THE PENTAGON

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National Officers

Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

L. P. Woods – In Memoriam

On February 26 we were saddened by the death of our friend Professor L. P. Woods, one of the founders of Kappa Mu Epsilon. The Fraternity and a great number of students will feel his loss most keenly. He was an inspiration to every person with whom he came in contact. A friendly hand, a cheery smile, a glowing heart, and a truly dedicated spirit passed from among us.

Mr. Woods was a native of Tennessee but moved to Mississippi at an early age. As an undergraduate he attended Transylvania College and Arkansas University. He received his B.A. and M.S. degrees from the University of Arkansas and did graduate work at the University of Texas, George Peabody College, Columbia University, and the University of Missouri.

He was a member of the Oklahoma Education Association and the National Education Association and filled offices in both of these organizations. Other professional and scientific groups of which he was a member include Kappa Delta Pi, Sigma Xi, and Pi Mu Epsilon. He was a member of Sigma Alpha Epsilon social fraternity and was listed in "Who's Who in American Education."

He taught in the public schools at Greenwood, Arkansas, and Fort Towson and Stigler, Oklahoma, before coming to Northeastern State College in 1921. He was at first head of the training school and later became head of the mathematics department. He was dean of men from 1929 utnil 1947. Last year he was selected as head of the newly organized Science Division but did not accept because of ill health.

Mr. Woods organized a Mathematics Club in 1926 which was very active until it was transformed into a part of a national organization in 1931. When Mr. Woods was doing graduate work toward a Ph.D. degree at the University of Missouri in 1928-29 he met Miss Kathryn Wyant who was also working toward her Ph.D. in mathematics. After completing her degree Miss Wyant came to Northeastern State College to teach in 1930. Dr. Wyant under Mr. Woods guidance began to draw up plans for a national mathematics fraternity on the undergraduate level. In April, 1931, Mr. Woods and Dr. Wyant invited representatives from Pittsburg State Teachers College, Pittsburg, Kansas, and Iowa State Teachers College, Cedar Falls, Iowa, to meet in Tahlequah to establish the first chapter of Kappa Mu Epsilon.

Next only to his family and his church, Kappa Mu Epsilon has been of great concern to him. He has watched Kappa Mu Epsilon grow with pride and has welcomed each change wholeheartedly if it was for the improvement of Kappa Mu Epsilon. He held each member in highest esteem and was always telling of interesting and happy incidents which had occurred in the lives of Kappa Mu Epsilon members.

To be associated with Professor Woods was as wonderful an experience as a person could have. He was truly an outstanding educator, leader, and citizen. He was a great mathematician, a real Christian gentleman, and a guiding inspiration to every student who knew him. Although he has joined the Chapter Immortal, he still lives in the hearts and lives of all Kappa Mu Epsilon members and of the thousands of Northeastern students who still call him "Dean." Each one of us—associates, students, friends—have in some way caught the spark of his inspiring enthusiasm. Through us he still lives and will live for a long time.

—Raymond Carpenter

"Every man who rises above the common level has received two educations: the first from his teachers; the second, more personal and important, from himself."

—Edward Gibson

A "Self-Service Laundry" Problem

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As everyone knows who has taken out, singly and at random, the socks from the perfectly-shuffled batch that a self-service laundry customarily produces, an exasperatingly large number of socks must be drawn before a "match" occurs. This suggests the problem: *If, from n different pairs of like items, k single items are withdrawn in succession and at random, and if* P(*k*) *is the probability that a pair is completed for the first time on the kth draw, for what value of k is* P(*k*) *largest?*

Proceeding to the solution, we find first that $P(2) = [1/(2n - 1)]$, since, the first item having been withdrawn, there is only one among the remaining $2n - 1$ which matches it. Next, $P(3) = [(2n - 2)/(2n - 1)][2/(2n - 2)]$, the first fraction representing the probability that the second draw does *not* match the first, and the second fraction representing the probability that the third draw matches *one or the other* of the first and second. Similarly, $P(4) = [(2n - 2)/(2n - 1)][(2n - 4)/(2n - 2)][3/(2n - 3)]$. And, in general, $P(k) = [(2n - 2)/(2n - 1)][(2n - 4)/(2n - 2)]\cdots[(2n - 2k + 4)/(2n - k + 2)][(k - 1)/(2n - k + 1)]$, which, when simplified, becomes

$$P(k) = 2^{k-2}[(n - 1)(n - 2)\cdots(n - k + 2)(k - 1)] / [(2n - 1)(2n - 2)\cdots(2n - k + 2)(2n - k + 1)]$$
$$= [2^{k-2}][k - 1][(n - 1)!(2n - k)!] / [(n - k + 1)!(2n - 1)!].$$

In order to maximize $P(k)$, we set up the inequality

$$(1) \qquad P(k)/P(k - 1) \lesseqgtr 1,$$

to which each of the following forms, derived in sequence, is equivalent. (We use the fact that $0 < k \leqq 2n$.)

$$[(2)(k - 1)(n - k + 2)]/[(k - 2)(2n - k + 1)] \lesseqgtr 1,$$

$$(2) \qquad k^2 - 3k + 2(1 - n) \gtreqless 0$$

(to pass from which to the next inequality, see the remarks following (3)),

$$(3) \qquad k \underset{<}{\geqq} [(3 + \sqrt{8n+1})/2]$$

In deriving (3) from (2), we observe that (2) is an inequality to be satisfied by ordinates to the parabola $y = x^2 - 3x + 2(1 - n)$ where $x = k$ and k is surely positive; the quantity on the right-hand side of (3) is simply the positive x-intercept of this parabola.

Returning to (1), we see that (3) means that $P(k)$ is *not* maximum for $k > [(3 + \sqrt{8n+1})/2]$ nor for $k < [(3 + \sqrt{8n+1})/2] - 1$, so that the maximum occurs for k between these two values (inclusive). Hence (and by examination of (1) in case of equality), we see that $P(k)$ *is greatest for k equal to the greatest integer not greater than* $[(3 + \sqrt{8n+1})/2]$ *and, if* $[(3 + \sqrt{8n+1})/2]$ *is itself an integer, for both this value and one integer less.*

Thus, from 5 pairs of socks I must pull 4 individuals more often than any other number to get my first match. And from 10 pairs I will get my first match most often (and equally often) on the fifth or sixth draw.

Since a match is certain to be gotten for some k in the range $2 \leqq k \leqq n + 1$, a curious by-product of our work with $P(k)$ is that

$$(4) \quad 1 = \sum_{k=2}^{n+1} P(k) =$$

$$\frac{(n-1)!}{(2n-1)!} \sum_{k=2}^{n+1} [2^{k-2}][k-1][(2n-k)!/(n-k+1)!]$$

There are various problems associated with this one, of which the following are a few.

(i) What is the value of k if duplication is to have occurred with, say, 95% certainty in the first k draws?

(ii) How can relation (4) be deduced directly (as from known combinatorial formulas)?

(iii) What is the story if we start with n different *triplets* of like items and seek the most probable value of k for duplication? For triplication?

(iv) What are the corresponding stories for q-tuplets?

Undertaking a Graduate Mathematics Program

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This article is an attempt to answer some of the questions proposed by students about the various aspects of a program in graduate mathematics. It is hoped that it, along with other sources of information (*e.g.* [8]), will help the reader to understand some of the major points of such a program. The guiding principle throughout has been an enumeration of facts that the writer himself wanted to know before entering the graduate school.

1. Reasons for studying graduate mathematics. Anyone wishing to teach mathematics easily realizes the advantages of having an advanced degree. Moreover, a substantial background in graduate mathematics gives the teacher a better grasp of an over-all picture of the general mathematics program. Also, private industry now is offering special inducements to obtain the services of the person who has done graduate study. (For further information see [11].) Finally, one may simply develop a liking for mathematics and continue to study it beyond the undergraduate level.

2. The selection of a university or college. An important factor in determining what institution of higher learning a student in mathematics should select is the area of his interest: pure mathematics, applied mathematics, or education. Decide upon a school offering a maximum opportunity for developing this interest. A consultation with a favorite mathematics teacher where one is presently studying may be of help. He can suggest several institutions to which the student may write for information.

3. Financial aid. Fortunately, there are several sources from which the graduate student can receive financial assistance. Many of the larger universities have teaching assistantships which offer financial aid in the form of partial or total tuition exemption and, in addition, possibly give a small salary for teaching various lower division undergraduate mathematics courses. (Such teaching is valuable experience as well as a good review of elementary mathematics.) There are also research grants and scholarships offered by the universities, by the government, and by private industry. Summer-time work in industry can be of help. These are but a few of the sources. It might be a good idea to find out details of the availability of assistance well in advance of entrance to the graduate school.

4. Preparatory undergraduate courses. One should select an undergraduate program which will prepare him for his graduate studies. In mathematics, it should include the customary courses of calculus, advanced calculus, solid analytic geometry, differential equations, and college physics. The following subjects are also appropriate: elementary modern algebra, vector analysis, differential geometry, statistical analysis, and mathematical physics.

Also in his program, the student should include an appropriate foreign language. Much of today's mathematics, not in English, is being written in Russian, German, or French (possibly in that order!). At least one of these should be undertaken before entering the graduate school so that the student is already familiar with the process of learning a foreign language. Some institutions require the student to pass an examination in one foreign language before receiving a master's degree. Most institutions require examinations in two foreign languages for the doctorate.

5. Indications of mathematical maturity. Toward the end of a student's undergraduate work, there are several things which indicate that he is acquiring a more mature mathematical outlook. First is his interest in mathematics. He wishes to increase his knowledge of the subject. Second, the student really enjoys making original (at least to him) discoveries. He develops his intuition by applying vague ideas to some problems (it may not be new) to obtain in concrete form a new and interesting method of solution.

Progress in the study of advanced mathematics ultimately will lead the student into a realm in which he can see no immediate application to everyday life. This introduces the third point: no longer does the maturing student ask, "What is this good for?" Highly theoretical thinking has its place in this modern age. The atomic bomb was once merely mathematical theory which later crystallized into reality. Even the idea of an earth satellite was once an abstract notion (entertained as early as the seventeenth century.) There must always be someone who is pushing off into the unknown, in order that there be something new which can be applied to practical usage.

Finally, and not the least in importance, is the regular practice of self-improvement. Each day, a portion of time should be devoted to study. Learn in the quiet of one's sanctuary the methods and results of those who have left behind their mathematical legacies for posterity. If possible, attend lectures and club meetings on imporant mathematical topics.

6. About graduate mathematics subjects. There are two reasons for not giving a description of these. For one thing, it would probably not be understandable to the reader. For another, it would not be an accurate description of the mathematics courses offered by every university. An accompanying chart, however, lists some of the basic subjects, but even these will vary from place to place.

Subjects in mathematics might be classified either as *channel* subjects or as *fringe* subjects. The channel subject is a basic one; it is a trunk from which stem many important branches of mathematics. The fringe subject is not so much needed in a further study of mathematics outside of its own domain. Nevertheless, it has certain intrinsic values. Basic algebra and number theory are examples of a channel subject and a fringe subject, respectively.

Set theory is an excellent example of a channel subject taken from graduate mathematics. (On set theory see [1], [2], [3], [7], and [10].) However, some parts of it are being included more and more in some of the undergraduate subjects, which seems to indicate a modern trend in the undergraduate program. Set theory leads the way to two more channel subjects: modern algebra and set topology. (On modern algebra see [1], [10], and [15]; on topology see [5], [9], [10], and [13].) Classical geometry, untouched by set theory, was once doing a thriving business, among other things, of examining surfaces by use of the calculus. (On geometry see [5].) Now its modern counterpart, modern geometry, has joined hands with topology and to some extent with algebra. (See [5] on topology for examples of this.) Hence, set theory has made inroads there.

7. On grades. The grades obtained by the student while in the undergraduate school will be a decisive factor in allowing him to enter a graduate school. None of these should be below "B" and over half should be "A".

Those same high grades will not come so easily in the graduate school. There, talent is widespread and competition is keen. (The beginning graduate student will have to adjust to this situation!)

8. Obtaining an advanced degree. There are usually three degrees: Master of Science, Master of Arts, and Doctor of Philosophy. However, some institutions offer only one Master's degree, and some do not offer the doctorate at all. For specific information on these, it is advisable to consult the catalogue of the institution concerned.

A thesis is required for the doctorate and sometimes for the master's. It is a written report on a specific research problem carried out by the graduate student. Some institutions offer at the master's level a program in which the student does not write a thesis but must earn more credits in course work in lieu of the thesis.

Finally, there are examinations of various kinds, the number, form, and scope of which are set by the institution. There is usually some sort of qualifying examination; and if a thesis is required, some final examination on it must be passed.

9. After receiving the degree. What happens now depends upon the graduate's interests. The two most common choices are to teach or to work for private industry or the government. The department of mathematics or a placement service at many institutions compiles lists of available opportunities in both.

