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Kappa Mu Epsilon, national honorary mathematics society, was founded in 1931. The object of the fraternity is fourfold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; and to provide a society for the recognition of outstanding achievements in the study of mathematics at the undergraduate level. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

The Logarithmic Spiral

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One of the most fascinating curves in mathematics is the *logarithmic Spiral* (Fig. 1). The purpose of this paper is to investigate and compile all of the interesting properties of this spiral that are to be found in the literature. The main part of the paper will be a discussion of these properties and a few simple applications. This will be followed by a brief historical sketch.

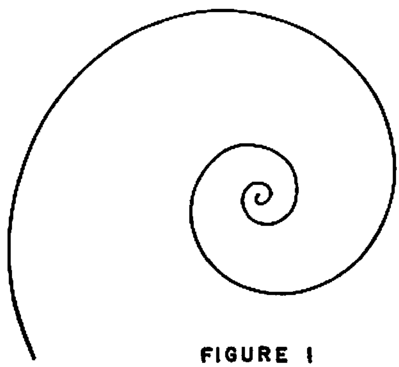

FIGURE 1

We shall begin with the simplest definition of the logarithmic spiral—the definition which displays the most outstanding single property of the curve. The logarithmic spiral is a curve cutting radius vectors from a fixed point at a constant angle. This property has led many to favor the name *equiangular spiral* for the curve. The constant angle property also is the basis of many applications of this curve in nature. It is said that insects, because of the compound structure of their eyes, keep a light which they are approaching at a constant angle to their line of flight. Continually adjusting their path to this constant angle, they approach the light along a logarithmic spiral.

The two most common forms of equation for the logarithmic spiral follow immediately from the definition. Consider the "right triangle" having for its hypotenuse the element of arc and for one leg the increment of the radius vector (Fig. 2); then

$$\tan \phi = r\, d\theta/dr;$$

whence

$$dr/r = \cot \phi \cdot d\theta$$

and, integrating,

$$\log r = \theta \cdot \cot \phi + \log a;$$

or

(1) $$r = ae^{k\theta},$$

where

$$k = \cot \phi.$$

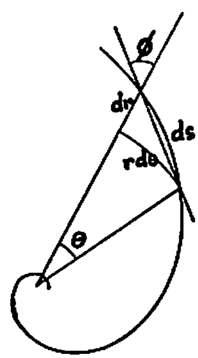

FIG. 2

From equation (1) it is readily seen that the form of the spiral is wholly dependent upon k and independent of a. Thus, for small changes in the constant angle ϕ, there are large changes in the form of the spiral. Suppose the distance between two successive convolutions to be one inch. Then, if the constant angle of the spiral were 80°, the distance to the next convolution would be about three inches; if it were 70°, the distance to the next convolution would be about ten inches; and if it were 60°, the distance to the next convolution would be nearly four feet. If the angle were 17°, the next convolution would be some 15,000 miles away [11, p. 792].*

Further information as to the shape of the spiral is obtained from equation (1). At $\theta = 0$, $r = a$, and it is seen that r diminishes for negative values of θ, but does not vanish until θ becomes negatively infinite; hence, the pole is an asymptotic point. As the spiral gets larger the curvature diminishes, the radius of curvature continually increasing.

*Numbers in brackets refer to the references cited at the end of the paper.

From the triangle of Fig. 2,

$$dr/ds = \cos\phi,$$

from which, by integration,

$$r = s\cos\phi,$$

where s is the length of the curve from the pole. Hence, this length is equal to the polar tangent PT (Fig. 3).

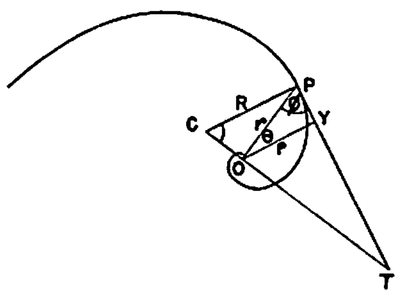

FIG. 3

By adding successive equal increments h to the polar angle θ, it is seen from (1) that each radius vector will bear the ratio e^{kh} to the one previous. That is, the lengths of radius vectors form a geometric progression if their central angles are in arithmetic progression. Thus, the pole and any two points on the curve uniquely determine the spiral, since the bisector of the angle made by the radius vectors is a mean proportional between their lengths.

If $k = \cot\phi = 1$, the ratio of radius vectors corresponds to a number and the angle between them to its natural logarithm, whence the more common name for the curve.

The area swept out by the radius vector from $r = 0$ to $r = r$ is given by the integral

$$\begin{aligned} K &= \tfrac{1}{2}\int_{-\infty}^{\theta} r^2\,d\theta \\ &= (a^2/4k)\int_{-\infty}^{\theta} e^{2k\theta}2k\,d\theta \\ &= (a^2/4k)e^{2k\theta} \\ &= r^2/4\,\cot\phi \\ &= \tfrac{1}{2}\,[\tfrac{1}{2}r(r\tan\phi)] \end{aligned}$$

Therefore, this area is seen to equal half of the area of the triangle formed by the radius vector to the point $r = r$, the tangent at that point, and the perpendicular to the radius vector at the pole (Fig. 3, triangle OPT).

A remarkable property of the spiral, and one that has attracted much attention to it, is its so-called "perpetual renascence." By this is meant that many of the derived curves are also equal logarithmic spirals. A few of these will be mentioned only briefly while others of them will be discussed in more detail.

The caustics by reflection and refraction, with light source at the pole, are equal logarithmic spirals.

The orthoptic curve (the locus of points from which two mutually orthogonal tangents can be drawn to the spiral) is again a logarithmic spiral.

In order to investigate the geometric inverse of the logarithmic spiral we obtain the equation of the derived curve from the expression

$$r\,r' = c^2,$$

where r' is the radius vector to a point on the inverse curve and c is the radius of the circle of inversion. Then,

$$r' = c^2/r = (c^2/a)e^{-k\theta},$$

and it is seen that the inverse curve is an equal logarithmic spiral asymptotic to the same point, unwinding in the opposite direction.

In order to investigate the evolute of the spiral, we first find the center of curvature. Let $R =$ the radius of curvature $= r\,dr/dp$, where $p = OY$ is the perpendicular distance from the pole to the tangent (Fig. 3). Now

$$p = r \sin\phi,$$

whence

$$dr/dp = \csc\phi.$$

Therefore,

$$R = r\,dr/dp = r\csc\phi = CP,$$

and the center of curvature is at C, the intersection of the polar normal and the polar subtangent.

Since PC is tangent to the evolute at C and angle $OCP = \phi$, the first evolute and all succeeding evolutes are equal spirals with the same asymptotic point. Here OC becomes the radius vector of C.

Conversely, all of the involutes of the logarithmic spiral are again equal spirals.

It can be shown that the axis of the spiral must be rotated through an angle $\pi/2-(\log_e k)/k$ to be brought into coincidence with the axis of its evolute. The proof of this is neither long nor especially difficult.

The logarithmic spiral can even be made to coincide with its own evolute; namely, if

$$\pi/2 - (\log_e k)/k = 2n\pi, \quad n \text{ any integer.}$$

Thus, in Figure 4, P_1 is a point on the first evolute and P_2 is a point on the second evolute. The condition for this "auto-evolute" will be fulfilled for any integral value of n, and it is seen that there is a different solution for each of these values. For $n = 1$, by numerical calculation we find ϕ to be approximately $75°$. This special spiral is named after Bernoulli and will be mentioned later in connection with one of its applications.

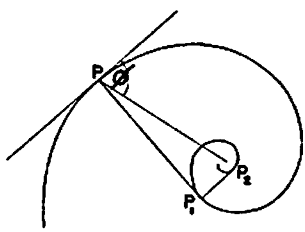

FIG. 4

A fundamental property of the pedal curve is that the angle between the radius vector and the tangent at a point on the pedal is equal to the angle between the radius vector and the tangent at the corresponding point of the original curve. Thus, it is immediately seen that the first positive pedal of the logarithmic spiral, as well as all succeeding pedals, is an equal logarithmic spiral with the same asymptotic point.

Haton de la Goupilliere proved that the logarithmic spiral is the only curve whose pedal with respect to a given pole is an equal curve which can be brought into coincidence with the first by rotation about that pole.

To assist in further investigation of the properties of the spiral, we will here give an alternate definition of the spiral showing the manner in which the spiral is described:

> If an extended straight line turn uniformly about its extremity, a point which is carried along it, with a celerity proportioned to the distance from that centre, will describe the logarithmic spiral [6, p. 430].

It will be noticed that this point can be either advancing or receding along the line.

From motion as in the definition above, describing the logarithmic spiral $r = ae^{k\theta}$, it is seen that the velocity at any point is given by

$$\begin{aligned} v &= \sec\phi \cdot dr/dt, \\ &= \sec\phi \cdot \cot\phi \cdot ae^{k\theta} \cdot \alpha, \\ &= \alpha \csc\phi \cdot ae^{k\theta}, \end{aligned}$$

where $\alpha = d\theta/dt$ is a constant. Thus, the locus of the extremities of succeeding velocity vectors laid off from the pole (the hodograph of the logarithmic spiral about its pole) is an equal spiral. It can be shown that the hodograph is obtained from the original spiral by rotating it about the pole through a constant angle ϕ if the spiral is described by a point advancing along a uniformly turning line or through an angle $\pi - \phi$ if the point is receding along the line. The direction of this rotation is the same as that of the uniformly turning line.

As a result of a proposition in Newton's *Principia* (Book I, Proposition 9), it develops that if the force of gravity had been inversely as the cube, instead of the square, of the distance, the planets would have all shot off from the sun in "diffusive logarithmic spirals."

Several properties of the spiral appear in a little known paper by James Clerk Maxwell, published when its author was only eighteen years of age [7, pp. 4-29]. Among them are the following:

"If any curve be rolled on itself, and the operation be repeated an infinite number of times, the resulting curve is the logarithmic spiral." The curve which being "rolled on itself traces itself is the logarithmic spiral."

"When a logarithmic spiral rolls on a straight line the pole traces a straight line which cuts the first line at the same angle as the spiral cuts the radius vector." Among later results, two are interesting because of their similarity to the two quoted above from Maxwell. They are:

(1) The limit of a succession of involutes of any given curve is a logarithmic spiral [12, pp. 208-9].

(2) If a logarithmic spiral rolls on a straight line, the locus of its center of curvature at the point of contact is another straight line.

The logarithmic spiral is the stereographic projection of a loxodrome (the spherical curve which cuts all meridians at a constant angle; the course of a ship holding a fixed direction on the compass) from one of its poles onto the equator.

Since the circle also has a constant angle between the tangent at a point and the radius vector to that point, it is interesting to view the circle as a special case of the logarithmic spiral. For this special case the constant angle ϕ is 90° (i.e., $k = \cot\phi = 0$), and the equation of the spiral becomes

$$r = a,$$

the equation of a circle of radius a about the pole.

