THE PENTAGON

Volume XVII | Fall, 1957 | Number 1

CONTENTS

National Officers

C. C. RICHTMEYER - - - - - *President*
Central Michigan College, Mt. Pleasant, Michigan

R. G. SMITH - - - - - *Vice-President*
Kansas State Teachers College, Pittsburg, Kansas

LAURA Z. GREENE - - - - - *Secretary*
Washburn Municipal University, Topeka, Kansas

M. LESLIE MADISON - - - - - *Treasurer*
Colorado A & M College, Fort Collins, Colorado

FRANK HAWTHORNE - - - - - *Historian*
The State Education Department, Albany, N.Y.

CHARLES B. TUCKER - - - - *Past President*
Kansas State Teachers College, Emporia, Kansas

Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

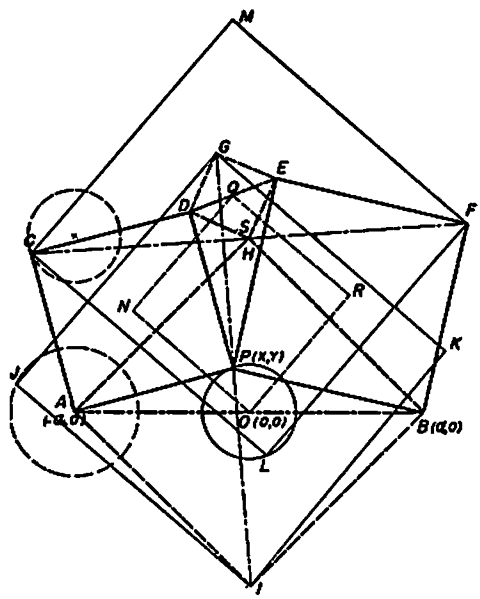
Invariants of Four Squares*

ADRIAN NIKITINS

Student, Southwest Missouri State College

This paper has been prepared in response to a challenge given by Professor Norman Anning in an article on "Four Squares" which appeared in the Spring, 1951, issue of THE PENTAGON. Some properties of four squares were given without proof and a challenge given to students to discover others.

I have taken an approach somewhat different from that of Professor Anning and have established essentially all the relations stated in his article and have discovered some additional ones. A number of invariant properties appeared in the course of the investigation.



If a circle of arbitrary radius r is constructed with its center at the midpoint of a line segment, then the sum of the squares of the

* A paper presented at the 1956 National Convention of Kappa Mu Epsilon and awarded first place by the Awards Committee.

distances from any point on the circle to the ends of the line segment is a constant. Let $2a$ be the length of the line segment. Set up a rectangular coordinate system such that the origin lies at the midpoint of the given line segment of length $2a$ and such that the x-axis coincides with it. Construct any circle of arbitrary radius r with its center at the origin.

Construct two squares, $APDC$ and $PBFE$, as shown in the figure, with AP and PB as sides. Construct two other squares, $DSEG$ and $AIBH$, with DE and AB as diagonals. These are the four basic squares whose invariant properties are to be investigated.

By means of the distance and slope formulas of analytic geometry the coordinates of the vertices of the squares were determined. The coordinates of these vertices are as follows:

$A(-a, 0)$	$J(-a + x - y, x + y)$
$B(a, 0)$	$K(a + x + y, -x + y)$
$C(-a - y, a + x)$	$L(-x, -y)$
$D(x - y, a + x + y)$	$M(x, 2a + y)$
$E(x + y, a - x + y)$	$N[(-a + x - y)/2, (a + x + y)/2]$
$F(a + y, a - x)$	$O(0, 0)$
$G(2x, a + 2y)$	$P(x, y)$
$H(0, a)$	$Q(x, a + y)$
$I(0, -a)$	$R[(a + x + y)/2, (a - x + y)/2]$

By the methods of analytic geometry the following invariant properties have been established:

1) The centers O, R, Q, and N of the four given squares are vertices of another square.

2) If a square is constructed with CF as a diagonal it will be homothetic to the square $ORQN$ with P as the homothetic center and the homothetic ratio 2:1.

3) If a square is constructed with IG as a diagonal it will be homothetic to $ORQN$ with homothetic center H and homothetic ratio 2:1.

4) The vertices of the squares move on circles whose radii have a constant ratio to the radius r of the given circle.

a) Vertices C, M, and F move on circles with radii r.

b) Vertices J, K, D, and E move on circles with radii $r\sqrt{2}$
c) Vertex G moves on a circle with radius $2r$.
d) Vertices N and R move on circles with radii $r/\sqrt{2}$

5) The centers of the circles upon which the vertices of the squares move are independent of r and depend only on the length of AB.

6) As the segment OP rotates clockwise about the point O, a radius drawn to a vertex from the center of rotation of that vertex moves in the same direction and through the same number of degrees.

7) The angle is constant between OP and the radius drawn to a vertex from a center of rotation of that vertex. Depending on the vertex, the constant is $0°$, $45°$, or $90°$. For example SG is parallel to OP, the radius drawn from the center of rotation of C to the vertex C is perpendicular to OP, but the angle between OP and the radius drawn from the center of rotation of J to the vertex J is $45°$.

8) The vertex S of square $DSEG$ always coincides with the vertex H of square $AIBH$.

9) The diagonal of the square $DSEG$ is of constant length and equal to the diameter of the given circle O.

10) The sum of the areas of the two squares $AIBH$ and $DSEG$ equals the sum of the variable areas of the two squares $APDC$ and $PBFE$.

11) CF is equal and perpendicular to IG.

12) S is the midpoint of CF and P is the midpoint of IG.

13) With respect to triangles ABP and PED, the altitude from P in each triangle coincides with the median from P in the other triangle.

14) Triangles ABP, AHD, and BHE have equal areas.

These are some of the properties of the four squares. Perhaps you will try to discover additional ones. I do not claim to have exhausted all of the possibilities.

An Invitation*

(Opportunities in Teaching Mathematics in Secondary Schools)

Sister Mary Neal Moran, B.V.M.

Student, Mundelin College

With the current prevalence of headlines like, "Mathematical Science and the Manpower Shortage"[1] and "The Growing Emphasis on Science and Mathematics in Industry,"[2] it is scarcely necessary to add emphasis to our increasing national need for competence and excellence in mathematics. It is not that our schools have begun to decline or that our capable youth steer clear of challenging courses. An analysis of the records of the 1953 and 1956 winners in the Annual National Honor Society Program reveals a small but steady increase of interest in scientific and technical courses and careers among students ranking above the 99.9 percentile.[3] But yesterday's resources will not answer today's—and even less tomorrow's—needs. In spite of present effort there is evidence to support the claim that "Russia is winning the cold war of the classroom."[4] America needs an army of technological manpower; she needs dependable ranks of regulars as well as gifted generals; the understanding and skill of numbers of technicians no less than the creative insight of designers.

The more clearly this need becomes defined, the more steadily is the focus sharpened on the students, faculties, and curricula of our high schools. At its own level the college builds on foundations which the high school has proportioned. The strength and the weakness of its work depend in large measure on that of its educational predecessors. And that work in turn rests to an inviting degree with the individual high school teacher. Of course he cannot create ability where none exists, nor force interest absorbed elsewhere. But it is a fact of universal experience that the minds and hearts committed to his direction are at a stage in their development when they are acutely sensitive to influence and encouragement. He can meet in

* This paper was awarded first prize in the essay contest sponsored jointly by KME and STIP.

1 R. J. Seeger, The Mathematics Teacher, January, 1957.
2 Andrew Luff, School Science and Mathematics, March, 1956.
3 M. H. Ahrendt, "Mathematics and Science," NEA Journal, February, 1957, 109-10.
4 Lewis L. Strauss, Chairman, Atomic Energy Commission, quoted in A. Luff, op. cit., p. 242.

part the peculiar needs of the adolescent and the young adult—fostering the self-confidence of secure knowledge, the humility of recognized truth, and the joy of intellectual insight—and simultaneously enjoy the satisfaction of helping to develop that wide potential we call personality.

Part of their need is that they fail to perceive any need, and their development is so gradual as to be imperceptible over short steps of time. That's why teaching worthy of the name demands devotion. A teacher is someone who has learned something worth sharing and who has the love and patience to work at sharing it, even with those who don't know they need it, and will only much later, or perhaps never, admit their indebtedness. But he is also someone who knows first-hand one of the keenest of human satisfactions: that of directing and encouraging the progress of a mind from stumbling unsureness to the security of precise thought.

One of the specific aims of the high school mathematics curriculum is this refinement of thought: the development of the ability intellectually to x-ray situations and arguments, deftly to determine their skeletal support, and accurately to analyze their strength. Students have to be won to mathematics, but the subject has naturally powerful drawing cards. In the hands of an enthusiastic and understanding teacher, they can stimulate simultaneously the interest in mathematics we need as a nation, the intellectual development possible to our students, and the personal satisfaction earned by our teachers.

Much is being done today toward re-evaluating and restructuring the high school mathematics program with a view to the fuller realization of this aim. Traditional course sequence and content, as well as methods, have come in for close appraisal and sometimes radical shifting. Studies of the mathematical readiness of college entrants have pointed up the need for a more unified program and one better orientated to the college work for which it is prerequisite. Concepts separately developed must be sufficiently integrated at the pre-college level to give a meaningful background to subsequent specialization and to form minds receptive to the implications of concepts introduced later. The dilemma of poorly-prepared students with its train of make-shift solutions—*e.g.*, remedial classes and diluted courses—can only be finally answered by a high school curriculum geared to eliminate, or at least to minimize, this inadequacy. A major corporation offers one solution to the problem:

> The Carnegie Corporation of New York has given the University of Illinois $277,000 to revamp high-school mathematics teaching. A project headed by Max Beberman has during the last four years evolved a more sophisticated mathematics curriculum, which is already being tested in five Illinois and Wisconsin schools. . .they are teaching algebra and analytic geometry in the freshman year. Sophomores get a new course called theory of sets. . .Juniors learn about complex numbers and elementary calculus. The mathematics course for seniors is still being planned.[5]

However; the most comprehensive program planned, the one most attentive to the separate factors involved and the specific goals established, will correct nothing in the hands of an uninterested and, hence, uninspiring teacher. What we need is large-scale propaganda—the propaganda of effective teachers who will combine enthusiasm with patience, who will appreciate the poetry as well as the power of mathematics, who will bring to the bare bones of textual material all that their personal resources and those of the school and community can provide in the way of appropriate and appealing context; teachers, finally, who are convinced that "it is something fine to be a mathematics teacher."[6]

A wealth of opportunity awaits them. A nation pacing the world in technological advances awaits the insight no computer can produce to sound the resources it has tapped. A curriculum outgrown by the culture it has produced waits to be revamped or perhaps even supplanted. Young minds, who neither guess nor appreciate their capacities, wait the direction and encouragement of devoted teachers.

---

[5] Scientific American, August, 1956, p. 50.
[6] R. E. Langer, "Time is Running Out," The Mathematics Teacher, October, 1956, p. 424.

REFERENCES

Ahrendt, M. H., "Mathematics and Science," *NEA Journal*, February, 1957.

Cooper, T. S., "The Technical Manpower Shortage—An Answer From the High-School," *The Mathematics Teacher*, October, 1956.

Hach, Alice, "Removing the 'Dis' from Dislike for Mathematics," *The Mathematics Teacher*, January, 1957.

Hartung, M. L., "A New Day?" *The Mathematics Teacher*, December, 1956.

Jones, P. S., "The History of Mathematics As a Teaching Tool," *The Mathematics Teacher*, January, 1957.

Langer, R. E., "Time is Running Out," *The Mathematics Teacher*, October, 1956.

Levy, N., "Toward Discovering Creativity," *The Mathematics Teacher*, January, 1957.

Luff, A., "The Growing Emphasis on Science and Mathematics in Industry," *School Science and Mathematics*, March, 1956.

"Math Demonstrates It Still Appeals to Youngsters," *Business Week*, March 31, 1956.

Parrish, C. E., "Junior High-School Mathematics and the Manpower Shortage," *The Mathematics Teacher*, December, 1956.

Schaaf, W. L., "Careers for the Mathematician," *The Mathematics Teacher*, December, 1956.

Scientific American, August, 1956.

Seeger, R. J., "Mathematical Science and the Manpower Shortage," *The Mathematics Teacher*, January, 1957.

Time, June 18, 1956, p. 74.

"For many parts of nature can neither be invented with sufficient subtilty nor demonstrated with sufficient perspicuity nor accommodated unto use with sufficient dexterity, without the aid and intervening of mathematics."

—LORD BACON

The Calendar

GLENN WARREN THORNTON
Student, University of New Mexico

The calendar is a system of dividing and recording time. All calendars recognize two natural divisions of time—the day and the year. The day is taken to be the length of time or period in which the earth makes a complete revolution about its axis. The year is the period in which the earth makes its complete elliptical orbit about the sun. This so-called solar year contains 365.242216 days. When the day is subdivided into hours, minutes, and seconds, this amounts to 365 days, 5 hours, 48 minutes, and 46 seconds. There are two types of months—sidereal and synodic. The sidereal month is determined by a star and the moon and is of length 27 days, 7 hours, 43 minutes, and 11.5 seconds. The synodic month is determined by the sun and the moon and varies in length between 29.26 and 29.80 days. The mean synodic month is taken to be 29.530588 days. Twelve of such months do not equal a solar year but only 354.3670 days. The week is entirely a man-made division of time and not based on any natural event.

There are three types of calendars. The lunar type is based on lunar phases or apparent motions of the moon. The solar type is based on the apparent movement of the sun. The third type known as the luni-solar type is based upon a combination of solar and lunar phenomena.

The Egyptians were among the first to devise a calendar. They based their year on the appearance of the Dog Star, Sirius, in the eastern sky just before sunrise. Later they based their calendar on the sun itself. Their solar calendar had twelve months each of 30 days, making a year of 360 days. The Egyptian astronomers were of course aware that a 360-day year was too short and did not agree with the seasons. They corrected this discrepancy by adding five days at the end of every year as special feast days. This five-day intercalary period was known as the "five epagomenal days" period. The earliest date known in history by the Egyptian calendar is 4236 B.C.