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(Continued on page 130)

SOME OF THE MAIN SUBJECTS IN MATHEMATICS*
(Fringe subjects are enclosed in brackets [ ]; see section 6)

LEVEL		FIELD: GEOMETRY	ALGEBRA	ANALYSIS
HIGH SCHOOL		Plane Geometry [Solid Geometry]	Elementary Algebra Intermediate Algebra	
HIGH SCHOOL / UNDERGRADUATE		Trigonometry		
UNDERGRADUATE	LOWER DIVISION	Plane Analytic Geometry [College Geometry]	College Algebra (incl. some Theory of Equations)	Calculus
UNDERGRADUATE	UPPER DIVISION (given limited graduate credit)	Solid Analytic Geometry [Non-Euclidean Geometry] [Projective Geometry] Differential Geometry	[Theory of Numbers] Elementary Theory of Matrices Higher Algebra Elementary Set Theory and Mathematical Logic	Differential Equations Advanced Calculus [Partial Differential Equations]
GRADUATE	BEGINNING	Differential Geometry (incl. Tensor Analysis)	General Set Theory [Foundations of Mathematics] Modern Algebra Theory of Fields	Metric Spaces, Measure Theory (Real Variable) Complex Variable Set Topology
GRADUATE	ADVANCED	[Riemannian Surfaces] [Metric Differential Geometry] Theory of Manifolds [Lie Groups]	[Theory of Algebras] Topological Groups and Linear Spaces [Boolean Algebra] Algebraic Topology [Homological Algebra]	[Advanced Set Topology] Banach and Hilbert Spaces [Spectral Theory] [Advanced Functional Analysis]

*Another column on STATISTICS has not been compiled.

Five Mutually Tangent Spheres

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A region of mathematics that has not been explored deeply, but one that has a wealth of simple and complex relationships is that of spheres and tetrahedrons. This paper will develop a formula relating the radii of five mutually tangent spheres.

It is possible to have any three spheres tangent to each other. Let us think of the centers of these three spheres as being in a horizontal plane. It is possible to place a fourth sphere above the three mutually tangent spheres so that it will be tangent to each of them. The only restriction is that the fourth sphere be large enough so that it does not fall through the space between the three spheres. If the fourth sphere is small enough so that it does not project above a plane tangent above to the first three spheres, a fifth sphere can be placed so that it will be tangent to the upper sides of the four given spheres. If the fourth sphere does project above the plane tangent to the first three, then a fifth and largest sphere can enclose the given four spheres so that it will be internally tangent to all of them. A limiting case exists when the fourth sphere is also tangent to the plane tangent to the first three. Then the fifth sphere becomes a plane surface, *i.e.*, a sphere with an infinite radius.

Therefore, for any four given spheres, there are two other spheres tangent to them, both externally, or one small one externally and one larger one internally.

Let the radii of the four given spheres be a, b, c, and d. Of the two spheres which can be tangent to them let the smaller have a radius of e. Let the larger one which could be tangent either externally or internally have a radius of f.

From Figures I and II, $x_a = a$, $x_b = b$, $y_a = 0$, $y_b = 0$, $z_a = z_b = z_c = 0$. By the formula for the distance between two points in 3-space:

(1) $(a + b)^2 = (a + b)^2$.

(2) $(a + c)^2 = (a - x_c)^2 + y_c^2$.

(3) $(a + d)^2 = (a - x_d)^2 + y_d^2 + z_d^2$.

(4) $(a + e)^2 = (a - x_e)^2 + y_e^2 + z_e^2$.

(5) $(b + c)^2 = (b + x_c)^2 + y_c^2.$
(6) $(b + d)^2 = (b + x_d)^2 + y_d^2 + z_d^2.$
(7) $(b + e)^2 = (b + x_e)^2 + y_e^2 + z_e^2.$
(8) $(c + d)^2 = (x_c - x_d)^2 + (y_c - y_d)^2 + z_d^2.$
(9) $(c + e)^2 = (x_c - x_e)^2 + (y_c - y_e)^2 + z_e^2.$
(10) $(d + e)^2 = (x_d - x_e)^2 + (y_d - y_e)^2 + (z_d - z_e)^2.$

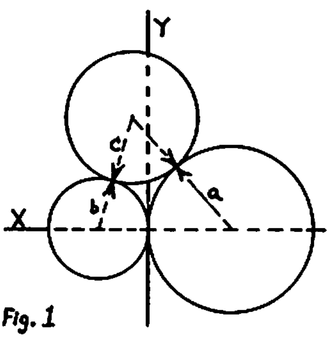

Fig. 1

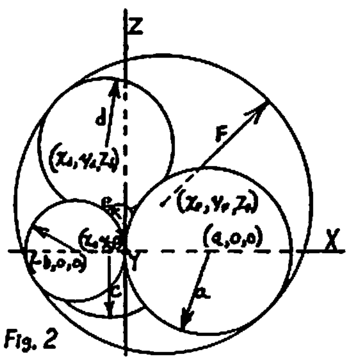

Fig. 2

For the internally tangent sphere f,

(11) $(f - a)^2 = (x_f - a)^2 + y_f^2 + z_f^2.$
(12) $(f - b)^2 = (x_f + b)^2 + y_f^2 + z_f^2.$
(13) $(f - c)^2 = (x_f - x_c)^2 + (y_f - y_c)^2 + z_f^2.$
(14) $(f - d)^2 = (x_f - x_d)^2 + (y_f - y_d)^2 + (z_f - z_d)^2.$

From (2) and (5),

(15) $$x_c = c(b - a)/(b + a).$$

From (3) and (6),

(16) $$x_d = d(b - a)/(b + a).$$

From (4) and (7),

(17) $$x_e = e(b - a)/(b + a).$$

Subtracting (3) from (8), and solving for $(y_c y_d)$,

(18) $$y_c y_d = [2/(a + b)^2][ab(a + b)(c + d) - cd(a^2 + b^2)].$$

Similarly, from (4) and (9),

(19) $$y_c y_e = [2/(a + b)^2][ab(a + b)(c + e) - ce(a^2 + b^2)].$$

Solving (2) for y_c^2,

(20) $$y_c^2 = 4abc(a + b + c)/(a + b)^2.$$

Solving (3) for z_d^2,

(21) $$z_d^2 = [4abd(a + b + d)/(a + b)^2] - y_d^2.$$

Solving (4) for z_e^2,

(22) $$z_e^2 = [4abe(a + b + e)/(a + b)^2] - y_e^2.$$

Expanding (10) and substituting for z_e^2 and z_d^2 from (3) and (4),

(23) $$-x_e x_d - ed + ad + ax_d + ae + ax_e - y_e y_d = z_e z_d \quad \text{or}$$

$$[2/(a + b)^2][ab(a + b)(d + e) - ed(a^2 + b^2)] - y_e y_d = z_e z_d.$$

Squaring both sides, substituting for z_e^2 and z_d^2 from (21) and (22) and eliminating the y's we obtain,

(24) $$e^2[a^2b^2c^2 + a^2b^2d^2 + a^2c^2d^2 + b^2c^2d^2 - abcd(ab + ac + ad + bc + bd + cd)] - e(abcd)(abc + abd + acd + bcd) + a^2b^2c^2d^2 = 0.$$

Let $g = (abc + abd + acd + bcd)$.

$k = (ab + ac + ad + bc + bd + cd)$.

$n = (a^2b^2c^2 + a^2b^2d^2 + a^2c^2d^2 + b^2c^2d^2)$.

$j = abcd$.

Then $$g^2 = n + 2jk.$$

Rewriting (24) in terms of g, k, n, and j,

$$e^2(n - jk) - ejg + j^2 = 0.$$

And solving for e by the quadratic formula

(25) $$\begin{aligned} e &= [jg \pm \sqrt{(jg)^2 - 4j^2(n - jk)}]/[2(n - jk)]. \\ &= [j/2][g \pm \sqrt{g^2 - 4(n - jk)}]/(n - jk) \\ &= j[g \pm \sqrt{3(g^2 - 2n)}]/(3n - g^2). \end{aligned}$$

It should be noted that equation (24) is symmetrical with respect to a, b, c, and d. Also if the fourth sphere does not project above the plane tangent to the first three, then spheres d and e are similarly placed, and it can be seen that d can be exchanged for e, and e for d in equation (24). This can also be proved by solving (24) for d in terms of a, b, c, and e. From this it can be reasoned that e can be interchanged for any of the variables. Or, if any four radii are given, the quadratic in the unknown will have the form of (24) and will be symmetrical with respect to the four radii.

If sphere d does not project above a plane tangent to the upper surfaces of the three given spheres, then it can be seen that the equations for e would be the same whether e was a small sphere in the middle, or a much larger one, on the opposite side of d. Therefore, the solution to (24) will give both possibilities, the large and the small, by considering the plus and the minus signs. This is the general solution for the case when both the spheres e and f are tangent externally to the other four spheres. Letting e represent the smaller radius obtained by using the minus sign, and f the larger radius,

(26) $e, f = [j/2][g \pm \sqrt{g^2 - 4(n - jk)}]/(n - jk)$.

In the limiting case when sphere d is tangent to a plane which is tangent to the upper surfaces of the spheres a, b, c, then the value of f will be infinite. In this case the denominator of (26) will go to zero, or $n = jk$. The value of e can then be determined by properly evaluating the indeterminate form. Since $n = jk$, $f = \infty$, and $e = (j/2)(g - \sqrt{g^2})/(n - n)$, but $g^2 = n + 2jk = 3n$, or $g = \sqrt{3n}$, so,

(27) $e = (j/2)(\sqrt{3n} - \sqrt{3n})/(n - n)$
$= (j\sqrt{3}/2\sqrt{n})(n - n)/(n - n)$
$= (j/2)\sqrt{3/n} = (j/2)\sqrt{3/(jk)} = \sqrt{(3j)/(4k)}$

As radius d increases still more, or the denominator of the right member of equation (26) becomes negative, then radius f will become finite. To derive the correct expression for f in this case, equations (4), (7), (9), and (10) would have to be slightly altered. Basically these equations equate the sum of two radii to the analytical expression for the length of the line connecting the centers of these two spheres. For the case where the largest sphere f would be internally tangent to the other spheres, the difference rather than the sum of the two radii would have to be used in equations (4), (7), (9), and (10). This is given by equations (11) through (14). But since all of these expressions are squared, substituting $-e$ for $+e$ or $-f$ for $+f$ would be mathematically equivalent to using the difference of the two radii. Then in equation (26), it would be $-f$ that would equal the whole expression when the denominator became negative. Whenever equation (26) is solved for f (or the $+$ sign in the numerator is used) if the denominator is negative, then the whole expression should be multiplied by a minus one to make it positive. Therefore, equation (26) represents all possible values of e and f.

Wonders of "i"

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Let us say fractions were invented to indicate the quotient of two integers. For example, 5 divided by 3 is 5/3. As far as I know, there was no resistance to accepting fractions into the number system. Taking another step, we see that the symbol "$\sqrt{\ }$" was invented to indicate the principal square root of a number. It is indeed clear that the positive square root of 4 is 2. Let us stick to positive numbers for the time being. Then the square root of 5 is $\sqrt{5}$. Man seemingly did not resist the acceptance of these numbers, called irrational. Man's resistance to accepting extension of the number system started with the introduction of negative numbers. But soon the negative numbers made their place in arithmetic. In order to have subtraction always possible we define negative numbers. For example, $3 - 8 = -5$. The introduction of negative numbers brings another problem; that is, how to take square roots of negative numbers. For a mathematician this is not a problem at all. He creates another symbol i which he defines to be equal to $\sqrt{-1}$. For him this is just as interesting and usable as -5. At the time of their introduction there was a great resistance to accepting the square roots of negative numbers. As a matter of fact, numbers containing i were named *imaginary*. This is a very bad name for them. What is so real about the symbol 5 which is not real about the symbol i? Anyhow, we continue to use the old names, and we classify numbers as *real* or *imaginary* numbers.

Now if a and b are two real numbers, we call $a + ib$ a complex number. For example, $3 + 5\sqrt{-1}$ is a complex number. The arithmetic of complex numbers is the same as that of real numbers.

Let us remember a few little ideas from our analytic geometry. We know that $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ is the distance between the points (x_1, y_1) and (x_2, y_2). Also let us recall that $y = mx + b$ is the equation of a straight line whose slope is m. The third fact we would like to keep in mind is that for the two lines $y = m_1x + b$ and $y = m_2x + b$; the condition $m_1m_2 = -1$ implies that these two lines are perpendicular.

(Continued on page 88)

Mathematical Approximations Employed In The Physical Sciences

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Not only are various types of approximations often used in the physical sciences, but the theories which form the foundations of the sciences are themselves approximations to nature. These theories are useful only in so far as they may be used to predict with a specified or sufficient degree of accuracy and precision the phenomena which man observes. If more precise agreement is desired between the theoretical effects and the actually perceived effects, refinements of the existing theories must be made or new theories must be devised. As greater precision and accuracy in the observation and measurement of phenomena are attained, more adequate theories must be evolved to account for that which is observed and measured.

Because the theoretical considerations of the advanced aspects of the physical sciences are often marked by the complicated and unwieldy mathematics, approximations are frequently used for practical applications. Such approximations are made both in the derivation of significant formulas and in the evaluation of formulas. When approximations are made in the derivation of a formula, the range of useful employment of the formula is generally limited. Obviously, such a formula will give a close approximation to the phenomenon involved only when the phenomenon is restricted within the bounds of the validity of the approximation. For this reason approximations often limit the application of a formula considerably.

Approximations frequently involve the dropping of terms whose numerical values are small compared with the other terms to which they are to be added. An example which may be taken is a parallel resistance circuit composed of two resistances, one considerably larger than the other. The formula relating the equivalent resistance of any parallel system of two resistances to the individual resistances is $1/R = 1/R_1 + 1/R_2$ where R is the equivalent resistance and R_1 and R_2 are the two resistances in the circuit. If R_2 is much larger than R_1, the term $1/R_2$ is considerably smaller than $1/R_1$ and as a rough approximation may be neglected. If, for example, R_1 is 10 ohms and R_2 is 1000 ohms, $1/R_1$ is then 0.1 and

$1/R_2$ is 0.001. The reciprocal of R is then equal to $0.1 + 0.001$. This may be approximated as 0.1, giving R as 10 ohms. If a more accurate answer is desired, the second term must be included.