An important application of the logarithmic spiral is in the field of engineering. Since the radius of curvature of the spiral increases continually, the spiral can be fitted to any portion of a given curve. This fact, together with the property of the auto-evolute of Bernoulli, provides a method for the rapid and easy determination of the center of curvature of any portion of a curve. There are at least two plastic curves of this type available at the present time.

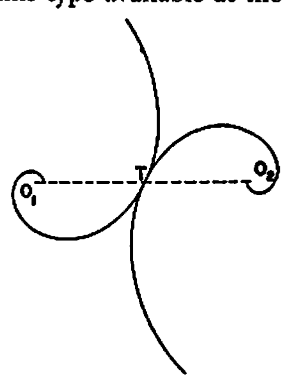

FIG. 5

Consider two equal spirals which are tangent at point T (Fig. 5). Since the common tangent must make equal angles with the two radius vectors, this point T must be on the line connecting the two poles O_1 and O_2. Now let each curve rotate about its own pole in such a way that they remain tangent. The tangent point will move along the line O_1O_2. After a certain rotation, let T_1' and T_2' become tangent points. The change in length of the first radius vector r_1 will be proportional to the distance s_1 between T and T_1' along the first curve, and the change in length of the second radius vector r_2 is similarly proportional to the distance s_2 between T and T_2' along the second curve. If the poles of the two curves are fixed, the sum of the radius vectors must be constant, and r_1 and r_2 will change by equal, though opposite, amounts. Therefore, s_1 and s_2 are equal, and the two curves will roll without sliding. We may note that the angular velocities of these two curves differ, and that even the ratio of the two angular velocities is not constant, so that wheels of this form may be used to transform uniform rotation into nonuniform rotation or, conversely, to correct for the nonuniformity of a rotation.

If all tangents to a logarithmic spiral are drawn from a fixed point R, the line OR is seen to subtend the fixed angle ϕ from all tangent points on one side of the line, and the angle $\pi - \phi$ on the other side of the line. Hence, the pole, the fixed point R, and all of the tangent points are situated on a circle. This simple property can be utilized to determine the pole of a spiral when given only a portion of the curve. It is necessary only to construct any two of these circles, their intersection determining the pole.

HISTORICAL NOTE

Descartes first discussed the logarithmic spiral in connection with a problem in dynamics in a letter to Mersenne in 1638. He showed that radius vectors at equal angles to one another at the pole are in continued proportion; and that distances measured along the curve from the origin and intercepted by any radius vectors, as P, P', are porportional to the radius vectors OP and OP'. Hence, the logarithmic spiral was the first transcendental curve to be rectified. It follows that sectors cut off by successive radii, at equal angles to one another, are in every way similar to one another; and the quest of Descartes for a growing curve produced a curve which grows continuously without ever changing its shape.

Torricelli studied this spiral and obtained expressions for areas and lengths of arc. John Wallis and Sir Christopher Wren also arrived at similar results.

Jaques Bernoulli (1691-93) and Collins (at an earlier date) also studied the spiral. Bernoulli noticed several of the properties of the curve, including some of the properties of "perpetual renascence." The spiral so delighted him that he requested, in imitation of Archimedes, that the logarithmic spiral be engraved on his tomb, along with the inscription *Eadem Mutata resurgo* (I shall arise the same, though changed).

Bernoulli gave to the spiral the name which is most commonly used, the *logarithmic spiral* (1691). Other names displaying certain properties of the spiral are: *geometrical spiral,* by P. Nicholas in 1693; and *proportional spiral,* by E. Halley in 1696. R. Cotes, referring to Descartes' original conception, termed it the *equiangular spiral* in 1719.

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Consequences of $A^2 + B^2 = C^2$

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If $$A^2 + B^2 = C^2$$

then

1) $$A^4 + B^4 + C^4 = (B^2 - A^2 - AB)^2 + (2AB)^2 + (B^2 - A^2 + AB)^2$$

2) $$A^8 + B^8 + C^8 = (B^2 - A^2 - AB)^4 + (2AB)^4 + (B^2 - A^2 + AB)^4.$$

Substituting $D = B^2 - A^2 - AB$, $E = 2AB$, and $F = B^2 - A^2 + AB$ in 1) and 2) respectively we obtain

3) $$A^4 + B^4 + C^4 = D^2 + E^2 + F^2 \text{ and}$$

4) $$A^8 + B^8 + C^8 = D^4 + E^4 + F^4.$$

For any arbitrary k

5) $$3[(ABC)^4 - (DEF)^2] = [(A^4 + k)^3 + (B^4 + k)^3 + (C^4 + k)^3] - [(D^2 + k)^3 + (E^2 + k)^3 + (F^2 + k)^3] \text{ and}$$

6) $$3(ABC)^4 = [(A^4 + k)^3 + (B^4 + k)^3 + (C^4 + k)^3] - [(k^3 + 2(G + k)^3]$$

where

$$G = A^4 + B^2C^2 = B^4 + A^2C^2 = C^4 - A^2B^2.$$

If we let $H = ABC$ it follows that

7) $$-A^6 - B^6 + C^6 = H^2 + H^2 + H^2 = 3H^2$$
$$(-A)^6 \cdot (-B)^6 \cdot (C)^6 = H^2 \cdot H^2 \cdot H^2 = H^6$$

9) $$(-A)^{12} + (-B)^{12} + C^{12} - 2G^3 = 3H^4$$

10) $$-A^6 - B^6 + C^6 + 3(2H^2) = 3H^2 + (3H^2 + A^6) + (3H^2 + B^6) + (3H^2 - C^6)$$

11) $$A^{12} + B^{12} + C^{12} + 3(2H^2)^2 = 3H^4 + (3H^2 + A^6)^2 + (3H^2 + B^6)^2 + (3H^2 - C^6)^2$$

12)
$$-A^{18} - B^{18} + C^{18} + 3(2H^2)^3$$
$$= 3H^6 + (3H^2 + A^6)^3 + (3H^2 + B^6)^3 + (3H^2 - C^6)^3$$

13)
$$A^{24} + B^{24} + C^{24} + 3(2H^2)^4$$
$$= 3H^8 + (3H^2 + A^6)^4 + (3H^2 + B^6)^4 + (3H^2 - C^6)^4$$

14)
$$\tfrac{1}{2}(A^4 + B^4 + C^4) = G$$

15)
$$\tfrac{1}{2}(A^8 + B^8 + C^8) = G^2$$

If
$$S = \tfrac{1}{2}(A + B - C)$$

16) $A^4 + B^4 + 2[(SA)^2 + (SB)^2 + (AB)^2] = C^4 + 2(SC)^2$

also for
$$n = 1, 2, 3, 4$$

17)
$$(2A + 2C)^n + [2(A + B + C)]^n + 2(B - A + C)^n + 2(B - A + 3C)^n$$
$$= [2(C - A)]^n + 2(A + B + C)^n + [2(B - A + C)]^n + 2(A + B + 3C)^n$$

For $A = 3$, $B = 4$, and $C = 5$ the relations expressed by 17) become:

$$8 + 2(3) + 12 + 2(8) = 2 + 2(6) + 6 + 2(11)$$
$$8^2 + 2(3)^2 + 12^2 + 2(8)^2 = 2^2 + 2(6)^2 + 6^2 + 2(11)^2$$
$$8^3 + 2(3)^3 + 12^3 + 2(8)^3 = 2^3 + 2(6)^3 + 6^3 + 2(11)^3$$
$$8^4 + 2(3)^4 + 12^4 + 2(8)^4 = 2^4 + 2(6)^4 + 6^4 + 2(11)^4$$

Mathematics, like all other subjects, has now to take its turn under the microscope and reveal to the world any weaknesses there may be in its foundations.

—F. W. WESTAWAY

Paradox Lost–Paradox Regained

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Once upon a time there lived a little man who delighted in confusing people—no, he wasn't a professor, but rather one of those students who spends his lecture periods searching for the slightest opportunity to ask a question that will stump the professor and thus openly embarrass him. But of all such students that have existed through the ages, this "blue ribbon" grand champion managed not only to stump his professor but all who followed him for centuries.

Approximately 475 B.C., this little man known as Zeno of Elea, a student of Parmenides, confounded the mathematical world with his paradoxes. Four of the eight paradoxes preserved for us in the writings of Aristotle and Simplicius are concerned with motion and are of special significance to us as mathematicians because of their appalling effect upon the development of higher mathematics. The paradoxes arose from the logical difficulties encountered by the ancient Greeks in their attempt to express their intuitive ideas with respect to ratio and proportion of line segments, which they vaguely recognized as continuous—in terms of the integers which are discrete. Because of this discreteness of their numbers, the Greeks were completely baffled by the resulting inconsistencies and abandoned anything of the infinite, forestalling the development of the calculus for almost 2000 years.

These paradoxes, which had such a retarding effect on the development of mathematics are referred to as *The Arrow, The Stadium, The Dichotomy,* and *The Achilles.*

The Arrow: Anything occupying space equal to itself is at rest; but this is true of the arrow at every moment of its flight. Therefore the arrow does not move.

The Stadium: Space and time being assumed to be made up of points and instants, let there be given three parallel rows of points, *A*, *B*, and C. Let C move to the right and *A* to the left at the rate of one point per instant, both relative to *B*; but then each point of *A* will move past two points of C in an instant, so that we can subdivide this, the smallest interval of time; and this process can be continued *ad infinitum,* so that time can not be made up of instants.

The Dichotomy: Before an object can traverse a given distance, it must first traverse half this distance; before it can cover half, however, it must cover one quarter; and so on *ad infinitum*. Therefore, since the regression is infinite, motion is impossible, inasmuch as the body would have to traverse an infinite number of divisions in a finite time.

The Achilles: Assume a tortoise to have started a given distance ahead of Achilles in a race. Then by the time Achilles has reached the starting point of the tortoise, the latter will have covered a certain distance; in the time required by Achilles to cover this additional distance, the tortoise will have gone a little farther; and so on *ad infinitum*. Since this series of distances is infinite, Achilles can never overtake the tortoise.

In the last two Zeno denies the possibility of infinite subdivision, while in the former he requires just such a subdivision. Thus Zeno tried to show that the assumption of the existence of such an infinitude of things in time and space carried with it serious inconsistencies.

These paradoxes may be grouped in still another manner. Let us consider the geometrical aspects of the first two while classing the latter ones as "the possibility of motion."

Under this grouping we may observe that the *Arrow* can be geometrically interpreted as a line segment consisting of points of zero length. Zeno's assertion is that the sum of these zero lengths can be nothing other than zero, *i.e.*, no motion when interpreted in the light of our hypothesis. Thus Zeno, conceiving of a line as the summation of discrete points and thereto applying the laws of finite numbers, arrived at this paradoxical result.

It was Cantor who isolated the crux of this problem in distinguishing between a denumerable and a non-denumerable infinity of points. Surely Zeno would have been astounded, as are most students, with his encounter with Cantor's bland assertion, "There are as many points on a line two centimeters in length as one one centimeter in length."

