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CONTENTS

National Officers

Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Coordinate Transformations*

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The undergraduate mathematics student often ponders the derivations of some of the more complex coordinate systems. He notes that many facets, such as the orthogonality of intersections of the coordinate axes of these systems, seem too propitious to have happened by chance. That there is a unifying principle behind these systems is well known to the student of complex variables. It is our purpose to point out this principle to students who as yet have not studied complex variables.

1. Necessary concepts of complex variables. In real variables we denote y as a function of x by $y = f(x)$ if for every value of x one or more values of y are defined. Similarily, if $z = x + iy$ and $w = u + iv$ are two complex variables such that for every value of z in the complex plane one or more values of w are defined, we denote w as a function of z by $w = f(z)$.

It is evident that all such expressions $w = f(z)$ will be eventually reducible to the form

$$(1) \qquad w = u(x,y) + iv(x,y)$$

where u and v are functions of the real variables x and y. We will be immediately interested in whether or not these functions are analytic, i.e. possess derivatives in neighborhoods about a point. It is found in the study of complex variables that the Cauchy-Riemann conditions,

$$(2) \qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x},$$

are both necessary and sufficient for the existence of the derivative.

2. Conformal mapping. Obtaining a geometrical representation of a function $w = f(z)$ is inhibited by the fact that we now have

* Prepared in a National Science Foundation Undergraduate Research Participation Program at Colorado State University by Mr. Strand under the direction of Professor Stein.

four variables, the two independent variables x and y and the two dependent variables u and v. To do so we use the correspondence between points of two planes, the z-plane in which the point $x + iy$ is plotted and the w-plane in which the point $u + iv$ is plotted. Thus, as x and y define various curves in the z-plane u and v will in turn define other curves in the w-plane.

The preceding discussion has been leading to the following result: *All analytic functions map conformally.* This means that angles of intersections of curves are preserved under the analytic transformation $w = f(z)$. For example, since the lines $x = a$ and $y = b$ intersect orthogonally in the z-plane, the intersection of the corresponding curves in the w-plane will also be orthogonal under the analytic transformation, $w = f(z)$.

3. Coordinate systems. Deriving new coordinate systems now become a routine job. All we need to do is to use analytic functions of z as our mapping functions. For example, all powers of z are analytic and each will define a conformal mapping. Specifically, let

(3) $$w = z^2 = (x + iy)^2 = x^2 - y^2 + 2ixy = u + iv.$$

Therefore,

$$u = x^2 - y^2 \quad \text{and} \quad v = 2xy.$$

Using Equations (2), it is obvious that the function is analytic. It is easily shown that, if we let $x = a$ and eliminate the parameter y, we

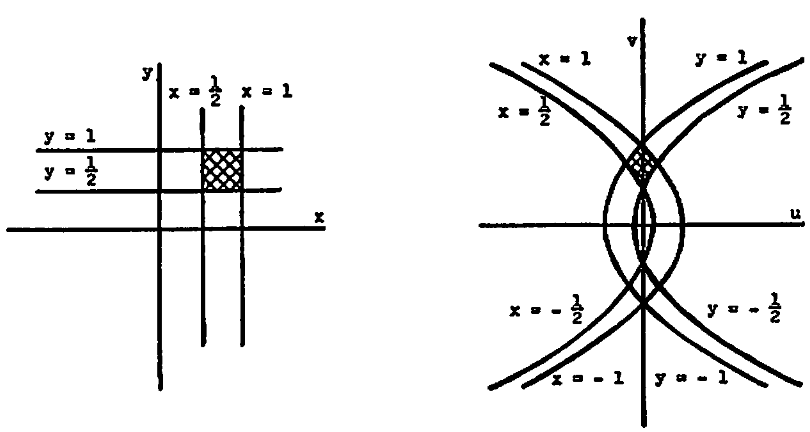

Fig. 1

obtain half of a parabola in the w-plane. The other half is obtained by letting $x = -a$. We also obtain parabolas by letting y assume constant values. An illustration of this mapping is shown in Figure 1. The fact that we have a new parabolic coordinate system is obvious when we note that we can determine any point in the w-plane by the intersection of two arcs of parabolas, which are in turn determined by x and y, i.e. the new coordinates.

We can derive the conventional polar coordinates in this fashion by letting

$$(4) \qquad w = e^z = e^{x+iy} = e^x(\cos y + i \sin y) = u + iv.$$

We now have

$$u = e^x \cos y \quad \text{and} \quad v = e^x \sin y,$$

and it is readily shown that $w = e^z$ is likewise an analytic function. If we let $x = a$ we have

$$u = e^a \cos y \quad \text{and} \quad v = e^a \sin y,$$

or

$$(5) \qquad u^2 + v^2 = e^{2a},$$

a circle whose radius is e^a and whose center is at the origin of the w-plane. If we let $y = b$ we have

$$u = e^x \cos b \quad \text{and} \quad v = e^x \sin b,$$

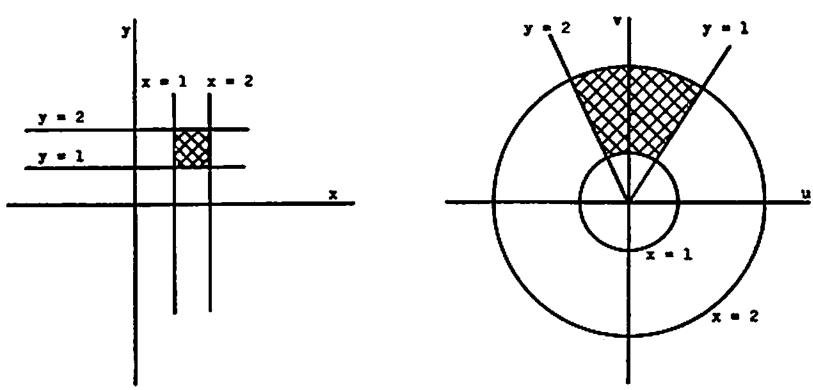

Fig. 2

or

(6) $$v = (\tan b)\, u,$$

a straight line. Thus, we have the polar coordinates in guise. The coordinates x and y are merely the radial distance from the origin and the angle of inclination respectively. This mapping is illustrated in Figure 2.

As a final, more complex, example we consider the mapping described by the analytic function

(7) $$\cosh z = \cosh x \cos y + i \sinh x \sin y$$

from which

$$u = \cosh x \cos y \quad \text{and} \quad v = \sinh x \sin y.$$

In this case we will obtain a coordinate system composed of confocal ellipses and hyperbolas as shown in Figure 3.

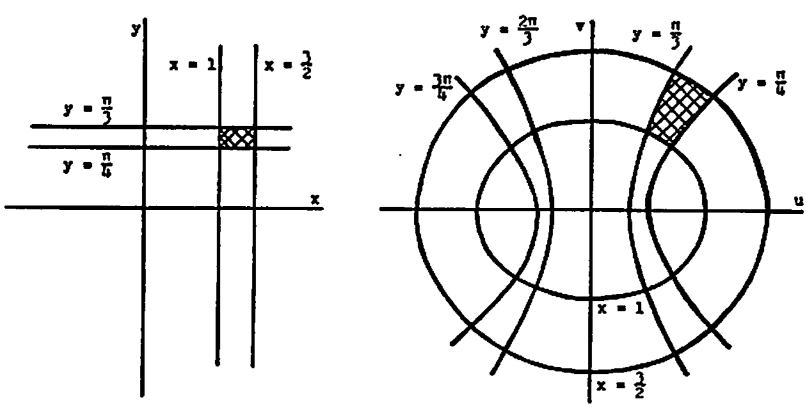

Fig. 3

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A Note on a Particular Class of Determinants

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The value of a determinant of order $n>2$ is zero if its elements satisfy the relation, $x_{jk} = j^m + k^q$, where j and k are the row and column indices of the elements and m and q are any constants whatever.

Proof: Suppose that A is such a determinant. Take an element $x_{r+h,p}$ and another element x_{rp} from a different row but from the same column, p. Their difference is $(r + h)^m - r^m$. Now take two other elements, one from row $r + h$ and the other from row r but both from the same column, t, different from p. Their difference is also $(r + h)^m - r^m$. Thus

$$A = [(r + h)^m - r^m]\, M$$

where M is a determinant having a row all elements of which are equal to one.

Since the two rows were chosen arbitrarily and since $n>2$, it is obvious that the same procedure may be applied to any two rows of M other than the row whose elements are all equal to one. Let s be the difference of the corresponding elements of two such rows of M.

Then

$$A = [(r + h)^m - r^m]\, sN$$

where N is a determinant having two rows all elements of which are equal to one. Therefore $N = 0$ and consequently $A = 0$.

Example: For $m = 3$, $q = 2$, and $n = 3$, we have

$$\begin{vmatrix} 2 & 5 & 10 \\ 9 & 12 & 17 \\ 28 & 31 & 36 \end{vmatrix} = 0.$$

The Life of Sir Isaac Newton

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In 1642 when England was in a turmoil because of the civil conflict between Parliament and King Charles I, little did she realize that in her midst in the tiny hamlet of Woolsthorpe in Lincolnshire a genius was born who would alter the course of history. In this period of unrest, young Isaac Newton was born on December 25, 1642 to Isaac Newton and the former Hannah Ayscough. For generations his father's family had farmed the small Woolsthorpe estate while his mother came from Market-Overton in Rutlandshire.

Newton was a sickly child. He had been born prematurely several months after the death of his father. In 1645 Reverend Barnabas Smith married his mother but Newton was raised by his maternal grandmother and his uncle, Reverend Ayscough of Burton Coggles. His paternal relations neglected him in his childhood and showed interest only after he had become famous.

When he was twelve he was to attend the King's School in Grantham but since it was too long a walk he boarded with an apothecary and his wife. At first he was not an outstanding student and had little interest in his studies because he was curious about other matters. It was not until a classmate of higher academic standing kicked him in the stomach that he became so infuriated that he not only sought physical revenge but scholastic superiority as well and did not relent until he was first in rank in the school. He was basically a gentle boy who derived more enjoyment from inventing new games than playing with the rougher boys. Because of this gentle and generous nature, a characteristic which endured all through his life, he preferred the friendships of the girls in his classes.

With the death of his stepfather fifteen year old Newton returned home to manage the estate for his mother upon her request. Agriculture held no interest for him, however, and after doing more harm then good he was sent back to King's School by willing relatives. After preparing for three years for further schooling he was admitted to Trinity College at Cambridge at the age of nineteen to dedicate his life to science.

When Newton first got to Cambridge he found he was no longer considered an outstanding student. The extent of his knowledge was merely par with most of the students and in fact was especially deficient in mathematics. He had some advantages over his adversaries, however, in that his early rural environment had enabled him to develop mentally and physically without unnecessary tensions and since he had started when he was older he was just at the peak of fullest intellectual development. He was released from attending several courses in which he independently mastered the topics and was therefore free to delve into those branches of science which interested him.

His first revolutionary discoveries were influenced by the first book he read at college, Kepler's "Optics." His studies in astronomy made him aware of his lack of knowledge of geometry but being a practical man he felt that Euclid was "merely trifling." He changed his opinion soon afterwards when he failed to receive a scholarship because of his poor knowledge of geometry.

At Trinity, Newton was a sizar, "a student required to perform various services in return for his tuition and necessaries of life, or 'commons' as they are still called"[1] since his mother could not finance his education. This man who contributed so much to mathematics was not interested in mathematics in these early years. He was more interested in the natural pheonomena about him and never tired of experimenting and trying to understand some of nature's mysteries.

When twenty-two he had already received his Bachelor of Arts degree and had studied algebra and developed his binomial theorem, the generalization of which was:

$$(a + b)^n = a^n + \frac{n}{1} a^{n-1}b + \frac{n(n-1)}{1 \cdot 2} a^{n-2}b^2 + \frac{n(n-1)\ (n-2)}{1 \cdot 2 \cdot 3} a^{n-3}b^3 + \cdots$$

where if n is a positive integer $1,2,3, \cdots$, the series automatically terminates after $n+1$ terms but if n is not a positive integer, the series does not terminate and the usual method of proof is inapplicable.[2]

The next year the Great Plague swept over England and in order not to endanger the professors and students, the university was closed for a two year period. During this interval he devoted himself to pure mathematics to develop differential calculus and his theory of fluxions, "the rates of flowing or changing quantities."

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1 S. Brodetsky, *Sir Isac Newton* (London: Methuen & Co. Ltd., 1929), p. 15.
2 E. T. Bell, *Men of Mathematics* (New York: Simon and Schuster, 1937), p. 97.

The problem of finding the area enclosed by a piece of a curve and straight lines faced even the Greeks.

> "The area bounded by a closed curve may be considered to lie between the areas of two polygons, one inscribed and the other circumscribed to the curve. • • • Let the number of sides be increased indefinitely. The areas of the two polygons approach one another indefinitely, giving ultimately the exact area of the curve itself."[3]

However, this method was only applicable in a few cases. Kepler worked on the problem and although he was only partially successful, his work inspired Cavalieri to continue the search. Cavalieri suggested that an infinite number of points made a curve, an infinite number of curves made a surface, and an infinite number of surfaces made a volume. No general rule was discovered until John Wallis found one dealing with a class of areas defined by curves in which one co-ordinate is proportional to a power of the other co-ordinate.[4]

Wallis's method was still restricting, however, and more complicated problems still remained unsolved. Using his binomial theorem, Newton developed a law which enabled him to work with problems with complicated roots. He went further to conceive of length, area, and volume as things growing or increasing by infinite units instead of fixed quantities. He did not publish his development of differential and integral calculus but continued and developed the second fluxion or repeated differentiation. The only written accounts at that time were some brief notes for his own use and twelve applications of the method.

