.

In Chapters 2, 3, and 4 properties of the mean, variance, and percentiles are effectively developed by making graphic use of the physical analogy to a weighted plank and the position of the fulcrum balancing it. The numerical examples are clear and very simple. Adhering to their stated philosophy, the authors emphasize the verbal statements of definitions and properties, and only after discussing applications in each chapter is any mention made of notation. As with the remainder of the book, there are few formulas and virtually no mathematical derivations. Extensive exercises end each chapter. While none is especially difficult, few have the simplicity of the examples and some are rather time consuming. Frequent reference is made in the exercises to data from past chapters. To the student who wants a good supply of exercises for developing computational skills, he will be well satisfied.

Chapters 5, 6, and 7 treat grouped data. The discussion of discrete and continuous variables and the graphs and illustrations are well presented. Two informal but particularly good chapters on the theoretical normal distribution and the central limit theorem follow. The authors make excellent use of numerous and much needed illustrations at this point. It is surprising how many textbooks written on this subject and at this level fail to take full advantage of illustrations to convey properties and concepts.

The treatment of probability is sparse. Only one chapter deals with probability directly, and there rather exclusively as it applies to the normal distribution. The stronger the student's feeling for probability, the greater will be his understanding of the ideas and interpretations of the next three chapters on decision making and risk, hypothesis testing, and estimation. These chapters read well and lay an adequate groundwork for statistical inference. The chapter on hypothesis testing handles key questions simply yet thoroughly.

A shorter course in elementary statistics would probably end at this point in the book. For a longer course, additional chapters are included on the t-distribution, the chi-square distribution, certain nonparametric tests, and regression and correlation. This last topic, covered in a three chapter sequence, is well organized, clearly written, and especially well illustrated making it one of the highlights of the book.

—Evan M. Maletsky
Montclair State College

Fifty Mathematical Puzzles and Oddities, Nicholas E. Scripture, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1963, 83 pp., $2.50.

From Great Britain comes this small book providing a collection of "oddments" selected from the fields of arithmetic, algebra, and geometry. Although of possible interest to the layman, the topics selected are quite traditional and likely to be quite familiar to any student of mathematics.

Examples of the items included are magic squares, repeating decimals, the standard "proof" that $2 = 1$, patterns for construction of the Platonic solids, binary notation, and the Mobius Strip. The oddities and puzzles that are included are neither new nor presented in any unusual manner which would merit consideration by any but the layman or possibly by junior high school students. (Answers to all puzzles are included in the text.)

A Book of Mathematical Reasoning Problems, Fifty Brain-Twisters, D. St. P. Barnard, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1963. 109 pp., $2.50.

This collection of puzzles is taken from the author's weekly column "Brain-Twister" which appears in Great Britain's "Observer." The puzzles are interesting and not the usual run-of-the-mill variety found in many other such collections.

One feature of this text that students of mathematics will find of interest is a section inserted between the problems and their solutions entitled "Leads." The reader is referred to this section for clues to the solution of problems if he experiences difficulty in getting started. Here, as the author states, the reader "will find a suggestion or two which, while not divulging the answer, may enable him to mount a renewed attack on the question."

Finally, the author writes: "If that should fail there remains the full solution at the back of the book as an insurance against insomnia."

The text has value as an aid to the development of skills at problem-solving, enhanced by the section on clues. As such it merits the attention of students of mathematics and laymen alike.

—MAX A. SOBEL
Montclair State College

BOOKS RECEIVED

Models for Production and Operations Management, Elwood S. Buffa, John Wiley & Sons, Inc., New York, 1963, 632 pp., $9.25.

The Discrete Maximum Principle, Liang-Tseng Fan and Chiu-Sen Wang, John Wiley & Sons, Inc., New York, 1964, 158 pp., $5.75.

The Paradox of Pleasure and Relativity, D. G. Garan, Philosophical Library, Inc., New York, 1963, 499 pp., $6.00.