Another example in which a term is small enough to be neglected for most practical purposes when added to a large term is offered by optics. It is desired to find the distance d from a point on a spherical lens to the flat surface on which the lens is resting as a function of the distance r from the center of the lens (see Diagram 1). The quantity d can be determined from the equation ob-

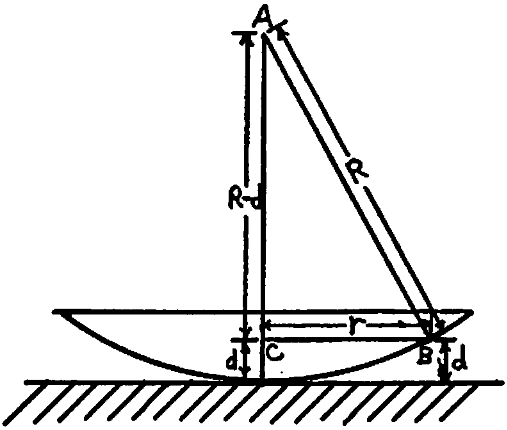

DIAGRAM I.—CROSS SECTION OF LENS.

tained by the Pythagorean theorem for the right triangle ABC,

$$(1) \qquad R^2 = (R - d)^2 + r^2,$$

which on expanding and rearranging becomes $2Rd = r^2 + d^2$, a quadratic in d. In the region that r is small, d^2 is much less than r^2 and as an approximation may be neglected leaving a linear equation in d which may be solved to give

$$(2) \qquad d = r^2/(2R)$$

This solution is valid over the range that the parabola defined by (2) closely approximates the circle represented by the cross section of the lens. This equation is considerably easier to handle than the quadratic equation.

There are two expansions that are used quite frequently to obtain approximations. They are the binomial and the Taylor expansions. The binomial expansion is the representation of a binomial raised to some power and may be expressed by:

$$(a + b)^n = a^n + (n/1)a^{n-1}b + [(n)(n - 1)]/[1 \cdot 2]\, a^{n-2}b^2 + [(n)(n - 1)(n - 2)]/[1 \cdot 2 \cdot 3]\, a^{n-3}b^3 + \cdots$$

The Taylor expansion is the representation of a function by an infinite power series in $x - a$ and may be represented by:

$$f(x) = f(a) + f'(a)(x - a) + [1/2!]f''(a)(x - a)^2 + [1/3!]f'''(a)(x - a)^3 + \cdots$$

Of particular interest is the Maclaurin series, which is the Taylor series with a equal to zero. The Maclaurin series is most often used in approximation work for the functions e^x, sin x, and cos x:

$$e^x = 1 + x + [1/2!]x^2 + [1/3!]x^3 + [1/4!]x^4 + \cdots$$
$$\sin x = x - x^3/3! + x^5/5! - x^7/7! + \cdots$$
$$\cos x = 1 - x^2/2! + x^4/4! - x^6/6! + \cdots$$

When x is small compared with unity, each of these functions may be approximated by the first few terms since the remaining terms will be very small compared with the first few. Thus sin x is often approximated by x for small x and the cos x is approximated by 1 or by $1 - (x^2/2)$ depending on both the magnitude of x and the accuracy and the precision desired.

The binomial expansion may be used to give a closer approximation to the resistance problem than was obtained by completely neglecting the smaller resistance. First it is noted that R may be solved for in the original equation to give $R = R_1R_2/(R_1 + R_2)$. If the numerator and denominator are divided by R_2, $R = R_1/(1 + R_1/R_2)$ is obtained which can be rewritten as $R = R_1(1 + R_1/R_2)^{-1}$. The term on the right can be expanded by means of the binomial expansion: $R = R_1[1 - R_1/R_2 + (R_1/R_2)^2 + \cdots]$. The Taylor expansion may also be used to give the same series. Since R_1 is much less than R_2, R_1/R_2 will be small compared to unity and R_1/R_2 raised to a positive power will be smaller yet. If all the terms in R_1/R_2 are neglected, the same rough approximation that was considered earlier is obtained. If all the terms but the first two are neglected, a reasonably good approximation is obtained; if more terms are kept, the approximation improves still further. If R_1 and R_2 are again given the values 10 ohms and 1000 ohms, the true value for the equivalent resistance is 9.900990099... ohms. If only the first term of the expansion is kept, 10 ohms and an error of 1% may be obtained. If the second term is included in the calculations, 9.900 ohms and an error of .01% will be obtained. This approximation is close enough for almost all practical circumstances which may arise. If a more accurate answer is desired, more terms must be retained.

The lens problem may also be solved by judicious use of the Taylor expansion. This time the variable θ instead of r is used (see Diagram 2). θ, like r, must be kept small if the approximations are

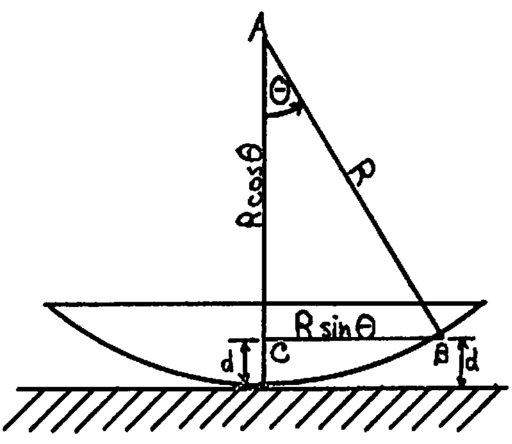

DIAGRAM II.- CROSS SECTION OF LENS

to be valid. The distance d may be seen to be equal to $R - R\cos\theta$. If $\cos\theta$ is approximated by only the first term of the Taylor expansion, the value obtained for d is zero. This result approximates the true value for d only when θ is very small. If, on the other hand, $\cos\theta$ is approximated by the first two terms of the expansion, d is given by $R\theta^2/2$. From the expansion of $\sin x$ it is evident that θ may be approximated by $\sin\theta$ which is seen from the figure to be r/R. If this value is substituted into the equation for θ, one obtains $d = r^2/2R$, which is exactly the approximation obtained by use of the Pythagorean theorem.

The various expansions can greatly simplify the mathematics of some other equations and formulas which are obtained in the sciences. Take, for example, a body falling through a viscous medium which exerts a resistive force directly proportional to the speed of the body through the viscous medium, $F = kv$. The distance y that the body has dropped in time t is found to be $y = (g/k)t - (g/k^2)(1 - e^{-kt})$ where g is the acceleration of gravity. This is a rather unwieldy equation. A much simpler relationship can be deduced for this distance for certain circumstances. If the viscous medium through which the ball is falling is air, k will be very small; and if t is sufficiently small, kt may be made smaller than one. If the exponential term is expanded, the above equation becomes

$$y = (g/k)t - (g/k^2)(1 - 1 + kt - [1/2!]k^2t^2 + [1/3!]k^3t^3 - [1/4!]k^4t^4 + \cdots)$$

which upon collecting terms and rearranging becomes

$$y = (1/2)gt^2 - (1/6)kgt^3 + (1/24)k^2gt^4 + \cdots$$

The first term may be seen to be just that distance which a body would drop if the medium exerted no resistive force. The other terms represent the distance lost due to the presence of the medium. It must be remembered that this equation has limited application since the approximation made in its derivation are valid only when the specified conditions are satisfied.

Many other examples may be given to demonstrate further the limitations and values of mathematical approximations in the sciences. As has been demonstrated in each of the foregoing examples, it is in general true that approximations limit the applications of the resultant equations. Because of this fact it is not always advantageous to make approximations, and if they are made, great care must be taken to employ the approximations only within the limits which are set by the accuracy and precision desired.

Mathematical approximations, if used wisely, will not limit the accuracy and precision of the results. This is true since two other types of approximations are tacitly, but necessarily, made in the solution of any physical problem. The first type concerns approximations which are made in the measurements of the phenomena. The accuracy and precision of this type of approximation is limited by the measuring device and the person operating the device and is thus limited only by the technology and engineering involved. The second type of tacitly assumed approximations is far more fundamental and of much greater interest to the scientist. These approximations are those which are inherent in the theories which attempt to describe observed phenomena mathematically. As was mentioned previously, physical theories can only approximate phenomena, and the degree of accuracy and precision involved is dependent upon the closeness with which the theory represents the workings of nature.

(Continued from page 82)

Now if we bring some complex numbers in, we will find that lines represented by the equation $y = ix + b$ are self-perpendicular and that the distance between any two points on these lines is zero.

Consider

$$y = ix + b.$$

It is clear that the slope is i and

$$i \cdot i = i^2 = (\sqrt{-1})^2 = -1.$$

Therefore, $y = ix + b$ is perpendicular to itself.

Now let (x_1, y_1) and (x_2, y_2) be any two points on $y = ix + b$; *i.e.*, $y_1 = ix_1 + b$, $y_2 = ix_2 + b$.

The distance between these two points is

$$d = \sqrt{(x_1 - x_2)^2 + [(ix_1 + b) - (ix_2 + b)]^2} = 0$$

These lines are called *isotropic* lines of the plane.

Kaleidoscopic Geometry

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The interest in this geometry lies mainly in the form of the graphs of some common equations as compared with their Cartesian form. This geometry is based on the reflections obtained by placing a regular triangular prism upon a sheet of paper. The prism is formed by two congruent mirrors and a blank, the faces of the mirrors reflecting toward the interior of the prism. When the mirrors are viewed from any point in the plane bisecting the dihedral angle between them, the kaleidoscopic effect is evident. The definitions have purposely been kept to a minimum since the main idea is to investigate the graphs of curves. Many of the principles and definitions of Euclidean geometry have been retained.

The model of this plane geometry is a regular hexagon, consisting of an equilateral triangle and its five reflections in the mirrors (See Figure 1).

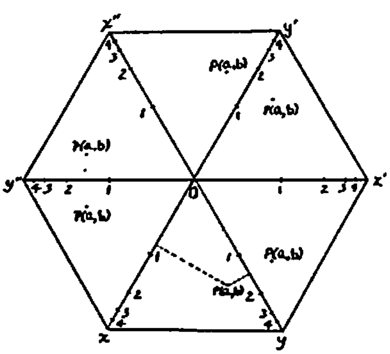

Figure 1

The plane is divided into six equilateral triangles. Each triangle will be called a sextant. A single point will be reflected by the mirrors to give five other points in the plane, one in each of the remaining sextants. This is the basis for the definition,

A k-point will consist of six points, one in each sextant, similarly located.

A point on a line connecting the center of the hexagon to a vertex will be reflected only twice so there will be three and only three points. To have six points, each will be counted as belonging to both of the adjacent sextants. The center of the hexagon will be considered as belonging to each of the sextants and having a multiplicity of six.

There are many ways of defining a coordinate system for this geometry. One of these ways is given. Let the center of the hexagon be the origin and the six lines joining the center to the vertices of the hexagon be the axes. Reflections will separate these lines into two classes. One class will be the x-axes and the other class will be the y-axes. The x-axes and the y-axes will alternate and each sextant will have both an x-axis and a y-axis. These axes will be designated as the x, x', x'', y, y', y'' axes. The xoy-sextant will be called the *primary sextant.*

In defining a number scale each axis will be considered as having a positive direction from the center to the vertex of the hexagon. Let the length of the axis be a unit length (Euclidean) then a distance of n kaleidoscopic units from the origin is the Euclidean distance d where $d = 1 - 1/2^n$. Thus each vertex of the hexagon has a coordinate which is infinite, and the sides of the hexagon join k-points at infinity. The coordinate lines will be the Euclidean normals to the axes. The normal to the x-axis at a distance of a units from the origin will be denoted by $x = a$. Normals to the y-axis will be similarly denoted. The k-point (a, b) is the intersection of the normals $x = a$ and $y = b$.

It is not necessary to have negative values for coordinates in order to locate any point in the plane. Negative numbers are not defined in this geometry. There are k-points with real coordinates which do not intersect within the sextant, such as $(1/2, 2)$. If the k-point (a, b) cannot be plotted within the primary sextant it is called *imaginary.*

If the locus of a point is plotted in one sextant this locus will be reproduced in each sextant. If the path of this moving point is followed until it strikes an axis it will be reflected into the next sextant at that point, making an angle with the normal in the new sextant equal to the angle made with the normal in the former sextant (angle of incidence). Quite often it may occur that a curve will strike and seemingly end at a base (perimeter) line. This does not mean that the curve ends at this point. From the kaleidoscopic model it is seen that the curve appears in each sextant. When a curve reaches the base line in any sextant it will return from the base line in the next sextant and will be said to be continuous. A curve that is continuous in one sextant has the properties of a closed curve in the Euclidean plane.

We are now ready to examine the graphs of equations plotted on the kaleidoscopic model. Given any two k-points in the plane, if a straight (Euclidean) line is drawn connecting the representation of these two k-points in the primary sextant and is extended in both directions, it will intersect at least two of the following: the x-axis, the y-axis, the base line. In any case it will be reflected in the remaining sextants and will be continuous. To plot $f(x, y) = 0$ in kaleidoscopic geometry it is necessary to plot only in the primary sextant.