According to the Ptolemaic system the earth was the center of the universe and the heavenly bodies revolved around it with uniform motion in circular orbits. Copernicus disagreed with this theory and, despite public disapproval, felt that the earth was neither stationary nor the center of the universe, but rather that the sun was the center of the universe and the earth, along with the other planets, revolved around it. Newton proved Copernicus to be correct with such discoveries as the discs of Saturn, Mars, and Jupiter, and the satellites of Jupiter.

John Kepler did not believe that the planets followed circular

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[3] Brodotsky, op. cit., p. 23.
[4] Ibid., pp. 25, 26.

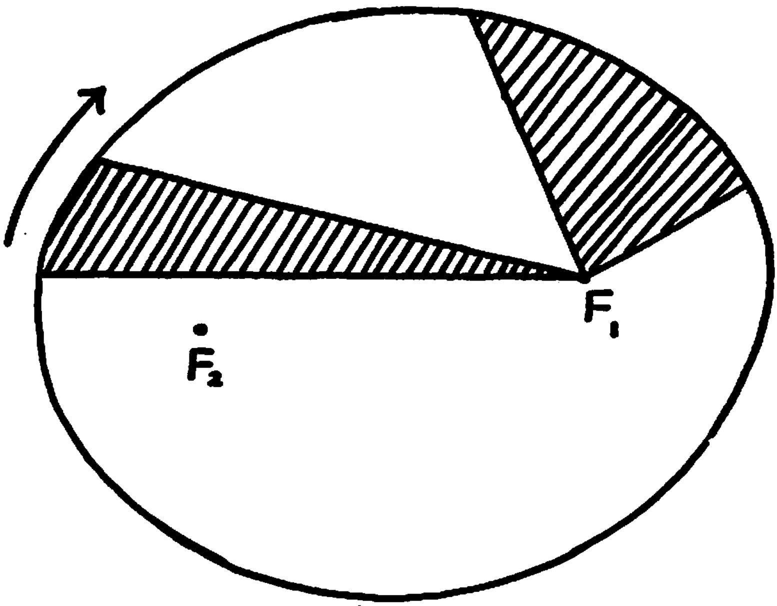

FIG. I

orbits and worked on his theory until in 1609 he discovered what are now known as Kepler's laws of planetary motion:

"I. The orbit of each planet is an ellipse with the sun at one focus.

II. The speed of the planet varies in such a way that the line joining the center of the planet and the center of the sun sweeps out equal areas in equal times.

III. The cubes of the semimajor axes of the elliptical orbits are proportional to the squares of the times for the planet to make a complete revolution about the sun."[5]

When Galileo experimented in the field of dynamics, the quantitative relationships between forces and motions, he discovered that heavier objects did not fall faster than lighter ones as they

[5] Alphous W. Smith and John N. Cooper, The Elements of Physics (New York: McGraw - Hill Book Company, Inc., 1957), p. 68.

neared the earth if both were heavy enough to make the air resistance negligible, and that the increase in speed was directly related to the time, or it "accelerated uniformly."

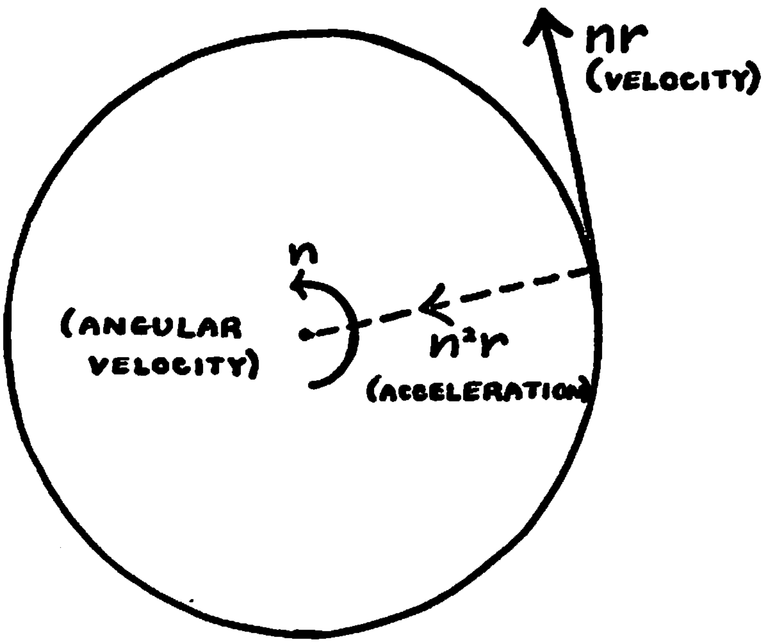

FIG. II

It is told that one day Newton saw an apple fall from a tree in his garden and reasoning that it was due to the earth's gravitational pull, he went on to develop his law of gravitation. Newton had been thinking about the problem and wondered what effect distance would have on the gravitational attraction and whether this force could also explain the motion of the moon around the earth.

Although the planets move on ellipses at varying velocities, these ellipses may be considered almost circles and the velocities vary so slightly that for rough approximation they may be considered constant.

> "This means that for radius r and angular velocity n the force per unit mass of the moving body must be the speed rn into the rate at which the direction of motion ro-

tates, namely, n. Hence the force per unit mass must be n^2r."[6]

At the age of twenty-three, he used Kepler's third law to show the inverse square law—that $\frac{n^2}{r}$ was proportional to $\frac{1}{r^2}$.

He then reasoned that the gravitational pull on the apple and moon also followed this law. From this he calculated the acceleration of the moon to be 0.00775 feet per second. Kepler had calculated 0.00895, however, and thinking that he had failed he did not announce his discovery. It was not until sixteen years later that he discovered the difference in values was due to the fact that the value he had used for the radius of the earth had been too small and that his discovery had been valid. Figure III illustrates Newton's Law of Gravitation.

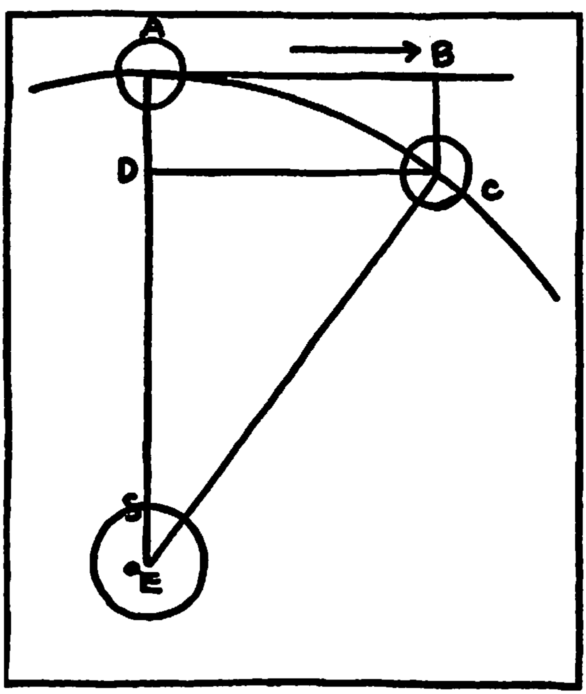

FIG. III

[6] Brodetsky, op. cit., p. 48.

"*E* represents the earth and *A* the moon. Were the earth's pull on the moon to cease, the moon's inertia would cause it to take the tangential course, *AB*. On the other hand, were the moon's motion to be stopped for an instant, the moon would fall directly towards the earth, along the line *AD*. The moon's actual orbit, resulting from these component forces, is *AC*. Let *AC* represent the actual flight of the moon in one minute. Then *BC*, which is obviously equal to *AD*, represents the distance which the moon virtually falls toward the earth one minute. • • • Another computation showed that this is the distance that the moon would fall towards the earth under the influence of gravity, on the supposition that the force of gravity decreases inversely with the square of the distance; the basis of comparison being furnished by falling bodies at the surface of the earth."[7]

Because of the Great Plague, Newton had been forced to abandon his work in optics temporarily. In 1666 he discovered why the refracting telescopes in use were defective and set about to construct a telescope using the principle of reflection instead. In a telescope the image of a distant object is seen through two lenses, being first formed by the object lens and then viewed by the eyepiece. Galileo's telescopes were very defective instruments and the cause was taken to be spherical aberration. Descrates thought that the cause of the defect was the spherical shape of the lenses and ground new ones called Cartesian ovals. These, however, failed to make any improvement.

Through experimentation Newton discovered that there were two causes of imperfection in refracting telescopes, spherical aberration and chromatic aberration, which was distortion due to color. He had passed a circular beam of sunlight through a prism. It was already known that if this beam emerging from the prism was shown on a screen different colors would appear rather than white, but Newton also noted that the beam was no longer circular but that it was about five times longer than it was wide. He found that chromatic aberration was not produced by reflection and proceeded to make a reflecting telescope.

Newton returned to Cambridge when it was reopened in 1667

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7 Henry Smith Williams, *A History of Science* (New York: Harpers & Brothers, 1904), p. 244.

and was elected Minor Fellow of Trinity College. A year later he took his M.A. degree and was elected Major Fellow. Late in 1668 he completed the first reflecting telescope which was six inches long and had an object-glass of about one inch diameter.

He gave his first paper about fluxions to one of his professors, Issac Barrow, who sent them to another mathematician, John Collins. If either man had realized their importance and had had them published then, a bitter debate regarding the true discoverer could have been avoided forty-two years later.

Cambridge honored Newton on October 29, 1669 by appointing him to the position of Lucasian professor. The Royal Society learned that he had perfected a reflecting telescope and asked if he would allow them to examine it. This resulted in his election to the society on January 11, 1672. One of the first papers that he sent to the society pertained to his discovery of the spectrum and the nature of color, for now he was able to explain why the beam of light was extended. White light was composed of a number of different colored lights which each had its own degree of refraction when passed through a prism. He made the mistake of believing that all prisms would give a spectrum of equal length when actually the length of the spectrum depends on the kind of glass used, but this error was not corrected until several generations later.

Often Newton did not sign his name to his own published works because he did not like the attention, both favorable and unfavorable, that it drew to himself. He was often in poor health, partly because of his insufficient diet when he did find time to eat. The King granted him a dispensation for which he had applied so that he could keep his Fellowship after it expired. To go into the orders as an alternative would have meant giving up much of his freedom of thought in scientific matters. He and other scientists in the Royal Society were also excused from paying their dues of a shilling a week which they could not afford.

Newton continued to experiment with optics until 1678. During this period he formulated his theory of fits which stated that the properties at all points of a ray of light cannot be the same. Since he was an extremely practical man he would not accept Hooke and Huygens' wave-theory of light and because of his scientific reputation it was forgotten and not brought to light again till more than a century later.

While corresponding with Leibniz, the German mathematician, Newton discovered that Leibniz had developed a differential calculus which was the same as his fluxions. There was only a difference in notation with Leibniz using $\frac{dy}{dx}$ and Newton using $\dot{y}$. Newton still did nothing pertaining to his development and instead continued his work on gravitation. Hooke had sent a paper to the Royal Society in 1666 concerning the variation of the gravitation of the earth with distance from the surface but was unable to offer a proof or even an assumption until 1679. Newton had proved it thirteen years before but had not published his findings.

Oldenburg died in 1678 and Hooke succeeded him as Secretary of the Royal Society. Newton's next contribution to the society was a paper suggesting a proof for the rotation of the earth. Newton planted the seed but Hooke tilled the soil and finally reaped the harvest. He made some corrections in Newton's initial suggestion and developed the first ocular proof of the earth's rotation. Newton had also thought that the path of a body attracted by the earth's gravitation would be a spiral curve. Hooke disagreed with this and thought it would be an ellipse but was not able to prove his theory. Newton, however, following Hooke's idea was able to prove that the path was an ellipse with the center of attraction as one of the foci.

Not until 1682 did Newton become aware of the fact that Picard's measurements for the number of miles per degree latitude was 69.1 miles instead of 60 miles. This made a difference in the value he had used for the radius of the earth and made his earlier calculations correct.

Edmund Halley had also discovered the inverse square law eighteen years after Newton had developed the only existing mathematical proof. When he heard of Newton's calculations he took a personal interest in their publication. Newton finally presented the Royal Society with a paper on planetary motion about the same time.

Newton obtained a great deal of help pertaining to necessary astonomical facts related to his gravitation from John Flamsteed, head of the Greenwich Observatory. In April, 1686, Halley presented Newton's "Philosophies Naturalis Principia Mathematica" or "The Mathematical Principles of Natural Philosophy" to the Royal Society. Since the Society was not able to finance the printing of the book, Halley took on the responsibilty himself. There was so much unpleasantness because others claimed they had discovered the inverse square law that Newton did not want to continue on his "De Mundi Systemate" which gave many applications of his law, but Halley convinced him to finish.

Newton had never married but rather had devoted himself to science. The first edition of the "Principia," written and published in Latin, appeared in 1687. This contained Newton's development of his three laws of motion:

"I. Every body continues in its state of rest or of uniform velocity in a straight line unless it is compelled to change that state by the application of some unbalanced external force.

II. The acceleration of a body is directly proportional to the unbalanced force acting upon it and is inversely proportional to the mass of the body.