The people who lived in ancient Babylonia also developed a calendar. Their calendar had twelve months of twenty-nine or thirty days each to correspond with periods of the moon. In order to correct the differences in the seasons which accumulated after several years, they inserted an intercalary month into their year from time to time.

The Greek year included twelve months alternating twenty-nine and thirty days. As a result, the Greek year was only 354 days long, eleven and one-fourth days short of the true solar year. To make up this difference an extra month was added every other year and then omitted every eight or nine years.

The earliest known Roman calendar had a year made up of ten months based on the changing appearance of the moon. The Roman calendar was dated from the founding of the city of Rome, in 753 B.C. by the calendar we use. There were many changes in the Roman year. At one time there were five months of 31 days, six months of 29 days, and one month of 28 days. This total of 355 days in the Roman year had to be supplemented every second year by an extra month and even this was not nearly accurate enough.

There have been numerous calendar changes throughout history. In 367 B.C. the Babylonian system began a nineteen year cycle. They would have twelve years of twelve months and seven years of thirteen months. This method made their calendar much more accurate, in fact, the error amounted to only one day in two centuries.

The Egyptians were still short almost a quarter of a day every year, so in 238 B.C. Ptolemy III (Euergetes) proposed that an additional day be inserted every four years, thus making the feast period six days rather than five days as in the other years. His plan was defeated. In 46 B.C. Julius Caesar ordered a new calendar. Upon the advice of an Alexandrian astronomer Sosigenes, he accepted the solar calendar of Egypt with two changes. He distributed the last five days of the year (the festival period in Egypt) more evenly throughout the calendar and he adopted Ptolemy's previously rejected extra leap-year day by adding it to February every four years. His calendar was put into effect in 45 B.C., but he changed the year 46 B.C. by inserting Mercedonius, a month of 23 days, and two more months making the year 445 days long in order to bring the calendar into step with the seasons. This year was known as the "year of confusion."

The Julian calendar provided for a year that lasted 365¼ days. This year was actually about 11 minutes and 14 seconds longer than the solar year. This difference led to a gradual change in the date of the equinoxes. About 1580 the spring equinox fell on March 11, ten days earlier than it should have occurred. So in 1582 Pope Gregory

XIII, determined to correct the difference in sun and calendar time, dropped ten days from October. By this arrangement the day following October 4 became October 15 and thus the next equinox was restored to its proper date. Because the Julian calendar year was a few minutes longer than the solar year, it gained a day every 128 years. To correct this difference, the Gregorian calendar dropped the extra day in February in century years which cannot be divided by 400. Therefore 1600, 2000, etc. are leap years but 1700, 1800, 1900 are not leap years. But even this is not entirely correct for there is an error in the fourth decimal place which amounts to one day in about 3300 years.

The Gregorian calendar was adopted almost immediately by all the Catholic nations of Europe. The German states retained the Julian calendar until 1700, England did not change until 1752 (in which year the American colonies also changed), and Russia adopted the Gregorian calendar in 1918. This is the calendar that most of the civilized world has used for nearly 400 years.

The Hebrew calendar began with the Creation, which is supposed to have taken place 3,760 years and three months before the Christian Era. To find the year in this calendar, 3761 must be added to the date in the Gregorian calendar. The Hebrew year ordinarily consists of twelve months alternately of thirty and twenty-nine days, based on the moon. Seven times during nineteen years an extra or embolismic month of twenty-nine days is inserted between two of the months and at the same time the month preceding the new month is given thirty instead of the usual twenty-nine days.

The Islamic calendar begins with Mohammed's flight from Mecca in A.D. 622 by the Gregorian calendar. The Islamic year consists of twelve months totaling 354 days and its New Year's Day moves completely backward through the seasons in the course of 32½ years. The Islamic calendar divides the years into cycles 30 years long. During each cycle, 19 years are regular years of 354 days and 11 years have an extra day. This system is off only one day in 2670 years with respect to the moon.

The early Christian calendar had no week nor names for the days. The week with its day names was introduced in the year 325 A.D. by the Emperor Constantine of Rome.

Even since the Gregorian calendar has been in use there have been attempts to reform the calendar. The most popular of recently suggested reforms are the thirteen-month calendar and the World Calendar. The thirteen-month calendar plan was published in 1849 by Auguste Comte, a French philsopher. His plan was to have 13 months of 28 days (the nearest thing to one lunation) and there was to be one blank day at the end of each regular year and two blank days at the end of each leap year. This was an attempt to combine solar and lunar elements, but apparently Comte failed to realize that even if 13 months of 28 days were 13 "moons," the blank days would throw the calendar out of gear again. After the first world war, George Eastman revived this proposal and maintained that it would be more suited for a businessman's calendar. However, the plan has given way to a more popular calendar reform—the World Calendar.

The World Calendar Plan proposes that there be twelve months, four of 31 days and eight of 30 days plus year-end Worldsday and a leap-year day following June every fourth year. This would make every year the same— every date would always fall on the same day of the week. This can be done by making February 30 days long and April 31 days long and changing March, May, and August from 31 to 30 days each. The number of days in each of the equal quarters would be 91 and each quarter would contain 13 weeks or three months and be identical in form. Also each month would have 26 weekdays plus Sundays. Other aspects of the calendar, such as the fixing of certain holidays, have been proposed. Under the World Calendar, Christmas would always fall on Monday, Labor Day would be set at September 4, and it has been proposed that Easter be set on April 8. There have been numerous other suggestions such as placing nearly all of the present holidays on Mondays so that holiday periods would contain three days.

This World Calendar and the thirteen-month calendar were the only two proposed calendar reforms the League of Nations did not reject when investigating the changing of the calendar. At the present the United Nations is considering only the World Calendar. The United States has refused to support the calendar reform in the United Nations and it has been said that if the United States would support it, it would probably go into effect in 1961. The arguments for this calendar reform are that is will make life much simpler, save business time and money, and will help toward unifying the world. The arguments against it include that it will cause confusion, will cost too much time and money to popularize it, is still weighed down by imperfections, and that it will disrupt our economy. Many people are willing to see another calendar change, but many are not.

While the Gregorian Calendar is still in use, it is sometimes convenient to know a method for finding the day of the week on which a certain date fell or will fall. The system which I shall describe will work for any date after 1582. For the system to be as efficient and as easy as one can get it, the days of the week are designated by the numerals 0, 1, 2, 3, 4, 5, and 6; where Sunday is represented by 0, Monday by 1, Tuesday by 2, etc. Since the months may begin on any day of the week, the months will be given characteristic numbers that will indicate the day of the week it began on with January 1 arbitrarily set on Sunday. This process gives to the months the following characteristics: January-0, February-3, March-3, April-6, May-1, June-4, July-6, August-2, September-5, October-0, November-3, and December-5. Then by designating the year and century by the correct number, one can make the computations as if the first day of the year were on Sunday. In the adjustment one must take into account the number of leap days occurring. When this is done one can set up the formula

$$(A + B + [C/4] + [D/4] + E + F - 1)/7 = Q + R/7.$$

(See footnote).

where A = the year, $(100C + D)$, *e.g.*, if $A = 1957$, $C = 19$ and $D = 57$; $B = 3C$; $[C/4]$ = leap year characteristic; $[D/4]$ = the number of leap days; E = the month characteristic; F = the day of the month; Q = the whole number of the quotient and R = the remainder; then R corresponds to the day of the week on which the date fell. For 1957, $(A + B + [C/4] + [D/4] - 1)/7 = Q + 1/7$; then 1957 has a characteristic of 1. If the month characteristic and the day of the month be added to this 1, the day of the week is obtained by considering the remainder after this sum is divided by seven. When a person has had practice with the system and has memorized the month characteristics, it is very easy to amaze people with this knowledge.

The symbol [C/4] indicates that only the integral part of the quotient is to be used.

When C is a multiple of 4 and D = 0 subtract 1 from R for any date preceding February 29 of that year.

SOURCES

Achelis, Elizabeth. *The World Calendar for The Mathematician,* The World Calendar Association, Inc., New York, New York.

American Mercury, Nov., 1953.

Case, John J., "Seasoning for the Calendar," *Science,* 122:648, October 7, 1955.

Johnson, Jotham, "Calendar," *The Encyclopedia Americana.*

Journal of Calendar Reform, Vol. 4, 23, 24, 25.

Nations Buisness, Vol. 43.

Popular Astronomy, Vol. 31, 35.

Popular Science, Vol. 166.

Scholastic, Vol. 65.

School Science and Mathematics, Vol. 23, 26.

Sollenberger, Paul, "Calendar," *The World Book Encyclopedia.*

United States Department State Bulletin, Vol. 32.

Wilson, P. W., *The Romance of the Calendar.*

"The invention of the symbol $\equiv$ by Gauss affords a striking example of the advantage which may be derived from an appropriate notation and marks an epoch in the development of the science of arithmetic."

—G. B. Mathews

Arithmetical Congruences with Practical Applications*

CAROL LAW
Student, Mt. Scholastica College

When we say that something is congruent, we mean that it is in agreement or in harmony with something else. In geometry we speak of congruent triangles and designate their harmonious qualities by the equal and similar signs $\cong$. In theory of numbers congruence still has the same connotation in that groups of numbers have some relation to each other. We are often concerned here with properties of numbers that are true for a whole class of integers differing from each other by multiples of a certain integer. For example, the squares of odd integers have the property that when divided by 8 the remainder is 1. This property holds for all odd numbers which is a class of numbers differing from each other by multiples of 2. Another example: when we square any number having 6 as the last digit, the product will also end in 6. Thus we see another property holding for a whole class of numbers. The consideration of these properties holding for all integers differing from each other by a multiple of a certain integer leads to a notion of congruence.

Carl Friedrich Gauss, who lived from 1777 to 1855, is credited with the foundation of congruences. He was a mathematical genius, especially with the theory of numbers as is shown in his great number classic, "Disquisitiones arithmeticae." His love for numbers is exemplified further in his well-known statement: "Mathematics is Queen of the Sciences, but Arithmetic is the Queen of Mathematics."

It is this topic, arithmetical congruences, which he treated so well, that will concern us now. Just what do we mean by arithmetical congruences? Two integers a and b whose difference $a - b$ is divisible by a given number m (not zero) are said to be congruent for the modulus m or simply modulo m. This means simply that two numbers are congruent if they have the same remainder when divided by the same number. This is expressed $a \equiv b \pmod{m}$ where the congruent sign is written $\equiv$. For example, $17 \equiv 5 \pmod{12}$ or $9^2 \equiv 1 \pmod{5}$. Note that the congruence is true whether you subtract the integers first or reduce first. To show that $9^2 \equiv 1 \pmod{5}$:

* A paper presented at the 1956 National Convention and awarded third place by the Awards Committee.

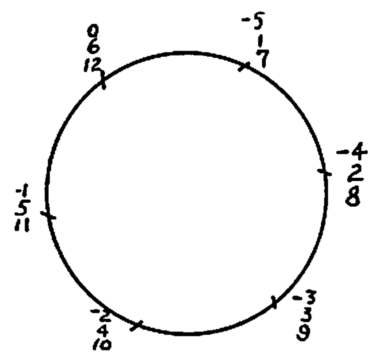
Subtracting first:

$9^2 - 1 = 81 - 1 = 80 = 5(16)$.
Then, by the definition,
$9^2 \equiv 1 \pmod 5$.

Reducing first:

$9^2 = 81 = 5(16) + 1$.
$1 = 5(0) + 1$.
Since 9^2 and 1 have the same remainder when divided by 5,
$9^2 \equiv 1 \pmod 5$.

A geometric interpretation of congruences is helpful.



Notice that the numbers at any one point in the figure are congruent with each other modulo 6. For example, $-5 - 7 = -12 = 6(-2)$. Therefore, $-5 \equiv 7$ (*mod* 6).

An application in daily life of this geometric interpretation is the clock. The hands indicate the hour modulo 12. We can find an extension of this idea used in telling time in the army. Whereas we start over with 1:00, 2:00, etc. after reaching midnight, soldiers begin with 0100 and continue to 2400 for an entire day.

For example: $0100 \equiv 1300$ (*mod* 12).

Congruences have many of the properties of equalities but some are quite different. The reader should be able to verify the following:

a) For any modulus m, $a \equiv a \pmod m$.
b) If $a \equiv b \pmod m$, then $b \equiv a \pmod m$.
c) If $a \equiv b \pmod m$ and $b \equiv c \pmod m$,
then $a \equiv c \pmod m$.
d) If $a \equiv b \pmod m$ and $c \equiv d \pmod m$,
then $a \pm c \equiv b \pm d \pmod m$.

e) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$.

f) If $a \equiv b \pmod{m}$, then $a^n \equiv b^n \pmod{m}$ where n is a positive integer.

g) A common factor n cannot be removed from $na \equiv nb \pmod{m}$ unless n is prime to m. If n is a divisor of m, then $a \equiv b \pmod{p}$ where $np = m$.

h) If $a \equiv b \pmod{m}$ and d is a divisor of m, then $a \equiv b \pmod{d}$.

i) If $a \equiv b$ with respect to moduli $m_1, m_2, \cdots m_n$ then $a \equiv b \pmod{l}$ where l is the greatest common factor of $m_1, m_2, \cdots m_n$.

Let us see now how we can apply these principles. Suppose we want to find if 47 is a factor of $2^{23} - 1$. To obtain 2^{23} would constitute a bit of work but with congruences the solution is simple.

$$
\begin{aligned}
2^6 &\equiv 17 \pmod{47}. \\
(2^6)^2 &\equiv 17^2 \pmod{47}. \\
2^{12} &\equiv 289 \pmod{47}. \\
2^{12} &\equiv 7 \pmod{47}. \\
2^6 \cdot 2^{12} &\equiv 17(7) \pmod{47}. \\
2^{18} &\equiv 119 \pmod{47}. \\
2^{18} &\equiv 25 \pmod{47}. \\
2^5 \cdot 2^{18} &\equiv 32(25) \pmod{47}. \\
2^{23} &\equiv 800 \pmod{47}. \\
2^{23} &\equiv 1 \pmod{47}. \\
\therefore\ 2^{23} - 1 &\equiv 0 \pmod{47}.
\end{aligned}
$$

The same method would be used to obtain the remainder when dividing such a number as 5^{65} by 127.