Let us designate coordinates for the primary sextant in the kaleidoscopic geometry by x and y and coordinates in the Cartesian system by X and Y. Then let us superimpose the kaleidoscopic coordinate system upon the Cartesian coordinate system so as to have a common origin, with the x-axis coinciding with the X-axis and the y-axis lying in the first quadrant of the Cartesian coordinate system. The following equations may be used to transform coordinates and loci from one system to the other:

$$\begin{aligned} X &= 1 - 1/2^x, \\ Y &= (1/\sqrt{3})(1 - 1/2^{y-1} + 1/2^x), \\ x &= -\ln(1 - X)/\ln 2, \\ y &= [\ln 2 - \ln(2 - X - \sqrt{3}Y)]/\ln 2. \end{aligned}$$

Using these equations of transformation the linear equation

$$x + y = -\ln c/\ln 2$$

in kaleidoscopic coordinates becomes the hyperbola

$$X^2 + \sqrt{3}XY - 3X - \sqrt{3}Y = 2(c - 1)$$

in Cartesian coordinates which upon rotation and translation of axes can be put in the form

$$3X'^2 - Y'^2 = 4c.$$

Thus the family of linear equations $x + y = -\ln c/\ln 2$ in kaleidoscopic geometry becomes a family of concentric hyperbolas with a common transverse axis, an eccentricity of 2, and common asymptotes (See Figure 2).

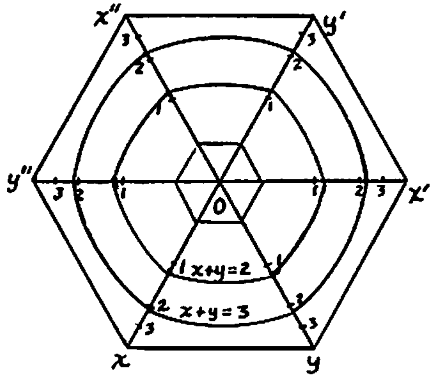

Figure 2

Below are listed the equations of some curves in kaleidoscopic form followed by the equations of the same curves in Cartesian form:

kaleidoscopic form

$x + y = -\ln c/\ln 2$

$y - x = \ln c/\ln 2$

$y = 2x + \ln 4c/\ln 2$

$e^{2(1-y)\ln 2} + e^{-2x\ln 2} = e^{\ln c}$

Cartesian form

$X^2 + \sqrt{3}XY - 3X - \sqrt{3}Y = 2(c-1)$
(hyperbola)

$(c-2)X + c\sqrt{3}Y = 2(c-1)$
(straight line)

$[X + (c-1)]^2 = -2c\sqrt{3}[Y - (c+2)/(2\sqrt{3})]$
(parabola)

$2X^2 + 2\sqrt{3}XY + 3Y^2 - 6X - 4\sqrt{3}Y = c - 5$
(ellipse)

The two linear equations $2x - 3y = 1$ and $2x + 2y = 3$ do not intersect since the point (1.1, .4) is imaginary. This produces the peculiar situation of two linear curves that do not intersect even though they have different slopes (in the usual meaning).

A curve is distorted from its graph in the Euclidean plane due to:

(1) the addition of new axes,
(2) not using negative values,
(3) the metric defined on the axes,
(4) the reflections.

The graphs of quadratic equations in x and y become quite complicated. A few simple examples will be given. The graph of $y^2 = x$ is shown in Figure 3.

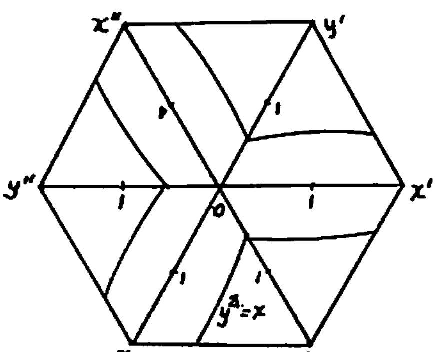

Figure 3

The graphs of some other simple quadratic forms are shown in Figure 4. The curves are plotted in the first sextant only.

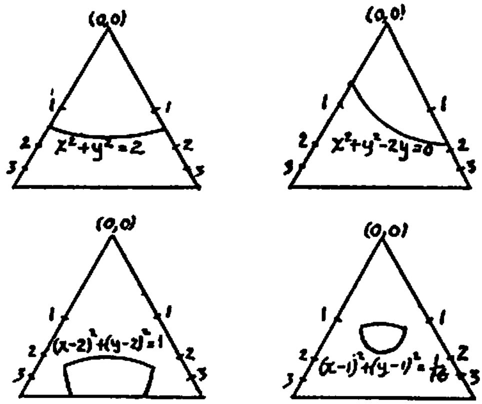

Figure 4

There is no claim on the part of the writer of this paper that the subject has been exhausted. Perhaps you, the reader, will want to make similar investigations or extend the results of this paper.

Editorial Note —

Any further investigation along the lines suggested by this article will be welcomed for consideration for publication in THE PENTAGON.

"Round numbers are always false."

—SAMUEL JOHNSON

Career Opportunities for the Student of Mathematics

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1. Introduction. Since mathematics is the tap root of the tree of knowledge, "student of mathematics" may be considered a phrase which could be applied to almost anyone—from the manual laborer to the mathematician. However, to limit the scope of this article, only those whose life work is most closely related to mathematics shall be considered fully. Before passing on to the main body of this article, some remarks should be made concerning the opportunities of the high school student who has pursued enough courses in mathematics to be interested in a career which entails some mathematics. There is, of course, a multitude of ordinary jobs which require general mathematical knowledge and skill. These jobs range from the business world to the farm. In business, we find the bookkeepers and clerical workers who must not only have achieved proficiency in the computational methods of mathematics but also must know something of the language of mathematics. Calculating machines, mathematical tables, and graphs are everyday tools to these people. The "Guidance Pamphlet in Mathematics for High School Students," which was the final report of the Commission on Post-War Plans of the National Council of Teachers of Mathematics, lists well over fifty apprenticeable trades that require from two to six years of mathematics. Basic to the former is a good working knowledge of business arithmetic. All of these occupations mentioned might include people under the category "students of mathematics." Mathematics required for these positions is a little more than the mathematics needed for everyday life.

What are the future vocational opportunities for the college student of mathematics? Roughly speaking, we may group professional mathematicians into two classes: (1) Those who pursue fields of pure mathematics, and (2) those who pursue fields of applied mathematics.

The pure mathematicians, in general, will consist of those who work in the foundations and fundamental subjects which at the time might not have applications to any of the sciences. This group would include the researchers of our leading colleges and universities as well as a limited number of pure researchers in industry and government. Many leading industrial organizations, such as Bell Laboratories, today carry on programs in basic research. The misconception that the pure mathematician is one who must satisfy his mathematical curiosity merely by playing with principles and laws of mathematics while idling away his time in some "Ivory Tower" away from the world, is slowly disappearing. The fact that industry has recognized the need for basic research in mathematics without any immediate application, has helped to bring about the passing of this illusion. Basic research has been and probably will continue to be considered the forerunner of applied mathematics.

The applied mathematicians, the second group, are those who work at problems which pertain directly to the needs of business, industry, government, insurance, and the sciences. Here the laws and formulas derived by the pure mathematicians and by the applied mathematicians are put to work to solve the immediate problems of the times. More often than not, the artificial demarcation separating the pure and the applied mathematicians is poorly made. For the pure mathematicians very often develop mathematics for which an application can be immediately found although this was not their primary objective just as the applied mathematician serving the needs of particular problems may contribute something entirely new to the field of mathematics. Such a contribution may well be the fundamental concept underlying a field of pure mathematics.

Another way by which we may group mathematicians is according to the type of employers which can be found, such as the federal government; colleges, universities, and other educational institutions; and industrial laboratories. Today the competition to obtain men trained in mathematics has reached a high peak. It is not easy to determine exactly how many trained mathematicians are employed by each one of the major groups above. Although some statistics can be found, many factors seem to confuse the issue. This is due to the fact that job descriptions which appear with erroneous titles such as engineer, analyst, physicist, and the like, are in reality jobs held down by people trained as mathematicians. The title *mathematician* is very often not applied to the position *mathematician*. There is a tendency for this to be less true, however, since industry and government alike are beginning to recognize the need for professional people trained primarily in the field of mathematics. This can be exemplified by the following comparison. In the final report of the Commission on Post-War Plans of the National Council of Teachers of Mathematics, a table appeared describing the scientific population by field and place of employment in 1947 (see Table 1.) This estimate showed no mathematicians in the industrial laboratories. Two reasons given for this being that probably only a small number was actually employed by industry and the other being the condition previously mentioned, that few research workers have the title mathematician. Today, however, leading industries have departments composed solely of mathematicians.

SCIENTIST POPULATION BY FIELD AND PLACE OF EMPLOYMENT (Estimate, 1947)[1]

Field of Work	Federal Gov't	Colleges and Universities	Industrial Laboratories	Total
Agricultural Sciences	7,900	4,700	——	12,600
Biological Sciences	1,200	5,900	1,750	8,850
Medical Sciences	550	13,500	250	14,300
Physical Sciences	8,000	17,500	27,500	53,000
Physics	1,750	3,900	2,800	
Chemistry	2,600	5,800	22,000	
Mathematics	750	5,800	——	
Other	2,900	2,000	2,700	
Engineering	11,400	9,000	21,500	41,900
Miscellaneous	950	——	5,800	6,750
Total	30,000	50,600	56,800	137,400

Table 1

Some of the major fields of employment for the mathematician will now be considered.

2. The Teaching Profession. Probably the greatest number of mathematicians are employed as teachers of mathematics. According to Walter James Greenleaf, in his book *Occupations and Careers* the largest group of professional workers is the group of teachers (See Table 2). Of course, it would be somewhat difficult to estimate how many of these teachers are teachers of mathematics. Mathematics teachers find employment in the elementary schools, high schools, junior colleges, colleges, and graduate schools. Of the 1,120,000

[1] Commission on Post-War Plans, "Guidance Report," The Mathematics Teacher, 1947, p. 17.

teachers employed in the elementary schools and high schools, Greenleaf estimates that only 286,000 are men. It is here that women mathematicians find their greatest employment opportunity. Of the 125,000 college teachers, 96,000 are men. Therefore, we would expect only a few women mathematicians to be employed in this category as compared to the number of men.

PROFESSIONS WITH LARGEST NUMBER OF WORKERS (1950)[2]

Teachers	1,120,000
Engineers	525,000
Nurses	398,000
Physicians	192,000
Lawyers	180,000
Clergymen	165,000
Musicians	153,000
College Instructors	125,000
Dentists	75,000

Table 2

The typical college instructor and professor would necessarily have obtained a doctor's degree or be well along the road to the completion of that degree. The chance of obtaining a college position without a master's degree is very slight. A list of the major graduate schools in 1947 offering doctorates in mathematics follows:

LIST OF SEVERAL GRADUATE SCHOOLS OFFERING DOCTORATES IN MATHEMATICS[3]

Calif.	Berkeley	University of California
	Pasadena	California Institute of Technology
	Stanford	Stanford University
Colo.	Boulder	University of Colorado
Conn.	New Haven	Yale University
D. C.	Washington	Catholic University
	Washington	George Washington University
Ill.	Chicago	Northwestern
	Chicago	University of Chicago
	Urbana	University of Illinois
Iowa	Ames	Iowa State College
	Iowa City	State University of Iowa

[2] Walter James Greenleaf, Occupations and Careers, McGraw-Hill Book Co., 1955.
[3] Commission on Post-War Plans, op. cit., p. 23.

Ind.	Bloomington	Indiana University
	Notre Dame	University of Notre Dame
	Lafayette	Purdue University
Kan.	Lawrence	University of Kansas
Ky.	Lexington	University of Kentucky
La.	Baton Rouge	University of Louisiana
Md.	Baltimore	Johns Hopkins University
	College Park	University of Maryland
Mass.	Cambridge	Harvard University
	Cambridge	Mass. Institute of Technology
	Cambridge	Radcliffe College (women)
Mich.	Ann Arbor	University of Michigan
Minn.	Minneapolis	University of Minnesota
Mo.	Columbia	University of Missouri
	St. Louis	St. Louis University
	St. Louis	Washington University
Neb.	Lincoln	University of Nebraska
N. J.	Princeton	Princeton University
N. Y.	Ithaca	Cornell University
	New York	Columbia University
	Troy	Rensselaer Polytechnique Institute
	New York	New York University
N. C.	Durham	Duke University
	Chapel Hill	University of North Carolina
Ohio	Cincinnati	University of Cincinnati
	Columbus	Ohio State University
Pa.	Philadelphia	Bryn Mawr (women)
	Philadelphia	University of Pennsylvania
	Pittsburgh	University of Pittsburgh
R. I.	Providence	Brown University
Tenn.	Nashville	George Peabody College
Tex.	Austin	University of Texas
	Houston	Rice Institute
Va.	Charlottesville	University of Virginia
Wash.	Seattle	University of Washington
W. Va.	Morgantown	University of West Virginia
Wis.	Madison	University of Wisconsin
	Milwaukee	Marquette University

For mathematics teachers in high schools, starting salaries will vary from state to state. However, the minimum starting salaries are now generally above the $3,000 level and are approaching the $4,000 level. Maximum salaries range close to the $7,000 mark with $8,600 being the top regular maximum salary in the nation. These maximum salaries are usually obtained by yearly increments.

The majority of these increments are now about $200. Other increments may be expected by the high school teacher of mathematics when he has earned his master's degree and additional graduate credits.

For the college teacher of mathematics starting salaries will vary with academic background and experience. The steps towards a full professorship start with the instructors position, followed by the assistant professorship, and later the associate professorship.