Let AB and CD be two unequal and parallel segments. Let P be the intersection of the two lines AC and BD. The pencil of lines determined by P will set up a one-to-one correspondence between the points of AB and CD.

Zeno's idea was that there must surely be twice as many points on a line two centimeters long as on a line one centimeter long.

We see that unfortunately there is no definition for the sum of a non-denumerable infinity of entities analogous to our definition of the sum of a finite number of entities. This analogy falsely made was the pitfall of Zeno.

Thus, as the measure of a non-denumerable infinity of entities can be finite, the flight of the arrow consists of time intervals of zero duration whose sum is a finite value. *I.e.*, motion *is*, as is daily perceived.

In the *Arrow* Zeno defies the summation of an infinity of points while in the *Stadium* he refutes infinite subdivison. Here again he was unable to comprehend the denseness of instants in time due to his belief in the discreteness and order of these instants. That there is no such thing as a "next" instant in time or a "next" point in space is the basis for the refutation of this paradox as it was the former.

In the *Dichotomy* and the *Achilles* Zeno bases his argument that motion cannot begin, or if begun cannot be completed in a finite time, on the infinite subdivision of motion which HAS ALREADY TAKEN PLACE!! One cannot justify the impossibility of motion by an argument which necessitates not only motion but its completion in a finite time.

Here, as before, Zeno leans on his intuition and runs into the problem of denseness. It is not true, as he assumed, that event B is later than event A because of the occurrence of intermediary events in a discrete ordering, but rather that a suitable definition of later event must be chosen to meet the requirement of a dense order of events. Thermodynamics does just this in its second law which defines time, order, and direction in terms of entropy states.

Thus Zeno's paradoxes resolved themselves into problems of continuity, the infinite, and the infinitesimal. And, as shown, in the 19th century with the establishment of the real number theory and a rigorous formulation of the fundamental concepts of the calculus by Cantor, Weierstrass, and Dedekind the paradoxes were labeled fallacies and mathematicians rejoiced in paradox lost. But the paradoxes were no sooner laid aside than serious questions were again raised by prominent logicians about the turn of the century. Thus for such men as Bertrand Russell and L. E. J. Brouwer paradox has been regained although it need not bother us in our mathematical work for it is merely an indication that the foundation upon which our mathematical thought is based may still be profitably studied.

Logicians, such as Russell, have propounded many paradoxes. Such statements as,

All statements on this board are false; and
All generalities are false, including this one,

are examples of this type of paradox.

Mathematics abounds in interesting fallacies and seeming paradoxes. Thus one can "prove" that the area of an infinite number of points is equal to the area of a single point.[1]

Construct a square *ABCD* with diagonal *AC* and the quadrant of a circle having *AB* as radius and center *A*. Draw any line *PR* parallel to *AD*, intersecting the quarter-circle *DB* at *M* and the diagonal at *N*. Construct, with center at *P*, a circle of radius *PN*, (this circle will pass through *A* as *PR* is parallel to *AD* and $\angle PAN = \pi/4$), another with radius *PM*, and another with radius *PR*. Denote *AM* as r_1, *MP* as r_2, and *PA* as r_3. Then A_1, the area of the small shaded circle is

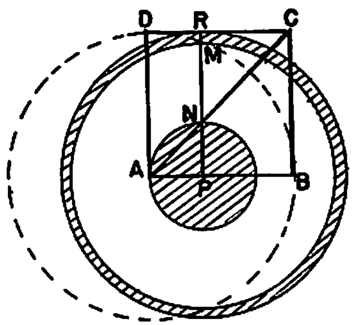

Fig. 1

$$A_1 = \pi(PN)^2 = \pi(PA)^2 = \pi r_3^2.$$

A_2, the area of the ring, is

$$\begin{aligned} A_2 &= \pi(PR)^2 - \pi(PM)^2 \\ &= \pi(AM)^2 - \pi(PM)^2 \\ &= \pi r_1^2 - \pi r_2^2. \end{aligned}$$

Considering triangle *AMP*,

$$r_3^2 = r_1^2 - r_2^2;$$

$$\pi r_3^2 = \pi r_1^2 - \pi r_2^2;$$

i.e., $$A_1 = A_2.$$

Therefore the shaded areas are at all times equal. Let *PR* approach *AD*. When they coincide, the small circle becomes point *A* and the

[1]To Zeno this would not have presented any contradiction, but in the light of our previous arguments it becomes a fallacious equality.

ring becomes the circumference of the circle having PR as a radius. Therefore from our calculations, the area of a single point is equal to that of an infinite number of points contained in the circumference.

In contrast are geometrical fallacies which generally arise from the fallacious location of points or other incorrect assumptions. One of the best known of these is the apparently sound geometrical proof that all triangles are isosceles.

We are given $\triangle ABC$; point O, the intersection of the bisector of A and the perpendicular bisector of BC; $OE \perp AB$; and $OF \perp AC$. Proof:

$$EO = OF.$$
$$\triangle AEO \text{ congruent to } \triangle AFO \text{ implies } AE = AF.$$
$$\triangle EOB \text{ congruent to } \triangle FOC \text{ implies } EB = FC.$$
$$AE + EB = AF + FC.$$
$$AB = AC.$$

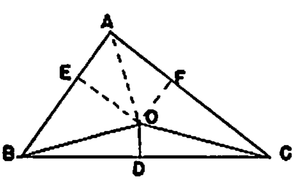

Fig. 2

Therefore all triangles are isosceles.

But suppose that point O does not lie within the triangle. Redraw the figure; this time placing the point O outside the triangle.

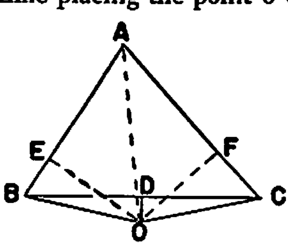

Fig. 3

It is easily verified using this figure that all the statements in the above proof are true and that the same conclusion may be drawn.

A final light touch is an extremely simple example of an arithmetical fallacy.

Let x = the weight of an elephant.
y = the weight of a gnat.
$x + y = 2v$, where v is a constant.

Then,
$$x = 2v - y,$$
$$x - 2v = -y.$$

Multiplying member by member,

$$x^2 - 2vx = y^2 - 2vy,$$
$$x^2 - 2vx + v^2 = y^2 - 2vy + v^2,$$
$$(x - v)^2 = (y - v)^2,$$
$$x - v = y - v,$$

i.e.,
$$x = y.$$

Therefore the elephant and the gnat weigh the same.

I guess gnats and elephants are about my speed, but I hope that logicians such as Russell can someday wholly and satisfactorily present us with *paradox lost.*

Editorial Note. Miss Steinbeck has graciously left for the student the discovery of the fallacies in the "proofs" that she has given.

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Many eminent men of science have been bad mathematicians. . .

—Bertrand Russell

Watch That Check – It Might Bounce!

DANA R. SUDBOROUGH
Faculty, Central Michigan College

Is "casting out nines" a bona fide check on arithmetical computations? May a calculation in algebra be verified by substituting a specific number for the variable involved? (This is the kind of check the writer refers to—not those sent to him by subscribers of THE PENTAGON.)

First, for the benefit of readers who may not be acquainted with the method of checking called "casting out nines", we briefly review the procedure. We shall define the "residue" of an integer to be the remainder obtained when the integer is divided by 9, and we point out the fairly well-known fact that this residue may be most readily obtained by dividing 9 into the sum of the digits of the integer instead of the integer itself. Now the method of checking, simply expressed, consists of replacing the integers of the problem by their respective residues and performing the same operations on these residues. Then, if the residue of the answer obtained in the checking operation is equal to the residue of the original answer, the original computation is supposedly correct.

For example, consider the following multiplication:

```
   6497
  × 516
  38982
  6497
32395
3343452
```

The residues of the factors are 8 and 3, respectively, and the residue of their product, 24, is 6, which is also the residue of the product obtained above. Was the original multiplication correctly performed? No, and unfortunately the error happened to be a multiple of 9.

As an example of a different sort of checking, consider the following problem from elementary algebra:
Multiply $2x - 6$ by $3x + 11$. Now suppose the student writes the following "identity":

$$(2x - 6)(3x + 11) = 5x^2 + 4x - 17$$

To check the result, one writer of a fairly recent college algebra textbook would, apparently, advise the student to substitute for x some specific number other than 0 or 1. Then, if the left member reduces to the same value as the right, the above "identity" is "indicated to be correct". But the reader may find it interesting to substitute 7 for x in the foregoing case. Unfortunately, the parabolas $y = 6x^2 + 4x - 66$ and $y = 5x^2 + 4x - 17$ intersect at the point (7, 256).

Construction of the Circle of Inversion

Shirley T. Loeven
Student, Central Missouri State College

An application of a theorem from plane geometry gives an easy construction of the circle of inversion. This construction is as follows:

If C and C′ are any two inverse curves with respect to the center of inversion O, draw any line m through O which intersects C and C′ at P and P′, respectively. P and P′ will be inverse points.

The problem is to construct the circle of inversion. To contruct a circle two things are necessary—the center and the radius. In this problem, the center O is given so the problem is now to find the radius.

The radius of inversion is equal to the square root of the product of the two line segments from the center of inversion to any two inverse points. Therefore, the radius of inversion r equals the square root of the product of the two line segments OP and OP'.

The theorem mentioned above can now be applied.

The tangent from an external point is the mean proportional between the external segment of any secant drawn through the external point and the complete secant.

To use this theorem it is necessary to draw a circle through P and P'. There are any infinitely many circles through P and P' and each will give the same result, but the easiest one to use is the one whose center bisects line PP'.

Line m is now a secant of the circle. If the tangent is constructed from O to the circle, it will be the radius of inversion.

Mathematical Notes

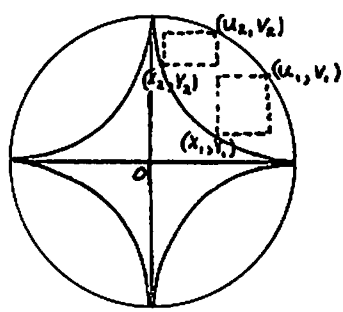
Merl Kardatzke
Student, Anderson College

Pseudo Circles

For some purposes, graphing is not used to find accurate values but rather as an inquiry into the characteristics of an equation. This means that we may make substitutions into our equation to change the variables in order to make the nature of the curve more apparent.

In many equations such as $x^{2/3} + y^{2/3} = 1$ we may make the substitutions $u = x^{1/3}$ and $v = y^{1/3}$ to get the simple equation $u^2 + v^2 = 1$ which may be easily drawn.

Then by reversing the substitution $x = u^3$, $y = v^3$ and applying these equations to conveniently selected points $(u, v) \longleftrightarrow (u^3, v^3) = (x, y)$ we get a fair idea of the curve.



This curve may be plotted with as many points as one has patience to determine by methods of cubing lengths geometrically.