Arithmetical congruences are useful in the solution of linear, quadratic, and cubic congruences with one unknown. A method of solving linear congruences when the modulus is small is simply by trial and error—substituting numbers for x and seeing if they satisfy the congruence. However, there is a limit to the number of numbers that need to be substituted. Mathematicians have figured out that only those positive and negative integers whose numerical value $\leqq [m/2]$[1] need to be tried. Every number with numerical value $> [m/2]$ will be congruent to a number whose numerical value

[1] The symbol [m/2] is used to indicate the integral part of the quotient.

$\leqq [m/2]$, modulo m. For instance, to solve the linear congruence $2x + 1 \equiv 0 \pmod 5$ only numbers whose numerical value $\leqq 2$ need to be tried. We find $+2$ is a solution. Any integer of the form $2 + 5k$ where k is any integer will be a solution.

The trial and error method can be used for the solution of a quadratic congruence; however, factorization may be used to simplify the solution.

$$
\begin{aligned}
x^2 + 6x + 8 &\equiv 0 \pmod 7,\\
(x + 4)(x + 2) &\equiv 0 \pmod 7,\\
(x + 4) &\equiv 0 \pmod 7,\\
x &\equiv 3 \pmod 7.\\
(x + 2) &\equiv 0 \pmod 7,\\
x &\equiv -2 \pmod 7.
\end{aligned}
$$

Usually the human mind attempts to find some practical application of knowledge. I have developed an original idea which I think is interesting and perhaps useful to some; in my case, to the marching band director. Let us take a very simple quadratic congruence and illustrate how it might be employed in figuring marching band formations.

Let us suppose that the college band of St. Benedict's College were marching down Pittsburg's football field. Mr. Roark, St. Benedict's band director, decides to have his band march on the field in a perfect square formation; that is, as many rows as columns. Then at the middle of the field he wants them to go into a *P* formation, for Pittsburg. To make a *P* he wants 9 men for the loop and 15 for the stem. He wants to enclose the P with rows of men, 15 per row. How many men would he need? We could set up the following relation:

$$
\begin{aligned}
x^2 - 9 &\equiv 0 \pmod{15},\\
(x - 3)(x + 3) &\equiv 0 \pmod{15},\\
x - 3 &\equiv 0 \pmod{15}.\\
x &\equiv +3 \text{ or } -12 \pmod{15}.\\
x + 3 &\equiv 0 \pmod{15},\\
x &\equiv -3 \text{ or } +12 \pmod{15}
\end{aligned}
$$

Using 12 as a solution, we have,

$$
\begin{aligned}
144 - 9 &\equiv 0 \pmod{15}\\
135 &\equiv 0 \pmod{15}
\end{aligned}
$$

Now he has 12 rows and 12 columns of men marching down the field. Nine of these will be used for the loop of the *P* which leaves

135 for the stem and the rows. By dividing 135 by 15 we see that there are 9 rows of 15 men in each.

Cubic congruences may also be solved by trial and error. However, with a congruence such as $x^3 + 19x^2 + x + 23 \equiv 0 \pmod{42}$ such a process would be quite tedious. Cubic congruences may be solved by use of the Chinese Remainder Theorem.[2]

Throughout my paper, I have attempted to give practical applications; but perhaps one of the most practical and fascinating applications of congruences is in working calendar problems.

You have heard about people being "walking dictionaries." You can become a "walking calendar." You can figure out the days of the week on which certain dates fall; certain dates of the month on which certain days fall; the day on which you were born; the day of your next birthday; or even dates on which Easter falls. One day in history class when we were discussing the second World War, my professor asked me, "When did the United States declare war on Japan?" I hesitated momentarily, then answered emphatically, "On Monday, December 8, 1941." He looked at me surprised to think that I remembered even the day of the week. He really would have been surprised if he had known that I had quickly applied a mathematical formula. This is what I did:

f = day of week beginning with Sunday, 0; Monday, 1; etc.
k = day of month.
m = number of month beginning with March, 1; etc.; Jan., 11; and Feb., 12.
D = last two digits of year.
C = first two digits of year.

$$f \equiv k + [2.6m - 0.2] + D + [D/4] + [C/4] - 2C \pmod{7}.$$[3]

$$f \equiv 8 + [(2.6)(10) - 0.2] + [41/4] + [19/4] - 2(19) \pmod{7}.$$

$$f \equiv 8 + 25 + 41 + 10 + 4 - 38 \pmod{7},$$

$$f \equiv 88 - 38 \pmod{7},$$

$$f \equiv 50 \pmod{7},$$

$$f \equiv 1 \pmod{7}.$$

Monday is the required day of the week.

---

[2] Harriet Grittin, Elementary Theory of Numbers, pp. 77-80.
[3] This symbol [2.6m — 0.2] indicates the integral part of the expression.

I'm sure there must be some application also in the social world. Let's say you're out with the favorite boyfriend or girlfriend in the car—the moon is shining like a silver dollar—and the subject, guess what! Finally you agree—yes, June is the month and Saturdays are the only days you'll consider. But what date shall we set? Here you are in the middle of nowhere—a calendar not within 10 miles and at such a vital moment! But, yes, you had studied congruences. On what dates do Satudrays fall?

$$
\begin{aligned}
k &= ? \\
f &= -1 \\
m &= 4 \\
C &= 19 \\
D &= 57
\end{aligned}
$$

$$
\begin{aligned}
-k &\equiv [(2.6)(4) - 0.2] + 57 + [57/4] + [19/4] - 2(19) \\
&\quad - (-1)\ (mod\ 7), \\
-k &\equiv 10 + 57 + 14 + 4 - 38 + 1\ (mod\ 7), \\
-k &\equiv +\ 48\ (mod\ 7). \\
k &\equiv 1\ (mod\ 7).
\end{aligned}
$$

The remainder, 1, indicates the first day of the month. Simply add 7 days on to this date each time to find the other Saturdays. You found that in June, 1957, Saturdays fall on the first, the eighth, the fifteenth, the twenty-second, and the twenty-ninth. You applied your formula and the wedding date was set.

I hope my paper has served to introduce congruences to many and has been of interest to all. I have found congruences to be interesting and satisfying; and, as I pointed out in my last example, they may be a help or a hindrance, depending in this case, upon your attitude toward the married or single state of life!

BIBLIOGRAPHY

Courant and Robbins, *What is Mathematics?* Oxford University Press, 1941. pp. 31-40.

Grittin, Harriet, *Elementary Number Theory,* pp. 77-80.

Reid, Legh Wilbur, *The Elements of the Theory of Algebraic Numbers.*

Uspensky and Heaslet, *Elementary Number Theory.* New York and London: McGraw-Hill Book Company, 1939. pp. 126-58, 206-11.

Motions of A Space Satellite[1]

CHARLES TRAUTH

Student, Kansas State Teachers College, Emporia

I am submitting this paper realizing the probable availability of more accurate material as the International Geophysical Year grows nearer. At present, the periodical material is somewhat stereotyped and inconclusive. The introductory section on the satellite and rocket will be shorter when more definite information is available.

Although this paper is primarily concerned with the motion of an earth satellite, I should like to present a brief description of the International Geophysical Year satellite.[2] Brooks and Perkins is manufacturing a satellite with dimensions as follows: skin thickness—0.02 in.; diameter—20 in.; weight empty—4 lbs. It is made of 95% magnesium, about 3½% aluminum, and about 1% zinc. The skin and interior bracing (tubing) are magnesium. The skin is screwed to the tubing, and the satellite will be sealed by welding or with a mechanical joint. The shell will have a tolerance of 4 micro inches and will be polished to a bright reflective surface.[3]

The satellite may be expected to carry instruments capable of measuring:

1) cosmic rays and high energy particles,
2) solar corpuscular radiation,
3) micro meteors,
4) solar ultraviolet and x-ray,
5) magnetic field and Stormer ring current,
6) hydrogen in interplanetary space,
7) atomic, ionic, and molecular masses in the ambient air.[4]

The internal temperature of the satellite in space is unpredictable. It might range from below 0° F. to 300° or 400° F.[5]

---

1 A paper presented at the 1956 National Convention of KME and awarded second prize by the Awards Committee.

2 There have been many recent articles concerning this, published within the last year, which give conflicting information. However, the most recent seem to indicate that the description above is accurate.

3 Aviation Week, 63: 14-15.

4 Sky and Telescope, 15: 112, Jan., 1956.

5 Sky and Telescope, 15: 247, Mar., 1956.

A more recent report from the Naval Research Laboratory indicates that the satellite will contain an inert gas and carry:

1) a solar cell and some chemical source of electric power,
2) an ion chamber to register ultraviolet radiation,
3) several thermistors to take the inner and outer temperatures (they will register from minus 200° F. to 300° F.)
4) a Nichrome erosion gauge to measure the effects of meteoric dust,
5) a "Minitrack" transmitter to radio data to earth,
6) a pressure gauge which will act if the shell is punctured by a meteorite,
7) various memory units and amplifiers.[6]

The rocket which carries the satellite to its orbit will be a three-stage rocket, and according to Dr. M. W. Rosen, of the Naval Research Laboratories, it will be 72 ft. long, 45 in. at greatest diameter, and have a gross weight of about 11 tons.[7]

The first stage of the rocket will be similar to the "Viking" rocket. It will have 27,000 lbs. thrust, fire for 140 sec., use liquid oxygen as an oxidizer and use ethyl alcohol, gasoline, and silicone oil as fuel. Turbine-driven pumps will transfer fuel from tanks to motor. It will be built by General Electric Company.

The second stage will use fuming nitric acid as its oxidizer and unsymmetrical dimethyl-hydrazine as fuel. It will have a pressurized fuel system (using pressurized Helium and will be built by Aerojet General Corporation.

The third stage will use a solid propellant.[8] [9]

The first stage will provide 15% of the orbital velocity and most of the energy to raise the satellite to orbital altitudes.[10] The second unit will contain a "system of inertial references and will direct the flexible main power plants of the first and second stages to produce pitch and yaw control."[11] It will furnish 32% of the orbital velocity. The third stage is unguided and will be contained within

---

[6] Popular Science, 170: 144, Jan., 1957.
[7] H. P. Steier, American Aviation, 19: 38, April 9, 1956.
[8] At present, I can find no more information as to the nature of the 3rd stage fuel.
[9] Aviation Week, 64: 37, Jan. 30, 1956.
[10] Aviation Week, 65: 38, Aug. 27, 1956.
[11] Ibid.

the second stage under a protective nose cone which is to be jettisoned after second-stage firing. The third stage will have a stabilizing spin, be directed by the second stage, and provide 50% of the orbital velocity.[12]

[13]Theoretically the vehicle is to be launched vertically, tilt gradually along its course until it falls into an orbit somewhere between 200 and 400 miles from the earth's surface at a speed of 18,000 mph. The first stage is supposed to cut off at an altitude of 30 to 40 miles at 3500 mph; the second, 130 miles at 11,000 mph, and it will coast to an altitude of approximately 300 miles where the third stage will fire sending the satellite on its 18,000 mph spinning trajectory.[14]

The rocket will follow the path illustrated by the following diagram.[15]

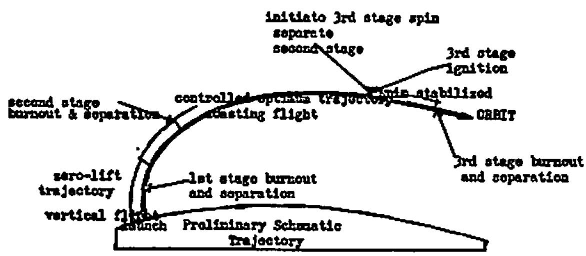


Fig. 1

The rocket will be fired from Patrick Air Force Base in Florida, southeast such that it makes a 30°-35° angle with the equator. This angle enables the rocket to take advantage of the earth's rotation and will assure that the satellite always passes above the region of the earth bounded by latitudes 40°N. and 40°S.[16]

The projection of the satellite's path on the earth will intersect the equator further westward on each revolution of the satellite. The points where the satellite crosses the equator are called nodes. Figure

---

[12] Ibid.
[13] The following information is somewhat general, but seems to include most of the values given in various other sources.
[14] **Aviation Week**, 65: 30-31, Oct. 15, 1956.
[15] **American Aviation**, op. cit.
[16] **Sky and Telescope**, op. cit., Mar., 1956.

2 shows geometrically the projection of the satellite's path on the earth as it crosses the equator further west each revolution. This is called regression of the nodes.

Now, if this path deviates from the equator by only a small distance, ε, this will lead to an expression for $\Delta\phi$ when the satellite makes a complete ε oscillation. The satellite's projection will cross the equator short of $\phi = 2\pi$ by the amount $\Delta\phi = 2\pi - T$, where T represents the azimuthal period of the satellite.

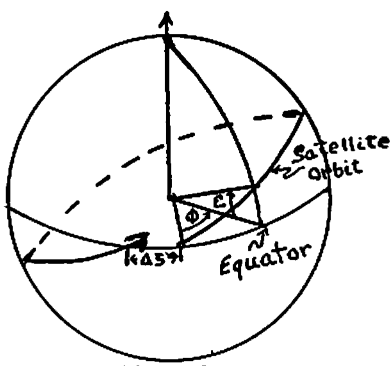


Fig. 2

Therefore for a regression of the nodes

$$\Delta s = R\Delta\phi = R[2\pi - (2\pi/\omega]; \text{ R is the radius of the earth.}$$

Now let us use the value $\omega = [1 + 2JGMR^2/r_0p^2]^{1/2}$, the development of which is outside the scope of this paper.[17] J is a constant with a value of $(1.637 \pm 0.004) \times 10^{-3}$ as determined by Jeffrey.[18] Other notation standard.