3. The Actuarial Profession. An actuary is a mathematician who is employed by a group to study data about deaths, accidents, thefts, and various other conditions which affect insurance. The actuarial field is small and expanding at the present time, with approximately 800 actuaries in the United States and Canada. The field is strictly regulated by the Society of Actuaries, an organization set up to determine the mathematical requirements and standards for actuarial jobs. To become a Fellow Actuary, one must pass a series of eight written examinations which are given yearly by the Society of Actuaries. The first five examinations constitute Associateship, with the remaining three being necessary for Fellowship. A brief description of the subjects covered follows:

SYLLABUS OF EXAMINATIONS[4]

Associateship-Parts 1 to 5

Part	Time Allowed	Subjects
1	One hour	Language aptitude
2	Three hours	General mathematics (Algebra, trigonometry, coordinate geometry, differential and integral calculus)
3	Three hours	Special mathematics (Finite differences, probability, statistics)
4	Six hours	a) Compound interest and annuities-certain b) Life contingencies, including multiple decrement functions
5	Six hours	a) Construction of mortality and disability tables, including the elements of graduation b) The sources and characteristics of the principal mortality and disability tables (including the methods used in the construction and graduation of such tables) and of the principal mortality and disability investigations

[4] Society of Actuaries, "Requirements for Admission," Chicago, Illinois, November, 1952.

Part	Time Allowed	Subjects
		c) Selection of risks, including underwriting of disability and additional accidental death benefits, insurance of under-average lives, and premiums for extra hazards

Fellowship—Parts 6 to 8

Part	Time Allowed	Subjects
6	Six hours	a) Gross premiums for insurance, annuities, disability, and other benefits
		b) Valuation of the liabilities of life insurance organizations
		c) Nonforfeiture values and change or reinstatement of life insurance contracts
7	Six hours	a) Analysis and distribution of surplus
		b) Life insurance accounting
		c) Investment of life insurance funds and valuation of assets
8	Six hours	a) Life insurance law
		b) Employee retirement plans
		c) Group insurance
		d) Social insurance and allied programs
		e) Actuarial phases of agency problems

Every large insurance company contains an actuarial department in which these mathematicians may work. The New York and Connecticut area employs the largest number of actuaries. Federal agencies such as the Railroad Retirement Board and the Social Security Board also employ actuaries, as do private institutions. Actuaries employed by private institutions are usually employed in the role of consultants to determine pensions, annuities, retirement benefit plans, accident rates, population growth, and similar studies.

In order to prepare for a career as an actuary, one should of course prepare himself in the field of mathematics. There are only a few universities which offer complete training in actuarial mathematics. Among these are the University of Michigan, University of Wisconsin, Pennsylvania State University, and Occidental College in the United States, as well as the University of Toronto and Manitoba University in Canada. Larger insurance companies in the New York area conduct their own training programs to fulfill the needs of young actuarial trainees.

Although both men and women are eligible to take the examinations, the possibilities for women are limited. Many of the problems of an actuary are more in the realm of business than mathematics and for this reason it is the policy of several insurance companies to discourage women from obtaining Fellowships in the Actuarial Society. Usually women do not attempt to pass examinations beyond the preliminary examinations which consist of the first three parts.

Starting salaries in the actuarial field for persons who have graduated with a degree in mathematics and have passed the first two examinations is approximately $350 per month. With the additional third examination the starting salary approaches $400 per month. The Society of Actuaries estimates that the average Fellow Actuary earns well above $8,000 per year.

4. Industrial Mathematicians. Industry is demanding more and more scientists and engineers with widely-varying specializations, aptitudes and levels of training. Many industries, such as the Bell Telephone Laboratories, are realizing the need for mathematicians as such, not as engineering aides or assistants. In the Bell Telephone Laboratories, for example, mathematicians are not dispersed through the technical departments, but are organized into a separate department of their own. This mathematical department operates with the rest of the laboratories in the role of a service agency and as a collaborator on projects. Some of the fields of work in which these mathematicians engage include such old fields as network, transmission, and propagation theory and such newer fields as information theory, non-linear equations, electronics, and the study of control systems. Problems in classical and modern analysis, the algebras, logic, modern probability and statistics, game theory, and fields such as numerical analysis are investigated by these mathematicians. Of the new employees hired yearly approximately one half are hired on the bachelor's level and the remainder have master's or doctor's degrees.

One of the chief industries to employ mathematicians today is the aircraft industry. The aircraft industry is relying more and more upon the high-speed, large-scale digital computers and analogue computers. With the development of these computing machines has come a revision of the study of numerical analysis. Large scale computers such as the ERA 1103 (Univac Scientific Research Computer) and the IBM 701 have provided many challenging opportunities for the mathematician. Mathematical investigations which may have otherwise taken years of concentrated study may now be solved in a relatively short time. The following is a chart taken from a report by

Charles R. Strang to the Joint Aiee Ire Computer Conference which will show the type of problems in which mathematicians engage in the aircraft industry.

SOME TYPICAL AIRCRAFT ENGINEERING PROBLEMS[5]

Method of Solution / Design Problem	Simultaneous Equations	Differential Equations	Matrix Algebra	Harmonic Analysis	Statistics and Probability	Non-Linear Algebraic Equations
Acoustical Studies						XX
Aerodynamic Performance	XX					
Aerodynamic Stability		XX	XX			
Aeroelastic Studies	XX	XX				
Airfoil Pressure Distributions				XX		
Autopilot Design		XX				
Catapult Launch Analysis		XX				XX
Continuous Beam Analysis	XX		XX			
Control System Transfer Functions				XX		
Flutter Analysis	XX	XX	XX			
Fuselage and Wing Section Analyses	XX		XX			
Landing Gear Spin-up Analysis		XX				
Lofting Calculations						XX
Miscellaneous Curve Fitting	XX		XX			XX
Miscellaneous Data Reduction	XX		XX	XX	XX	
Missile Tactical Employment Studies		XX			XX	
Radome Design			XX			
Supercharger Vane Design		XX				
Thermodynamic Analysis						XX
Trajectories of Airplanes and Missiles		XX				
Wing Spanwise Life Distribution	XX					

Industry may also employ the mathematician in the role of statistician and quality control specialist. The government is one of the chief competitors of industry for qualified statisticians. Business and economic agencies also employ numerous statisticians for purchasing, advertising, and marketing analysis. Statisticians usually

[5] Charles R. Strang, "Computing Machines in Aircraft Engineering," Joint Aiee Ire Computer Conference, December, 1951.

work in some applied field of mathematics such as economics, finance, physics, biochemistry, chemistry sociology, psychology, agriculture, and education. The statistician may be engaged to plan and direct statistical research projects, or he may serve as a consultant for complex administrative or research problems and carry out technical statistical studies.

According to a questionnaire sent out in October 1954 by Arnold Lee Janousek and Charles R. Deeter[*] to 122 firms employing mathematicians, mathematicians are needed in diversified fields of knowledge as well as in research in pure mathematics. According to the 75% reply obtained, mathematicians are employed in four major types of employment: (1) computational, (2) statistical, (3) actuarial, and (4) research. Mathematicians were found to be needed in these fields at all educational levels. Only the atomic energy field had no opening at the master's level. An explanation for this fact may be that the type of work involved in this field was either at such a high level as to require the work of Ph.D.'s or was the type of general calculational work which might be done by mathematicians at the bachelor's level. The investigators of this study found that starting salaries varied, but on the average a mathematician with a bachelor's degree might expect approximately $350 a month, $425 per month at the master's level, and $500 per month at the Ph.D. level. After ten years of work, the anticipated salary range was $10,000 to $12,000 per year.

Aside from the salary consideration, industry is offering many other inducements to the mathematician. The tuition-refund program, which the majority of the industries have in operation, might be considered to be the most influential of these, encouraging the college graduate to continue his studies at the expense of his employer. Company-paid retirement plans and accident and sickness benefits are added inducements.

5. Government Mathematicians. The federal government, through the U. S. Civil Service Commission, Washington, D.C., employs almost every type of mathematician for work in the Ordnance, Signal Corps, and Engineering Corps of the Department of Defense; in the Naval Research Laboratories; the FBI; the Bureau of Standards; the Coast and Geodetic Survey; and other departments of the government. The government employs mathematicians in the ca-

[*] Arnold Lee Janousek and Charles R. Deeter, "Job Opportunities in Mathematics," The Pentagon, Fall, 1955.

pacity of statisticians and actuaries as well. The Civil Service Commission regularly announces available positions in mathematics. Openings are filled by competitive examinations or evaluation of qualifications or both.

The General Schedule Wage Scale set up by the Civil Service Commission ranges from GS-1 to GS-18. Salary is determined by the GS rating. Mathematicians with a college degree start at the GS-5 level which is approximately $4,500 a year. After six months' experience the employee is eligible for a GS-7 rating which is approximately $5,000 a year. After an additional year's experience he is eligible for a GS-9 rating. In many cases additional educational credits may be substituted for experience. Aside from salary increments according to the GS level, there are additional increments contained in each GS level. Salaries at the top of the General Schedule Wage Scale are at the $12,000 mark.

6. Summary. The well-qualified mathematician of today virtually has a choice of any one of a number of diversified organizations in which he may find satisfying employment. The duties and responsibilities he must fulfill vary immensely with the choice of his field. However, the choice is here today as never before. The demand for mathematicians heightens every day. This article has attempted to present a brief description of the various opportunities for the college graduate with a mathematics major.

"It is a pleasant surprise to him [the pure mathematician] and an added problem if he finds that the arts can use his calculations, or that the senses can verify them, much as if a composer found that the sailors could heave better when singing his songs."

—George Santayana

The Problem Corner

EDITED BY J. D. HAGGARD

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond the calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before October 1, 1958. The best solutions submitted by students will be published in the Fall, 1958, issue of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to J. D. Haggard, Department of Mathematics, Kansas State Teachers College, Pittsburg, Kansas.

PROBLEMS PROPOSED

111. *Proposed by C. W. Trigg, Los Angeles City College.*

The letters in (HI)(VE) = BBB represent distinct digits, some four of which are consecutive. Decode the equation.

112. *Proposed by the Editor.* (From *The American Mathematical Monthly.*)

For what positive values of a is $\log_a b < b$ for all positive b?

113. *Proposed by the Editor.* (From *The Mathematics Teacher.*)

Find two similar triangles which are non-congruent but have two sides of one equal to two sides of the other.

114. *Proposed by the Editor.* (From Robinson's *Mathematical Recreations,* 1851.)

Professor E. P. B. Umbugio has recently been strutting around because he hit upon the solution of the fourth degree equation which results when the radicals are eliminated from the equation:

$$x = (x - 1/x)^{1/2} + (1 - 1/x)^{1/2}$$

Deflate the professor by solving this equation using nothing higher than quadratic equations.

115. *Proposed by R. G. Smith, Kansas State Teachers College, Pittsburg.*

Given two points A and B in three dimensional Euclidean space with distances of a and b respectively from a line l, show that no point interior to the segment AB has distance to l greater than max (a, b).

SOLUTIONS

106. In an isosceles triangle ABC, $AB = BC = b$, $AC = a$, $\angle ABC = 20°$. Prove $a^3 + b^3 = 3ab^2$.

Solution by Stephen Strom, Bronx High School of Science, Bronx, New York.

Since the triangle is isosceles drop a perpendicular from B to the base AC, then

$$\sin 10° = a/2b,$$

Using the trigonometric identity

$$\sin 3\theta = 3 \sin\theta - 4 \sin^3\theta$$

which with $\theta = 10°$ gives $\sin 30° = 3(a/2b) - 4[a/(2b)]^3$

but $\sin 30° = 1/2$, so, $1/2 = 3a/(2b) - a^3/2b^3$

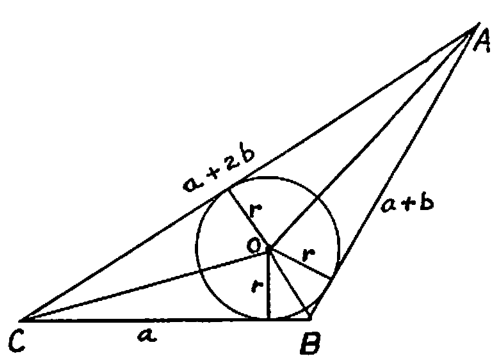
or $a^3 + b^3 = 3ab^2$.

Also solved by Ernest Isaacs, California Institute of Technology, Pasadena, California; Louis L. Hungate, Texas Technological College, Lubbock, Texas; Mark Bridger, Bronx High School of Science, Bronx, New York; Vernon Powers, Kansas State Teachers College, Pittsburg, Kansas; Phillip Griffiths, Wake Forest College, Winston Salem, North Carolina.

109. The lengths of the sides of a triangle are in arithmetic progression. Prove that the radius of the inscribed circle is one third the length of one of the altitudes.

Solution by Mark Bridger, Bronx High School of Science, Bronx, New York.

Consider $\triangle ABC$ with sides a, $a + b$, $a + 2b$, and inscribed circle with center at O and radius r.



Area of $\triangle BOC = ra/2$
Area of $\triangle BOA = (r/2)(a + b)$
Area of $\triangle AOC = (r/2)(a + 2b)$

Therefore, the area, K, of $\triangle ABC = (r/2)(a + a + b + a + 2b)$
$= (3r/2)(a + b)$ or $r = 2K/[3(a + b)]$

Let H be the altitude to side AB, then
$K = (H/2)(a + b)$

Substituting this value of K into the last equation above gives
$r = H/3$.

Also solved by Vernon Powers, Kansas State Teachers College, Pittsburg, Kansas.

110. Prove that if a, b, c, d are positive numbers and the system of inequalities $ax - by < 0$, $dy - cx < 0$, $x > 0$, $y > 0$, has a solution, then $ad - bc < 0$, and conversely.

Solution by Vernon Powers, Kansas State Teachers College, Pittsburg, Kansas.

a, b, c, d are each positive, and the inequalities $x > 0$, $y > 0$, $ax - by < 0$, $dy - cx < 0$ have a solution. Thus:

$ax < by$ and $dy < cx$,
so that by multiplication we get $adxy < bcxy$
or $ad - bc < 0$.

