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Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

Areas of Simple Polygons*

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Everyone can find the area of a square or a triangle, and there is little difficulty in finding the area of various regular polygons. However, let us consider the polygon drawn in Figure one. We see that finding the area would not be difficult but somewhat cumbersome. The polygon is easily divided into four areas (see dotted line Figure one), three of which are triangles and the fourth a rectangle. The areas of these smaller polygons can be easily found and their sum would equal the area of the original polygon.

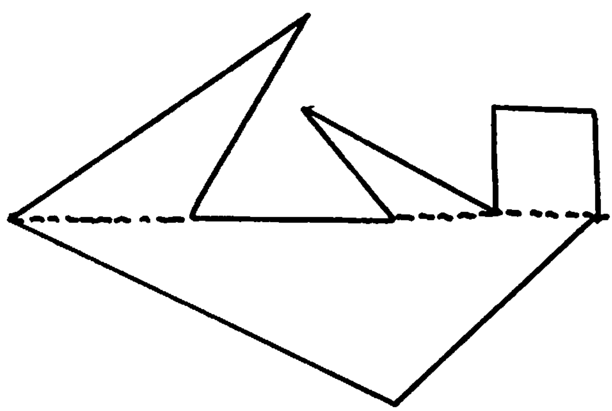

FIGURE ONE

Consider this polygon drawn on a square lattice as shown in Figure two. A square lattice is the set defined as all points of the plane whose cartesian coordinates are integers. We will now find the areas of the three small triangles and the rectangle, the sum of which is $9\frac{1}{2}$ square units. There is an easier method of doing this which I would like to introduce at this time. To all the interior lattice points

*A paper presented at a Regional Convention of KME at Washburn University, Topeka, Kansas on March 26, 1960.

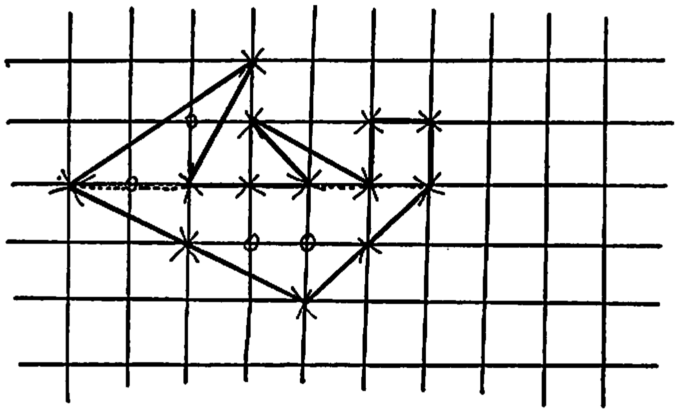

FIGURE TWO

(circled interior points) we will add the number of lattice points on the border (marked with X in the figure) divided by two, and from this sum we will subtract one and we arrive at the same answer, 9½ square units.

A theorem found in *Mathematical Snapshots* by Steinhaus states this, however the proof is not given. I wish to prove this theorem for any simple polygon.

THEOREM: The area of any simple polygon having vertices that are points of a square lattice is equal to the number of interior lattice points of the simple polygon plus one half the number of lattice points on the border minus one.

If I is the number of interior lattice points and B is the number of lattice points on the border, the theorem, stated as a formula, is:

$$A = I + \tfrac{1}{2}B - 1.$$

Before we continue, I would like to define a simple polygon. A simple polygon is a closed figure, each side being a straight line, with no sides having any other points in common with any other side of the polygon except vertices. When traversed in a counter-clockwise direction the area is always on the left and the algebraic sum of the deflection angles is 360 degrees.

First let us consider (Figure three) a rectangle whose vertices are lattice points and whose sides are parallel to the coordinate axes. Let the length of the rectangle be L units and the width W

units. Then the area would be LW square units. On the boundary there are $2L + 2W$ lattice points. There are $L - 1$ columns of interior lattice points with $W - 1$ lattice points in each column. Thus there are $(L - 1)(W - 1)$ interior lattice points. We may now verify that the theorem holds for this rectangle, since

$$\begin{aligned} A &= I + \tfrac{1}{2}B - 1 \\ &= (L-1)(W-1) + \tfrac{1}{2}(2L + 2W) - 1 \\ &= LW. \end{aligned}$$

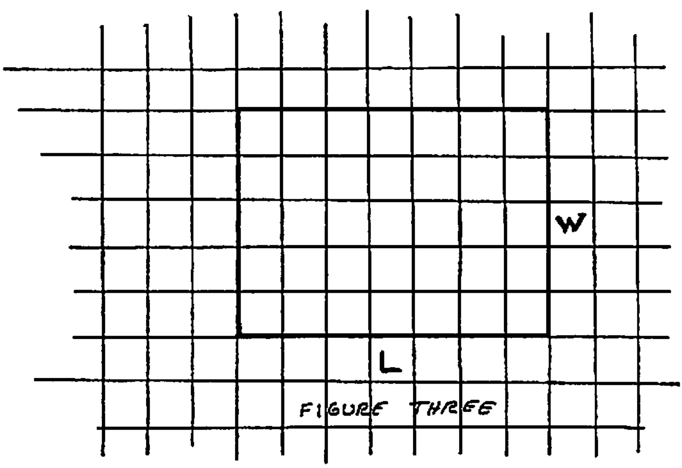

FIGURE THREE

Now let us consider (Figure four) a right triangle determined by drawing the diagonal of a rectangle. The area of the triangle would be one half the area of the rectangle, or

$$A_t = \tfrac{1}{2}(I + \tfrac{1}{2}B - 1),$$

where A_t is the area of the right triangle.

Let I_t be the number of interior lattice points, B_t the number of lattice points on the border, and d the number of lattice points contained on the diagonal, excluding the end points. Then

$$I_t = \tfrac{1}{2}(I - d) \quad \text{and} \quad B_t = \tfrac{1}{2}B + d + 1$$

which implies that

$$I = 2I_t + d \quad \text{and} \quad B = 2B_t - 2d - 2.$$

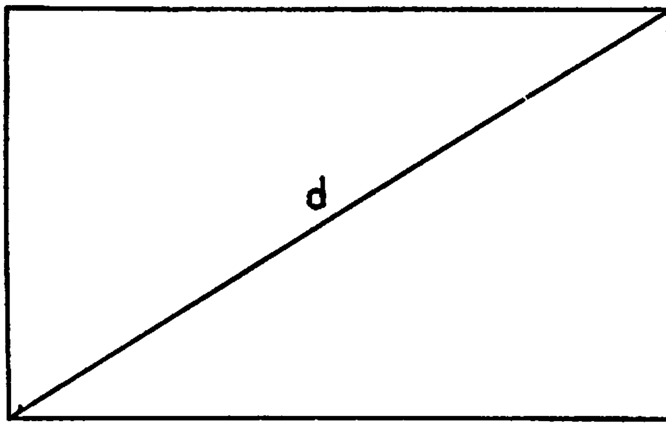

FIGURE FOUR

From this we obtain

$$\begin{aligned} A_t = \tfrac{1}{2}A &= \tfrac{1}{2}(I + \tfrac{1}{2}B - 1) \\ &= \tfrac{1}{2}[(2I_t + d) + \tfrac{1}{2}(2B_t - 2d - 2) - 1] \\ &= I_t + \tfrac{1}{2}B_t - 1. \end{aligned}$$

Thus we have verified that the theorem holds for such a right triangle. Let us now consider (Figure five) any triangle in any position. Let A be the area of the rectangle, B the number of lattice points on the border of the rectangle, and I the number of lattice points in the interior of the rectangle. First let us consider a triangle with no sides parallel to the coordinate axes. For $i = 0,1,2,3$, let T_i represent the triangles, A_i the areas of the triangles, B_i the number of lattice points on the border of the triangles, and I_i the number of interior lattice points. Then

$$I = \sum_{i=0}^{3} I_i + B_0 - 3$$

$$B = \sum_{i=1}^{3} B_i - B_0$$

$$A = I + \tfrac{1}{2}B - 1$$

$$= \left(\sum_{i=0}^{3} I_i + B_0 - 3\right) + \tfrac{1}{2}\left(\sum_{i=1}^{3} B_i - B_0\right) - 1.$$

Thus

$$\begin{aligned} A_0 &= A - \sum_{i=1}^{3} A_i \\ &= \left(\sum_{i=0}^{3} I_i + B_0 - 3\right) \\ &\qquad + \tfrac{1}{2}\left(\sum_{i=1}^{3} B_i - B_0\right) - 1 - \left(\sum_{i=1}^{3} I_i + \tfrac{1}{2} \sum_{i=1}^{3} B_i - 3\right) \\ &= I_0 + \tfrac{1}{2} B_0 - 1. \end{aligned}$$

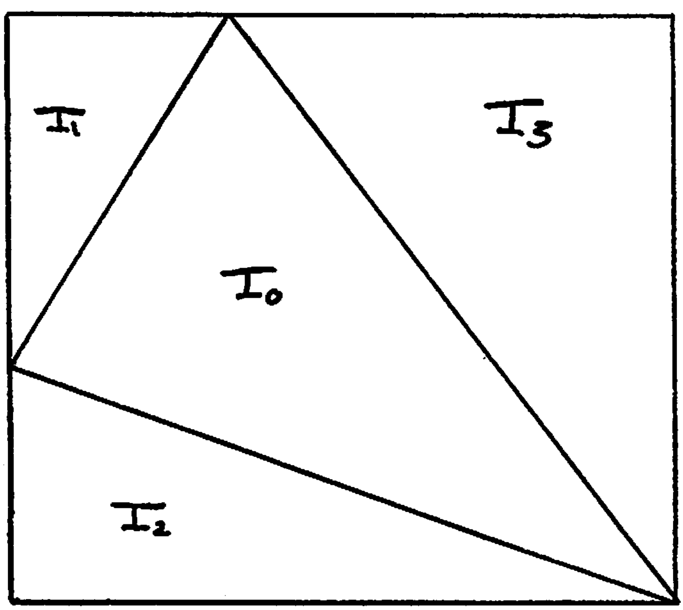

FIGURE FIVE

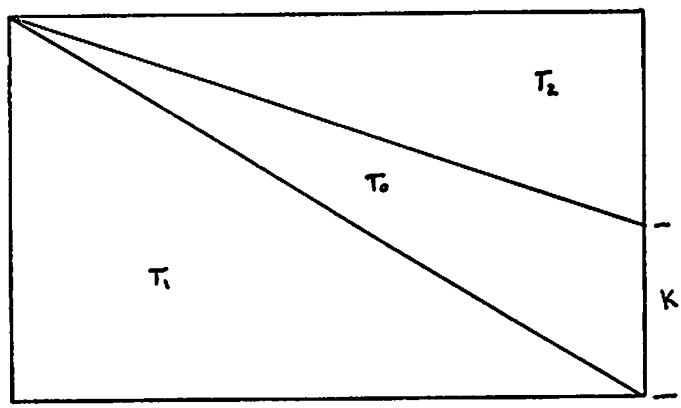

FIGURE SIX

This verifies the theorem for a triangle with no sides parallel to the coordinate axes. Now let us consider (Figure six) a triangle with one side parallel to the coordinate axes. We use the same notation as before, except that now $i = 0,1,2$. Let k be the number of lattice points, excluding the end points, on the side of T_0 parallel to the coordinate axes. Then

$$I = \sum_{i=0}^{2} I_i + B_0 - k - 3$$

$$B = \sum_{i=1}^{2} B_i - B_0 + 2k + 2,$$

and

$$A_0 = A - \sum_{i=1}^{2} A_i$$

$$= I + \tfrac{1}{2}B - 1 - \sum_{i=1}^{2} A_i$$

$$= \sum_{i=0}^{2} I_i + B_0 - k - 3 + \tfrac{1}{2}\Big(\sum_{i=1}^{2} B_i - B_0 + 2k + 2\Big)$$
$$- 1 - \Big(\sum_{i=1}^{2} I_i + \tfrac{1}{2} \sum_{i=1}^{2} B_i - 2\Big)$$
$$= I_0 + \tfrac{1}{2} B_0 - 1,$$

Thus verifying the theorem for a triangle with one side parallel to a coordinate axis. Therefore the theorem holds for a triangle in any position.

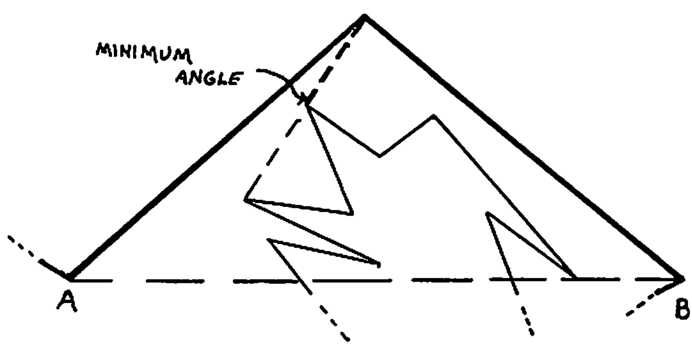

FIGURE SEVEN

To complete the proof we will introduce three lemmas.