Assuming a circular orbit such that $p = v_0r_0$, and $GM/r_0v_0^2 = 1$, the following equation is obtained,

$$\Delta s = 2\pi JR(1 - 2h/R) \quad \text{where } h = r_0 - R = 200 \text{ miles.}$$

This equation yields $\Delta s = 2\pi(1.637)(3964)[1 - 2(200)/3964] \times 10^{-3}$, which is equal to 36.8 miles (slide rule accuracy).

---

[17] Journal of Applied Physics, Oct., 1956, p. 1142.
[18] Ibid.

The period, then, for precession would be equal to the circumference of the earth, $2\pi R$, divided by the amount precessed each oscillation, Δs, (giving the no. of oscillations) times the period of the satellite—which has been estimated at 1.51 hours. Therefore,

$$\begin{aligned} P &= 2\pi RT/\Delta s \\ &= 2\pi(3964)(1.51)/3.68 = 1025 \text{ hours (slide rule accuracy)} \end{aligned}$$

This is approximately 42.7 days.

All of this was based on the assumption that the orbit is circular and that ε would be small. However, since this is not always the case, one should consider the motions for an elliptical orbit when ε is not small necessarily.

The nodes are still shifting westward, but simultaneously the longest axis of the elliptical orbit will turn eastward in its own plane. This is illustrated in Fig. 3, where the apogees are shifting eastward.

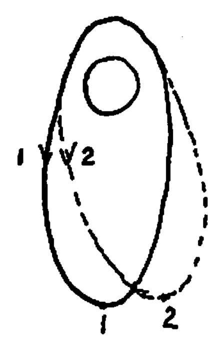


Fig. 3

This eastward motion is at very nearly the same rate as the precession of the nodes when one is considering natural satellites that move nearly in the equatorial planes of their planets.

However, L. E Cunningham of the Smithsonian Astrophysical Observatory points out that the situation will be quite different for an artificial satellite with expected inclination of about 40°.[10]

Mr. Cunningham goes on to say that this eastward drift of the satellite's orbit will be considerably slower than the westward preces-

[10] *Sky and Telescope*, 15: 57, Dec., 1956.

sion of the nodes due to the inclination of the orbit. In fact, he states that were the inclination to be greater than about 45° both motions would be westward, and that for an inclination of about 39° the daily eastward motion (in right ascension) of the sun and apogee are equal, so the same point on the satellite's orbit would be observable in a given twilight zone.

Geometrically, the first of these statements can be shown by Fig. 4, which shows that node 1 has precessed westward to 2, while the apogee 3 has moved eastward to 4.

In the case where the inclination is 45° or more, the Δs would be so great that the paths would not cross and thus the apogee would have to move westward.

Fig. 4

The latter, for a 39° inclination, may be explained geometrically in Fig. 5, which shows the earth, sun, and satellite in greatly distorted relationships.

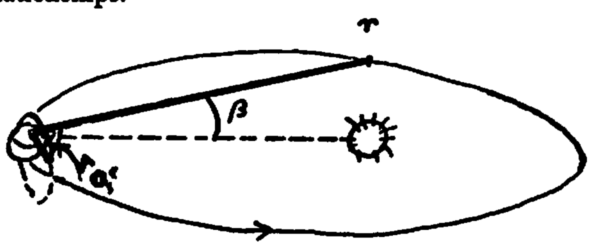


Fig. 5

The right ascension of the sun is essentially the angle β between the sun and the vernal equinox, and the right ascension of the satellite is essentially the angle α.

Therefore if the inclination of the satellite's orbit is 39°, the change in β (due only to the earth's motion about the sun) will equal the change in α (due to the earth's motion and the eastward motion of the satellite's orbit).

In this orbit (of 39° inclination), the observations of the satellite would tend to cluster around two points of the orbit. If this were too concentrated, the accuracy of orbit determinations would be greatly decreased. It is interesting to note that the satellite will be fixed at nearly this angle, but ". . .there will not be the necessary degree of accuracy to cause. . ."[20] the above conditions.

Eventually, the satellite will be slowed down at the perogee of its orbit due to the earth's atmosphere, such that the apogee will decrease and the orbit will become more and more spherical until the satellite spirals into the earth's atmosphere and burns.

There remains another phase of the satellite's motion to be discussed—the perturbation of the satellite's orbit due to the nearness of the satellite to the earth. The analytical discussion of this is outside the scope of this paper. It will suffice to say that the perturbations will be very large and that the mathematics in setting up the equations of motion is involved and laborious.

Within the next year, who knows what valuable information may be obtained from the space that man has long dreamt of reaching. Only two hundred miles—and yet, in a sense—another world.

[20] Ibid.

"The nineteenth century which prides itself upon the invention of steam and evolution, might have derived a more legitimate title to fame from the discovery of pure mathematics."

—Bertrand Russell

Opportunities in Teaching Mathematics in Secondary Schools*

ROSEMARY KOTESA
Student, College of St. Francis

The opportunities offered to a mathematics teacher will be considered in this paper in view of the general teaching profession with applications to mathematics. The subject will be approached in the light of opportunities for intellectual and cultural growth, personal development, and positive influence upon the students by the mathematics teacher.

"Teaching is the profession that makes all professions possible."[1] The secondary teacher is the greatest single factor in high school education. His influence permeates the thought and behavior of the boys and girls associated with him. The secondary teacher has the opportunity to build a bridge between youth and maturity.

This positive influence of the mature upon the immature is of primary importance in the field of teaching mathematics. Due to lack of competent mathematics teachers with a genuine interest in the subject, the influence upon youth has been to steer them away from mathematics courses, mathematics careers, and appreciation of mathematics in daily living instead of toward it. New York's Board of Education reported that only eighty-eight candidates filed for six-hundred fifty teaching positions in mathematics in secondary schools. Many of the candidates admitted that they acquired the attitude to hate mathematics as a subject from their secondary mathematics teachers.[2] Thus the opportunity to break down the barriers of hatred surrounding high school mathematics courses and false conceptions of the value of mathematics is one of the first challenges the teacher must meet.

In preparing for a profession in mathematics, the student should realize that teaching offers the opportunity of working with a subject he loves and the opportunity to impart some of his love and understanding of the subject to others. He has been offered the opportunity

---

* This paper was awarded second prize in the essay contest sponsored jointly by KME and STIP.
1 The Mathematics Teacher, May, 1955, p. 343.
2 Scholastic Teacher, "Most Hated Subject", September, 27, 1955, p. 1.

to take the living mind and mold it through the influence of his personality and the subject matter he has to offer.

Today there exists more of a controversy over the content of our mathematics courses than any other subject taught in the high school. The individual mathematics teacher has a great opportunity to make a contribution to the development of a course of study in mathematics designed to fulfill the needs of every student whether for preparation for a career or merely preparation for daily living. It has been claimed that mathematics teachers have done next to nothing to inform and convince either students or parents and citizens of the community that mathematics means something in their lives. The teacher's position with his students and in the community presents the opportunity to really sell the subject of mathematics.

In many professions we note that after a few years on the job the work becomes routine and interests become stagnant. However, since mathematics itself is neither dull nor routine, the teaching of it always brings new experiences with the pupils and new problems and new solutions with the material. Working with others on the teaching staff, the mathematics teacher has the opportunity to share in their knowledge and to broaden his interests. He can continue to develop in his field throughout his career and keep up an active enthusiasm for problems of the mind.

The high school teacher—in this case the mathematics teacher—usually has an important place in community life. The role of the high school teacher is enhanced by the great faith placed in him by the vast majority of parents throughout the country. An alert and enthusiastic teacher with wide social and professional interests can wield a wholesome influence in his community.

Our cultural and economic progress has resulted primarily from effective teaching. The part that mathematics has played in this progress is indispensible. Modern living requires scientific knowledge by the lay person; this technical age requires trained scientists and mathematicians. Interest, encouragement, and guidance in such fields must be created and stimulated by the mathematics teacher. This opportunity is passed up by many mathematics teachers who fail to challenge their students and train youth to carry on the position mathematics has attained today.

It is the role of the secondary mathematics teacher to inspire and impress on the students that mathematics becomes their ready servant rather than a problem in itself. He must challenge his superior students. As a result, in the future, he may have the opportunity to see each of these students as a mathematician in industry, a scientist, or following in his footsteps as a mathematics teacher. He must interest the dullest pupils of his class so that they may at least take pleasant memories and a sincere appreciation of the beauty and worth of the subject from the class. With both types of students the teacher must recognize his opportunities and take advantage of them.

The mathematics teacher by the very nature of the subject he is teaching helps his students to think clearly, to abstract, to use fundamental skills, and to organize materials—not just for his course in mathematics, but for life. The skillful teacher has an opportunity to develop those attitudes, understandings, ideals, and appreciations which should result from a study of mathematics.

The teaching field of mathematics in the secondary school promises job security for the present and even more in the future. To those entering into this field of teaching mathematics or already in it, Mr. G. Seidel offers the following advice:

> You are on the firing line of a technological America. Keep your sights high so that the students you develop today will be capable, well-trained, hard-working, well-balanced men and women of tomorrow. Be sure that the torch of learning you hand to them will be burning as brightly or brighter than it was when you first took hold of it and dedicated your life to the great and proud profession of teaching and inspiring the youth of America.[3]

[3] George R. Seidel, The Mathematics Teacher, "Math—A Language", April, 1955, p. 217.

BIBLIOGRAPHY

Crow, Lester and Alice, *High School Education,* New York: The Odyssey Press, 1951.

Seidel, George, *The Mathematics Teacher,* "Math—A Language", April, 1955.

The Mathematics Teacher, May, 1955.

Scholastic Teacher, "Most Hated Subject", September 27, 1955.

What Should a Good Teacher of Mathematics Know?

Gertrude V. Pratt
Faculty, Central Michigan College

I am a teacher and I help train young people to become teachers of mathematics. At the beginning of each college year I recount what I believe to be the requisites of a good teacher, not only to check up on myself but also that I may be better prepared for my mission in developing good teachers. Upon all of us depends the instruction of thousands of boys and girls in the secondary schools of our nation. Will you join me in a little self-analysis? This article is written for those of you who plan to teach.

1. **A good teacher of mathematics should know his subject.** He should know mathematics far beyond anything he expects to teach. If for nothing else, his education should be of such a nature that he will have a feeling of confidence when he is teaching, a feeling of adequacy, a feeling that he knows more than what is in the textbook. But more than this, such a preparation makes him better able to organize his courses to fit the future needs of those under his instruction, whether those needs be college preparatory, business, mechanics and shop work, or just for adequate living in our time.

Knowing a great deal about his field enables a teacher to point out the interrelations between arithmetic, algebra, geometry, and trigonometry so that they become an integrated whole in the minds of the students and not isolated pigeonholes of instruction.

2. **He should know what mathematics is and how it came about.** One needs to know something of the history of mathematics in order to truly understand mathematics. When we realize how very, very old it is; when we study its development throughout the ages—then and only then can we appreciate its beauty and its power. The respect that we feel for something we had hitherto taken for granted cannot help but increase our efficiency as teachers.

Let me illustrate. There are many students in college today training to be elementary teachers who cannot perform simple fundamental operations. Furthermore, they are allergic to arithmetic. They are elements of a vicious cycle. Somewhere along the line, they have been taught by teachers who disliked arithmetic and slighted it, and subconsciously they are prepared to go out and do the same thing.

It is amazing how one can recondition these people by exposing them to the history of numbers while teaching them some simple arithmetic. Gradually they come to realize that there weren't always numbers and number symbols; that there was a beginning and a reason for their creation. They begin to understand what a number means in our system with its base of ten. They have fun making up number systems with other bases and learn to add, subtract, multiply, and divide, using bases other than ten. In this manner, they develop an understanding of why we perform the fundamental operations as we do; these are no longer mechanical processes for them; they have meaning. Arithmetic begins to make sense for them and they begin to like to work with numbers.

Such a unit also includes an examination of the ancient Egyptian, Babylonian, Mayan, Roman, and Greek number systems. They then understand why the abacus came into being, and they learn to add and subtract on it. As a result of this study they get an appreciation of the Hindu-Arabic numeral system: its zero as a place-holder for the empty rod on the abacus; its notion of place-value which enables us to write any number, large or small, by means of nine digits and a zero.

These things should be known by every teacher of mathematics. Secondary school teachers would be wise to familiarize themselves with the development of algebra and geometry.

3. **He should know the uses of mathematics in the affairs of men.** A successful teacher knows the fields in which mathematics is useful and necessary. He consciously uses this knowledge in his teaching in order that specialists not be lost to our country and to our way of life because of lack of proper guidance. We cannot help but be aware that trained mathematicians and scientists are in great demand. Everywhere we see evidences of the efforts that are being put forth by our federal government, educational associations, the National Science Foundation, and big industry in general.

Remember that behind the artisan there is a chemist; behind the chemist there is a physicist; behind the physicist there is a mathematician. Our universe is governed by physical laws. Were it not for the formulas worked out by mathematicians in the last century, we would not have automobiles, airplanes, radio, television, radar, atomic energy, etc., today. Even to know advanced psychology and sociology, one must know mathematics. We could add such diverse fields as biology, the graphic arts, music, and philosophy, to the list of areas requiring mathematical knowledge.

Mathematics helps the scientist and the engineer when he experiments in an effort to solve problems; it gives them the language for expressing and treating these problems. It enables the engineer to predict conclusions; he uses mathematical formulas and tables to determine what size beams he must have in building a skyscraper. The industrialist, by use of mathematical methods, determines before installing new machines what they will cost and what savings in the cost of production they will effect. The occurrence of an eclipse of the sun or moon or the appearance of a comet in the sky are far less astonishing than the astronomer's ability to predict the time of its occurrence.