Conversely we suppose a, b, c, d are positive with $ad - bc < 0$, and we are to show that there exists x and y such that if $x > 0$, and $y > 0$, then $ax - by < 0$ and $by - cx < 0$.

Between any two unequal positive numbers such as ad and bc there is always a positive number; for example, their mean M. Thus:

$ad < M < bc$

Let $y = M > 0$, and $x = bd > 0$, then $y/x = M/bd$, $M = ybd/x$.
$ad < (ybd/x)$ and $(ybd/x) < bc$
which gives $ax - by < 0$ and $dy - cx < 0$.

Also solved by Ernest Isaacs, California Institute of Technology, Pasadena, California.

The Mathematical Scrapbook

EDITED BY J. M. SACHS

The solution of the difficulties which formerly surrounded the mathematical infinite is probably the greatest achievement of which our age has to boast.

—BERTRAND RUSSELL, *The Study of Mathematics*, 1910

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The new mathematics is a sort of supplement to language, affording a means of thought about form and quantity and a means of expression, more exact, compact, and ready than ordinary language. The great body of physical science, a great deal of essential facts of financial science, and endless social and political problems are only accessible and only thinkable to those who have a sound training in mathematical analysis.

—H. G. WELLS, *Mankind in the Making*, 1904

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John Wallis (1616-1703) invented the symbol ∞ for infinity.

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THE CASE OF THE CURIOUS VASE. Can you imagine a vase having a small enough volume so that full, it contains less water than an ordinary drinking glass, and yet large enough so that all of the paint in the world will not suffice to paint it? Consider the solid of revolution obtained by revolving the hyperbola $xy = 1$ about the x-axis. Set up the integrals for the volume of this solid between $x = 1$ and $x = h$ and for the surface area between the same limits. Allow h to approach infinity and see if this vase satisfies the conditions. (Let the common unit on the coordinate axes be one inch. It has been suggested that the surface area integral yields readily to the substitution $u = 1/x^2$. Can you do this integration by other methods or other substitutions also?)

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In the history of mathematics, the approach to problems has sometimes been divided into three main categories, the rhetorical, the syncopated, and the symbolical. In the first, the problem is attacked through the medium of ordinary language, either spoken or written. In the second we get a mixture of language approach and symbols which are likely to be based on these abbreviations. The last approach is through symbols with the language playing a minor role if, indeed, any direct role at all.

In our times it seems scarcely necessary to state reasons for preferring the symbolical to the other two approaches. Essentially we expect more, in matters of logic and in the application of logical method to mathematics and science, from symbolism than from rhetoric. We expect more in the way of clarity in expressing and elaborating logical sequences. We expect economy in expression in which the heart of the matter lies exposed, stripped of the frills and ambiguities of the language of words. We have even come to expect that the generality and abstractness of the symbols may cause us to see relations obscured by language. Not all people in all times have agreed to these stated expectations.

Stimulated by the final sections in the last chapter of Cajori's *History of Mathematical Notations,* Vol. 1, the Editor of the Scrapbook has attempted to collect a few historical notes and quotations indicating some of the development of the use of symbolism.

In the middle 17th century Thomas Hobbes wrote,
". . .So there is double labour of the mind, one to reduce your symbols to words, which are also symbols, another to attend to the ideas which they signify. Besides if you consider how none of the ancients ever used any of them in their published demonstrations of geometry, nor in their books of arithmetic. . .you will not, I think, for the future be so much in love with them."[1]

It is perhaps an interesting commentary on the development of symbolism to note that it was far easier to find apologists than critics.

Two hundred years later George Boole published a small book. Of this book Bertrand Russell has said, "Pure mathematics was discovered by Boole in a work called *The Laws of Thought*. . .. His work was concerned with formal logic, and this is the same thing as mathematics."[2]

Most of the following illustrations deal with the use of symbolism in facilitating the use of the techniques of mathematics. However, as the quotation from Leibniz indicates, men were concerned with a symbolic language long before the time of the ingenious Boole. Much of the material contained herein came from Cajori's book or from sources referred to in that book.

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1 Sir William Molesworth, The English Works of Thomas Hobbes, Vol. VII, London, 1845.
2 Bertrand Russell, International Monthly, 1901.

The first printed edition of Euclid's *Elements* and the earliest translations of Arabic algebras into Latin contained little or no mathematical symbolism. During the Renaissance the need of symbolism disclosed itself more strongly in algebra than in geometry. During the sixteenth century European algebra developed symbolisms for the writing of equations, but the arguments and explanations of the various steps in a solution were written in the ordinary form of verbal expression.

—CAJORI, *A History of Mathematical Notations*

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Luca Pacioli in his *Summa* published in Venice in 1494 used *co.*, an abbreviation for the word *cosa* meaning *thing*, for the unknown quantity in an equation as we use the letter x or y. He used *ce.*, for *censo,* as we would use x^2; the symbol *Rx* was used for the word *radix* to indicate roots. The plus and minus *piu* and *meno* in Italian were indicated by the letters *p* and *m* with curved overlines which were dropped at a later date. The equation $x^2 - 3x + 1 = 5$ would have been written as 1.*ce.m* 3.*co.p*.1. — 5. The dash was used by Pacioli as an equals sign. This kind of mixture, abbreviated rhetoric plus symbols which rarely meant the same thing from page to page of a single author much less from one author to another, seems to have been characteristic of the 15th and 16th century algebra. The symbols + and − appeared in Germany in the late 15th century. The competition between these symbols and the letters *p* and *m* lasted almost 200 years.

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Curcus Mathematicus (1634) written by Pierre Herigone showed a great inventive spirit on the part of the author as well as a great determination to further the cause of the symbol. Herigone used symbolism on Euclid as well as in his algebra. Many of the symbols shown were not his inventions but were adopted from colleagues and predecessors. Some of the symbols have persisted to this day, some have not; but Herigone is an important figure for his ingenuity. In his book, he says, "I have invented a new method of making demonstrations brief and intelligible, without the use of any language."

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In his *La Geometrie* in 1637, René Descartes used exponents much as we do today, followed Thomas Harriot's lead in the use of small letters, used a stylized *æ*, the first two letters of the word *aequalis* for an equals sign, and adapted the German radical sign to its present form. Here we find an appreciation of the generality of the symbolic approach as the symbolism of algebra is applied liberally to geometry.

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Descartes says, "Often it is not necessary thus to draw the lines on paper, but it is sufficient to designate each by a single letter. Thus to add the lines BD and GH, I call one *a* and the other *b*, and write $a + b$. Then $a - b$ will indicate that *b* is subtracted from *a*; *ab* that *a* is multiplied by *b*; a/b that *a* is divided by *b*; *aa* or a^2 that *a* is multiplied by itself; a^3 that this result is multiplied by *a*, and so on indefinitely. Again if I wish to extract the square root of $a^2 + b^2$, I write $\sqrt{a^2 + b^2}$; · · ·. Here it must be observed that by a^2, b^3, and similar expressions, I ordinarily mean only simple lines, which, however, I name squares, cubes, etc., so that I may make use of the terms employed in algebra."

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The Englishman, William Oughtred, invented or promulgated more than a hundred symbols and played a great role in the development and popularity of symbols. In his own words chosen from *The Key of the Mathematicks* (1647), ". . .Which Treatise being not written in the usual synthetical manner, nor with verbous expressions, but in the inventive way of Analitice, and with symboles or notes of things instead of words, seemed unto many very hard; though indeed it was but their owne diffidence, being scared by the newness of the delivery; and not any difficulty in the thing it selfe. For this specious and symbolical manner neither racketh the memory with multiplicity of words, nor chargeth the phantasie with comparing and laying things together; but plainly presenteth to the eye the whole course and processe of every operation and argumentation."

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. . .This manner of setting downe theoremes, whether they be Proportions, or Equations, by Symboles or notes of words, is most excellent, artificiall, and doctrinall. Wherefore I earnestly exhort every one, that desireth though but to looke into these noble Sciences

Mathematicall, to accustome themselves unto it: and indeede it is easie, being most agreeable to reason, yea even to sence. And out of this working may many singular consectaries be drawn: which without this would, it may be, for ever lye hid.

—WILLIAM OUGHTRED, *Circles of Proportion*, 1632

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The influence of the mathematics of Leibniz upon his philosophy appears chiefly in connection with his law of continuity and his prolonged efforts to establish a LOGICAL CALCULUS. . .to find a Logical Calculus (implying a universal philosophical language or system of signs) is an attempt to apply in theological and philosophical investigations an analytic method analogous to that which proved so successful in Geometry and Physics. It seemed to Leibniz that if all the complex and apparently disconnected ideas which make up our knowledge could be analysed into their simple elements, and if these elements could each be represented by a definite sign, we should have a kind of "alphabet of human thoughts." By the combination of these signs (letters of the alphabet of thoughts) a system of true knowledge would be built up, ... Thus it seemed to Leibniz that a synthetic calculus, based upon a thorough analysis, would be the most effective instrument of knowledge that could be devised. "I feel," he says, "that controversies can never be finished, nor silence imposed upon the Sects, unless we give up complicated reasonings in favor of simple calculations, words of vague and uncertain meanings in favor of fixed symbols."

—ROBERT LATTA, *Leibniz, The Nonadology*, 1898

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Johann Heinrich Rahn in his book "Teutsche Algebra" published in Zurich in 1629, first used the symbol ÷ for *division* in print, the symbol * for *multiplication* and the familiar ∴ for *therefore.*

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John Wallis adopted the new symbolism, especially the innovations of Oughtred, and used them extensively. Opposition to his use of symbols was expressed with great determination by the English philosopher Thomas Hobbes, ". . .And for your Conic Sections, it is so covered over with the scab of symbols, that I had not the patience to examine whether it be well or ill demonstrated. . . .Symbols are poor unhandsome, though necessary scaffolds of demonstration.

. . .Symbols though they shorten the writing, yet they do not make the reader understand it sooner than if it were written in words." (Quotation from *The English Works of Thomas Hobbes,* 1845)

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The Elements of all Sciences ought to be handled after the most simple Method, and not to be involved in Symbols, Notes, or obscure Principles taken elsewhere.

—JOHN KEILL, 1713

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That which renders Logic possible, is the existence in our minds of general notions, our ability to conceive of a class, and to designate its individual members by a common name. The theory of Logic is thus intimately connected with that of language. A successful attempt to express logical propositions by symbols, the laws of whose combinations should be founded upon the laws of the mental processes which they represent, would, so far, be a step toward a philosophical language. . .. The generality of the method will even permit us to express arbitrary operations of the intellect, and thus lead to general theorems in logic analogous, in no slight degree, to the general theorems of ordinary mathematics. . ..

—GEORGE BOOLE, *Mathematical Analysis of Logic*

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All modern higher mathematics is based on a calculus of operations, on laws of thought. All mathematics was so in reality; but the evolvers of the modern higher calculus have known that it is so. Therefore elementary teachers who, at the present day, persist in thinking about algebra and arithmetic as dealing with laws of numbers, and about geometry as dealing with laws of surface and solid content, are doing the best that in them lies to put their pupils on the wrong track for reaching in the future any true understanding of higher algebras. Algebras deal not with laws of number, but with such laws of the human thinking machinery as have been discovered in the course of investigations on numbers.

—M. E. BOOLE, *Logic of Arithmetic,* 1903

The Book Shelf

EDITED BY R. H. MOORMAN

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor R. H. Moorman, Box 169-A, Tennessee Polytechnic Institute, Cookeville, Tennessee.

Remarks on the Foundations of Mathematics, by Ludwig Wittgenstein, (Translated by G. E. M. Anscombe), the Macmillan Company (60 Fifth Avenue) New York, 1956, xxxviii + 400 pp., $5.75.

For the student of mathematics, philosophy, and logic this is an excellent reference work, and it is an absorbing book to challenge the general reader with some degree of interest and background in mathematical thinking.

The volume is published in the original German with the accompanying English translation by G. E. M. Anscombe. The editors, G. H. von Wright, R. Rhees and Anscombe, have done a fine piece of work in selecting, classifying, and chronologically arranging some particular topics from Wittgenstein's extensive writings. They state in the editors' preface that this was not an easy and satisfying task. Their excellent preface and table of contents help the reader preview some notions and survey the intent of the writer.

Compiled from manuscripts of Wittgenstein written between 1937 and 1944, the book's central theme is the philosophy of mathematics and logic. The work, however, is divided into five principal parts with each part subdivided into numbered topics, ideas, questions, or fragments of information. Two appendices accompany Part I.

Among the topics of Part I, the nature of proof, mathematical belief, logical inferences, and compulsion seem to lead to the conclusion that the mathematician is an inventor, not a discoverer. To illustrate, in a particular system of symbolism (Russell's) an unprovable, but necessarily true proposition may be constructed. That is, a proposition may be proved to be unprovable. Part II is primarily concerned with a somewhat confusing approach to explaining a variety of techniques in proofs, calculation, and experiment, followed by a discussion of the real proof as being a proof of consistency. Parts III and IV lead into more fascinating realms of the nature and interrelationship of philosophy, symbolic logic, and mathematical concepts. Empirical propositions, calculation and experiment, contradiction and consistency, and the role of language get an extensive treatment in Part V.

Perhaps Wittgenstein, in his own words, summarizes his purpose in his writings. "The philosopher is the man who has to cure himself of many sicknesses of the understanding before he can arrive at the notions of the sound human understanding. If in the midst of life we are in death, so in sanity we are surrounded by madness."

—E. F. WARD
Tennessee Polytechnic Institute

The Teaching of Mathematics, Issued by the Incorporated Association of Assistant Masters in Secondary Schools, Cambridge Press, American Branch (32 East 57th Street) New York, 1957, ix + 231 pp., $3.00.