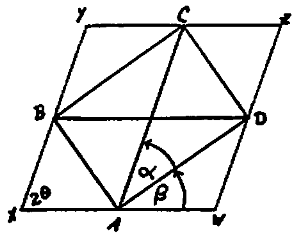
Other substitutions may be made where certain other fractional powers or even powers are involved.

Circles are not the only basic figure which may be used for this method. Thus we have pseudo parabolas, pseudo hyperbolas, and many others.

An Application of the Rhombus to Trigonometry

Prove: $\sin 2\theta = 2 \sin\theta \cos\theta$ where $0 < 2\theta < \pi$

Construct rhombus $wxyz$ with sides equal to unity and the angle at x equal to 2θ. Construct points A, B, C, D as midpoints of wx, xy, yz, zw respectively. Draw all lines joining A, B, C, D. The area of *wxyz* = sin 2θ. By plane geometry we know: (1) *ABCD* is

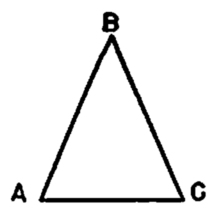


a rectangle, and (2) the area of *ABCD* = ½ the area of *wxyz*. Therefore, the area of *ABCD* = (*CD*)(*AD*) = sin α cos α. Since *AC* is parallel to *xy*, $\alpha + \beta = 2\theta$. But $\alpha = \beta$ and hence $\alpha = \theta$. We now have sin 2θ = area of *wxyz* = 2(area of *ABCD*) = 2 sin α cos α = 2 sin θ cos θ which was to have been shown. By basic trigonometry the formula may be extended to any angle.

Two Triangles in One

Theorem: If two sides of a triangle are equal, the angles opposite are equal.

Given: $AB = BC$
Prove: $\angle A = \angle C$



Proof:

1. $AB = CB$	1. Given
2. $BC = BA$	2. Identical lengths as in (1)
3. $\angle B = \angle B$	3. Identity
4. ΔABC is congruent to ΔCBA	4. S.A.S. = S.A.S.
5. $A = C$	5. Corresponding parts of congruent triangles are equal.

The converse of this theorem may be proved in similar fashion.

Creating Interest for Gifted Students

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Faculty, Queens College

Some of my colleagues, as well as college students, might be interested in knowing that a few students of mine became interested in finding their own proofs of some theorems. In fact some of their proofs are so elegant that they are worthy of attention.

THEOREM: Let a and b be real numbers, and let x be a real variable. Then $ax + b$ has the same sign as a for $x > -b/a$ and it has different sign from a for $x < -b/a$.

PROOF (1): This proof is given by Frank Miller, a freshman of UCLA. Let $x > -b/a$. Then $x = -b/a + c$ where $c > 0$. Therefore $ax + b = a(-b/a + c) + b = ac$ which has the same sign as a. Similarly let $x < -b/a$. Then $x = -b/a - c$ where $c > 0$. Therefore $ax + b = a(-b/a - c) + b = -ac$ which has different sign from a.

PROOF (2): This proof is supplied by Jess Liss, a freshman of Queens College. Suppose $f = ax + b$. Then $f/a = x + b/a$. Since $x = -b/a$ makes f/a equal to zero, $x > -b/a$ makes $f/a > 0$, *i.e.*, f and a have the same signs, and $x < -b/a$ makes $f/a < 0$, *i.e.*, f and a have opposite signs.

Both proofs are quite elegant because they avoid the use of inequalities.

Studying relations between the sides and the angles of a triangle ABC, we ran into the law of tangents which was stated without proof since the formulas for addition of angles were presented in the next chapter. The theorem was given to the class as an exercise. Two geometric proofs were supplied which were quite interesting.

A PROOF OF THE LAW OF TANGENTS: This is by Joe VanEpps, a freshman of the University of Idaho. Let ABC be a triangle (Fig. 1). Without loss of generality we can suppose $a > b$. Choose D and E on CB so that $AC = CD = CE$. The triangle AED is a right triangle at A because $AC = ½ED$. Erect a perpendicular to CB at B. This perpendicular intersects the line EA at F. We observe the following:

The four points A, F, B, and D are on a circle.

$$EB = a + b, DB = a - b.$$

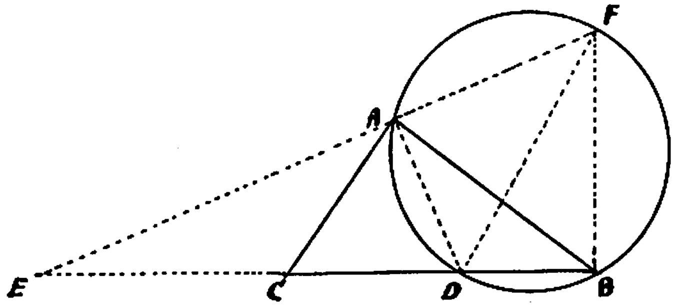


Fig. 1

$\angle EFB = ½(A + B)$ because angle C is the exterior angle of the isosceles triangle ACE, and therefore $\angle AEC = ½C$. $\angle DFB = ½(A - B)$ since $\angle AFD = B$. They are inscribed angles with common arc AD. Therefore,

$$\angle DFB = \angle EFB - \angle AFD = ½(A + B) - B = ½(A - B).$$

Thus, $\tan ½(A + B) = EB/FB = (a + b)/FB$, $\tan ½(A - B) = DB/FB = (a - b)/FB$. Consequently,

$$\tan ½(A + B)/\tan ½ (A - B) = (a + b)/(a - b).$$

ANOTHER PROOF OF THE LAW OF TANGENTS: This is the proof by Lee Davison, a freshman of the University of Idaho. Let ABC be a triangle so that $a > b$ (Fig. 2). Choose CF = CE = b

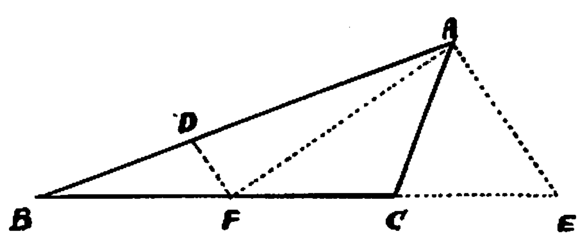


Fig. 2

as in Figure 2 and draw FD parallel to EA. The triangle AFE is a right triangle because $AC = \frac{1}{2}EF$. We observe:

$$BE = a + b, BF = a - b.$$

$\angle ACE = A + B$ since it is the exterior angle of the triangle ABC. On the other hand $\angle ACE$ is the exterior angle of the isosceles triangle ACF. Therefore, $\angle AFE = (A + B)/2 = \angle FAC$. $\angle DAF = (A - B)/2$ because $\angle DAF = A - \angle FAC = A - (A + B)/2 = (A - B)/2$. Now $\tan (A - B)/2 = DF/AF$ and $\tan (A + B)/2 = AE/AF$. But $[DF/AF]/[AE/AF] = DF/AE = BF/BE = (a - b)/(a + b)$ which proves the theorem.

From the axiomatic point of view, mathematics appears thus as a storehouse of abstract forms—the mathematical structures; and it so happens—without our knowing why—that certain aspects of empirical reality fit themselves into these forms, as if through a kind of preadaptation. Of course, it cannot be denied that most of these forms had originally a very definite intuitive content; but, it is exactly by deliberately throwing out this content, that it has been possible to give these forms all the power which they were capable of displaying and to prepare them for new interpretations and for the development of their full power.

It is only in this sense of the word "form" that one can call the axiomatic method a "formalism". The unity which it gives to mathematics is not the armor of formal logic, the unity of a lifeless skeleton; it is the nutritive fluid of an organism at the height of its development, the supple and fertile research instrument to which all the great mathematical thinkers since Gauss have contributed, all those who, in the words of Lejeune-Dirichlet, have always labored to "substitute ideas for calculations."

—N. Bourbaki
Translated by A. Dresden
American Mathematical Monthly
Vol. 62 (1950)

Order Among Complex Numbers

MERL KARDATZKE
Student, Anderson College

Order (relative precedence) is defined:

O_1: If $a \neq b$, then $a < b$ or $b < a$
O_2: If $a < b$ and $b < c$, then $a < c$

There is order among the real numbers or any subset thereof. This order seems intuitive because of our ideas of quantity. We must not let quantity or magnitude be our only means of putting numbers in order.

I. Semicountable Complex Numbers

Rule of Order: Where $z = x + 1/(1 + e^{-y})$ and x, y are integers $(x + iy)$ is ordered if $(x + iy)$ shall have the same order as z.

Proof: For every $x + iy$ there exists a distinct z; z is real; hence z is ordered; therefore $x + iy$ is ordered. This formula is just as good if y assumes any real value. We can extend the formula to cases where x is countable. If we let $z = r + 1/(1 + e^{-y})$, there exists a one-to-one correspondence between z and $x + iy$, where r is the ordinal of x, x is countable, and y is real.

Similarly, x may assume any real value if y is countable and $x + iy$ may be ordered by the same formula by interchanging x and y.

Thus we see that we may order all semicountable complex numbers. As we have seen semicountable means complex numbers with either the real or the imaginary coefficient countable.

Now we shall order the general complex numbers. First we shall order them by a rule compatible with the rule for the semicountable complex numbers but not by formula. Second we give a rule of order, by a formula, which gives a different concept of order.

II. Rule One for Ordering General Complex Numbers

Since $[x_1 + iy_1 = x_2 + iy_2]$ if and only if $[x_1 = x_2, y_1 = y_2]$, we may state a rule for ordering the general complex numbers as follows: (R_1) $[x_1 + iy_1 < x_2 + iy_2]$ if and only if (1) $x_1 < x_2$, or (2) $x_1 = x_2$ and $y_1 < y_2$.

Proof that the rules of order O_1 and O_2 are satisfied: O_1 becomes: If $[x_1 + iy_1 \neq x_2 + iy_2]$ then $[x_1 + iy_1 < x_2 + iy_2]$ or $[x_2 + iy_2 < x_1 + iy_1]$. This is true for if $[x_1 + iy_1 \neq x_2 + iy_2]$ then (1) $x_1 < x_2$ or $x_2 < x_1$ or (2) $x_1 = x_2$ and $y_1 < y_2$ or $y_2 < y_1$ which may be grouped as follows:

$$\begin{bmatrix} (1)\ x_1 < x_2 \\ \text{or}\ (2)\ x_1 = x_2,\ y_1 < y_2 \end{bmatrix} \text{ or } \begin{bmatrix} (1)\ x_2 < x_1 \\ \text{or}\ (2)\ x_2 = x_1,\ y_2 < y_1 \end{bmatrix} \text{ if and}$$

only if $[x_1 + iy_1 < x_2 + iy_2]$ or $[x_2 + iy_2 < x_1 + iy_1]$ respectively, by R_1.