III. Whenever one body exerts a force upon a second body, the second body exerts an equal and opposite force on the first."[8]

Book I, "De Motu," discussed his method of fluxions, the effects of forces on different bodies, and the cases of open paths where a body may not return to the same point. He then proved that the same force will be in effect on any point on the surface of a sphere if the mass of the sphere is symmetric with respect to the center, just as if this mass were concentrated at the center. Finally he discussed the attraction of bodies other than spheres and his now non-accepted corpuscular theory of light.

Book II, "De Motu Corporum," concerned motion encountering resistance produced by the medium which it was in, the study of wave motion in fluids, and disproved the Vortex Theory.

Book III, "De Mundi Systemate," contained Newton's statement that "like effects in nature are produced by like causes" and the generalization of the law of universal gravitation: the force between two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. $F = G\frac{m_1 m_2}{r^2}$, where G is the universal gravitational constant. He showed that the variation of the gravitational pull of the earth at different point on its surface was due to the flattening at its poles caused by the rotation of the earth. He explained how the tides were produced by the gravitational attraction of the sun and moon on the earth, and finally proved that a comet moving in space moves in a long ellipse which approaches closely to a parabola.

8 Smith and Cooper, op. cit., pp. 53, 56, and 59.

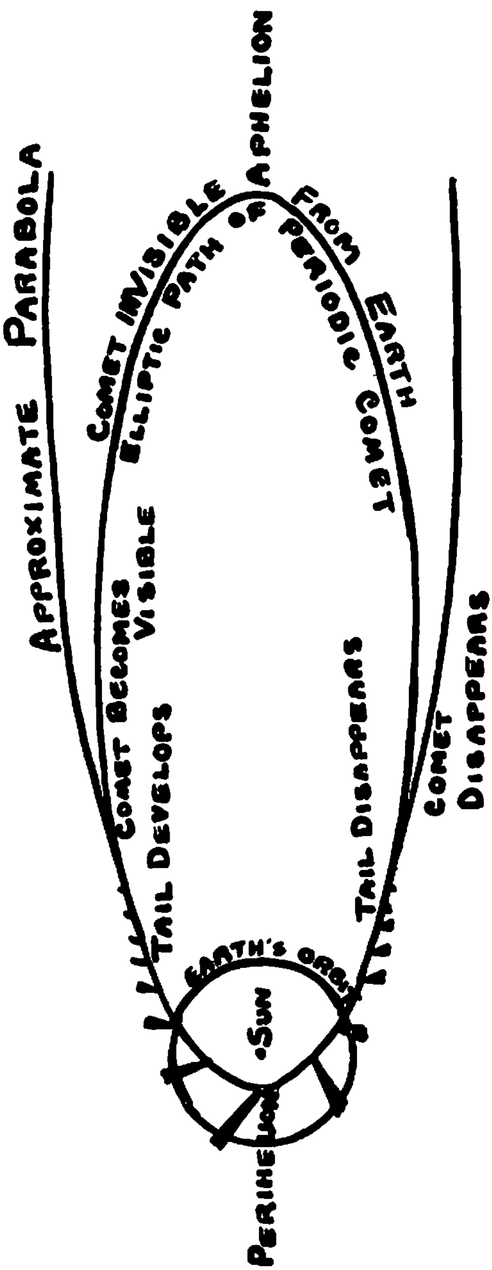
APPROXIMATE PARABOLA
APHELION
COMET INVISIBLE FROM EARTH
ELLIPTIC PATH OF PERIODIC COMET
COMET BECOMES VISIBLE
TAIL DEVELOPS
TAIL DISAPPEARS
COMET DISAPPEARS
EARTH'S ORBIT
SUN
PERIHELION

FIG. IV

Using this information Halley studied all the valid reports on comets which had appeared that he could find and discovered that ones in 1531, 1607, and 1682 had similar parabolic paths. Halley was convinced this was the same comet and with Newton's help calculated that it would reappear about the end of 1758 or the beginning of 1759. Halley's Comet appeared within one month of the predicted time and later with better agreement in 1835 and 1910.

Some of Newton's theories met with opposition in most of Europe but were quickly accepted in England. The "Principia" was taught at Cambridge three years after its first publication, at a time when the book was in great demand. He then set aside his scientific work temporarily and was a member of the House of Commons for a year. Although nominated, Newton's failure to receive the position of Head of King's College in Cambridge was a great disappointment.

Following his mother's death, Newton's interest turned to theology, the greatest of his works being his letters to Richard Bentley. In answer to Lucretius who said that the gravitational attraction between the matter distributed in space would form the universe without divine assistance, Newton replied that matter would have not formed solar systems with planets in orbits around suns without the "aid of Divine will and design."

The last thirty-one years of his life were occupied by public affairs. English silver coinage had depreciated so in value that the Bank of Amsterdam refused to accept it. Charles Montague appointed Newton Warden of the Mint in 1696 to issue new coins and remove the old ones from circulation. Newton accomplished this task in three years and was soon appointed Master of the Mint, a position which he held till he died.

With his new wealth he enjoyed helping his relatives, and his niece Catherine took care of him in London and was hostess to his guests for many years. In 1699 he became a foreign associate to the French Academy and two years later he reigned as Lucasian professor and gave up his Trinity fellowship.

He sent a description of a sextant to Halley in 1700 but no one recognized its importance until thirty years later when it was reinvented by John Hadley. He published two other important theories in 1701. The first was his law of cooling which stated that the rate of cooling of a warm body is proportional to the difference between the temperatures of the medium and the body, and the second was that a body's temperature remains constant as it melts or evaporates. Two years later he was elected President of the Royal Society and in 1705 Queen Anne conferred knighthood upon Newton.

Newton would not let his book "Optics" be published until one year after Hooke died. This book also contained two mathematical works, finding the area of closed curves using fluxions and an account of seventy-two cubic curves. Many thought he had taken Leibniz's work and rewritten it with different notation.

In 1710 the Royal Observatory was placed under the supervision of a board, with Newton as its head. Newton and Flamsteed engaged in such a quarrel that they ceased corresponding.

Because Newton's time was occupied, Roger Cotes, a mathematician, prepared the second edition of the "Principia" for printing. About this same time an open dispute began as to whether Newton or Leibniz had first invented calculus. From 1708 both sides tried to discredit the other and before it was over it not only involved Newton and Leibniz, but also John Keill, the Royal Society, John Bernoulli, the Princess of Wales, and even the King. It probably would have continued longer but Leibniz died in November, 1716. All this could have been avoided if Newton and his friends had published his works soon after he formulated them. Priority had always been established not by the date of discovery but rather by the date of publication.

At the age of eighty Newton's health began to be affected. He secured the help of Henry Pembeton, another young scientist, to publish the third edition of the "Principia." He continued as Master of the Mint but Conduitt, who had married Newton's niece Catherine Barton, really did the main work. The third edition appeared in 1726 and signified Newton's last contribution to science.

In his last years he studied chronology. His largest work in this field was his "Chronology of Ancient Kingdoms." On March 4, 1727, he became seriously ill and died in the early hours of March 20, 1727, at the age of eighty-five. His body lay in state in the Jerusalem Chamber and was buried in Westminster Abbey.

> "I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."[9]
>
> —Isaac Newton

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9 Bell, op. cit., p. 90.

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Williams, Henry Smith. *A History of Science.* New York: Harpers & Brothers, 1904. pp. 225-251.

The figures for this article were adapted from the following sources:

Figure I. Smith, Alpheus W. and Cooper, John N. *The Elements of Physics.* New York: McGraw - Hill Book Company, Inc., 1957. p. 68.

Figure II. Brodetsky S. *Sir Isaac Newton.* London: Methuen & Co. Ltd., 1929. p. 49.

Figure III. Williams, Henry Smith. *A History of Science.* New York: Harpers & Brothers, 1904. p. 244.

Figure IV. Brodetsky, op. cit., p. 117.

Every one knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions. . . .A curve is the totality of points, whose coordinates are functions of a parameter which may be differentiated as often as may be required.

—F. KLEIN

Graphical Solution of Quartic Equations*

S. McDowell Steele, Jr.
Student, Kansas State College of Pittsburg

There are several solutions of quartic equations, but most of them require a great deal of arithmetic and are quite time consuming. While, by its very nature, a graphical solution cannot be as accurate as some of the others, it requires somewhat less time. The computer has made the solution of the quartic a speedy process by any method, and there are limited applications of the graphical solution. However, as mathematicians, we first think of the mathematics involved and secondly of the possible applications.

As the topic suggests, we shall solve an equation of this type: $x^4 + px^3 + qx^2 + rx + s = 0$, where p, q, r, and s are constants.

In the equation which will be derived, there will be no third degree term; therefore, we must eliminate the third degree term in any equation which we wish to solve. This can be done by making the substitution $x = (z - p/4)$. This substitution may be derived from the binomial theorem.

We shall solve our equation by measuring the ordinates of intersections of two curves, specifically: the parabola, $y^2 = 2x$, and the general circle, $(x - h)^2 + (y - k)^2 = r^2$. First we express one variable in terms of the other.

$$x = \frac{y^2}{2}$$

Then substitute this into the equation of the circle to obtain

$$(1) \qquad (\frac{y^2}{2} - h)^2 + (y - k)^2 = r^2$$

We now have solved our two equations simultaneously; therefore, any y value that satisfies one of them will satisfy the other. Expanding (1), clearing of fractions and collecting like terms we have the equation

$$(2) \qquad y^4 + 4(1 - h)y^2 - 8ky + 4(h^2 + k^2 - r^2) = 0$$

(Note: As was previously mentioned, we have no third degree term.)

* Presented at the Kappa Mu Epsilon National Convention, Emporia, Kansas, April 21-22, 1961.

We will now introduce a variable in the form of t^4 which, as we shall see later, will allow us to vary the radius and throw the center of the circle to some desirable point so that it will not cut the parabola at angles that are too acute for an accurate reading. We now have

(3)

$$\left(\frac{y}{t}\right)^4 + \frac{4(1-h)}{t^2}\left(\frac{y}{t}\right)^2 - \frac{8k}{t^3}\left(\frac{y}{t}\right) + \frac{4(h^2+k^2-r^2)}{t^4} = 0$$

Equation (3) is the equation we shall equate to any fourth degree equation which we desire to solve.

Now, why will this work? Because equation (1) is the simultaneous solution of a specific parabola and a general circle, and because any y that will satisfy one of these equations will also satisfy the other two.

The general form of equation (3) is:

(4) $$z^4 + az^2 + bz + c = 0.$$

If we set $z = \frac{y}{t}$ and equate the constants of any equation in the form of (4) to those in (3), we have

(5) $$a = \frac{4(1-h)}{t^2}$$

(6) $$b = \frac{-8k}{t^3}$$

(7) $$c = \frac{4(h^2+k^2-r^2)}{t^4}$$

Solving (5), (6), and (7) for h, k, and r, respectively, we have the coordinates of the center of our circle and its radius.

(8) $$h = \frac{4-at^2}{4}$$

(9) $$k = -\frac{bt^3}{8}$$

(10) $$r = \sqrt{h^2+k^2-\frac{ct^4}{4}}$$

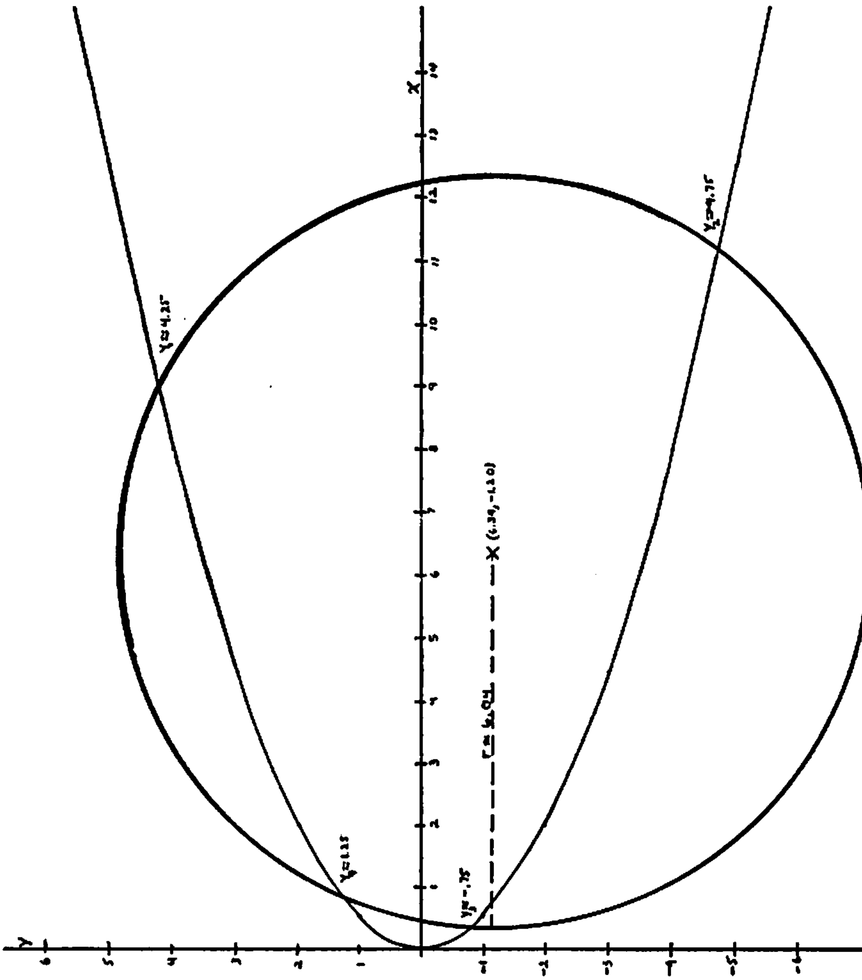

Figure 1.