LEMMA 1: Every simple polygon has at least one internal angle less than 180 degrees.

We can prove this by the definition of a simple polygon which states to traverse a simple polygon in a counter-clockwise direction always keeping the area on the left, you must make at least one left turn, that is, a deflection angle to the left. The angle thus formed will be less than 180 degrees.

LEMMA 2: Every m-sided simple polygon can be separated into two simple polygons of less than m sides.

Lemma 1 states that there exists at least one angle in every simple polygon which is less than 180 degrees. We will draw (Figure seven) the triangle determined by this angle and its including sides

(Figure seven, connect A to B). If this triangle does not have any vertices of the polygon on its boundary or in its interior, we have divided the given polygon into a triangle and a polygon of fewer sides than the given polygon. If there are vertices of the polygon interior to the triangle or on the third boundary (Figure seven, light broken line) drawn in the preceeding step then we choose the vertex interior to the triangle or on the third boundary which will give a minimum angle when a line is drawn from it to the vertex of the given angle. If there are two or more vertices on this terminal side of the minimum angle, we shall choose the vertex nearest the vertex of the originally chosen angle and draw the line to it. (Figure seven, bold dashed line) This line will divide the original polygon, for no other side of the polygon can have a point in common with the line thus drawn.

LEMMA 3: If two simple polygons have a side in common and if the formula holds for each, then the formula will hold for the simple polygon formed by deleting the common side.

Let P_1 be one polygon, P_2 be the other polygon, P the polygon formed by deleting the common side of P_1 and P_2 (Figure eight),

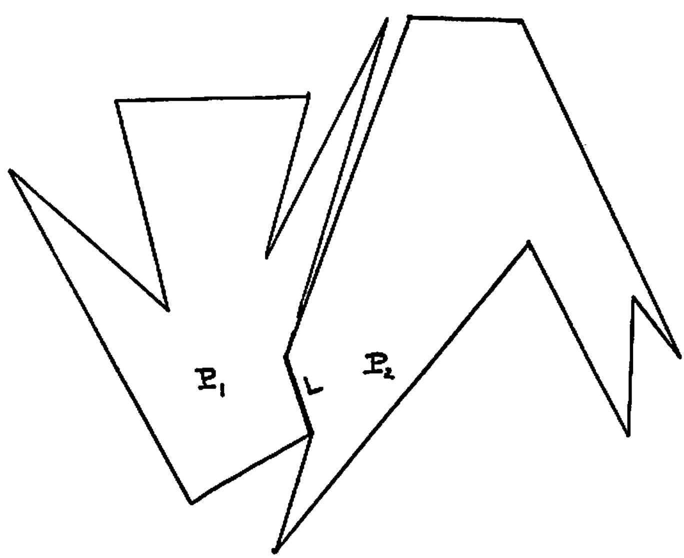

FIGURE EIGHT

and let A_1, A_2, and A be the areas of P_1, P_2 and P respectively, and L the number of lattice points on the common side of P_1 and P_2, excluding the vertices. As before, I_1, I_2, and I are the number of interior lattice points, while B_1, B_2, and B are the boundary lattice points. Since

$$\begin{aligned} I &= I_1 + I_2 + L \\ B &= B_1 + B_2 - 2L - 2, \end{aligned}$$

then

$$\begin{aligned} A &= A_1 + A_2 = (I_1 + \tfrac{1}{2}B_1 - 1) + (I_2 + \tfrac{1}{2}B_2 - 1) \\ &= (I_1 + I_2 + L) + \tfrac{1}{2}(B_1 + B_2 - 2L - 2) - 1 \\ &= I + \tfrac{1}{2}B - 1. \end{aligned}$$

Therefore the formula is valid for a simple polygon formed by the deletion of the side common to two polygons.

We have shown that every simple polygon has at least one internal angle less than 180 degrees, that every m-sided simple polygon can be separated into two simple polygons of less than m-sides, and that if two simple polygons have a side in common and if the formula holds for each, then the formula will hold for the simple polygon formed by deleting the common side.

Now we conclude that the formula is valid for any simple polygon. Consider a simple polygon with $n + 2$ sides. For $n = 1$, the polygon is a triangle and we have shown that the formula holds in this case. If there are $m + 2$ sides, we can divide it into two simple polygons each with less than $m + 2$ sides and we have shown that if the formula holds for each of these simple polygons of less than $m + 2$ sides, it will be valid for the $(m + 2)$-sided polygon. Hence by the second principle of mathematical induction, the formula must hold for a simple polygon of $n + 2$ sides where n is any positive integer.

The whole of Mathematics consists in the organization of a series of aids to the imagination in the process of reasoning.

—A. N. Whitehead

Changing Concepts in the Teaching of Mathematics*

KENNETH L. MARSI
Faculty, Fort Hays Kansas State College

First of all, I would like to take this opportunity to congratulate those of you who have just been initiated into Kappa Mu Epsilon. You are to be commended for your excellence in mathematics. But, lest you consider yourselves full-fledged mathematicians at this point, I will hasten to say that you have only seen a dim outline of mathematics and should be awed by the fact that there is still more to mathematics than you could ever hope to learn in several lifetimes. I hope that your initiation into Kappa Mu Epsilon may serve as an incentive, not only to the continuation of good work, but to its steady improvement. Whether you intend to make a career for yourself in mathematics or to use mathematics as a tool in furthering knowledge in the natural, physical or social sciences, its future usefulness to you is being determined largely by your willingness to master the subject right now.

At the moment I am thinking of the scientist who needs mathematics to help him design experiments and classify and interpret experimental data. Often he will avoid a mathematical approach to experimentation because he has found that he either does not possess the sound understanding of mathematical concepts or does not have a knowledge of the kind of mathematics essential for this. Instead, he will take the long way around by performing an unnecessary number of experiments when a mathematical anaylsis of the problem at the outset would have saved him much time. This has happened to many who, in preparing for a career in science, have not foreseen the need for mathematics in their careers, or who, in their learning, did not get understanding.

I would like to use a few minutes of your time to discuss a subject which has been of major concern to our nation in the past three or four years, mathematics and science education—in particular, mathematics education.

The launching of the first earth satellite by the USSR in Octo-

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*An address to the members and initiates of the Fort Hays Kansas State College Chapter of Kappa Mu Epsilon, March 16, 1960.

ber, 1957, revealed a state of affairs already familiar to many scientists, but unsuspected by the public at large; namely, that a real crisis existed in mathematics and science education. The USSR had not created the crisis, but simply awakened most of us to the realization that we had entered a new era of technology and were not prepared to meet its demands. Public dissatisfaction with an educational system which would allow the Russians to outperform us became widespread, and many suggestions were offered in the heat of criticism which lacked sound judgment and workability. Although the shouting has since died away, this does not alter the fact that certain changes in our mathmematics and scientific curricula are still necessary. Equally important, the recruitment of more and better qualified students into these professions is imperative if we are to exercise world leadership in this new technological age.

Twenty years ago and earlier, those holding advanced degrees in mathematics had practically no choice at earning a livelihood, they either taught or starved—or did both.

How times have changed can be borne out by figures which cite the decreasing numbers of qualified mathematicians available for high school and college teaching—this, in the face of rapidly growing enrollments at our educational institutions. In 1950, for example, 4,618 college graduates were prepared to teach mathematics. In 1957 the figure dropped to 2,892, of whom only two-thirds actually went into teaching. Industry has been competing too successfully for mathematicians. The lure of higher salaries, regular working hours, exciting and interesting research and opportunities for professional growth have been more than most mathematicians have been able to resist. This is not to say that we must counsel our students of mathematics against entering industry. The scientific revolution demands their services, and these demands must be met if scientific progress is to continue in the areas of peacetime uses of atomic energy, space exploration, medicine, national defense, and so forth.

Aside from trying to keep ahead of the Russians, which is in itself a questionable goal, three notable facts suggest a need for re-evaluating mathematics instruction in our high schools and colleges: first, this lack of enough good mathematicians for filling teaching and industrial positions; second, the increasing role of mathematics in industry and technology in general; and third, the fact that there is a great disparity between the state of mathematical research and course content in mathematics in our schools.

Ordinarily, the law of supply and demand regulates fairly well the numbers of persons entering a given profession. For example, when there is a need for engineers, engineering enrollments in colleges generally rise. The lack of enough good mathematicians may, therefore, be symptomatic of a poor quality of high school mathematics instruction which is failing to captivate the interest and imagination of our youth. Inspired teaching, an exciting approach to the subject, and demonstration of what can be done in mathematics as a career, are important techniques in attracting young people into this area of knowledge. Most mathematicians feel that the old ways of teaching mathematics account for some of the apathy toward mathematics among high school students. It does not tell the students what mathematics is all about and does not give them any real understanding of the principles of the subject, they say. There is too much emphasis on mechanics and not enough on meaning. It has the same content today as it did 100 years ago and leaves out many new ideas and discoveries. And, say some, the old way of teaching the subject has managed to make mathematics about the most unpopular of all branches of learning. Therefore, misconceptions that it is a dead subject and that nothing remains to be done in mathematics are prevalent. Actually it is a live and vital subject, and there are limitless possibilities for innovation and discovery in the field.

What are some of the new uses of mathematics which are creating so many job opportunities for mathematicians? The social sciences represent one of the most important of these new fields of application. Here trends and opinions are estimated by processing large masses of data, usually obtained by sampling. Data are summarized, relationships established and inferences drawn within the framework of theories of probability and the science of statistics. The same is true of psychology and sociology which often use statistical methods and other advanced mathematical techniques in research. In economics the theory of games is an example of a branch of mathematics put to use solving problems related to competition and cooperation. In industry, statistical quality control has meant significant savings in the manufacture of many products. Another aspect of the demand for mathematical manpower is the automatic digital computers. Although they offer quick and accurate solutions to problems formerly requiring months or years of computation by a single individual, each automatic computer needs, on the average, 10 mathematicians to attend it, serving as programmers, coders, analysts, supervisors and so on. There are more than 3,000 automatic computers in existence.

This means jobs for 30,000 mathematicians right there, considerably more than the combined membership of the American Mathematical Society and the Mathematical Association of America.

To meet these changing needs and to uphold high standards in mathematics, the Commission on Mathematics was formed five years ago by the College Entrance Examination Board. As a result of study and committee work, the Commission has outlined a nine-point proposal which it is hopeful will be gradually worked into the secondary schools. The nine points are:

1. Strong preparation in both concepts and skills in the high school so the student entering college may enroll in calculus and analytic geometry. This would mean that college instructors who are now having to teach remedial courses in mathematics would be made available to teach college-level courses,
2. Understanding the role of deductive reasoning in algebra—as *well* as in geometry,
3. A study of the properties of natural, rational, real and complex numbers.
4. Use of unifying ideas—sets, variables, functions, and relations,
5. Treatment of inequalities along with equations,
6. Revision of geometry to include some analytics and the essentials of solid geometry. The Commission advocates discontinuing solid geometry as a separate course in high schools,
7. Introduction of fundamental trigonometry, centered on coordinates, vectors and complex numbers in the junior year of high school,
8. Emphasis on polynomial, exponential, and circular functions in the senior year,
9. An introduction to either probability and statistics or modern algebra in the senior year, but not necessarily the teaching of calculus.

The Commission realizes that it will take a long while for most of these recommendations to be incorporated into the high school curriculum. School boards and school administrators will have to be educated to the necessity for these changes, and many mathematics teachers will, in a sense, need to be re-educated. The National Science Foundation is sponsoring 400 institutes in mathematics this year. These institutes are providing one of the more effective ways for bringing about the teaching of modern mathematics in our high school.

In summary, the future for a career in mathematics is bright indeed. That nearly all areas of knowledge will continue to rely even more heavily on mathematics for clarifying and solving their own problems is a certainty. Although the mathematics profession is faced with problems of recruiting many more good students, producing more competent teachers, and modernizing course offerings in view of modern trends, there are hopeful signs that these problems are gradually being met.

Thirteenth Biennial Convention

April 21-22, 1961

The thirteenth biennial convention of Kappa Mu Epsilon will be held on the campus of Emporia State Teachers College, Emporia, Kansas on April 21-22, 1961. Students are urged to prepare papers to be considered for presentation at the convention. Papers must be submitted to Dr. Ronald G. Smith, National Vice-President, Kansas State College, Pittsburg, Kansas, before February 1, 1960. For complete directions with respect to the preparation of such papers, see pages 122-123 of the Spring 1960 issue of THE PENTAGON.

I hope that every chapter will be well represented at the convention.