O_2 becomes: If $[x_1 + iy_1 < x_2 + iy_2]$ and $[x_2 + iy_2 < x_3 + iy_3]$ then$[x_1 + iy_1 < x_3 + iy_3]$. $[x_1 + iy_1 < x_2 + iy_2]$ if and only if (1) $x_1 < x_2$, or (2) $x_1 = x_2$ and $y_1 < y_2$; $[x_2 + iy_2 < x_3 + iy_3]$ if and only if (1) $x_2 < x_3$, or (2) $x_2 = x_3$ and $y_2 < y_3$ by R_1.

This gives combinations: $[x_1 < x_2 < x_3]$, $[x_1 < x_2 = x_3]$, $[x_1 = x_2 < x_3]$, and $[x_1 = x_2 = x_3$ and $y_1 < y_2 < y_3]$ which imply either (1) $x_1 < x_3$ or (2) $x_1 = x_3$ and $y_1 < y_3$ which, in turn, implies $[x_1 + iy_1 < x_3 + iy_3]$ by R_1.

III. Rule Two for Ordering General Complex Numbers

We shall now put a general complex number $x + iy$ in one-to-one correspondence with a real number z.

The absolute value of x and y may be represented as the infinite decimal:

$$|x| = x_1\, x_2\, x_3 \cdots x_n\, .\, x_{n+1}\, x_{n+2} \cdots$$

$$|y| = y_1\, y_2\, y_3 \cdots y_n\, .\, y_{n+1}\, y_{n+2} \cdots$$

n = the number of significant digits in x or y before the decimal (whichever) is greater), zero being filled in, starting x_1 or y_1 for as many values as is necessary. For example if $y = -56234.18$, we shall write $x = 00012.12$. We shall let $z = qx_1y_1x_2y_2x_3y_3 \cdots x_ny_n.x_{n+1}y_{n+1} \cdots$ where q is the quadrant number (Arabic numeral) when labeled in the usual way.

Thus in our example z = 40,506,021,324.1182.

For every $(x + iy)$ there exists a distinct z.
For example if $x + iy = 5 + 3i$, z = 153.0 or if $x + iy = -200.01 + 0.01\, i$, z = 2,200,000.0011.

Conclusion

We have ordered complex numbers $(x + iy)$. Thus we have shown that a pair of coordinates (x, y) may be put in one-to-one correspondence with a single coordinate. This when applied to a two-dimensional surface means we may represent the points of such a surface on a line. A three-dimensional space may, in turn, be made to correspond to a line.

In other words (w, x, y)$\longleftrightarrow$z

where $[z_1 < z_2]$ if and only if (1) $w_1 < w_2$; (2) $w_1 = w_2$ and $x_1 < x_2$; (3) $w_1 = w_2$, $x_1 = x_2$, and $y_1 < y_2$.

We can easily show that n—dimensional space may be put in corespondence with points on a line.

"It is impossible not to feel stirred at the thought of the emotions of men at certain historic moments of adventure and discovery—Columbus when he first saw the Western shore, Pizarro when he stared at the Pacific Ocean, Franklin when the electric spark came from the string of his kite, Galileo when he first turned his telescope to the heavens. Such moments are also granted to students in the abstract regions of thought, and high among them must be placed the morning when Descartes lay in bed and invented the method of co-ordinate geometry."

—Alfred North Whitehead

The Problem Corner

EDITED BY FRANK C. GENTRY

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tool beyond the calculus. Although new problems are preferred, old problems of particular interest or charm are welcome provided the source is given. Solutions of the following problems should be submitted on separate sheets of paper before October 1, 1957. The best solutions submitted by students will be published in the Fall, 1957, number of THE PENTAGON with credit being given for other solutions received. To obtain credit a solver should affirm that he is a student and give the name of his school. Address all communications to the new Problem Corner Editor, J. D. Haggard, Department of Mathematics, Kansas State Teachers College, Pittsburg, Kansas.

PROBLEMS PROPOSED

101. *Proposed by Frank Hawthorne, New York State Education Department, Albany, New York.*

Show that if a gun be fired (in vacuo) from a point P on a plane hillside of inclination θ, the points of extreme range form an ellipse with P as the "uphill" focus, with the eccentricity equal to $\sin \theta$, and with one directrix of the ellipse in the directrix plane of the paraboloid of revolution which is the envelope of the trajectories. (Note: See the author's article, "Projectile Geometry," in the Spring, 1954, number of THE PENTAGON.)

102. *Proposed by J. L. Brenner, Stanford Research Institute.*

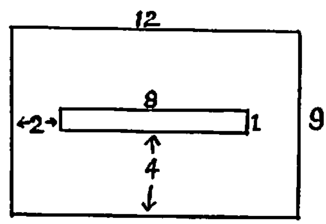
Let $x^2 + cx + d$ be a quadratic polynomial with zeros a and b. If ε is a positive number, show that the roots of the equation $x^2 + cx + d = \varepsilon^2$ can be so arranged that one root is within ε of a and the second root is within ε of b.

103. *Proposed by Glenn W. Thornton, Student, University of New Mexico, Albuquerque, New Mexico.*

A military ambulance traveling at the average speed of 12 miles per hour sent on ahead a motorcyclist who could travel with twice the speed of the ambulance. A half hour later it was found necessary to revise the message, and a second motorcyclist was sent to overtake the first. The second messenger returned to the ambulance in 45 minutes. What was his average speed while delivering the message?

104. *Proposed by Vaughn Hopkins, Student, Central Missouri State College, Warrensburg, Missouri.*

Cut the slotted rectangle shown below into two pieces which may be placed together to form a square 10 inches on a side.



105. *Proposed by Roger Entringer, Graduate Student, University of New Mexico, Albuquerque, New Mexico.*

If p is a prime and $m + n = p - 1$, (m, n non-negative), prove that p divides $m!\, n! + (-1)^n$.

Note by the Editor. No satisfactory solutions have been received for the following problems: Nos. 84, 85, 86, 87 published in the Fall, 1955, number of THE PENTAGON; No. 91, Spring, 1956; Nos. 99, 100, Fall, 1956.

SOLUTIONS

97. *Proposed by Harvey Fiala, Student, North Dakota State College, Fargo, North Dakota.*

If two circles, one having a radius of 1 inch and the other a radius of 1 light year, each have their circumferences increased by 6 feet, what is the difference in the increase in the radii of the two circles.

Solution by Vaughan D. Hopkins, Central Missouri State College, Warrensburg, Missouri.

If r is the radius and C the circumference of a circle, then $r = C/2\pi$ and $dr = dC/2\pi$. Hence the increase in radius of either circle varies directly with the increase in circumference, so the difference in the increase in the radii is zero. In fact $dr = 6/2\pi$ feet $= 0.96$ feet approximately for either circle.

Also solved by Richard Ross, Southwest Missouri State College, and Neal Durkin, Chicago Teachers College.

98. *Proposed by the Editor.* (From Robinson's *Progressive Higher Arithmetic,* 1871).

A general forming his army into a square had 284 men remaining; but increasing each side by one man he wanted 25 men to complete the square. How many men had he?

Solution by Jerry Thompson, Texas Technological College, Lubbock, Texas.

Let x be the number of men on each side of the first square. Then the total number of men is represented by $x^2 + 284$ or by $(x + 1)^2 - 25$. Setting these two expressions equal to each other and solving the resulting equation leads to $x = 154$ and $(154)^2 + 284 = 24{,}000$, the total number of men.

Also solved by Vaughen D. Hopkins, Central Missouri State College; and Richard Ross, Southwest Missouri State College.

The theory of invariants sprang into existence under the strong hand of Cayley, but that it emerged finally a complete work of art, for the admiration of future generations of mathematicians, was largely owing to the flashes of inspiration with which Sylvester's intellect illuminated it."

—P. A. MACMAHON

The Mathematical Scrapbook

EDITED BY J. M. SACHS

Pure mathematics proves itself a royal science both through content and form, which contains within itself the cause of its being and its methods of proof. For in complete independence mathematics creates for itself the object of which it treats, its magnitude and laws, its formulas and symbols.

—E. DILLMAN

=Δ=

Those of us who are familiar with the courses taught in the American schools, the texts used for these courses and the various topics covered in these courses are sometimes led into the false belief that it was ever so. Oh, we know that there have been changes in stress and some material has gradually disappeared while other material has edged into the small gaps. But in general I think we are prone to think the pattern rather rigid. The editor, as guilty as most in the above named misconception, derived a great deal of enjoyment from reading Bulletin No. 18 of the Bureau of Education of the Department of Interior. This bulletin, issued in 1924 and written by L. G. Simons, traces the history of the introduction of algebra into the American schools. The following material is drawn from this bulletin.

With the American University modeled on the English University, the introduction of courses in algebra and the publication of texts in England in the late 17th Century brought this material to the American Universities early in the next century. Isaac Newton's ARITHMETICA UNIVERSALIS, a work of algebra and theory of equations appeared in 1707 and was published in English in 1720.

The earliest records of algebra in the Americas comes from notebooks. Often these contained material or problems from the popular English texts. There is one such notebook written by James Diman who was later the librarian of Harvard College. This notebook is dated 1730 and was probably from lectures prepared by Isaac Greenwood, a brilliant student of mathematics at Harvard and then a professor of mathematics and natural philosophy at Harvard. (Greenwood was later dismissed from Harvard for ". . .many acts of gross intemperance, to the dishonor of God and the great hurt and reproach of the society." There seems to be complete agreement as to genius of Greenwood, but it would seem from the wording above that his passion for mathematics was not as great as his taste for good New England rum.)

There is another notebook almost identical with Diman's written by Samuel Langdon, later president of Harvard, dated 1739. There seems little doubt that both of these were taken from the lectures of Greenwood. From the topics covered in these notebooks it would seem that algebra was not taught for the practical everyday usage which prompted so much of the instruction in arithmetic. This seems the proper place to give a quotation, undoubtedly from Greenwood's lectures since it appears in both notebooks. Under the heading: *The Method of Resolving Algebraical Questions,* we find, "This part of Algebra is wholly arbitrary & everyone is left to himself to pursue his own particular Genius and way of thinking, which is so far from being a Defect *y*ᵗ it is one of *y*ᵉ Chief Excellencies of this Science, which may from hence not unjustly be called a sublime way of Reasoning."

A set of problems used at the College of New Jersey, now Princeton University, in the latter part of the 18th Century seem to be drawn from two English texts, *Elements of Algebra,* by Saunderson and *Arithmetick,* by Hill. The name and notebook of Professor Robert Patterson appear in connection with 18th century algebra at the University of Pennslyvania. The notebooks of Robert Brooke, the five-volume hand-printed set by Thomas Sullivan, and the notebook of Nathaniel Bowditch are part of the material available for the period. The custom of keeping manuscript notebooks gradually died out as printed books became commonplace in America. With the popular printed text, the notebook became a supplement instead of being the principal source of information.