As an example, we shall now solve the equation.

$$x^4 - 3x^3 - 18x^2 + 40x = 0$$

Let $x = z - \frac{p}{4} = z + \frac{3}{4}$.

$$z^4 - 21.38z^2 + 9.63z + 20.88 = 0$$

Let $t = 1$. Then

$$h = \frac{4 + 21.38(1)^2}{4} = 6.34$$

$$k = -\frac{9.63(1)^3}{8} = -1.20$$

$$r = \sqrt{(6.34)^2 + (-1.20)^2 - \frac{20.88(1)^4}{4}} \doteq 6.04$$

Figure 1 shows the parabola $y^2 = 2x$ and the circle with center at $(6.34, -1.20)$ and radius 6.04. The y coordinates of the four points of intersection are the solution to the quartic in z since $z = y/t$ and $t = 1$. To obtain the solution of the original equation, we know that $x = z + 3/4$. Thus

$$\begin{array}{ll} z_1 = 4.25 & x_1 = 4.25 + .75 = 5 \\ z_2 = -4.75 & x_2 = -4.75 + .75 = -4 \\ z_3 = -.75 & x_3 = -.75 + .75 = 0 \\ z_4 = +1.25 & x_4 = 1.25 + .75 = 2 \end{array}$$

In Figure 2 we have an interesting demonstration of the importance of t. On this graph are three solutions of the equation

$$x^4 - 1.04x^2 + .04 = 0$$

When $t = 1$:

$$h = 1.26$$
$$k = 0.00$$
$$r \doteq 1.26$$

When $t = 2$:

$$h = 2.04$$
$$k = 0.00$$
$$r \doteq 2.00$$

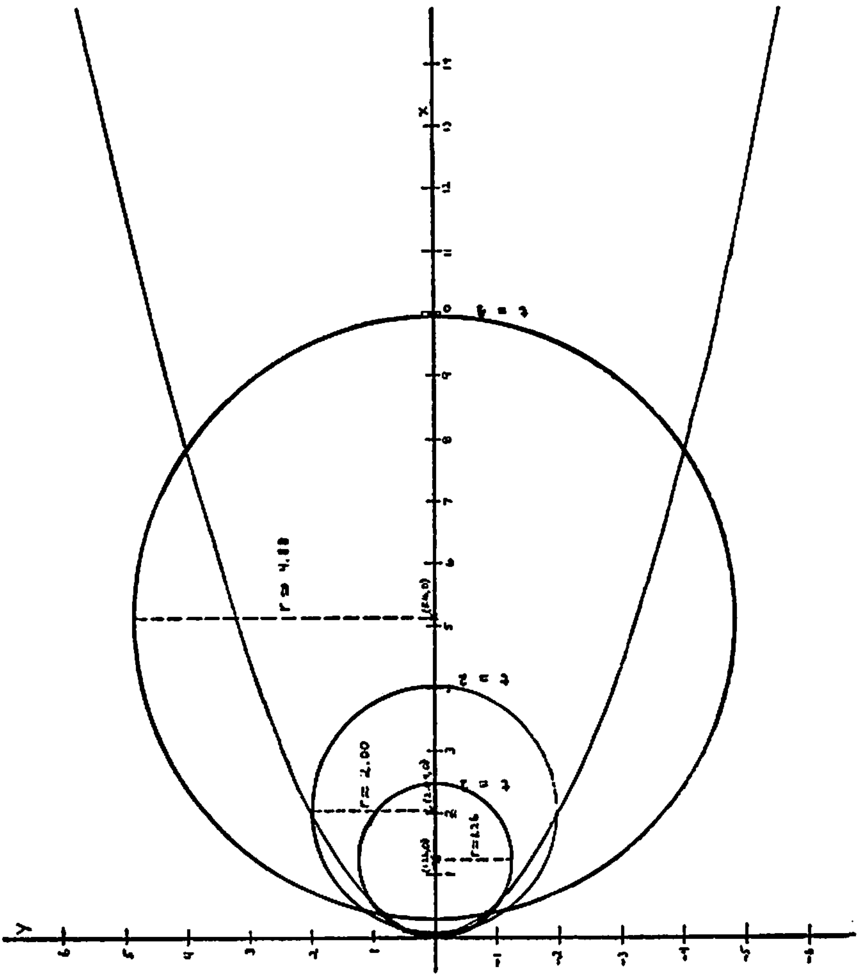

Figure 2.

When $t = 4$:

$$h = 5.16$$
$$k = 0.00$$
$$r \doteq 4.88$$

In all three cases the roots are:

$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = .2$$
$$x_4 = -.2$$

Acknowledgments: I would like to express my appreciation to R. W. Hart, the typists of the mathematics department, and all others in the mathematics department who gave so willingly of their time.

There is probably no other science which presents such different appearances to one who cultivates it and to one who does not, as mathematics. To this person it is ancient, venerable, and complete; a body of dry, irrefutable, unambiguous reasoning. To the mathematician, on the other hand, his science is yet in the purple bloom of vigorous youth, everywhere stretching out after the "attainable but unattained" and full of the excitement of nascent thoughts; its logic is beset with ambiguities, and its analytic processes, like Bunyan's road, have a quagmire on one side and a deep ditch on the other and branch off into innumerable by-paths that end in a wilderness.

—C. H. Chapman

Some Aspects of Geometrical Probability

Anthony Pettofrezzo
Faculty, Montclair State College, N.J.

The classical definition of mathematical probability is due to Bernoulli: If an experiment has n mutually exclusive, equally likely outcomes for which m outcomes can be associated with event A, then the a priori probability of event A is m/n.

The application of the classical definition involves the consideration of the totality of relevant possibilities as some finite number. However, we often need to consider problems in which the events belong to a continuous field with the relevant possibilities being unlimited. While no new principle of probability is involved the methods of application of the basic principle vary. A geometric approach, often employing the integral calculus, is especially useful.

The simplest of geometrical conditions to consider involve the position of points on a line or in a plane, or the position of points, lines, and planes in space. The following problem illustrates the technique of employing geometry in the calculation of probabilities.

Consider a line segment of length L. If two points are chosen at random, what is the probability that the three segments determined will form a triangle? In order to answer this question we shall consider two random variables x and y as representing the distances of the two points from the two extremities of the line segment of length L as shown in Figure 1.

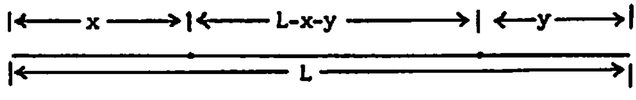

Figure 1.

Now, $0 < x < L$, $0 < y < L$, and $x + y < L$. Consider x and y as the coordinates of a point in the plane with the rectangular cartesian coordinate system for reference. The points representative of the possible values of x and y are contained in an area which is an isosceles right triangle with side L since $x + y < L$. (Figure 2).

Now, if the segments x, y, and $L - x - y$ are to form a triangle, then the sum of any two segments must be greater than the

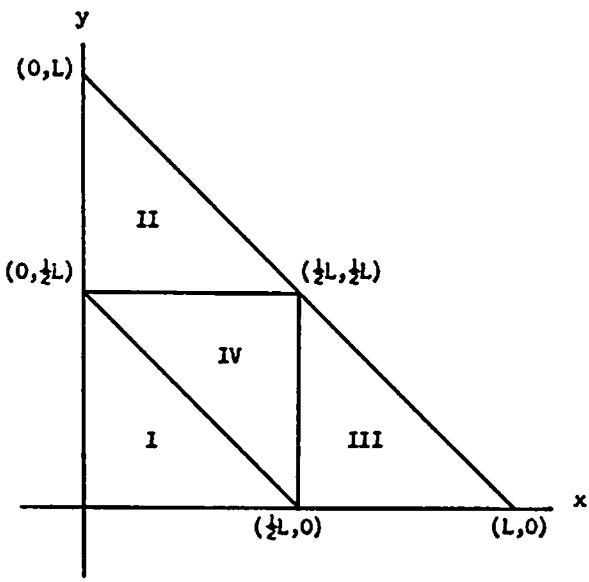

Figure 2.

third. Hence

$$x + y > L - x - y$$
$$2(x + y) > L$$
$$x + y > \tfrac{1}{2}L$$

which eliminates points in the area marked *I*. Also,

$$x + (L - x - y) > y$$
$$L - y > y$$
$$L > 2y$$
$$\tfrac{1}{2}L > y$$

which eliminates points in the area marked *II*. Finally

$$y + (L - x - y) > x$$
$$\tfrac{1}{2}L > x$$

which eliminates points in the area marked *III*. The favorable points are contained in the area marked *IV*. Since *I*, *II*, *III*, *IV* represent the areas of congruent triangles, the desired probability is ¼, the ration of area *IV* to the sum of the arease of *I*, *II*, *III*, and *IV*.

The greatest difficulty in handling problems in terms of geometric probability consists in determing the independent variable to be used in expressing the probability. For example, consider a chord drawn at random in a given circle. What is the probability that it is at least as long as the radius? Since the circle is symmetric with respect to the center, the direction of the chord is immaterial. The length of the chord L depends upon the distance d from the center as shown in Figure 3.

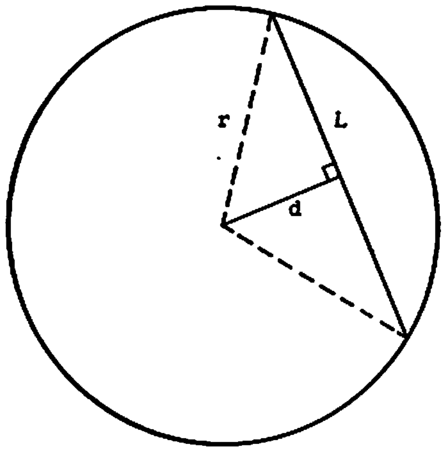

Figure 3.

The chord will be at least as long as the radius if $d \leq \frac{1}{2}\sqrt{3}r$. Since d ranges from 0 to r, the probability desired is

$$\frac{\frac{1}{2}\sqrt{3}r}{r} = \frac{1}{2}\sqrt{3} \doteq 0.866.$$

However, consider the problem in the following manner. The first point of intersection of the chord with the circle is immaterial in determining the probability desired. Since all positions of the second intersection are equally likely, all angles between the chord and the radius are equally likely (Figure 4). If $L \geqq r$, then $\theta \leqq \pi/3$. Considering the angle as being on either side of the diameter shown in Figure 4, the desired probability that $L \geqq r$ is

$$\frac{2\ \pi/3}{\pi} \doteq 0.667.$$

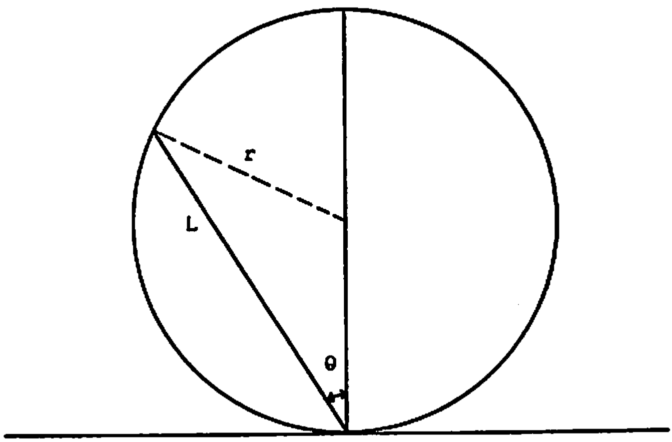

Figure 4.

Which answer is correct? This is Bertrand's famous paradox. The difficulty lies in the fact that we really have here two different problems. For example, consider this probability empirically. The first case is similar to our throwing a cardboard disk of diameter $2r$ on a plane with ruled parallel lines $2r$ units apart. One and only one line would cut the disk at some distance from the center. The empirical probability would support our first analysis. However, consider a cardboard disk free to rotate about a fixed point on a line. Spun in a random fashion, the disk would stop on the line covering a line segment which could represent a chord of the circle. The frequency ratios of length greater than r to the number of trials would approach the mathematical probability value calculated by our second analysis. We can see that answers given to problems in geometrical probability should be subject to considerable suspicion. The best we can do in problems of this type is to make the best possible judgment as to the nature of the independent variables. In some cases support by experimentation may be useful.

The most famous and oldest probability problem (1733) involving a geometric situation is Buffon's Needle problem. Consider a board ruled with a set of parallel lines d units apart. A fine needle of length $L \leqq d$ is thrown down on the board. What is the probability that the needle will intersect one of the lines? We can approach this problem either empirically or analytically. However the analytic

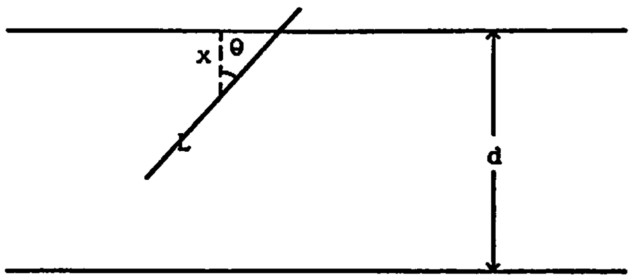

Figure 5.

approach will involve the use of some calculus. We can determine the position of the needle by the distance x of its midpoint from the nearest line and the smallest angle θ which this line makes with the needle as shown in Figure 5.