Diophantine Geometry, Serge Lang, John Wiley & Sons, Inc., Interscience Division, New York, 1962, 170 pp., $7.45.

Readings in Mathematical Psychology, Volume II, Edited by R. Duncan Luce, Robert R. Bush, and Eugene Galanter, John Wiley & Sons, Inc., New York, 1965, 568 pp., $8.95.

Three-Dimensional Problems in the Theory of Elasticity, A. I. Lur'e, John Wiley & Sons, Inc., Interscience Division, New York, 1964, 493 pp., $16.00.

Local Rings, Masayoshi Nagata, John Wiley & Sons, Interscience Division, New York, 1962, 234 pp., $$11.00.

Diophantine Approximations, Ivan Niven, John Wiley & Sons, Interscience Division, New York, 1963, 68 pp., $5.00.

Problems in the Sense of Riemann and Klein, Josip Plemelj, John Wiley & Sons, Interscience Division, New York, 1964, 173 pp., $8.00.

Contributions to Order Statistics, edited by Ahmed E. Sarhan and Bernard G. Greenberg, John Wiley & Sons, Inc., New York, 1962, 467 pp., $11.25.

Theory of Relationships, Sanford L. Silverman and Martin G. Silverman, Philosophical Library, Inc., New York, 1963, 111 pp., $6.00.

Concise Dictionary of Atomics, Alfred Del Vecchio, Philosophical Library, Inc., New York, 1964, 262 pp., $10.00.

CORRECTION

The description of the book reviewed on page 53-54 of the Fall, 1964 issue of *The Pentagon,* XXIV, No. 1, was incorrectly given. It should have read: *Tables of Series, Products, and Integrals,* I. M. Ryshik and I. S. Gradstein, New York: Plenum Press, 1963, x + 438 pp., $15.00.

Installation of New Chapters

EDITED BY SISTER HELEN SULLIVAN

OKLAHOMA BETA CHAPTER

University of Tulsa, Tulsa, Oklahoma

Oklahoma Beta Chapter was installed on May 3, 1964 by Dr. Carl V. Fronabarger, Past President of Kappa Mu Epsilon. The installation was held in Sharp Chapel on the University of Tulsa campus at 3:00 p.m.

Following the installation ceremony Dr. Fronabarger gave a brief history of Kappa Mu Epsilon. A tea for the initiates and their guests followed the formal ceremonies. Oklahoma Alpha, the first chapter of Kappa Mu Epsilon, sent a beautiful floral arrangement which was used in connection with the tea. Mr. Carpenter and other representatives from Oklahoma Alpha were present for the installation ceremony.

Charter members are: Steve Atiyah, Carolyn Axton, Pamella Bedford, George W. Bright, Thomas W. Cairns, James H. Chafin, William R. Chichester, Fred J. Clare, John W. Conwell Jr., K. Wiley Cox, Patricia Sue Curby, Michael Del Casino, Michael C. Ellis, John Finck, Warren B. Garrison, Said H. Ghachem, Ann Gibbons, Margaret L. Gibson, James W. Gresham IV, Raymond B. Heath, Kenneth W. Hennigan, Bill D. Johnson, Dianne Krumme, John Lafferty, David Lawson, Margaret P. Leach, Randal H. Lefler, J. Larry Martin, Charles G. McConnell, James C. McGill, Ricardo A. Morales, Necmittin Mungan, William J. Osher, Richard Parker, Joe N. Pelton, Veril L. Phillips, Ralph C. Raynolds, Janet Ries, Martha Roberts, Jerry L. Roger, James W. Scheer, Karen Spradling, Stephen E. Szasz, David E. C. Teagarden, Lindon Thomas, John F. Vaughn, Ralph W. Veatch, Tommy D. Weathened, John Welge, Robert D. Wright, Charles C. Wu, and Ann L. Ziemer.