One rather unlikely source of information on the introduction of algebra teaching comes from the public press. The following advertisements are self explanatory: N. Y. Gazette, Jan. 7, 1734 ". . .At the said School are Taught all the Branches of the Mathematicks, Geometry, Algebra, Geography, and Merchant's Bookkeeping after the most perfect manner."

N.Y. Evening Post, June 15, 1747 "Arithmetic, Vulgar, Decimal and Algebra carefully and exactly taught by Joseph Blancherd."

Pennsylvania Gazette Oct. 1, 1728 ". . .Algebra, or the *Doctrine of AEquations, Simple, Quadratick, Cubic,* & c., . . ."

=Δ=

"The true mathematician is always a good deal of an artist, an architect, yes, of a poet. Beyond the real world, though perceptibly connected with it, mathematicians have intellectually created an ideal world, which they attempt to develop into the most perfect of all worlds, and which is being explored in every direction. None has the faintest conception of this world, except he who knows it."

—A. Pringsheim

=Δ=

"Heliodorus says that the Nile is nothing else than the year, founding his opinion on the fact that the letters nu, epsilon, iota, lambda, omicron, sigma as the name is spelled in Greek are respectively the numbers 50, 5, 10, 30, 70, 200. The sum of these numbers is 365."

—From Littell's Living Age

"As lightning clears the air of impalpable vapours, so an incisive paradox frees the human intelligence from the lethargic influence of latent and unsuspected assumptions. Paradox is the slayer of Prejudice."

—J. J. Sylvester

=Δ=

Mary is twice as old as Ann was when Mary was as old as Ann is now. If Mary's age in years is a two digit number with the ten's digit equal to one-half the unit's digit, what are the possibilities for the ages of Mary and Ann? All ages are an integral number of years.

=Δ=

"Herigone adopted the symbol $<$ for "angle" in 1634. Harriot had already used this symbol (1631) for "less than." This ambiguity persisted into the eighteenth century even though the modification for the angle symbol into the familiar $\angle$ was suggested by Oughtred in 1657.

—From Cajori, *A History of Mathematical Notations.*

=Δ=

"A friend of mine has a flower-garden—a very pretty one, though no great size—"

"How big is it?" said Hugh.

"That's what you have to find out!" Balbus gayly replied. "All I tell you is that it is oblong in shape—just half a yard longer than its width—and that a gravel-walk, one yard wide, begins at one corner and runs all round it."

"Joining into itself?" said Hugh.

"Not joining into itself, young man. Just before doing that, it turns a corner, and runs round the garden again, alongside of the first portion, and then inside that again, winding in and in, and each lap touching the last one, till it has used up the whole of the area."

"Like a serpent with corners?" said Lambert.

"Exactly so. And if you walk the whole length of it, to the last inch, keeping in the centre of the path, it's exactly two miles and half a furlong [a furlong is 220 yards]. Now you find out the length and breadth of the garden."

"You said it was a flower-garden?" Hugh inquired, as Balbus was leaving the room.

I did," said Balbus.

"Where do the flowers grow?" said Hugh. But Balbus thought it was best not to hear the question.

Perhaps you agree with Balbus that it is best to ignore Hugh's last question but can you find the dimensions of the garden?

—From *A Tangled Tale*
LEWIS CARROLL

=Δ=

"The faculty of resolution is possibly much invigorated by mathematical study, and especially by that highest branch of it which, unjustly, merely on account of its retrograde operations, has been called, as if par excellence, analysis."

—EDGAR A. POE

=Δ=

"Some persons have contended that mathematics ought to be taught by making the illustrations obvious to the senses. Nothing can be more absurd or injurious: it ought to be our never-ceasing effort to make people think, not feel."

—S. T. COLERIDGE

The Book Shelf

EDITED BY R. H. MOORMAN

From time to time there are published books of common interest to all students of mathematics. It is the object of this department to bring these books to the attention of readers of THE PENTAGON. In general, textbooks will not be reviewed and preference will be given to books written in English. When space permits, older books of proven value and interest will be described. Please send books for review to Professor R. H. Moorman, Box 169-A, Tennessee Polytechnic Institute, Cookeville, Tennessee.

Cryptanalysis, a Study of Ciphers and their Solution (formerly published under the title of *Elementary Cryptanalysis*), Helen Fouche Gaines, Dover Publications, Inc. (920 Broadway) New York 19, 1956, 237 pp., $1.95 paperbound, $3.95 clothbound.

Cincinnati, November 26 (AP). "A cryptic forearm tattoo today stumped police as they sought the identity of a man about 65 years old who collapsed and died here Saturday on a streetcar. He appeared to be of Slavic extraction. The tattoo was:

K J K I R N X S
2 E R A K O M P
B A A A
H O B O K O B
A H N E P
1909."

Ten years ago the above item in a New York newspaper sent this reviewer scurrying on a hunt for a cryptographic volume that might help solve the cipher and expose long-sought spies, reveal long-hidden diplomatic incidents and military secrets, and disclose intrigue against the Allies, circa 1914. The reviewer found the volume—*Elementary Cryptanalysis*—and promptly forgot the press item in the vast new world of serious puzzles that the book had opened to him. The volume, indeed, was his cryptographic teething ring; and the current Dover reprint retains all the allure of the original.

For an elementary work, the volume is remarkably complete, discussing topics ranging from concealment ciphers and various forms of simple substitution ciphers, to the more advanced multi-alphabet ciphers (Vigenere, Beaufort, Gronsfeld, Porta, etc.) and polygram and fractional substitution. The book touches on other ciphers with names fearsome to the uninitiated—quagmires, mixed alphabets, bifid, multifid—but for a mathematician the charm and interest of the volume lie in its painstaking mathematical approach to the subject.

Cryptography here is examined as the study of behavior of letters under conditions varying from normal to abnormal; and its observational and statistical "theorems" are carefully explained. They go from the familiar, basic frequency list "E T A O N I R S H. . ." to Commandant Bassieres' "theorems" on autoencipherment, in which the message itself is the key for the cipher.

Theory is illustrated, step by step, and then applied, quite often from different viewpoints. The method is good, and the reviewer can testify that the book is, indeed, its own fine instructor. It is true that there have been in the pages of the American Cryptogram Association's journal, *The Cryptogram*, many improvements in solving techniques since the original was published, but this does not now invalidate the theory. The present volume is augmented by solutions to 165 of the 167 cipher examples appearing at the end of the chapters. (The two remaining examples answer the tyro's usual question: "Are there unsolvable ciphers?" There are two of these.)

In this connection there is an oversight in the fact that the reprint does not include solutions to the anagramming exercises in Chapter VII on pages 64 and 65, and solutions to the exercises in Chapter XVIII on page 183, which deal with recovery of mixed alphabets. Omitted from the list of solutions is the keyword to problem 39—JERKINESS—and there is a typographical error in the keyword to problem 163, given as "chikory" instead of "hickory." Though the copyright notation at the front of the book declares this reprint to be a "corrected edition," the reviewer has been unable to find such corrections, the plates apparently all having been made by the photographic process. The format is smaller than the original.

The publisher has dropped the word "elementary" from the original title and now claims the volume is "intermediate." The reviewer must take issue with this. Excellent as the methodology is and advanced as some of the material appears to be, the book is even more elementary today than when it first appeared, for now it must be viewed against the development of electronic ciphers.

But all of these are admittedly small matters. From a larger viewpoint, a mathematician looking for a related hobby would be well rewarded in getting this book. Serious cryptography is close enough to mathematics to be profitably fascinating to mathematicians, and far enough removed from it to provide them a fresh point of reference. But a warning goes with this recommendation: cryptography is time-consuming, raising a challenge that is almost hypnotic in intensity. In any event it is good to have "Elcy" back in print. A generation of amateur cryptographers grew up on this volume, some even becoming "pros" during the war. In the several years past the book has been missed. There is no doubt that the reqrint will again enliven interest in this important subject.

—SAMUEL SESSKIN
"New York Mirror"

Engineering Mathematics, Kenneth S. Miller, Rinehart and Company, Inc. (232 Madison Ave.) New York, 1956, xii + 417 pp., $6.50.

This text is aimed at the first year graduate engineering student with prerequisites being elementary calculus and a "smattering" of differential equations. This being the case, schools wishing to strengthen their undergraduate program could well consider this book for use in the curriculum of the junior or senior years.

An unusual feature is the early (first chapter) development of determinants and matrices. Modern notation is employed extensively to make the proofs reasonably brief. The succeeding four chapters are concerned with topics usually found in an advanced calculus text. Chapter II is a collection of non-elementary functions. The author succeeds in producing a considerable degree of continuity in the presentation by employing functions already introduced in the derivation of further relations. In fact, Miller comments that "No topic is introduced merely for the purpose of exhibiting mathematical gymnastics."

An engineering flavor is introduced by the use of $j = \sqrt{-1}$ as well as certain terminology in the statement of illustrative examples. In many cases, however, the tacit assumption is made that the reader is familiar with its meaning. Also one might question the use of x as both the variable and upper limit of integration in the definition of such functions as erf (x) and Si (x).

Chapter III, entitled *Linear Differential Equations,* covers the subject quite adequately. Included is the Green's function solution of the non-homogeneous linear differential equation in terms of the homogeneous solutions. The general solutions of the equation

$$x^2y'' + xp_1(x)y' + x^2p_2(x)y = 0$$

are developed by the method of Frobenius and the results applied to the equations of Legendre, Hermite, and Bessel.

Fourier analysis and the Laplace transform are the topics of the next two chapters. The discussion is definitely slanted toward its practical use with several illustrations and applications of each new idea being presented. An opinion of the level of the presentation may be gleaned from the fact that the Gibb's phenomenon is not mentioned while the Fourier integral and transform are given in considerable detail.

The type of material presented in Chapter VI deviates rather sharply from that of the previous chapters. Electrical network theory is discussed with apparently two objectives in mind. The material is given in sufficient detail so that the student should obtain a sound introduction to the theory. It also serves admirably to illustrate the value of matrix theory, Fourier analysis, and the Laplace transform. Several properties of the delta function are derived in a reasonably careful manner. The author mentions some difficulties and refers them to the theory of distributions.

The contents of the last chapter, "Random Functions," are more mathematical in nature. The foundations of probability are presented in a rather concise and abstract form. However, the author succeeds in discussing such topics as expected value for discrete and continuous distributions, characteristic functions, multidimensional distributions and stochastic processes in an assimilable style. The student having no experience with probability would likely need some guidance through this material. The type of reasoning required is quite in contrast with that required for the familiar work of the previous chapters.

The title *Engineering Mathematics* may include such topics as vector analysis and complex function theory. The omission of these may well be a commendable feature since there are well written texts on each of these subjects and the inclusion of an adequate discussion of complex functions would cause the book to be of a size rather unhandy for a text. On the other hand, the vector analysis might well have been a useful and enlightening preliminary chapter to the matrix theory.

The book is printed in an open style with all equations displayed and well spaced. A brief introduction to each section is designed to motivate the student by explaining what is to be accomplished and how it relates to other material. The reviewer finds the text quite readable and well organized—a statement which cannot be made concerning several competing books.