CARL V. FRONABARGER
National President

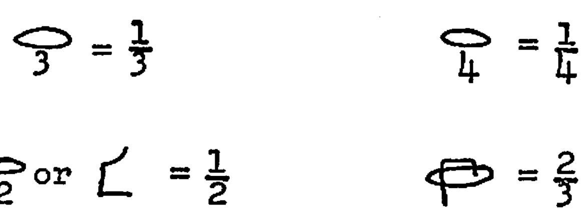
Rules of False and Double False*

A. Matthew Bazik

Student, Illinois State Normal University

Much of our knowledge of early methods of solving equations comes to us by way of papyri upon which early mathematicians as well as others wrote problems and their solutions. Development of many of the early methods of solution were discovered by a trial-and-error method.

The need for such methods as these grew out of the fact that there was little developed in the matter of symbolism. There is some symbolism in Egyptian algebra, but this was not a good one. We find such a symbolism in the Rhind papyrus for plus and minus. The first of these symbols represents a pair of legs walking from right to left, the normal direction for Egyptian writing, and, the other, a pair of legs walking from left to right. Symbols were also used for "equals" and for the unknown. To further point up the need for a better symbolism, we need only to look at the unit fractions denoted by Egyptian hieroglyphs by placing an elliptical symbol above the denominator number. A special symbol was used also for the exceptional ⅔ and another symbol sometimes appeared for ½. These symbols are shown below in association with some modern numerals:



The operation of the Rule of False in solving such an equation as $ax + bx = c$ is rather simple and proved very practical to early peoples. To solve the equation $x + \frac{1}{7}x = 24$, the unknown number is assumed to be some number, say 7. Then the left-hand side of the

*Presented at a Regional KME Convention at Illinois State Normal University on May 14, 1960.

equation will be 8, and the unknown of the equation x, is the same multiple of 7 that 24 is of the guessed number, 8.

A problem found in a papyrus of about 1950 B.C. contains the following problem in which we may easily apply our knowledge of quadratics; however, early mathematicians used the Rule of False to obtain a solution. The problem is that "a given surface of 100 square units shall be represented as the sum of two squares whose sides are to each other as 1:3/4." Here we have $x^2 + y^2 = 100$ and $x = 3y/4$. We may take $y = 4$, then $x = 3$, but this will make $x^2 + y^2 = 25$, rather than 100. We must therefore correct x and y by doubling the initial values, obtaining $x = 6$ and $y = 8$.

To illustrate both the practicallity of the rule and also the simplicity of it when applied to problems involving fractions, we may use the following problem as an example. "A cistern may be emptied by each of three pipes in 2, 3, and 4 hours, respectively. What is the number of hours needed if all three pipes are opened at once?" The solution would be as follows. If the time were 12 hours, the first pipe would empty the cistern 6 times, the second 4 times, and the third 3 times. Thus, in 12 hours the pipes would empty the cistern 13 times. Accordingly, they would empty it once in 12/13 hours or in 55 5/13 minutes.

An idea closely akin to the Rule of False for the solution of the general equation $ax + b = 0$ is the Rule of Double False. In this rule two guesses are made as to what the variable of the equation is, noting the error due to each guess. We may denote our guesses by g_1 and g_2, and our resulting failures due to these guesses by f_1 and f_2, respectively. Thus our resulting equations would be

$$ag_1 + b = f_1$$

and

$$ag_2 + b = f_2.$$

After observing how these guesses could be used in finding the true value of the variable, a formula was devised to solve the general equation $ax + b = 0$. This formula is

$$x = \frac{f_1 g_2 - f_2 g_1}{f_1 - f_2}.$$

When this formula was originally developed, it was done so by chance. However, today the formula can be derived quite elegantly by setting equal to zero the eliminant of

$$\begin{aligned} ax + b + 0 &= 0 \\ ag_1 + b - f_1 &= 0 \\ ag_2 + b - f_2 &= 0, \end{aligned}$$

which is

$$\begin{vmatrix} x & 1 & 0 \\ g_1 & 1 & -f_1 \\ g_2 & 1 & -f_2 \end{vmatrix}.$$

For example we may take the equation $5x - 10 = 0$. Making two guesses as to the value of x, say $g_1 = 3$ and $g_2 = 1$, then $f_1 = 5$ and $f_2 = -5$, and

$$x = \frac{5 \cdot 1 - (-5)3}{5 - (-5)} = 2$$

Both of these ancient rules seem rather needless when viewed by today's student of elementary algebra; however, much appreciation could be gained by the student if he were made aware of the time taken to develop the field of algebra as studied today. Many students of elementary algebra still accept the quadratic formula, for instance, without proof just as early peoples accepted formulas developed by others, not curious of why they were valid or why they worked.

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The essence of mathematics lies in its freedom.

—GEORGE CANTOR

The Nomogram*

BEVERLY BOUTELL
Student, Central Michigan University

Nomography is the graphical representation of mathematical laws and relationships. The field, itself, is broad and new to the mathematical world. The study of nomography includes delving into the understanding and practical use of diagrams. These diagrams express mathematical relationships in such a way that in some cases by simply connecting two points it is possible to solve complicated problems. The study of this subject dates back to the middle of the last century and most of the works are in French.

The main purpose of a nomogram, or alignment chart as it is sometimes called, is its practical use. The construction of a basic nomogram is quite simple. In fact, most of them consist of a number of lines either curved or straight, which are graduated to represent the variables in a specific formula. There are many types of nomograms which have been developed to solve practically every equation possible.

An example of a very simple nomogram is a chart comparing the Fahrenheit to the Centigrade scale in temperature readings. Another common nomogram is a ruler with the metric system on one side and the English system of measurement on the other. In a specific equation such as $x + y = z$, an alignment chart can be developed quite easily. Figure I shows a trapezoid $ABCD$ with EF drawn parallel to and midway between the bases. AB and CD have been graduated using the same scale and EF has been graduated using a scale one half as large. Since this is a trapezoid, by geometry it is true that:

$$EF = \frac{AB + CD}{2} \quad \text{or} \quad 2EF = AB + CD$$

As shown in Figure I, let $AB = x$, $CD = y$, and $EF = z$. On AB let the point x_0 receive the value of distance 0, let x_1 be equal to the distance $\overline{x_1x_0}$, and x_2 equal the distance $\overline{x_2x_0}$. Similarly on the CD scale, let y_0 receive the value of distance 0, let y_1 be equal to the distance $\overline{y_1y_0}$, and y_2 equal the distance $\overline{y_2y_0}$. On the EF scale k_0 equals the distance 0, k_1 equals $\overline{k_1k_0}$, and k_2 equals $\overline{k_2k_0}$. Therefore,

*A paper presented at a Regional Convention of KME at Illinois State Normal University, Normal, Illinois on May 13, 1960.

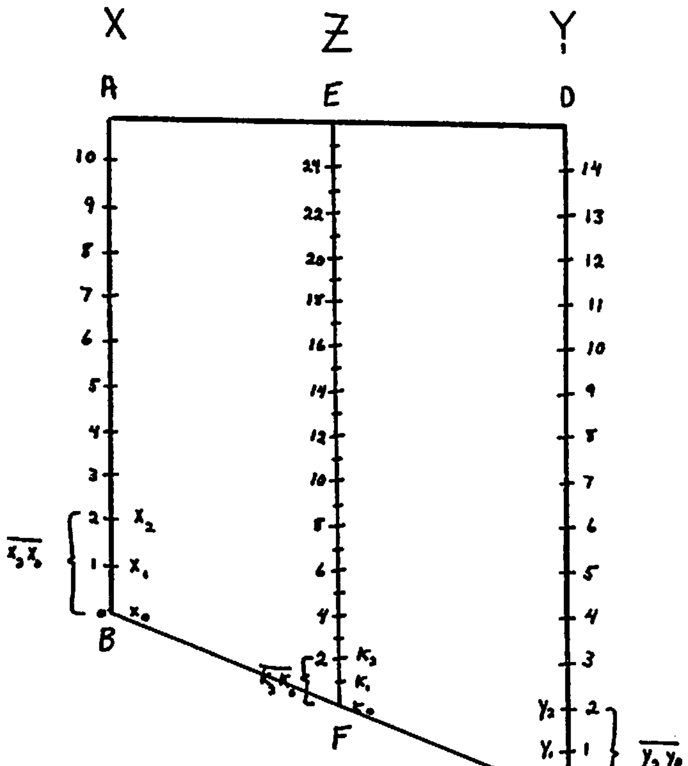


Figure I

the distance from $\overline{BFC}$ can be described as the value from it to the calibration needed. $2EF = AB + CD$ can now be written as $2k_i = x_i + y_i$. Since the median was graduated using a scale one half as large as either of the two bases and the median is the z variable, then the values of k are related to the values of z by the equation $k = \frac{1}{2}z$. Then $2k_i = z_i$, and since $2k_i = x_i + y_i$, by the transitive law $x_i + y_i = z_i$. Taking a specific example, if $x = 4$, and $y = 3$, connect 4 and 3 on the respective scales with a straightedge and the answer 7 will appear on the z scale.

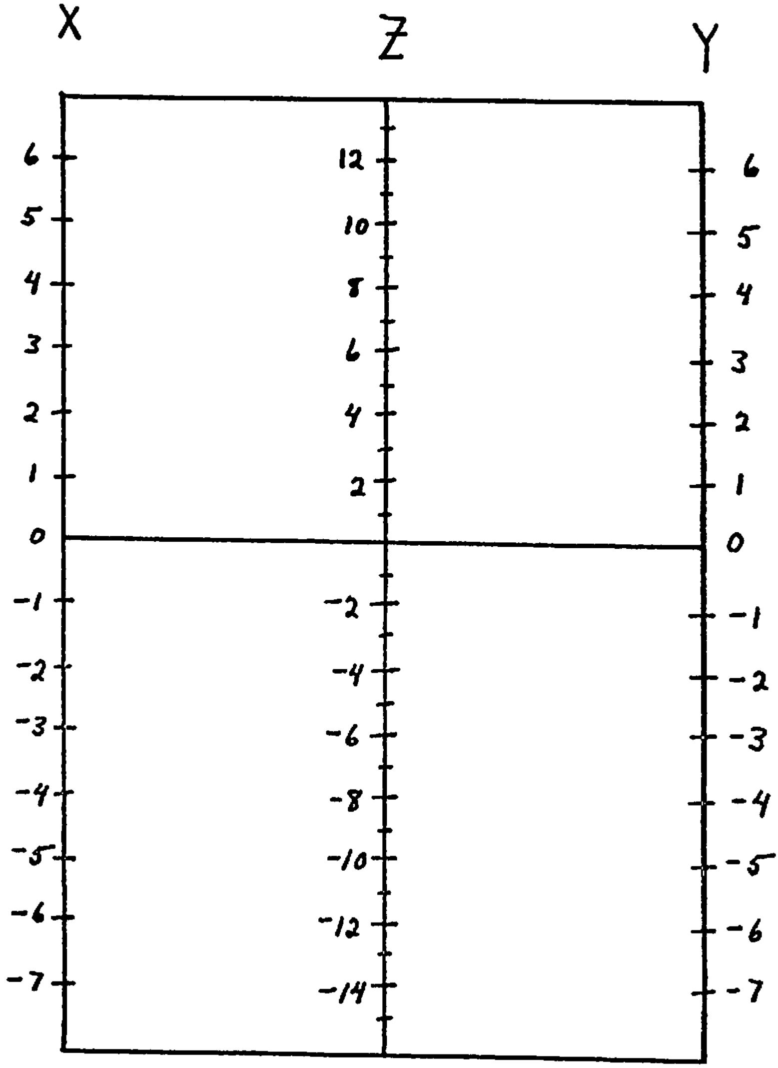


Figure II

A rectangle as in Figure II is a specific case of the same problem and in this particular figure the values have been given as positive or negative. For instance, let $x = -4$ and $y = -3$, the answer -7 appears on the z scale. Since $x + y = z$, it is true that $z - x = y$. Therefore, on this type of nomogram subtraction may also be shown. As an example let $z = 1$ and $x = -2$, the answer 3 is shown on y.

In Figure III we have essentially the same nomograph, but this time we multiply instead of adding or subtracting. In this case the product of x and y always equals z. The vertical lines *AB* and *CD* instead of being divided equally as they were before, are divided, so that they form two vertical and equal logarithmic scales. Each distance on *EF* is half the corresponding distance on *AB* or *CD*. The mathematics involved in this scale is also very simple. Let some $f(a) \cdot g(b) = k(c)$. Then $\log f(a) + \log g(b) = \log k(c)$. If *AB* would equal $\log f(a)$, *CD* equal $\log g(b)$, and *EF* equal $\log k(c)$, then by the transitive law, $AB + CD = EF$. Therefore, if x is a function on *AB*, y a function on *CD* and z a function on *EF*, by adding the sides of the figure as was done before, we are actually multiplying x by y and obtaining the answer z. Taking a specific example, let $x = 2$ and $y = 4$, the answer 8 appears on the z scale. Using this same nomogram, division is also possible by subtracting the logarithmic values. This type of nomogram is similar in theory to a slide rule which is, actually, a nomogram in itself.