If the American who reads *The Teaching of Mathematics* is unfamiliar with the organization of English public schools, he will undoubtedly be startled when he reads in the table of contents "Calculus in the Fifth Year." At this point in his reading, then, it would be wise for him to acquaint himself with the British system of public education.

In this survey, the first general one on the teaching of mathematics in England since 1919, the masters concentrate their attention on mathematics in the secondary grammar school. In this type school the average pupil, who differs in ability and purpose from his peers in the secondary modern and the secondary technical school, will be preparing for the professions and/or the university. At the age of sixteen, after he has spent five years in the grammar school, he may obtain the General Certificate of Education at the Ordinary Level if he writes passing papers on the examinations conducted by one of the regional university boards. He may, however, stay three years longer in the sixth form to prepare for the G. C. E. at the Advanced or Scholarship Level.

Repeated reference is made to the effects of the external examination system on both the mathematics curriculum and teaching methods. Teachers are urged to resist the natural temptation to let the examination syllabus become the teaching syllabus. It is specifically pointed out that topics from statistics, among others, are not generally found in the mathematics curriculum because the "examination papers in mathematics do not encourage the teaching of this kind of applied arithmetic."

In the middle school, mathematics instruction for all follows a concentric development with arithmetic, algebra, geometry, and trigonometry being taught each year to pupils in homogeneous sets. Since the main purpose of instruction in the fifth year is to integrate material previously studied, little new subject matter, except very elementary calculus, is introduced. The masters recommend better correlation of the four main branches in all years than now exists.

The American teacher will be particularly interested in the mathematics work of the sixth form. It is here that the capable, interested few have the opportunity to learn, under the careful supervision of a mathematics specialist, topics in algebra, trigonometry, analytics, Euclidean and non-Euclidean geometry, and calculus that the American youth doesn't encounter until he enters college.

Degrees of accuracy, the correlation of mathematics with other subjects, and the environmental approach are some of the general teaching topics to which the longest chapter is devoted. Another chapter contains detailed descriptions of an ideal mathematics room and a combination classroom and laboratory. Suggested equipment for this room ranges from single pulleys and spring balances to such "oddments" as string and sealing wax.

Indeed, topics discussed throughout the book range from decimalization of £ 1 to work on orthogonal and coaxial circles; from the slow revelation of the nature of the deductive process to the examination of logical foundations of nonmetrical geometry; from the utilitarian value of practical work to the philosophical aspect of mathematics.

—EMMA W. GARNETT, *Graduate Student*
George Peabody College for Teachers

History of Analytic Geometry, by Carl B. Boyer, *Scripta Mathematics,* Yeshiva University (Amsterdam Avenue at 186th Street) New York, 1956, iii + 291 pp., $6.00.

This is a well-written book which makes interesting reading even for the person who has very little training in mathematics. It could very profitably be used as supplementary reading for college courses in the history of mathematics. The text contains several illustrative examples, but it is not overburdened with long, detailed explanations of theory. There are many geometric figures which make the text easy to follow and give the book a pleasing and mathematical appearance. The book is printed in very clear type, the sentences are easily read, and the paragraphs are well developed.

The book contains nine chapters presented in chronological order and covers a period of about 4000 years. Chapter I discusses the works of the Babylonians and Egyptians during the period 1600-1400 B.C. It treats such topics as the value of π used by the Babylonians and the concept of coordinates which these early people possessed. The latter part of the chapter is devoted to the works of such early Greeks as Eudoxus, Zeno, Thales, and Pythagoras.

Chapter II, "The Alexandrian Age," is concerned with the works of such great Greek mathematicians as Archimedes, Apollonius, and Euclid. It gives good discussions of Euclid's *Elements* and the *Conic Sections* of Apollonius.

The next two chapters deal with developments in analytic geometry during the Middle Ages and the period immediately preceding the actual invention of analytic geometry. The Hindu, Arab, and Byzantine contributions are discussed in relation to the Middle Ages while the works of such early mathematicians as Viète are discussed in the chapter, "The Early Modern Prelude."

Probably the most important chapter in the book is the one dealing with the works of the inventors of analytic geometry—Fermat and Descartes. Fermat's *Introduction to Plane and Solid Loci* is treated in this chapter and Descartes's *La Géométrie* is given an excellent discussion.

The remaining four chapters bring the reader up to the most recent developments in analytic geometry beginning with the works of Newton, l'Hospital, Bernoulli, and Euler. Chapter IX, "The Golden Age," gives discussions of all the important discoveries which brought analytic geometry to the stage of development which is familiar to us today.

As Boyer states in the preface, this book is practically free of biographical materials and peculiarities of terminology, and much sociological background has been omitted. The book contains only materials that would ordinarily be taught in a college course in analytic geometry.

An extensive bibliography, given at the end of the book, is divided into two parts. The primary sources are arranged roughly in chronological order while the secondary sources are arranged alphabetically. The bibliography contains those available foreign works dealing with the history of analytic geometry as well as articles written in English. This bibliography will be invaluable to serious students of mathematics who would like to do more extensive work in the history of analytic geometry.

—OTIS McCOWAN, *Student*
Tennessee Polytechnic Institute

BOOKS RECEIVED BY THE BOOK SHELF EDITOR—

The editor welcomes suggestions in regard to which of these books the readers of THE PENTAGON would like to see reviewed in future issues.

Applied Probability, Volume VII, Proceedings of the Seventh Symposium in applied Mathematics of the American Mathematical Society, L. A. MacColl, McGraw-Hill Book Company (330 West 42nd Street) N. Y., 1957, v + 104 pp., $5.00.

Atoms and Energy, H. S. W. Massey, Philosophical Library (15 East 40th Street) New York, 1956, i + 174 pp., $4.75.

Advanced Calculus, R. Creighton Buck, McGraw-Hill Book Company (330 West 42nd Street) New York, 1956, viii + 423 pp., $8.50.

Basic Mathematics for Radio and Electronics, F. M. Colebrook and J. W. Head, Philosophical Library (15 East 40th Street) New York, 1957, 359 pp., $6.00.

An Analytical Calculus, E. A. Maxwell, Cambridge University Press (32 East 57th Street) New York, 1957, viii + 288 pp., $4.00.

Calculus, Jack R. Britton, Rinehart and Company (232 Madison Avenue) New York, 1956, ii + 584 pp., $6.50.

Calculus Refresher for Technical Men, A. Albert Klaf, Dover Publications (920 Broadway) New York, 1956, i + 431 pp., $1.95.

College Plane Geometry, Edwin M. Hennerling, John Wiley and Sons (440 Fourth Avenue) New York, 1958, ix + 310 pp., $4.95.

Commutative Algebra, Volume I, Oscar Zariski and Pierre Samuel, D. Van Nostrand Company (120 Alexander Street) Princeton, N. J., 1958, xi + 329 pp., $6.95.

Computing With Desk Calculators, Walter W. Varner, Rinehart and Co. (232 Madison Avenue) New York, 1957, viii + 108 pp., $1.95.

Economic Models, E. F. Beach, John Wiley and Sons (440 Fourth Avenue) New York, 1957, vii + 227 pp., $7.50.

Electrical Applied Physics, N. F. Astburry, Philosophical Library (15 East 40th Street) New York, 1957, xi + 241 pp., $10.00.

Experimental Designs, William G. Cochran and Gertrude M. Cox, John Wiley and Sons (440 Fourth Avenue) New York, 1957, xiv + 617 pp., $10.25.

Finite-Dimensional Vector Spaces, Paul R. Halmos, D. Van Nostrand (120 Alexander Street) Princeton, N. J., 1958, viii + 200 pp., $5.00.

Games and Decision, R. D. Luce and H. Raiffa, John Wiley and Sons (440 Fourth Avenue) New York, 1957, xi + 506 pp., $8.75.

(The) Generation of Electricity by Wind Power, E. W. Golding, Philosophical Library (15 East 40th Street) New York, 1955, iii + 318 pp., $12.00.

History of Mathematics, Florian Cajori, Macmillan Company, (60 Fifth Avenue) New York, 1919, i + 516 pp., $6.00.

Insight, A Study of Human Understanding, Bernard J. F. Lonergan, Philosophical Library, (15 East 40th Street) New York, 1957, xxx + 784 pp., $10.00.

Intermediate Algebra, Paul K. Rees and Fred W. Sparks, McGraw-Hill Book Company (330 West 42nd Street) New York, 1957, iii + 306 pp., $3.90.

(An) Introduction to the Foundations and Fundamental Concepts of Mathematics, Howard Eves and C. V. Newsom, Rinehart and Company (232 Madison Avenue) New York, 1958, xv + 363 pp., $6.75.

(An) Introduction to Probability Theory and Its Applications, William Feller, John Wiley and Sons (440 Fourth Avenue) New York, 1957, x + 461 pp., $10.75.

Introduction to Statistical Analysis, W. J. Dixon and F. J. Massey, Jr., McGraw-Hill Book Co. (330 West 42nd Street) New York, 1957, vii + 488 pp., $6.00.

Introduction to Statistical Reasoning, Philip J. McCarthy, McGraw-Hill Book Company (330 West 42nd Street) New York, 1957, vii + 488 pp., $6.00.

(The) Leibniz-Clarke Correspondence, H. G. Alexander, Philosophical Library (15 East 40th Street) New York, ii + 200 pp., $4.75.

Linear Algebra for Undergraduates, D. C. Murdoch, John Wiley and Sons (440 Fourth Avenue) New York, 1957, xi + 239 pp., $5.50.

Marine Electrical Practice, G. O. Watson, Philosophical Library (15 East 40th Street) New York, 1957, viii + 325 pp., $12.00.

Mathematics and Wave Mechanics, R. H. Atkin, John Wiley and Sons (440 Fourth Avenue) New York, 1957, xv + 348 pp., $6.00.

Mathematics for Science and Engineering, Philip L. Alger, McGraw-Hill Book Company (330 West 42nd Street) New York, 1957, xi + 360 pp., $5.50.

Mathematics: Magic and Mystery, Martin Gardner, Dover Publications (920 Broadway) New York, 1956, ii + 176 pp., $1.00.

Matrix Calculus, E. Bodewig, North-Holland Publishing Company, Amsterdam, and Interscience Publishers, New York, 1956, iii + 334 pp., $7.50.

Modern Geometry, Claire Fisher Adler, McGraw-Hill Book Company (330 West 42nd Street) New York, 1958, xiv + 215 pp., $6.00.

Modern Mathematics for the Engineer, Louis N. Ridenour, McGraw-Hill Book Company (330 West 42nd Street) New York, 1957, i + 490 pp., $6.00.

Nonparametric Methods in Statistics, D. A. S. Frasser, John Wiley and Sons (440 Fourth Avenue) New York, 1957, i + 299 pp., $8.50.

Numerical Analysis, Kaiser S. Kinz, McGraw-Hill Book Company (330 West 42nd Street) New York, 1957, xv + 381 pp., $8.00.

(The) Oscilloscope at Work, A. Haas and R. W. Hallows, Iliffe and Sons, Ltd., London, and Philosophical Library (15 East 40th Street) New York, 1954, i + 171 pp., $10.00.

Precision Electrical Measurements, Philosophical Library (15 East 40th Street) New York, 1956, ix + 361 pp., $12.00.

Principles and Techniques of Applied Mathematics, Bernard Friedman, John Wiley and Sons (440 Fourth Avenue) New York, 1956, ii + 315 pp., $8.00.

Queues, Inventories and Maintenance, Philip M. Morse, John Wiley and Sons (440 Fourth Avenue) New York, 1958, ix + 202 pp., $6.50.

Radio Aids to Air Navigation, J. H. H. Grover, Philosophical Library (15 East 40th Street) New York, 1957, x + 138 pp., $6.00.

Reason and Chance in Scientific Discovery, R. Taton, Philosophical Library (15 East 40th Street) New York, 1957, 171 pp., $10.00.

Rocket, Sir Philip Joubert, Philosophical Library (15 East 40th Street) New York, 1957, 190 pp., $6.00.

Scientific French, William N. Locke, John Wiley and Sons (440 Fourth Avenue) New York, 1957, iv + 112 pp., $2.25.

Scientific German, George E. Condoyannisk, John Wiley and Sons (440 Fourth Avenue) New York, 1957, x + 164 pp., $2.50.

Spectroscopy at Radio and Microwave Frequencies, D. J. E. Ingram, Philosophical Library, (15 East 40th Street) New York, 1956, ii + 332 pp., $15.00.

Statistical Analysis of Stationary Time Series, Ulf Grenander and Murray Rosenblatt, John Wiley and Sons (440 Fourth Avenue) New York, 1957, vii + 300 pp., $11.00.

Symposium on Monte Carlo Methods, Herbert A. Meyer (editor), John Wiley and Sons (440 Fourth Avenue) New York, 1956, v + 381 pp., $7.50.

Transistor A. F. Amplifiers, D. D. Jones and R. A. Hilbourne, Philosophical Library (15 East 40th Street) New York, 1957, vii + 152 pp., $6.00.

Trigonometry Refresher for Technical Men, A. Albert Klaf, Dover Publication, (920 Broadway) New York, 1956, iii + 629 pp., $1.95.

Understanding Arithmetic, Robert L. Swain, Rinehart and Company (232 Madison Avenue) New York, 1957, xxi + 264 pp., $4.75.

Vector Analysis, Louis Brand, John Wiley and Sons (440 Fourth Avenue) New York, 1957, xii + 282 pp., $6.00.

Vector Spaces and Matrices, Robert M. Thrall and Leonard Torneeim, John Wiley and Sons (440 Fourth Avenue) New York, 1957, xii + 318 pp., $6.75.