—FRANK J. PALAS
Southern Methodist
University

Topology, E. M. Patterson, Interscience Publishers, Inc. (250 Fifth Ave.) New York, 1956, 127 pp., $1.55.

This book was written by E. M. Patterson, lecturer in mathematics at St. Andrews University, to serve as a textbook in introductory topology. It was originally published by Oliver and Boyd in Edinburgh. It is geared to the better students in mathematics at the senior college level. It is brief and written in non-technical language as much as possible. It does not take up the more modern aspects of topology since its purpose is to lay a foundation for more advanced study and to develop an interest in topology in the curious student.

The first chapter is introductory and explores the basic ideas of topology. Topology is defined to be the study of situation and continuity. Topological equivalence is demonstrated by moulding and remoulding a piece of plasticine into various shapes without making breaks or joins. Special topological surfaces, such as the Mobius strip, the torus, and the Klein bottle are illustrated in this section in pictures and by identification of the points of a rectangle. One of the unsolved problems of topology, the four-color problem, is discussed here. The problem is this—in the making of maps, if no two bounding divisions are to be the same color, it seems that four different colors would be sufficient for a map. It can be shown that three colors are sufficient in most cases and it has been proved that five are sufficient in all cases. Theoretically it seems possible that the number can be reduced to four, but the proof has never been found.

Chapter II deals with topological space in a general form. Various theorems dealing with sets, which are collections of objects determined by some property, and topological spaces are stated and proved in this section. A group of exercises using these theorems is listed at the end of the chapter.

Specific types of topological space, such as Hausdorff spaces, are discussed in the third chapter and various theorems are demonstrated concerning open sets, convergence, compactness, and other topics which are defined and discussed briefly.

Homotopy is discussed thoroughly in the next chapter. Two subspaces are said to be homotopic if one can be transformed into the other by continuous deformation.

The last two chapters deal with algebraic topology. The author states that the main idea of algebraic topology is concerned with homology, the process of dividing up space into pieces which are topologically equivalent, or homeomorphic, with the interior of a triangle or its analogues in other dimensions.

The senior college student will gain much from reading this book on his own. To understand it thoroughly, however, it would be necessary to have supervised study. Most of the topics require knowledge of advanced courses in mathematics for clarity.

—Betty Donath
Tennessee Polytechnic
Institute

Electrical Interference, A. P. Hale, Philosophical Library, Inc. (15 E. 40th St.) New York, 1956, vii + 122 pp., $4.75.

Complaints of interference with radio and television reception mount every year as the rapid growth of the radio and television industry continues and more and more radios and televisions are sold. The chief cause of dissatisfaction is with the wide range of high frequency noise, normally the result of operating electrical machinery, neon lights, diathermy apparatus, ham-radio equipment, and industrial electronic equipment.

As the author states, literature on electrical interference is at present scarce, being largely in the form of papers in technical journals. Specialist engineers have gained most of their knowledge from practical experience and have rarely put it into print. This book has accordingly been written from a thoroughly practical point of view. It covers the causes of interference, the effects of interference, receiving antennas, measurement of interference levels, location of sources of interference, avoidance of interference, basic filters, safety, practical filters, and Faraday cages.

Whether called upon to design electrical apparatus conforming to legal requirements relating to interference suppression or to trace and eliminate existing interference, electrical engineers and service technicians will find this practical book invaluable in their work.

Since mathematics is finding increasing use in connection with the problems of radio and television, any person whose major interest is mathematics could profit by understanding some of the general principles given in this book.

—ROBERT Q. CHILDRESS
Tennessee Polytechnic
Institute

Introduction to Mathematical Logic, Volume I, Alonzo Church, Princeton University Press (Princeton, New Jersey) 1956, x + 372 pp., $7.50.

This is the first volume of a planned two-volume set and covers the propositional calculus and functional calculi of first and second orders.

In an excellent introductory chapter, the author develops the formalized language so essential to the analysis of propositions and proof by form as abstracted from matter. To make clear the failure of the natural languages, examples are given such as the following comparison of two arguments that have the same linguistic form: (1) "I have seen a portrait of John Wilkes Booth; John Wilkes Booth assassinated Abraham Lincoln; thus I have seen a portrait of an assassin of Abraham Lincoln;" and (2) "I have seen a portrait of somebody; somebody invented the wheeled vehicle; thus I have seen a portrait of an inventor of the wheeled vehicle." Similar examples are given illustrating Frege's theory of proper names, the name relation, denotation and concepts.

Other explanations in the introduction are of constants and variables, functions, propositions, truth values, and propositional functions, symbols, connectives, operators, quantifiers, the logistic method, syntax, and semantics, as used in the formalized language.

Though *Introduction to Mathematical Logic* would not be chosen as a text for undergraduate study, it could be very useful as a reference book. This use is enhanced by extensive footnotes, and especially by an index of definitions and an index of authors cited throughout the book.

—JAMES M. DORAN
Tennessee Polytechnic
Institute

The Philosophy of Mathematics, Edward A. Maziarz, Philosophical Library (15 East 40th Street) New York, 1950, viii + 286 pp., $4.00.

This is a most valuable work of scholarship, distinguished by exhaustive treatment, and showing evidence that it was indeed a "labor of love" on the part of the author. Its scope can best be indicated by the chapter headings: The Problem of the Philosophy of Mathematics; The Historical Relation Between Mathematics and Philosophy; Ancient Conceptions; The Gartesian Era; British Empiricism; Idealism and Positivism; Contemporary Directions; The Distinction of Speculative Sciences; Nature of Mathematical Abstraction; Mathematical Abstraction; and Contemporary Mathematics.

This book is, however, one for the relatively sophisticated reader. The apparatus of scholarship, so evident throughout, would make it seem rather forbidding to a tyro in either mathematics or philosophy, or worse still, in both.

The present reviewer is especially appreciative of the critical bibliography and feels that scholarship is indebted to the author for its construction. This bibliography will be of important service to future scholars in these fields. The careful study of "intuitionism" vs. "formalism" is especially significant for the contemporary mathematician.

—F. C. Ogg
Bowling Green State University

Editorial Note. A number of people have suggested that we review *The World of Mathematics* by James R. Newman. The book editor has written to the publisher to request a review copy, but the company can not furnish it to THE PENTAGON. If any one will write a review of this four-volume work without receiving complimentary review copies, the bookshelf editor will be delighted to publish it.

Installation of New Chapter

EDITED BY MABEL S. BARNES

THE PENTAGON is pleased to report the installation of Indiana Gamma Chapter of Kappa Mu Epsilon.

INDIANA GAMMA CHAPTER

Anderson College, Anderson, Indiana

Indiana Gamma Chapter was installed at Anderson College on April 5, 1957, by Dr. Harold D. Larsen of Michigan Alpha. Dr. Larsen is chairman of the Mathematics Department at Albion College, and was national vice-president from 1949 to 1951 and editor of *The Pentagon* from 1943 to 1952.

The fifteen charter members initiated were: Patricia Arnold, Norman Burd, Hubert Dixon, Carl Foley, Thomas Harbron, Louise Johnson (faculty), Merl Kardatzke, Terry Magsig, Harry Nachtigall, Gloria Olive (faculty), Herman Reichenbach (faculty), Paul Saltzmann, Joann Snook, Kenneth Swick, Myron Williams.

A banquet was held at the Top Hat Restaurant, with Myron Williams of Kingston, Jamaica, acting as master of ceremonies. Dr. Larsen addressed the group on the subject "Some Famous Problems of Mathematics". Harry Nachtigall and Norman Burd composed a song "Hail to Kappa Mu Epsilon" for the occasion. President and Mrs. John A. Morrison of Anderson College were special guests. Indiana Alpha was represented by Dr. J. E. Dotterer and Professor A. E. Baumgart.

We welcome Indiana Gamma and wish to express our pleasure at their association with us.

Kappa Mu Epsilon News

EDITED BY FRANK HAWTHORNE, HISTORIAN

In preparation for homecoming this year, active members of **Alabama Beta** extended invitations to all former members to attend a coffee hour during the activities of the day. Much interest was demonstrated in writing invitations, making place cards, displaying materials, and sharing responsibilities of the coffee hour. Representatives from fifteen years of the chapter's history attended.

The members of **California Alpha** have shown a great interest in applied mathematics this year. They had as a guest speaker Mr. Don Furth from Douglas Aircraft Company for a talk entitled "Computers and Computer Engineering." A field trip is planned through the El Segundo refinery of the Standard Oil Company.

California Beta cooperated with Occidental College, The Actuarial Club of Los Angeles, and the Southern California Council of Teachers of Mathematics in sponsoring a mathematics field day on March 2, 1957. Over 350 students from 75 southern California high schools participated, and some applications (unfortunately) had to be denied.

The events were a leap frog relay (a two-part written examination taken by partners), a mad hatter marathon (a rapid calculation and estimation contest), a chalk talk derby (a speech contest), and three kinds of individual games—three-dimensional tic-tac-toe, nim, and five-in-a-row. Twenty-five KME members helped plan and run the events. The day was a great success and plans are being made for another field day next year.

Donna Mae Sorensen won the KME Freshman Mathematics Award for 1955-56. She was among the sixteen new members initiated on February 21, 1957.

Illinois Alpha has spent a busy and memorable year, taking its place in the activities of Illinois State Normal University's Centennial Year. The Homecoming Breakfast, honoring alumni, was one of the special events of the year. Mr. W. D. Ashbrook of the Industrial Arts Department was the principal speaker.

Highlighting one of the regular chapter meetings was a discussion of mathematical logic titled, "Looking Under Geometry." At the annual Spring Banquet in April, Dr. Bruce E. Meserve, State Teachers College, Upper Montclair, New Jersey, was our guest speaker. Dr. Henry Van Engen, former National President of KME, also spoke briefly about the National Fraternity.

Illinois Beta was host to the Eastern Sectional Conference on the Teaching of Mathematics sponsored by the Illinois Council of Teachers of Mathematics on April 10. Dr. Lester R. Van Deventer was chairman of the conference.

Dr. Ruth Rasmusen became the sponsor of **Illinois Gamma** upon the death of Dr. Norman Goldsmith.

On March 12 **Illinois Delta** broadcast "Kappa Mu Epsilon Presents" over Joliet's own station WJOL. It depicted a typical meeting of the local chapter.

Indiana Beta took a field trip to the Naval Avionics Facilities at Indianapolis in January.

On March 4, 1957, **Kansas Gamma** gave a fifteen minute program on KFEQ-TV, St. Joseph, Missouri. The theme of this program was "Math and You." The purposes of Kappa Mu Epsilon were illustrated by showing the personal, cultural, and practical values of mathematics to the general public.

Michigan Beta revised its local by-laws. There will be no further fines for unexcused absences from meetings. An "Integration Party" welcomed back student teachers of mathematics who had been externing. In conjunction with the departments of Psychology, Education, and Mathematics, the chapter sponsored two lectures and an exhibit by the Do-All Company titled: "The Story of Measurement."