The theory of the following nomograms will not be shown because most of the ideas are quite elementary. In Figure IV we have a nomogram which can solve very simple quadratic equations. The familiar formula for solving the quadratic is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

but by using a nomogram and simply by connecting two points with a straightedge the equation can be solved without having to substitute specific values for a, b, and c. In the general quadratic equation $ax^2 + bx + c = 0$, letting a always equal one, b will be the line on the left of the figure, c is the line on the right. Connecting b and c, the solution can be read on the curved line x. The theory involved in determining the graduations of the lines is proved using analytic geometry. The intersection of the straight line with the curved line in Figure IV represents the solution of the equation $x^2 - 5x + 6 = 0$.

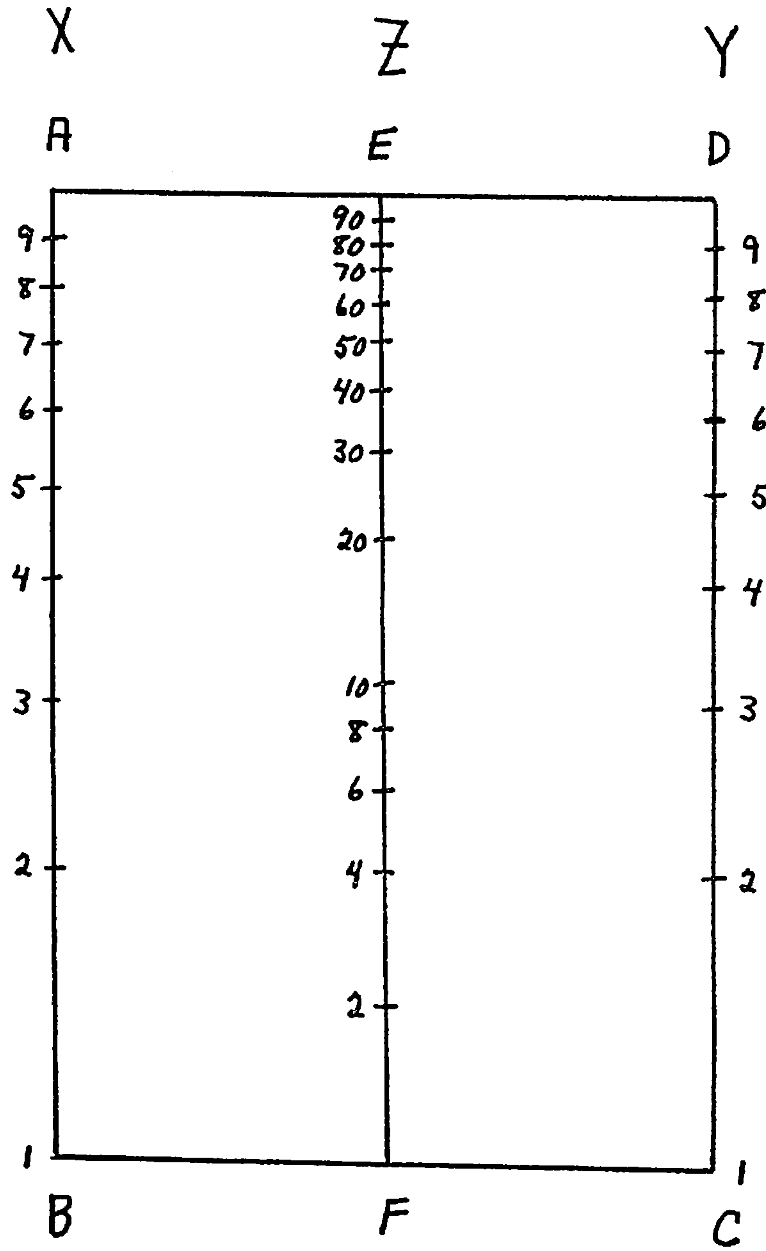


Figure III

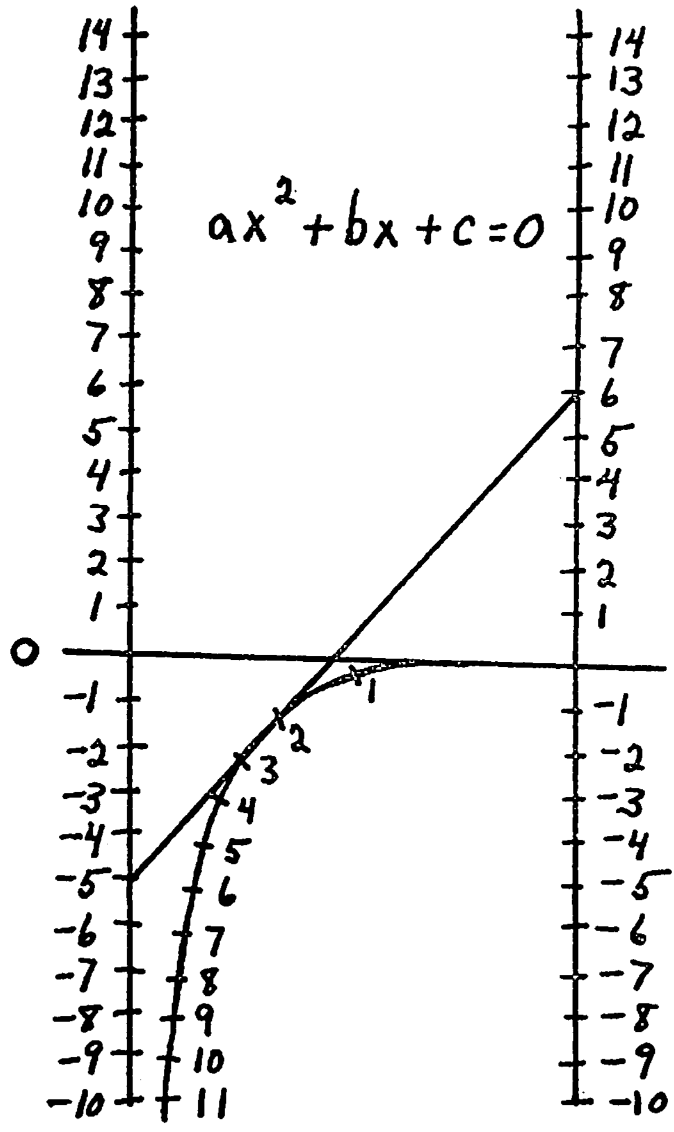


Figure IV

In this case, $b = -5$, $c = 6$, and the solution on x is seen to be $+2$ and $+3$. This is a very simple nomogram for quadratic equations and is quite limited, giving only positive roots. All types of quadratic equations could be solved using nomograms but some of them would involve the use of parabolas, ellipses, and other mathematical figures.

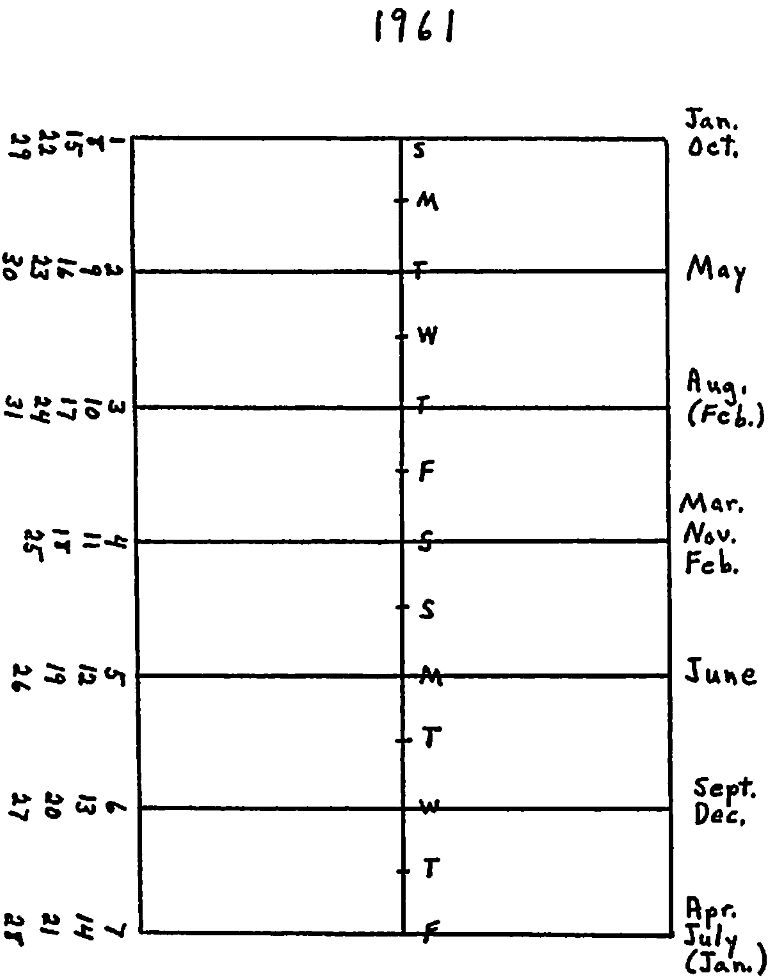


Figure V

In Figure V we have a perpetual calendar created and copyrighted by Harry D. Ruderman in *Fun with Mathematics*. In this diagram you can tell the day of the week on which any specific date will fall. The center line gives the days of the week and lies equidistant from the outside lines. The months are on the right with those in parentheses representing the months in leap year, and the dates are on the left line. This particular diagram only works for 1961. To change it to any other year it is necessary to change the day on which January 1 falls. As a suggestion as to how to determine the day on which January 1 will fall, reference is made to the October, 1957 issue of the *Mathematics Teacher* and the article, "A Mental Calendar", by Rev. Brother Leo. As an example of this specific perpetual calendar as a nomogram, consider the date June 28 and the day on which it falls. It is merely necessary to connect June in one column with 28 in the other and the day is seen to be Wednesday. This is certainly much easier than having to figure out anything mathematically.

The theory of the nomogram is actually far from simple. Some of the more basic ideas have been shown. However, a complete treatment of the subject draws upon every aspect of analytic, descriptive, and projective geometry, the several fields of algebra and many other mathematical areas of study.

I don't know what I may seem to the world, but, as to myself, I seem to have been only as a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

—I. Newton

Determination of A Parametric Formula for Great Circle Curves

CHARLES W. BACHMAN
Dow Chemical Company, Midland, Michigan

The determination of a series of points on a great circle course has been the task of navigators for centuries. The advent of light weight spherical structures now has the architect seeking solutions to the same problem.

This paper is concerned with a means of obtaining the cartesian coordinates (x, y, z) of a series of points on a great circle curve defined by two points on the surface of a sphere. The solution collapses when the points are on opposite sides of the sphere.

The parameter s defines the proportional distance along a great circle. If **A** is the initial point $(s = 0)$, then **B** is the second point $(s = 1)$. Values of s greater than one are proportionally beyond point **B**. Negative values of s turn backward away from **B**. There is no limit to the value of s, plus or minus, as the vector **C** turns repeatedly around the sphere. A spherical radius of unity will be used in the derivation to simplify the equations. However, the general case is easily written and the parametric equation is not limited to unity radius.

Consider the vector space of dimension three. Two independent vectors define a plane which is a subspace of two dimensions. Any vector **C** in this plane can be defined as a linear combination of the two defining vectors, i.e.

$$\mathbf{C} = a\mathbf{A} + b\mathbf{B} \qquad (1)$$

Equation (1) will serve as the basis for determining the vector **C**, whose elements are the cartesian coordinates of the point desired.