The new chapter's officers are:

President	Veril Phillips
Vice President	Joe Pelton
Secretary	Martha Roberts
Treasurer	Pam Bedford
Faculty Sponsor	Warren B. Garrison
Corresponding Secretary	Ralph W. Veatch

Kappa Mu Epsilon News

EDITED BY J. D. HAGGARD, HISTORIAN

Florida Alpha, Stetson University, DeLand.

Some of the activities of the chapter during this school year have been: Initiated ten new members on January 11, sponsored distinguished visiting lecturers, visited the Radio Corporation of America installation in Cocoa Beach, Florida, held a second initiation banquet for new members in March.

Illinois Alpha, Illinois State University, Normal.

Professor Francis Florey, who received his B.A. degree from Augustana in 1958 and an M.A. from the University of Illinois in 1962, is the new sponsor of the chapter. Professor Downing, the former sponsor of the chapter, is spending a year at the University of Illinois on a National Science Foundation Fellowship. Another faculty member and a supporter of Kappa Mu Epsilon, Professor Rowe, has been awarded a National Science Foundation Fellowship and will do work on his doctorate degree at Florida State University.

Illinois Beta, Eastern Illinois University, Charleston.

Our chapter meets each month with the Math Club in addition to several separate business meetings. We are conducting an examination in elementary calculus and awarding the Kappa Mu Epsilon Calculus prize to the winner in honor of Dr. Lester Van Deventer. The high-light of the year is the initiation ceremony and banquet for the new members to which an outstanding guest speaker is invited.

Indiana Gamma, Anderson College, Anderson.

Joseph Heffelfinger, who was president of the local chapter of Kappa Mu Epsilon last year, is now a graduate assistant at Michgan State University.

Eleven new members were initiated this year, bringing the active total membership to seventy-six.

Professor Mahlon M. Day, lecturer for the Mathematical Association of America spoke on "Smoothness and Rotundity" and was guest of honor at a Kappa Mu Epsilon Tea.

Illinois Delta, College of St. Francis, Joliet.

At our monthly meeting this year we have centered our attention on professional opportunities for women in mathematics. We have had guest lecturers from nearby industries and corporations such as Illinois Bell Telephone and John Hancock Insurance. At our March meeting, five candidates presented short mathematical papers of their own choice and were subsequently initiated into the chapter.

Kansas Alpha, Kansas State College, Pittsburg.

The activities of the year began with a picnic in October. In November we initiated twenty-one new members and at the meeting Richard Thompson spoke on "Exponential Order". Professor H. D. Brunk of the University of Missouri spoke to the chapter on December 10 on the topic, "Geometric Approach to Probability". On February 25, the new chapter sponsor, Mr. Bryan Sperry, spoke on "Ruled Surfaces". On March 25 we held a second initiation and on April 19, Dr. L. M. Blumethal, of the University of Missouri, spoke on "The Golden Age of Mathematics — Today".

Kansas Beta, Kansas State Teachers College, Emporia.

Regular monthly meeting with guest speakers were held throughout the year. We initiated a total of seventeen new members at ceremonies which included banquets.

Our chapter donated $100 to the Second Century Club at Kansas State Teachers College. Social Activities included a spring picnic and a Christmas Party.

Our chapter had a delegation in attendance at the National Kappa Mu Epsilon Convention held this April in Fort Collins, Colorado.

Kansas Gamma, Mount St. Scholastica College, Atchison.

Kansas Gamma initiated fourteen new members on September 20, 1964 and inducted eighteen pledges on October 5, 1964. The twenty fifth anniversary of the installation of Kansas Gamma will be observed on May 8 with a banquet and program at which charter members and past presidents will be honored.

Social events have included the chili supper given by actives for pledges, the Wassail Bowl Christmas party, the pledge party and program for actives.

Papers at the regular meetings have included topics in statistics, topology, lattice theory, number theory, non-Euclidean geometry, projective geometry, quarternions, and curriculum change in elementary and secondary schools. Professor William Scott of the University of Kansas was the visiting lecturer on February 2.

Ten members were in attendance at the National Convention in Fort Collins, Colorado, in April.