With the invention of the microscope, a whole new field opened up to biologists. The biological sciences benefit from the mathematical way of thinking. Quantitative studies extended to botany determined the rate of absorbtion and evaporation of water by plants. Such studies in physiology resulted in the ability to measure temperature and blood pressure. Quantitative studies of the flow of water in pipes were believed to cover the case of blood flowing in arteries and veins. Proof that the blood circulates around the body before returning to the heart reinforced this view because it likened the body to a pumping plant in action.

The study of light explains much of the sense of sight while the study of sound clarifies problems involving the sense of hearing. In fact, years ago, the famous Dr. William Osler said that medicine would become a science when man learned to count.

We should not close this brief resumé of the ways by which mathematics contributes to our culture without mentioning the part played by statistics—the art of reasoning by figures. The world is our laboratory; we gather statistics on what has occurred. Then we obtain laws and express them by mathematical formulas. Statistics is an effective aid in the study of population changes, stock-market quotations, unemployment, wage scales, cost of living, birth and death rates, extent of drunkenness and crime, distribution of physical characteristics and intelligence, and the incidence of diseases. It is the basis of life insurance, social security systems, medical treatments, and government policies. Even the business man uses statistical methods to locate his best markets, to control manufacturing processes, to test the effectiveness of his advertising, and to gauge interest in a new product.

The theory of probability is a branch of mathematics that is fundamental in the theory of gases, the insurance business, quality control in the manufacturing business, and the physics of the atom. The ordinary person is interested in this theory only to the extent of finding the probability of whether or not an event will happen to him; precise numerical probabilities are necessary as a basis for decisions on large commercial, engineering, or medical ventures.

Mendel's Law on the inheritance of physical characteristics was the result of an experiment, the results of which were the same as the associations of heads and tails in throwing two coins. The knowledge obtained from this experiment on green and yellow peas became valuable in the study of human heredity, horticulture, and animal husbandry, as well as in other fields.

I recommend that you read "Mathematics in Western Culture" by Kline, should you desire further amplifications of the areas mentioned above or further fields to be explored. This is an important phase of teacher education today. It is important not only in the teacher's job of guiding future mathematicians but also in his role of developing in each student an appreciation of and a respect for mathematics as a useful art and an understanding of what it contributes to the era in which he lives.

4. **A good teacher of mathematics should know how the mind comes to grasp mathematics and to use it.** One of the foundation stones in my philosophy of teaching is that every secondary school teacher should know how the arithmetic processes are taught in the grade schools. This is necessary for the understanding of the discipline of mathematics as a whole and for an orientation to the constant re-teaching he must do. We should not be impatient as we do this re-teaching but rather welcome it as an opportunity to set some youngster on the way to loving and enjoying mathematics because he now undersands it. Establish a concrete, meaningful, approach for each fundamental process in arithmetic. Surround yourself with visual aids; crude, homemade ones will suffice. You will be happy with the progress that some of your problem pupils will make.

In algebra, teaching a child to rationalize and formulate his own rules makes for re-rationalization when the rule has been forgotten and must be discovered anew. Not only is it a lot more fun than just memorizing rules and then applying them, but it establishes a pattern of thought. Once you have established the inductive approach in your teaching, you will be amazed and pleased to find that some of your students will leap ahead of you and make their own generalizations. They are thinking for themselves.

Discovery in the field of geometry makes each of us feel like Columbus! Let's not just prove a theorem in geometry; let's discover the fact first because it is fun and because it stimulates us to investigate further that we may make new discoveries. The analytic approach to a problem fosters the habit of analysing any situation that arises and helps the student to organize the facts at hand and draw logical conclusions.

5. **We should know the objectives for the course we teach** in addition to the purposes for teaching mathematics in general and for all teaching in the secondary schools. We can find discussions on such aims in any good book on "Teaching Mathematics in the Secondary Schools." We educate in our schools today many students who will not become specialists in mathematics. It is necessary that we give each the mathematical training which will meet the ordinary demands of his child world and his adult world as well.

In general, let us say that the purpose for teaching our subjects through the plane geometry should emphasize attitudes, interpretations, organization of materials, thought processes and the like, more than skills and complete mastery of subject matter. We leave such for the specialist who takes the advanced courses in high school. But each teacher must have a philosophy of teaching and know where he wants to go in each course.

6. **A good teacher knows that even though he has a degree from a college or university his education is never completed.** We never remain static; we either improve or we regress. This means summer school, work-shops, conferences, and on-the-job training. Above all, it means an awareness of human frailty and the too-easy way of staying in a rut. It means a will for improvement, self-study, and experimentation.

In addition to courses in education aimed at the improvement of teaching techniques, we should study mathematics at the graduate level. Many teachers fail to grow mathematically. Too few of our college graduates are earning their advanced degrees in mathematics. We cannot otherwise meet the challenge of our discipline in the light of all the new developments in mathematics. Mathematics is a dynamic, growing science and the good teacher keeps up-to-date.

One way of staying abreast of the times is to belong to the National Council of Teachers of Mathematics and attend its conferences. Membership also makes available to the teacher its publications, *The Mathematics Teacher, The Arithmetic Teacher,* and *The*

(Continued on page 52)

The Problem Corner

EDITED BY J. D. HAGGARD

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond the calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets before March 1, 1958. The best solutions submitted by students will be published in the Spring, 1958, issue of THE PENTAGON, with credit being given for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to J. D. Haggard, Department of Mathematics, Kansas State Teachers College, Pittsburg, Kansas.

PROBLEMS PROPOSED

Each of the proposed problems in this issue of THE PENTAGON *is taken from an examination given in one of the public secondary schools of Russia.*

106. In an isosceles triangle ABC, AB = BC = b, AC = a, $\angle$ ABC = 20°. Prove that $a^3 + b^3 = 3ab^2$.

107. Prove that the g.c.d. of the sum of two numbers and their l.c.m. is equal to the g.c.d. of the numbers themselves.

108. Show that $1/2 + 1/3 + \cdots + 1/n$ cannot be an integer for any integer n.

109. The lengths of the sides of a triangle are in arithmetic progression. Prove that the radius of the inscribed circle is one third the length of one of the altitudes.

110. Prove that if a, b, c, d are positive numbers and the system of inequalities $ax - by < 0$, $-cx + dy < 0$, $x > 0$, $y > 0$, has a solution, then $ad - bc < 0$, and conversely.

Note by the Editor. No satisfactory solutions have been received for the following problems: Nos. 84, 85, 86, 87 published in the Fall, 1955, number of THE PENTAGON; No. 91, Spring, 1956; Nos. 99, 100, Fall, 1956; Nos. 101, 102, 105, Spring, 1957.

SOLUTIONS

103. *Proposed by Glenn W. Thornton, Student, University of New Mexico, Albuquerque, New Mexico.*

A military ambulance traveling at the average speed of 12 miles per hour sent on ahead a motorcyclist who could travel with twice the speed of the ambulance. A half hour later it was found necessary to revise the message, and a second motorcyclist was sent to overtake the first. The second messenger returned to the ambulance in 45 minutes. What was his average speed while delivering the message?

Solution by Louis L. Hungate, Texas Technological College, Lubbock, Texas.

Let x denote the speed of the second motorcycle, in mph and t the time in hours required to overtake the first motorcycle.

The first motorcycle is 6 miles ahead of the second when the second departs, thus:

$$tx = 6 + 24t$$

The ambulance will have traveled 9 miles during the 45 minutes the second cyclist is gone, thus the total miles traveled by the second cyclist is

$$tx + tx - 9 \quad \text{and also} \quad (3/4)x$$

from which we get

$$t = (3x + 36)/(8x)$$

Substituting this value of t into the above equation yields the quadratic

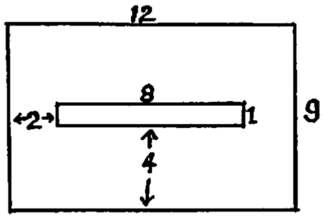
$$x^2 - 28x - 288 = 0$$

which has the positive solution of 36 mph.

Also solved by Betty Baker, Chicago Teachers College, Loia McInally, Wayne State University, Darcy Sullivan, Michigan State University, and Robert Gailor Justice, Tennessee Polytechnic Institute.

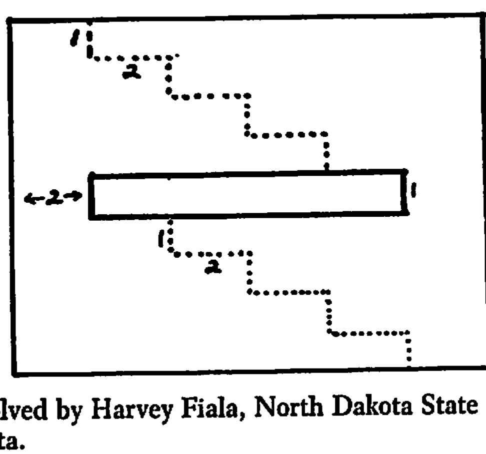
104. *Proposed by Vaughn Hopkins, Student, Central Missouri State College, Warrensburg, Missouri.*

Cut the slotted rectangle shown below into two pieces which may be placed together to form a square 10 inches on a side.



Solution by Loia McInally, Wayne State University, Detroit, Michigan.

Cut along the dotted lines, then translate the right hand piece up 1 unit and left 2 units.



Also solved by Harvey Fiala, North Dakota State College, Fargo, North Dakota.

The Mathematical Scrapbook

EDITED BY J. M. SACHS

"Formal thought, consciously recognized as such, is the means of all exact knowledge; and a correct understanding of the main formal sciences, Logic and Mathematics, is the proper and only safe foundation for a scientific education."

—ARTHUR LEFEVRE

=Δ=

Professor C having spent a delightful afternoon looking up references to his ancestors in the local library, came up with the following puzzle:

"I had two parents, four grandparents, eight great grandparents, etc. If we go back n generations then I had 2^n ancestors. But $2^{30} = 1,073,741,824$. Thirty generations is a little less than a thousand years. What a crowded place our little planet must have been in the year 957!"Can you solve the professor's population puzzle?

=Δ=

"The study of mathematics is apt to commence in disappointment. The important applications of the science, the theoretical interest of its ideas, and the logical rigour of its methods, all generate the expectation of a speedy introduction to processes of interest. We are told that by its aid the stars are weighed and the billions of molecules in a drop of water counted. Yet, like the ghost of Hamlet's father, this great science eludes the efforts of our mental weapons to grasp it—" 'Tis here, 'tis there, 'tis gone"—and what we do see does not suggest the same excuse for illusiveness as sufficed for the ghost, that it is too noble for our gross methods. "A show of violence," if ever excusable, may surely be "offered" to the trivial results which occupy the pages of some elementary mathematical treatises.

The reason for this failure of the science to live up to its reputation is that its fundamental ideas are not explained to the student disentangled from the technical procedure which has been invented to facilitate their exact presentation in particular instances. Accordingly, the unfortunate learner finds himself struggling to acquire a knowledge of a mass of details which are not illuminated by any general conception. Without a doubt, technical facility is a first requisite for valuable mental activity: we shall fail to appreciate the rhythm of Milton, or the passion of Shelley, so long as we find it necessary to spell the words and are not quite certain of the forms of the individual letters. In this sense there is no royal road to learning. But it is equally an error to confine attention to technical processes, excluding the consideration of general ideas. Here lies the road to pedantry."

—A. N. Whitehead

=Δ=

I hope that posterity will judge me kindly, not only as to the things which I have explained, but also as to those which I have intentionally omitted so as to leave to others the pleasure of discovery.

—Rene Descartes

=Δ=

In this International Geophysical Year it seems appropriate to devote a portion of this issue to the subject of map projection. The problem of mapping the earth onto a flat sheet of paper has interested man for many years. Let us look at the subject of projections briefly. It can be shown that no map of the globe preserves distance. Some preserve angle, some area, some are better approximations than others in these characteristics as well as in preservation of general shape. Considering the earth to be a sphere, we can represent the coordinates of a point on the surface of the earth by the equations:

$$x = r \cdot \cos u \cdot \cos v$$
$$y = r \cdot \sin u \cdot \cos v$$
$$z = r \cdot \sin v$$

where u represents longitude and v latitude. If we were to map the earth onto a cylinder, tangent to the earth at the equator, by means of projections in planes perpendicular to the polar axis, then the point on the earth having coordinates (u, v) would map into a point on the cylinder (α, β). With the Greenwich Meridian mapping into the β-axis and the equator mapping into the α-axis, the new plane coordinates would be expressible in terms of the old coordinates by means of the equations:

$$\alpha = -ru$$
$$\beta = r \cdot \sin v$$

This kind of map would not be particularly useful. By means of the fundamental forms from differential geometry, it can be shown to preserve area while it distorts distance, angle, and general shape quite badly. A variation on this projection is obtained by equal spacing of the parallels of latitude. The equal-area property is lost and, while the distortion of the high latitudes is reduced, the projection has little practical application.

Suppose we were to project on this tangent cylinder by means of a perspective from the center of the sphere. The equations giving the new plane coordinated in terms of the old would then be:

$$\alpha = -ru$$
$$\beta = r \cdot \tan v$$

Again this map is not particularly useful but it leads to a variation which is both useful and interesting historically.

If we ask how we can modify the equations of this projection to get a map which preserves angle we are led to the equations:

$$\alpha = -ru$$
$$\beta = r \cdot \ln (\sec v + \tan v).$$

This is the well-known Mercator Projection, invented by Gerhard Kramer who was born in Flanders in 1512. Kramer, whose Latin surname was Mercator, published his system of mapping the globe in 1569. His methods were approximations and the method was put on an exact mathematical basis in 1599 by Edward Wright of Cambridge in a publication called, "Certaine Errors in Navigation." The importance of this projection cannot be overestimated. At the same time it must be stated that it is not good for all purposes. It preserves angle and it maps a "rhumb line" or "loxodrome" (a line which cuts all meridians on the sphere at constant angle; *i.e.*, the line which would represent a constant compass course for plane or ship) into a straight line. This last property makes it extremely useful for navigation. However, it distorts distance and area quite badly away from the equator.