It is known from analytical geometry that the angle between two unit vectors, $\mathbf{A} = (x_1, y_1, z_1)$ and $\mathbf{B} = (x_2, y_2, z_2)$, can be determined by equation (2).

$$x_1x_2 + y_1y_2 + z_1z_2 = \cos\phi \qquad (2)$$

The dot product of two unit vectors is a scalar equal to the cosine of the angle between them.

$$\mathbf{A} \cdot \mathbf{B} = \cos \phi \tag{3}$$

$$\mathbf{A} \cdot \mathbf{C} = \cos \psi \tag{4}$$

The vector **C** is in the plane formed by vectors **A** and **B**. The parameter s is defined by formula (5). Thus s is a

$$s\phi = \psi \tag{5}$$

measure of the proportional distance of vector **C** with respect to the vectors **A** ($s = 0$) and **B** ($s = 1$). Equation (5) may be substituted into equation (4) to give

$$\mathbf{A} \cdot \mathbf{C} = \cos s\phi \tag{6}$$

Vector **C**, as defined by equation (1), is substituted into equation (6) giving

$$a\,(\mathbf{A} \cdot \mathbf{A}) + b\,(\mathbf{A} \cdot \mathbf{B}) = \cos s\phi \tag{7}$$

The dot product $\mathbf{A} \cdot \mathbf{A}$ is equal to unity because the vector **A** is of unit length. The dot product $\mathbf{A} \cdot \mathbf{B}$ is equal to $\cos \phi$ as defined by equation (3). Therefore, equation (7) may be simplified giving

$$a + b \cos \phi = \cos s\phi \tag{8}$$

The vector **C** is also of unit length. It is a radius of the sphere. Therefore

$$\mathbf{C} \cdot \mathbf{C} = 1 \tag{9}$$

Substituting **C** as defined in equation (1) gives

$$(a\,\mathbf{A} + b\,\mathbf{B}) \cdot (a\,\mathbf{A} + b\,\mathbf{B}) = 1 \tag{10}$$

Expanding equation (10) gives

$$a^2\,(\mathbf{A} \cdot \mathbf{A}) + 2ab\,(\mathbf{A} \cdot \mathbf{B}) + b^2\,(\mathbf{B} \cdot \mathbf{B}) = 1$$

or

$$a^2 + 2ab \cos \phi + b^2 = 1 \tag{11}$$

Equations (8) and (11) are two simultaneous equations that may be used to determine a and b as functions of ϕ and s. If a^2 as determined in equation (8), is substituted into equation (11) a new equation is determined.

$$\begin{aligned} &\cos^2 s\phi - 2b \cos s\phi \cos \phi + b^2 \cos^2 \phi + b^2 \\ &\quad + 2b \cos s\phi \cos \phi - 2b^2 \cos^2 \phi = 1 \end{aligned} \tag{12}$$

grouping terms

$$b^2 (1 - \cos^2 \phi) + \cos^2 s \phi = 1$$

thus

$$b^2 \sin^2 \phi = \sin^2 s\phi$$

or

$$b = \pm \frac{\sin s\phi}{\sin \phi} \qquad (13)$$

Substituting this expression (13) for b in equation (8) yields the equation

$$a = \cos s\phi \pm \left(\frac{\sin s\phi}{\sin \phi}\right) \cos \phi \qquad (14)$$

These values of a and b, when substituted back into equation (1), give a parametric equation for **C** in terms of **A**, **B**, ϕ, and s.

$$\mathbf{C} = \left[\cos s\phi - \left(\frac{\sin s\phi}{\sin \phi}\right) \cos \phi\right] \mathbf{A} + \left(\frac{\sin s\phi}{\sin \phi}\right) \mathbf{B} \qquad (15)$$

$$\mathbf{C} = \left[\cos s\phi + \left(\frac{\sin s\phi}{\sin \phi}\right) \cos \phi\right] \mathbf{A} - \left(\frac{\sin s\phi}{\sin \phi}\right) \mathbf{B} \qquad (16)$$

Equations (15) and (16) are two versions of the same equation. The negative root obtained in equation (13) results in equation (16) where the vector turns negatively or away from **B**. This can be seen by substituting equation (17) into equation (16) to yield equation (15) with negative values of s.

$$- \sin s\phi = \sin (- s)\phi \qquad (17)$$

The angle ϕ was related to vectors **A** and **B** by equation (2). This is for a unity radius sphere. A general equation for a sphere of any radius is

$$\frac{x_1x_2 + y_1y_2 + z_1z_2}{x_1^2 + y_1^2 + z_1^2} = \cos \phi \qquad (18)$$

Equation (18) should be used to determine ϕ with respect to the cartesian coordinates of a sphere of any radius.

Derivation of Sums of Powers of Positive Integers*

NORMAN SELLERS
Student, Kansas State Teachers College

For many years the mind of the mathematician has been teased by the problem of finding an expression for the summation of the powers of numbers. The purpose of this paper is to develop a method of deriving expressions giving the summation of the first n integers raised to a positive integral power.

For example, the sum of the first n positive integers is given by the formula

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

This may be proved by the process of mathematical induction. We will show that if the formula is true for any integer, which will be represented by $n - 1$, then it must hold for the following integer, n. If

$$\sum_{i=1}^{n-1} i = \frac{(n-1)[(n-1)+1]}{2},$$

then

$$\begin{aligned} \sum_{i=1}^{n} i &= \sum_{i=1}^{n-1} i + n \\ &= \frac{(n-1)[(n-1)+1]}{2} + n \\ &= \frac{(n-1)n}{2} + n = \frac{n^2 - n + 2n}{2} = \frac{n(n+1)}{2}. \end{aligned}$$

Since the formula is obviously true when $n = 1$, it follows that it is true when $n = 2, 3, 4, \cdots$. This proves that the relationship is

*Presented at a Regional KME Convention at Washburn University, Topeka, Kansas, March 26, 1960.

valid for any positive integer, n. The same type of proof may apply to the identities

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

and

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

The difficulty in producing these identities lies not in proving their validity, but rather in deriving a suitable expression for the summation.

In order to discover suitable expressions for the summations for any power summation let us first analyze the form of the three summation functions given above. It is first noticed that the sum is given by an algebraic polynomial with the constant term missing. Next it is noticed that this polynomial is divided by an integer. The degree of the polynomial appears to be one greater than that of the summation terms.

The Riemann summations replacing integrals provides a good suggestion that the degree of this polynomial should be one greater than the summation term exponents. As an example, let $y = x^m$. Then an area under the graph is given by

$$A = \int_0^a x^m dx = \frac{a^{m+1}}{m+1}.$$

Or it may be found by

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} x_i \,\Delta x_i.$$

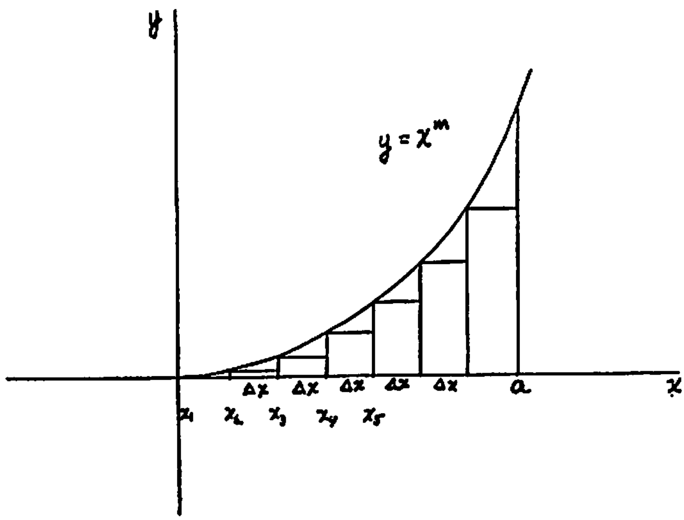
We may use equal subdivisions (see figure) so that $\Delta x_i = a/n$. Let

$$x_i = (i-1)\Delta x_i = (i-1)a/n.$$

Substituting these gives

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} (i-1)^m \frac{a^{m+1}}{n^{m+1}}.$$

For this limit to be a definite value other than zero, we know that



$\sum\limits_{i=1}^{n} (i-1)^m$ must be of degree $m+1$ in n. For more information on Reimann summations, refer to Johnson and Kiokemeister, *Calculus with Analytic Geometry*, Allyn and Bacon, 1957, pages 234 to 240.

Using these observations, let us assume that

$$\sum_{i=1}^{n} i^k = \frac{a_1 n + a_2 n^2 + \cdots + a_{k+1} n^{k+1}}{b} .$$

We will represent the expression on the right with $f_k(n)$. It is evident that

$$f_k(n) + (n+1)^k = f_k(n+1).$$

These two equations provide the basis for deriving $f_k(n)$ for any k.

We shall now use these facts to derive $f_4(n)$. Since

$$f_4(n) = \frac{a_1 n + a_2 n^2 + a_3 n^3 + a_4 n^4 + a_5 n^5}{b},$$

then

$$\frac{a_1n + a_2n^2 + a_3n^3 + a_4n^4 + a_5n^5}{b} + (n+1)^4 =$$

$$\frac{a_1(n+1) + a_2(n+1)^2 + a_3(n+1)^3 + a_4(n+1)^4 + a_5(n+1)^5}{b}$$

Expanding and simplifying yields

$$\begin{aligned} a_5n^5 + n^4(a_4+b) + n^3(a_3+4b) + n^2(a_2+6b) + n(a_1+4b) + b \\ = a_5n^5 + n^4(5a_5+a_4) + n^3(10a_5+4a_4+a_3) \\ + n^2(10a_5+6a_4+3a_3+a_2) \\ + n(5a_5+4a_4+3a_3+2a_2+a_1) \\ + (a_5 + a_4 + a_3 + a_2 + a_1). \end{aligned}$$

Since this equation is to be an identity for all n, the coefficients of like powers of n for both members are identical. This gives rise to 5 equations in 6 unknowns. The symmetry of these equations is readily seen by writing the system in the following manner:

$$\begin{aligned} a_4 + b &= 5a_5 + a_4 \\ a_3 + 4b &= 10a_5 + 4a_4 + a_3 \\ a_2 + 6b &= 10a_5 + 6a_4 + 3a_3 + a_2 \\ a_1 + 4b &= 5a_5 + 4a_4 + 3a_3 + 2a_2 + a_1 \\ b &= a_5 + a_4 + a_3 + a_2 + a_1 \end{aligned}$$

or

$$\begin{aligned} b &= 5a_5 \\ 4b &= 10a_5 + 4a_4 \\ 6b &= 10a_5 + 6a_4 + 3a_3 \\ 4b &= 5a_5 + 4a_4 + 3a_3 + 2a_2 \\ b &= a_5 + a_4 + a_3 + a_2 + a_1 \end{aligned}$$

Comparing this set of equations with the table of binomial coefficients strongly suggests that a pattern exists for any k. These equations may be solved for all the a's in terms of b. A series of simple algebraic substitutions readily produces the solution:

$$a_5 = b/5, \quad a_4 = b/2, \quad a_3 = b/3, \quad a_2 = 0, \quad a_1 = -b/30$$

We may now choose $b = 30$ to remove all fractions. Then

$$a_5 = 6, \quad a_4 = 15, \quad a_3 = 10, \quad a_2 = 0, \quad a_1 = -1$$

Therefore

$$f_4(n) = \frac{6n^5 + 15n^4 + 10n^3 - n}{30}$$

$$= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}.$$

Now, to generalize the derivation for $f_k(n)$ for any k, the set of equations giving the coefficients in $f_k(n)$ may be written directly from the form of the set of equations found when $k = 4$. That is, notice the coefficients of the b's reading vertically are those of a binomial expansion to the fourth power. Likewise the coefficients of the a_5's are the binomial coefficients of a binomial expansion to the fifth power. Or, in general, the coefficients of the b's are the binomial coefficients of a binomial expansion to the kth power. The subscript on the a's gives the power of the binomial expansion from which the coefficients for the particular column are chosen.

The author, using this method of calculation, has derived the summation identities for k ranging from 1 to 8. The additional formulas for $k = 5$, 6, 7, and 8 are given below.

$$\sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$$

$$\sum_{i=1}^{n} i^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-n+1)}{42}$$

$$\sum_{i=1}^{n} i^7 = \frac{n^2(n+1)^2(3n^4+6n^3-n^2-4n+2)}{24}$$

$$\sum_{i=1}^{n} i^8 =$$

$$\frac{n(n+1)(2n+1)(5n^6+15n^5+5n^4-15n^3-n^2+9n-3)}{90}$$

It takes about 30 minutes to compute one of these identities but it will take a few sleepless nights to discover the single general expression for the summation for any k.

Editor's Note: It took the genius of a Bernoulli and the invention of the Bernoulli numbers to write a general formula for any integer k. James Bernoulli's *Ars Conjectandi,* published in 1713, eight years after his death, contained the required generalization, but there was no hint as to how it was obtained. Proofs have been given since that time, and many other uses of the Bernoulli numbers have been found in mathematics.

The sums of powers of integers has always been an intriguing topic. For readers who are interested in another approach, see Yates, Robert C. "Sums of Powers of Integers", *The Mathematics Teacher,* LII (April 1959), pages 268-271. Also see the solution to Problem 132 in The Problem Corner, pages 38-40 of this issue of the PENTAGON for an inductive method of determining $f_k(n)$ when $f_1(n), f_2(n), \cdots, f_{k-1}(n)$ are known.

The student of mathematics often finds it hard to throw off the uncomfortable feeling that his science, in the person of his pencil, surpasses him in intelligence—an impression which the great Euler confessed he often could not get rid of. This feeling finds a sort of justification when we reflect that the majority of the ideas we deal with were conceived by others, often centuries ago. In a great measure it is really the intelligence of other people that confronts us in science.

—Ernest Mach

The Problem Corner

EDITED BY J. D. HAGGARD

The problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond the calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1961. The best solutions submitted by students will be published in the Spring, 1961, issue of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to J. D. Haggard, Department of Mathematics, Kansas State College, Pittsburg, Kansas.

PROPOSED PROBLEMS

136. *Proposed by Jimmy M. Rice, Fort Hays Kansas State College, Hays, Kansas.*

A schoolboy on Neptune solves the quadratic equation $x^2 - 10x + 31 = 0$ and finds the roots to be 5 and 8. What is the base of the number system he is using?

137. *Proposed by Robert Myers, Chicago Teachers College.* (From *Introduction to Theory of Equations,* Conkwright)

Determine p and q so that 5 will be a double root of $x^4 - 9x^3 + px^2 + qx + 25 = 0$.