Louisiana Beta, University of Southwest Louisiana, Lafayette.

The local chapter of Kappa Mu Epsilon assisted with the annual convention of Mu Alpha Theta held on the University campus. Duane Blumberg, a former member of the chapter, is now in graduate school at the University of Wisconsin and Willis Bourque, also a former member, is in graduate school at Louisiana State University.

Dr. Merlin M. Ohmer, Faculty sponsor, has just written a book *Elementary Contemporary Mathematics* to be published by Blaisdell Publishing Company Another book entitled, *Elementary Contemporary Algebra,* is just now ready for distribution by the same publishers. Dr. Ohmer is also a visiting lecturer for the Mathematical Association of America.

Mr. Henry Pellerin, chapter president, will speak at the Judice High School, Mu Alpha Theta Chapter, next month; John Peck spoke to the same group in February.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore.

Nine students were initiated into membership in the chapter in the spring of 1964. The programs this year have included such topics as the following: "The Curious Helix" by Sr. Marie Augustine; a career panel consisting of a systems analyst, a statistician, an actuary, and a teacher; "A Study of Intuitionism" by Sue Albert; "Linear Programming" by Jacqueline Prucha; "The Theory of Networks" by Mary Teresa Flippen, "The Loxodromic Subgroup of the Group of Mobius Transformations" by Sylvia Smardo.

At the May meeting we plan to initiate eight new members. After the invited address on "Bourbaki" by Professor Dagamar Henney of the University of Maryland, a buffet dinner will follow.

Michigan Alpha, Albion College, Albion.

Our program for the year has included such activities as: a Mathematical Mixer; "The Coordinatization of a Finite Geometry" by Professor Ronald Fryxell; "Computer Application to Automobile Design" by Mr. Robert B. McLean, Methods Department of Ford Motor Company; pledge papers followed by an initiation; field trip to Consumer's Power Company; presentation of papers by Senior Honors Students; picnic and election of officers for 1965-66.

Michigan Beta, Central Michigan University, Mount Pleasant.

The primary project this year is to acquaint the high school seniors in this area of the state with the mathematics program at Central Michigan University.

Missouri Beta, Central Missouri State College, Warrensburg.

The Chapter meets on the third Tuesday of each month. On October 20, 1964, we had twelve initiates. Each meeting since then has been given over to the reading of papers by the initiates. Presently we are planning for the presentation of a paper at the National Meeting at Fort Collins and one at the meeting of Missouri Academy of Science in St. Louis.

Missouri Gamma, William Jewell College, Liberty.

This past semester our local Kappa Mu Epsilon chapter ranked third of all campus organizations in grade point average with a 3.4 out of a possible 4.0

We initiated sixteen new members at a banquet meeting in March at which Father Doyle from Rockhurst College spoke on "Topics in Non-Euclidean Geometry".

During the school year we enjoyed a guest speaker from Park College who spoke on "Cardinal Numbers". Student papers presented during the year include: "Pursuit Curves", "Three Point Method of Iteration of Finding Roots of Function", and "The Cafeteria Diet Problems". This last problem was a cooperative effort of the group and can be formulated somewhat as follows: Determine the daily diet which satisfies all nine nutrient requirements and is at the same time least expensive. The simplex method of linear programming was used with very interesting and surprising results. Using one approach the basic foods were potatoes, spinach, and soybeans at a cost of $137.74 a year per person.

Nebraska Beta, Kearney State College, Kearney.

We are conducting free help sessions for students in pre-calculus mathematics courses.

Eleven new students were initiated into the chapter on January 7. They were as follows: Constance Daniels, Richard L. Ender, Donald E. Gardner, Ervin K. Huffman, DuWayne Johnson, Caroll Kinnaman, Gary Maas, Dennis R. McGraw, Thomas E. Martin, Karen Peterson, and Elmer Wall.

We gave two $25 scholarships this year. Peggy Miller received one first semester and Gary Maas received one second semester.

Nebraska Gamma, Chadron State College, Chadron.