So far all of the projections described have been projections onto a tangent cylinder or modifications of such projections. It is also possible to map a section of the globe onto a tangent plane, usually as a perspective. I shall not attempt equations for this type of projection but just describe results. One of the commonest types is Gnomonic Projection where the point for the perspective is the center of the sphere. Any great circle maps into a straight line. The parallels of latitude map into complex curves unless the point of tangency is one of the poles. Points on the earth at a given distance from the point of tangency lie on a circle on the map. This last property makes the map useful in showing airline distances from a particular location. The navigator uses this map as a supplement to his Mercator charts as he plots dog-leg approximations to great circle paths.

In the Stereographic Projection the point for the perspective is the opposite end of the diameter from the point of tangency of the tangent plane. This type dates back to Hipparchus (160-125 B.C.). In this map general shape is fairly well preserved but area and distance are badly distorted if we move far away from the point of tangency.

One of the most versatile men in the history of cartography was Johann Heinrich Lambert (1728-1777) who made perhaps the first general investigations into the theories of map projection in addition to his valuable contributions in pure mathematics, physics, and astronomy. The projection which carries his name is based on mapping a section of the globe onto a cone which cuts the globe along two parallels of latitude. This map has proved to be an exceedingly good approximation particularly in mapping areas, such as the United States, which have larger east-west than north-south spreads. There are many variations on the cone map. The cone can be tangent along a parallel instead of cutting the surface in two parallels. In polyconic projection a map is assembled from a number of conical pieces.

The air age makes it necessary to consider maps which were unknown or impractical in the age of sailing ships. The space age which seems to be in its infancy will bring new problems. So far we have wrestled with the problem of mapping a curved two-space onto a flat two-space. In the not-too-distant future we may be faced with the problem of mapping a three-space onto a two-space. It is not inconceivable that man may travel far enough into space so that he will have to consider the question of whether that three-space is flat or curved as he tries to map it.

If any reader is interested in the development of map projections, he might start his reading with "Elements of Map Projection" by Charles Deetz and Oscar S. Adams, Special Publication No. 68, United States Government Printing Office. Discussions on maps are particularly appropriate program topics at this time.

"A peculiar beauty reigns in the realm of mathematics, a beauty which resembles not so much the beauty of nature and which affects the reflective mind, which has acquired an appreciation of it, very much like the latter."

—E. E. Kummer

=Δ=

Do you feel that because of your interest in mathematics you must excel at the bridge table or on the chess board? If you have had feelings of inadequacy on these scores be sure to read the following from the writings of the celebrated and versatile Henri Poincare.

". . .According to this, the special aptitude for mathematics would be due only to a very sure memory or to a prodigous force of attention. It would be a power like that of the whist-player who remembers the cards played; or, to go up a step, like that of the chess-player who can visualize a great number of combinations and keep them in his memory. Every good mathematician ought to be a good chess-player and conversely; likewise he should be a good computer. Of course that sometimes happens; thus Gauss was at the same time a geometer of genius and a very precocious and accurate computer.

But there are exceptions; or rather I err; I cannot call them exceptions being more than the rule. Gauss it is, on the contrary, who was an exception. As for myself, I must confess, I am absolutely incapable even of adding without mistakes. In the same way I should be but a poor chess-player; I would perceive that by a certain play I should expose myself to a certain danger; I would pass in review several other plays, rejecting them for other reasons, and then finally I should make the move first examined, having meantime forgotten the danger I had foreseen."

=Δ=

Three cubes have edges which are consecutive integers. The largest cube is placed on a table; the intermediate cube is placed on top of the largest cube; the smallest cube is placed on top of the intermediate cube. Can you find the lengths of the sides of the cubes if the total exposed area is 1604 square inches? (Do you have to make any assumptions about the placement or position of the cubes?)

—Wilson Junior College Mathematics Tournament

The Book Shelf

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor R. H. Moorman, Box 169-A, Tennessee Polytechnic Institute, Cookeville, Tennessee.

Numerical Analysis (Proceedings of the Sixth Symposium in Applied Mathematics of the American Mathematical Society, Vol. 6). Edited by John H. Curtiss, McGraw-Hill (330 West 42nd Street) New York, 1956, 303 pp., $9.75.

This volume contains nineteen papers that were presented at the Sixth Symposium in Applied Mathematics of the American Mathematical Society. The Symposium was held at Santa Monica City College, Santa Monica, California., in August, 1953.

One of the most enlightening facts to be obtained from this volume is the indication of the tremendous growth in numerical analysis and, in particular, the application of large scale computers to mathematical problems in various fields. The growth and diversity have been at least exponential since 1953. For an excellent discussion of the background and developments that led to a meeting such as the Sixth Symposium, the reader is referred to the review of this same volume by A. S. Householder in the *American Mathematical Monthly,* April, 1957.

This book would not appeal to the beginner who is primarily interested in a uniform treatment of numerical analysis, nor would it serve the experienced computer as a handy reference of techniques. Rather, the series of papers discusses various specialized problems from an advanced and abstract point of view. The mathematician or physicist who would like to learn of the types of problems arising in numerical analysis and, in particular, the sometimes surprising problems that can be solved by computers will find this volume very appealing.

Three of the papers, one by Bergman, one by Clutterham and Taub, and one by Frankel, deal with the more classical problem of solving partial differential equations. The paper by Bergman discusses the method of the kernel function for solving boundary value problems. Through the use of large scale computers Bergman indicates how whole classes of problems can be solved by construction of certain tables. Clutterham and Taub exhibit a numerical scheme employed on the problem of Mach reflection as applied to the University of Illinois Digital Computer, the ILLIAC. The paper by Frankel discusses the stability of certain difference equations used to approximate partial differential equations.

Four of the papers, one by Fischbach, one by Hestenes, one by Young and one by Rosenbloom, are concerned with the problem of solving systems of linear algebraic equations. The paper by Fischbach actually discusses two gradient methods, the methods of steepest descent and of conjugate gradients. These two methods are then applied to the solution of a system of linear algebraic equations. The paper by Hestenes uses the conjugate gradient method for solving linear systems. The paper by Rosenbloom applies the method of steepest descent. The paper by Young attacks the problem using successive iterations.

Four of the papers (Bruch, Emma Lehmer, Tompkins, and Taussky) discuss computational problems as concerning number theory. The paper by Emma Lehmer gives an account of the experiences one has in coding for a computer. The myth of the Giant Brain with powers of thought and learning is exploded. A person with knowledge of elementary number theory could very well follow this excellent discussion and gain understanding of how a problem of this nature could be prepared for a digital computer.

Three of the papers investigate the problem of approximation. A paper by Hastings, Hayward, and Wong has as its purpose the demonstration of the approximate type of parametric form for use in a given instance of approximation. A paper by Sard deals with function space approximations, and one by Walsh discusses approximation of polynomials.

Two of the papers give accounts of new and interesting fields of mathematics. A paper by Bellman gives some computational problems in the theory of dynamic programming. The aim of this theory is to translate problems in the fields of policies, strategies, scheduling, etc., into functional equations which can be attacked analytically. A paper by Motzkin discusses the assignment problem.

There are three papers yet to be mentioned. One by Wielandt gives error bounds for eigenvalues of symmetric integral equations.

One by Warschawski gives some results in numerical methods of conformal mapping. A paper by Wasow discusses transformations to normal distributions.

—Charles L. Bradshaw
Computation Laboratory
Army Ballistic Missile Agency

Introduction to Operations Research, C. West Churchman, Russell L. Ackoff, and E. Leonard Arnoff, John Wiley and Sons (440 Fourth Avenue) New York, 1957, 640 pp., $12.

One book in almost every field seems good enough to get stamped as the "bible" of that field. This book, in the field of operations research, is destined to be the "bible" in its field.

As the title says, this is an introduction to operations research. It seems, however, to have as much appeal for the practicing operations researcher as for the beginner. The information included in this book consists mainly of clear and complete descriptions of the present methods in practical operations research (commonly referred to as O. R.) through the medium of actual case histories. These case histories are very well picked and provide an excellent insight into the objectives and methods of an O. R. study. In fact, the case histories themselves would provide reasons sufficient for the necessity of having this book in the library of both the practicing O. R. man and the beginner.

The first part of the book describes the nature of operations research. It gives as a working definition the following:

> Operations research (O. R.) is the application of scientific methods, techniques, and tools to problems involving the operations of a system so as to provide those in control of the system with optimum solutions to the problems.

The second part of the book deals with the phase of O. R. which is that of formulating the problem to be solved. This phase is perhaps the key to every operations research study and the exposition presented is unusually clear. It is followed very effectively with the construction of the model of the system under study and then by several individual chapters on special techniques employed in solutions.

The techniques that are presented are those which are applied to inventory problems, allocation problems, queing or waiting time problems, competitive problems, and replacement problems. Most

O. R. studies are within the above mentioned categories and, for the most part, an entire systems study usually overlaps into several of these types. All the presently used techniques are discussed here, with fine illustrations. The treatment of linear programming is excellent.

The book is not written with the main emphasis on the philosophy of the methods used but rather more on the practical side of operations research. The mathematical treatments can be understood by one with no more background than a knowledge of calculus. It is noteworthy here to say that several chapters have been included in which the mathematical aspects are held to a minimum; these chapters provide a good picture of O. R. and its value to business for people on a management level, for example.

Overall, the book is very well written and organized. Other features included are: an excellent bibliography after each chapter; many graphs and tables; a wide variety of case histories; and a chapter on how one chooses, trains, and organizes an O. R. team.

A minor criticism might be its lack of treatment of the uses of electronic data processing machines that play such an important role in many of the solutions to O. R. problems. The authors do state, however, that since so much literature is already available on this subject that complete details are omitted and probably rightly so.

The book could serve very nicely as a textbook although no specific exercises are included.

—C. E. Shotwell
Applied Science Representative
IBM Corporation

Automation: Its Purpose and Future, Magnus Pyke, Philosophical Library (15 East 40th Street) New York, 1957, 191 pp., $10.

Dr. Pyke's book is not a highly technical treatment of the subject of automation. The purpose of the book appears to be to show, in a general way and in terms that permit the reader with a minimum scientific knowledge to understand, what is happening in the field and why it is important.

In the opening chapter the author reviews some of the new things that are being done automatically but shows that automation is not simply a continuation of the industrial revolution, since automatic control as well as operation is implied in the new word "automation."

Since the electronic computer is the device which makes the modern automatic factory different from the mechanical marvels of previous years, one chapter is devoted to this subject. The treatment is historical and general rather than highly mathematical and technical.

Following these introductory chapters are accounts of the present status and future possibilities of automation in the mass-production industries, chemistry, and the petroleum industry. The rapid appearance of automatic systems in offices, banks, and insurance companies is discussed. The possibilities of automation in transportation, food production and catering, and even in the housewife's shopping are presented. There is also one brief chapter on guided missiles.

Throughout the entire book the economic and sociological implications of automation are presented and analyzed. The author is optimistic about both of the above effects of the new revolution.

Although the book deals largely with British industry and society, the average American educator, scientist, economist, sociologist, or businessman should find it interesting and profitable reading.

—Wallace S. Prescott
Tennessee Polytechnic Institute

Functional Mathematics, Book 1, William A. Gager, Mildred H. Mahood, Carl N. Shuster, and Franklin W. Kokomoor, Charles Scribner's Sons (597 Fifth Avenue) New York, 1953, xiv+434 pp., $3.05.

Functional Mathematics, Book 2, William A. Gager, Charlotte Carlton, Carl N. Shuster, and Franklin W. Kokomoor, Charles Scribner's Sons (597 Fifth Avenue) New York, 1953, xv+447 pp., $3.05.

Functional Mathematics, Book 3, William A. Gager, Lilla C. Lyle, Carl N. Shuster, and Franklin W. Kokomoor, Charles Scribner's Sons (597 Fifth Avenue) New York, 1955, xiii+481 pp., $3.20.

Functional Mathematics, Book 4, William A. Gager, Luther J. Bowman, Carl N. Shuster, and Franklin W. Kokomoor, Charles Scribner's Sons (597 Fifth Avenue) New York, 1956, xiv+578 pp., $3.40.

The series is the result of an effort to combine the potentialities of arithmetic, algebra, geometry, and trigonometry into a single force for meeting the quantitative problems of our modern world. It has been recognized in America for half a century that the organization of each mathematical subject into a separate and compact body free of explicit relationships to the other mathematical subjects has definite weaknesses. From about 1900 to the present many attempts have been made to fuse the mathematical subjects or certain parts of them. The fusion has been pretty completely accomplished for grades seven and eight. Most senior high school mathematics textbooks, even though carrying the title "algebra" or "geometry," contain some topics from one or more of the other subjects.

There have been many single volumes published for a one-year course at the ninth-grade or higher level and a few two-year programs for senior high schools have appeared in textbook form. To the reviewer's knowledge the series being reviewed is the only one yet published for grades nine through twelve.

Philosophy, objectives, and guiding principles are stated with much care by the authors in the foreword of each book. Most of the foreword in Books 2, 3, and 4 is identical with that in Book 1, down to the part which describes the contents of the book. Books 1, 2, and 3 contain topics from arithmetic, algebra, geometry, and trigonometry while Book 4 includes, in addition, topics from statistics, analytic geometry, and the calculus.

The authors have attempted to develop the topics spirally. With some topics this has been done in an excellent manner—not only reviewing and extending a topic repeatedly in one book but also meeting the same topic again in another book at levels of greater difficulty in other applications. An example of this development is the treatment of slope which appears at least once in all four books and in three separate places in Book 3, with the more extensive discussion in Books 1 and 3. Few topics appear in all four books but many may be found in two or three. Approximate numbers are treated with especial care. In addition to the spiral development of the basic ideas, attention is given to correct application of the computational processes wherever operations are to be performed with numbers which are not exact.

The desirability of the spiral development need not be argued and a series of books such as this has greater possibilities for such organization than books based upon the conventional subject matter arrangement. Completeness of coverage and continuity are other values to be expected in such a series.