138. *Proposed by Mark Bridger, Columbia University, New York, N.Y.*

Professor Umbugia has had his pool table inscribed in the first quadrant of a cartesian coordinate system, with a corner pocket located at the origin and a coordinate axis along two of the sides. In a game with one of his students the Professor finds his ball on the spot (14,8). Taking careful (though inaccurate) aim he hits the ball in such a way that it misses every other ball, bouncing off the x and y axes in that order, and finally coming to rest at the spot (7,12). To cover up his "scratch," the Professor asks: How far has the ball traveled?

139. *Proposed by Paul R. Chernoff, Harvard University, Cambridge, Massachusetts.*

Evaluate $\sum_{p=0}^{n} \binom{2n+1}{2p}$, where $\binom{m}{n} = \dfrac{m!}{n!(m-n)!}$

140. *Proposed by the Editor.* (From *The American Mathematical Monthly*).

Evaluate the infinite product

$$\prod_{n=1}^{\infty}\left(1+\frac{1}{a_n}\right) \text{ where } a_1 = 1,\ a_n = n(a_{n-1}+1).$$

SOLUTIONS

131. *Proposed by Mary Sworske, Mount Mary College, Milwaukee, Wisconsin.*

Find three natural numbers a, b, c such that $a^n + b^n + c^n$ is an integral multiple of 18 for any natural number n.

Solution by Richard Wright, Kansas State Teachers College, Emporia, Kansas.

We examine the natural numbers 9, 6, 3. For $n = 1$, $9^1 + 6^1 + 3^1 = 18 \cdot 1$; for $n = 2$, $9^2 + 6^2 + 3^2 = 18 \cdot 7$. Now consider any $n > 2$.

$$\begin{aligned} 9^n + 6^n + 3^n &= (2 \cdot 9 \cdot 9 \cdot 9^{n-2})/2 + (2 \cdot 2 \cdot 3 \cdot 6 \cdot 6^{n-2})/2 \\ &\quad + (2 \cdot 3 \cdot 3 \cdot 3^{n-2})/2 \\ &= (18/2)(9 \cdot 9^{n-2} + 4 \cdot 6^{n-2} + 3^{n-2}) \end{aligned}$$

It remains to be shown that the expression in the second parenthesis is even. For any $n > 2$, $9 \cdot 9^{n-2}$ is odd and 3^{n-2} is odd, while $4 \cdot 6^{n-2}$ is even. Thus the sum of two odd numbers and an even is even.

Also solved by Patrick Boyle, San Jose State College, San Jose, California; Mark Bridger, Columbia University, New York, N.Y.; William O'Brien, Harvey Mudd College, Claremont, California.

Editorial note: The above problem asked for a triple of numbers satisfying a certain condition. Several such correct triples were submitted, but only one is used in the solution given above.

132. *Proposed by Don Hayler, Pomona College, Claremont, California.*

Consider the series

$$S_k = 1^k + 2^k + 3^k + \cdots + n^k = \sum_{x=1}^{n} x^k$$

For

$$k = 1, S_1 = \frac{n(n+1)}{2}$$

For

$$k = 2, S_2 = \frac{n(n+1)(2n+1)}{6}$$

Compute S_3 and S_4.

Solution by Paul R. Chernoff, Harvard University, Cambridge, Massachusetts.

By the binomial theorem,

$$(x-1)^k = x^k + \sum_{r=1}^{k} (-1)^r \binom{k}{r} x^{k-r}$$

Then

$$\sum_{x=1}^{n} [x^k - (x-1)^k] = \sum_{x=1}^{n} \sum_{r=1}^{k} (-1)^{r+1} \binom{k}{r} x^{k-r}$$

Evaluating the left hand side and reversing the order of the finite sums on the right side, we have

$$n^k = \sum_{r=1}^{k} \left[(-1)^{r+1} \binom{k}{r} \sum_{x=1}^{n} x^{k-r} \right]$$

$$= \sum_{r=1}^{k} (-1)^{r+1} \binom{k}{r} S_{k-r}$$

Hence,

$$S_{k-1} = \frac{1}{k}\left[n^k + \sum_{r=2}^{k} (-1)^r \binom{k}{r} S_{k-r} \right]$$

Setting $k = 4$ in this formula, and replacing S_2 by the value given in the problem, we obtain

$$S_3 = 1/4 \left[n^4 + 6\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n \right]$$

$$= \frac{n^2(n+1)^2}{4}$$

Similarly, making use of S_3 and taking $k = 5$ we obtain

$$S_4 = \tfrac{1}{30}[6n^5 + 15n^4 + 10n^3 - n]$$
$$= \tfrac{1}{30}[n(n+1)(2n+1)(3n^2+3n-1)]$$

Also solved by Patrick Boyle, San Jose College, San Jose, California; Mark Sworske, Mount Mary College, Milwaukee, Wisconsin.

134. *Proposed by the Editor* (From *The American Mathematical Monthly*).

Evaluate the determinant:

$$|A| = \begin{vmatrix} n & n-1 & \cdots & n-m \\ n-1 & n & \cdots & n-m+1 \\ n-2 & n-1 & \cdots & n-m+2 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ n-m & n-m+1 & \cdots & n \end{vmatrix}$$

Solution by Mark Bridger, Columbia University, New York, N.Y.

The determinant is of order $m + 1$. Now subtract the mth column from the $(m + 1)$th, the $(m - 1)$th from the mth, and so on to the 1st from the 2nd. Note that the 1st column remains unchanged. We obtain

$$|A| = \begin{vmatrix} n & -1 & -1 & \cdots & -1 & -1 \\ n-1 & 1 & -1 & \cdots & -1 & -1 \\ n-2 & 1 & 1 & \cdots & -1 & -1 \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ n-m & 1 & 1 & \cdots & 1 & 1 \end{vmatrix}$$

Now add the $(m + 1)$th row successively to the 1st, 2nd, 3rd and so on to the mth rows, obtaining

$$|A| = \begin{vmatrix} 2n-m & 0 & 0 & \cdots & 0 & 0 \\ 2n-m-1 & 2 & 0 & \cdots & 0 & 0 \\ 2n-m-2 & 2 & 2 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ 2n-2m+1 & 2 & 2 & \cdots & 2 & 0 \\ n-m & 1 & 1 & \cdots & 1 & 1 \end{vmatrix} .$$

Then taking the product of the diagonal elements we obtain

$$|A| = (2n - m) \cdot 2^{m-1}.$$

Also solved by Paul R. Chernoff, Harvard College, Cambridge, Massachusetts; William O'Brien, Harvey Mudd College, Claremont, California; R. C. Weger, William Jewell College, Liberty, Missouri; Margaret Salmor, Pomona College, Claremont, California.

Pure mathematics is a collection of hypothetical, deductive theories, each consisting of a definite system of primitive, undefined, concepts or symbols and primitive, unproved, but self-consistent assumptions (commonly called axioms) together with their logically deducible consequences following by rigidly deductive processes without appeal to intuition.

—G. D. Fitch

The Mathematical Scrapbook

EDITED BY J. M. SACHS

No more impressive warning can be given to those who would confine knowledge and research to what is apparently useful, than the reflection that conic sections were studied for eighteeen hundred years merely as an abstract science, without regard to any utility other than to satisfy the craving for knowledge on the part of mathematicians, and that then at the end of this long period of abstract study, they were found to be the necessary key with which to attain the knowledge of the most important laws of nature.

—A. N. WHITEHEAD

$=\triangle=$

Can you find integral values which satisfy the equalities $a^2 = b^3 = c^4$? Since $c = b^{3/4}$, b must be a fourth power. Let $b = d^4$. Then $a^2 = b^3 = d^{12}$ and $a = d^6$. Thus for any integer d, $a = d^6$, $b = d^4$, $c = d^3$ will be integral solutions of the given equalities. Are there any other integral values which satisfy this relation?

Under what conditions will there be integral solutions for the relation $a^n = b^m = c^p$ with $n < m < p$ and m, n, and p positive integers?

$=\triangle=$

We see that experience plays an indispenable role in the genesis of geometry; but it would be an error thence to conclude that geometry is, even in part, an experimental science.

—H. POINCARE

$=\triangle=$

Everyone is familiar with the famous Konigsberg Bridge Problem and the solution which leads to the following generalization:

It is possible to trace a closed path without retracing any part if and only if (i) All vertices have an even number of paths, or (ii) Two vertices have an odd number of paths and the other vertices have an even number of paths. By a vertex is meant the intersection of two or more paths. In the case of all even vertices, we can begin at any one and that leaves an odd number of paths for that one. We will pass through all of the other vertices using two paths, one coming and one going, for each passage. We may also pass through the starting vertex, if there were more than two paths there to begin with. Finally we will return to the start. If we are careful not to use up our final path back to the start too soon we can trace any figure with all vertices even, however complicated. In the case of two odd vertices we start at one and finish at the other.

Now let us look at the problem of tracing paths from a slightly different angle. What can we say about this problem with a fixed number of vertices? If we have one vertex then each path must consist of a loop returning to this vertex. Thus no matter how many paths, a closed figure with one vertex has *all* even vertices and is traceable. What about two vertices? If the figure is to be closed then there must be at least two paths. Adding an additional path makes both vertices odd. Adding a fourth makes them both even, etc. Thus with two vertices, both are even or both are odd hence the figure is always traceable. With three vertices we must have at least three paths to get a closed figure, the simplest being a triangle. This has all even vertices. Adding another path we will make two of the three odd. Adding still another path we will end up with all even or two odd and one even. Can you argue that this must be so? Thus adding paths with three vertices does not change the fact that the figure is traceable.

With four vertices the problem gets more interesting. The smallest number of paths would be four. This would be traceable. Can you verify or disprove the following conjectures about the four vertex figure:

(i) It is possible to add as many paths as one wishes and keep the figure traceable if the vertices for the new paths are properly chosen.

(ii) If at any step in the process of adding new paths, all vertices are even then adding one more path leaves the figure traceable no matter which two vertices are used for that new path.

(iii) If at any step in the process of adding new paths, two vertices are even and two are odd then it is possible to get a non-traceable figure by adding one more path if the vertices are properly chosen.

(iv) It is not possible to have a single odd vertex.

Readers may be interested in looking at five or more vertices also and may wish to ask if it is ever possible to have a single odd vertex or a single even vertex. One way of attacking this problem might be to consider the effect of an additional vertex, e.g. having a solution for three vertices, what is the effect of adding a fourth. This brings up another question which may be of interest. If these are plane figures can every vertex be connected with every other vertex without adding new intersections (vertices)?

=Δ=

Whoever despises the high wisdom of mathematics nourishes himself on delusion and will never still the sophistic sciences whose only product is an eternal uproar.

—DA VINCI

=Δ=

If we know the sum of the squares of two integers and also their product then by solving the product equation and substituting into the sum of the squares we get, after clearing of fractions, a quartic equation (a quadratic in the square of one of the variables). This problem can be solved however without a quartic.
Given:

$$a^2 + b^2 = \mathrm{m}$$
$$ab = k$$

Therefore

$$a^2 + 2ab + b^2 = m + 2k \quad \text{and} \quad a^2 - 2ab + b^2 = m - 2k$$

so

$$(a + b) = \sqrt{m + 2k} \quad \text{and} \quad (a - b) = \sqrt{m - 2k}.$$

From this we get

$$a = (\sqrt{m + 2k} + \sqrt{m - 2k})/2 \quad \text{and}$$
$$b = (\sqrt{m + 2k} - \sqrt{m - 2k})/2.$$

Example:

$$a^2 + b^2 = 218 \text{ and } ab = 91$$

so

$$a^2 + 2ab + b^2 = 400 \text{ and } a^2 - 2ab + b^2 = 36.$$

Thus

$$a + b = 20 \text{ and } a - b = 6$$

so

$$a = 13 \text{ and } b = 7.$$

Have we really managed to solve a quartic by solving a quadratic? Can you find the flaw in the statement that we have done so above?

=Δ=

Can you apply the method just described to the problem of finding a rational number if we are given the sum of the number and its reciprocal and told that the rational is in lowest terms? In doing this you will have the material for proving that the sum of a rational in lowest terms and its reciprocal (also in lowest terms) will again be in lowest terms.

=Δ=

Whatever can be an object of scientific thinking in general, becomes amenable to the axiomatic method—and so mediately to mathematics—as soon as it becomes ripe for the construction of a theory. By penetrating to ever deeper-lying levels of axioms we acquire ever deeper insights into the essence of scientific thinking. itself. Thus we become steadily more conscious of the unity of our knowing.

—D. HILBERT

=Δ=

The following is an excerpt from a fascinating document contributed to the editor's collection by a friendly statistician. The document is meant to apply to statistical design and quality control but the mathematical representation seems worth repeating here. There is no source indicated on the material so credit cannot be given.