The two variables x and θ vary independently in such a manner that $0 \leqq x \leqq \frac{1}{2}d$ and $0 \leqq \theta \leqq \frac{1}{2}\pi$. The domain of x and θ is rectangle, $OABC$, where $OA = \frac{1}{2}d$ and $OC = \frac{1}{2}\pi$ (Figure 6).

The needle will intersect the nearest line in Figure 5 if $x < \frac{1}{2}L \cos \theta$. Values of x and θ which satisfy this relationship are found in the shaded area (OCD) in Figure 6. The required probability that the needle intersect a line is equal to

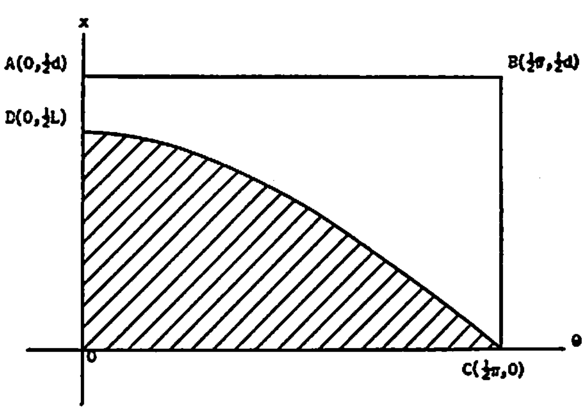

Figure 6.

$$\frac{\text{Area } OCD}{\text{Area } OABC} = \frac{\frac{1}{2}L \int_0^{\frac{1}{2}\pi} \cos\theta \, d\theta}{\frac{1}{2}\pi \cdot \frac{1}{2}d}$$

$$= \frac{2L}{\pi d}.$$

Notice if $L = d$, the resulting probability is $2/\pi$. Many mathematicians have empirically calculated π by actually throwing a thin needle onto a board of ruled parallel lines the length of the needle apart. Their data would support our hypothesis in deriving the expression for the desired probability.

Several types of variations on Buffon's problem are possible. For example consider the needle greater than the distance between the set of rulings. In Figure 7 the domain of admissable points are found in rectangle $OABC$ whose area is $\frac{1}{4}d\pi$. Those points considered favorable are contained in the shaded area since $x < \frac{1}{2}L \cos\theta$ and $x < \frac{1}{2}d$. The θ coordinate of the intersection of the line $x = \frac{1}{2}d$ and the curve $x = \frac{1}{2}L \cos\theta$ is given by

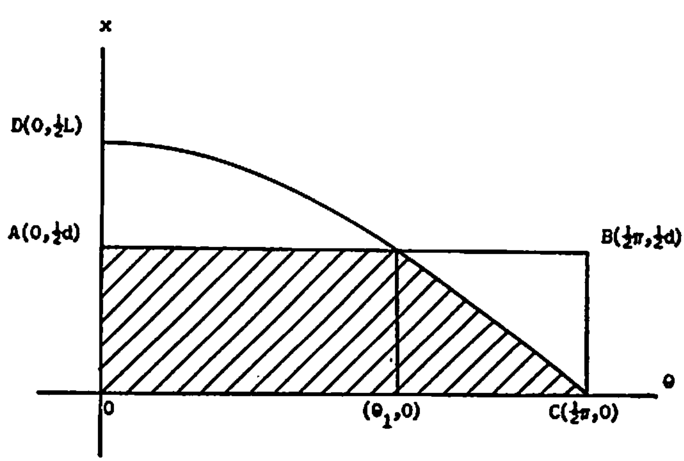

Figure 7.

$$\begin{aligned} \tfrac{1}{2}L \cos \theta_1 &= \tfrac{1}{2}d \\ \cos \theta_1 &= d/L \\ \theta_1 &= \arccos(d/L) \end{aligned}$$

Hence the probability of intersection is given by

$$\frac{\tfrac{1}{2}d\theta_1 + \int_{\theta_1}^{\frac{1}{2}\pi} \tfrac{1}{2}L \cos \theta \, d\theta}{\tfrac{1}{4}d\pi} = \frac{2\theta_1}{\pi} + \frac{2L}{\pi d} \sin \theta \Big]_{\theta_1}^{\frac{1}{2}\pi}$$

$$= \frac{2\theta_1}{\pi} + \frac{2L}{\pi d}(1 - \sin \theta_1)$$

where $\theta_1 = \arccos(d/L)$.

Mathematical science is in my opinion an individual whole, an organism whose vitality is conditioned upon the connection of its parts. For with all the variety of mathematical knowledge, we are still clearly conscious of the similarity of the logical devices, the relationship of the ideas in mathematics as a whole and the numerous analogies in its different departments. We also notice that, the farther a mathematical theory is developed, the more harmoniously and uniformly does its construction proceed, and unsuspected relations are disclosed between hitherto separated branches of the science. So it happens that, with the extension of mathematics, its organic character is not lost but manifests itself the more clearly.

—D. Hilbert

The Problem Corner

EDITED BY J. D. HAGGARD

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before October 1, 1962. The best solutions submitted by students will be published in the Fall, 1962, issue of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to J. D. Haggard, Department of Mathematics, Kansas State College, Pittsburg, Kansas.

PROPOSED PROBLEMS

151. *Proposed by Sam Sessken, Hofstra College, Hempstead, New York.*

The algebra teacher restricts the use of logarithms to log 3 and log. 2. Find log 5 accurate to the nearest thousandths.

152. *Proposed by C. W. Trigg, Los Angeles City College.*

Find a set of three-digit numbers, each of which is a permutation of the same three digits, which when divided by the sum of the digits yields two pairs of consecutive integers.

153. *Proposed by Mary Sworske, Mount Mary College, Milwaukee, Wisconsin.* (From *The Theory of Numbers,* by Burton Jones)

A woman with a basket of eggs was knocked down by a bicycle. In presenting the bill to the rider's father, she said she did not know how many eggs she had, but when she counted them two at a time there was one egg left, and similarly when she counted them three, four, five, and six at a time; but in sevens there were not any left. What is the smallest number of eggs she could have had?

154. *Proposed by the Editor.* (From *The American Mathematical Monthly*)

Find a number 1*abcde* so that 3 times that number gives *abcde*1.

155. *Proposed by Sam Sessken, Hofstra College, Hempstead, New York.*

In the set of natural numbers, suppose $N = 2n + 1$, show that the sum, S, of all the products whose divisors total N is $2(1^2 + 2^2 + \cdots n^2)$. In other words show

$$\sum_{i=1}^{n} [(2n + 1)i - i^2] = 2 \sum_{i=1}^{n} i^2$$

SOLUTIONS

146. *Proposed by Fredric Gey, Harvey Mudd College, Claremont, California.*

Sum the double series:

$$\sum_{k=n}^{\infty} \sum_{l=n}^{\infty} \frac{a^{l-n} x^l}{2^{k-n} (l-n)!}$$

Solution by Robert Kurosaka, State University of New York, Albany.

$$\sum_{k=n}^{\infty} \sum_{l=n}^{\infty} \frac{a^{l-n} x^l}{2^{k-n} (l-n)!} = \sum_{k=n}^{\infty} \left[\sum_{l=n}^{\infty} \frac{a^{l-n} x^l}{2^{k-n} (l-n)!} \right]$$

$$= \sum_{k=n}^{\infty} \left[\frac{1 \cdot x^n}{2^{k-n}\, 0!} + \frac{a x^{n+1}}{2^{k-n}\, 1!} + \frac{a^2 x^{n+2}}{2^{k-n}\, 2!} + \frac{a^3 x^{n+3}}{2^{k-n}\, 3!} + \cdots \right]$$

$$= \sum_{k=n}^{\infty} \frac{x^n}{2^{k-n}} \left[1 + \frac{ax}{1!} + \frac{(ax)^2}{2!} + \frac{(ax)^3}{3!} + \cdots \right] = \sum_{k=n}^{\infty} \frac{x^n}{2^{k-n}} e^{ax}$$

$$= x^n e^{ax} \sum_{k=0}^{\infty} (1/2)^k = 2x^n e^{ax}$$

Also solved by Phil Huneke, Pamona College, Claremont, California; Norman Nielsen, Pamona College, Claremont, California; Perry Smith, Albion College, Albion, Michigan.

147. *Proposed by Rickey M. Turkel, Hofstra College, Hempstead, New York.*

How many different paths can be traced through the following diagram, spelling the word "TRIGONOMETRY"?

```
                      T
                    T R T
                  T R I R T
                T R I G I R T
              T R I G O G I R T
            T R I G O N O G I R T
          T R I G O N O N O G I R T
        T R I G O N O M O N O G I R T
      T R I G O N O M E M O N O G I R T
    T R I G O N O M E T E M O N O G I R T
  T R I G O N O M E T R T E M O N O G I R T
T R I G O N O M E T R Y R T E M O N O G I R T
```

Solution by Perry Smith, Albion College, Albion, Michigan.

Since the diagram is symmetrical, let us consider the vertical "TRIGONOMETRY" and everything to its left. Beginning with the letter "Y" and working backward through the word, we have two choices for R's; for each R we have two choices for a T; for each T two choices for an E, etc. Hence there is a total of $1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{11} = 2048$ paths. From the vertical "TRIGONOMETRY" and to the right we would obtain another 2048 paths. The vertical path has now been counted twice; therefore, we have $2^{11} + 2^{11} - 1 = 4095$ possible paths.

Also solved by Phil Huneke, Pamona College, Claremont, California; Owen Kardatyke, Anderson College, Anderson, Indiana; Norman Nielsen, Pamona College, Claremont, California; Roger Richards, Westminster College, New Wilmington, Pennsylvania; Sam Sessken, Hofstra College, Hempstead, New York.

148. *Proposed by C. W. Trigg, Los Angeles City College.*

Two numbers whose three digits are consecutive integers have the property that the number and a permutation of its digits can each be represented as the sum of two cubes. Identify the permutation.

Solution by Norman Nielsen, Pamona College, Claremont, California.

Since the desired numbers contain three digits, we are restricted to cubes of numbers less than 10, since $10^3 = 1000$, a four-digit number.

x:	1	2	3	4	5	6	7	8	9
x^3:	1	8	27	64	125	216	343	512	729

An addition table for these cubes will show 32 sums with the necessary three digits, but only four of these sums contain consecutive digits. They are:

$$243 = 216 + 216 = 6^3 + 6^3$$
$$576 = 64 + 512 = 4^3 + 8^3$$
$$432 = 27 + 216 = 3^3 + 6^3$$
$$756 = 27 + 729 = 3^3 + 9^3$$

Thus 243 and 432; 576 and 756 are the desired permutations.

Also solved by Dee Fuller, Davidson College, Davidson, North Carolina; Phil Huneke, Pamona College, Claremont, California; Owen Kardatyke, Anderson College, Anderson, Indiana; Roger

Richards, Westminster College, New Wilmington, Pennsylvania; Sam Sessken, Hofstra College, Hempstead, New York.

149. *Proposed by Jim Brooking, State University of New York, Albany.*

Find any solution to the equation:

$$\frac{\arcsin x}{\arccos x} = \arctan x$$

Solution by Phil Huneke, Pamona College, Claremont, California.

Let

$$\arctan x = y \quad \text{or} \quad \tan y = x$$
$$\arccos x = z \quad \text{or} \quad \cos z = x.$$

Then

(1) $$\tan y = \cos z$$

and

$$\frac{\arcsin x}{z} = y$$
$$\arcsin x = yz$$
$$\sin yz = x$$

(2) $$\sin yz = \cos z.$$

A solution to (1) and (2) is $z = \frac{1}{2}\pi$, $y = 0$, which gives $x = 0$ as a solution to the original equation.

Also solved by Owen Kardatyke, Anderson College, Anderson, Indiana; Norman Nielson, Pamona College, Claremont, California; Roger Richards, Westminster College, New Wilmington, Pennsylvania.

150. *Proposed by the Editor.*

The last proposition of the ninth book of Euclid's *Elements* states that "If $2^n - 1$ is a prime number, then $2^{n-1}(2^n - 1)$ is a perfect number." Show that a necessary condition for $2^n - 1$ to be prime is that n be prime.

Solution by Owen Kardatyke, Anderson College, Anderson, Indiana.

The problem is to show that:

If $2^n - 1$ is prime, then n is prime.

Assume n is not prime, then write $n = ab$. $1 < a < n$, $1 < b < n$. Thus $2^n - 1 = 2^{ab} - 1 = (2^a)^b - 1$

$$= (2^a - 1)(2^{a(b-1)} + 2^{a(b-2)} + \cdots + 2^a + 1).$$

Contradicting $2^n - 1$ being prime, therefore n is prime.

Also solved by Bob Bailey, William Jewell College, Liberty, Missouri; Dee Fuller, Davidson College, Davidson, North Carolina; Phil Huneke, Pamona College, Claremont, California; Joseph Neisendorfer, Tilden Technical High School, Chicago, Illinois; Perry Smith, Albion College, Albion, Michigan.

Editor's Note: The Problem Corner is in need of good problems to propose.

Those skilled in mathematical analysis know that its object is not simply to calculate numbers, but that it is also employed to find the relations between magnitudes which cannot be expressed in numbers and between functions whose law is not capable of algebraic expression.

—AUGUSTIN COURNOT

The Mathematical Scrapbook

EDITED BY J. M. SACHS

Now more than ever there exists the danger of frustration and disillusionment unless students and teachers try to look beyond mathematical formalism and manipulation and to grasp the real essence of mathematics.