One is impressed by the rather wordy appearance of the text and wonders if the amount of material to be read may be a handicap in a time when we hear so much from teachers about the reading difficulties of high school pupils. The developmental material is in the nature of a monologue addressed to the student, interspersed with questions designed to awaken interest, stimulate thinking, or direct attention to a certain objective, and is generally rather well done. New mathematical terms are introduced with care and applied immediately; the nonmathematical vocabulary should not give average students appreciable difficulty. Occasional instances of statements whose meaning probably would not be clear to the inexperienced student are to be found.

There is an excellent and unusually detailed table of contents and an excellent index in each book. No answers are provided in the texts and no other teaching aids accompany the texts. However, answer books are available to the teacher. The socialized arithmetic usually found in ninth grade "general mathematics" and occasionally in a one- or two-semester course at the eleventh or twelfth grade level is distributed through the four books. Books 1 and 2 seem to be appropriate for average or superior ninth and tenth grade groups. If all eleventh and twelfth graders were enrolled in mathematics, many would find some of Book 3 and much of Book 4 difficult.

It is the reviewer's opinion that the series is a long stride in the right direction of organizing mathematics in a manner that it can funcion better in the lives of the learners; that the mathematics needed in a problem situation can be called upon whether it be arithmetic, algebra, geometry, or trigonometry; that the simpler and needed topics can be developed first, leaving those which can be postponed until the student is more mature; and that at any grade level one can teach the mathematics which is determined to be most valuable to the pupil without the restriction of subject boundaries.

The amount of information in these books is great and the scope is broad. It may well be that too much has been attempted at the expense of thoroughness. Certainly it would be a fine improvement to have all young people entering college and adult life thoroughly competent (as a minimum) in the topics of Book 1. With justifiable confidence the authors ask the students on page 421 of Book 2 to check their mathematical capabilities against the twenty-nine elements of functional competence set forth by the Post-War Plans Commission in the "Guidance Pamphlet."

The reviewer realizes that the use of a book with a class of students is the best basis for evaluation and wishes he might have that experience with this series. He hopes that the books may be tested in service and new efforts made to revitalize the high school mathematics program through continuous study and improvement of textbooks. The authors of this series are to be congratulated for their contribution.

—E. HOWARD MATTHEWS
Southwest Missouri State College

Can you determine the conditions on the coefficients so that the cubic whose equation is $y = x^3 + 3bx^2 + 3cx + d$ shall have its maximum where $x = b$ and its minimum where $x = d$?

(Continued from page 36)

Mathematics Student Journal. The Council also publishes other books and pamphlets which are valuable aids to all teachers.

I believe that we all feel that today the teaching of mathematics is a challenge more than ever before in the history of our educational system. Are we ready to accept the responsibility of that challenge?

Installation of New Chapter

EDITED BY MABEL S. BARNES

THE PENTAGON is pleased to report the installation of New York Beta Chapter of Kappa Mu Epsilon.

NEW YORK BETA CHAPTER

Albany State Teachers College, Albany, New York

New York Beta Chapter was installed at Albany State Teachers College on May 16, 1957, by Mr. Frank Hawthorne, National Historian of Kappa Mu Epsilon.

A banquet, preceded by an informal reception, was held at the University Club. Robert Stimson then introduced Mr. Hawthorne, who conducted the installation ceremonies, assisted by Dr. Violet Larney.

The 83 charter members initiated were: W. Adams, E. Allen, S. Alquire, B. Andy, J. Bailey, R. Beaver, A. Bednarek, H. Berleth, E. Bills, M. Bullock, J. Burt, E. Butler, P. Carapellucci, C. Carpenter, S. Chandler, D. Davis, W. De Groat, G. De Long, M. De Santa, B. Dezendorf, K. Dicks, J. Downes, M. Fox, R. Gardner, E. Glass, M. Goldstein, R. Guzewich, G. Harris, C. Haughey, M. Hess, C. Hutt, M. Johnson, J. Kiehle, J. Kilroy, R. Kopecek, M. LaCave, H. LaDuke, V. Larney, A. Leahy, C. Lester, A. Lewis, B. Lindeman, S. Lister, R. Luippold, W. Mackie, B. Marsh, C. Maxson, C. McDuffee, M. Meiser, J. Merritt, J. Mooney, B. Moore, T. Mullen, A. Oatman, D. Olechna, C. Payne, D. Peck, R. Pfeiffer, J. Powers, D. Purfield, J. Rookwood, L. Root, P. Schaefer, M. Schlotthauber, J. Slezak, L. Smith, R. Snider, E. Steifeld, R. Stimson, D. Stokes, B. Tackabury, N. Turner, D. Vradenburg, B. Wales, W. Walter, D. Warmuth, L. Wells, R. Wiggin, J. Wilcox, G. Wilder, E. Wilson, K. Wooster, A. Wooton.

The responsibilities of the offices of New York Beta were accepted by: Clinton Carpenter, president; Dolores Peck, vice-president; Mary Bullock, secretary; Patricia Carapellucci, treasurer; and Elizabeth Glass, corresponding secretary.

Mr. Hawthorne addressed the new chapter on the subject "Projectile Geometry."

We are very happy to welcome our large new chapter, and we wish it success in all its activities.

Kappa Mu Epsilon News

EDITED BY FRANK HAWTHORNE, HISTORIAN

The Eleventh Biennial Convention of Kappa Mu Epsilon was held at Kansas State Teachers College, Pittsburg, Kansas, on April 25, 26, 27, 1957, with Kansas Alpha as host.

Two hundred seven members of Kappa Mu Epsilon were registered as follows:

Chapter	No. of Students	No. of Faculty	Chapter	No. of Students	No. of Faculty
Alabama Beta	2		Michigan Alpha	1	
California Alpha	1		Michigan Beta	5	2
California Beta	1		Michigan Gamma	2	
Colorado Alpha	1	1	Missouri Alpha	10	1
Illinois Alpha	4	1	Missouri Beta	4	1
Illinois Gamma	6	2	Missouri Gamma	2	1
Indiana Beta	5		Missouri Epsilon	12	1
Iowa Alpha	7	2	New Mexico Alpha	3	
Iowa Beta	2	1	Nebraska Alpha	13	1
Kansas Alpha	40	6	N. Carolina Alpha	4	1
Kansas Beta	10	3	Ohio Alpha		1
Kansas Gamma	7	1	Oklahoma Alpha	6	1
Kansas Delta	6	4	Pennsylvania Alpha	3	
Kansas Epsilon	7	1	Texas Alpha	4	1
Louisiana Beta	3	1	Wisconsin Alpha	3	

THURSDAY, APRIL 25, 1957

Registration took place in the Lounge of the Student Center with Kansas Alpha in charge.

Members of the National Council met in the Conference Room at the Student Center.

At 8 p.m. the students and faculty members met in the Twilight Lounge for a "Get Acquainted Session."

FRIDAY, APRIL 26

8:00 to 9:00 a.m. registration took place in Music Hall.

At 9:00 the first General Session was held in Music Hall Auditorium with C. C. Richtmeyer, National President, presiding.

The address of welcome was given by President Ross H. Hughes of Kansas State Teachers College. The response was given by Professor Jerome M. Sachs, National Vice President of KME. Music was furnished by the College Choir, Professor Richard Smith, Conductor. At the first business meeting roll was called by Laura Z. Greene, National Secretary, and C. C. Richtmeyer greeted the new chapters.

At 10:15 a.m. the following student papers were read:

1. *The Calendar,* Glenn W. Thornton, New Mexico Alpha, University of New Mexico
2. *A New Look at Pascal's Triangle,* Sandra Ladehoff, Iowa State Teachers College
3. *Invariants of Four Squares,* Adrian Nikitins, Missouri Alpha, Southwest Missouri State College
4. *The Derivation of Some Mathematical Terms,* Marcia Caton, Michigan Alpha, Albion College
5. *Chemistry and Mathematics,* Daniel S. Dudley, Iowa Beta, Drake University

At 1:30 p.m. the following student papers were read in the Student Center Ballroom:

1. *Ruled Surfaces,* Beverly Kos, Illinois Gamma, Chicago Teachers College
2. *Motions of a Space Satellite,* Charles Trauth, Kansas Beta, Kansas State Teachers College, Emporia
3. *Arithmetical Congruences with Practical Applications,* Carol Law, Kansas Gamma, Mt. St. Scholastica College

At 3:00 p.m. was "Let's Exchange Ideas" by the Faculty Section in the Twilight Lounge. Professor Dana L. Sudborough of Michigan Beta presided and Professor Frank Hawthorne acted as secretary. The following topics were discussed by the group:

(1) Help sections.
(2) Separate mathematics clubs vs. Kappa Mu Epsilon.
(3) High school contests.
(4) Publishing a newsletter.
(5) Merit award to student whose contribution to Kappa Mu Epsilon is outstanding.

In the Student Section Lynn Bartlett of Kansas Delta presided and Joan Carvalho of Kansas Gamma served as secretary. The following topics were discussed by the group:

(1) Attendance.
(2) Banquets and picnics.
(3) Requirements for membership.
(4) Programs.
(5) Association with other campus clubs.
(6) Dues and finances.
(7) Help sections.

At 6:00 p.m. a banquet was held in the dining room of the Student Center. Professor J. D. Haggard, Kansas Alpha, presided. The invocation was given by Dean Mahan, and Professor R. G. Smith introduced the guests. An introduction of each chapter with a response from each chapter was given. Music was furnished by Kansas State Teachers College. Professor Arthur Bernhart of the University of Oklahoma spoke on "Language and Numbers." Friday night at 8:00 p.m. Fun Night was held in the ballroom.

SATURDAY, APRIL 27

At 9:00 a.m. the following student papers were read:

1. *Fermat's "Last Theorem,"* Lawrence Arnold, California Beta, Occidental College
2. *Types of Approximations Made in Physics,* Robert Diebold, New Mexico Alpha, University of New Mexico
3. *The Structure and Form of Human Bone as Governed by Mathematical Laws,* David J. O'Mara, Iowa Beta, Drake University

At 10:30 a.m. the second business meeting was held in the ballroom. The business included reports of the "Let's Exchange Ideas" sections, both Faculty and Student, and reports of the national officers. All reports were filed with the secretary. The Auditing Committee, consisting of Professors Orville Etter, Basil Gillam, and Charles Kelley, reported that the Treasurer's books were in order.

The following motion of Professor Madison, chairman of the committee on transportation, was passed.

"Any chapter whose distance from the convention site is of the order of 1,000 miles or more, which has been active, and needs financial aid to send a delegate to the National Convention, may submit a letter of application not less than 60 days prior to convention time to the National President for such aid. Carbon copies of this letter will be sent to the National Secretary and the National Treasurer who together with the National President will make such allowances as they consider justified."

Sister Jeanette reported for the committee on constitutional revision. Since the committee recommended changes, it was agreed that the secretary should send copies of the proposed changes to the chapters for a vote.

Invitations for the 1959 convention were extended by Ohio Alpha and Texas Alpha.

At 12:15 p.m. there was a luncheon in the college cafeteria.

At 1:00 p.m. the third business meeting was held in the college auditorium. Professor C. T. McCormick reported for the Awards Committee consisting of: Professor Clyde T. McCormick, Illinois Alpha; Professor Raymond Carpenter, Oklahoma Alpha; Professor Elizabeth Woolridge, Nebraska Alpha; Kay Cunningham, Wisconsin Alpha; Bethsebe Lou Hoskin, Illinois Alpha; Joan Pawelski, Nebraska Alpha. The committee reported its decision to award copies of *The Mathematical Dictionary* as prizes for first, second, and third places, respectively, to:

Adrian Nikitins, Missouri Alpha, Southwest Missouri State College, for his talk on *Invariants of Four Squares.*

Charles Trauth, Kansas Beta, Kansas State Teachers College, Emporia, for his talk on *Motions of a Space Satellite.*

Carol Law, Kansas Gamma, Mt .St. Scholastica College for her talk on *Arithmetical Congruences with Practical Application.*

A two-year subscription to *The Pentagon* was given to each of the students presenting a paper at the convention.

In the absence of Professor Tucker, Professor Eberhart presented the report for the Nominating Committee, naming the following nominees:

President:
C. C. Richtmeyer, Michigan Beta
Clyde McCormick, Illinois Alpha

Vice-President:
Floyd Helton, Missouri Epsilon
R. G. Smith, Kansas Alpha

Secretary:
Laura Z. Greene, Kansas Delta

Treasurer:
M. Leslie Madison, Colorado Alpha
Harold Tinnappel, Ohio Alpha

Historian:
Frank Hawthorne, State Education Dept., N.Y.
Frank Gentry, New Mexico Alpha

Professor Eberhart moved that the report be accepted. Motion carried. Professor Richtmeyer called for nominations from the floor. There were none. Professor Kelley moved that the nominations be closed. The motion was seconded and carried. The voting delegates were given ballots.

Professor Helton gave the report for the resolutions committee consisting of: Professor Floyd Helton, Missouri Epsilon; Professor John K. Baumgart, Indiana Alpha; Professor Richard Crumley, Iowa Alpha; Professor D. V. LaFrenz, Missouri Gamma; Roger L. Brockmeyer, Iowa Alpha; Larry Edwards, Missouri Epsilon; Nill Mahler, Missouri Gamma; Phil Shellhaas, Indiana Alpha.

"Whereas this, the Eleventh Biennial Convention of Kappa Mu Epsilon assembled, finds absent from its sessions three persons who for so many conventions past have been in the forefront of its leadership, be it resolved that through the National Secretary we convey to Professor L. P. Woods, Miss E. Marie Hove, and Professor Charles B. Tucker our regret that they are not with us and our hope that each of them will soon be restored to good health.

Whereas the enjoyment of the activities of this convention has been a continuous function of time since our arrival on the campus, be it resolved that we express our appreciation:

1. To the host chapter, Kansas Alpha, and to Kansas State Teachers College, Pittsburg, Kansas, for their fine hospitality, for their provision of excellent facilities for our convention, for the

music, the food and refreshments, and all the many things that contribute to the success of a meeting such as this.