"Years ago when the universe was relatively easy to explain the famous Finagle Constant K_f was introduced so that $x' = x + K_f$, where x represents a measured variable, x' its theoretical counterpart and K_f is arbitrary. Later as difficulties compounded F_f, the fudge factor appeared and $x' = F_f x + K_f$ was used as an aid and comfort to those in distress. In World War II the multiplicity of experiments made a stronger influence imperative and some unsung hero rose to the occasion with the diddle factor F_d so that it was now possible to use $x' = F_d x^2 + F_f x + K_f$. This helped a lot. It is felt that for the present reality can be brought into reasonable agreement with theory by the use of these three constants and no further extension in this direction is anticipated in the immediate future. It seems wise to point out however that there is a difference in the structure and thus the use of the three constants. The Finagle Constant changes the universe to fit the equation. The fudge factor changes the equation to fit the universe. Finally the diddle factor changes both just enough to insure an adequate fit somewhere about half way between. This sacrifices both reality and theory and is known as statistical fence sitting.

=Δ=

Again, a touch of humor (strange as the contention may seem) in mathematical works is not only possible with perfect propriety, but very helpful; and I could give instances of this even from the pure mathematics of Salmon and the physics of Clerk Maxwell.

—G. MINCHIN

=Δ=

What's wrong here? Can you find the flaw in this "obviously" incorrect argument?

$$1/(1+x) = 1 - x + x^2 - x^3 + \cdots,$$
$$1/(1+x+x^2) = 1 - x + x^3 - x^4 + \cdots.$$

Let $x = 1$ to obtain

$$1/2 = 1 - 1 + 1 - 1 + \cdots,$$
$$1/3 = 1 - 1 + 1 - 1 + \cdots,$$

hence

$$1/2 = 1/3.$$

=Δ=

Mathematics has beauties of its own—a symmetry and proportion in its results, a lack of superfluity, an exact adaptation of means to ends, which is exceedingly remarkable and to be found elsewhere only in the works of greatest beauty.

—J. W. A. YOUNG

=Δ=

In mathematics as in other fields, to find one self lost in wonder at some manifestation is frequently the half of a new discovery.

—P. G. L. DIRICHLET

The Book Shelf

EDITED BY H. E. TINNAPPEL

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor Harold E. Tinnappel, Bowling Green State University, Bowling Green, Ohio.

Naive Set Theory, Paul R. Halmos, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1960, vii + 102 pp., $3.50.

All of the present revision movements in regard to Junior and Senior High School and the Undergraduate College curricula of mathematics advocate strongly the early introduction of set language and concepts. The apparent purpose of such recommendations is to shed some much needed light upon those mathematical concepts and relations which occur at the very basis of our discipline, but which used to be taken very much for granted in the classical approaches to the various school mathematics. As a consequence, there has arisen an urgent demand for a concise and authoritative account of the principles of Set Theory. Professor Halmos' small (102 pages) book, entitled *Naive Set Theory*, fills this need admirably. Though it was written primarily "to tell the beginning student of advanced mathematics the basic set-theoretic facts of life" (from the preface), it can be read and understood, in most of its sections, by interested mathematics teachers of Junior and Senior High School levels with great benefit for themselves and for their students. The treatment is axiomatic to a certain extent; the language is informal and frequently delightful. The first 5 sections are basic and of general introductory value; they are essential to anybody who wishes to talk set language correctly. The once feared Russell Paradox is treated set-theoretically in a clear and precise manner in one brief paragraph right at the outset (page 6). The basic concept of "singleton" is clarified to complete satisfaction (page 10). Variations in symbolism and terminology, both of which abound in Set Theory, are briefly stated and sometimes evaluated. The author's own choice is generally for maximum economy and clarity of symbolism. Section 6, on "Ordered Pairs," is extremely illuminating and dispells certain widely held misconceptions. Section 11, 12, and 13, entitled

"Numbers," "The Peano Axioms," and "Arithmetic," respectively, provide a solid foundation for ordinary arithmetic and number systems. This foundation is extended in Sections 24 and 25 under the headings of "Cardinal Arithmetic" and "Cardinal Numbers," both of which provide fundamental information and insights.

Many important proofs are either given or sketched out; others are left to the reader. Like any good and solid mathematical book, *Naive Set Theory* must be worked rather than read. Paper and pencil are indispensable. While, in the author's own words, the book has nothing to offer for the professional expert in the field, it is a reading "must" for all of those mathematics students and mathematics teachers who find themselves as yet in a somewhat lesser classification. This little book is "modern" in the very best sense of this much misused term.

—Bernard H. Gundlach
Bowling Green State University

Testing Statistical Hypotheses, E. L. Lehmann, John Wiley & Sons, Inc., (440 Fourth Avenue) New York, 1959, $11.00.

The purpose of this book is to give a systematic account of the mathematical theory of hypothesis testing in which tests are derived as solutions of clearly stated optimum problems, and of the closely related theory of confidence sets. The principal applications of these theories are given, including one- and two-sample problems concerning normal, binomial, and Poisson distributions. There is also a brief treatment of non-parametric tests and of some aspects of the analysis of variance and of regression analysis.

All statistical concepts are developed from the beginning but the abstract level of the treatment makes previous experience with statistical methods desirable before using the book. The natural mathematical framework used for a systematic treatment of hypothesis testing is the theory of measure in abstract spaces. The author employs the Neyman-Pearson formulation mainly, but he presents a discussion of the concepts of general decision theory.

The chapter titles indicate the scope of the topics covered: (1) The General Decision Problem; (2) The Probability Background; (3) Uniformly most Powerful Tests; (4) Unbiasedness: Theory and First Applications; (5) Unbiasedness: Applications to Normal Distributions, Confidence Intervals; (6) Invariance; (7) Linear Hypotheses; (8) The Minimax Principle.

At the end of each chapter there is a substantial problem list keyed to the various sections into which the chapter is divided. There is also given at the end of each chapter an annotated list of references. The book is one of the Wiley Publications in Statistics.

—David M. Krabill
Bowling Green State University

Regression Analysis, E. J. Williams, John Wiley & Sons, Inc., (440 Fourth Avenue) New York, 1959, $7.50.

The material of this book is based on the author's long experience in analyzing experimental results in the fields of agriculture, biology, chemistry, and forestry. It is intended primarily for research workers in the experimental sciences. Most of the examples are from the biological sciences. There are no problem lists.

The author assumes that the reader is acquainted with general statistical methods and he presents no basic statistical theory. Since the book is addressed to experimenters rather than to mathematicians, a minimum of mathematical technique is employed. The author attempts to point out when a particular method is appropriate, how to make most effective use of the data, and the precautions to take in interpreting data.

The book contains eleven chapters. The first four deal with the determination of various regression relations. Chapter 5 treats of the choice among various regression formulas, while Chapter 6 discusses the use of the regression equation in estimation. Chapter 7 presents a brief treatment of the analysis of covariance. The analysis of covariance is applied in the remaining chapters in the development of significance tests for various multivariate problems. Heterogeneous data are treated in Chapter 8, and simultaneous regression equations in Chapter 9. The final two chapters treat discriminant functions and functional relations. The author distinguishes clearly between regression and functional relationships.

The book is intended to be within the reach of experimenters and practical statisticians generally. It would not be suitable as a textbook but it could be used in an advanced course in applied statistics for supplementary material in the practical applications of statistics. The notation employed is sometimes cumbersome. For example, S is frequently, but not consistently used to denote summation. Also, superscripts are employed in place of subscripts interchangeably and without warning.

—David M. Krabill
Bowling Green State University

Modern Electronic Components, G. W. A. Dummer, Philosophical Library, Inc., (15 East 40th Street) New York 16, 1959, 472 pp., $15.00.

In the preface the author makes the following statements:

"This book has been written to provide a reasonable comprehensive summary of essential data on electronic components . . . .

"Many users of components may appreciate . . . a volume in which all the characteristics of the commonly used components are now required to be used under many arduous environmental conditions and several chapters have, therefore, been written to cover those aspects in reasonable detail . . . .

"It is hoped that the information in this book will help the user to an understanding of the basic characteristics of the components and enable him to choose the best component for his particular purpose.

"This is the first comprehensive book of its kind to be published in the world, and it is hoped it will fulfill a useful function in the technical literature . . . ."

The author is the Head of Components Research, Development and Testing at the Royal Radar Establishment, Ministry of Supply, Great Britain. He is, therefore, uniquely situated to provide the information contained within the book.

Except for electronic tubes and transistors, practically all types of components for use in electronic circuits are described. The general and specialized characteristics of these components, the design considerations and requirements for operation under extreme conditions of temperature, humidity, pressure, mechanical vibration, and the areas of particular application together with limitations for the various components, are briefly described. For many of the components, a brief sketch of manufacturing procedures is provided. Diagramatic sketches and photographs are shown and these are invariably of British manufacture. So far as the reviewer was able to observe, no components of United States manufacture are either described or pictured. This raises the question as to the value of this book for United States readers. In general, however, most of the components are sufficiently similar so as to make comparison rather simple.

Many of the chapters are completed with a selected bibliography, which in most cases will be of very limited value unless the reader has access to a library containing a large selection of British electrical industry and electrical periodicals. This is true since a relatively few references are made to United States publications such as "Electronics," and "The Journal of the Bureau of Standards."

A rather interesting chapter is the one in which the author considers future developments and component design. In this chapter he bases his discussion primarily on improvements which are required and possible ways in which these may be effected, provided the limitations of available material are reduced.

Since this book is so recent a publication, it is regrettable that more emphasis is not given to sub-miniature requirements for transistor-circuit components. The chapter devoted to these requirements appears to be rather sketchy and since it is likely that applications in this field will increase at a greatly accelerated rate, it would have improved the book to have had this field covered in greater detail.

In spite of the criticisms voiced above, particularly with respect to the bibliography and lack of United States component research, the reviewer believes that this book can serve a very useful purpose to the individual responsible for the design of electronic circuits. The use of the book as collateral reading with the large amount of information available from manufacturers' catalogues should enable him to make a more judicious choice of a given component.

The book also should be welcome in the technical library of any Physics or Electrical Engineering Department offering courses in electronics where students would have the opportunity to build up a greater background of knowledge of the various components used.

—Willard E. Singer
Bowling Green State University

There is no science which teaches the harmonies of nature more clearly than mathematics, . . .

—Paul Carus

Installation of New Chapters

EDITED BY MABEL S. BARNES

THE PENTAGON is pleased to report the installation of four new chapters.

OHIO DELTA CHAPTER

Wittenberg University, Springfield, Ohio

A dream was realized on Friday, April 29, 1960, when the Wittenberg Mathematics Society became the Ohio Delta Chapter of Kappa Mu Epsilon. The installation was conducted by Professor Harry Mathias of Ohio Alpha, Bowling Green State University. The candidates for membership were presented by Professor Norman E. Dodson. Warren Bosch discussed the purpose of Kappa Mu Epsilon, and Lynn Meister explained the meaning of the various parts of the crest.

After the installation the group went to the Rustic Inn for the installation banquet. Professor Mathias gave the address, in which he welcomed the new chapter and recounted the history of Kappa Mu Epsilon. Professor Dodson read several letters of congratulation from various chapters and national officers. Dr. Raymond Krueger, Chairman of the Mathematics Department, greeted the group, and Dr. John N. Stauffer, Dean of the University and former mathematics professor, welcomed the organization to the campus and challenged the members to take part in the rapid development of modern mathematics.

The Wittenberg Mathematics Society was conceived in the minds of Professor Dodson and Honor Student George Lindamood in the fall of 1958. To some extent it was a subgroup of the Engineering Society. During the year 1959-60 three members, George Lindamood, Warren Bosch, and Lee Simon, built an electric analog computer. The Mathematics Society set high standards with the goal of ultimately becoming a chapter of Kappa Mu Epsilon, and it grew in stature and membership. Now its new connection has added further stature to the organization and has given impetus to still more and better programs.

The officers are Warren Bosch, president; Bert Price, vice-president; Lynn Meister, secretary; Lee Simon, treasurer; Norman Dodson, faculty sponsor; and Henry Diehl, corresponding secretary.

The other charter members are Robert Baer, Virginia Banet, George Benson, Kathryn Case, Nancy Danford, Terry Deems, Martha Drake, George Fisher, Robert Flectner, Jack Flinner, Dennis Flood, James Flora, Paul Hagelberg, Mrs. Maurice Hanes, Joan Hendrickson, Norman Heyerdahl, James Hutchison, Richard Lamka, Jean Lannert, George Lindamood, Richard Little, Mary McCarty, Janice Nelson, Karin Nilsson, Ruth Philip, Thomas Prior, Janice Reller, Timothy Riggle, Mary Rilling, Karen Sohner, Karen Spriegel, John Stedke, and David Wulff.

Wittenberg University is a Lutheran school, founded in 1845. There are five constituent divisions in the University, the largest and central one being the College of Liberal Arts. The present enrollment is approximately 1500 students with 130 faculty members.

FLORIDA ALPHA CHAPTER

Stetson University, DeLand, Florida

Sigma Alpha Omega of Stetson University became Florida Alpha Chapter of Kappa Mu Epsilon on May 26, 1960.