Directions for Papers to be Presented at the Kappa Mu Epsilon Convention

BOWLING GREEN, OHIO

May 8 and 9, 1959

A most important part of the convention program will be the presentation of papers by student members of KME. It is essential that chapter sponsors and members begin now to plan for papers to be presented at the next convention. Each student should choose the field of mathematics of greatest interest to him and then search for a topic area suitable for a paper. Faculty sponsors should counsel with students in the selection of topics and encourage competition within the chapter in order that high-caliber papers will result.

Who may submit papers: Any member may submit a paper for use on the convention program. Papers may be submitted by graduates and undergraduates; however, undergraduates will not compete against graduates. Awards will be granted for the best papers presented by undergraduates. If enough papers are presented by graduates, special awards may be given for their best paper or papers.

Subject: The material should be within the scope of the understanding of undergraduates, preferably the undergraduate who has completed differential and integral calculus. The Selection Committee will naturally favor papers that are within this limitation and which can be presented with reasonable completeness within the time limit prescribed.

Time limit: The usual time limit should be twenty minutes but it may be extended to thirty minutes on recommendation of the Selection Committee.

Paper: The paper to be presented or a rather complete outline of it must be submitted to the Selection Committee accompanied by a description of charts, models, or other visual aids that are to be used in presenting the paper. A carbon copy of the complete paper may be submitted if desired. Each paper must indicate that the author is a member of KME and whether he is a graduate or an undergraduate student.

Date and place due: The papers must be submitted before February 1, 1959, to the office of the National Vice-President.

Selection: The Selection Committee will choose about eight papers from those submitted for presentation at the convention. All other papers will be listed on the convention program.

Prizes:

1. The authors of all papers presented will be given a two-year extension of their subscription to THE PENTAGON.
2. Authors of the two or three best papers presented by undergraduates, according to the judgment of a committee composed of faculty and students, will be awarded copies of the James' *Mathematical Dictionary,* suitably inscribed.
3. If a sufficient number of papers submitted by graduate students are selected for presentation, then one or more similar prizes will be awarded for the best paper or papers from this group.

Ronald G. Smith
National Vice-President
Kansas State Teachers College
Pittsburg, Kansas

"The die is cast; I have written my book; it will be read either in the present age or by posterity, it matters not which; it may well await a reader, since God has waited six thousand years for an interpreter of his words."

—JOHANN KEPLER

Kappa Mu Epsilon News

EDITED BY FRANK HAWTHORNE, HISTORIAN

California Beta cooperated with Occidental College, the Los Angeles Actuarial Club, and the Southern Section of the California Mathematics Council in sponsoring the second annual Mathematics Field Day on March 1, 1958. Five hundred students from over one hundred southern California high schools participated.

Charles F. Barlow won the KME Freshman Mathematics Award during the year.

Continuing their studies in mathematics are the following 1957 graduates: Lawrence Arnold, at the University of California, Berkeley, on a National Science Foundation Fellowship; Marilyn Cottle, at the Sorbonne, Paris, on a Fulbright Scholarship; and John Dauwalder, University of Utah, on a teaching assistantship. Continuing in physics are Millard Mier at Bryn Mawr and Neville W. Reay at the University of Minnesota.

Former students who are members of **Illinois Delta** were invited to a Mathematics Homecoming Day, March 16, 1958. After an introduction by chairman Marlene Schaab, Sister M. Claudia, O.S.F., welcomed the group. Graduate members discussed topics related to their mathematical professions. A tour of the new scholastic wing of the College followed.

Indiana Alpha holds joint meetings every two weeks with the mathematics club, Chi Psi.

In March, **Indiana Beta** visited Allison's "Powerama."

Members of **Kansas Alpha** cooperated with the students from the Physical Science Department in sponsoring a weekly seminar on electronic computers. During the first semester, the mathematics students conducted the study on digital computers and during the second semester, the science students discussed analog computers.

Mr. H. Franklin Lanier, who received his undergraduate degree in 1940 and the master's degree in 1941 from Kansas State Teachers College, Pittsburg, and was a member of KME, is now manager of the guided missiles electronics department of Goodyear Aircraft Corporation. At the Commemoration Day convocation, he was presented one of the two awards which are given each year by the College to former students for meritorious achievement in their professions.

Betty Jo Fliginger and Lewis Bertalotto received awards as outstanding senior girl and boy of the 1958 class.

Kansas Beta will be host to a regional KME convention on May 10, 1958. Registration will begin at 9:00 a.m. and the convention will adjourn about 4:00 p.m. There will be a banquet at noon. Invitations have been extended to the following chapters in a six-state area:

Iowa Alpha	Iowa State Teachers College	Cedar Falls
Iowa Beta	Drake University	Des Moines
Kansas Alpha	Kansas State Teachers College	Pittsburg
Kansas Gamma	Mt. St. Scholastica College	Atchison
Kansas Delta	Washburn University	Topeka
Kansas Epsilon	Fort Hays State College	Hays
Missouri Alpha	Southwest Missouri State College	Springfield
Missouri Beta	Central Missouri State College	Warrensburg
Missouri Gamma	William Jewell College	Liberty
Missouri Epsilon	Central College	Fayette
Nebraska Alpha	State Teachers College	Wayne

It is expected that approximately 120 will attend. The program will include a number of papers from the various participating chapters, a business meeting, and an "Idea Exchange" session.

This year's programs at **Kansas Gamma** centered around the theme, "Insights into Modern Mathematics." In March, the chapter visited Midwest Research Plant and Linda Hall Technical Library in Kansas City, Missouri.

Missouri Alpha honored Dr. Lawrence E. Pummill with a banquet upon his retirement. Dr. Pummill served as chapter sponsor for twenty-five years and also served as National Treasurer.

Interest in the regular help sessions for mathematics students is at an all-time high.

Missouri Beta sponsored two open meetings concerned respectively with some of the theoretical and applied aspects of mathematics. A delegation from the chapter attended the Mid-America Electronics Convention of IRE, held at Kansas City in November and visited the Midwest Research Institute and Linda Hall Library in March.

Each initiate is required to present a paper to the chapter during his first year.

New Mexico Alpha reports the establishment of a departmental library in the Mathematics Office Building, each member of the department lending any pertinent books or periodicals he owns to the library. The chapter has voted the expenditure of $100 from its treasury for purchase of books not otherwise available.

On March 28 members of the chapter attended a three-hour lecture and demonstration of electronic computers at Sandia Base, the A.E.C. installation at Albuquerque.

Changes of position reported by members of **Virginia Alpha** include:

Mr. H. M. Linette, from Mathematics Department, V.S.C., to Mathematician, Romo and Woolridge, San Diego, California.

Benjamin Williams, from Computation Division, Directorate of Management Analysis, USAF, Washington, D.C. to Mathematician, Convair, San Diego, California.

Dr. J. M. Hunter became Dean of Virginia State College during this academic year.

Gladys Brown West and L. J. Griffin are mathematicians at the Naval Proving Ground, Dahlgren, Virginia.

R. E. Galloway is enrolled as a graduate student at Catholic University and is working as a mathematician in the Washington, D.C., area.

Dr. W. E. Williams spent part of last year in Costa Rica where he served as labor consultant to the government. Mr. C. E. Taylor spent last year in Africa where he was Visiting Professor of Physics at Yoba Technical Institute, Nigeria.

Members of **Ohio Gamma** now have access to a Differential Analyzer. This analog computer, originally developed by Vannevar Bush, was constructed at the Lewis Flight Propulsion Laboratory of the National Advisory Committee for Aeronautics at Cleveland from which it was obtained by Baldwin-Wallace College.

The annual mathematics contest for high school students who have completed five semesters of mathematics will be held again this year by **Wisconsin Alpha.** The six students ranking highest in the contest receive pins and the highest ranking girl, a scholarship to the college. The school with the highest score (this score being the sum of the scores of three individuals chosen previously to represent their school) receives a plaque to be held by the school until the time of the next contest. One hundred and forty contestants from twenty-six different schools are expected to compete this year.

Program Topics

(School Year 1957-58)

Illinois Delta, College of St. Francis, Joliet

The International Geophysical Year, Sr. M. Crescentia, O.S.F.
An Attempt at Squaring the Circle, Marlene Schaab
Infinity, Rose Mary Kotesa
A Glance at Number Theory, Sr. M. Claudia, O.S.F.
Nomography, Dorothy Pulo
Paradoxes of Zeno, Angela Rudolphi
An Introduction to Zero, Number, and Symbol, Anna M. Di Monte

Indiana Alpha, Manchester College, North Manchester

The Satellite, Professor John Baumgart
Count Buffon's Needle, Paul Masterson
Perfect Numbers, Glenn Schmucker
The Treasure Hunt, Wendall Dilling
Velocity of Escape, Alan Frantz
Hydroponics and Supersonics, Professor John Dotterer
Pseudo Logarithms, Dr. Harold Larsen

Indiana Beta, Butler University, Indianapolis

Relativity, Vyron Klassen
Mechanical Integrators, Gordon Sawrey
Fundamental Theorem of Algebra, Ruth Ann Clark
Vector Analysis, Professor W. H. Bessey
Mathematics in Medicine, James Kriner
Gauss, Prince of Mathematicians, Vuryl Klassen
U. S. Naval Observatory, Mr. F. A. Graf

Iowa Alpha, Iowa State Teachers College, Cedar Falls

The Integraph, Ronald Moehlis
The Theory of Games, Susan Rock
Genaille's Rods, David Koos
We Bow to You, Marilyn Hala
Mathematical Transformation of Musical Themes, Jerry Janssen
A Rapid Method of Extracting Square Roots, Jack Daniels
Nomograms, Wayne Hascall
Non-Euclidean Geometry, Carol Hatch

Kansas Alpha, Kansas State Teachers College, Pittsburg

Linear Programming, Professor Frank German
Guided Missile Program at Goodyear Aircraft, H. Franklin Lanier

Kansas Gamma, Mount St. Scholastica College, Atchison

A New Look at Number, Dorothy Schmedding
Traditional Algebra vs. Study of Structure, Sr. Ambrose Aubry
Some Views of Topology, Dorothy Schiedeler
Traditional Geometry or Invariance? Mary Ann Ginaine
Probability, Olga Gronniger
Mathematical Systems, Mary Syron
Puzzles, Tricks, and Games Involving Mathematical Principles, Mary Yim
New Mathematics, J. G. Kemeny

Kansas Delta, Washburn University, Topeka

Space Travel, Gus Babb, Jr.
Science and Theology, Rev. Henry Breul
25-point Geometry, Mr. Terry McAdam

Michigan Beta, Central Michigan College, Mount Pleasant

How a Child Forms Number Concepts, Neal Dow
Some Original Ideas on Infinity, Donald Lamphere
Some Introductory Ideas in Topology, Donald Neuville

Missouri Alpha, Southwest Missouri State College, Springfield

How Many Triangles? Dr. Carl V. Fronabarger
Mathematical Recreations, Barbara Clinger
Linear Programming, Jerry Beckerdite
Theory of Games, Robert Pearce

Missouri Beta, Central Missouri State College, Warrensburg

Tables and Table-Making, Mr. Yudell Luke
Some of IBM's Contributions to the Satellite Program, Mr. George M. Brooks

Ohio Gamma, Baldwin-Wallace College

Analog Computers, Mr. Gino Coviello
Demonstration of Differential Analyzer, Dr. Dean L. Robb
The Laplace Transform, Professor Robert E. Schlea

Oklahoma Alpha, Northeastern State College, Tahlequah

Complex Numbers, John Hagan
Soap Bubbles and Surfaces, John Eichling
Combinations and Probability, Jack Witt
Number Systems, W. A. Hamilton
On Squaring a Circle — An Approximation, Mr. Carpenter
Extracting Roots of Numbers by Binomial Expansion, Mr. Carpenter
On Trisecting An Angle, Gerald Vaughn
The Development of Our Number System

Virginia Alpha, Virginia State College, Petersburg, Virginia.

Calculation of the Number of Isomers for Monohydric Alcohols, Dr. T. N. Baker

Jordan Curve Theorem, Mrs. B. A. Harrington

Infinite Sets, Dr. L. S. Hunter

A Note on Matrices, Miss R. Richardson

A Sequence of Triangles, Mrs. P. A. Smith

Exact and Inexact Differentials, Mr. J. H. Trotter

Wisconsin Alpha, Mount Mary College, Milwaukee, Wisconsin.

Prime Numbers, Marie Hagner

The International Geophysical Year and Our Satellite Program, Mr. Edward Schreck

Determinants, Mary Sworske

Methods of Subtraction, Mary Beth Gryga

(Continued from page 76)

9. B. Meserve, "Topology for Secondary Schools," *The Mathematics Teacher,* 46:465-474, 1953.
10. National Council of Teachers of Mathematics, *Insights into Modern Mathematics,* 1957, pp. 36-64, 100-144, 306-335.
11. M. Rees, H. Brinkmann, Z. Mosesson, S. Schelkunoff, and S. Wilks, "Professional Opportunities in Mathematics," *American Mathematical Monthly,* January, 1954.
12. M. Richardson, *Fundamentals of Mathematics,* MacMillan, 1941.
13. A. Tucker and H. Bailey, "Topology," *Scientific American,* 182:18-24, January, 1950.
14. M. Weiss, *Higher Algebra for the Undergraduate,* John Wiley and Sons, 1949.

"The point about zero is that we do not need to use it in the operations of daily life. No one goes out to buy zero fish. It is in a way the most civilized of all the cardinals, and its use is only forced on us by the needs of cultivated modes of thought."

—Alfred North Whitehead