Michigan Gamma is conducting its meetings on a three-week rotational plan. One week is utilized for chapter business with a lecture by one of the members, one week for a problem session, and the third for a talk by a member of the faculty.

As an added incentive to student effort, the mathematics department conducts annually an honors examination. First and second places in 1957 went to Charles Conley and Eva Kuhn, respectively, both members of KME.

Dr. Walter Hoffman, KME Alumnus, has been appointed "Manager of Computational Services" at the Wayne State University computation laboratory, the home of UDEC.

Missouri Beta will again honor the outstanding boy and girl mathematics students of the freshman class.

New Jersey Beta gave a banquet in honor of Dr. David R. Davis who resigned as faculty member and head of the mathematics department. He was presented with a portable radio and jewelry.

Tennessee Alpha is sponsoring a scholarship to be awarded to the winner of the Upper Cumberland regional contest of the statewide mathematics competition. Each member is contributing one dollar. The scholarship, amounting to $120, will cover all the fees at Tennessee Tech.

The students of **Ohio Alpha** are conducting "help sessions" on both the freshman and sophomore level. Andrew P. Ogg, past president, won a National Research Scholarship in Mathematics and is now studying at Harvard.

Pennsylvania Beta will join with the other scientific clubs of the college for an annual banquet at which each of the clubs will hold separate formal inductions.

Wisconsin Alpha is emphasizing among its members the Essay Contest on "Opportunities in Teaching Mathematics in Secondary Schools" and the upcoming National Convention. Plans also have been made for the annual mathematics contest set for April 13.

Program Topics

(School Year 1956-57)

California Alpha, Pomona College
- *The Theory of Games,* Dr. Chester G. Jaeger
- *Boolean Algebra,* Graham Wallace
- *Computers and Computing Engineering,* Donald Furth

Illinois Beta, Eastern Illinois State College, Charleston
- Movies on astronomy, arranged by Dr. Davis
- *The Seven Bridges of Konisberg,* Dr. Ringenberg
- *Sidelights on Euler's Theorem for Polyhedra,* Dr. Van Deventer
- *History of Some Units of Measure,* Sherrill Harrold
- *Some Aspects of the Trachtenberg System of Computation,* Gilbert Rainey
- Discussion of mathematical opportunities

Illinois Gamma, Chicago Teachers College
- *Paradox Lost, Paradox Regained,* Erwin Marks

Illinois Delta, College of St. Francis, Joliet
- *Applied Geometry—Through Problems from Geometrical Optics,* Sister M. Crescentia
- *Geometric Problems Lead to Algebraic Statements,* Dorothy Pulo
- *Departure from Euclidean Geometry,* Vivian Makowski

Development of Non-Euclidean Geometry, RoseMary Kotesa
Introduction to Projective Geometry, Sister M. Claudia
Study of the Golden Section and its Application to Nature, Sister M. Ursuline

Indiana Beta, Butler University
The Mathematics of High Fidelity, Robert M. Gasper
How to Draw a Straight Line, James Fulton
The Abacus, Janet Crull
Rockets and Guided Missiles, Lloyd W. Stark

Iowa Alpha, Iowa State Teachers College
Four Color Problem, Roger Brockmeyer
New Meaning for Old Symbols, John Shuler
Nim, Sandra Ladehoff
Some Early Egyptian Mathematics, Oma Chody
One Plus One, 1200 Times a Second, Dale Bird

Iowa Beta, Drake University, Des Moines
IBM's Electronic Business Machine "705", Russel Thurau
Computer Memories, John Niccum
Mathematical Notation and its History, John Flitte
The Abacus, Steve Ashford
The Relation of Mathematics and Mechanics to Bone Structure, Jerry O'Mara

Kansas Gamma, Mount St. Scholastica College, Atchison
Home Ownership, Sister Jeanette, OSB
Stocks and Bonds, J. Henry
Problem Workshop, Carol Law, Joan Carvalho, Marilyn Zimmerman
Wassail Bowl Christmas Party
Income Tax, Julia Handke, Dorothy Schmedding
Logarithms, Barbara Rentchler
Problem Workshop, Dorothy Schmedding, Judith Bock, Mary Syron
Problem Workshop, Sr. Benedict Joseph, DC, Julia Handke, Barbara Rentchler
Research in Modern Age, Marilyn Zimmerman, Mary Anne Ginaine
Technical report of opportunities in mathematics experienced by graduates, Dorothy Schmiedeler, Dolores Ready, Carol Law

Michigan Beta, Central Michigan College, Mount Pleasant
How to Lie with Statistics, Lois Sudborough
Time and its Measurement, Oliver Porter
Factoring Trinomials-Novel Methods, Norma Fultz

Michigan Gamma, Wayne State University
Boolean Algebra, Student committee
Differential Equations and Applications, Faculty
Geometry, Faculty
Probability, Faculty

Missouri Beta, Central Missouri State College, Warrensburg
The Graduate Program in Mathematics at Central Missouri State College, Dr. Reid Hemphill
The Life of Gauss, Wallace Grififth
Flexigons, Hubert Kienberger
Unusual Personalities of Mathematics, Linda Land

New Jersey Beta, State Teachers College, Montclair

Problems in Industry, Paul Clifford
High Speed Computing, George W. Kays

Ohio Alpha, Bowling Green State University, Bowling Green

An Experimental Approach to Teaching Elementary Mathematics, Dr. Bernard H. Gundlach
Some Properties of the Normal Distribution, Dr. Leon H. Harter
Finite Difference Solutions, Ross Cornell
Operations Research, Dr. E. Leonard Arnoff

Pennsylvania Beta, La Salle College, Philadelphia

The Laplace Transform and its Applications, Thomas Devlin
Theory of Groups, Brother Brendan Gregory
The Galois Theory of Equations, Brother Damian
Nomography, Joseph Liebsch
Game Theory and Linear Programming, Brother Damian
Some Applications of Point Set Theory, Dr. Robert Putnam

South Carolina Alpha, Coker College

History of Mathematics, Elinor Askins

Virginia Alpha, Virginia State College, Petersburg

Numerical Solutions of Differential Equations, Janie L. Cooper
The Cubic Equation, Gene A. Dimmie
The Gamma Function, Robert R. Edmonds
Special Chemistry Problem Involving Differential Equations, Blondell Hudson
The Simple Pendulum, Dr. J. M. Hunter
Infinity, Dr. R. R. McDaniel
Linear Programming, Benjamin Williams
Mathematical Aspects of a Problem in Economics, Dr. W. E. Williams

Wisconsin Alpha, Mount Mary College

Permutations, Ilene Victory
Multiplication Short Cuts, Gwen Petretti

How to Draw A Straight Line

A

Lecture on Linkages

By

A. B. KEMPE, B. A.,

Of the Inner Temple, Esq.;

Member of the Council of the London Mathematical Society;

and late scholar of Trinity College, Cambridge.

London:

Macmillan and Co.

1877

ACTIVE CHAPTERS of KAPPA MU EPSILON*

Chapter	Location	Installation Date
Oklahoma Alpha	Northeastern State College, Tahlequah	April 18, 1931
Iowa Alpha	State Teachers College, Cedar Falls	May 27, 1931
Kansas Alpha	State Teachers College, Pittsburg	Jan. 30, 1932
Missouri Alpha	Southwest Missouri State College, Springfield	May 20, 1932
Mississippi Alpha	State College for Women, Columbus	May 30, 1932
Mississippi Beta	State College, State College	Dec. 14, 1932
Nebraska Alpha	State Teachers College, Wayne	Jan. 17, 1933
Illinois Alpha	Illinois State Normal University, Normal	Jan. 26, 1933
Kansas Beta	State Teachers College, Emporia	May 12, 1934
New Mexico Alpha	University of New Mexico, Albuquerque	March 28, 1935
Illinois Beta	Eastern Illinois State College, Charleston	April 11, 1935
Alabama Beta	State Teachers College, Florence	May 20, 1935
Alabama Gamma	Alabama College, Montevallo	April 24, 1937
Ohio Alpha	Bowling Green State University, Bowling Green	April 24, 1937
Michigan Alpha	Albion College, Albion	May 29, 1937
Missouri Beta	Central Missouri State College, Warrensburg	June 10, 1938
South Carolina Alpha	Coker College, Hartsville	April 5, 1940
Texas Alpha	Texas Technological College, Lubbock	May 10, 1940
Texas Beta	Southern Methodist University, Dallas	May 15, 1940
Kansas Gamma	Mount St. Scholastica College, Atchison	May 26, 1940
Iowa Beta	Drake University, Des Moines	May 27, 1940
New Jersey Alpha	Upsala College, East Orange	June 3, 1940
Tennessee Alpha	Tennessee Polytechnic Institute, Cookeville	June 5, 1941
New York Alpha	Hofstra College, Hempstead	April 4, 1942
Michigan Beta	Central Michigan College, Mount Pleasant	April 25, 1942
Illinois Gamma	Chicago Teachers College, Chicago	June 19, 1942
New Jersey Beta	State Teachers College, Montclair	April 21, 1944
Illinois Delta	College of St. Francis, Joliet	May 21, 1945
Michigan Gamma	Wayne University, Detroit	May 10, 1946
Kansas Delta	Washburn Municipal University, Topeka	March 29, 1947
Missouri Gamma	William Jewell College, Liberty	May 7, 1947
Texas Gamma	Texas State College for Women, Denton	May 7, 1947
Wisconsin Alpha	Mount Mary College, Milwaukee	May 11, 1947
Texas Delta	Texas Christian University, Fort Worth	May 13, 1947
Ohio Gamma	Baldwin-Wallace College, Berea	June 6, 1947
Colorado Alpha	Colorado A & M College, Fort Collins	May 16, 1948
California Alpha	Pomona College, Claremont	June 6, 1948
Missouri Epsilon	Central College, Fayette	May 18, 1949
Mississippi Gamma	Mississippi Southern College, Hattiesburg	May 21, 1949
Indiana Alpha	Manchester College, North Manchester	May 16, 1950
Pennsylvania Alpha	Westminster College, New Wilmington	May 17, 1950
North Carolina Alpha	Wake Forest College, Wake Forest	Jan. 12, 1951
Louisiana Beta	Southwest Louisiana Institute, Lafayette	May 22, 1951
Texas Epsilon	North Texas State College, Denton	May 31, 1951
Indiana Beta	Butler University, Indianapolis	May 15, 1952
Kansas Epsilon	Fort Hays Kansas State College, Hays	Dec. 6, 1952
Pennsylvania Beta	La Salle College, Philadelphia	May 19, 1953
California Beta	Occidental College, Los Angeles	May 28, 1954
Virginia Alpha	Virginia State College, Petersburg	Jan. 29, 1955
Indiana Gamma	Anderson College, Anderson	April 5, 1957

* Listed in order of date of installation.