At the banquet preceding the installation, Mr. Emmett Ashcraft, who was the guiding hand behind Sigma Alpha Omega, gave a very interesting talk on the background of the organization and its history at Stetson. Letters from National Officers of Kappa Mu Epsilon were read, including a letter from the past National President, Charles B. Tucker. Letters of congratulation from nineteen chapters were acknowledged.

David Hayes was awarded a Kappa Mu Epsilon key for submitting the best mathematical essay during the year. The outgoing secretary of Sigma Alpha Omega, Sue Boren, was honored for her accomplishments and also was awarded a key.

The climax of the evening was the very impressive installation ceremony, conducted by Dr. Gene W. Medlin, Chairman of the Department of Mathematics at Stetson University, with the assistance of Mr. Oscar Jones, also of the Department of Mathematics.

Charter members are John B. Adams, Mary Stone Adams, Mary Ann Aiken, Ann Severance Booth, Nancy Sue Boren, Kathleen Damewood, William Dolbier, Diane Drummond, Lewis James Edgemon, Dalton Epting, Mary Louise Goslin, David Hayes, Dale Keiter, Jo Lysek, Judith McKenzie, Carol Christie Moore, Jackie Pitts, Judy Scudder, Donna Stevens, Jerry Glenn Tate, and Nancy Walker; and from the faculty Mr. Emmett S. Ashcraft, Dr. Elizabeth Boyd, General C. H. Chorpening and Colonel Wayland H. Parr.

The officers are David Hayes, president; Mary Adams, vice-president; Nancy Walker, secretary; Jerry Glenn Tate, treasurer; Dr. Gene W. Medlin, faculty sponsor; and Colonel Wayland H. Parr, corresponding secretary.

INDIANA DELTA CHAPTER

Evansville College, Evansville, Indiana

Indiana Delta Chapter was installed on May 27, 1960, by the National President, Dr. Carl V. Fronabarger. The ceremony took place in the Club Rooms of the Student Union. The candidates for membership were presented by Dr. Philip Kinsey, Professor of Chemistry at Evansville College. Dr. Kinsey was a charter member of Indiana Alpha at Manchester College.

Following the installation a banquet was held for members and guests in the Dining Room of the Student Union. After the banquet Mr. V. C. Bailey, faculty sponsor of Indiana Delta, traced the history of the Mathematics Club from its origin ten years ago as a problem solving group to its affiliation with Kappa Mu Epsilon. Dr. Fronabarger spoke on the topic "Large Oaks From Small Acorns Grow". In his address he compared the growth and the reasons for growth of the city of Evansville and Evansville College with those of Kappa Mu Epsilon.

Charter members are Gene W. Bennett, Charles Bertram, Bruce Brazelton, Hardy Curd, Larry Fowler, Stephen E. Heeger, Paul Herr, John J. Hicks, William Jones, Arthur Kushner, John Paul Lobeck, C. Douglas McConnell, Tommy Milton, David Montgomery, Brownie Muir, William Muir, Marjorie Olson, Robert W. Polz, Dolores Reasor, Carlos H. Seltzer, Brady Shafer, Thomas Sharpe, Sylvia Skinner, and George W. Stephenson; and from the faculty Mr. V. C. Bailey, Mr. C. W. Buesking, Dr. Ralph H. Coleman, Dr. Ray T. Dufford, Dr. Edgar McKown, and Dr. Traver C. Sutton.

The officers are Gene W. Bennett, president; G. Douglas McConnell, vice-president; Marjorie Olsen, secretary; Dolores Reasor, treasurer; Mr. C. W. Buesking, corresponding secretary; and Mr. V. C. Bailey, faculty sponsor.

Evansville College was founded as Moore's Hill College in

1854, but was moved to Evansville in 1919. The combined enrollment of both Day and Evening Divisions is now in excess of 3000 students. Its faculty and administrative staff number over 125. The College offers curricula leading to degrees in liberal arts, engineering, education, and business.

OHIO EPSILON CHAPTER

Marietta College, Marietta, Ohio

Ohio Epsilon Chapter was installed on Saturday, October 29, 1960 at Marietta College, Marietta, Ohio. Mr. Harry Mathias, past National Vice President of Kappa Mu Epsilon was the installing officer. Mr. Mathias is a member of the faculty at Bowling Green University.

The installation took place on Saturday afternoon at 4:00 p.m. in the Charles Otto Lounge of the Gilman Student Center at Marietta College. Fifteen students and four faculty members were initiated. The members are as follows: Dr. Theodore Bennett, William Brastow, Rosalie Brum, Mr. Richard Cherril, Sarah DeCoster, Judith Fisher, James Gaal, Richard Givens, Richard Gluckstern, Robert Havran, William Hazlett, Sally Heckert, Mr. Raymond Huck, Dr. Paul Hutt, William MacDonald, Robert Monter, Margaret Orcutt, Lynn Roux, Harvey Tapolow.

The officers installed at the ceremonies were James Gaal, President; Richard Givens, Vice President; Judith Fisher, Secretary; Raymond Huck, Corresponding Secretary; Robert Havran, Treasurer; Theodore Bennett, Faculty Adviser.

Following the initiation ceremonies, a business meeting was held. During this meeting the club officially adopted the local By-Laws. An informal get-to-gether was held prior to the banquet.

The banquet was held in the dining room of the Gilman Student Center.

Mr. Mathias addressed the group on the purposes of the establishment of Kappa Mu Epsilon and showed the growth rate in the past few years.

We are happy to welcome the four new chapters and to wish them well.

Kappa Mu Epsilon News

EDITED BY FRANK C. GENTRY, HISTORIAN

REGIONAL CONFERENCE, Washburn University, Topeka, Kansas.

A Regional Conference for the states of Kansas, Missouri, and Nebraska was held with Kansas Delta at Washburn University, Topeka, Kansas, on March 26, 1960. The five Kansas chapters, four Missouri chapters, and two Nebraska chapters were all represented. The program was modeled after the National Convention Program with two sessions for student papers, a discussion session and a luncheon. Dr. J. M. Sachs of Illinois Gamma was guest speaker for the luncheon. His subject was "Revolution and Counter-Revolution".

REGIONAL CONFERENCE, Illinois State Normal University, Normal, Illinois.

The first Regional Conference for the states of Illinois, Indiana, Michigan and Wisconsin was held with Illinois Alpha at Illinois State Normal University, Normal, Illinois, on May 13-14, 1960. Three of the four chapters in Illinois, two of the three chapters in Indiana, one of the two chapters in Michigan and the Wisconsin chapter were represented. The seven student papers on the program were presented by representatives of six of the participating chapters. At a banquet on Friday evening, Dr. Herbert Wills, York Community High School, Elmhurst, Illinois was the guest speaker. Dr. Wills is closely associated with the University of Illinois Committee on School Mathematics.

Alabama Beta, Florence State College, Florence.

Our chapter sponsors a coffee-hour at each Homecoming so that old members may get together. We initiated twenty new members in March for a total of 312. Six of last year's members were offered assistantships and scholarships for graduate study during the current year. A faculty member has a National Science Foundation Faculty Fellowship and is at Vanderbilt University this year. Dr. George Walden, Research Chemist for Chemstrand Corporation and a former member of our chapter was guest speaker at a recent meeting.

Illinois Alpha, Illinois State Normal University, Normal.

Six new members were initiated last year. Four of our programs consisted of student papers, two were by faculty members and the other two involved lectures by visitors. Dr. W. T. Reid of the University of Iowa spoke on "The Solution of Certain Mathematical Problems with Soap Films" and Dr. David Blackwell, Traveling Lecturer for the Mathematical Association of America, spoke on "Theory of Games".

Iowa Beta, Drake University, Des Moines.

Our chapter had six regular meetings and an initiation banquet during 1959-60. Most of the papers were given by pledges. Their subjects were: "The Debicon", "Roman Numerals and the Roman Abacus", "Logic and Mathematics", "An Application of the Theory of Games", "Bohr's Formula on the Energy of the Electron", "The Use of Statistics" and "Continued Fractions". Officers for the year were: Mary Laughlin, Pres.; Roger Rosenberry, V. Pres.; Sonya Mikkelson, Sec.; Dr. B. E. Gillam, Treas.; Dr. E. L. Canfield, Cor. Sec.

Kansas Alpha, Kansas State College of Pittsburg.

Our chapter held four meetings in 1959-60, at each of which we listened to one student paper and one paper by a member of the Mathematics Staff. We also had Dr. M. R. Hestenes, MAA Visiting Lecturer from the University of California at Los Angeles, for a lecture, "The Quickest and the Least". Officers for the year were: William E. Coffelt, Pres.; Kenneth Feurborn, V. Pres.; Bing Wong, Sec.; Max Steele, Treas.; Prof. Helen Kriegsman, Cor. Sec. Our chapter held two picnics with the Mathematics and Physical Science Departments.

Kansas Gamma, Mount St. Scholastica College, Atchison.

This year our chapter has fourteen members including Sister Helen Sullivan, O.S.B., who has been a member of KME for twenty years. We plan to induct 22 pledges in October, many of whom have expressed a desire to major in mathematics. Our theme for the year is KME's Double Refracting Prism. We plan a trip to Kansas City this year to visit Linda Hall Technical Library of Midwest Research. Kansas Gamma publishes "THE EXPONENT, a quarterly newsletter. The 1960 Spring issue, a three page mimeographed paper, contains on its front page two articles headed, "Sister Helen, Two Members to Take Roles at Regional Conference" and "Kansas Gamma Plans Dinner to Celebrate Twentieth Birthday".

Plans Dinner to Celebrate Twentieth Birthday".

Mississippi Gamma, Mississippi Southern College, Hattiesburg.

Our chapter will hold its annual steak cook-out in October.

Missouri Alpha, Southwest Missouri State College, Springfield.

We had Professor L. M. Blumenthal of the University of Missouri as guest lecturer last April. His subject was "Some Curious New Theorems of Elementary Geometry". Each year a member of our chapter, who in the judgment of the organization and the faculty, has made the greatest contribution to the society, is honored by having his name placed on a merit award plaque and given a KME key. David A. Kelley received this award in 1960.

Missouri Beta, Central Missouri State College, Warrensburg.

Each year our chapter prepares a Christmas letter consisting of the compilation of letters from former members telling about their families, jobs, etc. Our programs are presented by initiates, graduate students and guest speakers from industrial firms.

Nebraska Alpha, Wayne State Teachers College, Wayne.

Our chapter initiated 16 new members last year giving us a total active membership of 34. Twenty-two members attended the Regional Conference at Washburn University where David Helm won first prize for his paper, "Quadrispace". The Mathematics and Science Societies hold an annual joint banquet in the spring.

Nebraska Beta, Nebraska State Teachers College, Kearney.

Thirty-two students are eligible to join our chapter this year. Eight members who graduated last year are now employed as teachers, most of them in Nebraska schools. Three others are doing graduate work at the Universities of Illinois, Kansas and Nebraska.

New Mexico Alpha, University of New Mexico, Albuquerque.

Our chapter initiated 32 new members last year for a total membership of 707 in twenty-five years. We held five regular meetings with two papers by new members of the Mathematics Staff and three by student members.

Ohio Gamma, Baldwin-Wallace College, Berea.

We initiated 11 new members last year. In October we held a joint meeting with the Science Seminar to hear Dean John R. Dunning of the College of Engineering, Columbia University. In January, Mr. George Moshos of N.A.S.A., Cleveland, discussed the

Univac 1103 computer. In February, Mr. George O'Brien, a Baldwin-Wallace alumnus, and presently teaching mathematics at Roehm Junior High School in Berea presented "Teaching Tips for Mathematics Teachers". Later, Ken Hovey, one of Mr. O'Brien's ninth grade pupils, presented a paper on topology which he had prepared for his mathematics class.

Pennsylvania Gamma, Waynesburg College, Waynesburg.

In addition to seven regular meetings with papers by faculty and student members, we held a banquet in May at which time Mr. Bernard Last, of the Rockwell Meter Plant, spoke on "Mathematics in Industry". Our officers for 1960-61 are: Stephen Priselac, Pres.; William Roos, V. Pres.; Ruth Moredock, Sec.; Harold Hartley, Treas.; Prof. Lester T. Moston, Cor. Sec.

Wisconsin Alpha, Mount Mary College, Milwaukee.

Our chapter sponsored a mathematics contest for high school students in the vicinity, as it has for the last six years. Six students and two faculty members attended the Regional Conference at Illinois State Normal University. Mary Sworske read a paper entitled "The Construction of Logarithms" which was prepared by Mary Staudemaier.

It is true that mathematics, owing to the fact that its whole content is built up by means of purely logical deduction from a small number of universally comprehended principles, has not unfittingly been designated as the science of self-evident (Selbstverstandlichen). Experience however, shows that for the majority of the cultured, even of scientists, mathematics remains the science of the incomprehensible (Unverstandlichen).

—Alfred Pringsheim