We initiated two students and two faculty during the first semester and ten students during the second semester.

Our chapter entered a skit in the annual Blue Key Revue on March 12. This is the first year we have entered the Revue.

Some of the program topics given at meeting this year include: "Finite Geometry" by Milt Eloe; "Magic Squares and Number Bases" by Pat Barry; and "Computer Programming" by Karen Kruse.

We have recently revised and updated our constitution, adding what amendments we felt were needed and revising some of the out-of-date articles. One item was changing the institution's name to Chadron State College.

The chapter acted as coordinator at the recent Inter-High School Scholastic Contest held March 19.

New York Epsilon, Ladycliff College, Highland Falls.

On March 1, 1965, a symposium was held in the Ladycliff College auditorium. The conference was divided into three parts namely "Modern Mathematics", "Careers in Mathematics", and displays on the various careers. Members of Kappa Mu Epsilon and the Mathematics Club were available to answer questions. Students and faculty members of the high schools in the surrounding area were guests of the Mathematics Department of Ladycliff.

Ohio Gamma, Baldwin-Wallace College, Berea.

Since the last Ohio Gamma news item, we have had the following program topics: "History of Calculus" by Terry Hull; "Finite Differences and Integration by Austin Miller; "Mathematics and Music" by Sue Hubbard; "Number Systems" by Joe Freeman; "Topology" by Robert Vicek; "Educational Systems in England" by Mr. Colin Turner, an English exchange teacher; "New Teaching Programs and the Revolution in Mathematics" by Donna Phelps; "Infiinite Primes" by George Trever; "Trachtenberg System of Rapid Calculation" by Marie Haushalter; "Mathematics of Warfare" by Richard Bohrer.

On October 14, 1964, we initiated fifteen new members, bringing our active total to thirty-eight and the all time list of members to two hundred and fifty-five.

Oklahoma Alpha, Northeastern State College, Tahlequah.

Two highlights of the school year for the Oklahoma Alpha Chapter were the formal initiations held during the Fall and Spring semesters. At the Fall initiation, held for 22 initiates, the sponsors and their wives served refreshments for the members, mathematics faculty and their wives. During the Spring, we held our initiation in conjunction with our annual Kappa Mu Epsilon Founders Day Banquet. Guest speaker, Dr. Emmit Wheat, Professor of Mathematics at Northeastern, spoke on the subject, "The Relation Between Music and Mathematics".

Pennsylvania Beta, La Salle College, Philadelphia.

Professor Robert Z. Norman, Dartmouth College, gave a Mathematical Association of America lecture on "Representable Numbers" and met informally with the students and faculty of the mathematics department. Other papers given during the year include: "Game Theory" by Henry Potoczny, "Probability" by James Filliben, "Topology" by Thomas Devlin.

Mr. James Filliben has a graduate fellowship at Princeton University and Mr. Henry Potoczny has an assistantship at Villanova University for 1965-66.

Tennessee Beta, East Tennessee State University, Johnson City.

The chapter met in January for a dinner meeting at the home of Mrs. Lora McCormick. The president, Linda Green, presided over a short business session at which time the chapter voted to give $50 to the T. C. Carson Loan Fund at East Tennessee State University. Dean Ella Ross showed pictures of her trip to Russia last summer.

Virginia Beta, Radford College, Radford.

Mr. Whitney Johnson of Virginia Polytechnic Institute's computing center lectured at our regular meeting and conducted a tour of the computing facilities

Wisconsin Alpha, Mount Mary College, Milwaukee.

On January 13, the chapter enjoyed a panel discussion on "Experiences During Student Teaching in the Milwaukee Public Schools." Panelists were: Phyllis Bruni, Mary Alice Inzeo, Karen Kindel, Karen Pfersch, Marsha Schmitt, and Barbara Stengal. On February 10, a program on "Data Processing" was jointly given by Miss Carol Zaffrann and Mr. Gutterman from Northwestern Mutual Life Insurance Company.