2. To each of the national officers whose work, always in addition to a full load of responsibility elsewhere, provides the direction and continuity that has kept KME a growing fraternity.

3. To Professor Arthur Bernhart for his interesting, entertaining and informative address on *Language and Number*.

4. To the Editor and Staff of *The Pentagon* whose work has contributed so much to our pride in KME.

5. To the students who prepared and presented papers, without which part of our program there would be little occasion for our convening.

6. To all those and to many more unnamed who have worked in many ways to make this convention the success it has been, the delegates to the Eleventh Biennial Convention of Kappa Mu Epsilon say 'Thank you.' "

Professor Helton moved the adoption of the report and Professor McCormick seconded the motion. The motion carried.

Professor Richtmeyer paid tribute to the work of Professor J. Sachs, retiring Vice-President, and Professor Sudborough, who has resigned as Business Manager of *The Pentagon*.

Professor William Perel of the tellers committee reported the election of the following officers:

President, C. C. Richtmeyer, Michigan Beta; Vice President, R. G. Smith, Kansas Alpha; Secretary, Laura Z. Greene, Kansas Delta; Treasurer, M. Leslie Madison, Colorado Alpha; Historian, Frank Hawthorne, State Education Dept., N. Y. Professor O. J. Peterson installed the newly-elected officers. The meeting was adjourned at 1:45 p.m.

REPORT OF THE NATIONAL PRESIDENT

The last two years have brought increasing enrollments and increased interest in mathematics and science on all our campuses. These increases have been reflected in the activities of our fraternity. During this biennium a total of 1328 students have been initiated into Kappa Mu Epsilon compared with 1168 in 1953-55 and 1133 in 1951-53. With the accelerating enrollments and with the addition of new chapters, I predict that the next biennium will show at least 1500 initiates.

Two new chapters have been approved prior to this convention. Indiana Gamma was installed at Anderson College on April 5 of this year. Dr. Larsen, the installing officer, reports a very enthusiastic charter group. The New York Beta chapter will be installed at Albany State College for Teachers early in May. The installing officer will be our National Historian, Mr. Frank Hawthorne. Petitions are on file from two additional groups and inquiries have been received from a number of others. We now have 51 active chapters and I anticipate at least three or four new chapters in the next biennium.

As we increase in size and number of chapters, we shall need to give increased attention to regional meetings in the even-numbered years. You will recall that Past President Tucker made such a recommendation at the 1955 convention, that a committee in regional meetings was appointed, and that at least one regional meeting was held in 1956. I recommend that this committee be continued and that additional regional meetings be held in the spring of 1958.

With continued growth the work of your national officers becomes correspondingly greater. In particular, the duties of the National Secretary and National Treasurer increases in almost direct proportion to the increase in membership. The cooperation of all corresponding secretaries in easing the burden is earnestly requested. In the not-too-distant future we may need to supply funds to hire a part-time or full-time executive secretary to handle many of the details of the national organization. The positions of editor and business manager of *The Pentagon* also entail many hours of work on the part of those individuals and eventually some way should be found to give them some paid assistance and relief from the time-consuming details of their jobs. We have been very fortunate to have had the services of dedicated people who have been willing to make great sacrifices to carry on the work of the fraternity. As the burdens of these offices increase, however, we may find it increasingly difficult to find people who can and will devote the time and energy necessary.

I should like to take this opportunity to pay tribute to the many people who have made this biennium a successful one for the fraternity. In particular, I should like to commend Miss Green and Mr. Madison who have been very efficient in carrying out the duties of the important offices of National Secretary and National Treasurer. My thanks also to Professor Hawthorne for his work as National Historian and to Professor Sachs as Vice-President. I am especially grateful to our Past President, Professor Tucker, who has been exceedingly helpful in giving me counsel and advice. To Professor Fronabarger, Editor of THE PENTAGON, and Professor Sudborough, Business Manager, my appreciation for a job well done.

I also wish to thank all of the corresponding secretaries and faculty sponsors who have unselfishly given so much of their time to the fraternity. It is at the chapter level where the real work of the fraternity is carried on, and without the cooperation and diligent effort of these people the fraternity could not exist.

To all of you my thanks for having helped make possible a successful biennium for Kappa Mu Epsilon.

—Cleon C. Richtmeyer

REPORT OF VICE-PRESIDENT

The most attractive part of the report of a Vice-President is its brevity. This is in the nature of things, vice-presidents being excess baggage in most cases.

Perhaps there is one item in my report which deserves consideration. I was charged by the President with the responsibility of forming a committee to aid in communications and arrangements for those chapters interested in even-year regional conventions. In attempting to discharge this responsibility I contacted faculty members and asked how many chapters were interested in regional conventions. Getting an up-to-date list of faculty sponsors and corresponding secretaries took time and I must confess that I was somewhat remiss in getting out my letters of inquiry promptly. The response was quite varied. I am afraid that the chapters which already had plans for regional conventions were the only ones able to go ahead. What I am trying to say is that from my experience, I doubt the practicality of having the matter of regional conventions handled through a national committee. Those chapters involved in regional committee activities were able to handle regional conventions.

I recommend that before this convention adjourns, we find a few minutes to congregate in regional groups. In these brief meetings we can determine the interest in regional conventions and form regional committees to handle arrangements if such interest is shown. I would be reluctant to try to designate geographical areas as I would probably be unfamiliar with the transportation problems involved. Instead, if this meets with the approval of the President and the membership, I would like to suggest that a roll call of chapters represented at the convention be made and that each chapter then indicate its interest in a regional convention. The representatives of chapters interested could then meet and group themselves into practical geographical units. Those who have been so successful with regional meetings could give good advice to the rest of us.

—J. M. Sachs

REPORT OF THE NATIONAL SECRETARY

Since the last convention of Kappa Mu Epsilon two new chapters have been approved. Indiana Gamma was installed at Anderson College in Anderson, Indiana, April 5, 1957. Professor Harold Larsen, a former Vice-President from Michigan Alpha, served as the installing officer.

New York Beta will be installed May 16, 1957, at New York State College for Teachers at Albany by Mr. Frank Hawthorne, National Historian.

Kappa Mu Epsilon now has 51 active chapters and four inactive chapters, making a total of 55 chapters in 24 states.

One thousand three hundred twenty-eight initiations were received during the biennium prior to April 1, 1957, bringing the total membership to more than 12,000.

The number of initiation ceremonies per chapter has varied from one to seven this biennium. New Mexico Alpha initiated seventy-nine during the biennium with Texas Alpha, Kansas Alpha, and Kansas Beta ranking high.

—Laura Z. Greene

REPORT OF THE NATIONAL HISTORIAN

The duties of the National Historian include the preservation of topical material about the fraternity, the editing of the Kappa Mu Epsilon News section of THE PENTAGON, and assisting the Editor of THE PENTAGON. In the last of these three roles, I have carried on considerable correspondence with Professor Fronabarger and have referred two papers. The distance between New York and Springfield, Missouri, has limited the amount of direct assistance.

(Continued on page 64)

FINANCIAL REPORT OF THE NATIONAL TREASURER
April 3, 1955 — April 4, 1957

Cash on hand April 4, 1955				$4786.15
Receipts from chapters				
Initiates (1327 at $5 and 1 by advanced payment)		$6635.00		
Miscellaneous (Supplies, installations, advance payments, etc.		290.39		
			$6925.39	
Miscellaneous receipts				
Interest on bonds		150.30		
Balfour Company (Commissions)		174.50		
The Pentagon (Surplus)		375.00		
Matured U. S. Bonds		1400.00		
Refund Michigan Sales Tax		17.67		
			2117.47	
Total Receipts				9042.86
Total receipts plus cash on hand				$13829.01
Expenditures (1955-1957)				
National Convention, 1955				
Paid to chapter delegates	$1519.91			
Officers' Expenses	477.40			
Miscellaneous (Speaker, convention programs)	126.06			
		$2123.37		
Balfour Company (membership certificates, stationery, etc.)		569.06		
Pentagon (Printing and mailing 4 issues)		3105.65		
Purchase U. S. Bonds		1500.00		
Bond for Treasurer		62.50		
National Office Expense		424.57		
Refund Poudre Valley National Bank (Improperly executed check)		5.00		
Michigan Sales Tax		210.22		
Total expenditure			$8000.37	
Cash balance on hand April 3, 1957				$5828.64
Bonds on hand April 4, 1957				3000.00
Total assets as of April 4, 1957				$8828.64
Net gain for the period	$1042.49			

—M. Leslie Madison

Four issues of THE PENTAGON have appeared since our last convention. In the Fall, 1955, number, a general report of the tenth biennial convention filled the Kappa Mu Epsilon News section. The Spring, 1956, issue marked the silver anniversary of the fraternity and KME News carried accounts of the successes of some of our outstanding alumni. The Fall, 1956, issue reported on the regional meeting at Missouri Gamma and included items of news about individual chapters as well as program topics. Similar items will be published in the Spring, 1957, edition.

Considerations of space and consistency have sometimes resulted in a considerable shortening of some of the reports submitted from some chapters. For this I must apologize but take full responsibility. I am sure that I have sometimes deleted important items when facing a deadline in time and several thousand miles of distance.

—FRANK HAWTHORNE

REPORT OF THE EDITOR OF THE PENTAGON

I would like to take this opportunity to thank all who have assisted in the preparation of copy for THE PENTAGON during the past two years. Frank Hawthorne, National Historian, formerly on the faculty of Hofstra College and now with the State Department of Education, Albany 1, New York, has edited the KME News. The Book Shelf during the major portion of the biennium was edited by Rex Depew of Florence Teachers College, Florence Alabama. He is now with the IBM Corporation. The new Editor is R. H. Moorman of Tennessee Polytechnic Institute, Cookeville. The Installation of Chapters reports have been edited by Mabel Barnes of Occidental College, San Francisco, California. Jerome Sachs, formerly of Chicago Teachers College and now Dean of Southeast Chicago City Junior College has served as Editor of the Mathematical Scrapbook. Frank C. Gentry of the University of New Mexico has been the Editor of the Problem Corner. Beginning with the Fall, 1957, issue of THE PENTAGON, the Problem Corner will be edited by J. D. Haggard of Kansas State College, Pittsburg. Special thanks are due to E. Howard Matthews of Southwest Missouri State College who has so ably assisted with the proofreading of manuscripts, galley proofs, and page proofs. Dana Sudborough, the Business Manager, has also frequently helped with the reading of page proofs.

In the four most recent issues of THE PENTAGON, including the one that will come out early in May, there have been twenty-three articles published. Sixteen of these were written by students and seven by faculty members or others. This is about the same ratio of student papers to the total number of papers which has existed since I took over the editorship in 1952. It is my hope that this ratio will continue or increase. The encouragement of publication of student papers is in keeping with the purposes and ideals of KME. THE PENTAGON is a magazine for students.

There are areas in which there is student interest but no articles will likely be written by students because of their lack of information and experience. Articles by faculty members or others in these areas are most welcome. Any article of interest to undergraduate students is solicited. Faculty members at the various institutions should encourage their students, as a part of their educational process, to contribute articles. The rewards for preparing a paper and seeing it in print are great. I appeal to each of you present to contribute to THE PENTAGON by sending in articles for publication and solutions to problems in the Problem Corner.

—Carl V. Fronabarger

REPORT OF THE BUSINESS MANAGER OF THE PENTAGON

. . .I have been asked to give you an address, but a much more important address is the one which you give me. Or let me say this another way: if the Business Manager of THE PENTAGON were to be adopted by a tribe of American Indians, his new name might well be He-Who-Wants-Your-Correct-Address.

I have just a few samples of typical cases here to illustrate why some people do not receive a magazine they are entitled to have. Let me pick out a single one and tell you the complete story of it. Remember, the story is precisely the same on many others; and there are others, similar stories that we haven't time to tell.

The subscription card came to my office, and I typed the exact, same address on a mailing envelope. A copy of THE PENTAGON was then placed in the envelope and mailed at a postage cost of 5 cents. But after about 2 weeks the envelope and contents were returned to us by the Post Office Department, marked "No such street number" and "Unknown." This return, by the way, costs us another

5 cents in postage, and so now, if we add the value of the envelope to the postage that has been spent on this one copy, we find that an amount of money equal to approximately 25% of the value of the copy itself has been spent by Kappa Mu Epsilon for nothing.

Let us now change the subject slightly. A question I frequently hear is: Do you have to be a member of Kappa Mu Epsilon in order to receive issues of THE PENTAGON regularly? The answer is "no." Any human being anywhere on Earth may subscribe to our magazine at a cost of $2 for 2 years — BUT his correct address must be in our files.

When a magazine is returned to us in the manner I just described, that's the end of it. We make no further move on your behalf until you register a complaint with us. In this connection let me remind you that postcards are still cheap.

When you join Kappa Mu Epsilon, $2 of your entry fee is automatically used to buy you 4 issues of our magazine. So you paid for it. But to receive it you must give us an address at which this third class mail will be accepted and held for you. There is no forwarding service for this. That is why your corresponding secretary probably had you sign up with your home address rather than a temporary, college one. But I gravely suspect that what may happen at home sometimes is that Mom or Dad have not been notified by you of the situation, and your magazine is discarded.

. . .All those who have presented papers at the convention will have their subscription extended two years. This is with the compliments of the National Council. On the other hand, those who have an article printed in THE PENTAGON receive 5 copies of that single issue but not the 2-year extension.

. . .It has been an honor and a very valuable learning experience to serve you as Business Manager of THE PENTAGON. . ..

—Dana R. Sudborough

Charles B. Tucker
Past National President

Cleon C. Richtmeyer
National President

R. G. Smith
National Vice-President

Laura Z. Greene
National Secretary

M. Leslie Madison
National Treasurer

Frank Hawthorne
National Historian

Carl V. Fronabarger
Pentagon Editor

Wilbur J. Waggoner
Business Manager of
The Pentagon

1957 Convention