—R. COURANT

=△=

Given any four numbers u, v, w, and x, can you find a y such that

$$\frac{u+y}{v+y} = \frac{w+y}{x+y}?$$

Are there any exceptions? I think you will find that the exceptions are more interesting that the solution for y.

=△=

. . .the development of the quantum theory has several times suggested that mathematicians may be called upon to devise a geometry in which there are no points. It may (or it may not) turn out that the process of infinite subdivision of chunks of space which is supposed to yield the concept of a point is physically impossible. If we come to such a geometry without points and without infinite re-subdivision, it is likely that we will continue to use analytic methods and coordinate systems. But our coordinate systems will no longer be one-to-one correspondences between points and sets of numbers. They will be relations of some other type between sets of numbers and the entities which they describe.

—O. VEBLEN

=△=

Suppose we are working in a notation base m, where of course, m is an integer. If N is also an integer will you always obtain the same remainder when N is divided by $(m-1)$ and when the sum of the digits in N is divided by $(m-1)$?

Example: $m = 7$; $\frac{163}{6} = 21 + \frac{4}{6}$; remainder is 4.

$1+6+3 = 13$

$\frac{13}{6} = 1 + \frac{4}{6}$; remainder is 4.

Can you make a general proof of this? Are there any restrictions on the integer m or on the integer N? What would this mean for $m = 2$? How is this related to casting out nines?

=△=

It is a truth very certain that, when it is not in our power to determine what is true, we ought to follow what is most probable.

—R. DESCARTES

The great Rene Descartes regards as true the statement from the quotation above, ". . . when it is not in our power to determine what is true, we ought to follow what is most probable." This can be the basis of a rewarding argument about why this statement is regarded as true and how to turn this statement upon itself.

=△=

In discussing the Russell type paradox of the kind illustrated by considering the possible truth of three statements like the following:

1. Grass is pink;
2. The moon is green;
3. All three of these statements are false;

a group of mathematics students raised some interesting questions. Suppose we have a finite collection of statements with at least one contradiction. Can we eliminate the contradiction by deleting one or more of the statements? That is, can we assign the value T to all of the statements in a proper subset of the original set of statements? Can we do this in more than one way? The students soon discovered that they could deal with some of these questions in examples by using the concept of logical equivalence.

Example:

p	Grass is pink.
q	The moon is green.
$p \to q$	If grass is pink then the moon is green.
$\sim q \to p$	If the moon is not green then grass is pink.

Clearly deleting the last statement would allow us to assign the value T to p, q, and $p \to q$ without contradiction. Could we gain our end by deleting p? Can you convince yourself that if we allow p to be false, we can have the other three statements as true without contradiction? Can we do this by deleting q? If p is T and $p \to q$ is T

then can q be F? Can we delete $p \rightarrow q$? If p and q are T what about $p \rightarrow q$? Try to construct other examples and see where they lead.

=Δ=

. . . For, whereas all previous discoveries of inadequate undergirdings seemed capable of being remedied, a remarkable result due to Kurt Godel in 1931 established the existence of undecidable propositions in any mathematical system of any depth. That is, Godel established that there were propositions which could be neither proved nor disproved within the system. . . . Thus it now seems clear that it is impossible to develop mathematics into a complete, consistent system. In mathematics we cannot heed the counsel of the Biblical parable and build our house upon the solid rock. Whether we like it or not, there is some sand in the foundations of mathematics.

—I. Niven

=Δ=

Edmund Halley, 1658-1744, was a scholar whose writing spanned a spectrum from the classics to mortality tables. It is interesting to note that one of his mathematical papers turned the attention of Joseph Louis Lagrange from the classics to mathematics. One can speculate on whether this was the necessary spark to bring out the mathematical genius of Lagrange or whether that great man would have turned to mathematics without that spark or perhaps any spark.

=Δ=

Triangular numbers in sequence can be formed by starting with 1 and adding successive elements of the arithmetic progression, 2, 3, 4, 5, ... Thus we obtain 1, 3, 6, 10, 15, ... Square numbers can be obtained by starting with 1 and adding successive elements of the arithmetic progression 3, 5, 7, 9, ... Thus we obtain 1, 4, 9, 16, 25, ... Pentagonal numbers can be obtained by starting with 1 and adding successive elements of the arithmetic progression 4, 7, 10, 13, ... Thus we obtain 1, 5, 12, 22, 35, ... Can you state the rule of formation for hexagonal numbers? How about a generalization for polygonal numbers, say n-gonal numbers?

To take this a step further, we see that any square number is the sum of two triangular numbers; any pentagonal number is the sum of a square number and a triangular number. Is it true that any n-gonal number is the sum of an $(n-1)$-gonal number and a triangular number? If so can we express a pentagonal as the sum of three triangulars and a hexagonal as the sum of four triangulars?

Are the triangulars all distinct? Can you generalize for an n-gonal number?

A pyramidal number is a pyramid of polygonal numbers all of the same kind. For example the pyramid made up of the triangular numbers 1, 3, 6, 10 yields the pyramidal number 20. Can the summing idea suggested for the polygonal numbers be extended to pyramidal numbers?

=Δ=

It is impossible not to feel stirred at the thought of the emotions of men at certain historic moments of adventure and discovery—Columbus when he first saw the Western shore, Pizarro when he stared at the Pacific Ocean, Franklin when the electric spark came from the string of his kite, Galileo when he first turned his telescope to the heavens. Such moments are also granted to students in the abstract regions of thought, and high among them must be placed the morning when Descartes lay in bed and invented the method of co-ordinate geometry.

—A. N. WHITEHEAD

=Δ=

One of the most difficult distinctions for non-mathematicians to make is between the statements, "*B* follows *A*", and "*A* causes *B*." It is possible of course that *B* may follow *A* in a cause and effect relationship but this is not necessary. The possibility which cannot be ignored is that *B* and *A* are concomitants, perhaps both effects from a common cause. As an absurd example let us consider the following statistical evidence (made up out of whole cloth). A careful statistical study made over several years shows that the total number of dollars per week spent on ice cream in a certain midwest city is relatively constant during the winter months. In the spring this total rises week by week towards a peak reached in late summer. Shortly after this rise begins, the total number of dollars per week spent on sunburn lotions—almost nil during the winter—begins a steady rise to a peak in late summer. Conclusion: the more ice cream consumed, the more sunburn lotion used. Does this mean that the consumption of ice cream causes sunburn?

=Δ=

A thorough advocate in a just cause, a penetrating mathematician facing the starry heavens, both alike bear the semblance of divinity.

—GOETHE

Directions for Papers to be Presented at the Fourteenth Biennial Kappa Mu Epsilon Convention

NORMAL, ILLINOIS
April 8-10, 1963

As has been the case at previous conventions, a significant feature of this convention will be the presentation of papers by student members of KME. To ensure high quality papers, it would be wise for chapter advisors and members to start plans now for papers to be presented at the convention. The student should investigate areas in mathematics of greatest interest to him and select a topic suitable for a paper. The practice of scheduling these student papers in programs of the local chapter yields double value—it will enrich the programs of the local chapter and it will give the student experience in presenting a paper preparatory to his presentation before the national convention.

Who may submit papers: Any member may submit a paper for use on the convention program. Papers may be submitted by graduates and undergraduates; however, undergraduates will not compete against graduates. Awards will be granted for the best papers presented by undergraduates. Special awards may be given for the best papers presented by graduates, if a sufficient number are presented.

Subject: The material should be within the scope of the understanding of undergraduates, preferably the undergraduate who has completed differential and integral calculus. The Selection Committee will naturally favor papers that are within this limitation and which can be presented with reasonable completeness within the time limit prescribed.

Time Limit: The usual time limit is twenty minutes but this may be changed on recommendation of the Selection Committee.

Paper: The paper to be presented or a complete outline of it must be submitted to the Selection Committee accompanied by a description of charts, models, or other visual aids that are to be used in presenting the paper. A carbon copy of the complete paper may be submitted if desired. Each paper must indicate that the author is a member of KME and whether he is a graduate or an undergraduate student.

Date and Place Due: The papers must be submitted before February 1, 1963, to the office of the National Vice-President.

Selection: The Selection Committee will choose about eight papers for presentation at the convention. All other papers will be listed by title on the convention program.

Prizes:

1. The author of each paper presented will be given a two-year extension of his subscription of *The Pentagon.*
2. Authors of the two or three best papers presented by undergraduates, according to the judgment of a committee composed of faculty and students will be awarded copies of the James' *Mathematical Dictionary,* suitably inscribed.
3. If a sufficient number of papers submitted by graduate students are selected for presentation, then one or more similar prizes will be awarded for the best paper or papers from this group.

Harold Tinnappel
National Vice-President
Bowling Green State University
Bowling Green, Ohio

A peculiar beauty reigns in the realm of mathematics, a beauty which resembles not so much the beauty of art as the beauty of nature and which affects the reflective mind, which has acquired an appreciation of it, very much like the latter.

—E. E. Kummer

The Book Shelf

EDITED BY H. E. TINNAPPEL

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Harold E. Tinnappel, Bowling Green State University, Bowling Green, Ohio.

College Algebra, Fourth Edition, Paul K. Reis and Fred W. Sparks, McGraw-Hill Book Company, Inc., (330 West 42nd Street) New York, 1961, 438 pages, $6.50.

According to the preface, this text is designed for the "usual three-semester-hour course" in college algebra. The first four chapters contain a comprehensive review of high school algebra, with an abundance of supplementary exercises. An informal style is used and the authors have attempted to aid the student in developing a systematic method of problem solving by providing illustrative examples and by giving detailed instructions for attacking different types of problems. Marginal notes indicate these solution steps as well as defined terms and laws. All marginal notes and other aids are printed in red. The glossary gives a complete list of definitions.

The traditional method of presenting mathematics is used exclusively. The concept of sets is omitted entirely, even though in the glossary, function is defined as "a set of ordered pairs (x,y) so related that to each x there corresponds at least one y." However, in the chapter on "Functions and Graphs," the definition reads, "One variable is function of a second if at least one value of the first is determined whenever a value is assigned to the second."

The treatment of inequalities comes late in the book; it is brief and inadequate.

Additional chapters cover ratio, proportion, and variation, complex numbers, higher degree equations, logarithms, progressions, mathematical induction, binomial theorem, compound interest and annuities, permutations and combinations, probability, determinants, and partial fractions.

The organization of the book and the great number of topics covered allow great flexibility in adapting the material to the needs of the class being taught. Numerous "drill" problems are provided in each set of exercises; answers to three problems out of each four are given. Some problems similar to those needed in analytic geometry and calculus have been included.

The book is attractive; the graphs are especially pleasing because of the introduction of the red color; and the material is well organized. It should be a useful textbook in the classroom in which conventional techniques are used.

—Elizabeth T. Wooldridge
Wayne State Teachers College

Elementary Algebra for College Students, Irving Drooyan and William Wooton, John Wiley & Sons, Inc., (440 Fourth Avenue) New York 16, 1961, 272 pp., $4.95.

Under the present educational system in this country, many students begin their serious study of mathematics at the college level. This is indeed unfortunate. However, due to the efforts of Irving Drooyan, William Wooton and many others like them, texts for these beginning students are available.

I have just finished reading such a text. *Elementary Algebra for College Students* was written by Drooyan and Wooton of Los Angeles Pierce College. In this text, algebra is presented as generalized arithmetic. This is especially appropriate for the beginner since it affords some feeling of familiarity with the operations performed in the algebra. Frequently, a new topic is begun by considering well-chosen arithmetic examples. The sequence of topics is traditional, beginning with a discussion of the natural numbers. The traditional sequence ends with a chapter discussing the solution of quadratic equations by formula. Added to these nine chapters is a tenth which is especially good. It contains information about the structure of the number system of mathematics which provokes enough thought to challenge the beginner to continue studying mathematics.

The most obvious omission of the text is the absence of formal proof. The authors are aware of this omission but feel that an intuitive approach is better for the type of student who would use such a text. In this writer's experience, they are quite correct in this assumption. One might well question the absence of the distributive law from discussion until the fourth chapter. It could be used advantageously in an earlier discussion of signed numbers.

In summary, perhaps the greatest compliment that can be paid a textbook is that it is self-teaching. *Elementary Algebra for College Students* contains an abundant number of sample problems of varied difficulty. Coupled with the sample problems are precise topical discussions. The text merits the above-mentioned compliment.

—Daley Walker
Central Methodist College

Elementary Analysis, H. C. Trimble and F. W. Lott, Jr., Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1960, 621 pp., $6.95.

This text for college freshmen contains the material for a four- or five-hour course with just the right touch of the integration of algebra, trigonometry and coordinate geometry. The authors seem aware of the need in most colleges and universities, due a good bit to irregular students, to keep the three disciplines somewhat separated in the text. However, when elementary functions are covered, the introduction to the circular functions is made along with the algebraic function. The number line and some coordinate geometry as a tool is introduced early in the algebra. The logarithmic and exponential functions are covered thoroughly at the end of the algebra and at the beginning of the trigonometry.

The use of sets to distinguish between different kinds of numbers used for analysis is an excellent feature of the book. Also, the time spent on the Basic Properties (axioms) of the real numbers is good. The set motation is used to good advantage but is not overdone. Some teachers may like more use of Universe, Domain, and Range. The book is well written in understandable words and is not verbose. It is reasonably rigorous and free from loose statements and should be appealing to both the student and the teacher.

There are no tables in the book, which the authors feel is not a bad feature because all students should have a good handbook and learn to use it. Answers are not included. Perhaps a teacher's manual is available.

The text seems to meet the demands of the modern approach to the teaching of mathematics, with emphasis on concepts as well as sufficient drill for learning the mechanics of manipulations. A course from the book should certainly meet its main objective of getting students ready for calculus as well as being most satisfactory as a terminal course.

The preface is well written and seems to be an accurate description of the book and its objectives.

—Walter C. Butler
Colorado State University

Introduction to Geometry, H. S. M. Coxeter, F. R. S., John Wiley & Sons Inc., (440 Fourth Avenue) New York 16, 1961, 443 pp., $9.95.

That a new awakening has occured in the field of geometry is beyond dispute. In this renascence, it is particularly encouraging to teachers of college geometry to find the recent book, *Introduction to Geometry*, by the internationally known scholar and geometer, H. S. M. Coxeter, professor of mathematics at the University of Toronto. The comprehensive nature of this excellent work makes it (or portions of it) desirable for many different classes of students. Actually, college mathematics departments will find it indispensable for courses such as: Advanced Topics in Euclidean Geometry; Foundations in Geometry; Combinatorial Topology; Differential Geometry.

While it is termed an "Introduction", the reader will be pleasantly surprised to find a complete, rigorous treatment and a detailed emphasis on many kinds of geometry as well as many unusual and varied applications.

Parts I and II (the first eleven chapters) treat of topics frequently referred to as "advanced Euclidean geometry". Students who have had the usual elementary training in Euclidean geometry and some knowledge of coordinate geometry will find this section which is devoted to symmetries, isometries, glide-reflections and inversions highly informative as well as interesting. Secondary school geometry teachers could also draw much profit from a mastery of this section.

In Part III, the author treats the two distinct geometries which may be extracted from Euclid's postulates. These two geometries are; *Absolute Geometry*, which rests on the first four postulates of Euclid (completely ignoring the fifth), and *Affine Geometry* in which the unique parallel plays a leading role while the third and fourth postulates of Euclid are passed over. Part III also includes a good exposition of both *Projective Geometry* and *Hyperbolic Geometry*. This section of the book which covers four chapters is highly recommended for a course in *Foundations*.

The remaining portions of the book are devoted to topics which might well be used for separate courses, namely *Differential Geometry* and *Topology*.

To the student of modern geometry who is interested in the hierarchy of geometries as well as the classification of these various geometries from the viewpoint of invariants by use of transformation groups this book will be an invaluable reference. The spirit of Klein's

Erlanger Program is manifest throughout (since the unifying concept underlying the whole work is symmetry) and it is the contention of the author that it is only since that famous address that "affine geometry has been recognized as a self-contained discipline." (p. 191)

The book abounds in excellent illustrations and drawings. Copious references appear throughout the book. Many well chosen exercises that are both highly interesting and unusual are to be found in each chapter. Frequent, appropriate and well documented quotations add much to the general appeal of the entire work.

It is the author's stated purpose "to revitalize this sadly neglected subject"—geometry. In the opinion of the reviewer, he has administered a most potent "shot in the arm" to the present day mathematician. We can only hope that it "takes" and that its effects will be fully felt.

—SISTER HELEN SULLIVAN, O.S.B.
Mount St. Scholastica College

A Second Course in Number Theory, Harvey Cohn, John Wiley & Sons, Inc., (440 Fourth Ave.) New York 16, 1962, 276 pp., $8.00.

As the title indicates, this book is intended for a course to follow a sound first course in elementary number theory. With such a first course and some knowledge of algebra and analysis, the student or reader should have little difficulty in proceeding from the 18th-century backgrounds of quadratic reciprocity on to the twentieth century frontiers of modern mathematics.

Professor Cohn offers what is probably more than enough for a second-semester course, but he aims for the student on completing the course to have acquired "an appreciation for the historical origins of linear algebra, for the zeta-function tradition, for ideal class structure and for genus theory."

With its central idea, the role of number theory as a "foundation head of algebra and analysis," the book is a most welcome addition to the library of any student or teacher of mathematics eager for further insight into the inherent unity of mathematics.

The exposition contains five parts: 1) An introductory survey proposing the main ideas to be developed; 2) Background material consisting of a review of elementary number theory, some algebraic number theory, group theory, and relations among them; 3) Ideal theory in quadratic fields, of which two chapters are devoted to the problem of unique factorization before taking up norms, ideal classes, and class structure in quadratic fields; 4) Applications of ideal theory introducing analysis to number theory; and 5) Concluding survey giving the new directions number theory is taking.

Of special value to the interested reader is the section, Bibliography and Comments which Professor Cohn has divided into "Some Classics Prior to 1900" and "Some Recent Books (after 1900)" which are followed by special references by chapter. In addition, the book contains an appendix of three tables: 1) Minimum prime divisors of numbers not divisible by 2, 3, or 5 from 1 to 18,000; 2) Power residues for primes less than 100; and 3) Class structures of quadratic field of $\sqrt{m}$ for m less than 100 for both imaginary and real fields.

With over 200 exercises, many of them numerical, the book offers ample material for a second course in number theory. Its best feature is the emphasis on the close relation of both analysis and algebra to number theory as brought out in the 19th-century advances in the field.

The exposition is clear and precise, the type readable, the format attractive. This is a book which number theory enthusiasts will welcome.

—Sister Malachy Kennedy, O.S.B.
Mount St. Scholastica College

Elementary Statistics, Paul G. Hoel, John Wiley and Sons, Inc., (440 Fourth Avenue) New York 16, 1960, vii + 261 pp., $5.50.

This is another of the many texts written for a course in "Introduction to Statistics" or "Principles of Statistics." The author states that the book is designed as a one-semester course for students with only a limited background in mathematics. A student who *knew* second year high-school algebra would have no difficulty with the mathematical level of the text.

The first nine chapters cover the usual material included in books of this type. The topics are: descriptive statistics, probability, theoretical frequency distributions, sampling, estimation, testing hypothesis, correlation, and regression. Special topics included in the last four chapters for a more extensive course are the Chi-Square distribution, nonparametric tests, analysis of variance, time series and index numbers. The problems are ample in number, well selected, and appropriate to the level of the text. Appendices necessary to the solution of the problems are included as well as answers to the odd-numbered problems.

The reviewer was impressed by the fact that descriptive statistics was adequately covered in just twenty-four pages. The author was interested in the student studying statistical inference as soon as possible. The chapter on probability uses the equally likely and relative frequency definitions of probability without using set notation. The reviewer felt that the chapter on probability was adequate to give the student a "taste" of probability, without swamping this level of student.

Professor Hoel's approach on using the normal approximation for binomial probabilities is excellent. He gives the student a feel for the "why" of this approximation. The reviewer was also impressed by Professor Hoel's strong statement on the necessity of a random sample in making probability statements. Many researchers who have only one or two courses in statistics tend to disregard this important fact. The author's approach to interval estimation and tests of hypothesis which put the emphasis on principles of statistical inference seem appropriate for a text of this nature.

This text is well-written, concise, and the examples are well chosen from various areas of applications of statistical analysis. A teacher who is selecting a text for a first course in elementary statistics should consider this text among its many competitors.

—Wilbur J. Waggoner
Central Michigan University

Analysing Qualitative Data, A. E. Maxwell, (Methuen's Monographs on Applied Probability and Statistics) John Wiley & Sons, Inc., (440 Fourth Avenue) New York 16, 1961, 163 pp., $3.00.

This monograph, one of a series on applied probability and statistics devoted to recent developments in this area, is primarily concerned with the up-to-date techniques of Chi-Square tests of association and goodness of fit. Some consideration, in addition, is given to rank correlation techniques, miscellaneous tests of significance, classification procedures based on Baye's Theorem and decision theory, and item analysis and the construction of attitude scales.

Years ago, Granville's *Differential and Integral Calculus* was considered the Bible of Calculus. This monograph might be considered a Bible of Chi-Square Tests. It is unique in its collection of useful techniques that the author has gathered from recent statistical publications in both book and journal form and combined in a concise, carefully written, and well organized piece of work that reads smoothly from start to finish. It is made interesting by practical problems taken from medical and sociological studies that illustrate each technique, problems that are worked out in detail.

References, some of which include further reading suggestions, are listed at the end of each chapter.

The book is one that should be available to all students of statistics and at the finger tips of everyone working with the analysis of qualitative data.

—Nura D. Turner
State University of New York
College of Albany

Analogue and Digital Computers, M. G. Say, Philosophical Library, Inc., (15 East 40th Street) New York 16, 1960, viii + 308, $15.00.

The motivation of this book is best described in its preface: "It is easy to forget that the widespread adoption of analogue and digital computers has occurred in a single decade. Such a rapid evolution of a new group of devices has, naturally, been accompanied by a stream of publications marking each advance in technology or application. Almost without exception, however, such papers and articles assume a specialist knowledge on the part of the reader. We were glad, therefore, to accept the invitation of the publishers to act as advisory editors of a book *aimed at presenting the basic material on the design and application of both analogue and digital computing systems.*"

This volume is composed of ten chapters, beginning with a chapter serving as an introduction to computers and terminating with programming methodology. Each chapter is an independent entity written by someone in England who is quite familiar with the subject and who is well known to the mathematics and computing fraternities in that country. The chapters are not organized in the same manner, but this lack of style and material does not detract from continuity for the reader. The book may be divided into three major sections. The first section describes the analogue computers, the second section describes digital computers, and the third serves an an introduction to computer programming.

In discussing each type of computer the authors treat computer number systems, primarily binary and binary-coded-decimal; computer circuitry, with emphasis on transistor and diode switching circuits; and the fundamental of logical design. Also described under digital computers are the arithmetic element, the memory element, input-output devices, and the control element. The basic functions of the digital computer are covered, as are the characteristics of such components as magnetic-cores, drums and tape, punched cards and paper tape, and other components common to the computer industry. Each section of the computer is described from both a systems and components viewpoint.

This book will give a reader with moderate maturity in mathematics and electronic engineering but with no prior knowledge of computers a very intelligible idea of the basis, nature, and uses of the analogue and digital computers. The topics treated have been wisely selected and their treatment is orderly and clear. Those requiring a more detailed insight of particular aspects will find numerous references in the text. This is a book that should be in the hands of everyone who intends to do any serious work on a computer as well as the mathematician and physicist to round out effectively their computer "know-how."

—Alphonso J. Di Pietro
Eastern Illinois University

The Impact of the New Physics, Frank Hinman, Philosophical Library, Inc., (15 East 40th Street) New York 16, New York, 1961, 174 pp., $4.50.

Frank Hinman's *The Impact of the New Physics* is readily seen to be a work intended for the layman. It professes to tell what mankind has learned about the universe surrounding him in his relentless struggle towards truth. The book is fashioned in a logical order, starting with the beginnings of the macrocosm and leading up to the development of man's most precious gift, his mind. The book ends in the sixth chapter on a note of optimism: "The training and exercise of that human sensation called consciousness may transform the lives of men; this scientific revolution will open the road widely to a spiritual revolution by giving a clear understanding of the human mind with its progressive potentialities."

The first three chapters of the book are devoted to explaining inorganic evolution; it is here that I find my only criticism of the book, careless usage. As examples, I may mention Dr. Hinman's statement "A proton is 2,000 times *bigger* than an electron, . . ." (p. 15), and the statement "Of the salts, sodium and chlorine are in highest percentage, . . ." (p. 72), and the statement "The force of gravity and atmospheric pressure are less on top a high mountain than at sea level and so is temperature but these variations are not enough to change the state of elements, some are gaseous, only *one*, is liquid. . ." (p. 27), or finally the statement "Therefore, particles at high speed travel as waves." (p. 12). With regard to spelling, I find the following: "Edington," "Farraday," and "chlorophyl." All three of these may be acceptable, but they are certainly not prevalent.

The remaining three chapters clearly show that Dr. Hinman can rest on much firmer ground among the biological sciences. Chapter four gives a fine discussion of the living processes in general in all of the Phyla, and this discussion includes such topics as the role of enzymes and the nervous processes. Chapter five presents an enlightening resume of language, its defects and disturbances, and its significance. The concluding chapter explores for the reader the potentialities of man's brain by briefly discussing such topics as personality, subconscious activities, and belief in the supernatural.

In general, the style of the book was consistent, although in places the reading was irregular when too many ideas were crowded into a single sentence. Dr. Hinman is a staff member of the California School of Medicine.

—Joseph B. Dence
Bowling Green State University

Radioactive Substances, Marie Curie, Philosophical Library Inc., (15 East 40th Street) New York 16, 1961, 94 pp., $2.75.

Radioactive Substances is a translation from the French of the classical thesis presented to the Faculty of Science in Paris by the distinguished Marie Curie, the only winner of two Nobel prizes.

The ninety-four pages of research contained in this volume cover a period of over four years of work, extending from the investigations of Becquerel rays (the emissions of thorium and uranium and their salts) to the variations of activity of radium salts after solution and after heating. When analyses of uraninite (pitchblende) made it apparent that small amounts of a very active substance were present, the Curies in conjunction with M. Bémont succeeded in isolating from one ton of ore concentrate 0.2 g. of radium chloride plus a small amount of the element polonium. Chapter two details the complex process that was used in fractionating the chlorides of polonium and radium; a subsequent determination of the atomic weight (225) of radium convinced the Curies that radium was a new element.

In chapter three the various properties of the new radioactive substances were investigated. The radiation of radium was found to consist of *emanation* (now known to be radon) plus numerous rays: alpha-rays, which are slightly deflected in a magnetic field; beta-rays, which are deflected to a greater extent in the opposite direction; gamma rays, similar to the Rontgen rays and unaffected in a magnetic field. Other properties discussed are the penetrating power of rays from radium through different layers and at different distances, the law of absorption of polonium rays and of the absorbable rays of radium, the coloration of glass in the vicinity of radium, the spontaneous evolution of heat (estimated by M. Curie to be approximately 100 calories per gram-atom of radium), and the production of fluorescence in zinc sulfide and other compounds by radium rays.

The final chapter records the Curies' work on induced radioactivity and their investigation of radiation within a confined space. The chapter concludes with a discussion of the variations of activity of radium salts on solution and on heating, and with the theory of radioactivity proposed by M. Curie and Debierne.

Certain materials in the book presages events which were to be realized in the near future: namely, the statement that the mass of moving particles increases with their velocity, and the use of radium in medical treatments.

I find the book quite informative and interesting, even though it is not written in a smoothly readable style. The work should be a part of every physicist's library.

—JOSEPH B. DENCE
Bowling Green State University

Dictionary of Mechanical Engineering, Alfred Del Vecchio, Philosophical Library, Inc., (15 East 40th Street) New York 16, 1961, $6.00.

This modern, comprehensive dictionary brings to its readers over 2400 prime definitions in the fields of architecture, automatic controls, engineering mechanics, fuels and combustion, and power plants along with related definitions in the fields of basic electricity, heat treatment of metals, basic mathematics, and welding.

The choice of words to be entered and defined gives good coverage to the fields concerned and all the definitions are written in an easy to understand manner. Considerable effort has been made to give definite meaning to numerous scientific terms which do not have precise definitions. In the preface the author states "It is only natural that an inquisitive and active mind seek out precise definitions to scientific terms. Unfortunately, this is not always possible. Many scientific terms have three or more dimensions and it is difficult to depict in the mind's eye beyond the third dimension. Hence, the definitions in this category become intangible and must be presented as properties. For instance the term MOMENT OF INERTIA as used archicture or engineering mechanics has units of inches to the fourth power (in.)^4. This is defined as MEASURE OF THE RIGIDITY OF A STRUCTURAL MEMBER. There is no tangible definition here which can be depicted. Hence we must study the term as a property and content ourselves with the definition that it is a measure of something".

In addition to many terms which do not have precise definitions, the writer has covered others which are troublesome to science and perhaps should never have been allowed to develop. Examples of these include BOILER HORSEPOWER, which has nothing to do with horsepower; TONS OF REFRIGERATION, which has nothing to do with weight; and SPECIFIC GRAVITY, which has nothing to do with gravity.

The book should be a welcome reference in any mechanical engineering or industrial-technical library as it would be useful for the practicing engineer, engineering students and teachers, as well as skilled workers in the fields covered.

—Robert W. Innis
Bowling Green State University

It has come to pass, I know not how, that Mathematics and Logic, which ought to be but the handmaids of Physic, nevertheless presume on the strength of the certainty which they possess to exercise dominion over it.

—Francis Bacon

Installation of New Chapters

EDITED BY SISTER HELEN SULLIVAN

NEW YORK DELTA CHAPTER

Utica College of Syracuse University, Utica, N.Y.

On April 13, 1961, the Delta chapter of Kappa Mu Epsilon was installed at Utica College of Syracuse University in Utica, New York. Formal installation took place at a dinner at Garramone's restaurant. Mr. Frank Hawthorne, supervisor of mathematics education for the State Education Department, represented the national organization as installation officer. In the afternoon Mr. Hawthorne spoke on "Projectile Geometry" at an assembly in the college lounge.

Installed as officers were: Andrew J Kennedy, Jr., President, James A. Sapanara, Vice-President, Donald F. Fama, Secretary and Frederick D. Schmandt, Treasurer. The new members included David L. Fama, Sally Graudons, Grace Lovecchio, Raymond Mellen, Theresa Rugari, John D. Vadney, William J. Kelly, William L. Warmuth, Ronald Ferris, Dennis Maynard, and Gerald VanHatten.

Mr. Daniel Goss and Mr. Thomas J. Burke, both assistant professors of mathematics at Utica College, were installed as Advisor and Corresponding Secretary, respectively.

NEW YORK EPSILON CHAPTER

Ladycliffe College, Highland Falls, New York

New York Epsilon Chapter was installed on Tuesday, February 27, 1962, at Ladycliffe College in Highland Falls. Professor Loyal F. Olmann of Hofstra College, former national Treasurer of Kappa Mu Epsilon, was the installing officer.

The installation took place at 4:15 p.m. in the college auditorium. Charter members are the following: Nancy Alderdice, Marion Beck, Patricia Boyne, Laura H. Connolly, Margaret Gibson, Dorothy Gilman, Gail Cluff Mee, Sheila Murray, Maureen O'Halloran, Jeanne O'Rourke, Patricia Stephens, Rosalie Sucato, Frances Vassalo, Mary Louise Wilson, Sister Rose Marian, O.S.F., and Sister Dennis, O.S.F. Officers installed at the ceremonies were: Nancy Alderdice, President; Marion Beck, Recording Secretary; Sister Rose Marian, O.S.F., Faculty Moderator; Sister Dennis, O.S.F., Corresponding

Secretary. Following the installation ceremonies, a banquet was served to the Charter Members and their Guests.

Ladycliffe College is a four year Liberal Arts College for Women under the direction of the Franciscan Sisters. The College with its thirty-two acre campus is located on the West Bank of the Hudson River, one mile south of the United States Military Academy at West Point.

The object of pure Physic is the unfolding of the laws of the intelligible world; the object of pure Mathematic that of unfolding the laws of human intelligence.

—J. J. SYLVESTER

The great notion of Group, . . .though it had barely merged into consciousness a hundred years ago, has meanwhile become a concept of fundamental importance and prodigious fertility, not only affording the basis of an imposing doctrine—the Theory of Groups—but therewith serving also as a bond of union, a kind of connective tissue, or rather as an immense cerebro-spinal system, uniting together a large number of widely dissimilar doctrines as organs of a single body.

—C. J. KEYSER

Kappa Mu Epsilon News

EDITED BY FRANK C. GENTRY, HISTORIAN

Alabama Beta, Florence State College, Florence.

Miss Orpha Ann Culmer retired at the close of the 1961-62 school year. She had been Chairman of the Department of Mathematics since 1920 and Corresponding Secretary of Alabama Beta since its founding in 1935. She was National Historian of Kappa Mu Epsilon from 1939 to 1943. The Historian's files show a great deal of evidence that she spent many hours keeping the records of the various chapters up to date. At the annual commencement program in June, Miss Culmer was presented a citation for meritorious service by President E. B. Norton of Florence State College and a merit service pin by the graduating class. Hers will be the first name entered on the Roll of Honor Plaque presented to the college by the senior class.

Dr. Burton Jones, of the University of Colorado, was a guest speaker on our campus in October. Mrs. Jean T. Parker is our new corresponding secretary.

California Alpha, Pomona College, Claremont.

We initiated 19 new members in November. We had a student demonstration of the Milliken Laboratory Planetarium as one of our programs this year.

California Gamma, California State Polytechnic College, San Louis Obispo.

Mr. Victor Azgapetian, Systems Engineer of Servomechanisms, Inc., was guest speaker at a banquet honoring 11 initiates in November. We expect to have Dr. Saunders MacLane, of the University of Chicago, and Dr. Tom M. Apostle, of California Institute of Technology, as guest speakers this spring.

Colorado Alpha, Colorado State University, Fort Collins.

We plan to revise our initiation procedure this year by substituting a banquet for the traditional chilli supper. We expect to initiate 53 new members bringing our total membership to 454.

Illinois Beta, Eastern Illinois University, Charleston.

We initiate new members only once a year in May. Last year we had 32 qualify for membership.

Illinois Gamma, Chicago Teachers College, Chicago.

We initiated 16 new members at a luncheon meeting in March.

Illinois Delta, College of St. Francis, Joliet.

We are devoting all of our meetings this year to an intensive study of number systems.

Indiana Gamma, Anderson College, Anderson.

We have 9 new members this year. Professor Saunders MacLane representing the Mathematical Association of America visited us this year. His lectures were entitled; "Discovery", "What is Topology" and "The p-adic Numbers". We honor the senior, junior and sophomore students having the highest grade average in mathematics by naming them president, vice-president and secretary respectively of our chapter.

Kansas Alpha, Kansas State College of Pittsburg, Pittsburg.

The Robert Miller Mendenhall Memorial Award, a Kappa Mu Epsilon key, is presented each spring to each of those seniors having the highest grade rating in mathematics for their four years of college work. Last spring the awards went to Kenneth Feuerborn and Bing Wong. This year the awards were received by Joan Petty, Dorothy Geier and Albert Cummings. Bing Wong is a graduate assistant at the University of Illinois this year.

Kansas Beta, Kansas State Teachers College, Emporia.

Kansas Beta is very pleased and happy to have a new sponsor and head of the Mathematics Department this year. He is Dr. Marion Emerson from Missouri Alpha. We initiated 36 new members this year. We visited Kansas State University to inspect their IBM 650 and 1620 computers.

Kansas Delta, Washburn University, Topeka.

We initiated 18 new members at a dinner meeting in November.

Kansas Gamma, Mount St. Scholastica College, Atchison.

The regional conference for this area will be held on our campus the first week-end in April. We are expecting a large attendance. Professor Samuel Eilenberg, of Columbia University, was our guest lecturer last fall. We visited the Mid-West Research Laboratories and the Linda Hall Library in Kansas City in November. An undergraduate research program in mathematics is being introduced this year by our chapter.

Michigan Alpha, Albion College, Albion.

Michigan Beta, Central Michigan University, Mount Pleasant.

Our chapters will jointly sponsor the regional conference for this area in April at St. Mary's Lake, Battle Creek, Michigan. Dr. Phillip S. Jones, of the University of Michigan and President of the National Council of Teachers of Mathematics, will be our guest speaker. We expect representatives from the Illinois, Indiana, Michigan, Ohio, and Wisconsin chapters.

Mississippi Gamma, University of Southern Mississippi, Hattiesburg.

We expect to have Dr. J. D. Mancill of the University of Alabama, as guest speaker for our annual banquet in April.

Missouri Alpha, Southwest Missouri State College, Springfield.

Each year a member of the local chapter, who in the judgment of the organization and the faculty, has made the greatest contribution to the organization, is honored by having his name placed on a merit award plaque and given a Kappa Mu Epsilon key. Glen H. Bernet, Jr. was the recipient of this honor in 1961-62.

Missouri Beta, Central Missouri State College, Warrensburg.

In addition to regular monthly meetings, we had Dr. Reid Hemphill, Director of the Graduate Division of the college, as guest speaker. Seven new members were initiated last fall.

Nebraska Beta, Nebraska State Teachers College, Kearney.

We have 30 active members this year. An honorary membership in our chapter was presented to our retiring college president, Herbert L. Cushing. Our chapter is conducting help sessions for mathematics students again this year.

New Jersey Alpha, Upsala College, East Orange.

We have 17 student members including 7 initiates this year.

New Jersey Beta, Montclair State College, Montclair.

Mr. Paul Clifford discussed with us the work he did in preparing to teach on the T. V. Continental Classroom. Mrs. Howden spoke on the influence of mathematics in music. Our senior members presented a panel discussion on their experiences in practice teaching.

New Mexico Alpha, University of New Mexico, Albuquerque.

Dr. Arthur Steger, who has returned from a year's leave spent at the Royal College of Science and Technology at Glasgow, Scotland discussed his experiences as a mathematics teacher in Great Britain. Mr. Charles Lehman, of the Los Alamos Scientific Laboratories, discussed his work on the perspective projection of a cube onto a plane and allowed us to view some of his sketches through a stereoscope.

New York Alpha, Hofstra College, Hempstead.

We are celebrating our 20th year in Kappa Mu Epsilon. Eight new members were initiated at the annual banquet last April bringing our total membership to 358.

Ohio Alpha, Bowling Green State University, Bowling Green.

Among our members who are recent graduates, three are doing graduate work in mathematics at the University of Illinois and one at the University of North Carolina. We also have 5 members studying medicine or chemistry at various universities.

Ohio Gamma, Baldwin-Wallace College, Berea.

Among our unusual programs this year was one based on the Bell Telephone Laboratories record "Music from Mathematics". We also saw "The Day before Tomorrow", a film about scientists at work in the Army's Ballistic Research Laboratories at Aberdeen, Maryland. We visited Parma Research Center of Union Carbon and Carbide Corporation.

Ohio Epsilon, Marietta College, Marietta.

We initiated 16 new members this year giving us a total active membership of 26. Two of our members were initiated into Phi Beta Kappa last spring.

Oklahoma Alpha, Northeastern State College, Tahlequah.

Our chapter organizes the Mathematics Club each year, made up of students not yet eligible for membership in the chapter. The two groups hold joint meetings.

Pennsylvania Alpha, Westminster College, New Wilmington.

Thomas S. Mansell, one of our members, was awarded a Rotary Foundation Fellowship for advanced study abroad during the current school year. He is among 134 outstanding graduate students from 32 countries who have received grants this year from Rotary International.

Texas Epsilon, North Texas State University, Denton.

We expect to initiate 23 members this year. We had those students preparing to take the Putnam Competition present a program of problems they had solved.

Wisconsin Alpha, Mount Mary College, Milwaukee.

A number of our members attended a series of six lectures on modern mathematics given by Professor Don Lichtenberg, of the University of Wisconsin. We had 110 students from more than 20 high schools participate in our annual mathematics contest. Several of our programs were devoted to the presentation of mathematical films.